Utrecht University

Institute of Theoretical Physics

Master's Thesis

The one-loop correction to gravitons in de Sitter space induced by massive scalars

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Abstract

The status of understanding quantum loop corrections of cosmological perturbations is in its infancy. In this Master's thesis we investigate the graviton self-energy one-loop corrections induced by a massive, non-minimally coupled scalar field during an Inflationary period produced by a de Sitter background. We fully renormalize this result by the addition of four counterterms, two of which, the Ricci scalar square and the Weyl tensor square, agree with the massless renormalization performed by 't Hooft and Veltman on a Minkowski background. This generalizes the work done by Park and Woodard, who performed this procedure for the massless, minimally coupled case also on de Sitter.

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Introduction

One of the biggest questions facing modern Cosmology is the origin of the Large Scale Structure (LSS) Fig. 1, i.e. why do we observe a coherent structure of galaxy clusters at very large scales [1–4], as opposed to a uniform distribution. Such a structure suggests at a mechanism to induce density perturbations in the very early universe, able to survive and influence this structure today. The theory that fits best with the data [5–8] is Inflation, a period of rapid accelerating expansion in the early universe that enabled quantum fluctuations to enlargen and act as a seed to grow into the structure we observe.

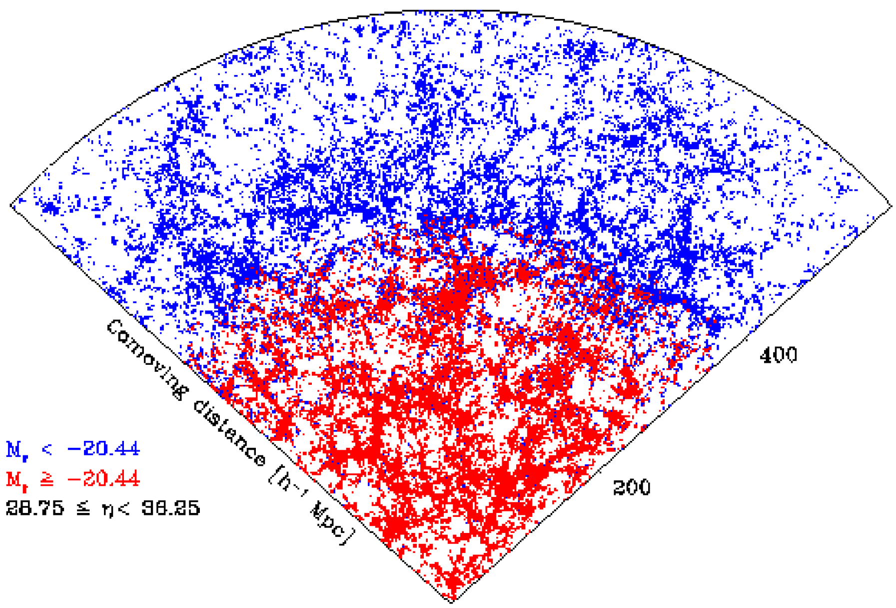

Figure 1: Large Scale Structure of galaxy clusters from the Sloan Digital Sky Survey (SDSS) [4]. The colour indicates relative density of galaxy clusters at different luminosity scales, while the white patches are comparatively empty *void*-space.

The theory of Cosmological Inflation was first introduced by Starobinsky and Guth[9–11] and improved shortly after by Linde, Albrecht and Steinhardt [12, 13]. The theory was originally proposed as a means to explain the Horizon, Flatness and Monopole problems in the observations of Cosmology at the time, but was also used to explain LSS after it was introduced [14–19]. Around the time that this theory was being developed, it was predicted that such an Inflationary period will generate primordial gravitational waves [20]. With the recent development of gravitational wave detectors at LIGO [21] we hope that direct observations of the so-called B-modes of a gravitational wave signal would cement the position of an Inflation Epoch into the early universe [22].

The recent announcements from the International Pulsar Timing Array (PTA) collaborations [23] (made up from: NANOGrav [24, 25], EPTA [26–30] and Parkes Observatory [31]) along with the Chinese PTA announcement [32], provide strong evidence for a Gravitational Wave Background (GWB). The origin of this signal is thought to be made from the mergers of supermassive black holes and possibly at least part of this signal could come from primordial gravitational wave generated during Inflation, evidence for such would cement Inflation into standard Cosmology theories.

In this thesis we are interested primarily on the quantum loop corrections of the graviton propagator that, via Inflation, could grow to a scale to be observable in LSS, in the Cosmic Microwave Background (CMB) or even in the new GWB. We consider a small perturbation of a background metric $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$, $|h_{\mu\nu}| \ll 1$ then the tree-level propagation of these gravitational waves is described by the Linearised Einstein Equation

$$\mathcal{L}^{\mu\nu\rho\sigma} h_{\rho\sigma} = \frac{1}{8\sqrt{\pi G}} T^{\mu\nu}, \tag{0.1}$$

with $\mathcal{L}$ the Lichnerowicz operator, and is the result of a linear expansion of Einstein Field Equation [33–35]. We are interested in the quantum-loop corrections to this equation

$$\mathcal{L}^{\mu\nu\rho\sigma} h_{\rho\sigma} - \int d^D x' \left[{}^{\mu\nu}\Sigma^{\rho\sigma}\right](x;x') h_{\rho\sigma}(x') = 8\sqrt{\pi G} T^{\mu\nu}, \tag{0.2}$$

where $\left[{}^{\mu\nu}\Sigma^{\rho\sigma}\right]$ is the graviton-self energy term bitensor, written in this notation to reinforce the notion that it acts as a tensor for μ, ν indices on the left, at x, and as a separate tensors for ρ, σ on the right, at x'. The ultimate goal for this thesis is to calculate the one-loop contribution to this self-energy, due to a Massive, Non-Minimally Coupled Scalar Field (MvNMCS)

Some of the earliest work looking at quantum loop-corrections to the graviton come from 't Hooft and Veltman's famous 1974 paper [36], where they looked at the one-loop corrections arising from a charged, Massless Minimally Coupled Scalar field (MMCS). They found that by adding two counterterms, corresponding to the Ricci scalar and Ricci tensor squares, they could remove the divergences of the theory at this one-loop order. We will be able to compare their counterterm coefficients to those that we find in Section 4.4. This was further expanded to the two loop level by Goroff and Sagnotti [37, 38].

More recently, work have previously been done by Park and Woodard in [39], where they considered a real MMCS on a de Sitter background. They employ gauge invariance and the implied tranversality of the self-energy to fix the tensor form and then identify the trace of the divergent terms with that of the counterterms. This method is very calculation heavy which were performed primarily on a computer, thus making comparison to our work difficult in certain places.

This was then extended to a non-minimally coupling, but on a Minkowski background by Marunovic and Prokopec [40]. Due to the much simpler terms one finds by taking the Minkowski limit, this work acts as a very useful aid and guideline for the methods we use. We will follow their initiative and perform our renormalization procedure first in Minkowski, and then compute the de Sitter corrections in order to finish the calculations.

Inflation provides a perfect test bed for looking at the effects that primordial particles have on Cosmological perturbations. Glavan, Prokopec and collaborators [41–43] looked at the backreactions coming from various scalar fields during inflation, and their late time effects. They found some evidence for contributions to Dark Energy and Dark Matter from these scalars. Prokopec, Tsamis and Woodard also worked on a stochastic theory of Inflation for Scalar Quantum Electrodynamics (SQED) in [44–46]. Prokopec also worked on non-scalar dynamics, by considering Majorana Fermions [47]. Recenetly, Glavan and Prokopec [48] looked at fluctuations involving a massless non-minamally coupled scalar field. Other work that has been done in this area by Park, Prokopec and Woodard [49], where they looked at the implications these quantum corrections have on the gravitational potentials.

This provides us with ample motivation to further investigate the graviton interactions during Inflation. We will consider a de Sitter space, with a de Sitter invariant scalar propagator, and assume $m^2 - \xi R > 0$ (where we use the positive coupling in the action and so $\xi < 0$, with a conformal coupling at $\xi = -1/6$). The limit as this goes to 0 acts quite differently than to the full MMCS [39], due to the addition of a de Sitter breaking term $\propto \ln(aa')$ in the propagator.

This thesis will be laid out in the following manner: First we will provide a background for Inflationary Cosmology that will be useful as an introduction to the concepts used throughout this thesis. We will then derive the Chernikov-Tagirov [50] propagator using two methods, first via direct application of the Hypergeometric equation, and then separately by splitting the scalar field equation of motion into Fourier modes and solving the resulting Bessel integrals. We then calculate the one-loop self-energy of the graviton in a Minkowski background, and renormalize the the result via dimensional regularization. For this reason, the number of dimensions, D, will be kept general, before we complete this Renormalization procedure. Finally we repeat the calculation for a de Sitter background by looking at the H^2 suppressed corrections to finish our calculation.

1 Inflationary Cosmology

Most modern Cosmological models are built from the *Cosmological Principle* [51]. It states that the Universe at large scales ($\gtrsim$ 100 Mpc) looks the same to all observers, ie. it is Isotropic (rotationally invariant) and Homogeneous (translationally invariant). The universe however is not static, but observations [5, 52] indicate that galaxies further away from us recede from us at higher velocities, $v = H_0 d$, which is known as Hubble's Law, with the proportionality constant, Hubbles Paramater. This was then formulated independently by Friedmann [53, 54] and Lemaître in the 1920s [55, 56] into the theory that the universe is expanding. Together, this cosmology model became the Friedmann-Lemaître-Robertson-Walker (FLRW) metric

$$ds^2 = -dt^2 + a^2(t)\left(dr^2 + S_\kappa^2(r)d\Omega^2\right) \tag{1.1}$$

with $a(t)$ the scale factor characterising the rate of expansion, and $S_\kappa(r)$ is the degree of freedom left in the metric that depends on whether the universe is flat, open or closed (i.e. hyperbolic or spherical)[1]. Observations tells us that the universe appears to be approximately spatially-flat [5–7], such that we can consider $S_\kappa(r) = r$ to recover spherically-flat coordinates.

Upon solving the vacuum Einstein field equations for this metric, one obtains the two Friedmann equations$H = \dot{a}/a$,

$$H^2 = \frac{8\pi G}{3}\rho_i(t) + \frac{\Lambda c^2}{3}, \tag{1.2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left[\rho_i(t) + 3\frac{P}{c^2}\right] + \frac{\Lambda c^2}{3}, \tag{1.3}$$

where we sum over the various density contributions ρ_i, which couple differently with the scale factor. These equations give rise to solutions for the scale factor during periods dominated by one of the energy densities. For example during a radiation dominated period, where $\rho_{\text{rad}} \propto a^{-4}$, the scale factor becomes $a \propto \sqrt{t}$. Currently observations indicate that the universe is dominated by a cosmological constant, [5–7], corresponding to an accelerating expansion (Planck's observed values for the fraction of energy density of cosmological constant $\Omega_{0,\Lambda} = 0.6935 \pm 0.0072$, and matter $\Omega_{0,m} = 0.3065 \pm 0.0072$ most of which is dark matter).

This roughly describes the standard cosmology model of the universe, known as ΛCDM. However there are some *fine-tuning* issues that remain. First, the Horizon problem reflects the fact that the Cosmic Microwave Background (CMB) appears to be nearly perfectly uniform, and yet distant points in space should be causally disconnected. Second is the flatness problem, which arises from the simple fact that the Friedmann equations dictate that a flat universe is an unstable fixed point, and yet we see that our universe is approximately flat instead of being dominated by a curvature 'energy density'. These would require very specific initial conditions, and we would need to consider why the universes took these specific initial conditions as opposed to any others. Finally, the magnetic monopole problem [57]

[1] One should be careful with the word *flat*, as occasionally this means we are considering Minkowski space, as opposed to the shape of the space as it is meant here.

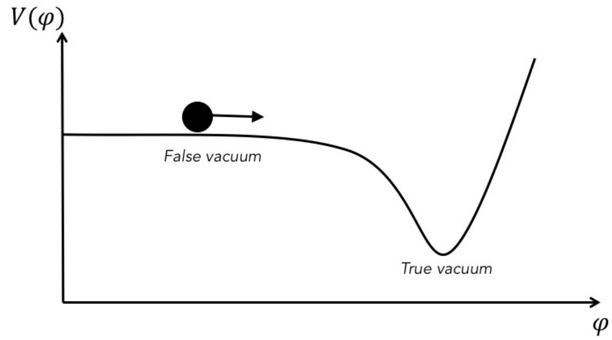

Figure 2: An example of the single field scalar potential for slow-roll inflation, "rolling" down from the initial unstable false vacuum, to the true vacuum, where reheating occurs [59].

is that the predicted production of magnetic monopoles during the early universe should be sufficiently high as to be observable today, despite no such observational evidence.

The theory of Inflation, a period of exponential expansion in the early universe, provides a neat explanation to resolve these issues. This period of rapid expansion allows for points that were once causally connected to move outside of this connectedness, and explain the Horizon problem, and would also explain the flatness, with this expansion driving the universe to be very spatially flat. Rapid expansion also serves to cool the universe, reducing the production of magnetic monopoles [12]. Estimations place the duration of this period to be approximately 60 e-foldings [19] (i.e. the universe scaled a factor of $e^{60} \equiv 10^{26}$ during the inflation, which is posited to have taken $\sim 10^{-36}s$).

In Guth's original description [11], a scalar field (originally the Higgs field was considered but was later abandoned) was allowed to tunnel through a false vacuum to the true vacuum to achieve the required inflation. The Hubble parameter became constant value, with a purely exponential scale factor,

$$a(t) = e^{Ht}. \tag{1.4}$$

This *old*-Inflation was left with some major issues, one being no mechanism for ending the Inflation and thus the classical *Friedman* style expansion could not occur, and no way for the phase transition bubbles walls to uniformly coalesce, which would break isotropy and homogeneity. Linde, and independently afterwards by Albrecht and Steinhardt, quickly resolved these issues with the introduction of a slow-roll parameter $\epsilon = -\dot{H}/H$, which would be very small during the required Inflationary period, allowing the scalar factor to be quasi-exponential, while eventually transitioning the expansion out of this Inflationary epoch, and into the desired *Friedman*-type expansion period. This "slow-roll Inflation" can be achieved by considering a Coleman-Weinberg potential [58] (see Fig. 2 for an example of such a potential, and an illustration for what "slow-roll" means).

This is what is known as Single Field Inflation, as only one scalar field is required to govern the Inflation. There are other such as Multi-field Inflations, but they are typically less favoured for their increased complexity. A good introduction to Inflation can be found in [22, 60].

Before moving forward, we shall briefly recall several components from General Relativity (GR) that will be useful for understanding this thesis. For a proper introduction to GR, see Carrol [61], or a more in depth look can be found in Misner, Thorne and Wheeler [62]. We have already seen the first of our important objects, the metric tensor itself $g_{\mu\nu}$, which comes directly from the line element (such as Eq. (1.1)) with $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$. Second there are objects stemming from the *covariant*-derivative, defined to transform under Lorentz transformations *correctly*, i.e.

$$\nabla_\mu V^\nu = \partial_\mu V_\nu + \Gamma^\nu_{\mu\alpha}V^\alpha, \tag{1.5}$$

where[2] $\Gamma^\alpha_{\mu\nu}$ is the Levi-Civita connection fixed by metric compatibility $\nabla_\alpha g_{\mu\nu} = 0$, and considering a torsion free theory $\Gamma^\alpha_{\mu\nu} = \Gamma^\alpha_{\nu\mu}$, to

$$\Gamma^\alpha_{\mu\nu} = g^{\alpha\beta}\big(\partial_\mu g_{\nu\beta} + \partial_\nu g_{\mu\beta} - \partial_\beta g_{\mu\nu}\big).$$

All of the geometrical information for our space is contained in the Riemann tensor

$$R^\rho{}_{\mu\sigma\nu} = \nabla_\sigma\Gamma^\rho_{\mu\nu} - \nabla_\nu\Gamma^\rho_{\mu\sigma} = \partial_\sigma\Gamma^\rho_{\mu\nu} - \partial_\nu\Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\sigma\lambda}\Gamma^\lambda_{\mu\nu} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma} \tag{1.6}$$

which has a number of important symmetries. Written with a lowered first index we can write $R_{\rho\mu\sigma\nu} \equiv R_{[\rho\mu][\sigma\nu]} \equiv R_{\sigma\nu\rho\mu}$, where the square parentheses indicate antisymmetry about those indices, i.e. exchanging these indices will induce a minus sign (and round brackets indicate symmetry, and exchanging does not change the sign)[3]. Unlike in Minkowski space where derivatives always commute, the presence of the extraneous connection in the covariant derivative imparts the following commutation rule[4]

$$\big[\nabla_\mu, \nabla_\nu\big]V^\alpha = R^\alpha{}_{\lambda\mu\nu}V^\lambda. \tag{1.7}$$

From the Riemann tensor we can extract specific parts of the geometric information, first by way of the taking the trace of the first and third indices, we define the Ricci tensor, which describes the deformation of objections by the curvature of space-time

$$R_{\mu\nu} = R^\rho{}_{\mu\rho\nu} = \partial_\rho\Gamma^\rho_{\mu\nu} - \partial_\nu\Gamma^\rho_{\mu\rho} + \Gamma^\rho_{\rho\lambda}\Gamma^\lambda_{\mu\nu} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\rho}, \tag{1.8}$$

and, taking a second trace to define the Ricci scalar, which characterizes the curvature of the space-time

$$R = R^\mu{}_\mu = R^{\rho\mu}{}_{\rho\mu}.$$

These two objects contain all of the trace-information for the Riemann tensor, but there is also traceless information potentially missing. This is encapsulated in the Weyl Tensor, which by definition is made to be traceless itself

$$C^\rho{}_{\mu\sigma\nu} = R^\rho{}_{\mu\sigma\nu} - \frac{2}{(D-2)}\Big(\delta^\rho_{[\sigma}R_{\nu]\mu} + R^\rho_{[\sigma}g_{\nu]\mu}\Big) + \frac{2}{(D-1)(D-2)}g_{\mu[\nu}g_{\sigma]\rho}R. \tag{1.9}$$

[2]For lowered indices there is a minus on the connection term, covariant derivatives of higher order tensors are defined analogously.

[3]We use the normalization convention $T_{(\alpha\beta)} = \frac{1}{2!}\left(T_{\alpha\beta} + T_{\beta\alpha}\right)$ and $T_{[\alpha\beta]} = \frac{1}{2!}\left(T_{\alpha\beta} - T_{\beta\alpha}\right)$ etc.

[4]For lowered indices there is a minus sign in front of the Riemann term.

This contains the shape deformation information of the Riemann tensor, and can be thought of as a way of expressing the angle preserving nature of the curvature. The Weyl tensor is also defined with the same symmetries as the Riemann tensor. There is also a particularly useful Ricci decomposition of the Riemann tensor square (see Section 1.G. of [63] for details), that allow us to relate the square of these three tensors rather neatly

$$|C_{\rho\mu\sigma\nu}|^2 = |R_{\rho\mu\sigma\nu}|^2 - \frac{4}{(D-2)}|R_{\mu\nu}|^2 + \frac{2}{(D-1)(D-2)}R^2. \tag{1.10}$$

We have seen several objects which we describe as *tensors*, though strictly speaking we consider tensor-fields. We normally think of a tensor field as an space-time dependent object with (m, n) *valence*, meaning it has m-lowered indices, and n-upper indices. A Tensor is defined by the way in which it transforms under a coordinate change

$$T_{\mu_1 \ldots \mu_m}{}^{\nu_1 \ldots \nu_n}(x) = \frac{\partial x^{\mu_1'}}{\partial x^{\mu_1}} \cdots \frac{\partial x^{\mu_m'}}{\partial x^{\mu_m}} \frac{\partial x^{\nu_1}}{\partial x^{\nu_1'}} \cdots \frac{\partial x^{\nu_n}}{\partial x^{\nu_n'}} T_{\mu_1' \ldots \mu_m'}{}^{\nu_1' \ldots \nu_n'}(x). \tag{1.11}$$

We can think of this tensor as matching a matrix-like object to every space-time point, the temperature at every point in a room is an example of a (0,0) Tensor field i.e. a scalar field; while the velocity of particles in a fluid could be described by a (0,1) Tensor field, i.e. a vector. Thus when we define the covariant derivative to transform correctly, we mean such that it transforms as a tensor.

In a similar vein, we will want some way of describing relations between different space-time points, thus, we define a *bitensor* to be an object that depends on two space-time points, and transforms independently as a tensor at each point (potentially with two separate valences). The details behind this concept are aptly summarised by Allen and Jacobson [64]. A simple example for a bitensor that we will be interested in is the distance between each set of pairs of points in a space, which would be a biscalar. One important factor of bitensors that we will need to be careful with, is the preservation of how they transform, we will need to ensure that for example an object that transform as a $(2, 0)$ tensor at x, and as a scalar at x' does not become a scalar at x and a $(2, 0)$ tensor at x' during our calculation, as could happen if we are not careful with certain limits.

There are of course some trivial examples of bitensors, such as the product of two tensors at different points, but for non-trivial bitensors are non-local in nature, meaning they cannot be separated into two distinct local objects. The core bitensor of this thesis is the graviton self energy

$$\left[{}_{\mu\nu}\Sigma_{\rho\sigma}\right](x; x'),$$

written in this notation to emphasise that the μ, ν indices are associated with the point x and ρ, σ with x' (we will keep to this convention throughout this thesis). We will further drop the x, x' dependance when it is clear from the context what the points should be.

We will now explore the specifics of the model we will be working with in this thesis. In particular, we will consider the maximally symmetric de Sitter space for our model of inflation in an FLRW metric, which aligns with Guth's original model [11]. We will then introduce the matter content of our Massive, Non-Minimally Coupled Scalar field.

2 Our Model

We consider a period of Inlfation described by a background de Sitter space, which means we take the de Sitter limit of slow roll inflation, i.e. $\epsilon \to 0$, which simplifies our calculation. For a similar treatment, without this limit, see Janseen et al. [65], where they consider a MMCS with a constant $\epsilon > 1$ (which induces a decelerating expansion). Throughout this thesis, we will keep the dimension D general, so as to allow for dimensional regularization during the renormalization procedure.

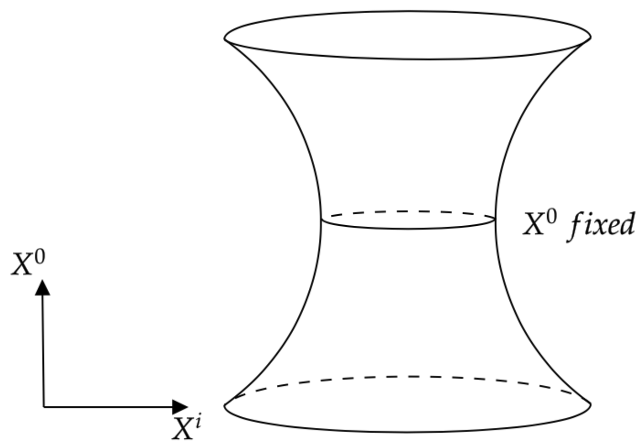

Figure 3: dS_D embedding in $\mathbb{R}^{1,D}$, circles about this hyperboild as indicated are hypersurfaces of fixed time X^0. The central *throat* represents the causal past a singularity between an expanding and contracting universe.

De Sitter space is defined as the unique maximally symmetric (meaning they contain the maximum number of symmetries a space can have: $\frac{1}{2}D(D+1)$) with constant positive curvature [5] $R > 0$, and belongs to the $SO(1, D)$ symmetry group. We can represent our D-dimensional de Sitter space as a hyperboloid embedding of a $D+1$-dimensional Minkowski space, with the constraint equation

$$-(X^0)^2 + \sum_i^D (X^i)^2 = \text{constant}, \tag{2.1}$$

which is shown in Fig. 3. We will consider the planar coordinates (t, x^i), $i, j = 1, ...D-1$ [66], defined by

$$X^0 = \sinh(t) + \frac{H}{2} x_i x^i e^{-t}, \quad X^i = x^i e^{-t}, \quad X^D = \cosh(t) - \frac{H}{2} x_i x^i e^{-t}, \tag{2.2}$$

$$ds^2 = -dt^2 + (e^{Ht})^2 dx_i dx^i, \tag{2.3}$$

[5]the others are Anti-de Sitter, with constant negative curvature, and Minkowski with vanishing curvature

which has an additional constraint $X_0 + x_d = \sinh(t) + \cosh(t) > 0$, thus restricting the coordinates to the upper half of the embedding. This is known as the Poincaré patch of de Sitter, and matches to the flat FLRW metric with a scale factor of $a(t) = e^{Ht}$, and thus also corresponds to a space containing only a Cosmological constant. For some general reviews and literature on de Sitter see [66–71], while for a closer look at de Sitter as it relates to the graviton propagator see the discussions by Miao et al. and Morrison [33–35].

We can also rewrite our metric into conformal time η, via $dt = ad\eta$

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 = a(\eta)^2\big(-d\eta^2 + d\vec{x}^2\big), \tag{2.4}$$

such that the metric becomes a conformally rescaled Minkowski metric $g_{\mu\nu} = a^2\eta_{\mu\nu}$. We can also define a conformal, or coming Hubble parameter $\mathcal{H} = (\frac{da}{d\eta})/a \equiv Ha = -\frac{1}{\eta}$. Throughout this thesis, primarily make use of these conformal coordinates, and write $a \equiv a(\eta)$ and $a' \equiv a(\eta')$ for the scale factor at two conformal times. The curvature terms are given by

$$\Gamma^\rho_{\mu\nu} = H^2 a\big(\delta^0_\mu\delta^\rho_\nu + \delta^0_\nu\delta^\rho_\mu - \eta^{\rho0}\eta_{\mu\nu}\big), \tag{2.5}$$

$$R_{\rho\mu\sigma\nu} = 2H^2 g_{\mu[\nu}g_{\sigma]\rho}, \tag{2.6}$$

$$R_{\mu\nu} = H^2(D-1)g_{\mu\nu}, \tag{2.7}$$

$$R = H^2 D(D-1). \tag{2.8}$$

Solving the vacuum Einstein equation gives the Cosmological constant for this space

$$\Lambda = \frac{1}{2}(D-2)(D-1)H^2 \tag{2.9}$$

We now want to introduce the matter content. We consider a MvNMC scalar field, described by the general action

$$S[\phi, g_{\mu\nu}] = \int d^Dx\sqrt{-g}\Big\{-\frac{1}{2}\partial_\mu\phi\partial_\nu\phi g^{\mu\nu} - V(\phi) + \xi R\phi^2\Big\}. \tag{2.10}$$

As we will be interested in diagrams with no scalar interactions, we can freely set $V(\phi) = \frac{1}{2}m^2\phi^2$. By varying this action with respect to the scalar field via $\frac{\delta\phi(x')}{\delta\phi(x)} := \delta^D(x-x')$ we find the scalar field Equation of Motion

$$\frac{\delta S}{\delta\phi(x)} = -\int d^Dx'\partial'_\nu\Big\{\sqrt{-g(x')}g^{\mu\nu}(x')\partial'_\mu\phi(x')\delta^D(x-x')\Big\}$$
$$+\int d^Dx\sqrt{-g(x')}\Big\{\Box'\phi(x') - \frac{dV}{d\phi} + \xi R\phi(x')\Big\}\delta^D(x-x') \tag{2.11}$$
$$= \sqrt{-g(x)}\Big(\Box\phi(x) - m^2 + \xi R\Big). \tag{2.12}$$

The first term in Eq. (2.11) vanishes on the boundary[6]. Now we demand that the variation of the action vanishes and we find the equation of motion for $\phi(x)$, known as the Klein-Gordon equation

$$\sqrt{-g(x)}\,\big(\Box\phi(x) - m^2 + \xi R\big) = 0. \tag{2.13}$$

[6]We assume that we can set the boundary to be sufficiently separated from x such that the Dirac delta function vanishes

3 The Scalar Propagator

In this section, we derive the Chernikov-Tagirov scalar propagator [50]. We will start by quantizing our scalar field $\phi(x)$, along with the canonical momentum $\Pi(x)$

$$\Pi(x) := \frac{\delta S}{\delta \partial_0 \phi(x)} = \int d^D x' \sqrt{-g(x')} \left\{ -\frac{\delta \partial_\mu \phi(x')}{\delta \partial_0 \phi(x)} \partial_\nu \phi g^{\mu\nu}(x') \right\} \tag{3.1}$$

$$\frac{\delta \partial_\mu \phi(x)}{\delta \partial_0 \phi(x')} = \delta^0_\mu \delta^D(x - x') \tag{3.2}$$

$$\Pi(x) = -\sqrt{-g} g^{0\nu} \partial_\nu \phi(x) \xrightarrow{FLRW} a^{D-2} \partial_\eta \phi(x). \tag{3.3}$$

Now we want to promote $\phi(x)$ and $\Pi(x)$ to operators, $\hat{\phi}(x)$ and $\hat{\Pi}(x)$ by promoting their Poisson bracket to an equal time commutation relation:

$$\left[\hat{\phi}(\eta, \vec{x}), \hat{\Pi}(\eta, \vec{x}')\right] = i\hbar \delta^{D-1}(\vec{x} - \vec{x}'), \tag{3.4}$$

$$\Rightarrow \left[\hat{\phi}(\eta, \vec{x}), -\sqrt{-g} g^{0\nu} \partial_\nu \hat{\phi}(\eta, \vec{x}')\right] = a^{D-2} \left[\hat{\phi}(\eta, \vec{x}), \partial_\eta \hat{\phi}(\eta, \vec{x}')\right] = i\hbar \delta^{D-1}(\vec{x} - \vec{x}'). \tag{3.5}$$

Next we want to compute the scalar propagator for our theory. In doing so we acknowledge that de Sitter space is an out of equilibrium system (due to the nature of expansion). We will employ Wightman functions $\Delta^{(+)}$ and $\Delta^{(-)}$, with the so called in-out formalism, but there are others that are one could choose which give some additional insight. The primary alternative is the in-in formalism, known as the Schwinger-Keldysh (SK) formalism. This makes use of an imaginary time contour with $\Re(t) \in [-\infty, t_c] + [t_c, t_c, -\infty]$ [72–77], and is also particularly useful for solving the linearised Einstein equation.

3.1 Equation of Motion

With both the equation of motion for $\hat{\phi}(x)$, and the commutation relation between $\hat{\phi}(x)$ and $\hat{\Pi}(x)$, we can find a propagator Equation of Motion by starting with the definition of the propagator as the time ordered two point function:

$$i\,\Delta(x; x') := \langle \Omega | T \hat{\phi}(x) \hat{\phi}(x') | \Omega \rangle \tag{3.6}$$

$$= \theta(\Delta x^0) \langle \Omega | \hat{\phi}(x) \hat{\phi}(x') | \Omega \rangle + \theta(-\Delta x^0) \langle \Omega | \hat{\phi}(x') \hat{\phi}(x) | \Omega \rangle \tag{3.7}$$

$$= \theta(\Delta x^0) \Delta^{(+)}(x; x') + \theta(-\Delta x^0) \Delta^{(-)}(x; x'). \tag{3.8}$$

We introduce the Wightman functions $\Delta^{(\pm)}(x; x')$, and recall that $\theta(x)$ is the Heaviside step function, defined with $H(0) = \frac{1}{2}$. We take the ansatz that the equation of motion has the same form as the scalar field itself

$$\sqrt{-g} \left(\Box - M^2 \right) i\,\Delta(x; x') = \hbar\, i\, \delta^D(x - x'). \tag{3.9}$$

where $M^2 = m^2 - \xi R$ is the effective mass. Now let's act this first on the positive frequency Wightman function $\Delta^{(+)}$

$$\Box \left(\theta(\Delta x^0)\, i\,\Delta^{(+)} \right) = \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \left[\theta(\Delta x^0)\, i\,\Delta^{(+)} \right] \right), \tag{3.10}$$

All of the terms where both of the derivatives do not act on $\theta(\pm\Delta x^0)$, will vanish, along with the M^2 term, by employing the Klein-Gordon equation onto the Wightman functions $(\Box - M^2)\Delta^{(\pm)} = 0$. We also note that $\partial_\nu\left(\theta(\pm\Delta x^0)\right) = \pm\delta^0_\nu\delta(\Delta x^0)$, which gives

$$\partial_\nu\left(\theta(\Delta x^0)\right) i\,\Delta^{(+)} + \partial_\nu\left(\theta(-\Delta x^0)\right) i\,\Delta^{(-)} = \delta^0_\nu\delta(\Delta x^0)\left(i\,\Delta^{(+)} - i\,\Delta^{(-)}\right) \tag{3.11}$$

$$= \langle\Omega|[\hat{\phi}(x^0,\vec{x}), \hat{\phi}(x^0,\vec{x}')]|\Omega\rangle = 0. \tag{3.12}$$

Thus, we are only left with the first derivative acting on the $\Delta^{(\pm)}$, and the second acting on the $\theta(\pm\Delta x^0)$ and our final result is

$$\sqrt{-g}\left(\Box - M^2\right) i\,\Delta(x;x') = \delta(\Delta x^0)\langle\Omega|[\sqrt{-g}g^{0\nu}\partial_\nu\hat{\phi}(x^0,\vec{x}), \hat{\phi}(x^0,\vec{x}')]|\Omega\rangle \tag{3.13}$$

$$= \hbar\, i\,\delta^D(x - x'). \tag{3.14}$$

On the second line we made use of the commutation relation between $\hat{\phi}(x)$ and $\hat{\Pi}(x)$, and obtain the correct form for the equation of motion.

We will now show two methods for solving this equation, first by changing our equation into the form of the hypergeometric equation, and second by expanding the equation into Fourier modes.

3.2 Hypergeometric Equation

We start by performing a change of variables for our equation of motion, into $\bar{y}(x;x')$ (related to the geodesic distance $l(x;x')$ of de Sitter space $\bar{y}(x;x') = 4\sin\left(\frac{1}{2}Hl(x;x')\right)$)

$$\bar{y}(x,x') = a(\eta)a(\eta')H^2\Delta x^2 = aa'H^2\left(-\Delta\eta^2 + ||\Delta\vec{x}||^2\right). \tag{3.15}$$

The bar indicates that we have not yet introduced an $i\,\varepsilon$-prescription. We can easily rewrite our propagator equation, and expand into derivatives of $\bar{y}$ (see Appendix A for more details). First we note the expansion of the d'Alembertian operator in the FLRW metric is $\Box\phi = a^{-2}\left(-\partial^2_\eta + \nabla^2 - (D-2)\mathcal{H}\partial_\eta\right)\phi$.

$$a^{D-2}\left\{-\partial^2_\eta + \nabla^2 - aH(D-2)\partial_\eta - a^2M^2\right\} \tag{3.16}$$

$$= a^{D-2}\left\{\eta^{\mu\nu}\left(\frac{\partial^2\bar{y}}{\partial x^\mu\partial x^\nu}\frac{d}{d\bar{y}} + \frac{\partial\bar{y}}{\partial x^\mu}\frac{\partial\bar{y}}{\partial x^\nu}\frac{d^2}{d\bar{y}^2}\right) + aH(D-2)\delta^0_\nu\eta^{\mu\nu}\frac{\partial\bar{y}}{\partial x^\mu}\frac{d}{d\bar{y}} - a^2M^2\right\} \tag{3.17}$$

$$= a^D H^2\left\{\left(4\bar{y} - \bar{y}^2\right)\frac{d^2}{d\bar{y}^2} + D\left(2 - \bar{y}\right)\frac{d}{d\bar{y}} - \frac{M^2}{H^2}\right\}. \tag{3.18}$$

Next we apply another change of coordinates $\bar{y} = 4z$, so that our propagator equation becomes

$$\sqrt{-g}\left(\Box - M^2\right) i\,\Delta(x;x') = a^D H^2\left[z\left(1-z\right)\frac{d^2}{dz^2} + \left(\frac{1}{2}D - Dz\right)\frac{d}{dz} - \frac{M^2}{H^2}\right] i\,\Delta(x;x'). \tag{3.19}$$

We start by trying the naive solution: $\sqrt{-g}\,(\Box - M^2)\,F^{\text{naive}}(z) = 0$, i.e. by ignoring the $\delta^D(x - x')$ of the propagator equation (In order to switch to the full propagator solution, we will have to add an $i\,\varepsilon$-prescription to $y(x;x')$, which will be explained below)

$$\left[z\,(1-z)\,\frac{d^2}{dz^2} + \left(\frac{1}{2}D - Dz\right)\frac{d}{dz} - \frac{M^2}{H^2}\right]F^{\text{naive}}(z) = 0. \tag{3.20}$$

This has the same form as the hypergeometric equation (see section 9.15 and in particular Eq. (9.151) in [78]), which has two linearly independent solutions u_1 and u_2

$$z(1-z)\frac{d^2 f}{dz^2} + (\gamma - (\alpha+\beta+1)z)\frac{df}{dz} - \alpha\beta f = 0. \tag{3.21}$$

$$\Rightarrow \alpha + \beta + 1 = D, \quad \alpha\beta = \frac{M^2}{H^2}, \tag{3.22}$$

$$\beta^2 - (D-1)\beta + \frac{M^2}{H^2} = 0, \tag{3.23}$$

$$\Rightarrow \beta = \frac{D-1}{2} \pm \sqrt{\left(\frac{D-1}{2}\right)^2 - \frac{M^2}{H^2}}, \tag{3.24}$$

$$\nu^2 \coloneqq \left(\frac{D-1}{2}\right)^2 - \frac{M^2}{H^2} = \left(\frac{D-1}{2}\right)^2 - \frac{m^2}{H^2} + \xi D(D-1). \tag{3.25}$$

Note that the plus-minus option gives α and β. In Eq. (9.152.7), they then list that in the special case, as we have, that $\gamma = \frac{1}{2}(\alpha+\beta+1)$, the solutions are of the form

$$u_1 = {}_2F_1\left(\alpha, \beta; \frac{1}{2}(\alpha+\beta+1); z\right), \tag{3.26}$$

$$u_2 = {}_2F_1\left(\alpha, \beta; \frac{1}{2}(\alpha+\beta+1); 1-z\right). \tag{3.27}$$

$$\text{Hypergeometric Equation: } {}_2F_1\left(\alpha, \beta; \gamma; z\right) = \sum_{n=0} \frac{(\alpha)_n(\beta)_n}{(\gamma)_n}\frac{z^n}{n!}, \tag{3.28}$$

$$\text{Pochhammer Symbol: } (\alpha)_n \coloneqq (\alpha)(\alpha+1)\ldots(\alpha+n-1) = \frac{\Gamma(\alpha+n)}{\Gamma(\alpha)}, \quad (\alpha)_0 \coloneqq 1. \tag{3.29}$$

The hypergeometric function in this series representation is convergent for $|z| < 1$, which we analytically continue it with a branch cut between the poles at $z = 1, \infty$. Because of this, we will add an $i\varepsilon$-prescription to our invariant distance quantity $\bar{y} \to y$ in order to change from the naive solution to that of the propagator. We make use of the following prescription

$$y = H^2 a(\eta)a(\eta')\Delta x_F^2(x;x'), \tag{3.30}$$

$$\Delta x_F^2 = -(|\Delta\eta| - i\varepsilon)^2 + ||\Delta\vec{x}||^2. \tag{3.31}$$

We now have our two solutions, which we linearly combine with coefficients $A, B \in \mathbb{C}$, to give the scalar propagator as

$$i\,\Delta(x;x') = \quad A \cdot {}_2F_1\left(\frac{D-1}{2}+\nu, \frac{D-1}{2}-\nu; \frac{D}{2}; \frac{y}{4}\right) + B \cdot {}_2F_1\left(\frac{D-1}{2}+\nu, \frac{D-1}{2}-\nu; \frac{D}{2}; 1-\frac{y}{4}\right). \tag{3.32}$$

We now want to make sure our solution is in Hadamard form, which means that the only singularities involved in our propagator should be at the lightcone $\Delta x_F^2 \to 0$. For us that means $y \to 0$. As the hypergeometric function contains poles at $z = 1$, the singularity obtained in the term of A is at $y = 4$, known as the antipodal singularity on de Sitter, and can be obtained by setting $\eta = -\eta$. Thus, we discard this term, setting $A = 0$. The other term is in the correct form, and we can expand it via Eq. (9.131.2) in [78], to give

$$i\,\Delta(x;x') = B \cdot \left\{ \frac{\Gamma\left(\frac{D}{2}\right)\Gamma(1-\frac{D}{2})}{\Gamma(\frac{1}{2}-\nu)\Gamma(\frac{1}{2}+\nu)} {}_2F_1\left(\frac{D-1}{2}+\nu, \frac{D-1}{2}-\nu; \frac{D}{2}; \frac{y}{4}\right) + \left(\frac{y}{4}\right)^{1-\frac{D}{2}} \frac{\Gamma\left(\frac{D}{2}\right)\Gamma\left(\frac{D}{2}-1\right)}{\Gamma\left(\frac{D-1}{2}+\nu\right)\Gamma\left(\frac{D-1}{2}-\nu\right)} {}_2F_1\left(\frac{1}{2}+\nu, \frac{1}{2}-\nu; 2-\frac{D}{2}; \frac{y}{4}\right) \right\} \tag{3.33}$$

Now again comparing against Eq. (3.28) we see our solution does have the Hadamard form, with the $(\frac{y}{4})^{1-\frac{D}{2}}$. We can now evaluate B, by comparing with the D-dimensional massless propagator for Minkowski Eq. (C.19)

$$B = \hbar \frac{H^{D-2}}{(4\pi)^{D/2}} \frac{\Gamma(\frac{D-1}{2}+\nu)\Gamma(\frac{D-1}{2}+\nu)}{\Gamma(\frac{D}{2})}. \tag{3.34}$$

And all together our equation for the scalar field propagator is

$$i\,\Delta(x;x') = \hbar \frac{H^{D-2}}{(4\pi)^{D/2}} \frac{\Gamma\left(\frac{D-1}{2}+\nu\right)\Gamma\left(\frac{D-1}{2}-\nu\right)}{\Gamma\left(\frac{D}{2}\right)} {}_2F_1\left(\frac{D-1}{2}+\nu, \frac{D-1}{2}-\nu; \frac{D}{2}; 1-\frac{y}{4}\right). \tag{3.35}$$

So far we have worked in D-dimensions, but we will want to take the limit $D \to 4$ after dimensional regularization. Thus we require the propagator equation to be regular in this limit, which we show explicitly in Appendix C.2, and gives the limit

$$\lim_{D\to 4} i\,\Delta(x;x') = \frac{\hbar H^2}{(4\pi)^2} \left\{ \frac{4}{y} + \sum_{n=0} \frac{(\frac{1}{2}+\nu)_n(\frac{1}{2}-\nu)_n}{(2)_n n!} \left(\frac{y}{4}\right)^n \times \left[\ln\left(\frac{y}{4}\right) + \psi\left(\frac{3}{2}+\nu+n\right) + \psi\left(\frac{3}{2}-\nu+n\right) - \psi\left(1+n\right) - \psi\left(2+n\right) \right] \right\}. \tag{3.36}$$

3.3 Fourier Mode Expansion

For this section, we follow very closely with [65], with a slight difference in that for they include a slow-roll parameter $\epsilon > 0$, and use a massless, minimally coupled scalar. However, this slow-roll parameter acts similarly to a mass term in our equation of motion, allowing us to follow in the same steps. Let's now perform a Fourier transform on our field $\hat{\phi}(x)$ into the momentum space field operator $\hat{\phi}(\eta, \vec{k})$

$$\hat{\phi}(\eta, \vec{x}) = \int \frac{d^{D-1}k}{(2\pi)^{D-1}} \left\{ e^{i\vec{k}\cdot\vec{x}} \phi(\eta, \vec{k}) \right\}. \tag{3.37}$$

$$\hat{\phi}^\dagger(\eta, \vec{k}) = \hat{\phi}(\eta, -\vec{k}) \qquad \text{as } \hat{\phi}^\dagger(x) = \hat{\phi}(x), \text{ i.e. } \hat{\phi}(x) \text{ is hermitian}, \tag{3.38}$$

$$[\partial_\eta^2 + (D-2)aH\partial_\eta + k^2 + a^2M^2]\hat{\phi}(\eta, \vec{k}) = 0. \tag{3.39}$$

This is a useful form, but by bringing in a factor of $a^{(D-2)/2}$ into the $\hat{\phi}$, and recall we define $\nu^2 = \left(\frac{D-1}{2}\right)^2 - \frac{M^2}{H^2}$, we can simplify this expression to

$$\left(\partial_\eta^2 + k^2 - \frac{\nu^2 - \frac{1}{4}}{\eta^2}\right)\left(a^{(D-2)/2}\hat{\phi}(\eta, \vec{k})\right) = 0. \tag{3.40}$$

We now want to solve this equation, first we will expand $\hat{\phi}(\eta, \vec{k})$ into creation and annihilation operators for our Fock Space, $\hat{\alpha}^\dagger(\vec{k})$ and $\hat{\alpha}(\vec{k})$

$$\hat{\phi}(\eta, \vec{k}) = \phi(\eta, k)\hat{\alpha}(\vec{k}) + \phi^*(\eta, k)\hat{\alpha}^\dagger(\vec{k}), \tag{3.41}$$

where we used the isotropy of $\phi(\eta, k)$ to remove the dependence on the direction of $\vec{k}$ (which comes from requiring the state to be rotational symmetric). Our creation and annihilation operators are defined in the usual manner, such that we define our vacuum state[7] $|\Omega\rangle$ to be a Gaussian state, and to obey:

$$\hat{\alpha}(\vec{k})\,|\Omega\rangle = 0 \quad (\forall\vec{k}) \tag{3.42}$$

This then reduces our equation of motion for our mode functions $\phi(\eta, k)$ and $\phi^*(\eta, k)$ to

$$\left(\partial_\eta^2 + k^2 - \frac{\nu^2 - \frac{1}{4}}{\eta^2}\right)\left(a^{(D-2)/2}\phi(\eta, k)\right) = 0 \tag{3.43}$$

and analogous for ϕ^*. These equations have solutions known as Hankel's functions, denoted as $H_\nu^{(1/2)}(z)$, which are Bessel functions of the third kind, linear combinations of the first two kinds. We could have expressed this solution in terms of the Bessel functions, but in the UV limit (large $\vec{k}$) this solution reduces to plane waves, while we want a solution in the form of a travelling wave.

[7] A more general Gaussian state would be defined with $[a(\vec{k})\hat{\alpha}(\vec{k}) - b(\vec{k})\alpha(\vec{k})\hat{\alpha}^\dagger(\vec{k})]\,|\Omega\rangle = 0$, with $a(\vec{k}), b(\vec{k}) \in \mathbb{C}$

We can find this solution in section 8.4 and 8.5: Eq. (8.491.5) in [78]:

$$\phi(\eta,k)=\sqrt{\frac{\pi}{4\mathcal{H}}}a^{-(D-2)/2}H_\nu^{(1)}\left(\frac{k}{\mathcal{H}}\right). \tag{3.44}$$

We have $(H_\nu^{(1)}(\frac{k}{\mathcal{H}}))^*=H_\nu^{(2)}(\frac{k}{\mathcal{H}})$ for real $\nu,\frac{k}{\mathcal{H}}$, this is the case if $\left(\frac{D-1}{2}\right)^2>\frac{M^2}{H^2}$, which we will assume for now, and can analytically continue later. Decomposing our propagator into Wightman functions again, we have

$$i\,\Delta(x;x')\coloneqq\theta(\Delta\eta)\,i\,\Delta^{(+)}(x;x')+\theta(-\Delta\eta)\,i\,\Delta^{(-)}(x;x'), \tag{3.45}$$

$$i\,\Delta^{(+)}(x;x')=\frac{\pi}{4}\sqrt{\frac{1}{\mathcal{H}\mathcal{H}'}}[a(\eta)a(\eta')]^{-(D-2)/2}\int\frac{d^{D-1}k}{(2\pi)^{D-1}}e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}H_\nu^{(1)}\left(\frac{k}{\mathcal{H}}\right)H_\nu^{(2)}\left(\frac{k}{\mathcal{H}'}\right), \tag{3.46}$$

$$i\,\Delta^{(-)}(x;x')=\frac{\pi}{4}\sqrt{\frac{1}{\mathcal{H}\mathcal{H}'}}[a(\eta)a(\eta')]^{-(D-2)/2}\int\frac{d^{D-1}k}{(2\pi)^{D-1}}e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}H_\nu^{(1)}\left(\frac{k}{\mathcal{H}'}\right)H_\nu^{(2)}\left(\frac{k}{\mathcal{H}}\right). \tag{3.47}$$

Now we want to solve these integrals. We do this by changing to generalised spherical polar coordinates in $D-1$ spatial dimensions, and then use Eq. (8.411.7) and (6.578.10) from [78] to solve the spherical and radial integrals respectively (with φ is defined as the angle between $\vec{k}$ and $\vec{r}=\vec{x}-\vec{x}'$).

Consider the following general integral:

$$I=\int\frac{d^{D-1}k}{(2\pi)^{D-1}}e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}f(k)=\frac{1}{(2\pi)^{D-1}}\int d\Omega^{D-2}\int_0^\infty dk k^{D-2}e^{ikr\cos(\varphi)}f(k) \tag{3.48}$$

$$=\frac{1}{(2\pi)^{D-1}}\int d\Omega^{D-3}\int_0^\infty dkk^{D-2}f(k)\int_0^\pi d\varphi e^{ikr\cos(\varphi)}\sin^{2(\frac{D-3}{2})}(\varphi) \tag{3.49}$$

$$=\frac{1}{2^{D-2}\pi^{\frac{D-1}{2}}}\int_0^\infty dkk^{D-2}f(k)\frac{J_{\frac{D-3}{2}}(kr)}{(\frac{1}{2}kr)^{\frac{D-3}{2}}}. \tag{3.50}$$

We used (8.411.7) for the φ integral, and the relation given in Appendix C for the remaining angular part. Now we can compare this with Eq. (3.46), and we obtain

$$i\,\Delta^{(+)}(x;x')=\frac{2^{\frac{D-3}{2}}\pi^{3/2}}{(4\pi)^{D/2}}\frac{[a(\eta)a(\eta')]^{-(D-2)/2}}{\sqrt{\mathcal{H}\mathcal{H}'}r^{\frac{D-3}{2}}}\int_0^\infty dkk^{\frac{D-3}{2}+1}J_{\frac{D-3}{2}}(kr)H_\nu^{(1)}\left(\frac{k}{\mathcal{H}}\right)H_\nu^{(2)}\left(\frac{k}{\mathcal{H}'}\right) \tag{3.51}$$

To solve this integral we want to use Eq. (6.578.10) (along with the identities in Eq. (8.407.1/2) and (9.131.1)) in [78], however, this integral is only defined for $\mathrm{Im}(-\Delta\eta)>|\mathrm{Im}(r)|$. Thus, we must add an $i\varepsilon$-prescription: $\eta\to\eta-\frac{i\varepsilon}{2}$ and $\eta'\to\eta'+\frac{i\varepsilon}{2}$, so that $\Delta\eta\to\Delta\eta-i\varepsilon$ and the condition for the integral is satisfied. Then our equation for the Wightman function becomes

$$i\,\Delta^{(+)}(x;x')=\frac{2^{\frac{D+1}{2}}}{\pi(4\pi)^{D/2}}\frac{[a(\eta)a(\eta')]^{-(D-2)/2}}{\sqrt{\mathcal{H}\mathcal{H}'}r^{\frac{D-3}{2}}}\int dkk^{\frac{D-3}{2}+1}J_{\frac{D-3}{2}}(kr)K_\nu\left(-ik\left(\frac{1}{\mathcal{H}}+\frac{i\varepsilon}{2}\right)\right)K_\nu\left(ik\left(\frac{1}{\mathcal{H}'}-\frac{i\varepsilon}{2}\right)\right). \tag{3.52}$$

This integral is in now in correct form to be solved, and we have defined a convenient invariant distance

$$y_{-+} = H^2 a(\eta)(\eta')(\Delta x^{(+)})^2, \tag{3.53}$$

$$(\Delta x^{(+)})^2 = \left(-\left(\Delta\eta - i\varepsilon\right)^2 + ||\Delta\vec{x}||^2\right). \tag{3.54}$$

Eq. (6.578.10) expresses this integral Eq. (3.52) in terms of Associated Legendre functions of the Third Kind $P_\nu^\mu(z)$, which are then expressed as hypergeometric functions in Eq. (8.702). Utilizing these, we find the result for our positive frequency Wightman function to be

$$i\,\Delta^{(+)}(x;x') = \hbar\frac{H^{D-2}}{(4\pi)^{D/2}}\frac{\Gamma\left(\frac{D-1}{2}+\nu\right)\Gamma\left(\frac{D-1}{2}-\nu\right)}{\Gamma\left(\frac{D}{2}\right)} \tag{3.55}$$

$$ {}_2F_1\left(\frac{D-1}{2}+\nu, \frac{D-1}{2}-\nu; \frac{D}{2}; 1-\frac{y_{-+}}{4}\right). \tag{3.56}$$

In performing this integral, we have assumed ν to be real, i.e. $(\frac{D-1}{2})^2 \geq \frac{M^2}{H^2}$. If this is not the case, then we would need to analytically continue our solution.

For the negative frequency Wightman function, we take all of the same steps, except that in the $i\varepsilon$-prescription, we must flip the signs (as the Hankel functions are switched). Thus we have $\eta \to \eta + \frac{i\varepsilon}{2}$ and $\eta' \to \eta' - \frac{i\varepsilon}{2}$, such that we now have: $\Delta\eta^2 \to (\Delta\eta + i\varepsilon)^2$, and we define our invariant distance in a similar manner.

$$y_{+-} = H^2 a(\eta)(\eta')(\Delta x^{(-)})^2 = [y_{-+}]^* i, \tag{3.57}$$

$$(\Delta x^{(-)})^2 = \left(-\left(\Delta\eta + i\varepsilon\right)^2 + ||\Delta\vec{x}||^2\right) = [(\Delta x^{(-)})^2]^*, \tag{3.58}$$

and our result for the negative frequency integral is

$$i\,\Delta^{(-)}(x;x') = \hbar\frac{H^{D-2}}{(4\pi)^{D/2}}\frac{\Gamma\left(\frac{D-1}{2}+\nu\right)\Gamma\left(\frac{D-1}{2}-\nu\right)}{\Gamma\left(\frac{D}{2}\right)}$$

$$ {}_2F_1\left(\frac{D-1}{2}+\nu, \frac{D-1}{2}-\nu; \frac{D}{2}; 1-\frac{y_{+-}}{4}\right), \tag{3.59}$$

$$i\,\Delta^{(+)}(x;x') = [i\,\Delta^{(-)}(x;x')]^*. \tag{3.60}$$

Finally, we can our two Wightman propagator solutions, using Eq. (3.45) and the properties of the $\theta(\pm\Delta\eta)$ functions. We again define an invariant distance, this time for the full propagator

$$y = H^2 a(\eta)a(\eta')\Delta x_F^2, \tag{3.61}$$

$$\Delta x_F^2 = -(|\Delta\eta| - i\varepsilon) + ||\Delta\vec{x}||^2. \tag{3.62}$$

and our scalar field propagator is

$$i\,\Delta(x;x') = \hbar\frac{H^{D-2}}{(4\pi)^{D/2}}\frac{\Gamma\left(\frac{D-1}{2}+\nu\right)\Gamma\left(\frac{D-1}{2}-\nu\right)}{\Gamma\left(\frac{D}{2}\right)}{}_2F_1\left(\frac{D-1}{2}+\nu, \frac{D-1}{2}-\nu; \frac{D}{2}, 1-\frac{y}{4}\right), \tag{3.63}$$

which agrees with the propagator we found for via the hypergeometric equation in Eq. (3.35). We can also rewrite this by separating out the Hadamard pole

$$i\Delta(x;x') = \frac{\hbar H^{D-2}}{(4\pi)^{D/2}}\left(\Gamma\left(\frac{D}{2}-1\right)\left(\frac{y}{4}\right)^{1-\frac{D}{2}} + f_0(y)\right), \tag{3.64}$$

$$f_m(y) = \sum_{n=m}^{\infty}\Bigg[\frac{\Gamma\left(\frac{D-1}{2}\pm\nu\right)\Gamma(1-\frac{D}{2})}{\Gamma(\frac{1}{2}\pm\nu)}\frac{\left(\frac{D-1}{2}\pm\nu\right)_n}{\left(\frac{D}{2}\right)_n n!}\left(\frac{y}{4}\right)^n + \frac{\Gamma\left(\frac{D}{2}-1\right)\left(\frac{1}{4}-\nu^2\right)}{2-\frac{D}{2}}\frac{\left(\frac{3}{2}\pm\nu\right)_n}{n!(n+1)\left(3-\frac{D}{2}\right)_n}\left(\frac{y}{4}\right)^{n+2-\frac{D}{2}}\Bigg]. \tag{3.65}$$

3.4 Minkowski Limit

Before moving on, we can take the Minkowski limit ($H \to 0$) of the scalar propagator

$$i\,\Delta(x;x') = \frac{\hbar m^{D-2}}{(2\pi)^{D/2}}\frac{1}{\left(m\sqrt{\Delta x^2}\right)^{(D-2)/2}}K_{\frac{D-2}{2}}\left(m\sqrt{\Delta x^2}\right), \tag{3.66}$$

$$K_\nu(z) = \frac{\Gamma\left(-\nu\right)}{2^{\nu+1}}\sum_{n=0}^{\infty}\frac{(z/2)^{2n+\nu}}{(1+\nu)_n\, n!} + \frac{\Gamma\left(\nu\right)}{2^{\nu+1}}\sum_{n=0}^{\infty}\frac{(z/2)^{2n-\nu}}{(1-\nu)_n\, n!}, \tag{3.67}$$

$$\frac{1}{z^{\frac{D-2}{2}}}K_{\frac{D-2}{2}}(z) = \frac{\Gamma\left(1-\frac{D}{2}\right)}{2^{D/2}}\sum_{n=0}^{\infty}\frac{(z/2)^{2n}}{\left(\frac{D}{2}\right)_n n!} + \frac{\Gamma\left(\frac{D}{2}-1\right)}{2^{D/2}}\sum_{n=0}^{\infty}\frac{(z/2)^{2n+2-D}}{\left(2-\frac{D}{2}\right)_n n!}, \tag{3.68}$$

where $K_\nu(z)$ is the Bessel function of the second kind. See Eq. (4.2) and (4.3) of [79] for a similar derivation for this result. To make this equation more analogous to our de Sitter version, let's define[8] Minkowski form of $y(x;x') = m^2\Delta x^2$). We can then rewrite the Minkowski propagator equation as

$$i\,\Delta(x;x') = \frac{\hbar m^{D-2}}{(4\pi)^{D/2}}\sum_{n=0}^{\infty}\left\{\frac{\Gamma\left(1-\frac{D}{2}\right)}{\left(\frac{D}{2}\right)_n n!}\left(\frac{y}{4}\right)^n + \frac{\Gamma\left(\frac{D}{2}-1\right)}{\left(2-\frac{D}{2}\right)_n n!}\left(\frac{y}{4}\right)^{n+\frac{2-D}{2}}\right\} \tag{3.69}$$

$$= \frac{\hbar m^{D-2}}{(4\pi)^{D/2}}\left\{\Gamma\left(\frac{D}{2}-1\right)\left(\frac{y}{4}\right)^{1-\frac{D}{2}} + f_0(y)\right\} \tag{3.70}$$

$$f_m(y) = \sum_{n=m}^{\infty}\left\{\frac{\Gamma\left(1-\frac{D}{2}\right)}{\left(\frac{D}{2}\right)_n n!}\left(\frac{y}{4}\right)^n + \frac{\Gamma\left(\frac{D}{2}-1\right)}{\left(2-\frac{D}{2}\right)\left(3-\frac{D}{2}\right)_n (n+1)!}\left(\frac{y}{4}\right)^{n+\frac{4-D}{2}}\right\} \tag{3.71}$$

[8] Do not take this definition to literally, as this is simply to enforce the same dimensionless argument for our propagator.

4 Graviton Self Energy in a Minkowski Background

Now that we have the scalar propagator, we are able to compute the primitive graviton self-energy. Our goal for this section is to find the divergent contributions to the self-energy from the Feynman diagrams in Fig. 4, i.e. the 3-point and 4-point contributions. We then perform dimensional regularization to remove these divergences, and renormalize our theory by adding four counterterms. We will first perform these steps using a Minkowski background so as to fix the counterterms and their coefficients. Then in Section 5, we will repeat these steps on a de Sitter background, where we will be able to focus on the H^2 and higher suppressed terms.

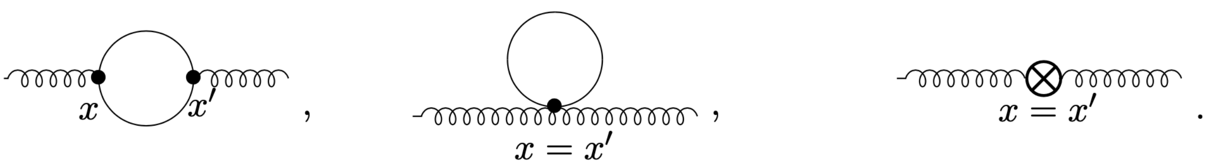

Figure 4: The one-loop Feynman diagrams to the graviton self energy: the 3-point vertex (non-local), the 4-point vertex (local) and the counterterm vertex contributions

Consider a perturbation to the background metric of the form $g_{\mu\nu} \to \eta_{\mu\nu} + \delta g_{\mu\nu}$. By demanding that the Kronecker delta δ^{μ}_{ρ} is invariant, we find the inverse metric perturbation to be related by

$$\delta g_{\mu\nu} = -g_{\mu\rho} g_{\nu\sigma} \delta g^{\rho\sigma}. \tag{4.1}$$

We detail the variations for other curvature terms in Section D. We use these rules to expand our Action around this perturbation, and perform functional variation to find the 3-point and 4-point contributions. The functional derivative with respect to the metric on Minkowski is given by:

$$\frac{\delta g^{\rho\sigma}(x')}{\delta g^{\mu\nu}(x)} = \delta^{\rho}_{(\mu}\delta^{\sigma}_{\nu)}\delta^{D}(x - x'), \tag{4.2}$$

The first two diagrams in Fig. 4 are defined via the functional derivatives of the action, the 3-point contribution is given by the product of first order functional derivatives, each representing one of the vertices with one graviton propagator, while the 4-point is given by the second order functional derivative, representing the vertex with two graviton propagators:

$$\left[{}_{\mu\nu}\Sigma_{\rho\sigma}\right](x;x') = \frac{1}{\sqrt{-g(x)}\sqrt{-g(x')}}\left\langle \mathcal{T}^{*}\left\{ \frac{i}{\hbar^2}\frac{\delta S[\phi, g_{\alpha\beta}]}{\delta g^{\mu\nu}(x)}\frac{\delta S[\phi, g_{\alpha\beta}]}{\delta g^{\rho\sigma}(x')} + \frac{1}{\hbar}\frac{\delta^2 S[\phi, g_{\alpha\beta}]}{\delta g^{\mu\nu}(x)\delta g^{\rho\sigma}(x')}\right\}\right\rangle. \tag{4.3}$$

The $\mathcal{T}^{*}$ indicates a time-ordered product, where every vertex derivative is pulled outside of the time ordering, this will then generate another term which we will be removed by our counterterm diagram. We will now look at the 3-point and 4-point contributions separately.

4.1 4-Point Contribution

The 4-point diagram, also known as the local contribution, is defined by:

$$
\begin{aligned}
\big[{}_{\mu\nu}\Sigma^{\text{4Pt}}_{\rho\sigma}\big](x;x') &= \frac{1}{\hbar}\frac{1}{\sqrt{-g}\sqrt{-g'}}\left\langle \mathcal{T}^* \frac{\delta^2 S_\phi}{\delta g^{\mu\nu}(x)\delta g^{\rho\sigma}(x')}\right\rangle \\
&= \frac{1}{\hbar}\Bigg\{ -\frac{1}{2}\left(\frac{1}{4}\eta_{\mu\nu}\eta_{\rho\sigma} + \frac{1}{2}\eta_{\mu(\rho}\eta_{\sigma)\nu}\right)\eta^{\alpha\beta}\langle \mathcal{T}^*\partial_\alpha\phi(x)\partial_\beta\phi(x)\rangle \\
&\quad + \frac{1}{4}\Big(\eta_{\mu\nu}\langle\mathcal{T}^*\partial'_\rho\phi(x)\partial'_\sigma\phi(x)\rangle + \eta_{\rho\sigma}\langle\mathcal{T}^*\partial_\mu\phi(x)\partial_\nu\phi(x)\rangle\Big) \\
&\quad - \frac{1}{2}m^2\langle\mathcal{T}\phi^2\rangle\left(\frac{1}{4}\eta_{\mu\nu}\eta_{\rho\sigma} + \frac{1}{2}\eta_{\mu(\rho}\eta_{\sigma)\nu}\right) - \frac{1}{2}\xi\langle\mathcal{T}\phi^2\rangle\mathcal{L}_{\mu\nu\rho\sigma}\Bigg\}\delta^D(x-x'), \qquad (4.4)
\end{aligned}
$$

where $\mathcal{L}_{\mu\nu\rho\sigma}$ is the Lichnerowicz operator, and is related to to the second order variation of the Ricci scalar up to a minus sign,

$$
\mathcal{L}_{\mu\nu\rho\sigma} = \eta_{\mu\nu}\eta_{\rho\sigma}\partial^2 - \frac{1}{2}\left(\eta_{\mu\nu}\partial'_\rho\partial'_\sigma + \eta_{\rho\sigma}\partial_\mu\partial_\nu\right) - \frac{1}{2}\left(\eta_{\mu)(\rho}\eta_{\sigma)(\nu}\partial^2 + 2\eta_{\mu)(\rho}\partial'_{\sigma)}\partial_{(\nu}\right). \qquad (4.5)
$$

To evaluate the correlators in Eq. (4.4), we recall that in dimensional regularization D-dependent powers of Δx^2 vanish in the coincidence limit, such that we find

$$
\begin{aligned}
\langle\mathcal{T}\phi^2\rangle &= \lim_{x'\to x} i\,\Delta(x;x') = \frac{\hbar m^{D-2}}{(4\pi)^{D/2}}\Gamma\left(1-\frac{D}{2}\right) \qquad (4.6)\\
&= \frac{\hbar m^{D-4}}{(4\pi)^{D/2}}\frac{2m^2}{(D-4)} - \frac{\hbar m^2}{(4\pi)^2}\left(\psi(1)+1\right) \qquad (4.7)\\
&\equiv \frac{\hbar m^{D-4}\Gamma\left(\frac{D}{2}-1\right)}{(4\pi)^{D/2}}\frac{2m^2}{(D-4)} - \frac{\hbar m^2}{(4\pi)^2}\left(2\psi(1)+1\right), \qquad (4.8)\\
\langle\mathcal{T}^*\partial_\alpha\phi(x)\partial_\beta\phi(x)\rangle &= \lim_{x'\to x}\partial_\alpha\partial'_\beta\, i\,\Delta(x;x') = \frac{\hbar m^{D-2}}{(4\pi)^{D/2}}\Gamma\left(-\frac{D}{2}\right)\frac{1}{2}m^2\eta_{\alpha\beta} \qquad (4.9)\\
&= -\frac{\hbar m^D}{(4\pi)^{D/2}}\frac{1}{2}\frac{\eta_{\alpha\beta}}{(D-4)} - \frac{\hbar m^2}{(4\pi)^2}\frac{1}{4}\left(2\psi(1)+3\right)\eta_{\alpha\beta} \qquad (4.10)\\
&\equiv -\frac{\hbar m^{D-4}\Gamma\left(\frac{D}{2}-1\right)}{(4\pi)^{D/2}}\frac{m^4}{2}\frac{1}{(D-4)}\eta_{\alpha\beta} - \frac{\hbar m^2}{(4\pi)^2}\frac{1}{4}\left(3\psi(1)+3\right)\eta_{\alpha\beta} \qquad (4.11)
\end{aligned}
$$

The divergent terms are those proportional to $\delta^D(x-x')/(D-4)$. As the main purpose for this Minkowski calculation is to prepare us for the de Sitter case, we primarily focus on the divergent terms here. Together they give the 4-point contribution:

$$
\begin{aligned}
\big[{}_{\mu\nu}\Sigma^{\text{4Pt}}_{\rho\sigma}\big](x;x') &= \frac{m^{D-4}\Gamma\left(\frac{D}{2}-1\right)}{(4\pi)^{D/2}}\frac{1}{(D-4)}\left\{-\frac{1}{4}m^4\eta_{\mu\nu}\eta_{\rho\sigma} - \xi m^2\mathcal{L}_{\mu\nu\rho\sigma}\right\} \\
&\quad + \textit{finite-terms}. \qquad (4.12)
\end{aligned}
$$

4.2 3-Point Contribution

The 3-point diagram is defined by

$$
\begin{aligned}
\big[{}_{\mu\nu}\Sigma^{\rm 3Pt}_{\rho\sigma}\big](x;x') &= \frac{1}{\hbar^2}\frac{1}{\sqrt{-g}\sqrt{-g'}}\left\langle \mathcal{T}^*\, i\,\frac{\delta S_\phi}{\delta g^{\mu\nu}(x)}\frac{\delta S_\phi}{\delta g^{\rho\sigma}(x')}\right\rangle^{\text{divergent-terms}} \\
&= \frac{i}{4\hbar^2}\Bigg\{\left(\frac{1}{2}\eta_{\mu\nu}\eta^{\alpha\beta}-\delta^\alpha_{(\mu}\delta^\beta_{\nu)}\right)\left(\frac{1}{2}\eta_{\rho\sigma}\eta^{\gamma\delta}-\delta^\gamma_{(\rho}\delta^\delta_{\sigma)}\right)\langle\mathcal{T}^*\partial_\alpha\phi(x)\partial_\beta\phi(x)\partial'_\gamma\phi(x')\partial'_\delta\phi(x')\rangle \\
&\qquad + \frac{1}{2}m^2\left[\eta_{\mu\nu}\left(\frac{1}{2}\eta_{\rho\sigma}\eta^{\gamma\delta}-\delta^\gamma_{(\rho}\delta^\delta_{\sigma)}\right)\langle\mathcal{T}^*\partial'_\gamma\phi(x')\partial'_\delta\phi(x')(\phi(x))^2\rangle+\dots\right] \\
&\qquad + \frac{1}{4}m^4\langle\mathcal{T}(\phi(x))^2(\phi(x'))^2\rangle \\
&\qquad - \xi\Bigg[\left(\frac{1}{2}\eta_{\mu\nu}\eta^{\alpha\beta}-\delta^\alpha_{(\mu}\delta^\beta_{\nu)}\right)D'_{\rho\sigma}\langle\mathcal{T}^*\partial_\alpha\phi(x)\partial_\beta\phi(x)(\phi(x'))^2\rangle+\dots \\
&\qquad\qquad + \frac{1}{2}m^2\Big(\eta_{\mu\nu}D'_{\rho\sigma}+\eta_{\rho\sigma}D_{\mu\nu}\Big)\langle\mathcal{T}(\phi(x))^2(\phi(x'))^2\rangle\Bigg] \\
&\qquad + \xi^2 D_{\mu\nu}D'_{\rho\sigma}\langle\mathcal{T}(\phi(x))^2(\phi(x'))^2\rangle\Bigg\},
\end{aligned}
\tag{4.13}
$$

where $D_{\mu\nu}=\partial_\mu\partial_\nu-g_{\mu\nu}\partial^2$ comes from the first variation of the Ricci scalar up to a minus sign. We Wick contract these four point correlators[9],

$$
\langle\mathcal{T}(\phi(x))^2(\phi(x'))^2\rangle = 2\left(i\,\Delta(x;x')\right)^2, \tag{4.14}
$$

$$
\langle\mathcal{T}^*\partial_\alpha\phi(x)\partial_\beta\phi(x)(\phi(x'))^2\rangle = 2\partial_{(\alpha}\, i\,\Delta(x;x')\partial_{\beta)}\, i\,\Delta(x;x'), \tag{4.15}
$$

$$
\langle\mathcal{T}^*\partial_\alpha\phi(x)\partial_\beta\phi(x)\partial'_\gamma\phi(x')\partial'_\delta\phi(x')\rangle = 2\partial_{\alpha)}\partial'_{(\gamma}\, i\,\Delta(x;x')\partial'_{\delta)}\partial_{(\beta}\, i\,\Delta(x;x'), \tag{4.16}
$$

and plugging these into Eq. (4.13), we can express the 3-point contribution in terms of the scalar propagator

$$
\begin{aligned}
\big[{}_{\mu\nu}\Sigma^{\rm 3Pt}_{\rho\sigma}\big](x;x') &= \frac{i}{4\hbar}\Bigg\{2\partial_{\mu)}\partial'_{(\rho}\, i\,\Delta(x;x')\partial'_{\sigma)}\partial_{(\nu}\, i\,\Delta(x;x') \\
&- \left(\eta_{\mu\nu}\partial^\alpha\partial'_{(\rho}\, i\,\Delta(x;x')\partial'_{\sigma)}\partial_\alpha\, i\,\Delta(x;x')+\dots\right)+\frac{1}{2}\eta_{\mu\nu}\eta_{\rho\sigma}\partial^\alpha\partial'^\gamma\, i\,\Delta(x;x')\partial'_\gamma\partial_\alpha\, i\,\Delta(x;x') \\
&+ m^2\left[\eta_{\mu\nu}\eta_{\rho\sigma}\partial^\alpha\, i\,\Delta(x;x')\partial_\alpha\, i\,\Delta(x;x')-\big(\eta_{\mu\nu}\partial'_\rho\, i\,\Delta(x;x')\partial'_\sigma\, i\,\Delta(x;x')+\dots\big)\right] \\
&+ \frac{1}{2}m^4\eta_{\mu\nu}\eta_{\rho\sigma}\left(i\,\Delta(x;x')\right)^2 \\
&- \xi\Bigg[\big(\eta_{\mu\nu}D'_{\rho\sigma}+\eta_{\rho\sigma}D_{\mu\nu}\big)\partial^\alpha\, i\,\Delta(x;x')\partial_\alpha\, i\,\Delta(x;x')-2\big(D'_{\rho\sigma}\partial_\mu\, i\,\Delta(x;x')\partial_\nu\, i\,\Delta(x;x')+\dots\big) \\
&\qquad + m^2\Big(\eta_{\mu\nu}D'_{\rho\sigma}+\eta_{\rho\sigma}D_{\mu\nu}\Big)\left(i\,\Delta(x;x')\right)^2\Bigg] \\
&+ 2\xi^2 D_{\mu\nu}D'_{\rho\sigma}\left(i\,\Delta(x;x')\right)^2\Bigg\},
\end{aligned}
\tag{4.17}
$$

[9]Wick contraction across the same position vanishes.

where the $\ldots$ represents a symmetric term with $(\mu,\nu,x)\leftrightarrow(\rho,\sigma,x')$. Now we need identities for the product of derivatives of the propagators. We will detail briefly the process for extracting derivatives here.

The terms we are interested in contain various powers of $y=m\Delta x^2$, which are divergent if they are non-integrable in D=4 dimensions. This is the case for y^{-2} type terms. We first extract derivatives via

$$\left(\frac{y}{4}\right)^{-\alpha}=\frac{2}{m^2(\alpha-1)(2\alpha-D)}\partial^2\left(\frac{y}{4}\right)^{1-\alpha}, \tag{4.18}$$

until all divergent terms are in the form of y^{2-D}, at which point we extract one more d'Alembert and pick up a $(D-4)$ pole,

$$\left(\frac{y}{4}\right)^{2-D}=\frac{2}{m^2(D-3)(D-4)}\partial^2\left(\frac{y}{4}\right)^{3-D}. \tag{4.19}$$

We can then add the massless propagator equation of motion in order to isolate the poles onto Dirac deltas,

$$\frac{\partial^2}{m^2}\left(\frac{y}{4}\right)^{3-D}=\frac{\partial^2}{m^2}\left[\left(\frac{y}{4}\right)^{3-D}-\left(\frac{y}{4}\right)^{1-\frac{D}{2}}\right]+\frac{(4\pi)^{D/2}}{m^D\Gamma\left(\frac{D}{2}-1\right)}\,i\,\delta^D(x-x')\Bigg\}. \tag{4.20}$$

This procedure is given in more detail in Appendix B.1. The final propagator identities that we are interested in, up to the finite terms, are

$$\begin{aligned}\Big(i\,\Delta(x;x')\Big)^2 &= \frac{\hbar m^{2D-4}\Gamma^2\left(\frac{D}{2}-1\right)}{16\pi^D}\frac{\partial^2}{2(D-3)(D-4)}\frac{(4\pi)^{D/2}}{m^D\Gamma\left(\frac{D}{2}-1\right)}\,i\,\delta^D(x-x')\\ &\quad+\textit{finite-terms},\end{aligned} \tag{4.21}$$

$$\begin{aligned}\partial_\mu\, i\,\Delta(x;x')\partial_\nu\, i\,\Delta(x;x')_{\text{div}} &= \frac{\hbar m^{2D-4}\Gamma^2\left(\frac{D}{2}-1\right)}{16\pi^D}\frac{1}{4(D-3)(D-4)}\\ &\quad\times\left\{\frac{1}{2}\frac{(D-2)}{(D-1)}\partial_\mu\partial_\nu+\frac{1}{2}\frac{1}{D-1}\eta_{\mu\nu}\partial^2-m^2\eta_{\mu\nu}\right\}\frac{(4\pi)^{D/2}}{m^D\Gamma\left(\frac{D}{2}-1\right)}\,i\,\delta^D(x-x')\\ &\quad+\textit{finite-terms},\end{aligned} \tag{4.22}$$

$$\begin{aligned}\partial_\mu\partial'_\rho\, i\,\Delta(x;x')\partial_\nu\partial'_\sigma\, i\,\Delta(x;x')_{\text{div}} &= \frac{\hbar m^{2D-4}\Gamma^2\left(\frac{D}{2}-1\right)}{16\pi^D}\frac{1}{4(D-3)(D-4)}\\ &\times\Bigg\{\frac{1}{8}\frac{D(D-2)}{(D+1)(D-1)}\partial_\mu\partial_\nu\partial'_\rho\partial'_\sigma+\frac{1}{8}\frac{D}{(D+1)(D-1)}\left(\eta_{\mu\nu}\partial'_\rho\partial'_\sigma+\eta'_{\rho\sigma}\partial_\mu\partial_\nu\right)\partial^2\\ &+\frac{1}{2}\frac{1}{(D+1)(D-1)}\eta_{\mu)(\rho}\partial'_{\sigma)}\partial_{(\nu}\partial^2+\frac{1}{2}\frac{1}{(D+1)(D-1)}\left(\frac{1}{4}\eta_{\mu\nu}\eta'_{\rho\sigma}+\frac{1}{2}\eta_{\mu\rho}\eta_{\nu\sigma}\right)\partial^4\\ &+m^2\Bigg[-\frac{1}{4}\frac{(D-2)}{(D-1)}\left(\eta_{\mu\nu}\partial'_\rho\partial'_\sigma+\eta'_{\rho\sigma}\partial_\mu\partial_\nu\right)-\frac{1}{D-1}\left(\frac{1}{4}\eta_{\mu\nu}\eta'_{\rho\sigma}+\frac{1}{2}\eta_{\mu\rho}\eta_{\nu\sigma}\right)\partial^2\\ &\qquad-\frac{1}{(D-1)}\eta_{\mu)(\rho}\partial'_{\sigma)}\partial_{(\nu}\Bigg]+m^4\left(\frac{1}{4}\eta_{\mu\nu}\eta'_{\rho\sigma}+\frac{1}{2}\eta_{\mu\rho}\eta_{\nu\sigma}\right)\Bigg\}\frac{(4\pi)^{D/2}}{m^D\Gamma\left(\frac{D}{2}-1\right)}\,i\,\delta^D(x-x')\\ &+\textit{finite-terms}.\end{aligned} \tag{4.23}$$

Now we are ready to expand these terms in our 3-Point contribution Eq. (4.17). This Equation has a number nice symmetries that one can employ to speed up the gathering of divergent terms. For example any term $\propto g_{\mu\nu}$ in $\eta^{\rho\sigma}\partial_\mu\partial'_\rho i\Delta(x;x')\partial'_\sigma\partial_\nu i\Delta(x;x')$ will contribute finitely to the self energy, due to the coefficients in front of it and the second trace in Eq. (4.17) coming together as $\propto (D-4)$.

4.3 Divergent terms in Vertex Function

Together, the divergent terms for the 4-Point and 3-Point diagrams are given by

$$\left[{}_{\mu\nu}\Sigma^{\text{4Pt}}_{\rho\sigma}\right](x;x') = \frac{m^{D-4}\Gamma\left(\frac{D}{2}-1\right)}{(4\pi)^{D/2}}\frac{1}{D-4}\left\{-\frac{1}{4}m^4\eta_{\mu\nu}\eta_{\rho\sigma} - m^2\xi\mathcal{L}_{\mu\nu\rho\sigma}\right\}\delta^D(x-x') \tag{4.24}$$

$$\begin{aligned}\left[{}_{\mu\nu}\Sigma^{\text{3Pt}}_{\rho\sigma}\right](x;x') = \frac{i}{4}\Bigg\{& 2\partial_{\mu)}\partial'_{(\rho}\, i\,\Delta(x;x')\partial'_{\sigma)}\partial_{(\nu}\, i\,\Delta(x;x') - \Bigg(\eta_{\mu\nu}\partial^{\alpha)}\partial'_{(\rho}\, i\,\Delta(x;x')\partial'_{\sigma)}\partial_{(\alpha}\, i\,\Delta(x;x') + \ldots\Bigg) \\ &+ \frac{1}{2}\eta_{\mu\nu}\eta_{\rho\sigma}\partial^\alpha\partial'^\gamma\, i\,\Delta(x;x')\partial'_\gamma\partial_\alpha\, i\,\Delta(x;x') + \frac{1}{2}m^4\eta_{\mu\nu}\eta_{\rho\sigma}\left(i\,\Delta(x;x')\right)^2 \\ &+ m^2\Bigg[\eta_{\mu\nu}\eta_{\rho\sigma}\partial^\alpha\, i\,\Delta(x;x')\partial_\alpha\, i\,\Delta(x;x') - \left(\eta_{\mu\nu}\partial'_\rho\, i\,\Delta(x;x')\partial'_\sigma\, i\,\Delta(x;x') + \ldots\right)\Bigg] \\ &- \xi\Bigg[\left(\eta_{\mu\nu}D'_{\rho\sigma} + \eta_{\rho\sigma}D_{\mu\nu}\right)\partial^\alpha\, i\,\Delta(x;x')\partial_\alpha\, i\,\Delta(x;x') - 2\left(D'_{\rho\sigma}\partial_\mu\, i\,\Delta(x;x')\partial_\nu\, i\,\Delta(x;x') + \ldots\right) \\ &\qquad + m^2\Bigg(\eta_{\mu\nu}D'_{\rho\sigma} + \eta_{\rho\sigma}D_{\mu\nu}\Bigg)\left(i\,\Delta(x;x')\right)^2\Bigg] \\ &+ 2\xi^2 D_{\mu\nu}D'_{\rho\sigma}\left(i\,\Delta(x;x')\right)^2\Bigg\},\end{aligned} \tag{4.25}$$

Now we evaluate this out fully, where again we only consider those terms $\propto 1/(D-4)$, as the others will contribute finitely. We first must expand the $m^{D-4} = \mu^{D-4}(1+(D-4)\ln(m^2/\mu^2)/2)$, introducing a mass scale μ, then we can express these divergences as

$$\begin{aligned}\left[{}_{\mu\nu}\Sigma^{\text{div}}_{\rho\sigma}\right](x;x') = \frac{\mu^{D-4}\Gamma\left(\frac{D}{2}-1\right)}{16\pi^{D/2}}\Bigg\{& \\ &\frac{-1}{8(D+1)(D-1)(D-3)(D-4)}\Bigg[\frac{(D-2)}{(D-1)}\partial_\mu\partial_\nu\partial'_\rho\partial'_\sigma + \frac{1}{(D-1)}\Bigg(\eta_{\mu\nu}\partial'_\rho\partial'_\sigma + \eta_{\rho\sigma}\partial_\mu\partial_\nu\Bigg)\partial^2 \\ &\qquad - \frac{1}{(D-1)}\eta_{\mu\nu}\eta_{\rho\sigma}\partial^4 + \Bigg(\eta_{\mu)(\rho}\eta_{\sigma)(\nu}\partial^4 + 2\eta_{\mu)(\rho}\partial'_{\sigma)}\partial_{(\nu}\partial^2\Bigg)\Bigg] \\ &+ \frac{1}{4(D-3)(D-4)}\left(\frac{1}{2}\frac{(D-2)}{(D-1)} + 2\xi\right)^2 D_{\mu\nu}D'_{\rho\sigma} \\ &- \frac{m^2}{2(D-3)(D-4)}\left(\frac{1}{2}\frac{(D-2)}{(D-1)} + 2\xi\right)\mathcal{L}_{\mu\nu\rho\sigma} \\ &+ \frac{m^4}{2(D-3)(D-4)}\left(\frac{1}{4}\eta_{\mu\nu}\eta_{\rho\sigma} + \frac{1}{2}\eta_{\mu)(\rho}\eta_{\sigma)(\nu}\right)\Bigg\}\delta^D(x-x'),\end{aligned} \tag{4.26}$$

4.4 Renormalizing the Minkowski Case

As we do not have any external scalar propagators, the terms already in our action cannot contribute to the renormalization procedure. To renormalize our theory at one-loop order, we add the following counterterms

$$S_{\text{ct}}[g_{\alpha\beta}] = \int d^D y\sqrt{-g}\Bigg\{c_1|C_{\gamma\alpha\delta\beta}|^2 + c_2 R^2 + c_3 m^2 R + c_4 m^4\Bigg\}, \tag{4.27}$$

with c_i arbitrary coefficients that we wish to fix. As all of the curvature terms vanish on Minkowski, we only need the first variation of these terms.

$$\begin{aligned}\frac{\delta^2}{\delta g^{\mu\nu}(x)\delta g^{\rho\sigma}(x')}\int d^D y\sqrt{-g}\Bigg\{c_1|C_{\gamma\alpha\delta\beta}|^2 + c_2R^2 + c_3m^2R + c_4m^4\Bigg\} = \sqrt{-g}\sqrt{-g'}\Bigg\{\\ c_1\frac{2(D-3)}{(D-2)}\Bigg[\frac{(D-2)}{(D-1)}\partial_\mu\partial_\nu\partial'_\rho\partial'_\sigma + \frac{1}{(D-1)}\left(g_{\rho\sigma}\partial_\mu\partial_\nu\partial^2 + g_{\mu\nu}\partial'_\rho\partial'_\sigma\partial^2\right)\\ - \frac{1}{(D-1)}g_{\mu\nu}g_{\rho\sigma}\partial^4 + \Big(g_{\mu\rho}g_{\nu\sigma}\partial^4 + 2g_{\mu\rho}\partial_\nu\partial'_\sigma\partial^2\Big)\Bigg]\\ + 2c_2\Bigg[\partial_\mu\partial_\nu\partial'_\rho\partial'_\sigma - \Big(g_{\mu\nu}\partial'_\rho\partial'_\sigma\partial^2 + g'_{\rho\sigma}\partial_\mu\partial_\nu\partial^2\Big) + g_{\mu\nu}g'_{\rho\sigma}\partial^4\Bigg]\\ + c_3m^2\Bigg[-\frac{1}{2}g_{\mu\nu}g'_{\rho\sigma}\partial^2 + \frac{1}{2}\Big(g_{\mu\nu}\partial'_\rho\partial'_\sigma + g'_{\rho\sigma}\partial_\mu\partial_\nu\Big) + \frac{1}{2}\Big(g_{\mu\rho}g_{\nu\sigma}\partial^2 + 2g_{\mu\rho}\partial'_\sigma\partial_\nu\Big)\Bigg]\\ + c_4m^4\left[\frac{1}{4}g_{\mu\nu}g_{\rho\sigma} + \frac{1}{2}g_{\mu\rho}g_{\nu\sigma}\right]\Bigg\}\frac{\delta^D(x-x')}{\sqrt{-g}}\end{aligned} \tag{4.28}$$

We can now compare this against Eq. (4.26) to find the counterterm coefficients:

$$c_1 = \frac{\mu^{D-4}\Gamma\left(\frac{D}{2}-1\right)}{16\pi^{D/2}}\frac{-(D-2)}{16(D+1)(D-1)(D-3)^2(D-4)} \sim -\frac{1}{\varepsilon}\frac{1}{120} \tag{4.29}$$

$$c_2 = \frac{\mu^{D-4}\Gamma\left(\frac{D}{2}-1\right)}{16\pi^{D/2}}\frac{1}{8(D-3)(D-4)}\left(\frac{1}{2}\frac{(D-2)}{(D-1)} + 2\xi\right)^2 \sim \frac{1}{\varepsilon}\frac{1}{72} \tag{4.30}$$

$$c_3 = \frac{\mu^{D-4}\Gamma\left(\frac{D}{2}-1\right)}{16\pi^{D/2}}\frac{1}{2(D-3)(D-4)}\left(\frac{1}{2}\frac{(D-2)}{(D-1)} + 2\xi\right) \sim \frac{1}{\varepsilon}\frac{1}{6} \tag{4.31}$$

$$c_4 = \frac{\mu^{D-4}\Gamma\left(\frac{D}{2}-1\right)}{16\pi^{D/2}}\frac{1}{2(D-3)(D-4)} \sim \frac{1}{\varepsilon}\frac{1}{2}. \tag{4.32}$$

After the $\sim$, we have dropped the common $\mu^{D-4}\Gamma(\frac{D}{2}-1)/(16\pi^{D/2})$ and expressed $D = 4+\varepsilon$ in order to compare against 't Hooft and Veltman's result in [36]. To do so, we must convert their Ricci tensor square counterterm to our Weyl square. Consider the Gauss-Bonnet term, this provides a variation proportional to $D-4$, so its contribution can be set to 0:

$$G^2 = |R_{\mu\rho\nu\sigma}|^2 - 4|R_{\mu\nu}|^2 + R^2 \sim 0 \tag{4.33}$$

$$\Rightarrow |R_{\mu\rho\nu\sigma}|^2 \sim 4|R_{\mu\nu}|^2 - R^2 \tag{4.34}$$

$$|C_{\mu\rho\nu\sigma}|^2 = |R_{\mu\rho\nu\sigma}|^2 - \frac{4}{(D-2)}|R_{\mu\nu}|^2 + \frac{2}{(D-1)(D-2)}R^2 \tag{4.35}$$

$$= \frac{4(D-3)}{(D-2)}|R_{\mu\nu}|^2 - \frac{D(D-3)}{(D-1)(D-2)}R^2 \tag{4.36}$$

$$\xrightarrow{D=4} = 2\left(|R_{\mu\nu}|^2 - \frac{1}{3}R^2\right) \tag{4.37}$$

Finally, 't Hooft and Veltman have in Eq. (3.34) of [36]

$$\Delta\mathcal{L} = \frac{\sqrt{g}}{\varepsilon}\left(\frac{1}{72}R^2 + \frac{1}{60}\left(|R_{\mu\nu}|^2 - \frac{1}{3}R^2\right)\right) \to \frac{\sqrt{g}}{\varepsilon}\left(\frac{1}{72}R^2 + \frac{1}{120}C^2\right) \tag{4.38}$$

which match our counterterms coefficients above. Note that we can interpret this equivalence by noting that we most include all counterterms possible. There are 5 independent structures, the Riemann square, the Ricci tensor square, the Ricci scalar square, Ricci m^2 and m^4. The Gauss-Bonnet term removes one of these, and we can convert another to the Weyl square simply due to the easier calculation involved (Weyl square vanishes on de Sitter).

We are now ready to move onto the de Sitter case, and check if our terms can be renormalized there.

5 Graviton Self-Energy in a de Sitter Background

Now that we have correctly renormalised the Minkowski self-energy, we want to generalize the result to a de Sitter background. The expression for the graviton self energy at one loop is given in the same form as for Minkowski

$$\left[{}_{\mu\nu}\Sigma_{\rho\sigma}\right](x;x') = \frac{1}{\sqrt{-g(x)}\sqrt{-g(x')}}\left\langle \mathcal{T}^* \left\{ i\,\frac{\delta S^{(3)}}{\delta g^{\mu\nu}(x)}\frac{\delta S^{(3)}}{\delta g^{\rho\sigma}(x')} + \frac{\delta^2 S^{(4)}}{\delta g^{\mu\nu}(x)\delta g^{\rho\sigma}(x')}\right\}\right\rangle. \tag{5.1}$$

The $\mathcal{T}^*$ indicates that we pull derivatives outside of the time ordering again.

Before we compute these terms, we need to know how to take functional derivatives with respect to the metric in de Sitter space. Unlike in Minkowski, taking the naive Kronecker deltas in the definition results in an ambiguity in the ordering of covariant derivatives. Instead we apply

$$\frac{\delta g^{\alpha\beta}(y)}{\delta g^{\mu\nu}(x)} = \left[{}_{(\mu}g^{\alpha}\right](x;y)\left[{}_{\nu)}g^{\beta}\right](x;y)\delta^D(x-y) \tag{5.2}$$

where we introduce this bitensor term $\left[{}_{\mu}g^{\alpha}\right](x;y)$. This is defined such that in the coincidence limit, it reduces to the ordinary metric, and in the Minkowski limit to the Minkowski metric:

$$\left[{}_{\mu}g_{\rho}\right](x;x') = -\frac{1}{2H^2}\nabla_\mu\nabla'_\rho y, \tag{5.3}$$

where the prime indicates a derivative at x'. We detail some of the primary identities involving this bitensor in Appendix A.1, and additional information an be found in literature [64, 80, 81]. As it will be clear in context, we drop the position argument. Further throughout this section, we assume that μ,ν are symmetrised, as are ρ,σ, as this can be difficult to represent with bitensors.

Analogously to Minkowski, we define two operators which come from the first and second variation of he Ricci scalar,

$$\mathcal{P}_{\mu\nu} = \nabla_\mu\nabla_\mu - g_{\mu\nu}\Big(\Box + H^2(D-1)\Big), \tag{5.4}$$

$$\begin{aligned}\mathcal{P}_{\mu\nu\rho\sigma} = {}& \frac{1}{2}g_{\mu\nu}g'_{\rho\sigma}\Box - \frac{1}{2}\Big(g_{\mu\nu}\nabla'_\rho\nabla'_\sigma + g'_{\rho\sigma}\nabla_\mu\nabla_\nu\Big) - \frac{1}{2}\Big(\left[{}_{\mu}g_{\rho}\right]\left[{}_{\nu}g_{\sigma}\right]\Box + 2\left[{}_{\mu)}g_{(\rho}\right]\nabla'_{\sigma)}\nabla_{(\nu}\Big) \\ & - \frac{(D-4)(D-1)}{4}H^2 g_{\mu\nu}g_{\rho\sigma} - \frac{(D-2)(D-1)}{2}H^2 g_{\mu\rho}g_{\nu\sigma}.\end{aligned} \tag{5.5}$$

The $\mathcal{P}_{\mu\nu\rho\sigma}$ is similar to the Lichnerowicz operator: the derivative terms match, while the H^2 terms do not. If we shift the Ricci scalar by $R\to R-2\Lambda$, then the second order variation matches the Lichnerowicz operator, as is done in [39] (see Section D for more details on the variation of curvature terms).

Now we proceed as in the Minkowski case by considering the 4-point and 3-point contributions separately. Most of the of the analysis is the same, with the addition of H^2, H^4 corrections that appear. As such we have three remaining independent terms to renormalize: H^2-double derivative, H^4 and H^2m^2 terms.

5.1 4-Point Contribution

$$\begin{aligned}
\left[{}_{\mu\nu}\Sigma^{\text{4Pt}}_{\rho\sigma}\right](x;x') &= \left\langle \mathcal{T}^* \frac{\delta^2 S_\phi}{\delta g^{\mu\nu}(x)\delta g^{\rho\sigma}(x')} \right\rangle^{\text{divergent-terms}} \\
&= \frac{1}{\hbar}\Bigg\{ -\frac{1}{2}\left(\frac{1}{4}g_{\mu\nu}g_{\rho\sigma} + \frac{1}{2}g_{\mu(\rho}g_{\sigma)\nu}\right)\eta^{\alpha\beta} \lim_{x'\to x} \partial_\alpha\partial'_\beta\, i\,\Delta(x;x') \\
&+ \frac{1}{4}\left(g_{\mu\nu} \lim_{x'\to x} \partial_\rho\partial'_\sigma\, i\,\Delta(x;x') + g_{\rho\sigma} \lim_{x'\to x} \partial_\mu\partial'_\nu\, i\,\Delta(x;x')\right) \\
&- \frac{1}{2}m^2 \lim_{x'\to x} i\,\Delta(x;x')\left(\frac{1}{4}g_{\mu\nu}g_{\rho\sigma} + \frac{1}{2}g_{\mu(\rho}g_{\sigma)\nu}\right) - \frac{1}{2}\xi \lim_{x'\to x} i\,\Delta(x;x')\mathcal{P}_{\mu\nu\rho\sigma}\Bigg\}\frac{\delta^D(x-x')}{\sqrt{-g}}
\end{aligned} \tag{5.6}$$

As in Minkowski we use the vanishing of D-dependant powers of y in dimensional regularization in the coincident limit, to find

$$\begin{aligned}
\lim_{x'\to x} i\,\Delta(x;x') &= \frac{\hbar H^{D-2}}{(4\pi)^{D/2}} \frac{\Gamma\left(\frac{D-1}{2} \pm \nu\right)\Gamma(1-\frac{D}{2})}{\Gamma(\frac{1}{2}\pm\nu)} \\
&= \frac{\hbar H^{D-2}}{(4\pi)^{D/2}}\left\{ \frac{2\Gamma\left(\frac{D}{2}-1\right)}{(D-3)(D-4)}\left(\frac{1}{4}-\nu^2\right) + \left(\frac{1}{4}-\nu^2\right)\left(1 - 2\psi(1) + 2\psi\left(\frac{1}{2}\pm\nu\right)\right) + 1 \right\}.
\end{aligned} \tag{5.7}$$

$$\begin{aligned}
\lim_{x'\to x} \partial_\alpha\partial'_\beta\, i\,\Delta(x;x') &= \frac{\hbar H^{D-2}}{(4\pi)^{D/2}} \frac{\Gamma\left(\frac{D-1}{2}\pm\nu\right)\Gamma(1-\frac{D}{2})}{\Gamma(\frac{1}{2}\pm\nu)} \frac{\left(\frac{D-1}{2}\pm\nu\right)}{\left(\frac{D}{2}\right)} \frac{-1}{2} H^2 g_{\alpha\beta} \\
&= \frac{\hbar H^D}{(4\pi)^{D/2}} \frac{\Gamma\left(\frac{D}{2}-1\right)}{(D-3)(D-4)}\left(\frac{1}{4}-\nu^2\right)\left(\frac{9}{4}-\nu^2\right)\frac{1}{2}g_{\alpha\beta} \\
&+ \frac{\hbar H^4}{(4\pi)^2}\left\{\left(\frac{1}{4}-\nu^2\right)\left(\frac{9}{4}-\nu^2\right)\left[-\frac{1}{2}\psi(\frac{1}{2}\pm\nu) - \frac{1}{2}\psi(1)\right] + \frac{3}{2}\left(\frac{1}{4}-\nu^2\right) + \frac{1}{2}\left(\frac{9}{4}-\nu^2\right)\right\}.
\end{aligned} \tag{5.8}$$

All together the 4-point contribution gives

$$\begin{aligned}
\left[{}_{\mu\nu}\Sigma^{\text{4Pt}}_{\rho\sigma}\right](x;x') &= \frac{H^{D-4}\Gamma\left(\frac{D}{2}-1\right)}{(4\pi)^{D/2}} \frac{1}{(D-3)(D-4)}\left(\frac{1}{4}-\nu^2\right) \\
&\times \Bigg\{ H^2\xi\Big[-\frac{1}{2}g_{\mu\nu}g'_{\rho\sigma}\Box + \frac{1}{2}\left(g_{\mu\nu}\nabla'_\rho\nabla'_\sigma + g'_{\rho\sigma}\nabla_\mu\nabla_\nu\right) + \frac{1}{2}\left(\left[{}_\mu g_\nu\right]\left[{}_\nu g_\sigma\right]\Box + 2\left[{}_\mu g_\nu\right]\nabla'_\sigma\nabla_\nu\right) \\
&- (D-1)H^2\left[{}_\mu g_\nu\right]\left[{}_\nu g_\sigma\right]\Big] - \frac{1}{4}m^2H^2 g_{\mu\nu}g_{\rho\sigma}\Bigg\}\delta^D(x-x') + \textit{finite-terms}
\end{aligned} \tag{5.9}$$

5.2 3-Point Contribution

Repeating the steps from Minkowski, with the same Wick contractions, the 3-point contribution can be expressed in terms of the scalar propagator

$$
\begin{aligned}
\left[{}_{\mu\nu}\Sigma^{\text{3Pt}}_{\rho\sigma}\right](x;x') = \frac{i}{4\hbar^2}\Bigg\{ & 2\partial_{\mu)}\partial'_{(\rho}\, i\,\Delta(x;x')\partial'_{\sigma)}\partial_{(\nu}\, i\,\Delta(x;x') \\
& - \Bigg(g_{\mu\nu}\partial^\alpha\partial'_{(\rho}\, i\,\Delta(x;x')\partial'_{\sigma)}\partial_\alpha\, i\,\Delta(x;x') + \ldots \Bigg) + \frac{1}{2} g_{\mu\nu}g_{\rho\sigma}\partial^\alpha\partial'^\gamma\, i\,\Delta(x;x')\partial'_\gamma\partial_\alpha\, i\,\Delta(x;x') \\
& + (m^2 - \xi R)\Big[g_{\mu\nu}g_{\rho\sigma}\partial^\alpha\, i\,\Delta(x;x')\partial_\alpha\, i\,\Delta(x;x') - \big(g_{\mu\nu}\partial'_\rho\, i\,\Delta(x;x')\partial'_\sigma\, i\,\Delta(x;x') + \ldots \big) \Big] \\
& + \frac{1}{2}(m^2 - \xi R)^2 g_{\mu\nu}g_{\rho\sigma}\left(i\,\Delta(x;x')\right)^2 \\
& - \xi\Big[\big(g_{\mu\nu}\mathcal{P}'_{\rho\sigma} + g_{\rho\sigma}\mathcal{P}_{\mu\nu} \big)\partial^\alpha\, i\,\Delta(x;x')\partial_\alpha\, i\,\Delta(x;x') - 2\big(\mathcal{P}'_{\rho\sigma}\partial_\mu\, i\,\Delta(x;x')\partial_\nu\, i\,\Delta(x;x') + \ldots \big) \\
& \qquad + (m^2 - \xi R)\Big(g_{\mu\nu}\mathcal{P}'_{\rho\sigma} + g_{\rho\sigma}\mathcal{P}_{\mu\nu} \Big)\left(i\,\Delta(x;x')\right)^2 \Big] \\
& + 2\xi^2 \mathcal{P}_{\mu\nu}\mathcal{P}'_{\rho\sigma}\left(i\,\Delta(x;x')\right)^2 \Bigg\}.
\end{aligned}
\tag{5.10}
$$

Now we wish to evaluate these propagator terms, which is outlined in Appendix C, and remains the exact same as in Minkowski. The resulting propagator identities, again only focusing on the divergent terms here, are

$$
\Big(i\,\Delta(x;x') \Big)^2 = \frac{\hbar^2 H^{D-4}\Gamma\left(\frac{D}{2}-1\right)}{(4\pi)^{D/2}} \frac{2}{(D-3)(D-4)} \frac{\delta^D(x-x')}{\sqrt{-g}} + \textit{finite-terms} \tag{5.11}
$$

$$
\begin{aligned}
\partial_\mu\, i\,\Delta(x;x')\partial_\nu\, i\,\Delta(x;x') = & \frac{\hbar^2 H^{D-4}\Gamma\left(\frac{D}{2}-1\right)}{(4\pi)^{D/2}} \frac{1}{(D-3)(D-4)} \\
\times & \left\{ \frac{1}{2}\frac{(D-2)}{(D-1)}\nabla_\mu\nabla_\nu + \frac{1}{2}\frac{1}{(D-1)} g_{\mu\nu}\Box - \frac{1}{2}(D-2)H^2 g_{\mu\nu} - \left(\frac{1}{4} - \nu^2\right) H^2 g_{\mu\nu} \right\} \frac{\delta^D(x-x')}{\sqrt{-g}} \\
& + \textit{finite-terms}
\end{aligned}
\tag{5.12}
$$

We express the much longer $\partial_\mu\partial'_\rho\, i\,\Delta(x;x')\partial_\nu\partial'_\sigma\, i\,\Delta(x;x')$ term in Eq. (C.35). At this point we are ready to renormalize these terms, which must match the counterterms we found in Minkowski. As in that case, these steps are rather involved, though many terms can be neglected from the divergences after they pick up $(D-4)$ factors. Further, in Minkowski we found that all of the non-minimal coupling dependance came in the came in a so-called conformal-form, such that we can rewrite the following

$$
\left(\frac{1}{4} - \nu^2\right) = m^2 - \frac{1}{2}D(D-1)V, \tag{5.13}
$$

where $V \equiv \frac{1}{2}\frac{(D-2)}{(D-1)} + 2\xi$ is the conformal prefactor.

5.3 Renormalizing the de Sitter Case

Gathering together the 4-point and 3-point contributions, we can write the divergent terms as the following (where as in Minkowski, we replace $H^{D-4} = \mu^{D-4}(1+(D-4)\ln(H^2/\mu^2)/2)$)

$$
\begin{aligned}
\left[{}_{\mu\nu}\Sigma^{\rm div}_{\rho\sigma}\right](x;x') = \frac{\mu^{D-4}\left(1+\frac{1}{2}\ln(H^2/\mu^2)\right)\Gamma\left(\frac{D}{2}-1\right)}{16\pi^{D/2}}\Bigg\{ \\
&\frac{-1}{8(D+1)(D-1)(D-3)(D-4)}\Bigg[\frac{(D-2)}{(D-1)}\nabla_\mu\nabla_\nu\nabla'_\rho\nabla'_\sigma + \frac{1}{(D-1)}\Big(g_{\mu\nu}\nabla'_\rho\nabla'_\sigma\Box + g'_{\rho\sigma}\nabla_\mu\nabla_\nu\Box'\Big) \\
&\quad - \frac{1}{(D-1)}g_{\mu\nu}g'_{\rho\sigma}\Box\Box' + \Big(\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\Box\Box' + 2\left[{}_\mu g_\rho\right]\nabla'_\sigma\nabla_\nu\Box'\Big) \\
&\quad - 2H^2\Big(g_{\mu\nu}\nabla'_\rho\nabla'_\sigma + g'_{\rho\sigma}\nabla_\mu\nabla_\nu\Big) - H^2 g_{\mu\nu}g'_{\rho\sigma}\Box \\
&\quad - H^2(D-2)\Big(\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\Box + 2\left[{}_\mu g_\rho\right]\nabla'_\sigma\nabla_\nu\Big) + 2H^4(D-1)g_{\mu\nu}g'_{\rho\sigma}\Bigg] \\
&+ \frac{1}{4(D-3)(D-4)}\left(\frac{1}{2}\frac{(D-2)}{(D-1)}+2\xi\right)^2\Bigg[-2\nabla_\mu\nabla_\nu\nabla'_\rho\nabla'_\sigma + 2\Big(g_{\mu\nu}\nabla'_\rho\nabla'_\sigma\Box + g_{\rho\sigma}\nabla_\mu\nabla_\nu\Box'\Big) \\
&\quad - 2g_{\mu\nu}g_{\rho\sigma}\Box\Box' - H^2(D-2)(D-1)\Big(g_{\mu\nu}\nabla'_\rho\nabla'_\sigma + g'_{\rho\sigma}\nabla_\mu\nabla_\nu\Big) \\
&\quad + H^2(D-4)(D-1)g_{\mu\nu}g_{\rho\sigma}\Box - H^2D(D-1)\Big(\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\Box + 2\left[{}_\mu g_\rho\right]\nabla'_\sigma\nabla_\nu\Big) \\
&\quad - H^4\Big(\frac{1}{4}D^2(D-1) - 2(D-1)^3\Big)g_{\mu\nu}g'_{\rho\sigma} \\
&\quad - H^4\Big(\frac{1}{2}D^2(D-1) + 2D(D-1)^2\Big)\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\Bigg] \\
&- \frac{m^2}{2(D-3)(D-4)}\left(\frac{1}{2}\frac{(D-2)}{(D-1)}+2\xi\right)\mathcal{P}_{\mu\nu\rho\sigma} \\
&+ \frac{m^4}{2(D-3)(D-4)}\left(\frac{1}{4}g_{\mu\nu}g_{\rho\sigma} + \frac{1}{2}\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\right)\Bigg\}\delta^D(x-x'). \qquad (5.14)
\end{aligned}
$$

In Appendix D we derive the full variation of the counterterms and we use the same coefficient as in Minkowski

$$c_1 = \frac{\mu^{D-4}\Gamma\left(\frac{D}{2}-1\right)}{16\pi^{D/2}}\frac{-(D-2)}{16(D+1)(D-1)(D-3)^2(D-4)}, \qquad (5.15)$$

$$c_2 = \frac{\mu^{D-4}\Gamma\left(\frac{D}{2}-1\right)}{16\pi^{D/2}}\frac{1}{8(D-3)(D-4)}\left(\frac{1}{2}\frac{(D-2)}{(D-1)}+2\xi\right)^2, \qquad (5.16)$$

$$c_3 = \frac{\mu^{D-4}\Gamma\left(\frac{D}{2}-1\right)}{16\pi^{D/2}}\frac{1}{2(D-3)(D-4)}\left(\frac{1}{2}\frac{(D-2)}{(D-1)}+2\xi\right), \qquad (5.17)$$

$$c_4 = \frac{\mu^{D-4}\Gamma\left(\frac{D}{2}-1\right)}{16\pi^{D/2}}\frac{1}{2(D-3)(D-4)}. \qquad (5.18)$$

Thus we find that the counterterm contribution is given by:

$$
\begin{aligned}
&\frac{1}{\sqrt{-g}\sqrt{-g'}}\frac{\delta^2}{\delta g^{\mu\nu}(x)\delta g^{\rho\sigma}(x')}\int d^Dy\sqrt{-g}\Big\{c_1|C_{\gamma\alpha\delta\beta}|^2+c_2R^2+c_3m^2R+c_4m^4\Big\}\\
&=\frac{\mu^{D-4}\Gamma\left(\frac{D}{2}-1\right)}{16\pi^{D/2}}\Bigg\{\\
&\frac{1}{8(D+1)(D-1)(D-3)(D-4)}\Bigg[\frac{(D-2)}{(D-1)}\nabla_\mu\nabla_\nu\nabla'_\rho\nabla'_\sigma+\frac{1}{(D-1)}\Big(g_{\mu\nu}\nabla'_\rho\nabla'_\sigma\Box+g'_{\rho\sigma}\nabla_\mu\nabla_\nu\Box'\Big)\\
&\qquad-\frac{1}{(D-1)}g_{\mu\nu}g'_{\rho\sigma}\Box\Box'+\Big(\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\Box\Box'+2\left[{}_\mu g_\rho\right]\nabla'_\sigma\nabla_\nu\Box'\Big)\\
&\qquad-2H^2\Big(g_{\mu\nu}\nabla'_\rho\nabla'_\sigma+g'_{\rho\sigma}\nabla_\mu\nabla_\nu\Big)-H^2g_{\mu\nu}g'_{\rho\sigma}\Box\\
&\qquad-H^2(D-2)\Big(\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\Box+2\left[{}_\mu g_\rho\right]\nabla'_\sigma\nabla_\nu\Big)+2H^4(D-1)g_{\mu\nu}g'_{\rho\sigma}\Bigg]\\
&-\frac{1}{4(D-3)(D-4)}\left(\frac{1}{2}\frac{(D-2)}{(D-1)}+2\xi\right)^2\Bigg[-2\nabla_\mu\nabla_\nu\nabla'_\rho\nabla'_\sigma+2\Big(g_{\mu\nu}\nabla'_\rho\nabla'_\sigma\Box+g_{\rho\sigma}\nabla_\mu\nabla_\nu\Box'\Big)\\
&\qquad-2g_{\mu\nu}g_{\rho\sigma}\Box\Box'-H^2(D-2)(D-1)\Big(g_{\mu\nu}\nabla'_\rho\nabla'_\sigma+g'_{\rho\sigma}\nabla_\mu\nabla_\nu\Big)\\
&\qquad+H^2(D-4)(D-1)g_{\mu\nu}g_{\rho\sigma}\Box-H^2D(D-1)\Big(\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\Box+2\left[{}_\mu g_\rho\right]\nabla'_\sigma\nabla_\nu\Big)\\
&\qquad-H^4\Big(\frac{1}{4}D^2(D-1)-2(D-1)^3\Big)g_{\mu\nu}g'_{\rho\sigma}\\
&\qquad-H^4\Big(\frac{1}{2}D^2(D-1)+2D(D-1)^2\Big)\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\Bigg]\\
&+\frac{m^2}{2(D-3)(D-4)}\left(\frac{1}{2}\frac{(D-2)}{(D-1)}+2\xi\right)\mathcal{P}_{\mu\nu\rho\sigma}\\
&-\frac{m^4}{2(D-3)(D-4)}\left(\frac{1}{4}g_{\mu\nu}g_{\rho\sigma}+\frac{1}{2}\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\right)\Bigg\}\delta^D(x-x'),
\end{aligned}
\tag{5.19}
$$

which exactly removes all of our divergent terms.

Finally we want to obtain the renormalized one-loop self-energy. First, there are the terms $\propto\ln(H^2/\mu^2)$ in our divergent term. Then there are the non-local finite terms that come directly from the $i\,\Delta(x;x')$ identities. Finally there are the local terms (i.e. $\propto\delta^4(x-x')$) and come from terms that are expanded around $D=4$ (which thus give $(D-4)/(D-4)=1$ contributions to the finite self-energy). Note that as all of the divergences are extracted, we can evaluate $D=4$ at this stage. The non-local terms are immediate to compute and is given by

$$
\begin{aligned}
&\left[{}_{\mu\nu}\Sigma_{\rho\sigma}\right]^{\text{finite-non-local}}_{\text{1-loop}}(x;x') = \frac{iH^4}{4(4\pi)^4} \\
&\times \Bigg\{ \nabla_\mu\nabla_\nu\nabla'_\rho\nabla'_\sigma \Bigg[\left(\frac{1}{15}+\frac{2}{3}\xi+2\xi^2\right) g(y) + \left(\frac{2}{3}+12\xi-\frac{m^2}{H^2}\right)\left(\frac{1}{4}-\nu^2\right)\ln\frac{y}{4} \\
&\qquad - \left(\frac{1}{3}+4\xi\right)\left(\frac{1}{4}-\nu^2\right)\frac{y}{4} + 2\xi^2\left(\frac{8}{y}f_0(y)+f_0^2(y)\right)\Bigg] \\
&\quad + \left(g_{\mu\nu}\nabla'_\rho\nabla'_\sigma\square + g'_{\rho\sigma}\nabla_\mu\nabla_\nu\square'\right)\Bigg[-\left(\frac{1}{20}+\frac{2}{3}+2\xi^2\right) g(y) \\
&\qquad + \left(\frac{1}{2}+2\xi\right)\left(\frac{1}{4}-\nu^2\right)\frac{y}{4} - \left(\frac{1}{2}+2\xi^2\right)\left(\frac{8}{y}f_0(y)+f_0^2(y)\right)\Bigg] \\
&\quad + g_{\mu\nu}g'_{\rho\sigma}\square\square'\Bigg[\left(\frac{1}{20}+\frac{2}{3}+2\xi^2\right) g(y) + \left(\frac{1}{8}+\xi+2\xi^2\right)\left(\frac{8}{y}f_0(y)+f_0^2(y)\right) \\
&\quad + \frac{1}{60}\Big(\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\square\square' + 2\left[{}_\mu g_\rho\right]\nabla\sigma'\nabla_\nu\square\Big) g(y) \\
&\quad + H^2\left(g_{\mu\nu}\nabla'_\rho\nabla'_\sigma + g'_{\rho\sigma}\nabla_\mu\nabla_\nu\right)\Bigg[-\left(\frac{1}{6}+2\xi+6\xi^2\right) g(y) - \left(\frac{1}{6}+\xi\right)\left(\frac{1}{4}-\nu^2\right) g(y) \\
&\qquad + \frac{1}{2}\frac{m^2}{H^2}\left(\frac{1}{4}-\nu^2\right)\frac{y}{4} + 2\xi\frac{4}{y} - 6\xi^2\left(\frac{8}{y}f_0(y)+f_0^2(y)\right)\Bigg] \\
&\quad + H^2 g_{\mu\nu}g'_{\rho\sigma}\square\Bigg[\left(\frac{19}{60}+4\xi+12\xi^2\right) g(y) - \frac{1}{12}\left(\frac{1}{4}-\nu^2\right) g(y) \\
&\qquad - (1+4\xi)\left(\frac{1}{4}-\nu^2\right)\frac{y}{4} + \left(3\xi+12\xi^2\right)\left(\frac{8}{y}f_0(y)+f_0^2(y)\right)\Bigg] \\
&\quad - H^2\Big(\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\square + 2\left[{}_\mu g_\rho\right]\nabla\sigma'\nabla_\nu\Big)\left[\frac{1}{30}+\frac{1}{3}\left(\frac{1}{4}-\nu^2\right)\right] g(y) \\
&\quad + H^2\left[{}_\mu g_\rho\right]\nabla\sigma'\nabla_\nu\Bigg[-8\left(\frac{1}{4}-\nu^2\right)\frac{\ln(y/4)}{y} - 4\left(\frac{1}{4}-\nu^2\right)^2\frac{\ln(y/4)}{y} \\
&\qquad - 2\frac{4}{y} - 4\left(\frac{1}{4}-\nu^2\right)\frac{4}{y} + \frac{1}{3}\left(\frac{1}{4}-\nu^2\right)^2\frac{4}{y} \\
&\qquad + \left(\frac{1}{4}-\nu^2\right)\left(\frac{9}{4}-\nu^2\right)\left(-\frac{5}{6}+2\psi(1)-\psi\left[\frac{1}{2}\pm\nu\right]\right)\frac{4}{y}\Bigg]
\end{aligned}
$$

$$+ H^4 g_{\mu\nu} g_{\rho\sigma} \Bigg[\left(\frac{3}{5} + 6\xi + 18\xi^2\right) g(y) + \left(1 + 5\xi + \frac{1}{4}\frac{m^2}{H^2}\right)\left(\frac{1}{4} - \nu^2\right) g(y)$$

$$+ \left(-6\xi + \frac{1}{2}\frac{m^2}{H^2}\right)\left(\frac{1}{4} - \nu^2\right)\frac{y}{4} + 18\xi^2\left(\frac{8}{y} f_0(y) + f_0^2(y)\right)\Bigg]$$

$$+ H^4 \left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\left(-6\xi + \frac{1}{2}\frac{m^2}{H^2}\right)\left(\frac{1}{4} - \nu^2\right) g(y) + 4\nabla_\mu \nabla'_\rho \frac{4}{y}\nabla_\nu \nabla'_\sigma f_2(y)$$

$$+ 4\left(\frac{1}{4} - \nu^2\right)\nabla_\mu \nabla'_\rho \ln(y/4)\nabla_\nu \nabla'_\sigma f_1(y) + \nabla_\mu \nabla'_\rho f_1(y)\nabla_\nu \nabla'_\sigma f_1(y)$$

$$+ \left[-\left(\frac{1}{2} + 2\xi\right) g_{\mu\nu}\Box - 6\xi H^2 g_{\mu\nu} + 2\xi \nabla_\mu \nabla_\nu\right]\left(2\nabla'_\rho \frac{4}{y}\nabla'_\sigma f_1(y) + \nabla'_\rho f_1(y)\nabla'_\sigma f_1(y)\right)$$

$$+ \left[-\left(\frac{1}{2} + 2\xi\right) g'_{\rho\sigma}\Box - 6\xi H^2 g'_{\rho\sigma} + 2\xi \nabla'_\rho \nabla'_\sigma\right]\left(2\nabla_\mu \frac{4}{y}\nabla_\nu f_1(y) + \nabla_\mu f_1(y)\nabla_\nu f_1(y)\right)\Bigg\}. \tag{5.20}$$

The local finite terms take some more effort to find, as they are the divergent terms that pick up a factor of $(D-4)$ such to make the term finite.

$$\left[{}_{\mu\nu}\Sigma^{\text{4Pt-finite-local}}_{\rho\sigma}\right](x;x')$$

$$= \frac{1}{16\pi^2}\Bigg\{\frac{1}{120}\left[H^2\Big(g_{\mu\nu}\nabla'_\rho\nabla'_\sigma + g_{\mu\nu}\nabla'_\rho\nabla'_\sigma\Big) + 2H^2\left[{}_\mu g_\rho\right]\nabla_\nu\nabla'_\sigma - 3H^4 g_{\mu\nu} g'_{\rho\sigma}\right]$$

$$+ \frac{(1+6\xi)^2}{6} H^4 g_{\mu\nu} g'_{\rho\sigma} + \frac{(1+6\xi)}{12} m^2 H^2 g_{\mu\nu} g'_{\rho\sigma}$$

$$+ \frac{1}{24}\left(\frac{1}{4} - \nu^2\right)\Bigg[3(4\xi - 1)H^4 g_{\mu\nu} g'_{\rho\sigma} - H^2\Big(\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\Box + 2\left[{}_\mu g_\rho\right]\nabla_\nu\nabla'_\sigma\Big)$$

$$- 6H^4\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\Bigg]$$

$$+ \frac{1}{4}\left[\left(\frac{1}{4} - \nu^2\right)\left(1 - 2\psi(1) + \psi(\tfrac{1}{2} \pm \nu^2)\right) + 1\right]$$

$$\times \Bigg[\left(\frac{1}{4} + \xi\right) H^2 g_{\mu\nu} g'_{\rho\sigma}\Box + 2\xi H^2\Big(\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\Box + 2\left[{}_\mu g_\rho\right]\nabla_\nu\nabla'_\sigma\Big)$$

$$- 6\xi H^4\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\Bigg]\Bigg\}\frac{\delta^4(x - x'}{\sqrt{-g}} \tag{5.21}$$

6 Discussion and Conclusion

In this thesis, we successfully completed the renormalization procedure for the graviton one-loop self-energy, $\left[{}_{\mu\nu}\Sigma_{\rho\sigma}\right]^{\text{ren}}_{\text{one-loop}}(x;x')$, on a Minkowski, and a de Sitter background. We did this by adding four counterterms to our theory, $|C_{\gamma\alpha\delta\beta}|^2$, R^2, Rm^2, m^4. The coefficients for the massless terms correspond exactly to those found by 't Hooft and Veltman [36] and also by Park and Woodard [39].

In order to complete this renormalization procedure, we first needed to derive the scalar propagator in the massive, non-minimal coupling case, known as the Chernikov-Tagirov Propagator in Section 3,

$$i\,\Delta(x;x') = \frac{\hbar H^{D-2}}{(4\pi)^{D/2}} \frac{\Gamma\left(\frac{D-1}{2}+\nu\right)\Gamma\left(\frac{D-1}{2}-\nu\right)}{\Gamma\left(\frac{D}{2}\right)} {}_2F_1\left(\frac{D-1}{2}+\nu, \frac{D-1}{2}-\nu; \frac{D}{2}, 1-\frac{y}{4}\right). \tag{6.1}$$

This propagator is de Sitter invariant, and we assume $m^2 - \xi R > 0$, unlike in the massless, minimally coupled case found in Park and Woodard's work, where their propagator contains a de Sitter breaking term $\propto \ln(aa')$.

Our steps were then to evaluate the divergent terms found in the 4-point and 3-point Feynman diagrams. We did this first on a Minkowski background in Section 4 and then again on de Sitter in Section 5. This allowed us to first fix the counterterm coefficients in Minkowski and then compute the H^2 suppressed terms in de Sitter. The 4-point contribution requires the second order variation of the Ricci scalar, given in Appendix D, along with the curvature variations for the counterterms. These variations make it clear that we need to include the bitensor $\left[{}_{\mu}g_{\rho}\right](x;x')$ in our calculations in order to maintain the bitensorial form of the self-energy, as without it, an ambiguity arises in how to order derivatives, giving rise to ill-defined results.

The 3-point contribution involves taking products of derivatives of this propagator. We used then isolated all derivatives onto a Dirac delta proportional to $1/(D-4)$, by extracting derivatives. This procedure is detailed in Appendix B, and is then implemented in Appendix C.4 to obtain the divergent and finite terms for required propagator derivatives.

Finally to obtain the fully renormalized one-loop graviton self-energy, we gathered together both the local and non-local finite terms. This requires us to be careful when we computed the divergent terms, as certain $D-4$ expansions were required to gather terms into the correct Renormalizable form.

Due to time constraints, we were unable to apply this result and this work acts only to derive the result. However, there are a number of interesting directions future work could go. The most obvious next step is to solve the one-loop quantum corrected linearised Einstein field equation

$$\mathcal{L}^{\mu\nu\rho\sigma}h_{\rho\sigma} - \int d^Dx'\left[{}^{\mu\nu}\Sigma^{\rho\sigma}\right](x;x')h_{\rho\sigma}(x') + \mathcal{O}(h^2) = 8\pi G T^{\mu\nu}, \tag{6.2}$$

to find the effect of massive non-minimally coupled scalars on dynamical gravitons at one-loop level, and potentially to further study static mass and quadrupole influences. This was previously done in the massless, non-minimal coupling case [77], where no effects were found, however the additional massive and non-minimal coupling terms in our case could provide these effects. To solve this, one should employ the Schwinger-Keldysh formalism [49, 82, 83], by replacing our self-energy with its retarded form

$$\left[{}^{\mu\nu}\Sigma^{\rho\sigma}\right](x;x') \to \left[{}^{\mu\nu}\Sigma^{\rho\sigma}_{\text{ret}}\right](x;x') \equiv \left[{}^{\mu\nu}\Sigma^{\rho\sigma}_{++}\right](x;x') + \left[{}^{\mu\nu}\Sigma^{\rho\sigma}_{+-}\right](x;x'). \tag{6.3}$$

Where we obtain this $\pm$ prescription by changing the $i\,\varepsilon$-prescription used in the invariant distance $y(x;x')$

$$y_{++}(x;x') = H^2 aa' \left(-(|\eta-\eta'| - i\,\varepsilon)^2 + ||\Delta\vec{x}||^2\right) \equiv y(x;x'), \tag{6.4}$$

$$y_{+-}(x;x') = H^2 aa' \left(-(\eta-\eta' + i\,\varepsilon)^2 + ||\Delta\vec{x}||^2\right), \tag{6.5}$$

$$y_{-+}(x;x') = H^2 aa' \left(-(\eta-\eta' - i\,\varepsilon)^2 + ||\Delta\vec{x}||^2\right), \tag{6.6}$$

$$y_{--}(x;x') = H^2 aa' \left(-(|\eta-\eta'| + i\,\varepsilon)^2 + ||\Delta\vec{x}||^2\right). \tag{6.7}$$

It was also found [49] that for the massless minimally coupled case, there was an secular correction of the form $\ln(a)$ to the gravitational scalar potentials, such to be an effective screening of the newtons constant

$$G \to G\left(1 - \frac{\hbar}{c^5}\frac{GH^2}{30\pi}\ln(a)\right). \tag{6.8}$$

This effect is suppressed by the very small coefficient during primordial inflation, but for a long lasting inflationary epoch these effects could grow to become a significant contribution. The effects of the additional mass, and non-minimal coupling terms could give additional similar effects in our case.

It would also be interesting to repeat our calculation in a number of different ways. First, recall that we employed a strictly positive effective mass, $m^2 - \xi R > 0$, but in the massless, minimally coupled case, $m^2 - \xi R = 0$, the scalar propagator picks up an additional de Sitter breaking term. This means that we cannot take the limit $m^2 - \xi R \to 0^+$, however in the negative effective mass case, $m^2 - \xi R < 0$, you do pick up this additional symmetry breaking term. It would be interesting to investigate this limiting procedure from below and compare against the fully massless, minimally coupled case [39].

Finally, while the results in de Sitter space are very useful, it would be more insightful to look at $\epsilon > 0$ slow roll inflation, as de Sitter does not capture all of the relevant effects [48]. The propagator equation is not expected to deviate very much. Janssen et. al. [65] found that for a massless, minimally coupled scalar, the additional slow-roll parameter acted as an effective mass term in the equation of motion for the scalar field, appearing as $\nu = \frac{D-1-\epsilon}{2(1-\epsilon)}$ in the same propagator form as we found.

A Identities involving the Invariant Distance y

A very useful quantity for us is the invariant distance $y(x;x')$, which is related to the de Sitter geodesic distance $l(x;x')$ as

$$y(x;x') = 4\sin^2\left(\frac{1}{2}Hl(x;x')\right). \tag{A.1}$$

We wish to study both the classical, $\bar{y}(x;x')$ and the quantum $y(x;x')$ forms, which are related as:

$$\bar{y}(x;x') = H^2 a(\eta)a(\eta')\Delta x^2 = \mathcal{H}\mathcal{H}'\Delta x^2 \tag{A.2}$$
$$y(x;x') = H^2 a(\eta)a(\eta')\Delta x_F^2 = \bar{y}(x;x') + 2i\varepsilon\,\mathrm{sgn}(\Delta\eta)\Delta\eta \tag{A.3}$$

Note that $y \to \bar{y}$ in the $\varepsilon \to 0$ limit, and so we only care about this distinction when we are dealing with propagator poles, as elsewhere the ε will not be present.

We will need a variety of derivatives of y, noting that $y(x;x')$ is a biscalar, and so the first covariant derivative w.r.t. each coordinate will be a normal derivative. To do this we look at derivatives of the Feynman propagator first: $\Delta x_F^2 = -(|\Delta\eta| - i\varepsilon)^2 + ||\Delta\vec{x}||^2$. Then using property that $\partial_x\,\mathrm{sgn}(x) = 2\delta(x)$ we can calculate the derivatives acting on it, and obtain the following identities:

$$\partial_\mu\left(\Delta x_F^2\right) = 2\Delta x_\mu + 2\delta_\mu^0 i\varepsilon\,\mathrm{sgn}(\Delta\eta), \tag{A.4}$$
$$\partial^\mu\left(\Delta x_F^2\right) = 2\Delta x^\mu - 2\delta_0^\mu i\varepsilon\,\mathrm{sgn}(\Delta\eta), \tag{A.5}$$
$$\partial^2\left(\Delta x_F^2\right) = 2D + 4\delta_0^\mu\delta_\mu^0 i\varepsilon\delta(\Delta\eta), \tag{A.6}$$
$$\partial_\mu\left(\Delta x_F^2\right)\partial^\mu\left(\Delta x_F^2\right) = 4\left(\Delta x^2 + \varepsilon^2 + i\varepsilon(\Delta x^\mu\delta_\mu^0 - \Delta x_\mu\delta_0^\mu)\right) = 4\Delta x_F^2. \tag{A.7}$$

This allows us to find

$$\nabla_\mu y = \partial_\mu y = \delta_\mu^0\mathcal{H}y + 2\Delta x_\mu\mathcal{H}\mathcal{H}' + 2i\varepsilon\,\mathrm{sgn}(\Delta\eta)\mathcal{H}\mathcal{H}'\delta_\mu^0, \tag{A.8}$$
$$\nabla_\mu\nabla_\nu y = \mathcal{H}^2\left(\eta_{\mu\nu}(2-y) + 4i\varepsilon\delta(\Delta\eta)\delta_\mu^0\delta_\nu^0 - 2i\varepsilon\,\mathrm{sgn}(\Delta\eta)\eta_{\mu\nu}\mathcal{H}'\right), \tag{A.9}$$
$$\frac{\Box}{H^2}y = D(2-y) - 4i\varepsilon\delta(\Delta\eta) - 2i\varepsilon\,\mathrm{sgn}(\Delta\eta)D\mathcal{H}', \tag{A.10}$$
$$g^{\mu\nu}\partial_\mu y\partial_\nu y = H^2\left(4y - y^2 - 4i\varepsilon\,\mathrm{sgn}(\Delta\eta)\mathcal{H}'y\right). \tag{A.11}$$

Let's gather together a list of identities we can make from these, setting $y = \bar{y}$, i.e. $\varepsilon = 0$

$$\nabla_\mu\nabla_\nu\frac{y}{4} = H^2 g_{\mu\nu}\left(\frac{1}{2} - \frac{y}{4}\right) \tag{A.12}$$
$$\nabla_\mu\nabla'_\rho y = \mathcal{H}^2\left(\delta_\mu^0\delta_\rho^0 - \frac{1}{2}Ha\delta_\mu^0\Delta x_\rho + \frac{1}{2}Ha\delta_\rho^0\Delta x_\mu - \frac{1}{2}\eta_{\mu\rho}\right), \tag{A.13}$$
$$\nabla'_\rho\nabla'_\sigma\frac{y}{4} = H^2 g'_{\rho\sigma}\left(\frac{1}{2} - \frac{y}{4}\right) \tag{A.14}$$
$$\frac{\Box}{H^2}\frac{y}{4} = D\left(\frac{1}{2} - \frac{y}{4}\right) \tag{A.15}$$
$$\nabla_\mu\frac{y}{4}\nabla^\mu\frac{y}{4} = H^2\left(\frac{y}{4} - \left(\frac{y}{4}\right)^2\right) \tag{A.16}$$

(A.17)

Assuming de Sitter invariance, the chain rule gives us

$$\partial_\mu = \frac{\partial y}{\partial x^\mu}\frac{d}{dy} \tag{A.18}$$

$$\partial^2 = \eta^{\mu\nu}\left(\frac{\partial^2 y}{\partial x^\mu \partial x^\nu}\frac{d}{dy} + \frac{\partial y}{\partial x^\mu}\frac{\partial y}{\partial x^\nu}\frac{d^2}{dy^2}\right) \tag{A.19}$$

We can then use $\Delta x^2 = \eta^{\mu\nu}\Delta x_\mu \Delta x_\nu \Rightarrow \partial_\sigma(\Delta x^2) = 2\Delta x_\sigma$ to evaluate these

$$\frac{\partial y}{\partial x^\mu} = 2aa'H^2\Delta x_\mu + \delta^0_\mu aHy, \tag{A.20}$$

$$\frac{\partial^2 y}{\partial x^\mu \partial x^\nu} = 2aa'H^2\eta_{\mu\nu} + 2aa'H^2aH\left(\delta^0_\nu \Delta x_\mu + \delta^0_\mu \Delta x_\nu\right) + 2\left(aH\right)^2 \delta^0_\mu \delta^0_\nu y, \tag{A.21}$$

$$\frac{\partial y}{\partial x^\mu}\frac{\partial y}{\partial x^\nu} = \left(2aa'H^2\Delta x_\mu + \delta^0_\mu aHy\right)\left(2aa'H^2\Delta x_\nu + \delta^0_\nu aHy\right) \tag{A.22}$$

$$= 4\left(aa'H^2\right)^2 \Delta x_\mu \Delta x_\nu + 2aa'H^2aH\left(\delta^0_\nu \Delta x_\mu + \delta^0_\mu \Delta x_\nu\right)y + \left(aH\right)^2 \delta^0_\mu \delta^0_\nu y^2. \tag{A.23}$$

Finally gathering all of the terms, and after some careful manipulation, we find

$$\sqrt{-g}\left(\Box - M^2\right) = a^D H^2\left\{\left(4y - y^2\right)\frac{d^2}{dy^2} + D\left(2 - y\right)\frac{d}{dy} - \frac{M^2}{H^2}\right\}. \tag{A.24}$$

Now let's take a deeper look at some identities and contractions under the coincidence limit. Note that the order of prime and unprimed derivatives on a biscalar or bitensor are independent.

$$\frac{y}{4}\frac{\delta^D(x-x')}{\sqrt{-g}} = 0 \tag{A.25}$$

$$\nabla_\mu \frac{y}{4}\frac{\delta^D(x-x')}{\sqrt{-g}} = \frac{y}{4}\nabla_\mu \frac{\delta^D(x-x')}{\sqrt{-g}} = 0 \tag{A.26}$$

$$\nabla_\mu \frac{y}{4}\nabla_\nu \frac{\delta^D(x-x')}{\sqrt{-g}} = -\frac{1}{2}H^2 g_{\mu\nu}\frac{\delta^D(x-x')}{\sqrt{-g}} \tag{A.27}$$

$$\nabla_\mu \nabla_\nu \frac{y}{4}\frac{\delta^D(x-x')}{\sqrt{-g}} = \frac{1}{2}H^2 g_{\mu\nu}\frac{\delta^D(x-x')}{\sqrt{-g}} \tag{A.28}$$

$$\frac{y}{4}\nabla_\mu \nabla_\nu \frac{\delta^D(x-x')}{\sqrt{-g}} = \frac{1}{2}H^2 g_{\mu\nu}\frac{\delta^D(x-x')}{\sqrt{-g}} \tag{A.29}$$

$$\frac{y}{4}\square \frac{\delta^D(x-x')}{\sqrt{-g}} = H^2\frac{D}{2}\frac{\delta^D(x-x')}{\sqrt{-g}} \tag{A.30}$$

$$\nabla_\mu \frac{y}{4}\square \frac{\delta^D(x-x')}{\sqrt{-g}} = -H^2\nabla_\mu \frac{\delta^D(x-x')}{\sqrt{-g}} \tag{A.31}$$

$$\nabla_\mu \frac{y}{4}\nabla_\nu \frac{y}{4}\square \frac{\delta^D(x-x')}{\sqrt{-g}} = \frac{1}{2}H^4 g_{\mu\nu}\frac{\delta^D(x-x')}{\sqrt{-g}} \tag{A.32}$$

$$\nabla_\mu \frac{y}{4}\nabla_\alpha \nabla_\beta \frac{\delta^D(x-x')}{\sqrt{-g}} = -H^2 g_{\mu(\alpha}\nabla_{\beta)}\frac{\delta^D(x-x')}{\sqrt{-g}} \tag{A.33}$$

$$\nabla_\alpha \frac{y}{4}\nabla'_\gamma \nabla'_\delta \frac{\delta^D(x-x')}{\sqrt{-g}} = H^2 \left[{}_\alpha g_{(\gamma}\right] \nabla'_{\delta)}\frac{\delta^D(x-x')}{\sqrt{-g}} \tag{A.34}$$

A.1 The Bilocal Metric

Much of the ideas behind this section are described in detail in [64] and [80], though be aware, they typically use a slightly different definition which causes some of the following identities to differ.

Let x, x' be two space-time points and $y = H^2 a a' \Delta x^2$ be the invariant distance between them. It is useful to distinguish between them in the space-time indices. To do this, we associate μ, ν with x and ρ, σ with x'. We define the bitensor

$$\left[{}_\mu g_\rho\right](x;x') = -\frac{1}{2H^2}\nabla_\mu \nabla'_\rho y = -\frac{1}{2H^2}\partial_\mu \partial'_\rho y \tag{A.35}$$

In the coincidence limit, we can apply A.13 to get the metric tensor, i.e.

$$\lim_{x'\to x}\left[{}_\mu g_\rho\right](x;x') = g_{\mu\rho} \tag{A.36}$$

though one should be careful with this limit, as in certain expressions it is vital to retain the bitensorial form. In principle this should only be applied after we integrate over a Dirac delta, or similarly take this coincidence limit. This limit also implies that in the Minkowski limit, $H \to 0$, this bitensor becomes the Minkowski metric. We should use the following identity when we want to exchange the location of a derivative

$$\left[{}^{\mu}g_{\rho}\right]\nabla_{\mu}\frac{\delta^{D}(x-x')}{\sqrt{-g}} = -\nabla'_{\rho}\frac{\delta^{D}(x-x')}{\sqrt{-g}} \tag{A.37}$$

It is also important to commute these bitensors through covariant derivatives, as this appear next to the Dirac delta in the functional derivative definition. The identities we need are

$$\Box\left(\left[{}_{\mu}g_{\rho}\right]\left[{}_{\nu}g_{\sigma}\right]\frac{\delta^{D}(x-x')}{\sqrt{-g}}\right) = \left[{}_{\mu}g_{\rho}\right]\Box\frac{\delta^{D}(x-x')}{\sqrt{-g}} + 2H^{2}\left[{}_{\mu}g_{\rho}\right]\left[{}_{\nu}g_{\sigma}\right]\frac{\delta^{D}(x-x')}{\sqrt{-g}} \tag{A.38}$$

$$\nabla'_{\sigma}\nabla_{\nu}\left(\left[{}_{\mu}g_{\rho}\right]\frac{\delta^{D}(x-x')}{\sqrt{-g}}\right) = \left[{}_{\mu}g_{\rho}\right]\nabla'_{\sigma}\nabla_{\nu}\frac{\delta^{D}(x-x')}{\sqrt{-g}} - H^{2}g_{\mu\nu}g'_{\rho\sigma}\frac{\delta^{D}(x-x')}{\sqrt{-g}} \tag{A.39}$$

$$\begin{aligned}\Box\Box'\left(\left[{}_{\mu}g_{\rho}\right]\left[{}_{\nu}g_{\sigma}\right]\frac{\delta^{D}(x-x')}{\sqrt{-g}}\right) = \Big\{&\left[{}_{\mu}g_{\rho}\right]\left[{}_{\nu}g_{\sigma}\right]\Box\Box' + 4H^{2}\left[{}_{\mu}g_{\rho}\right]\left[{}_{\nu}g_{\sigma}\right]\Box' \\ &- 8H^{2}\left[{}_{\mu}g_{\rho}\right]\nabla'_{\sigma}\nabla_{\nu} + 4H^{4}g_{\mu\nu}g'_{\rho\sigma} + 4H^{4}\left[{}_{\mu}g_{\rho}\right]\left[{}_{\nu}g_{\sigma}\right]\Big\}\frac{\delta^{D}(x-x')}{\sqrt{-g}}\end{aligned} \tag{A.40}$$

$$\begin{aligned}\nabla'_{\sigma}\nabla_{\nu}\Box'\left(\left[{}_{\mu}g_{\rho}\right]\frac{\delta^{D}(x-x')}{\sqrt{-g}}\right) = \Big\{&\left[{}_{\mu}g_{\rho}\right]\nabla'_{\sigma}\nabla_{\nu}\Box' - H^{2}g_{\mu\nu}g'_{\rho\sigma}\Box' \\ &- 2H^{2}\left(g'_{\rho\sigma}\nabla_{\mu}\nabla_{\nu} + g_{\mu\nu}\nabla'_{\rho}\nabla'_{\sigma}\right) + H^{4}(D-1)g_{\mu\nu}g'_{\rho\sigma} + H^{2}\left[{}_{\mu}g_{\rho}\right]\nabla'_{\sigma}\nabla_{\nu}\Big\}\frac{\delta^{D}(x-x')}{\sqrt{-g}}\end{aligned} \tag{A.41}$$

$$g^{\mu\nu}\left[{}_{\mu}g_{\rho}\right]\left[{}_{\nu}g_{\sigma}\right]\frac{\delta^{D}(x-x')}{\sqrt{-g}} = g'_{\rho\sigma}\frac{\delta^{D}(x-x')}{\sqrt{-g}} \tag{A.42}$$

$$g^{\mu\nu}\left[{}_{\mu}g_{\rho}\right]\nabla_{\nu}\frac{\delta^{D}(x-x')}{\sqrt{-g}} = -\nabla'_{\rho}\frac{\delta^{D}(x-x')}{\sqrt{-g}} \tag{A.43}$$

$$g^{\mu\nu}\left[{}_{\mu}g_{\rho}\right]\nabla_{\nu}\nabla'_{\sigma}\frac{\delta^{D}(x-x')}{\sqrt{-g}} = -\nabla'_{\rho}\nabla'_{\sigma}\frac{\delta^{D}(x-x')}{\sqrt{-g}} + DH^{2}g'_{\rho\sigma}\frac{\delta^{D}(x-x')}{\sqrt{-g}} \tag{A.44}$$

$$\begin{aligned}g^{\mu\nu}\left[{}_{\mu}g_{\rho}\right]\nabla_{\nu}\nabla'_{\sigma}\Box\frac{\delta^{D}(x-x')}{\sqrt{-g}} =& -\nabla'_{\rho}\nabla'_{\sigma}\Box\frac{\delta^{D}(x-x')}{\sqrt{-g}} + (D+2)H^{2}\nabla'_{\rho}\nabla'_{\sigma}\frac{\delta^{D}(x-x')}{\sqrt{-g}} \\ &+ (D+2)H^{2}g'_{\rho\sigma}\Box\frac{\delta^{D}(x-x')}{\sqrt{-g}} - D^{2}H^{4}g'_{\rho\sigma}\frac{\delta^{D}(x-x')}{\sqrt{-g}}\end{aligned} \tag{A.45}$$

$$g^{\mu\nu}\left[{}_{\mu}g_{\rho}\right]\left[{}_{\nu}g_{\sigma}\right]\Box'\frac{\delta^{D}(x-x')}{\sqrt{-g}} = g'_{\rho\sigma}\Box\frac{\delta^{D}(x-x')}{\sqrt{-g}} - 2H^{2}g'_{\rho\sigma}\frac{\delta^{D}(x-x')}{\sqrt{-g}} \tag{A.46}$$

$$\begin{aligned}g^{\mu\nu}\left[{}_{\mu}g_{\rho}\right]\left[{}_{\nu}g_{\sigma}\right]\Box\Box'\frac{\delta^{D}(x-x')}{\sqrt{-g}} =& g'_{\rho\sigma}\Box\Box'\frac{\delta^{D}(x-x')}{\sqrt{-g}} - 4H^{2}g'_{\rho\sigma}\Box\frac{\delta^{D}(x-x')}{\sqrt{-g}} \\ &- 8H^{2}\nabla'_{\rho}\nabla'_{\sigma}\frac{\delta^{D}(x-x')}{\sqrt{-g}} + 4(D+1)H^{4}g'_{\rho\sigma}\frac{\delta^{D}(x-x')}{\sqrt{-g}}\end{aligned} \tag{A.47}$$

B Extracting Derivatives

Consider f to be a differentiable function of $y = H^2 aa' \delta x^2$, on de Sitter. We can derive the d'Alembert operator acting on $f(y)$ as

$$\frac{\square}{H^2} f(y) = \frac{1}{H^2}\Big(\square y f'(y) + g_{\mu\nu}\partial_\mu y \partial_\nu y f''(y)\Big), \tag{B.1}$$

$$= (4y - y^2) f''(y) + D(2-y) f'(y) - 4i\varepsilon\delta(\Delta\eta) f'(y) - 2i\varepsilon\, \mathrm{sgn}(\Delta\eta)\mathcal{H}'\big(2y f''(y) + D f'(y)\big). \tag{B.2}$$

This is a very useful equation which forms the basis for the extracting derivatives or d'Alembertians process. Consider $f(y) = \left(\frac{y}{4}\right)^{\alpha}$

$$\frac{\square}{H^2}\left(\frac{y}{4}\right)^{1-\alpha} = (\alpha-1)(D-\alpha)\left(\frac{y}{4}\right)^{1-\alpha} + \frac{1}{2}(D-2\alpha)(1-\alpha)\left(\frac{y}{4}\right)^{-\alpha} + i\varepsilon\Big[\delta(\Delta\eta)(\alpha-1) - \frac{1}{2}\,\mathrm{sgn}(\Delta\eta)(\alpha-1)(\alpha-\frac{D}{2})\Big]\left(\frac{y}{4}\right)^{-\alpha}. \tag{B.3}$$

The $\varepsilon\delta(\Delta\eta)(\Delta x_F^2)^{-\alpha}$ term generates a Dirac delta in the $\varepsilon \to 0$ limit if $\alpha = D/2$, and is regular otherwise. Thus for $\alpha \neq D/2$ we can take discard the ε terms and find:

$$\left(\frac{y}{4}\right)^{-\alpha} = \frac{-2}{(\alpha-1)(D-2\alpha)}\frac{\square}{H^2}\left(\frac{y}{4}\right)^{1-\alpha} + \frac{2(D-\alpha)}{D-2\alpha}\left(\frac{y}{4}\right)^{1-\alpha}, \qquad (\alpha \neq D/2) \tag{B.4}$$

$$\frac{\square}{H^2}\left(\frac{y}{4}\right)^{1-\frac{D}{2}} = \frac{(4\pi)^{D/2}}{\Gamma(\frac{D}{2}-1)H^D}\frac{i\delta(x-x')}{\sqrt{-g}} + \frac{1}{4}D(D-2)\left(\frac{y}{4}\right)^{1-\frac{D}{2}}. \tag{B.5}$$

We use these two identities to rewrite some powers of y in order to isolate the divergences onto Dirac deltas. It is convenient to gather the finite terms into

$$g(y) = -4\frac{\square}{H^2}\frac{\ln(y/4)}{y} + 8\frac{ln(y/4)}{y} - \frac{4}{y}, \tag{B.6}$$

which appears when expanding $\left(\frac{y}{4}\right)^{2-D}$ about $D = 4$. We then find the following

$$\left(\frac{y}{4}\right)^{2-D} = \frac{2}{(D-3)(D-4)}\frac{(4\pi)^{D/2}}{\Gamma\left(\frac{D}{2}-1\right)H^D}\frac{i\delta^D(x-x')}{\sqrt{-g}} + g(y) \tag{B.7}$$

$$\left(\frac{y}{4}\right)^{1-D} = \frac{4}{(D-2)^2(D-3)(D-4)}\frac{(4\pi)^{D/2}}{\Gamma\left(\frac{D}{2}-1\right)H^D}\left\{\frac{\square}{H^2}\frac{i\delta^D(x-x')}{\sqrt{-g}} - (D-2)\frac{i\delta^D(x-x')}{\sqrt{-g}}\right\} + \frac{1}{2}\frac{\square}{H^2}g(y) - g(y) \tag{B.8}$$

$$\left(\frac{y}{4}\right)^{-D} = \frac{8}{D(D-1)(D-2)^2(D-3)(D-4)}\frac{(4\pi)^{D/2}}{\Gamma\left(\frac{D}{2}-1\right)H^D} \times\left\{\frac{\square}{H^2}\frac{\square'}{H^2}\frac{i\delta^D(x-x')}{\sqrt{-g}} - (D-2)\frac{\square}{H^2}\frac{i\delta^D(x-x')}{\sqrt{-g}}\right\} + \frac{1}{12}\frac{\square}{H^2}\frac{\square'}{H^2}g(y) - \frac{1}{6}\frac{\square}{H^2}g(y) \tag{B.9}$$

We would like an expression for the product of two derivatives of general powers of y, so we consider

$$\nabla_\mu\nabla_\nu\left(\frac{y}{4}\right)^{-(\alpha+\beta)} = \frac{(\alpha+\beta)(\alpha+\beta+1)}{\alpha\beta}\partial_\mu\left(\frac{y}{4}\right)^{-\alpha}\partial_\nu\left(\frac{y}{4}\right)^{-\beta} \tag{B.10}$$

$$+ (\alpha+\beta)\left(\frac{y}{4}\right)^{-(\alpha+\beta+1)}\nabla_\mu\nabla_\nu\frac{y}{4} \tag{B.11}$$

Rearranging, and making use of Eq. (B.4), we find

$$\partial_\mu\left(\frac{y}{4}\right)^{-\alpha}\partial_\nu\left(\frac{y}{4}\right)^{-\beta} = \frac{\alpha\beta}{(\alpha+\beta)(\alpha+\beta+1)}\nabla_\mu\nabla_\nu\left(\frac{y}{4}\right)^{-(\alpha+\beta)} + \frac{\alpha\beta}{D-2(\alpha+\beta+1)}H^2 g_{\mu\nu}\left\{\frac{-1}{(\alpha+\beta)(\alpha+\beta+1)}\frac{\square}{H^2}+1\right\}\left(\frac{y}{4}\right)^{-(\alpha+\beta)} \tag{B.12}$$

In a similar manner, one can also show

$$\begin{aligned}\partial_\mu\partial'_\rho\left(\frac{y}{4}\right)^{-\alpha}\partial_\nu\partial'_\sigma\left(\frac{y}{4}\right)^{-\beta} &= \frac{\alpha\beta(\alpha+1)(\beta+1)}{(\alpha+\beta)(\alpha+\beta+1)(\alpha+\beta+2)(\alpha+\beta+3)}\Bigg\{\nabla_\mu\nabla_\nu\nabla'_\rho\nabla'_\sigma\left(\frac{y}{4}\right)^{-(\alpha+\beta)}\\
&+ (\alpha+\beta)\big[H^2 g_{\mu\nu}\nabla'_\rho\nabla'_\sigma + H^2 g'_{\rho\sigma}\nabla_\mu\nabla_\nu\big]\left(\frac{1}{2}\left(\frac{y}{4}\right)^{-(\alpha+\beta+1)} - \left(\frac{y}{4}\right)^{-(\alpha+\beta)}\right)\\
&- (\alpha+\beta)^2 H^4 g_{\mu\nu}g'_{\rho\sigma}\left(\frac{1}{2}\left(\frac{y}{4}\right)^{-(\alpha+\beta+1)} - \left(\frac{y}{4}\right)^{-(\alpha+\beta)}\right)\\
&+ \frac{1}{2}(\alpha+\beta)(\alpha+\beta+1)H^4 g_{\mu\nu}g'_{\rho\sigma}\left(\frac{1}{2}\left(\frac{y}{4}\right)^{-(\alpha+\beta+2)} - \left(\frac{y}{4}\right)^{-(\alpha+\beta+1)}\right)\Bigg\}\\
&+ \left[-2\frac{\alpha\beta(\alpha+1)(\beta+1)}{(\alpha+\beta+1)(\alpha+\beta+2)(\alpha+\beta+3)} + \frac{\alpha\beta}{2(\alpha+\beta+1)}\right]H^2\left[{}_\mu g_\rho\right](x;x')\nabla_\nu\nabla'_\sigma\left(\frac{y}{4}\right)^{-(\alpha+\beta+1)}\\
&+ \frac{1}{2}\frac{\alpha\beta(\alpha+1)(\beta+1)}{(\alpha+\beta+2)(\alpha+\beta+3)}H^4\left[{}_\mu g_\rho\right](x;x')\left[{}_\nu g_\sigma\right](x;x')\left(\frac{y}{4}\right)^{-(\alpha+\beta+2)}\end{aligned} \tag{B.13}$$

B.1 Minkowski Equivalents

In this subsection we will briefly detail some of the equivalent formulae for the Minkowski *invariant distance* $y = m^2\Delta x^2$. For a similar treatment, see Marunovic and Prokopec [40], where they detailed this procedure for a massless minimally coupled scalar on a Minkowski background. They do not employ this dimensionless y parameter.

$$\partial_\mu y = 2m^2\Delta x_\mu = -\partial'_\mu y \tag{B.14}$$

$$\partial_\mu\partial_\nu y = 2m^2\eta_{\mu\nu} \tag{B.15}$$

$$\partial_\mu\partial'_\rho y = -2m^2\eta_{\mu\rho} \tag{B.16}$$

For the extraction of derivatives procedure, it is much more simple in Minkowski.

$$\left(\frac{y}{4}\right)^{-\alpha} = \frac{2}{m^2(\alpha-1)(2\alpha-D)}\partial^2\left(\frac{y}{4}\right)^{1-\alpha} \tag{B.17}$$

Then we also have, from the massless, minimally coupled propagator equation

$$\frac{\partial^2}{m^2}\left(\frac{y}{4}\right)^{1-\frac{D}{2}} = \frac{(4\pi)^{D/2}}{m^D\Gamma\left(\frac{D}{2}-1\right)}\, i\,\delta^D(x-x') \tag{B.18}$$

This allows us to extract d'Alembertians onto $\left(\frac{y}{4}\right)^{3-D}$ and then expand around $D-4$ according to:

$$\frac{\partial^2}{m^2}\left(\frac{y}{4}\right)^{3-D} = \frac{\partial^2}{m^2}\left[\left(\frac{y}{4}\right)^{3-D} - \left(\frac{y}{4}\right)^{1-\frac{D}{2}}\right] + \frac{(4\pi)^{D/2}}{m^D\Gamma\left(\frac{D}{2}-1\right)}\, i\,\delta^D(x-x')\Bigg\}, \tag{B.19}$$

We then define $g(y) = -4\partial^2\frac{\ln(y/4)}{m^2 y}$ so that we get

$$\left(\frac{y}{4}\right)^{2-D} = \frac{1}{2(D-3)(D-4)}\frac{\mu^{D-4}4\pi^{D/2}}{\Gamma\left(\frac{D}{2}-1\right)}\, i\,\delta^D(x-x') + g(y) \tag{B.20}$$

$$\left(\frac{y}{4}\right)^{1-D} = \frac{1}{m^2(D-2)^2(D-3)(D-4)}\partial^2\frac{(4\pi)^{D/2}}{m^D\Gamma\left(\frac{D}{2}-1\right)}\, i\,\delta^D(x-x') + \frac{1}{2m^2}\partial^2 g(y) \tag{B.21}$$

$$\left(\frac{y}{4}\right)^{-D} = \frac{2}{m^4 D(D-1)(D-2)^2(D-3)(D-4)}\partial^4\frac{(4\pi)^{D/2}}{m^D\Gamma\left(\frac{D}{2}-1\right)}\, i\,\delta^D(x-x') + \frac{1}{12m^4}\partial^4 g(y) \tag{B.22}$$

Similar to the de Sitter case, we want general formula for products of single and double derivatives of powers of y. The single derivative is straightforward

$$\partial_\mu \left(\frac{y}{4}\right)^{-\alpha} \partial_\nu \left(\frac{y}{4}\right)^{-\beta} = \frac{\alpha\beta}{(\alpha+\beta)(\alpha+\beta+1)} \left\{ \partial_\mu \partial_\nu + \frac{1}{2(\alpha+\beta+1)-D} \eta_{\mu\nu} \partial^2 \right\} \left(\frac{y}{y}\right)^{-(\alpha+\beta)} \tag{B.23}$$

$$\begin{aligned} \partial_\mu \partial'_\rho \left(\frac{y}{4}\right)^{-\alpha} \partial_\nu \partial'_\sigma \left(\frac{y}{4}\right)^{-\beta} = \frac{\alpha\beta(\alpha+1)(\beta+1)}{(\alpha+\beta)(\alpha+\beta+1)(\alpha+\beta+2)(\alpha+\beta+3)} \Bigg\{ \partial_\mu \partial_\nu \partial'_\rho \partial'_\sigma \left(\frac{y}{4}\right)^{-(\alpha+\beta)} \\ + \frac{1}{2} m^2 (\alpha+\beta) \big(\eta_{\mu\nu} \partial'_\rho \partial'_\sigma + \eta'_{\rho\sigma} \partial_\mu \partial_\nu \big) \left(\frac{y}{4}\right)^{-(\alpha+\beta+1)} \\ + m^4 (\alpha+\beta)(\alpha+\beta+1) \left(\frac{1}{4} \eta_{\mu\nu} \eta'_{\rho\sigma} + \frac{1}{2} \eta_{\mu)(\rho} \eta'_{\sigma)(\nu} \right) \left(\frac{y}{4}\right)^{-(\alpha+\beta+2)} \\ + \left[\frac{(\alpha+\beta)(\alpha+\beta+2)(\alpha+\beta+3)}{2(\alpha+1)(\beta+1)} - 2(\alpha+\beta) \right] m^2 \eta_{\mu\rho} \partial'_\sigma \partial_\nu \left(\frac{y}{4}\right)^{-(\alpha+\beta+1)} \Bigg\} \end{aligned} \tag{B.24}$$

C Scalar Propagator Calculations

This Appendix is a collection of more in depth looks at some of the calculations involving the scalar propagator

C.1 D-Dimensional Minkowski Propagator Calculations

Let's briefly investigate the propagator solution for a D-dimensional Minkowski massless scalar field. The Equation of Motion then looks like:

$$\partial^2(i\Delta(x;x')) = i\hbar\delta^D(x-x'). \tag{C.1}$$

We want to show the following form does indeed satisfy this equation, and determine the normalization constant A_0:

$$i\Delta(x;x') = A_0\left(\frac{1}{\Delta x_F^2}\right)^{\frac{D}{2}-1}, \quad \Delta x_F^2 = -(|\Delta x^0| - i\varepsilon)^2 + ||\Delta\vec{x}||^2. \tag{C.2}$$

As indicated, we add an $i\varepsilon$-prescription, in order to avoid the pole at $x = x'$, and then we will take the limit of $\varepsilon \to 0$ in order to generate the $\delta^D(x-x')$. Plugging this into Eq. (C.1), we find

$$\partial^2(i\Delta(x;x')) = A_0\left(\frac{D}{2}-1\right)\left(\frac{1}{\Delta x_F^2}\right)^{\frac{D}{2}-2}\left\{-\frac{-\partial^2(\Delta x_F^2)}{(\Delta x_F^2)^2} + \left(\frac{D}{2}\right)\frac{\partial_\mu(\Delta x_F^2)\partial^\mu(\Delta x_F^2)}{(\Delta x_F^2)^3}\right\} \tag{C.3}$$

$$= A_0\left(\frac{D}{2}-1\right)\left(\frac{1}{\Delta x_F^2}\right)^{\frac{D}{2}-2}\left\{-\frac{2D - 4i\varepsilon\delta(\Delta x^0)}{(\Delta x_F^2)^2} + \left(\frac{D}{2}\right)\frac{4\Delta x_F^2}{(\Delta x_F^2)^3}\right\} \tag{C.4}$$

$$= A_0\left(\frac{D}{2}-1\right)\left(\frac{1}{\Delta x_F^2}\right)^{\frac{D}{2}}\left(4i\varepsilon\delta(\Delta x^0)\right). \tag{C.5}$$

We can check to make sure that this is a Dirac delta in the $\varepsilon \to 0$ limit, by acting it on a test function and thus we can find A_0:

$$\int d^Dx'\{\partial^2(i\Delta(x;x'))f(x')\} := i\hbar f(x), \tag{C.6}$$

$$\lim_{\varepsilon\to 0} A_0\left(\frac{D}{2}-1\right)4i\int d^Dx'\left\{\frac{\varepsilon\delta(\Delta x^0)}{(\Delta x_F^2)^{\frac{D}{2}}}f(x')\right\} \tag{C.7}$$

$$= \lim_{\varepsilon\to 0} A_0\left(\frac{D}{2}-1\right)4i\int d^{D-1}x'\left\{\frac{\varepsilon}{(||\Delta\vec{x}||^2+\varepsilon^2)^{\frac{D}{2}}}f(x^0,\vec{x}')\right\}. \tag{C.8}$$

Let $\vec{r} = \vec{x} - \vec{x}' = \Delta\vec{x}$ and perform a change of variables, Taylor expanding $f(x^0, \vec{x}-\vec{r})$ about $\vec{r}$:

$$\lim_{\varepsilon\to 0} A_0\left(\frac{D}{2}-1\right)4i\int d^{D-1}\vec{r}\left\{\frac{\varepsilon}{(||\vec{r}||^2+\varepsilon^2)^{\frac{D}{2}}}\left(f(x) - r_i\partial^i f(x) + \frac{1}{2}r_i r_j\partial^i\partial^j f(x) + \dots\right)\right\}. \tag{C.9}$$

Then substituting $\vec{r} = \varepsilon\vec{\rho}$

$$\lim_{\varepsilon\to 0} A_0 \left(\frac{D}{2} - 1\right) 4i \int d^{D-1}\vec{\rho} \left\{ \frac{1}{(||\vec{\rho}||^2 + 1)^{\frac{D}{2}}} \left(f(x) - \mathcal{O}(\varepsilon)\right) \right\} \tag{C.10}$$

$$= A_0 \left(\frac{D}{2} - 1\right) 4if(x) \left[\frac{\pi^{\frac{D}{2}}}{\Gamma\left(\frac{D}{2}\right)} \frac{2^{2\left(\frac{D-1}{2}\right)-1}\Gamma\left(\frac{D}{2}\right)\Gamma\left(\frac{D-1}{2}\right)}{\sqrt{\pi}\Gamma\left(D-1\right)} \right], \tag{C.11}$$

where to compute this integral, we switched to polar coordinates and used the generalised formula for D dimensions. This generalization is given in terms of angles $\theta_i, i > 0$ defined on an interval $[0, \pi]$ and θ_0 along $[0, 2\pi)$, as well as a radial $r = ||\vec{x}||$.

$$\int d^{D-1}x = \int d\Omega^{D-2} \int_0^\infty dr = \int d\theta_{D-3} \dots d\theta_1 d\theta_0 \sin^{D-3}(\theta_{D-3}) \dots \sin(\theta_1), \tag{C.12}$$

$$\int d\Omega^{D-2} = \frac{2\pi^{\frac{D-1}{2}}}{\Gamma\left(\frac{D-1}{2}\right)} = \frac{2(4\pi)^{\frac{D}{2}-1}\Gamma\left(\frac{D}{2}\right)}{\Gamma\left(D-1\right)}. \tag{C.13}$$

Thus if we have an integral dependent on one angle φ, we can extract it by letting $\theta_{D-3} = \varphi$ (and assuming $D > 3$)

$$\int d\Omega^{D-2} f(\varphi) = \int d\Omega^{D-3} \int_0^\pi d\varphi \sin^{D-3}(\varphi) f(\varphi), \tag{C.14}$$

$$\int d\Omega^{D-3} = \frac{2\pi^{\frac{D-2}{2}}}{\Gamma\left(\frac{D-2}{2}\right)} = \frac{2(4\pi)^{\frac{D-3}{2}}\Gamma\left(\frac{D-1}{2}\right)}{\Gamma\left(D-2\right)}. \tag{C.15}$$

We can also use Eq. (8.335.1) from [78], with $x = \frac{D-1}{2}$, to reduce this fraction

$$\Gamma(2x) = \frac{2^{2x-1}}{\sqrt{\pi}} \Gamma(x) \Gamma\left(x + \frac{1}{2}\right), \tag{C.16}$$

$$A_0 \left(\frac{D}{2} - 1\right) 4if(x) \left[\frac{\pi^{D/2}}{(\frac{D}{2} - 1)\Gamma\left(\frac{D-2}{2}\right)} \right] := i\hbar f(x), \tag{C.17}$$

$$\Rightarrow A_0 = \hbar \frac{\Gamma(\frac{D-2}{2})}{4\pi^{D/2}}, \tag{C.18}$$

$$i\Delta(x; x') = \hbar \frac{\Gamma(\frac{D-2}{2})}{4\pi^{D/2}} \left(\frac{1}{\Delta x_F^2}\right)^{\frac{D}{2}-1}. \tag{C.19}$$

C.2 $D \to 4$ limit of Scalar Propagator

We would like to ensure that the $D \to 4$ limit of our proposed scalar propagator is well defined. Here is the equation we want to check (denoting $\Gamma(a \pm b) = \Gamma(a + b)\Gamma(a - b)$ for brevity):

$$\begin{aligned} i\Delta(x; x') = &\frac{\hbar H^{D-2}}{(4\pi)^{D/2}} \cdot \left\{ \frac{\Gamma\left(\frac{D-1}{2} \pm \nu\right)\Gamma(1 - \frac{D}{2})}{\Gamma(\frac{1}{2} \pm \nu)} {}_2F_1\left(\frac{D-1}{2} + \nu, \frac{D-1}{2} - \nu; \frac{D}{2}; \frac{y}{4}\right) \right. \\ &\left. + \left(\frac{y}{4}\right)^{1-\frac{D}{2}} \Gamma\left(\frac{D}{2} - 1\right) {}_2F_1\left(\frac{1}{2} + \nu, \frac{1}{2} - \nu; 2 - \frac{D}{2}; \frac{y}{4}\right) \right\}. \end{aligned} \tag{C.20}$$

We can expand these Hypergeometric functions, rearranging some of the terms and extracting the $n = 0$ term from the second ${}_2F_1$. Then recasting the index $n \to n + 1$ (and employing $(a \pm b)_n = (a + b)_n(a - b)_n$) our equation becomes

$$i\Delta(x;x') = \frac{\hbar H^{D-2}}{(4\pi)^{D/2}} \left\{ \Gamma\left(\frac{D}{2} - 1\right) \left(\frac{y}{4}\right)^{1-\frac{D}{2}} \right.$$

$$\left. + \sum_{n=0} \left[\frac{\Gamma\left(\frac{D-1}{2} \pm \nu\right) \Gamma(1 - \frac{D}{2})}{\Gamma(\frac{1}{2} \pm \nu)} \frac{\left(\frac{D-1}{2} \pm \nu\right)_n}{\left(\frac{D}{2}\right)_n n!} \left(\frac{y}{4}\right)^n + \frac{\Gamma\left(\frac{D}{2} - 1\right)\left(\frac{1}{4} - \nu^2\right)}{2 - \frac{D}{2}} \frac{\left(\frac{3}{2} \pm \nu\right)_n}{n!(n+1)\left(3 - \frac{D}{2}\right)_n} \left(\frac{y}{4}\right)^{n+2-\frac{D}{2}} \right] \right\}. \tag{C.21}$$

For later convenience we define $f_m(y)$ as this sum starting at $n = m$, so that we can write

$$i\Delta(x;x') = \frac{\hbar H^{D-2}}{(4\pi)^{D/2}} \left(\Gamma\left(\frac{D}{2} - 1\right) \left(\frac{y}{4}\right)^{1-\frac{D}{2}} + f_0(y) \right) \tag{C.22}$$

We are interested in the terms in this $f_0(y)$, as the other term is perfectly regular in $D \to 4$. In all of the following, we will keep up to linear terms in $D - 4$, first we can express the general expansion of Γ functions and Pochhammer symbols as z_D around the point z_4:

$$\Gamma(z_D) = \Gamma(z_4)\left(1 + \psi(z_4)(z_D - z_4)\right) \tag{C.23}$$

$$(z_D)_n = (z_4)_n\left(1 + (\psi(z_4 + n) - \psi(z_4))(z_D - z_4)\right), \tag{C.24}$$

where here the $\psi(z)$ function is defined as $\psi(z_0) = \frac{d}{dz}\ln(\Gamma(z))|_{z=z_0}$. Each term we need to take the limit of, contains: $\pm\frac{D}{2} \Rightarrow z_D - z_4 = \pm\frac{D-4}{2}$. Now expanding each part of the first term inside the sum of Eq. (C.21) we get

$$\frac{(\frac{1}{4} - \nu^2)(\frac{3}{2} \pm \nu)_n}{(2)_n n!} \left(\frac{y}{4}\right)^n \left(\frac{2}{D-4} - \psi(1) - 1 + \psi\left(\frac{3}{2} \pm \nu + n\right) - \psi\left(\frac{3}{2} \pm \nu\right) \right.$$
$$\left. + \psi\left(\frac{3}{2} \pm \nu\right) - \psi(2 + n) + \psi(2) \right), \tag{C.25}$$

where we define $\psi(z \pm x) = \psi(z + x) + \psi(z - x)$. For the second term we get the same prefactor

$$\frac{(\frac{1}{4} - \nu^2)(\frac{3}{2} \pm \nu)_n}{(2)_n n!} \left(\frac{y}{4}\right)^n \left(-\frac{2}{D-4} + \ln\left(\frac{y}{4}\right) - \psi(1 + n) - \psi(1) + \psi(1) \right). \tag{C.26}$$

Finally our expression for the scalar propagator in $D = 4$ dimensions is

$$\lim_{D \to 4} i\Delta(x;x') = \frac{\hbar H^2}{(4\pi)^2} \left\{ \frac{4}{y} + \sum_{n=0} \frac{(\frac{1}{2} + \nu)_n (\frac{1}{2} - \nu)_n}{(2)_n n!} \left(\frac{y}{4}\right)^n \right.$$
$$\left. \times \left[\ln\left(\frac{y}{4}\right) + \psi\left(\frac{3}{2} + \nu + n\right) + \psi\left(\frac{3}{2} - \nu + n\right) - \psi(1 + n) - \psi(2 + n) \right] \right\}. \tag{C.27}$$

C.3 Scalar Propagator at Coincidence

The coincidence limit for the scalar propagator can be obtained from Eq. (3.35), and utilising the fact that in dimensional regularization, D-dependent powers of y vanish at coincidence. This results in

$$\lim_{x'\to x} i\,\Delta(x;x') = \frac{\hbar H^{D-2}}{(4\pi)^{D/2}}\frac{\Gamma\left(\frac{D-1}{2}\pm\nu\right)\Gamma\left(1-\frac{D}{2}\right)}{\Gamma\left(\frac{1}{2}\pm\nu\right)} \tag{C.28}$$

$$= \frac{\hbar H^{D-2}}{(4\pi)^{D/2}}\Bigg\{\frac{2\Gamma\left(\frac{D}{2}-1\right)}{(D-3)(D-4)}\left(\frac{1}{4}-\nu^2\right) + \left(\frac{1}{4}-\nu^2\right)\left(1-2\psi(1)+2\psi\left(\frac{1}{2}\pm\nu\right)\right)+1\Bigg\} \tag{C.29}$$

C.4 Derivatives of the Scalar Propagator

Recall that the scalar propagator was found to be

$$i\Delta(x;x') = \frac{\hbar H^{D-2}}{(4\pi)^{D/2}}\left(\Gamma\left(\frac{D}{2}-1\right)\left(\frac{y}{4}\right)^{1-\frac{D}{2}} + f_0(y)\right) \tag{C.30}$$

$$f_m(y) = \sum_{n=m}^{\infty}\Bigg[\frac{\Gamma\left(\frac{D-1}{2}\pm\nu\right)\Gamma(1-\frac{D}{2})}{\Gamma(\frac{1}{2}\pm\nu)}\frac{\left(\frac{D-1}{2}\pm\nu\right)_n}{\left(\frac{D}{2}\right)_n n!}\left(\frac{y}{4}\right)^n + \frac{\Gamma\left(\frac{D}{2}-1\right)\left(\frac{1}{4}-\nu^2\right)}{2-\frac{D}{2}}\frac{\left(\frac{3}{2}\pm\nu\right)_n}{n!(n+1)\left(3-\frac{D}{2}\right)_n}\left(\frac{y}{4}\right)^{n+2-\frac{D}{2}}\Bigg] \tag{C.31}$$

To calculate these identities, we must make use of a process known as extracting derivatives, which is detailed in Appendix A. For the propagator square we find

$$(i\Delta(x;x'))^2 = \frac{2}{(D-3)(D-4)}\frac{\Gamma\left(\frac{D}{2}-1\right)\hbar^2 H^{D-4}}{(4\pi)^{D/2}}\frac{i\delta^D(x-x)}{\sqrt{-g}} + \frac{\hbar^2 H^4}{(4\pi)^4}\left(g(y)+\frac{8}{y}f_0(y)+f_0^2(y)\right) \tag{C.32}$$

Next we consider the derivative of each scalar propagator in the square. We need to apply several more renditions of this extracting d'Alembertians operation, giving

$$\partial_\mu i\Delta(x;x')\partial_\nu i\Delta(x;x') = \frac{\hbar^2\Gamma\left(\frac{D}{2}-1\right)H^{D-4}}{(4\pi)^{D/2}}\frac{1}{(D-3)(D-4)}$$
$$\times\left\{\frac{1}{2}\frac{(D-2)}{(D-1)}\nabla_\mu\nabla_\nu + \frac{1}{2}\frac{1}{(D-1)}g_{\mu\nu}\Box - \frac{1}{2}(D-2)H^2 g_{\mu\nu} - \left(\frac{1}{4}-\nu^2\right)H^2 g_{\mu\nu}\right\}\frac{i\delta^D(x-x')}{\sqrt{-g}}$$
$$+\frac{\hbar^2 H^4}{(4\pi)^4}\Bigg\{\left(\frac{1}{6}\nabla_\mu\nabla_\nu + \frac{1}{12}g_{\mu\nu}\Box - \frac{1}{2}H^2 g_{\mu\nu} - \frac{1}{2}\left(\frac{1}{4}-\nu^2\right)H^2 g_{\mu\nu}\right)g(y)$$
$$+\left(\frac{1}{4}-\nu^2\right)H^2 g_{\mu\nu}\frac{4}{y} - \left(\frac{1}{4}-\nu^2\right)\nabla_\mu\nabla_\nu\frac{4}{y} + 2\partial_{(\mu}\frac{4}{y}\partial_{\nu)}f_1(y) + \partial_\mu f_0\partial_\nu f_0\Bigg\} \tag{C.33}$$

Now for the second derivative form, $\partial_\mu \partial'_\rho i\Delta(x;x')\partial'_\sigma \partial_\nu i\Delta(x;x')$. Solving this makes use of Eq. (B.13). We start with

$$\partial_\mu \partial'_\rho i\Delta(x;x')\partial'_\sigma \partial_\nu i\Delta(x;x') = \frac{\hbar^2 H^{2D-4}}{(4\pi)^D}\Gamma^2\left(\frac{D}{2}-1\right)\Bigg\{$$

$$\partial_{\mu)}\partial'_{(\rho}\left(\frac{y}{4}\right)^{1-\frac{D}{2}}\partial'_{\sigma)}\partial_{(\nu}\left(\frac{y}{4}\right)^{1-\frac{D}{2}} - 4\frac{\left(\frac{1}{4}-\nu^2\right)}{(D-4)}\partial_{\mu)}\partial'_{(\rho}\left(\frac{y}{4}\right)^{1-\frac{D}{2}}\partial'_{\sigma)}\partial_{(\nu}\left(\frac{y}{4}\right)^{2-\frac{D}{2}}$$

$$+ 4\frac{\left(\frac{1}{4}-\nu^2\right)\left(\frac{9}{4}-\nu^2\right)}{(D-4)(D-6)}\partial_{\mu)}\partial'_{(\rho}\left(\frac{y}{4}\right)^{1-\frac{D}{2}}\partial'_{\sigma)}\partial_{(\nu}\left(\frac{y}{4}\right)^{3-\frac{D}{2}} + 4\frac{\left(\frac{1}{4}-\nu^2\right)^2}{(D-4)^2}\partial_{\mu)}\partial'_{(\rho}\left(\frac{y}{4}\right)^{2-\frac{D}{2}}\partial'_{\sigma)}\partial_{(\nu}\left(\frac{y}{4}\right)^{2-\frac{D}{2}}\Bigg\}$$

$$+ \frac{\hbar^2 H^{2D-4}}{(4\pi)^D}\Bigg\{4\Gamma\left(\frac{D}{2}-1\right)\Gamma\left(1-\frac{D}{2}\right)\frac{\left(\frac{D-1}{2}\right)^2-\nu^2}{D}\frac{\Gamma\left(\frac{D-1}{2}\pm\nu\right)}{\Gamma\left(\frac{1}{2}\pm\nu\right)}\partial_{\mu)}\partial'_{(\rho}\left(\frac{y}{4}\right)^{1-\frac{D}{2}}\partial'_{\sigma)}\partial_{(\nu}\left(\frac{y}{4}\right)$$

$$+ 2\Gamma\left(\frac{D}{2}-1\right)\partial_{\mu)}\partial'_{(\rho}\left(\frac{y}{4}\right)^{1-\frac{D}{2}}\partial'_{\sigma)}\partial_{(\nu}f_2(y) - 2\Gamma\left(\frac{D}{2}-1\right)\frac{\left(\frac{1}{4}-\nu^2\right)}{(D-4)}\partial_{\mu)}\partial'_{(\rho}\left(\frac{y}{4}\right)^{2-\frac{D}{2}}\partial'_{\sigma)}\partial_{(\nu}f_1(y)$$

$$+ \partial_{\mu)}\partial'_{(\rho}f_1(y)\partial'_{\sigma)}\partial_{(\nu}f_1(y)\Bigg\}, \tag{C.34}$$

where we expanded $\partial_\mu \partial'_\rho f_0(y)\partial'_\sigma \partial_\nu f_0(y)$ to extract divergences. Next we must evaluate Eq. (B.13) for five sets of α, β values.

The final result, summing all of the terms together is:

$$\partial_\mu \partial'_\rho i\Delta(x;x')\partial'_\sigma \partial_\nu i\Delta(x;x') = \frac{\hbar^2 H^{D-4}}{(4\pi)^{D/2}} \frac{\Gamma\left(\frac{D}{2}-1\right)}{(D-3)(D-4)} \Bigg\{$$

$$\frac{1}{8}\frac{D(D-2)}{(D+1)(D-1)}\nabla_\mu\nabla_\nu\nabla'_\rho\nabla'_\sigma + \frac{1}{8}\frac{D}{(D+1)(D-1)}\left[H^2 g_{\mu\nu}\nabla'_\rho\nabla'_\sigma + H^2 g'_{\rho\sigma}\nabla_\mu\nabla_\nu\right]\frac{\Box}{H^2}$$
$$-\left(\frac{1}{8}\frac{D(D-2)}{(D+1)} + \frac{1}{4}\left(\frac{1}{4}-\nu^2\right)\frac{(D-2)}{(D-1)}\right)\left[H^2 g_{\mu\nu}\nabla'_\rho\nabla'_\sigma + H^2 g'_{\rho\sigma}\nabla_\mu\nabla_\nu\right]$$
$$+\frac{1}{8}\frac{1}{(D+1)(D-1)}H^4 g_{\mu\nu}g'_{\rho\sigma}\frac{\Box}{H^2}\frac{\Box'}{H^2} + \frac{1}{4}\frac{1}{(D+1)(D-1)}H^4\left[{}_\mu g_\rho\right]\left[{}_\rho g_\sigma\right]\frac{\Box}{H^2}\frac{\Box'}{H^2}$$
$$-\frac{1}{8}\frac{((D+1)(D-2)+D(D-1))}{(D+1)(D-1)}H^4 g_{\mu\nu}g'_{\rho\sigma}\frac{\Box}{H^2} - \frac{1}{4}\frac{1}{(D-1)}\left(\frac{1}{4}-\nu^2\right)H^4 g_{\mu\nu}g'_{\rho\sigma}\frac{\Box}{H^2}$$
$$-\frac{1}{4}\frac{(D-2)}{(D+1)(D-1)}H^4\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\frac{\Box}{H^2} - \frac{1}{2}\frac{1}{D-1}\left(\frac{1}{4}-\nu^2\right)H^4\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\frac{\Box}{H^2}$$
$$+\frac{1}{2}\frac{1}{(D+1)(D-1)}H^2\left[{}_{\mu)}g_{(\rho}\right]\nabla'_{\sigma)}\nabla_{(\nu}\frac{\Box}{H^2} - \frac{1}{2}\frac{(D-2)}{(D+1)(D-1)}H^2\left[{}_{\mu)}g_{(\rho}\right]\nabla'_{\sigma)}\nabla_{(\nu}$$
$$-\frac{1}{(D-1)}\left(\frac{1}{4}-\nu^2\right)H^2\left[{}_{\mu)}g_{(\rho}\right]\nabla'_{\sigma)}\nabla_{(\nu} + \frac{1}{8}\frac{D(D-1)(D-2)}{(D+1)}H^4 g_{\mu\nu}g'_{\rho\sigma}$$
$$+\frac{1}{2}\frac{(D-2)^2}{(D-1)}\left(\frac{1}{4}-\nu^2\right)H^4 g_{\mu\nu}g'_{\rho\sigma} + \frac{1}{2}\frac{(D-2)}{(D-1)}\left(\frac{1}{4}-\nu^2\right)H^4\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]$$
$$+\left(\frac{1}{4}-\nu^2\right)\left(\frac{9}{4}-\nu^2\right)\left[\frac{1}{8}\frac{D}{(D-1)}H^4 g_{\mu\nu}g'_{\rho\sigma} + \frac{1}{4}\frac{D}{(D-1)}H^4\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\right]$$
$$+\left(\frac{1}{4}-\nu^2\right)^2\left[\frac{1}{8}\frac{(D-2)}{(D-1)}H^4 g_{\mu\nu}g'_{\rho\sigma} + \frac{1}{4}\frac{(D-2)}{(D-1)}H^4\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\right]\Bigg\}\frac{i\,\delta^D(x-x')}{\sqrt{-g}}$$
$$+\frac{\hbar^2 H^4}{(4\pi)^4}\Bigg\{\frac{1}{30}\nabla_\mu\nabla_\nu\nabla'_\rho\nabla'_\sigma g(y) + \left(H^2 g_{\mu\nu}\nabla'_\rho\nabla'_\sigma + H^2 g'_{\rho\sigma}\nabla_\mu\nabla_\nu\right)\left(\frac{1}{60}\frac{\Box}{H^2}g(y) - \frac{1}{10}g(y)\right)$$
$$+H^4 g_{\mu\nu}g'_{\rho\sigma}\left(\frac{1}{240}\frac{\Box}{H^2}\frac{\Box'}{H^2}g(y) - \frac{11}{120}\frac{\Box}{H^2}g(y) + \frac{3}{10}g(y)\right)$$
$$+\frac{1}{120}\Big(\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\left(\Box\Box' - 2H^2\Box\right) + 2\left[{}_\mu g_\rho\right]\nabla'_\sigma\nabla_\nu\left(\Box - 2H^2\right)\Big)g(y)$$
$$+\left(\frac{1}{4}-\nu^2\right)\Bigg[-\frac{1}{6}\nabla_\mu\nabla_\nu\nabla'_\rho\nabla'_\sigma\frac{4}{y} + \left(H^2 g_{\mu\nu}\nabla'_\rho\nabla'_\sigma + H^2 g'_{\rho\sigma}\nabla_\mu\nabla_\nu\right)\left(-\frac{1}{12}g(y) + \frac{1}{6}\frac{4}{y}\right)$$
$$+H^4 g_{\mu\nu}g'_{\rho\sigma}\left(-\frac{1}{6}\frac{4}{y} - \frac{1}{24}\frac{\Box}{H^2}g(y) + \frac{1}{3}g(y)\right) - \frac{1}{6}H^2\left[{}_{\mu)}g_{(\rho}\right]\nabla'_{\sigma)}\nabla_{(\nu}g(y)$$
$$-\frac{1}{12}H^2\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\left(\Box - 2H^2\right)g(y)\Bigg]$$
$$+\left[-\frac{1}{2}\left(\frac{9}{4}-\nu^2\right) - \frac{3}{2}\left(\frac{1}{4}-\nu^2\right)\right]H^2\left[{}_{\mu)}g_{(\rho}\right]\nabla'_{\sigma)}\nabla_{(\nu}\frac{4}{y}$$

$$
\begin{aligned}
&+\left(\frac{1}{4}-\nu^2\right)\left(\frac{9}{4}-\nu^2\right)\Bigg[-\frac{1}{3}\nabla_\mu\nabla_\nu\nabla'_\rho\nabla'_\sigma\ln(y/4)+\frac{1}{6}H^2\Big(g_{\mu\nu}\nabla'_\rho\nabla'_\sigma+g'_{\rho\sigma}\nabla_\mu\nabla_\nu\Big)\frac{4}{y}\\
&\qquad+\frac{1}{12}H^4\Big(g_{\mu\nu}g'_{\rho\sigma}+2\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\Big)g(y)-\frac{1}{6}H^4 g_{\mu\nu}g'_{\rho\sigma}\frac{4}{y}-2H^2\left[{}_{\mu)}g_{(\rho}\right]\nabla'_{\sigma)}\nabla_{(\nu}\frac{\ln(y/4)}{y}\\
&\qquad+\left(\psi(1)-\frac{5}{12}-\frac{1}{2}\psi\left(\frac{1}{2}\pm\nu\right)\right)H^2\left[{}_{\mu)}g_{(\rho}\right]\nabla'_{\sigma)}\nabla_{(\nu}\frac{4}{y}\Bigg]\\
&+\left(\frac{1}{4}-\nu^2\right)^2\Bigg[-\frac{1}{6}\nabla_\mu\nabla_\nu\nabla'_\rho\nabla'_\sigma\ln(y/4)+\frac{1}{12}H^2\Big(g_{\mu\nu}\nabla'_\rho\nabla'_\sigma+g'_{\rho\sigma}\nabla_\mu\nabla_\nu\Big)\frac{4}{y}\\
&\qquad+\frac{1}{24}H^4\Big(g_{\mu\nu}g'_{\rho\sigma}+2\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\Big)g(y)-\frac{1}{12}H^4 g_{\mu\nu}g'_{\rho\sigma}\frac{4}{y}+\frac{1}{6}H^2\left[{}_{\mu)}g_{(\rho}\right]\nabla'_{\sigma)}\nabla_{(\nu}\frac{4}{y}\Bigg]\\
&+2\partial_\mu\partial'_\rho\frac{4}{y}\partial_\nu\partial'_\sigma f_2(y)+2\left(\frac{1}{4}-\nu^2\right)\partial_\mu\partial'_\rho\ln(y/4)\partial_\nu\partial'_\sigma f_1(y)+\partial_\mu\partial'_\rho f_1(y)\partial_\nu\partial'_\sigma f_1(y)\Bigg\}
\end{aligned}
\tag{C.35}
$$

It is more useful to write the divergent part in this way, splitting the H^2, m^2 dependences

$$
\begin{aligned}
\partial_\mu\partial'_\rho i\Delta(x;x')\partial'_\sigma\partial_\nu i\Delta(x;x') &= \frac{\hbar^2 H^{D-4}\Gamma\left(\frac{D}{2}-1\right)}{(4\pi)^{D/2}}\frac{1}{(D-3)(D-4)}\\
\times\Bigg\{&\frac{1}{8}\frac{D(D-2)}{(D+1)(D-1)}\nabla_\mu\nabla_\nu\nabla'_\rho\nabla'_\sigma+\frac{1}{8}\frac{D}{(D+1)(D-1)}\left[g_{\mu\nu}\nabla'_\rho\nabla'_\sigma+g'_{\rho\sigma}\nabla_\mu\nabla_\nu\right]\square\\
&+\frac{1}{8}\frac{1}{(D+1)(D-1)}g_{\mu\nu}g'_{\rho\sigma}\square\square'+\frac{1}{4}\frac{1}{(D+1)(D-1)}\Big(\left[{}_\mu g_\rho\right]\left[{}_\rho g_\sigma\right]\square\square'+2\left[{}_{\mu)}g_{(\rho}\right]\nabla'_{\sigma)}\nabla_{(\nu}\square\Big)\\
&+H^2\Bigg[-\left(\frac{1}{8}\frac{D(D-2)}{(D+1)}\right)\Big(g_{\mu\nu}\nabla'_\rho\nabla'_\sigma+g'_{\rho\sigma}\nabla_\mu\nabla_\nu\Big)\\
&\qquad-\frac{1}{8}\frac{((D+1)(D-2)+D(D-1))}{(D+1)(D-1)}g_{\mu\nu}g'_{\rho\sigma}\square\\
&\qquad-\frac{1}{4}\frac{(D-2)}{(D+1)(D-1)}\Big(\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\square+2\left[{}_{\mu)}g_{(\rho}\right]\nabla'_{\sigma)}\nabla_{(\nu)}\Big)\Bigg]\\
&+H^4\left[\frac{1}{8}\frac{D(D-1)(D-2)}{(D+1)}g_{\mu\nu}g'_{\rho\sigma}\right]\\
&+\left(\frac{1}{4}-\nu^2\right)\Bigg[H^2\Bigg(-\frac{1}{4}\frac{(D-2)}{(D-1)}\Big(g_{\mu\nu}\nabla'_\rho\nabla'_\sigma+g'_{\rho\sigma}\nabla_\mu\nabla_\nu\Big)-\frac{1}{4}\frac{1}{(D-1)}g_{\mu\nu}g'_{\rho\sigma}\square\\
&\qquad-\frac{1}{2}\frac{1}{D-1}\Big(\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\square+2\left[{}_{\mu)}g_{(\rho}\right]\nabla'_{\sigma)}\nabla_{(\nu}\Big)\Bigg)\\
&\qquad+H^4\left(\frac{1}{4}\frac{2(D-2)^2+D}{(D-1)}g_{\mu\nu}g'_{\rho\sigma}+\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\right)\Bigg]\\
&+\left(\frac{1}{4}-\nu^2\right)^2 H^4\left(\frac{1}{4}g_{\mu\nu}g'_{\rho\sigma}+\frac{1}{2}\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\right)\Bigg\}\frac{i\,\delta^D(x-x')}{\sqrt{-g}}\\
&+\textit{finite-terms}
\end{aligned}
\tag{C.36}
$$

D Curvature Variations

This section deals with the variations induced in Curvature tensors due to adding a perturbation of a background metric $g_{\mu\nu} \to g_{\mu\nu} + \delta g_{\mu\nu}$. Standard variational principle gives

$$\delta g_{\mu\nu} = -g_{\mu\rho} g_{\nu\sigma} \delta g^{\rho\sigma}, \tag{D.1}$$

$$\delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g} g_{\alpha\beta} \delta g^{\alpha\beta}. \tag{D.2}$$

The curvature tensors/objects we are interested in are defined bellow, with their value on de Sitter:

$$\sqrt{-g} = (-\det(g_{\mu\nu}))^{1/2} = a^D, \tag{D.3}$$

$$\Gamma^\rho_{\mu\nu} = \frac{1}{2} g^{\rho\sigma}(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} + \partial_\sigma g_{\mu\nu}) = aH(\delta^0_\mu \delta^\rho_\nu + \delta^0_\nu \delta^\rho_\mu - \eta^{\rho 0}\eta_{\mu\nu}), \tag{D.4}$$

$$R^\rho{}_{\mu\sigma\nu} = 2\partial_{[\sigma}\Gamma^\lambda_{\nu]\mu} + 2\Gamma^\rho_{\alpha[\sigma}\Gamma^\alpha_{\nu]\mu} = H^2(g_{\rho\sigma}g_{\mu\nu} - g_{\rho\nu}g_{\mu\sigma}) = 2H^2 g_{\rho[\sigma}g_{\nu]\mu}, \tag{D.5}$$

$$R_{\mu\nu} = g^{\rho\sigma} R_{\rho\mu\sigma\nu} = H^2(D-1) g_{\mu\nu}, \tag{D.6}$$

$$R = g^{\mu\nu} R_{\mu\nu} = H^2 D(D-1), \tag{D.7}$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = H^2 \frac{(D-1)(2-D)}{2} g_{\mu\nu}, \tag{D.8}$$

$$C^\rho{}_{\mu\sigma\nu} = R^\rho{}_{\mu\sigma\nu} - \frac{2}{(D-2)}\left(\delta^\rho_{[\sigma} R_{\nu]\mu} + g_{\mu[\nu} R_{\sigma]}{}^\rho\right) + \frac{2}{(D-1)(D-2)} \delta^\rho_{[\sigma} g_{\nu]\mu} R \tag{D.9}$$

$$= 0 \text{ on de Sitter} \tag{D.10}$$

To find the first and second variations of these quantities, we follow the procedure in Section 35.14 of [62].

$$\delta R^\gamma{}_{\alpha\delta\beta} = \nabla_\delta \delta\Gamma^\gamma_{\beta\alpha} - \nabla_\beta \delta\Gamma^\gamma_{\delta\alpha}, \tag{D.11}$$

$$= \frac{1}{2} g^{\gamma\lambda}\Big(- g_{\alpha\mu}g_{\lambda\nu}\nabla_\delta\nabla_\beta - g_{\beta\mu}g_{\lambda\nu}\nabla_\delta\nabla_\alpha + g_{\alpha\mu}g_{\beta\nu}\nabla_\delta\nabla_\lambda$$
$$+ g_{\alpha\mu}g_{\lambda\nu}\nabla_\beta\nabla_\delta + g_{\delta\mu}g_{\lambda\nu}\nabla_\beta\nabla_\alpha - g_{\alpha\mu}g_{\delta\nu}\nabla_\beta\nabla_\lambda\Big)\delta g^{\mu\nu}, \tag{D.12}$$

$$\delta R_{\alpha\beta} = \nabla_\lambda \delta\Gamma^\lambda_{\beta\alpha} - \nabla_\beta \delta\Gamma^\mu_{\mu\alpha} = \left(-g_{\alpha)\mu}\nabla_\nu\nabla_{(\beta} + \frac{1}{2} g_{\alpha\mu}g_{\beta\nu}\Box + \frac{1}{2} g_{\mu\nu}\nabla_\beta\nabla_\alpha\right)\delta g^{\mu\nu}, \tag{D.13}$$

$$\delta^2 R^\gamma{}_{\alpha\delta\beta} = \left(\frac{1}{2}g_{\delta\kappa}\nabla^{[\lambda}\delta g^{\gamma]\kappa} - \frac{1}{4}\nabla_\delta \delta g^{\gamma\lambda}\right)\left(g_{\alpha\mu}g_{\lambda\nu}\nabla_\beta + g_{\beta\mu}g_{\lambda\nu}\nabla_\alpha - g_{\alpha\mu}g_{\beta\nu}\nabla_\lambda\right)\delta g^{\mu\nu}$$
$$- \left(\frac{1}{2}g_{\beta\kappa}\nabla^{[\lambda}\delta g^{\gamma]\kappa} - \frac{1}{4}\nabla_\beta \delta g^{\gamma\lambda}\right)\left(g_{\alpha\mu}g_{\lambda\nu}\nabla_\delta + g_{\delta\mu}g_{\lambda\nu}\nabla_\alpha - g_{\alpha\mu}g_{\delta\nu}\nabla_\lambda\right)\delta g^{\mu\nu}$$
$$+ \delta g^{\gamma\lambda}\left(g_{\alpha\mu}g_{\lambda\nu}\nabla_{[\delta}\nabla_{\beta]} + g_{\delta]\mu}g_{\lambda\nu}\nabla_\alpha\nabla_{[\beta} - g_{\delta]\mu}g_{\alpha\nu}\nabla_\lambda\nabla_{[\beta}\right)\delta g^{\mu\nu} \tag{D.14}$$

$$\delta^2 R_{\alpha\beta} = \frac{1}{4}g_{\mu\rho}g_{\nu\sigma}\nabla_\alpha \delta g^{\rho\sigma}\nabla_\beta \delta g^{\mu\nu} + \frac{1}{2}g_{\beta\nu}g_{\mu\rho}g_{\alpha\sigma}\nabla^\lambda \delta g^{\mu\nu}\nabla_\lambda \delta g^{\rho\sigma} - \frac{1}{2}g_{\beta\mu}g_{\alpha\rho}\nabla_\sigma \delta g^{\mu\nu}\nabla_\nu \delta g^{\rho\sigma}$$
$$+ \frac{1}{2}\delta g^{\mu\nu}\left(g_{\mu\rho}g_{\nu\sigma}\nabla_\alpha\nabla_\beta + g_{\alpha\rho}g_{\beta\sigma}\nabla_\mu\nabla_\nu - g_{\mu\rho}g_{\alpha\sigma}\nabla_\nu\nabla_\beta - g_{\mu\rho}g_{\beta\sigma}\nabla_\nu\nabla_\alpha\right)\delta g^{\rho\sigma}$$
$$- \frac{1}{2}\nabla_\nu \delta g^{\mu\nu}\left(g_{\mu\rho}g_{\alpha\sigma}\nabla_\beta \delta g^{\rho\sigma} + g_{\mu\rho}g_{\beta\sigma}\nabla_\alpha \delta g^{\rho\sigma} - g_{\alpha\rho}g_{\beta\sigma}\nabla_\mu \delta g^{\rho\sigma}\right)$$
$$+ \frac{1}{4}g_{\mu\nu}\nabla^\lambda \delta g^{\mu\nu}\left(g_{\lambda\rho}g_{\alpha\sigma}\nabla_\beta \delta g^{\rho\sigma} + g_{\lambda\rho}g_{\beta\sigma}\nabla_\alpha \delta g^{\rho\sigma} - g_{\alpha\rho}g_{\beta\sigma}\nabla_\lambda \delta g^{\rho\sigma}\right) \tag{D.15}$$

In order to take the functional derivatives, we employ

$$\frac{\delta g^{\alpha\beta}(y)}{\delta g^{\mu\nu}(x)} = \left[{}_\mu g^\alpha\right](x;y)\left[{}_\nu g^\beta\right](x;y)\delta^D(x-x') \tag{D.16}$$

We must be careful to ensure the bitensorial form is unbroken during our calculations, and so we do not change the space-time point of μ,ν or ρ,σ derivatives/metrics. This leaves us with the following identities to change the position of a derivative

$$\left[{}_\mu g_\rho\right]\left[{}^\mu g_\sigma\right]\frac{\delta^D(x-x')}{\sqrt{-g}} = g_{\rho\sigma}(x')\frac{\delta^D(x-x')}{\sqrt{-g}} \tag{D.17}$$
$$\left[{}^\mu g_\rho\right]\nabla_\mu \frac{\delta^D(x-x')}{\sqrt{-g}} = -\nabla'_\rho \frac{\delta^D(x-x')}{\sqrt{-g}}, \tag{D.18}$$
$$\left[{}^\mu g_\rho\right]\nabla_\mu\nabla_\nu \frac{\delta^D(x-x')}{\sqrt{-g}} = \left(-\nabla'_\rho\nabla_\nu - H^2\left[{}_\nu g_\rho\right]\right)\frac{\delta^D(x-x')}{\sqrt{-g}}, \tag{D.19}$$
$$\Rightarrow \nabla_\mu\nabla_\nu\left(\left[{}^\mu g_\rho\right]\left[{}^\nu g_\sigma\right]\frac{\delta^D(x-x')}{\sqrt{-g}}\right) = \nabla'_\rho\nabla'_\sigma \frac{\delta^D(x-x')}{\sqrt{-g}}. \tag{D.20}$$

Once this is understood, the variation follows quickly. We want the Ricci scalar, the Ricci Scalar square and the Weyl Tensor variations for our counterterms

$$\frac{\delta^2}{\delta g^{\mu\nu}(x)\delta g^{\rho\sigma}(x')}\int d^D y\sqrt{-g}R = \sqrt{-g(x)}\sqrt{-g(x')}\Big\{$$
$$\frac{(D-4)(D-1)}{4}H^2 g_{\mu\nu}g'_{\rho\sigma} + \frac{(D-2)(D-1)}{2}H^2\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right] - \frac{1}{2}g_{\mu\nu}g'_{\rho\sigma}\square$$
$$+ \frac{1}{2}\left(g_{\mu\nu}\nabla'_\rho\nabla'_\sigma + g'_{\rho\sigma}\nabla_\mu\nabla_\nu\right) + \frac{1}{2}\left(\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\square + 2\nabla'_{\sigma)}\nabla_{(\nu}\left[{}_{\mu)}g_{(\rho}\right]\right)\Big\}\frac{\delta^D(x-x')}{\sqrt{-g}}, \tag{D.21}$$

$$\frac{\delta^2}{\delta g^{\mu\nu}(x)\delta g^{\rho\sigma}(x')}\int d^D y\sqrt{-g}R^2 = \sqrt{-g(x)}\sqrt{-g(x')}\Bigg\{$$
$$+ 2\nabla_\mu\nabla_\nu\nabla'_\rho\nabla'_\sigma - 2\Big(g_{\mu\nu}\nabla'_\rho\nabla'_\sigma\Box + g'_{\rho\sigma}\nabla_\mu\nabla_\nu\Box'\Big) + 2g_{\mu\nu}g'_{\rho\sigma}\Box\Box'$$
$$+ H^2\Bigg[(D-2)(D-1)\Bigg(g_{\mu\nu}\nabla'_\rho\nabla'_\sigma + g'_{\rho\sigma}\nabla_\mu\nabla_\nu\Bigg) - (D-4)(D-1)g_{\mu\nu}g'_{\rho\sigma}\Box$$
$$+ D(D-1)\Bigg(\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\Box + 2\left[{}_\mu g_\rho\right]\nabla'_\sigma\nabla_\nu\Bigg)\Bigg]$$
$$H^4\Bigg[\Bigg(\frac{D^2(D-1)^2}{4} - 2(D-1)^3\Bigg)g_{\mu\nu}g'_{\rho\sigma}$$
$$+ \Bigg(\frac{1}{2}D^2\left(D-1\right)^2 - 2D(D-1)^2\Bigg)H^4\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\Bigg]\Bigg\}\frac{\delta^D(x-x')}{\sqrt{-g}} \tag{D.22}$$

$$\frac{\delta^2}{\delta g^{\mu\nu}(x)\delta g^{\rho\sigma}(x')}\int d^D y\sqrt{-g}C^2 = \sqrt{-g(x)}\sqrt{-g(x')}\frac{2(D-3)}{(D-2)}\Bigg\{$$
$$\frac{(D-2)}{(D-1)}\nabla_\mu\nabla_\nu\nabla'_\rho\nabla'_\sigma + \frac{1}{(D-1)}\Big(g_{\mu\nu}\nabla'_\rho\nabla'_\sigma\Box + g'_{\rho\sigma}\nabla_\mu\nabla_\nu\Box'\Big)$$
$$- \frac{1}{(D-1)}g_{\mu\nu}g'_{\rho\sigma}\Box\Box' + \Big(\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\Box\Box' + 2\left[{}_\mu g_\rho\right]\nabla'_\sigma\nabla_\nu\Box'\Big)$$
$$- 2H^2\Bigg(g_{\mu\nu}\nabla'_\rho\nabla'_\sigma + g'_{\rho\sigma}\nabla_\mu\nabla_\nu\Bigg) - H^2 g_{\mu\nu}g'_{\rho\sigma}\Box$$
$$- H^2(D-2)\Big(\left[{}_\mu g_\rho\right]\left[{}_\nu g_\sigma\right]\Box + 2\left[{}_\mu g_\rho\right]\nabla'_\sigma\nabla_\nu\Big) + 2H^4(D-1)g_{\mu\nu}g'_{\rho\sigma}\Bigg\}\frac{\delta^D(x-x')}{\sqrt{-g}} \tag{D.23}$$

Note that we can actually express this R^2 as a linear combination of the following:

$$R^2 = \Big[R - D(D-1)H^2\Big]^2 + 2D(D-1)H^2\Big[R - (D-2)(D-1)H^2\Big]$$
$$- \Big[(D(D-1)H^2)^2 - 2D(D-1)^2(D-2)H^4\Big], \tag{D.24}$$

which aligns in the counterterms used in [39], and allows one to make use of only the first variation of the Ricci scalar for the first term, the Lichnerowicz operator for the second and the variation of the volume element $\sqrt{-g}$ for the third term.

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