Universiteit Utrecht

# Internship and Thesis Report
# (45 ECTS)

# Diffusion of the diffusion curve: A research on the S-curves in relation to technological clusters

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June 2010

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## Abstract

The aim of this research is to understand the diffusion of products belonging to different cluster technologies in terms of the available chaotic and stochastic models. After reviewing the relevant mathematical models, we categorize their use according to selected technology clusters. Furthermore, the development and adjustment of mathematical models is compared with relevant changes of the products/services, Roger's communication channels, needs, demands.

## Key Words

Innovation diffusion models, Innovation diffusion, Roger's communication channels, Technology clusters

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# 1 Introduction

Suppose a traveler from France that is influenced from an advertisement and reviews in the internet about a compact new innovation device i.e. a GPS. The traveler discusses his wish to buy the product with friends from the local skiing organization that may advise him on the usefulness and qualifications of the GPS systems. After this process he buys the product and you decide to go on a skiing tour to Finland and use the device. In Finland he passes through a trial period where he learns how to benefit from the GPS device and he shows it to other skiers. The demonstration of the device finds some skiers very enthusiastic about its usefulness that they express their wish to buy the same device once they get back into their countries or in Finland. After this skiing tour the same traveler goes back to France and describes to his friends in his blog his adventure and experiences. He mentions what he accomplished with the help of the GPS device and writes a review about it. Some of his friends from other countries write him an email and want to know more about the device, so, he provides them with information. Afterwards he goes for another skiing tour in the Alps. The reactions about the device are similar like before and more people want to know about it and the process repeats. Later in the same year the same person goes for a summer cruise in Greek islands and he takes the GPS device with him to test it in the sea. It proves that the device is functioning effectively and it is helpful but add-on software specialized on nautical maps and terms would be more beneficial. He comments on that and publishes it in his blog. The example above can be a short summary of the story of an innovation diffusion product and the ways one can adopt it and/or influence its adoption and further development. Someone may wonder what innovation diffusion is and when did this come up?

After 1950's there were some attempts to explain innovation diffusion. A well-known researcher, Rogers M. Everett, was the first to write a book about the diffusion of innovation (1962). Rogers defined *Innovation* as 'an idea, practice, or object that is perceived as new by an individual or other unit of adoption (Rogers, 1985, p.11) and *Diffusion* as 'the process by which an innovation is communicated through certain channels over time among the members of a social system'. Rogers continues that diffusion is 'a special type of communication, in that the messages are concerned with new ideas' and that '*Communication* is a process in which participants create and share information with one another in order to reach a mutual understanding' (Rogers, 1983, p.5). In 1961, Mansfield hypothesized that the rate of diffusion is a 'function of the extend of economic advantage of innovation, the amount of investment required to adopt the innovation and the degree of uncertainty associated with the innovation' (Mahajan V. and Peterson R. A., 1985). Later, Griliches (1957), Robinson and Lakhani (1975) and Brown (1981) proposed a supply and demand rationale as a diffusion explanation. Furthermore, Casetti and Semple (1969) and Sahal (1981) pointed the learning perspective of innovation diffusion. Other scholars like Hagerstrand (1967) and MacKenzie and Bernhardt (1972) supported an information transfer explanation. Soon after, Blackman (1974) and Sharif and Kabir (1976) used a technological substitution frame to describe innovation diffusion. Rogers (1983) offered another approach on the same topic: a communications based theory. All the above come to verify that variables from various sets like economic and social are important to explain an innovation diffusion process.

Innovation diffusion might grow or suddenly pause. The latter might be due to a technological discontinuity. When it comes to explain technological discontinuities and the emergence of new innovations, innovation management theories provide us with a number of different approaches. A few examples are: Schumpeter's "creative destruction" (Schumpeter, 1942), the Incremental – Radical innovation dichotomy (Abernathy, 1978, Abernathy and Utterback, 1978), Incremental – Breakthrough innovations (Tushman and Anderson, 1986) Continuous – Discontinuous technological changes (Tushman and Anderson, 1986), and the Henderson - Clark model on "Architectural innovation" (Henderson R. and Clark K., 1990). Even if all the theories above can provide a good explanation for which company is, under certain circumstances, in a better position to innovate, they are less able to predict when a technological discontinuity will take place (Levy D., 1994). A useful tool that is used by many researchers and firms is the S-curve. The S-curve framework seems to be an approach able to both analyze technological cycles and predict when the introduction, the adoption and the maturation of innovations occur. Some researchers claim that the S-curve framework, despite its shortcomings, is able to do this and this will be a core element of this thesis.

To further explain the association of S-curve framework and technological discontinuities the following graph is provided. When a technology reaches saturation (S-curve limit) a new technology initiates and starts growing. What lies between the first and the second technology is the phase of the technological discontinuity of the first technology. That discontinuity helps the transition from the first technology to the second. An example is the transition from the sound cassettes to the compact discs. The discontinuation of the analog sound (cassettes) and the choice of companies to promote the digital sound products (CD's) gradually lead to the termination of any analog technology and the raise of the digital era. A formal explanation is given by McGrath who states that 'a transition to a different technology that has a higher theoretical performance limit is generally termed and graphically depicted as a "discontinuity' (McGrath, 1998).

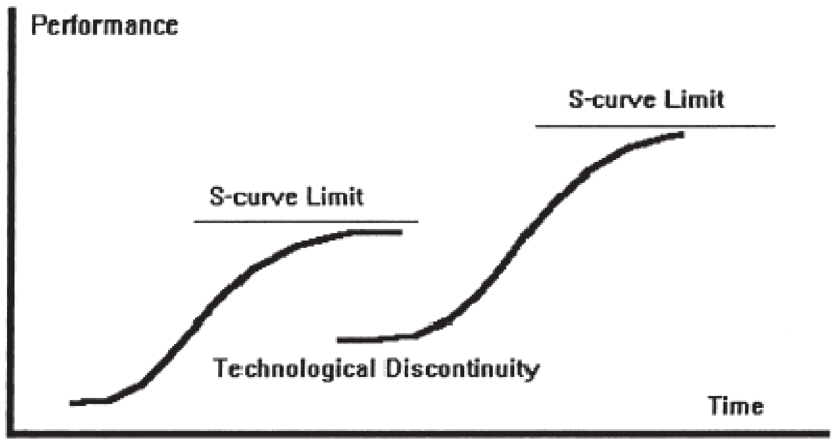


**Figure 1: Technology S-curves and discontinuity (McGrath, 1998)**

An additional approach also mentions that discontinuity is 'regularly depicted as a family of s-curves that is, on the whole, collectively discontinuous from the dominant design (McGrath, 1998). That definition is useful when different technologies in their infancy compete with each other on which performs better and could be the winning technology. A nice example is given in McGrath(1998) and it concerns the different technologies of electric vehicle batteries.

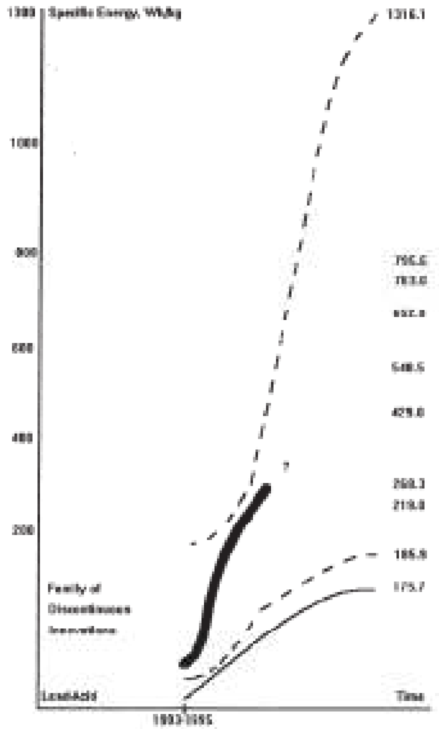


Figure 2: The discontinuity of a family of S-curves

The significance of S-curves is also pointed in Merino (1990). The author admits that like Foster and many other researches discussed, ‘coping with technological discontinuities is one of the most difficult challenges confronting corporations’ (Merino, 1990). A well known example is the transitions from vacuum tubes to transistors where the companies that dominated in the transistors did not survive due to their discontinuity because they did not recognize soon enough the technological changes that occurred. Merino (1990) also states the importance of the S-curve as a method ‘that can help a company manage its R&D and cope with potential technological discontinuities’.

The S-shaped curve maybe used to replicate and forecast the start-up of industrial innovations (Naim and Towill, 1993). The S-curve is distinguished in three main phases:

1. The infancy phase, where a performance of an innovation is initially improving and early adopter occur

2. The growth stage, where a rapid improvement happens and the majority of adopters uses the innovation
3. The maturity phase, where an innovation performance reaches a maximum limit and a decline stage may be followed

The following graphical representation shows these phases as well as the very early phases of innovation diffusion. All steps are mapped to social and economic conditions (Beal and Bohlen). What we can observe is that at the initial phase of innovation creation innovators are the main actors. Then a small number of early adopters start accepting or rejecting an innovation and this is a critical point. Later if the product is accepted, we have an increase of the adopters that keeps growing to majority phase where the innovation has accelerated its diffusion. At the end when the innovation product reaches saturation, only few non-adopters are left.

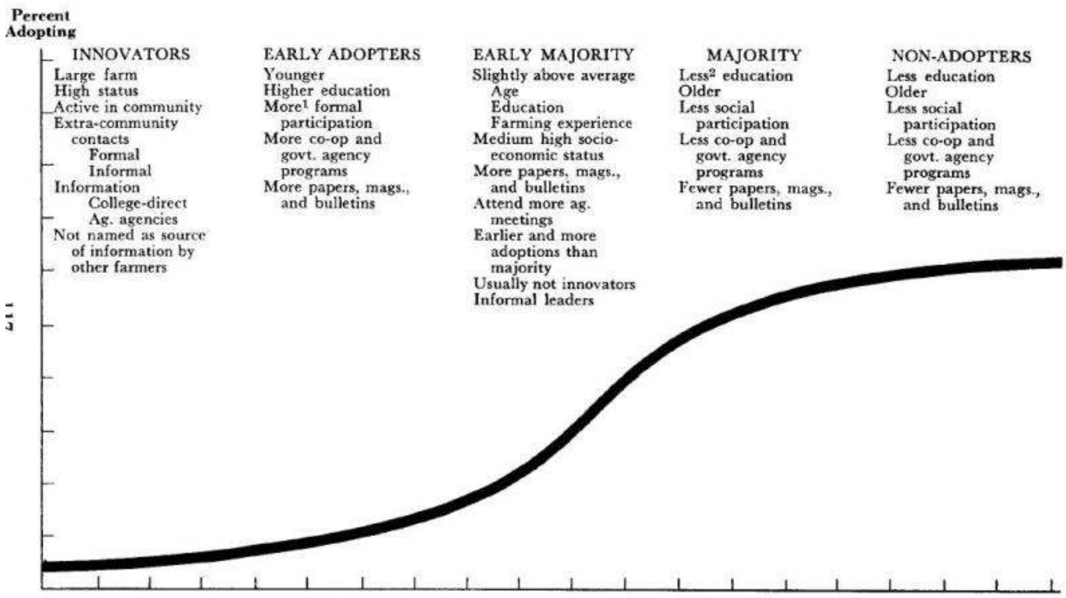


**Figure 3: The phases of innovation diffusion (Beal and Bohlen)**

Another graphical representation of innovation diffusion is also provided by Rogers (1962, 1983, p.10) where he plots the four main elements in the innovation diffusion process (figure 2).

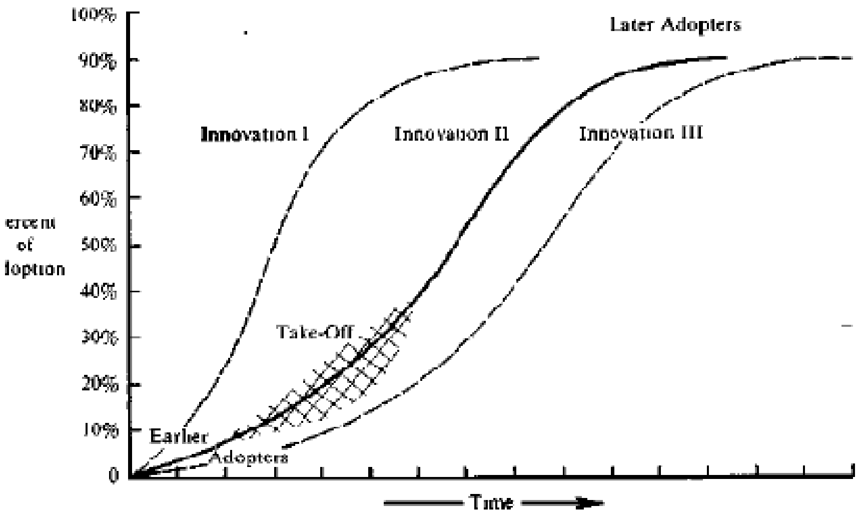


**Figure 4: Diffusion is the process by which (1) an innovation (2) is communicated through certain *channels* (3) over *time* (4) among the members of a *social system. (Source: Rogers, 1983, p.11)***

When it comes to a technology, the main method to measure and plot its diffusion is in relation to its performance and effort. The next graph shows a technology's development which follows the same S-curve and reaches a limit.

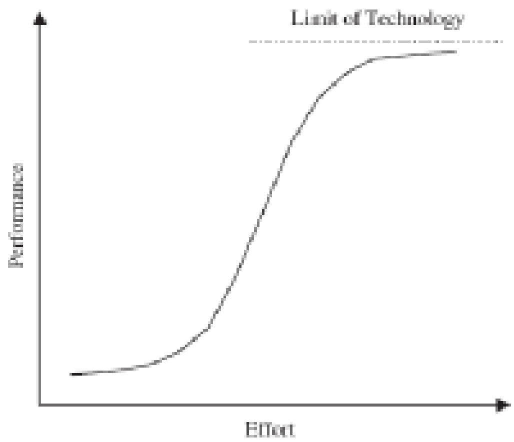


**Figure 5: A typical S-curve of a technology life cycle (Source Schilling M. A. and Esmundo M. (2009))**

From previous research it is shown that what lies at the beginning and at the end of an S-curve is chaos, while in the between, there is order (Modis T. and Debecker A., 1992). Considering the deterministic characteristics of chaos and its sensitivity to initial conditions, it is interesting to examine its variance and duration on different technologies: high versus low technologies in relation to their correlation with other technologies, or/and stand alone products. The case of a technology lifecycle being affected by other technologies has an impact on its development. We should insert here the term of technology clusters given by Rogers (1983, p. 14) and that is 'A *technology cluster* consists of one or more distinguishable elements of technology that are perceived as being closely interrelated'. Suppose we have a family of i={1, 2, 3, 4} technologies i.e. Xi={X1, X2, X3, X4}. Given that technologies X1, X2, X3, X4, can affect the development of technology A, the winner technology among the Xi family, will define the initial conditions of the affected technology A. Let's say that technology A is the digital media storage disc (DVD). There are several technologies that affected its format and development. X1 is the physical shape that a DVD should have, a round thin disc that is familiar to the users after the CD dominance. X2 is the materials used to manufacture the DVDs as it should be compatible with existing hardware technology (laser beams). X3 is the volume capacity of a DVD to allow recording and selling a movie or a PC game to be compatible with the demands of the entertainment industry. X4 is the quality specified with ISOs that the DVD should fulfill. Each of the Xi technologies had to compete with other technologies to become the standards. So is technology A (DVDs) as it had to compete with other technologies that considered the fulfillment of the Xi technologies. As a consequence, when it comes to the creation of the S-curve of a product or technology, competition is a critical factor.

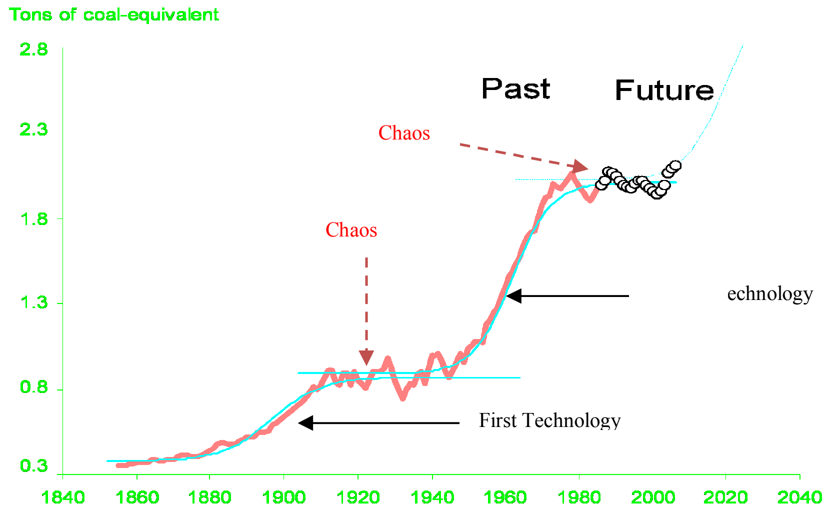


**Figure 6: Growth alternates with chaos, Per Capita Annual Energy Consumption Worldwide (Source: Modis 1998)**

There are different mathematical functions that have been used to model diffusion processes. However, because any unimodal distribution can generate an S-curve, it is often not possible to empirically determine which function best describes a specific competing trend. In this direction many attempts are made to develop a theory based 'diffusion models' for analyzing and modeling the spread of an innovation over time considering numerous factors (Mahajan V. and Peterson R. A., 1985).

Another motivation for further research on the innovation diffusion process in regard to S-curve plotted equations is the criticisms of Martino (1972) and Sahal (1977) that the assumption of an S-curve pattern is naïve. Furthermore as pointed in Bernhardt and Mackenzie (1972) and Brown (1975) there was not a clear and satisfactory explanation of either to use a specific mathematical function against another since many could generate the same S-curve.

## 1.1 Research Question

As I have presented, during the last six decades, researchers have tried to analyze innovation diffusion. That might have occurred on the basis of observations showing a products specific lifetime and the launch of a new one as a successor. Others addressed the necessity to systematically analyze the factors that affected the innovation diffusion, something that Rogers did. Furthermore, sometimes observations showed that some products never diffused maybe due to keen competition or non stable social and economical conditions. The development of the theoretical framework of innovation diffusion gave birth to a practical mathematical modeling that had to include the variables that described the process. As a consequence, we inquire into the development of the mathematical modeling of innovation diffusion (the stochastic or chaotic equations) that had to adjust to the theoretical innovation diffusion principles to describe the diffusion of products.

Additionally, on a firm level, the successful mapping of technology and its products on an S-curve may be useful for its future, because by understanding the dynamics of innovation in its market space is important in terms of businesses challenges, structure and metrics (Modis T., 1998, Kaplan S., 2009). Continuing, 'the firm's awareness of its S-curve can represent a more accurate notion of what its future may be like' (Zawislak P. et.al, 2009). Supporting and developing a technology or adopting a new one is vital. Practically, a firm can assess its actual innovation potential, by tracing its position on the curve (Zawislak P. et.al, 2009). A firm can either remain sustaining a technology's development or chose to 'jump' over another technology's S-curve. Some firms decide to follow a new technology at early stages, before the whole S-curve forms while others wait to do so when the new technology reaches a satisfactory market potential. When a product is at the first stages of its development the S-curve model can predict its start of growth rate, its penetration in the market place, take over time and saturation. For these reasons, knowing which is the optimum innovation diffusion model applicable for different technologies, is advantageous. Furthermore, the successful map of the state of a product on an S-curve affects the managerial and strategic decisions of a firm (Modis T, 1998, Kaplan S., 2009).

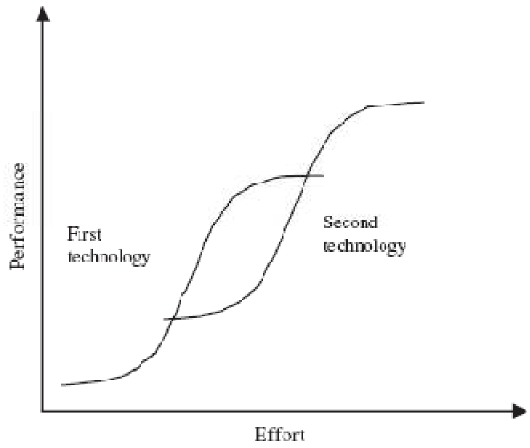


**Figure 7: A firm should choose if it will continue using the first technology or make use (jump) of the second one (Source Schilling M. A. and Esmundo M. (2009))**

Considering the importance of innovation diffusion, its basis on mathematical modeling and the ability to plot it on a Cartesian coordinate system, the goal of the thesis is to review the virtue of an S-curve framework as an innovation diffusion scheme.

The research question that arises is:

RQ: How do stochastic or chaotic models, which result in an S-curve, describe the different products that belong to different technology clusters?

## 1.2 Justification

An argumentation about the choice of the innovation diffusion models review and analysis on different products includes the following reasons. Diffusion is an important aspect of innovation since 1960's and until now the topic of innovation diffusion has been researched by many scholars. For instance, the search engine "Web of Science" returns for the term 'innovation diffusion' more than 3500 studies, with more than 35000 citation number. Traditionally, innovation diffusion has been modeled with S-curve plots. While insight into the process of diffusion has increased also the research on the models of innovation diffusion have progressed. In this thesis a review on the progress in modeling innovation diffusion and an assessment whether important progress has been made would be presented.

Another motivation for preferring this topic is that it bridges two important disciplines: mathematics and innovation management. It is very interesting to map mathematical models with innovation because they are practical tools useful for all innovation managers. Furthermore, the S-curve framework has a strategic implication and managerial relevance on firm level. That is related to optimal entry timing of an innovative product or even its launch in different markets. For example a firm might research and develop on an innovative project but launch it only when it is an appropriate time. In the field of policy making, innovation diffusion models are useful as they could provide policy makers with the appropriate timing for subsidies, creation or re-design of policies. Market planners are also extremely interested in diffusion models as they want to find the optimal choices in terms of timing, technologies to promote, advertising amount and means, and more. After all, this is in agreement with the reasoning that lies behind innovation: knowledge diffusion, use of diverse scientific fields and so on.

Originality

The aim of this research is to understand the diffusion of products belonging to different cluster technologies in terms of the available chaotic and stochastic models. After reviewing the relevant mathematical models, we categorize their use according to selected technology clusters. Furthermore, the development and adjustment of mathematical models is compared with relevant changes of the products/services, communication channels, needs, demands.

The first relevant paper found -is written in 1993 (Parker P. M., 1993) - uses empirical evidence to choose among twenty four innovation diffusion models but it is restricted into an extension of a basic model. Furthermore, the data used in the previous paper are chronologically limited and the products examined are only consumer durables. A recent paper (Meade and Islam, 2006) that presents a 25-year review of modeling and forecasting of innovation diffusion only illustrates four main categories: the diffusion of a single innovation in a single market, the modeling of diffusion across several countries, the modeling of diffusion across several generations of technology and multi-technology models.

Although there are some reviews on the subject,

**There are two novelties in our study:**

**1) the review of the selected relevant mathematical models is the most comprehensive and updated to our knowledge.**

**2) the link of innovation diffusion models with the Rogers communication channels in the frame of technology clusters.**

<u>Usefulness</u>

Our work provides a clear categorization of various stochastic and chaotic models that are or may be used in business. Usually managers apply a method without being 100% sure that an alternative method might work better with their data. Some managers might misuse a model because they don't fully exploit its capabilities. Although a model might sometimes "best fit" for a specific case, an analyst may still not be able to use it to predict a technology's saturation level because of not being able to interpretate his models parameters in the real world (Naim and Towill, 1993). As a result, a clear categorization that will be done with this research might clarify and help them to use the optimum model for their case.

## 1.3 Outline

This thesis is organized as follows: section 2 provides the theoretical framework of the thesis. The methodology follows in section 3. Then, in section 4, the various different application cases are presented, where models which have been used on various products in the literature and their advantages and disadvantages are discussed. In section 5, the results of this study are presented which shows the mathematical model of innovation diffusion that can describe the products/clusters. The mathematical diffusion models are discussed to assess how progress in modeling contributes to the understanding of diffusion. In section 6, conclusions are drawn and an answer to the research question is provided. In section 7 the results and discussion, in which some gaps or problems faced are written as well as future possible steps needed in the area of the innovation diffusion modeling are proposed. At the end of this study, a very detailed informative section 9 (appendix), contains the mathematical models of innovation diffusion found in the literature. This is chosen because all the models used in the application cases sections are based on the models included in the appendix section.

# 2 Theoretical Framework

## 2.1 The S-Curve Framework

Based on Rogers (1983), the communication process, as defined in our introduction, among innovation adopters occurs via *Communication channels* that are split into two main categories:

i. **According to their sources:**

*Localite* channels are internal sources of information within the same system.

*Cosmopolite* channels are those from outside the social system being investigated

ii. **According to their nature:**

*Mass media* channels are all those means of transmitting messages that involve a mass medium (i.e. radio, television, internet[1]) which enable a source of one or a few individuals to reach an audience of many (Rogers, 1983,p.198).

*Interpersonal* channels involve a face-to-face exchange between two or more individuals (Rogers, 1983, p.198). Scholars name this interpersonal communication as a *mouth-effect*.

Interpersonal channels may be either localite or cosmopolite, while mass media ones are almost entirely cosmopolite (Rogers, 1983, p.200).

The communication channels are very important and helpful method to distinguish who and how is affecting the innovation diffusion process. As a consequence, the S-curve will be influenced by alterations on the communication channels. But what is the S-curve framework and how does an S-curve plot look like?

The S-Curve is a mathematical model which is being applied to a variety of diverse fields: physics, biology, economics and management. In the innovation management field

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[1] Internet was not mentioned in Rogers book of 1983 edition, but we included it in the examples due to its current role

the S-Curve illustrates the introduction, growth and maturation of innovations as well as the technological cycles that most industries experience. In the early stages of a technological development radical innovations take place, while in the subsequent growth phase, incremental innovations occur; eventually, saturation takes place. As stated by Freeman and Soete, 'as in the product life cycle model, the path of such successive incremental innovations from introduction to maturity of any particular technology, could be represented in the familiar S-shape fashion' (2000, p.357). The plot of S-curve is presented in figure 1. The model also has plenty of empirical evidence: it was exhaustively studied within many industries including semiconductors, telecommunications, and so on.

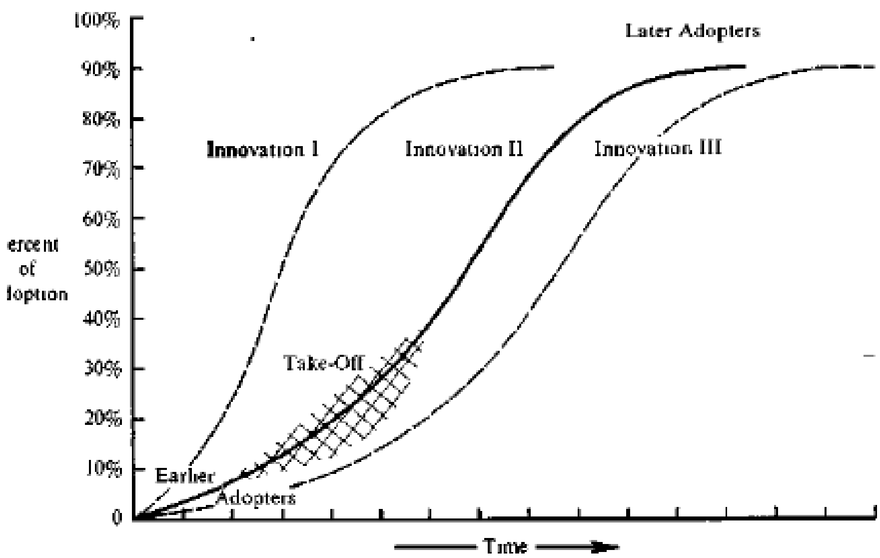



Figure 8: The S-curve plot of Innovation Diffusion (Source: Rogers, 1983, p.11)

The S-Curve is a mathematical model which is being applied to a variety of diverse fields: physics, biology, economics and management. In the innovation management field the S-Curve illustrates the introduction, growth and maturation of innovations as well as the technological cycles that most industries experience. In the early stages of a technological development radical innovations take place while in the subsequent growth phase, incremental innovations occur. Eventually, saturation takes place. The model also has plenty of empirical evidence: it was exhaustively studied within many industries including semiconductors, telecommunications, and so on.

The S-curve is usually represented as the variation of performance in function of the time/effort. There are also other possible metrics, like the number of inventions, the number of employees, the level of the overall research, or the profitability associated with the innovation. A critical point is to consider the cases where different performance parameters tend to be used over different phases of the innovation, as a result the outcomes may get mixed together, or one parameter will end up influencing the outcome of another. This links back to the different categories of competition and the modification of the type of competition that might happen, i.e. in the early stages of civil aircrafts, emphasis was given on their speeds while later on their fuel consumption.

## 2.2 Stochastic and Chaotic models

In the literature there is a specific mathematical function of which the S-curve constitutes a special case: the logistic function. Different initial values or additions of parameters will impact on the form of the S-curve over time as the resulting technological innovation stems from a particular mix of initial conditions, random events and long-term trends. These values are connected to the nature of technologies. Finding the parameters which best fit the model will result in a robust model and good argumentations. This research will try to find various stochastic or chaotic models that define S-curves for different clusters of technologies. The more accurate the model is, the better the performance of the S-curve is.

Before any analysis it is crucial to understand the foundation of the innovation diffusion models both conceptual and mathematical. A complete presentation starting from the fundamental model and ending with more complex ones is given in the appendix.

## 2.3 Comments on parameter estimation

One of the most critical procedures that affects a models' performance is the choice of the proper parameter estimation method. In the absence of historical or time-series data parameters can be estimated by means of certain-innovation specific analogues or expert judgments. If historical or time-series data are available, parameters can be estimated by means of standard but often nonlinear least squares (NLS) estimation procedure suggested by Srinivasan and Mason (1986) or through the maximum likelihood estimation (MLE) procedures proposed by Schmittlein and Mahajan (1982). Bass (1969) also proposed the ordinary least squares (OLS) estimation procedure for obtaining model parameters. The last method, OLS, is applicable when the process is observed at equally spaced time points and is based on the discrete function of the Bass model and is the simplest of all estimation methods (Dalal and Weerahandi, 1995). A method used is to use OLS to obtain some initial values for parameters in other more complex methods. When few data points are available, initial parameter values can be updated and revised as new data become available by using adaptive or Bayesian estimation procedures.

Mahajan and Sharma (1986), accounting the shortcomings of Bass proposal for parameter estimation, suggested a simple algebraic estimation procedure. This method is applicable not only for the Bass model parameter estimation but also for the other diffusion models. Mahajan and Sharma (1986) procedure requires knowledge of the occurrence of the point of inflection based on actual data, analogous products, or management judgments. Furthermore this method does not employ period by period time-series data and it may not provide the best fit to the data. On the other hand the method can be helpful when data up to the point of inflection are missing and when absence or limited data are given.

An alternative method used for the parameter estimation of diffusion models is the Augmented Kalman Filter with Continuous State and Discrete Observations (AKF(C-D)) (Xie et.al, 1997). This procedure claims better applicability to the models and is independent of the constraints of the models structure or the parameters nature. More specifically, it can be applied either in time-invariant and time-variant models and it can also be used in both cases of a solvable and unsolvable diffusion models. AKF(C-D) approach can be used for estimating parameters that are deterministic or stochastic and observation errors can be incorporated (Xie et.al, 1997). The algorithm of AKF(C-D) is a Bayesian updating procedure. A comparison of this method in relation to the other methods showed that in most of the cases it out performs them.

# 3 Methodology

In this section, the methodology used for the analysis of the research topic is discussed.

The method used for this research is a combination of literature reviews and desk research. On the one hand, we refer to the mathematical based literature and, on the other hand, to the innovation literature. Scientific literature is found through the university's digital library, Omega, Scopus and Web of Science. The most frequently cited papers were traced - note that more than 35000 citations are found for the term 'innovation diffusion' in the Web of Science database. The criteria for choosing the most useful papers within this set are many. Firstly we use the book by Mahajan and Robertson (1985) on innovation diffusion models and delineate our choice of models on those who are broadly cited and applied in journals. Therefore, not all the models, mentioned in the book of Mahajan and Robertson (1985), are selected, but those who are used extensively or be under research and development by many researchers (like the fundamental model of Bass). Since the book was written many years ago, we found new extended models that are not in the book. Secondly the papers are chosen according to their usability and applicability on Rogers's categorization on communication channels. The name of the author and the quality of the magazines found are an important factor for choosing the papers. An example of the procedure according to the first two criteria is the following: in the Omega search engine, using the terms 'bass model AND innovation diffusion' returns more than 500 results. The choice on which ones to analyze is done according to the model (i.e. Bass) journals published (i.e. Technological forecasting and social change, International journal of research in marketing, etc) the authors and the topic. Thirdly, we focus on mathematical innovation diffusion models according to their contribution in the modeling literature, showed also by querying a model in the scientific literature. The data are organized in a database where a separation of different technological clusters is made. For instance, a high tech product like the iPhone may show a different diffusion pattern that a low tech consumer product like a set of cultery. Due to the amount of information available and the importance of some innovation products in our everyday life, the following technological categories (clusters) have been chosen:

- Consumer products
- High-tech products
- High-tech services

- Energy consumption
- Renewable energy performance

The reasoning behind consumer products has to do with the availability of information and with the interest to examine products that are massively used, thus familiar to consumers. High-tech products are also widely used and seem to have conquered the marketplace and integrated into our everyday lives. Data about high-tech products during the last decades are also available and have been used by scholars in innovation diffusion modeling. High-tech services are following the high-tech products in time, and have been a promising rapidly growing field based on innovation creation and diffusion. The focus on the energy sector is also due to its importance in society and climate change. This sector has showed that during its implementation it has been characterized by important social and technological changes. Plenty of people with various backgrounds and jobs are interested on the energy sector particularly in renewable energy.

A simple database is created to categorize the use of S-curves found in the literature. When a paper describes an S-curve model, the first action is to check if the communication channels are part of its analysis. If a communication channel is not clearly mentioned or is not part of the analysis or cannot be defined logically, it is excluded from the research. Then we examine the model's mathematical formula and if it is contributing in the innovation diffusion modeling by proposing any additional factors related to the communication channels. Then we continue with the parameter estimation method of the models. We are interested to examine if and how the same innovation diffusion model with the same dataset is performing when different parameter estimation methods apply. This is to categorize the applicability of innovation diffusion models. Furthermore, we examine the models chosen for different products. Lastly, to track the evolving research agenda, comments and future suggestions would be kept in the file. After that, we denote and discuss the application of the S-curve model. The comparison among the models found for each category are based on a table that contains all the important information in a glance: technology cluster, products name or service, mathematical model, parameter estimation method, type of model according to communication channels, comments and applicability of the model.

The models were expected to be different in their formula or sensitivity of parameter estimation method, as the complexity of the product and the communication channels are varying among the technology clusters. For example, a black and white TV was only sold in big megastores and the communication channels were mostly local, while now a 3D TV is sold not only in stores but also in the internet and we have access to global communication channels. Furthermore, we can compare the models based on their extensions that would probably have been made; for example one parameter in the one cluster category might be not enough in the other again due to the development of communication channels.

Afterwards the records of the database of technological clusters are mapped to the best fitting chaotic or stochastic model.

## 4 Comparing S-curves

In the literature of innovation diffusion models, there are several cases that try to clarify what is the best model for each case. In this section a presentation of various applications of the models presented is performed.

We recommend the reader to turn back to appendix for a more detailed presentation of the innovation diffusion models found in the literature. We should point a reference about a well known model, Bass model, and its strong correlation on the Rogers' conceptual framework. Robertson et al. (2007) states that the Bass model is build on the Rogers' conceptual framework by developing a mathematical model that captures the non-linear structure of S-shaped curves. Furthermore, as seen in the appendix (section 8), the general type of the fundamental innovation diffusion model is:

$\frac{dN(t)}{d(t)} = g(t)\left(\overline{N} - N(t)\right)$, $N_{t=t_0} = N_0$, where the term

$g(t)$: $is\ the\ coefficient\ of\ diffusion$, that depends on the

- Nature of innovation
- Communications channel employed
- Social system attributes

The different types of Rogers communication channels, would alter the form of the term $g(t)$ and its parameter estimation methods. Maybe when $g(t)$ is not balanced to explain both Interpersonal and Mass media communication channels the model is not performing well. Perhaps in some cases, only one of the communication channels should be studied and considered, while the other type should not be present at all. Since many different hypotheses can be drawn, we chose to examine case by case the presence of the communication channels and comment on the models applicability for the products analyzed.

A simple and quick notification to remember the above correlations is the following:

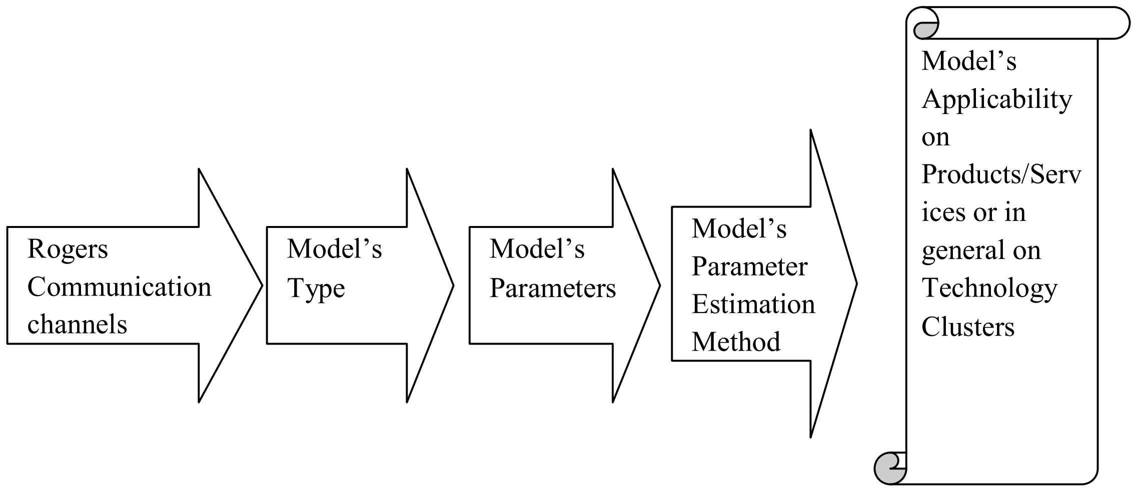


**Table: 1**

| Case number | Date/Authors | Applications-Case Studies | Model | Parameters estimation method |
|---|---|---|---|---|
| 1 | Schmittlein and Mahajan (1982) | Clothes Dryers Room Air Conditioners Color T.V. Dishwashers | Bass | Maximum Likelihood Estimation (MLE) |
| 2 | Srinivasan and Mason (1986) | Clothes Dryers Room Air Conditioners Color T.V. Dishwashers | Bass | Nonlinear Least Squares (NLS) approach |
| 3 | Kennendy (1991) | Cable T.V. subscription Clothes Dryers | Bass mixed model | |
| 4 | Sharma and Bhargava (1994) | Air conditioners, Clothe Dryers, Color T.V., Ultra Sound, Mammography, CT Head Scanner, CT Body Scanner, | Modified Mansfield internal model and modified flexible NSRL and NUI models | Categorization of adopters that influence potential adopters is weighted via a weight factor w |
| 5 | Dalal and Weerahandi (1995) | The penetration of telephone answering machines in the US. | Bass | MLE and Weighted least squares estimation methods |
| 6 | Giovanis and Skiadas (1999) | Electricity consumption in Greece and US | Stochastic Logistic | MLE and multiplicative noise estimation parameter |
| 7 | Karmeshu and Goswami (2001) | Black and white T.V. sets in India | Based on TPD (two-point distribution) and FPT (first passage time) | NLS, VFSR (very fast simulated re-annealing) technique |
| 8 | Hirooka (2003) | 17 products: 5 bulk chemicals, 4 engineering plastics, 6 electric appliances, crude steel, automobiles (all data except one concern the Japanese market) | Logistic | A linear correlation of the Fisher-Pry plot was examined, diffusion coefficient α was determined |
| 9 | Smith and Song (2004) | New Internet grocery-shopping service in Philadelphia from May 1997 till January 2001. Total number of zip codes=46, adopters=1288 | Bass, spatial mixture model | MLE requires large sample size. Alternatively an EM estimation algorithm is constructed and Bayesian. Both are good for small sample sizes. |
| 10 | Goswami and | Cloth dryers (US), | Bass, NUI and | NLS |

| | | | | |
|---|---|---|---|---|
| | Karmeshu (2004) | room AC(US), black and white TV(US and India), color TV (US) | PVRD (parameter variability randomness in diffusion) models | |
| 11 | Everdingen et al. (2005) | The diffusion of Internet access at home and mobile telephony among households in 15 EU | An extension of a model by Putsis et al. (1997) | An extension of the augmented Kalman Filter with Continuous States and Discrete observations (AKF(C-D)) |
| 12 | Gutierrez et al. (2005) | Forecasting the total natural-gas consumption in Spain in 1973-2000 | Gompertz | MLE in continuous sampling and extended method of linear SDE with multiplicative noise to the case of a non-linear SDE with multiplicative noise (white) |
| 13 | Robertson et al. (2007) | Segmental new-product diffusion of residential broadband services in UK | Gompertz | NLLS estimator |
| 14 | LIAO and XU (2007) | Telephone subscribers of urban and rural areas in China | They constructed their own model with migration between two different colonies | Probability estimation |
| 15 | Tseng and Hu (2009) | Inventory of cars in The Netherlands (1965-1989), Cellular phones in Portugal (1995-2000) and Worldwide PC demand (1981-1999) | Bass based | Fuzzy regression |
| 16 | Schilling and Esmundo (2009) | Wind and Geothermal energy technologies and trajectories performance (9 countries) | Pearl curve | Linear regression |
| 17 | Putsis et al. (1997) | VCRs 1997-1990, Microwave ovens 1975-1990, CD players 1984-1993, PCs 1981-1991, for 10 EU nations | Bass based, an extension | NLS |

## 4.1 Analysis on reviewed papers

Case number 1:

| Case no. | Authors | Product or Service | Model | Parameter estimation method | Communication channels |
|---|---|---|---|---|---|
| 1 | Schmittlein and Mahajan (1982) | Clothes Dryers<br>Room Air Conditioners<br>Color T.V.<br>Dishwashers<br>Medical equipment | Bass | Maximum Likelihood Estimation (MLE) | Interpersonal, Localite |

The first paper chosen for our analysis is cited 113 times (source: Google Scholar) and is published in the first volume of the Marketing Science journal. In this paper, Schmittlein and Mahajan (1982) apply the Bass model. It is one of the first publications that focused on the Innovation Diffusion Models and aimed to try the Bass model and suggest additional tests of the model on products under different conditions. The paper does not focus on the model itself but on the models parameter estimation method; Maximum Likelihood Estimation (MLE). The applications that are used can be separated into two categories; consumer products (clothes dryers, room air conditions, color TVs, dishwashers) and specialized medical equipment (radiological such as ultrasound, ct head-scanner, mammography). The first category of products was bought by households while the second by hospitals in USA. MLE is tested for its validity in comparison with Ordinary Least Squares (OLS) parameter estimation method.

In this paper it is shown that MLE permits the computation of approximate standard errors for the parameters p,q,m and can verify the required sample size that will allow the forecast of the adoption level of the innovation to “any desired accuracy”.

The results are good but open to discussion in terms of their ability to support the argument that this is the best model and parameter estimation method. For instance, their attempt seems to perform better in fitting measures and one-step ahead forecasting of an innovative product acceptance; thus, diffusion. Furthermore, they consider only sampling errors and not any other kind of error (Srinivasan and Mason, 1982). Additionally, it is pointed that a "…tractable method for incorporating…" direct word of mouth effects should be developed.

The Bass model used, is stated by the following equation:

$\frac{dN(t)}{dt} = \left(p + \frac{q}{m}N(t)\right)(m - N(t))$, where, $N(t)$ is the cumulative number of adopters at time t,$m$ is the maximum number of adopters, $p$ , $q$ are the coefficients of innovation and imitation respectively. A more detailed presentation of the Bass model is given in the appendix section.

The following equation represents the cumulative distribution function of adoption time for an individual chosen at random from the population:

$$F(t) = \frac{c(1 - e^{-bt})}{(1 + ae^{-bt})}$$

In the previous equation the following substitutions are set; $a \equiv \frac{q}{p}$ , $(a \geq 0)$, $b \equiv (p + q)$, $(b \geq 0)$ ,the sample size of adopters is $M$ and the expected number of eventual adopters is $cM$as the probability of eventually adopting is represented by $c$ $(0 \leq c \leq 1)$. The estimators of $p, q, m$ obtained by MLE are expressed by the following equations:

$$\hat{p} = \frac{\hat{b}}{(\hat{a} + 1)}, \hat{q} = \frac{\hat{a}\,\hat{b}}{(\hat{a} + 1)}, \widehat{m} = \hat{c}M$$

When the OLS method is used in comparison to MLE, the equation and parameters were:

$E\big(X(t)\big) = \left(p + \frac{q}{m}N(t-1)\right)(m - N(t-1))$ and $X(t) = a_1 + a_2N(t-1) + a_3N^2(t-1) + \varepsilon(t)$, where $a_1 = pm, a_2 = (q - p), a_3 = -{}^{q}/_{m}, E[\varepsilon(t)] = 0, Var[\varepsilon(t)] = \sigma^2 and\ \varepsilon(t_i)$ is independent of $\varepsilon\big(t_j\big)$ for $i \neq j$. The parameters estimations are:

$$\hat{p} = \frac{\widehat{a_1}}{\widehat{m}}, \hat{q} = -\widehat{m}\widehat{a_3}\ and\ \widehat{m} = \frac{(-a_2 - \sqrt{a_2^2 - 4a_1a_3})}{2\,a_3}$$

Concerning the empirical data used in all the products, the time series used for the consumer products considered only the first years of sales growth to avoid re-purchases or replacements. The length of the years and criteria of choices are not mentioned.

It is important to point that the estimated of p and q are not strictly comparable because they derive from different models.

Back to the analysis, the Bass model with MLE seemed to be correct for the dishwashers. For the clothes dryers, air conditions and color TVs, MLE provided a better fit to the data and the one-step ahead sales forecasts were predicted with validity. The data for the consumer products were survey data. In the cases of the medical products, the MLE possessed correct signs and conceivable values, concluding a good fit to the adoption data. The data for the medical equipment was sample data of hospitals. A logical consequence of the observations showed that more data by lengthening the data collection period or collecting more data for the same could increase the reliability of the parameters estimations and creation of confidence regions. In all cases the individual adoption times are not known making the use of a histogram with the number of individuals falling in each time interval necessary to fit the data.

Focusing on the communication channels by Rogers (1983), this paper is interesting as it includes in its analysis a mass media communication channel itself (Color TV's). This excludes the mass media communication channels as a mean to transfer information for products. Considering the time period of the diffusion of all the consumer products of the paper, it is more probable that the diffusion of the innovations occurred via *interpersonal communication channels*. Since the only information is based on the Statistical Abstracts of the United States, and the records demonstrate the diffusion of the products all over USA, a distinction of the communication channels according to their sources is risky. Probable among the medical products diffused in hospital, some might suggest that the communication channels are localite, because the products are very specialized and the hospitals are a close market. For the house consumer products, an argument to support the diffusion of the products via *localite communication channels* is based on the habit of consumers to 'advertise' in their local community the products they purchase. We should not forget that at that time there were no mass media expanded and broadly used in the households and organizations as they are nowadays.

Case number 2:

| Case no. | Authors | Product or Service | Model | Parameter estimation method | Communication channels |
|---|---|---|---|---|---|
| 2 | Srinivasan and Mason (1986) | Clothes Dryers<br>Room Air Conditioners<br>Color T.V.<br>Dishwashers | Bass | Nonlinear Least Squares (NLS) approach | Interpersonal, Localite |

Our second choice of papers chosen for our analysis is cited 195 times (source: Google Scholar) and is published in the fifth volume of the Marketing Science journal. In this paper, Srinivasan and Mason (1986) apply a Bass model of Innovation Diffusion and focus on the fit and validity of a parameter estimation method; Nonlinear Least Squares (NLS). This can be characterized as a logical reaction to the Schmittlein and Mahajan (1982) paper. The reason is that Srinivasan and Mason (1986) apply the same model on the same products with the Schmittlein and Mahajan (1982) paper, in order to combine the MLE method to NLS and prove NLS's effectiveness.

The mathematical expression of the model used is the same as in the case 1 Schmittlein and Mahajan (1982). The additional element is the $u_i$ error term in the equation that expresses the sales $X(i)$ in the i-th time interval $(t_{i-1}, t_i)$ considering that the number of the ultimate adopters is m. This is written by the equations:

$X(i) = m[F(t_i) - F(t_{i-1})] + u_i$ and $X(i) = m\left[\frac{1-e^{-(p+q)t_i}}{1+\frac{q}{p}e^{-(p+q)t_i}} - \frac{1-e^{-(p+q)t_{i-1}}}{1+\frac{q}{p}e^{-(p+q)t_{i-1}}}\right] + u_i$ for $i = 1,2,\dots,T$ and $varu_i = \sigma^2$.

According to the authors, $u_i$ may be considered to represent the net effect of sampling errors, or the misspecification of the density function, or the impact of variables like economic situation, technological development, advertising, pricing or in few words the effect of marketing and competition.

Concerning the parameter estimations and standard errors this paper shows that the NLS method leads to accurate results. Regarding the fit statistics, in this paper compared to the one of Schmittlein and Mahajan (1982), are calculated by using only actual and fitted number of adopters for each time period and not that with combination of the actual and fitted number of non adopters remaining. On the topic of the one-step ahead forecasts, the NLS and MLE methods provide similar results with NLS performing better in two consumer products and MLE in the remaining two. Pertaining to the one-step ahead forecasts for the medical equipment due to the small sample size, MLE accounted la larger to NLS standard error in seven out of twelve cases. To conclude, NLS was expected to perform better than MLE in statistics. Another important conclusion it concerns the number of years of data required for NLS and MLE including data on peak sales. It seems that for the products used, MLE looks more accurate when the available data are about the first four years of a products life. On the contrary, NLS looks more accurate when the available data are about the first eight years of a products life.

The authors let an open window for future studies on additional product diffusion models and comparison by using MLE and NLS parameter estimation methods.

The argumentation, on which communication channels are applied here and why, is the same with the analysis on the first case paper. That is because in this paper the products and the datasets for these products diffusion are the same with the first paper by Schmittlein and Mahajan (1982). As a consequence we propose the use of *interpersonal communication channels* for the consumer products and the use of *localite communication channels* for the medical equipment.

<u>Case number 3</u>:

| Case no. | Authors | Product or Service | Model | Parameter estimation method | Communication channels |
|---|---|---|---|---|---|
| 3 | Kennedy (1991) | Cable T.V. subscription<br>Clothes Dryers | Bass mixed model | Mixed estimation technique | Interpersonal<br>Localite |

This paper is based on the Bass model and is cited 6 times according to Google Scholar. It focuses on the problematic aspect of the 'managerial intuition' which is not correctly estimated usually in the model. The term managerial intuition refers to extraneous various sources that specify the parameter values. The aim is to 'extend the mixed estimation technique to handle stochastic prior information' as an acknowledgement that the validity of the value of the managerial intuition is uncertain. There are mostly some not so reliable methods applied to calculate the managerial intuition (term m in the model that defines the total number of first purchasers). These are the Bayesian method, or the use of a common value for similar products (i.e. consumer products) when the value seems to work for the most cases applied. The aim of this paper is to show that the technique proposed is a good and simple to compute alternative to the Bayesian method.

Mathematically speaking, the equation used is the typical Bass model (see appendix). The additive value of the paper is the presentation of their proposed parameter estimation method.

There are two applications mentioned. The first illustrates Cable Tv's Subscribers in USA during the period 1962-1965. A comparison among the traditional method and the proposed one, shows that the latter provides good results and can also predict the sales peak year. The second example analyses a typical consumer product studied from many scholars; the clothe dryers. The choice is based upon the findings that for this case a 'published "expert advice" could be found'. The goal is to show how this approach can be efficient in forecasting even in the case where the expert advice is absence. In the second application the approach is not so successful maybe as the authors' state 'because of the lack of the expert advice'.

The restricted number of the applications cannot judge the validity or not of the mixed estimation attempt. More examples are needed to be tested.

We base our choice to choose *interpersonal and localite communication channels* as a mean to transfer information to adopters, for the applications mentioned in this paper, because again as in the previous papers, the data are taken from the Statistical Abstract of USA for the years 1962-1965 (for the cable TV subscribers) and 1949-1955 (for the consumer household products). As a result, the purchasers of consumer products are mostly affecting their close social cycle personally by sharing their opinion on the product. Furthermore, the cable TV subscribers are studied when the TV diffusion was in its early phases. So, the communication channel probable was not a mass media but again based on a person by person experience.

Case number 4:

| Case no. | Authors | Product or Service | Model | Parameter estimation method | Communication channels |
|---|---|---|---|---|---|
| 4 | Sharma and Bhargava (1994) | Air conditioners, Clothe Dryers, Color T.V., Ultra Sound, Mammography, CT Head Scanner, CT Body Scanner, | Modified flexible NSRL and NUI models | Categorization of adopters that influence potential adopters is weighted via a weight factor w | NSRL – Interpersonal NUI-Mix of Interpersonal and Mass media |

This paper is cited 5 times (source: ISI Web of Science) and it is printed in a prestigious journal: Technological Forecast and Social Change. Sharma and Bhargava (1994) propose two models based on two already known flexible diffusion models: NSRL and NUI (see appendix). Their focus is on the non equal weightage of adopters of a product during the introduction and diffusion of a new innovative product. Specifically, they question the constancy and homogeneity of the internal influence coefficient over the groups of adopters who adopt the product at various temporal stages in the NSRL and NUI models.

In mathematical terms, the number of adopters is represented by the equation
$N(t) = n(t) + wn(t-1) + w^2 n(t-2) + \cdots + w^{t-2} n(2) + w^{t-1} n(1)$ where the new term w is a weight factor $0 < w < 1$ ,
and not by the $N(t) = n(t) + n(t-1) + n(t-2) + \cdots + n(2) + n(1)$. As a consequence, the models proposed are written:

$\frac{dN}{dt} = q\left[\frac{1}{M}\left\{\sum_{i=0}^{i=t-1} n(t-i)w^i\right\}\right]^{\delta}(M-N)$ and is the NSRL modified model of this paper

$\frac{dN}{dt} = \left(p + q\left[\frac{1}{M}\left\{\sum_{i=0}^{i=t-1} n(t-i)w^i\right\}\right]^{\delta}\right)(M-N)$ and is the NUI modified model of this paper.

According to the authors, in comparison to other scholars, they treat the social group of adopters ‘as being uniform and introduce heterogeneity in the context of category of adopters n(1),n(2),…,n(t)’ while the others ‘divide total adopters into social groups and treat each group separately’.

Sharma and Bhargava (1994) test their models by using already known and used by previous scholars data of certain consumer products in USA; air conditioners, clothe dryers, color TV's, ultrasound, mammography, ct head scanner and ct body scanner.

The advantages of using the proposed models are briefly that

i. the conversion factor (the terms multiplied with the term (M-N)) increases slowly and then instead of reaching a maximum value (saturation) falls to zero (in NSRL modified model) or concentrates to a much lower value (in NUI modified model)
ii. they closely picks up the market growth trends
iii. the coefficient term q can take values higher than one and still pertain that the overall conversion factor is still less than one

A probable disadvantage is how to value the weighting term w. As the authors point out, 'lower w might give a good fit but it may also give a very large and sometime unrealistic M'. The value w equal to ¼ is the one the author use in this paper because they assume that the influence of previous years adopters is less influential in time depth. They also state that the choice of which is the best value for w depends on the form of data sets and the researchers understanding of growth. It should be noted here that even the first argument sounds logical, the second is very subjective.

In overall, from this paper the market potential of the proposed model is larger in all cases. Markets ups and downs are very closely picked up. Adjusted $R^2$ are better. One-step forecasts are not so good but a five year forecast looks realistic. The authors let an open window for further research for those who will study software product diffusion by using their equations but also consider the learning time someone needs.

In terms of Rogers's communication channels, what the authors do is to propose their model as a result of the NSRL and NUI modifications. These modifications focus on the coefficient of the internal influence parameter of the adopters (q). That means the center of attention is the interpersonal contacts among the adopters. By definition the NSRL model is mapped to interpersonal communication channels and the NUI to both interpersonal and mass media communication channels. Sharma and Bhargava (1994) here modify the internal influence coefficient in both of their proposed models but do not alter the mass media coefficient. That does not entail that the authors' approach does not include the mass media communication channels in the NUI modified model but since mass media communications coefficient is not a point of interest, it maintains as in the original NUI model. To sum up, the NSRL modified model is based on *interpersonal communications channels* and the NUI on a combination of both *interpersonal and mass media communication channels*.

Case number 5:

| Case no. | Authors | Product or Service | Model | Parameter estimation method | Communication channels |
|---|---|---|---|---|---|
| 5 | Dalal and Weerahandi (1995) | The penetration of telephone answering machines in the US. | Bass | MLE and Weighted least squares estimation methods | Not clear but can be both Interpersonal and Mass media |

This paper is cited 4 times according to Google Scholar search engine and is published in an early volume of the Marketing Letters Journal. In this paper, Dalal and Weerahandi (1995) focus on the Bass model estimation parameters. More specifically, they readdress the pitfall of the assumption made in Bass model that 'lead to specific stochastic processes rather than deterministic models' and that 'statistical inferences concerning the model are usually based on the deterministic analog of the process' (p.123). As a consequence they support that in practice the estimators used (they don't mention where they had read that) 'tend to have poor performance'. Due to the fact that exact MLEs of parameters are found only for very small number of adopters$(\bar{N})$, Dalal and Weerahandi consider the probability $(p_m(t))$ an non adopter at time $\tau$ to become an adopter at time $\tau + t$. Briefly, what the authors propose is to try to improve the parameter estimation methods by incorporating the following; consider the covariance structure in a nonlinear weighted least squares regression (NWLS) and use the likelihood function to compute MLEs of the parameters. Mathematical details on the procedure the authors followed to build their method are out of the scope of this research.

The application that is used is the penetration of telephone answering machines in the US during the period 1978-1987. Even though statistical data of sales were missing for the years before 1978, Bass model provides the feature to study the new adopters by accounting only the available data. The performances of the estimation methods choose were test for various values of the parameters.

The conclusions from this paper are that when $\overline{N}$ is treated as an unknown parameter the estimation is very sensitive to the observed data and estimation method. For short terms it does not make a difference. For long term $\overline{N}$ needs to be determined exogenously. We should point that the authors did not account changes such as price, or customer demographics, product attributes and furthermore dimensions of a products diffusion process. In overall this paper is using a well-known model and just tests some modifications of a known parameter estimation method.

This paper does not mention clearly its correlation with Rogers's communication channels. The focus is mostly on the parameter estimation method and not on the terms of the model. The analysis of the application is based on the Bass model which incorporates both interpersonal and mass media communication channels. In this case, the diffusion of the telephone answering machines during the 1978-1987 implies that there were both interpersonal and mass media means of influence in the society. The telephone answering machines is a product that can be used either in a house or in a firm. Some adopters could have been aware of the product from their friends and neighbors, collaborators, customers, suppliers, while others from a newspaper, TV or radio. Taking into consideration our arguments and way of thinking, we conclude that the communication channels are both affecting the diffusion process.

Case number 6:

| Case no. | Authors | Product or Service | Model | Parameter estimation method | Communication channels |
|---|---|---|---|---|---|
| 6 | Giovanis and Skiadas (1999) | Electricity consumption in Greece and US | Stochastic Logistic | MLE and multiplicative noise estimation parameter | Probable interpersonal. Not so clear. |

The sixth paper of our study is written by Giovanis and Skiadas (1999) and published in a very high-status journal: Technological forecast and social change. The number of citations is 10 according to ISI Web of Science search engine (April 2010).

As Giovanis and Skiadas (1999) state in their paper, the objective here is to formulate a stochastic logistic model derived from the logistic growth model, that can describe an innovation diffusion process implementing the disturbances from the environment (socioeconomic factors affecting the adopters). The perspective of the stochastic approach of the logistic model recognizes the necessity to include in long-term forecasts the existence of rapidly changing internal and external factors affecting the innovation diffusion process. Obviously Giovanis and Skiadas (1999) base their research on the logistic model (see appendix) and include additional estimation methods needed for their study of the stochastic model. The deterministic logistic model assumes that the innovation diffusion process occurs in a stable and finite environment and that the remaining width of growth is known since the saturation value and current size of the process is also known (p.236). In order to support their idea to focus on the stochastic logistic model, the authors compared their estimating results from the stochastic model with those from the deterministic model. They also try to predict the performance of the adoption process 'by defining a sub domain in which all trajectories should belong with a predefined probability'. It is crucial to reference a statement made by Clarke (1973) and written in this paper: ' this kind of forecasting is especially vital in the field of technological forecasting since it is more useful not to try to describe the future, but to define the boundaries within which possible futures must lie'.

The mathematical form of the equation in this paper is:

$\frac{df(t)}{dt} = g(f(t), p)\left[h(F) - h\big(f(t)\big)\right] + g(f(t), p)\, u(t)$ where the term $F - f(t)$ represents the known remaining adoptions, $u(t)$ is the random fluctuations because of the action of many uncontrollable factors, $F - f(t) + q\, u(t)$ is the real remaining adoptions of the product, $u(t)$ is a one-dimensional white noise process and finally $q$ is the parameter controlling the power of the noise. The equation is finally formed as:

$df(t) = b\,\frac{f(t)}{F}\,\big(F - f(t)\big)\,dt + q\,f(t)\,dW(t)$, where $c = \frac{b\,q}{F}$ and $W(t)$ is a one-dimensional Wiener process. The solution of the previous equation and used in the paper is given by the type:

$$f(t) = \frac{f_0 e^{\left\{\left(b - \frac{c^2}{2}\right) t + c\,W(t)\right\}}}{1 + b\,\frac{f_0}{F} \int_0^t e^{\left\{\left(b - \frac{c^2}{2}\right) s + c\,W(s)\right\}}\,ds}$$

There are two methods used to estimate the terms b, F and c respectively. The first is the MLE and the second is the multiplicative noise estimation parameter.

The application of this approach is the electricity consumption in US and Greece for 28 years (the exact dates are not mentioned) and the forecasting period is 6 years.

The results showed that the saturation level predicted from stochastic model is larger from the deterministic one for both US and Greece. The results are good in overall as the model as the real data are between the upper and lower limits of the measurements. The reader can look at the figure 1 of this paper where for the values $F = 50, b = 0.05, c = 0.002\ and\ f(t_0) = 2$, the solution of the stochastic logistic model has a sigmoid form.

The overall picture of the paper in relation to Rogers's communication channels is not so clear. By default, a model based on the logistic equation, uses interpersonal communication channels to describe the innovation diffusion. So we expect to find interpersonal communication channels as a mean for the electricity diffusion, indicated by its consumption. More houses adopting electricity, the more consumption is measured. In the paper the authors do not report the timeline of their analysis but just the duration of prediction (6 years) and the data (in years) on which they were based on (25 and 28 years for Greece and USA respectively). Our suggestion for the type of the communication channels used is localite and their categorization according to their nature, *interpersonal*. We could have been more confident on our choice if we attained more information from the authors. The parameter estimation method could be an additional indicator for the choice of the communication channel, but its stochastic type with the combination of MLE method is not helpful into this direction.

Case number 7:

| Case no. | Authors | Product or Service | Model | Parameter estimation method | Communication channels |
|---|---|---|---|---|---|
| 7 | Karmeshu and Goswami (2001) | Black and white T.V. sets in India | Based on TPD (two-point distribution) and FPT (first passage time), Bass based | Stochastic, random choices NLS, VFSR (very fast simulated re-annealing) technique | Mixed |

The paper by Karmeshu and Goswami (2001) is cited 3 times (source: Google scholar) and published in a very specialized journal: IMA (Institute of Mathematics and its Applications) Journal of Management Mathematics.

That paper uses more complex way to study the life cycle patterns that occur when heterogenous groups of adopters are studied. They are also interested on issues related to the first passage time (FPT) in relation to market penetration in the same population of adopters. As the authors support, the stochastic behavior of the context of innovation diffusion is due to the' intrinsic, environmental and population variability' (p.108) an argument that strengthens the focus on the heterogenous populations. Most of the research was done before this paper only on homogenous populations implying the adopters to be identical in terms of parameters of the model, something not so realistic.

The Bass model is used as a basis to formulate the model used in this paper. The stochastic model that the authors use for the innovation diffusion focus on the randomness of parameters due to the heterogeneity mentioned before. The reader can look at the paper for a detailed presentation of the model which is out of the scope of this study. The authors provide figures that show their approach (TPD extention) outperforms the Bass and NUI models and is efficiently simulating the actual data. TPD framework was developed in 1975 by Rosenblueth and later extended to study stochastic evolution of dynamic systems with random parameters (in 1987). That extended TPD is employed here in 'a dynamic setting for analysis of moments of the adoption process' (p.112).

The application that is used to demonstrate the approach of this paper is the diffusion of black and white televisions in India, a common durable product. The time period covered is 1971-1998. The Bass and NUI models are clearly not satisfactory to demonstrate the diffusion of the selected product. The proposed one is better for far from homogenous populations (p.124). The results also show that while in homogenous populations the pattern of the life cycle curves generated is uni-modal, in the case of heterogenous populations it can be both uni-modal or bi-modal. Some scholars referenced also in this paper query if the appearance of both uni and bi modal life cycle curves is due to the heterogeneity of the population or due to repeat purchase of the same product. Another issue is the 'strategic allocation of resources for monitoring and controlling the external source' i.e. the heterogenous population.

In overall this paper alerts the reader for the importance to consider the context of the population that adopts an innovation and affects its diffusion. The form of the bi-modal pattern supports the necessity for a stochastic approach on the model. Open inquiries mentioned if answered will strengthen the argumentations of the authors. Further test on the same approach for more durable products would be an asset.

The interesting part of this paper is its correlation to Rogers communication channels. As the model used is Bass based, by default it posses both types of communication channels. The terms $\alpha$ and $\beta$ in the model represent the coefficients of external and internal influence respectively. The choice of the authors is to treat them as random constant coefficients. An interesting attempt made by the authors is to test their model under two special conditions: when external influence is only present ($\beta$=0) and when internal influence is only present ($\alpha$=0). From the analysis it seemed that the influence of the external source (*cosmolite communication channels*) grew more than the internal (*localite*) along the ten year period examined. That was partly due to the improved 'relay/transmission infrastructure in the country' (p.123). The heterogeneity of the adopting population also justifies the influence of both communication channels considered that some adopters were influenced by word of mouth and others by other adopters. We support that as the application cases are black and white TV's, the communication channel is not a *mass media* one but mostly an *interpersonal*.

<u>Case number 8</u>:

| Case no. | Authors | Product or Service | Model | Parameter estimation method | Communication channels |
|---|---|---|---|---|---|
| 8 | Hirooka (2003) | 17 products: 5 bulk chemicals, 4 engineering plastics, 6 electric appliances, crude steel, automobiles (all data except one concern the Japanese market) | Logistic | A linear correlation of the Fisher-Pry plot was examined, diffusion coefficient α was determined | Mostly mass media |

This paper is cited 11 times according to the Web of Science search results and is published quite recently (2003) in the Journal of evolutionary economics.

In this paper, Hirooka (2003), demonstrates that the diffusion of many innovative products obey the S-curve of the logistic equation (see appendix). He recognizes that under a stable economy the explanation of a product's diffusion through the logistic equation is valid, but this diffusion is easily disturbed by economic turbulences. The author explains the diffusion of products that characterize the Kondratiev waves and presents the graphical representation of those diffusions pointing that they form an S-shaped curve. In these graphs, after the recession, the diffusion of the product resumes and takes the same slope of the logistic curve as before the recession. This supports that the product diffusion has its own inherent trajectory with a definite diffusion coefficient. The locus of technological development also follows a logistic equation. As the author claims, Kondratiev waves are also presented to help explain the findings of causal relation between bubble economy and depressions with innovations. The focus of this study is to 'determine where the diffusion curves of innovation products are located on the Kondratiev waves' (p.556). The results show that 'the diffusion timing of various innovations [among those studied] always gathered at the upswings of the Kondratiev business cycles' (p.558). According to the author, the previous fact may also imply that the 'diffusion of innovation products is retarded over the recession and resumes at the next upswing' (p.559).

In relation to the products cluster examined, the study shows that 'chemical products, crude steel, and automobiles are slow diffusing products, while' on the contrary, electric appliances are 'fast diffusing products'. That according to the author is logically explained due to their nature because consumer products are usually easily recognized by potential users 'at a glance' while most of the future applications of chemicals are not.

Briefly, one of the conclusions is the physical nature of innovation diffusion meaning that it is a physical phenomenon with its own 'inherent diffusion coefficient' (p.572). Another conclusion is the nonlinearity of innovation diffusion (p.572) and the necessity to study it with more complex models that might be able to integrate some characteristics of economic turbulence. That sounds to be a very difficult task for researchers even until now, a challenge. Lastly, there is a causal relation between innovation and economic development (p.572) as the one fosters the other.

In terms of Roger's communication channels, this paper does not focus on them. What can be concluded is that for many of the products like automobiles, we do expect the presence and influence of *interpersonal channels*. For mass production industrial products related to the infrastructure of a nation, the communication channels seem no to affect their diffusion i.e. railways. But the use of the infrastructure i.e. trains, is for sure based on *interpersonal and mass media communication channels*. The same story goes for the chemical industry and its products i.e. plastics. The use of the logistic equation is not so helpful because it is mostly based on mass media communication channels. So we conclude that the authors did not count the interpersonal communication channels as they focused mostly on the technologies and not on the products because their concern was on the innovation diffusion, innovation clusters and their relation to Kondratiev waves and business cycles.

Case number 9:

| Case no. | Authors | Product or Service | Model | Parameter estimation method | Communication channels |
|---|---|---|---|---|---|
| 9 | Smith and Song (2004) | New Internet grocery-shopping service in Philadelphia from | Spatial mixture model | MLE requires large sample size. Alternatively an | Interpersonal, Localite, Mass media, Internet |

| | | | | | |
|---|---|---|---|---|---|
| | | May 1997 till January 2001. Total number of zip codes=46, adopters=1288 | | EM estimation algorithm is constructed and Bayesian. Both are good for small sample sizes. | |

The choice of this paper is due to its interesting topic to approach innovation diffusion. It is a quite recent one, numbering only one citation (source: ISI Web of Science /two citations in Google scholar) and perhaps this is because it is published in a non-typical managerial journal: Geographical Analysis.

Assumptions implicit in this model are: the spatial-mixture and the constant-population. Demographic data like: access to off-line retail stores, access to the internet and attractiveness of on-line shopping are considered. The model is limited in scope: two possible extensions could be made. First is to relax the constant-population assumption and second is involve real-time considerations with a Markov chain.

A recent paper is the one by Smith and Song (2004) in which the main interest is to model the diffusion of a new product or technical innovation (as they call the internet service they studied). The approach they follow is based on a spatial mixture model of innovation diffusion. That is done by considering that an adopter in a region (named r), would be influenced by his interactions (inside region r) with current adopters in neighboring regions or all other attributes of individuals again inside the region r that may influence their adoption propensity. Then the probabilistic mixture of the two previous processes results to the spatial diffusion model that is proposed. With this approach the authors combine the likelihood of adoption due to spatial contacts and intrinsic effects. While for the spatial process the steps and logic followed are clear, for the intrinsic process the factors that can distinguish individuals of region r from others could be their income and age. The factors that can influence the innovation itself can be local advertisement and media information in area r.

The equation of the model proposed is expressed by the following line:

$p_n(r|f_n) = \lambda p_c(r|f_n) + (1-\lambda)p_0(r), r \in R$, where

$p_c(r|f_n) = \sum_{s\in R} \frac{M_r e^{-\theta c_{sr}}}{\sum_{v\in R} M_v e^{-\theta c_{sv}}} f_{ns}, r \in R$, and

$$p_0(r) = \frac{M_r e^{\sum_{j=1}^{J} \beta_j x_{rj}}}{\sum_{s\in R} M_s e^{\sum_{j=1}^{J} \beta_j x_{sj}}}, r \in R$$

Given that $p_0(i)$denotes the probability that the individual i in region r is the first adopter, then the corresponding regional event probability is $p_0(r)$. $M_r$ denotes the population size of region r. $\beta_j$ are coefficients assumed to be common in all regions. $x_{rj}$ are the intrinsic to the individuals factors in region r and serve to distinguish them from the other individuals (p.121). $c_{sr}$ is the contact cost from region s to r. $\theta$ reflects the relative importance of each type of cost. $f_n$ denotes the corresponding relative-frequency distribution for the n$^{th}$ adoption event. Finally $\lambda$ is the mixture probability where, $\lambda \in (0,1)$.

The assumptions made in this spatial-mixture process model are that the adoptions are treated first as either a process due to contact effects or not. That determines which distribution is used ($p_c$ or $p_0$) and sampled to determine the region in which the new adoption occurs (p.123). Another assumption is that the regional number of potential adopters ($M_r$) is treated as constant. Furthermore, the number of actual adopters is a small proportion of the total regional population to allow them to be considered constant. In the model there are only relative population sizes that need to be considered as the doubling of the population sizes does not affect the adoption process.

The parameter estimation method is chosen carefully by showing first that MLE approach would require large sample sizes. Therefore, the authors develop two other estimation approaches to achieve more reasonable results for small sample sizes (EM and Bayesian-MAP). This is crucial in cases where it is more important to focus on the early stages of an adoption process (p. 120). The EM algorithm has the advantage of 'building in a natural constraint on λ with no additional modeling assumptions' while the Bayesian 'turns out to be more attractive from a practical viewpoint, but does require additional modeling assumptions' (p.131). An advantage of the EM algorithm is that it can estimate the parameters (β, λ, θ), even when there are missing data. EM is also simpler in calculation.

The application that is used is about a small data set about the adoption of a new internet grocery-shopping service (Netgrocer.com) by consumers in the Philadelphia metropolitan area. The selection of the area is made by using zip codes (numbered 46). This is the area r mentioned before. The time period studied is May 1997 to January 2001. The contact costs,$c_{sr}$, were taken to be 'linear in distance between centroids of zipcode areas' (p.140). Populations were assumed to be the same since the 1999 demographic data for all the years. The adopters were separated into categories according to their age, origin, educational level, number of people in a household. The results showed that the word-of-mouth contacts were an important component of new-adoption behavior. Contacts proved to be very sensitive to distance and they occur inside the zip code areas selected. Adoptions might have resulted from direct exposure to internet advertising as well. Accessibility to the usual groceries shops decreases the likelihood of adoption of the on-line grocery service. Another conclusion was that the zip code areas are too large to capture diffusion effects in detail (i.e. heterogeneity of population). Some possible extensions that can be made in the model in order to widen its applicability: first to relax the constant population assumption and allow the dynamic change of populations and second, to insert the time variant by transforming the model into a Markov process. The latter will allow the research on questions that are interested on the number of adopters during a specific time period, or when the adoptions will reach a proportion of the saturation level.

The paper offers a rich argumentation on which communication channels are used. This is due to the nature of the study: internet grocery diffusion. So the *mass media communication channel* here is the internet. The authors also include the *interpersonal communication channel* in the form of *mouth effect* between individuals that pass the news from one to another. The study is carried in a small spatial area so the presence or not of the mass media or interpersonal communication channel affection on the diffusion is more tractable. The outcome is the interpretation of a *mixture of both communication channels* in a local area. Both direct contacts in spatial and regional level are captured.

The estimated results showed that for the case chosen, the word of mouth contacts may be a significant factor of the innovation diffusion. Furthermore, the interpersonal contacts on such a new internet based service are sensitive to distance because people would prefer a typical grocery nearby unless few clicks in a computer promise to offer the same products. The presence of the mass media mean, i.e. internet, is also very critical. The group of individuals with an available internet access, in areas close to universities and university campus, are taking advantage of the innovation and help its diffusion. On the contrary, the individuals without an internet access due to their income, age, ethnic and racial backgrounds, etc, seemed not to adopt the innovation. In that case the non existence of the mass media communication channel is extremely critical. As a consequence, the weight of the interpersonal communication channels and their influence upon the internet grocery diffusion is based upon the availability of internet or not. That points the supremacy of the mass media communication channels against the interpersonal ones when the use of the interpersonal contacts depends on the presence of the mass media mean. If the mass media were for instance advertising posters or articles in newspapers, then that argument would have been less strong. For the diffusion model itself, it is one of the few that touches the challenging and difficult topic of the spatial diffusion of an innovation. The integration of both types of communication channels is very useful to draw conclusions. The focus on the statistical properties of the parameters of the model is very analytical and strong.

<u>Case number 10</u>:

| Case no. | Authors | Product or Service | Model | Parameter estimation method | Communication channels |
|---|---|---|---|---|---|
| 10 | Goswami and Karmeshu (2004) | Cloth dryers (US), room AC(US), black and white TV(US and India), color TV (US) | Bass, NUI and PVRD (parameter variability randomness in diffusion) models | NLS | Localite and Cosmolite |

The choice to include this paper in our research is due to its application cases. Furthermore, it uses a new approach to describe the diffusion of these products and it is published in a prestigious journal (Technological and Social Change) and is cited 4 times (source: ISI Web of Science).

In this paper, Goswami and Karmeshu (2004), apply a Parameter variability randomness in diffusion model (PVRD). While usually some models predefine that the population of the adopters is homogenous, this study examines the validity of the PVRD model for real data sets. PVRD incorporates the heterogeneity of the population in the model and encompasses “the variable influence of the intensity of transmission of the source and the intensity of the transmission between individuals” (p.706).

The parameter estimation method of the model is the Nonlinear least squares (NLS) one, based on a simulated annealing (SA) algorithm. As the authors point, ‘the objective of parameter estimation in case of the PVRD model is to capture the variability of parameters, which would reflect the heterogeneity present in the social group’ (p.709).

PVRD builds on the Bass model (see appendix), with the difference that while in Bass model the α and β coefficients are constant (external and internal influence), here the same parameters α and β, are rendered as random variables (random constant coefficients). The formula of the PVRD equation is:

$X(t|\alpha,\beta) \equiv \frac{1-A_1 e^{-(\alpha+\beta)t}}{1+A_2 e^{-(\alpha+\beta)t}}$ , where, $A_1 = \frac{\alpha(1-X_{0)}}{\alpha+\beta X_0}$ and $A_2 = \frac{\beta(1-X_{0)}}{\alpha+\beta X_0}$, $X(t_0 = 0) = X_0$

This model can be projected backwards to the Bass model by assuming that the α and β sources of influence are invariable.

The applications that are used to validate the PVRD model are consumer products in US and India: Cloth dryers (US) (1949-1965), room AC(US) (1949-1961), black and white TV (US(1947-1965) and India(1981-1998)), color TV (US(1963-1979)). The authors illustrate the success of the PVRD model against the Bass and NUI model, by providing a graphical representation of the prediction of the diffusion of these products. In the graphs, the plots of PVRD, Bass and real data values, indicate that PVRD outperforms Bass model.

In overall the NLS parameter estimation method for the PVRD model performed satisfactory for the products chosen. As the authors state, their model should be further tested across real data sets of various products in developed and developing countries. An interesting remark is the need to investigate the issue of segmentation of heterogeneous populations of adopters. Furthermore more research can be carried out on the error terms of the parameter estimation method.

The overall picture from this paper is that both types of *localite* and *cosmolite communication channels* are present. This can be based on the argument that the parameters of the model are representing the internal and external sources influences on the diffusion of the innovations studied. The difference among other studies is that here both parameters are taken as random variables. Since the years of the analysis for consumer products in USA are about TV's in their early phase (black and white), we expect the interpersonal communication channels to be stronger. For the same product in India, the data analysis are based on more recent years (1981-1998) so we expect again the interpersonal communication channels to be more influential because of the different market, economical and social dynamics among USA and India.

<u>Case number 11</u>:

| Case no. | Authors | Product or Service | Model | Parameter estimation method | Communication channels |
|---|---|---|---|---|---|
| 11 | Everdingen et al. (2005) | The diffusion of Internet access at home and mobile telephony among households in 15 EU countries in the period 1990-1999. 44.000 phone interviews were executed in 130 regions. | An extension of a model by Putsis et al. (1997) (Putsis et al. model is a generalized Bass model). The model's results are compared to Bass and Putsis et al. model. | An extension of the augmented Kalman Filter with Continuous States and Discrete observations (AKF(C-D)) | Interpersonal and Mass media |

The paper that is chosen is very interesting due to its application cases (high tech services) and its new approach on the parameter estimation method. It is published recently in the International Journal of Research Marketing and cited 5 times (source: ISI Web of Science).

This study, by Everdingen et. al. (2005), introduces a forecasting cross-population adaptive innovation diffusion model. The objective of this research is to generate forecasts of a product's diffusion early in the diffusion process. The model used is an extension of a model by Putsis et al. (1997), which is a generalized Bass model. The choice to use the Putsis et. al. (1997) model is based on its appropriateness in an international diffusion with cross-country interactions setting. The contribution of the model of this paper is that it reformulates the Putsis et. al. (1997) model and enables it to include time-varying parameters and apply a sample matching procedure including a cross-country interaction effect in the diffusion process. The latter justifies the term adaptive in the model proposed in this paper. Another contribution is that they involve another parameter estimation method (AKF(C-D)) as in Putsis et. al. (1997) the NLS technique is used. We should comment here that the implementation of another estimation method is logical due to the unsuitability of NLS method for a cross-population study.

While there had been few attempts to model the innovation diffusion considering cross-country interactions, these usually describe only a one-way direction between a lead country in which the innovation first launches and lag countries that later receive the same innovation. Another issue that this study overcomes is the non realistic assumption that the influence of adopters on potential ones is equal within their own country and other countries. The matching criteria for the matching of the sample sizes of the model are the social system size, the long-term penetration ceiling and the time of origin intercept.

The non-detailed formula of the differential equation that describes the model of this paper is:

$n(t) = \frac{dN(t)}{dt} = f(N(t), \beta_c(t), \beta_p(t), \beta_q(t), \Phi(t), t_0)$, where n(t) and N(t) are Kx1 vectors with the speed of adoption and the total number of adopters in each of the K countries, $\beta_c(t)$ is the penetration ceiling, $\beta_p(t)$ is the external influence adoption, $\beta_q(t)$ is the imitation adoption, $\Phi(t)$ and $t_0$ give the mixing probabilities and the time of introduction of the product for each country.

The AKF(C-D) parameter estimation method is preferred due to its applicability to differential diffusion models, its ability to follow the changes in parameters over time and incorporation of observation errors in the estimation process. In overall, AKF(C-D) as the authors point, 'seems to be the most appropriate adaptive estimation method available' (p.297).

The applications of the proposed model are on high-tech services: the penetration of Internet access and mobile telephony among households in 15 EU countries in the period 1990-1999.

Results showed that this model outperforms the other two. The model also satisfactory forecasted the penetration of the services in all countries using a one, two and three year ahead forecasts. The latter means that only the information on a time period t was used to forecast the time periods t+1, t+2, t+3. That was an advantage against the other models since the forecasts were based on only few observations. The consideration of effect of cross-country population mixing on the forecasting performance of the model also revealed that in EU the country-borders still affect the diffusion of innovations and that countries should not be treated independently when a new product/service is launched in EU. Another important indication is that within an EU country influence between the diffusion processes exist while on the other hand EU is still not a one unified big market. For the Internet case, its use was more affected by the network effects within and across countries in comparison to the case of the mobile phone telephony. This is explained easily if one considers that the use of email, social network applications, etc among users that have internet access and communicate demand an internet access, while the presence of the traditional phones makes sometimes some people to be reached not necessarily on or through a mobile phone. The estimations also implied that there are different speeds of innovation diffusion of the cases chosen among different countries.

The proposed model's results are also compared to Bass and Putsis et al. model. The comparison confirmed the necessity to include the mutual influence on the diffusion process across the countries and the need of an adaptive model. Concerning the parameter estimation method, since NLS needs at least two observations, it could not provide forecasts for the initial two years after the introduction of a product. The Putsis et. al. model needed at least three observations for each country in order to estimate its parameters. That was not always possible and the Putsis et. al. model was unable to generate forecasts. The latter, strengthens the arguments of the authors of this paper.

Closing, this paper confronts with a difficult topic, the forecasting of cross-population innovation diffusion in a multicultural and diverse region (EU). The adaptive model seems to be satisfactory under the support of the AKF(C-D) parameter estimation model. More applications in the same region should be performed to provide researchers with more information.

One of the few papers that clearly states the implementation of both *interpersonal* and *mass media communication channels* is this one by Everdingen et al. (2005). The authors indicate that during the moment of a product's launch the value parameter of the internal influence coefficient representing the interpersonal communication channels influence is set to null. That is because logically thinking a product at that specific moment the potential adopters may be influenced only by external sources like mass media. Then, after the launch of the product, the value of the interpersonal communication channels is increasing in the form of word of mouth effect. So as the time passes, the influence of internal and external influences to adopters is a combination of both interpersonal and mass media communication channels. We should note that as the authors admit that one their limitations in this research was not to use covariates (exogenous variables) in their empirical tests i.e. money spent on the advertisement of a product.

<u>Case number 12</u>:

| Case | Authors | Product or | Model | Parameter | Communication |
|---|---|---|---|---|---|

| no. | | Service | | estimation method | channels |
|---|---|---|---|---|---|
| 12 | Gutierrez et al. (2005) | Forecasting the total natural-gas consumption in Spain in 1973-2000 | Gompertz | MLE in continuous sampling and extended method of linear SDE with multiplicative noise to the case of a non-linear SDE with multiplicative noise (white) | Not necessary to examine |

The following paper is mostly worthy for its cases described and the models parameter estimation method applied. It is published in the Journal Applied Energy and is already cited 17 times (source: ISI Web of Science).

The paper by Gutierrez et al. (2005) studies the forecast of the total (domestic and industrial) natural-gas consumption in Spain during the time period 1973-2000. The objective of this attempt is to examine if the Gompertz-type stochastic innovation diffusion model can be applied for the energy case.

The mathematical formula of the stochastic Gompertz innovation diffusion process is:

$dx(t) = (ax(t) - bx(t)\log x(t))dt + cx(t)dw(t), x(s) = x_s,$ where $c > 0, x_s \in \mathbb{R}_+^*$ and w is a one-dimensional Wiener standard process.

The estimations of the terms α and b of the model are performed with the MLE method. The estimation of the noise coefficient c is performed by using a non-linear SDE with multiplicative noise approach.

The results of the proposed model are also compared to other two: the stochastic logistic and the stochastic lognormal.

The results showed that a good description of the series and good short-medium term forecasts (1998-2000) were obtained. The year by year short-term conditioned forecasts of the trend are better. For the period inquired, the Gompertz model is found better than the logistic (diffusion-innovation) and the lognormal (diffusion-non innovation) models.

More research can be done by incorporating exogenous factors and other socio-economic variables.

In overall the forecast is good but more factors should be implemented in the future. That would also raise questions on the ability of the specific stochastic Gompertz model to handle various inputs.

Theoretically the Gompertz model assumes that the communication channels used are interpersonal. Although this is not mentioned in the paper, it is not the core element of it. Since the cases studied are about natural-gas consumption it is easy to hypothesize that usually individuals influence each other and adopt a new form of energy. We should also point that usually in the initial process the mass media encourage, promote, advertize the use of the new energy with advertisements or publications of states policy prices. The year of publication and the data of the paper allow us to skip any discussion on the type of the communication channels since the focus is on the model. Energy in EU is after all a common good available almost everywhere during the last 50 years and more.

<u>Case number 13</u>:

| Case no. | Authors | Product or Service | Model | Parameter estimation method | Communication channels |
|---|---|---|---|---|---|
| 13 | Robertson et al. (2007) | Segmental new-product diffusion of residential broadband services in UK | Gompertz | NLLS estimator | Not mentioned |

This is a paper written very recently, in the journal Telecommunications Policy. We can't judge it according to citations because it is still early. The case examined is very interesting and useful for our study, as well as the model and parameter estimation method.

Here Robertson et al. (2007) present a segmental diffusion model based on Gompertz innovation diffusion model. Apart from the interest on examining the applicability of the Gompertz model the authors aim to study the effect of a household income to its propensity to adopt the product. In this case the product is the diffusion of residential broadband services in UK. The authors test the model both for the segmental and for the total diffusion of the same service.

The mathematical formula of the Gompertz model adjusted for the segmental use in this paper is:

$N_i(t) = e^{-\alpha e^{-\beta_i t}}$ , where $N_i(t)$ is the cumulative proportion of adopters from segment $i$at time $t$. An assumption made is that each segment remains a constant proportion $a_i$ of the full system. In this case, the segment is defined by the outcome of the households. The parameter estimation method used was the NLLS (non-linear least squares).

For the application of this study, postal surveys were sent to households in July-August 2005 with 20% response (1221 replies). Non response biases were encountered by the National Statistics Office and Department of Education (for socio-demographic data: income). The time length in which the service was adopted was selected among: less than 3 months, 3-6 months, 6-12 months, 1-2 years, 2-3 years, more than 3 years. The income rates were: Missing income, under £10.000, £10.000-£14.999, £15.000-£24.999, £25.000-£39.999, and finally above £40.000.

The results of this study showed that the broadband penetration levels were income sensitive. The plot of the model is steeper for higher income categories. The results were compared against the actual data from the national statistics as given from British Telecom PLC. The approach the authors followed was able to predict accurately the segmental and national diffusion of the broadband internet among the different segments of household incomes. The higher household incomes had faster adoption curves than the lower ones.

As the authors point, their research can be valuable to innovation policy makers as well as market planners. A future study can segment the households by using different measures like social class, education, geographic region and more. A critical point is that the model does not account the probable transition of a household from one segment to another.

As in the other Gompertz model, there was not an indication on which communication channels are used. By default, we should expect only interpersonal, but due to the nature of the technology examined and the time of research data, we are confident that the mass media channel should have also affected somehow the process of residential broadband services adoption. In that case, mass media could be TV's, radio, newspapers, telephones. Another thought is that since the application under research is a mass media mean, theoretically it is fine to focus only on interpersonal communication channels. Remembering that interpersonal communication channels can be localite or cosmolite, then it is sensible to think a combination of both types affecting the adoption of the service. All the above lead us to the conclusion that the Gompertz model might not be the optimum one to be used in this application case in terms of Roger's communication channels.

Case number 14:

| Case no. | Authors | Product or Service | Model | Parameter estimation method | Communication channels |
|---|---|---|---|---|---|
| 14 | LIAO and XU (2007) | Telephone subscribers of urban and rural areas in China | They constructed their own model with migration between two different colonies | Probability estimation | Interpersonal and mass media |

Although the next study is very interesting for its application and model approach, it is published in an engineering journal; Systems Engineering – Theory& Practice in both English and Chinese but a citation report is not found.

This paper by Liao and Xu (2007) tries to show in a realistic manner, that the innovation diffusion among the populations of two colonies is a dynamic process. That is based on the comment that adopters belong to populations that are converted in terms of their economic conditions. The aim of the paper is to construct an innovation diffusion model with its 'population increasing and the conversion between two different colonies and attempts to find the principles of innovation diffusion' (p. 66).

The conceptual model and hypotheses on which the authors build their model is interesting and detailed. The mathematical formulas of the diffusion model are:

$$\dot{Q}_1 = (\beta_{10} - \beta_{11})Q_1 - (d_{10} + d_{11})Q_1^2 + \theta_2 Q_2 - \theta_1 Q_1$$

$$\dot{N}_1 = a_1 A_1 + b_1 N_1 A_1 - (\beta_{11} + d_{11} Q_1)N_1 - e_1 N_1 - \theta_1 N_1 + k_2 \theta_2 N_2$$

$$\dot{Q}_2 = (\beta_{20} - \beta_{21})Q_2 - (d_{20} + d_{21})Q_2^2 + \theta_1 Q_1 - \theta_2 Q_2$$

$$\dot{N}_2 = a_2 A_2 + b_2 N_2 A_2 - (\beta_{21} + d_{21} Q_2)N_2 - e_2 N_2 - \theta_2 N_2 + k_1 \theta_1 N_1$$

Where $N_i$ are the number of users in colony $i$ at time $t$ and $A_i(t)$ is the number of non-users in colony $i$ at time $t$. The term $a_1$ is the probability for non-users to become users caused by medium, $b_1$ is the probability for non-users to become users caused by oral communication between users and non-users. The terms $(\beta_{11} + d_{11} Q_1)N_1$ represent the decreasing number of users due to natural death. The terms $e_1 N_1$ are for the users that have to give up using the innovation for some reason and caused by member shifting. The terms $\theta_1 N_1$ are for the users shifting out of colony 1 and$k_2 \theta_2 N_2$ are the users shifting from colony 2 into colony 1. Lastly, $\theta_i$ is the probability of members of colony $i$ converting into another one and $k_i$ is the probability of users to continue using the innovation in colony $i$ after converting into another colony.

The model is applied to the data of telephone subscribers of urban and rural areas in China. The results showed that 'the subscribers will reach their maximal market potential whether there are members shifting between town and rural area or not'.

The authors compared their model for the same data, with Bass model and it proved that their approach describes more efficiently the diffusion since the Bass model 'may magnify the potential of innovation'. Some future research can be on the improvement of data collection and key factors such as the economic environment living level and the effect of expense decreasing on communication.

In relation to Roger's communication channels the paper is an excellent example of the use of both types of interpersonal and mass media types of communication channels. The foundation of the model that the authors build, apart from being new, is based on the broadcasting channels and ways, oral communications that occur among adopters and non adopters. It is also confirmed that between urban and rural populations in China, there are great differences and importance rates in communication channels; in the cities the adopters are affected from both types of communication channels while in the country the interpersonal communications are more common. Furthermore, the results indicate that the necessity and popularity of the mass media communication channels in a city are more critical and better than the ones in the country.

Case number 15:

| Case no. | Authors | Product or Service | Model | Parameter estimation method | Communication channels |
|---|---|---|---|---|---|
| 15 | Tseng and Hu (2009) | Inventory of cars in The Netherlands (1965-1989), Cellular phones in Portugal (1995-2000) and Worldwide PC demand (1981-1999) | Bass based Quadratic – interval model Results were compared with other models | Fuzzy regression | Both types |

This is one of the latest papers in the innovation diffusion literature published in the journal Expert Systems with Applications. We did not expect to find any citations because it is still early. The applications and model used in this paper allowed its sorting in our analysis.

This is a very recent paper by Tseng and Hu (2009) The authors use as a basis the Bass model and construct a quadratic-interval Bass diffusion model which is a combination of the Bass model with fuzzy regression. The argument on using fuzzy regression lies on its ability to describe the relationships between dependent and independent variables and the demand of less data -in comparison to other models- to generate realistic estimates of parameters. The main difference of the fuzzy regression theory among other parameter estimations is that it any ‘residuals between estimators and observations are produced by uncertainties in the model parameters rather than by measurement errors and a possibility distribution is used to deal with practical observations' (p.8497).

The performance of the proposed model is compared with three others: the Gompertz, the logistic, the quadratic-interval logistic and the Bass model. The applications cases upon which the models are compared are the Inventory of cars in The Netherlands (1965-1989), Cellular phones in Portugal (1995-2000) and Worldwide PC demand (1981-1999). All became high-tech mass consumer products.

For the Inventory of cars in The Netherlands, results showed that the quadratic-interval Gompertz model has a better prediction capability in terms of a larger confidence interval translated into the prediction of the best and worst possible sales amount. For the cellular phone subscribers in Portugal the results demonstrated that the quadratic-interval Bass model has a better performance. Lastly, for the worldwide PC demand the Gompertz and quadratic-interval Gompertz model parameter estimation is not significant and these models are not tested. The result showed that the quadratic-interval Bass model has an 'encouraging' predictive ability. To conclude, the comparisons of the models on forecasting performance does not convince that one model outperforms the others as performance depends on the time series pattern.

The authors remark that their approach – the quadratic Bass model – should not be used when there is a high variability in data. On the contrary, when data are not sufficient, quadratic-interval diffusion models 'are potentially useful tools' (p.8502). Another comment is that the quadratic-interval diffusion models 'reflect the concept of possibility not probability' (p.8502). Future work can be done on applications on the same time series pattern and compare the models chosen again.

The quadratic-interval Bass model used here is by default based on both types of communication channels. Even that the authors just mention the definition of communication channels by Roger's, they do not provide in depth detail about the form of the communication channels integrated into their model but mostly focus on the model itself. From a critical look, it is also obvious that from the dates of the data analyzed (last decades) and the geographical setting chosen (The Netherlands, Portugal, Worldwide) the mass media and interpersonal communication channels are very strong.

Case number 16:

| Case no. | Authors | Product or Service | Model | Parameter estimation method | Communication channels |
|---|---|---|---|---|---|
| 16 | Schilling and Esmundo (2009) | Wind and Geothermal energy technologies and trajectories performance (9 countries) | Pearl curve | Linear regression | Not neccessary |

The main reason to choose this article lies on its topic (S-curves) and applications. It is published recently in the Energy Policy journal and is still not cited.

This paper is an interesting case of application on a very modern topic: renewable energy diffusion. More specific, Schilling and Esmundo (2009) focus on the wind and geothermal energy technologies and trajectories. They plot the performance of a technology against the capital or effort invested. With the S-curve plots the authors try to provide insight on the fact that some technologies over take others and at the end dominate. They base their arguments on literature that describes how investments affect the improvement of a technology among energy technologies. Other factors that can limit the use of an S-curve as a prescriptive tool are that the true limits of a technology are unknown, the shape of an S-curve might be changed due to unexpected changes in the market, of because firms might overcome barriers that set some limitations to the development a technology.

The equation used in the paper is the Pearl curve with the form:

$y = \frac{L}{1+ae^{-bi}}$ , y refers to the expected limit of performance, i to cumulative investment. The term α is a coefficient that determines the height of the curve and b its slope.

Renewable and fossil fuel technologies are compared. Limitations of this analysis are: the difficulty to estimate the costs of energy production accurately, the determination of a good investment to cost relationship. The analysis on the cumulative R&D investments on wind energy showed that comparing the plots of the predicted against actual performance indicates that we are in the dominant design or era of incremental change state of a technology cycle. For the geothermal case, the analysis suggested that it may still be in a very early state of development. The results of the fossil fuel for the same time period as the other renewable energies (1980-2005) do not lead to an S-curve plot. That is logical considering that fossil fuels had been used since 1875. So, what the plot showed was just a proportion of the S-curve.

A key factor in plotting effort or capital investment over time against a products performance is that obscure S-curve might appear. If the amount of effort alters over time, the resulting curve might flatter out quicker or not flatten at all (p.1768).

In overall, the results reveal important implications for the industry and government, since usually the policies or subsidies a government designs, may affect the acceptance of a technology by the industry. Furthermore, the plots provide a general insight into the total R&D investment limit, up to which a technology would be exploitable. For instance, ‘if 50kwh per dollar is an appropriate limit for wind energy, the regression coefficients suggest that this limit should be reached by the time cumulative investment reaches 6 billion dollars’ and if 100kwh per dollar is an appropriate limit for the geothermal energy, this will become ‘less expensive than current fossil fuels at a cumulative investment of 9.8 billion dollars’ (p.1779). Closing, the results also suggested that even though fossil fuel technologies seem to attract more governmental investments, their performance is declining. On the other hand, wind and geothermal energy technologies are underfunded even though they can be superior to fossil fuels technologies. Incumbent firms seem to invest in fossil fuel technologies due to their ‘asset positions and strategic commitments in fossil energy sources than renewable’ ones (p.1780). New entrants are into the energy sector might benefit more from investment in wind or geothermal energy compared to fossil, biomass or solar technologies.

As expected like in similar cases, the communication channels for a mass good like energy it mostly counts on national policies, funding, incentives and private firms initiatives for investments in R&D. As a consequence Roger's communications channels role is not primary and is not under analysis.

<u>Case number 17</u>:

| Case no. | Authors | Product or Service | Model | Parameter estimation method | Communication channels |
|---|---|---|---|---|---|
| 17 | Putsis et al. (1997) | VCRs 1997-1990, Microwave ovens 1975-1990, CD players 1984-1993, PCs 1981-1991, for 10 EU nations | Bass based, an extension | NLS | Localite and Cosmolite |

The last paper of our choice has been written in the Marketing Science journal and is cited 86 times. It is one of the studies that initiated other authors to apply more ideas in the same topic.

This paper addresses the importance of interaction within and across countries that affects the adoption of a new product. This pattern of interaction (called 'mixing') refers to communication. The focus of this paper is a 'comprehensive flexible theory of interaction' not yet (till 1997) discussed (p. 355). This framework includes multiple simultaneous interactions.

The applications examined are consume products: VCRs 1997-1990, Microwave ovens 1975-1990, CD players 1984-1993, PCs 1981-1991, for 10 EU nations (UK, Germany, France, Italy, Spain, Belgium, Denmark, Netherlands, Sweden, and Austria). A reason for choosing those products was that they were introduced at the same date in these countries.

Prior to research, the authors estimated the Bass model results on each of the 10 countries. The results indicated that important cross-country influences were not addressed and important covariates that influenced the sales of a product in two countries at the same time were missing. The type of mixing that occurs not only among populations but also countries plays a significant role on the diffusion pattern. This research considers three possible mixing patterns: pure segregation, random mixing, and

Bernoulli noise. Future country launch product will benefit not only from prior adoption in countries with high contact rates but also form countries with high rates of external contact: i.e. focusing first on Germany, Italy, France and Spain will maximize adoption in subsequent countries (Denmark, Netherlands, Austria, Sweden).

A future study can extend some elements of this research like: include optimal sequential introduction times, optimal advertising, distribution support across countries, optimal pricing, impact of regional media that go beyond national boundaries, importance of availability of complementary products, cross-country correlations in pricing and advertising, etc. The authors also point that they have not exhausted potential mixing behaviors. Further research on that can be done.

Closing, this paper offered another approach to the innovation diffusion modeling. For example other scholars used the proposed framework as a reference and extended it (case 12). Specifically in case 12 presented in this study, the extension model was tested on high-tech services.

In this study Roger's communication channels are extensively examined. The mixing of behavior translated into Roger's definitions, is the mix of both *interpersonal* (word of mouth effect) and *mass media communication channels*. Furthermore this is expanded not only to one nation but among many nations with different contact rates to their neighboring countries. Adopters in one country are susceptible to interpersonal interactions from within their country as well as from out of their country. An assumption used is that at the time of a product's launch, none of the individuals has adopted the product and that can be done at that point only by advertising (mass media). Media intensity in a country affects the volume of interpersonal communications in a disproportionate manner. A social factor like the outcome of an adopter is also pointed as a feature able to determine the information-seeking behavior and susceptibility of potential adopters. Additionally, the graphical representation of the sales of a product due to *internal and external interpersonal channels* is importance as it shows the weighting of each interpersonal influence. Furthermore, for some countries the internal to external communication channels are differently weighted. A country with strong influence on nearby countries in terms of communication channels will be the one where the product should be launched and then the internal communication channels of the other countries will take action. Finally, the *localite* and *cosmolite communication channels* are both present in this paper.

# 5 Results

This section presents the results based on the findings of this study. The following table categorizes the innovation diffusion of products studied and presented in the previous section (chapter 4). More specifically, the table gathers all the information attained from the analysis of the seventeen papers that provided data about innovation diffusion models and their applications. This will provide a clear picture of which model better describes the innovation diffusion of specific product categories (consuming, high-tech, etc). The applicability of each model is based on the comments written in each of the studies examined.

The table has six columns: technology clusters, products or services, innovation diffusion models and the parameter estimation method used, Rogers's communication channels, basic comments and applicability. Products are grouped which belong to the same technology cluster and are analyzed in a paper. Then the models that are used and the parameter estimation method of the models coefficients are defined. We examined each model for a correlation with Roger's communication channels and if that is not clearly stated we scanned the paper for indications that help support an argument. Then, after careful consideration of the cases data, information, graphs, analysis, we state if the model is applicable for the specific product/service of the cluster. Sometimes the authors of a paper support their approach by combining their results with another paper for the same products. That combination helps us strengthen the applicability or not of a model under specific conditions (i.e. one parameter estimation method against the other). We chose to state that a model is applicable for a specific product/service member or a cluster, and we wrote 'yes' in the cell of the table, when the results of the papers analyzed clearly show that the model performs successfully on predicting the innovation diffusion. Sometimes in the papers the authors demonstrate the efficiency of the model by plotting ex-post data indicating the almost identical points on the plot between actual data and predicted values of the model. Following the same logic, we wrote 'no' in the cell of applicability column when a model is clearly not able to predict the innovation diffusion. That is usually extracted from the results of the papers under analysis after comparisons and from any plots included. When there is still room for improvement, which is stated in the cell as future work that is needed to be done or more applications that should be tested.

Table: 2

| Technology Cluster | Product or Service | Models / Parameter estimation | Rogers Communication Channel | Comments | Applicability |
|---|---|---|---|---|---|
| Consumer products | Cloth Dryer, Room Air conditioners, Color T.V., BW T.V., Dishwashers | Bass / MLE | Interpersonal, Localite | Considers only sampling errors | No |
| | | Bass / NLS | Interpersonal, Localite | Considers all errors | Maybe |
| | VCRs, Microwaves, CDs, PCs | Bass extension / NLS | | Extensions should be included | Future elements should be accounted |
| | Cloth Dryer, Room Air conditioners, Color T.V. | Modified Mansfield, NSLR, NUI | NSLR →Interpersonal NUI → Both Interpersonal and Mass media | The market potential is larger, market ups and downs are very closely picked up. One-step forecasts are not good but a five year forecast looks reasonable. Adjusted $R^2$is better | No |
| | | Bass, PVRD / NLS | Localite and Cosmolite | NLS | |
| | BW T.V. | NUI, TPD, FPT / NLS, VFSR | A mixture of both types | TPD is better when population is far from homogeneous | Future research should be done |
| | Cloth Dryer | Bass mixed | Interpersonal, Localite | Not successful maybe due to lack of expertise | No |
| | Refrigerators, Color T.V., Microwave, VTR, Facsimile | Logistic / Linear correlation | In this case, mostly mass media | Sensitive to economic turbulence | Yes but under stability performs better |
| | Telephone answering machines | Bass / MLE with weighted least squares | Should be both types | $\overline{N}$ can be very sensitive to observation data and | Future cases should be applied |

| | | | | | |
|---|---|---|---|---|---|
| | | | | estimation method or to exogenous factors. | |
| | Inventory of cars and PC demands | Bass, Gompertz, Bass (plain and extensions) / Fuzzy | Both types | There is high variability and under some conditions a model works or not | Gompertz is ok for cars the rest not. The Bass is ok for PC the rest not. |
| High tech products | Ultra Sound, Mammography, CT Head Scanner, CT Body Scanner, | Modified Mansfield, NSRL and NUI / weight factor | NSLR →Interpersonal NUI → Both Interpersonal and Mass media | The market potential is larger, market ups and downs are very closely picked up. One-step forecasts are not good but a five year forecast looks reasonable. Adjusted $R^2$is better | No |
| High tech. service | Cable T.V. subscription | Bass mixed | Interpersonal, Localite | Can predict sales peak year | Yes |
| | Internet access and mobile phones | Putsis extension model / Kalman filter | Interpersonal, Mass media | | Yes |
| | Telephone subscriptions, Internet access | Bass | Interpersonal, Mass media | | No |
| | Cellular phones | Bass, Gompertz | Both types | The Bass is better than Gompertz when compared but in another study Bass is not good | Probable no |
| | Telephone subscriptions | (a new model)/probability estimation | Interpersonal and mass media | | Good but future cases should be applied |
| | Broadband service | Gompertz / NLLS | Not mentioned | | Good but more can be incorporated |
| | Internet grocery service | Bass spatial mixed / MLE, ED algorithm, Bayesian | Interpersonal Localite, Mass media Internet | Needs to be extended | More factors can be included in future |
| Energy | Electricity | Stochastic Logistic / | Probable | The first is ok | Good |

| | | | | | |
|---|---|---|---|---|---|
| consumption | | MLE+noise and SDE+noise | interpersonal, not so clear | | |
| | Natural Gas | Stochastic Gompertz/MLE+noise and SDE+noise | Not necessary | Is better than Logistic with SDE+noise | Good |
| Energy performance | Wind and Geothermal / | Pearl curve / Linear regression | Not necessary | There are some limitations | Maybe but more factors should be evaluated |

## 5.1 Consumer products

As we can observe from the previous table, for consumer products there are many models applied and none is 100% able to provide an actual forecast. There are always minor or major diverges resulting from either the model itself or parameter estimation. There are models with one, two, three or more parameters that increase the factors they incorporate. That results to very complex models that lack a precise measurement or parameter estimation. It is a vicious cycle.

The most common model applied in the category of consumer products is the Bass model. This model incorporates by definition both types of communication channels; interpersonal and mass media. When the products are a mass media mean itself, the model is mostly based on interpersonal communication channels. That occurred clearly in the cases of the TV's diffusion. Researchers tried different parameter estimation methods in their approaches but none concentrated on a better balance among the interpersonal and mass media communication channels. After the first attempts to model innovation diffusion of consumer products with the Bass models it seems that the scholars realized the need to extend it. The outcome was the Bass extension model that tried to fulfill more requirements and overcome practical problems. It looks like the problem of the non applicability of the simple Bass model in most of the cases and the open window for improvement might have been generated from the non direct consideration of the communication channels.

Closing, it seems that there is not a winning model that can predict accurately the diffusion of consumer products when not all the relevant parameters are considered. What the models did offer is a discussion and trials on many alternatives and a quite close estimation of the diffusion. In most of the cases that was enough. We should also consider that the papers we investigated where wrote many years ago when the available communication channels were limited in number and penetration. Furthermore, the data used for the analysis by the scholars were usually from the US Statistical Office without a categorization of how they were measured.

## 5.2 High tech products

For high tech products the results indicate that there is still not a unique precise innovation diffusion modeling method to apply, just like the consumer products. It is found that for a group of four high tech products mostly two flexible internal and mixed models were used. We would expect more models to be tested considering more dynamic factors to be accounted. Even though the use of mixed models predefines the involvement of interpersonal and mass media communication channels, the authors were not focusing on that. The main area of importance was the ability to forecast the market potential and not the way through which that could be achieved; the communication channels. A reason for not focusing on the communication channels could be due to the expertise of those products and the small group of people that would be interested on those four high tech products. We should remind that the products analyzed are considered high tech between specific time intervals. When the technology changes or develops, the same products are not high-tech anymore. Based on the literature, the Ultra Sound, Mammography, CT Head Scanner, CT Body Scanner, were considered extremely high-tech products during the 1965-1980.

## 5.3 High tech services

Continuing for high tech services, it is though encouraging that there are several examples found and those on average in the telecommunications sector. The models applied seem to work satisfactory but more elements and factors should be included to improve their efficiency. Maybe the reason for the good performance of the models in this cluster is due to the availability of defining and tracking of multi communication channels (phone interviews, internet polls, etc) regardless of the location of the consumer or the service. The most common model used is the Bass model and its extensions while there are also some attempts to compare the results of Bass models with other models already known (Gompertz) or design new ones. As a consequence both types of communication channels are accounted for . That can be due to the parameter estimations applied that can better incorporate the importance and influence of the interpersonal and mass media communication channels. Considering that the diffusion of a service is strongly based on the articulation among adopters that describe it and the marketing approaches to promote it, the results seem logical.

## 5.4 Energy

### 5.4.1 Consumption

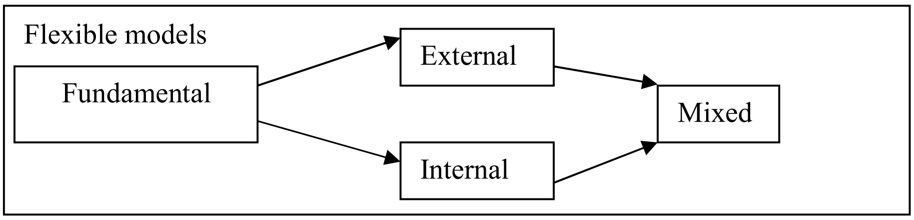
Energy consumption is a crucial research topic that attracted the attention of scholars. In general, the better the model, the better the forecasting for future programming of energy production and consumption is. It is pleasant to find that the stochastic models applied (Logistic and Gompertz) seem to be successful. It is also good that the authors of two different papers choose two different models but the same parameter estimation method when analyzed their energy cases. That helps us to define which model is better; stochastic Gompertz. An interesting point is the lack of the reference and probable necessity of the communication channels. It seems that this cluster was not in need of the communication channels as of the policies set by governments. Having a complete overview would demand more models to be tested for the specific class.

### 5.4.2 Performance

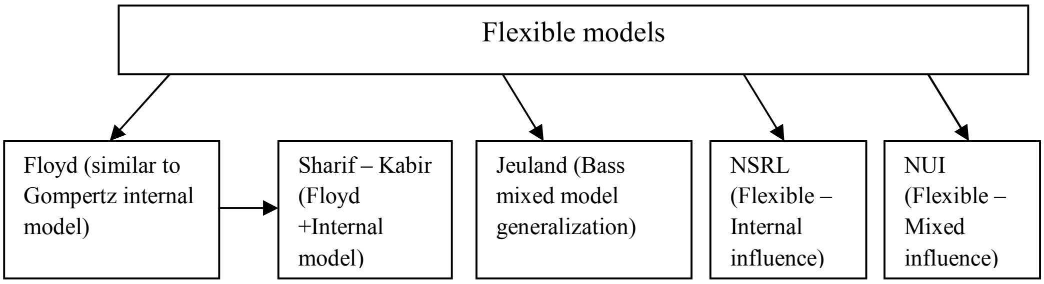
Another interesting category is the energy performance of two of the main renewable energy forms; wind and geothermal. Unlike the consumer and high tech product and services, we did not found a Bass model type to be used. On the contrary, a Pearl curve model was used, showing that it can estimate the diffusion of wind and geothermal energy. Additionally the research illustrated that more factors in the process should be evaluated, screening that there are some limitations needed to overcome. A way to bypass that is to apply and try several models that would be able to track and include the characteristics each of the types of the energy sector. That can be done for many new forms of energy, mostly renewable –solar, sea wave, biomass, etc. A reason that may justify the delay of research on new forms of energy and the modeling of their diffusion is that some of them are still in infancy.

## 5.5 The development of the models and their correlation with the communication channels

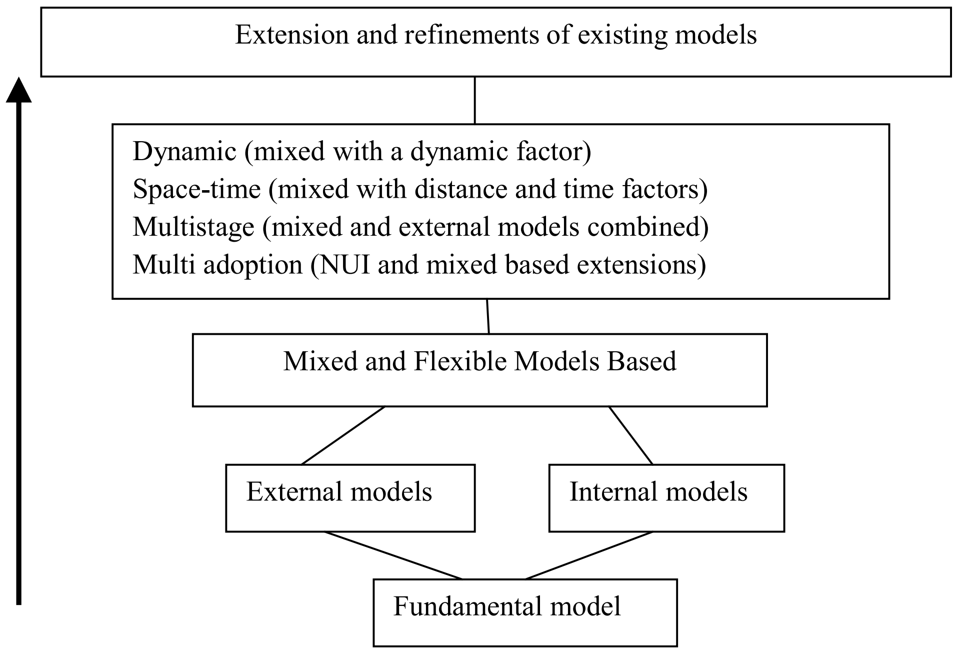
Based on the models presented in this report, one can observe the successive development of these models. Starting from the fundamental model in which the three different coefficients outcome three different approaches (effect of media, communication channels): the external, internal and mixed models occurred.



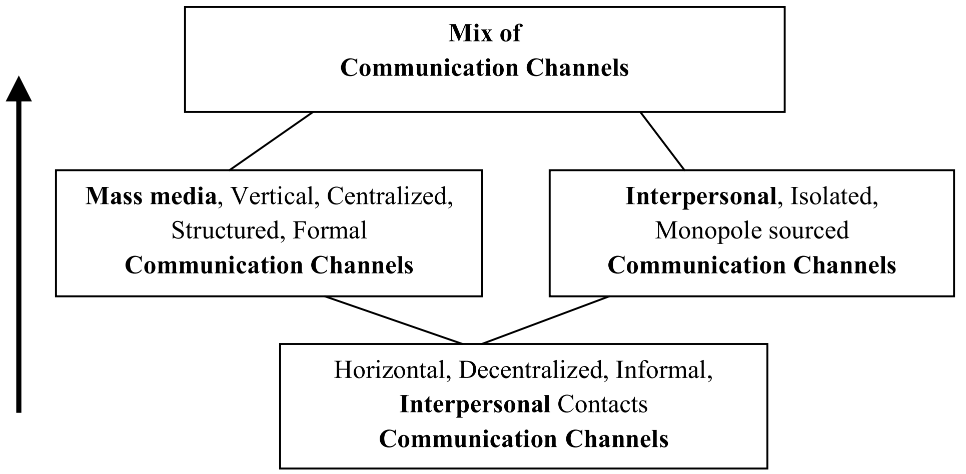

Observing the need for more flexibility, scholars tested many combinations of the external, internal and mixed models. That created a new category: flexible models.




Following, as the models were tested more extensively extensions and refinements of existing models were needed (Mahajan V. and Peterson R. A., 1985). Each new idea was again based on the existing models with something new to propose. For instance, setting as dynamic the number of maximum adopters of a mixed model creates a dynamic model. Considering distance and time factors in a mixed model, generated a space-time model. Combining mixed and external models Dodson and Muller (1978) created a new model. Tapiero (1983) also formed a model as a generalization of the Dodson and Muller one. Finally in the multi-adoption approach, Mahajan et.al (1983) generated an extension of the NUI model. All these are reflected in the following graph.




The results of this report based on multiple cases examined by scholars show that there is not a specific model that can accurately (error free) predict innovation diffusion for many categories of products, like a pattern. More explicitly, each time one wants to describe the innovation diffusion of a product should consider which model is the most appropriate to use and which parameter estimation method would be the proper. In order to do so, first one should define the class in which this product belongs, then identify the factors (spatial scale, time scale) affecting the diffusion process, then whether there are factors that should be considered together (multi- adoption, change agents, multi-stage) and finally to examine whether all these can be incorporated in a model.




Trying to follow the development of the innovation diffusion models according to Rogers's communication channels, the following table can be drawn starting from bottom to top from the simple to more complex models and communication channels:

<table>
<tr><th colspan="2">Communication Channels</th><th colspan="2">Models</th></tr>
<tr><td colspan="2"></td><td colspan="2">Dynamic influence models and extensions of flexible models<br>Dodson and Muller, Tapiero, etc</td></tr>
<tr><td colspan="2"></td><td colspan="2">Flexible influence models<br>NSRL(Mansfield modified), NUI (Bass modified), Floyd, Sharif-Kabir, Jeuland</td></tr>
<tr><td colspan="2">Combination (or Mix) of Communication Channels</td><td colspan="2">Mixed influence models<br>Bass, Lawton</td></tr>
<tr><td>Interpersonal, Isolated, Monopole sourced Communication Channels</td><td>Mass media, Vertical, Centralized, Structured, Formal Communication Channels</td><td>Internal influence models<br>Mansfield</td><td>External influence models</td></tr>
<tr><td colspan="2">Horizontal, Decentralized, Informal, Interpersonal Contacts Communication Channels</td><td colspan="2">Internal influence models<br>Gompertz, Logistic</td></tr>
</table>

What works for a consumer's product might not be adequate to describe the process of a high-tech service, or new forms of energy and so on. What complicates the process is the difficulty to incorporate appropriately in the models all the factors that affect the innovation diffusion. Take for instance a very common scenario that happens regularly: a traveler from Greece that adopts an innovation in The Netherlands, stays in Benelux for few months, then travels back to Greece through Italy and during this process communicates with friends from Spain through the internet. Trying to describe and predict the innovation diffusion of the product that the traveler posses would be an extremely difficult process as it involves media, communication channels, spatial, time, space, dynamic, etc considerations.

# 6 Conclusions and Discussion

After presenting the conclusions of our research, some thoughts and arguments are discussed.

Since Bass proposed his model for a mathematical explanation of innovation diffusion, there is plenty of research focused on this topic. After applying Bass model in plenty of applications, researchers realized that the model needed to be improved. The need for better forecasting accuracy gave rise to more realistic mathematical models which are increasingly complex, and therefore face computability difficulties. That occurs in two ways: the increase of the models variables and the formulation of more accurate estimation parameters methods. The more variables able to be incorporated the better the model is. The best fit estimation method that will define the parameters of the models, the best the results are. The more the applications on a technology or product/service innovation are tested by extended forms of models, the better the outcomes are.

For different clusters of technology, we found a reasonable number of studies that used stochastic or chaotic models to describe the diffusion of products and services. Almost all of the approaches resulted in an S-curve graph that dominated as a virtue of the diffusion of innovations. The models used described the innovations as an accumulation of internal or/and external factors that after the creation of the innovation, affected its early adoption, its market growth and finally saturation. As time was passing and the social and innovation systems evolved, the models needed to be mixed and become more flexible. The need for further improvement and map with real life scenarios further developed the models. Multi adoption and multistage models were defined. But innovation did not occur only in isolation. New space-time models [Appendix 7.3] may come through with more refined dynamic models. The result is a group of extended and refinements of old-based models with alter approaches. All these new models required new parameter estimation methods.

It is also found that in relation to consumer products and services, regardless of their cluster, marketing methods seem to control the speed of diffusion of an innovation since the degree and timing of the insertion of a mass media communication channel in the adoption process can control the diffusion process. For the diffusion of the energy sector and its clusters (i.e. renewable), the models showed that the communication channels are not as influential as the national policies and incentives. In such cases the R&D and money/effort graphs guide the process.

The importance of S-curves is high for many scholars, even though some do not seem to have paid much attention in it. Some scholar's support that it is not the innovators that change the world but the users of the innovations: the adopters (Schrage, 2004). The importance of innovation diffusion is also considered as the dynamic driving today's world and tomorrows (Schrage, 2004). Moore's Law has been characterized as a typical example of a classic S-curve where the performance follows an S shape over time, reaching saturation (maturity) (Bowden, 2004). That statement might sound an exaggeration but it can be justified by reminding that for many of the equations we examined (i.e. Gompertz or Bass for $q=0$) we get an exponential equation. Probable many authors of the paper are not so familiar with all the equations and characterize the S-curve as exponential just due to its graph[2]. Continuing, it is stated that the S-curve encompasses Moore's Law is a 'composite of multiple S-curves associated with technologies such as: light source, image projection, optics, tools, etc. (Bowden, 2004).

[2] A similar characterization of exponential S-curve is found also in websites where managers post their opinions (i.e. http://www.circleid.com/posts/the_exponential_s_curve_hastens_decline_of_asset_values/)

Concerning the models, we should point here that still some ex post applied models show that they can be efficient in predicting innovation diffusion, showing their usefulness. These methods are used quite successfully by managers to draw a picture of a product's diffusion. However as technologies develop and complex products evolve, the innovation diffusion process becomes more complicated. Additionally, some models were initially designed for population growth applied in biology and their use in product diffusion falls out of their scope. So, their improvement and refinement to fit in the product diffusion process was a procedure that took some time after many tests and errors. Furthermore, some models were developed to cover specific needs many years ago and have to be updated as the time scales for innovation diffusion have been decreased. The models may be significantly improved mathematically, but improvements should be application driven.

Innovation managers may also find some points which are not sufficiently covered in the models discussed. For instance, as Nooteboom (1994) pointed out, even though the diffusion of innovations has been extensively discussed, there is limited discussion of innovation diffusion models in relation to firm size. Another remark for future research is the relation of innovation diffusion with mergers and acquisitions. Industries face difficulties to accurately assess the position of a technology on the S-curve (Modis, Bowden 2004). Moreover, future research can focus on the statistical properties of the models mentioned along with their structure (number of parameters and underlying assumptions) and also the technology applied, timeline, social and economic conditions that occurred at the time when a technology was examined.

In the high tech cluster, even though some products now do not sound so high-tech, we did not fall into the trap to account them simple. We categorized them according to the era of their diffusion and the time the study was performed.

Further research should be performed on high tech products that have conquered our everyday life (i.e. iphones, mp3 players). Compared to the consumer products, there were not plenty of applications found. Maybe this is because high technology products (as we define them today) were developed recently and historical data and all factors are still not available. They are plenty of products to be tested but sometimes the data are not available due to firms' competition and thus secrecy. A closer communication among marketing researchers and innovation scholars could help define which communication channels are preferred, why, and on what extend do the marketers control the diffusion process.

Continuing, it would be very interesting to apply the models for more renewable energy forms. The delay may be due to the early phase in which some energy forms still are.

A future investigator could also consider the development of the communication channels since the time Roger's defined them. Nowadays the internet is dominating in the mass media communications and as a paradox it also encourages interpersonal communication. An email communication from a friend to the other is a typical example. That did not occurred with a newspaper or a TV, so maybe more modern approaches can be studied in innovation modeling.

The application of innovation curves in project management and in particular in project management development is not discussed. A relevant overview and a discussion on when is it optimum to release a product and what could occur if not so is provided by (Mahajan and Muller, 1996). Recent research is also carried out on the Japanese videogame console market showing the introduction of various competing consoles, their discontinuation and the launch of new versions of the consoles in the market (Zawislak et.al, 2009).

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# 8 Appendix

## 8.1 The fundamental diffusion model

The general type of this model is:

$\frac{dN(t)}{d(t)} = g(N(t))\left(\overline{N} - N(t)\right), N_{t=t_0} = N_0$, (eq.1)

$N(t)\ is\ the\ cumulative\ number\ of\ adopter\ at\ time\ t$

$$N(t) = \int_{t_0}^{t} n(t)dt,\ \ n(t){:}\ non\ cumulative\ number\ of\ adopters$$

$\overline{N}{:}\ the\ total\ number\ of\ potential\ adopters\ at\ time\ t$

$\frac{dN(t)}{d(t)}{:}\ the\ rate\ of\ diffusion\ at\ time\ t$

$N_0{:}\ the\ cumulative\ number\ of\ adopters\ at\ time\ t_0$

$g(t){:}\ is\ the\ coefficient\ of\ diffusion$ that depends on the

- Nature of innovation
- Communications channel employed
- Social system attributes

The trick here is the interpretations of the $g(t)$ function. Some scholars account it as a probability of an adoption at time t resulting that $g(t)\left(\overline{N} - N(t)\right)$ represents the expected number of adopters at time t, n(t). If n(t) is viewed as the number of social system members transferred from potential adopters to adopters then $g(t)$is considered a transfer mechanism. To conclude $g(t)$ is seen as a function either of time or number of previous adopters.

In this section $g(t)$ will be further analyzed as a function of the number of adopters. In that case we may have:

$$g(t) = \begin{cases} a \\ bN(t) \\ a + bN(t) \end{cases},a, b\ are\ model\ coefficients\ or\ parameters \quad \text{(eq. 1')}$$

Placing the various values of $g(t)$ to eq. 1, the result is the categorization of three different diffusion models.

### 8.1.1 Comments

Designing a model that is precise and considers all the possible parameters is not easy and usually not applicable. This is why they are simplifying assumptions designed to facilitate analytical solutions to the models. One such assumption is that the diffusion process is *binary* (Sharif and Ramanathan, 1981): members of a social system either adopt the innovation or not. Another assumption of the fundamental model is that adoption is treated as a discrete rather than continuous event. As a consequence, the fundamental diffusion model does not take into account stages in the adoption process (e.g., awareness, knowledge, etc).

Second, the fundamental diffusion model is based on the assumption that there is a distinct and constant ceiling $\overline{N}$, on the number of potential adopters in the social system and that this ceiling is either known or can be estimated. The size of the social system is considered to be finite and fixed. Consequently, the fundamental diffusion model is *static*; the social system is not allowed to increase (grow) or decrease in size during the course of the diffusion process (Mahajan and Peterson, 1978; Sharif and Ramanathan, 1981).

Third, the fundamental diffusion model only permits one adoption by an adopting unit. Multiple adoptions by a single adopting unit (e.g., repeat purchasing of a product) are not permitted. A simultaneous assumption is that an adoption cannot be repealed. There is no provision in the model for discontinuance of an innovation once it has been adopted.

Fourth, in the internal-influence and mixed-influence models, the term $N(t)[\overline{N}-N(t)]$ implies that there is complete mixing of social system members. In other words, it is assumed there is complete, pairwise interaction between prior adopters of an innovation and potential adopters. Furthermore, because

$N(t)=\sum_{j=1}^{t}\big(N(j)-N(j-1)\big)=\sum_{j=1}^{t}n(j)$, or

$N(t)\big[\overline{N}-N(t)\big]=\big(n(1)+n(2)+\cdots+n(t)\big)[\overline{N}-N(t)]$

it is assumed that the effect of interaction between prior and potential adopters is identical, regardless of time of adoption and time of interaction. Hence, internal influence represented by $\big(n(1)\big)[\overline{N}-N(t)]$ is equivalent to that represented by $\big(n(t)\big)[\overline{N}-N(t)]$. The coefficient of internal influence is assumed to be temporally independent fixed or constant over time. A related assumption is that the external-influence parameter does not change over the course of the diffusion process: it is fixed or constant.

In addition, an implicit assumption is that the innovation itself does not change over the diffusion process. This means, for example, that in the case of a new technology, modifications would not take place during the diffusion process. Moreover, the innovation is assumed to be independent of other innovations. Thus, adoption of the innovation does not complement, substitute for, detract from, or enhance the adoption of any other innovation (and vice versa).

A sixth, implicit assumption is that the geographical boundaries of the social system do not change over the diffusion process; the innovation is confined to one geographical area. In other words, the spatial diffusion of an innovation is not considered in the fundamental model.

Finally, when applying the fundamental diffusion model, a global assumption is that all relevant information about the diffusion process has been "captured" by the model. Thus, when forecasting the sales of a product, for example, it is assumed that all relevant information as to marketing strategies employed, activities of competitors, and the like is represented in the model, usually through the term$N(t)$. Generally speaking, application of the fundamental diffusion model requires the common forecasting assumption that the past can be used to predict the future.

## 8.2 The external influence diffusion model

The form of the mathematical function is: $\frac{dN(t)}{d(t)} = a\left(\overline{N} - N(t)\right)$ (eq.2)

This model represents the effect of the mass media communications on the diffusion process, the influence of governmental agencies, sales people and the effect of channels of communication (vertical-centralized-structured-formal). Plotting the previous model allows us to observe that it does not have a point of infection (or tipping point).

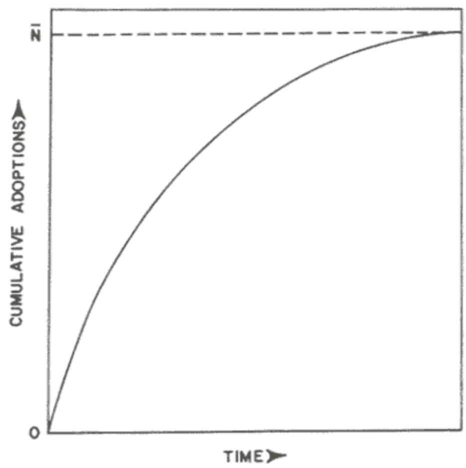


**Figure a.1: The external influence diffusion model (Source: Mahajan V. and Peterson R. A., 1985)**

Comments: This model does not attribute any diffusion to interaction between prior adopters and potential adopters.

It is appropriate when members of a social system are isolated (do not interact), or for innovations not complex and/or subject to interpersonal communications, or adequate to information about the innovations is only available from a source external to the social system.

There is a paradox pointed by Philips (2007) that the ‘logistic curve can be the maximum-entropy model (i.e., the information-theoretically optimal model) for a data set that has an obvious knee, even though the logistic curve has no knee’.

## 8.3 The internal influence model or Logistic equation

The form of the Logistic function is: $\frac{dN(t)}{d(t)} = bN(t)\left(\overline{N} - N(t)\right)$ (eq.3)

This approach was proposed by many scholars as listed in Hirooka M (2003) (Griliches (1957), Mansfield (1961,1963,1969), Mercalfe (1970), Fisher and Pry (1971), Nakicenovic and Grubler (1991), Modis(1992), Marchetti (1997, 1988, 1995, 1996) etc.). It represents the effect of horizontal channels of communication, decentralized channels or communication and unstructured or informal channels of communication.

The coefficient $b$ is a constant defined as an index of imitation or internal influence. It reflects the interaction of prior adopters $N(t)$ with potential adopters$\left(\overline{N} - N(t)\right)$.

Plotting the model we get the following S-curve looking graph with inflection point when the total number of potential adopters reaches the value of 50%.

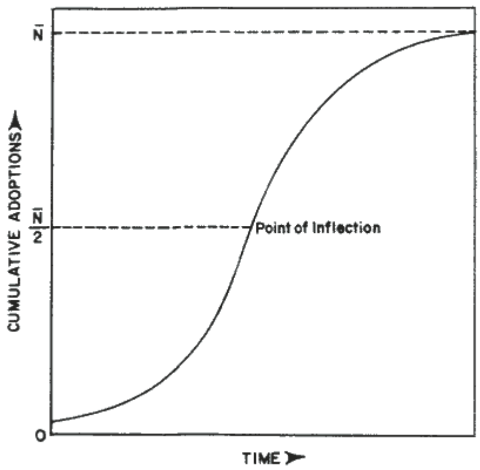


**Figure a.2: The internal influence diffusion model or logistic (Source: Mahajan V. and Peterson R. A., 1985)**

Comment: This model is based on the principle that diffusion occurs only through interpersonal contacts or social interactions between prior-potential adopters. It is most appropriate when an innovation is complex and socially visible. This equation describes the diffusion of an innovation mostly in a sound economy. This is difficult to find nowadays as there are many economic turbulences (Hirooka, 2003).

It is critical also to present an additional differential equation used for population growth estimations but under some additions can be also used in the literature of project management and in particular in project development: it is Prigogine's equation (Prigogine, 1980) with the mathematical form,

$$\frac{dX}{dt} = bX(N - X) - \bar{d}X$$

In a recent paper (Cioffi, 2005), this equation is used as a basis for the creation of another equation as a tool for managing projects:

$y(\beta) \equiv y_\infty \frac{1-e^{-8r_{0.67}\beta}}{1+\gamma e^{-8r_{0.67}\beta}}$, where $\beta \equiv \frac{t}{t_1}$, $r = \frac{2}{3}r_{0.67}$

$t_1$ is the total duration of the project

r defines the rise of the S-curve in the middle third part of it

### *8.3.1 The Gompertz model*

A very well known model of this category is the Gompertz model. This is a broadly used model in technological forecasting. Its mathematical type is:

$\frac{dN(t)}{d(t)} = bN(t)[\ln \overline{N} - \ln N(t)]$ or $y(t) = ae^{be^{ct}}$

The solution of the first derivative of this equation results the inflection point value which is $37\overline{N}$ and is obvious in the plot that follows.

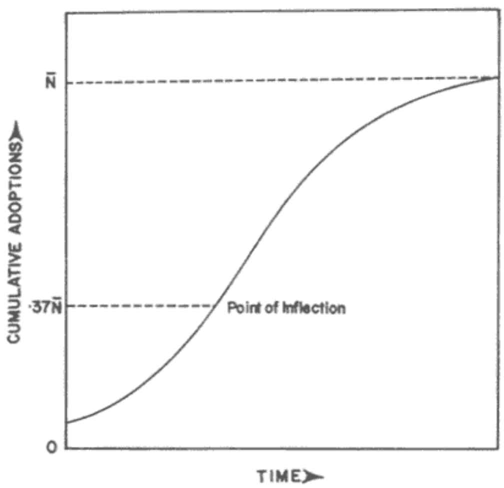


**Figure a.3: The Gompertz model (Source: Mahajan V. and Peterson R. A., 1985)**

However, the Gompertz curve does not show the initial slow growth, thus not being very useful when one wants to model this effect (Cioffi, 2005).

## 8.4 Mixed - influence diffusion models

The mathematical type of mixed –influence models is:

$\frac{dN(t)}{d(t)} = \left(a + bN(t)\right)\left(\overline{N} - N(t)\right)$ (eq.4)

These kinds of models are the most widely used. They are used for forecasting long-term sales of consumer durable products. The mixed model is modified by many scholars and is used to investigate the impact of location, simulate the effect of certain internal and external influence, forecast the market potential of solar technology, study the diffusion of educational innovations, etc.

Its plot results to a generalized Logistic curve, which shape is determined by both a and b.

### 8.4.1 The Bass model

The Bass model is a differential equation stating that the rate of the adoption of innovation is proportional to the number of the potential adopters that still haven't adopted it and to the sum of an innovation and an imitation coefficient. Innovation coefficient measures the propensity of potential adopters to become adopters and the imitation one measures the propensity of potential adopters to imitate previous adopters. Bass model is build on the Rogers' conceptual framework by developing a mathematical model that captures the non-linear structure of S-shaped curves (Robertson et al. 2007). Roger's suggested that different types of consumers enter the market at different stages of a product lifecycle (Roger, 1962).

The mathematical type of the Bass (1969) model is:

$n(t) = \frac{dN(t)}{dt} = p[m - N(t)] + \frac{q}{m}N(t)[m - N(t)] = pm + (q - p)N(t) - \frac{q}{m}N^2(t)$, where p and q are the coefficients of innovation and imitation respectively. N(t) and n(t) are the cumulative and non cumulative number of adopters in time t. The adopters due to mass media are represented by the term $p[m - N(t)]$ while the adopters due to interpersonal communications are represented by $\frac{q}{m}N(t)[m - N(t)]$.

Comment: It is possible to calculate an exact and general tipping point for this model (Philips, 2007). It is the first time period in which the total number of adopters exceeds the ration of p to q (Philips, 2007). Switching off advertising two periods before reaching the saturation level, the tipping point postpones achievement of this saturation (Philips, 2007).

### 8.4.2 The Lawton example (1979)

Lawton modified the valued of a and b of the mixed model and expressed them through a single rate parameter $p^*$. That made him able to express differences in diffusion patterns as a single rate parameter $p^*$. He used the following transformations:

$a = p^* \frac{N_0}{\bar{N} - N_0}$, $b = p^* \frac{1}{\bar{N} + N_0}$, $\frac{a}{b} = N_0$.

Lawton calculated that the value of $p^*$ for industrial innovations is $p^* = 0.66$ and for consumer product innovations$p^* = 0.50$.

## 8.5 Flexible diffusion models

Due to the absence of flexibility of the previous models and to the restrictions or assumptions they make, there are several flexible models build. The most important ones are described below.

### *8.5.1 The Floyd model*

As the name of the model declares, Floyd (1968) tried to empirically 'fit' certain observed diffusion data. The mathematical type of this model is:

$$\frac{dF}{dt} = bF(1-F)^2$$

His model is a non symmetric and possesses a fixed point of inflection at F* = 0.33. Hence, in these regards the Floyd model is similar to the Gompertz formulation of the internal-influence diffusion model.

### *8.5.2 The Sharif-Kabir model*

Sharif and Kabir (1976) combined an internal influence logistic model and Floyd's one to create their model. The mathematical type of this model is:

$$\frac{dF}{dt} = \frac{bF(1-F)^2}{1-F(1-\sigma)}$$

Although the Sharif-Kabir model can accommodate symmetric as well as nonsymmetric diffusion patterns, it produces a point of inflection that must be in the range 0.33 ≲ 0.5.

### *8.5.3 The Jeuland model*

Jeuland (1981) proposed a generalization of the Bass model based on the following assumptions:

- external influence in the diffusion process relates to the potential adopter's propensity to adopt the innovation
- the population of potential adopters is heterogeneous with respect to the propensity to adopt
- propensity to adopt varies according to a gamma distribution

The mathematical type of the model is:

$$\frac{dF}{dt} = (a+bF)(1-F)^{(1+\gamma)}$$

It is noticeable here that when $\gamma$=0 the model reduces to Bass model. When $\alpha$=0 and $\gamma$=1, it reduces to the Floyd model.

Although the Jeuland model can accommodate symmetric as well as nonsymmetric diffusion patterns, it cannot yield a point of inflection beyond a 50% adoption level.

### *8.5.4 The NSRL and NUI models*

Easingwood et al. (1981, 1983) proposed two flexible versions of the fundamental diffusion model: the Non-symmetric Responding Logistic (NSRL) model a flexible internal-influence model and the Non-Uniform Influence (NUI) model, a flexible mixed-influence model. The purpose of these models was to overcome an inherent limitation of the fundamental diffusion model the assumption that the impact of internal influence between adopters and potential adopters remains constant over the entire diffusion process (i.e., the coefficient of internal-influence, b, is a time-invariant constant). For most innovations, this assumption is questionable because the impact of internal influence is likely to change, either increasing or decreasing, as the diffusion process unfolds.

NUI and the NSRL models can both accommodate symmetric as well as nonsymmetric diffusion patterns. In addition, the point of inflection can occur at any time during the diffusion process.

The mathematical types of these models are:

NUI , $\frac{dF}{dt} = \left(a + bF^{\delta}\right)(1 - F)$

NSRL, $\frac{dF}{dt} = \left(bF^{\delta}\right)(1 - F)$

### *8.5.5 The Von Bertalanffy model*

A less known mode is created by von Bertalanffy (1957) and its mathematical form is :

$$\frac{dF}{dt} = \frac{b}{1-\theta} F^{\theta}\left(1 - F^{(1-\theta)}\right)$$

### 8.5.6 Comments for flexible diffusion models

Flexible diffusion models allow the generalized S-shaped diffusion curve to be symmetrical as well as nonsymmetrical, with the point of inflection responding to the diffusion pattern instead of being determined a priori. Although these models can be adjusted by means of the nonlinear or maximum likelihood procedures, they require estimation of an additional parameter. For example, in addition to a, b, and $\overline{N}$, the NUI model requires estimating d and the Jeuland model requires estimating g. Similarly, in addition to b and $\overline{N}$, the Sharif-Kabir, NSRL, and the Von Bertalanffy models require estimating s, d, or q, respectively. Hence, all flexible diffusion models achieve their flexibility by requiring estimation of an additional parameter. As a consequence of their flexible nature, though, it is possible to develop a taxonomy of diffusion patterns because the models produce diffusion curves that mirror, rather than "force" the shape of the underlying diffusion data (Mahajan and Peterson, 1985). Despite the increased flexibility for capturing diffusion patterns, flexible models are also characterized by the same seven assumptions underlying the fundamental diffusion models. The next section presents diffusion models that address some of these assumptions.

## 8.6 Comments about the fundamental and flexible models

There are seven important and sometimes restricting assumptions describing both fundamental and flexible models. These are:

- The diffusion process is considered binary
- There is a fixed ceiling on the number of potential adopters
- There is only one adoption by an adopting unit
- There is a complete mixing of prior and potential adopters with model parameters constant over the diffusion process
- The innovation is independent of all other innovations
- The geographical boundaries of the system do not change
- All relevant information about the diffusion process is captured by the model

## 8.7 Extensions and refinements of models

Due to the seven assumptions characterizing the fundamental and flexible diffusion models, efforts are made to overcome this. That is by inserting-forming more diffusion models of these types:

- Dynamic
- Multi-innovation
- Space and time
- Multistage
- Multiadoption
- Incorporating influencing or change agents

### *8.7.1 Dynamic*

Mahajan and Peterson (1978) consider that the number $\overline{N}(t)$ of potential adopters in reality is not fixed. Setting $\overline{N}(t) = f(\underline{S}(t))$ which is a vector of potentially relevant exogenous and endogenous variables controllable as well as uncontrollable affecting$\overline{N}(t)$. Then we have the equation:

$$\frac{dN(t)}{d(t)} = \big(a + bN(t)\big)(f(\underline{S}(t)\text{-N (t))}$$

A plot of this model shows clearly the dynamic flexibility of this model.

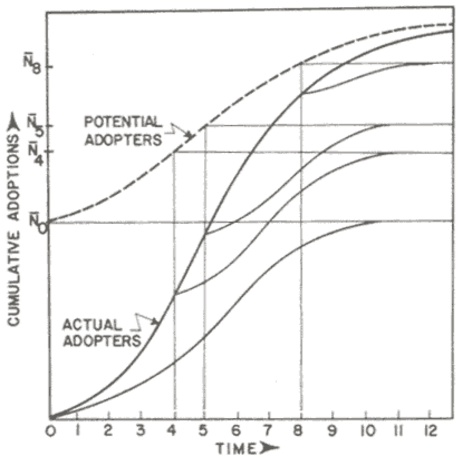


**Figure a.4: A dynamic model (Source: Mahajan V. and Peterson R. A., 1985)**

Other dynamic diffusion models have been developed by Chow (1967), Lackman (1978), Dodson and Muller (1978) and Sharif and Ramanathan (1981). Examining the natural growth of computers by means of a Gompertz internal-influence model, Chow argued that the number of computer adoptions was influenced by a change-price reduction effect. Lackman (1978) used a Gompertz-based dynamic model formulation when studying the growth of a new plastic product in the automotive industry:

### 8.7.2 *Multi-innovation*

Models of this category examine the interrelation of innovations $N_1, N_2$. Peterson and Mahajan (1978) have identified four categories of innovation interrelationships that can affect the adoption rate as well as the cumulative number of adoptions of an innovation. These are the following interrelationships:

- Independent of each other in a functional sense, but adoption of one may enhance adoption of others (e.g., modular housing units and electric trash compactors).
- Complementary: increased adoptions of one innovation result in increased adoptions of other innovations (e.g., washers and dryers).
- Contingent: adoption of one innovation (e.g., computer software) is conditional on (usually previous) adoption of other innovations (e.g., computer hardware).
- Substitutes: increased adoptions of one innovation result in decreased adoptions of other innovations (e.g., black and white versus color television sets)

Multi-innovation diffusion models can easily be used to test hypothesized relationships between innovations. For example, when comparing the sales growth rates of color and black and white television sets from 1959 to 1973, Peterson and Mahajan (1978) found that although a substitution-model specification was not appropriate for color television sets, it improved model fit for black and white television sets significantly. In other words, substitution was unidirectional: The growth in sales of color television sets had a substitution effect on the sales growth of black and white television sets although the converse was not true. If anything, the sales growth of black and white television sets slightly *complemented* that of color sets.

Other researchers have addressed the nature of innovation interdependencies in the context of "competitive independence." This is the notion that an innovation is only offered by a single organization or, if more than one organization does offer it, the organizations' innovation offerings have no impact on each other. Examples of research refuting this notion, in the area of business, include the work of Eliashberg and Jeuland (1982), Rao and Bass (1985), Clarke and Dolan (1984), Mate (1982), Teng and Thompson (1983), and Fershtman et al. (1983). The major thrust of these cited works seems to be toward examination of pricing or advertising strategies of the competing firms and their impact on "market equilibrium."

### *8.7.3 Space and time models*

Although innovations diffuse simultaneously in space and time, most research focus on spatial dimension. Usually adopters are affected by mass media and interpersonal contact. For example three empirical regularities occurred:

- S-curve
- A hierarchical effect (diffusion from large to small centers)
- A neighborhood effect (diffusion is expected to proceed in a wave like fashion outward an urban center first from nearby then to remote locations)

A graphical representation of such a model is the following:

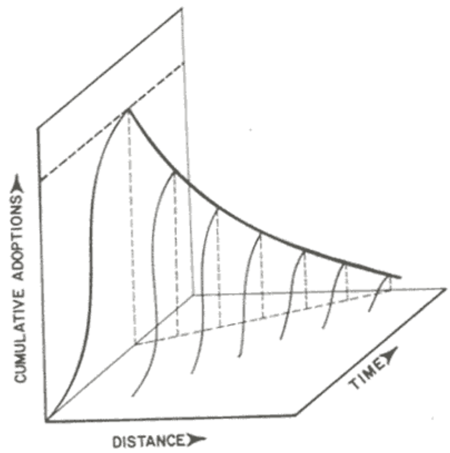


**Figure 1: A space-time model (Source: Mahajan V. and Peterson R. A., 1985)**

A mathematical form of such a model is suggested by Mahajan and Peterson (1979) and is:

$$N = f(x,t); \frac{\partial N}{\partial y} = 0$$

$$\frac{\partial N(x,t)}{\partial t} = \big(a(x) + b(x)N(x,t)\big)[\overline{N}(x) - N(x,t)]$$

Mahajan and Peterson (1979) applied the model on the adoption of tractors in 25 states in the central agricultural production region of US in 1920-1964.

### 8.7.4 Multistage model

Rogers (1983: 165), pointed that in practice an adopting unit may pass through a series of stages in the innovation-decision process. Towards this direction, attempts are made to extend two-stage models to incorporate the multistage (or polynomial) nature of the diffusion process. Some of these models are suggested by Midgley (1976), Dodson and Muller (1978), Sharif and Ramanathan (1982) and Mahajan, Muller, and Kerin (1984).

#### 8.7.4.1 The Dodson and Muller model

Dodson and Muller (1978) hypothesized that because of advertising and word-of-mouth influence, uninformed social system members first become potential adopters (customers) and then current adopters (customers). In the presence of competing innovations (e.g., brands), current adopters can either readopt (repurchase) the same innovation or by adopting competing innovations return to potential adopter status. Finally, because of forgetting, current adopters and potential adopters can become members of the uninformed group.

Let $x(t)$ = number of social system members who are unaware of the innovation at time t,
$y(t$ = number of social system members who are aware of the innovation at time t but still have not adopted it and,
$z(t)$= number of current adopters who have adopted the innovation. If it is assumed that the total population of the social system M remains constant over time, then at any time t the equations are:

$$x(t) + y(t) + z(t) = M$$
$$\frac{dy}{dt} = \beta x(y + z) + \mu x - \gamma y,$$

where the first term indicates the increase in the number of potential adopters due to the interaction between uninformed social system members and potential and current adopters; the second term, μx, represents the increase due to external influences (i.e., advertising). The third term, gy, denotes the decrease in the number of potential adopters due to the transfer of potential adopters to current adopters.

$$\frac{dz}{dt} = \gamma y$$

<u>Comment</u>

The Dodson-Muller model uses a mixed-influence approach to represent the flow of uninformed social system members to potential adopters and the external-influence model to represent the flow of potential adopters to current adopters. The multistage diffusion model of Dodson and Muller assumes that an individual's experience with the innovation is communicated positively through word-of-mouth. This assumption is tenuous because communicators of the innovation experience may transfer favorable, unfavorable, or indifferent messages to others. The multistage models proposed by Midgley (1976), Sharif and Ramanathan (1982) and Mahajan et al. (1984) attempt to relax this assumption in their formulations. An extension of the Dodson-Muller model, divides the potential adopters (customers) and current adopters (triers) into two groups, each based on the positive or negative nature of communicated information. Mahajan et al. have applied their model to forecast attendance for the movie "Gandhi" in the Dallas, Texas area.

##### 8.7.4.2 The Tapiero (1983) model

Tapiero (1983) proposed a stochastic model that is based on explicit assumptions regarding word of mouth effects and response to advertising. He states that it is a generalization of earlier ones like the Dodson and Muller (1978) model that are obtained as special mean-evolution cases. The methodology includes several classes of consumers (unaware, aware, buyers), interaction effects between these consumers and a stochastic framework to asses the risk implications of advertising policies. The hypotheses on which the theoretical framework is build are formulated in probability terms and by adopting Markovian assumptions.

#### 8.7.5 Multi-adoption diffusion models

Multi adoption models describe the phenomenon where a buyer of an innovation may be a first-time buyer or a repeat one. Considering that many product-innovations that are introduced into the market are repurchasable, sellers of these innovations are interested in predicting the successive increase in number of adopters due to repeat buyers even more than the number of first-time buyers. Given the satisfaction of first time buyers at the initial adoption stage, repeat buyers tend to become heavy users of a product innovation. To conclude, for non repurchasable product-innovations (e.g., many consumer durables), the purpose of a diffusion model is to depict the first-purchase diffusion curve while for repurchasable products (e.g., packaged goods) the purpose is also to model the repeat purchase diffusion curve.

##### 8.7.5.1 The Lilien et al. (1981), Mahajan et al. (1983) and Dodson-Muller (1978) models

Lilien et al. (1981) and Mahajan et al. (1983) proposed models that use early diffusion data and explicitly consider word-of-mouth communication in their formulations to forecast repeat purchase. Another model is also suggested by Dodson and Muller (1978) focusing on the repeat-purchase case.

The Lilien et al. (1981) and Mahajan et al. (1983) models tried to forecast the sales of ethical drugs and their mathematical forms are the following:

Lilien : $N(t+1) = a(t)\left(\overline{N} - N(t)\right) + b\left(N(t) - N(t-1)\right)\left(\overline{N} - N(t)\right) + c(t)N(t)$

Mahajan: $N(t+1) = a\left(\overline{N} - N(t)\right) + b\left(\frac{N(t)}{\overline{N}}\right)^{\delta}\left(\overline{N} - N(t)\right) + cN(t)$

The Dosdson-Muller (1978) model includes repeat purchasing and forgetting. They assumed that the population is constant in a social system. The mathematical type of this model is written as a system of two equations:

$$N(t+1) = \gamma\left(\overline{N}(t) - N(t)\right) + cN(t), \textit{ where } c=\frac{1}{fq}$$
$$\overline{N}(t) = \mu\left(M - \overline{N}(t-1)\right) + \beta\overline{N}(t-1)\left(M - \overline{N}(t-1)\right) + K\overline{N}(t-1), \textit{ where } K = \frac{1}{q}$$

The parameter f is a constant forgetting factor and K is a constant switching factor reflecting adoption of a competitive product.

Comments

In Lilien et al. model, a, is the coefficient of external influence as a function of a firm's promotional (e.g., detailing) efforts and the coefficient of retention, c, is as a function of competitors' promotional (detailing) efforts. With regard to the interaction effect (the second term), because N(t) can be greater than or less than N(t-1), they assume that at any time t the number of potential adopters, $(\overline{N} - N(t))$, can be influenced only by the additional number of adopters who adopt between time t and (t 1)as compared to *all* of the adopters, N(t), as assumed in Mahajan et al. equation or those who *stop* repurchasing the product between time t and (t-1) as reflected by (N(t)-N(t-1)).

Mahajan et al. models is a direct extension of the NUI model. Assuming a constant population of potential adopters, the first term in Mahajan's et al. equation represents the number of adopters at time (t + 1) due to external influence, the second term denotes adopters due to word-of-mouth communication, and the third indicates the fraction of adopters in period t who continue to adopt in period (t + 1). The constant c is an index or coefficient of retention.

In Dosdson-Muller (1978) model, the first equation reflects adopters due to external influence and the second reflects adopters at time t who continue to repurchase at time (t + 1). Because $\mathrm{cN(t)} = \mathrm{N(t)/fN(t)qN(t)}$, the number of repurchasers at time $(\mathrm{t}+1)1)$ is obtained by substracting the number of social system members who forget about the product, $\mathrm{fN(t)}$, and the number who switch to a competitive product, $\mathrm{qN(t)}$), from $\mathrm{N(t)}$. The dynamic market potential $\overline{\mathrm{N}}(\mathrm{t})$ in the second equation includes newly awares due to advertising (first term), newly awares due to word of mouth (second term) and potential adopters who do not switch over to a competitive product as reflected by $\mathrm{K} = \frac{1}{\mathrm{q}}$.

We should remark here that the the three repeat-purchase diffusion models do not distinguish between repeat adopters in terms of the number of times they have repurchased the product. That is, the models ignore the "depth" of repeat buying; they do not distinguish between first repeaters, second repeaters, and so on.

### 8.7.6 Influencing – Change agents diffusion models

Change agents are maybe the most central actors in an organizational network as they can promote the diffusion of an innovation by influencing a group of members to adopt it (Maienhofer and Finholt, 2002). The efficiency of change agents depends on the selection of the right targets for their efforts.

In that kind of models the attempt is to incorporate diffusion strategies in the diffusion models. These strategies are mainly drawn by economists, technological forecasters, and market researchers. There are scholars who studied the relationship between policies and diffusion rates.