Planning small scale agriculture to optimize nutritional intake for households living on small daily budgets

Thesis Project Report

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1 Introduction

1.1 Malnutrition in the world

With a growing world population and no increase in food security, one in three people experienced food insecurity in 2021 [20]. Improving food security requires financing and policy changes on many levels. In regions facing crises or conflict it can be hard to introduce policy changes on government level but according to '2022 Global Hunger Index' [1], initiatives on a more local level or in local/decentralized government can still be successful in reducing malnutrition in that area.

In this project, the term *malnutrition*, always refers to *undernutrition* which is defined as a deficiency in one ore more of the nutrients that the human body needs to survive and thrive.

The only way to avoid *undernutrition* is frequent intake of the right quantities of nutrients. The most common nutrient deficiencies globally are protein-, iron-, vitamin A-, zinc-, and iodine deficiency [18, 20].

Nutrient requirements are not equal for all people. Very similar individuals can have very different dietary requirements [22, 23]. Estimation of nutrient requirements for an individual is not a trivial task. Diets are especially complex and varied for children and pregnant women and they are particularly affected by *undernutrition* [11, 20].

The term *full nutrition* is used throughout this project to indicate that all nutrients are available in the required quantities to achieve normal health for an individual. *Full nutrition* implies the absence of *malnutrition*.

1.2 Subsistence

Small scale agriculture on the local level is still very important in rural areas. In many developing countries a significant part of the population is involved in intensive subsistence farming [17]. In this type of agriculture, smallholders produce food for their survival on land that is typically smaller than 2 ha [10]. The choice of crops to grow mainly depends on the nutritional needs of the household. Market value of crops is less important but planting of *cash crops* from which harvest can be traded can still be very usefull. Subsistence farms require a diverse crop selection to generate a nutritious diet for the household. This in turn requires good planning, some knowledge of nutrition and efficient land use.

1.3 Computational perspective

The aim of this research project is to use a computational point of view to look at crop choices and land use in relation to nutrition for smallholder subsistence farmers. A model could calculate the best layout of farm land by optimizing nutritional output of the farm by planting crops that match the nutritional requirements of the household. Algorithms could provide assistance in the decision making processes for subsistence farmers to help them in growing crops that benefit nutrition.

Mathematical optimization techniques have been used many times to evaluate decisions in the agricultural process [8, 6]. Often the goal is to increase profit while maintaining healthy farm soil or efficient water usage. Models are often made to resemble real farms which makes it easier to implement results to then test if model predictions are accurate. A good example is the case study of a farm in New York as described in a paper by Liang, Wai Hui and You [8]. Efficient crop planning, the subject of this thesis, is essentially a land use allocation optimization problem. The objective of increasing nutritional value of the farm yield is a single objective with many criteria. Many vitamins and nutrients have to be taken into account. We are dealing with *Multi-Criteria Decision Making*. A review paper from 2018 on Multi-Criteria optimization techniques for agricultural land use allocation [6] identifies three categories of Multi-Criteria Decision Analysis (MCDA) in the literature [9, 24]: 1. *Multi-objective Decision making* (*MODM*), which involves design problems solved with continuous mathematical optimization methods where the solutions are not previously known. There often are conflicting objectives which result in trade-offs. 2. *Multi-Attribute Decision making* (*MADM*), where a solution for an evaluation problem is chosen by a discrete method from one of the candidate solutions. 3. *Multi-Criteria Decision Aid* (*MCD*A*)[1], containing uncertainty or vague information. Handling the vagueness is generally solved with fuzzy programming [4]. Not all problems fit in these categories and other approaches of distinguishing between models exist, but categorizing them does help us explore the different kind of problems that one can encounter.

Algorithmic applications for farming are often used on large farms but they have also been applied to subsistence-oriented agriculture [3, 12]. A notable example of this is the 2015 paper by Niragira *et al.* [12], which uses a mixed integer linear programming model to optimize nutrition and profit for farms of different scales in Burundi. The group of smallest farms has only 0.05 ha of farm land per member of the household. These small farms were modeled with three scenarios: 1. In the first scenario farm output value is maximized with land, labor and capital as limited resources required to produce. 2. Then a subsistence constraint was added. The farm has to produce enough food so that the household can live of the farm. 3. In the last version of the model, subsistence was made seasonal, meaning that food needs to be available in all seasons. The model succeeds in finding a feasible solution for *nearly landless* farms in the first two scenarios. The third scenario leads to infeasibility due to low production in the dry season. If you need to buy food during the dry season then you have to fund that somehow. Strategies to improve monetary income from the farm mostly lead to infeasibility if applied to such a small scale. This implies that farmers with very little land do not have the luxury to implement strategies that lead to more profitable agriculture. One example of a failing strategy is specialization. Subsistence farming means that all types of nutrients are produced. Doing this requires growing more than just one crop type. Achieving subsistence contradicts specialization.

A more recent paper from Burundi by the same authors incorporates risk in their model [13]. They also use availability of storage as a relaxation to the frequency of food production. This solves infeasibility due to lacking production in the dry season. The paper concludes that most farmers can improve the state of nutrition in their household by growing fewer crops in a more optimal combination.

1.4 Trade-offs

Sometimes when not enough resources are available, a farm's production simply cannot support a diet without any nutritional deficiency. It may be possible to improve on a state of *malnutrition* by producing the most important nutrients in a balanced way. Solving *malnutrition* entirely is not an option as this would require resources that aren't available. Changing which nutrients are prioritised may however improve overall health. This requires a change in attitude to a situation where achieving *full nutrition* is not a hard constraint but a goal to work towards. Planting a crop that may be high in protein reduces the area available to plant crops that are high in iron. These trade-offs, efficiency frontiers, or Pareto frontiers can be explored by varying weights of objectives in Multi-objective optimization [6]. Solutions created with different weights in the objective function can clearly illustrate which objectives lead to trade-offs. This only works in convex solution spaces. Kennedy *et al.* [7] showed that exploration of trade-offs helps in identifying where a small loss in one

[1] The star "*" on MCDA* was added by the authors to avoid confusion between abbreviations for Multi-Criteria Decision Analysis (MCDA) and Multi-Criteria Decision Aid (MCDA*) which is a form of MCDA. The star is not a reference to a footnote, hence this footnote to avoid confusion.

of the objectives can lead to big wins on other fronts. An optimization function with static weights would not find those solutions.

1.5 Goal

The subject of this research project was largely inspired by an initiative of Village of Peace [5] in Afghanistan. Village of Peace, or VoP for short, is an NGO that attempts to help break the spiral of poverty, violence and injustice in Afghanistan by investing in education, employment, medical care and agriculture. VoP organizes a project that helps widows and orphans by housing them in communities together. In these communities small scale agriculture is practiced to make the community more sustainable and self sufficient. This also has an educational component since most small scale farming in Afghanistan is done very traditionally without the benefits that modern agricultural techniques provide.

In this thesis an algorithm is designed for the crop planting multiple-criteria decision problem that subsistence farmers face. The algorithm should aid in the process of deciding which crops to plant for intensive subsistence farming on a very small patch of land. With farming on such a small scale, positioning of plants becomes very important and space availability can no longer be simplified by using surface areas. Instead our algorithm must find solutions for a two-dimensional puzzle that keeps changing over time.

An overview of the problem is given in section 2 where the objective and scope of the project are formulated based on a case study from Afghanistan. We present a mixed integer linear program in the model formulation in section 3. Some concepts at the core of this model are explained in section 4. The output generated by the model is presented and analyzed in the results in section 5. We then close on some conclusions, discussion and recommendations in section 6.

2 Problem description

The concept of communities in Afghanistan is used as a case study and starting point to develop a model to assist in decision making for crop selection in intensive subsistence agriculture on a very small scale. This problem description is based on the case study of a community where $20m^2$ gardens are used to support widows and the children in their care.

In this case study, a cropping plan must be constructed for the garden, adhering to the following objectives, prioritized in descending order:

1. Produce a harvest that contains all nutrients to achieve a diet with *full nutrition* for a single person.
    - If achieving *full nutrition* is infeasible, minimize nutrient deficiencies.
2. Maximize profit by producing *cash crops*.

The case study is modeled as a two-dimensional fitting problem with the added dimension of time.

2.1 Core Definitions

Let there be a garden G of width w^{G} and length l^{G} for which we make a cropping plan. A cropping plan P contains the total quantity of plants Q_c^{tot} of crop c that is planted in garden G over the *problem duration* $\mathrm{t}^{\mathrm{total}}$ spanning from day $\mathrm{t}^{\mathrm{start}}$ to day $\mathrm{t}^{\mathrm{end}}$.

2.1.1 Quantities

The cash crop quantity $Q_c^{¤}$[2], specifies the number of plants of crop c, that is grown for their market value. These cash crops are not consumed and therefore do not contribute to nutrition intake. The number of plants from which all harvest is eaten for their nutritional value, is given by Q_c^{nut}. The total quantity of plants of a crop c is given by Q_c^{tot}.

2.1.2 Space & Location

Plants are fitted in the garden so that they do not overlap. Each plant of type c takes up a square area of w_c by l_c. Plants are positioned in garden G grouped by their type c, in rectangular crop areas, as is common practice in gardening. An area A_c, where crops of type c are planted, has a width and length that are equal to a multiple of w_c and l_c respectively.

Let x_c^{min}, x_c^{max}, y_c^{min}, and y_c^{max} be the coordinates of the four borders of the *crop area* A_c in which crop c is planted.

2.1.3 Seasons

Crops of type c are all planted on the same day t_c^{planted}. All plants of type c need to grow for $\mathrm{t}_c^{\mathrm{grow}}$ days after which they are harvested. Plants need to be put in the ground during periods of the year that allow them to develop and produce harvest. These periods differ between *crop types*. We can start planting crops of type c on day $\mathrm{t}_c^{\mathrm{earliest}}$ but not before. They must be havested no later than day $\mathrm{t}_c^{\mathrm{latest}}$.

[2]Currencies have political connotations. To avoid that problem the symbol for unspecified currency '¤' is used to denote that something is about monetary value.

2.1.4 Harvest & Nutrition

The average mass that harvesting of a single plant of type c yields, is given by M_c and the nutritional content of 100g of that harvest is given by $\mathrm{N}_{c,n}$ for any nutrient n. Counting the nutritional value of all harvest from Q_c^{nut} plants for each crop c, will lead to the total amount of nutrients produced N_n^{tot}. The nutritional minimum $\mathrm{N}_n^{\text{AnnualMin}}$ is the amount of nutrients of type n required for subsistence. Realism is increased if nutritional requirements are enforced for more granular periods so that nutritional food is available year round. Yearly nutritional minimums $\mathrm{N}_n^{\text{AnnualMin}}$ are therefore spread out over P^{num} periods where time granularity P^{num} is 1, 2, 3, or 4. Nutrition acquired from harvesting a crop in period $p \in \mathrm{P}^{\text{num}}$, only counts for nutritional targets of period p.

2.1.5 Value

The value of 100g of harvest of crop c is given by V_c. Any space that is not needed for nutritional crops can be used for growing *cash crops*.

3 Model Formulation

Building on terminology defined in section 2 an integer linear programming model is introduced that designs a cropping plan P for a small subsistence garden G.

A complete list of definitions for all variables, constants, constraints, and objective functions, can also be found in appendix A.

3.1 Decision variables

Let integer decision variable $t_c^{\text{planted}} \in [\mathrm{t}_c^{\text{earliest}}, \mathrm{t}^{\text{end}} - \mathrm{t}_c^{\text{grow}}]$ denote the day on which all plants of crop c are planted, where constant $\mathrm{t}_c^{\text{earliest}} \in [0, \mathrm{t}^{\text{end}}]$ defines the earliest day on which crop c can be planted. Upper bounds are formed by two constants: final day of the problem t^{end} and the number of days of growing time $\mathrm{t}_c^{\text{grow}}$ that plants of crop c need before they can be harvested.

Rectangular cropping areas for each crop c, are defined by their 4 borders. Border locations are given by the 4 continuous decision variables $x_c^{\min}$, $x_c^{\max}$, $y_c^{\min}$, and $y_c^{\max}$ that are bounded by the dimensions of the garden given by constants w^{G} and l^{G}:

$$0 \leq x_c^{\min} \leq x_c^{\max} \leq \mathrm{w}^{\mathrm{G}} \quad \forall c \in C \tag{1}$$

$$0 \leq y_c^{\min} \leq y_c^{\max} \leq \mathrm{l}^{\mathrm{G}} \quad \forall c \in C \tag{2}$$

Based on the time of planting and available surface area defined by the aforementioned border variables, we introduce auxiliary variables to aid in tracking for each crop c:

1. the quantity of plants in the garden,
2. the time period p in which they are harvested, and
3. the quantity of those plants from which all harvest is used to reach nutrition & subsistence goals for each period p.

3.1.1 Used crops and crop quantity

We introduce the binary auxiliary variable U_c to indicate if a crop c is used in a solution. The integer variable $Q_c^{\text{tot}} \in [0, \mathrm{q}_c^{\max}]$ denotes the number of plants that are in use, where the constant $\mathrm{q}_c^{\max}$ defines an upper limit for the number of plants of crop c that may be planted. The crop quantity Q_c^{tot} is non-zero if, and only if, U_c is equal to 1. This is enforced by the constraints in equations (3) and (4).

$$Q_c^{\text{tot}} \geq U_c \tag{3}$$

$$Q_c^{\text{tot}} \leq U_c \cdot \mathrm{q}_c^{\max} \tag{4}$$

$$Q_c^{\text{tot}} = \sum_{i=0}^{\mathrm{M}_c^{\mathrm{x}}-1} 2^i x_{c,i} y_c \qquad \forall c \in C \tag{5}$$

$$y_c^{\max} - y_c^{\min} + \epsilon^{\leftrightarrow} \cdot U_c \geq (\mathrm{l}_c + \epsilon^{\leftrightarrow}) \cdot y_c \tag{6a}$$

$$x_c^{\max} - x_c^{\min} + \epsilon^{\leftrightarrow} \cdot U_c = (\mathrm{w}_c + \epsilon^{\leftrightarrow}) \cdot \sum_{i=0}^{\mathrm{M}_c^{\mathrm{x}}-1} 2^i \cdot x_{c,i} \tag{6b}$$

$$\text{where } \mathrm{M}_c^{\mathrm{x}} = \left\lfloor \log_2 \left(\mathrm{w}^{\mathrm{G}} / \mathrm{w}_c + 1 \right) \right\rfloor + 1$$

The crop quantity Q_c^{tot}, is derived from surface areas. The product of the width and length of an area, both measured by how many plants of crop c would fit, is computed with a piece-wise linear approximation as shown in equation (29). For any bit i of the integer value for width of an area of crop c, a binary variable $x_{c,i}$ is introduced. The product of width and length is then computed by linearizing products of binary variables with the integer variable y_c that represents the length of the area. The products of variables $x_{c,i}$ and y_c require linearization. Appendix A.6.1 demonstrates the steps involved.

Width and length of the area for crop c, are linked to border variables $x_c^{\min}$, $x_c^{\max}$, $y_c^{\min}$, and $y_c^{\max}$ by equations (6a) and (6b). The width and length of a single plant of crop c is given by the constants l_c and w_c. The constant $\epsilon^{\leftrightarrow}$ is used to introduce some padding between plants.

3.1.2 Quantity of crops per period

The integer variable $Q_{c,p}^{\mathrm{nut}}$ is introduced to denote the amount of plants of crop c, that is planted for nutrition, and that is harvested in a period $p \in \mathrm{P}^{\mathrm{num}}$. Equation (7) constrains the amount of plants that was planted to producte nutrition by the total amount of crops that is produced. The binary variable $H_{c,p}$ indicates if a crop is harvested in period p, based on the required growing time $\mathrm{t}_c^{\mathrm{grow}}$, and the time of planting t_c^{planted}. Equations (8) and (9) define the relationship between the three auxiliary variables. A linearization of equation (9) is provided in appendix A.6.2.

$$Q_c^{\mathrm{nut}} \leq Q_c^{\mathrm{tot}} \qquad \forall c \in C \tag{7}$$

$$Q_c^{\mathrm{nut}} \equiv \sum_{p}^{\mathrm{P}^{\mathrm{num}}} Q_{c,p}^{\mathrm{nut}} \tag{8}$$

$$Q_{c,p}^{\mathrm{nut}} \equiv Q_c^{\mathrm{nut}} \cdot H_{c,p} \tag{9}$$

Equations (10) and (11) define the relationship between $H_{c,p}$ and the time of planting of a crop. These constraints ensure that crop c is planted at a point in time that results in them being harvested during period p if $H_{c,p}$ is equal to 1.

$$t_c^{\mathrm{planted}} + \mathrm{t}_c^{\mathrm{grow}} \geq \left(\frac{\mathrm{t}^{\mathrm{total}}}{\mathrm{P}^{\mathrm{num}}} \cdot p \right) \cdot H_{c,p} + 1 \tag{10}$$

$$t_c^{\mathrm{planted}} + \mathrm{t}_c^{\mathrm{grow}} \leq \left(\frac{\mathrm{t}^{\mathrm{total}}}{\mathrm{P}^{\mathrm{num}}} \cdot (p+1) \right) + \left(\mathrm{t}^{\mathrm{total}} - \frac{\mathrm{t}^{\mathrm{total}}}{\mathrm{P}^{\mathrm{num}}} \cdot (p+1) \right) \cdot (1 - H_{c,p}) + 1 \tag{11}$$

3.1.3 Nutrition targets

The continuous variable $F_{p,n} \in [0, 1]$ denotes the relative part of the nutritional goal that was not produced for period p and nutrient n. If all nutritional targets are achieved then $F_{p,n}$ is zero for any p and n.

3.1.4 Time & space overlapping indicators

Binary variable $\Lambda^{\mathrm{u,t}}_{c_1,c_2}$ indicates when for two crops c_1 and c_2, the following two conditions hold:

1. Both crops exist in the garden simultaneously.
2. Both crops are used in the solution

When $\Lambda^{\mathrm{u,t}}_{c_1,c_2}$ is equal to 1, spacial overlapping must not occur between crops c_1 and c_2. The crops must however have overlap in their growing periods.

If two periods overlap then the end of the first period must be later than the start of second period and vice versa. We introduce binary variables $O^1_{c_1,c_2}$ and $O^2_{c_1,c_2}$ to denote the occurance of these two conditions:

1. $O^1_{c_1,c_2}$: Crop c_1 is harvested after having planted crop c_2
2. $O^2_{c_1,c_2}$: Crop c_2 is harvested after having planted crop c_1

If both conditions are true then there exists at least one day on which the garden contains plants of both crop c_1 and c_2 simultaneously.

Overlap between two axis-aligned rectangular shapes in two-dimensional space can be avoided in 4 different ways, as there are two ways to avoid overlap between two ranges in a single dimension. Consider the case of the single dimension:

The binary auxiliary variable $\delta_{c_1,c_2,k}$ is introduced to indicate for a crop c_1 and c_2, the occurance of the 4 ways to avoid overlap, indexed by k. Here $\delta_{c_1,c_2,k}$ is 0, when overlap preventing situation k is happening, or 1 when the situation does not occur. These variables will be used to activate overlapping constraints in section 3.2.2.

We now proceed to define a two-step modeling appraoach.

3.2 Step 1: Nutrition

In the first variant of the model, the goal is to minimize malnutrition. An optimal output is either a cropping plan that achieves *full nutrition* goals for each nutrient in each period, or a plan that gets as close as possible if achieving *full nutrtition* is found to be infeasible.

3.2.1 Objective function

$$\max \sum_{p=0}^{\mathrm{P^{num}}} \sum_{n=0}^{|N|} F_{p,n} \tag{12}$$

The first objective function, shown in equation (12) aims to reduce nutrient deficiency by maximizing over $F_{p,n}$ which represents the relative succesfulness at achiving minimum targets for any period p and nutrient n. *Full nutrition* is achieved if $F_{p,n}$ reaches a value of 1, for each p and n.

3.2.2 Constraints

Crops & Nutrition

$$\sum_{c=0}^{|\mathrm{C}|} \mathrm{N}_{c,n} \cdot \mathrm{M}_c \cdot Q^{\mathrm{nut}}_{c,p} \geq \frac{\mathrm{N}^{\mathrm{AnnualMin}}_n}{\mathrm{P^{num}}} \cdot F_{p,n} \qquad \forall n \in N \quad \forall p \in \{1, \dots, \mathrm{P^{num}}\} \tag{13}$$

The constraint for reaching nutritional requirements $\mathrm{N}^{\mathrm{AnnualMin}}_n$ for a nutrient $n \in \mathrm{N}$ is given by equation (13).

Planting & Harvesting time limits can be set for each crop. The constants $\mathrm{t}_c^{\mathrm{earliest}}$ and $\mathrm{t}_c^{\mathrm{latest}}$ define the range of days for which a crop c is allowed to be in the garden. The following constraints enforce these time limits for any crop c that is active in the solution:

$$t_c^{\mathrm{planted}} \geq \mathrm{t}_c^{\mathrm{earliest}} \cdot U_c \tag{14}$$

$$\mathrm{t}_c^{\mathrm{latest}} \cdot U_c \geq t_c^{\mathrm{planted}} + \mathrm{t}_c^{\mathrm{grow}} \qquad \forall c \in C \tag{15}$$

Fit cropping areas in space & time

$$x_{c_1}^{\max} + \epsilon^{\longleftrightarrow} \leq (x_{c_2}^{\min} + \mathrm{w}^{\mathrm{G}} \cdot \delta_{c_1,c_2,0}) \tag{16}$$

$$x_{c_2}^{\max} + \epsilon^{\longleftrightarrow} \leq (x_{c_1}^{\min} + \mathrm{w}^{\mathrm{G}} \cdot \delta_{c_1,c_2,1}) \tag{17}$$

$$y_{c_1}^{\max} + \epsilon^{\longleftrightarrow} \leq (y_{c_2}^{\min} + \mathrm{l}^{\mathrm{G}} \cdot \delta_{c_1,c_2,2}) \tag{18}$$

$$y_{c_2}^{\max} + \epsilon^{\longleftrightarrow} \leq (y_{c_1}^{\min} + \mathrm{l}^{\mathrm{G}} \cdot \delta_{c_1,c_2,3}) \tag{19}$$

$$\sum_{k=0}^{3} \delta_{c_1,c_2,k} \leq 4 - \Lambda_{c_1,c_2}^{\mathrm{u,t}} \qquad \forall \{c_1, c_2 | c_1 \prec c_2\} \in C \tag{20}$$

Crops c_1 and c_2 must not overlap in space and time. The overlapping constraint in equation (20) enforces this by counting for each the axis, the two ways in which two shapes could have overlap using the conditional constraints in equations (16) to (19). If crops c_1 and c_2 do not exist in the garden during overlapping time frames then $\Lambda_{c_1,c_2}^{\mathrm{u,t}}$ will be equal to 0 which will relax the overlapping constraint.

$$t_{c_1}^{\mathrm{planted}} + \mathrm{t}_{c_1}^{\mathrm{grow}} \geq t_{c_2}^{\mathrm{planted}} - \mathrm{t}^{\mathrm{total}} \cdot (1 - O_{c_1,c_2}^1) \tag{21a}$$

$$t_{c_2}^{\mathrm{planted}} + \mathrm{t}_{c_2}^{\mathrm{grow}} \geq t_{c_2}^{\mathrm{planted}} - \mathrm{t}^{\mathrm{total}} \cdot (1 - O_{c_1,c_2}^2) \tag{21b}$$

$$t_{c_1}^{\mathrm{planted}} + \mathrm{t}_{c_1}^{\mathrm{grow}} \leq t_{c_2}^{\mathrm{planted}} + \mathrm{t}^{\mathrm{total}} \cdot O_{c_1,c_2}^1 \tag{21c}$$

$$t_{c_2}^{\mathrm{planted}} + \mathrm{t}_{c_2}^{\mathrm{grow}} \leq t_{c_1}^{\mathrm{planted}} + \mathrm{t}^{\mathrm{total}} \cdot O_{c_1,c_2}^2 \qquad \forall \{c_1, c_2 | c_1 \prec c_2\} \in C \tag{21d}$$

$$\Lambda_{c_1,c_2}^{\mathrm{u,t}} \leq U_{c_1} \cdot U_{c_2} \cdot O_{c_1,c_2}^1 \cdot O_{c_1,c_2}^2 \qquad \forall \{c_1, c_2 | c_1 \prec c_2\} \in C \tag{22}$$

If two crops are in the garden at the same time then indicator variables O_{c_1,c_2}^1 and O_{c_1,c_2}^2, need to both be true, as is enforced by the conditional constraints in equations (21a) to (21d).

A more detailed description of the concepts that lead to space and time constraints is given in section 4.1.1 and a set of constraints that linearizes equation (22) can be found in appendix A.5.3.

3.3 Step 2: Market Value

The model as formulated in section 3.2 produces a solution with minimal nutritional deficiency. In the best case this means that there is no deficiency. This implies that a relaxation of the nutrition constraint in equation (13) is not required to get a feasible solution. If full nutrition was not achieved then the $F_{p,n}$ of the previous solution are used to fix the minimum nutritional output to an achievable level while optimizing for market value.

3.3.1 Objective function

The second goal of our models is to optimize for market value. Market value maximization is achieved by equation (23).

$$\max \sum_{c=0}^{|C|} \left(Q_c^{\text{tot}} - Q_c^{\text{nut}}\right) \cdot \mathrm{V}_c \tag{23}$$

3.3.2 Constraints

The same constraints used in section 3.2 are used without change. One addition is made that changes the functioning of the nutritional constraint in equation (13) by fixing auxiliary variable $F_{p,n}$:

$$F_{p,n} \geq F_{p,n}^{\text{previous}} \qquad \forall p \forall n \tag{24}$$

If full nutrition is achieved by the model in the first step (section 3.2), then the constraint in equation (24) makes the nutrition constraint in equation (13) enforce full nutritional requirements. If minization over $F_{p,n}$ did not result in full nutrition then the current degree of malnutrition is maintained during value maximization.

4 Modeling Concepts & Linearizations

In this section we provide some background on modeling choices for the model defined in section 3. The

4.1 Fitting in 3 dimensions

Cropping areas are not allowed to overlap in both time and space at the same time. In our model we defined a variable $\Lambda^{\mathrm{u,t}}_{c_1,c_2}$ to denote if two crops are both in use, and overlapping in time with each other. Time is a single dimension. Fitting in space contains two dimensions. Together they form a 3-dimensional space where overlap is not allowed. Lets first look at one dimensional overlap.

4.1.1 Overlap in 1 dimension

Consider a situation where two ranges do not overlap. Theorem 1 shows that there are two configurations in which this happens:

Theorem 1 *Let $A = [a_1, a_2]$ and $B = [b_1, b_2]$ denote two ranges that have no overlap, indicating that A entirely precedes B or B entirely precedes A:*

1. $a_2 < b_1$ ***or***
2. $b_2 < a_1$

No overlap:

$$A \cap B \equiv \emptyset \iff (a_2 < b_1) \vee (b_2 < a_1) \tag{25}$$

If two ranges overlap, then neither of the cases in theorem 1 is encountered. Negating equation (25) yields:

$$A \cap B \neq \emptyset \iff \neg\,(\,(a_2 < b_1) \vee (b_2 < a_1)\,) \tag{26}$$

Theorem 2 (De Morgan's theorem) *The negation of a disjunction is the conjunction of the negations:* $\neg(Y \vee Z) \iff (\neg Y) \wedge (\neg Z)$

The negation of disjunction in equation (26) can be transformed into a conjunction of negations using De Morgan's theorem 2. This yields equations without any strict inequalities:

Corollary 2.1 *For ranges $A = [a_1, a_2]$ and $B = [b_1, b_2]$:*

$$A \cap B \neq \emptyset \iff (a_2 \geq b_1) \wedge (b_2 \geq a_1)$$

The two parts of the conjunction in corollary 2.1 are used to form conditional constraints activated by binary variables $O^1_{c_1,c_2}$ and $O^2_{c_1,c_2}$. The conjunction of $O^1_{c_1,c_2}$ and $O^2_{c_1,c_2}$ is formed by the $\Lambda^{\mathrm{u,t}}_{c_1,c_2}$ variable, indicating that two crops exist in overlapping time frames.

4.1.2 Overlap in 2 dimensions

Overlap in two dimensions builds on the one-dimensional case.

Theorem 3 *Let Q and R denote two rectangles. Rectangles Q and R do not overlap. This implies that their ranges in the x-axis and y-axis do not overlap:*

$$Q \cap R \equiv \emptyset \iff (Q_x \cap R_x \equiv \emptyset) \vee (Q_y \cap R_y \equiv \emptyset) \tag{27}$$

Overlap in two dimensions occurs if overlap is happening in both the x, and y dimensions. Equation (27) from theorem 3 shows there only needs to be one axis without overlap to avoid overlapping Q and R.

Corollary 3.1 *Each rectangle consists of a range along the x-axis and y-axis:*

- $Q_x = [q_{x1}, q_{x2}]$, $Q_y = [q_{y1}, q_{y2}]$
- $R_x = [r_{x1}, r_{x2}]$, $R_y = [r_{y1}, r_{y2}]$

Substituting equation (27) *with equations from theorem 1 and filling in the above range variables yields:*

$$Q \cap R \equiv \emptyset \iff (q_{x2} < r_{x1}) \vee (r_{x2} < q_{x1}) \vee (q_{y2} < r_{y1}) \vee (r_{y2} < q_{y1}) \tag{28}$$

Using the overlapping rules for two ranges from section 4.1.1 once for each dimension, leads to a disjunction of 4 inequalities in corollary 3.1. These 4 inequalities are used as conditional constraints powered by the binary auxiliary variables $\delta_{c_1,c_2,k}$ where the sum over $\delta_{c_1,c_2,k}$ for $0 \leq k < 4$ has to be less or equal to 3 to avoid overlapping. If the sum is less than 4, then one of the inequalities in equation (28) is active, leading to avoidance of overlap in 2D space. This concept is applied by the constraint in equation (20) in the previous section.

4.2 Crop quantity derived from surface area

Surface area of of a rectangle is computed with by the product of width and length. Both width and length are variables so this product has to be linearized.

The following linearization was inspired by the approach used by D'Ambrosio, Lodi and Martello [2]. In our case the linearization is not an approximation since the step size used is equal to the size of a plant.

Crop quantity Q_c^{tot} is equal to the product of width x and length y (both measured in multiples of crop size w_c) of a cropping area. A piece-wise linear approximation is introduced where each bit of the integer width x, is represented by a binary variable $x_{c,i}$. The width of a cropping area is equal to $\sum_{i=0}^{\mathrm{M}_c^{\mathrm{x}}} 2^i \cdot x_{c,i}$ where $\mathrm{M}_c^{\mathrm{x}}$ is equal to the number of bits required to express the number of plants that would fit in the total width of the garden. The number of crops in the y-direction is measured by an integral variable y_c.

$$Q_c^{\text{tot}} = \text{plants-wide} \cdot \text{plants-long} = \sum_{i=0}^{\mathrm{M}_c^{\mathrm{x}}-1} 2^i \cdot x_{c,i} \cdot y_c = \sum_{i=0}^{\mathrm{M}_c^{\mathrm{x}}-1} 2^i \cdot z_{c,i} \tag{29}$$

Linearization of $x_{c,i} \cdot y_c$ in equation (29) is done by substitution with $z_{c,i}$ and adding constraints:

$$z_{c,i} \leq y_c^{\mathrm{ub}} \cdot x_{c,i} \tag{29a}$$

$$z_{c,i} \geq y_c^{\mathrm{lb}} \cdot x_{c,i} \tag{29b}$$

$$z_{c,i} \leq y - y_c^{\mathrm{lb}} \cdot (1 - x_{c,i}) \tag{29c}$$

$$z_{c,i} \geq y - y_c^{\mathrm{ub}} \cdot (1 - x_{c,i}) \tag{29d}$$

The use of this piece-wise linear technique requires the introduction of $\mathrm{M}_c^{\mathrm{x}}$ binary variables. For each crop c this comes down to $\left\lceil \log_2 \left(\mathrm{w}^{\mathrm{G}} / \mathrm{w}_c + 1 \right) \right\rceil$ binary variables for a garden of width w^{G} and plant width w_c.

5 Results

The model described in section 3 is capable of generating cropping plans for a full year. It optimizes production for market value while maintaining nutritional requirements for each period.

5.1 Case study of a 20 square meter garden

The scenario of our case study is modeled by a garden of 4 meters wide and 5 meters long. Crop repeats are set to 2, so planting for the same crop may be done at two moments in time at most. The time granularity is set to 3. A time granularity of 3 enforces the nutrition constraint for three periods seperately, that each span 1/3rd of the total time duration. This means that all harvested crops in a period should together contain enough nutrients to match or exceed minimum nutrition values that would be required for the duration of that period. The full list of parameters, including information on nutrition, market value, and the used crops, is provided in appendix B.1.

Results of running the two step model of section 3, are visualized in figure 1. The crop plan is plotted for each day on which it changes. A time-linear representation is shown in figure 2.

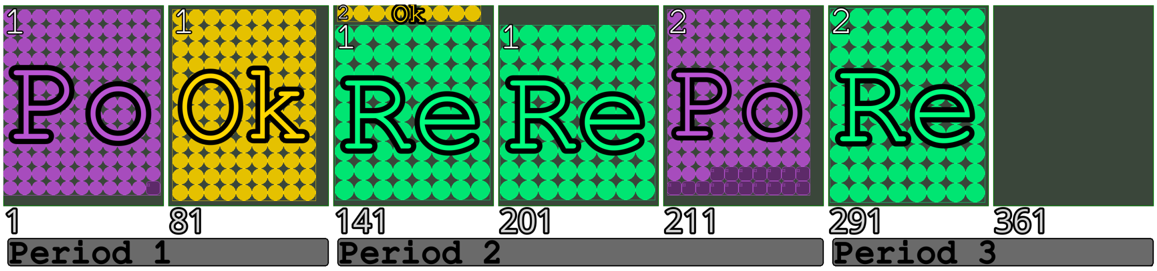


Figure 1: Cropping plan with 3 nutritional periods for a garden of 4m×5m. A map of the garden is drawn for each day on which the cropping plan changes in the garden layout. A plant is shown as circle if it's harvest is used for nutrition and as a square if all produce is sold on markets. The day number is displayed below each garden render, and the horizontal bars below that indicate in which nutritional period p each day is. Each crop is color coded as, and abbreviated by: **Eg**: Eggplant, **Le**: Lettuce, **Ok**: Okra, **Po**: Potatoes, **Sw**: Sweetpotatoes, **To**: Tomatoes, **Zu**: Zucchini, **Re**: Redbellpepper

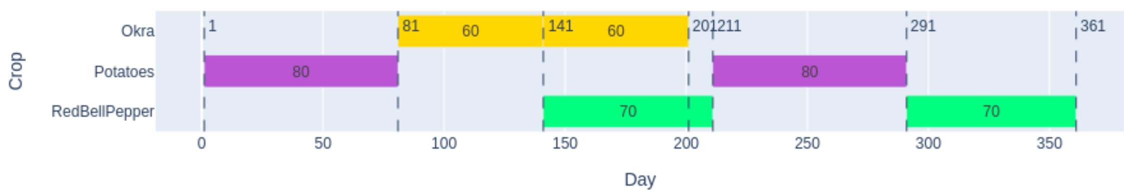


Figure 2: Timeline of planting and harvesting

From looking at our plots for space (figure 1) and time (figure 2) we can see that crops are harvested in each period and that there was not a lot of time and space left over for planting cash crops. The harvest of a total of 18 potato plants is used to create monetary value (the square plants in the bottom right of day 1 and 211 in figure 1). All other plants contribute to nutritional targets. Table 1 list the choosen number of plants of each crop that are planted according to the output cropping plan. This table also contains the market value (for unspecified currency unit ¤) that is produced for each crop. The production of nutrients per period and the required nutrient production per period to reach *full nutrition*, are displayed in table table 2.

Crop	**Quantity 1**	**Quantity 2**	**Quantity total**	**Market value generated (¤)**
Eggplant	0	0	0	0
Lettuce	0	0	0	0
Okra	108	9	217	0
Potatoes	143	130	173	26892
Sweetpotatoes	0	0	0	0
Tomatoes	0	0	0	0
Zucchini	0	0	0	0
Redbellpepper	72	80	152	0

Table 1: Crop quantities planted and market value generated

Nutrient	**Goal per period (g)**	**Produced p1**		**Produced p2**		**Produced p3**	
		mass (g)	**goal**	**mass (g)**	**goal**	**mass (g)**	**goal**
A	0.0852	6.7958	√	0.244	√	5.6089	√
B1	0.1338	0.7779	√	0.4143	√	0.6894	√
C	5.4750	138.60	√	203.94	√	292.06	√
Carbs	28591	572800	√	18437	64.5%	461759	√
E	1.825	0	0%	1.8202	99.7%	2.0224	√
Iron	2.190	2.1922	√	1.4913	68.1%	2.1925	√
Protein	5597	67180	√	4371.3	78.1%	54612	√
Zinc	0.9733	2.1214	√	1.2483	√	1.9442	√

Table 2: Nutritients produced in grams per period. Full nutrition is achieved if production matches or exceeds the minimum requirement for all nutrients in all periods. Production values that do not reach this target are colored red.

Full nutrition was not achieved with the output cropping plan since there is a nutritional production deficit in two of the three periods (shown in red in table 2).

These results were generated in 40 minutes (20 minute time limits for each step) with *Gurobi Optimizer version 11.0.1 build v11.0.1rc0* [19], using default settings, running on an AMD Ryzen 5800x CPU locked to 4.7 GHz with 32 GB of ram. Step 1 (nutrition) may just be optimal with an optimality gap of 6.87%. Step two reached an optimality gap of over a 1000% before the timelimit was reached. Convergence in step 2 is very slow when *full nutrition* was not achieved in step 1.

Linear programming models are generated using *Python-MIP* [16] in Python 3.11.2 [15]. Reproducibility is ensured by using virtual environments supplied by *Pipenv* [14]. More practical information on parameter configuration and running the model can be found in appendix B. Raw model output data is included under appendix C. All code and configuration files used to generate these results, are available on Github [21].

5.2 Sensitivity analysis

A model such as the one constructed during this thesis can be used to analyze properties of the problem. If we find that the solution is very sensitive to specific variables, then this could provide insight into the problem in the real world. We now consider some testing scenario's to demonstrate this process of discovery.

5.2.1 Impact of Garden Size & Time Granularity on *full nutrition* feasibility

The feasibility of achieving *full nutrition* is highly dependent on the size of the garden and on the granularity of the nutrition constraint. Small gardens may not produce enough nutrients and a high granularity of the constraint is expected to reduce efficiency due to a preference for more frequent nutritional output over relatively bigger nutritional output at lower frequencies.

Gardens are tested in 4 variants, once for each value of timegranularity P^{num}. When P^{num} equals 1, the full year is a single period. All produced nutrition counts towards the yearly target. A P^{num} of 2, splits the year in two periods of 6 months. Nutrients in harvests from period 1 cannot help to reach nutrition goals for period 2 or vice versa. The same principle extends to cases with a P^{num} of 3 and 4.

Testing with 4 different *timegranularities* is repeated for growing garden sizes until all nutritional targets can be met without compromise for all *timegranularities.* Results for testing step 1 of the model (step 2 is skipped for value optimization is not relevant here) are presented in table 3:

Size $[m^2]$	**Nutrient**	**Timegranularity** (P^{num}) **1**	**2**	**3**	**4**
4×5	A	1	1,1	1,1,1	1,1,1,1
	B1	1	1,1	1,1,1	1,1,1,1
	C	1	1,1	1,1,1	1,1,1,1
	Carbs	1	1,1	1,1,0	1,0,1,0
	E	0	0,0	0,1,0	0,0,0,1
	Iron	0	0,1	1,0,1	0,0,1,0
	Protein	1	1,1	1,1,1	1,0,1,1
	Zinc	1	1,1	1,1,1	1,1,1,1
4×6	A	1	1,1	1,1,1	1,1,1,1
	B1	1	1,1	1,1,1	1,1,1,1
	C	1	1,1	1,1,1	1,1,1,1
	Carbs	1	1,1	1,1,0	1,0,1,0
	E	0	0,0	0,1,0	0,0,0,1
	Iron	1	0,1	1,1,1	0,1,0,0
	Protein	1	1,1	1,1,1	1,1,1,1
	Zinc	1	1,1	1,1,1	1,1,1,1
4×7	A	1	1,1	1,1,1	1,1,1,1
	B1	1	1,1	1,1,1	1,1,1,1
	C	1	1,1	1,1,1	1,1,1,1
	Carbs	1	1,1	1,0,1	0,1,1,0
	E	1	1,1	0,0,1	0,1,1,0
	Iron	1	1,1	0,1,1	0,1,1,1
	Protein	1	1,1	1,1,1	1,1,1,1
	Zinc	1	1,1	1,1,1	1,1,1,1
5×5	A	1	1,1	1,1,1	1,1,1,1
	B1	1	1,1	1,1,1	1,1,1,1
	C	1	1,1	1,1,1	1,1,1,1
	Carbs	1	1,1	1,1,0	1,0,1,0
	E	0	0,1	0,1,1	0,0,0,1
	Iron	1	0,1	0,1,1	0,0,0,1
	Protein	1	1,1	1,1,1	1,1,1,1
	Zinc	1	1,1	1,1,1	1,1,1,1
5×6	A	1	1,1	1,1,1	1,1,1,1
	B1	1	1,1	1,1,1	1,1,1,1
	C	1	1,1	1,1,1	1,1,1,1
	Carbs	1	1,1	1,1,1	1,0,1,1
	E	1	1,1	0,1,0	1,0,1,0
	Iron	1	1,1	0,1,0	1,1,1,1
	Protein	1	1,1	1,1,1	1,1,1,1
	Zinc	1	1,1	1,1,1	1,1,1,1
⋮	⋮	⋮	⋮	⋮	⋮

Size $[m^2]$	**Nutrient**	**Timegranularity** (P^{num}) **1**	**2**	**3**	**4**
5×7	A	1	1,1	1,1,1	1,1,1,1
	B1	1	1,1	1,1,1	1,1,1,1
	C	1	1,1	1,1,1	1,1,1,1
	Carbs	1	1,1	1,1,1	1,1,1,1
	E	1	1,1	0,0,0	1,0,1,0
	Iron	1	1,1	1,1,1	1,1,1,1
	Protein	1	1,1	1,1,1	1,1,1,1
	Zinc	1	1,1	1,1,1	1,1,1,1
6×6	A	1	1,1	1,1,1	1,1,1,1
	B1	1	1,1	1,1,1	1,1,1,1
	C	1	1,1	1,1,1	1,1,1,1
	Carbs	1	1,1	1,1,1	1,1,1,1
	E	1	1,1	1,1,0	1,0,1,0
	Iron	1	1,1	1,1,0	1,1,1,1
	Protein	1	1,1	1,1,1	1,1,1,1
	Zinc	1	1,1	1,1,1	1,1,1,1
6×7	A	1	1,1	1,1,1	1,1,1,1
	B1	1	1,1	1,1,1	1,1,1,1
	C	1	1,1	1,1,1	1,1,1,1
	Carbs	1	1,1	1,1,1	1,1,1,1
	E	1	1,1	1,1,1	0,1,1,0
	Iron	1	1,1	1,1,1	0,1,1,1
	Protein	1	1,1	1,1,1	1,1,1,1
	Zinc	1	1,1	1,1,1	1,1,1,1
7×7	A	1	1,1	1,1,1	1,1,1,1
	B1	1	1,1	1,1,1	1,1,1,1
	C	1	1,1	1,1,1	1,1,1,1
	Carbs	1	1,1	1,1,1	1,1,1,1
	E	1	1,1	1,1,1	1,1,1,1
	Iron	1	1,1	1,1,1	1,1,1,1
	Protein	1	1,1	1,1,1	1,1,1,1
	Zinc	1	1,1	1,1,1	1,1,1,1

Table 3: Binary achievement of nutritional targets for each nutrient for each period, tested for varying garden sizes and timegranularities. Results for each time granularity are given in 4 columns. Within each colum are 1,2,3 and 4 colums of binary numbers indicating succes or failure to reach the nutritional target for period 1,2,3 or 4, for a specific nutrient.

Higher *timegranularities* result in tighter nutritional constraints throughout the year. In a 6×7 garden for example, *full nutrition* is achieved when testing with a timegranularity of 1, 2, or 3, but not with 4. With 4 periods per year, the cropping plan does not produce enough vitamin E in periods 1 and 4 and and not enough Iron in period 1.

Feasibility of reaching *full nutrition* is quickly increased when gardens get larger. The inverse is true for timegranularity. Achieving nutrition goals is more difficult with more nutritional periods. Feasibility of reaching *full nutrition* quickly increases for *timegranularities* 1 and 2 when garden size is increased. All gardens larger than 5×6 achieve *full nutrition*. A much larger garden is required to achieve this with 3 or 4 nutritional periods in a year. *Full nutrition* with 3 or 4 periods is finally achieved by gardens with dimensions of 6×7 and 7×7 respectively.

It is worth noting that relative achievement of nutritional targets is modeled by continuous variables. The number of fully achieved targets is therefore not a good metric for solution quality. The binary representation of table 3 can display zeros for multiple nutrients even if *full nutrition* is almost reached. Take for example the case study results where Vitamin E reached 99.7% of the nutritional goal in period 2 (see: table 2) which would be represented by a binary variable with a value of zero in table 3. Solutions that achieve targets fully for all but one nutrient and solutions that fail to fully achieve a single nutritional target, may have identical objective values. The binary representation of achieving targets is merely used to show in which configurations full nutrition can be achieved.

Size $[m^2]$	**Statistic**	**Timegranularity** ($\mathbf{P^{num}}$)			
		1	**2**	**3**	**4**
4×5	Runtime (s)	601.31	601.32	601.38	601.53
	Optimality gap (%)	4.31	4.13	6.11	8.39
4×6	Runtime (s)	601.31	601.35	601.5	601.5
	Optimality gap (%)	0.78	0.84	4.37	5.81
4×7	Runtime (s)	1.73	1.98	601.41	601.53
	Optimality gap (%)	0.00	0.00	3.70	3.98
5×5	Runtime (s)	601.37	601.4	601.62	601.51
	Optimality gap (%)	0.14	0.70	4.18	5.41
5×6	Runtime (s)	1.82	1.93	601.59	601.46
	Optimality gap (%)	0.00	0.00	2.39	3.41
5×7	Runtime (s)	1.81	2.03	601.46	601.93
	Optimality gap (%)	0.00	0.00	0.84	3.57
6×6	Runtime (s)	1.73	1.98	601.63	601.75
	Optimality gap (%)	0.00	0.00	0.43	3.08
6×7	Runtime (s)	1.79	1.89	5.25	601.58
	Optimality gap (%)	0.00	0.00	0.00	1.22
7×7	Runtime (s)	1.84	1.94	3.56	22.2
	Optimality gap (%)	0.00	0.00	0.00	0.00

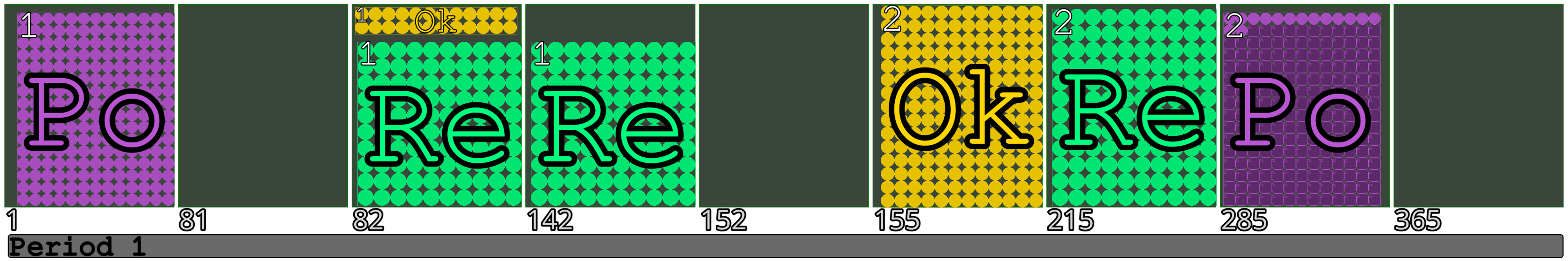
Table 4: Runtime and optimality gap of each test for garden size and time granularity. Optimality gaps larger than zero are written in red ink to emphasize that these testcases were not solved to an optimal result before the time limit.

For each problem instance that is tested, the nutritional model is allowed to run no more than 10 minutes after which the best result is selected. The runtime and optimality gap of the solution that is used are listed in table 4. Problems where full nutrition is feasible are often solved to optimality in a few seconds. The solutions from the runs where a time limit is reached are sometimes optimal as well, as their optimality gap is mostly quite small and always below 8.4%.

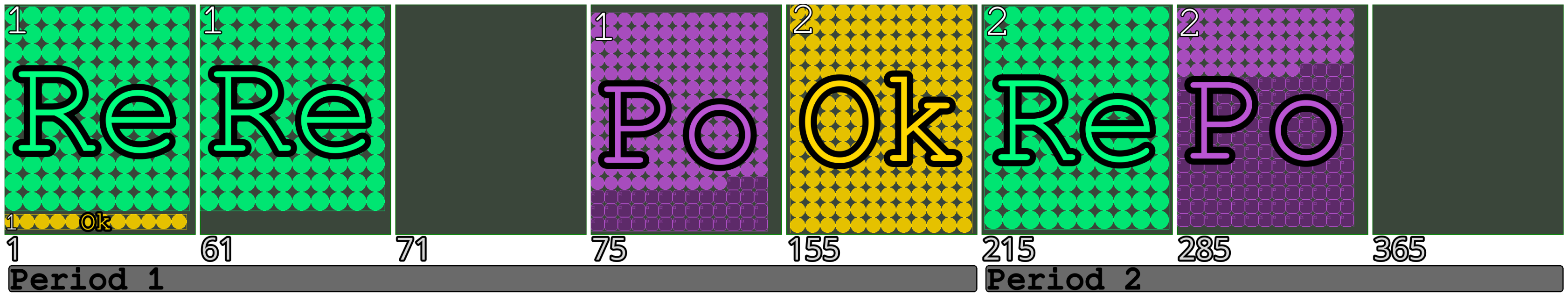
5.2.2 Opportunity for use of cash crops

The case study of 20 square meters in section 5.1, showed that there are opportunities to plant cash crops, even if nutritional goals are not fully achievable. This opportunity was there somewhat artificially formed by shape constraints of cropping areas and by the fact that the same crop may only be planted twice. Even with added opportunities, cases occur where not a single cash crop will be present in the cropping plan. An example of this is also shown in figure 3.

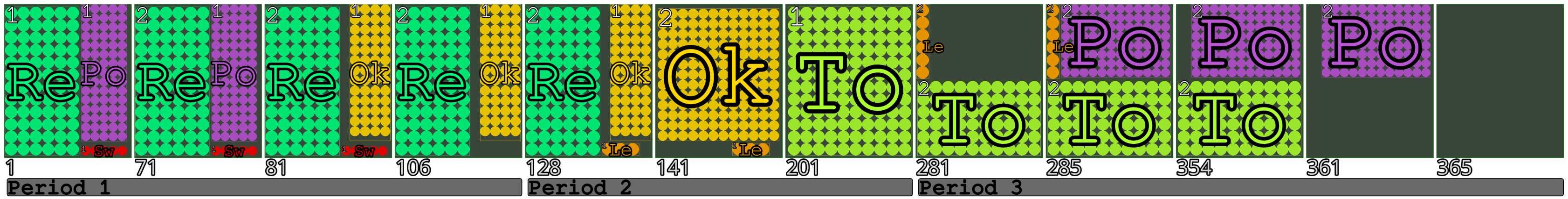
(a) Timegranularity of 1



(b) Timegranularity of 2



(c) Timegranularity of 3



(d) Timegranularity of 4

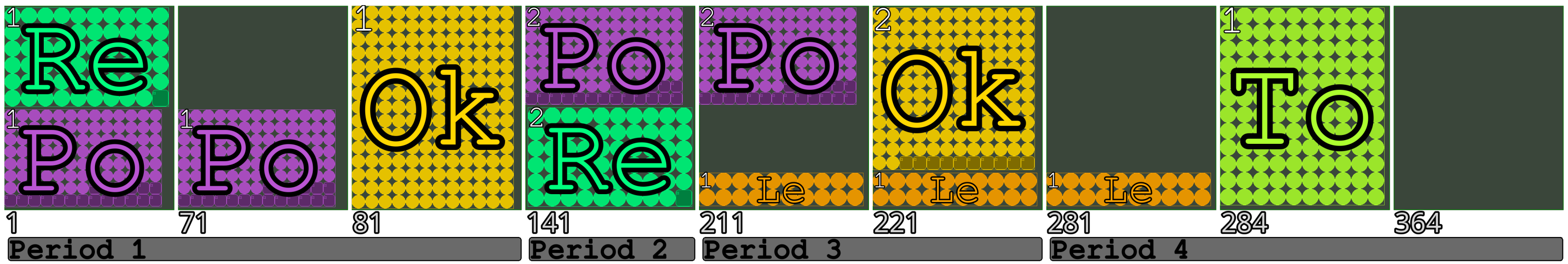


Figure 3: Value optimization scenarios in cropping plans for gardens of 5 by 6 meter with timegranularities of 1,2,3 and 4. Scenario's a & b reached *full nutrition* but this was not achieved for c or d. In scenario c, no improvement was found using step 2, meaning that no ways were found to add cash crops, without damaging nutritional results.

Two effects are found to influence the opportunity for usage of cash crops in generated cropping plans:

1. Cash crops are rarely used when nutritional targets are not achieved for many nutrients at the same time. Most crops contain small quantities of a lot of nutrients, making each plant of any crop useful in reducing malnutrition. Plants are therefore not used as cash crops as soon as they can contribute even the most insignificant amount to nutritional goals.

2. With higher *timegranularities*, opportunities for planting cash crops are created for each period for which *full nutrition* is achieved. This results in multiple possibilities for cash crops where no chance for planting cash crops would exist in the same scenario with a timegranularity of 1. This effect can be observed in figure 3d.

The runtime and optimality gap for each of the 4 tests are listed in table 5. Termination of the first step in the model was again swift if *full nutrition* was easily achieved. If optimality was not reached before the time limit then the gaps still became narrow within seconds. Value optimization is not proven to be optimal for any of the 4 cases and optimality gaps are much higher.

Timegranularity (P^{num})	**Optimality gap 1**	**Runtime 1**	**Optimality gap 2**	**Runtime 2**
1	0.00%	0s	191%	800s
2	0.00%	0s	182%	800s
3	2.39%	600s	-	800s
4	3.41%	600s	805%	800s

Table 5: Optimality gaps and runtimes for 4 cropping plans for a garden of 5 by 6 meters.

5.3 Complexity

The computational complexity of the model is most impacted by the number of crops. Each added crop generates 8 big-M constraints for each crop it could overlap with.

Crop quantity is derived from the surface area. Computing surface area requires a piecewise linear construct to remove the non-convex bilinear terms. This adds $\left\lceil \log_2 \left(\mathrm{w}^{\mathrm{G}} / \mathrm{w}_c + 1 \right) \right\rceil$ binary and integer variables and constraints for each possible cropping area. The number of cropping areas is equal to the number of crops multiplied by the number of times a crop may be planted.

6 Conclusion & Discussion

The aim of this research project was to model the crop planting multiple-criteria decision problem after a subsistence farm with a very small patch of land and to develop an algorithm that could aid in the decision process.

The resulting model uses a two step approach. In the first step nutrient shortages are minimized. The second step then maximizes market value of the cropping plan while maintaining the nutritional levels achieved by step 1.

We show that the mixed-integer linear programming model designed during this research project is capable of generating cropping plans that are optimal in their nutritional output for scenarios where *full nutrition* is possible. Even if optimality is not reached before a time limit, the algorithm provides high quality solutions early on, as reflected by convergence to single digit optimality gaps in the first seconds of commencing step 1 of the model. Step 2 often finds ways to improve financially even in cases with small solution spaces caused by unmet nutritional targets.

This work has some limitations. The proposed model is capable of producing optimal solutions for both the nutrition and value step. The computational complexity of the model limits the number of crops, garden sizes, *timegranularities* and the number of nutrients that can be tracked. Slow convergence of step 2 is a problem for larger test cases.

Simulation of gardens require many parameters. The better these parameters correspond to the real world, the more accurate model predictions are expected to be. The user is therefore encouraged to review these parameters carefully. In this work, data for testing of the model has been collected from many sources with the goal of providing enough realism to be able to demonstrate how the developed model could provide insight into a real world problem. The data collected is not representative of any real-world situation. No conclusions should therefore be drawn about real-world feasibility of subsistence, based on modeling results generated from our example data.

There are opportunities for research. Measures to improve balance between levels of nutrients and between nutrition levels over time would further improve the practicality and usability of cropping plans for cases where *full nutrition* is not feasible. This comes down to a more non-linear approach to different severities of nutrition deficiency.

Income from market value optimization may be used to purchase food. This would allow for more specialization and it could dampen the effect of nutritional deficiencies in the cropping plan. Incorporating risk analysis in the decision making process may help in producing more robust solutions where some efficiency may be sacrificed for a higher chance of successful harvests or a reduced dependence on market stability.

Acknowledgement

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Special thanks to Dr. Jaco Smit for providing me with his valuable knowledge on nutrition and subsistence agriculture in Afghanistan. The case study of a $20m^2$ garden in Afghanistan was inspired by his work done in affiliation with Village Of Peace [5].

A Model Specification

All constraints, objective functions, variables and constants are listed here in a central place.

A.1 Constants

- M_c^x Minimum number of bits required to express the integer value of how many times the width of a plant of crop c w_c would fit in the width of the garden w^G.
- M_c^y Minimum number of bits required to express the integer value of how many times the size of a crop l_c would fit in the length of the garden l^G.
- l_c Length of a single plant of crop c
- w_c Width of a single plant of crop c
- $N_{c,n}$ The mass of nutrient n contained in 100 grams of harvest yield for crop c
- $\epsilon^{\leftrightarrow}$ How much empty space must be used between two plants of the same crop. (Currently hard-coded to 1cm)
- $\epsilon^{\longleftrightarrow}$ How much empty space must be used between cropping areas. (Currently hard-coded to 10cm)
- V_c Value of 1g of harvest from of crop c.
- C The collection containing all crops that are used.
- l^G The length of the garden in meters
- w^G The width of the garden in meters
- t_c^{grow} The time in days it takes after planting before crop c can be harvested.
- t_c^{latest} Latest day on which crop c must be harvested
- M_c The average mass of the yield that a single plant of crop c produces when it is harvested.
- q_c^{max} Upper bound on the number of plants that may be planted for crop c.
- $t_c^{earliest}$ Earliest day on on which planting of crop c may take place.
- t^{end} The last day of the period that is modeled. (Hardcoded to 365)
- t^{start} The day of the total time to be modeled. (Hardcoded to 1)
- t^{total} The total duration for which a croppin gplan is designed. This is set to a full year, or 365 days.
- P^{num} Granularity of the nutrition constraint. This value can be 1, 2, 3 or 4, indicating in how many periods the year is split for which nutrition targets must be met.

A.2 Continuous Variables

A.2.1 Variables for crop counts

1. Q_c^{tot}: Total number of plants for *crop type* c, used in cropping plan
2. Q_c^{nut}: Total number of plants for *crop type* c, used in cropping plan

Where for both Q_c^{tot} and Q_c^{nut}:

- lb = $\mathrm{q}_c^{\mathrm{min}}$
- ub = $\mathrm{q}_c^{\mathrm{max}}$

A.2.2 Variables for minimum and maximum x and y position for $1 <= i <= |CropTypes|$

These variables specify the borders of the areas where crops of each crop type reside.

1. x_c^{min}
2. x_c^{max}
    - lb = 0
    - ub = w^{G}
3. y_c^{min}
4. y_c^{max}
    - lb = 0
    - ub = l^{G}

A.3 Binary Variables

A.3.1 Variables for crop usage X

1. U_c for c in the range of crop counts

A.3.2 AND variables

- $\Lambda_{c_1,c_2}^{\mathrm{u,t}}$: Crops overlap in time and space (based on endOneMTstartTwo, endTwoMTstartOne)
- $\Lambda_{c_1,c_2}^{\mathrm{u}}$: Crops are both being used
- $\Lambda_{c_1,c_2}^{\mathrm{t}}$: Crops have overlapping growing periods
- O_{c_1,c_2}^{1}: End time of crop c_1 is later than start of c_2
- O_{c_1,c_2}^{2}: End time of crop c_2 is later than start of c_1

A.3.3 Variables for crop type separation delta for 2D fitting of areas of crops

1. $\delta_{c_1,c_2,k}$
    - $c1$ in the range of crop counts
    - $c2$ in the range of ub for each crop
    - $k \in [0, 1, 2, 3]$

A.3.4 Variables used for area computation of cropType

1. $x_{c,i}$
2. $z_{c,i}$

Where for each:

- c in the range of crop counts
- i,j in range of nr of bits required to express integer of max size of *crop type* c in direction (x and y respectively) if c uses all garden space in that direction.

A.4 Integer Variables

A.4.1 Variables used for area computation of cropType

1. y_c

A.4.2 Growing period

- t_c^{planted}: day on which croptype c is planted

A.4.3 Variables for crop counts

1. Q_c^{tot}: Total number of crops of each type crop type c, used in solution
   - lb = $\mathrm{q}_c^{\text{min}}$
   - ub = $\mathrm{q}_c^{\text{max}}$
2. Q_c^{nut}: Number of crops from Q_c^{tot}, that are reserved for nutrition. (So not used for value optimization)
   - lb = $\mathrm{q}_c^{\text{min}}$
   - ub = $\mathrm{q}_c^{\text{max}}$
   - where: $Q_c^{\text{nut}} \leq Q_c^{\text{tot}}$

A.5 Constraints

For each *crop type* c:

$$Q_c^{\text{nut}} \leq Q_c^{\text{tot}} \qquad \forall c \in C \tag{7}$$

$$Q_c^{\text{tot}} \geq U_c \tag{3}$$

$$Q_c^{\text{tot}} \leq U_c \cdot \mathrm{q}_c^{\text{max}} \tag{4}$$

$$t_c^{\text{planted}} \geq \mathrm{t}_c^{\text{earliest}} \cdot U_c \tag{14}$$

$$\mathrm{t}_c^{\text{latest}} \cdot U_c \geq t_c^{\text{planted}} + \mathrm{t}_c^{\text{grow}} \tag{15}$$

A.5.1 Nutrition Constraints

$$\sum_{c=0}^{|\mathrm{C}|} \mathrm{N}_{c,n} \cdot \mathrm{M}_c \cdot Q^{\mathrm{nut}}_{c,p} \geq \frac{\mathrm{N}^{\mathrm{AnnualMin}}_n}{\mathrm{P}^{\mathrm{num}}} \cdot F_{p,n} \qquad \forall n \in N \quad \forall p \in \{1, \ldots, \mathrm{P}^{\mathrm{num}}\} \tag{13}$$

Optionally if nutrition was not feasible, fix current nutri values for value optimisation:

$$F_{p,n} \geq F^{\mathrm{previous}}_{p,n} \qquad \forall p \forall n \tag{24}$$

A.5.2 Fitting

Fitting crops within areas of cropTypes without overlap

A.5.3 Both crops in use space-time

1. Define two variables to mean that overlap in time is happening if they are both TRUE and both FALSE if no overlap occurs:

$$t^{\mathrm{planted}}_{c_1} + \mathrm{t}^{\mathrm{grow}}_{c_1} \geq t^{\mathrm{planted}}_{c_2} - \mathrm{t}^{\mathrm{total}} \cdot (1 - O^1_{c_1,c_2}) \tag{21a}$$
$$t^{\mathrm{planted}}_{c_2} + \mathrm{t}^{\mathrm{grow}}_{c_2} \geq t^{\mathrm{planted}}_{c_2} - \mathrm{t}^{\mathrm{total}} \cdot (1 - O^2_{c_1,c_2}) \tag{21b}$$
$$t^{\mathrm{planted}}_{c_1} + \mathrm{t}^{\mathrm{grow}}_{c_1} \leq t^{\mathrm{planted}}_{c_2} + \mathrm{t}^{\mathrm{total}} \cdot O^1_{c_1,c_2} \tag{21c}$$
$$t^{\mathrm{planted}}_{c_2} + \mathrm{t}^{\mathrm{grow}}_{c_2} \leq t^{\mathrm{planted}}_{c_1} + \mathrm{t}^{\mathrm{total}} \cdot O^2_{c_1,c_2} \tag{21d}$$

2. Define binary variable $\Lambda^{\mathrm{u,t}}_{c_1,c_2}$ to indicate if crops are both in garden at overlapping time, meaning that spacial overlap should be checked.

$$\Lambda^{\mathrm{u,t}}_{c_1,c_2} \leq U_{c_1} \cdot U_{c_2} \cdot O^1_{c_1,c_2} \cdot O^2_{c_1,c_2} \qquad \forall \{c_1, c_2 | c_1 \prec c_2\} \in C \tag{22}$$

In practice this is linearized by the following constraints:

(a) Crops have overlapping time periods:

$$\Lambda^{\mathrm{u,t}}_{c_1,c_2} \leq O^1_{c_1,c_2} \tag{22a}$$
$$\Lambda^{\mathrm{u,t}}_{c_1,c_2} \leq O^2_{c_1,c_2} \tag{22b}$$

(b) Both crops in use in solution:

$$\Lambda^{\mathrm{u,t}}_{c_1,c_2} \leq U_{c_1} \tag{22c}$$
$$\Lambda^{\mathrm{u,t}}_{c_1,c_2} \leq U_{c_2} \tag{22d}$$

(c) Crops are used in cropping plan and exist in overlapping time periods if:

$$\Lambda^{\mathrm{u,t}}_{c_1,c_2} \geq U_{c_1} + U_{c_2} + O^1_{c_1,c_2} + O^2_{c_1,c_2} - 3 \tag{22e}$$

A.5.4 Fit areas of crops in total area of garden

Overlapping constraints between areas of each crop type. No overlap is allowed in space if they are also overlapping in time.

For each crop type $c1 \in [1, \dots, \mathrm{q}_{c_1}^{\max}]$ where and $c2 \in [c1+1, \dots, \mathrm{q}_{c_2}^{\max}]$:

$$x_{c_1}^{\max} + \epsilon^{\longleftrightarrow} \leq (x_{c_2}^{\min} + \mathrm{w}^{\mathrm{G}} \cdot \delta_{c_1,c_2,0}) \tag{16}$$
$$x_{c_2}^{\max} + \epsilon^{\longleftrightarrow} \leq (x_{c_1}^{\min} + \mathrm{w}^{\mathrm{G}} \cdot \delta_{c_1,c_2,1}) \tag{17}$$
$$y_{c_1}^{\max} + \epsilon^{\longleftrightarrow} \leq (y_{c_2}^{\min} + \mathrm{l}^{\mathrm{G}} \cdot \delta_{c_1,c_2,2}) \tag{18}$$
$$y_{c_2}^{\max} + \epsilon^{\longleftrightarrow} \leq (y_{c_1}^{\min} + \mathrm{l}^{\mathrm{G}} \cdot \delta_{c_1,c_2,3}) \tag{19}$$
$$\sum_{k=0}^{3} \delta_{c_1,c_2,k} \leq 4 - \Lambda_{c_1,c_2}^{\mathrm{u,t}} \tag{20}$$

A.6 Linearizations & auxiliary variable constraints

A.6.1 Counting crops by area

Inspired by piecewise linear approximation method in D'Ambrosio, Lodi and Martello [2]. In our case the linearization is not an approximation since the step size used is equal to the size of a plant.

Crop quantity Q_c^{tot} is derived from surface areas. The product of width x and length y (both measured in multiples of crop size w_c) is computed with a piece-wise linear approximation with binary variable $x_{c,i}$ and integer variable y_c:

$$Q_c^{\mathrm{tot}} = \text{plants-wide} \cdot \text{plants-long} = \sum_{i=0}^{\mathrm{M}_c^{\mathrm{x}}-1} 2^i \cdot x_{c,i} \cdot y_c = \sum_{i=0}^{\mathrm{M}_c^{\mathrm{x}}-1} 2^i \cdot z_{c,i} \tag{29}$$

$$\text{Where } \mathrm{M}_c^{\mathrm{x}} = \left\lfloor \log_2 \left(\mathrm{w}^{\mathrm{G}} / \mathrm{w}_c + 1 \right) \right\rfloor + 1$$

Linearization of $x_{c,i} \cdot y_c$ in equation (29) is done by substitution with binary variable $z_{c,i}$ and adding constraints:

$$z_{c,i} \leq y_c^{\mathrm{ub}} \cdot x_{c,i} \tag{29a}$$
$$z_{c,i} \geq y_c^{\mathrm{lb}} \cdot x_{c,i} \tag{29b}$$
$$z_{c,i} \leq y - y_c^{\mathrm{lb}} \cdot (1 - x_{c,i}) \tag{29c}$$
$$z_{c,i} \geq y - y_c^{\mathrm{ub}} \cdot (1 - x_{c,i}) \tag{29d}$$

The value for y_c^{ub} is given by dividing the garden size in the y direction by the size w_c a single plant of crop c.

The number of crops of type c fitting an area in two axis, computable from binary variables $x_{c,i}$ and y_c, is linked to border coordinates $x_c^{\min}$, $x_c^{\max}$, $y_c^{\min}$, and $y_c^{\max}$ by equations (6a) to (6b):

$$y_c^{\max} - y_c^{\min} + \epsilon^{\leftrightarrow} \cdot U_c \geq (\mathrm{l}_c + \epsilon^{\leftrightarrow}) \cdot y_c \tag{6a}$$
$$x_c^{\max} - x_c^{\min} + \epsilon^{\leftrightarrow} \cdot U_c = (\mathrm{w}_c + \epsilon^{\leftrightarrow}) \cdot \sum_{i=0}^{\mathrm{M}_c^{\mathrm{x}}-1} 2^i \cdot x_{c,i} \tag{6b}$$

A.6.2 Crops harvested in periods

$$t_c^{\text{planted}} + \mathrm{t}_c^{\text{grow}} \geq \left(\frac{\mathrm{t}^{\text{total}}}{\mathrm{P}^{\text{num}}} \cdot p \right) \cdot H_{c,p} + 1 \tag{10}$$

$$t_c^{\text{planted}} + \mathrm{t}_c^{\text{grow}} \leq \left(\frac{\mathrm{t}^{\text{total}}}{\mathrm{P}^{\text{num}}} \cdot (p+1) \right) + \left(\mathrm{t}^{\text{total}} - \frac{\mathrm{t}^{\text{total}}}{\mathrm{P}^{\text{num}}} \cdot (p+1) \right) \cdot (1 - H_{c,p}) + 1 \tag{11}$$

$$Q_{c,p}^{\text{nut}} \leq H_{c,p} \cdot \mathrm{q}_c^{\max} \tag{9a}$$

$$Q_{c,p}^{\text{nut}} \leq Q_c^{\text{nut}} \tag{9b}$$

$$Q_{c,p}^{\text{nut}} \geq Q_c^{\text{nut}} - \mathrm{q}_c^{\max} \cdot (1 - H_{c,p}) \tag{9c}$$

$$Q_c^{\text{nut}} \equiv \sum_{p}^{\mathrm{P}^{\text{num}}} Q_{c,p}^{\text{nut}} \tag{8}$$

A.7 Objective functions

Objective function for step 1 (Nutrition optimization):

$$\max \sum_{p=0}^{\mathrm{P}^{\text{num}}} \sum_{n=0}^{|N|} F_{p,n} \tag{12}$$

Objective function for step 1 (Value optimization):

$$\max \sum_{c=0}^{|C|} \left(Q_c^{\text{tot}} - Q_c^{\text{nut}} \right) \cdot \mathrm{V}_c \tag{23}$$

B Model Configuration Files

Input specifications for all testcases used in this thesis listed here. The written out config files are also available (along with model implementation) on Github [21].

B.1 Configuration 1

The configuration of the scenario used in the two-step optimization for the case study in section 5.1.

B.1.1 Crops & Nutrition constants

```
[Mass Units]
kg = 1000
100g = 100
g = 1
mg = 0.001
µg = 0.000001

[Lettuce]
area = 0.225
marketvalue = 0.30
yield = 900 g
daystomaturity = 73
plantearlyest = 1
harvestlatest = 365
carbohydrates = 2.23 g
sugars = 0.94 g
dietaryfiber = 1.1 g
fat = 0.22 g
protein = 1.35 g
a = 166.0 µg
b1 = 0.057 mg
b2 = 0.062 mg
b5 = 0.15 mg
b6 = 0.082 mg
b9 = 73.0 µg
c = 3.7 mg
e = 0.18 mg
k = 102.3 µg
calcium = 35.0 mg
iron = 1.24 mg
magnesium = 13.0 mg
manganese = 0.179 mg
phosphorus = 33.0 mg
sodium = 5.0 mg
potassium = 238 mg
zinc = 0.2 mg

[RedBellPepper]
area = 0.225
marketvalue = 0.30
yield = 1600 g
daystomaturity = 70
plantearlyest = 1
harvestlatest = 365
carbohydrates = 4.64 g
sugars = 2.4 g
dietaryfiber = 1.8 g
fat = 0.13 g
protein = 0.9 g
a = 157 µg
b1 = 0.055 mg
b2 = 0.142 mg
b3 = 1.0 mg
b6 = 0.3 mg
b9 = 47 µg
c = 142.0 mg
e = 1.58 mg
calcium = 6.0 mg
iron = 0.35 mg
magnesium = 11.0 mg
manganese = 0.122 mg
phosphorus = 27.0 mg
potassium = 213.0 mg
sodium = 3.0 mg
zinc = 0.2 mg

[Okra]
area = 0.15
marketvalue = 0.20
yield = 1500 g
daystomaturity = 60
plantearlyest = 1
```

```
harvestlatest = 365
carbohydrates = 7.46 g
sugars = 1.48 g
dietaryfiber = 3.3 g
fat = 0.19 g
protein = 1.9 g
a = 36 μg
b1 = 0.2 mg
b2 = 0.06 mg
b3 = 1.0 mg
b9 = 60 μg
c = 23.0 mg
e = 0 g
k = 31.3 μg
calcium = 82.0 mg
iron = 0.62 mg
magnesium = 57.0 mg
phosphorus = 61.0 mg
potassium = 299 mg
zinc = 0.58 mg

[Potatoes]
area = 0.12
marketvalue = 0.30
yield = 4980 g
daystomaturity = 80
plantearlyest = 1
harvestlatest = 365
carbohydrates = 81.0 g
sugars = 3.7 g
dietaryfiber = 10.5 g
fat = 0.4 g
protein = 9.5 g
a = 961 μg
b1 = 0.11 mg
b2 = 0.11 mg
b3 = 1.5 mg
b6 = 0.29 mg
b9 = 6 μg
c = 19.6 mg
e = 0 g
calcium = 5.0 mg
iron = 0.31 mg
magnesium = 22.0 mg
manganese = 0.14 mg
phosphorus = 44.0 mg
potassium = 379.0 mg
sodium = 4.0 mg
zinc = 0.3 mg

[Tomatoes]
area = 0.225
marketvalue = 0.05
yield = 1300 g
daystomaturity = 80
plantearlyest = 1
harvestlatest = 365
carbohydrates = 3.9 g
sugars = 2.6 g
dietaryfiber = 1.2 g
fat = 0.2 g
protein = 0.9 g
a = 42.0 μg
b1 = 0.03757 mg
b2 = 0.019 mg
b3 = 0.594 mg
b5 = 0.089 mg
b6 = 0.08 mg
b9 = 15.0 μg
c = 14.0 mg
e = 0.54 mg
k = 7.9 μg
calcium = 10.0 mg
iron = 0.27 mg
magnesium = 11.0 mg
manganese = 0.114 mg
phosphorus = 24.0 mg
sodium = 5.0 mg
potassium = 237 mg
zinc = 0.17 mg

[SweetPotatoes]
area = 0.12
marketvalue = 0.05
yield = 2250 g
daystomaturity = 105
plantearlyest = 1
harvestlatest = 365
carbohydrates = 2.7 g
sugars = 6.5 g
dietaryfiber = 3.3 g
fat = 0.15 g
protein = 2.0 g
a = 961 μg
b1 = 0.11 mg
```

```
b2 = 0.11 mg
b3 = 1.5 mg
b6 = 0.29 mg
b9 = 6 µg
c = 19.6 mg
e = 0 g
calcium = 38 mg
iron = 0.69 mg
magnesium = 27 mg
manganese = 0.5 mg
phosphorus = 54 mg
potassium = 475 mg
sodium = 36 mg
zinc = 0.32 mg

[Zucchini]
area = 0.3
marketvalue = 0.10
yield = 800 g
daystomaturity = 48
plantearlyest = 1
harvestlatest = 365
carbohydrates = 2.69 g
sugars = 1.71 g
dietaryfiber = 1 g
fat = 0.36 g
protein = 1.14 g
a = 56 µg
b1 = 0.035 mg
b2 = 0.024 mg
b3 = 0.51 mg
b5 = 0.288 mg
b6 = 0.08 mg
b9 = 28 µg
c = 12.9 mg
e = 0 g
k = 4.2 µg
calcium = 18 mg
iron = 0.37 mg
magnesium = 19 mg
manganese = 0.173 mg
phosphorus = 37 mg
potassium = 264 mg
sodium = 3 mg
zinc = 0.33 mg

[Eggplant]
area = 0.75
marketvalue = 0.10
yield = 2200 g
daystomaturity = 80
plantearlyest = 1
harvestlatest = 365
carbohydrates = 5.88 g
sugars = 3.53 g
dietaryfiber = 3 g
fat = 0.18 g
protein = 0.98 g
a = 0 g
b1 = 0.039 mg
b2 = 0.037 mg
b3 = 0.649 mg
b5 = 0.281 mg
b6 = 0.084 mg
b9 = 0.022 µg
c = 2.2 mg
e = 0.3 mg
k = 3.5 µg
calcium = 9 mg
iron = 0.23 mg
magnesium = 14 mg
manganese = 0.232 mg
phosphorus = 24 mg
potassium = 229 mg
zinc = 0.16 mg

[Drawing]
enabledraw = True
colortype = hex
colours = 0000ff,ffa500,ffd700,ba55d3,00ff7f,ff0000,adff2f,ff00ff,1e90ff,fa8072,dda0dd,87ceeb,ff1493,7fffd4,2e8b57,7f0000,808000,000080
```

B.1.2 Model Input Constants

```
[Solver Parameters]
solver = GRB
valueoptafternutri = True
```

```
valueoptafternutrifailure = True
timeoutvalueafter = 1200
timeoutnutriafter = 1200

[Value Selection]
name = Default
crops = Eggplant,Lettuce,Okra,Potatoes,SweetPotatoes,Tomatoes,Zucchini,RedBellPepper
maxtimescropused = 2
timegranularity = 3
nutrients = Carbohydrates,Protein,Iron,A,B1,C,Zinc,E

[Daily nutrimin]
carbohydrates = 235 g
protein = 46 g
iron = 18 mg
a = 700 µg
b1 = 1.1 mg
c = 45 mg
e = 15 mg
zinc = 8 mg
fat = 15 g

[Resources]
resources = Area
gardenwidth = 4
gardenheight = 5
```

B.2 Configuration 2

B.2.1 Crops & Nutrition constants

All nutritional values and constants remain the same compared to the previous configuration. See values listed under appendix B.1.1.

B.2.2 Model Input Constants

Constants for tests of nutritional step 1 are listed below. The values for *gardenwidth*, *gardenheight* and *timegranularity* vary between test cases as described in section 5.2.1.

```
[Solver Parameters]
solver = GRB
valueoptafternutri = False
valueoptafternutrifailure = False
timeoutvalueafter = 0
timeoutnutriafter = 600

[Value Selection]
name = Default
```

```
crops =
↪  Eggplant,Lettuce,Okra,Potatoes,SweetPotatoes,Tomatoes,Zucchini,RedBellPepper
maxtimescropused = 2
timegranularity = ?
nutrients = Carbohydrates,Protein,Iron,A,B1,C,Zinc,E

[Daily nutrimin]
carbohydrates = 235 g
protein = 46 g
iron = 18 mg
a = 700 µg
b1 = 1.1 mg
c = 45 mg
e = 15 mg
zinc = 8 mg
fat = 15 g

[Resources]
resources = Area
gardenwidth = ?
gardenheight = ?
```

C Output

C.0.1 Case study

Raw model output on crop quantities, growing periods and the nutritional results for the test of the case study section 5.1:


```
{
    "name": "Value_maximization (with
↪   minimised nutri-deficiency)",
    "resultConclusion": {
        "Nutrition_maximization":
↪   "FEASIBLE but Not achieved",
        "Value_maximization":
↪   "undefined",
        "Value_maximization (with
↪   minimised nutri-deficiency)":
↪   "FEASIBLE"
    },
    "total_crops": 542,
    "total_space": "84.51 m2",
    "selected_crop_amounts": {
        "Eggplant": 0.0,
        "Lettuce": 0.0,
        "Okra": 108.0,
        "Potatoes": 143.0,
        "RedBellPepper": 72.0,
        "SweetPotatoes": 0.0,
        "Tomatoes": 0.0,
        "Zucchini": 0.0,
        "Eggplant2": 0.0,
        "Lettuce2": 0.0,
        "Okra2": 9.0,
        "Potatoes2": 130.0,
        "RedBellPepper2": 80.0,
        "SweetPotatoes2": 0.0,
        "Tomatoes2": 0.0,
        "Zucchini2": 0.0
    },
    "crop_crow_time_range": [
        [
            -1,
            -1
        ],
        [
            -1,
            -1
        ],
        [
            81.0,
            141.0
        ],
        [
            1.0,
            81.0
        ],
        [
            141.0,
            211.0
        ],
        [
            -1,
            -1
        ],
        [
            -1,
            -1
        ],
        [
            -1,
            -1
        ],
        [
            -1,
            -1
        ],
        [
            -1,
            -1
        ],
        [
            141.0,
            201.0
        ],
        [
            211.0,
            291.0
        ],
        [
```

```
            291.0,
            361.0
        ],
        [
            -1,
            -1
        ],
        [
            -1,
            -1
        ],
        [
            -1,
            -1
        ]
    ],
    "space_by_crop_type": {
        "Eggplant": "0.0 m2",
        "Lettuce": "0.0 m2",
        "Okra": "16.2 m2",
        "Potatoes": "17.16 m2",
        "RedBellPepper": "16.2 m2",
        "SweetPotatoes": "0.0 m2",
        "Tomatoes": "0.0 m2",
        "Zucchini": "0.0 m2",
        "Eggplant2": "0.0 m2",
        "Lettuce2": "0.0 m2",
        "Okra2": "1.3499999999999999
↪   m2",
        "Potatoes2": "15.6 m2",
        "RedBellPepper2": "18.0 m2",
        "SweetPotatoes2": "0.0 m2",
        "Tomatoes2": "0.0 m2",
        "Zucchini2": "0.0 m2"
    },
    "yield_by_crop_type": {
        "Eggplant": "0.0 g",
        "Lettuce": "0.0 g",
        "Okra": "162000.0 g",
        "Potatoes": "712140.0 g",
        "RedBellPepper": "115200.0 g",
        "SweetPotatoes": "0.0 g",
        "Tomatoes": "0.0 g",
        "Zucchini": "0.0 g",
        "Eggplant2": "0.0 g",
        "Lettuce2": "0.0 g",
        "Okra2": "13500.0 g",
        "Potatoes2": "647400.0 g",
        "RedBellPepper2": "128000.0 g",
        "SweetPotatoes2": "0.0 g",
        "Tomatoes2": "0.0 g",
        "Zucchini2": "0.0 g"
    },
    "value_by_crop_type": {
        "Eggplant": 0.0,
        "Lettuce": 0.0,
        "Okra": 0.0,
        "Potatoes": 1494.0,
        "RedBellPepper": 0.0,
        "SweetPotatoes": 0.0,
        "Tomatoes": 0.0,
        "Zucchini": 0.0,
        "Eggplant2": 0.0,
        "Lettuce2": 0.0,
        "Okra2": 0.0,
        "Potatoes2": 25398.0,
        "RedBellPepper2": 0.0,
        "SweetPotatoes2": 0.0,
        "Tomatoes2": 0.0,
        "Zucchini2": 0.0
    },
    "nutrition_required_per_period": {
        "A": 0.08516666666666667,
        "B1": 0.13383333333333333,
        "C": 5.4750000000000005,
        "Carbohydrates":
↪   28591.666666666668,
        "E": 1.825,
        "Iron": 2.1900000000000004,
        "Protein": 5596.666666666667,
        "Zinc": 0.9733333333333333
    },
    "nutrition_produced_per_period": {
        "0": {
            "A": 6.7958076,
            "B1": 0.777876,
            "C": 138.60336,
            "Carbohydrates": 572799.6,
            "E": 0.0,
            "Iron": 2.192196,
            "Protein": 67180.2,
            "Zinc": 2.1214799999999996
        },
        "1": {
            "A": 0.244044,
```

```
            "B1": 0.41436000000000006,
            "C": 203.94899999999998,
            "Carbohydrates":
↪   18437.579999999998,
            "E": 1.82016,
            "Iron": 1.4912999999999998,
            "Protein": 4371.3,
            "Zinc": 1.2483000000000002
        },
        "2": {
            "A": 5.6088914,
            "B1": 0.6894140000000001,
            "C": 292.05704000000003,
            "Carbohydrates":
↪   461758.60000000003,
            "E": 2.0224,
            "Iron": 2.192494,
            "Protein": 54612.3,
            "Zinc": 1.9442199999999998
        }
    },
    "nutrition_deficit_per_period": {
        "0": {
            "A": -6.710640933333333,
            "B1": -0.6440426666666667,
            "C": -133.12836000000001,
            "Carbohydrates":
↪   -544207.9333333333,
            "E": 1.825,
            "Iron":
↪   -0.0021959999999996427,
            "Protein":
↪   -61583.53333333333,
            "Zinc": -1.1481466666666664
        },
        "1": {
            "A": -0.15887733333333334,
            "B1": -0.2805266666666667,
            "C": -198.474,
            "Carbohydrates":
↪   10154.08666666667,
            "E": 0.0048399999999999554,
            "Iron": 0.6987000000000005,
            "Protein":
↪   1225.3666666666668,
            "Zinc": -0.2749666666666669
        },
        "2": {
            "A": -5.523724733333333,
            "B1": -0.5555806666666667,
            "C": -286.58204,
            "Carbohydrates":
↪   -433166.93333333335,
            "E": -0.19740000000000024,
            "Iron":
↪   -0.002493999999999552,
            "Protein":
↪   -49015.63333333334,
            "Zinc": -0.9708866666666666
        }
    }
}
```

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