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Master's Thesis

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Environmental Value Adjustments for Asset Pricing

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Abstract

In this thesis, we explore how environmental aspects can be incorporated into asset valuation in the financial sector to internalise climate change economically. We propose several Environmental Value Adjustment (EVA) models and demonstrate their application by financial institutions and authorities to mitigate climate-related risks. EVAs are expressed in terms of an Environmental Impact Factor (EIF), quantifying the climate impact of assets relative to their exposure. We show that EVA formulation in terms of EIF allows seamless integration into existing xVA models. We introduce a Climate Risk Value Adjustment (CRVA) based on climate risk-related adjustments to the CVA of assets, addressing uncertainties associated with physical climate risk over a long time horizon.

Additionally, we examine the current infrastructure of the Voluntary Carbon Market (VCM) and identify issues that prevent it from being deployed as a climate change mitigation mechanism. As the VCM matures, financial institutions can achieve net zero by offsetting financed emissions using voluntary carbon credits. We propose a Financed Emissions Value Adjustment (FEVA) model that incorporates these costs into asset valuation based on the Partnership for Carbon Accounting Financials (PCAF) standard. Finally, we define a stochastic control problem to identify cost-minimising carbon credit buying strategies, which we solve using a least squares Monte Carlo approach. We assess its effectiveness and limitations, particularly in conjunction with the proposed higher-dimensional VCM model.

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- Nibble meeting for the Mathematical Institute, Utrecht University (4-10-2024)
- Dutch Math Finance Afternoon, TU Delft (13-12-2024)
- Pricing Model Validation & Financial Engineering Quantitative Seminar, Rabobank (8-1-2025)
- Markets Knowledge Session, Rabobank (28-3-2025)

I am grateful to the organisers for the opportunity to share my work and exchange ideas on the topic at these events.

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CONTENTS

Notation and abbreviations

Important abbreviations

Abbreviation	Definition	First appearance
EVA	Environmental Value Adjustment	Section 1.3
EIF	Environmental Impact Factor	Section 1.3
GHG	GreenHouse Gas	Chapter 1
CO2e	CO2 equivalent	Chapter 1
ESG	Environmental, Social & Governmental	Section 1.2
PCAF	Partnership of Carbon Accounting Financials	Section 1.2
xVA	Value Adjustment[1]	Section 1.1
CCR	Counterparty Credit Risk	Section 1.1
CVA	Credit Value Adjustment	Section 1.1
ECL	Expected Credit Losses	Section 1.1
OTC	Over-The-Counter	Section 1.1
NGFS	Network for Greening the Financial System	Section 2.1
CSRD	Corporate Sustainability Reporting Directive	Section 2.2
CEP	Carbon Equivalence Principle	Section 2.2
SPC	Shadow Price of Carbon	Section 2.3
SCC	Social Cost of Carbon	Section 2.3
ICP	Internal Carbon Price	Section 2.3
VCM	Voluntary Carbon Market	Chapter 3
OU	Ornstein-Uhlenbeck	Section 3.2
CTD	Cheapest-To-Deliver	Section 3.3
LGD	Loss Given Default	Section 4.1
NHPP	Non-Homogeneous Poisson Process	Section 4.1
CRVA	Climate Risk Value Adjustment	Section 4.2
GBM	Geometric Brownian Motion	Section 5.1
FEVA	Financed Emissions Value Adjustment	Section 5.2
CCA	Continuous Carbon Annuity	Section 5.4
FBVA	Financed Biodiversity Value Adjustment	Section 5.5
LSMC	Least Squares Monte Carlo	Section 6.3

[1] The x can represent various types of Value Adjustments

List of mathematical symbols

Symbol	Definition
$\mathbb{R}$	Set of real numbers
$\mathbb{N}$	Set of natural number
$\mathbb{Z}_n$	Set of integers $\{0, 1, \ldots, n-1\}$
$[n]$	Set of integers $\{1, 2, \ldots, n\}$
$\mathcal{S}_+$	Nonnegative elements of a totally ordered set: $\{s \in \mathcal{S} \mid s \geqslant 0\}$
$\mathcal{S}_{++}$	Postive elements of a totally ordered set: $\{s \in \mathcal{S} \mid s > 0\}$
$L^2(\mathcal{D})$	Space of square-integrable functions on a measure space[2] $\mathcal{D}$: $\{Y : \mathcal{D} \to \mathbb{R} \mid \int_{\mathcal{D}} Y(x)^2 \, \mathrm{d}\mu(x) < \infty\}$
$\mathcal{C}(\mathcal{D})$	Set of continuous functions on a domain $\mathcal{D}$
$\langle X, Y \rangle(t)$	Cross-variation of the stochastic processes $X(t)$ and $Y(t)$
$(v_1, \ldots, v_d)^T$	d-dimensional column vector with entries $v_1, \ldots, v_d$
$\|\| \cdot \|\|$	Euclidean norm operator
$(\,\cdot\,)^+$, $(\,\cdot\,)^-$	Positive part operator $\max(\,\cdot\,, 0)$ and negative part operator $\min(\,\cdot\,, 0)$ respectively
$\lceil \cdot \rceil$	Ceiling operator
$\Phi(\,\cdot\,)$	Cumulative density function of the standard normal distribution
$\mathcal{N}(\mu, \sigma^2)$	Normally distributed variable with mean μ and variance σ^2
T	Finite time horizon
t	Time variable in $[0, T]$, usually in years
N	Number of paths in a Monte Carlo simulation
m	Number of steps in a discretization of $[0, T]$
$(\Omega, \mathcal{F}, \mathbb{P})$	The sample space, event space & probability measure of a probability space respectively
$\mathbb{1}_A$	Indicator function of an event $A \in \mathcal{F}$
$\mathcal{F}(t)$	Natural filtration of the filtered probability space
$\mathbb{E}_t[\,\cdot\,]$	Conditional expectation $\mathbb{E}[\,\cdot \mid \mathcal{F}(t)]$
$\mathbb{V}\mathrm{ar}_t[\,\cdot\,]$	Conditional variance $\mathbb{V}\mathrm{ar}[\,\cdot \mid \mathcal{F}(t)]$
$\mathbb{P}_t[\,\cdot\,]$	Conditional probability $\mathbb{P}[\,\cdot \mid \mathcal{F}(t)]$
d	Dimension of the VCM model
ρ	Correlation matrix
$W(t)$	Wiener process
$C_i(t)$, $X_i(t)$	Price and log-price of voluntary carbon credit type i, respectively
$H(t)$	CTD price process in the VCM model
r	Instantaneous interest rate
$V(t, T)$	Risk-free value of an asset with maturity T
$E(t)$	Exposure of an asset
t_D	Default time of a counterparty
$\lambda(t)$	Default rate of a counterparty
$\mathcal{G}(t)$	Enlarged filtration
$\widetilde{V}(t, T)$	CCR-adjusted value of an asset with maturity T *
$\hat{\cdot}$	Climate risk-adjusted variable[3]
$\lambda^{\mathrm{CR}}(t)$	Climate risk adjustment to the default rate
$I(t)$	Emission intensity of a counterparty
$C(t)$	Carbon price in the FEVA model
$\widehat{V}'(t, T)$	Financed emissions- and climate risk-adjusted value of an asset with maturity T
Ξ, $\xi(t)$	State space and state space process respectively
$\mathbf{b}(t, \xi(t))$	Feedback control law
$L(t)$, U	Lower bound process and upper bound on the cumulative bought emissions $B(t)$ respectively

[2]The measure is denoted by μ. For $\mathcal{D} \subseteq \mathbb{R}^n$, we let μ be the Borel measure on $\mathbb{R}^n$.
[3]For example, $\widehat{V}(t, T)$ denotes the climate risk-adjusted value of an asset with maturity T

CHAPTER 1

INTRODUCTION

Climate change is considered the greatest threat to humanity (particularly to developing nations) [180, 64] and the most notable way climate change emerges is the rising global surface temperature, which on average was 1.1° C higher during 2011-2020 than during 1850-1900, with devastating consequences that only worsen with further temperature increases: weather and climate extremes and rising sea levels have adverse impacts on food and water security, human health and cause substantial and sometimes irreversible damages to ecosystems [64, 203].

It is widely accepted that rising global temperatures are caused by human activities, particularly their consequent GreenHouse Gas (GHG) emissions [64, 168], which are usually expressed in tonnes CO2 equivalent[1] (CO2e) [80]. To mitigate climate change, the Paris Agreement was adopted at the 2015 United Nations Climate Change Conference, improving on its predecessor, the Kyoto Protocol from 1997 [188]. The Paris Agreement recognises that urgent changes are required and aims to keep global surface temperatures less than 2°C higher than over 1850-1900 and to pursue efforts to achieve a temperature increase of at most 1.5° to significantly reduce the risks and impacts of climate change. [170]. Following the Paris Agreement, the International Panel on Climate Change concluded that reaching CO2e neutrality[2] (net zero) by 2050 is necessary to achieve the goal of 1.5° [119]. Since then, reaching net zero has become a central topic in climate change mitigation.

Despite the threat that climate change poses, society struggles to deal with climate change since it is a global externality, which is defined as follows:

Definition 1.1 (Externality)**.** *From an economical perspective, an externality is a phenomenon of which the costs, benefits and drivers are not adequately captured in the financial market. The effects of externalities affect parties other than those who cause them in the first place. An externality is called global if its effects have global impact. These effects can be positive or negative [127].*

A negative externality can have problematic consequences since, by definition, the parties that cause them have no financial incentive to stop doing so. In the past, there has been limited success in dealing with global economic externalities, but the exceptions found their success through international cooperation and governance [158]. To internalise climate change, global cooperation and coordinated regulations are therefore considered crucial, which is why the Paris Agreement was a crucial step towards internalising climate change [188].

[1] See Section 2.2.2 for more details.
[2] CO2e neutrality is achieved when any CO2e emissions are offset by projects that remove CO2e from the atmosphere.

1.1 Asset pricing and value adjustment models

Counterparty credit risk was another example of a global externality before the global credit crisis of 2007-2008 and is defined as follows:

Definition 1.2 (Counterparty Credit Risk). *Counterparty Credit Risk (CCR) is the risk to the holder of a financial instrument that the counterparty will not be able to meet contractual payment obligations during the duration of the contract. If this happens, we say that the counterparty defaults [101].*

Definition 1.1 was applicable to CCR prior to the credit crisis, since potential losses from counterparty default were not adequately incorporated into asset valuation and risk management practices. Assets were often priced by discounting expected future cash flows to the present with insufficient focus on CCR, but the crisis revealed that CCR was not adequately internalised in asset pricing and introduced strict regulations to better recognise the price of CCR associated with assets [47]. This price tag forms an adjustment to the value of an asset to incorporate the possibility of counterparty default, which can be formulated as follows:

$$\text{Risk-adjusted value} = \text{Risk-free value} - \text{Value adjustment}, \tag{1.1}$$

where the risk-free value is obtained using a present value method that does not incorporate the possibility of counterparty default, and the risk-adjusted value is obtained by a present value method that does incorporate the possibility of counterparty default. This makes the value adjustment the present value of expected future losses on the asset from counterparty default. We make a distinction between assets in the trading book and in the banking book of banks and specifically focus on assets with finite maturity (see Remark 1.1).

Assets on the trading book of banks were priced using present value methods prior to the crisis, namely the risk-neutral valuation framework as introduced by Black and Scholes [38]. As a consequence of the crisis, required adjustments to the risk-neutral pricing framework to mitigate CCR were introduced in the third Basel Accord, which came into force in 2012 [18]. These adjustments come in the form of Value Adjustment (xVA) models, which have the purpose of internalising a given external aspect into asset valuation and are commonly applied to uncollateralised over-the-counter (OTC) trades [209, 184]. The Credit Value Adjustment (CVA) is the xVA that internalises CCR and plays the role of the value adjustment in Equation (1.1) and can be seen as the price of CCR [209, 184]. Under Basel 3, the CVA was introduced as an additional charge to a counterparty when a trade takes place to incorporate CCR in the price [224, 184]. Basel 3 also introduced regulatory capital requirements related to the CVA over trading books [18]. Moreover, the CVA of assets must be continuously monitored and hedged by the xVA desk of banks under Basel 3, which they do with credit instruments such as credit default swaps [101, 230].

For assets on the banking book of banks, losses from counterparty default were only recognised once the default occurred under the market standard prior to the crisis, even if default was already expected [47]. The International Financing and Reporting Standard (IFRS) 9 was developed in response to the crisis and came into force in 2014 [40]. Under IFRS 9, the forward-looking Expected Credit Loss (ECL) was introduced to incorporate future losses from counterparty default in asset valuation, where it plays the role of the value adjustment in Equation (1.1) [27, 104]. The introduction of ECL also allows regulators to mitigate CCR by establishing hedging and capital requirements based on forward-looking information [47]. Even though value adjustments are primarily applied in the context of uncollateralised OTC trades on the trading book, we emphasise that similar principles can also be applied to general assets to incorporate an externality into broader asset valuation.

In summary, both the ECL and the CVA function as a value adjustment to present value pricing methods to internalise CCR, for assets on the banking book and trading book of banks, respectively. In this thesis, we will consider three purposes that xVAs have based on how the ECL and particularly the CVA are used to internalise CCR in the financial sector. First of all, they represent the fair value for an asset that the involved parties agree on when they both incorporate CCR in the valuation process. Secondly, the CVA and ECL play an important role in risk management. The CVA can be continuously hedged by a financial institution if the appropriate credit instruments are available, thereby neutralising the impact of CCR on the portfolio. Lastly, regulatory capital requirements are imposed on financial institutions based on the total CVA and

ECL on their books. We go into more detail on these three applications of the CCR value adjustments in Subsection 4.1.6.

Value adjustments are primarily applied to derivatives on the trading book that are traded OTC [184, 162]. In Subsection 4.1.1, we illustrate how collateralisation decreases the extent in which credit risk is transmitted between the parties involved in a contract. For the remainder of this thesis, we will therefore primarily focus on uncollateralised assets that are traded OTC because, but also address how the proposed principles and resulting models apply to more general assets.

Remark 1.1 (Finite-maturity assets)**.** *In this thesis, we focus on the valuation process of financial instruments between two parties that have some finite maturity. These are contracts with an end date that prescribe the exchange of capital at predetermined moments between two parties. We will refer to the involved parties as the holder and the counterparty from now on. Examples of contracts with finite maturity include loans, bonds and derivatives such as options, swaps, futures and forwards or more exotic derivatives [162, 115]. We will use the term asset to refer to contracts with a finite maturity unless specified otherwise, even though there are various assets without a financial maturity, such as stocks, real estate, physical and digital currencies, perpetual bonds and commodities. Although these instruments can also carry risks that can be reflected in their value, we restrict our focus on contracts between two parties for the remainder of this thesis with the aim of quantifying how the impact of externalities is transmitted between the parties involved in a contract.*

1.2 Climate change and the financial sector

In Figure 1.1, the relationship between the financial sector and climate change is illustrated in a simplified manner. In this section, the illustrated aspects will briefly be described to clarify the role of the financial sector in addressing climate change. In Chapter 2, we will delve deeper into the relevant aspects of this relationship as depicted in Figure 1.1.

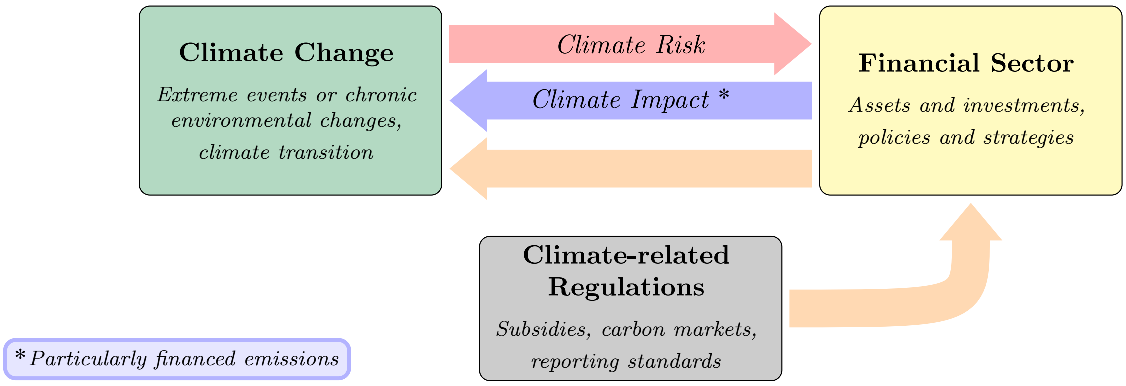


Figure 1.1: *Simplified illustration of the bidirectional relation between climate change and the financial sector, including how climate-related regulations indirectly impact climate change. The most relevant aspects for this thesis of the various components are listed, including the impact of climate risk (See Chapter 4) and financed emissions (See Chapter 5) from climate change and the financial sector on each other.*

On the one hand, assets and investments of financial institutions may be subject to the consequences of climate change, which can generally be categorised into physical consequences (either acute events or chronic environmental changes) and consequences related to the transition to a climate-friendly economy [19, 61, 203]. These consequences can harm corporates across the entire economy, particularly in vulnerable sectors or geographical locations [16]. This can in turn have a negative impact on the effectiveness of investments of financial institutions, leading to value deterioration or even increased CCR from the consequences of climate change [51, 211, 8]. Financial institutions can also be forced to adapt their strategies and policies to the transition to a climate-friendly economy, potentially leading to operational risks [102, 8]. In this thesis, we will limit our attention to the impact of climate change on CCR out of these topics, in line with the CCR-related xVA models that are introduced in Section 1.1. We start with a further investigation of climate risk drivers and their impact on counterparty credit risk in the financial sector in Section 2.1, before we formulate a climate-related adjustment to existing CCR frameworks in Section 4.2.

On the other hand, the climate can also be affected by the assets and investments of financial institutions, most notably if they cause GHG emissions. Generally, investments in activities with limited or even positive climate impact tend to yield lower returns than traditional emission-intensive investments, as illustrated in Figure 1.2. This makes climate change a negative externality and implies that financial incentives are necessary to internalise the climate impact associated with investments. In Section 2.2, we explore various current measures on this topic.

It should be noted that the relationship with climate change in Figure 1.1 is applicable not only to the financial sector, but to all human beings to some extent. Climate change can have adverse impacts on human health, food security and infrastructure [64], but humans also impact the climate with their daily decisions, albeit with negligible impact, meaning that the relationship is generally limited to the red arrow in Figure 1.1. For the financial sector however, the story is slightly different: with their substantial pool of capital, financial institutions influence the entire global economy and therefore a large fraction of global GHG emissions and other ways that human activities impact the climate [169], as illustrated in Figure 1.2. This means that more than for any other sector, the impact on the climate, represented by the blue arrow in Figure 1.1, is very important to consider for financial institutions.

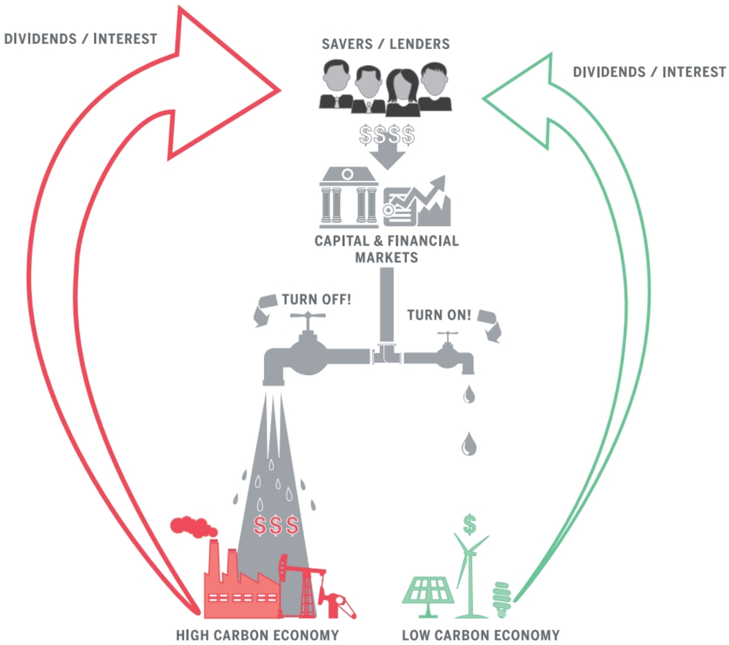


Figure 1.2: *Illustration of (1) the influence of the financial sector on the climate impact of the economy and (2) the fact that high carbon investments yield higher returns, meaning that action is required to shift towards low carbon investments [169]. This also illustrates that climate change is an externality.*

Their substantial influence on climate change puts the financial sector in a unique position of power when it comes to dealing with this externality. Not only do the choices of financial institutions indirectly impact their own capital (as illustrated by the bidirectionality of the relation in Figure 1.1), but the financial sector also carries the responsibility to make ethical and sustainable decisions on the topic of climate change[3].

[3]This perception is based on the famous belief that with great power comes great responsibility.

Environmental, Social and Governmental (ESG) topics have therefore emerged in recent years in financial decision-making [121], but there are concerns about whether or not financial institutions are acting adequately on this responsibility of dealing with this externality [169]. Sceptics sometimes accuse companies of *greenwashing*, which is the practice of misleading the public to believe that a party is making more efforts on the topic of sustainability than it actually is [140].

Concerns and issues like these ultimately stem from the fact that climate change is an externality and illustrate that regulations on this topic have to be imposed on the system to successfully deal with this externality. In Figure 1.1, the indirect impact of ESG regulations in the financial sector on the climate is illustrated. Given the crucial role of CO2e emissions in climate change [64], so-called *carbon pricing* schemes are considered the most effective regulations to internalise climate change and are defined as follows:

Definition 1.3 (Carbon pricing). *Carbon pricing is a tool for regulators to internalise CO2e emissions. Parties that fall under a carbon pricing scheme are charged by the regulator for the amount of CO2e they emit [158].*

Carbon pricing is a direct and cost-effective method to economically internalise the external costs of CO2e emissions because it creates price signals to both consumers and producers, incentivising emission reductions across the entire value chain [43]. Carbon pricing schemes decrease the cost-efficiency of carbon-intensive investments, reducing the imbalance illustrated in Figure 1.2. Regulators often use the generated revenue from the covered CO2e emissions to offset these emissions or invest in broader ESG themes to neutralise the negative climate impact of charged emitters. However, the generated revenue is sometimes also used for more general purposes, such as cuts in other taxes [17, 7, 74]. Moreover, it creates market incentives to accelerate the climate transition [158, 169]. This is a clear example of how ESG regulations can indirectly have a positive impact on the climate, as illustrated in Figure 1.1. In Section 2.3, we will further explore regulated carbon pricing schemes.

Moreover, we will explore the voluntary carbon market in Chapter 3, which is an emerging carbon market without a central regulator where participation is not obligatory in contrast to regulated carbon pricing schemes. The voluntuary carbon market is considered a crucial component of the road to a net zero economy, as relying on companies to abate all emissions inside their value chain is not feasible[199, 189]. Most companies already include voluntary carbon credits in their net zero strategies, but the quality issues surrounding credit-generating projects have prevented them from contributing significantly to climate change mitigation [206, 212], as we will discuss in more detail in Subsection 3.1.2.

In line with the necessity for regulations and global cooperation when dealing with externalities, the transition to a net zero economy has to be facilitated and catalysed by universal frameworks and standards for measuring and reporting climate risk and impact. Standardised measurement and reporting improve transparency throughout the economy, making them key components in addressing climate change [169, 213]. The GHG Protocol provides a fundamental example of such a standardised framework, which helps companies address and manage their impact on the climate through their GHG emissions. Standardised, consistent and transparent GHG accounting and reporting are crucial components for voluntary and mandatory GHG programs, such as carbon pricing schemes [223]. The GHG Protocol provides a comprehensive framework for measuring, reporting and managing GHG emissions by categorising emissions across the value chain into different scopes, which will be explored in Section 2.2.

Building on the GHG Protocol, the Partnership for Carbon Accounting Financials (PCAF) has developed a standard specifically to measure and report emissions that are associated with investments, which are called *financed emissions*. For financial institutions, financed emissions are the primary source of GHG emissions and therefore the most relevant source of their impact on the climate. Before the development of this standard however, inconsistent assessments and a lack of standardisation hampered the transparency and accountability across the financial sector on the topic of the transition to net zero [165]. Financed emissions are a primary focus of this thesis and will be further explored in Section 2.2 and Chapter 5, where we propose a model with the purpose of internalising financed emissions.

1.3 Contribution and thesis structure

This thesis aims to explore how value adjustments can be applied in the context of environmental topics in the financial sector. Similarly to how a CVA adjusts the value of assets by internalising CCR (see Equation (1.1)), we introduce Environmental Value Adjustment (EVA) models to internalise different aspects of the relation between the financial sector and climate change, which are illustrated in Figure 1.1. We give the following general Definition of EVAs.

Definition 1.4 (Environmental Value Adjustment)**.** *An Environmental Value Adjustment (EVA) model is an xVA model that adjusts the value of individual assets by incorporating a given environmental aspect. The general form of an EVA of an asset is*

$$\text{Adjusted value} = \text{Financial value} - \text{EVA}, \tag{1.2}$$

where the financial value of an asset is computed without incorporating the given environmental aspect, and 'adjusted' refers to the value of the asset if the given environmental aspect is incorporated into the financial valuation process.

Equation (1.2) is the environmental variant of Equation (1.1), considering a given environmental aspect instead of CCR. We will generally formulate EVAs as an integral over the lifetime of an asset where we multiply the exposure by a so-called Environmental Impact Factor (EIF) for a given considered environmental aspect. The EIF represents the currently external environmental costs associated with an asset, relative to the financial exposure. An EVA therefore represents the total environmental costs of the given asset, which are internalised by introducing the EVA as a value adjustment. In Section 4.3, we formalise the definition of a general EVA model and a corresponding EIF.

When we introduce specific EVA models, we will explore the following research questions based on the purposes of general value adjustments that we listed in Section 1.1.

Research question 1. *How can EVAs be formulated and applied to incorporate environmental aspects into the asset valuation process?*

For an xVA to represent an adjustment to the fair value of an asset, all parties involved in a trade should agree on the assumptions that result in an environmental adjustment to the fair value of an asset. This is not trivial when incorporating environmental aspects into asset valuation because climate change is an externality, so the answer to Research Question 1 is highly dependent on the underlying assumptions. We will also demonstrate that the formulation of an EVA in terms of an EIF allows seamless integration in existing xVA frameworks.

Research question 2. *How can EVAs be used in risk management by a financial institution?*

We introduce EVA models that can be used by a financial institution to manage given aspects of climate risk or their climate impact (see Figure 1.1). Generally, xVA models themselves can in turn also induce various risks to the (adjusted) value on the books of financial institutions, particularly if they rely on simplifying assumptions. xVAs can also be sensitive to changes in the underlying prices. We explore these topics in the context of EVAs, particularly focussing on wrong-way risk and hedging of EVAs.

Research question 3. *How can EVAs be applied by regulators to internalise environmental aspects across the financial sector?*

Even though the perspective of authorities and governing bodies is not the primary focus of this thesis, we will explore the role that they can play in establishing EVA models and their underlying assumptions. We will take an example from existing regulations related to xVAs.

The structure of this thesis is shown in Figure 1.3. In Chapter 2, we explore the different topics that we will apply to define the different EVA models in this thesis. The three different topics are indicated by different colours in both Figures 1.3 and 1.1 and roughly speaking create three different storylines throughout this thesis.

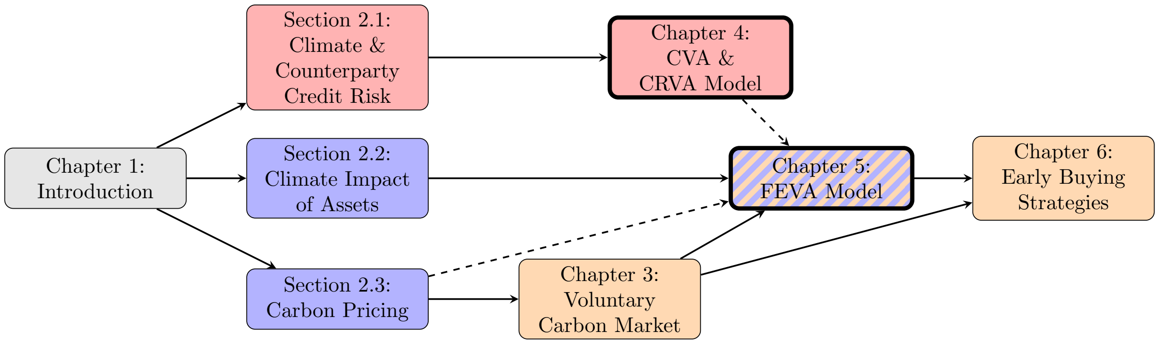


Figure 1.3: *Sketch of the structure of this thesis. In Chapter 2, we further explore different aspects of the relation between climate change and the financial sector, as illustrated in Figure 1.1. This creates three different storylines throughout the thesis, indicated by the three different colours corresponding to Figure 1.1. The two EVA models that we define in this thesis are defined in the accentuated chapters.*

We start by exploring climate risk and the extent to which it is already internalised in the financial sector in Section 2.1. We find that additional climate risk assessments can be necessary to incorporate into regular credit risk frameworks, which we explore in Chapter 4. First, we define a mathematical formulation of a CVA framework in Section 4.1 and then introduce climate risk-related adjustments to the parameters and introduce an EVA based on the adjustment of the CVA parameters in Section 4.2. This EVA is called the Climate Risk Value Adjustment (CRVA) and quantifies the expected losses from climate-related risk factors that are not incorporated in regular credit risk frameworks. We further discuss the interpretation and purpose of the CRVA in Section 4.2, answering Research Questions 1, 2 and 3.

In Section 2.2, we explore various existing regulations that aim to internalise the climate impact of assets. Given their crucial role in climate change, we then focus on GHG emissions and how they are measured and reported, putting emphasis on the role that financed emissions play in the financial sector. In Section 2.3 and Chapter 3, we explore various perspectives on carbon pricing. By associating a price with GHG emissions, we construct an EVA model in Chapter 5 that represents the expected total costs of the financed emissions associated with an asset. This Financed Emissions Value Adjustment (FEVA) model is based on the PCAF Standard for financed emissions [165]. We then discuss the application and interpretation of this EVA model in light of Research Questions 1, 2 and 3. We also propose under what assumptions the same underlying principles that we use can also be applied to other aspects of environmental impact than GHG emissions[4] and how this can lead to EVA models for other environmental aspects, of which the application and interpretation are analogous to the FEVA model.

Lastly, we explore how various aspects of the voluntary carbon market can be applied to minimise the costs associated with offsetting (financed) emissions and thereby achieving net zero.

Research question 4. *How can the various characteristics of the voluntary carbon market be applied to minimise the costs of offsetting emissions?*

In particular, we investigate the facts that (1) the voluntary carbon market consists of different unique types of credits, and (2) that credits do not have to be used immediately when being bought, meaning that proactive buying strategies can be explored to minimise costs.

In Chapter 3, we introduce voluntary carbon credits and under what conditions they are a viable instrument for corporates, and financial institutions in particular, to offset their emissions and achieve net zero. If financial institutions are obliged to achieve net zero, or do so voluntarily, the price of voluntary carbon credits

[4] For example, damage to biodiversity, water usage, etc.

represents the costs associated with their (financed) emissions under these assumptions. With this perspective on carbon pricing, the FEVA that we define in Chapter 5 represents the expected costs of achieving net zero by offsetting the financed emissions associated with an asset using voluntary carbon credits. In Section 3.3, we investigate the impact on these costs of the possibility to choose between different unique types of carbon credits, analogously to the cheapest-to-deliver aspect that arises regularly in the context of bond futures or collateralisation [109, 222]. We find that a cheapest-to-deliver aspect leads to a significant decrease in the expected future price of carbon credits and therefore the FEVA, as demonstrated in Section 5.3.

In Chapter 6, we define a stochastic control problem to minimise the costs of offsetting financed emissions by allowing carbon credits to be bought before they are used instead of at the exact moment that they are used. By doing so, the costs of achieving can potentially be decreased compared to the FEVA that is defined in Chapter 5. We solve the control problem by introducing a dynamic programming approach based on Bellman's principle of optimality [28], making use of a least squares Monte Carlo approach to approximate the conditional expectations that arise in our proposed approach. We solve this control problem in a number of numerical experiments in Section 6.4 and show that this approach can indeed result in a decrease in costs compared to the FEVA. We conclude by discussing the implications of this control problem and the cheapest-to-deliver aspect on the application of the FEVA in practice, in light of Research Question 1.

CHAPTER 2

CLIMATE CHANGE AND THE FINANCIAL SECTOR

In this chapter, we further investigate the most relevant aspects of the relation between the financial sector and climate change, as illustrated in Figure 1.1. Climate change impacts the financial sector in several ways, of which we focus on Counterparty Credit Risk (CCR) in Section 2.1. In Section 2.2, we analyse the impact that the financial sector has on climate change through their investments and explore the current landscape on the topic of *green finance*, which is the field of financial instruments with a positive impact on the climate. We explore mechanisms that create financial incentives for climate-positive investments, focussing on GHG emissions and financed emissions in particular. Then, we investigate several perspectives on carbon pricing in Section 2.3, examining the effectiveness of current carbon pricing schemes in internalising climate change.

2.1 Climate change and counterparty credit risk

Although climate change primarily affects vulnerable regions and sectors [64], its effects spread throughout the economy, thereby posing a risk to the financial sector [169, 69]. Despite the emphasis on credit risk in the financial world since the credit crisis in 2007-2008 and the subsequent third Basel Accord that focusses on strengthening risk frameworks among other topics [18], there are concerns as to whether or not climate risk is adequately incorporated in current credit risk assessments [19]. For example, the European Central Bank reported that around 60% of European banks did not incorporate climate stress testing into their credit risk models [16]. These findings are in line with the growing concerns that financial markets must act more adequately in responding to and managing systemic risks in general. These concerns have grown in the wake of the credit crisis, which was caused by credit risk not being adequately incorporated into financial decision-making [169]. The credit crisis illustrates the dangers of not dealing adequately with an externality and only taking action when it is too late with improved global standards and regulations, which emphasises the urgency of internalising climate change.

For financial institutions, climate risk can have various negative consequences, including increasing value losses over their investments (market risk) or it can even lead to counterparties not being able to meet payment obligations (credit risk). Financial institutions can also be forced to adjust their strategies and policies based on regulatory pressure (operational risk) or societal pressure (reputational risk) [19, 8, 20, 193]. Given our focus on xVA models for OTC trades, for which credit risk is referred to as CCR, we limit our attention to the impact of climate change on CCR from now on. The impact of climate risk on existing xVA models can be directly quantified, as we demonstrate in Section 4.2. Even though climate-related market risk also impacts the value of assets, this can be complex to model given that climate change can induce financial losses across the entire value chain, see for example [69, 6].

2.1.1 Climate risk taxonomy

The financial risks that were mentioned at the start of this section arise from climate-related risk factors, of which the effects impact financial institutions through so-called *transmission channels*, which can either be microeconomic (e.g. impact on individual people, corporates, assets and their counterparties) or macroeconomic (impact on governmental policies, market prices and other macroeconomic variables) [203, 19, 100, 61]. We describe the climate risk taxonomy that is used throughout the financial world, illustrating how the different climate-related risk factors can impact CCR using examples. Climate risks can be divided into two categories, each consisting of a number of subtypes:

1. **Physical risk** is the risk of changes in weather and climate. These can lead to value losses and even default, but counterparties of financial institutions are generally more vulnerable to physical risk than financial institutions, as there is often limited direct impact on their offices, data centres and employees [169]. The following two subtypes are recognised:
   (a) **Acute** physical risk is the risk of acute extreme climate events, such as floods, wildfires, storms, heatwaves or extreme precipitation. These can abruptly disrupt operation of counterparties of a financial institutions' trades. For example, this can suddenly cause unexpected losses or liquidity problems to the counterparty, potentially resulting in default.
   (b) **Chronic** physical risk is caused by long-term shifts of environments, e.g. rising sea levels or average temperatures. These can also negatively impact corporate operations (particularly in the agricultural sector [19]), potentially causing them to default.
2. **Transition risk** is the risk of not adapting to the societal transition to a climate-friendly economy. The climate transition demands the most action from, and therefore poses the greatest risk to, high-emitting corporates, potentially even leading to financial default. Transition risk can therefore increase the CCR associated with trades with high-emitting counterparties. The following four drivers of transition risk are generally recognised:
   (a) **ESG regulations** can induce additional costs to be covered in the form of fees or fines. The risk of failing to comply with future ESG regulations such as carbon pricing schemes is generally associated with high-emitting sectors and corporates [51]. Despite the focus on regulations related to reducing GHG emissions since the Paris Agreement in 2015 [170], other regulations such as required ESG reporting can also induce transition risk.
   (b) **Technological advancements** can play an important role in climate change mitigation, particularly on the topic of energy efficiency and sustainability. This leads to transition risk for corporates whose business models rely on technologies that are expected to be superseded by more climate-friendly technologies.
   (c) **Market shifts** can also result from the climate transition. Shifts can occur on both the demand side (e.g. changing customer preferences) or the supply side (e.g. increased production costs) of the market for goods, services and commodities. This leads to transition risk for parties whose competitiveness is negatively affected by potential climate-related market shifts.
   (d) **Reputational risk** can also arise for corporates as a result of the climate transition. Consumer and investor sentiment might shift as society becomes more aware of the impacts of climate change and the need or action, resulting in potential reputational damage to those who fail to adapt.

   It should be noted that these subcategories of transition risk can apply and result in financial risks to corporates across the entire economy, including the financial sector. However, we limit our attention in the remainder of this thesis to the impact on default risk of counterparties of a financial institution's trades to assess the impact of the climate transition on value adjustments, as discussed earlier.

2.1.2 Modelling climate risk factors

Historical data alone are considered insufficient to model climate change, since the complex interactions between climate risk factors and how they result in financial risk are likely not yet fully understood given the unprecedented nature of climate risk [76, 18]. Particularly over longer timescales, there are large uncertainties in (1) the future path of both climate change and the global economy and (2) the dynamics of the interaction between different environmental and economical aspects, e.g. the impact of future GHG emissions on future global warming, the impact of global warming on the frequency and severity of extreme events and geopolitical and macroeconomic responses to future changes in the climate [22]. The fact that these interactions are not yet well understood on longer timescales creates an additional layer of uncertainty in climate risk assessments [21], making climate risk a systemic risk to the financial sector. Without adequate assessments, this systemic risk can be underestimated [19, 24, 66, 8].

Due to the large uncertainties in the interaction between different variables in climate risk forecasting, modelling their financial consequences is generally done for a fixed set of scenarios with given climate variable paths [52]. These scenarios are modelled using so-called integrated assessment models that attempt to incorporate environmental, societal and economic features into a single model [217]. Given the large uncertainties on the topic of climate change, financial institutions formed the Network for Greening the Financial System (NGFS) in 2017 to centralise the incorporation of climate change into the financial sector and developed the so-called *NGFS scenarios* in 2020 as a standardised climate risk framework [100]. These scenarios are developed from a number of integrated assessment models and provide a common basis for climate risk assessments and disclosure across the financial sector [19, 151]. The NGFS scenarios are divided into four categories as illustrated in Figure 2.1 and demonstrate a trade-off between physical and transition risks, depending on the pace and extent of global cooperation to mitigate climate change [100]. If the climate transition does not continue, the degree of physical risk will increase over time. On the other hand, embracing the climate goals of the Paris Agreement will limit physical risk but can only take place if the economy adapts to the emerging climate change mitigation strategies, inducing transition risk. The climate transition should take place in an orderly manner to limit transition risk, which emphasises the need for global cooperation [158]. The NGFS scenarios are particularly useful for scenario analysis and stress testing in climate risk assessments [100, 69, 6]. In Section 4.2, we will demonstrate how climate risk assessments can be incorporated into regular CCR frameworks, resulting in an adjustment to the financial value of an asset.

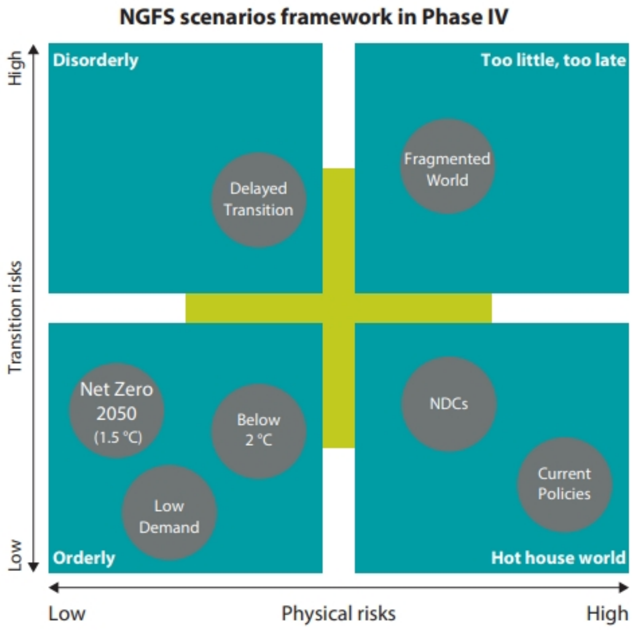


Figure 2.1: *The seven NGFS scenarios from Phase IV divided into four categories, based on the level of physical and transition risks per scenario [100].*

2.2 Climate impact of assets

In this section, we explore several existing regulations, initiatives and frameworks on the topic of climate impact. We start by exploring attempts to internalise broader environmental aspects through *green* (climate-friendly) finance and address the difficulties that arise when general environmental aspects are incorporated in financial decision-making. We provide our reasoning to focus on GHG emissions for the remainder of this thesis and give a detailed description of the measurement and reporting framework that is used across the industry. We conclude by introducing financed emissions, a specific subcategory of emissions that is particularly significant for financial institutions [165].

2.2.1 Green finance

As illustrated in Figure 1.2, the financial market receives funding through savers and lenders, and can choose how they impact the climate through their investments and also by altering market behaviour through financial incentives for sustainable products [121, 169]. Despite the importance of investing in climate change mitigation and positive impact on the climate, doing so is currently only directly financially incentivised through regulations and funding from willing investors who prioritise ESG performance over financial returns[1] and accept a lower return on their investments[2]. However, this does not mean that there is no market for ESG-linked funding: due to a growing demand for action on the topic from financial stakeholders and broader society, companies and financial institutions are investing more and more in bonds and loans that are *sustainability-linked* or labelled *'green'*. The label 'sustainability-linked' or 'ESG-linked' refers to general purpose loans and bonds of which the contractual terms are related to performance on some predetermined ESG key performance indicators of the borrower [143]. On the other hand, the label 'green' is used for bonds and loans that are based on the use of proceeds. For loans, it is more common to be ESG-linked than green, and for bonds it is the other way around [134, 86]. Until recently, a problem with the green label was that there was no centralised regulation or framework to determine whether or not a financed project deserves the green label. A common definition helps investors compare green products and helps borrowers to appropriately give their activities the green label, while the lack of a common definition results in the opportunity for greenwashing [163].

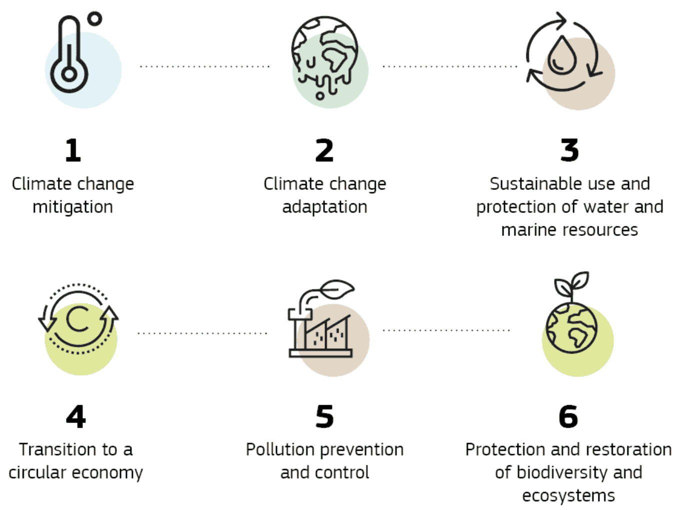


Figure 2.2: *The six environmental goals of the EU taxonomy [178].*

The EU Green Bond Standard attempts to centralise the definition of the label 'green' [179]. A financed

[1] As explained in this article and this article for example.

[2] However, other potential reasons for companies for investing in climate change mitigation include (1) mitigation of transition risk, (2) market competitiveness and reputation and (3) internal stakeholder pressure or goals.

activity can be labelled 'green' if it is aligned with the EU taxonomy, which is one of the main accelerators towards a climate-positive economy in the EU and was introduced in 2020 following the European Green Deal [178]. The taxonomy classifies an activity as green if it contributes to one of the six goals laid by the regulation and does no significant harm to the others (see Figure 2.2). The EU taxonomy is expanded on a regular basis, but as of now, the largest part of the EU taxonomy concerns climate change mitigation and adaption, which are considered the most urgent topics. Using the EU taxonomy, European regulations introduced a new key performance indicator that financial institutions must report on: the Green Asset Ration, which is the fraction of assets in its portfolio that are eligible for and aligned with the EU taxonomy [46]. Such centralised regulations make sustainability-linked and green products become more and more popular and therefore help to internalise climate change [86].

However, it is argued that a green label alone is not helpful to internalise climate change mitigation in the financial world and especially not to achieve net zero, as comparing the emissions of assets requires quantitative variables instead of a binary label [132]. Rather than internalising climate change with add-ons based on the green label or key performance indicators, Kenyon, Berrahoui and Macrina go as far as arguing that tracking CO2e flows is necessary to reach net zero [132]. They introduce the *Carbon Equivalence Principle* (CEP) which states that CO2e flows should be internalised by explicitly treating CO2e as a commodity, allowing market mechanisms to determine the costs of emissions and therefore the price of climate change mitigation. This principle achieves the goal of incorporating CO2e emissions in financial decision-making and therefore internalises climate change mitigation, but internalising other climate-related topics (e.g. the other goals of the EU Taxonomy) via a similar principle is less urgent and straightforward for the following three reasons.

Most importantly, CO2e emissions from human activities are the main contributor to climate change [64]. Moreover, other climate topics (e.g. biodiversity and local pollution) are more complex to quantify and compare across the economy. This is not the case for CO2e emissions because every tonne of CO2e has approximately the same impact on climate change, regardless of where the emissions take place [146]. Given the previous two reasons, CO2e emissions also receive the most attention in green finance and regulations. This leads to significantly more data available on CO2e emissions of companies than on other climate topics, which helps financial institutions and authorities in establishing strategies and measures on the topic [178, 73].

An important example of how data availability and comparability can be improved by regulations is the Corporate Sustainability Reporting Directive (CSRD) that the EU introduced in 2022, which requires large and listed companies (not necessarily from the financial world) to report annually in detail on ESG topics like CO2e emissions, sustainable activities and climate risks among other topics [73]. Although regulations like these do not directly incentivise green finance, they play a significant role in internalising climate change by improving transparency on the topic [116, 213].

2.2.2 Measuring and reporting CO2e emissions

For the three reasons that were mentioned in Subsection 2.2.1, we will focus on CO2e emissions when assessing climate impact in the remainder of this thesis. In the remainder of this section, we will further explore the definition of CO2e and the global standard on measuring and disclosing emissions in order to understand how they can be economically internalised.

To quantify emissions, GHGs are converted to CO2 equivalent (CO2e) using their global warming potential, which is the most popular emission metric and is periodically revised. It represents the impact on global warming of a tonne of the specific GHG compared to the impact of a tonne of CO2 over a given period. Table 2.1 shows the global warming potential for a selection of GHGs over a period of 100 years, which is a metric that is commonly used to convert GHG emissions to CO2e [91].

Putting emphasis on the necessity of global cooperation is crucial for adequate climate change mitigation [158], banks formed the Net Zero Banking Alliance (NZBA) in 2021 as a step towards the goals of the

Table 2.1: *The Global Warming Potential (GWP) of a selection of GHGs over a 100-year period [91, 60]. There is a distinction between fossil and non-fossil methane (CH_4).*

GHG	CO_2	CH_4 (fossil)	N_2O	HFC-134a	HFC-125	PFC-14	PFC-116	SF_6	NF_3
GWP-100	1.00	27.0 (29.8)	273	1530	3740	7380	12400	24300	17400

Paris agreement. This initiative was formed[3] as a group of leading global banks that currently cover 41% of global banking assets [133], with the aim of achieving net zero by 2050 [171]. A standardised global carbon accounting framework is crucial in achieving net zero, and this is exactly what the *Greenhouse Gas Protocol* is. The GHG Protocol provides a framework that describes how a company should measure and disclose the CO2e emissions it causes, making a distinction between different scopes of emissions [223], as illustrated in Figure 2.3:

- **Scope 1** emissions (also called *direct emissions*) of a company come directly from their properties, e.g. in the manufacturing process. The sectors with on average the highest scope 1 emissions include utilities, materials and energy [95].

- **Scope 2** emissions of a company are the emissions generated by producing the electricity, steam, heating and cooling that they use. Scope 2 emissions are called *indirect emissions* because they occur at the facility where the energy is produced and not directly at the facility where the energy is used.

- **Scope 3** emissions of a company are the emissions across their value chain and are also called indirect emissions. These can come in various shapes, depending on the sector and product/service that a company sells. The GHG Protocol defines 15 categories of indirect emissions that contribute to scope 3. The categories are shown in Figure 2.3. For financial institutions, the emissions from scopes 1 and 2 are generally limited and the main source of emissions are their investments, which is category 15 of scope 3 [169].

- **Avoided** emissions[4] are so-called *negative emissions* and should be reported separately from direct and indirect emissions. If a company takes action to reduce or reduce the total emissions across its value chain compared to a scenario in which this action is not taken (which is called the *baseline scenario*), the resulting difference in emissions is called avoided emissions if the net difference is positive, i.e.

$$\text{Avoided emissions} = (\text{Baseline emissions} - \text{Actual emissions})^+. \tag{2.1}$$

  The avoided emissions directly depend on the assumed baseline scenario, which leaves room for uncertainty, interpretability and controversy on the topic, particularly if data collection is an obstacle in establishing baseline scenarios [147, 164]. Consistent, complete and transparent standards are required to make accounting of avoided emissions credible and as objective as possible [185].

- **Removals** are another type of negative emissions and should also be reported separately from the other emission scopes. Emission removals come from projects that sequester CO2e from the atmosphere, either using nature-based or technological methods. Emission removals generally do not occur within the value chain of most companies, but financial institutions can contribute to emission removals through their investments [165].

Remark 2.1 (Double counting of emissions)**.** *It should be noted that the standard from the GHG Protocol intentionally results in double counting. For example, electricity that a company uses generates emissions that fall under their scope 2, but the same emissions fall under scope 1 of the company that generated the electricity. Similarly, if a financial institution funds a company, all three scopes of the company's emissions contribute to the scope 3 emissions of the financial institution. Double counting can particularly arise when financial institutions invest in multiple parties inside a value chain. In this case, double counting can be minimised by detailed, consistent and transparent accounting across the entire value chain [165].*

[3] At the moment of writing, the Net Zero Banking Alliance is facing significant challenges as major US and EU banks reconsider their membership and climate ambitions following Trump's return to the White House and withdrawal from the Paris agreement.

[4] Avoided emissions are sometimes referred to as Scope 4 emissions.

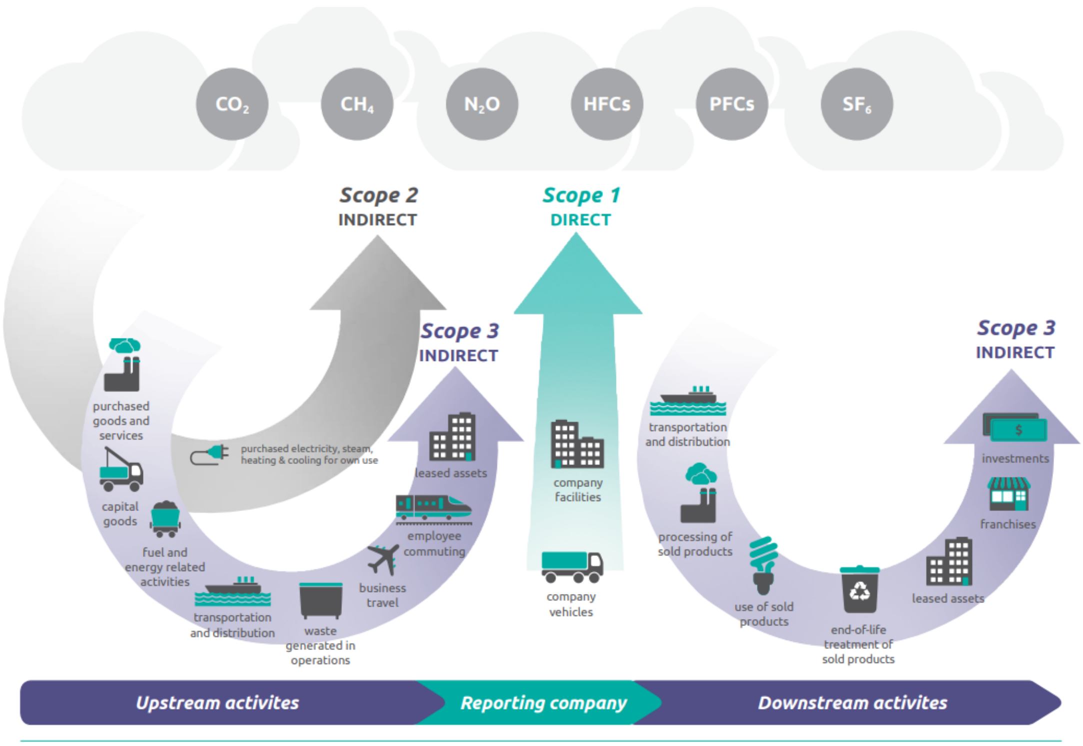


Figure 2.3: *Overview of the sources and scopes of CO2e emissions according to the GHG protocol [99].*

Investments are the most relevant source of indirect emissions for financial institutions and are called *financed emissions*. The Partnership for Carbon Accounting Financials (PCAF) is a global partnership of financial institutions that has developed a detailed framework specifically for assessing and disclosing financed emissions that is consistent with the GHG protocol [165], which is instrumental in adequate assessments of the climate impact from investments. The PCAF standard is based on the idea that an investor contributes to the same fraction of its counterparty's emissions as it contributes to their total value[5], which can be formulated as

$$\text{Financed emissions} = \frac{\text{Value of investment}}{\text{Total counterparty value}} \cdot \text{Counterparty emissions.}$$

This will be explicitly formulated in Chapter 5, where we define an EVA model based on financed emissions. Although most CO2e accounting standards (including the PCAF Standard) focus on *absolute emissions* (scope 1, 2 and 3 combined), we will also describe how negative emissions can be incorporated into the financed emissions framework.

2.3 Carbon markets and carbon pricing

As we already explored in Chapter 1, carbon pricing is considered the most effective method to internalise CO2e emissions (see Definition 1.3) [158]. Before we apply the principle of carbon pricing to financed emissions in Chapter 5 to construct an EVA model to internalise them, we will explore various perspectives on carbon pricing that can be used for the EVA model in this section. The purpose of the model that we define in Chapter 5 determines what perspective on carbon pricing will be suitable to use.

The simplest carbon pricing mechanism is a *carbon tax*, where the basic idea is that regulators directly tax the emissions of covered companies, forcefully putting a price tag on climate change mitigation. In this case,

[5]The meaning of 'total value' differs per context (e.g. enterprise value including cash or total debt & equity), as we will explain in Chapter 5.1.

the regulator needs to determine what price level is necessary to achieve the desired emissions reduction [43]. There are currently 75 carbon pricing mechanisms in operation globally that cover roughly 20% of global emissions [17], most notably the Emission Trading System (ETS) in the EU that was already established in 2005 and surpassed in 2021 as largest carbon pricing scheme when China deployed its own emissions trading scheme [1]. The EU ETS is a so-called *cap and trade* system [74], which sets a cap on the total allowed amount of CO2e emissions inside its scope. Covered companies can then trade the right to emit, leading to a market price of CO2e [214]. In this sense, it does the exact opposite of a carbon tax, which puts a price tag on CO2e, leading to a reduction of the amount of emissions.

In a cap and trade system, the allowed emissions of covered entities are expressed in *allowances*: one allowance gives the right to emit one tonne CO2e, and companies must surrender enough allowances to account for their emissions, otherwise they are fined. Companies can buy allowances from the regulator and are generally free to trade them or carry them over to the next year. It should be noted however that often only emissions above a predetermined threshold are covered by a cap and trade system and only the most heavy-polluting sectors are covered, which is the case in the EU for example [74]. Depending on the sector, companies can also receive allowances for free based on the idea that not all emissions can be avoided. These measures are generally taken to gain political support and avoid that covered parties relocate to other jurisdictions[6], even though they compromise the environmental goals of the cap and trade system [43, 7]. Despite the free allocation of allowances and the limited number of covered sectors, the EU ETS covers roughly 40% of the emissions in the EU in total and auctioning allowances has generated the EU over 175 billion EUR. The EU ETS cap is also lowered annually, making it an effective emissions reduction mechanism [214].

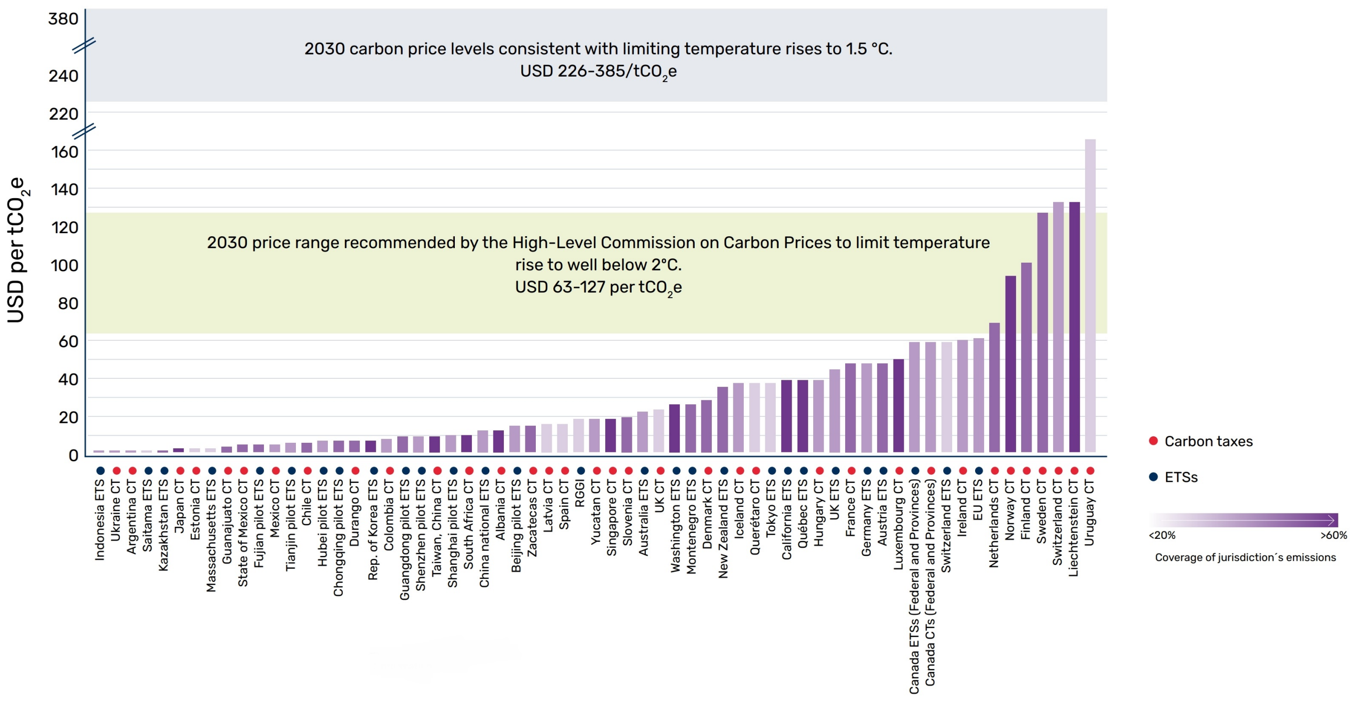


Figure 2.4: *Current carbon prices under different schemes, with the price ranges (in 2024 USD) indicated that are necessary to limit global warming to 1.5° and 2° [17]. Most carbon prices are currently not high enough to sufficiently cut CO2e emissions to reach global temperature goals.*

Both carbon tax and cap and trade systems induce additional costs on a national or sector level over CO2e emissions to internalise them, most often focussing on direct emissions. However, most current carbon mech-

[6]The importance of globally coordinated regulations comes up when assessing potential pitfalls of carbon pricing schemes, such as companies relocating to other jurisdictions.

anism prices (including the EU ETS, for example) are lower than the carbon price that the International Panel on Climate Change estimates to be necessary to limit global warming to 1.5° [17], as summarised in Figure 2.4. This concept is called the global *Shadow Price of Carbon* (SPC), which is the global uniform price required of one tonne of CO2e to achieve the objectives of the Paris agreement. The principle of shadow pricing can also be applied to achieve different environmental goals or to limit emissions within a jurisdiction. Calculated values of the global SPC can vary widely depending on the underlying method, model, assumptions, and discount factor, but are generally higher than the prices in Figure 2.4 [216]. Due to a lack of global pricing mechanisms, the actual global carbon price turns out to be at most a few EUR [158], meaning that the carbon prices in Figure 2.4 effectively stem from national regulations rather than internal motivation to cut CO2e emissions or from global regulations. The discrepancy between the global SPC and the market price of carbon indicates that climate change is indeed still a global externality and that global cooperation and regulations are necessary to achieve the goals of the Paris agreement.

Remark 2.2 (Carbon leakage)**.** *It should be noted that for a carbon pricing scheme to function, covered entities should not push their emissions beyond the border of the carbon pricing scheme, for example by relocating their facilities to different countries. This would be a form of carbon leakage and means that the carbon pricing scheme is not effective even though it reduced total emissions within its jurisdiction. This illustrates the need for global cooperation in dealing with externalities.*

Another perspective on carbon pricing is the *Social Cost of Carbon* (SCC), which corresponds to the economical costs of the additional damage that an additional tonne of emitted CO2e would cause. Similarly to the SPC, this price tag depends very much on the underlying method, model, assumptions, and discount factor. Controversially, only economic impact is valued directly and social impact is not. For example, reduced food security is included in the SCC because it can lead to lower productivity and higher food prices, and not because the health of vulnerable people is negatively affected [159, 215, 167]. Estimates of the SCC vary widely, but can be as high as 1000 USD [35]. Another popular measure to internalise climate change is using an *Internal Carbon Price* (ICP), which is done by a significant fraction of large companies around the world. Using an ICP means that a company voluntarily puts an internal virtual price tag on CO2e emissions that is taken into account in decision-making processes. The main motivation of an ICP is often strategic risk management and the level of this price tag is often driven by expectations about future regulations that are used in an internal shadow pricing process, often resulting in prices that are higher than the current carbon price of the regulations they fall under [208].

In Chapter 5, we define an EVA model based on the principle of carbon pricing. A credible and useful value adjustment model should not be constructed based on subjective assumptions and large uncertainties, since all involved parties in a trade should agree on the fair value that is determined from the xVA model. Subjectivity and uncertainty in an xVA model also prevent it from playing a significant role in risk management, or being useful for establishing regulations based on the xVA. For this reason, we proceed to focus on carbon pricing schemes in the remainder of this thesis, i.e. cap and trade systems or carbon taxes, and we do not focus on the carbon pricing perspectives of shadow pricing, the social cost of carbon and internal carbon pricing. Before we proceed to define EVA models however, we explore the voluntary carbon market in Chapter 3 as a different perspective on carbon pricing.

CHAPTER 3

VOLUNTARY CARBON MARKET (VCM)

In this chapter, we introduce the Voluntary Carbon Market (VCM), an emerging market where negative emissions are traded in the form of *voluntary carbon credits*. This is a fundamentally different perspective on carbon pricing than the ones that were introduced in Section 2.3. We introduce carbon credits in Section 3.1 and provide arguments for participation in the VCM, as well as the issues in the current market environment. A more complete overview of the supply side of the VCM is given in Appendix C. We also describe the issues that are currently present in the VCM and outline the steps that are believed to be necessary for the VCM to mature.

In Section 3.2, we propose a model for the matured VCM, consisting of a small number of different credit types, each modelled using a stochastic process. In Section 3.3, we describe that a cheapest-to-deliver aspect naturally arises when we consider our proposed VCM model. We then demonstrate that this cheapest-to-deliver aspect will generally lead to a significant decrease in the expected price of carbon credits, meaning that it can not be ignored when we explore how the VCM can be used to offset CO2e emissions and define EVA models in this thesis. We will apply this market model in Chapter 5, where we define an EVA model to internalise financed emissions by associating the cost of voluntary carbon credits with them, motivated by the fact that financed emissions can be offset by buying a high-quality carbon credit. In Chapter 6, we define a stochastic control problem to minimise the costs of compensating financed emissions with voluntary carbon credits by making use of the fact that carbon credits can be bought *before* they need to be retired.

3.1 Introducing the VCM

Voluntary carbon credits are digital certificates that represent the ownership of a tonne of negative emissions and are generated by voluntary projects. In line with the scopes of the GHG protocol that we explored in Subsection 2.2.2, we can differentiate between avoidance credits (generated by projects that reduce or avoid CO2e emissions, e.g. renewable energy or avoided deforestation projects) and removal credits (generated by projects that remove CO2e from the atmosphere, e.g. forestry or direct air capture projects). We will use *abatement* as an umbrella term for negative emissions in this thesis, i.e. emissions avoidance/reduction and removal combined. Companies can contribute to CO2e abatement by buying credits from voluntary CO2e abatement projects via a so-called *carbon registry*, thereby financing the abatement project. However, a bought credit can only be counted as contributing to CO2e abatement once it is *retired*, after which point the credit can not be sold on. Once the ownership of the credit is definitive and the credit can no longer be traded, it can be counted as a committed investment in the corresponding CO2e abatement project.

The voluntary carbon market differs from compliance markets such as cap and trade systems because negative emissions are traded [189], making it a unique carbon pricing mechanism. Unlike allowances from a cap and trade system, different voluntary carbon credits can not be treated as identical instruments since every credit-generating project has its own unique characteristics. More details on the unique characteristics and issues associated with individual projects are given in Appendix C. Some voluntary CO2e abatement projects are perceived as more likely to achieve the promised negative emissions than others, which we will refer to as a difference in *quality* of the generated credits, which is also reflected in market price differences between lower- and higher-quality credits. This is fundamentally different from a cap and trade system, where each allowance represents the right to emit one tonne of CO2e emissions and can therefore be treated as identical.

3.1.1 Participating in the VCM

Another difference between compliance markets and the VCM is that participation in the VCM is not mandatory. However, the demand for voluntary carbon credits comes from companies that participate in the market for various reasons, as illustrated in Figure 3.1. Most market participants use carbon credits as a part of their own carbon strategy, meaning that they buy carbon credits as a cost-efficient method to work towards their voluntary emission reduction goals. Achieving net zero, despite being an ambitious target, is the most popular voluntary emissions reduction goal among VCM participants. Other common reasons to buy carbon credits are to uphold and embody their company values and by supporting sustainable development and/or local communities, which often comes from a sense of responsibility of the company. Lastly, some companies buy carbon credits as a branding tool to try to stand out between competitors, but also create more awareness and engage customers in positive change. Surprisingly, hardly any of the companies surveyed specified that they use carbon credits as a tool to mitigate reputational risk. A possible explanation is that carbon credits can carry significant reputational risk in the current intransparent market environment if they are not of high quality [142, 59, 2].

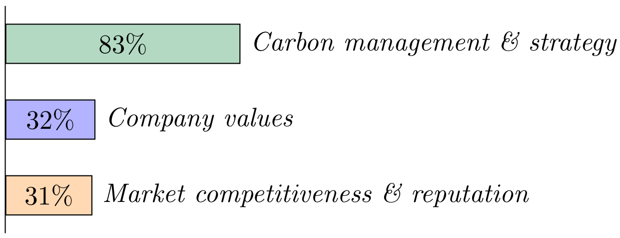


Figure 3.1: *Chart showing the percentage of VCM participants that buy carbon credits for the given reasons, based on a survey among 186 companies [142].*

It should be noted that there are debates about whether or not negative emissions and particularly emission removals are a valid method of climate change mitigation, as opponents suggest that more effort should be put towards the mitigation of absolute emissions instead of negative emission projects [10, 212]. Relying on investments in negative emissions through voluntary carbon credits can be perceived as negligent or even as greenwashing, particularly if low-quality credits are used [199, 12]. Voluntary carbon credits can also not be used to directly offset absolute emissions under CO2e accounting and reporting standards such the GHG Protocol, the PCAF Standard and the CSRD [223, 165, 73]. Instead, voluntary carbon credits and other climate change mitigation activities have to be reported separately. Voluntary carbon credits can generally also not be used to offset emissions under regulated carbon pricing schemes, although there are some small exceptions such as Singapore's carbon tax system[1], for example. Another notable carbon market is the Carbon Offsetting and Reduction Scheme for International Aviation (CORSIA) that the International Civil Aviation Industry introduced in 2021. This offsetting scheme is unique because it is the first scheme that resembles a global carbon pricing measure and it concerns an emissions-heavy sector [79]. Until 2026, governments can decide to participate voluntarily, in which case airlines from those countries are required to offset their CO2e emissions that exceed a baseline level using carbon credits that meet a certain quality

[1] See this article about Singapore's carbon pricing scheme.

standard[2]. This means that CORSIA is the only regulated carbon pricing scheme that is primarily focused on voluntary carbon credits.

In Appendix C, we give more details on the supply side of the VCM, including an outline of the general lifecycle of carbon credits, a categorisation of different types of credits and a taxonomy of risk factors that can negatively affect the effectiveness of CO2e abatement projects. This Appendix gives a good overview for readers who are interested in more details on the VCM, but it suffices for now to emphasise that each carbon credit-generating project has its own unique characteristics, including the described risk factors that result in large quality differences between individual carbon credits. The lack of contribution to climate change mitigation of the majority of carbon credits leads to the possibility of greenwashing and is one of the primary reasons for the current poor reputation of the VCM [199, 25]. In the next subsection, we outline the steps that are believed to be necessary to solve these problems and turn the VCM into a reliable instrument for companies to mitigate climate change.

3.1.2 Problems in the current market environment

The VCM is considered necessary to achieve net zero, as relying on companies to abate all emissions inside their value chain is not feasible. Most companies already include voluntary carbon credits in their net zero strategies, but the quality issues surrounding credit-generating projects have prevented them from contributing significantly to climate change mitigation [206, 212]. The generally low prices of carbon credits[3] are considered to reflect the inefficiency of the market that prevents the development of high-quality projects [25]. The inefficiency of the VCM stems from its decentralised and unregulated nature and a current lack of transparency, as most trades occur OTC. The lack of transparency makes the VCM a 'lemon market', i.e. a market with information asymmetry, affecting the perceived quality of carbon credits and the market as a whole [71].

To increase the efficiency and integrity of the VCM, it is considered crucial to introduce universal quality standards across all carbon registries to combat the difficulties in quality assessments across different carbon registries and to standardise the monitoring, reporting and verification processes. Quality assessments from independent parties are also considered crucial and should be incentivised by policy makers. High quality standards will solve the quality issues in the VCM, provided that they are universal and monitored by third parties. All of this should be accommodated by technological improvements that improve the efficiency of CO2e abatement projects or the monitoring, reporting and verification processes [197]. Universal standards also allow companies to report on the use of carbon credits in a consistent way, facilitating more transparency and combating greenwashing concerns. These transparency measures will lead to a higher demand for high-quality credits, improving the effectiveness of the VCM in climate change mitigation [189, 25].

Another problem with the current market infrastructure is the OTC nature of most marketplaces, leading to opacity and counteracting price discovery processes. A centralised marketplace is deemed crucial for the VCM to mature, as its opacity currently even dissuades companies from participating in the market in the first place [39]. Facilitated by unified quality standards, standardised reference contracts are likely to emerge in a more transparent market. Even though it causes credits to lose their unique project-specific properties, standardisation (also referred to as commoditisation) of carbon credits is seen as necessary to scale up the VCM [55, 189, 39]. Secondary markets should also be established to further improve liquidity of the VCM and lower the volatility. Spot and futures reference contracts already exist, but the lack of standardisation of carbon credits leads to a lack of demand [189, 198]. Appropriate integration in regulated carbon pricing schemes (as seen in CORSIA and Singapore's carbon tax system for example) can also improve the liquidity and maturity of the VCM, but should be accompanied by high quality standards to maintain their effectiveness in climate change mitigation and avoid greenwashing accusations [189, 25].

[2]From the CORSIA website.

[3]The volume-weighted average price of carbon credits has been reverting around an average price of below 10 USD, as from MSCI's Volume Weighted Carbon Credit Price Index.

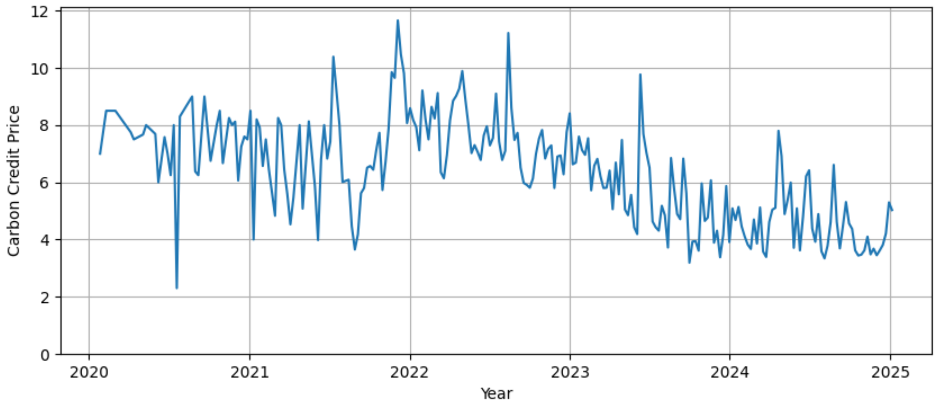


Figure 3.2: *The Volume Weighted Carbon Credit Price Index of MSCI over the period 2020-2024. It can be observed that carbon credit prices have been reverting around an average value of less than 10 USD.*

3.2 Modelling the VCM

There is a large supply in the VCM of low-quality credits that do not make the promised contribution to climate change mitigation, which is one of the primary reasons for the current poor reputation of the VCM and results in arguably low average prices [199, 25], as shown in Figure 3.2. For carbon credits to be considered valid and additional, it is believed that their prices should be between 20 USD and 50 USD, which is significantly higher than the current average market price of below 10 USD [25]. Because of the wide discrepancy in quality between different credits, accurate and transparent quality assessments are instrumental in the current market environment for price discovery to take place, but the opacity in the market and particularly the lack of independent quality assessments leads developers to charge arbitrary prices in the absence of a centrally regulated marketplace, reducing trust in the market from the demand side [39, 25]. We conclude that the VCM is currently not sufficiently mature to be modelled mathematically in the absence of liquidity, transparency and therefore the lack of well-functioning supply and demand dynamics. We propose that the best attempt at modelling the current market revolves around modelling the perceived quality of individual credits relative to some benchmark credit. Independent quality assessments can be incorporated in modelling the perceived quality. However, we will focus on modelling the VCM assuming a more transparent and liquid market than is currently the case, resulting in a more efficient and mature market.

From now on, we will assume that due to the emergence of unified quality standards and standardised reference contracts in the future, the VCM will eventually come to resemble other commodity markets. Even though the commoditisation of credits will result in them losing some project-unique characteristics, this change is considered necessary for more liquidity and efficiency in the market. This means that pricing will follow the law of supply and demand [39, 189]. In this matured market environment, carbon credit prices will behave similarly to other commodities, even though they do not represent a physical product in contrast to other commodities. Commodity spot prices typically tend to revert around an equilibrium price that is governed by the level of supply and demand [101], as can also be observed in Figure 3.2.

Unlike most other commodities, carbon credits do not carry storage costs, which means that their price is not influenced by restrictions on the storage or transmission of the commodity [144]. This also means that the price of carbon credits will never become negative. These observations also apply to allowances in cap and trade systems, of which prices tend to display mean-reverting behaviour for these reasons [89, 32] and follow a positively skewed distribution [187]. Although little research has been done on modelling the prices of individual voluntary carbon credits to the best of our knowledge, we hypothesise that these characteristics are also applicable to a matured and commoditised VCM because of the observed similarities.

Based on these observations, we choose to model the price of voluntary carbon credits with an exponential Ornstein-Uhlenbeck (OU) process, also known as a Schwartz one factor model [192, 96]. This model results in a mean-reverting and nonnegative price process with positive skewness.

To describe the matured and commoditised VCM, we consider a flexible market model consisting of $d \in \mathbb{N}$ different categories of commoditised credits, meaning that all credits in a given category are perceived as equivalent. For example, this could be a highly standardised market of dimension $d = 2$, consisting of (1) nature-based credits and (2) technology-based credits. On the contrary, it could also be a more granular and therefore higher-dimensional market consisting of (1) African REDD+ projects, (2) South-American REDD+ projects, (3) North-American REDD+ projects, (4) European sustainable agriculture projects, etc. for example[4]. The granularity of the market depends on the balance between standardisation and attention to project-unique properties in the market.

3.2.1 Stochastic market model preliminaries

To model carbon credit prices and other stochastic processes in this thesis, we define a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and an appropriate finite time horizon $T \in \mathbb{R}_{++}$ to model their prices using stochastic processes. Here, $\Omega \neq \emptyset$ is called the sample space, $\mathcal{F}$ is a σ-algebra of subsets of Ω, and $\mathbb{P}$ is a probability measure. For a given stochastic process[5] $\{X(t)\}_{t \in [0,T]}$, we say that a filtration[6] $\{\mathcal{F}(t)\}_{t \in [0,T]}$ contains all publicly known information about $\{X(t)\}_{t \in [0,T]}$ if the stochastic process is adapted to the filtration, meaning that the random variable $X(t)$ is $\mathcal{F}(t)$-measurable for all $t \in [0,T]$. In the remainder of this thesis, we will continue to work with the filtration $\{\mathcal{F}(t)\}_{t \in [0,T]}$ that is generated by the stochastic processes that we consider, which means that the stochastic processes that we define are by construction adapted to the filtration $\{\mathcal{F}(t)\}_{t \in [0,T]}$. For simplicity, we will denote stochastic processes and filtrations by $X(t)$ and $\mathcal{F}(t)$ instead of using set notation from now on. For more technical details on probability spaces, filtrations and stochastic processes, see [194, 160].

The following two definitions are necessary for the stochastic VCM model that we propose:

Definition 3.1 (Wiener Process)**.** *A one-dimensional Wiener process (also called a Brownian motion) is an $\mathbb{R}$-valued stochastic process that satisfies the following properties [162]:*

1. $W(0) = 0$ *almost surely.*
2. $W(t)$ *is almost surely continuous.*
3. $W(t)$ *has independent increments: For* $0 \leqslant t_1 < t_2 < t_3 < t_4 \leqslant T$*, the increments* $W(t_4) - W(t_3)$ *and* $W(t_2) - W(t_1)$ *are independent.*
4. *The increments of* $W(t)$ *are normally distributed with*

$$W(t_2) - W(t_1) \sim \mathcal{N}(0, t_2 - t_1), \tag{3.1}$$

for $0 \leqslant t_1 < t_2 \leqslant T$.

Definition 3.2 (Correlation matrix)**.** *A d-dimensional correlation matrix ρ is a matrix $\rho \in \mathbb{R}^{d \times d}$ that satisfies the following properties [111]:*

1. *ρ is positive semidefinite, i.e.* $v^T \rho v \geqslant 0$ *for all* $v \in \mathbb{R}^d$*. This implies that ρ is symmetric.*
2. $\rho_{i,i} = 1$ *for all* $i \in [d]$.

Any matrix $\rho \in \mathbb{R}^{d \times d}$ that satisfies these properties has elements $\rho_{i,j} \in [-1, 1]$ for all $i, j \in [d]$.

[4] See Appendix C for an overview of the different types of voluntary carbon credits.

[5] A stochastic process is a collection of random variables on the sample space Ω that is indexed by a time variable $t \in [0,T]$.

[6] A filtration is a collection $\{\mathcal{F}(t)\}_{t \in [0,T]}$ of sub-σ-algebras of $\mathcal{F}$ that is indexed by a time variable, such that $\mathcal{F}(t) \subseteq \mathcal{F}(s)$ if $t \leqslant s$ for $t, s \in [0,T]$.

Given a d-dimensional correlation matrix ρ, we define $W(t) := (W_1(t), \ldots, W_d(t))^T$ to be a d-dimensional correlated Wiener process, which is defined as follows:

Definition 3.3 (Correlated Wiener process)**.** *A d-dimensional correlated Wiener process with correlation matrix ρ is an $\mathbb{R}^d$-valued stochastic process $W(t) := (W_1(t), \ldots, W_d(t))^T$ such that for all $i \in [d]$, $W_i(t)$ is a one-dimensional Wiener process, and the cross-variation of those processes is $\langle W_i, W_j \rangle(t) = \rho_{i,j} t$ for all $j \in [d]$ [162].*

A d-dimensional correlated Wiener process can be constructed from a given correlation matrix ρ and d independent one-dimensional Wiener processes $\widetilde{W}_i(t)$ (where $i \in [d]$) using the Cholesky decomposition of the correlation matrix ρ, which is given by a lower triangular matrix $L \in \mathbb{R}^{d \times d}$ such that

$$LL^T = \rho.$$

Any positive semidefinite matrix ρ admits a Cholesky decomposition, which is unique if ρ is positive definite [97]. For the Cholesky decomposition of a general correlation matrix, see [145]. The correlated Wiener process is constructed using the Cholesky decomposition by

$$W(t) := L\widetilde{W}(t), \tag{3.2}$$

where $\widetilde{W}(t) := (\widetilde{W}_1(t), \ldots, \widetilde{W}_d(t))^T$.

3.2.2 Correlated exponential Ornstein-Uhlenbeck process

Given $d \in \mathbb{N}$ different categories of commoditised credits that form the VCM model, we define d correlated exponential OU processes to model the prices of all carbon credit categories. For $i \in [d]$, we define

$$C_i(t) = e^{X_i(t)}, \tag{3.3}$$

where $X_i(t)$ is an OU process that models the log-prices of carbon credit category i. $X_i(t)$ satisfies the following Stochastic Differential Equation (SDE):

$$\mathrm{d}X_i(t) = \theta_i(\mu_i - X_i(t))\,\mathrm{d}t + \sigma_i\,\mathrm{d}W_i(t), \tag{3.4}$$

with $X_i(t_0) := x_i \in \mathbb{R}$ for some $t_0 \in [0, T]$. The parameters $\mu_i \in \mathbb{R}$ and $\theta_i, \sigma_i \in \mathbb{R}_{++}$ are called the long-term mean, mean-reverting speed and volatility of the i'th log-price process, respectively. Here, $W(t) = (W_1(t), \ldots, W_d(t))^T$ is a d-dimensional correlated Wiener process with correlation matrix ρ with respect to the filtration generated by $W(t)$, meaning that the cross-variation between process i and j is given by

$$\langle W_i, W_j \rangle(t) = \rho_{i,j} t, \tag{3.5}$$

for all $j \in [d]$. With the initial value $X_i(t_0) = x_i$ for $t_0 \in [0, T]$ given, the solution of SDE (3.4) for $t \in [t_0, T]$ is given by [162]

$$\begin{aligned} X_i(t) &= e^{-\theta_i(t-t_0)} x_i + (1 - e^{-\theta_i(t-t_0)})\mu_i + \sigma_i \int_{t_0}^{t} e^{-\theta_i(t-t_0-s)}\,\mathrm{d}W_i(s) \\ &= e^{-\theta_i(t-t_0)} x_i + (1 - e^{-\theta_i(t-t_0)})\mu_i + \frac{\sigma_i}{\sqrt{2\theta_i}} W_i(1 - e^{-2\theta_i(t-t_0)}), \end{aligned} \tag{3.6}$$

meaning that for all $t \in [t_0, T]$, $X_i(t)$ is normally distributed conditional on $\mathcal{F}(t_0)$:

$$X_i(t) \sim \mathcal{N}\left(e^{-\theta_i(t-t_0)} x_i + (1 - e^{-\theta_i(t-t_0)})\mu_i\,,\ \frac{\sigma_i^2}{2\theta_i}(1 - e^{-2\theta_i(t-t_0)}) \right). \tag{3.7}$$

Moreover, the cross-variation between the i'th and j'th log-price process (for $i, j \in [d]$) is given by

$$\langle X_i, X_j \rangle(t) = \sigma_i \sigma_j \langle W_i, W_j \rangle(t) = \sigma_i \sigma_j \rho_{i,j} \tag{3.8}$$

and their correlation is given by

$$\text{corr}_{t_0}(X_i(t), X_j(t)) := \frac{\mathbb{C}\text{ov}_{t_0}(X_i(t), X_j(t))}{\sqrt{\mathbb{V}\text{ar}_{t_0}(X_i(t))\mathbb{V}\text{ar}_{t_0}(X_j(t))}} = \rho_{i,j}\frac{2\sqrt{\theta_i\theta_j}}{\theta_i+\theta_j}\frac{1-e^{-(\theta_i+\theta_j)(t-t_0)}}{\sqrt{(1-e^{-2\theta_i(t-t_0)})(1-e^{-2\theta_j(t-t_0)})}}, \quad (3.9)$$

which is constant in time iff $\theta_i = \theta_j$, otherwise it decreases in time [222]. Here, $\mathbb{C}\text{ov}_{t_0}(\cdot,\cdot)$ denotes the covariance conditional on $\mathcal{F}(t)$. In this case, the correlation is $\text{corr}_{t_0}(X_i(t), X_j(t)) = \rho_{i,j}$.

Since $X_i(t) = \log(C_i(t))$ is normally distributed for all $i \in [d]$ and $t \in [t_0, T]$, the carbon credit prices are lognormally distributed with the same parameters, which are shown in Equation (3.7). Since a random variable $Y \in L^2(\Omega)$ such that $Y \sim \text{Lognormal}(\mu', \sigma'^2)$ has mean and variance [162]

$$\mathbb{E}[Y] = e^{\mu'+\frac{\sigma'^2}{2}},$$

and

$$\mathbb{V}\text{ar}[Y] = (e^{\sigma'^2}-1)e^{2\mu'+\sigma'^2},$$

we have

$$\mathbb{E}_{t_0}[C_i(t)] = e^{e^{-\theta_i(t-t_0)}x_i+(1-e^{-\theta_i(t-t_0)})\mu_i+\frac{\sigma_i^2}{4\theta_i}(1-e^{-2\theta_i(t-t_0)})}, \quad (3.10)$$

and

$$\mathbb{V}\text{ar}_{t_0}[C_i(t)] = (e^{\frac{\sigma_i^2}{2\theta_i}(1-e^{-2\theta_i(t-t_0)})}-1)e^{2e^{-\theta_i(t-t_0)}x_i+2(1-e^{-\theta_i(t-t_0)})\mu_i+\frac{\sigma_i^2}{2\theta_i}(1-e^{-2\theta_i(t-t_0)})}, \quad (3.11)$$

where $0 \leqslant t_0 < t \leqslant T$.

3.3 Cheapest-To-Deliver (CTD) aspect

In the current market infrastructure, voluntary carbon credits can generally not be considered a valid instrument to offset CO2e emissions due to the general and project-specific quality issues associated with them, as well as the general issues with the market, as explained in more detail in Appendix C and Subsection 3.1.2 respectively. Not only does the majority of carbon credits actually not represent a tonne of CO2e abatement and are therefore not appropriate instruments for offsetting emissions, they can therefore also be perceived as greenwashing instruments, inducing reputational risk. However, in a matured VCM with d commoditised credit categories that are all of sufficiently high quality, these issues are no longer applicable. In this case, all categories of carbon credits can be used interchangeably to offset CO2e emissions, each credit representing precisely one tonne of CO2e abatement. The choice of interchangeable credit types to offset a tonne of CO2e emissions creates an optionality: the selection of the optimal credit type, which we refer to as a cheapest-to-deliver aspect:

Definition 3.4 (Cheapest-to-deliver). *The term Cheapest-To-Deliver (CTD) refers to the option to choose an instrument out of a predetermined set of interchangeable instruments that can be used to fulfil contractual requirements. With this optionality, the instrument with minimal costs will be chosen.*

The CTD aspect commonly appears in the context of bond futures. The seller of a future that is settled by physical delivery can typically choose which bond to deliver out of a pool of eligible bonds, based on a predetermined set of criteria. The seller can maximise profit by choosing the bond that maximises the profit from delivery [109, 110, 195]. The CTD aspect also appears when different collateral securities can be used to reduce counterparty credit risk (see Subsection 4.1.1). The costs of collateralisation must be taken into account in the valuation of an asset [124] and can be minimised by the seller by choosing the optimal collateral security [222, 110]. Minimising collateralisation costs maximises the profit of the asset seller, which can also be passed on to the buyer in the form of a lower funding value adjustment [9].

3.3.1 Relative impact of the CTD aspect

In the context of minimising collateralisation costs, the CTD aspect started to receive attention when interest rates decreased and profit margins deteriorated as a result, which made financial institutions look for additional sources of profitability which were considered less relevant beforehand [222, 34]. In the context of the d-dimensional VCM model that we proposed in Section 3.2, the significance of the optionality to choose between different credit types $i \in [d]$ when applying the VCM model in practice should be examined. The optionality can potentially reduce the costs of offsetting CO2e emissions, but a high-dimensional correlated VCM model can also complicate calculations, particularly in the context of Monte Carlo simulations.

In the context of the d-dimensional VCM model, a tonne of CO2e emissions can be offset at time $t \in [0, T]$ for the CTD price, which is defined as

$$H(t) := \min(C_1(t), \ldots, C_d(t)) = e^{\min(X_1(t), \ldots, X_d(t))}. \tag{3.12}$$

The CTD aspect can result in a significant price decrease compared to the average prices of individual instruments if they are modelled with volatile processes. A direct consequence of Jensen's inequality is that

$$\mathbb{E}_{t_0}[H(t)] \leqslant \min(\mathbb{E}_{t_0}[C_1(t)], \ldots, \mathbb{E}_{t_0}[C_d(t)]), \tag{3.13}$$

where $t_0 < t \leqslant T$. We examine the profitability of the CTD aspect by calculating the expected CTD price at time $T \in \mathbb{R}_{++}$ conditional on $\mathcal{F}_0$ using an exact Monte Carlo simulation of a d-dimensional exponential OU process using Equations (3.7) and (3.9), where the parameters $\theta_i := \theta \in \mathbb{R}_{++}$, $\mu_i := \mu \in \mathbb{R}$,$\sigma_i := \sigma \in \mathbb{R}_{++}$ and $x_i := x \in \mathbb{R}$ are identical for each process. Moreover, we assume that the correlation matrix ρ has a so-called AR(1) correlation structure [166] for simplicity, meaning that

$$\rho_{i,j} := {\rho_0}^{-|i-j|}, \tag{3.14}$$

for $i, j \in [d]$. The correlation matrix is therefore defined by a single parameter ρ_0.

We then compute the *relative CTD price* R_{CTD}, which we define by

$$R_{\text{CTD}} := \frac{\mathbb{E}_0[H(T)]}{\mathbb{E}_0[C_1(T)]}, \tag{3.15}$$

where $\mathbb{E}_0[C_1(T)]$ is given by Equation (3.10) and represents the price of a voluntary carbon credit without the CTD aspect. Because we consider parameters that are identical for all processes i, the first process can be chosen without loss of generality. We compute the relative CTD price for various values of the parameters d, ρ_0 and σ. The results are shown in Figures 3.3 and 3.4.

In Figures 3.3 and 3.4, it can be observed that the CTD aspects has a significant impact on the price of voluntary carbon credits, particularly in higher-dimensional markets, when there is a lower correlation between different processes or when the price processes are more volatile. Since the different types of carbon credits can be used interchangeably, the correlation between their price processes will generally be relatively high. Nevertheless, it can be observed in Figure 3.3 with $\rho_0 = 0.8$ that the CTD aspect is generally significant, even for a relatively small number of credit types d.

These observations justify our attention to the CTD aspect in our proposed VCM model, even if they possibly complicate calculations. We will use the CTD price in Equation (3.12) when we associate costs with assets of offsetting the associated financed emissions, as we will demonstrate in Chapter 5. The offsetting costs can be minimised by choosing the cheapest available carbon credit type, resulting in a lower charge in the form of a value adjustment. In Chapter 6, we will further explore how the costs of offsetting financed emissions can be lowered by defining a stochastic control problem that we solve using Monte Carlo simulations of the d-dimensional VCM model.

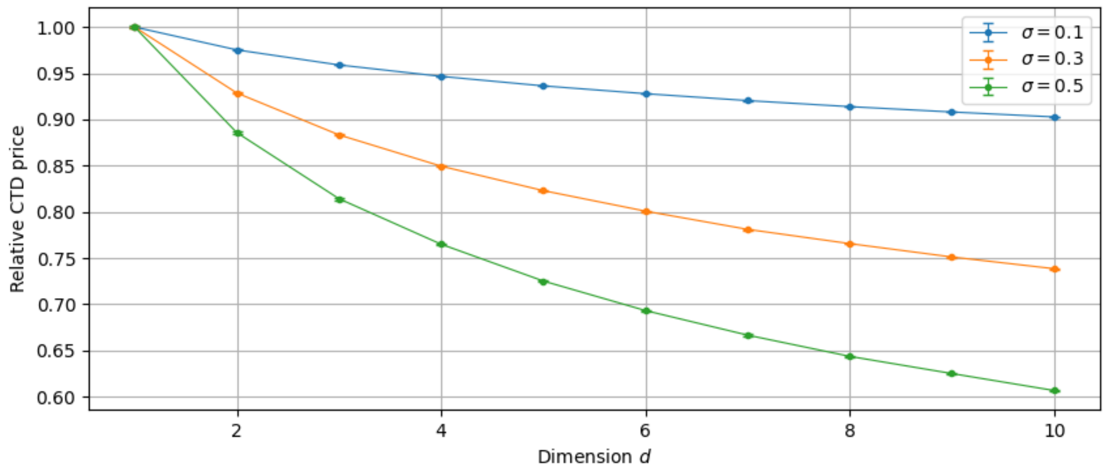


Figure 3.3: *The relative CTD price R_{CTD} decreases as the dimension d of the model increases and the volatility σ increases. The relative CTD price and its standard deviation are calculated from a Monte Carlo simulation with 10^5 samples of $H(T)$, with parameter configuration $T = 10$, $\mu = x = 0$, $\theta = 0.5$ and $\rho_0 - 0.8$.*

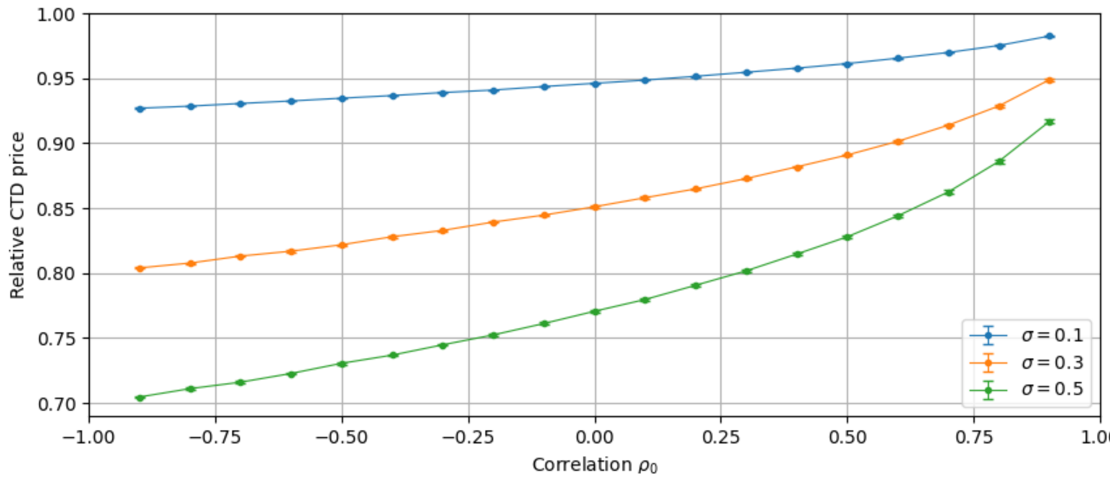


Figure 3.4: *The relative CTD price R_{CTD} increases as the correlation parameter ρ_0 of the model increases and the volatility σ decreases. The relative CTD price and its standard deviation are calculated from a Monte Carlo simulation with 10^5 samples of $H(T)$, with parameter configuration $d = 2$, $T = 10$, $\mu = x = 0$ and $\theta = 0.5$.*

CHAPTER 4

CLIMATE RISK EVA MODEL

In this chapter, we start by formulating a framework to incorporate Counterparty Credit Risk (CCR) into the asset valuation framework in Appendix A, resulting into a Credit Value Adjustment (CVA). The purpose of introducing the CVA model in this Section 4.2 is twofold: first of all, we illustrate how value adjustments can be introduced in asset valuation frameworks to incorporate a given externality. Moreover, we formulate how a climate risk-related adjustment to the parameters in the framework can be introduced to adjust the CVA in Section 4.2, based on the discussion in Subsection 2.1.2 that additional climate risk assessments can be necessary to incorporate in regular credit risk frameworks [19, 69]. We define this adjustment to the CVA to be the Climate Risk Value Adjustment (CRVA) and discuss its possible interpretations and purposes in Section 4.2.3.

With the CRVA being the first Environmental Value Adjustment (EVA) model that we introduce in this thesis, we introduce a more general formulation of EVA models in terms of an Environmental Impact Factor (EIF) in Section 4.3, and describe how this allows EVAs to be integrated into existing xVA frameworks seamlessly. With this formulation, we lay the foundation for the EVA models that we define in Chapter 5.

4.1 Credit Value Adjustment (CVA) framework

Classic risk-neutral asset pricing takes place by discounting future cash flows that are prescribed by the contract between its holder and the counterparty, using a (possibly stochastic) interest rate $r(t)$ [37]. In this thesis, we will assume that the interest rate $r(t) = r \in \mathbb{R}_{++}$ is constant for simplicity, since our main focus will not be on modelling interest rates.

Despite its simplicity, the assumption that the interest rate is constant generally does not lead to significantly less accurate prices [136], particularly for short-term and not too complicated instruments and under stable economic conditions. It also turns out that constant interest rates are able to describe prices modelled using stochastic interest rates as long as there is no correlation between the interest rate and the priced assets [220]. Stochastic interest rate processes $r(t)$ are necessary when pricing interest rate instruments or other more complex products, when assessing prices and risk over longer timescales or when modelling processes that are correlated with the interest rate. In cases where even more precision is required to price more complicated interest rate derivatives, short-rate models are not sufficient [221], for example because more flexibility is required to describe the term structure of yield curves with interest rate models. In this case, so-called Libor market models or standard market models can be used [115], but this is beyond the scope of this thesis.

The risk-neutral value of a general asset at time t is the total expected value of the prescribed cash flows discounted to time $t \in [0, T]$ and is denoted by $V(t, T)$, which can be calculated by

$$V(t,T) = \mathbb{E}_t \left[\sum_{\substack{\tau \in \mathcal{T} \\ \tau > t}} e^{-r(\tau - t)} \widetilde{C}_\tau \right] = \sum_{\substack{\tau \in \mathcal{T} \\ \tau > t}} \mathbb{E}_t \left[e^{-r(\tau - t)} \widetilde{C}_\tau \right], \tag{4.1}$$

as explained in more detail in Section A.1. Here, $\mathcal{T} \subset [0, T]$ is the set of moments of the prescribed cash flows, and $\widetilde{C}_\tau \in L^2(\Omega)$ denotes the value of the cash flow at time $\tau \in \mathcal{T}$. We will generally assume the time variables to be expressed in years, unless specified otherwise. For the remainder of this thesis, we assume that the price processes that are used to determine the value $V(t, T)$ are given, and $\mathcal{F}(t)$ is the filtration generated by these price processes, meaning that $V(t, T)$ is adapted to the filtration $\mathcal{F}(t)$.

4.1.1 Exposure, collateralisation and loss given default

Since the credit crisis in 2007-2008 and the subsequent third Basel Accord, it has become market practice to take CCR into account in asset management and pricing. The *exposure* of an asset is a widely used measure to quantify the potential losses in the event of a counterparty default and is mathematically defined as follows:

Definition 4.1 (Exposure, Expected Exposure)**.** *For an uncollateralised asset with maturity T we define the exposure to the counterparty at time t as*

$$E(t) := (V(t,T))^+, \tag{4.2}$$

where $V(t, T)$ is the value of the asset, assuming no CCR. The exposure $E(t)$ represents the maximum amount that the holder of an asset stands to lose if the counterparty defaults at time t. The exposure $E(t)$ is a stochastic process adapted to $\mathcal{F}(t)$. For $t_0 \in [0, T]$, the expected exposure discounted to time t_0 is defined as

$$\mathrm{EE}_{t_0}(t) := \mathbb{E}_{t_0} \left[e^{-r(t-t_0)} E(t), \right] \tag{4.3}$$

for $t \in [t_0, T]$ and is $\mathcal{F}(t_0)$-measurable.

Collateralisation is the most common method to mitigate CCR since the credit crisis and is often even mandated for trades between financial institutions [18, 135]. If collateral is posted by the counterparty of a contract, the holder of an asset will receive the collateral if the counterparty defaults, which means that the value of the asset $V(t, T)$ is (at least partially) recovered in the case of counterparty default [23, 101]. This can explicitly be included into the definition of the exposure (Equation (4.2)):

$$E(t) = (V(t,T) - \mathrm{col}(t))^+, \tag{4.4}$$

where $\mathrm{col}(t)$ denotes the value of the posted collateral at time t, as adapted from [227]. It immediately follows that for general well-collateralised assets (i.e. $\mathrm{col}(t) \approx V(t, T)$), the exposure is approximately zero. This completely mitigates CCR since the holder of an asset does not suffer losses in the case of counterparty default [23], meaning that classic risk-neutral pricing can take place with an appropriate discount rate [227]. Although collateral agreements often involve negotiated thresholds or minimum transfer amounts that lead to imperfect collateralisation and thereby result in some CCR [101, 54], we primarily focus on fully uncollateralised assets in our credit risk framework from now on, adapting Equation (4.2) as the definition of exposure. However, collateralisation can always be introduced by adapting Equation (4.4) as the definition of the exposure instead of Equation (4.2) since credit risk is expressed in terms of the exposure as we will demonstrate in the remainder of this Section.

Moreover, we do not consider more complex structures of potential collateral agreements and capital flows associated with assets that can arise in practice for simplicity. In doing so, we limit our attention to the impact that the possibility of counterparty default has on asset valuation. When taking the perspective of the holder of an asset, we also exclude the possibility of our own default. Including the possibility of self-default leads to the formulation of a Debit Value Adjustment (DVA), which is equivalent to the CVA from the perspective of the counterparty and is crucial to establish a fair price of the asset that the two parties in a trade agree on [162, 184]. When a party has to hold collateral or capital as a result of a trade, they can also charge the counterparty the costs that are associated with funding the collateral or capital, resulting in different types of xVAs [101, 184]. However, these considerations are beyond the scope of this thesis as we primarily focus on the CVA associated with an asset to illustrate how xVAs can be formulated and to incorporate climate risk in credit risk frameworks.

Remark 4.1 (Netting)**.** *Netting should also be mentioned for completeness, which is the practice of offsetting multiple trades between parties with each other to decrease the total exposure over the trades. This is another popular measure to mitigate CCR, but only some so-called homogeneous trades can legally be netted [101]. Mathematically, the exposure of two netted trades (with values $V_1(t,T)$ and $V_2(t,T)$ respectively) between two parties are netted becomes*

$$E_{\text{netting}}(t) = (V_1(t,T) + V_2(t,T))^+ \leqslant (V_1(t,T))^+ + (V_2(t,T))^+ = E_1(t) + E_2(t),$$

where $E_1(t)$ and $E_2(t)$ are the exposures of the individual assets [162], since $(x+y)^+ \leqslant x^+ + y^+$ for $x, y \in \mathbb{R}$. This illustrates that the exposure of assets can be decreased by netting them when applicable, in turn mitigating the associated CCR.

Even without any CCR mitigation practices applied, the value of an asset might be partially recovered by the counterparty and paid to the holder when default occurs, as illustrated by the following definition.

Definition 4.2 (Loss Given Default)**.** *The Loss Given Default (LGD) is a fraction in $[0,1]$ that represents the fraction of the uncollateralised value of an asset that can not be recovered in the case of a counterparty default [191].*

The LGD associated with an asset can depend on various factors, including but not limited to its value $V(t,T)$ and the seniority of the asset, which is the counterparty's priority of payment, meaning that an asset with higher seniority will have a lower LGD in the event that the counterparty defaults [191, 93]. In the remainder of this thesis, we assume a constant LGD for simplicity, as is market practice [126, 150].

4.1.2 Default processes and filtration switching

The asset valuation framework in Section A.1 assumes that the counterparty will always be able to pay the prescribed cash flows to the holder, but this assumption does not necessarily hold in practice, as illustrated by Definition 1.2 of CCR. We introduce the following random variable to model CCR.

Definition 4.3 (Default time)**.** *For a given counterparty of a contract, we define the positive random variable t_D to be the default time. The counterparty can not meet their contractual payment obligations after this moment.*

We define a generalisation of a Poisson process to model the default time t_D:

Definition 4.4 (Non-Homogeneous Poisson Process, Intensity Process)**.** *A Non-Homogeneous Poisson Process (NHPP) with intensity process $\lambda(t)$ is an integer-valued stochastic process $N(t)$ that can be constructed from a standard Poisson process[1] $\bar{N}(t)$ by*

$$N(t) = \bar{N}\left(\int_0^t \lambda(u)\,\mathrm{d}u\right), \tag{4.5}$$

for $t \in [0,T]$. Here, $\lambda : [0,T] \to \mathbb{R}_+$ is a nonnegative, right-continuous and piecewise continuous deterministic function, which we call the intensity process of the NHPP.

[1] A standard Poisson process is a Poisson process with a constant intensity of 1. The moment of the first jump of a standard Poisson process follows an exponential distribution with rate 1.

A nonnegative random variable τ is modelled by an NHPP $N(t)$ if it is defined as

$$\tau := \inf\{t \in [0,T] \,|\, N(t) > 0\}, \tag{4.6}$$

meaning that it is defined as the moment of the first jump of the NHPP [44]. If $N(T) = 0$, we say that $\tau = \infty$.

For the remainder of the thesis, we will model the default time t_D of the counterparty of an asset using an NHPP $N(t)$ with a deterministic intensity process $\lambda(t)$, which we refer to as the default rate. If we define $\Lambda : [0,T] \to \mathbb{R}_+ : t \mapsto e^{\int_0^t \lambda(u)\,\mathrm{d}u}$, we find from Equation (4.5) that for $t_D \in [0,T]$, we have

$$\text{The first jump of } N \text{ is at time } t_D \iff \text{The first jump of } \bar{N} \text{ is at time } \Lambda(t_D),$$

meaning that the random variable $\Lambda(t_D)$ then follows an exponential distribution with rate 1. Since the intensity process is nonnegative, the function $\Lambda(t)$ is non-decreasing. From the definition in Equation (4.6), it follows directly[2] for $t \in [0,T]$ that $\{t_D \leqslant t\} = \{\Lambda(t_D) \leqslant \Lambda(t)\}$ and therefore

$$\mathbb{P}_0[t_D \leqslant t] = \mathbb{P}_0[\Lambda(t_D) \leqslant \Lambda(t)] = 1 - e^{-\Lambda(t)},$$

so the density functions of the random variable t_D are given by

$$F_{t_D}(t) := \mathbb{P}_0[t_D \leqslant t] = 1 - e^{-\int_0^t \lambda(s)\,\mathrm{d}s}, \tag{4.7}$$

and

$$f_{t_D}(t) = \frac{\mathrm{d}}{\mathrm{d}t} F_{t_D}(t) = \lambda(t) e^{-\int_0^t \lambda(s)\,\mathrm{d}s}, \tag{4.8}$$

where $t \in [0,T]$. By construction, we have

$$\mathbb{P}_0[t_D = \infty] = \mathbb{P}_0(\bar{N}(\Lambda(T)) = 0) = e^{-\Lambda(T)} = e^{-\int_0^t \lambda(s)\,\mathrm{d}s}.$$

When taking conditional expectations at time t, it should also be taken into account whether counterparty default has already occurred ($t_D \leqslant t$) and if so, what the value of t_D is, which we refer to as the *default information* available at time t. The filtration $\mathcal{F}(t)$ generated by the price processes that we considered until now includes the default-free information[3], and the filtration $\{\sigma(N(s) \,|\, 0 \leqslant s \leqslant t)\}_{t \in [0,T]}$ contains the default information. We introduce the following filtration to include the default information available at time t into $\mathcal{F}(t)$.

Definition 4.5 (Enlarged filtration)**.** *In contrast to the filtration $\mathcal{F}(t)$ that does not include information on the default time t_D , we define the enlarged filtration $\{\mathcal{G}(t) \,|\, t \in [0,T]\}$ by*

$$\mathcal{G}(t) := \sigma(\mathcal{F}(t) \cup \sigma(N(s) \,|\, 0 \leqslant s \leqslant t)), \tag{4.9}$$

which is sometimes referred to as the σ-hull of $\mathcal{F}(t)$ and $\sigma(N(s) \,|\, 0 \leqslant s \leqslant t)$, and sometimes denoted by $\mathcal{F}(t) \vee \sigma(N(s) \,|\, 0 \leqslant s \leqslant t)$ [190, 123].

This means that the filtration $\mathcal{G}(t)$ is generated by the price processes and the default process $N(t)$. In practice, it can be more convenient to compute conditional expectations from the default-free filtration $\mathcal{F}(t)$ instead of the enlarged filtration $\mathcal{G}(t)$, even though $\mathcal{F}(t)$ does not necessarily include information on default events during $[0,t]$ [44]. The filtration switching formula allows easy switching between the conditional expectations $\mathbb{E}_t[\,\cdot\,] = \mathbb{E}[\,\cdot \,|\, \mathcal{F}(t)]$ and $\mathbb{E}[\,\cdot \,|\, \mathcal{G}(t)]$.

[2] This can be proven by contraposition: if $\Lambda(t) > \Lambda(t_D)$ for $t \neq t_D$, then $N(t) = \bar{N}(\Lambda(t)) \geqslant \bar{N}(\Lambda(t_D)) > 0$ follows from Equations (4.5) and (4.6) and the fact that a Poisson process is non-decreasing. Because t_D is by construction the smallest t that satisfies $N(t) > 0$, we have $t > t_D$.

[3] Technically, the filtration $\mathcal{F}(t)$ only contains no default information under the assumption of independence from $\sigma(N(s) \,|\, 0 \leqslant s \leqslant t)$, which will be assumed in Section 4.1.4. However, we will refer to it as default-free.

Theorem 4.1 (Filtration Switching Formula). *If $Y \in L^2(\Omega)$ is a square-integrable random variable, then [123]*

$$\mathbb{1}_{t_D>t}\mathbb{E}[Y|\mathcal{G}(t)] = \mathbb{1}_{t_D>t}\frac{\mathbb{E}_t[\mathbb{1}_{t_D>t}Y]}{\mathbb{P}_t[t_D>t]}, \tag{4.10}$$

for a given $t \in [0,T]$, assuming that $\mathbb{P}_t[t_D > t] \neq 0$.

The proof of Theorem 4.1 is given in Section B.2. Typically, we will compute conditional expectations when we consider cash flows that will occur at some time $\widetilde{T} \in (t,T]$, which will only take place when the counterparty does not default at time $t_D \in (t,\widetilde{T})$. In this case, we can formulate the cash flow as $Y = \mathbb{1}_{t_D \geqslant \widetilde{T}}\widetilde{Y}$ for some $\mathcal{F}(\widetilde{T})$-measurable random variable $\widetilde{Y}$. In this case, we obtain the version from [44] of the filtration switching formula that can be applied to compute the expectation of cash flows, which reads

$$\mathbb{E}[\mathbb{1}_{t_D\geqslant\widetilde{T}}\widetilde{Y}|\mathcal{G}(t)] = \mathbb{1}_{t_D>t}\frac{\mathbb{E}_t[\mathbb{1}_{t_D\geqslant\widetilde{T}}\widetilde{Y}]}{\mathbb{P}_t[t_D>t]}, \tag{4.11}$$

since $\mathbb{1}_{t_D>t}\mathbb{1}_{t_D\geqslant\widetilde{T}} = \mathbb{1}_{t_D\geqslant\widetilde{T}}$ and therefore

$$\mathbb{1}_{t_D>t}\mathbb{E}[\mathbb{1}_{t_D\geqslant\widetilde{T}}\widetilde{Y}|\mathcal{G}(t)] = \mathbb{E}[\mathbb{1}_{t_D>t}\mathbb{1}_{t_D\geqslant\widetilde{T}}\widetilde{Y}|\mathcal{G}(t)] = \mathbb{E}[\mathbb{1}_{t_D\geqslant\widetilde{T}}\widetilde{Y}|\mathcal{G}(t)].$$

The interpretation of Equation (4.11) is that expected cash flows $Y = \mathbb{1}_{t_D\geqslant\widetilde{T}}\widetilde{Y}$ should be computed at time t without incorporating any available information on possible counterparty default events during $[0,t]$, after which the conditional expectation given all available information can be computed from Equation (4.11).

Notice that since the default process is independent of the filtration $\mathcal{F}(t)$ by construction, it follows from the properties of the conditional expectation that $\mathbb{P}_t[t_D \in B] = \mathbb{P}_t[t_D \in B]$ for any Borel set $B \subseteq \mathbb{R}$. In particular, this means that

$$\mathbb{P}_t[t_D > t] = 1 - \mathbb{P}_t[t_D \leqslant t] = e^{-\int_0^t \lambda(s)\,\mathrm{d}s}$$

can be substituted in Equations (4.10) and (4.11). In the remainder of this thesis, we will continue to compute conditional expectations using the filtration $\mathcal{F}(t)$ for convenience. This results in the so-called *pre-default values* of the considered variables, which are generally the object of interest when working with forward-looking measures such as xVA models, meaning that we restrict our attention to the event $\{t_D > t\}$. To incorporate default information into conditional expectations with filtration $\mathcal{G}(t)$, the filtration switching formulas in Equations (4.10) and (4.11) can then be applied. This is illustrated in Remark 4.4 for the CVA model that we formulate.

Remark 4.2 (Exponential distribution). *If an NHPP has a constant default rate $\lambda(t) = \lambda > 0$ for all $t \in [0,T]$, it becomes a regular Poisson process with intensity λ, in which case the modelled default time t_D follows an exponential distribution with rate λ.*

Remark 4.3 (Cox process). *An NHPP can be generalised in a natural way by letting the intensity process $\lambda(t)$ be stochastic, in which case the process $N(t)$ is called a Cox process. Cox processes are called doubly stochastic because of this additional source of randomness and are used to price more complicated credit derivatives, similar to how stochastic short-rate models are used to price complicated interest rate derivatives [44]. The default process $\lambda(t)$ is usually included in the processes that generate the default-free filtration[4] $\mathcal{F}(t)$, in which case the default probability calculations become more involved. For example, this results into*

$$\mathbb{P}_t[t_D > s] = \mathbb{E}_t[e^{-\int_0^s \lambda(u)\,\mathrm{d}u}] = e^{-\int_0^t \lambda(u)\,\mathrm{d}u}\,\mathbb{E}_t[e^{-\int_t^s \lambda(u)\,\mathrm{d}u}],$$

for $t \in [0,T]$ and $s \in [t,T]$, which is $\mathcal{F}(t)$-measurable instead of deterministic, as it would be for a deterministic default rate. Moreover, applying the filtration switching formula in Equation (4.10) yields

$$\mathbb{E}[\mathbb{1}_{t_D>s}|\mathcal{G}(t)] = \mathbb{1}_{t_D>t}\frac{\mathbb{E}_t[\mathbb{1}_{t_D>s}]}{\mathbb{P}_t[t_D>t]} = \mathbb{1}_{t_D>t}\frac{e^{-\int_0^t \lambda(u)\,\mathrm{d}u}\,\mathbb{E}_t[e^{-\int_t^s \lambda(u)\,\mathrm{d}u}]}{e^{-\int_0^t \lambda(u)\,\mathrm{d}u}} = \mathbb{1}_{t_D>t}\mathbb{E}_t[e^{-\int_t^s \lambda(u)\,\mathrm{d}u}]$$

as the resulting survival probability until time $s \in [t,T]$ conditional on all information available at time $t \in [0,T]$. Because complicated credit derivatives are not the primary focus of this thesis, we will not consider Cox processes and continue to model default processes using deterministic default rates $\lambda(t)$.

[4] This seems counter-intuitive, but is justified by the observation that the process $\lambda(t)$ does not include information on whether counterparty default has occurred or not, but only contains information about the likelihood of default.

4.1.3 Credit risk parameter estimation

Financial institutions take different approaches to estimate the credit risk parameters LGD and $\lambda(t)$ (the parameters that are needed to model the NHPP $\lambda(t)$, to be precise) for different purposes.

In risk management, the so-called *real-world* probability measure is typically used in asset valuation, meaning that credit risk parameters are calibrated to historical default data and other counterparty-specific characteristics [14]. These are used to determine so-called *credit ratings* of borrowers and represent their credibility. Credit ratings can also be obtained from external credit rating agencies [98]. Moreover, additional scenario analysis also often takes place in credit risk assessments to assess credit risk under particular assumptions, often in the form of stress testing [107]. The calibration of credit risk parameters to real-world data is done primarily in ECL calculations under IFRS 9 [27, 29].

On the other hand, derivative pricing is performed primarily under the so-called *risk-neutral* probability measure. Credit risk parameters are derived from market prices of credit instruments such as credit default swaps, of which the prices reflect the market expectations of credit risk parameter values [114, 137, 90]. These instruments can also be used to hedge the risk that a counterparty defaults [44, 176, 219] and are a useful tool in internalising CCR.

Advanced credit risk assessments are often based on a combination of the real world and risk-neutral measures, particularly if the financial institution is subject to regulations on the subject [44, 162, 18]. For the remainder of the thesis, we will not explicitly address or make assumptions on the nature of the probability measure that we use to keep flexibility in the proposed environmental value adjustment models, allowing financial institutions to incorporate the models in existing climate and credit risk frameworks. We assume that LGD and the NHPP $\lambda(t)$ can be obtained from credit risk frameworks without making assumptions on the probability measure $\mathbb{P}$ under which they are obtained. However, we assume that the applied credit risk framework does not explicitly include additional climate risk assessments such as scenario analyses. This explicit assumption is necessary to illustrate the impact of climate risk assessments on risk management in Section 4.2.

4.1.4 CVA model formulation

From now on, we will refer to $V(t,T)$ as the *CCR-free value* of the asset since CCR is not incorporated into the valuation framework in Section A.2. Assuming that the exposure of an asset can be modelled and the default rate $\lambda(t)$ of the counterparty and LGD known, the impact of CCR on the valuation of an asset can be quantified using the following xVA model. Its derivation are given in Section A.2.

Definition 4.6 (Credit Value Adjustment, CCR-adjusted value)**.** *The CCR-adjusted value $\widetilde{V}(t,T)$ of an asset with CCR-free value $V(t,T)$ at time t is given by*

$$\widetilde{V}(t,T) := V(t,T) - \text{CVA}(t,T), \tag{4.12}$$

where the Credit Value Adjustment (CVA) is defined as

$$\text{CVA}(t,T) := LGD \cdot \mathbb{E}_t \left[\mathbb{1}_{t_D \leqslant T} \cdot e^{-r(t_D - t)} E(t_D) \right]. \tag{4.13}$$

The stochastic processes $\text{CVA}(t,T)$ and $\widetilde{V}(t,T)$ are adapted to the filtration $\mathcal{F}(t)$ that is generated by the price processes that model the exposure, but does not take into account the default information that it available at time t.

Remark 4.4 (Filtration switching for value adjustments)**.** *We derive Definition 4.6 by taking default-free conditional expectations, but the derivation can also be performed with conditional expectations with respect to the enlarged filtration $\mathcal{G}(t)$. In this case, we define the filtration-switched CCR-adjusted value*

$$\widetilde{V}_{\mathcal{G}}(t,T) := V(t,T) - \text{CVA}_{\mathcal{G}}(t,T),$$

where the filtration-switched CVA is defined as

$$\mathrm{CVA}_{\mathcal{G}}(t,T) := LGD \cdot \mathbb{E}\left[\mathbb{1}_{t_D \leqslant T} \cdot e^{-r(t_D - t)} E(t_D) \middle| \mathcal{G}(t) \right].$$

The derivation is analogous to the one we provide in Section A.2. Applying the filtration switching formula in Theorem 4.1 results into

$$\mathbb{1}_{t_D > t} \mathrm{CVA}_{\mathcal{G}}(t,T) = \frac{\mathbb{1}_{t_D > t}}{\mathbb{P}_t[t_D > t]} \mathrm{CVA}(t,T), \tag{4.14}$$

indicating that including default information into $\mathrm{CVA}(t,T)$ *only requires scaling by a factor* $\frac{1}{\mathbb{P}_t[t_D > t]}$ *in the event that* $t_D > t$. *We emphasise that we are only interested in the CVA of an asset when the counterparty default has not yet occurred since it is a forward-looking measure for CCR, which is why we apply the indicator function* $\mathbb{1}_{t_D > t}$ *to* $\mathrm{CVA}_{\mathcal{G}}(t,T)$. *This illustrates the power of Theorem 4.1 in the context of forward-looking measures.*

The CVA of an asset at time t represents the expected value to be lost due to counterparty default assuming that this has not yet occurred, discounted to time t. It can be computed from the joint distribution of t_D and the price processes associated with the exposure, and these processes are often assumed to be independent of the NHPP that models the default time t_D for simplicity. This leads to a formulation of the CVA as an integral over time, as demonstrated in detail in Section A.2:

$$\begin{aligned} \mathrm{CVA}(t,T) =& LGD \int_t^T \mathrm{EE}_t(s) f_{t_D}(s)\,\mathrm{d}s \\ =& LGD \int_t^T \mathrm{EE}_t(s) \cdot \lambda(s) e^{-\int_0^s \lambda(u)\,\mathrm{d}u}\,\mathrm{d}s. \end{aligned} \tag{4.15}$$

Notice that the integral of the default intensity over $[0,s]$ in Equation (4.15) indicates that the $\mathcal{F}(t)$-measurable CVA does not incorporate the default information from $[0,t]$.

In financial risk management frameworks, the expected exposure in Equation 4.15 is often referred to as the Exposure At Default (EAD), and the integral over the probability density function results in the so-called Probability of Default (PD), which is a parameter commonly used alongside the LGD [101]. We illustrate the impact of CCR on asset valuation in Examples 4.2 and 4.3.

Remark 4.5 (Comparing the ECL to the CVA)**.** *Despite our focus on uncollateralised OTC trades in this Chapter, it should be emphasised that similar principles also apply to general assets. Similarly to the CVA, the Expected Credit Loss (ECL) is computed from the exposure, loss given default and probability of default. As mentioned in Subsection 4.1.3, one of the key differences between ECL and CVA computations in practice is how the credit risk parameters are obtained (real-world versus risk-neutral probability measure). Moreover, instead of modelling CCR over the continuous interval* $[0,T]$ *as is done to compute the CVA in Equation* (4.15)*, the ECL is typically computed under IFRS 9 on a discrete time grid* $\{t_0, t_1, \ldots, t_m\}$ *where* $t_j = j \cdot \Delta t$ *for* $j \in \mathbb{Z}_{m+1}$ *and* $m := \frac{T}{\Delta t}$ *is the number of steps. This assumes that* T *is a multiple of the time step* Δt*, which is typically one month* ($\Delta t = \frac{1}{12}$)*. This results in the following expression for the ECL [27, 47]:*

$$\mathrm{ECL}(t,T) = LGD \sum_{j=k}^{m} \mathrm{EE}_t(t_j) \cdot PD_t(t_{j-1}, t_j), \tag{4.16}$$

where k *is the smallest integer such that* $t \leqslant t_k$*, and* $PD_t(t_{j-1}, t_j) = \mathbb{P}_t[t_{j-1} < t_D \leqslant t_j]$ *is the probability of default during* $(t_{j-1}, t_j]$ *[47]. This is a finite Riemann sum corresponding to the integral in Equation* (4.15) *and results from the assumption that the LGD of an asset is constant and that the default process is independent of the price processes that drive the exposure, analogously to the assumptions that we make in this Section for the CVA of an asset.*

This illustrates that the reasoning that leads to the value adjustment in Definition 4.6 can also be applied to more general assets under IFRS 9 [40] such as bonds and business and private loans, even though we focus *on uncollateralised assets that are traded OTC. This should be kept in mind when we define the climate risk value adjustment in Section 4.2, where the principles presented are also not only applicable to uncollateralised OTC trades.*

4.1.5 Wrong-way risk in the CVA

The assumption of independence simplifies CVA calculations significantly in practice since the expected discounted exposure $EE_t(s)$ can be modelled, simulated and computed separately from the default process. However, assuming independence between the factors driving CCR and the exposure can be dangerous. Any asset whose value is positively correlated with the likelihood of default of the counterparty carries so-called *wrong-way risk*. Not modelling this phenomenon potentially leads to underestimation of the CVA because default risk mainly manifests when the exposure to the counterparty is high if this is the case. This is illustrated with the following example:

Example 4.1 (Wrong-way risk in a put option). *A simple example of an asset with potential for wrong-way risk is a put option on an asset that is financially linked with the counterparty,e.g. if the underlying asset is a stock of the counterparty. If the asset decreases in value, the value and exposure of the put option increase, but the counterparty's creditworthiness can deteriorate as a consequence of the value decrease. When possible, the counterparty could reduce its exposure to a potential value decrease by hedging this risk, in turn mitigating the wrong-way risk of the put option holder [101].*

Other typical examples of assets that carry wrong-way risk are cross-currency products such as FX swaps, particularly from the perspective of the party that receives a floating rate in the local currency of the counterparty [101]. Example 4.1 illustrates that wrong-way risk (and CCR in general) can be mitigated if parties hedge against their risk that can lead to their default. The counterpart of wrong-way risk is *right-way* risk, which occurs when exposure to a counterparty is adversely correlated with CCR. Obviously, this is a less problematic phenomenon since it generally results in lower exposure at default than modelled.

Remark 4.6 (Modelling wrong-way and right-way risk). *Wrong- and right-way risk are difficult to model since the relation between CCR and asset prices can be quite complicated and depend on both micro- and macroeconomic factors. A commonly used model is Vasicek's model for credit risk [210], which assumes that a counterparty defaults if the value of their assets (which are modelled using a geometric Brownian motion) drops below a certain value at maturity. It also elegantly incorporates correlation between different assets in the portfolio of the holder. The model does not account for the possibility of counterparty default before maturity however, and the assumed default process is designed specifically for assets based on stocks and is less realistic and applicable for general assets. For this reason, we choose to use our previous formulation of an independent default process in terms of a default time* t_D *modelled by an NHPP with default rate* $\lambda(t)$ *for the remainder of this thesis and ignore the wrong-way risk that it may induce.*

4.1.6 Applications of the CVA

Before integrating climate risk into the given CVA model, we explore how Research Questions 1, 2 and 3 can be answered in the context of CCR and briefly describe the role that the CVA and ECL play in asset pricing, risk management and regulatory requirements respectively.

In Section 1.1, we briefly introduced how xVAs adjust the fair value of assets by incorporating a given risk factor in the valuation process. In case of the CVA, additional expected costs associated with counterparty default are introduced to negatively adjust the asset value, as illustrated in Equation (4.12). When charging a fee of $\text{CVA}(0,T)$ to the counterparty in addition to paying the risk-free price $V(0,T)$ of the asset at the inception of the trade, the asset holder covers the expected losses due to counterparty default, discounted to $t=0$. By receiving a cash amount of $\text{CVA}(0,T)$ in addition to the asset, the expected total amount received from the trade over its lifetime is by construction $\widetilde{V}(0,T)+\text{CVA}(0,T)=V(t,T)$, taking CCR into account. The CVA thus represents the premium that the asset holder receives because he exposes himself to CCR through his investment in the asset to cover the expected losses. In the following Examples, we calculate the CVA for a European option and for a fixed rate bond. Then, we discuss the applications of the CVA.

Example 4.2 (CVA of a European Option). *We apply the framework to compute the CVA of a European call option, of which the CCR-free value $V(t,T) = e^{-r(T-t)}\mathbb{E}_t[(S(T)-K)^+]$ for $t < T$ is derived in Example A.1. Since this expectation is nonnegative, the expected exposure of the asset at time t for $s \in [t,T]$ is*

$$\begin{aligned}\mathrm{EE}_t(s) &= \mathbb{E}_t[e^{-r(s-t)}(V(s,T)^+)] \\ &= e^{-r(s-t)}\mathbb{E}_t[e^{-r(T-s)}\mathbb{E}_s[(S(T)-K)^+]] \\ &= e^{-r(T-t)}\mathbb{E}_t[(S(T)-K)^+] \\ &= V(t,T),\end{aligned}$$

making use of the tower property of iterated conditional expectations [162]. Moreover, we assume that the loss given default $LGD \in [0,1]$ and the default rate $\lambda(t) := \lambda \in \mathbb{R}_{++}$ are given constants. The CVA of the European option at time $t \in [0,T]$ can be derived from Equation (4.15)*:*

$$\begin{aligned}\mathrm{CVA}(t,T) =& LGD \int_t^T \mathrm{EE}_t(s)\cdot\lambda(s)e^{-\int_0^s \lambda(u)\,\mathrm{d}u}\,\mathrm{d}s \\ =& LGD \int_t^T V(t,T)\cdot\lambda e^{-\lambda s}\,\mathrm{d}s \\ =& LGD\cdot V(t,T)\int_t^T \lambda e^{-\lambda s}\,\mathrm{d}s \\ =& LGD\cdot V(t,T)\cdot(e^{-\lambda t}-e^{-\lambda T}),\end{aligned}$$

where it can be observed that $e^{-\lambda t}-e^{-\lambda T} = \mathbb{P}_t[t < t_D \leqslant T]$ is the probability of counterparty default during the remaining lifetime of the option, conditional on the default-free filtration.

Example 4.3 (CVA of a fixed rate bond). *We also apply the framework to compute the CVA of a fixed rate bond with predetermined payment dates $T_1 < T_2 < \cdots < T_m$, a fixed rate $K \in \mathbb{R}_{++}$ and a notional $N \in \mathbb{R}_{++}$, of which we derived the value in Example A.3. Under the assumption of a constant discount rate r, the CCR-free value is given by*

$$V(t,T) = e^{-r(T-t)}N + \sum_{i=k(t)}^{m} e^{-r(T_i-t)}NK(T_i-T_{i-1}), \tag{4.17}$$

where $k(t)$ denotes the smallest integer $k \geqslant 0$ such that $T_k \geqslant t$, and $T_0 := 0$ and $T := T_m$. Because of the constant interest rate, the value is a deterministic process. Moreover, $V(t,T) > 0$ because each term in Equation (4.17) *is positive. This means that the expected exposure at time $t \in [0,T]$ for $s \in [t,T]$ is given by*

$$\mathrm{EE}_t(s) = \mathbb{E}_t[e^{-r(s-t)}(V(s,T)^+)] = e^{-r(s-t)}V(s,T) = e^{-r(T-t)}N + \sum_{i=k(s)}^{m} e^{-r(T_i-t)}NK(T_i-T_{i-1}).$$

With the loss given default $LGD \in [0,1]$ and the default rate $\lambda(t)$ being given, the CVA of the fixed rate bond can be derived from Equation (4.15)*, which we demonstrate in Section B.3:*

$$\begin{aligned}\mathrm{CVA}(t,T) =& LGD\cdot e^{-r(T-t)}N(e^{-\int_0^t \lambda(u)\,\mathrm{d}u} - e^{-\int_0^T \lambda(u)\,\mathrm{d}u}) & (1)\\ &+ LGD\sum_{i=k(t)}^{m} e^{-r(T_i-t)}NK(T_i-T_{i-1})(e^{-\int_0^t \lambda(u)\,\mathrm{d}u} - e^{-\int_0^{T_i} \lambda(u)\,\mathrm{d}u}), & (2)\end{aligned}$$

where the two terms represent (1) the expected losses on the notional amount N to be received at maturity and (2) the expected losses upcoming payments of a fixed rate K over the notional.

It would be naive from the perspective of the asset holder to treat the charged CVA as a cash amount to be invested in a risk-free money account, since this leaves the total value of the portfolio vulnerable to default risk. Instead, the received CVA charge can be invested in credit instruments to hedge against CCR. In the case of a perfect hedge, the received CVA represents the expected costs of the hedging strategy [184, 128]. Over the past decades, a market for credit instruments has emerged, of which a credit default swap (CDS) is a basic example [149, 114]. When credit instruments related to the counterparty in a trade are available, the holder can use them to buy protection against the associated CCR. We illustrate their functioning by describing how a so-called *Postponed payoffs Running CDS* (PRCDS) works based on [44], but several types of slightly different CDS payoff structures exist in practice.

When a party buys a CDS on a third company (say, company C) from the seller over a notional amount, he makes payments at predetermined moments of a given spread over the notional amount, and he receives the notional amount[5] at the next moment of a predetermined payment if company C defaults. This makes the CDS function as an insurance for the buyer against the event that company C defaults [75]. The holder of an asset can hedge the associated CCR by holding CDSs on the counterparty so that their total notional amount equals the exposure of the asset [201]. Changes in the exposure should be handled using a dynamic hedging strategy [201, 33, 128]. We discuss this in the context of Examples 4.2 and 4.3.

In the context of the fixed rate bond in Example 4.3, the only source of uncertainty in the resulting future cash flows comes from the default process that models t_D. The holder can buy protection against this risk with the received CVA charge by buying CDSs with the same payment dates $T_1 < T_2 < \cdots < T_m$, such that their notional amounts equal the prescribed payments from the fixed rate bond. In this way, the total received amount from CDSs upon counterparty default at time t_D equals $E(T_{k(\hat{t}_D)})$ at time $T_{k(\hat{t}_D)}$ in the case of a PRCDS. This means that the total profits and losses of the asset holder are unaffected by the event of counterparty default with this hedging strategy. Since the exposure is deterministic, no dynamic hedging is required.

In this way, the initial CVA charge can be used to hedge the CCR associated with an asset using CDSs. Since an increase in the exposure $E(t)$ can lead to an increase in $\text{CVA}(t,T)$ when assets with a non-deterministic future value are considered, the CVA itself is also susceptible to market risk [31]. This risk can also be mitigated by hedging against an increase in exposure, as illustrated in [230]. In Example A.1, we demonstrate that the value $V(t,T)$ of a European option is a function of the current underlying asset price $S = S(t)$. The sensitivity of the CCR-adjusted option value is therefore given by

$$\frac{\partial}{\partial S}\widetilde{V}(t,T) = \frac{\partial}{\partial S}V(t,T) - \frac{\partial}{\partial S}\text{CVA}(t,T) = \Big(1 - LGD(1 - e^{-\lambda(T-t)})\Big)\frac{\partial}{\partial S}V(t,T), \tag{4.18}$$

making use of the result in Example 4.2. The so-called *delta hedging* strategy[6] offsets the sensitivity of the asset value to changes in $S(t)$ by buying minus the amount in Equation (4.18) of assets. Delta hedging is a fundamental hedging strategy and protects the profits and losses of asset holders from changes in the underlying asset value [162, 230, 67]. Sensitivities to other parameters can be hedged analogously, provided that appropriate hedging instruments are available.

Dynamic hedging strategies are required to rebalance the positions and ensure (practically) perfect protection against counterparty default. Financial institutions should always hold on to a capital buffer for protection against the sensitivities in the value of their assets, since it might be necessary to buy more hedging instruments in the future. Capital buffers are also necessary to protect against potential default-related losses if no hedging takes place. This illustrates that capital buffers can be useful to mitigate CCR, when hedging strategies are applied as well as when they are not.

Credit risk-related regulatory capital requirements can also be imposed by supervisors, as is the case under Basel 3 for example [18]. Examples of CCR-related capital requirements include that banks should hold on to a given fraction in capital of the CCR-weighted value of their assets and a capital buffer based on the sensitivities of the CVA to market risk factors. These requirements offer protection against CCR-related uncertainty in the profits and losses if financial institutions. A formulation of the capital requirements under Basel 3 can be found in [3]. Similar requirements are also imposed regarding the ECL under IFRS 9 [40].

[5] The received notional amount is multiplied by the loss given default

[6] The sensitivity $\frac{\partial V}{\partial S}$ of the asset to changes in the underlying asset value is generally denoted by Δ.

These requirements build resilience to CCR and other financial risks in the financial sector and function as an example of how regulators can guide financial institutions in incorporating externalities in their risk management [155]. However, we will not explicitly focus on specific regulatory requirements regarding credit risk or the market risk in xVAs furthermore, including how their market risk can be hedged.

4.2 Climate Risk Value Adjustment (CRVA)

As discussed in Subsection 2.1.2, there are concerns about how adequately climate risk is incorporated into credit risk frameworks. Transition risk is generally believed to be represented in market prices, but this does not hold for physical risk. This introduces the need for additional climate risk assessments regarding physical risk, but these are complicated due to the complexities and uncertainties of the interaction between global warming and the resulting extreme events, geopolitical dynamics and the climate transition of the financial sector, particularly over longer timescales.

In Subsection 4.2.1, we propose an extension of the CVA framework from Section 4.1 to incorporate an additional source of CCR. This results into an adjustment to the CVA, which we define as the Climate Risk Value Adjustment (CRVA), an EVA model that internalises climate risk. We illustrate the impact of CRVA on the value of assets in Examples 4.4 and 4.5. Then, we discuss the interpretation and purpose of the CRVA model in Section 4.2.3, and discuss how the formulation of the CRVA in terms of an environmental impact factor allows for a seamless integration of the EVA model into existing xVA frameworks.

4.2.1 Climate-adjusted CCR parameters

In Section 2.1.2, we discussed the necessity of additional climate risk assessments on top of current risk frameworks since climate change poses a systemic risk to the financial system [19, 169, 16, 175, 8]. As discussed in Subsection 4.1.3, credit risk parameters are usually obtained from real-world data or using a risk-neutral approach. We already discussed in Subsection 2.1.2 that climate risk forecasting with models based on historical data can result in underestimation of climate risk due to the unprecedented nature of climate risk and multiple dimensions of uncertainty, particularly when assessing physical risk over longer timescales [19, 76, 21, 8]. In this Subsection, we will argue that a risk-neutral approach to assess the impact of climate risk on CCR can also result in an underestimation of climate risk. Even though complete market theory would suggest that market prices should accurately reflect any available information on climate risks [122], there are concerns in reality on whether or not this is actually the case [78], with some going as far as comparing climate events with black swans[7] [42]. Because risk-neutral pricing is, by definition, less likely to identify a misalignment between market expectations and reality [50], an underestimation of climate risk can lead to an underestimation of CCR when credit risk parameters are estimated under the risk-neutral probability measure. This emphasises the need to internalise climate change into risk management across the financial sector.

It has been shown that climate risk and particularly physical risk are not reflected in market prices [41, 112], at least not to an economically significant extent [177]. In contrast to physical risk, the transition risk associated with future regulations on CO2e emissions is generally believed to be reflected in market prices, although there appears to be a correlation between the transition risk and the total emissions of counterparties rather than their carbon intensity[8] [41]. The transition risk associated with CO2e emissions has been investigated in considerable depth, some going as far as defining xVA frameworks specifically to capture the vulnerability of counterparties to changes in CO2e prices associated with future regulations [133]. There are fewer studies on other sources of transition risk, but the focus on CO2e emissions in transition risk assessments is justified given their crucial role in climate change and its mitigation [64].

Because both historical data and market prices can fail to accurately assess climate risk in current credit risk frameworks, additional climate risk modelling is required to accurately do so and fully internalise climate change. With 60% of European banks not integrating climate stress testing in their credit risk frameworks

[7] A *black swan* symbolises unforeseen events with high impact that people are only able to explain in hindsight.
[8] *Carbon intensity* is the amount of emissions of a company per unit of their value, as we will define in Chapter 5.

for example [16], additional climate risk assessments are still not common practice and the impact of climate change on financial assets is still not fully internalised. It is important to emphasise that the development of climate risk assessments requires well-defined, high quality frameworks and guidance from regulators, which illustrates the important role that the NGFS scenarios and similar initiatives play in climate risk assessments [100, 19]. However, even the NGFS scenarios do not sufficiently incorporate the possibility of complex feedback loop or fundamental system changes [175], highlighting the necessity to further develop climate risk models.

From now on, we assume that additional climate risk assessments are incorporated into the asset valuation process. This introduces an adjustment to the climate risk parameters LGD and $\lambda(t)$ in the introduced CVA framework (see Equation (4.15)), as discussed at the end of Subsection 4.1.3. Although climate risk might already be partially incorporated in the parameters LGD and $\lambda(t)$ (either implied by market prices or obtained from real-world data), it may not be fully incorporated into the CCR framework that we introduced in Section 4.1 yet. We refer to the non-incorporated part of climate risk as *external climate risk*, and define the following adjustments to the CCR parameters to quantify external climate risk.

Definition 4.7 (Climate-adjusted CCR variables)**.** *For a counterparty with default process $\lambda(t)$ to model CCR for a given asset, we define the intensity process $\lambda^{\mathrm{CR}}(t)$ to be the adjustment to $\lambda(t)$ that is required to describe any aspects of climate risk that are not internalised in the existing CCR framework. Analogously to Definition 4.4, the intensity process $\lambda^{\mathrm{CR}}(t)$ defines an NHPP by*

$$N^{\mathrm{CR}}(t) := \bar{N}'\left(\int_0^t \lambda(u)\,\mathrm{d}u\right), \tag{4.19}$$

where $\bar{N}'(t)$ is a standard Poisson process that is independent of the process $\bar{N}(t)$ that models t_D. $\bar{N}'(t)$ is also independent of the considered price processes. we define

$$\widehat{N}(t) := N(t) + N^{\mathrm{CR}}(t), \tag{4.20}$$

which is an NHPP with intensity process

$$\widehat{\lambda}(t) := \lambda(t) + \lambda^{\mathrm{CR}}(t). \tag{4.21}$$

With this NHPP, we model the climate-adjusted default time

$$\widehat{t}_D := \inf\{t \in [0,T] \mid \widehat{N}(t) > 0\}. \tag{4.22}$$

We also define $\widehat{LGD} \in [0,1]$ to be the climate-adjusted loss given default.

The NHPP $N^{\mathrm{CR}}(t)$ models counterparty default from external climate risk, meaning that the first jump of $\widehat{N}(t) = N(t) + N^{\mathrm{CR}}(t)$, which occurs at time $\widehat{t}_D$, comes either from the 'regular' default-modelling NHPP or from the external climate risk-related NHPP. Since the sum of two NHPPs is again an NHPP, with their combined intensity rate [68], it is justified that we refer to $\widehat{N}(t)$ and $\widehat{\lambda}(t)$ in Definition 4.7 as an NHPP and intensity process, respectively. We demonstrate this in Lemma 4.2.

Lemma 4.2 (Combining two Non-Homogeneous Poisson Processes)**.** *The construction in Definition 4.7 results into the fact that $\widehat{t}_D$ follows the probability distribution of a random variable that is modelled by an NHPP with intensity process $\widehat{\lambda}(t)$.*

Proof. We denote the moments of the first jumps of $N(t)$ and $N^{\mathrm{CR}}(t)$ by t_D and $t_D^{\mathrm{CR}} := \inf\{t \in [0,T] \mid N^{\mathrm{CR}}(t) > 0\}$, respectively. We observe that

$$\widehat{t}_D := \inf\{t \in [0,T] \mid N'(t) > 0\} = \inf\{t \in [0,T] \mid N(t) > 0 \text{ or } N^{\mathrm{CR}}(t) > 0\} = \min(t_D, t_D^{\mathrm{CR}}).$$

Because the NHPPs $N(t)$ and $N^{\mathrm{CR}}(t)$ are constructed from independent standard Poisson processes, we therefore have

$$\mathbb{P}_0[\widehat{t}_D > t] = \mathbb{P}_0[t_D > t \text{ and } t_D^{\mathrm{CR}} > t] = \mathbb{P}_0[t_D > t]\cdot\mathbb{P}_0[t_D^{\mathrm{CR}} > t] = e^{-\int_0^t \lambda(s)\,\mathrm{d}s}\cdot e^{-\int_0^t \lambda^{\mathrm{CR}}(s)\,\mathrm{d}s} = e^{-\int_0^t (\lambda(s)+\lambda^{\mathrm{CR}}(s))\,\mathrm{d}s},$$

where $t \in [0,T]$. This means that the density function of $\widehat{t}_D$ matches the density function of a random variable that is modelled using an NHPP with intensity process $\widehat{\lambda}(t) = \lambda(t) + \lambda^{\mathrm{CR}}(t)$, as given in Equation (4.7). The process $\widehat{\lambda}(t)$ is also a nonnegative,right-continuous and piecewise continuous deterministic function on $[0,T]$ because $\lambda(t)$ and $\lambda^{\mathrm{CR}}(t)$ are, which are the properties of an intensity process. □

In the proof of Lemma 4.2, we already derived the density function for $\widehat{t}_D$ on $[0,T] \ni t$. Analogously to Equations (4.7) and (4.8), it follows that

$$F_{\widehat{t}_D}(t) := \mathbb{P}_0[\widehat{t}_D \leqslant t] = 1 - e^{-\int_0^t \widehat{\lambda}(u)\,\mathrm{d}u}, \tag{4.23}$$

and

$$f_{\widehat{t}_D}(t) = \frac{\mathrm{d}}{\mathrm{d}t} F_{\widehat{t}_D}(t) = \widehat{\lambda}(t) e^{-\int_0^t \widehat{\lambda}(u)\,\mathrm{d}u}, \tag{4.24}$$

where $t \in [0,T]$. If $\widehat{t}_D \notin [0,T]$, we have $\widehat{t}_D$ by convention, so $\mathbb{P}_0[t_D = \infty] = e^{-\int_0^t \lambda(s)\,\mathrm{d}s}$. We will not describe the joint distribution of t_D and $\widehat{t}_D$ because it will not appear in the models that we define in this thesis, but it should be emphasised that $\widehat{t}_D \leqslant t_D$ by construction.

In Section 4.1, we introduced the enlarged filtration $\mathcal{G}(t) = \mathcal{F}(t) \vee \sigma(N(s) \,|\, 0 \leqslant s \leqslant t)$ that included the available information on t_D at time t. The climate-adjusted NHPP $\widehat{N}(t) = N(t) + N^{\mathrm{CR}}(t)$ that models $\widehat{t}_D$ is not adapted to the filtration $\mathcal{G}(t)$ because it does not contain information on the NHPP $N^{\mathrm{CR}}(t)$ because of the independence of $N^{\mathrm{CR}}(t)$ from the other processes. Therefore, we define the *climate-adjusted enlarged filtration*

$$\widehat{\mathcal{G}}(t) := \sigma(\mathcal{G}(t) \cup \sigma(N^{\mathrm{CR}}(s) \,|\, 0 \leqslant s \leqslant t)), \tag{4.25}$$

which is the σ-hill of the enlarged filtration and the σ-algebra generated by $N^{\mathrm{CR}}(t)$, and is sometimes denoted by $\widehat{\mathcal{G}}(t) = \mathcal{G}(t) \vee \sigma(N^{\mathrm{CR}}(s) \,|\, 0 \leqslant s \leqslant t)$. The filtration $\widehat{\mathcal{G}}(t)$ is therefore generated by the considered price processes and the NHPPs $N(t)$ and $N^{\mathrm{CR}}(t)$, and this filtration should be interpreted as containing all available information at time t, including information on the occurrence of an external climate-risk induced default event. By construction, the NHPP $\widehat{N}(t)$ is adapted to the filtration $\widehat{\mathcal{G}}(t)$. The filtration switching formula can also be applied to incorporate climate-adjusted default information by substituting $\widehat{\mathcal{G}}(t)$ for $\mathcal{G}(t)$ and $\widehat{t}_D$ for t in Equation (4.10). However, we will proceed to calculate CCR-related conditional expectations with respect to the default-free filtration $\mathcal{F}(t)$, as we explained in Remark 4.4.

In summary, the adjustment $\lambda^{\mathrm{CR}}(t)$ quantifies external climate risk, so that the CCR-adjusted parameters adequately internalise climate risk. The adjustment to the CCR variables can be obtained by climate-related scenario analysis and stress testing, for which the NGFS scenarios are generally used across the industry [100, 69, 6]. However, we do not further specify how the adjustment to the CCR variables should be obtained in this thesis and assume that the adjustment $\lambda^{\mathrm{CR}}(t)$ to the default rate $\widehat{\lambda}(t)$ and $\widehat{LGD} \in [0,1]$ are known. Notice that we do not include the possibility that external climate risk has a positive impact on the creditworthiness of the counterparty by assuming that $\lambda^{\mathrm{CR}}(t) \geqslant 0$, since this is a relatively unlikely scenario[9].

4.2.2 Climate-adjusted CVA and CRVA model

The climate-adjusted CCR variables $\widehat{\lambda}(t)$ and $\widehat{LGD}$ can be substituted for the regular CCR variables $\lambda(t)$ and LGD in the CVA framework from Section 4.1 in order to compute the CVA of an asset with climate risk fully internalised. This is introduced in the following definition, where we formulate the climate-adjusted CVA making use of the framework in Section A.2 with the climate-adjusted variables.

[9] If a counterparty is positively affected by the drivers of physical or transition climate risk, this will probably already included in market prices in practice.

Definition 4.8 (Climate-adjusted value and CVA). *We define $\widehat{V}(t,T)$ to be the climate-adjusted value of an asset at time t, which is the CCR-adjusted value with climate risk fully internalised in the CCR framework. Analogously to Definition 4.6, it is formulated as*

$$\widehat{V}(t,T) = V(t,T) - \widehat{\mathrm{CVA}}(t,T), \tag{4.26}$$

where

$$\widehat{\mathrm{CVA}}(t,T) := \widehat{LGD} \cdot \mathbb{E}_t \left[\mathbb{1}_{\hat{t}_D \leqslant T} \cdot e^{-r(\hat{t}_D - t)} E(\hat{t}_D) \right] \tag{4.27}$$

is the climate-adjusted CVA of the asset, with the exposure $E(t)$ defined as in Definition 4.1. The processes $\widehat{V}(t,T)$ and $\widehat{\mathrm{CVA}}(t,T)$ are adapted to the default-free filtration $\mathcal{F}(t)$.

Similarly to Section 4.1, we assume independence between the NHPP that models $\hat{t}_D$ and the price processes that model $V(t,T)$ and therefore the exposure $E(t)$, resulting into

$$\begin{aligned} \widehat{\mathrm{CVA}}(t,T) =& \widehat{LGD} \int_t^T \mathrm{EE}_t(s) f_{\hat{t}_D}(s) \, \mathrm{d}s \\ =& \widehat{LGD} \int_t^T \mathrm{EE}_t(s) \cdot \hat{\lambda}(s) e^{-\int_0^s \hat{\lambda}(u)\,\mathrm{d}u} \, \mathrm{d}s. \end{aligned} \tag{4.28}$$

Remark 4.7 (Climate risk and collateral). *Under given circumstances, the assumption of independence between climate-related default processes and the exposure can lead to wrong-way risk, particularly if there is collateral posted which is vulnerable to climate risk. A typical example of this phenomenon is a real estate loan where the building is posted as collateral. Even though real estate is typically seen as stable and high-quality collateral [125], it can lose its value due to extreme climate events, resulting into an increased exposure $E(t) = (V(t,T) - \mathrm{col}(t))^+$. This phenomenon should be addressed in the adjustment to the loss given default to avoid wrong-way risk. Because collateralised trades are not the primary focus of this thesis, we will proceed with under assumption of independence between the exposure and the default processes.*

With the climate-adjusted CVA model, we define an Value Adjustment (xVA) model that captures the impact of climate risk on the CVA associated with an asset, which we call the Climate Risk Value Adjustment:

Definition 4.9 (Climate Risk Value Adjustment). *The Climate Risk Value Adjustment (CRVA) of an asset with maturity T is the adjustment to the CCR-adjusted value when internalising external climate risk into the CCR framework:*

$$\mathrm{CRVA}(t,T) := \widetilde{V}(t,T) - \widehat{V}(t,T), \tag{4.29}$$

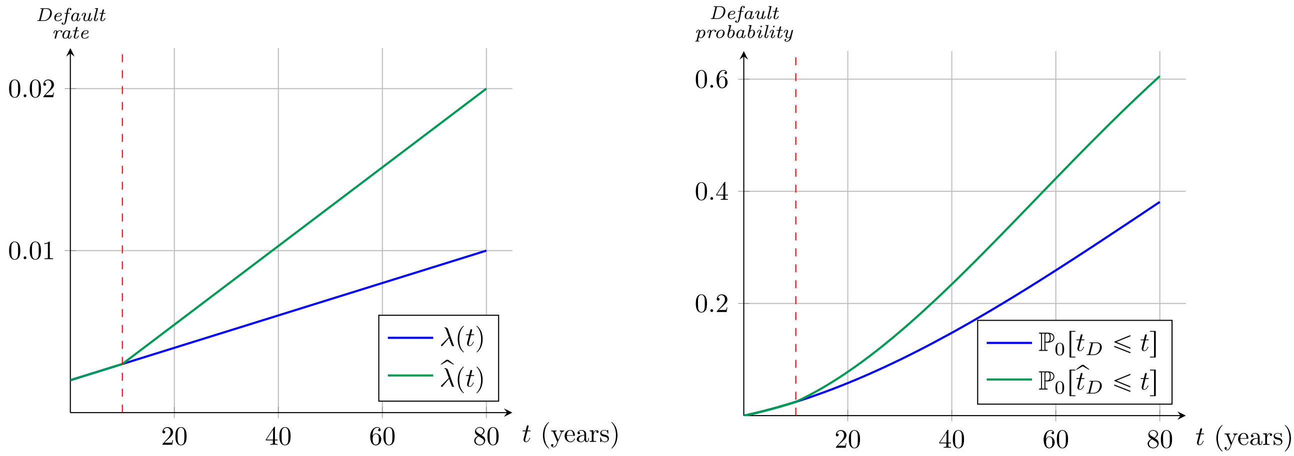
where $t \in [0,T]$. The regular CCR-adjusted value and climate-adjusted CCR-adjusted value are given by $\widetilde{V}(t,T)$ and $\widehat{V}(t,T)$ respectively. Both $\widehat{V}(t,T)$ and $\mathrm{CRVA}(t,T)$ are adapted to the default-free filtration $\mathcal{F}(t)$.

From this Definition and Equations (4.13) and (4.26), it immediately follows that the CRVA is the difference between the CVAs:

$$\mathrm{CRVA}(t,T) = \Big(V(t,T) - \mathrm{CVA}(t,T)\Big) - \Big(V(t,T) - \widehat{\mathrm{CVA}}(t,T)\Big) = \widehat{\mathrm{CVA}}(t,T) - \mathrm{CVA}(t,T), \tag{4.30}$$

It should be noted that a zero adjustment $\lambda^{\mathrm{CR}} : [0,T] \mapsto 0$ results into $\hat{\lambda}(t) = \lambda(t)$ and $N^{\mathrm{CR}}(t) = \bar{N}'(0) = 0$ for all $t \in [0,T]$, resulting into $\hat{t}_D = t_D$. Moreover, we have $\sigma(N^{\mathrm{CR}}(s) \,|\, 0 \leqslant s \leqslant t) = \{\varnothing, \Omega\}$ and hence $\widehat{\mathcal{G}}(t) = \mathcal{G}(t)$ for all $t \in [0,T]$. If $\widehat{LGD} = LGD$ as well, it follows that $\widehat{\mathrm{CVA}}(t,T) = \mathrm{CVA}(t,T)$ and $\widehat{V}(t,T) = \widetilde{V}(t,T)$, resulting into $\mathrm{CRVA}(t,T) = 0$. We draw the unsurprising conclusion that the CVA framework does not have to be adjusted for external climate risk if climate risk is adequately assessed in the regular CVA framework and the resulting parameters $\lambda(t)$ and LGD.

The idea of adjusting the CVA of an asset based on climate-associated effects that are not adequately captured in regular CCR models is not new: Kenyon and Berrahoui define a climate change value adjustment to internalise climate risk that is currently invisible assuming typical market practice [131]. Instead of directly introducing adjustments to the CCR parameters, they define an adjusted probability measure to compute the CVA. Similarly to Equation (4.21), the adjusted probability measure adds an additional term to the default rate. However, the regular CVA is computed using the risk-neutral probability measure, which assumes linear extrapolation of risk-neutral default rates after a time horizon of 10 years, based on the assumption that there is a lack of transactions with credit instruments with a maturity beyond 10 years. Because physical climate risk materialises over longer timescales [177, 21], Kenyon and Berrahoui also argue that this leads to an underestimation of credit risk for assets with longer maturity, demanding an adjustment to the default rate $\lambda(t)$ for $t > 10$ years. A simplified illustration of their approach is given in Figure 4.1.



(a) *Simple piecewise linear model for the regaular and climate-adjusted default rates $\lambda(t)$ and $\widehat{\lambda}(t) = \lambda(t) + \lambda^{\mathrm{CR}}(t)$.*

(b) *Cumulative density functions of the regular and climate-adjusted default times t_D and $\widehat{t}_D$ that result from the default rates in Figure 4.1a.*

Figure 4.1: *Illustration of the climate-related adjustment to the default rate, as proposed by Kenyon and Berrahoui [131]. In this example, the default rate $\lambda(t)$ is extrapolated linearly after the CDS trading horizon $t = 10$, which is indicated in red. However, the external climate risk-related default rate $\lambda^{\mathrm{CR}}(t)$ grows linearly after this in this example, indicating that climate risks materialise over longer timescales. However, the impact on the default probability is limited over the first few years after $t = 10$.*

Although credit instruments are indeed typically traded with maturities of ten years at most, instruments with longer maturities are becoming more liquid [94, 148] and the assumption of linear extrapolation imposes a strong restriction on the CCR model. With Equation (4.21) to obtain the CVA adjustment in Definition 4.9, we propose a more flexible approach that does not assume a credit instrument market horizon of 10 years or linear extrapolation of default rates, making the model more adaptable in existing credit risk frameworks. We demonstrate how external climate risk results into a CRVA in the case of a European option and a fixed rate bond in Example 4.4 and Example 4.5, respectively.

Example 4.4 (CRVA of a European option)**.** *In Example 4.2, we showed that the CVA of a European option with maturity T and constant default rate $\lambda(t) = \lambda$ is given by*

$$\mathrm{CVA}(t,T) = LGD \cdot V(t,T) \cdot (e^{-\lambda t} - e^{-\lambda T}),$$

where $V(t,T)$ is the default-free value of the option for $t \in [0,T]$. We consider a European option where the counterparty is a financial institution with very limited susceptibility to climate risk. We let the default rate and loss given default be $\lambda = 0.009$ and $LGD = 0.5$, and we let $\lambda^{\mathrm{CR}}(t) := 0.001$ and $\widehat{LGD} = LGD = 0.5$, so *that* $\widehat{\lambda}(t) = 0.01$. *This results into*

$$\widehat{\text{CVA}}(t,T) = \widehat{LGD} \cdot V(t,T) \cdot (e^{-\hat{\lambda}t} - e^{-\hat{\lambda}T}),$$

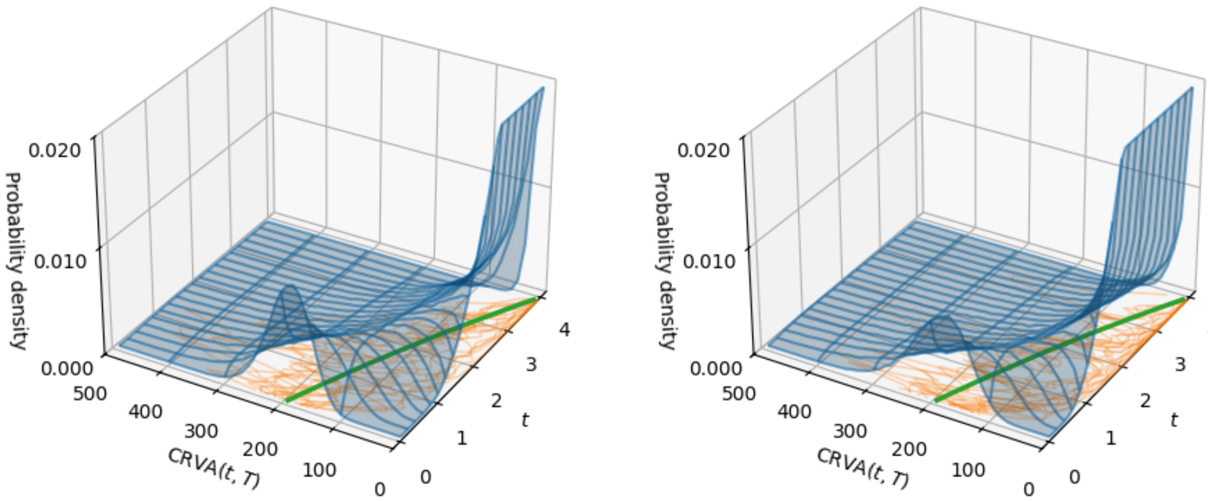
so that

$$\text{CRVA}(t,T) = \widehat{\text{CVA}}(t,T) - \text{CVA}(t,T) = LGD \cdot V(t,T)\Big(e^{-\hat{\lambda}t} - e^{-\hat{\lambda}T} - e^{-\lambda t} + e^{-\lambda T}\Big). \tag{4.31}$$

We simulate the value process of a European option contract with maturity $T = 4$ *years. The holder receives an number of European options such that their total default-free value is* $V(0,T) =$ €1 *million, and the initial CVA and climate-adjusted CVA for the given parameters are*

$$\left.\begin{aligned}\text{CVA}(0,T) &= €1796.76\\ \widehat{\text{CVA}}(0,T) &= €1996.01\end{aligned}\right\} \Rightarrow \text{CRVA}(0,T) = €199.24, \tag{4.32}$$

which is a relatively small fraction of the risk-free value $V(0,T)$. *We conclude that unsurprisingly, a small adjustment* $\lambda^{\text{CR}}(t)$ *to the default rate* $\lambda(t)$ *results in a relatively small CRVA. In Figure 4.2, we show how the CRVA of the option evolves over time.*



(a) *CRVA for an at the money option with* $K = 40$. ***(b)*** *CRVA for an out of the money option with* $K = 55$.

Figure 4.2: *Graph of the evolution of the probability distribution of* $\text{CRVA}(t,T)$ *over time* t *for an at the money option contract and an out of the money option contract. The distribution is obtained from a Monte Carlo simulation of* 10^5 *asset paths, using an Euler discretisation of* $[0,T]$ *with* 10^3 *steps. Twenty paths of* $\text{CRVA}(t,T)$ *are shown in orange in the horizontal plane, and* $\mathbb{E}_0[\text{CRVA}(t,T)]$ *is shown in green. The parameter configuration for the underlying asset (see Equation* (A.6)*) is* $\mu = r = 0.05$, $\sigma = 0.2$, *and* $S_0 = 40$.

Because the option contracts in Figure 4.2 both have a notional amount of $V(0,T) =$ €1 *million, their CRVA is identical, as computed from Equation 4.31, even though the individual options have different values (10.09 and 4.51 for* $K = 40$ *and* $K = 55$*, respectively). The distributions of their CRVAs over time are relatively similar, with the only difference being that the value and therefore the CVA of the out of the money option is likely to become zero. Two main observations can be made. Firstly, the uncertainty in* $\text{CRVA}(t,T)$ *initially grows over time, since the uncertainty in* $S(t)$ *and therefore in* $V(t,T)$ *grows over time. However, the CRVA approaches zero as* t *approaches the maturity* T. *This is because the probability of default in* $(t,T]$ *naturally decreases as* t *approaches* T. *This demonstrates that the CRVA of an asset, similarly to the CVA, is subject to changes in the value of the underlying asset, as well as the remaining duration of the contract.*

In contrast to the counterparty in Example 4.4 that is not very vulnerable to external climate risk, we consider an asset in Example 4.5 for which the counterparty is more vulnerable to external climate risk, despite carrying a green label.

Example 4.5 (CRVA of a fixed rate green bond)**.** *We consider a green bond issued by a hypothetical renewable energy company. The company is perceived by the market as having a high creditworthiness, but additional climate risk assessments indicate that their creditworthiness can suffer over longer timescales from extreme climate events, most notably because their offshore wind farms can be impacted by hurricanes.*

We consider a green bond that prescribes payments at a fixed rate $K \in \mathbb{R}_{++}$ *to the holder over a notional* $N \in \mathbb{R}_{++}$*, of which the CVA is derived in Section B.3:*

$$\begin{aligned} \mathrm{CVA}(t,T) &= LGD \cdot e^{-r(T-t)} N(e^{-\int_0^t \lambda(u)\,\mathrm{d}u} - e^{-\int_0^T \lambda(u)\,\mathrm{d}u}) \\ &\quad + LGD \sum_{i=k(t)}^{m} e^{-r(T_i-t)} NK(T_i - T_{i-1})(e^{-\int_0^t \lambda(u)\,\mathrm{d}u} - e^{-\int_0^{T_i} \lambda(u)\,\mathrm{d}u}), \end{aligned}$$

where the default rate $\lambda(t)$ *and loss given default* LGD *are given. The formula for the climate-adjusted CVA* ($\widehat{\mathrm{CVA}}(t,T)$) *is analogous, with* $\widehat{\lambda}(t)$ *and* $\widehat{LGD}$ *substituted for* $\lambda(t)$ *and* LGD*, respectively. The risk-free value* $V(t,T)$ *is given in Example A.3. To exaggerate the impact of increasing climate risk over longer timescales, we set* $LGD = \widehat{LGD} = 0.3$, $\lambda(t) := 0$ *and*

$$\widehat{\lambda}(t) = \lambda^{\mathrm{CR}}(t) := \alpha \cdot t, \tag{4.33}$$

where $\alpha \in \mathbb{R}_+$ *indicates the rate at which external climate risk grows over time. This results into* $\mathrm{CVA}(t,T) = 0$ *and therefore* $\mathrm{CRVA}(t,T) = \widehat{\mathrm{CVA}}(t,T)$*. To exaggerate the impact of climate risk on the value of the bond, we consider a relatively long maturity* $T = 20$ *years. In Figure 4.3, we show the CRVA and the climate-adjusted value of the bond.*

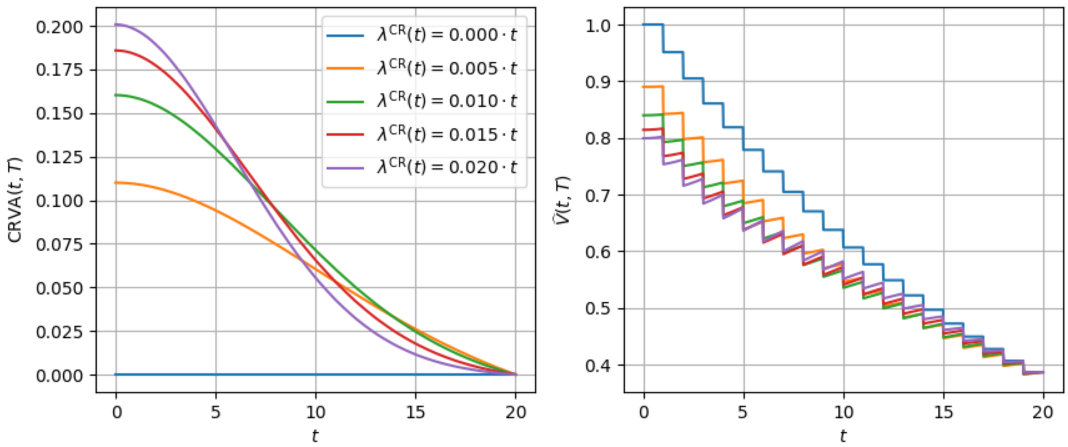


Figure 4.3: *Illustration of the impact of increasing external climate risk on the value of a fixed rate bond with payment dates* $T_i = i$ *for* $i = 0, 1, \ldots, m := 20$*. The left graph shows the CRVA over time for different values of* α *(see Equation (4.33)), and the right graph show the climate-adjusted value* $\widehat{V}(t,T) = V(t,T) - \mathrm{CRVA}(t,T)$ *over time, where* $t \in [0, 20]$ *is in years. The parameter configuration is* $r = 0.05$, $K = e^r - 1$ *and* $N = 1$*.*

In Figure 4.3, it can be observed that unsurprisingly, the initial CRVA grows with α*. Moverover,* $\mathrm{CRVA}(t,T)$ *continuously decreases over time, and it initially decreases at a higher rate for high values of* α*, since* $\mathbb{P}_0[\widehat{\tau}_D > t] = e^{-\int_0^t \widehat{\lambda}(u)\,\mathrm{d}u}$ *decreases at a higher rate. This results in large values of* α *yielding lower CRVAs towards maturity and illustrates that* $\mathrm{CRVA}(t,T)$ *only incorporates default-free information from* $\mathcal{F}(t)$ *and not from* $\widehat{\mathcal{G}}(t)$*.*

The risk-free value $V(t,T)$ decreases stepwise at the moments T_i of the coupon payments ($i = 1,\dots,m$). We conclude that the exaggerated default processes that we consider result in a very significant CRVA from external climate risk, despite its green label. We will discuss the implications for the CRVA of this green bond in Example 4.6 for a more realistic parameter configuration. We will then consider default processes $\lambda(t) = 0.005$ and $\lambda^{\mathrm{CR}}(t) = 0.005 \cdot \frac{t}{T}$, meaning that the adjusted default rate

$$\widehat{\lambda}(t) = 0.005\Big(1 + \frac{t}{T}\Big)$$

doubles over the lifetime of the bond. We consider the same set of parameters $LGD = \widehat{LGD} = 0.3$, $r = 0.05$ and $K = e^r - 1$, with a notional value of $N =$ €1 million, but now we consider a contract with annual payments until the maturity of $T = 4$ instead of 20 years. This results into

$$\left.\begin{aligned} \mathrm{CVA}(0,T) &= €5521.50 \\ \widehat{\mathrm{CVA}}(0,T) &= €8156.54 \end{aligned}\right\} \Rightarrow \mathrm{CRVA}(0,T) = €2635.04, \tag{4.34}$$

which is a significant adjustment to the CVA.

Despite its green label, climate change has a negative impact on the value of the hypothetical bond in Example 4.5 when we incorporate climate risk. However, given that the CVA and CRVA of an asset quantify the entire default risk over the period $[0,T]$, the CRVA of an asset can still be relatively small when external climate risk only materialises over the latest part of the lifetime of an asset, which can be the case in the framework that Kenyon and Berrahoui propose, as illustrated in Figure 4.1. Similarly, the doubling default rate over the lifetime of the asset in Example 4.5 does not directly translate into a doubling of the CVA.

4.2.3 Interpretation and purpose of the CRVA

The climate-adjusted CCR variables $\lambda^{\mathrm{CR}}(t)$ and $\widehat{LGD}$ are obtained from the necessary additional climate risk assessments and result into an adjustment to the CCR-adjusted value and therefore to the CVA, as illustrated by Equations (4.29) and (4.30). Following the additional climate risk assessments, the CVA of assets can be expressed as

$$\widehat{\mathrm{CVA}}(t,T) = \mathrm{CVA}(t,T) + \mathrm{CRVA}(t,T), \tag{4.35}$$

meaning that the adjusted CVA (denoted by $\widehat{\mathrm{CVA}}$) can be interpreted as having a component for 'regular CCR' (the orginal CVA) and a component that captures CCR that results from external climate risk (the CRVA). These components are illustrated in Figure 4.4. It should be noted that we assume that the CRVA is positive by taking this perspective. This is generally the case, but $\mathrm{CRVA}(t,T) \geqslant 0$ for $t \in [0,T]$ does not have to hold in rare cases, for example when $\mathbb{P}_t[\lambda^{\mathrm{CR}} \leqslant t]$ is sufficiently large, or when $\widehat{LGD} < LGD$.

To answer Research Question 1, we conclude that the CRVA is an additional adjustment to the fair value of an asset that is necessary to internalise external climate risk. The adjusted CVA can be interpreted as the associated costs and applied as a charge, exactly as we described in Subsection 4.1.6 without any special attention to the origin of these costs.

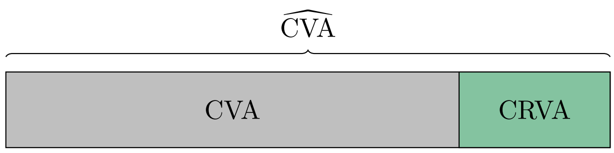


Figure 4.4: *The CRVA can be seen as a component of the adjusted CVA that represents climate risk-related CCR, while the original CVA represents is the component that represents regular CCR. The CRVA is assumed to be positive in this Figure, but this is not necessary.*

In the perspectives on climate risk assessments that we provided in Subsection 4.2.1, we discuss that the external climate risk that we defined is evaluated with additional climate-related scenario analyses and stress tests, in an attempt to mitigate the large uncertainties that are particularly present in physical climate risk assessments over longer timescales. This means that the magnitude of the CRVA of an asset is highly dependent on the future vulnerability of the counterparty to physical climate risk. OTC derivatives are generally traded between financial institutions with limited vulnerability to external climate risk (see Example 4.4), but an exception to this can arise in situations where trades take place with banks in vulnerable regions with high exposure to the local economy, for example [175]. In this case, an extreme climate event can cause a disruption to the local economy and thus potentially affect the credibility of the counterparty. Given the current uncertainties in climate risk modelling, the CRVA should be interpreted as a component of the CVA that carries relatively large uncertainties. This means that financial institutions should be cautious when entering trades with a large CRVA despite the fact that the CRVA represents the fair value of external climate risk.

In the remainder of this section, we describe how the CRVA can play a role in risk management given the relatively large uncertainties in climate risk assessments. If credit instruments such as CDSs are available for a counterparty, the adjusted CVA should be used to hedge using these instruments instead of the ‘regular CVA‘ to protect against all sources of default risk, both climate-related and from other sources. In this way, any source of default risk (including climate risk of a potentially highly uncertain magnitude) is nullified.

Hedging climate risk becomes more complicated when counterparty-specific instruments are not available. Literature suggests that climate risk can also be hedged with products whose payoffs specifically depend on climate risk-related variables such as global temperature, but general green assets also offer some protection against climate risk [183, 53, 5]. Climate risk-related credit instruments and other insurance mechanisms can also be used to mitigate external climate risk over a portfolio [139]. A more detailed investigation in the sources of climate risk that contribute to the CRVA can help to determine what type of hedging instruments should be used in the case that counterparty-specific credit instruments are not available. Overall, a sufficient amount of climate risk hedging instruments should be bought to offset the CRVA in order to mitigate the external climate risk over portfolio's, with the aim of protecting against the uncertainty in the magnitude of associated expected losses. This summarises the general approach that we propose risk managers should apply to internalise external climate risk using the CRVA over their assets.

As discussed, regulators and sector-led initiatives can mitigate external climate risk and reduce its inherent uncertainties by improving the quality and scope of climate risk models, such as the NGFS scenarios. Once climate risk is fully internalised in the financial sector and these uncertainties would disappear, the importance of the CRVA would diminish and it would not be necessary to differentiate between ’regular CCR’ and ’climate-risk induced CCR’ which done in Figure 4.4. Until external climate risk can be assessed more accurately, we suggest that regulators can impose regulatory capital requirements based on the CRVA to protect financial institutions from these uncertainties.

Similar to how the CVA and sensitivities of the CVA to market risk factors are used to impose capital requirements under Basel 3 [18], we suggest that regulators can require financial institutions to hold a given fraction of the CRVA over their assets as a capital buffer in addition to the capital requirements regarding $\widehat{\mathrm{CVA}}$. This results in additional protection against external climate risk compared to the protection against ‘regular CCR‘ of the Basel 3 capital requirements, which we argue is necessary due to the high degree of uncertainty in the magnitude of climate risk. Moreover, the sensitivities of the CRVA to both climate-related and financial risk factors should be assessed and incorporated into capital requirements to protect against the large uncertainty in climate risk, similarly to how the sensitivities of the CVA to market risk and other risk factors are assessed, monitored, hedged and incorporated into regulatory capital requirements [77, 31, 230, 18].

Example 4.6 (CCR and climate risk management)**.** *We consider three different scenarios in which a regulatory capital charge is defined for a book with a single asset and corresponding hedging instruments: scenario (1) in Table 4.1 corresponds to the European option in Example 4.4, and scenarios (2) and (3) correspond to the green bond in Example 4.5. As a simplified version of the capital requirements under Basel 3, we introduce a hypothetical capital requirement to protect a financial institution against uncertainty in the value* *on their books, defined by*

$$K_{\text{Req}} := \alpha|\text{CVA} - H_{\text{CCR}}| + \beta|\text{CRVA} - H_{CR}|, \tag{4.36}$$

where H_{CCR} *and* H_{CR} *denote the value of the hedging instruments for 'regular CCR' and for climate risk, respectively. The parameters* $\alpha \in \mathbb{R}_{++}$ *and* $\beta \in \mathbb{R}_{++}$ *represent the uncertainty in the regular CVA and in the CRVA, respectively. Since external climate risk carries a high degree of uncertainty, we let* $\beta > \alpha$. *Notice that these are not necessarily related to the capital ratios prescribed under Basel 3 [18], and that we ignore market risk here. For a formulation of the capital requirements under Basel 3, see [3] for example.*

We consider three scenarios with a book that consists of a single asset at $t = 0$. *The value adjustments* CVA, $\widehat{\text{CVA}}$ *and* CRVA *are defined as in Definitions 4.6, 4.8 and 4.9, respectively. The CVA charge is used to hedge the CVA using CDSs at* $t = 0$, *so* $H_{\text{CCR}} = \text{CVA}$. *In scenario (1) and (2) in Table 4.1, no protection against external climate risk is bought at* $t = 0$, *meaning that* $H_{\text{CR}} = 0$ *and the expected future losses over the book discounted to* $t = 0$ *are given by* CRVA. *In scenario (3) however, the CRVA charge is used to hedge the CRVA at* $t = 0$, *resulting in no expected losses. In this scenario, we have* $H_{\text{CR}} = \text{CRVA}$.

For all three scenarios, we have

$$K_{\text{Req}} = \alpha(\text{CVA} - H_{\text{CCR}}) + \beta(\text{CRVA} - H_{\text{CR}}) = \alpha(\widehat{\text{CVA}} - H) + (\beta - \alpha)(\text{CRVA} - H_{\text{CR}}), \tag{4.37}$$

where $H + H_{\text{CCR}} + H_{\text{CR}}$. *The resulting capital charge in the three considered scenarios is computed for* $\alpha = 0.3$ *and* $\beta = 0.5$. *When* $\beta > \alpha$, *this should be interpreted as that a ratio of the total unhedged (climate-adjusted) CVA should be kept as a capital buffer, and the additionally, an additional fraction of the unhedged CRVA should be kept to protect against the uncertainty in climate risk.*

Table 4.1: *Illustration of three different scenarios where a financial institution enters a trade and chooses to hedge the external climate risk associated with the asset at* $t = 0$. *This impacts the capital requirement* K_{Req}, *as defined in Equation 4.37. The asset in scenario (1) is a European option with little external climate risk (Example 4.4) and the asset in scenarios (2) and (3) is a green bond with significant climate risk (Example 4.5). All displayed amounts are in EUR. Notice that the total hedging costs and expected losses add up to* $\widehat{\text{CVA}}$.

Scenario	Climate risk	CRVA hedge at $t = 0$	$\widehat{\text{CVA}}$	CRVA	Hedging costs at $t = 0$	Expected future losses	K_{Req}
(1)	Low	No	1996.01	199.24	$H_{\text{CCR}} = 1796.76$	CRVA = 199.24	99.62
(2)	High	No	8156.54	2635.04	$H_{\text{CCR}} = 5521.50$	CRVA = 2635.04	1317.52
(3)	High	Yes	8156.54	2635.04	$H = 8156.54$	0	0

Example 4.6 illustrates that high capital buffers are required when significant external climate risk is associated with an asset and there is no hedge against the climate risk, because it results in a relatively large degree of uncertainty in the future losses, as demonstrated in Table 4.1. This can be mitigated by (1) improving the quality and quantity of climate risk assessments, (2) hedging the relevant risk factors that drive climate risk, or (3) avoiding trades with counterparties that are significantly affected by external climate risk.

4.2.4 Climate risk EIF

Assuming independence between the default process and the price processes, the CRVA of an asset can be expressed in terms of the default rate, loss given default and their climate-adjusted counterparts as follows, combining Equations (4.15) and (4.28):

$$\begin{aligned}\text{CRVA}(t,T) &= \widehat{\text{CVA}}(t,T) - \text{CVA}(t,T)\\ &= \widehat{LGD}\int_t^T \text{EE}_t(s)\cdot\hat{\lambda}(s)e^{-\int_0^s \hat{\lambda}(u)\,\mathrm{d}u}\,\mathrm{d}s - LGD\int_t^T \text{EE}_t(s)\cdot\lambda(s)e^{-\int_0^s \lambda(u)\,\mathrm{d}u}\,\mathrm{d}s,\end{aligned}$$

which results in the expression of the CRVA as a single integral:

$$\mathrm{CRVA}(t,T) = \int_t^T \mathrm{EE}_t(s)\left[\widehat{LGD}\cdot\widehat{\lambda}(s)e^{-\int_0^s\widehat{\lambda}(u)\,\mathrm{d}u} - LGD\cdot\lambda(s)e^{-\int_0^s\lambda(u)\,\mathrm{d}u}\right]\mathrm{d}s. \tag{4.38}$$

In this way, the CRVA can be separated into the expected exposure and a term that depends on the climate adjustment to the CCR framework. This term quantifies the impact of climate-related adjustments on the CCR framework per unit of exposure, and we define this to be the Environmental Impact Factor (EIF) of an asset that is associated with climate risk (see Definition 4.11 for the Definition of a general EIF).

Definition 4.10 (Climate Risk Environmental Impact Factor)**.** *For an asset with maturity T, we define the stochastic process* $\mathrm{EIF}^{\mathrm{CR}}(t)$ *for* $t\in[0,T]$ *as follows:*

$$\mathrm{EIF}^{\mathrm{CR}}(t) := \widehat{LGD}\cdot\widehat{\lambda}(t)e^{-\int_0^t\widehat{\lambda}(s)\,\mathrm{d}s} - LGD\cdot\lambda(t)e^{-\int_0^t\lambda(s)\,\mathrm{d}s}. \tag{4.39}$$

We call this process the Environmental Impact Factor (EIF) associated with climate risk. This factor represents the relative impact of external climate risk at time $t\in[0,T]$ on the valuation of assets, without incorporated default information.

For deterministic default rate models, the EIF is a deterministic process. This model naturally generalises to a stochastic process if the default rates $\lambda(t)$ and $\widehat{\lambda}(t)$ are Cox processes (see Remark 4.3), but this is beyond the scope of this thesis.

The EIF can also be rewritten using $\widehat{\lambda}(t) = \lambda(t) + \lambda^{\mathrm{CR}}(t)$:

$$\mathrm{EIF}^{\mathrm{CR}}(t) = \left(\widehat{LGD}\cdot\widehat{\lambda}(t) - LGD\cdot\lambda(t)e^{\int_0^t\lambda^{\mathrm{CR}}(s)\,\mathrm{d}s}\right)e^{-\int_0^t\widehat{\lambda}(s)\,\mathrm{d}s},$$

where the term

$$e^{-\int_0^t\widehat{\lambda}(s)\,\mathrm{d}s} = \mathbb{P}_0[\widehat{t}_D > t]$$

represents the probability of no counterparty default up to time t, incorporating external climate risk. The filtration switching formula in Theorem 4.1 can be applied to compute the expected environmental impact factor at time $s\in[t,T]$ in the event that $\widehat{t}_D > t$, conditioning on the available default information at time $t\in[0,T]$:

$$\begin{aligned}\mathbb{1}_{\widehat{t}_D>t}\mathbb{E}\Big[\mathrm{EIF}^{\mathrm{CR}}(s)\Big|\widehat{\mathcal{G}}(t)\Big] &= \mathbb{1}_{\widehat{t}_D>t}\frac{\mathrm{EIF}^{\mathrm{CR}}(s)}{\mathbb{P}_t[\widehat{t}_D>t]} = \mathbb{1}_{\widehat{t}_D>t}\frac{\left(\widehat{LGD}\cdot\widehat{\lambda}(s) - LGD\cdot\lambda(s)e^{\int_0^s\lambda^{\mathrm{CR}}(u)\,\mathrm{d}u}\right)e^{-\int_0^s\widehat{\lambda}(u)\,\mathrm{d}u}}{e^{-\int_0^t\widehat{\lambda}(u)\,\mathrm{d}u}}\\ &= \mathbb{1}_{\widehat{t}_D>t}\left(\widehat{LGD}\cdot\widehat{\lambda}(s) - LGD\cdot\lambda(s)e^{\int_0^s\lambda^{\mathrm{CR}}(u)\,\mathrm{d}u}\right)e^{-\int_t^s\widehat{\lambda}(u)\,\mathrm{d}u},\end{aligned}$$

since $\mathbb{E}_t[\mathrm{EIF}^{\mathrm{CR}}(s)] = \mathrm{EIF}^{\mathrm{CR}}(s)$. From Definition 4.7, we conclude that

$$\mathbb{1}_{\widehat{t}_D>t}\mathbb{E}\Big[\mathrm{EIF}^{\mathrm{CR}}(s)\Big|\widehat{\mathcal{G}}(t)\Big] = \mathbb{1}_{\widehat{t}_D>t}\left(\widehat{LGD}\cdot\widehat{\lambda}(s) - \frac{LGD\cdot\lambda(s)}{\mathbb{P}_t[t_D^{\mathrm{CR}}>s]}\right)\mathbb{P}[\widehat{t}_D>s|\widehat{\mathcal{G}}(t)]. \tag{4.40}$$

Given that the LGD times the default rate can be interpreted as the instantaneous expected losses per unit of exposure [44], which we refer to as the *relative instantaneous impact* of credit risk for now, we observe that this expression can be interpreted as follows:

The expected future impact at time $s>t$ of external climate risk on the valuation of an asset, conditional on the information at time $t\in[0,T]$, is given by the difference between the climate-adjusted relative instantaneous impact and the relative instantaneous impact as derived from regular CVA frameworks, given that no external climate risk-related default has occurred until time s. This difference has to be multiplied by the probability that the counterparty will not default until time s.

The EIF defined in Equation (4.39) can be substituted into Equation 4.38, resulting in an concise expression for the CRVA as an integral over the climate risk EIF, multiplied by the expected exposure:

$$\mathrm{CRVA}(t,T) = \int_t^T \mathrm{EE}_t(s) \cdot \mathrm{EIF}^{\mathrm{CR}}(s)\,\mathrm{d}s. \tag{4.41}$$

In the next Section, we define a general formulation of environmental value adjustments, generalising expression 4.41. This expression allows for seamless integration into existing xVA frameworks such as the CVA framework from Section 4.1, since these already involve the modelling of the (expected) exposure, in the form of Exposure At Default (EAD) models.

4.3 General EVA and EIF formulation

The CRVA of an asset quantifies how its value is adjusted by internalising external climate risk, as expressed in Definition 4.9. In other words, it internalises a given aspect of climate change into the asset valuation process. This makes it an example of an Environmental Value Adjustment (EVA) that we introduced in Section 1.3, which we defined using the following Equation (see Definition 1.4):

$$\text{Adjusted value} = \text{Financial value} - \text{EVA}.$$

In the case of the CRVA, external climate risk the considered environmental aspect that the EVA model internalises. The financial value and climate risk-adjusted value are $\widetilde{V}(t,T)$ and $\widehat{V}(t,T)$ respectively, and the EVA (given by the CRVA) represents the adjustment to the fair value of an asset from incorporating external climate risk. In Chapter 5, we will define another EVA model, which attempts to internalise financed emissions and discuss how other environmental aspects of the relation between climate change and the financial sector (see Figure 1.1) can be internalised using EVA models.

The EVA models for the environmental aspects that we suggest in this thesis can all be formulated as integrals over an asset's lifetime of the exposure of assets multiplied by an environmental impact factor corresponding to the environmental aspect, which we define as follows:

Definition 4.11 (General Environmental Impact Factor)**.** *For a given environmental aspect, the Environmental Impact Factor (EIF) associated with an asset represents the monetary value of the impact that is associated with the asset per unit of exposure per year. Here, we consider the impact on/from that specific environmental aspect. Mathematically, the EIF is a process* $\mathrm{EIF}(t)$ *that is adapted to the climate-adjusted enlarged filtration* $\widehat{\mathcal{G}}(t)$*.*

The EIF that we defined to quantify the impact of climate risk on the CVA of an asset in Definition 4.10 is an example of a general EIF. Similarly to how the CRVA of an asset can be computed by integrating the climate risk EIF of an asset multiplied by the exposure over its lifetime (see Equation (4.41)), a general EVA can be computed by applying the same method to the corresponding EIF:

$$\mathrm{EVA}(t,T) = \int_t^T \mathbb{E}_t\left[e^{-r(s-t)}E(s) \cdot \mathrm{EIF}(s)\right]\mathrm{d}s, \tag{4.42}$$

where $E(t)$ is the exposure associated with an asset, r is the interest rate and T is the maturity of the asset. Assuming independence between the price processes driving the exposure and the processes that model the EIF leads to

$$\mathrm{EVA}(t,T) = \int_t^T \mathrm{EE}_t(s) \cdot \mathbb{E}_t[\mathrm{EIF}(s)]\,\mathrm{d}s. \tag{4.43}$$

As described at the end of Subsection 4.2.4, this allows for seamless integration with existing xVA models, for which the (expected) exposure is already modelled. The EIF can then be separately modelled to compute the EIF in Equation (4.43), although the assumption of independence between can lead to wrong-way risk from underestimation of the EVA when the drivers of the EIF and the exposure are positively correlated.

Because the climate risk EIF is a deterministic process, Equation (4.43) simplifies to Equation (4.41) in the case of the EVA model for climate risk. In Chapter 5, we introduce an EVA model for assets to internalise associated financed emissions and formulate an EIF corresponding to this model, which is not necessarily deterministic. After doing so, we propose how other environmental aspects can be internalised by constructing EVA models that can be formulated analogously with Equation (4.43), highlighting the role of the corresponding EIFs.

CHAPTER 5

FINANCED EMISSIONS EVA MODEL

In this chapter, we focus on the impact that financial institutions have on the climate through their investments, particularly focusing on CO2e emissions motivated by their crucial role in climate change, as discussed in Section 2.2. Reporting and accounting standards regarding ESG topics are necessary to incorporate environmental impact in financial decision-making, of which the PCAF Standard for financed emissions is the most important example. In Section 5.1, we formulate a framework to model financed emissions based on the PCAF Standard [165] and the Carbon Equivalence Principle (CEP), which also describes how contribution to CO2e emissions is transmitted between parties involved in a trade [132].

We propose a Financed Emissions Value Adjustment (FEVA) model in Section 5.2 that adjusts the value of assets based on the assumption that there are costs associated with the financed emissions, which we illustrate with two examples in Subsection 5.3.1. We assume a hypothetical scenario in which financial institutions offset their financed emissions using voluntary carbon credits to achieve net zero, but other perspectives on carbon pricing are possible to establish a FEVA, as discussed in Subsection 5.3.2. We then propose how the FEVA model can be applied by financial institutions to internalise their impact on the climate from financed emissions, drawing comparisons to how the CVA is used to internalise CCR in the financial sector in Section 5.3. We specifically outline how the FEVA can be hedged in Section 5.4, demonstrating that this mitigates both the financed emissions of the asset holder and their sensitivity to the underlying price of CO2e.

Lastly, we describe how the principles that lead to the FEVA can also be applied in the context of other types of environmental impact that can be associated with investments in Section 5.5. We demonstrate this by defining an EVA model for ‘financed biodiversity damage‘ in Subsection 5.5.1 analogously to the FEVA framework in this chapter and demonstrate that EVA models for broader types of environmental impact can be formulated in terms of an EIF that is associated with an asset. We discuss that this formulation allows for seamless integration with existing xVA models, but can also result in wrong-way risk.

5.1 Financed emissions and the PCAF Standard

In this Section, we derive a model for financed emissions that is based on the PCAF Standard [165] and consistent with the CEP [132]. These are computed from the so-called attribution factor from the PCAF Standard and require the future CO2e emissions of the counterparty to be modelled. With these two variables being modelled in Subsections 5.1.1 and 5.1.2, we construct a model for financed emissions that also incorporates CCR in Subsection 5.1.3 to apply in the EVA model in Section 5.2.

5.1.1 PCAF attribution factor

Both the PCAF Standard and the CEP are formulated based on the following principle [165, 132]:

By having financial exposure to a counterparty, their emissions are also linked to the asset owner. In other words, the responsibility for climate impact is transmitted through the exposure.

This is analogous to CCR, since the default risk of a counterparty is transmitted to the asset holder through the exposure that they have to the counterparty. More concretely, the PCAF Standard and CEP state that the underlying assumption is that the CO2e emissions of entities are enabled by the total value of their assets, and the attribution of an instrument to the total CO2e emissions of an entity is equal to its attribution to the total value of their assets on their balance sheet, i.e. their total liabilities (debt) and equity combined [72]. This is captured in the following definition, which is analogous to the attribution factor in the PCAF Standard, and is in line with the CEP:

Definition 5.1 (Attribution Factor)**.** *The attribution factor associated with an asset is defined as*

$$AF(t) := \frac{E(t)}{TV(t)}, \tag{5.1}$$

where $E(t)$ denotes the exposure as defined in Definition 4.1, and $TV(t)$ denotes the total value of the counterparty.

Depending on the financial instrument, the interpretation of the total value of the counterparty can vary, as summarised in Table 5.1. If the use of proceeds is known, a more precise interpretation is given and only the emissions associated with the use of proceeds are considered.

Table 5.1: *Overview of different interpretations of the total value of a counterparty.*

Financing type	Interpretation of $TV(t)$
Private company finance	Total company equity + debt
Listed company finance	Enterprise value including cash
Project finance	Total project equity + debt
Real estate	Property value at origination
Motor vehicle loan	Vehicle value at origination
Sovereign debt	Purchase Power Parity-adjusted GDP

From now on, we refer to $TV(t)$ as the total value of the counterparty, even if a specific use of proceeds is known. We also assume that $TV(t)$ is modelled for $t \in [0, T]$ using $\mathbb{R}_{++}$-valued stochastic processes adapted to the filtration $\mathcal{F}(t)$, meaning that $AF(t)$ is also adapted to the filtration $\mathcal{F}(t)$ and finite-valued.

Expressing the attribution factor in terms of the exposure $E(t) = (V(t,T))^+$ results in the following implications. First of all, the attribution factor is floored at zero. This is a desirable property of the attribution factor, because a negative attribution factor would imply that a borrower (from whose perspective the value of an asset is negative) disables emissions of the lender by entering a contract, which is a questionable implication at best. Secondly, the total attribution factor over all debt and equity on the balance sheet of an entity is 1, resulting in consistent accounting of attribution factors without double counting.

The PCAF Standard does not explicitly address collateralisation and primarily focusses on uncollateralised assets [165]. The CEP states that climate impact does not transfer through fully collateralised assets, since an asset does not enable the counterparty to emit if the net position on the balance sheet is zero [132]. This is similar to how credit risk is not transmitted between parties through collateralised trades [101, 23]. Based on these observations, we proceed to focus on uncollateralised assets in the remainder of this thesis, analogously to Chapter 4.

5.1.2 Estimating counterparty emissions

For the counterparty of a given asset, we introduce a stochastic process to model their emissions. Later, we will propose how these can be internalised using an EVA model.

Definition 5.2 (Counterparty GHG emissions, carbon intensity)**.** *We define $G(t)$ to be the stochastic process that describes the GHG emissions of a counterparty that are not yet economically internalised (see Remark 5.1), expressed in tonnes of CO2e. This assumes that the counterparty does not default. We define the carbon intensity of the counterparty by*

$$I(t) := \frac{G(t)}{TV(t)}, \tag{5.2}$$

which represents the annual amount of CO2e emissions per unit of total value of the counterparty.

This metric is generally seen as a good indictor for the contribution of a company to the climate [229].For our purposes, we define the process $I(t)$ to be driven by the following SDE:

$$\mathrm{d}I(t) = \mu_I I(t)\mathrm{d}t + \sigma_I I(t)\mathrm{d}W_I(t), \tag{5.3}$$

where $\mu_I \in \mathbb{R}$ and $\sigma_I \in \mathbb{R}_{++}$ are the growth rate and volatility of the counterparty's emissions respectively. Moreover, $W_I(t)$ is a Brownian motion with respect to $\mathcal{F}(t)$. The initial value is given by $I(t_0) = I_0 \in \mathbb{R}_{++}$ for some $t_0 \in [0,T]$. Equation (5.3) describes a Geometric Brownian Motion (GBM). The SDE is solved by [161]

$$I(t) = I_0\, e^{(\mu_I - \frac{\sigma_I^2}{2})t + \sigma_I W_I(t)}. \tag{5.4}$$

Instead of this simple model for the carbon intensity $I(t)$, the intensity can also be computed from projections of the future emissions $G(t)$ and the total value $TV(t)$. A counterparty-agnostic model for $G(t)$ can be based on the so-called 'carbon law' [182], which proposes a global path towards net zero with absolute emissions being halved every decade. Country-, sector- or counterparty-specific information can be used to adjust the growth rate of the carbon intensity in Equation (5.3), e.g. sustainability targets of a counterparty, particularly their past emissions. Controversially, ESG scores have not been found to be a predictor for company emissions [205, 65] and should therefore not be used to adjust the emissions growth rate. More specific models for future counterparty emissions can also be used instead of a GBM, including negative emissions of the counterparty, which are not included in the suggested GBM in Definition 5.2.

Specifically, $G(t)$ can be modelled based on the past emissions of a counterparty. The PCAF Standard describes different methods to obtain estimates for past emissions of a company, labelled by the perceived quality of the estimates, as illustrated in Table 5.2, where the different data scores are explained in the case of corporate debt and equity. For different asset classes, data quality scoring can differ slightly, but the same principles apply when it comes to the way that past emissions are estimated [165]. Reported emissions are the preferred data source, and physical data (production, energy consumption, etc.) are used otherwise, making use of so-called emission factors, which are the average emissions associated with a unit of production or consumption. These can also be applied to financial data (e.g. revenue), but this is the least preferred method to estimate $G(t)$. Lower data scores indicate more accurate estimates of past emissions and allow for more accurate modelling of future emissions $G(t)$. Regulators also introduce reporting standards and frameworks to improve the quality and availability of ESG data, for example with the CSRD [73, 165].
Emissions are often estimated in practice based on assumptions and approximations, particularly in the case of complex value chains [223]. Transparency on the emissions of companies across the value chain are crucial in determining financed emissions, which is why regulations on climate reporting such as the CSRD help improve the accuracy in financed emissions calculations across the economy. This is necessary for regulators to establish regulations and frameworks, such as carbon pricing schemes or the EVA model that we propose in Section 5.2, on the topic of emissions and internalise climate change in doing so. From now on, we assume that future counterparty emissions $G(t)$ or their emission intensity $I(t)$ can be modelled by a stochastic process that is adapted by $\mathcal{F}(t)$.

[1] In this case, the fraction $\frac{G(t)}{TV(t)} = I(t)$ is determined with a sector-specific asset turnover ratio and emission factor.

Table 5.2: *Explanation of PCAF data quality scores for estimating absolute emissions. These estimation methods are specifically for corporate debt and equity and are slightly different for other asset classes [165].*

PCAF Score	Method to estimate counterparty emissions $G(t)$
1	Reported absolute emissions, verified by a third party.
2	Unverified reported absolute emissions.
3	Total production/energy consumption times emission factor.
4	Total revenue times sector-average emission intensity.
5	*No counterparty-specific estimate can be made*[1].

Remark 5.1 (Internalised emissions)**.** *As mentioned in Definition 5.2, we only focus on GHG emissions that need to be internalised. We will propose an EVA model in Section 5.2 with the purpose of internalising financed emissions, but it is possible that some of the absolute or negative emissions of a counterparty are already internalised. Examples of such situations include that (1) when a counterparty falls under a regulatory carbon pricing scheme, the covered absolute emissions are already internalised from an economical perspective since the counterparty is charged for them. (2) Negative emissions from a counterparty can be excluded if they already received financial incentives (e.g. subsidies, funding discounts) from external parties to finance the negative emissions, in which case it can be argued that they are already internalised. (3) If a counterparty generates voluntary carbon credits, the generated negative emissions are also already economically internalised because of the received funding.*

In situations like this, emissions that have already been internalised could be excluded from the model for $G(t)$ that will be used to calculate the EVA if the purpose of the EVA model is to internalise the covered emissions. A calculation of non-internalised emissions $G(t)$ is given in Example 5.1. From now on, we simply refer to $G(t)$ as the counterparty emissions, implying that their emissions that are already internalised are already excluded from the model.

5.1.3 Financed emissions model

We incorporate the possibility of counterparty default with the climate-adjusted default time $\hat{t}_D$, as introduced in Definition 4.7. Notice that we incorporate climate risk in the counterparty default model for completeness, incorporating all the forward-looking information available. The indicator function is included because there is no exposure and therefore no contribution to counterparty's emissions after they default.

Definition 5.3 (Financed Emissions)**.** *The financed emissions at time $t \in [0, T]$ associated with an asset with exposure $E(t)$ are given by*

$$FE(t) = \mathbb{1}_{\hat{t}_D > t} \cdot AF(t) \cdot G(t), \tag{5.5}$$

where $\hat{t}_D$ is the default time of the counterparty. $G(t)$ represents the GHG emissions of the counterparty that need to be considered for the financed emissions, and $AF(t)$ is the attribution factor of the asset.

The inclusion of the indicator function $\mathbb{1}_{\hat{t}_D > t}$ makes the process $FE(t)$ adapted to the filtration $\widehat{\mathcal{G}}(t)$, but not to the default-free filtration $\mathcal{F}(t)$.

We provide an example of a calculation of financed emissions in Example 5.1, where the financed emissions associated with three different counterparties are computed, adjusting for emissions that are already economically internalised by carbon credits or allowances, as discussed in Remark 5.1.

The Definitions in Equations (5.1) and (5.2) can be inserted in Equation (5.5) to obtain

$$FE(t) = \mathbb{1}_{\hat{t}_D > t} \cdot E(t) \cdot I(t). \tag{5.6}$$

In Equation (5.6), it can be observed that the financed emissions are proportional to exposure to a counterparty, implying that financial institutions can limit their financed emissions by limiting their exposure to entities with a high carbon intensity.

Example 5.1 (Financed emissions calculation)**.** *We provide an example that is adapted from the PCAF Standard document on financed emissions [165]. A financial institution invests in three companies: Company A is a forestry company. Company B is a large industrial company that falls under a cap and trade system and has to buy allowances for its scope 1 emissions. It also uses voluntary carbon credits to achieve its ESG goals. Company C builds renewable energy installations, accompanied by some forestry projects. Company A and C are able to sell voluntary carbon credits from their CO2e abatement activities. For a given year t, their CO2e accounting is summarised in Table 5.3. The emissions that are already internalised (see Remark 5.1) are subtracted from the net emissions to compute $G(t)$. The total financed emissions $FE(t)$ over al are 5010 tonnes of CO2e in this Example, computed from Equation* (5.5) *(assuming no counterparty defaults). Financial institutions are also required to report on the totals of the portfolio over the different scopes in the bottom row by the PCAF Standards [165].*

Table 5.3: *CO2e accounting for a given year of a portfolio of a financial institution that invests in three companies. The financed emissions per category of the portfolio are computed using the attribution factors. Additionally, the non-internalised emissions $G(t)$ are computed from the emission categories that the counterparties are required to report on [165]. All emissions are denoted in tonnes of CO2e, which is also the amount of CO2e that an allowance or voluntary carbon credit represents.*

Counterparty	Attribution factor $AF(t)$	Scope 1 emissions	Scope 2 emissions	Scope 3 emissions	Negative emissions	Retired allowances	Retired credits	Generated credits	Non-internalised emissions $G(t)$
Company A	0.1	1000	100	5000	20000	0	0	10000	-3900
Company B	0.25	20000	5000	30000	0	20000	25000	0	10000
Company C	0.2	5000	0	10000	1000	0	0	500	14500
Total financed emissions		6100	1260	10000	2200	5000	6250	1100	5010

Notice that Company B in Example 5.1, despite its emission-heavy nature, contributes less non-internalised total emissions than Company C because Company B retires carbon credits and complies with a cap and trade system to compensate financially for its emissions. Also note that the non-internalised negative emissions of Company A are limited because a large portion of the abated emissions are used to generate carbon credits. This illustrates the importance of paying attention to internalised emissions, as discussed in Remark 5.1. Not excluding these from the calculation of $FE(t)$ and therefore the EVA model that we define in Section 5.2 can result in these emissions being 'doubly internalised', e.g. because a counterparty would be required to buy allowances to cover their emissions, but then again be charged for these emissions in the form of the EVA, as we will explain in Section 5.3.2.

5.2 Financed Emissions Value Adjustment (FEVA)

In this section, we define an EVA model by associating costs with financed emissions. We let $C(t)$ be a general price tag associated with a tonne of (financed) CO2e emissions, which can be chosen based on the different perspectives on carbon pricing that we presented in Section 2.3 and Chapter 3. Given that voluntary carbon credits are increasingly considered by companies and regulators as a an option to offset CO2e emissions [199, 142], we assume for now that the carbon price $C(t)$ is the price of a voluntary carbon credit, and therefore the costs of offsetting one tonne of (financed) CO2e emissions assuming that the credit is bought and then retired immediately. It is possible that once the VCM matures (see Subsection 3.1.2), carbon credits become a valid instrument to achieve net zero for financial institutions to reach net zero. This can be done voluntarily, but it is also possible that regulators require companies to offset their financed emissions using high-quality carbon credits once the VCM matures in the future.

5.2.1 FEVA model formulation

The idea behind the EVA model that we define is as follows: multiplying the annual financed emissions $FE(t)$ associated with an asset by the costs $C(t)$ that we associate with financed emissions results in the annual costs that are associated with the financed emissions of an asset. This can be integrated over the lifetime of the asset to obtain the total costs of the financed emissions associated with an asset. In the following definition, we define the Financed Emissions Value Adjustment (FEVA) to be the expected value of these total costs, discounted to the present.

Definition 5.4 (Financed Emissions Value Adjustment)**.** *The Financed Emissions Value Adjustment (FEVA) of an asset with maturity T at time $t \in [0, T]$ is defined as*

$$\mathrm{FEVA}(t,T) := \mathbb{E}_t\left[\int_t^T e^{-r(s-t)} C(s) \cdot FE(s)\,\mathrm{d}s\right], \tag{5.7}$$

where r is the interest rate, $FE(s)$ denotes the financed emissions associated with an asset at time $s \in [t, T]$ (see Definition 5.3) and $C(s)$ denotes the costs associated with financed emissions. If the (climate) risk-adjusted value of the asset is given by $\widehat{V}(t, T)$ (see Definition 4.9), we define the financed emissions-adjusted value as

$$\widehat{V}'(t,T) := \widehat{V}(t,T) - \mathrm{FEVA}(t,T). \tag{5.8}$$

It should be noted that the costs $C(s)$ of offsetting a tonne of financed emissions at time $s \in [t, T]$ are discounted to time t to obtain the total expected costs of the financed emissions, discounted to the present time t.

It should be noted that a FEVA of zero does not necessarily imply that an asset achieves net zero or vice versa, as is illustrated in the following example:

Example 5.2 (Net zero is not equivalent to no FEVA)**.** *Consider an asset with maturity T and deterministic financed emissions. given by*

$$FE(t) = FE_0(1 - \frac{2t}{T}),$$

where $FE_0 \in \mathbb{R}_{++}$. The cumulative financed emissions over the lifetime of the asset are therefore given by

$$\int_0^T FE(t)\,\mathrm{d}t = 0.$$

However, when we assume for simplicity that the carbon credit price is mean-reverting and satisfies $\mathbb{E}_0[C(t)] = C_0 \in \mathbb{R}_{++}$, Equation (5.7) results into

$$\mathrm{FEVA}(0,T) = C_0 \cdot FE_0 \int_0^T e^{-rt}(1 - \frac{2t}{T})\,\mathrm{d}t > 0,$$

given that $r > 0$ (as illustrated by the differently-sized shaded regions in Figure 5.1). The discount factor affects the relative value differences of financed emissions between difference moments, which makes the FEVA not a suitable measure for directly assessing the total climate impact of assets.

Example 5.2 illustrates that the FEVA is not a suitable indicator for the cumulative financed emissions associated with an asset, but it also demonstrates that zero cumulative financed emissions do not imply that an asset is net zero at a given point in time. Until $t = \frac{T}{2}$, the asset owner should offset the financed emissions in order to achieve net zero. Only at precisely $t = \frac{T}{2}$ is the asset net zero, after which the asset results in negative financed emissions. The asset owner can then use these to offset other financed emissions on their books in order to achieve net zero. In conclusion, the FEVA is not an indicator of the current climate impact from financed emissions, but a forward-looking measure regarding the expected cumulative financed emissions and their total associated costs over the lifetime of an asset.

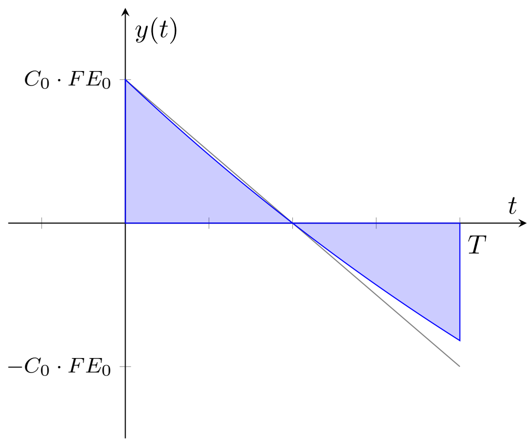


Figure 5.1: *Plot of the annual expected discounted annual costs associated with the financed emissions, given by* $y(t) := C_0 \cdot FE_0 \cdot e^{-rt}(1 - \frac{2t}{T})$. *The shaded region represents the cumulative expected costs associated with the financed emissions, i.e.* $\text{FEVA}(0,T) = \int_0^T y(t)\,\mathrm{d}t$. *The grey line represents the non-discounted expected costs to illustrate the impact of the discount factor* r.

5.2.2 Financed emissions EIF

From now on, we assume independence between the price processes driving the exposure and the other counterparty-specific processes for simplicity[2]. The wrong-way risk and right-way risk that this might induce is discussed in Subsection 5.2.3. We can substitute Equation (5.6) into the definition of the FEVA to obtain

$$\text{FEVA}(t,T) = \mathbb{E}_t\left[\int_t^T e^{-r(s-t)} C(s) \cdot \mathbb{1}_{\hat{t}_D > s} \cdot AF(s) \cdot I(s)\,\mathrm{d}s\right], \tag{5.9}$$

which we can rewrite as

$$\text{FEVA}(t,T) = \int_t^T \text{EE}_t(s) \cdot \mathbb{E}_t\left[\mathbb{1}_{\hat{t}_D > s} \cdot C(s) \cdot I(s)\right]\mathrm{d}s, \tag{5.10}$$

making use of the assumed independence and Fubini's Theorem (see Theorem B.1) and the independence of $E(s)$ from the other processes. This results in an expression for the FEVA of an asset as an integral over its lifetime of the expected exposure multiplied by another term, which was also the case for the formulation of the CRVA in Equation (4.38). Analogously to our formulation of the CRVA, we introduce the following Definition to quantify the environmental impact associated with an asset:

Definition 5.5 (Financed Emissions Environmental Impact Factor)**.** *For an asset with maturity* T*, we define the stochastic process* $\text{EIF}^{\text{FE}}(t)$ *for* $t \in [0,T]$ *as follows:*

$$\text{EIF}^{\text{FE}}(t) := \mathbb{1}_{\hat{t}_D > t} \cdot C(t) \cdot I(t). \tag{5.11}$$

We call this process the Environmental Impact Factor (EIF) associated with financed emissions, which represents the annual costs associated with financed emissions per unit of exposure associated with an asset. This process is adapted to the filtration $\widehat{\mathcal{G}}(t)$.

With this definition, the FEVA can be rewritten as

$$\text{FEVA}(t,T) = \int_t^T \text{EE}_t(s) \cdot \mathbb{E}_t[\text{EIF}^{\text{FE}}(s)]\,\mathrm{d}s, \tag{5.12}$$

[2]To be precise, we assume that the process $E(t)$ is independent from (1) the NHPP that models $\hat{t}_D$, (2) the price process $C(t)$ of financed emissions and (3) the emission intensity $I(t)$ of the counterparty.

which is analogous to the proposed formulation of a general EVA model in Equation (4.43). Notice that the EIF does not depend on the moment at which the FEVA is computed, in contrast to the climate risk EIF in Definition 4.10. By assuming pairwise independence between the default process that models $\widehat{t}_D$, $C(t)$ and $I(t)$, the expectation of the EIF can be rewritten as

$$\mathbb{E}_t\left[\mathrm{EIF}^{\mathrm{FE}}(s)\right] = \mathbb{E}_t\left[\mathbb{1}_{\widehat{t}_D>s}\right] \cdot \mathbb{E}_t\left[C(s)\right] \cdot \mathbb{E}_t\left[I(s)\right] = e^{-\int_0^s \widehat{\lambda}(u)\,\mathrm{d}u} \cdot \mathbb{E}_t\left[C(s)\right] \cdot \mathbb{E}_t\left[I(s)\right], \tag{5.13}$$

making use of the cumulative density function of $\widehat{t}_D$, as given in Equation (4.23). In Example 5.3, we investigate the wrong-way risk that arises by assuming independence between the carbon intensity and the carbon price.

5.2.3 Wrong-way risk and right-way risk in the FEVA

With the large number of variables involved in the FEVA, close attention should be paid to the dependences between the different processes in Equation (5.9). We assumed independence between the price processes that drive the exposure $E(t)$ and the other processes, but this can potentially lead to wrong-way risk if the asset is financially linked to the performance of the counterparty or the carbon price $C(t)$. We illustrate this by considering a European option, where the underlying asset is the counterparty's own stock.

In Example 4.1, we already discussed that assuming independence between the default process and the exposure can lead to wrong-way or right-way risk in the context of the CVA. In the context of the FEVA (which depends on $\mathbb{1}_{\widehat{t}_D>t}$, see Equation (5.9)), assuming independence between the exposure of an asset and the default process can lead to an underestimation (overestimation) of the value adjustment if the exposure is positively (negatively) linked to the general financial performance and particularly the credibility of the counterparty, as is the case for a call (put) option.

The relation between carbon intensity and financial performance of companies has been studied in numerous different contexts with varying results, finding that the relation depends on factors including but not limited to their total value and revenue, sector and region [41, 13, 103, 62]. Nevertheless, a negative correlation between carbon intensity and financial returns is generally observed. Further investigation of the relation between the carbon intensity and financial performance of counterparties is beyond the scope of this thesis, but deserves attention when it is appropriate to presume dependence between the exposure of an asset and the carbon intensity of the counterparty. The generally assumed negative correlation between the financial performance and carbon intensity of the counterparty leads to an underestimation (overestimation) of the FEVA and therefore results in wrong-way (right-way) risk in the case of a put (call) option, because an increase in carbon intensity is associated with an increase (decrease) in the value of the asset.

The impact of the carbon price $C(t)$ on the general financial performance of a counterparty can also be complex to model and depends on the applied perspective on the carbon price $C(t)$ that is used to construct the FEVA in Definition 5.4, which we investigated in Section 2.3 and Chapter 3. Regardless of the specific perspective on carbon pricing, an increase in the carbon price $C(t)$ will generally have a negative (positive) impact on a emissions-heavy (climate-friendly) counterparty, affecting the credibility as well as possibly the value of the asset if it is financially linked to the counterparty, as is the case for an option on the counterparty's stock. Moreover, an increase in the carbon price $C(t)$ will generally lead to a decrease in carbon intensity $I(t)$ if the carbon price affects the counterparty. The magnitude of the impact of carbon pricing on emissions has been widely studied[3] [58, 174] and illustrated in Example 5.3. In Figures 5.2a and 5.2b, we summarise our discussion on the dependencies between the different variables in Equation (5.9) and the implications of assuming pairwise independence. An underestimation (overestimation) of the FEVA leads to wrong-way (right-way) risk, of which we discuss the implications for how the FEVA should be applied in Section 5.3.

[3] In establishing carbon pricing schemes, regulators have to determine the required carbon price to achieve the environmental goals. This makes determining the impact of carbon pricing on emissions a central component in shadow pricing, which we discussed in Section 2.3.

	$E(t)$	$\mathbb{1}_{\hat{t}_D>t}$	$I(t)$	$C(t)$
$E(t)$				
$\mathbb{1}_{\hat{t}_D>t}$	⇑			
$I(t)$	⇓	⇓		
$C(t)$	⇓ (⇑)	⇓ (⇑)	⇓	

(a) Correlation if the exposure of an asset is positively correlated with the general financial performance of the counterparty.

	$E(t)$	$\mathbb{1}_{\hat{t}_D>t}$	$I(t)$	$C(t)$
$E(t)$				
$\mathbb{1}_{\hat{t}_D>t}$	⇓			
$I(t)$	⇑	⇓		
$C(t)$	⇑ (⇓)	⇓ (⇑)	⇓	

(b) Correlation if the exposure of an asset is negatively correlated with the general financial performance of the counterparty.

Figure 5.2: *Overview of the dependencies between the processes in the FEVA model, leading to wrong-way risk or right-way risk if two processes are assumed to be pairwise independent. If two processes are generally positively (negatively) correlated, assuming that they are independent will lead to an underestimation (overestimation) of the FEVA, which results in wrong-way (right-way) risk, as indicated with the arrow ⇑ (⇓). The relation between the carbon price $C(t)$ and the financial performance of the counterparty depends on whether they are emission-heavy or climate-friendly. The impact of $C(t)$ on $E(t)$ and $\mathbb{1}_{\hat{t}_D}$ is given in the Tables for an emission-heavy counterparty, and the impact for a climate-friendly counterparty is given in brackets.*

In Figure 5.2, it can be observed that assuming pairwise independence between the processes $\mathbb{1}_{\hat{t}_D>t}$, $I(t)$ and $C(t)$ that make up the EIF generally leads to overestimation of the FEVA and therefore leads to right-way risk. The dependency between $E(t)$ and $\text{EIF}^{\text{FE}}(t)$ is more ambiguous and is highly dependent on how the returns of the asset depend on counterparty-specific variables and on the carbon price. For the remainder of the thesis, we only consider assets with a financial value that has limited sensitivity to counterparty-specific variables such as the carbon intensity and credibility, as well as the carbon price $C(t)$. We will assume independence between $E(t)$ and the other processes, which results in limited wrong-way and right-way risk in the FEVA for the considered assets. In Example 5.3, we investigate the impact of dependencies between the other variables on wrong-way risk and right-way risk in the FEVA.

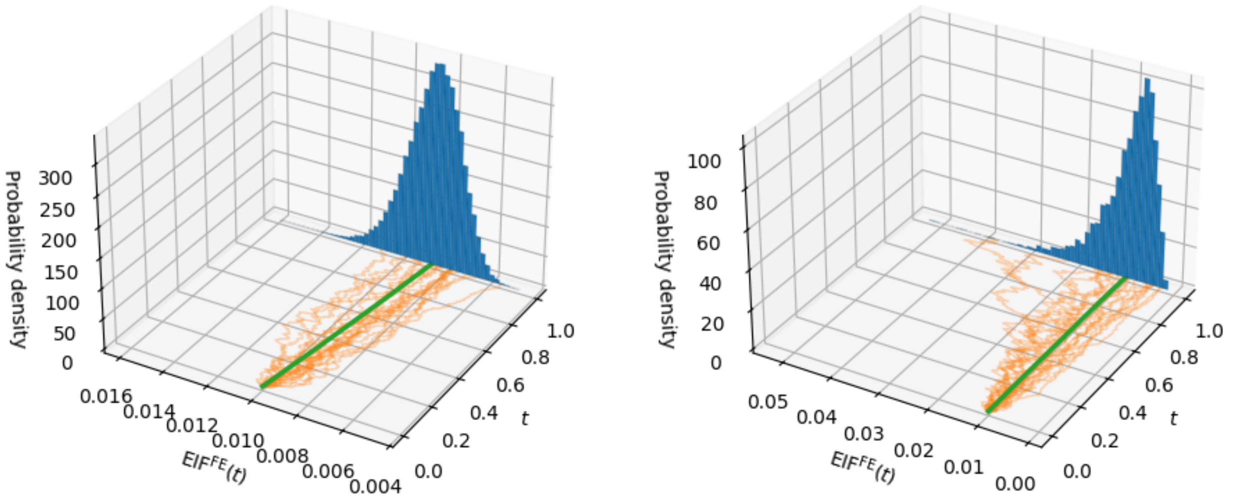
Example 5.3 (Dependency of carbon intensity on the carbon price)**.** *We illustrate the impact of the assumption of independence between $I(t)$ and $C(t)$ in this Example using a Monte Carlo simulation. We assume that the carbon price is given by the CTD price of a voluntary carbon market of dimension $d = 1$, meaning that the carbon price is $C(t) = H(t) = C_1(t) = e^{X_1(t)}$ (see Equation (3.12)), where $X_1(t)$ is an Ornstein-Uhlenbeck process. We model the emission intensity $I(t)$ of the counterparty using the geometric Brownian motion in Equation (5.3) and assume that the counterparty does not default, i.e. $\hat{\lambda}(t) = 0$ for all $t \in [0, T]$. We introduce interdependence by imposing correlation $\rho \in [-1, 1]$ between the Wiener processes that drive the processes $X_1(t)$ and $I(t)$, i.e. $\mathrm{d}W_1(t)\mathrm{d}W_I(t) = \rho \mathrm{d}t$. In Figure 5.3, the impact of ρ on the distribution of $\text{EIF}^{\text{FE}}(t) = C(t) \cdot I(t)$ is given*[4]*.*

We observe that the EIF, being the product of an exponential OU process and a GBM, follows a positively skewed distribution if $C(t)$ and $I(t)$ are independent (which is the case when $\rho = 0$). A negative correlation between the Wiener processes makes the tail of the distribution disappear and results in a lower expected EIF. This illustrates the wrong-way risk that can result from assuming independence between the carbon price and carbon intensity of a counterparty, although the impact of the correlation is significantly lower than one standard deviation of $\mathbb{E}_0[\text{EIF}^{\text{FE}}(t)]$.

5.3 Examples and interpretation of the FEVA

In this section, we give examples of the calculation of the FEVA of assets, applying the model to both the European option in Example 4.4 and the green bond in Example 4.5. Then, we discuss how the FEVA

[4] Notice that $\mathbb{1}_{\hat{t}_D>t} = 1$ for all $t \in [0, T]$, which simplifies the expression for $\text{EIF}^{\text{FE}}(t)$.



(a) *Correlation* $\rho = -0.9$ *results into* $\mathbb{E}_0[\mathrm{EIF}^{\mathrm{FE}}(T)] \approx 8.7 \cdot 10^{-3}$ *and* $\sqrt{\mathbb{V}\mathrm{ar}_0[\mathrm{EIF}^{\mathrm{FE}}(T)]} \approx 1.2 \cdot 10^{-3}$.

(b) *Correlation* $\rho = 0.9$ *results into* $\mathbb{E}_0[\mathrm{EIF}^{\mathrm{FE}}(T)] \approx 9.9 \cdot 10^{-3}$ *and* $\sqrt{\mathbb{V}\mathrm{ar}_0[\mathrm{EIF}^{\mathrm{FE}}(T)]} \approx 5.5 \cdot 10^{-3}$.

Figure 5.3: *Illustration of the impact of the correlation parameter* ρ *on the paths of* $\mathrm{EIF}^{\mathrm{FE}}(t) = C(t) \cdot I(t)$. *In the horizontal plane, 20 sample paths of* $\mathrm{EIF}^{\mathrm{FE}}(t)$ *are shown in orange, as well as* $\mathbb{E}_0[\mathrm{EIF}^{\mathrm{FE}}(t)]$ *as determined from the Monte Carlo simulation of* 10^5 *paths in green. The approximated probability density function of* $\mathrm{EIF}^{\mathrm{FE}}(T)$ *is shown in blue for* $T = 1$. *The parameter configuration is* $I_0 = 5 \cdot 10^{-4}$, $\mu_I = -0.07$, $\sigma_I = \sigma_1 = 0.3$, $\theta_1 = 0.5$, $\mu_1 = x_1 = \log(20)$ *and* ρ *is chosen differently in each of the simulations.*

can be interpreted, depending on the different perspectives on carbon pricing that we introduced in Section 2.3 and Chapter 3, focusing on the interpretation of $C(t)$ as the price of a voluntary carbon credit that is used to offset a tonne of financed CO2e emissions. We propose different purposes of the FEVA by making a comparison to how other xVA models are applied in practice (see Subsection 1.1), looking to answer Research Questions 1, 2 and 3.

5.3.1 FEVA for a green bond and a European option

In both Example 5.4 and Example 5.5, we let the emission intensity of the counterparty be given by the solution to the SDE of a GBM[5] in Equation (5.4):

$$I(t) = I_0\, e^{(\mu_I - \frac{\sigma_I^2}{2})t + \sigma_I W_I(t)}, \tag{5.14}$$

where $I_0 \in \mathbb{R}$ is the initial emission intensity of the counterparty. The parameters μ_I and σ_I represent the growth rate and volatility of the counterparty's future negative emissions, and $W_I(t)$ is a Wiener process, as in Equation (5.4). We let the carbon price $C(t)$ represent the costs of offsetting financed emissions using voluntary carbon credits, and we assume a VCM market model of $d = 1$ (see Section 3.2) to compute the FEVA, meaning that

$$C(t) = e^{X(t)}, \tag{5.15}$$

with

$$\mathrm{d}X(t) = \theta(\mu - X(t))\mathrm{d}t + \sigma \mathrm{d}W(t), \tag{5.16}$$

where $X(0) = x \in \mathbb{R}$ is the initial log-price, and $\theta, \sigma \in \mathbb{R}_{++}$ and $\mu \in \mathbb{R}$ are the mean-reverting speed, volatility and long-term mean of $X(t)$, respectively. The Wiener processes $W(t)$ and $W_I(t)$ are assumed to be independent. We assume independence between the default process and the other processes, and a constant

[5] Technically, $I(t)$ is not a GBM if we allow $I_0 \leqslant 0$ since a GBM is strictly positive, but we can apply Equation (5.14) to model emissions regardless of the sign of I_0.

climate-adjusted default rate $\widehat{\lambda}(t) = \widehat{\lambda} \in \mathbb{R}$ for simplicity.

In both examples, we investigate the costs of the financed emissions by defining the process

$$Y_0(t) := e^{-rt}E(t) \cdot \mathrm{EIF}^{\mathrm{FE}}(t), \tag{5.17}$$

and the random variable

$$\mathcal{Y} := \int_0^T Y_0(t)\,\mathrm{d}t, \tag{5.18}$$

such that

$$\mathrm{FEVA}(0,T) = \mathbb{E}_0\left[\int_0^T e^{-rt}E(t) \cdot \mathrm{EIF}^{\mathrm{FE}}(t)\right] = \mathbb{E}_0[\mathcal{Y}].$$

The stochastic process $Y_0(t)$ represents the annual costs of offsetting financed emissions, and the random variable $\mathcal{Y}$ represents the total costs of offsetting the financed emissions over the period $[0,T]$, so the distribution of $\mathcal{Y}$ gives information about the uncertainty in the FEVA, as illustrated in Examples 5.4 and 5.5.

Example 5.4 (FEVA of a green bond). *As in Example 4.5, we consider a green bond on a notional $N \in \mathbb{R}_{++}$ that prescribes coupon payments with a rate of $K \in \mathbb{R}_{++}$ to the holder on a set of moments $T_i = i$ for $i = 1,\ldots,T$. We consider a notional of $N =$ €1 million. We take the perspective of a financial institution that buys the bond to lower the financed emissions on their books, since the counterparty is an independent power producer that uses the bond to build an offshore wind park. As a result of the negative financed emissions, the financial institution has to buy fewer voluntary carbon credits to achieve net zero over their entire portfolio.*

We choose parameter values so that $I(t)$ stays around I_0 and is not too volatile. We investigate the distribution of the costs of the financed emissions associated with the green bond. The distribution of $Y_0(t)$ and $\mathcal{Y}$ are shown in Figure 5.4.

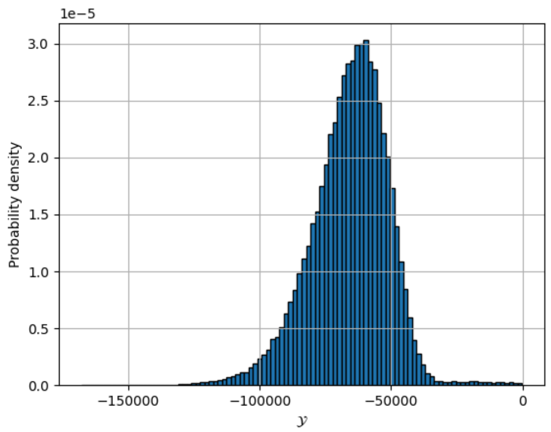
In Figure 5.4b, it can be observed that the process $Y_0(t)$ follows a bimodal distribution, with the peak at $Y_0(t) = 0$ coming from default events. In Figure 5.4a, it can be observed that the environmental impact $\mathcal{Y}$ follows an unimodal distribution, with the mean being $\mathrm{FEVA}(0,T) = \mathbb{E}_0[\mathcal{Y}] = -$€65949.57, which is a significant adjustment to the bond with risk-free value $V(0,T) =$ €1 million[6].

The green bond in Example 5.4 has a significant negative-valued FEVA at $t = 0$. This adjustment can be transmitted to the bond issuer in the form of an additional fee that is paid to the issuer when buying the bond, motivated by the fact that it creates negative financed emissions for the financial institution that buys the bond. The financial institution is willing to pay the additional fee of $-\mathrm{FEVA}(0,T)$, because they would need to buy more voluntary carbon credits to offset the financed emissions on their books if they would not have bought the green bond. In this way, the negative-valued FEVA forms an adjustment to the fair price of the green bond that both parties agree on.

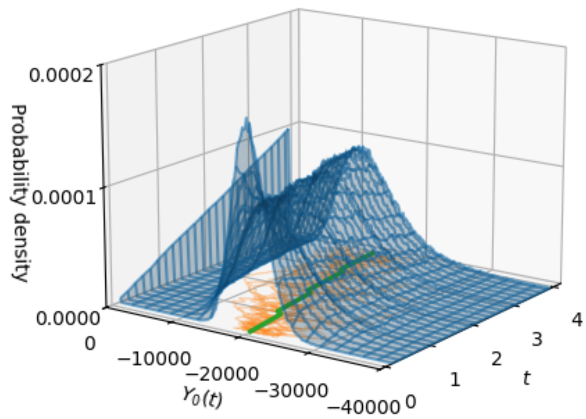
It should be noted that a bond issued by an independent power producer is an extreme example of an asset with high associated negative emissions, but the same principles apply to other investments in parties with negative emissions. In most cases, trades occur between parties that do not have a negative emission intensity, meaning that the FEVA will be a positive quantity and will therefore have a negative impact on the financed emissions-adjusted price $\widehat{V}'(t,T)$, as demonstrated in Equation (5.8). A general example of such a trade is given in Example 5.5.

Example 5.5 (FEVA of a European option). *We consider the European call option contract with a total risk-free value of $V(0,T) =$ €1 million that we considered in Example 4.4, and take the perspective of a financial institution that buys the option from another financial institution. Even though the emission intensity $I(t)$ of financial institutions is generally relatively low, the trade results in financed emissions to the*

[6]The interest rate $K = e^r - 1$ is chosen so that the initial risk-free value is exactly the notional N.



(a) The distribution of the total costs of the financed emissions $\mathcal{Y}$. For the given parameter configuration, we have $\text{FEVA}(0,T) = \mathbb{E}_0[\mathcal{Y}] = -€65949.57$.



(b) The distribution of the annual costs of the financed emissions $Y_0(t)$ over time. The average is shown in green, 20 paths are shown in orange.

Figure 5.4: *Illustration of the distribution of the costs of the financed emissions associated with the green bond, obtained from a Monte Carlo simulation of 10^5 paths of a discretisation of $[0,T]$ into 10^3 steps. The parameter configuration is $T = 4$, $r = 0.05$, $K = e^r - 1$, $\widehat{\lambda} = 0.005$, $I_0 = -0.001$, $\mu_I = 0$, $\sigma_I = 0.1$, $\theta = 0.5$, $\sigma = 0.3$, $\mu = x = \log(20)$.*

asset holder, which they offset using voluntary carbon credits.

We assume that the counterparty is an averagely-emitting financial institution that approximately decreases its emission intensity by a factor 2 each decade, i.e. $\mu_I = -0.07$. Analogously to Example 5.4, we investigate the distribution of the costs of the financed emissions associated with the green bond. The distribution of $Y_0(t)$ and $\mathcal{Y}$ are shown in Figure 5.5.

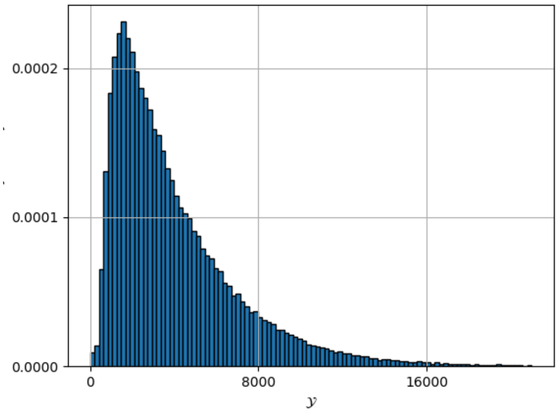
In Figure 5.5, it can be observed that the total environmental impact $\mathcal{Y}$ follows a positively skewed distribution, and the distribution of the annual environmental impact $Y_0(t)$ becomes wider over time. Most notably, the likelihood of vanishing financed emissions grows over time, which can either come from the exposure $E(t) = V(t,T)$ of the option vanishing as the underlying asset value decreases, or from a counterparty default. This results in a wider distribution of both $Y_0(t)$ for the option that we observed in Figure 5.4 for the green bond, of which the exposure is deterministic. This also results in a more skewed distribution of the total costs, as can be seen by comparing Figures 5.4a and 5.5a. The total FEVA associated with the asset is $\text{FEVA}(0,T) = \mathbb{E}_0[\mathcal{Y}] = €4041.37$, *which is an order of magnitude smaller in absolute value than the FEVA in Example 5.4, but still not negligible.*

If we introduce a VCM model of dimension d instead of one, the CTD aspect that we discussed in Section 3.3 can result in significantly lower prices of carbon credits, since now

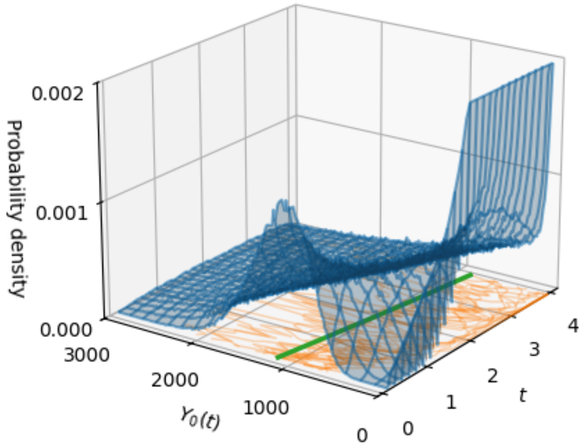
$$C(t) := H(t) = e^{\min(X_1(t),\ldots,X_d(t))},$$

where the log-prices $X_i(t)$ follow the dynamics in Equation (3.4). We assume an AR(1) correlation structure for the d-dimensional correlation matrix ρ with parameter ρ_0 (see Equation (3.14)). Moreover, we assume that the parameters $\theta_i = \theta$, $\mu_i = \mu$, $\sigma_i = \sigma$ and $x = x_i$ are identical for the three processes $X_i(t)$.

We observe that the distribution of the costs of the financed emissions in Figure 5.6 strongly resembles the distribution in Figure 5.5, but the CTD aspect results in an overall decrease of approximately 10%, which is in line with our findings in Section 3.3.



(a) The distribution of the total costs of the financed emissions $\mathcal{Y}$. For the given parameter configuration, we have $\text{FEVA}(0,T) = \mathbb{E}_0[\mathcal{Y}] = €4041.37$.



(b) The distribution of the annual costs of the financed emissions $Y_0(t)$ over time. The average is shown in green, 20 paths are shown in orange.

Figure 5.5: *Illustration of the distribution of the costs of the financed emissions associated with the European option, obtained from a Monte Carlo simulation of 10^5 paths of a discretisation of $[0,T]$ into 10^3 steps. The parameter configuration is $T = 4$, $r = 0.05$, $S_0 = K = 40$, $\sigma_S = 0.2$, $\widehat{\lambda} = 0.01$, $I_0 = 6 \cdot 10^{-5}$, $\mu_I = -0.07$, $\sigma_I = 0.1$, $\theta = 0.5$, $\sigma = 0.3$, $\mu = x = \log(20)$.*

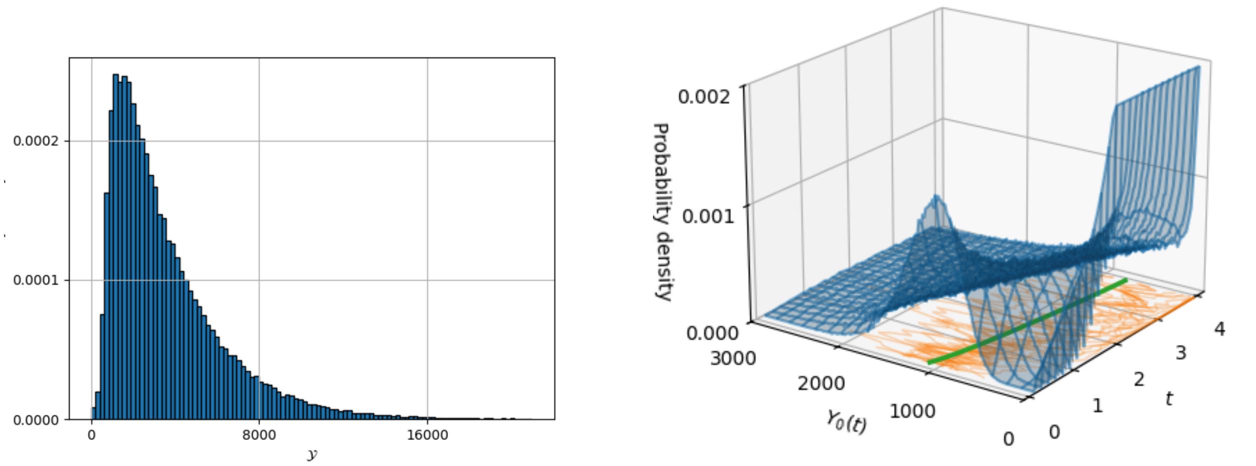
As observed in Example 5.5, the FEVA of a general trade between two financial institutions can be a quite significant value. In this specific example, the FEVA is approximately 0.4% of the total risk-free value $V(t,T) = €1$ million. The option buyer can charge this value to the counterparty when entering the contract at $t = 0$ and use the charged amount to offset the financed emissions over the duration of the contract using voluntary carbon credits. The CTD aspect of the d-dimensional VCM model can result in a significant decrease of the FEVA, namely approximately 10% in Example 5.5. This means that the opportunity to switch between different types of credits to use over the lifetime of the asset should be taken into account in the asset valuation process to decrease the FEVA and make it more attractive for counterparties to engage in business.

It can also be observed in Figures 5.4, 5.5 and 5.6 that the total future costs of offsetting, which are given by $\mathcal{Y}$ carry a large degree of uncertainty, particularly when the exposure also carries uncertainty as in the case of the European option. In Section 5.4, we demonstrate how this uncertainty can be reduced by hedging the FEVA, but we proceed to discuss the interpretation of the FEVA and how it can be integrated into financial decision-making.

5.3.2 Purpose of the FEVA

In this subsection, we propose different purposes of the FEVA by making a comparison to how other xVA models are applied in practice (see Subsection 1.1), looking to answer Research Questions 1, 2 and 3. We focus primarily on the interpretation of the carbon price $C(t)$ as the costs of offsetting financed emissions with a voluntary carbon credit, but keep in mind the different perspectives on carbon pricing that we discussed in Section 2.3.

Research Question 1 examines the applicability of EVAs in the valuation process of assets. If all parties involved in a trade agree that the financed emissions associated with an asset should be offset by the asset holder, either voluntarily or to comply with regulations, it is reasonable to interpret the FEVA of an asset as the adjustment to the fair value, because the FEVA by definition represents the expected total costs



(a) *The distribution of the total costs of the financed emissions* $\mathcal{Y}$*. For the given parameter configuration, we have* FEVA$(0,T) = \mathbb{E}_0[\mathcal{Y}] =$ €3633.09.

(b) *The distribution of the annual costs of the financed emissions* $Y_0(t)$ *over time. The average is shown in green, 20 paths are shown in orange.*

Figure 5.6: *Illustration of the distribution of the costs of the financed emissions associated with the European option, obtained from a Monte Carlo simulation of* 10^5 *paths of a discretisation of* $[0,T]$ *into* 10^3 *steps. The parameter configuration is identical to the one in Figure 5.5, but with a VCM market model with* $d = 3$ *and* $\rho_0 = 0.8$.

associated with offsetting the financed emissions associated with the asset[7]. A positive FEVA can therefore be charged to the counterparty when entering a contract at $t = 0$, similarly to how other xVAs are charged to the counterparty when entering a contract because they represent an adjustment to the fair value of the asset, while a negative FEVA results in an additional fee to be paid to the counterparty at $t = 0$.

This means that counterparties are financially incentivised to decrease their emission intensity, which has a positive impact on the climate. This illustrates how the FEVA can contribute to the internalisation of climate change by incorporating the climate impact of assets into the financial decision-making process. The importance of Remark 5.1 should be emphasised here: if internalised emissions are not excluded from the model for $FE(t)$, the implementation of a FEVA would result in these emissions being 'doubly internalised'. For example, charging the FEVA to a counterparty that has to buy allowances for their emissions means that they have to pay twice for their emissions, which will likely result in the counterparty disagreeing with the FEVA as a fair adjustment to the asset value that can be charged to them.

While there is currently no standardised approach in the financial sector to incorporate climate impact in financial decision-making, we argue that the FEVA is an adequate metric to quantify future climate impact and base financial incentives on since it is a standardised forward-looking metric, similar to how the CVA and ECL have become standardised forward-looking metrics to quantify counterparty credit risk. In this way, a FEVA can play a similar role in internalising climate change in financial decision-making. However, it should be emphasised that this relies on the assumption that all market participants agree that it is reasonable to charge the counterparty with the costs of offsetting financed emissions and thereby achieving net zero. Similarities can be drawn to how value adjustments associated with collateral funding costs have been introduced as a potential charge to counterparties [184, 101] after collateralisation has become more regulated under the third Basel accord [18], and has therefore become common practice. If offsetting financed emissions is not market practice on the contrary, charging a counterparty for the financed emissions that result from a trade

[7]It should be noted that this is true under the assumption that the voluntary carbon credits are bought at the exact moment that they are retired to offset financed emissions, and we explore the possibility of buying carbon credits *before* retiring them to offset emissions in Chapter 6.

in the form of a FEVA will disincentivise the counterparty to do business with the investee and they will turn to another financial institution. This illustrates that, assuming that financial institutions generally will not offset their financed emissions voluntarily out of goodwill, governing bodies have to establish regulations regarding financed emissions offsetting, for example with voluntary carbon credits, for this to become market practice.

Research Question 2 examines the role of EVAs in risk management, although we argue that the term ‘impact management‘ is also applicable when it comes to FEVA management practices[8]. Financial institutions can limit their negative climate impact by limiting the FEVA of assets on their books, similarly to how they can mitigate CCR by limiting the CVA on their books, although this does not imply anything about the climate impact of financial institutions at a given moment in time, as illustrated in Example 5.2. Given that the FEVA represents the expected total costs for a financial institution from the costs of offsetting their financed emissions, it is desirable for financial institutions to hedge the financed emissions over their portfolio to reduce the sensitivity of their total expected future costs to the price $C(t)$ of voluntary carbon credits, which we will further explain in Section 5.4. In this way, risk management and climate change mitigation go hand in hand under a carbon pricing scheme that covers financed emissions. It should also be noted that even though the negative impact of assets is mitigated in theory when a high-quality carbon credit is retired, there is still reputational risk of greenwashing accusations (as discussed in Section 3.1) when financial institutions invest in assets with a large FEVA, i.e. a large negative impact on the climate.

Research Question 3 approaches EVA models from the perspective of a regulator, drawing comparisons with existing regulations. Financed emissions and the FEVA in particular can be used by regulators to limit the climate impact of the covered financial institutions, e.g. by imposing that they hedge their FEVA, or at least partially do so. An upper bound on the financed emissions or the FEVA on the books of financial institutions is primarily a measure to limit their climate impact, but another consequence is that the sensitivity of the total climate-adjusted value over a portfolio to changes in the carbon price $C(t)$ is limited, as demonstrated in more detail in Section 5.4. We also propose that regulatory capital requirements can be established based on the unhedged FEVA over portfolios to ensure that financial institutions cover the uncertainty in the climate-adjusted value $\sum_i \widehat{V}_i' = \sum_i (\widehat{V}_i - \text{FEVA}_i)$ over the assets i over their portfolios. Similarly to the capital requirements of Basel 3, hedging the FEVAs can reduce the required capital buffer [3, 18].

In the remainder of this section, we discuss the purpose of FEVAs based on the different perspectives on carbon pricing from Section 2.3, focusing on the perspective of regulated carbon pricing schemes to begin with. If financial institutions are charged for the total financed emissions on their books under a regulated carbon pricing scheme, similar interpretations of the FEVA arise to the scenario in which financial institutions voluntarily offset their financed emissions. Firstly, the process $C(t)$ would have to be defined as the price per tonne of CO2e under the pricing scheme and modelled appropriately. For a tax, this is more straightforward, but pricing models for cap and trade systems are also widely studied in literature [138, 105, 152, 57]. From the perspective of a regulator, such a scheme would force covered institutions to incorporate their financed emissions into their decision-making, internalising the financed emissions[9].

If the financed emissions of a financial institution would be charged at the exact moment they are emitted, the FEVA in Equation (5.7) represents the expected total costs charged to the owner of an asset under the carbon pricing scheme, discounted to time t. If credits can be bought beforehand however, the approach in Chapter 6 can be applied to reduce the expected costs of compensating the financed emissions. Moreover, the hedging approach in Section 5.4 can also be applied to reduce the uncertainty in the future costs associated with financed emissions by reducing the sensitivity to the costs $C(t)$ of financed emissions. It should be noted that it is not very likely that financed emissions will be covered by carbon pricing schemes in the near future, given the difficulties that can arise when determining the absolute emissions of counterparties [165]. For this reason, carbon pricing schemes primarily cover emissions in scope 1 rather than indirect emissions, which are more difficult to determine [17].

[8]Remember that financed emissions are the primary form of climate impact of financial institutions, as illustrated in Figure 1.1.

[9]This assumes that the price per tonne of CO2e under this scheme is am appropriate price to internalise financed emissions.

Remark 5.2 (Other perspectives on carbon pricing). *At the end of Section 2.3, we discussed the uncertainties and subjectivities associated the other described perspectives on carbon pricing that we explored, and we concluded that this prevents these perspectives from being useful to define xVA models. However, the FEVA in Equation* (5.7) *can still be defined when the carbon price process $C(t)$ represents (1) an Internal Carbon Price (ICP), (2) the Social Cost of Carbon (SCC), or (3) a general different Shadow Price of Carbon that is established to be necessary to achieve a given environmental goal by creating financial incentives.*

In the case that a financial institution uses an ICP, the FEVA represents an adjustment to the value of an asset, as perceived by the financial institution themselves. This makes the FEVA highly subjective, meaning that it does not represent an adjustment to the fair value that the parties involved in a trade will agree on. It is also not useful for governing bodies to base regulations on, but the FEVA can still be used for internal risk management purposes, or rather in impact management. In this case, it represents the value of the financed emissions associated with assets, as perceived internally. Similarly, the uncertainty in an SCC makes it not suitable to construct a FEVA that represents an adjustment to the fair value of assets or that can be used to establish regulations. By letting $C(t)$ represent the SCC, the resulting FEVA represents the expected incurred social costs by the asset holder, which is a metric that can still be used for internal impact management purposes.

Lastly, we consider what the implications are of $C(t)$ being a shadow price that is necessary to achieve a given financial goal by incentivising the involved parties. In this case, the FEVA represents the total adjustment to the value of an asset that would be necessary to incentivise the involved parties in order to achieve the given goal. Unless this value adjustment is established and imposed by a regulator that covers both parties in a trade, they will not necessarily agree on the fair value of the FEVA. Establishing a carbon price $C(t)$ by shadow pricing is already done by regulators that establish carbon pricing schemes, and regulators can go a step further and impose regulations that require covered financial institutions to compute the FEVA over their assets, charge the FEVA of assets to the counterparty, or include the FEVA in their risk management practices, either by hedging the FEVA or holding on to regulated capital buffers.

5.4 General FEVA hedging approach

In this section, we will outline how the FEVA of an asset can be hedged to protect against uncertainty in the underlying carbon price $C(t)$. This requires investors to offset the financed emissions associated with an asset using some instruments that continuously generate negative financed emissions in the future. We introduce a stylised hypothetical asset that takes the most general shape of such a negative financed emissions-generating instrument, which we call a continuous carbon annuity, drawing comparisons with a continuous annuity given the continuous nature of the associated 'payments'.

Definition 5.6 (Continuous carbon annuity). *We define a Continuous Carbon Annuity (CCA) with maturity T as a contract that prescribes continuous negative emissions at a given rate $\nu(t)$, where $\nu : [0,T] \to \mathbb{R}$ is a deterministic function. By entering the contract, the counterparty is required to produce negative emissions on behalf of the owner of the contract at a rate of $\nu(t)$ tonnes of CO2e per year at time $t \in [0,T]$. We assume that the counterparty of the CCA does not default and does not produce other CO2e emissions, meaning that the financed emissions associated with a CCA are $-\nu(t)$ for $t \in [0,T]$.*

This hypothetical asset can be approximated in reality with appropriate instruments, e.g. (1) an investment in a project that generates negative financed emissions to the counterparty during the period $[0,T] \ni t$ at a rate of $\nu(t)$ tonnes of CO2e per year, or (2) a set of forwards or futures on voluntary carbon credits[10] with maturities during the period $[0,T]$, such that the settlements that occur in discrete time approximate the continuous rate $\nu(t)$.

[10] For the CCA to protect against changes in the underlying carbon price, both cash and physical settlement are appropriate. To achieve CO2e neutrality however, a credit or allowance has to be retired, so we assume that this happens upon settlement.

Since no cash flows are associated with a CCA, its risk-free financial value (not adjusted for climate impact) is in principle given by

$$V^{\text{CCA}}(t,T) = 0.$$

However, the holder of the contract will have to pay the counterparty for the emissions that are associated with the CCA. If the buyer and seller of a CCA agree on the price process $C(t)$ of a tonne of CO2e, the FEVA of the CCA with emissions rate $\nu(t)$ is given by

$$\text{FEVA}^{\text{CCA}}(t,T) = \mathbb{E}_t\left[\int_t^T e^{-r(s-t)}C(s)\cdot -\nu(s)\,\mathrm{d}s\right] = -\int_t^T e^{-r(s-t)}\mathbb{E}_t\left[C(s)\right]\cdot\nu(s)\,\mathrm{d}s, \tag{5.19}$$

because the financed emissions associated with a CCA are by construction $FE^{\text{CCA}}(t) = -\nu(t)$. Here, we applied Fubini's Theorem, as given in Theorem B.1. Hence, the financed emissions-adjusted value of the CCA is given by

$$V'^{\text{CCA}}(t,T) = V^{\text{CCA}}(t,T) - \text{FEVA}^{\text{CCA}}(t,T) = \int_t^T e^{-r(s-t)}\mathbb{E}_t\left[C(s)\right]\cdot\nu(s)\,\mathrm{d}s,$$

which is the fair value of the CCA so that the associated emissions are internalised.

We now demonstrate how the FEVA of an asset can be hedged using a CCA. We consider a portfolio that consists of a general asset with maturity T and associated financed emissions $FE(t)$ and a CCA with emissions rate $\nu(t)$ for some fixed deterministic function $\nu : [0,T] \to \mathbb{R}$. The financial value of the asset is given by $\widehat{V}(t,T)$ and is adjusted for (potentially climate-related) counterparty credit risk, following Equation 4.26, and the FEVA and financed emissions-adjusted value of the asset are given by $\text{FEVA}(t,T)$ and $\widehat{V}'(t,T) = \widehat{V}(t,T) - \text{FEVA}(t,T)$, respectively (see Definition 5.4). The financial value of the portfolio becomes

$$\Pi(t) = \widehat{V}(t,T) + V^{\text{CCA}}(t,T) = \widehat{V}(t,T),$$

and the financed emissions-adjusted value of the portfolio becomes

$$\begin{aligned}
\Pi'(t) :=& \widehat{V}'(t,T) + V'^{\text{CCA}}(t,T) \\
=& \widehat{V}(t,T) - \text{FEVA}(t,T) - \text{FEVA}^{\text{CCA}}(t,T) \\
=& \widehat{V}(t,T) - \mathbb{E}_t\left[\int_t^T e^{-r(s-t)}C(s)\cdot FE(s)\,\mathrm{d}s\right] + \mathbb{E}_t\left[\int_t^T e^{-r(s-t)}C(s)\cdot\nu(s)\,\mathrm{d}s\right] \\
=& \widehat{V}(t,T) + \int_t^T \mathbb{E}_t\left[e^{-r(s-t)}C(s)\cdot(\nu(s) - FE(s))\right]\mathrm{d}s,
\end{aligned}$$

making use of Fubini's Theorem (Theorem B.1). If we assume independence between the carbon price $C(t)$ and the financed emissions $FE(t)$ associated with an asset (of which we discussed the implications for wrong-way risk and right-way risk in Subsection 5.2.3), this results into

$$\Pi'(t) = \widehat{V}(t,T) + \int_t^T e^{-r(s-t)}\mathbb{E}_t\left[C(s)\right]\cdot\Big(\nu(s) - \mathbb{E}_t[FE(s)]\Big)\,\mathrm{d}s. \tag{5.20}$$

We investigate the sensitivity of the value of the portfolio to the carbon price at a fixed time t by introducing the operator $\frac{\partial}{\partial C}$, which takes the partial derivative of $\mathcal{F}(t)$-measurable quantities with respect to the current value of $C(t) = C$. In Remark 5.3, we discuss that the partial derivative is well-defined if the considered variable is sufficiently smooth as a function of $C(t)$, which we assume to be the case from now on.

Remark 5.3 (Partial derivatives of stochastic processes)**.** *If a random variable is $\mathcal{F}(t)$-measurable (e.g. because it is constructed from a conditional expectation $\mathbb{E}_t[\,\cdot\,]$), its value depends on the values of the current values of the state variables (which are also $\mathcal{F}(t)$-measurable) and can therefore be seen as a function of the current values of the state variables. This means that the partial derivative $\frac{\partial}{\partial C}$ of a variable is well-defined as long as it is sufficiently smooth as a function of the current carbon price $C = C(t)$ on the domain of C. We will assume that the considered processes are sufficiently smooth as a function of $C = C(t)$, although this is not necessarily true in the general framework that we consider.*

The sensitivity of the portfolio to the carbon price $C(t)$ is given by

$$\begin{aligned}\frac{\partial}{\partial C}\Pi'(t) &= \frac{\partial}{\partial C}\widehat{V}(t,T) + \int_t^T e^{-r(s-t)}\frac{\partial}{\partial C}\left[\mathbb{E}_t\left[C(s)\right]\cdot\Big(\nu(s)-\mathbb{E}_t[FE(s)]\Big)\right]\mathrm{d}s \\ &= \frac{\partial}{\partial C}\widehat{V}(t,T) + \int_t^T e^{-r(s-t)}\left[\frac{\partial}{\partial C}\mathbb{E}_t\left[C(s)\right]\cdot\Big(\nu(s)-\mathbb{E}_t[FE(s)]\Big) + \mathbb{E}_t\left[C(s)\right]\cdot -\frac{\partial}{\partial C}\mathbb{E}_t[FE(s)]\right]\mathrm{d}s,\end{aligned}$$

making use of the linearity and product rule of the partial derivative. We also make use of the fact that $\frac{\partial \nu(t)}{\partial C} = 0$ and of the Leibniz rule [172] to interchange the partial derivative and integration[11]. If we assume independence between the carbon price and the exposure $E(t)$, the default process that models $\widehat{t}_D$ and the carbon intensity $I(t)$ (of which we discussed the implications for wrong-way risk and right-way risk in Subsection 5.2.3), it follows that $\widehat{V}(t,T)$ and $\mathbb{E}_t[FE(s)]$ are independent of $C(t)$, and hence their partial derivatives with respect to $C = C(t)$ are zero. This simplifies the expression for the sensitivity of the financed emissions-adjusted value to the carbon price:

$$\frac{\partial}{\partial C}\Pi'(t) = \int_t^T e^{-r(s-t)}\frac{\partial}{\partial C}\mathbb{E}_t\left[C(s)\right]\cdot\Big(\nu(s)-\mathbb{E}_t[FE(s)]\Big)\mathrm{d}s, \tag{5.21}$$

which equals zero if

$$\nu(s) := \mathbb{E}_t[FE(s)], \tag{5.22}$$

for all $s \in [t,T]$, implying that the uncertainty in the carbon price at time t can be eliminated by buying a CCA with a prescribed rate ν of negative emissions given by Equation (5.22). Based on this observation, we suggest the following approach to hedge the FEVA over the lifetime of the considered asset.

Definition 5.7 (Hedging the FEVA at $t = 0$)**.** *The holder of an asset can hedge the associated FEVA as follows. When entering a trade, the holder pays the counterparty a price of*

$$\widehat{V}'(0,T) = \widehat{V}(0,T) - \mathrm{FEVA}(0,T)$$

as the fair value of the asset, adjusted for climate risk and the impact of the associated financed emissions. This means that on top of the payment of the climate risk-adjusted value $\widehat{V}(0,T)$, a charge of $\mathrm{FEVA}(0,T)$ *is received from the counterparty. This money is used to buy a CCA with prescribed negative emissions rate*

$$\nu(t) := \mathbb{E}_0[FE(t)], \tag{5.23}$$

for $t \in [0,T]$. In this way, the combined financed emissions-adjusted value of the asset and the CCA is insensitive to changes in the carbon price $C(t)$ at time $t = 0$ under the assumption of independence between the carbon price and the other involved processes, as illustrated in Equation (5.21)*.*

After buying a CCA with prescribed negative emissions rate $\nu(t) = \mathbb{E}_0[FE(t)]$ when entering the trade, the net financed emissions over the asset and CCA are $FE(t) - \mathbb{E}_0[FE(t)]$ for the remainder of the lifetime of the asset ($t \in [0,T]$). To illustrate the impact of the hedging strategy, we define $RC(0,T)$ to be the remaining total future costs associated with the net financed emissions (discounted to $t = 0$), i.e.

$$RC(0,T) = \int_0^T e^{-rt}C(t)\cdot\Big(FE(t)-\mathbb{E}_0[FE(t)]\Big)\mathrm{d}t, \tag{5.24}$$

of which the expected value is

$$\mathbb{E}_0[RC(0,T)] = \mathbb{E}_0\left[\int_0^T e^{-rt}C(t)\cdot FE(t)\,\mathrm{d}t - \int_0^T e^{-rt}C(t)\cdot\mathbb{E}_0[FE(t)]\,\mathrm{d}t\right] = \mathrm{FEVA}(0,T) - \mathrm{FEVA}^{\mathrm{CCA}}(0,T) = 0,$$

assuming independence between the $C(t)$ and the other processes, and making use of the tower property of iterated conditional expectations [162]. This can be obtained from Equations (5.7) and (5.19). This

[11]We implicitely assume that $\mathbb{E}_t[C(s)]$ is a sufficiently smooth function of the current carbon price $C(t)$ for all $s \in [t,T]$, which will generally be the case.

demonstrates that by buying a CCA, the holder of an asset can move the expected future costs to the present. Moreover, the smaller the future net financed emissions over the asset and CCA are, the smaller the sensitivity of the costs with respect to the carbon price $C(t)$ are.

We now consider the uncertainty of the position at a later time $t_0 \in (0, T)$. Over the period $[0, t_0]$, the net financed emissions $FE(t) - \mathbb{E}_0[FE(t)]$ incurred the expected costs (discounted to $t = 0$)

$$RC(0, t_0) = \int_0^{t_0} e^{-rt} C(t) \cdot \Big(FE(t) - \mathbb{E}_0[FE(t)]\Big) \, \mathrm{d}t,$$

which are not necessarily zero. Moreover, the sensitivity $\frac{\partial}{\partial C}\Pi'(t_0)$ is not necessarily zero any more (see Equation (5.21)), because $\mathbb{E}_{t_0}[FE(t)]$ is not necessarily equal to $\nu(t) = \mathbb{E}_0[FE(t)]$ for all $t \in [t_0, T]$ due to the volatility of the process $FE(t)$. The FEVA can be hedged again by buying an additional CCA with prescribed negative emissions rate

$$\nu'(t) := \mathbb{E}_{t_0}[FE(t)] - \mathbb{E}_0[FE(t)], \tag{5.25}$$

for $t \in [t_0, T]$, so that the net financed emissions over the asset and CCAs become

$$FE(t) - \mathbb{E}_{t_0}[FE(t)],$$

instead of

$$FE(t) - \mathbb{E}_0[FE(t)].$$

Although it is intuitively evident that the future uncertainty in the difference between $FE(t)$ and its conditional expectation decreases as it incorporates the information that becomes available over time, we formalise this idea in the following lemma.

Lemma 5.1 (Reducing future uncertainty). *Let $t_0, t_1, t_2 \in [0, T]$ such that $t_0 \leqslant t_1 \leqslant t_2$, and let $Y(t)$ be a stochastic process adapted to the filtration $\mathcal{F}(t)$. For all $t \in [t_2, T]$, we then have*

$$\mathbb{V}\mathrm{ar}_{t_0}\Big[Y(t) - \mathbb{E}_{t_2}[Y(t)]\Big] \leqslant \mathbb{V}\mathrm{ar}_{t_0}\Big[Y(t) - \mathbb{E}_{t_1}[Y(t)]\Big]. \tag{5.26}$$

The proof is given in Section B.4. Applied to the process $FE(t)$, Lemma 5.1 proves that the uncertainty in the net financed emissions over the asset and CCAs is generally reduced by rebalancing the position with the CCA with the rate in Equation (5.25) at time t_0, because

$$\mathbb{V}\mathrm{ar}_0\Big[FE(t) - \mathbb{E}_{t_0}[FE(t)]\Big] \leqslant \mathbb{V}\mathrm{ar}_0\Big[FE(t) - \mathbb{E}_0[FE(t)]\Big]. \tag{5.27}$$

Therefore, the uncertainty in the future offsetting costs $RC(t_0, T)$ will generally also be lower as a result of the rebalancing at time t_0. Moreover, the sensitivity of $\mathbb{E}_{t_0}[RC(t_0, T)]$ to the carbon price $C(t_0)$ is also eliminated.

The sketched approach of adjusting the negative financed emissions from CCAs is similar in nature to the process of delta hedging to eliminate the sensitivity of the value of a derivative to changes in the underlying asset value. Delta hedging thus reduces the variance of the resulting profits and losses, as illustrated in [162] for example. More details on hedging strategies are beyond the scope of this thesis, but it is good to emphasise that more rebalancing moments generally lead to less uncertainty in the resulting profits and losses by reducing the hedged sensitivity, meaning that more moments of buying additional CCAs to rebalance the hedge is generally advisable.

Even though this approach seems cumbersome to apply often, it is very scalable since one CCA can be bought to hedge the FEVA over an entire portfolio, as long as the entire portfolio is subject to the same carbon price $C(t)$. In this case, a CCA should be bought at time t_0 such that the total negative emissions from CCAs are given by

$$\nu(t) = \sum_i \mathbb{E}_{t_0}[FE_i(t)],$$

for $t \in [t_0, T]$, where T is an appropriate time horizon that exceeds all maturities and $FE_i(t)$ are the financed emissions associated with asset i in the portfolio. The summation runs over all assets i in the portfolio.

However, it should be kept in mind that a perfect hedge is nearly impossible in practice because the replication of CCAs, being hypothetical instruments, with existing instruments is only possible in theory. Moreover, the assumption of independence between the carbon price process $C(t)$ and the other involved processes can lead to wrong-way risk or right-way risk by resulting in an imperfect hedge if the approach given in this section is followed. This does not mean that the proposed strategy should not be implemented, since partial hedges can also effectively reduce the risks that a position carries [67].

5.5 Value adjustments for broader climate impact

In Section 2.2, we justified our focus on GHG emissions in this thesis by observing that (1) GHG emissions from human activities primarily contribute to climate change [64], (2) the metric CO2e makes GHG emissions relatively straightforward to quantify compared to other types of environmental impact [146], and (3) GHG emissions also receive more attention than other types of environmental impact across the sector and from regulators, which also results into more available data [73, 178]. However, other environmental aspects are also receiving more and more attention over the past years [225, 130], and the principles behind the PCAF Standard for financed emissions can also be applied to other types of environmental impact to a certain extent. In this section, we demonstrate what this would look like and discuss the issues that would be encountered when defining value adjustments for other types of environmental impact. To do so, we focus on internalising biodiversity by attempting to construct an EVA that captures the financed contribution or damage to biodiversity.

5.5.1 Financed Biodiversity Value Adjustment (FBVA)

In 2022, the Kunming-Montreal Global Diversity Framework was adopted at the United Nations Biodiversity Conference as a new set of goals to restore and protect the earth's biodiversity over the next decades [92]. The framework is built on a vision of 'Living in harmony with nature' by 2050 and is sometimes referred to as the 'Paris Agreement for nature' due to the similarly crucial role they play in their respective topics [200]. The Kunming-Montreal Framework takes a holistic approach and formulates a series of biodiversity-related targets and also formulates how these should be operationalised, proposing that biodiversity dependencies and impacts should be monitored, assessed and reported on by financial institutions. Moreover, it recognises the need to create financial incentives and mobilise financial markets to contribute on the topic [200, 113]. The framework was accompanied by a complementary monitoring framework that was adopted at the same conference, which addresses the shortcomings of its predecessors by improving the measurability of biodiversity [63].

Biodiversity finance, the sub-field of green finance that focusses on biodiversity, has only been gaining popularity over the past few years. Nevertheless, there is still a significant financing gap [70] as well as a research gap in biodiversity finance [87], including a lack of reliable and widely accepted metrics that are required for substantial progress in biodiversity finance [129]. This is not the case for climate change, which can be consistently quantified using the metric CO2e. Approaches to develop standardised and generalisable metrics are described in [218], which we refer to as a *biodiversity unit* from now on. Project developers can assess their climate impact by going through different stages as illustrated in Figure 5.7, which includes defining clear boundaries and variables of consideration to define a unit of nature, and measuring the impact of a project against a baseline scenario, similar to how voluntary CO2e abatement projects are developed and assessed. There is an emerging market for biodiversity credits [225, 218], which therefore faces problems similar to the voluntary carbon market, including the additional issue of limited comparability and interchangeability of biodiversity between different ecosystems. This poses practical and ethical questions regarding the development of biodiversity credit markets and whether they can be deployed to offset biodiversity losses elsewhere. Moreover, concerns regarding the legitimacy of credits as offsetting instruments are likely to lead to greenwashing concerns [117, 218].

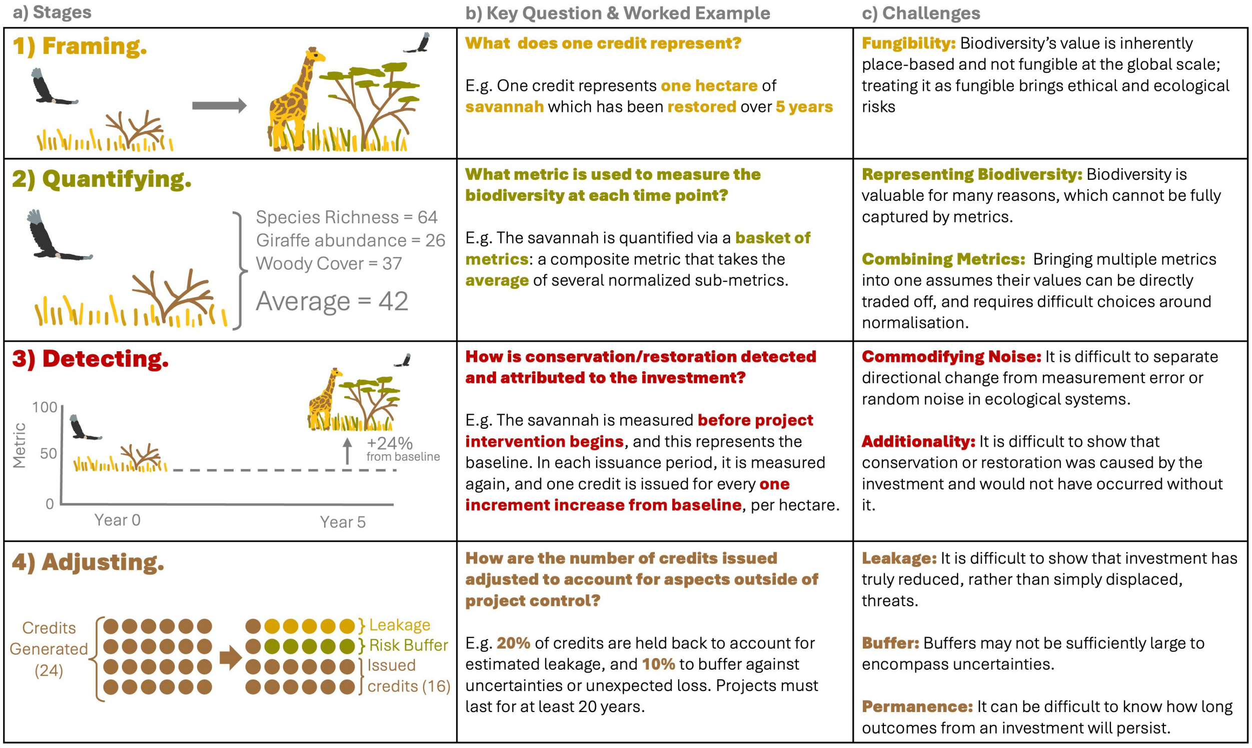


Figure 5.7: *Illustration of the potential stages involved in the process of issuing biodiversity credits, including the challenges in each stage [218].*

For the remainder of this section, we assume that there is a market for biodiversity credits where one credit corresponds to one biodiversity unit, which we assume to be a well-defined and fungible metric for biodiversity, viewing the credits as valid instruments to offset biodiversity damage. Similarly to the PCAF Standard for financed emissions, we assess the impact of financial institutions on biodiversity by defining the *financed biodiversity* associated with an asset. We do so by analogously defining the attribution factor $AF(t) = \frac{E(t)}{TV(t)}$ of an asset (see Equation 5.1), and we denote the damage to biodiversity of a counterparty by $G^{BD}(t)$, measured in biodiversity units per year. Positive impact on biodiversity will therefore result in $G^{BD}(t) < 0$. Analogously to the financed emissions framework, we exclude any impact on biodiversity for which the counterparty has received funding or has been charged because these impacts are already economically internalised. All considered processes are assumed to be modelled using stochastic processes adapted to the default-free filtration $\mathcal{F}(t)$, apart from the climate-adjusted default time $\widehat{t}_D$. Analogously to Definition 5.3, we introduce the following definition.

Definition 5.8 (Financed biodiversity damage)**.** *We define the financed biodiversity damage $FBD(t)$ associated with an asset as*

$$FBD(t) := \mathbb{1}_{\widehat{t}_D > t} \cdot AF(t) \cdot G^{BD}(t), \tag{5.28}$$

where the attribution factor $AF(t)$ is defined as in Definition 5.1 and $\widehat{t}_D$ is the climate-adjusted default time of the counterparty (see Definition 4.7). This process is adapted to the filtration $\widehat{\mathcal{G}}(t)$.

By letting $C^{BD}(t)$ be the stochastic process that describes the price of a biodiversity credit, we can construct an EVA model by attaching this price tag to the future financed biodiversity damage and integrating this quantity over the remaining lifetime of an asset, analogously to how the FEVA is defined:

Definition 5.9 (Financed Biodiversity Value Adjustment)**.** *The Financed Biodiversity Value Adjustment (FBVA) at time of an asset with maturity T is given for $t \in [0,T]$ by*

$$\mathrm{FBVA}(t,T) := \mathbb{E}_t\left[\int_t^T e^{-r(s-t)} FBD(s) \cdot C^{BD}(s)\,\mathrm{d}s\right], \tag{5.29}$$

where r is the constant interest rate. The FBVA of an asset represents the expected total costs of offsetting any financed impact on biodiversity associated with the asset using biodiversity credits, discounted to time t.

In other words, the FBVA represents the costs associated with internalising the impact on biodiversity of an asset. By assuming independence between the price processes driving the exposure and the other stochastic processes involved (which raises the same questions regarding wrong-way risk and right-way risk that we discussed in Subsection 5.2.3), it follows from Fubini's Theorem (see Theorem B.1) that

$$\mathrm{FBVA}(t,T) = \int_t^T \mathrm{EE}_t(s) \cdot \mathbb{E}_t\left[\mathbb{1}_{\hat{t}_D > s} \cdot C^{BD}(s) \cdot \frac{G^{BD}(s)}{TV(s)}\right] \mathrm{d}s,$$

where we again recognise the separation of the integral into the expected exposure and a climate impact-related term, analogous to Equation (5.12) for the FEVA and Equation (4.43) for a general EVA. We define the Biodiversity Damage Environmental Impact Factor (EIF) by the stochastic process

$$\mathrm{EIF}^{BD}(t) := \mathbb{1}_{\hat{t}_D > t} \cdot C^{BD}(t) \cdot \frac{G^{BD}(t)}{TV(t)}, \tag{5.30}$$

resulting into

$$\mathrm{FBVA}(t,T) = \int_t^T \mathrm{EE}_t(s) \cdot \mathbb{E}_t\left[\mathrm{EIF}^{BD}(s)\right] \mathrm{d}s, \tag{5.31}$$

which is perfectly analogous to Equations (5.12) and (4.43).

5.5.2 Constructing EVAs for general climate impacts

The FBVA and FEVA that we defined in this chapter illustrate that a value adjustment can be constructed for assets regarding any environmental impact associated with them, provided that two conditions are met. Firstly, the considered type of environmental impact of the counterparty in the trade should be quantifiable in a standardised metric that uses well-defined baseline scenarios, so that the considered environmental impact of financial entities can be assessed and compared between them. This requires universal measurement and reporting standards. Secondly, there should be a market for instruments that can be used to offset the considered type of environmental impact, such as voluntary carbon credits for CO2e emissions or biodiversity credits for biodiversity damage. The legitimacy of these instruments relies on a standardised metric that allows for quantitative comparability of the environmental impact between different locations.

When these two conditions are met for a given general aspect of environmental impact, we model the considered environmental impact of the counterparty in a trade with the stochastic process $G^X(t)$, and we model the price of the offsetting instrument with the process $C^X(t)$, both adapted to the filtration $\mathcal{F}(t)$. The superscript X refers to the given general aspect of environmental impact. We refer to the units of the standardised metric as *environmental unit* for now, and the units of $G^X(t)$ and $C^X(t)$ are given by environmental units per year and monetary units per environmental unit, respectively. For the given general aspect of environmental impact, we introduce the following definition to construct an EVA.

Definition 5.10 (EIF and EVA for general environmental impact)**.** *Consider a given aspect, denoted by X, of environmental impact associated with an asset with maturity T. Let $G^X(t)$ denote the amount of units of environmental impact of the counterparty, and let $C^X(t)$ denote the costs of offsetting a unit of environmental impact. The Environmental Impact Factor (EIF) for the given aspect is defined for*

$t \in [0, T]$ by

$$\mathrm{EIF}^X(t) := \mathbb{1}_{\hat{t}_D > t} \cdot C^X(t) \cdot \frac{G^X(t)}{TV(t)}, \tag{5.32}$$

where $\widehat{t}_D$ and $TV(t)$ are the climate-adjusted default time and total value of the counterparty, respectively. An Environmental Value Adjustment for the given environmental aspect is defined by

$$\mathrm{EVA}^X(t,T) := \mathbb{E}_t \left[\int_t^T e^{-r(s-t)} C^X(s) \cdot \mathbb{1}_{\hat{t}_D > t} \cdot G^X(t) \cdot \frac{E(t)}{TV(t)} \mathrm{d}s \right], \tag{5.33}$$

where r is the constant interest rate. The EVA represents the expected total costs associated with offsetting the financed environmental impact.

By assuming independence between the price processes that drive the exposure and the other stochastic processes, the EVA in Equation (5.33) can be rewritten using Fubini's Theorem (Theorem B.1) as

$$\mathrm{EVA}^X(t,T) = \int_t^T \mathrm{EE}_t(s) \cdot \mathbb{E}_t[\mathrm{EIF}^X(s)] \mathrm{d}s, \tag{5.34}$$

where the expected exposure $\mathrm{EE}_t(s)$ is defined in Definition 4.1. This construction is analogous to the construction of the EVA for financed emissions and financed biodiversity damage, since the term

$$\mathbb{1}_{\hat{t}_D > t} \cdot G^X(t) \cdot \frac{E(t)}{TV(t)}$$

in Equation (5.33) represents the financed environmental impact, analogously to Equations (5.5) and (5.28) for financed emissions and financed biodiversity damage respectively.

The formulation in Equation (5.34) of the general EVA in terms of the EIF and the expected financial exposure of assets allows financial institutions to easily assess and compare the environmental impact of different assets. Moreover, the EVA models can be integrated seamlessly into existing xVA models for which the exposure is already modelled, as discussed in Section 4.3. Analogously to the discussion in Section 5.3 on Research Questions 1, 2 and 3, we sketch how the general EVA in Equation (5.33) can be incorporated into financial decision-making in order to economically internalise the considered aspect of environmental impact.

Firstly, the general EVA is a forward-looking measure that represents the adjustment to the fair value of an asset when the costs of offsetting the financed environmental impact associated with the asset are incorporated in the valuation process. When incorporating a given environmental aspect into the valuation process of an asset, its adjusted value at time t is given by

$$\bar{V}'(t,T) := \bar{V}(t,T) - \mathrm{EVA}^X(t,T),$$

where $\bar{V}(t,T)$ denotes the financial value without the environmental aspect incorporated in the asset valuation process, of which the precise definition depends on the context[12]. We discuss the implications of incorporating multiple environmental aspects in the valuation process in Subsection 5.5.3. By charging the EVA to a counterparty when entering a trade, an investor can offset and in turn internalise its financed environmental impact.

Secondly, the general EVA can be incorporated into risk management practices of financial institutions. By limiting the total EVA on its books, a financial institution limits both the total financed environmental impact and the sensitivity of the total climate-adjusted value on their books to the price of offsetting instruments, as discussed in the specific case of an FEVA in Section 5.3. Furthermore, the sensitivity of the total climate-adjusted value on a book to the current price $C^X(t)$ of offsetting instruments can be reduced by hedging $\mathrm{EVA}^X(t,T)$, assuming that the unadjusted value $\bar{V}(t,T)$ of assets does not depend on $C^X(t)$. As

[12] It is possible that some other environmental aspects are already incorporated in the value $\bar{V}(t,T)$, similarly to Equation (5.8) where $\widehat{V}(t,T)$ already incorporates climate risk, but is adjusted for financed emissions

illustrated in Section 5.4 for the FEVA, a financial institution can hedge the general EVA by ensuring that for each future time t, the total expected financed environmental impact on their books at time t equals zero. However, this is based on the assumption that the financed environmental impact is independent of $C^X(t)$, which induces wrong-way risk, as discussed in Subsection 5.2.3. Moreover, this requires the existence of a well-established market of hedging instruments that can be used to make the net environmental impact on a book close to zero, as we outlined by introducing the hypothetical continuous carbon annuity in Section 5.4.

Lastly, regulators can impose capital requirements associated with the EVA on the books of covered financial institutions to further internalise financed environmental impact in their decision-making. These can be based on both the EVA and the sensitivities of the EVA to the variables in the model. By holding on to additional capital based on the total EVA on their books, financial institutions are protected from potential losses resulting from uncertainty in the variables or from wrong-way risk from inaccurate model assumptions.

In summary, the general EVA framework in this section can not only be applied to CO2e emissions or biodiversity damages that are associated with assets, but also to environmental aspects such as soil conservation near rivers to improve downstream water use [130, 118], wastewater treatment in wetlands [11] for which crediting systems are established, or issues that may receive more attention in the future, given the emergence of ESG topics in the financial sector. These emerging topics will likely go through similar phases as biodiversity finance could be going through. Firstly, they would start receiving more attention in the broader field of green finance (e.g. by receiving more attention in the EU Taxonomy [178]), and universal measurement and reporting standards and a credit market would be established later. If this would be the case, EVA models can be defined based on the framework in Definition 5.10 to incorporate these topics, and they can be economically internalised.

5.5.3 Interactions between different EVAs

When establishing xVA models, overlap between the incorporated aspects and double counting between the different xVAs should be avoided to minimise model risk [196, 101, 184]. Therefore, we pay attention to the different environmental aspects in Figure 1.1 that we incorporate into asset valuation using different EVA models. We start by examining the overlap between the (climate-related) counterparty credit risk and financed emissions to discuss possible problems with applying a CRVA and FEVA alongside each other in the valuation process of assets. It should be immediately noted that climate risk is incorporated with the indicator function $\mathbb{1}_{\hat{t}_D>t}$ into the financed emissions in Definition 5.3 since no financed emissions are associated with an asset if a counterparty defaults. In this way, no contradictory assumptions are made in establishing the C(R)VA and FEVA.

Moreover, if the counterparty in a trade is relatively likely to default, the C(R)VA of an asset will be relatively high, but the FEVA may be relatively low. This means that these effect potentially balance each other out in the net xVA, depending on their relative magnitude. This also applies to the wrong-way risk and right-way risk that may appear from the assumption of independence between the exposure and default processes, as discussed in Subsections 4.1.5 and 5.2.3. If the exposure and the credibility of the counterparty are positively (negatively) correlated, right-way (wrong-way) risk may appear in the C(R)VA and wrong-way (right-way) risk may appear in the FEVA from the assumption that the exposure and default processes are independent. This illustrates that the wrong-way and right-way risk that result in the different xVA models can also counteract each other, depending on their relative magnitude. Analogously to the FEVA, these arguments also apply to the relation between the C(R)VA and the FBVA or general EVA from Definition 5.10.

The only remaining potential overlap to discuss can occur between EVAs for different aspects of climate impact as defined in Definition 5.10, such as the FEVA and FBVA that we defined in this chapter. When incorporating different topics X such as financed emissions and biodiversity damage, in the asset valuation process, the total EVA is given by

$$\mathrm{EVA}^{\mathrm{Tot}}(t,T) = \sum_X \mathrm{EVA}^X(t,T) = \int_t^T \mathrm{EE}_t(s) \cdot \sum_X \mathbb{E}_t[\mathrm{EIF}^X(s)]\,\mathrm{d}s, \tag{5.35}$$

making use of Equation (5.34) under the assumption of independence between the processes driving the exposure and the other involved processes. The summation runs over the different topics. This illustrates that the different EVAs can be calculated separately from modelling the corresponding processes and the resulting environmental impact factor $\text{EIF}^X(t)$ from Equation (5.32). The climate-adjusted default process and total counterparty value are identical in each EIF, meaning that the only potential for overlap and double counting comes from overlap between the different considered aspects in the measurement and reporting standards that apply and are used in the corresponding credit markets.

This is a realistic concern due to the complexity of nature, making it difficult to separate different topics when assessing the environmental impact of investments. For example, nature-based CO2e abatement projects often also have a beneficial impact on biodiversity or even on local communities [202, 85]. Voluntary carbon credits from projects with such co-benefits currently trade at a premium compared to projects without co-benefits [142, 207], indicating that buyers pay for more than just CO2e abatement. When the voluntary carbon market will mature and a biodiversity credit market will emerge, these co-benefits will also have to be quantified to avoid potential overlap or double counting. A potential solution is that nature-based carbon projects which also have a positive impact on biodiversity issue both voluntary carbon credits and biodiversity credits, separating the topics in the issuance process. However, such an approach requires well-established, consistent and holistic measuring and reporting standards. This illustrates that the potential overlap between different environmental aspects and EVAs depends on the definition of the considered metrics. These concerns will have to be addressed in the future when other environmental credit markets are established.

For the remainder of this thesis, we again restrict our attention to financed emissions, given that universal metrics and credit markets have to be established for the other topics to be incorporated in asset valuation in the form of an EVA. In Chapter 6, we explore how the costs of offsetting financed emissions using voluntary carbon credits can be decreased compared to the FEVA by considering the possibility of buying credits before they have to be retired.

CHAPTER 6

EARLY BUYING STRATEGIES

In Chapter 5, we introduced a general FEVA framework and specified that we assume that it is market practice that financial insitutions offset their financed emissions using voluntary carbon credits for the remainder of this thesis. This scenario assumes that the VCM has evolved into a mature market consisting of $d \in \mathbb{N}$ types of high-quality voluntary carbon credits that can be interchangeably used to offset CO2e emissions. Financial institutions can use carbon credits to offset their financed emissions and achieve net zero by doing so. From now on, we assume that the financed emissions $FE(t)$ associated with an asset (see Equation (5.5)) are modelled and the CTD price (see Section 3.3) of carbon credits is given by

$$H(t) = e^{\min(X_1(t),\ldots,X_d(t))}, \tag{6.1}$$

where the processes $X_i(t)$ are correlated OU processes that describe the log-prices of carbon credits, as explained in Section 3.2. The FEVA of an asset can then be computed using a carbon price $C(t) = H(t)$ as follows (see Equation 5.12):

$$\text{FEVA}(t,T) = \int_t^T \text{EE}_t(s) \cdot \mathbb{E}_t[\text{EIF}^{\text{FE}}(s)]\,\mathrm{d}s,$$

where the EIF can be computed explicitly (see Equation (5.13)). We denote the climate-adjusted default rate process of the counterparty by $\widehat{\lambda}(t)$ and independence is assumed between the exposure, default process, carbon credit prices and the carbon intensity process $I(t)$, ignoring wrong-way risk for now. The implications of assuming independence are discussed in Subsection 5.2.3. This results into

$$\text{FEVA}(t,T) = \int_t^T \text{EE}_t(s) \cdot e^{-\int_t^s \widehat{\lambda}(u)\,\mathrm{d}u} \cdot \mathbb{E}_t\left[H(s)\right] \cdot \mathbb{E}_t\left[I(t)\right]\mathrm{d}s. \tag{6.2}$$

As discussed in Section 5.3, the FEVA represents the expected total costs of offsetting financed emissions by buying carbon credits at the exact moment that they occur. The FEVA can be incorporated into the price of an asset when two parties enter a trade as illustrated in Equation (5.8), since the FEVA represents additional costs to the holder that are associated with the trade. However, these additional costs could hold counterparties back and discourage them to enter the trade. Motivated by this, we explore strategies to decrease the costs associated with offsetting emissions in this Chapter. Instead of buying carbon credits at the exact moment that financed emissions occur, we explore the possibility of buying carbon credits *before* the financed emissions occur, holding on to them and then retiring the credits at the appropriate moment to offset the financed emissions. We refer to such a strategy as an *early buying strategy.* Early buying might be beneficial when the carbon credit price $H(t)$ is relatively low, resulting in lower total costs associated with offsetting financed emissions compared to the FEVA in Equation (6.2).

In Section 6.1 we describe this approach as a continuous stochastic control problem of minimising expected total costs subject to the constraint that the number of bought credits is always sufficient to offset the financed emissions, which are at first assumed to be deterministic. Then, we discretise this problem in Section 6.2 and describe how this problem can be solved with a dynamic programming approach using Bellman's principle of optimality [28]. This involves the computation of future conditional expectations, for which we propose a least squares Monte Carlo method based on the algorithm for American option pricing as suggested by Longstaff and Schwartz [141]. We also introduce an approach to account for heteroscedasticity that arises in least squares Monte Carlo methods [154] in Subsection 6.3.1. In Subsection 6.3.2 we describe the impact of introducing uncertainty in the financed emissions process $FE(t)$ on the formulation of the control problem. In Section 6.4, we apply this method and compare the resulting expected costs in this control problem to the FEVA in Equation (6.2) to explore the effectiveness of this method to decrease the total costs associated with offsetting financed emissions, particularly in the context of a higher-dimensional VCM model with a CTD aspect.

6.1 Continuous control problem

From now on, we denote the state space of the problem by Ξ, which is the Cartesian product of the state spaces of the variables that define the exposure $E(t)$, carbon credit price $C(t)$ and emissions intensity $I(t)$. We exclude the default process for convenience. Later, we will introduce another dimension to the state space, which corresponds to the cumulative number of credits bought at a given time. The state at time $t \in [0, T]$ is typically denoted by $\xi(t)$ and is a Ξ-valued stochastic process. With this notation, we emphasise that $\mathcal{F}(t)$ is the σ-algebra generated by the stochastic process $\xi(t)$. We also temporarily assume that the process $FE(t)$ is deterministic for simplicity. This will avoid complications with respect to uncertainty in the upper bound on the state variables that we define in Section 6.2.

6.1.1 Feedback control laws

To formulate the problem of offsetting financed emissions as a continuous control problem[1], we define a continuous control process $b(t)$ using a feedback control law $\mathbf{b}$. The feedback control law gives a buying rate at a given moment in a given state of the system, and the control process $b(t)$ then represents the resulting buying rate. Mathematically, these concepts are defined as follows:

Definition 6.1 (Feedback control law, control process). *For a given $t_0 \in [0, T]$, we refer to a function*

$$\mathbf{b} : [t_0, T] \times \Xi \to \mathbb{R}_+$$

as a feedback control law at time $t_0 \in [0, T]$. A feedback control law $\mathbf{b}$ defines a control process $b(t)$ for $t \in [t_0, T]$ by

$$b(t) := \mathbf{b}(t, \xi(t)), \tag{6.3}$$

which represents the buying rate in state $\xi(t)$ at time t. This stochastic process is adapted to the filtration $\mathcal{F}(t)$. We define the following set of feedback control laws that we consider:

$$\mathcal{B}_{t_0} := \{\mathbf{b} : [t_0, T] \times \Xi \to \mathbb{R}_+ \mid \text{The process } \int_{t_0}^{t} \mathbf{b}(s, \xi(s))\,\mathrm{d}s \text{ is well-defined}\}. \tag{6.4}$$

For the integral to be well-defined, the process $b(t) = \mathbf{b}(t, \xi(t))$ has to be integrable, and its discontinuities should have a total measure of zero on $[t_0, T]$.

We only consider the possibility of *buying* credits to offset financed emissions by restricting the range of the feedback control law to $\mathbb{R}_+$. Allowing for negative buying rates (i.e. short selling carbon credits) would create a control problem revolving around carbon credit trading strategies, but this is beyond the scope of

[1]'continuous' refers to the fact that carbon credits are bought continuously over the period $[0, T]$.

this thesis. We focus primarily on the impact of buying credits beforehand instead of at the moment when they need to be retired.

For a given control process $b(t)$, we define the following processes to formulate the restriction that a sufficient number of carbon credits should be bought to offset financed emissions during the lifetime of an asset.

Definition 6.2 (Cumulative number of credits bought)**.** *For a given stochastic process $b(t)$ for $t \in [t_0, T]$ that represents the carbon credit buying rate, we define the stochastic process $B(t)$ by the following SDE:*

$$\mathrm{d}B(t) = b(t)\mathrm{d}t, \tag{6.5}$$

with initial condition $B(t_0) = B_0 \in \mathbb{R}_+$. It represents the cumulative number of carbon credits bought until time $t \in [t_0, T]$, given that B_0 credits are bought at time t_0. We include $B(t)$ as a variable in the representation of the state space Ξ, which is driven by the control process $b(t)$. The process $B(t)$ is adapted to the filtration $\mathcal{F}(t)$ and the solution of the SDE is trivially given by

$$B(t) := B_0 + \int_{t_0}^{t} b(s)\mathrm{d}s. \tag{6.6}$$

Definition 6.3 (Cumulative financed emissions)**.** *We define a constraint on $B(t)$ using the process $L(t)$ which is defined by the SDE*

$$\mathrm{d}L(t) = FE(t)\mathrm{d}t, \tag{6.7}$$

with initial value $L(0) = 0$. This SDE is solved by

$$L(t) = \int_{0}^{t} FE(s)\mathrm{d}s, \tag{6.8}$$

which we refer to as the cumulative financed emissions until time $t \in [0, T]$ associated with an asset, where $FE(t)$ is the annual amount of financed emissions at time $t \in [0, T]$. The process $L(t)$ will be used to define a lower bound constraint on $B(t)$ in Equation (6.9). The process $L(t)$ is adapted to the filtration $\widehat{\mathcal{G}}(t)$ and is deterministic if the financed emissions $FE(t)$ are deterministic.

With these processes, we define the following lower bound constraint on the carbon credit buying strategies for a given process $L(t)$, given that $B(t_0) = B_0$:

$$B(t) \geqslant L(t) \qquad \text{for all } t \in [t_0, T], \tag{6.9}$$

meaning that the cumulative number of bought credits always has to exceed the cumulative financed emissions, ensuring that the financial institution always has enough credits available to retire the necessary number of carbon credits to offset financed emissions. We include this constraint in the set of feedback control laws as follows.

Definition 6.4 (Admissible feedback control law)**.** *For a deterministic constraint process $L(t)$, we refer to a feedback control law $\mathbf{b}$ as admissible at time $t_0 \in [0, T]$ if the resulting process $B(t) = B_0 + \int_{t_0}^{t} b(s)\mathrm{d}s = B_0 + \int_{t_0}^{t} \mathbf{b}(s, \xi(s))\mathrm{d}s$ satisfies $B(t) \geqslant L(t)$ for all $t \in [t_0, T]$. Here, $B_0 \geqslant L(t_0)$ is given. We define the set of admissible feedback control laws by*

$$\widetilde{\mathcal{B}}_{t_0}(B_0) := \{\mathbf{b} \in \mathcal{B}_{t_0} \mid \forall t \in [t_0, T] : B(t) \geqslant L(t)\}. \tag{6.10}$$

6.1.2 Optimal costs and the FEVA

Having formulated the hard constraint on the feedback control laws, we formulate the continuous control problem of minimising the total costs of offsetting financed emissions by

Definition 6.5 (Optimal costs). *We define the optimal costs to be the solution to the stochastic control problem*

$$J(t) := \inf_{\mathbf{b}\in\widetilde{\mathcal{B}}_t(B)} \mathbb{E}_t\left[\int_t^T e^{-r(s-t)} H(s)\cdot \mathbf{b}(s,\xi(s))\,\mathrm{d}s\right], \tag{6.11}$$

where the conditional expectation is taken with respect to the filtration $\mathcal{F}(t)$ that is generated by the state space process $\xi(t)$. Here, the value of the state variable $B := B(t) \geqslant L(t)$ is given, where the lower bound process $L(t)$ is derived from the state space process. The value of $J(t)$ represents the total expected future costs at time t of offsetting financed emissions following an optimal buying strategy, which is given by some feedback control law $\mathbf{b} \in \widetilde{\mathcal{B}}_t(B)$.

Although the expected costs $J(t)$ depend on the state variables at time t, we omit the dependence of J on the value of $\xi(t)$ in our notation, and implicitly include it in the expectation $\mathbb{E}_t$. Notice that this includes the current value of the state variable $B(t) = B$ and therefore also impacts the resulting buying strategies $b(t) = \mathbf{b}(t, \xi(t))$, since $\mathbf{b}$ must satisfy

$$B(s) = \int_t^s \mathbf{b}(u,\xi(u))\,\mathrm{d}u + B(t) \geqslant L(s), \tag{6.12}$$

for all $s \in [t, T]$. Under certain assumptions on the feedback control laws, the existence of a solution to Equation (6.11) can be proven (see for example [88, 181]), but this is beyond the scope of this thesis since we will proceed to discretise the problem in Section 6.2.

It should be noted that Equation (6.11) represents the costs associated with offsetting financed emissions when it is necessary to buy carbon credits to do so, i.e. when the financed emissions are positive. Offsetting negative financed emissions would be done by short selling carbon credits, but this is beyond the scope of this control problem, as established by the constraint $\mathbf{b}(t, \xi) \geqslant 0$ for all $(t, \xi) \in [t_0, T] \times \Xi$. In the remainder of this section, we relate the resulting expected costs $J(t)$ to the FEVA of an asset and discuss the implications. The following theorem relates the costs from the control problem in (6.11) to the FEVA of an asset (see Definition 5.4) in the case of nonnegative financed emissions.

Theorem 6.1 (Improvement over the FEVA). *For all $t \in [0, T]$, the optimal costs from the control problem in Equation* (6.11) *are lower than the FEVA associated with an asset, provided that $B(t) \geqslant L(t)$, i.e.*

$$J(t) \leqslant \mathrm{FEVA}(t, T).$$

Proof. Let $t \in [0, T]$ be fixed. For a given $s \in [t, T]$, we define the deterministic function $f_s : \Xi \to \mathbb{R}_+$ by $f_s(\xi) = FE(s)$, where $FE(s)$ is computed from the state variables from Equation (5.5) assuming that $\xi(s) = \xi$.

Theorem 6.1 can be proven by showing that $\mathbf{b}^* : [t, T] \times \Xi \to \mathbb{R}_+$, defined by

$$\mathbf{b}^*(s, \xi) = f_s(\xi) = (FE(s))^+, \tag{6.13}$$

is a well-defined feedback control law and therefore an element of $\mathcal{B}_t$, making use of the previously defined function f_s. Moreover, it is an admissible feedback control law since for all $s \in [t, T]$, we have

$$\begin{aligned} B(s) &= B(t) + \int_t^s b(u)\,\mathrm{d}u = B(t) + \int_t^s \mathbf{b}^*(u,\xi(u))\,\mathrm{d}u = B(t) + \int_t^s (FE(u))^+\,\mathrm{d}u \\ &\geqslant L(t) + \int_t^s FE(u)\,\mathrm{d}u = L(s), \end{aligned}$$

making use of the assumption that $B(t) \geqslant L(t)$ and the SDEs in Equations (6.5) and (6.7).

Since $\mathbf{b}^* \in \widetilde{\mathcal{B}}_t(B)$, we conclude that

$$\begin{aligned}\mathrm{FEVA}(t,T) &= \mathbb{E}_t\left[\int_t^T e^{-r(s-t)}H(s)\cdot FE(s)\,\mathrm{d}s\right] = \mathbb{E}_t\left[\int_t^T e^{-r(s-t)}H(s)\cdot \mathbf{b}^*(s,\xi(s))\,\mathrm{d}s\right] \\ &\geqslant \inf_{\mathbf{b}\in\widetilde{\mathcal{B}}_t(B)} \mathbb{E}_t\left[\int_t^T e^{-r(s-t)}H(s)\cdot \mathbf{b}(s,\xi(s))\,\mathrm{d}s\right] = J(t).\end{aligned}$$

□

In Section 2.3, we discussed that the FEVA associated with an asset can be charged to the counterparty as the costs of internalising the associated financed emissions. The control problem in Equation (6.11) revolves around minimising these costs by introducing the possibility of buying credits before they need to be retired. This means that their difference $\mathrm{FEVA}(0,T) - J(0) \geqslant 0$ (see Theorem 6.1) can be seen as an expected profit to the financial institution when entering a trade at $t = 0$, or they can charge the counterparty an amount of $J(0)$ instead of $\mathrm{FEVA}(0,T)$, making it more attractive to enter the trade for the counterparty.

6.1.3 Negative financed emissions

The interpretation of 'offsetting financed emission' depends on the sign of $FE(t)$ and should be addressed before we proceed to solve the control problem in Equation (6.11).

We start by observing that if $FE(t) \leqslant 0$ for all $t \in [0,T]$, we also have $\mathrm{FEVA}(t,T) \leqslant 0$ (see Equation 6.2) and the constraint in Equation (6.9) is automatically satisfied for $t_0 = 0$, since

$$L(t) = \int_0^t FE(s)\,\mathrm{d}s \leqslant 0 \leqslant \int_0^t b(s)\,\mathrm{d}s = B(t),$$

making use of the fact that the initial number of carbon credits bought is $B(0) = 0$, meaning that the control problem in Equation (6.11) is trivially solved by the feedback control law $\mathbf{b} : (t,\xi) \mapsto 0$, resulting in $J(t) = 0$. Indeed, since 'offsetting negative emissions' is equivalent with 'selling carbon credits', this situation is meaningless in the context of this control problem, where our interest is in determining the impact of optimal credit buying strategies on the FEVA of individual assets, formulated using feedback control laws $\mathbf{b} : [0,T] \times \Xi \to \mathbb{R}_+$. However, assets with negative financed emissions can still be used to offset other financed emissions on the books of a financial institution, as explained in Section 5.4.

If the financed emissions $FE(t)$ can both be negative and positive over the lifetime $[0,T]$ of an asset, complications can arise in the interpretation of the optimal costs $J(t)$ and how they are incorporated in asset valuation. If the financed emissions are initially positive but turn negative after a given point, this can result into $J(0) > 0$ (since the financed emissions initially required offsetting) but $\mathrm{FEVA}(0,T) < 0$ (if the financed emissions are at a later stage sufficiently negative over a given period). In summary, the interpretation and application of the control problem in Equation (6.11) in the valuation of general assets depends on how negative financed emissions should be treated.

For the remainder of this chapter, we assume that $FE(t) \geqslant 0$ for all $t \in [0,T]$ to investigate how the control problem can be incorporated into asset valuation for the simplest and most common case of assets with positive financed emissions.

6.2 Discrete control problem formulation

To approximate the continuous control problem in Equation (6.11), we introduce a discretisation of the time grid into $m \in \mathbb{N}$ equidistant steps by defining the grid $\{t_j\}_{j\in\mathbb{Z}_{m+1}}$ as follows for $j = 0, 1, \ldots m$:

$$t_j := j \cdot \Delta t, \tag{6.14}$$

where $\Delta t := \frac{T}{m}$. Moreover, we discretise the range of the feedback control laws and restrict our attention to integer amounts of carbon credits to be bought. This discretisation translates naturally into practice, since fractional amounts of carbon credits can currently not be bought. This means that we define the set of feedback control laws in the discretised context by

$$\mathcal{B}_{j_0} := \{\mathbf{b} : \{t_{j_0}, \ldots, t_m\} \times \Xi \to \mathbb{Z}_+\}, \tag{6.15}$$

where $j_0 \in \mathbb{Z}_{m+1}$ is a fixed index that represents the first time t_{j_0} at which a decision has to be made.

Definition 6.6 (Control process, Cumulative number of credits bought, Lower bound process)**.** *For a given $j_0 \in \mathbb{Z}_{m+1}$ and B_{j_0-1}, a feedback control law $\mathbf{b} \in \mathcal{B}_{j_0}$ defines a control process $b = (b_j)_{j\in\{j_0,\ldots,m\}}$, by*

$$b_j := \mathbf{b}(t_j, \xi(t_j)), \tag{6.16}$$

where $j = j_0, \ldots, m$. Moreover, it defines the cumulative number of credits bought $B = (B_j)_{j\in\{j_0,\ldots,m\}}$ by

$$B_j := B_{j_0-1} + \sum_{j'=j_0}^{j} b_{j'}, \tag{6.17}$$

where $j = j_0, \ldots, m$, and B_{j_0-1} represents the number of credits bought prior to time t_{j_0}. We also define a constraint on B using the lower bound process $L = (L_j)_{j\in\{j_0,\ldots,m\}}$ that is defined by

$$L_j := \lceil L(t_j) \rceil = \left\lceil \int_0^{t_j} FE(s)\,\mathrm{d}s \right\rceil, \tag{6.18}$$

where $j = j_0, \ldots, m$. The process L_j represents the minimum cumulative number of carbon credits B_j that have to be bought to offset the financed emissions at time t_j.

The processes b, B and L can be represented as $(m - j_0 + 1)$-dimensional vectors. For $j = j_0, \ldots, m$, the random variables b_j and B_j that are generated by a given feedback control law $\mathbf{b}$ are $\mathcal{F}(t_j)$-measurable, and L_j is even deterministic because we temporarily assumed that the process $FE(t)$ is deterministic. The ceiling operator in Equation (6.18) does not affect the solution to the control problem, since we restrict our attention to integer-valued feedback control laws $\mathbf{b}$ in this section.

We define the set of admissible feedback control laws for the discretised problem by

$$\widetilde{\mathcal{B}}_{j_0}(B_{j_0-1}) := \{\mathbf{b} \in \mathcal{B} \,|\, \forall j \in \{j_0, \ldots, m\} : L_j \leqslant B_j \leqslant U\}, \tag{6.19}$$

where it follows from Equations (6.16) and (6.17) that

$$B_j = B_{j_0-1} + \sum_{j'=j_0}^{m} \mathbf{b}(t_{j'}, \xi(t_{j'})).$$

Here, we also define

$$U := \max_{j\in\mathbb{Z}_{m+1}} L_j$$

as the maximum cumulative number of carbon credits that need to bought at any point. This additional constraint does not affect optimal admissible feedback laws $\mathbf{b} \in \widetilde{\mathcal{B}}_{j_0}(B_{j-1})$, as it is by construction never beneficial to buy more than U carbon credits in total if their prices are always nonnegative. Nevertheless, the restriction $B_j \leqslant U$ makes the solution space of feedback control laws significantly smaller. Since $FE(t)$ is a deterministic process, the upper bound $U \in \mathbb{Z}_+$ is also deterministic. Since we assume that $FE(t) \geqslant 0$, it follows that L_j is increasing in j and therefore $U = L_m$. We assume that the process L and the constant U are given in the remainder of this section.

Definition 6.7 (Optimal costs). *We define the optimal expected remaining costs J in the discrete setting to be the solution to the following control problem for $j = 0, 1 \ldots, m$:*

$$J(t_j, B_{j-1}) := \inf_{\mathbf{b} \in \tilde{\mathcal{B}}_{t_j}(B_{j-1})} \mathbb{E}_{t_j} \left[\sum_{j'=j}^{m} e^{-r(t_{j'}-t_j)} H(t_{j'}) b_{j'} \right], \tag{6.20}$$

where the variable $B_{j-1} \in \{L_{j-1}, \ldots, U\}$ is explicitly included as a variable in the control problem to indicate that the optimal remaining costs at time t_j depend on the number of credits that are already bought at times $t < t_j$. The process $H(t)$ represents the costs of buying a carbon credit and is given in Equation (6.1)*. We use the convention that $B_1 = 0$.*

For $j' \in \{j, \ldots, m\}$, the term $e^{-r(t_{j'}-t_j)} H(t_{j'})$ represents the cost per carbon credit bought at time $t_{j'}$, and hence the conditional expectation in Equation (6.20) represents the total expected future costs from buying carbon credits.

6.2.1 Recursive control via Bellman's principle of optimality

In the remainder of this section, we show that Equation (6.20) can be transformed into a recursive relation. We do this via Bellman's famous principle of optimality, which he formulated in 1957 in the context of control problems:

Lemma 6.2 (Bellman's principle of optimality). *An optimal policy has the property that whatever the initial state and initial decisions are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decisions [28].*

This principle is widely applied in the field of stochastic control theory, and we will use it to prove the following theorem:

Theorem 6.3 (Recursive control problem). *For given $j = 0, 1, \ldots, m-1$ and $B_{j-1} \in \{L_{j-1}, \ldots, U\}$, the control problem in Equation* (6.20) *can be reformulated as*

$$J(t_j, B_{j-1}) = \min_{b \in \mathcal{A}_j(B_{j-1})} \left\{ H(t_j) b + e^{-r\Delta t}\, \mathbb{E}_{t_j} \left[J(t_{j+1}, B_{j-1} + b) \right] \right\}, \tag{6.21}$$

where we define the set of admissible numbers of credits to buy $b \in \mathbb{Z}_+$ as

$$\mathcal{A}_j(B_{j-1}) := \{ b \in \mathbb{Z}_+ \mid L_j \leqslant B_{j-1} + b \leqslant U \}. \tag{6.22}$$

Proof. For $j \in \mathbb{Z}_m$ and $B_{j-1} \in \{L_{j-1}, \ldots, U\}$, Equation (6.20) can be rewritten as follows:

$$\begin{aligned} J(t_j, B_{j-1}) &= \inf_{\mathbf{b} \in \tilde{\mathcal{B}}_{t_j}(B_{j-1})} \mathbb{E}_{t_j} \left[H(t_j) b_j + \sum_{j'=j+1}^{m} e^{-r(t_{j'}-t_j)} H(t_{j'}) b_{j'} \right] \\ &= \inf_{\mathbf{b} \in \tilde{\mathcal{B}}_{t_j}(B_{j-1})} \left\{ H(t_j) b_j + e^{-r\Delta t} \mathbb{E}_{t_j} \left[\sum_{j'=j+1}^{m} e^{-r(t_{j'}-t_{j+1})} H(t_{j'}) b_{j'} \right] \right\}, \end{aligned}$$

which we rewrite using the tower property of iterated conditional expectations [162]:

$$J(t_j, B_{j-1}) = \inf_{\mathbf{b} \in \tilde{\mathcal{B}}_{t_j}(B_{j-1})} \left\{ H(t_j) b_j + e^{-r\Delta t} \mathbb{E}_{t_j} \left[\mathbb{E}_{t_{j+1}} \left[\sum_{j'=j+1}^{m} e^{-r(t_{j'}-t_{j+1})} H(t_{j'}) b_{j'} \right] \right] \right\}. \tag{6.23}$$

The last term in this expression represents the expected future costs at time t_{j+1}, given that the control process $b_{j'} = \mathbf{b}(t_{j'}, \xi(t'_j))$ is followed for $j' = j+1, \ldots, m$. This means that we can relate the control problem at time t_j to the problem at time t_{j+1} and apply Bellman's principle of optimality, as stated in Lemma 6.2.

In the context of the control problem in Equation(6.20), Lemma 6.2 states that given the optimal feedback law $\mathbf{b} \in \widetilde{\mathcal{B}}_{t_j}(B_{j-1})$, the restriction of the feedback law $\mathbf{b}$ to $\{t_{j+1}, \ldots, t_m\} \times \Xi$ solves the following control problem[2] for any value of the state vector $\xi(t_{j+1})$:

$$J(t_{j+1}, B_j) = \inf_{\mathbf{b}' \in \widetilde{\mathcal{B}}_{t_{j+1}}(B_j)} \mathbb{E}_{t_{j+1}} \left[\sum_{j'=j+1}^{m} e^{-r(t_{j'}-t_{j+1})} H(t_{j'}) \mathbf{b}'(t_{j'}\, \xi(t_{j'})) \right], \tag{6.24}$$

where $B_j = B_{j-1} + b_j = B_{j-1} + \mathbf{b}(t_j, \xi(t_j))$ is the number of cumulative credits bought prior to time t_{j+1}, meaning that the optimal feedback law $\mathbf{b} \in \widetilde{\mathcal{B}}_{t_j}(B_{j-1})$ results into

$$\mathbb{E}_{t_{j+1}} \left[\sum_{j'=j+1}^{m} e^{-r(t_{j'}-t_{j+1})} H(t_{j'}) b_{j'} \right] = J(t_{j+1}, B_{j-1} + b_j), \tag{6.25}$$

which can then be substituted into Equation (6.23):

$$J(t_j, B_{j-1}) = \inf_{\mathbf{b} \in \widetilde{\mathcal{B}}_{t_j}(B_{j-1})} \left\{ H(t_j) b_j + e^{-r\Delta t}\, \mathbb{E}_{t_j}\left[J(t_{j+1}, B_{j-1} + b_j) \right] \right\}. \tag{6.26}$$

It can be observed that the only decision that appears in the recursion is the number of credits b_j to buy at the present time t_j (notice that the admissible values of b_j satisfy $L_j \leqslant B_{j-1} + b_j \leqslant U$), and all future decisions in this control problem are captured in the control problem at time t_{j+1}. Instead of taking the infimum over $\widetilde{\mathcal{B}}_{t_j}(B_{j-1})$, we can therefore reformulate the control problem as follows:

$$J(t_j, B_{j-1}) = \min_{b \in \mathcal{A}_j(B_{j-1})} \left\{ H(t_j) b + e^{-r\Delta t}\, \mathbb{E}_{t_j}\left[J(t_{j+1}, B_{j-1} + b) \right] \right\},$$

where $\mathcal{A}_j(B_{j-1})$ is defined as in Equation (6.22). □

Notice that Theorem 6.3 does not include the case that $j = m$, in which case Equation (6.20) simplifies into

$$J(t_m, B_{m-1}) = \min_{b \in \mathcal{A}_m(B_{m-1})} \mathbb{E}_{t_m}\left[H(t_m) \cdot b \right] = \min_{b \in \mathcal{A}_m(B_{m-1})} \{ H(t_m) \cdot b \} = H(t_m) \cdot \min_{b \in \mathcal{A}_m(B_{m-1})} b,$$

making use of the fact that $H(t_m)$ is $\mathcal{F}(t_m)$-measurable and independent of b. This results by Equation (6.22) in

$$J(t_m, B_{m-1}) = H(t_m) \cdot (L_m - B_{m-1})^+. \tag{6.27}$$

Given the recursive relation in Equation (6.21) and the simple expression for $J(t_m, \cdot)$ in Equation (6.27), we propose a dynamic programming approach based on a backward induction scheme to compute the total expected costs at $t = 0$, which is a common approach for this type of control problem [28, 30]. The difficulties in recursively solving the minimisation problem arise from the computation of the conditional expectation

$$\mathbb{E}_{t_j}\left[J(t_{j+1}, k) \right], \tag{6.28}$$

where $j = m-1, m-2, \ldots, 0$ and $k := B_{j-1} + b$ represents the total numbers of credits bought after time t_j. This conditional expectation can be seen as a function of $\xi(t_j)$ since $\mathcal{F}(t_j)$ is generated by the state space process $\xi(t)$, assuming that the state space process is Markovian[3].

Since it is not feasible in practice to recursively derive this conditional expectation analytically, particularly when we introduce stochasticity in the financed emissions and therefore the lower bound process L (see Equation (6.18)) in Section 6.3.2, we approximate the conditional expectation using a Monte Carlo simulation. The idea of this approach is to simulate $N \in \mathbb{N}$ paths of the process $H(t)$ on the discretised time grid, and start the backward induction at time t_m using Equation (6.27).

[2] Technically, Bellman's principle states that the feedback law $\mathbf{b}$ is a solution to the control problem $J(t_{j+\ell}, B_{j-1+\ell})$ for all $\ell \in \{1, \ldots, m-j\}$ for given paths of $\xi(t)$ on $[t_j, t_{j+\ell}]$, and not only for $\ell = 1$.

[3] For Markovian processes, the future distribution of the process only depends on its current value. For non-Markovian processes, the conditional expectation also depends on the values of $\xi(t_{j'})$ for $j' \in \mathbb{Z}_j$

For $j = m-1, m-2, \dots, 0$, the conditional expectation in Equation (6.28) should be approximated inductively for all $k \in \{L_j, \dots, U\}$ along each path, and the optimal number b of credits to be bought at time t_j and the resulting value $J(t_j, B_{j-1})$ should be determined along each path depending on the number $B_{j-1} \in \{L_{j-1}, \dots, U\}$ of credits that are already bought before t_j. This can be done recursively, ending at $t_0 = 0$. Finally, the total expected costs at $t = 0$ of offsetting the associated financed emissions $FE(t)$ over the lifetime $[0, T] \ni t$ of an asset are given by $J(0, 0)$, since no credits are bought before $t = 0$. This is summarised in Algorithm 1.

Algorithm 1 High-level overview of the dynamic programming approach for the control problem in Equation (6.20).

Require: Number of paths N, number of steps m .
Require: Equidistant time grid $\{t_j \mid j \in \mathbb{Z}_{m+1}\}$ with m time steps of Δt, interest rate r.
Require: Tensor of Monte Carlo paths $\xi_i(t_j)$ of the state space, costs matrix $\mathbf{H}_i(t_j)$.
Require: Lower bound process $L \in \mathbb{Z}_+^{m+1}$, Upper bound $U \in \mathbb{Z}_+$.
▷ Where applicable: $i \in [N]$ and $j \in \mathbb{Z}_{m+1}$.

1: Initialise costs at maturity: $\mathbf{J}_i(t_m, k) \leftarrow \mathbf{H}_i(t_m)(L_m - k)^+$ for $k = L_{m-1}, \dots, U$ and $i \in [N]$.
2: $\mathbf{J}_i(t_j, U) \leftarrow 0$ for all $j \in \mathbb{Z}_{m+1}$ and $i \in [N]$.
3: **for** $j = m-1, \dots, 0$ **do**
4: $\quad \hat{\mathbf{y}}_i(k') \leftarrow$ Estimate of $\mathbb{E}_{t_j}\left[J(t_{j+1}, k') \mid \xi(t) = \xi_i(t_j)\right]$ for $i \in [N]$ and $k' = L_j, \dots, U$. ▷ See Section 6.3.
5: $\quad$ **for** $k = L_{j-1}, \dots, U$ **do** ▷ Determine the cost-minimizing strategy along each path.
6: $\quad\quad \mathcal{A}_j(k) \leftarrow \{b \in \mathbb{Z}_+ \mid L_j \leqslant k + b \leqslant U\}$.
7: $\quad\quad \mathbf{J}_i(t_j, k) \leftarrow \min\limits_{b \in \mathcal{A}_j(k)} \{\mathbf{H}_i(t_j) \cdot b + e^{-r\Delta t}\hat{\mathbf{y}}_i(k+b)\}$ for all $i \in [N]$.
8: $\quad$ **end for**
9: **end for**
10: **return** Expected optimal costs $\frac{1}{N}\sum_{i=1}^{N} \mathbf{J}_i(0, 0)$.

Approximating the conditional expectation of Equation (6.28) in the fourth step of Algorithm 1 using a Monte Carlo simulation is not trivial and requires a so-called least squares Monte Carlo method, which we propose in Section 6.3.

6.3 Least squares Monte Carlo approach

In this section, we propose a Least Squares Monte Carlo (LSMC) approach to approximate the conditional expectation in Equation (6.28). This involves a Monte Carlo simulation of $N \in \mathbb{N}$ paths of the state variables $\xi(t)$ on the discretisation of the interval $[0, T]$. As a variance reduction technique, we simulate the involved Wiener processes using antithetic sampling [106].

Since at step $j = m-1, \dots, 0$, the state variables $\xi(t_j)$ take different values along each of the $N \in \mathbb{N}$ paths in the Monte Carlo approach that we proposed at the end of Section 6.2, we only have one simulation of the distribution of $J(t_j, \cdot)$ conditional on $\xi(t_j)$ along each path, which does not yield accurate approximations of the conditional expectation. Therefore, we have to incorporate the cross-sectional information in the simulated paths to obtain a high-quality approximation. We do so by defining the following function for a given $j \in \mathbb{Z}_m$ and a given $k \in \{L_j, \dots, U\}$:

$$f_k : \Xi \to \mathbb{R} : \xi \mapsto \mathbb{E}[J(t_{j+1}, k) \mid \xi(t_j) = \xi]. \tag{6.29}$$

By its Definition (see Equation (6.7)), it can be observed that $J(t_{j+1}, k)$ depends on the future values of the state variables, which means that

$$f_k(\xi(t_j)) = \mathbb{E}[J(t_{j+1}, k) \mid \sigma(\xi(t_j))] = \mathbb{E}_{t_j}[J(t_{j+1}, k)], \tag{6.30}$$

assuming in the last step that the state space process $\xi(t)$ is Markovian, which is the case for the state space processes that we consider in this thesis. Based on the observation in Equation (6.30), we approximate the function f_k by performing least squares polynomial regression on the pairs

$$(\xi_i, y_i) := (\xi(t_j)(\omega_i), J(t_{j+1}, k)(\omega_i)) \in \Xi \times \mathbb{R},$$

where $\omega_i \in \Omega$ represents the i'th simulated Monte Carlo path, for $i \in [N]$.

The polynomial regression is performed with a set of $M \in \mathbb{N}$ continuous basis functions $\psi_1, \ldots, \psi_M \in \mathcal{C}(\Xi)$ of the state space Ξ, and the objective is to find the coefficients $\beta \in \mathbb{R}^M$ that define an approximating function

$$\widehat{f}_k : \Xi \to \mathbb{R} : \xi \mapsto \sum_{\ell=1}^{M} \beta_\ell \psi_\ell(\xi),$$

such that the total mean squared error is minimised:

$$\beta^* := \underset{\beta \in \mathbb{R}^M}{\arg\min} \sum_{i=1}^{N} \left[\sum_{\ell=1}^{M} \beta_\ell \psi_\ell(\xi_i) - y_i \right]^2 = \underset{\beta \in \mathbb{R}^M}{\arg\min} \sum_{i=1}^{N} \left[\sum_{\ell=1}^{M} \mathbf{A}_{i,\ell} \beta_\ell - y_i \right]^2 = \underset{\beta \in \mathbb{R}^M}{\arg\min} ||\mathbf{A} \cdot \beta - y||^2, \tag{6.31}$$

where the matrix $\mathbf{A} \in \mathbb{R}^{N \times M}$ is defined by $\mathbf{A}_{i,\ell} := \psi_\ell(\xi_i)$ for $i \in [N]$ and $\ell \in [M]$ to express the least squares regression problem more conveniently.

6.3.1 Heteroscedasticity

This LSMC approach is widely used in American option pricing [141, 153], where the heteroscedasticity of the errors in the conditional expectations have a negative impact on the accuracy of the results from the LSMC method, since it results in least squares regression estimates not being the best linear unbiased estimator[82].

Definition 6.8 (Heteroscedastic error)**.** *In the context of the LSMC method to approximate the conditional expectation in Equation* (6.30)*, we formulate the error in the conditional expectation by the conditional variance*

$$\mathcal{V}(\xi) := \mathbb{V}\mathrm{ar}[J(t_{j+1}, k) | \xi(t_j) = \xi], \tag{6.32}$$

and the error is called heteroscedastic if $\mathcal{V}$ is not constant as a function of $\xi \in \Xi$.

When performing regression, data points (ξ_i, y_i) with a high expected error $\mathcal{V}(\xi_i)$ should have a lower impact on the regression estimate, which is usually done using weighted least squares regression [154]. To correct for heteroscedasticity in the context of our control problem, we adapt the approach of [82] and perform weighted least squares regression, which requires an estimate of the conditional standard deviation $\sqrt{\mathcal{V}(\xi)}$, which we approximate by defining

$$\epsilon_i := |\widehat{f}_k(\xi_i) - y_i|, \tag{6.33}$$

and then performing a separate least squares regression on the pairs (ξ_i, ϵ_i) to obtain an estimate[4] $\widehat{\epsilon}_i$ for $\sqrt{\mathcal{V}(\xi)}$. This estimate is applied as a weight in the minimisation problem in Equation (6.31) by introducing

$$y_i' := \frac{y_i}{\widehat{\epsilon}_i} \qquad \text{and} \qquad \mathbf{A}'_{i,\ell} := \frac{\mathbf{A}_{i,\ell}}{\widehat{\epsilon}_i},$$

where $i \in [N]$ and $\ell \in [M]$. These weighted variables can then be used to obtain an improved estimate

$$\beta'^* := = \underset{\beta \in \mathbb{R}^M}{\arg\min} ||\mathbf{A}' \cdot \beta - y'||^2, \tag{6.34}$$

which defines the heteroscedasticity-corrected approximation $\widehat{f}'_k(\xi) := \sum_{\ell=1}^{M} \beta'^*_\ell \psi_\ell(\xi)$ for the conditional expectation function f_k. This weighted least squares approach is formulated in Algorithm 2.

[4]This estimate may have to be adjusted to ensure $\widehat{\epsilon}_i > 0$ or avoid numerical instability, as is done in Algorithm 2.

Algorithm 2 (Weighted) Least Squares Regression.

Require: Number of pairs N, boolean **WeightedRegression**.
Require: Numerical stability constant $\delta \in \mathbb{R}_{++}$ if **WeightedRegression** is true.
Require: Pairs $(\xi_i, y_i) \in \Xi \times \mathbb{R}$ to perform regression on. ▷ Here, $i \in [N]$.
Require: Set of M basis functions $\{\psi_1, \dots, \psi_M\} \subseteq \mathcal{C}(\Xi)$.

$\mathbf{A}_{i,\ell} \leftarrow \psi_\ell(\xi_i)$ for all $i \in [N], \ell \in [M]$.
$\beta^* \leftarrow \arg\min_{\beta \in \mathbb{R}^M} ||\mathbf{A} \cdot \beta - y||^2$. ▷ Least squares regression, where $y \in \mathbb{R}^N$.
if WeightedRegression then
    Define approximating function $\widehat{f} : \xi \mapsto \sum_{\ell=1}^M \beta_\ell^* \psi_\ell(\xi)$ on Ξ.
    $\epsilon_i \leftarrow |\widehat{f}(\xi_i) - y_i|$. ▷ Absolute error of the approximation.
    Perform regression on pairs (ξ_i, ϵ_i) for $i \in [N]$ using Algorithm 2, where $\Xi = \mathbb{R}$, an appropriate set of basis functions is chosen and **WeightedRegression** = false. ▷ One level of recursion.
    $\widehat{\epsilon}_i \leftarrow$ Regression estimate on input ξ_i for all $i \in [N]$.
    $\widehat{\epsilon}_i \leftarrow \max(\widehat{\epsilon}_i, \delta)$ for all $i \in [N]$. ▷ For numerical stability.
    $\mathbf{A}'_{i,\ell} \leftarrow \mathbf{A}_{i,\ell}/\widehat{\epsilon}_i$ and $y'_i \leftarrow y_i/\widehat{\epsilon}_i$ for all $i \in [N]$ and $\ell \in [M]$.
    $\beta^* \leftarrow \arg\min_{\beta \in \mathbb{R}^M} ||\mathbf{A}' \cdot \beta - y'||^2$. ▷ Least squares regression, where $y' \in \mathbb{R}^N$.
end if
Define approximating function $\widehat{f} : \xi \mapsto \sum_{\ell=1}^M \beta_\ell^* \psi_\ell(\xi)$ on Ξ.
return Regression estimates $\widehat{y}_i \leftarrow \widehat{f}(\xi_i)$ for $i \in [N]$.

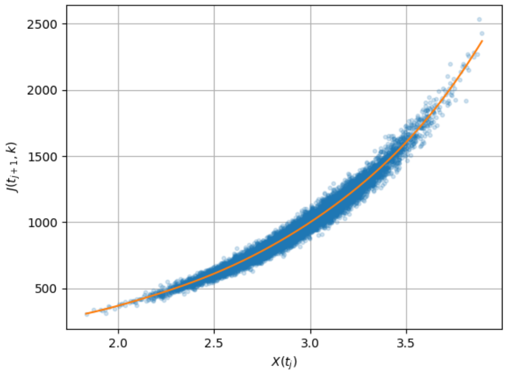
The proposed method of least squares regression in Algorithm 2 to approximate the conditional expectation in Equation (6.30) as a function of $\xi \in \Xi$ can be used to perform step 4 in Algorithm 1.

Before we implement Algorithm 1, we introduce the possibility of stochastic financed emissions in the next section.

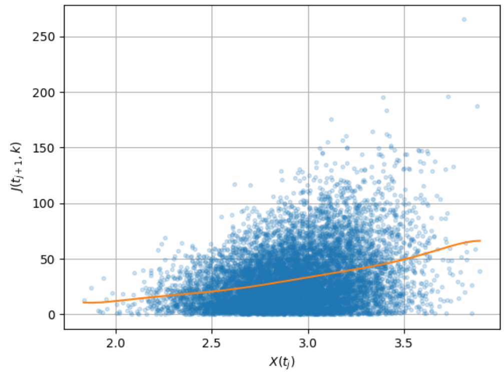
Remark 6.1 (Generalisation of the Longstaff-Schwartz algorithm). *The Longstaff-Schwartz algorithm for early-exercise option pricing can be formulated as a specific instance of the control problem in Equation* (6.20) *which can then be solved by Algorithm 3. Instead of the strategy b_j corresponding to an number of bought credits at time t_j, it represents the number of times that the option is exercised. We summarise the four adjustments that have to be made to transform the control problem in Equation* (6.20) *into the problem of American option pricing: (1) obviously, an option can only be exercised once, meaning that $U = 1$ is the appropriate upper bound on the total number of times $B_j = \sum_{j'=0}^{j} b_{j'}$ that an option can be exercised up to time t_j. (2) Since options do not have to be exercised, the lower bound on B_j is $L_j = 0$ for any j. (3) The process $H(t)$ should correspond to the payoff when the option is exercised on time t instead of the costs of buying a credit at time t, and (4) the feedback law* $\mathbf{b}$ *that solves the control problem in Equation* (6.20) *should then maximise the remaining payoff instead of minimise the remaining costs. This means that every minimum should be replaced by a maximum in Algorithm 3.*

6.3.2 Stochastic financed emissions

In Sections 6.1, 6.2 and 6.3, we assumed that the financed emissions $FE(t)$ and therefore the lower bound process were deterministic. Stochastic lower bounds on the feedback control law $\mathbf{b}$ introduce additional technicalities in the continuous control problem in Definition 6.5 and the existence of a solution. Similar continuous control problems are usually defined in a context where the control process takes values in a deterministic, compact space, see for example [88, 181]. These issues can be avoided by introducing soft constraints in the form of Lagrangian penalty functions [4, 81], for example, but these workarounds are beyond the scope of this thesis, since we focus on explicitly solving the discretised version of the control problem in Definition 6.5.



(a) *Scatter plot of the values of $J(t_{j+1}, k)$ along the Monte Carlo paths as a function of the values of $X_1(t_j)$ along the paths. The approximating function $\widehat{f}_k(\cdot)$ of the conditional expectation is shown in orange.*



(b) *Scatter plot of the error $|J(t_{j+1}, k) - f_k(X_1(t_j))|$ along the Monte Carlo paths, as a function of the values of $X_1(t_j)$ along the paths. The approximating function of $\sqrt{\mathcal{V}(\cdot)}$ is shown in orange.*

Figure 6.1: *Illustration of an arbitrary regression step in the LSMC approach (implemented in Algorithm 3), and the heteroscedasticity that can arise. 10000 regression points are shown in blue, and the approximating function, obtained from least squares regression, is shown in orange. In this case, the state space Ξ is one-dimensional and contains only the log-price of a carbon credit in a one-dimensional VCM model. It can be observed that the variance in the future costs increases as $X_1(t_j)$ increases, meaning that the error is heteroscedastic.*

Introducing uncertainty of $FE(t)$ and therefore the lower bound process L_j in the discretised control problem in Definition 6.7 also requires an adjustment to the problem formulation. Therefore, we repeat the definition of the lower bound in the discrete problem (see Equation (6.18)):

$$L_j := \lceil L(t_j) \rceil = \left\lceil \int_0^{t_j} FE(s)\,\mathrm{d}s \right\rceil,$$

where $j = j_0, \dots, m$. The lower bound L_j is now adapted to the filtration $\mathcal{F}(t_j)$. Moreover, the upper bound

$$U := \max_{j \in \mathbb{Z}_{m+1}} L_j \tag{6.35}$$

is $\mathcal{F}(t_m)$-measurable and path-dependent, meaning that the admissible feedback control laws set

$$\widetilde{\mathcal{B}}_{j_0}(B_{j_0-1}) := \{\mathbf{b} \in \mathcal{B} \,|\, \forall j \in \{j_0, \dots, m\} : L_j \leqslant B_j \leqslant U\}, \tag{6.36}$$

where $j_0 \in \mathbb{Z}_{m+1}$ and $\widetilde{\mathcal{B}}_{j_0}(B_{j_0-1}) \in \{L_{j-1}, \dots, U\}$ are path-dependent as well. Since U is not $\mathcal{F}(t_j)$-measurable for $j < m$, however, the set of admissible feedback controls in the control problem in Equation (6.20) is not well-defined at time t_j, meaning that the problem can not be solved. In other words, it is not known beforehand what the maximum number of credits to buy is when determining an optimal feedback control law. This problem can be circumvented by removing the upper bound constraint $B_j \leqslant U$ entirely, or equivalently letting $U = \infty$. However, the size of the solution space of admissible feedback control laws and therefore the computational complexity of the problem can be significantly reduced by imposing a reasonable upper bound $U \in \mathbb{N}$ such that

$$B_j \leqslant U,$$

for all $j \in \{j_0, \dots, m\}$. For the cases that the cumulative financed emissions $L(t)$ exceed the given upper bound U, we define

$$L_j := \min\left(\left\lceil \int_0^{t_j} FE(s)\,\mathrm{d}s \right\rceil, U \right), \tag{6.37}$$

meaning that the bought number of credits B_j does not have to exceed $\int_0^{t_j} FE(s)$ if the cumulative amount of financed emissions exceeds U at time t_j. In this way, the set $\mathcal{A}_j(B_{j-1})$ (see Equation (6.22)) is well-defined at time t_j since all variables in its definition are $\mathcal{F}(t_j)$-measurable, for $B_{j-1} = L_{j-1}, \ldots, U$. We propose an extension of Algorithm 1 in Algorithm 3, where we also explain the LSMC approach that we proposed in Section 6.3.

Algorithm 3 LSMC approach for the control problem in Equation 6.21.

Require: Number of paths N, number of steps m.
Require: Equidistant time grid $\{t_j \mid j \in \mathbb{Z}_{m+1}\}$ with m time steps of Δt, interest rate r.
Require: Tensor of Monte Carlo paths $\xi_i(t_j)$ of the state space process $\xi(t)$, including the process $L(t)$.
Require: Upper bound $U \in \mathbb{Z}_+$.
▷ Where applicable: $i \in [N]$ and $j \in \mathbb{Z}_{m+1}$.

Compute the costs matrix $\mathbf{H}_i(t_j)$ for $i \in [N]$ and $j \in \mathbb{Z}_{m+1}$.
$\mathbf{L}_i(t_j) \leftarrow \min\left(\lceil L(t_j)(\omega_i)\rceil, U\right)$ for $i \in [N]$ and $j \in \mathbb{Z}_{m+1}$. ▷ ω_i denotes the i'th Monte Carlo path.
$\widetilde{L}_j \leftarrow \min_{i \in [N]} \mathbf{L}_i(t_j)$ for $j \in \mathbb{Z}_{m+1}$. ▷ Lower bound in the induction steps.
Initialise feedback control law at maturity: $b_i(t_m, k) \leftarrow (\mathbf{L}_i(t_m) - k)^+$ for $k = 0, \ldots, U$ and $i \in [N]$.
Initialise costs at maturity: $\mathbf{J}_i(t_m, k) \leftarrow \mathbf{H}_i(t_m) b_i(t_m, k)$ for $k = 0, \ldots, U$ and $i \in [N]$.
$\mathbf{J}_i(t_j, U) \leftarrow 0$ and $b_i(t_j, U)$ for all $j \in \mathbb{Z}_{m+1}$ and $i \in [N]$. ▷ Trivial boundary case.
for $j = m-1, \ldots, 1$ **do**
  $x_i \leftarrow \xi_i(t_j)$ for all $i \in [N]$.
  for $k' = \widetilde{L}_j, \ldots, U-1$ **do** ▷ Approximate remaining costs $\mathbb{E}_{t_j}[\mathbf{J}_i(t_{j+1}, k')]$.
    $\mathcal{I}_{k'} \leftarrow \{i \in [N] \mid \mathbf{L}_i(t_j) \leqslant k'\}$.
    $y_i(k') \leftarrow \mathbf{J}_i(t_{j+1}, k')$ for all $i \in \mathcal{I}_{k'}$. ▷ Only regress on paths with admissible values of $B_j = k'$.
    Perform regression on pairs $(x_i, y_i(k'))$ for $i \in \mathcal{I}_{k'}$. ▷ See algorithm 2.
    $\hat{\mathbf{y}}_i(k') \leftarrow$ Regression estimate on input x_i for all $i \in \mathcal{I}_{k'}$.
  end for
  $\hat{\mathbf{y}}_i(U) \leftarrow 0$ for all $i \in [N]$. ▷ Trivial boundary case.
  for $i \in [N]$ **do** ▷ Determine the cost-minimizing strategy along each path.
    for $k \in \{\min(\mathbf{L}_i(t_{j-1}), U), \ldots, U-1\}$ **do**
      $\mathcal{A}_{i,j}(k) \leftarrow \{b \in \mathbb{Z}_+ \mid \min(\mathbf{L}_i(t_j), U) \leqslant k + b \leqslant U\}$.
      $b_i(t_j, k) \leftarrow \arg\min_{b \in \mathcal{A}_{i,j}(k)} \{\mathbf{H}_i(t_j) \cdot b + e^{-r\Delta t} \hat{\mathbf{y}}_i(k+b)\}$
      $\mathbf{J}_i(t_j, k) \leftarrow \mathbf{H}_i(t_j) \cdot b_i(t_j, k) + e^{-r\Delta t} \hat{\mathbf{y}}_i(k + b_i(t_j, k))\}$.
    end for
  end for
end for
$b_i(0,0) \leftarrow \arg\min_{b \in \mathbb{Z}_{U+1}} \left\{H(0) \cdot b + e^{-r\Delta t} \mathbf{J}_i(t_1, b)\right\}$ for all $i \in [N]$. ▷ Given that $\mathbf{H}_i(0) = H(0)$ for $i \in [N]$.
$\mathbf{J}_i(0,0) \leftarrow H(0) \cdot b_i(0,0) + e^{-r\Delta t} \mathbf{J}_i(t_1, b_i(0,0))$ for all $i \in [N]$.
return Expected optimal costs $\frac{1}{N}\sum_{i=1}^{N} \mathbf{J}_i(0,0)$ and optimal feedback control law $(b_i(j,k))_{i,j,k}$.

We explicitly include the cumulative financed emissions process $L(t) = \int_0^t FE(s)\,\mathrm{d}s$, which can be computed from the Monte Carlo simulation using the trapezoidal rule [226], in the state space process tensor because it contains necessary information in $\mathcal{F}(t)$ to determine optimal buying strategies. Moreover, we define a lower bound tensor $\mathbf{L}$ by

$$\mathbf{L}_i(t_j) \leftarrow \min\left(\lceil L(t_j)(\omega_i)\rceil, U\right),$$

where $\omega_i \in \Omega$ denotes the i'th Monte Carlo path, so that the optimal strategy to achieve the costs $J(t_j, B_{j-1})$ only has to be determined given admissible values of B_{j-1} along each path (see Equation (6.21)). However, the regression step in Algorithm 3 to approximate $\mathbb{E}_{t_j}[J(t_{j+1}, k')]$ requires cross-sectional information, but we choose to only regress on paths ω_i that follow from admissible feedback laws, i.e. on paths $i \in [N]$ such that $\mathbf{L}_i(t_j) \leqslant k'$. Numerical tests suggest that these paths do not improve the accuracy of the regression step, but reduce the computational efficiency. The smallest k' for which a least squares regression estimate for $\mathbb{E}_{t_j}[J(t_{j+1}, k')]$ has to be obtained in Algorithm 3 is given by

$$\widetilde{L}_j := \min_{i \in [N]} \mathbf{L}_i(t_j), \tag{6.38}$$

where $j \in \mathbb{Z}_m$.

Not only do we compute the expected costs in Equation (6.20) with Algorithm 3, we also approximate the optimal feedback control law along the Monte Carlo paths by defining $b_i(j,k)$ to be the optimal number $b \in \mathcal{A}_j(k)$ of credits to buy along the i'th Monte Carlo path, for $i \in [N]$, $j \in \mathbb{Z}_{m+1}$ and $k \in \{\min(\mathbf{L}_i(t_j), U), \ldots, U\}$. Following this feedback control law should theoretically result in the optimal costs $J(0,0)$ on average over the Monte Carlo paths in theory, but accumulating regression errors can result in this not being the case, as we discuss in Remark 6.3.

At the last step of the recursion (see Equation (6.21)), $J(0, B_{-1})$ only has to be computed for $B_{-1} = 0$. Moreover, it should be noted that we do not perform regression at the last step of the recursion, since the LSMC estimate of $\mathbb{E}_0[J(t_1, b)]$ for $b \in \mathcal{A}_j(0) = \mathbb{Z}_{U+1}$ is trivially the mean value of $J(t_1, b)$ along all paths. By not taking the average at the last step, which is common practice in the context of American option pricing [141], we obtain the variance of the total costs over the N paths.

6.3.3 Comparing buying strategies

With the LSMC approach sketched in Algorithm 3, we investigate the impact of early buying strategies on the costs of offsetting financed emissions. We will compare the expected costs $J(0,0)$ from optimal buying strategies (see Equation (6.7)) with the FEVA of an asset, but we also have to introduce another buying strategy to compare to. The FEVA and the continuous control problem are compared in Theorem 6.1, but a buying strategy such as $b_j = FE(t_j)$ is not possible in the discretised problem because we restrict feedback control laws $\mathbf{b}$ to take integer values only. Analogously to Equation (6.23), we define the following buying strategy and associated costs:

Definition 6.9 (Discrete no early buying strategy.). *We define a discrete feedback control* $\mathbf{b}_0 \in \widetilde{\mathcal{B}}_j(B_{j-1})$ *by*

$$\mathbf{b}_0(t_j, \xi(t_j)) = b_j := L_j - B_{j-1}, \tag{6.39}$$

where $j \in \mathbb{Z}_{m+1}$*, and* L *is defined as in Equation* (6.37)*. This is the feedback control law that represents a strategy of only buying carbon credits when it is required by the constraint* $B_j \geqslant L_j$*. This strategy therefore does not consider the possibility of buying credits in anticipation of future emissions.*

It should be noted that $\mathbf{b}_0$ indeed only takes on values in $\mathbb{Z}_+$ since $B_j = L_j$ follows for all $j \in \mathbb{Z}_{m+1}$, which follows from a simple proof by induction[5]. It can therefore directly be observed from Equation (6.20) that the costs from this strategy are given by

$$J_0(t_j, B_{j-1}) = \mathbb{E}_{t_j}\left[\sum_{j'=j}^{m} e^{-r(t_{j'}-t_j)} H(t_{j'}) b_{j'}\right] = \mathbb{E}_{t_j}\left[\sum_{j'=j}^{m} e^{-r(t_{j'}-t_j)} H(t_{j'})(L_{j'} - L_{j'-1})\right], \tag{6.40}$$

meaning that the feedback law $\mathbf{b}_0$ only prescribes buying credits at the moment when the discrete process L increases. We argue that this is the discrete variant of the continuous 'FEVA-replicating strategy' (see Equation (6.13)) with control process $b(t) = FE(t)$ in the continuous problem, and that the continuous limit of $J_0(t_j, B_{j-1})$ is given by the FEVA of an asset, since for $j \in \mathbb{Z}_{m+1}$, time t_j changes to $t \in [0,T]$, the summation over $j' = j, \ldots, m$ changes to an integral over $s \in [t,T]$ and $L_{j'} - L_{j'-1}$ changes to $\mathrm{d}L(s) = FE(s)\,\mathrm{d}s$, which results into

$$J_0(t) = \mathbb{E}_t\left[\int_t^T e^{-r(s-t)} H(s) \cdot FE(s)\,\mathrm{d}s\right],$$

[5] Initially, it follows that $L_0 = B_0 = 0$, and the equality $B_j = B_{j-1} + b_j = L_j$ can then be proven from Equation (6.39) for all $j \in \mathbb{Z}_{m+1}$.

which is exactly the definition of the FEVA. Therefore, we can interpret $J_0(0,0)$ as the discrete variant of the value adjustment FEVA$(0,T)$ in this context.

Remark 6.2 (Buying credits in batches). *In practice, VCM participants do not buy individual carbon credits, but rather buy credits in batches, often of 1000 credits per batch. In Section 6.4, we introduce a batch size $\mathcal{S} \in \mathbb{N}$ by restricting the feedback control laws $\mathbf{b}$, and therefore the processes b and B, to take values in $\{0, \mathcal{S}, 2\mathcal{S} \ldots, U\}$, and we assume for convenience that U is a multiple of $\mathcal{S}$. This restriction reduces the computational complexity of Algorithm 3 since the regression estimates $\widehat{\mathbf{y}}_i(k)$ of the conditional expectations of future costs are only required for k's that are multiples of $\mathcal{S}$. However, this restriction also effectively increases the lower bound process $L(t_j)$, since rounding $L(t_j)$ up to the next multiple of $\mathcal{S}$ does not affect the values that the admissible feedback laws $\mathbf{b} \in \widetilde{\mathcal{B}}_j(B_{j-1})$ can take, for $j \in \mathbb{Z}_{m+1}$. Because of the increased lower bounds from this restriction of a batch size, the introduction of a batch size $\mathcal{S}$ will generally result in higher expected costs of offsetting financed emissions, resulting from Algorithm 3.*

In the remainder of this chapter, we investigate the effectiveness of early buying strategies in reducing the costs of offsetting financed emissions associated with assets. We do so by solving the discrete control problem in Equation (6.21) using Algorithm 3 and compare the resulting costs to the costs J_0 that result from the strategy without early buying in Definition 6.9, which functions as a baseline.

6.4 Numerical results

We explore the impact of early buying strategies for two types of assets. Firstly, we consider a European option (see Example A.1), where the uncertainty in the exposure comes from the price of the underlying asset. Secondly, we consider a fixed rate bond (see Example A.3), which has a deterministic exposure profile. We do this to investigate the impact of uncertainty in the exposure profile on the effectiveness of early buying strategies. To amplify this distinction, we introduce uncertainty in the emission intensity of the counterparty of the European option, but assume that the emissions intensity of the counterparty of the fixed rate bond is deterministic for simplicity. In the case of the fixed rate bond, we ignore the possibility of counterparty default, resulting into the financed emissions being deterministic. These considerations are summarised in Table 6.1.

Table 6.1: *Overview of the two assets that are considered to produce the numerical results in Section 6.4. The European option represents an asset for which the financed emissions and therefore the lower bound $L(t)$ in the control problem carry uncertainty, while the fixed rate bond has deterministic financed emissions.*

Asset type	European option contract	Fixed rate bond
Exposure $E(t)$	$V(t, S(t))$ (Equation (A.7))	Deterministic (Equation (A.10))
Emission intensity $I(t)$	GBM (Equation (5.4))	$I(t) = I_0 e^{\mu_I t}$
Default process	NHPP with default rate $\widehat{\lambda}(t) = \widehat{\lambda}$	No counterparty default
Financed emissions $FE(t)$	$FE(t) = \mathbb{1}_{\hat{t}_D > t} \cdot E(t) \cdot I(t)$	$FE(t) = E(t) \cdot I_0 e^{\mu_I t}$
Lower bound $L(t)$	Stochastic	Deterministic

For all assets, we define $H(t)$ (see Equation (6.1)) to be the price of a carbon credit from a d-dimensional market model. Here, we let all log-price processes $X_i(t)$ be driven by an SDE with the same parameters $\theta_i = \theta$, $\mu_i = \mu$, $\sigma_i = \sigma$ and $X_i(0) = x$ for all $i \in [d]$. We investigate the impact of the CTD aspect on early buying strategies by considering a VCM model of $d = 1$, as well as a higher-dimensional market. In this case, we assume an AR(1) correlation structure (see Equation (3.14)) with correlation parameter $\rho_0 \in [-1, 1]$. We give the parameter configuration that we apply in all simulations in Table 6.2, and the asset-specific parameter configurations that are applied in all simulations are given in Table 6.3.

Table 6.2: *Parameter configuration that is used in all simulations in Section 6.4, for both the European option contract and the fixed rate bond.*

Parameter	Notional value	T	r	I_0	μ_I	θ	$\mu = x$	σ	ρ_0
Value	€1 million	4	0.05	$6 \cdot 10^5$	-0.07	0.5	log(20)	0.3	0.8

Table 6.3: *Parameter configuration that is used in all simulations in Section 6.4 that are specific to either the European option contract of the fixed rate bond.*

Asset	European option contract						Fixed rate bond	
Parameter	σ_I	σ_S	K	S_0	$\widehat{\lambda}$	U	K	U
Value	0.1	0.2	40	40	0.01	800	$e^r - 1$	200

Before investigating the optimal strategies, we investigate the lower bound process $L(t)$ for the two considered assets. In Figure 6.2, the deterministic process $L(t)$ for the fixed rate bond and the stochastic process $L(t)$ for the European option contract are shown. It can be observed that the lower bound is smooth, as it is the integral of the financed emissions process $FE(t)$. In the case of the European option contract, the resulting lower bound process follows a positively skewed distribution, which will generally result in the total costs of the bought carbon credits being positively skewed as well. By letting the upper bound on L_j ($j \in \mathbb{Z}_{m+1}$) be $U = 800$ in Algorithm 3, we allow the cumulative financed emissions to exceed the upper bound on B_j in a fraction of

$$\mathbb{P}_0[L(T) > U] \approx 0.01$$

of the Monte Carlo paths. Having to buy fewer carbon credits than $L(T)$ in total will result in lower optimal costs in this fraction of the Monte Carlo paths compared to if no upper bound U would have been present, resulting into the optimal strategy being perceived as more effective than it would be without this upper bound along approximately 1% of the paths. However, the computational complexity of the problem grows with the upper bound (see Remark 6.2), so we establish the upper bound $U = 800$ to make a balanced trade-off between accuracy and computational complexity.

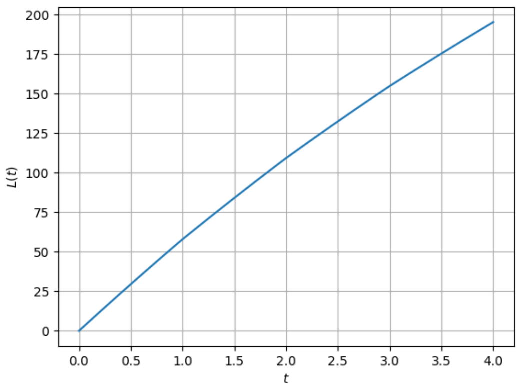
As the basis function for the state space of dimension $D \in \mathbb{N}$ in the regression step (see Algorithm 2), we define the basis functions $\psi_\ell : \Xi \to \mathbb{R}$ to be the ℓ'th entry of the following $\mathbb{R}^M$-valued function (where $M := 1 + 2D + \frac{(D+1)D}{2}$) for $\ell \in [M]$:

$$\psi(x) = (1, x_1, \ldots, x_D, x_1^2, x_1 x_2, \ldots, x_{D-1} x_D, x_D^2, x_1^3 \ldots, x_D^3)^T, \tag{6.41}$$

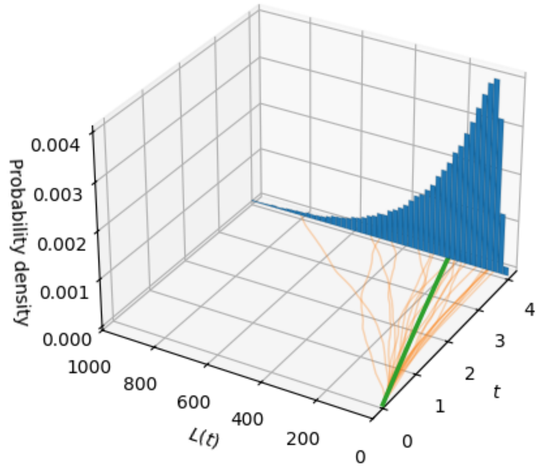
where x_i represents the i'th variable in the state space Ξ for $i \in [D]$. In other words, we consider univariate monomials of degree $\leqslant 3$ and all multivariate monomials of degree 2 to account for interaction between the state variables. We make this choice based on the observation of Longstaff and Schwartz [141], who suggest that the LSMC approach requires a small number of basis functions to achieve reasonable convergence and accurate results, and a limited number of cross-terms in higher-dimensional problems. Moreover, they find that the choice of basis functions for the state space does not significantly impact the convergence when comparing monomials, trigonometric functions, Laguerre polynomials and other types of basis functions. This result is also found in later studies [30, 153].

We investigate the effectiveness of early buying strategies by comparing three metrics for the given parameter configurations. Firstly, we compute the FEVA of the assets at time $t = 0$ from the financed emissions and the carbon credit price using Equation (5.7):

$$\text{FEVA}(0, T) = \mathbb{E}_0 \left[\int_0^T e^{-rt} H(t) \cdot FE(t) \, \mathrm{d}t \right],$$



(a) Deterministic lower bound process $L(t)$ for the fixed rate bond with annual payments with an interest rate of $K = e^r - 1$ over the notional value of the bond.



(b) Monte Carlo simulation of 10^5 paths of $L(t)$ for the European option contract, including its probability density function at $t = T$. Twenty paths are shown in orange, and $\mathbb{E}_0[L(t)]$ is shown in green.

Figure 6.2: *Illustration of the lower bound process $L(t)$ for both assets in Table 6.1. The parameter configuration is given in Table 6.2, and the European option contract parameters are given in Table 6.3. The estimates are obtained by numerical integration using the trapezoidal rule, where the interval $[0, T]$ is divided into $m = 200$ equidistant steps.*

where we compute the financed emissions from the exposure profile, emission intensity profile and the default process in the case of the European option contract. Secondly, we compute the costs $J_0(0, 0)$ from the no early buying strategy (see Equation (6.40)) as the discretised counterpart of $\text{FEVA}(0, T)$:

$$J_0(0, 0) = \mathbb{E}_0 \left[\sum_{j=1}^{m} e^{-rt_j} H(t_j)(L_j - L_{j-1}) \right],$$

where the discretised lower bound is defined by Equation (6.37), and we used the fact that $L_0 = 0$. Lastly, we compute the optimal buying strategy and the resulting costs at time $t = 0$, which are given by Equation (6.20):

$$J(0, 0) = \inf_{\mathbf{b} \in \widetilde{\mathcal{B}}_0(0)} \mathbb{E}_0 \left[\sum_{j=0}^{m} e^{-rt_j} H(t_j) b_j \right],$$

where

$$b_j := \mathbf{b}(t_j, \xi(t_j)) \tag{6.42}$$

is the control process that results from the feedback control law $\mathbf{b}$, which we restrict to take values that are multiples of the batch size $\mathcal{S} \in \mathbb{N}$, and $\widetilde{\mathcal{B}}_0(0)$ is defined in Equation (6.19). The optimal feedback control law is computed with Algorithm 3. We investigate the effectiveness of early buying strategies by comparing the costs $J(0, 0)$ to the costs $J_0(0, 0)$ that result from the no early buying strategy.

Remark 6.3 (Realised costs from optimal feedback laws)**.** *In theory, Algorithm 3 is used to determine the optimal feedback control law $\mathbf{b}^*$ that solves Equation (6.20), and also estimates the expected costs $J(0, 0)$ by iteratively computing the conditional expectation in Equation (6.21). In theory, the estimated expected optimal costs*

$$J_{\text{est}}(0, 0) := \frac{1}{N} \sum_{i=1}^{N} \mathbf{J}_i(0, 0) \tag{6.43}$$

should therefore correspond to the expected costs from the optimal feedback law $\mathbf{b}^*$ *that results from the algorithm. However, errors in the approximation of the conditional expectation in the regression step will accumulate with this approach, which can result in significant differences between the average resulting* $\mathbf{J}_i(0,0)$ *and*

$$J_{\text{real}}(0,0) = \frac{1}{N}\sum_{i=1}^{N}\left[\sum_{j=0}^{m} e^{-rt_j} H(t_j)(\omega_i)\cdot \mathbf{b}^*(t_j,\xi(t_j)(\omega_i))\right], \tag{6.44}$$

which are the average realised total costs along Monte Carlo paths $\omega_i \in \Omega$ *(for* $i \in [N]$*) from feedback control law* $\mathbf{b}^*$*.*

We investigate the impact of a higher-dimensional state space on the effectiveness of Algorithm 3 using a European option, where the state space Ξ corresponds to the processes $X(t) = (X_1(t),\ldots,X_d(t))^T$, $I(t)$ and $S(t)$. The expected costs from Algorithm 3 and the no early buying strategy in Definition 6.9 are shown in Figure 6.3.

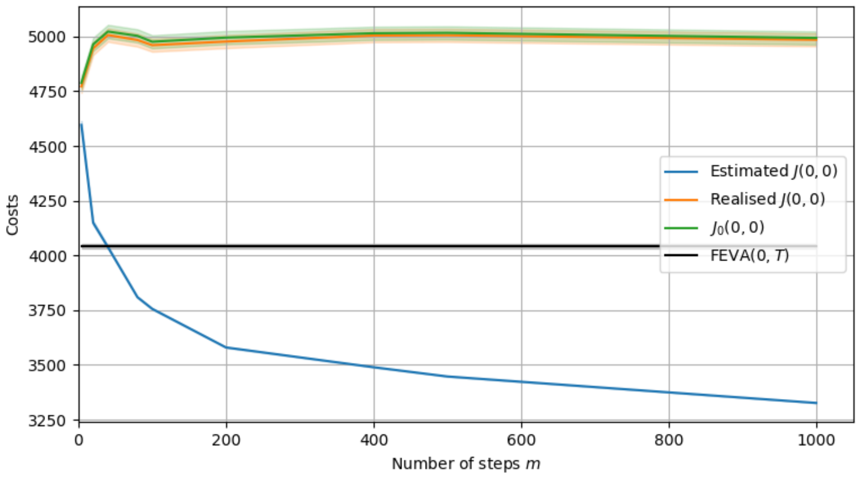


Figure 6.3: *Impact of the number of steps m in the discretisation of the interval* $[0,T]$ *on the expected costs resulting from Algorithm 3 with a batch size* $\mathcal{S} = 100$*, for the European option contract in Table 6.1. The resulting estimated costs* $J_{\text{est}}(0,0)$ *are shown in blue, but the resulting optimal feedback control law results on average in the realised costs* $J_{\text{real}}(0,0)$ *that are shown in orange, as given in Equation* (6.44)*. These realised costs lie within a standard deviation from the costs* $J_0(0,0)$ *in green that result from the no early buying strategy, as in Equation* (6.40)*. For comparison, the FEVA is shown in black, but does not depend on m. The standard deviations of the means are also shown as shaded regions. The results are obtained from* $N = 10^5$ *Monte Carlo paths, with a VCM market model of* $d = 1$ *dimension. The configuration of the other parameters for the simulations is given in Tables 6.2 and 6.3.*

In Figure 6.3, it can be observed that the average estimate $J_{\text{est}}(0,0)$ from Algorithm 3 diverges from the average realised costs $J_{\text{real}}(0,0)$ in Equation (6.44), which lie within one standard deviation of the expected costs $J_0(0,0)$ from the no early buying strategy in Definition 6.9. This indicates errors in the approximation of the conditional expectations in Equation (6.21), leading to a systemic underestimation of the expected future costs that accumulates in Algorithm 3, while the resulting optimal feedback law result in a minimal improvement over the costs from the no early buying strategy. We hypothesise that these errors come from errors in the regression step, and we also investigate the difference between the estimated optimal costs $J_{\text{est}}(0,0)$ and the realised costs $J_{\text{real}}(0,0)$ when using VCM market model of varying dimension $d \in \mathbb{N}$, applying the Algorithm to the fixed rate bond.

In Figure 6.4, it can be observed that the expected costs behave differently in the case of the fixed rate bond than they do for the European option contract. Firstly, the FEVA lies significantly below the expected costs $J(0,0)$ and particularly $J_0(0,0)$ in the discretised control problem of Definition 6.7, indicating that

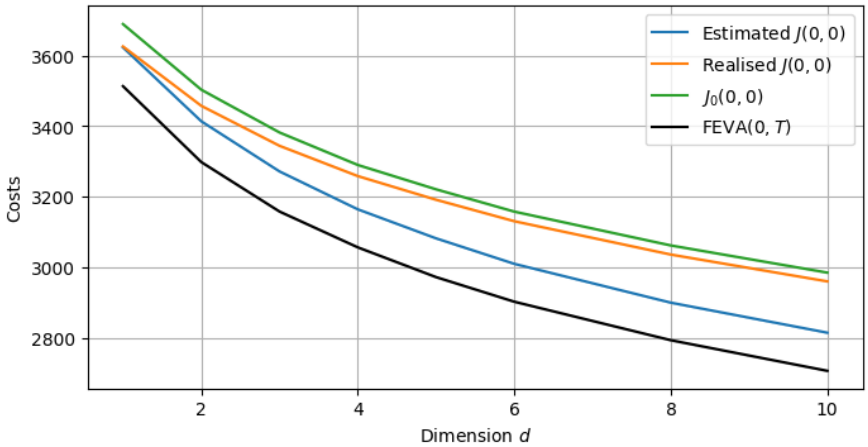


Figure 6.4: *Impact of the dimension $d \in \mathbb{N}$ of the VCM market model on the expected costs resulting from Algorithm 3 with a batch size $\mathcal{S} = 50$, for the fixed rate bond in Table 6.1. The resulting estimated costs $J_{\text{est}}(0,0)$ are shown in blue, and the resulting optimal feedback control law on average results in the expected costs $J_{\text{real}}(0,0)$ that are shown in orange. The costs in Equation (6.40) from the no early buying strategy are shown in green. For comparison, the FEVA is shown in black. The results are obtained from $N = 10^5$ Monte Carlo paths, on a discretisation of $[0,T]$ into $m = 200$ equidistant steps. The configuration of the other parameters for the simulations is given in Tables 6.2 and 6.3.*

the restriction of a batch size results into higher expected costs, as discussed in Remark 6.2. Moreover, the estimated expected costs $J_{\text{est}}(0,0)$ resulting from Algorithm 3 are significantly lower than the expected costs $J_0(0,0)$ from the no early buying strategy, and their relative difference increases as the dimension d of the VCM market model increases. However, the realised costs $J_{\text{real}}(0,0)$ from the feedback control law that results from Algorithm 3 diverge from the estimated costs $J(0,0)$, indicating that errors in the conditional expectation grow as the complexity of the regression step grows, i,e, as the dimension d of the VCM, and therefore the state space Ξ, grows. The average realised costs $J_{\text{real}}(0,0)$ from the obtained control feedback law are only marginally lower than the costs from the no early buying strategy in Definition 6.9, with their relative difference being approximately 1% for higher-dimensional VCM models.

We conclude that significant regression errors may arise if there is a large number of stochastic processes involved in the calculation of the FEVA, meaning that the dimension of the state space Ξ is large. This can result in an inaccurate estimate $J_{\text{est}}(0,0)$ of $J(0,0)$. Figures 6.3 and 6.4 illustrate that these estimates of $J(0,0)$ can differ significantly. In the remainder of this Section, we use Equation (6.44) as an approximation of $J(0,0)$ and restrict our attention to a VCM model of dimension $d = 1$, in which case the estimated and realised expected costs $J_{\text{real}}(0,0)$ generally lie within one standard deviation of each other. This indicates that the regression step in Algorithm 3 produces accurate approximations of the conditional expectation of future costs.

We also investigate the impact of the number of paths $N \in \mathbb{N}$ in the Monte Carlo simulation and the batch size $\mathcal{S} \in \mathbb{N}$ on the quality of the optimal strategy. In the context of pricing American options (see Remark 6.1), Longstaff and Schwartz proved that the value that results from the LSMC approach converges to the value of an American option [141], but a high number of Monte Carlo paths can be required for convergence in the context of a higher-dimensional problem, as is the case in the control problem in this Chapter. Moreover, the considered discretised control problem also introduces the limitation of a batch size $\mathcal{S}$, which also limits the convergence to the solution to the continuous control problem in Equation (6.11).

In Table 6.4, it can be observed that the estimated and realised costs $J(0,0)$ from Algorithm 3 generally correspond with each other: the estimated costs always are at most one standard deviation lower than the realised costs. This indicates that there is a limited error in regression step in Algorithm (3) that accumulates

Table 6.4: *Impact of the batch size $\mathcal{S}$ and the number of Monte Carlo paths N on the optimal costs $J(0,0)$ that result from Algorithm 3. Standard deviations of the means are also shown. For comparison, the costs $J_0(0,0)$ associated with the no early buying strategy and the FEVA are computed using 10^5 Monte Carlo paths. The control problem is solved for the fixed rate bond in Table 6.1 with a discretisation of $[0,T]$ into $m = 200$ equidistant steps, and a VCM model of dimension $d = 1$ is used. The parameter configuration is given in Tables 6.2 and 6.3.*

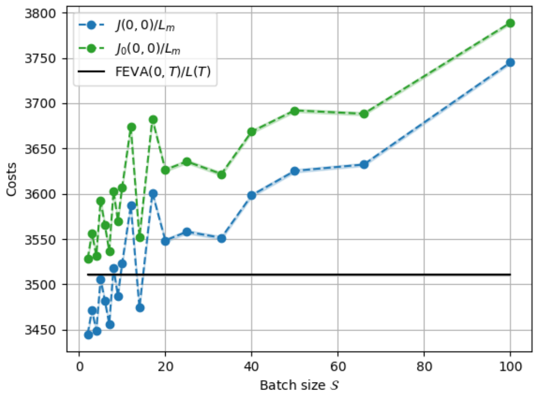
	$\mathcal{S} = 50$	$\mathcal{S} = 20$
FEVA$(0,T)$	3509.77 ± 1.97	
$J_0(0,0)$	3690.87 ± 1.83	3624.46 ± 1.94
$J(0,0)$ $(N = 10^3)$	Estimated: 3634.53 ± 3.32 Realised: 3636.76 ± 17.08	Estimated: 3540.54 ± 2.22 Realised: 3547.07 ± 17.87
$J(0,0)$ $(N = 10^4)$	Estimated: 3636.11 ± 1.04 Realised: 3638.55 ± 5.87	Estimated: 3546.22 ± 0.91 Realised: 3547.24 ± 6.04
$J(0,0)$ $(N = 10^5)$	Estimated: 3625.72 ± 0.32 Realised: 3627.96 ± 1.79	Estimated: 3543.73 ± 0.28 Realised: 3544.67 ± 1.91

over the backward induction steps, when applying the Algorithm to the fixed rate bond. This is in contrast to results for the European option contract, for which the errors accumulate over the backward induction steps, which is illustrated in Figure 6.3. Even for small numbers of paths N, the algorithm produces reasonable feedback control laws to improve over the costs $J_0(0,0)$ from the no early buying strategy, particularly for a smaller batch size $\mathcal{S}$. However, further investigation of convergence to the theoretical value $J(0,0)$ of the continuous problem in Equation (6.11) as $N \to \infty$ and $\mathcal{S} \to 0$ is beyond the scope of this Chapter. We further investigate the impact of the batch size on the expected costs in Figure 6.5 for simulations with $N = 10^5$ Monte Carlo paths.

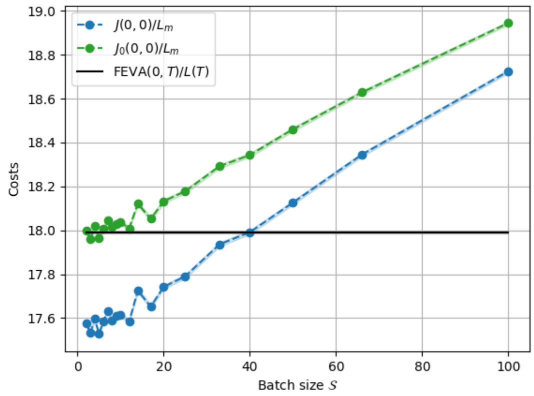
In Subfigure 6.5a, it can be observed that, unsurprisingly, the expected costs highly depend on the total number of credits that are bought. By letting $U = \mathcal{S}\lceil \frac{L(T)}{\mathcal{S}} \rceil = \widetilde{L}_m$, which is always deterministic in the case of the fixed rate bond, the cumulative number of bought credits over the period $[0,T]$ will always be U. By looking at the total expected costs per bought carbon credit and comparing these with the total FEVA per tonne of financed emissions, a better comparison between the strategies can be made. Under this assumption, it can indeed be observed that the expected costs $J_0(0,0)$ from the no early buying strategy approach FEVA$(0,T)$ under this metric as the batch size $\mathcal{S}$ approaches zero, which is consistent with the interpretation of $J_0(0,0)$ as the discretised counterpart of the FEVA, as discussed earlier in this Section.

We conclude by investigating how often an optimal feedback control law, as derived using Algorithm 3, identifies an opportunity to buy carbon credits before they are necessary to offset financed emissions. The distribution of the bought numbers of carbon credits that follow from the optimal feedback control law are shown in Figure 6.6.

In Subfigure 6.6a, it can be observed that it is generally relatively likely that it is optimal to only buy exactly the necessary number of credits to satisfy the constraint $B(t) \geqslant L(t)$, which is illustrated with the black line. However, when a 'deadline' to buy the next batch of credits approaches, it becomes more likely to be optimal to buy the next batch of credits before it is strictly necessary to do so, as illustrated by the slowly decreasing relative frequency of a fixed value of $B(t)$ as the process $L(t)$ approaches that value over time. However, the likelihood that it is optimal to buy the maximum number of credits necessary also increases over time, as illustrated with the turquoise line: the probability that $B(t) = U$ is even more than $\frac{1}{2}$ after $t = 3$ when following the optimal feedback control law. This indicates that buying early is often beneficial



(a) The expected costs from Algorithm (3) *and from the no early buying strategy. The irregularities arise because the total number of bought credits, which is U, differs per batch size.*



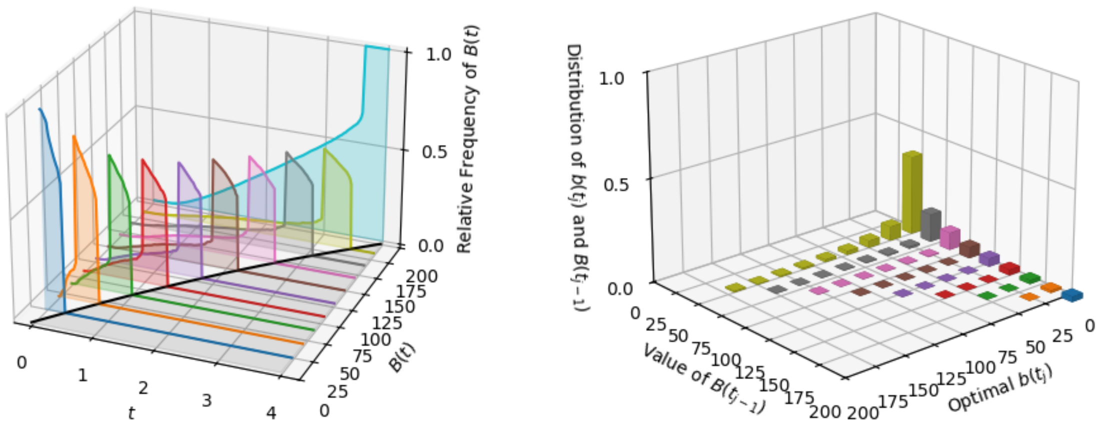
(b) The expected costs per bought carbon credit from Algorithm 3 and from the no early buying strategy, compared with the FEVA divided by the cumulative financed emissions.

Figure 6.5: *Impact of the batch size $\mathcal{S} \in \mathbb{N}$ on the expected costs of the optimal strategy and the no early buying strategy, denoted by $J(0,0)$ and $J_0(0,0)$, respectively. The considered asset is the fixed rate bond and the parameter configuration is given in Tables 6.2 and 6.3, but the value of U is chosen as the smallest multiple of $\mathcal{S}$ that exceeds $L(T)$. For comparison, the FEVA of the bond is also shown, but does not depend on $\mathcal{S}$. The interval $[0,T]$ is discretised into $m = 100$ equidistant steps, and a VCM model of dimension $d = 1$ is used.*

compared to the no early buying strategy in Definition 6.9.

In Subfigure 6.6b, the optimal buying strategy $b(t_j) \in \mathcal{A}_j(B_{j-1})$ is shown at the 33'rd step at time $t = t_j = 0.66$. The optimal number of credits to buy depends on the number of credits $B(t_{j-1})$ that are already bought before time t_j. It can be observed that it is not unlikely that $B(t_{j-1}) > 40$, which is the minimum number to satisfy the constraint $B(t_{j-1}) \geqslant L(t_{j-1})$. In these cases however, it is most likely that $b(t_j) = 0$ is the optimal number of credits to buy. Buying one or more batches of credits is optimal slightly more often when $B(t_{j-1}) = 40$, but it is also most likely optimal in this case not to buy carbon credits.

We conclude that under the right circumstances, Algorithm 3 is successful in identifying opportunities to buy carbon credits at an earlier moment than is necessary in order to minimise the expected costs. An optimal feedback control law can result in a significant reduction in the expected costs of offsetting financed emissions with carbon credits when compared to the FEVA, as suggested in Subsection 6.1.2. Figure 6.5 indicates that this is particularly the case when the batch size $\mathcal{S}$ is small and therefore does not form a significant restriction on the problem. However, a small batch size $\mathcal{S}$ increases the computational complexity of the algorithm, which also increases with the dimension of the state space Ξ. This makes the algorithm less suitable to apply to assets with several sources of uncertainty, and also makes the algorithm less applicable in the context of higher-dimensional VCM market models, as illustrated in Figure 6.4. Even though the CTD aspect in a higher-dimensional VCM model results in a significant decrease in costs, as illustrated in Figure 3.3, Algorithm 3 loses its effectiveness as a cost-decreasing method for higher-dimensional markets.



(a) *The relative frequency of the cumulative number of bought credits $B(t)$ for a fixed time t is plotted over time. Each colour represents how often a fixed value of $B(t)$ appears over time. The shaded region on the horizontal plane represents the region where the constraint $B(t) \geqslant L(t)$ is not satisfied.*

(b) *Distribution of the values $b(t_j)$ and $B(t_{j-1})$, for $j = 33$. It can be observed at time $t = t_j = 0.66$ that $B(t_{j-1}) = 40$ is the most likely number of carbon credits to have bought because the lower bound $L(t_j)$ is almost 40, and that $b(t_j) = 0$ is the optimal number of credits to buy at time t_j.*

Figure 6.6: *Illustration of the distribution of the numbers of credits that are bought when following an optimal feedback control law, as derived from 3. The results are obtained for a fixed rate bond from 10^5 Monte Carlo paths, with a discretisation of $[0, T]$ into $m = 200$ equidistant steps, with a batch size $\mathcal{S} = 20$ and a VCM model of dimension $d = 1$. The configuration of the other parameters is given in Tables 6.2 and 6.3.*

CHAPTER 7

DISCUSSION

7.1 Summary and conclusion

In this thesis, we explore various aspects of the relationship between the financial sector and climate change, as illustrated in Figure 1.1. We propose EVA models that aim to contribute to the internalisation of climate change in the financial sector, building on the general CVA framework that we defined in Section 4.1. We discuss our most important findings in light of Research Questions 1, 2, 3 and 4.

Climate Risk Value Adjustment (CRVA)

Given that climate risk, particularly physical climate risk over longer timescales, carries large uncertainties due to its complexity, it can have a significant impact on the asset valuation process and pose a systemic risk to the financial sector. Additional climate risk assessments on top of regular risk frameworks can be necessary to internalise climate risk when assets are vulnerable to physical climate risk over long timescales. Based on additional climate risk assessments to identify non-internalised climate risk, we propose a general framework to incorporate climate-related default events into regular CVA frameworks in Section 4.2, resulting into an adjustment to the default rate and loss given default. This results into an adjustment to the CVA, which we defined to be the CRVA of an asset. This CRVA model can be seen as a generalisation of the climate change value adjustment that Kenyon and Berrahoui propose [131].

In Example 4.4, we demonstrated that the CRVA of an asset is negligible if the impact of non-internalised climate risk on the counterparty is limited, for example when counterparties are not vulnerable to extreme events or environmental changes, as is generally the case for financial institutions. In Example 4.5, we demonstrated the impact of long-term increases of the default rate on the CRVA, showing that climate risk can have a negative impact on an asset value in the absence of adequate climate risk assessments, even if the asset has a green label.

In Subsection 4.2.3, we discussed the purpose of the CRVA based on Research Questions 1, 2 and 3. Given that the CRVA is an adjustment to the CVA of an asset, it should also be treated similarly to a CVA. It is an adjustment to the fair value of an asset that can be charged to the counterparty when entering a trade, and the charge can be used to hedge the CRVA. Protection against climate-related future losses could be bought in the form of CDSs when they are available, but specific climate-related instruments can also be used to buy insurance against climate-risk related losses after a detailed investigation on their drivers. Given that the CRVA represents a fraction of the CVA with potentially large uncertainty over longer timescales, it may be necessary to hold on to larger capital buffers when the CRVA is a relatively large fraction of the CVA, as illustrated in Example 4.6. Additional regulatory capital requirements may be imposed on financial institutions based on the unhedged CRVA to protect against the uncertainty associated with the CRVA.

Financed Emissions Value Adjustment (FEVA)

After identifying the importance of CO2e emissions and the effectiveness of carbon pricing schemes, we proposed an EVA model in Chapter 5 that internalises CO2e emissions associated with assets, by associating costs with financed emissions in a framework that is consistent with the PCAF Standard for financed emissions [165] and Kenyon's Carbon Equivalence Principle [132]. The FEVA model is defined in a context where a financial institution compensates its financed emissions using voluntary carbon credits, either voluntarily or because they are obliged to do so. However, the FEVA model can also be applied in a context where financial institutions are charged by regulators for their financed emissions in the form of a carbon tax or an ETS. If financial institutions are charged for their financed emissions, it is reasonable to assume that it is market practice to charge the FEVA to the counterparty when entering a trade, similar to how xVAs related to collateralisation are market practice. In the context of offsetting using voluntary carbon credits however, it is only reasonable to charge the FEVA to the counterparty as an adjustment to the fair value of an asset if it is market practice to compensate for financed emissions. Charging the FEVA of assets to the counterparty when entering a trade creates financial incentives for climate change mitigation, since the charge grows with the counterparty's emissions. For counterparties with negative emissions, the FEVA even becomes a discount on a trade.

Example 5.5 illustrates that the FEVA of a trade between financial institutions is generally significant, and that the future costs associated with the financed emissions can be volatile if the value of the underlying asset is volatile. However, Example 5.4 illustrates that the FEVA of a green bond can be negative if negative emissions are associated with the asset. We also demonstrated in Example 5.5 that a CTD aspect that arises from a higher-dimensional VCM model has a significant impact on the future price of a carbon credit and therefore on the FEVA. This higher-dimensional model is based on the commoditisation of the VCM, but the transparency and quality standards of the market must improve for this to happen and for carbon credits to be incorporated into financial decision-making.

The FEVA represents the expected future costs associated with the financed emissions and can be used to hedge against the uncertainty in the future costs. In Section 5.4, we proposed a FEVA hedging approach based on a continuous carbon annuity, a hypothetical asset that can be approximated with carbon credit forwards for example. By ensuring that the expected future financed emissions over a book are zero, the sensitivity of the total climate-adjusted value on the books to the carbon price is hedged, assuming independence between the financed emissions and the carbon price. Regulatory capital requirements can be imposed based on the FEVA if it is not hedged, but by continuously hedging the FEVA of assets, a financial institution achieves net zero and is simultaneously protected against carbon price risk. This demonstrates how the FEVA internalises climate impact, but can also play a role in climate risk mitigation, reflecting the complex relationship between the financial sector and climate change.

Optimising the costs of offsetting financed emissions

The FEVA represents the costs of offsetting financed emissions, and we explored how these costs can be minimised using early buying strategies in Chapter 6. Reducing the FEVA associated with an asset may make it more attractive to counterparties to do business if it is market practice to charge the FEVA to counterparties. We defined a stochastic control problem based on the idea that voluntary carbon credits can be bought before the emissions occur that they offset, instead of at the exact moment they occur. We formulated a control problem to explore the effectiveness of early buying strategies, based on a least squares Monte Carlo approach. We performed least squares regression to compute the conditional expectations of the future costs as a function of the state variables, also correcting for heteroscedasticity in the regression.

The proposed approach results in buying strategies that create a cost decrease of a few percent compared to a strategy without early buying, but mostly when performing regression on only one state variable. When other risk factors than the carbon credit price have to be modelled, particularly price processes that drive the exposure, the regression error becomes significant and accumulates over the backward induction steps in the proposed approach. The effectiveness of the approach is also limited by a restriction in the size of the batches in which carbon credits have to be bought, since this restriction reduces the space of possible strategies. However, the computational complexity grows as the batch size decreases, meaning that a trade-off between the resulting costs from the obtained strategy and the computational efficiency has to be made. All in all, the CTD aspect of a higher-dimensional VCM model and the proposed approach to determine early buying strategies may both be effective mechanisms to reduce the costs of offsetting financed emissions compared to the FEVA, particularly when a sufficiently small batch size is used to determine an optimal early buying strategy. However, the mechanisms are not complementary, since a higher-dimensional state space has a negative impact on the computational complexity or accuracy of the approach.

General EVA models

We also proposed an approach to construct EVA models for general environmental aspects in Section 5.5, based on the assumption of (1) the existence of standardised metrics to quantify the considered aspect and (2) the existence of a market for offsetting instrument for the considered aspect. Analogously to the derivation of the FEVA, we proposed a Financed Biodiversity Value Adjustment to illustrate the approach. We demonstrated that different EVA models can be combined to compute the total EVA of assets, which can be done by combining the EIFs for the different considered environmental aspects, assuming independence between the exposure of an asset and the other processes involved. This was illustrated in Equation (5.35) of the total EVA of an asset with maturity T at time $t \in [0, T]$:

$$\mathrm{EVA}^{\mathrm{Tot}}(t,T) = \sum_X \mathrm{EVA}^X(t,T) = \int_t^T \mathrm{EE}_t(s) \cdot \sum_X \mathbb{E}_t[\mathrm{EIF}^X(s)]\,\mathrm{d}s, \tag{7.1}$$

where the summation runs over all the considered environmental aspects to compute a value adjustment EVA^X for. Given that the exposure profiles of an asset are already modelled, which is generally already done in existing valuation and xVA frameworks, the EVA of an asset can be computed by modelling the EIFs for the different environmental aspects separately. The EVA models can then be seamlessly integrated in existing xVA frameworks that are also expressed in terms of the expected exposure $\mathrm{EE}_t(s)$ of the asset, such as the CVA that is expressed in terms of the exposure at default, for example.

However, the assumption of independence between the EIF and the exposure of an asset may lead to wrong-way risk. In Subsection 5.2.3, we discussed the wrong-way and right-way risk that can arise by assuming independence between the various processes involved in the calculation of the FEVA, and this discussion extends to EVAs that incorporate other aspects of climate impact. We established that the double counting when combining EVAs can arise when there is overlap between the considered topics in their respective credit markets or their quantification metrics. However, this will only become a concern in practice when standardised metrics and credit markets for multiple types of environmental impact mature, and if these aspects receive widespread attention across the financial sector to the point where deploying the EVA models in practice is feasible, which is far from being the case at the moment of writing.

7.2 Outlook and limitations

Since the urgency of addressing climate change has been recognised more in recent decades, it has emerged as a significant topic in financial decision-making over the recent years. However, due to the long-term nature of its consequences, it is susceptible to being pushed to the background in periods of short-term market stress[1], which hampers the development of climate-related regulations, frameworks and initiatives, which play a crucial role in establishing the EVA models in this thesis. In this section, we repeat and discuss the conditions and necessary assumptions under which the proposed EVA models and other methods can be deployed in the financial sector and applied to incorporate environmental topics into asset valuation and risk management. We also discuss general limitations of the models and methods that we proposed in this thesis.

[1]For example, over the last few months of writing this thesis, major US banks have withdrawn from the Net Zero Banking Alliance, threatening its future existence, and reporting requirements of the CSRD have been delayed by three years, which has adverse impact on the availability of ESG data in the coming years.

Climate Risk Value Adjustment (CRVA)

The CRVA is defined by the climate-adjustments to the CCR variables, the default rate and the loss given default, which have to be obtained by additional climate risk assessments on top of the regular CCR frameworks, such as scenario analysis and stress testing. Estimating physical climate risk can be challenging and result in large uncertainties due to the complex nature of modelling the complex and unprecedented dynamics between climate-related variables, particularly over longer timescales. This is why unified frameworks such as the NGFS scenarios are crucial in establishing a common ground to base climate risk assessments on. Establishing methods to obtain climate-related adjustments to the CCR parameters is a challenging topic for future research, which we consider to be necessary to deploy the CRVA model in practice. Given that this condition is met, appropriate risk management practices can be further established building on the outlined suggestions in this thesis, both from the perspectives of financial institutions and of governing bodies.

Financed Emissions Value Adjustment (FEVA)

The FEVA that we defined in Chapter 5 is based on the GHG Protocol and the PCAF Standard for financed emissions, which are constantly being extended to broaden their scope and provide a more complete guide to assess the climate impact of financial institutions and their counterparties. The FEVA relies on modelling the future CO2e emissions or emission intensity of the counterparty, which itself is already a very central topic in current research on climate impact of companies across the economy. We suggest that counterparty-specific characteristics can be incorporated in the process of determining the parameters in the GBM model that we proposed for the emission intensity in Equation (5.3), or an entirely different, more sophisticated model than a GBM may be used to model counterparty emissions. Data availability and quality are also crucial to determine financed emissions, so better measurement and reporting frameworks should be developed and deployed to enhance the applicability of the FEVA model.

Moreover, the proposed FEVA model relies on carbon credits being valid instruments to offset CO2e emissions, for which the quality and transparency issues of the VCM will have to be resolved first, as discussed in Subsection 3.1.2. Until then, using carbon credits as an instrument to achieve net zero will create a reputational risk of greenwashing acquisitions. The model for voluntary carbon credit prices that we proposed in Section 3.2 depends on the market structure that arises when the VCM matures. Modelling the VCM is a very relevant topic of future research for establishing a FEVA model, but this is currently very challenging due to the opaque nature of the market.

Most importantly, the FEVA can only be incorporated into the asset valuation process if it is market practice across the financial sector to compensate for financed emissions over portfolios, either voluntarily or as a result of regulations on the topic. The first scenario seems unlikely in the near future given that climate change is an externality, meaning that the FEVA framework can only be deployed if authorities establish regulations on the topic, for example by requiring financial institutions to offset their (financed) emissions using voluntary carbon credits or by establishing a carbon pricing scheme that covers financed emissions. Until this is the case, the applicability of the FEVA model is limited, particularly if financial institutions already measure and report the financed emissions and incorporate them into financial decision-making.

Optimising the costs of offsetting financed emissions

The control problem in Chapter 6 is an approach to minimise the expected costs associated with offsetting financed emissions compared to the FEVA. Therefore, the outlook and discussed limitations regarding the FEVA also apply to the applicability of the defined control problem. In particular, optimal carbon credit buying strategies to offset financed emissions should only be considered by financial institutions once the VCM has matured to avoid greenwashing accusations. In this case, the control problem should then be considered by parties that want to achieve net zero to minimise their costs of doing so.An interesting extension of the proposed approach is to offset financed emissions over entire portfolio's instead of the financed emissions associated with individual assets. However, assumptions would have to be made to reduce the dimensionality of the state space so that the LSMC approach yields accurate results. For example, by taking the expectation of the financed emissions, all uncertainty in the lower bound process in the control problem is removed.

As discussed in Section 7.1, the effectiveness of Algorithm 3 is not only limited by the dimension of the state space, but also by the restriction of a batch size in which credits have to be bought. Allowing to buy credits in smaller batch sizes results in lower expected costs, but increases the computational complexity of the problem. This trade-off limits the effectiveness of the LSMC approach as a method to decrease the costs of offsetting financed emissions compared to the FEVA.

In addition to antithetic sampling and weighted regression to correct for heteroscedasticity, other improvements to the LSMC approach can be introduced to improve the efficiency of Algorithm 3. In particular, methods to deal with the potentially high dimensionality of the state space should be explored, such as quasi-Monte Carlo sampling [49], the stochastic grid bundling method [120] or principal component analysis to effectively reduce the dimension of the state space [45]. Machine learning techniques such as reinforcement learning can also be applied in the context of investment strategies [228].

General EVA models

As mentioned in Section 7.1, EVA models can be developed for general aspects of climate change if standardised metrics are available to quantify a given aspect, and a well-functioning market of offsetting instruments exists for the considered aspect. As with the proposed FEVA model, the applicability of an EVA model for a given topic in the financial sector relies on the development of measurement and reporting frameworks on the topic, and a well-functioning, mature and transparent market of high-quality offsetting instruments. Unless these two conditions are met, the development and study of different EVA models and their applicability in the financial sector are not particularly relevant.

For the CRVA and FEVA models that we proposed, the assumption of independence between the exposure and the environmental aspects leads to a formulation of EVAs as an integral over the expected exposure and the EIF, as can be observed in Equation (7.1), allowing for seamless integration in existing xVA frameworks. However, this assumption limits the applicability of the models, since it can induce wrong-way risk if the asset performance is financially linked to the modelled variables in the EIFs, such as the (climate-adjusted) default process, carbon price or emission intensity. Moreover, the formulation of EVA models can be extended to be applicable to more complicated interest rate instruments by using a stochastic short-rate, Libor market models or standard market models instead of a constant interest rate, or to more complicated credit instruments by generalising NHPPs to Cox processes. Moreover, EVA models can naturally be extended to describe collateralised assets or netted trades. By incorporating the climate impact and vulnerability to climate risk of the asset holder, the EVA models can be extended to bilateral xVA models. This is analogous to how the possibility of self-default can be incorporated to transform a unilateral CVA into a bilateral CVA. The formulation of bilateral EVAs and assessment of their interpretation and applications is an interesting direction for future research and a natural next step in the development of EVA models.

APPENDIX A

DETAILS ON ASSET VALUATION & CVA

This appendix contains more details on the process of asset valuation and functions as a supplement to Chapter 4. In Section A.1, we introduce the principles behind asset valuation that we apply from Chapter 4 onwards, including a numéraire which grows at a short rate, which we assume to be deterministic and constant. Then, we provide a general asset valuation framework in terms of future expected discounted cash flows, including some illustrative examples. We use this framework to derive expression (4.13) of the CVA for a general asset in Section A.2. This framework is used in Section 4.2 to define a climate-adjusted CVA using climate-adjusted parameters, which leads to Definition 4.6 of the CRVA.

A.1 General asset valuation framework

A.1.1 Numéraire and interest rates

Under classic risk-neutral valuation principles, general assets are priced by computing the expected future cash flows that are discounted using a numéraire $M(t)$ that grows at the instantaneous interest rate $r(t)$ and is driven by the Stochastic Differential Equation (SDE)

$$\mathrm{d}M(t) = r(t)M(t)\,\mathrm{d}t, \tag{A.1}$$

which is solved by

$$M(t) = e^{\int_0^t r(s)\,\mathrm{d}s}. \tag{A.2}$$

The numéraire is also referred to as a money-savings account because it represents the concept of *time value of money*: as long as $r(t) > 0$, one unit of currency today is worth more today than in the future, because it can be invested in a money-savings account $M(t)$ for a risk-free rate of return $r(t)$ in the meantime. The money-savings account $M(t)$ therefore acts as a baseline value against which other values are compared. More precisely, cash flows can be discounted from time s to t (where $s, t \in [0, T]$) using a factor

$$\frac{M(t)}{M(s)} = e^{\int_s^t r(z)\,\mathrm{d}z}.$$

Remark A.1 (Risk-free money-savings account)**.** *Technically, an investment in a money-savings account still carries systemic risk if $r(t)$ is stochastic. However, we will always refer to the money-savings account as risk-free since systemic risk is unavoidable by definition. Mathematically, $M(t)$ is called risk-free because its SDE does not contain a diffusion term and its quadratic variation is therefore $\langle M, M\rangle(t) = 0$.*

In this thesis, we generally assume a constant deterministic interest rate $r(t) = r$ unless otherwise specified. This assumption was justified in the beginning of Section 4.1, and it results in a simplified expression for the numéraire

$$M(t) = e^{rt} \qquad \text{and} \qquad \frac{M(t)}{M(s)} = e^{r(t-s)}.$$

A.1.2 From cash flows to asset valuation

To valuate general assets, we consider the payments that the contract prescribes. We define $\mathcal{T} \subseteq [0, T]$ to be the set of predetermined moments at which the asset prescribes a cash flow, and let the random variable $\widetilde{C}_\tau \in L^2(\Omega)$ on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$ denote the cash flow prescribed by the asset at time $\tau \in \mathcal{T}$. For all $\tau \in \mathcal{T}$, the value of the cash flow C_τ has to be $\mathcal{F}(\tau)$-measurable, where $\mathcal{F}(t)$ is the filtration generated by the price processes involved in modelling the value of the cash flows. A positive cash flow indicates that the holder receives capital from the counterparty and a negative cash flow indicates a payment from the holder to the counterparty.

Definition A.1 (Value of an asset)**.** *For $t, t' \in [0, T]$ such that $t < t'$, we denote by $V(t, t')$ the expected value at time t of the sum of the discounted cash flows in $(t, t']$:*

$$V(t, t') := \mathbb{E}_t \left[\sum_{\substack{\tau \in \mathcal{T} \\ t < \tau \leqslant t'}} e^{-r(\tau - t)} \widetilde{C}_\tau \right] = \sum_{\substack{\tau \in \mathcal{T} \\ t < \tau \leqslant t'}} \mathbb{E}_t \left[e^{-r(\tau - t)} \widetilde{C}_\tau \right], \tag{A.3}$$

which is $\mathcal{F}_t$-measurable. The total financial value of the asset at time $t < T$ is then equal to the total discounted value of all future cash flows that it prescribes:

$$V(t, T) = \mathbb{E}_t \left[\sum_{\substack{\tau \in \mathcal{T} \\ \tau > t}} e^{-r(\tau - t)} \widetilde{C}_\tau \right] = \sum_{\substack{\tau \in \mathcal{T} \\ \tau > t}} \mathbb{E}_t \left[e^{-r(\tau - t)} \widetilde{C}_\tau \right]. \tag{A.4}$$

Notice that this value $V(t, T)$ does not incorporate counterparty credit risk or any environmental aspect. In the remainder of this Section, we provide some Examples of assets in our valuation framework.

A.1.3 Examples

Example A.1 (European option)**.** *Options are one of the most popular types of derivatives and come in various shapes and forms. The holder of a call (put) option has the right, but not the obligation, to buy (sell) the underlying asset for a predetermined strike price $K \in \mathbb{R}_{++}$, which provides insurance to the holder of an option against decreases (increases) in the future price of the underlying asset [115]. The simplest example is a European call (put) option, which gives the holder the right to buy (sell) an underlying asset for the strike price K on the expiration date $T \in \mathbb{R}_{++}$ of the contract. If the underlying asset is worth $S(T)$ at time T, the resulting cash flow is $C_T = S(T) - K$ if the holder would exercise the option, and $C_T = 0$ if he would not. Assuming that the holder maximises its profits, the cash flow at time T is therefore*

$$\widetilde{C}_T = (S(T) - K)^+,$$

which is the only prescribed moment of cash flows (i.e. $\mathcal{T} = \{T\}$). Therefore, the value of a European call option at time $t < T$ is

$$V(t, T) = \mathbb{E}_t \left[e^{-r(T-t)} (S(T) - K)^+ \right]. \tag{A.5}$$

Assets are usually modelled with a Geometric Brownian Motion (GBM) and valued under the risk-neutral probability measure, so we let $\mathbb{P}$ be the risk-neutral probability measure[1] in this example and in Example A.2. The risk-neutral dynamics of the asset price $S(t)$ are given by

$$\mathrm{d}S(t) = rS(t)\,\mathrm{d}t + \sigma_S S(t)\,\mathrm{d}W(t), \tag{A.6}$$

[1] Usually, $\mathbb{P}$ is the real-world probability measure and $\mathbb{Q}$ denotes the risk-neutral probability measure.

with initial condition $S(0) = S_0$ for some initial value $S_0 \in \mathbb{R}_{++}$ and volatility $\sigma_S \in \mathbb{R}_{++}$. The value of a European option can be found analytically by solving the Black-Scholes equation [38], which laid the foundations for modern derivative pricing theory. In this case, the value of the European call option is given by [162]

$$V(t,T) = S(t) \cdot \mathcal{N}(d_+) - Ke^{-r(T-t)}\mathcal{N}(d_-), \tag{A.7}$$

where

$$d_\pm := \frac{\log\left(\frac{S(t)}{K}\right) + (r \pm \frac{\sigma_S^2}{2})(T-t)}{\sigma_S\sqrt{T-t}}.$$

It can be observed that the value of an option is a function of the current value of the underlying asset $S(t)$.

Example A.2 (American option). *A more complicated example of an option to price is an American call option, which are mostly traded on exchanges. The owner of an American option can choose to buy the underlying asset for a predetermined strike price K at any moment $\tau \leqslant T$, which is called an early-exercise feature. The framework in this Section can be extended to incorporate an early-exercise feature. If the holder chooses at time $\tau \in [0,T]$ whether or not to exercise the option, the resulting cash flow is*

$$\widetilde{C}_\tau = (S(\tau) - K)^+.$$

The holder can freely choose the moment τ to maximise his return, so the value of the American call option at time $t < T$ is [157]

$$V(t,T) = \sup_{\tau \in [t,T]} \mathbb{E}_t\left[e^{-r(\tau-t)}(S(\tau)-K)^+\right], \tag{A.8}$$

where the expectation is taken under the risk-neutral probability measure. Determining the optimal exercise time τ is a stochastic control problem that can be solved via backwards induction, as proposed by Longstaff and Schwartz [141]. This involves discretising the interval $[0,T]$ into a time grid defined by $t_j := j \cdot \Delta t$ for $j \in \mathbb{Z}_{m+1}$, where $\Delta t = \frac{T}{m}$. The backwards induction starts at time $t_m = T$ with

$$V(t_m,T) = (S(T) - K)^+,$$

and the backwards induction is performed for $i = m-1, m-2, \ldots, 1, 0$ by

$$V(t_j,T) = \max\left\{(S(t_j)-K)^+, e^{-r\Delta t}\mathbb{E}_{t_j}[V(t_{j+1},T)]\right\},$$

indicating that the option holder assesses at each moment t_i whether waiting or exercising is expected to be more beneficial [162]. The challenges of this approach lie in the calculation of the future conditional expectation $\mathbb{E}_{t_j}[V(t_{j+1},T)]$, for which Longstaff and Schwartz propose a least squares Monte Carlo approach as an alternative method to finite difference and binomial techniques [141, 30].

In Section 6.2, we define a different stochastic control problem that also requires the computation of future conditional expectations. We demonstrate how the problem can be solved by an approach that is based on the least squares Monte Carlo method proposed by Longstaff and Schwartz, and we discuss in Remark 6.1 that the introduced control problem and dynamic programming approach to solve it can be seen as a generalisation of the problem of pricing American options.

Example A.3 (Fixed rate bond). *A fixed rate bond is a contract that prescribes coupon payments from a counterparty to the holder at a collection of predetermined moments $\mathcal{T} = \{T_1, T_2, \ldots, T_m\}$ of*

$$\widetilde{C}_{T_i} = \begin{cases} NK(T_i - T_{i-1}) & \text{for } i = 1,2,\ldots,m-1 \\ N(1 + K(T_i - T_{i-1})) & \text{for } i = m, \end{cases}$$

where K is the predetermined fixed rate, N is the notional (the face value at which interest rates are paid, which is paid back at maturity), and $T_0 := 0$. Notice that the maturity of the bond is $T := T_m$. This results *in the following value of a fixed rate bond at time $t < T$ [162].*

$$V(t,T) = M(t)\mathbb{E}_t\left[\frac{N}{M(T)} + \sum_{i=k}^{m}\frac{NK(T_i - T_{i-1})}{M(T_i)}\right], \tag{A.9}$$

where $k(t)$ denotes the smallest integer $k \geqslant 1$ such that $T_k \geqslant t$. Notice that we did not assume a constant interest rate r, since pricing a fixed rate bond under a constant interest rate model is trivial since all variables in Equation (A.9) *would be deterministic, resulting into*

$$V(t,T) = e^{-r(T-t)}N + \sum_{i=k(t)}^{m} e^{-r(T_i-t)}NK(T_i - T_{i-1}). \tag{A.10}$$

A.2 CVA model

In this section, we describe how Counterparty Credit Risk (CCR) can be incorporated into asset valuation for general assets. Even though we generally focus on uncollateralised assets in this thesis because collateralisation mitigates CCR[2], we explicitly include collateral in this section. We derive a formulation of a value adjustment in terms of the exposure (see Definition 4.1) of an asset. The value of the collateral can be set to zero in the resulting expressions to obtain a value adjustment for uncollateralised assets that is used in this thesis.

To incorporate CCR (see Definition 1.2) into the asset valuation framework, we introduce the default time t_D modelled by an NHPP (see Definition 4.4). Different scenarios take place depending on when default occurs: if $t_D > T$, all contractually prescribed cash flows take place and the value of the asset is not affected. If $t_D \leqslant T$ however, the prescribed cash flows up to time t_D take place, but the cash flows in $(t_D, T]$ will not take place any more. Depending on the value of the asset $V(t_D, T)$ and the posted collateral $\mathrm{col}(t_D)$ at that point, two different outcomes are possible.

1. If the value of the asset can not fully be recovered from the collateral (i.e. if $V(t_D) > \mathrm{col}(t_D)$), the holder of the contract receives the collateral with value $\mathrm{col}(t_D)$. Moreover, a fraction $1 - LGD$ of the uncollateralised value of the asset is also recovered (see Definition 4.2), meaning that the holder receives a total amount of
$$\mathrm{col}(t_D) + (1 - LGD)(V(t_D,T) - \mathrm{col}(t_D)).$$
2. If the value of the asset can fully be recovered from the collateral (i.e. if $V(t_D,T) \leqslant \mathrm{col}(t_D)$), the holder receives the total value $V(t_D,T)$ of the asset from the counterparty. This includes the scenario in which the asset value is negative, meaning that the holder owes the counterparty an amount of $-V(t_D,T)$. In this case, the counterparty receives this amount in order to meet as much of their payment obligations to other parties as possible[3].

These two situations can be combined into one Equation: the asset holder receives a total amount of

$$\widetilde{C}(t_D) := \min(V(t_D,T), \mathrm{col}(t_D)) + (1 - LGD)(V(t_D,T) - \mathrm{col}(t_D))^+ \tag{A.11}$$

upon counterparty default at time $t_D \leqslant T$. This can be rewritten as

$$\begin{aligned}
\widetilde{C}(t_D) &= \min(V(t_D,T), \mathrm{col}(t_D)) + (1 - LGD)\Big(\max(V(t_D,T), \mathrm{col}(t_D)) - \mathrm{col}(t_D)\Big) \\
&= \min(V(t_D,T), \mathrm{col}(t_D)) + \max(V(t_D,T), \mathrm{col}(t_D)) - \mathrm{col}(t_D) - LGD\Big(\max(V(t_D,T), \mathrm{col}(t_D)) - \mathrm{col}(t_D)\Big) \\
&= V(t_D,T) + \mathrm{col}(t_D) - \mathrm{col}(t_D) - LGD(V(t_D,T) - \mathrm{col}(t_D))^+ \\
&= V(t_D,T) - LGD \cdot E(t_D),
\end{aligned}$$

[2] This is consistent with the idea that, following the Carbon Equivalence Principle [132], climate impact transmission is limited by limiting the exposure to counterparties via collateralisation.

[3] With the value that the counterparty receives from its debtors, they can partly recover the uncollateralised part of their payment obligations. This explains why the loss given default over the uncollateralised value of an asset is not necessarily 1.

making use of Definition 4.1 of the exposure $E(t)$ and the fact that $\min(x,y)+\max(x,y)=x+y$ for all $x,y\in\mathbb{R}$.

We can put the considered scenarios together considering that cash flows at times $\tau\leqslant t_D$ are not affected by counterparty default at time t_D, resulting in the following CCR-adjusted value[4] $\widetilde{V}(t,T)$ of the asset at time $t\in[0,T]$:

$$\widetilde{V}(t,T) := \mathbb{E}_t\left[\mathbb{1}_{t_D>T}\sum_{\substack{\tau\in\mathcal{T}\\ \tau>t}} e^{-r(\tau-t)}\widetilde{C}_\tau + \mathbb{1}_{t_D\leqslant T}\left(\sum_{\substack{\tau\in\mathcal{T}\\ t<\tau\leqslant t_D}} e^{-r(\tau-t)}\widetilde{C}_\tau + e^{-r(t_D-t)}\overbrace{\Big(V(t_D,T)-LGD\cdot E(t_D)\Big)}^{=\widetilde{C}(t_D)}\right)\right]. \tag{A.12}$$

This can be simplified by rewriting the term

$$\begin{aligned}
\mathbb{E}_t\left[\mathbb{1}_{t_D\leqslant T}e^{-r(t_D-t)}V(t_D,T)\right] &= \mathbb{E}_t\left[\mathbb{1}_{t_D\leqslant T}e^{-r(t_D-t)}\mathbb{E}_{t_D}\left[\sum_{\substack{\tau\in\mathcal{T}\\ \tau>t_D}} e^{-r(\tau-t_D)}\widetilde{C}_\tau\right]\right]\\
&= \mathbb{E}_t\left[\mathbb{E}_{t_D}\left[\mathbb{1}_{t_D\leqslant T}\sum_{\substack{\tau\in\mathcal{T}\\ \tau>t_D}} e^{-r(\tau-t)}\widetilde{C}_\tau\right]\right]\\
&= \mathbb{E}_t\left[\mathbb{1}_{t_D\leqslant T}\sum_{\substack{\tau\in\mathcal{T}\\ \tau>t_D}} e^{-r(\tau-t)}\widetilde{C}_\tau\right],
\end{aligned}$$

where we made use of the tower property of iterated conditional expectations [162], which states that

$$\mathbb{E}[\mathbb{E}[\,\cdot\,|\,\mathcal{H}']\,|\,\mathcal{H}] = \mathbb{E}[\,\cdot\,|\,\mathcal{H}], \tag{A.13}$$

for any two sub-σ-algebras $\mathcal{H}',\mathcal{H}\subseteq\mathcal{F}$ of a probability space $(\Omega,\mathcal{F},\mathbb{P})$ such that $\mathcal{H}\subseteq\mathcal{H}'$. This results into

$$\begin{aligned}
\widetilde{V}(t,T) &= \mathbb{E}_t\left[\mathbb{1}_{t_D>T}\sum_{\substack{\tau\in\mathcal{T}\\ \tau>t}} e^{-r(\tau-t)}\widetilde{C}_\tau + \mathbb{1}_{t_D\leqslant T}\left(\sum_{\substack{\tau\in\mathcal{T}\\ t<\tau\leqslant t_D}} e^{-r(\tau-t)}\widetilde{C}_\tau + \sum_{\substack{\tau\in\mathcal{T}\\ \tau>t_D}} e^{-r(\tau-t)}\widetilde{C}_\tau - e^{-r(t_D-t)}LGD\cdot E(t_D)\right)\right]\\
&= \mathbb{E}_t\left[\mathbb{1}_{t_D>T}\sum_{\substack{\tau\in\mathcal{T}\\ \tau>t}} e^{-r(\tau-t)}\widetilde{C}_\tau + \mathbb{1}_{t_D\leqslant T}\sum_{\substack{\tau\in\mathcal{T}\\ \tau>t}} e^{-r(\tau-t)}\widetilde{C}_\tau - \mathbb{1}_{t_D\leqslant T}\cdot e^{-r(t_D-t)}LGD\cdot E(t_D)\right]\\
&= \mathbb{E}_t\left[\sum_{\substack{\tau\in\mathcal{T}\\ \tau>t}} e^{-r(\tau-t)}\widetilde{C}_\tau - \mathbb{1}_{t_D\leqslant T}\cdot e^{-r(t_D-t)}LGD\cdot E(t_D)\right]\\
&= V(t,T) - LGD\cdot\mathbb{E}_t\left[\mathbb{1}_{t_D\leqslant T}\cdot e^{-r(t_D-t)}E(t_D)\right],
\end{aligned}$$

making use of the fact that $\mathbb{1}_{t_D>T}+\mathbb{1}_{t_D\leqslant T}=1$. This results into the expression given in Definition 4.6 for the CCR-adjusted value of an asset:

$$\widetilde{V}(t,T) = V(t,T) - \mathrm{CVA}(t,T),$$

where the Credit Value Adjustment (CVA) is defined as

$$\mathrm{CVA}(t,T) := LGD\cdot\mathbb{E}_t\left[\mathbb{1}_{t_D\leqslant T}\cdot e^{-r(t_D-t)}E(t_D)\right].$$

[4]Notice that the conditional expectations are taken with respect to the default-free filtration $\mathcal{F}(t)$.

By assuming independence between the NHPP that models the default time t_D and the price processes that model the exposure, the CVA can be reformulated as follows by conditioning on t_D, which has probability density function$f_{t_D} : (0, \infty) \to \mathbb{R}_+$ conditional on the default-free filtration $\mathcal{F}(t)$:

$$\begin{aligned}
\mathrm{CVA}(t, T) &= LGD \int_t^\infty \mathbb{E}_t \left[\mathbb{1}_{t_D \leqslant T} \cdot e^{-r(t_D - t)} E(t_D) \,|\, t_D = s \right] f_{t_D}(s) \,\mathrm{d}s \\
&= LGD \int_t^\infty \mathbb{1}_{s \leqslant T} \cdot \mathbb{E}_t \left[e^{-r(s-t)} E(s) \right] f_{t_D}(s) \,\mathrm{d}s \\
&= LGD \int_t^T \mathbb{E}_t \left[e^{-r(s-t)} E(s) \right] f_{t_D}(s) \,\mathrm{d}s \\
&= LGD \int_t^T \mathrm{EE}_t(s) f_{t_D}(s) \,\mathrm{d}s,
\end{aligned}$$

allowing the calculation of the expected exposure $\mathrm{EE}_t(s)$ (see Definition 4.1) to be separated from the default process modelling.

APPENDIX B

VARIOUS PROOFS AND RESULTS

B.1 Fubini's Theorem for conditional expectations

As is typical in this thesis, we consider the probability space $(\Omega, \mathcal{F}, \mathbb{P})$, a time horizon $T \in \mathbb{R}_{++}$ and a state space $\Xi \subseteq \mathbb{R}^d$ of dimension $d \in \mathbb{N}$. We consider a regular stochastic process $\mathbf{X}(t)$ on $[0, T] \ni t$, meaning that for all $\omega \in \Omega$, the function $\mathbf{X}(\cdot)(\omega) : [0, T] \to \Xi$ is regular. Moreover, we define $\mathcal{F}(t)$ to be the filtration generated by the state space process $\mathbf{X}(t)$.

Theorem B.1 (Fubini's Theorem for conditional expectations)**.** *Let $\mathbf{X}(t)$ be a regular stochastic process that takes values in Ξ and let $f : \Xi \to \mathbb{R}$ be a regular function. Let $t_0, t_1, t_2 \in [0, T]$ such that $t_0 \leqslant t_1 \leqslant t_2$, and assume that for all $G \in \mathcal{F}$,*

$$\int_{[t_1,t_2]\times G} |f(\mathbf{X}(t)(\omega))| \,\mathrm{d}(t, \mathbb{P}(\omega)) < \infty, \tag{B.1}$$

which is an integral over the product measure on $[0, T] \times \Omega$. Then, we have

$$\mathbb{E}_{t_0}\left[\int_{t_1}^{t_2} f(\mathbf{X}(t))\,\mathrm{d}t\right] = \int_{t_1}^{t_2} \mathbb{E}_{t_0}[f(\mathbf{X}(t))]\,\mathrm{d}t, \tag{B.2}$$

i.e. the conditional expectation and integral operators can be interchanged.

Proof. Because of the assumptions of regularity and of finiteness in Equation (B.1), Fubini's Theorem [36] states that for all $G \in \mathcal{F}$, we have that

$$\int_{t_1}^{t_2}\int_G f(\mathbf{X}(t)(\omega))\,\mathrm{d}\mathbb{P}(\omega)\,\mathrm{d}t = \int_{[t_1,t_2]\times G} f(\mathbf{X}(t)(\omega))\,\mathrm{d}(t,\mathbb{P}(\omega)) = \int_G\int_{t_1}^{t_2} f(\mathbf{X}(t)(\omega))\,\mathrm{d}t\,\mathrm{d}\mathbb{P}(\omega). \tag{B.3}$$

By considering $G = \Omega$, we obtain

$$\int_{t_1}^{t_2} \mathbb{E}[f(\mathbf{X}(t))]\,\mathrm{d}t = \mathbb{E}\left[\int_{t_1}^{t_2} f(\mathbf{X}(t))\,\mathrm{d}t\right],$$

making use of the definition $\mathbb{E}[Y] = \int_\Omega Y(\omega)\,\mathrm{d}\mathbb{P}(\omega)$ for any random variable $Y \in L^2(\Omega)$. This is a version of Fubini's Theorem that states that the integration and expectation operator can be interchanged. This result can also be extended to the conditional expectation. Consider the σ-algebra $\mathcal{F}(t_0) \subseteq \mathcal{F}$, and define the stochastic process $Y(t) := \mathbb{E}_{t_0}[f(\mathbf{X}(t)]$ on $[t_0, T] \ni t$, which is again a random variable for a fixed t and satisfies

$$\int_G Y(t)\,\mathrm{d}\mathbb{P}(\omega) = \int_G f(\mathbf{X}(t))\,\mathrm{d}\mathbb{P}(\omega), \tag{B.4}$$

for all $G \in \mathcal{F}(t_0)$ by definition of the conditional expectation $\mathbb{E}_{t_0}[\,\cdot\,]$, and therefore also

$$\int_{t_1}^{t_2} \int_G Y(t)\,\mathrm{d}\mathbb{P}(\omega)\,\mathrm{d}t = \int_{t_1}^{t_2} \int_G f(\mathbf{X}(t))\,\mathrm{d}\mathbb{P}(\omega)\,\mathrm{d}t. \tag{B.5}$$

Equation (B.3) also holds if $Y(t)(\omega)$ is substituted for $f(\mathbf{X}(t)(\omega))$, since

$$\int_{[t_1,t_2]\times G} |Y(t)(\omega)|\,\mathrm{d}(t,\mathbb{P}(\omega)) = \int_{[t_1,t_2]\times G} |\mathbb{E}_{t_0}[f(\mathbf{X}(t)(\omega))]|\,\mathrm{d}(t,\mathbb{P}(\omega)) \leqslant \int_{[t_1,t_2]\times G} |f(\mathbf{X}(t)(\omega))|\,\mathrm{d}(t,\mathbb{P}(\omega)) < \infty,$$

by the properties of the conditional expectation [190]. Because the process $Y(t)$ is also regular, Fubini's Theorem [36] also applies and we have

$$\int_G \int_{t_1}^{t_2} Y(t)(\omega)\,\mathrm{d}t\,\mathrm{d}\mathbb{P}(\omega) = \int_{t_1}^{t_2} \int_G Y(t)(\omega)\,\mathrm{d}\mathbb{P}(\omega)\,\mathrm{d}t = \int_{t_1}^{t_2} \int_G f(\mathbf{X}(t))\,\mathrm{d}\mathbb{P}(\omega)\,\mathrm{d}t = \int_G \int_{t_1}^{t_2} f(\mathbf{X}(t))\,\mathrm{d}t\,\mathrm{d}\mathbb{P}(\omega)$$

for all $G \in \mathcal{F}$. Hence, the random variable $\int_{t_1}^{t_2} Y(t)\,\mathrm{d}t$, being $\mathcal{F}(t_0)$-measurable, satisfies the conditions of the conditional expectation of $\int_{t_1}^{t_2} f(\mathbf{X}(t))\,\mathrm{d}t$ given $\mathcal{F}(t_0)$ [190], hence

$$\mathbb{E}_{t_0}\left[\int_{t_1}^{t_2} f(\mathbf{X}(t))\,\mathrm{d}t\right] = \int_{t_1}^{t_2} Y(t)\,\mathrm{d}t = \int_{t_1}^{t_2} \mathbb{E}_{t_0}[f(\mathbf{X}(t)]\,\mathrm{d}t. \tag{B.6}$$

□

B.2 Filtration switching formula

In this section, we give the proof of Theorem 4.1, based on [123]. The filtration switching formula allows easy switching between the conditional expectations $\mathbb{E}_t[\,\cdot\,] = \mathbb{E}[\,\cdot\,|\,\mathcal{F}(t)]$ and $\mathbb{E}[\,\cdot\,|\,\mathcal{G}(t)]$, where $\mathcal{G}(t) := \sigma(\mathcal{F}(t) \cup \mathcal{H}(t))$ and $\mathcal{H}(t) := \sigma(N(s)\,|\,s \in [0,t])$ is the filtration generated by the NHPP $N(t)$.

Theorem B.2 (Filtration Switching Formula)**.** *If $Y \in L^2(\Omega)$ is a square-integrable random variable, then [123]*

$$\mathbb{1}_{t_D>t}\mathbb{E}[Y|\mathcal{G}(t)] = \mathbb{1}_{t_D>t}\frac{\mathbb{E}_t[\mathbb{1}_{t_D>t}Y]}{\mathbb{P}_t[t_D > t]}, \tag{B.7}$$

for a given $t \in [0,T]$, assuming that $\mathbb{P}_t[t_D > t] \neq 0$.

Proof. It can be observed that events $A \in \mathcal{G}(t)$ restricted to the event $\{t_D > t\}$ can purely be described in terms of the information in $\mathcal{F}(t)$: since $N(t)$ is zero if $t_D > t$, we have $H \cap \{t_D > t\} \in \{\varnothing, \{t_D > t\}\}$ for any $H \in \mathcal{H}(t)$, so it follows that $\mathcal{H}(t)$ restricted to $\{t_D > t\}$ is the trivial σ-algebra. For any event $G \in \mathcal{G}(t)$, its restriction $G \cap \{t_D > t\}$ therefore does not contain any information from $\mathcal{H}(t)$ and only contains information from $\mathcal{F}(t)$, meaning that there is some event $F \in \mathcal{F}(t)$ such that $G \cap \{t_D > t\} = F \cap \{t_D > t\}$, which therefore describes all the information in G restricted to the event $\{t_D > t\}$. This means that any $\mathcal{G}(t)$-measurable variable restricted to $\{t_D > t\}$ is also $\mathcal{F}(t)$-measurable, i.e. for any $\mathcal{G}(t)$-measurable variable X, there is an $\mathcal{F}(t)$-measurable variable $X' \in L^2(\Omega)$ such that $\mathbb{1}_{t_D>t}X = \mathbb{1}_{t_D>t}X'$.

The conditional expectation $\mathbb{E}[Y|\mathcal{G}(t)]$ is by definition $\mathcal{G}(t)$-measurable. Therefore, we can write $\mathbb{1}_{t_D>t}\mathbb{E}[Y|\mathcal{G}(t)] = \mathbb{1}_{t_D>t}Y'$ for some $\mathcal{F}(t)$-measurable random variable $Y' \in L^2(\Omega)$. Taking the conditional expectation with respect to $\mathcal{F}(t)$ on both sides yields

$$\mathbb{E}_t\Big[\mathbb{1}_{t_D>t}\mathbb{E}[Y|\mathcal{G}(t)]\Big] = \mathbb{E}_t\Big[\mathbb{1}_{t_D>t}Y'\Big],$$

which can be rewritten as

$$\mathbb{E}_t\Big[\mathbb{E}[\mathbb{1}_{t_D>t}Y|\mathcal{G}(t)]\Big] = Y'\,\mathbb{E}_t[\mathbb{1}_{t_D>t}],$$

making use of the fact that measurable variables can be taken out of a conditional expectation. The tower property of iterated conditional expectations [162] can be applied to obtain

$$\mathbb{E}_t[\mathbb{1}_{t_D>t}Y] = Y'\,\mathbb{P}_t[t_D > t],$$

meaning that Y' satisfies the definition of the conditional expectation of Y with respect to $\mathcal{F}(t)$, conditional on the event $\{t_D > t\}$ [48]. Dividing both sides by $\mathbb{P}_t[t_D > t] \neq 0$ results in the desired equality:

$$\mathbb{1}_{t_D>t}\mathbb{E}[Y|\mathcal{G}(t)] = \mathbb{1}_{t_D>t}Y' = \mathbb{1}_{t_D>t}\frac{\mathbb{E}_t[\mathbb{1}_{t_D>t}Y]}{\mathbb{P}_t[t_D > t]}.$$

□

B.3 CVA of a fixed rate bond

As demonstrated in Example 4.3, the expected exposure of a fixed rate bond with predetermined payment dates $T_1 < T_2 < \cdots < T_m$, a fixed rate $K \in \mathbb{R}_{++}$ and a notional $N \in \mathbb{R}_{++}$ is given by

$$\mathrm{EE}_t(s) = e^{-r(T-t)}N + \sum_{i=k(s)}^{m} e^{-r(T_i-t)}NK(T_i - T_{i-1}). \tag{B.8}$$

With the loss given default $LGD \in [0,1]$ and the default rate $\lambda(t)$ being given, the CVA of the fixed rate bond can be derived from Equation (4.15):

$$\begin{aligned}
\mathrm{CVA}(t,T) &= LGD\int_t^T \mathrm{EE}_t(s)\cdot\lambda(s)e^{-\int_0^s\lambda(u)\,\mathrm{d}u}\,\mathrm{d}s \\
&= LGD\int_t^T \left[e^{-r(T-t)}N + \sum_{i=k(s)}^{m} e^{-r(T_i-t)}NK(T_i - T_{i-1})\right]\cdot\lambda(s)e^{-\int_0^s\lambda(u)\,\mathrm{d}u}\,\mathrm{d}s \\
&= LGD\int_t^{T_{k(t)}} \left[e^{-r(T-t)}N + \sum_{i=k(t)}^{m} e^{-r(T_i-t)}NK(T_i - T_{i-1})\right]\lambda(s)e^{-\int_0^s\lambda(u)\,\mathrm{d}u}\,\mathrm{d}s \\
&\quad + LGD\sum_{j=k(t)+1}^{m}\int_{T_{j-1}}^{T_j}\left[e^{-r(T-t)}N + \sum_{i=j}^{m} e^{-r(T_i-t)}NK(T_i - T_{i-1})\right]\lambda(s)e^{-\int_0^s\lambda(u)\,\mathrm{d}u}\,\mathrm{d}s,
\end{aligned}$$

because $k(s) = j$ if $s \in (T_{j-1}, T_j]$ for $j \in [m]$. We split the integral over $[t, T_m] \ni s$ up into integrals over $[T_{j-1}, T_j]$ for $j \in \{k(t)+1, k(t)+2, \ldots, m\}$ and $[t, T_{k(t)}]$ to solve these individually because $\mathrm{EE}_t(s)$ is constant on these intervals. This results in integrals that take the shape[1] of

$$\int_{T_{j-1}}^{T_j}\lambda(s)e^{-\int_t^s\lambda(u)\,\mathrm{d}u}\,\mathrm{d}s,$$

which we solve by substituting

$$\Lambda(s) := \int_0^s \lambda(u)\,\mathrm{d}u \geqslant 0,$$

and therefore $\mathrm{d}\Lambda(s) = \lambda(s)\,\mathrm{d}s$, hence

$$\int_{T_{j-1}}^{T_j}\lambda(s)e^{-\int_0^s\lambda(u)\,\mathrm{d}u}\,\mathrm{d}s = \int_{\Lambda(T_{j-1})}^{\Lambda(T_j)} e^{-\Lambda}\,\mathrm{d}\Lambda = e^{-\Lambda(T_{j-1})} - e^{-\Lambda(T_j)}.$$

[1] The integral over $[t, T_{k(t)}]$ can be solved analogously.

This results into the summations

$$
\begin{aligned}
\mathrm{CVA}(t,T) &= LGD\left[e^{-r(T-t)}N + \sum_{i=k(t)}^{m} e^{-r(T_i-t)}NK(T_i-T_{i-1})\right](e^{-\Lambda(t)} - e^{-\Lambda(T_{k(t)})}) \\
&\quad + LGD \sum_{j=k(t)+1}^{m}\left[e^{-r(T-t)}N + \sum_{i=j}^{m} e^{-r(T_i-t)}NK(T_i-T_{i-1})\right](e^{-\Lambda(T_{j-1})} - e^{-\Lambda(T_j)}) \\
&= LGD\cdot e^{-r(T-t)}N\cdot\left[(e^{-\Lambda(t)} - e^{-\Lambda(T_{k(t)})}) + \sum_{j=k(t)+1}^{m}(e^{-\Lambda(T_{j-1})} - e^{-\Lambda(T_j)})\right] \\
&\quad + LGD\sum_{i=k(t)}^{m} e^{-r(T_i-t)}NK(T_i-T_{i-1})(e^{-\Lambda(t)} - e^{-\Lambda(T_{k(t)})}) \\
&\quad + LGD\sum_{j=k(t)+1}^{m}\sum_{i=j}^{m} e^{-r(T_i-t)}NK(T_i-T_{i-1})(e^{-\Lambda(T_{j-1})} - e^{-\Lambda(T_j)}).
\end{aligned}
$$

The order of summation over $(i,j)\in\mathbb{N}$ with $k(t)+1\leqslant j\leqslant i\leqslant m$ can be changed in the last term. We then recognise a telescopic series[2] in the summations over j:

$$
\begin{aligned}
\mathrm{CVA}(t,T) &= LGD\cdot e^{-r(T-t)}N\cdot\left[(e^{-\Lambda(t)} - e^{-\Lambda(T_{k(t)})}) + \overbrace{\sum_{j=k(t)+1}^{m}(e^{-\Lambda(T_{j-1})} - e^{-\Lambda(T_j)})}^{=\ e^{-\Lambda(T_{k(t)})} - e^{-\Lambda(T_m)}}\right] \\
&\quad + LGD\sum_{i=k(t)}^{m} e^{-r(T_i-t)}NK(T_i-T_{i-1})(e^{-\Lambda(t)} - e^{-\Lambda(T_{k(t)})}) \\
&\quad + LGD\sum_{i=k(t)+1}^{m} e^{-r(T_i-t)}NK(T_i-T_{i-1})\overbrace{\sum_{j=k(t)+1}^{i}(e^{-\Lambda(T_{j-1})} - e^{-\Lambda(T_j)})}^{=\ e^{-\Lambda(T_{k(t)})} - e^{-\Lambda(T_i)}} \\
&= LGD\cdot e^{-r(T-t)}N(e^{-\Lambda(t)} - e^{-\Lambda(T)}) + LGD\sum_{i=k(t)}^{m} e^{-r(T_i-t)}NK(T_i-T_{i-1})(e^{-\Lambda(t)} - e^{-\Lambda(T_i)}).
\end{aligned}
$$

We can substitute $\Lambda(s)=\int_0^s\lambda(u)\,\mathrm{d}u$ back into this Equation:

$$
\begin{aligned}
\mathrm{CVA}(t,T) &= LGD\cdot e^{-r(T-t)}N(e^{-\int_0^t\lambda(u)\,\mathrm{d}u} - e^{-\int_0^T\lambda(u)\,\mathrm{d}u}) \\
&\quad + LGD\sum_{i=k(t)}^{m} e^{-r(T_i-t)}NK(T_i-T_{i-1})(e^{-\int_0^t\lambda(u)\,\mathrm{d}u} - e^{-\int_0^{T_i}\lambda(u)\,\mathrm{d}u}).
\end{aligned}
\tag{B.9}
$$

This is the simplified expression for the CVA of a fixed rate bond, where the two terms represent the expected losses on the notional amount N to be received at maturity, and the expected losses upcoming payments of a fixed rate K over the notional, respectively. It should be noted that $e^{-\int_0^s\lambda(u)\,\mathrm{d}u}=\mathbb{P}_0[t_D>s]$ for $s\in[0,T]$. This means that the CVA of the payments depends on

$$
e^{-\int_0^t\lambda(u)\,\mathrm{d}u} - e^{-\int_0^s\lambda(u)\,\mathrm{d}u} = \mathbb{P}_0[t<t_D\leqslant s],
$$

where $s\in[t,T]$.

[2] A telescoping series is a sum over the differences over consecutive terms in another sequence, say $(a_i)_{i\in\mathbb{N}}$. Their partial sums simplify as follows: $\sum_{i=m}^{n}(a_i-a_{i+1})=a_m-a_{n+1}$ for integers $m\leqslant n\in\mathbb{N}$.

B.4 Proof of Lemma 5.1

In this section, we repeat and prove Lemma 5.1.

Lemma (Reducing future uncertainty)**.** *Let $t_0, t_1, t_2 \in [0, T]$ such that $t_0 \leqslant t_1 \leqslant t_2$, and let $Y(t)$ be a stochastic process adapted to the filtration $\mathcal{F}(t)$. For all $t \in [t_2, T]$, we then have*

$$\mathbb{V}\mathrm{ar}_{t_0}\Big[Y(t) - \mathbb{E}_{t_2}[Y(t)]\Big] \leqslant \mathbb{V}\mathrm{ar}_{t_0}\Big[Y(t) - \mathbb{E}_{t_1}[Y(t)]\Big]. \tag{B.10}$$

Proof. Let $t_0, t_1, t_2, t \in [0, T]$ such that $t_0 \leqslant t_1 \leqslant t_2 \leqslant t$ and observe that for $\tau \in \{t_1, t_2\}$, we have

$$\mathbb{V}\mathrm{ar}_{t_0}\Big[Y(t) - \mathbb{E}_{\tau}[Y(t)]\Big] = \mathbb{E}_{t_0}\Big[(Y(t) - \mathbb{E}_{\tau}[Y(t)])^2\Big] - \mathbb{E}_{t_0}\Big[Y(t) - \mathbb{E}_{\tau}[Y(t)]\Big]^2 = \mathbb{E}_{t_0}\Big[\mathbb{V}\mathrm{ar}_{\tau}[Y(t)]\Big], \tag{B.11}$$

since

$$\mathbb{E}_{t_0}\Big[(Y(t) - \mathbb{E}_{\tau}[Y(t)])^2\Big] = \mathbb{E}_{t_0}\Big[\mathbb{E}_{\tau}\Big[(Y(t) - \mathbb{E}_{\tau}[Y(t)])^2\Big]\Big] = \mathbb{E}_{t_0}\Big[\mathbb{V}\mathrm{ar}_{\tau}[Y(t)]\Big],$$

by the tower property of iterated conditional expectations [162] and the definition of the conditional variance, and

$$\mathbb{E}_{t_0}\Big[Y(t) - \mathbb{E}_{\tau}[Y(t)]\Big] = \mathbb{E}_{t_0}[Y(t)] - \mathbb{E}_{t_0}[\mathbb{E}_{\tau}[Y(t)]] = 0.$$

With Equation (B.11), we find that

$$\mathbb{V}\mathrm{ar}_{t_0}\Big[Y(t) - \mathbb{E}_{t_2}[Y(t)]\Big] = \mathbb{E}_{t_0}\Big[\mathbb{V}\mathrm{ar}_{t_2}[Y(t)]\Big] = \mathbb{E}_{t_0}\Big[\mathbb{E}_{t_1}\Big[\mathbb{V}\mathrm{ar}_{t_2}[Y(t)]\Big]\Big],$$

making use of the tower property of iterated conditional expectations [162]. From the law of total variance [56], we have

$$\mathbb{E}_{t_1}\Big[\mathbb{V}\mathrm{ar}_{t_2}[Y(t)]\Big] = \mathbb{V}\mathrm{ar}_{t_1}[Y(t)] - \mathbb{V}\mathrm{ar}_{t_1}\Big[\mathbb{E}_{t_2}[Y(t)]\Big] \leqslant \mathbb{V}\mathrm{ar}_{t_1}[Y(t)],$$

and therefore

$$\mathbb{V}\mathrm{ar}_{t_0}\Big[Y(t) - \mathbb{E}_{t_2}[Y(t)]\Big] = \mathbb{E}_{t_0}\Big[\mathbb{E}_{t_1}\Big[\mathbb{V}\mathrm{ar}_{t_2}[Y(t)]\Big]\Big] \leqslant \mathbb{E}_{t_0}\Big[\mathbb{V}\mathrm{ar}_{t_1}[Y(t)]\Big] = \mathbb{V}\mathrm{ar}_{t_0}\Big[Y(t) - \mathbb{E}_{t_1}[Y(t)]\Big],$$

again making use of Equation (B.11).

□

APPENDIX C

DETAILS ON THE SUPPLY SIDE OF THE VCM

In this appendix, we provide more details on the current state of the Voluntary Carbon Market (VCM), focussing on the supply side. This overview can be help readers who are not familiar with the VCM in understanding the unique characteristic of voluntary carbon credits. We start by describing the general lifecycle of carbon credits. The, we give a categorisation of the VCM by project type. Then, we provide a risk taxonomy that illustrate the different risk factors that can affect the effectiveness of voluntary CO2e abatement projects.

The lifecycle of carbon credits differs between project types and carbon registries, but we summarise the general lifecycle[1] in Figure C.1.

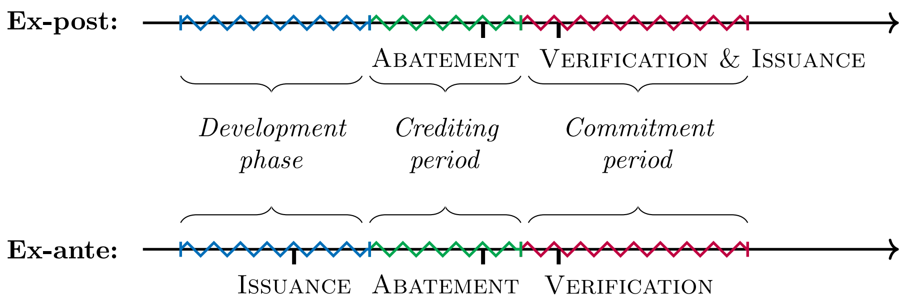


Figure C.1: *Illustration of the lifecycle of ex-post and ex-ante carbon credits. After the development phase, the CO2e abatement starts taking place during the crediting period. The abatement is verified afterwards, even though the project could still be in its commitment period. Ex-post credits are issued after the abatement is verified, and ex-ante credits are issued before the verification or even before the actual abatement.*

The development of carbon credit-generating projects starts with the design of the project and the selection of a so-called *carbon registry*, which is a party responsible for issuing the carbon credits and setting the quality standards that the project must meet [83]. After the project design is accepted and the project is developed, a third party is responsible for verifying that the project is executed according to plan. This is often referred to as Monitoring, Reporting and Verification (MRV) [59]. Once the development phase of the project is completed, CO2e abatement begins to take place. The same verification body will then confirm

[1]Based on the research of the Canadian Accounting Standards Board

how much CO2e is abated by the project following the standards of the carbon registry to determine the volume of CO2e abatement for which the project can be credited, which determines the number of carbon credits generated by the project. The project developer then receives a fee from the carbon registry, and the registry issues the carbon credits to the voluntary carbon market. This process repeats continuously during the crediting period of a project. In this way, credits are issued *after* the CO2e abatement takes place and is verified, at which point they can be bought and used immediately by participants in the carbon market. Carbon credits that follow this timeline are called *ex-post* credits. On the other hand, carbon credits can also be issued before CO2e abatement has taken place and is verified, in which case they are called *ex-ante* credits. The CO2e abatement still has to occur and be verified before its buyer can use the credits. In this way, a project receives financing before the crediting period, but this means that there is also extra risk associated with the carbon credit: if it turns out during MRV that the project does not go according to plan, the promised volume of CO2e abatement might not be achieved and the validity of the carbon credit can be affected.

Both ex-post and ex-ante credits can only be retired after the credited CO2e abatement is verified. Usually, this moment is referred to as the *vintage* of the carbon credit, but the precise definition of a credit's vintage can differ per carbon registry. However, carbon credits can even carry risk after their vintage, particularly in the case of nature-based projects: projects often have a commitment period after the crediting period during which MRV occurs and they still need to make sure that the CO2e abatement is not reversed. For example, afforestation projects remove CO2 from the atmosphere by planting trees and issue carbon credits once the trees have grown. However, projects must assure that the forest is not removed again during this commitment period, which would stop the the continuous CO2 removal by the trees. This means that both ex-post and ex-ante credits can carry risks that affect their validity.

The supply side of the VCM consists of almost 10,000 projects that contributed to a combined amount of over 2.2 Gt of negative CO2e emissions since 1996 [108]. Even though every voluntary CO2e abatement project has its own unique characteristics, credits are divided into nature-based and technology-based credits. Nature-based projects are based on natural processes that remove CO2e from the atmosphere, which are called *carbon sinks*. Projects that establish new carbon sinks generate emission removals by doing so, and projects that preserve existing carbon sinks generate avoided emissions. Technology-based abatement projects almost always reduce or avoid CO2e emissions. These types of projects can further be divided into different sectors, as illustrated in Table C.1 and Figure C.2. Different classifications are used across the industry, but we use the categorisation and database of the Berkeley Carbon Trading Project [173, 108]:

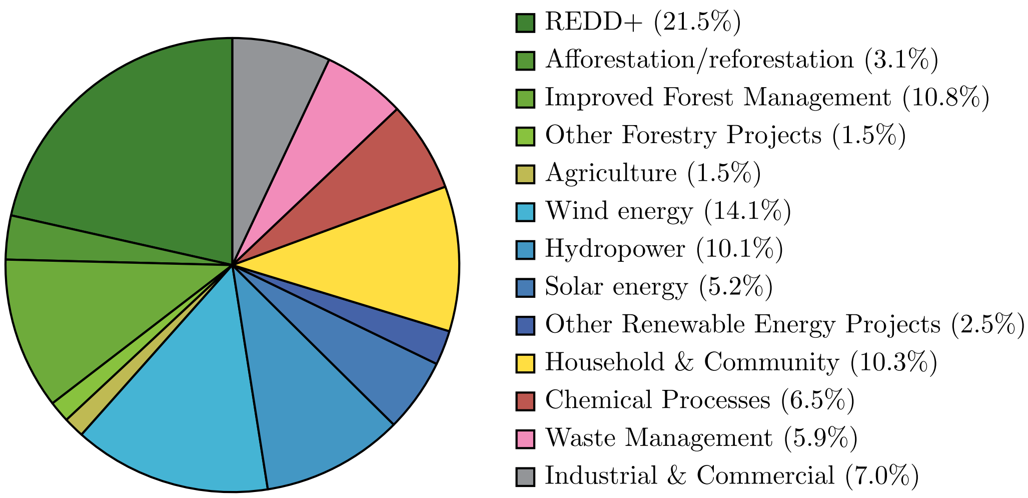


Figure C.2: *Supply of carbon credits since 1996, categorised by sector [108].*

- **Forestry & Land Use** projects either establish or preserve natural carbon sinks. Forestry projects are the most common, but other projects in this sector deal with grasslands, wetlands and mangrove ecosystems. REDD+ projects deserve special attention: the Reducing Emissions from Deforestation and forest Degradation (REDD+) framework was developed by the United Nations Framework Convention on Climate Change to combat deforestation and forest degradation, which contribute to around 25% of global CO2e emissions [156]. Despite their popularity, REDD+ projects can facilitate greenwashing by issuing illegitimate credits in the absence of transparent and accurate MRV, raising concerns about their effectiveness [12, 186, 26].

- **Agriculture** projects come in various shapes. Most credits in this sector are generated by installing biodigesters to capture methane from manure, but other types of projects include increasing the sustainability of rice cultivation or general agriculture. Some projects also use soil as a carbon sink by applying compost.
- **Renewable energy** projects install wind turbines, solar modules, hydroelectric power plant turbines and/or geothermal energy plants. These generate energy that can replace traditional energy sources, leading to avoided emissions.
- **Household & Community** projects reduce domestic emissions. The majority of these projects generate avoided emissions by installing efficient cookstoves to reduce the use of firewood, particularly in sub-Saharan Africa and India.
- **Chemical Processes** projects reduce emissions by either replacing GHGs by gases with a lower global warming potential or capturing GHGs in industrial processes, mainly in the United States. The majority of these projects are refrigerant related.
- **Waste Management** projects generate avoided emissions, with the exception of biochar projects that also generate emission removals by applying biochar to soil. The majority of waste management projects reduce emissions by destroying methane from landfills.
- **Industrial & Commercial** projects also generate avoided emissions, with the exception of Direct Air Capture (DAC) projects that directly remove CO2e from the atmosphere using chemical reactions, for example using CO2-absorbing concrete [197]. Other projects in this sector reduce direct emissions in industrial processes or transportation using various methods, such as methane destruction in mines, GHG leak detection and repair, capture of byproduct gas and heat or other energy efficiency-improving methods.

Table C.1: *An overview of the different sectors of voluntary carbon credit projects, including the total number of projects and issued credits. The sectors are also divided into nature-based projects and technology-based projects, indicated in green and orange respectively. In total, the 9,921 different projects generated 2,246,022,610 credits since 1996.*

Project Sector	Subsector	Type	# of Projects	# of Credits Issued
Forestry & Land Use	REDD+	Avoidance	298	482 537 161
	Afforestation/reforestation	Removal	554	70 314 725
	Improved Forest Management	Mixed	896	242 764 309
	Others	Mixed	189	34 111 470
Agriculture		Mixed	1042	33 260 434
Renewable Energy	Wind energy	Avoidance	1021	316 125 837
	Hydropower		532	226 741 651
	Solar energy		485	117 617 507
	Others		354	55 249 963
Household & Community		Avoidance	2799	231 798 064
Chemical Processes		Avoidance	519	145 056 798
Waste Management		Avoidance[2]	705	133 057 380
Industrial & Commercial		Avoidance[3]	527	157 387 311

Since every voluntary CO2e abatement project has its own unique characteristics, projects can face a wide variety of challenges that can affect their effectiveness. The validity of voluntary carbon credits is affected if the effectiveness of the corresponding project turns out to be lower than initially assumed and does not achieve the assumed amount of negative emissions. The issues associated with credit-generating projects can roughly be categorised following the taxonomy of risk factors that we present in this Subsection[4].

- One of the most significant risks of carbon credits is *overcrediting* risk, which is the risk that a project generates more carbon credits than the volume of CO2e it abated. Transparent and consistent MRV standards and frameworks are required to accurately quantify CO2e abatement, particularly for avoidance projects due to their dependence on baseline scenarios [164, 84]. In the absence of transparency measures, there is generally overcrediting risk associated with any carbon credit because all parties involved in the issuance of carbon credits (the project developer, the carbon registry and verification body) have an incentive to overstate the achieved CO2e abatement [25].
- Another significant risk is *permanence* risk, which is the risk that a project does not achieve its goals in the commitment period. This is mostly applicable to nature-based removal projects and forestry projects in particular. Their effectiveness is often threatened by wildfires and droughts [15], but mismanagement and political decisions can also cause permanence risk.
- One of the crucial aspects of credit-generating projects is that they are *additional*, meaning that the achieved CO2e abatement would not have occurred without the issuance of carbon credits from this project, which is related to the problem of accurate baseline setting [12]. To prove that the existence a project relies on funding from issued carbon credits, it needs to be ensured that the project is not already cost-competitive on itself and does not take place in response to regulations. Renewable energy projects are an example of a sector with high additionality risk depending on their location, because renewable energy sources are becoming increasingly cost-competitive, particularly in the presence of carbon pricing schemes [39, 204].
- The risk of *carbon leakage* was already mentioned in the context of carbon pricing schemes after Definition 1.3. This is the risk that a project pushes emissions beyond its boundaries instead of avoiding them, which can be illustrated with REDD+ projects. Avoided deforestation in one region can induce more deforestation in another, either because the deforestation activities are directly displaced or because the reduced supply of timber would indirectly increase the production in other areas [25].

All of these risk factors can affect the effectiveness of voluntary CO2e abatement projects and therefore the validity of the credits that they generate. Even though every project is unique and should be assessed on an individual level, most projects carry a combination of these risk factors, resulting in a high supply of various low-quality credits on the market. The lack of contribution to climate change mitigation of the majority of carbon credits leads to the possibility of greenwashing and is one of the primary reasons for the current poor reputation of the VCM [199, 25].

[2] Biochar projects generate removal credits.

[3] Carbon-absorbing concrete projects generate removal credits.

[4] This classification of risk factors is adapted from the methodology of BeZero, one of the largest independent carbon credit quality assessors in the market.

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