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USING THE ARTS TO ENRICH STUDENTS' PERCEPTIONS OF MATHEMATICS IN HIGHER EDUCATION.

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Abstract

Some students perceive mathematics as inflexible where it consists of purely the reproduction of a fixed set of rules and methods. Some also find it difficult to transfer the mathematics knowledge learnt in the classroom into their daily lives. To solve this, students must be able to attach meaning to what mathematics is and how it can be used in daily life. In this article it's proposed that a possible reason for such perceptions is the element of creativity being a neglected domain in mathematics education. To incorporate this element of creativity, the arts has been used as a context to teach higher level mathematics. In this study, it is evaluated whether this incorporation of art in mathematics education enriches students' perceptions of the subject. A pre-post questionnaire was administered to 35 bachelor students to measure change in their perceptions. Individual interviews were conducted with three students to follow their learning process and gradual perception change. Results showed that learning mathematics within this context of arts gradually changed students' perceptions of mathematics to more positive ideas of math as the course progressed. Towards the end of the course more students believed they had freedom to be creative, explore, and discover new things within mathematics education. This research promotes the facilitation of the arts as a context to teach mathematics education, highlighting that this context is a means for enriching students' perceptions of mathematics.

Introduction

For centuries the subject of mathematics has been deemed as the primary engine for the development of students' logical thinking and higher-order cognitive skills. However, the subject of mathematics is commonly seen as difficult, obscure, and of little interest, despite the many benefits mathematics knowledge brings (Hagan, 2020). The subject is seen to only have important applications for basic numerical skills and having little relevance to everyday life. Thus, some students consider mathematics as a domain comprising of merely abstract and disjointed principles, equations and formulas that have no concern with the needs of daily life, instead of viewing it as a tool able to be used in various aspects of life (Baki, 2014). So, it is not surprising that there are several myths associated with the subject of mathematics. These myths consist of views commonly expressed as: "mathematics is just computation", and "mathematics is only for clever people". (Sam & Ernest, 2000). Additionally, students with such perceptions tend to be unaware of the relation of mathematics to real life and find math-related activities meaningless and unnecessary also having poor academic performance in the subject itself.

However, the question may arise as to what causes such perceptions. There are of course different hypothesized reasons, namely: students' intrinsic motivation (Mariamah et al., 2012), engagement in the classroom (Attard, 2011), the lack of context (Vos, 2008), lack of creativity (Leikin & Pitta-Pantazi, 2012) in mathematics lessons, students' inability to link mathematics to their everyday life (Yavuz Mumcu,2018; Young-Loveridge et al. 2006), and the most frequently mentioned reason: a student's teacher (Mariamah et al., 2012; Paksu, 2008). Lester, Garofalo, and Lambdin Kroll (1989) also argued that for teachers to properly assess or evaluate students' mathematical knowledge, they must first be aware of students' perceptions.

This leads us to think on how mathematics knowledge is being transferred to students through teaching and what is the knowledge that students take away from a mathematics classroom. Teachers play a crucial role and are generally expected to adopt various methodologies when teaching mathematics, to keep content interesting. Teachers are essentially facilitators of change, and being able to engage students in mathematical activity is critical to active learning (Uysal, 2018). Teachers have the opportunity to change negative perceptions of a subject. Within teaching, subtle messages can be communicated about the nature of mathematics to potentially frame the way mathematics is perceived. The idea that mathematics is a subject for all pupils and not solely for persons who thrive to become mathematicians can be promoted, namely encouraging the idea of "mathematics for all" by Jan de Lange. Rethinking mathematics in a global context offers a twist on typical mathematical content, making the subject more applicable and meaningful for students (Norman, 2021). It is therefore important for mathematics educators to create connections that highlight how mathematics is related to everyday life and show it's applicability, since this is an element that many students are lacking.

To expand on this, it is argued that school mathematics offers students with inauthentic problems, which tend not to show the usefulness of mathematics in real life (Vos, 2008). Several studies have also shown that students are more motivated by authentic questions. So, the use of context, providing students with authentic problems akin to what can be found in real life scenarios are of great importance in mathematics education. However, it has also been argued that these misconceptions of mathematics may arise as creativity has been a neglected domain within mathematics education (Leikin & Pitta-Pantazi, 2012). When thinking of creativity, the main concept that comes to mind for most individuals is art. Therefore, Burnard et al (2016) proposed that art as a context for mathematical problem solving can be an advantageous starting point for bridging the avenues of creativity and mathematics. Within this study it will be analyzed whether students' perceptions of mathematics are enriched based on the inclusion of the arts in mathematics education.

Research has indicated that the areas of mathematics application expand as technology continues to advance (Bondi, 1991, Neyland, 1994, Restivo, van Bendegen, & Fischer, 1993). This in turn increases the need for individuals to possess mathematical knowledge as well as be able to use it in their daily lives. Nevertheless, it is seen that most students find it difficult to acquire mathematical skills needed to comprehend the subject, much less to see the usefulness of the subject in everyday life (Hagan, 2020).

One reason for this may be the fact that some teachers may stray away from developing students' understanding of how mathematics is used in everyday life and are more focused on students being able to find the correct answer to a mathematical problem (Paksu, 2008). With this focus teachers convey the belief that mathematics is divided into right and wrong answers (Balka, 1974; Ginsburg, 1996) and reject creative applications within mathematics. Therefore, being taught in this way reinforces student's conception of mathematics as solely an application of mathematical techniques, overemphasizing rules, algorithms, and convergent thinking to produce a single right answer (Mann, 2006). This then obstructs the true essence of mathematics, which is to promote creative thinking and not simply arriving to the "right answer" (Dreyfus & Eisenberg, 1966; Ginsburg, 1996).

Consequently, it is argued that there is a growing need for mathematical instruction that focuses on the creative process of performing mathematical problems (Lavicza et al, 2018). As Burnard et al (2016) stated art is a great context for linking the domains of creativity and mathematics, as art is commonly associated with encouraging individuals to find their own way as well as stimulating creative thinking. Learning through this context may challenge common misconceptions that creativity and mathematics are disconnected entities, but also challenging students' misconceptions of mathematics. Teaching mathematics through providing aspects of creativity gives students the opportunity to appreciate the beauty of mathematics (Mann, 2006).

But why this aspect of creativity? Creativity typically refers to the act of producing new ideas, approaches, or actions, and this can be done within the domain of mathematics. Liljedahl and Sriraman (2006) proposed that mathematical creativity can be defined as ‘‘the ability to produce original work that significantly extends the body of knowledge’’ or ‘‘opens up avenues of new views or questions within mathematics”. Thus, a student’s ability to produce mathematical ideas and solutions in new situations or the production of original solutions to previously learnt problems is considered to be an indicator of creativity (Leikin & Pitta-Pantazi, 2013). The facilitation of creativity can therefore act as a method to enrich students’ perceptions of mathematics. Silver (1997) suggests that if creativity is integrated within mathematics teaching at the college and pre-college levels, it is more likely for students to develop creative thinking skills as well as adopt a creative disposition toward mathematics. Thus, through fostering creativity, students’ perceptions of mathematics will be compelled/motivated to change.

This discourse for education to foster creativity is not a new topic of conversation, these ideas inspired the reform of STEM (science, technology, engineering, and mathematics) into STEAM (science, technology, engineering, arts, and mathematics) (Kang, 2019). It is suggested that to become an innovative and creative society we need to incorporate the arts in all STEM subjects, at all levels of education from primary schools to universities. Unlike abilities in mathematics or science, abilities associated with the arts are often considered to be nonreflective of intellectual ability. However, what many may not know is that the incorporation of arts tends to offer much support for students’ academic achievement (Clark & Zimmerman, 1998). As Pehkonen (1997) argues that creativity should be an intrinsic part of the “mathematics for all” program.

Consequently, arts in STEM education can enhance the academic performance of students and enrich students’ perceptions of the sciences. However, the impact of how the incorporation of arts can potentially change students' perceptions of mathematics specifically is still unknown. Therefore, the aim of this research is to examine the way in which students’ perceptions may change regarding applicability and the creative use of mathematics through using a concept widely known to all, the arts. Thus, aiming to change the perceptions of mathematics being an isolated discipline, consisting of purely calculations, definitions, and pointless procedures, through advocating mathematics' applicability. By demonstrating the creative side of the subject, it is possible to change perceptions of mathematics as being dull and inflexible, to further enrich perceptions. Through the incorporation of art, we expect that students' negative perceptions of mathematics to change, compelling them to have more positive views of the subject.

Theoretical Background

Students' Perceptions of Mathematics

Students' perceptions and attitudes towards math shape how they define mathematics and what they consider the role of mathematics to be in their life and in general. One's view transcends to their attitude towards mathematics, which plays a role in a student's learning process as perceptions and attitudes can affect their achievement within mathematics (Farooq & Shah, 2008). Despite the benefits that the study of Mathematics offers, it is commonly found to be a subject that many students dislike. As the way in which mathematics is presented in a classroom can sometimes alienate or deter students from mathematics and potentially skew their perceptions of the subject.

For many students the subject of mathematics is seen as difficult, abstract, and having little interest. They view math solely as a tool for basic numerical skills and having little relevance to everyday life. Most students unfortunately then find it difficult to acquire the mathematical skills and processes needed to see the usefulness of the subject within their everyday lives (Hagan et al., 2020). Some students argue that they hardly experience this usefulness of mathematics outside classroom walls, but instead interpret mathematics as an isolated discipline oriented on acquiring solutions for mathematical problems (e.g., how to solve quadratic equations).

A question that is frequently posed in mathematics classrooms is: "Why do I have to learn this?" The objective relevance of mathematics in society contrasts with its subjective irrelevance as perceived by many students and even by people who are supposed to know the role of mathematics in society (Hernandez-Martinez & Vos, 2017). The function that mathematics actually plays within society differs from the mathematics taught in a classroom. As a result, students experience a lack of connection, which Hernandez-Martinez and Vos refer to as students' subjective irrelevance in the subject of math. Research shows that large groups of students suffer from a paradoxical dilemma when encountering math: students may find mathematics unusable, while they are simultaneously aware of the usefulness of mathematics in the real world. So, what can solve this problem, the use of context as it is argued to reduce students' perception of mathematics as a remote body of knowledge (Boaler, 1993b).

Context and Problem Based Learning in Mathematics

This issue of students or individuals in general not being able to apply mathematics learnt in the classroom to the real world is not a new phenomenon. Since the late 1970's there has been an increasing number of reports regarding the school leavers' and adults' inability to transfer mathematics learned in school toward the 'everyday' use of mathematics (Boaler, 1993b). For each individual there seems to be a complex relationship between the world in which mathematics is developed and the world to which it is applied. Yet it is still believed that mathematics can be learned in school, embedded within any learning structure, and be applied to any situation in the real world (Boaler, 1993). Therefore it is argued that focusing on developing/cultivating this transferability in students not only prepares them for the specific content studied but by presenting 'real world' problems, it provides them with a bridge between the abstract role of mathematics in the classroom and their role as members of society (Broomes, 1989).

Boaler, (1993) states that moving away from abstract mathematics and towards mathematics in context is brought about to reflect real life problems and prepare students for the mathematical knowledge they would need when encountering particular issues in everyday lives. This allows mathematics teaching to stray away from focusing on abstract calculations and instead enriching mathematical understanding and perceptions of the subject. Thus contexts are considered a useful tool for mathematics learning. This is because through the use of contexts students' perception of mathematics as a remote body of knowledge reduces (Boaler, 1993). Mathematics teaching through contexts also provides students with strategies needed to solve mathematical problems (Van den Heuvel-Panhuizen, 1996), as well as enabling them to see the usefulness of mathematics for solving daily-life problems (De Lange, 1987).

Realistic Contexts to Immerse Students in Mathematical Activity

To dive a bit further, this type of analysis of real-world phenomena that is related to mathematics is called didactical phenomenology. According to Gravemeijer & Terwel (2000), Freudenthal emphasized the importance of phenomenological embedding of mathematical components. The goal of a phenomenological investigation is to find problem situations that evoke mathematical solution procedures. There is a teaching practice that does just this, as it is tailored to enabling students to see the realistic applicability of mathematics. This practice is called Realistic Mathematics Education.

Realistic Mathematics Education (RME) is an instructional approach where real life concepts are used to emphasize the applicability of mathematics to real life scenarios. RME builds an in-depth and long-term mathematical understanding by using contexts that students can make sense of and using mathematics within these contexts (Van den Heuvel-Panhuizen & Drijvers, 2014). One of the six principles encompassed within this approach of Realistic Mathematics Education is the reality principle. This principle is carried out in two ways. Firstly, to highlight the importance of the main goal within mathematics education, which is igniting/increasing students' ability to apply mathematics in solving "real-life" problems. Second, to show that within a mathematics classroom an educator should use problem situations that are meaningful to students, offering them the opportunity to attach meaning to the mathematical constructs they develop while solving problems. Another principle mentioned, is the activity principle. This is where students are treated as active participants in the learning process, which emphasizes that we learn best by 'doing' math.

Freudenthal argued that not all students are future mathematicians but through the use of RME the majority of students will be able to apply the mathematics within the RME approach to solve problems in everyday-life situations.

On the basis of this research, art is our context of realism, where art will be analyzed through this mathematical perspective/funnel and also to see how this approach contributes to students' learning and understanding of mathematics.

Incorporation of Arts in Academic Subjects

The arts are often viewed in terms of self-expression. Therefore, arts and academics are usually seen as mutually exclusive entities. Clark & Zimmerman (1998) claim that the arts offer all students support in their academic achievement. Their research has indicated that artistic ability and academic achievement work hand in hand. In a study of teenage students at a summer art institute, all participants reported positive reactions to schooling and had outstanding performance in academic subjects. This research highlighted the importance of educating students to use their imaginations and spatial abilities and not only relying on mathematical or verbal skills (taught in academic subjects). This also works the other way around, where it has been emphasized that even artists use mathematics, language arts, spatial skills, and personal skills, as well as visual thinking, not solely artistic/non-academic means to create a new piece. They have argued that the incorporation of the arts in all educational programs fosters great relevance for a holistic and comprehensive view on education. Awareness of the role of art in society is essential if students are to make aesthetic decisions about their environment. This has emphasized the importance of being able to depict things visually, as visual understanding is an indicator of understanding the world. Therefore, it is hypothesized that the incorporation of the arts in mathematics education will have positive effect on students' understanding of the world which links to their view of the use and applicability of mathematics in the real world through creative means, namely arts.

Arts and mathematics are usually taught in separate disciplines, and students therefore rely on the highly familiar knowledge and routines specific for these domains. Haylock (1987) and Plucker & Zabelina (2009) hypothesize that by integrating mathematics and arts education, students will be stimulated to integrate different conceptual understandings of both disciplines which can further increase students' willingness to create new things and attach deeper meaning to these domains of mathematics and art.

Research Question

It can be seen that these factors of creativity and applicability can be beneficial to mathematics education and that the incorporation of the arts can help implement these factors. However, through using realistic mathematics education, where the art is used as a tool of realism and didactical phenomenology as its core of instruction, students' perceptions may be able to gradually change, potentially enriching those perceptions. This new approach where art is the context in which mathematics instruction is given will hopefully change the perception that mathematics is disconnected from everyday life. Within this study it is hypothesized that students' perceptions of mathematics may be subject to change through this inclusion of the arts and creativity within mathematics teaching.

This study therefore investigates how the incorporation of arts when teaching mathematics may impact or potentially enrich students' perceptions of the subject at higher level education.

Therefore, this research is guided by the main research question:

Does the incorporation of the arts within the teaching of mathematics enrich non-science students' perceptions at higher level education?

This main question is elaborated in the following sub-questions:

- To what extent does non-science bachelor students' perceptions of mathematics change during an elective course on mathematics that includes the arts?
- What do students' report about the impact of the course, and in particular of the inclusion of the arts, on their perceptions of mathematics?

Methods

For this research a mixed-methods approach was used to ensure that the aforementioned research question and subquestions were answered. Quantitative and qualitative measures were conducted to produce proper research triangulation.

Participants and Research Design

The target group analyzed within this study were the students enrolled in the Bachelor course entitled 'Mathematics for poets, thinkers, doers' given at Utrecht University. The students participating in this research are non-science students, mainly from the the Faculty of the Humanities and Social Sciences and majority of them following the Bachelor program Liberal Arts and Sciences. These participants are used because they have all chosen to partake in this elective course that incorporates the arts within mathematics education. Furthermore, because they are students outside of the Faculty of Science, knowledge can be attained as to how the inclusion of the arts enriches their perceptions of mathematics if they are not mathematics majors themselves.

From the 35 students enrolled in this course, there were 27 of them who answered the pretest questionnaire, and 14 of these students who answered the posttest at the end of the course. There were 4 students who volunteered to partake in the individual interviews conducted for this study, where each interview participant signed a participation consent form to enable the researcher to report their statements within this study. Only 3 participants partook in the first round of interviews, namely P0 to P2. After the first round of interviews, one of the volunteers de-enrolled for the course, specifically participant P0. Therefore, the fourth volunteer was contacted and asked to partake in the upcoming rounds and answered questions from the first and second round in one sitting. So, participants P1 and P2 remained the same, whereas the fourth volunteer was labeled P3.

The course Mathematics for poets, thinkers, doers is an elective course especially intended for students from faculties other than the Faculty of Science. It is a course aimed to allow students to recognize and use mathematical structures and patterns in the world around them. Within the tripartite division of Poets (mathematics in language and art), Thinkers (philosophical side of mathematics) and Doers (mathematics in practice), this course shows the broad applicability of mathematics and the centuries-old wonders of the subject. Within this academic year the course facilitated more application of the arts for the main purpose of this research study. As the course highlighted the individual relationship between visual art, poetry, and nature/living things and mathematics. For example, displaying the famous painting like the Last Supper and pinpointing elements of mathematics within the painting, or bringing notice to the fact that within the structure of snowflakes there is geometry.

Examples of imagery resented in lecture can be found in Appendix III.

Research Instruments

Pre-/Posttest Questionnaire

Pre and posttest were conducted to assess whether students' perceptions of mathematics changed after taking the course. A multiple-choice questionnaire was used, that consisted of a Five Point Likert scale from Totally agree to Totally disagree. The complete questionnaire can be found in Appendix II. The questionnaire comprised of different domains regarding students' perceptions on the following:

School practices and their ideas of mathematics

The statements used in this section were adopted from a preexisting questionnaire by Schoenfeld (1989) within sections 3 and 7 of their questionnaire. These statements were asked to gain insight on what students thought of the mathematics education they've already experienced as well as statements that highlighted what students' ideas of mathematics was in general.

Two exemplarily statements within this domain are the following:

(1) The math I learnt in school is mostly facts and procedures that have to be memorized

(2) In mathematics you can be creative and discover things by yourself

The usefulness of mathematics and mathematics as a social activity

The statements in this section were adopted from another questionnaire within the research of Kislenko, Grevholm, & Lepik, (2007) as well as using the framework of "mathematics as a social activity" from 'The Students' Mathematics-Related Beliefs Questionnaire (MRBQ)' used in the research by Op 't Eynde & de Corte, (2003). These statements pinpointed students' interest in the subject and their thoughts regarding the usefulness of mathematics.

Two example statements within this domain are the following:

(1) Mathematics helps me to understand life in general

(2) Mathematics is boring

The last domain was the following:

The relationship between mathematics and arts and creativity

The two statements in this section were self-constructed by the researcher and supervisor of this study. This section was created to assess students' perceptions of the relationship between mathematics and arts and mathematics and the element of creativity within mathematics education. Statements were created to be as similar as possible to the previous sections in accordance with the additional element, namely the inclusion of the arts.

The two statements within this domain were:

(1) Mathematics stimulates creative thinking for solving questions

(2) Mathematical creativity is encouraged through the use of arts in mathematics education.

For this paper, only six statements from the questionnaire were assessed to gather results. Namely, one statement about students' ideas of mathematics, three statements within the domain of mathematics as a social activity, and the two statements looking into the relationship between mathematics and arts and mathematical creativity.

Interviews

Individual, semi-structured interviews were conducted throughout the duration of the course. There were three mini-interviews comprising of nine questions for each round. Interview questions were self-constructed. The questions were designed to ask students' what their prior mathematics education was before this course, their feelings when they thought of mathematics in school, their beliefs about mathematics learning, and their views about mathematics education. Additionally, they were asked questions related to the course, specifically what their expectations of the course were, their experiences within it, how they felt about the arts being included as the context in which they learn mathematics, and if the course and this incorporation of the arts caused their prior perceptions of mathematics to change.

However, within these self-constructed interviews, there were two question statements taken from the research of Sam and Ernest (2000) which asked students to describe their images of mathematics and what it's like learning mathematics. One or both question statements were in each round of the interviews. The two question statements were the following:

(1) Math is

____.

(2) Learning mathematics is like

____.

The questions from each interview round are given in full in the Appendix I.

Procedure

Pretest and posttest questionnaires were distributed to all students of the course to collect quantitative data regarding students' perceptions of mathematics. The pretest was given to students in their first lecture of the course, more specifically April 26th, 2022. Whereas the posttest was given to students within the last week of the course, namely at the end of June. The questionnaire was made using Google forms, where students were sent an invitation link to complete the form via email as well as being presented with a QR-code scan within their lecture breaks.

Within the first lecture when students were given the pretest, all students were asked if they would like to participate in individual interviews. For these individual interviews there were 3 rounds, with approximately a 4-week interval between each round, namely April 26th, May 31st, and June 21st. Students were sent reminders via email a week before every upcoming interview round to ensure their attendance and to confirm time and location of the interviews.

The interviews were held on a Tuesday afternoon after students' tutorial session and conducted in a general study area of the university. Therefore, in a familiar place for the students which had a less formal feel than the classroom setting to ensure students' comfort.

Data Analysis

The results from the pretest and the posttest were gathered separately. Results were then compared to see if there was a change in students' perceptions of mathematics due to taking the course and by the incorporation of the arts. The comparison was made through visual representations of pie charts.

The interviews were transcribed to have a written version of the data collected and statements were then sorted into different codes, enabling the researcher to quantify students' responses and to see how they are linked to their perceptions of mathematics. Patterns of any kind will be analyzed to study how perceptions are being changed.

For coding a top meets bottom approach was used, where both a top-down and bottom-up approaches were used. Additionally, a constant comparative method was carried out to sort and organize the raw text in order to make an interpretation (Glaser, 1965). All transcription files were also coded in NVivo. This is a qualitative data analysis computer software which helped the researcher to organize and analyze the non-numerical data of the interviews. Within this program the researcher was able to classify, sort and arrange information by examining relationships within the data.

Top-down approach

Within this part of the approach, the researcher decided on the topics and codes of interest that will be investigated prior to starting data analysis and these codes were based on the theory and literature. Thus, these codes were created before the interviews were conducted and created to be akin to categories of the questionnaires by Schoenfeld (1989), Kislenko, Grevholm, & Lepik, (2007) and Op 't Eynde & de Corte, (2003). Namely, the codes 'Ideas of Mathematics' and 'Mathematics as a Social Activity'.

Bottom-up approach

The starting point for this approach was the transcribed data and a blank sheet, where notes were made of the topics and codes that emerged or that became apparent whilst reading the transcription interview text. These codes included what students reported about the 'Effects of the Course', 'The Inclusion of the Arts' and 'Recommendations for the Future'.

An overview of all codes is presented below.

Ideas About Math	***Description***	***Example***
Positive ***Math as:*** ***Progressive*** ***Explorative*** ***Flexible***	Views that promote mathematics in an enthusiastic manner	*In math you can be creative and discover things by yourself*
Negative ***Math as:*** ***Restrictive*** ***Confined*** ***Rigid***	Views that depict mathematics in a dull manner	*In mathematics something is either right or it's wrong* *Math problems can only be done correctly in only one way*

Math as a Social Activity	***Description***	***Example***
Positive	Views that promote being enthusiastic about mathematics	*Mathematics is exciting and interesting* *Mathematics helps me to understand life in general.*
Negative	Views that convey disinterestedness in mathematics	*Mathematics is boring.*

Effects of the Course	***Description***	***Example***
Positive	Views that depict enrichment based on the course	I've realized sort of that it can be a lot more about creativity as well
Negative	Views that convey disinterest in mathematics due to the course	I don't feel like there's any new knowledge I've gained during this course

	Description	***Example***
Inclusion of the Art Elements	Remarks/examples including the general concept of art or views that are conveyed through arts	I've seen a lot of art come by in these presentations and things I thought, like, Oh, this is interesting, or that is interesting.
Recommendations	Remarks given from students in relation to how things can change in the future of the course and mathematics education in general	Explain the mathematical issue in what we've seen in the art, and after that we practice meeting immediately with it, I think it would have been more effective

Seconder Inter-reliability Check

A second coder inter-reliability check was conducted. This was where the second coder was given a code book explaining the way in which interviews were coded and to examine after coding 20-30% of interviews if the researcher and second coder came to the same results as to which statements should be coded in a particular category.

To decide which participant's interviews the second coder will code a die was rolled where numbers 1, 2, 3 represented participant 1, 2 and 3 respectively as well as number 4, 5, and 6.

The second coder started by reading the code book and then coding one the participant's first round of the interviews and the categories in which codes were placed were compared. A discussion was then had to sort out code categorization discrepancies. The second coder proceeded to code the selected participant's second and third interview, after comparing code categorization a second time both coders were able to reach a high level of agreement, namely a Kappa value of 0.82.

Results

As mentioned above, both quantitative and qualitative data results were collected from the pretest/posttest and individual interviews respectively.

Pretest and Posttest Results

The results of the pre/posttest are shown below. Where the percentages of students who agreed with the given statement in the pretest in comparison to the percentage of people who agreed with the same statement in the posttest can be seen. Note that when the percentage agreement is referred to; this is the addition of the students who reported to 'agree' or 'totally agree' with the given statement.

Students' Ideas of Mathematics

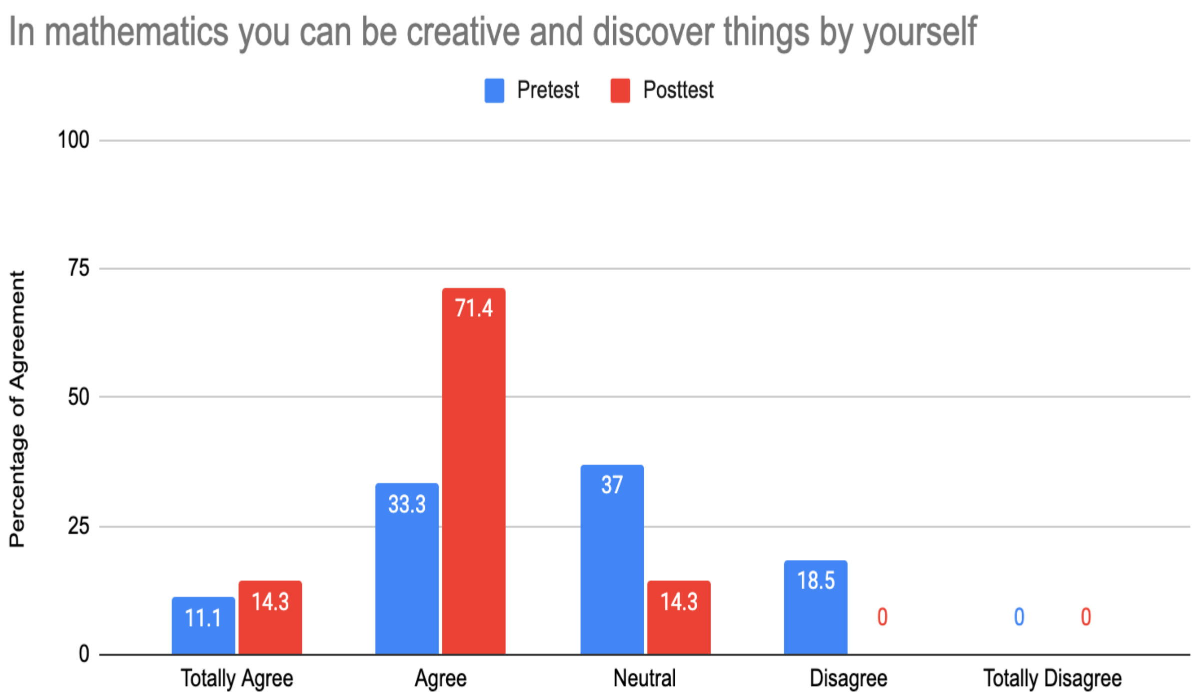

Figure 1. Percentage of Students' response to "Exploration in Math" on pre- and post-test questionnaire.

Figure 1 shows that more students viewed mathematics as a subject that they could be creative and discover things in once the course was completed. As there was 44.4% of students who agreed with this statement in the pretest in comparison to 85.7% agreement in the posttest.

Mathematics as a Social Activity

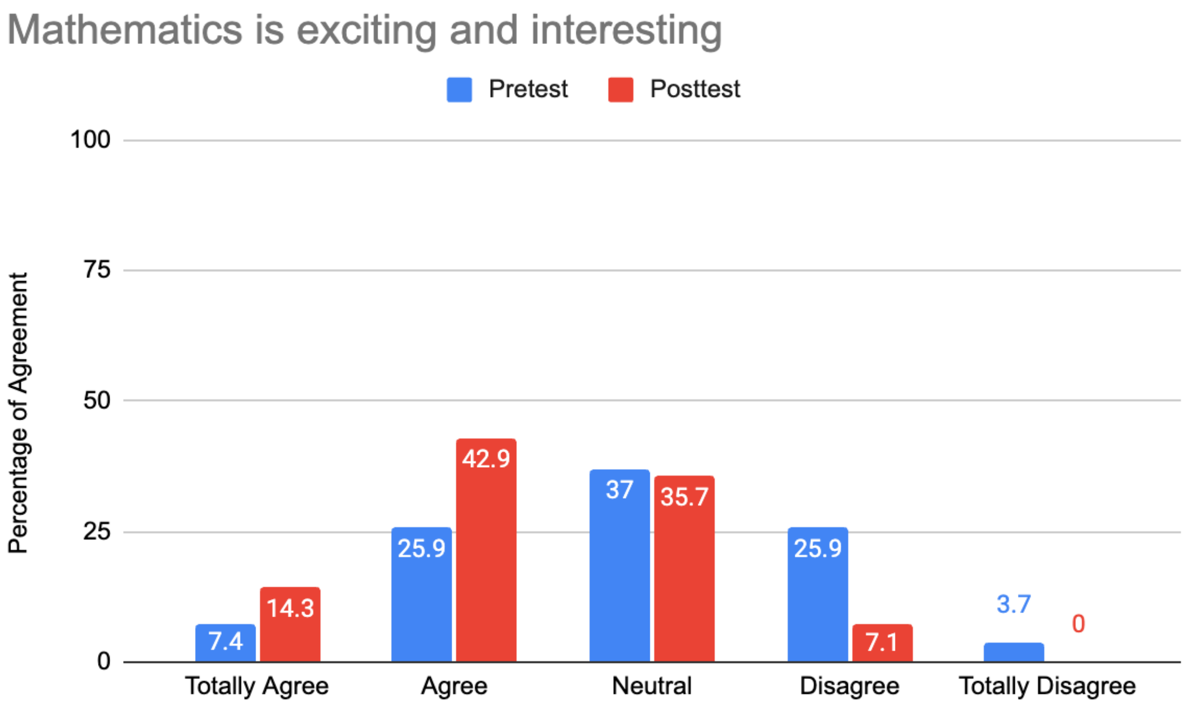

Figure 2. Percentage of Students who found Math Interesting.

It can be seen that more students found mathematics to be interesting and exciting after the course was completed. There was now less than 10% who disagreed with the posttest.

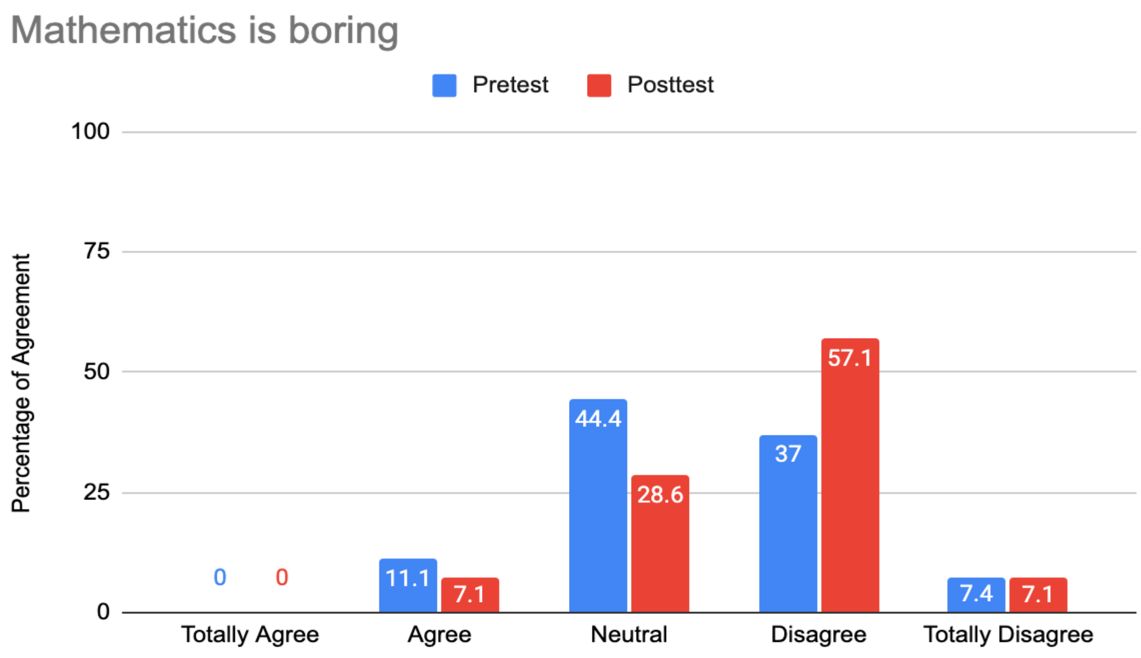

Figure 3. Percentage of Students' response to "Their Interest in Math" on pre- and post-test questionnaire.

This figure lies in agreement with Figure 2, as less students found mathematics to be a boring subject within posttest results. The increase in total disagreement (percentage of totally disagree and percentage of disagree) between the pretest and posttest indicates that less students find the subject of mathematics to be boring. This then points out that after partaking in this course or delving with the arts to be the context in which students learnt enabled for perceptions to change, where the subject of mathematics is of more interest to some.

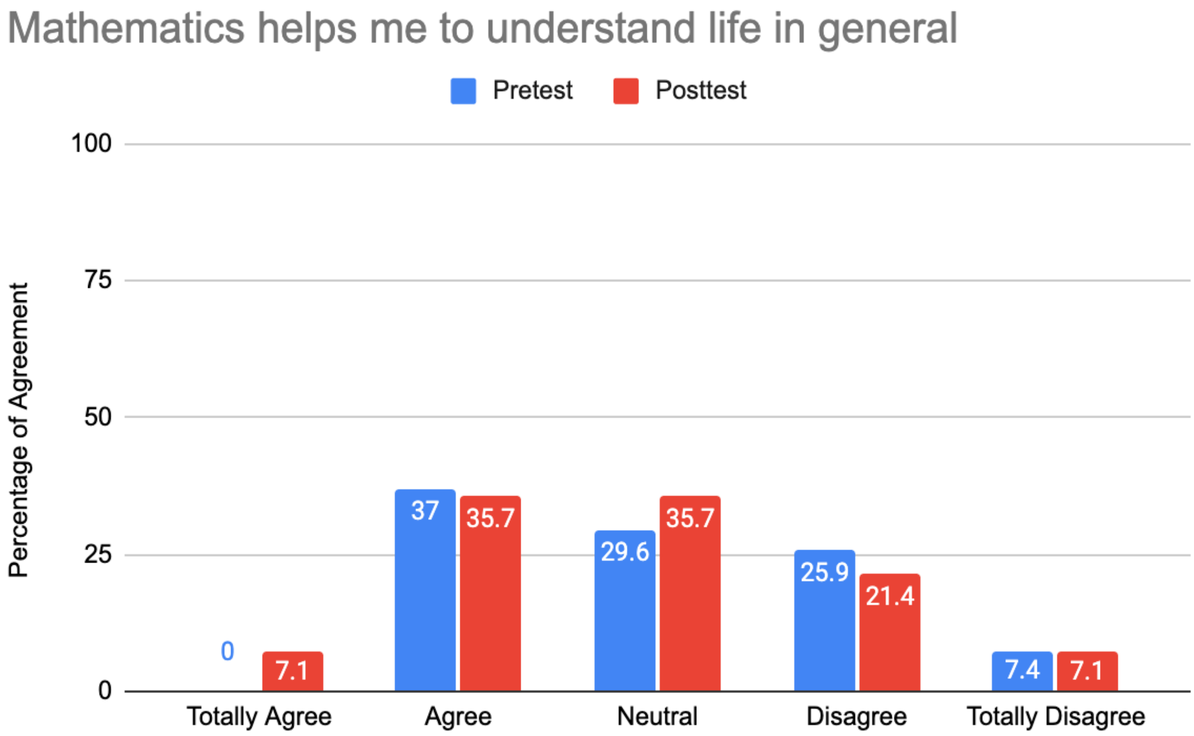

Figure 3. Percentage of Students' response to "The Usefulness of Math" on pre- and post-test questionnaire.

The statement assessed here is "Mathematics helps me to understand math in general". This figure shows that there was an increase in the percentage of students that agree with this statement, however this increase was not a large one. Namely, within the pretest only 37% of students agreed (percentage of students that totally agree and just agree) with this statement, whereas 42.8% agreed within the posttest. This was only a 5.8% increase, thus quite small but it showed that after partaking in this course students have changed their perceptions on the usefulness of mathematics in daily life.

Mathematics in relation to arts and creativity

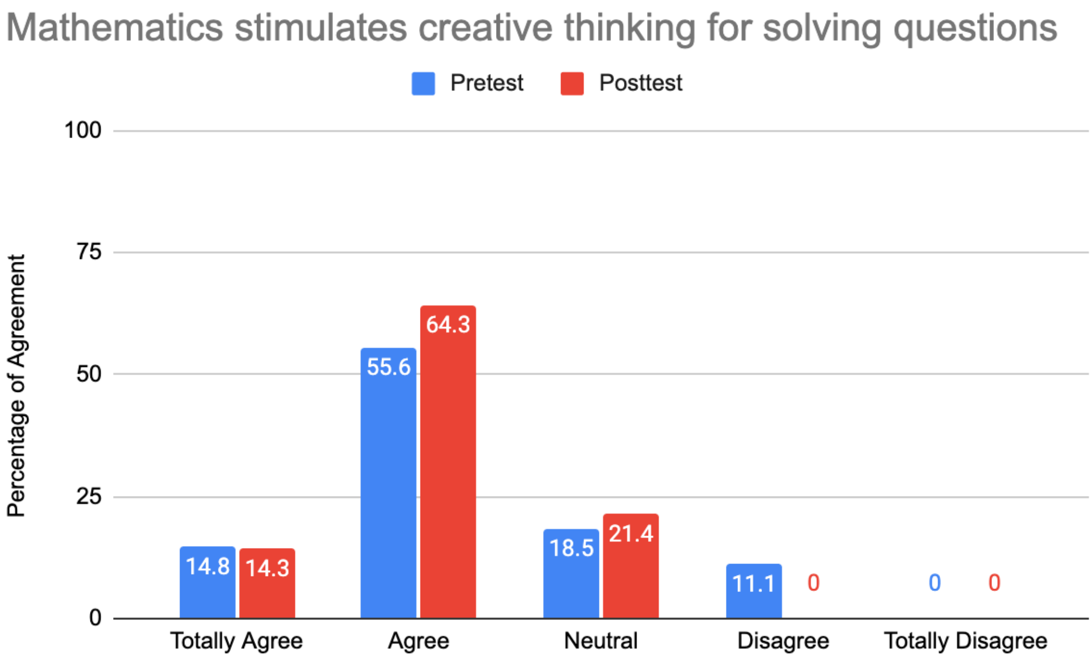

Figure 4. Percentage of Students' response to "Creative Thinking in Math" on pre- and post-test questionnaire.

Figure 5 shows the vast increase of students who reported in posttest that math can stimulate creative thinking, as majority of students now agree with this statement and there is no one that now disagrees.

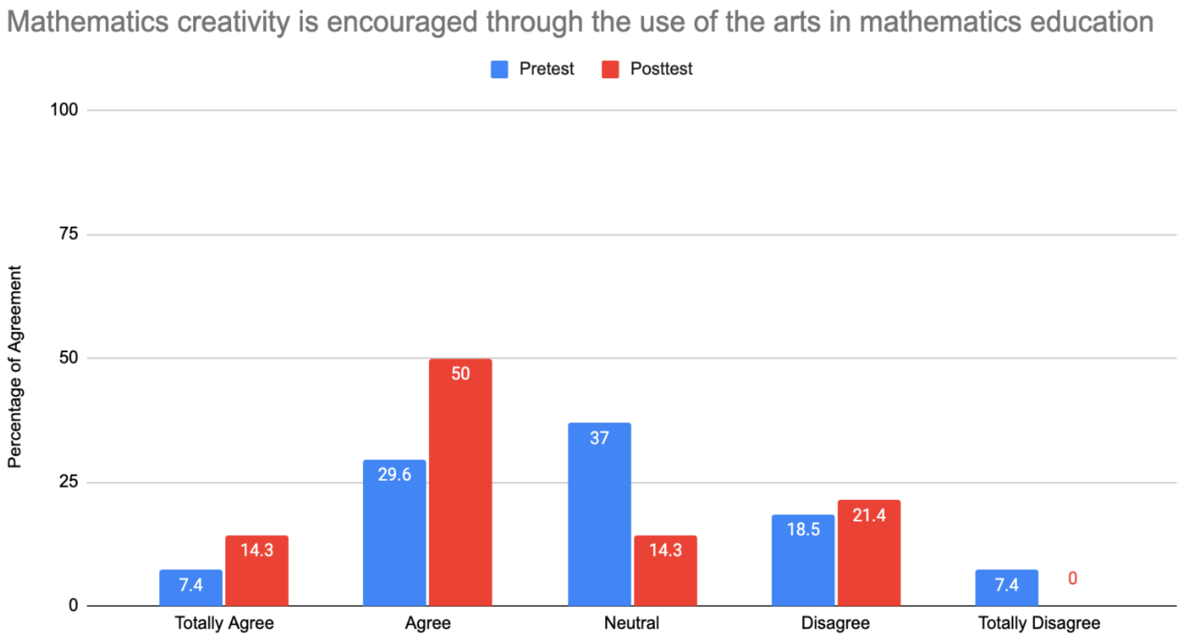

Figure 5. Percentage of Students' response to "The Arts within Math Education" on pre- and post-test questionnaire.

In Figure 6 it can be seen that majority students ascribed to the notion that the inclusion of art in mathematics education encourages mathematical creativity, which is an important find.

Additionally, it can be seen in Figures 5 and 6 that students' perceptions have positively changed regarding the relationship between mathematics and creative thinking as well as for the relationship between mathematics and the arts. This is because within both figures there is a higher percentage of students agreeing with the statements "Mathematics stimulates creative thinking for solving questions" and "Mathematical creativity is encouraged through the use of the arts in mathematics education" at the end of the course in comparison to the percentage agreement at the beginning of the course.

Interview Results

The interviews were conducted to assess if there was a gradual change in students' perceptions when taking the Mathematics for Poets, Thinkers and Doers, to further assess if using the arts as a context can enrich the way students think of the subject. The results of the interview will be discussed per domain/theme for all 3 rounds of interviews for all participants as a collective, then findings will be assessed individually.

Ideas of Mathematics

Round 1 (n=44)		Round 2 (n=36)		Round 3 (n=54)	
Positive	Negative	Positive	Negative	Positive	Negative
52%	48%	64%	36%	81%	19%

Table 1. Percentage of Statements Reported on "Students' Ideas of Mathematics"

Mathematics as a Social Activity

Round 1 (n=14)		Round 2 (n=6)		Round 3 (n=8)	
Positive	Negative	Positive	Negative	Positive	Negative
57%	43%	83%	17%	50%	50%

Table 2. Percentage of Statements Reported on "Students' Thoughts about Mathematics as a Social Activity"

Effects of the Course

Round 2 (n=24)		Round 3 (n=45)	
Positive	Negative	Positive	Negative
62.5%	37.5%	76%	24%

Table 3. Percentage of Statements Reported on "Students' Thoughts on the Effects of the Course"

Table 1 shows that within each round of interviews students reported more positive and less negative statements regarding their ideas of mathematics. More specifically, the percentage of positive statements reported became higher as the course progressed, where only 52% of reported statements were positive in the first round of interviews, in comparison to 82% within the last round.

The number of statements reported about their thoughts of 'Mathematics as a Social Activity' was not vast as the interview questions did not ask interviews much about their general interest in mathematics, but more about their thoughts of the subject in general and if this linked to this domain, then these statements were coded. In Table 2 it can be observed that there were generally more positive statements regarding mathematics as a social activity, linking to students' interest in the subject as well as what they thought of how mathematics is used in their everyday lives. However, towards the end of the course there was an equal percentage of positive and negative statements reported in this domain.

In Table 3 it is observed that there are more positive statements regarding the effects of the course than negative statements towards the end of the course. Even as the number of statements increased (from 24 to 45), the percentage in positive statements also increased.

To gain deeper insight let's look at each students' responses individually to highlight how perceptions may have changed.

Participant 1

Background. This participant completed Mathematics A at VWO level education in high school and is currently studying Liberal Arts and Sciences. At the beginning of the course this participant appeared to be very enthusiastic to start the course and showed great interest in this specific research.

Within Round One. In this round the participant reported to view mathematics as a language, where it may first be difficult to master but once mastered and understood, it is then easy to make use of it whenever you need to. They also expressed that solving mathematics questions was like solving a puzzle and racking the brain to find the right solution. They then added that these puzzles can vary in difficulty and enjoyment.

Here are some examples of what was reported in accordance with the following domains: the participant's ideas of math, how they view math as a social activity, and their opinion on the effects of the course and the inclusion of the arts.
Exemplary statements:
"Math is one of the languages of science, where you put information into numbers, it helps you to understand things."

"Math is a language with formulas and methods and for me once you use exactly the same method and formula with maybe different numbers you can get a solution."

"Math is very difficult to master. But it's basically learning to think in a certain way, a more structured and logical way."

"More recently, I found out that music and most art forms are also based around mathematics. Which kind of broaden the view as to what mathematics might be."

Within Round Two. After having partaking in the course for about a month, the student started to become a bit less excited about the course and content that had to be learnt. However, this time when reporting that learning math and learning a language is akin to one another, there was also indications that in order to learn math there needs to be motivation to do so, as well as effort must be put in. The participant explicitly stated:
"Mathematics in this course, for me is like learning Latin. Because it feels like it could be very understandable. I know that if I try my hardest and try my best and do a bit of research, I could understand the content. So, if I put in some extra effort, it wouldn't be a problem to solve any of these mathematical problems. But sometimes I just don't so it's quite difficult."

Another statement was:
"Math is quite difficult. But it can be very enjoyable and understandable if you but the effort in to learn it"

This can be seen as the social aspect of math, "enjoyment" in this case being dependent of understanding.

This participant continued to mention that in relation to this course, the lecture and tutorial sessions complemented each quite well. As information becomes clearer with the practice in tutorial sessions, in contrary to feeling lost and perceiving extreme difficulty to understand during a lecture. The student then went on to mention that they expected more creative elements of math to be included in the course. They also stated: *"the only real creative freedom you get is how you solve a question and what kind of tricks you want to apply. But the the fact that there is no set way to solve a math problem really allows the spirit of freedom and allows us to be a bit creative."*

This shows a shift in this participant's ideas of math, as this view is different than what was said in the first round, where doing mathematics was described as following steps which can only be regurgitated in one way.

This student stated that an effect of the course was realizing that math is a component of many things:

"Sometimes the explanations in lectures are very difficult. But I guess I'm sort of realizing that math can be secretly hidden in a lot of places"

Within Round Three. In this round, interviewees were asked to once again to describe what math is to them or what math is used for. This participant responded with the following statement:

"I would say math is a tool to think, a tool to organize, but also very much a tool to sort of be creative around, and just toy around with."

"I still think math is about a structured way of thinking but now also maybe a creative way too. To think of different ways to solve something yet there's both exploration and structure in that."

Here, one can see that there was a change in this participant though about math and what it can be used for. It was also interesting to see how this participant adopted a new way of viewing math.

Participant 1 reported:

"I now think math is like having two different sides of a coin. Because you've got the very structure side and we've also got the very creative side, while the side you use purely depends on what kind of reference you have. It's not so much that there just one thing but more about picking heads or tails. The choice is your own."

"I tend to be frustrated with all the equations we had to make. But then really enjoying the ones where you had something creative. For example, we had to make the Mobius strips. And I really enjoyed that exercise. But for example, there was one, or were a couple, just about fractions, chain fractions. And I absolutely hated that one. And it was really the combination of the two like them switching each other out really frustration and just enjoyment because it was easy and fun. Which made me sort of adopt this coin analogy!."

An example of the two questions being referred to can be seen in Appendix III.

In relation to our topics of discussion, this links both the students' ideas of math and their view of math as a social activity. When describing how they felt when math questions had to be done ('frustration', and 'enjoyment'), it can be inferred that this social aspect of math is dependent on what type of question needs to be done or if context is given. This is akin to what was reported in the second round, as this social aspect of math is dependent on something, but now this dependency is linked to the type of question a student is given and not just their understanding.

This participant also explicitly made mention of enjoying creative and artistic questions more than regular theoretical/didactic mathematics questions. Thus, when asking this participant what they thought of the context in which math was taught in this course, we got the following response:
"What worked very well for me in this course, was art. It's very visual, and it's very accessible. It's very easy to look at. art has a lot more of a fluid boundary."

Participant 1 really stood by their idea that math has two sides and that the subject calls for both creativity and rigidity.

Participant 2

Background. This participant did both Mathematics A and B at high school, one after the other and is currently studying Liberal Arts and Sciences, majoring in Dutch studies and Linguistics. At the start of the course, this participant made it clear that they compartmentalized the domains of Science and Humanities/Philosophy, because of the way of thinking these separate disciplines adopt.

Within Round One. When being asked about their thoughts about math and their feeling towards having to do math, this participant was very candid and responded as follows:
"I think Eww, oh no, not really my thing!"
"When getting exercises, I'm thinking, why am I doing this? I really think it's (math) interesting and doing exercises is of course useful, but I don't like it."

This participant further explained math to be a tool to:
"... understand how we work, and I think, in school, it's more to see the abstract point of view from things. To me I think it seems useless."

However, the usefulness of mathematics was dependent on certain things according to this participant. They expressed that in daily life situations, for example grocery shopping, the learning of math is indeed useful and necessary. Whereas when learning 'abstract things' such as the memorization of formulas and methods, they did not see the use of this. This use of methods was also described as fixed, leading them to categorize math as a disciple that does not really invoke aspects of freedom. However, these were the responses attained when the student was recalling prior experiences in math and thoughts before taking this course.

Although it was only the first week of the course this student expressed that they saw and is experiencing a difference in how math was taught in high school and how it is taught at university level within this course. This was expressed through the following statement:
"I think in back in high school, I don't think math is free, not within all the methods."
Further expressing that there is now more room for exploration:
"Now he (the lecturer) really wants to help us with understanding why people would do math, and why it's useful and where in the world we can see aspects of math."

There were obvious reflections of the effects the course, the lecturer, and the inclusion of the arts had on this participant, as they stated:
"I like it when in the lectures, he (the lecturer) gives examples of like, poetry or other things. I hope to see that more because that's really what I think is interesting, to not see mathematics as some abstract thing, but maybe a to be the ground of everything, and I hope to actually see that more and not seeing math as a side thing we have to learn."

Within Round Two. After partaking in the course for about a month, some of their views/ideas of mathematics did stay the same, but some did change and became less stringent.

Here are some example statements:
"I think within math, there is freedom in some ways. But there's still restriction in the way you solve things"

"I think in general when you learn about methods and have to repeat them, there's no freedom."

Thus, there was a realization of avenues of freedom within the subject, but not exactly within the way in which things are done but maybe somewhere else.

Additionally, they expressed that this realization was due to the course and what was being thought. It was even said:
"I think course gives me kind of wider perspective of what math is."

"Within this course they (lecturer and teaching assistants) are open for us to explore as non-mathematicians."

Within Round Three. In this round, interviewees were asked to describe what math is to them or what math is used for. This time a different response was given, implying that perceptions were changed and this participant indicating that the way you see math or choose not to see is it is all within your own personal reference frame. Thus, stating:
"Math can be everywhere if you want to see it."

"You can use math to explore a new way of thinking, a non-philosophical way"

It was observed that throughout the duration of the course this student's curiosity in the subject started to grow and accessing differences between the way in which students are guided to think in the sciences vs in philosophical studies. Participant 2 stated:
"I think it's really interesting to see the way of mathematicians think compared to my way of language people (humanities and lingistics)."

Statements mentioned when asked to reflect on thoughts of math prior the course to after:
"I think my thoughts of math have maybe changed in a positive way."

"I saw a lot of different ways to think about math and use it. I really like that the course was so applied."

"It was also really nice to see it (math) being used and applied in the art."

Participant 2's views of math really changed in a more positive way, where although the usefulness of the subject may be unclear to them, they now can see that mathematics does not exist within a vacuum and can be applied to many things. Thus, forcing first ideas of philosophy, art, and math as separate and unrelated domains to be challenged.

Participant 3

Background. This participant completed Mathematics B at HAVO level education when in high school and is studying Liberal Arts and Sciences. Based on previous education and personal thoughts, math was viewed as a means of only solving 'practical' issues if used outside of education. An example given was knowing the exact unit of how much water is needed to fill a swimming pool or knowing how much to change you'll receive when doing grocery shopping.

Within Round One. It was apparent that this participant viewed mathematics as a rigid and inflexible disciple based on the responses given when as to describe math and to what the subject of math can facilitate, for example, creativity or avenues of freedom.

The response to these questions was as follows:
"My one word to describe mathematics would be static."

"I think there's not much room for discussion within math as there's just one most efficient way to solve a problem."

This student did not explicitly say that they liked or disliked math, but rather expressed that enjoyment of the subject was dependent on understanding, as it was mentioned that:
"Math is only nice when you understand it."

Therefore, a similar view to what Participant 1 reported was found.

This student also thought that the purpose of math was also something that was not fixed but dependent on academic level. Further explaining this thought process, this is what was said:
"It (the purpose of math) really depends, as it changes because in primary school it's really the basics of knowing how to count. For example, knowing how many people are sitting in front of you and knowing how many chairs there are, this type of stuff, and knowing how to deal with money. Then I think in high school, it's more a basis of what you need to know for the university or after."

This participant having a flexible and pliable view on what math is, the purpose of it and even pertaining the feelings doing the subject invokes. Thus, it was hard to give a specific and rigid answer. When being asked about the link mathematics has regarding poets, thinkers and doers, a very detailed answer was given to each component. The response was:
"I don't think there's a correlation between poetry and math, but maybe some poets do use math in their poems. So, I guess there may be a link there but not in general. Doers, for example, architects, I think math is important, so I would say yes there is a relationship between the two. Lastly, thinkers. I mean, from philosophy I know that they think it aligns to math with rationalism and how there could be truth, the full truth and not only our perspective, so, thinkers, yes there is some correlation."

The participant showed much curiosity and mentioned just the nature of what this course intends to teach caused fascination. It was beginning of the course and the interest in finding out more was present. Quoting their words:
"I am pretty excited because I've never had a subject in high school or in primary school that combined art with mathematics, like the Islamic art or origami. I never linked it (art) to mathematics and then I saw this course and I thought, yeah, this should be pretty cool."

Within Round Two. Changes in the participant's perceptions of mathematics started to improve already and there was more of an awareness to mathematical applications in life. This was made evident by the following statements:
"I do think math is still used for practical things, but my perception of what math is used for, got broader. I know now it can be applied in different areas like arts and architecture. And there's math is things like snowflakes. I didn't know this before."

"Now with the course I can also see how it could be pretty cool to use it (math) in paintings and to think from a different perspective."

Realizations were being made about how one's brain is activated to find solutions to problems that are indirectly mathematical, like calculating how much change you should get back from a cashier. Thus, seeing that math is subtly in a lot of things we do.

They further made a comparison to what they learnt in high school and what they're learning now and noticing that math being taught was not just for preparation for further studies in the field. They enjoyed seeing how artists may have used math in their artwork and how one can use math outside of education for things that are not necessarily 'practical'.

It was mentioned that elements of creativity in this course added to this change in perception and feelings of occasional enjoyment in math. As math was something to be used to make poems or a TikTok video. But they highlighted that this is very seldom as in most math questions, especially the ones seen in high school education. This was said to be because the goal was usually to find the 'right answer' and steps need to be formulated in a certain way, so creativity was not stimulated nor was understanding a main priority.

Within Round Three. In this final round, the participant conveyed that they no longer see math to be solely rigid or 'static' in their words, but that there are indeed avenues of freedom, where students can explore, there is not one way to do things all the time. They mentioned that based on the vast about of information in the mathematics field, they no longer see math as restricted to practical matters and numbers and statistics.

However, there was still this sense of dependency when it can to enjoyment of the subject.
In the participants words:
"If I understand it math can be pretty fun but if it's too complicated, or I don't understand, or I feel pressured (in regards to time), or a combination of those three, I will hate it."

Regarding the effects of the course and the inclusion of the arts in the course, there was quite positive responses. Namely:
"Here (in the course) you can link it (math) to something (the arts), which gives it a deeper meaning when you learn it."

"Now with this course I can see how it could be pretty cool to use math to create paintings and how knowledge of math gives people a different perspective/reference frame."

"Things like the Last Supper painting, I didn't know there was this focal point in the middle with line drawn at a certain angle, which is math. Or for example, I didn't know there was

math in the structure of a snowflake, the golden ratio, the Michelangelo or Monalisa painting, and so many other things. And he (the lecturer) showed all of this in the lecturers."

Examples of what was being presented in lecturers can be found in Appendix III.

Generally, Participant 3 seemed to have these constant synapses of realizations throughout the course which caused their personal perceptions of math to change in a positive way.

Overall

Each student had a gradual change in having more positive perceptions of mathematics through the duration of the course. For example, Participant 1 first reported that mathematics is about putting things into numbers, which is a more passive function of mathematics. However, in the last round this participant highlights the element of creativity in mathematics. Additionally, this participant within the first round reported that mathematics can be difficult, however they then stray away from this idea and state that it can also be enjoyable with the inclusion of creativity.

Participant 2 highlighted how it was nice to see math being applied to different art forms such as poetry and seeing how math can be used in different domains. This then shows that their first perception of mathematics being useless, linking to the idea of not finding use for mathematics in daily life, has changed. However, this participant still thought that emphasis on using methods within the subject, limits freedom.

Participant 3 also showed a major shift in perception from seeing mathematics as a 'static' or rigid discipline to now recognizing avenues of math that are more flexible. Also, the realization that since math can be applied to so many things, it is indeed more adaptable than first perceived. This participant also highlighted the dependency of students' enjoyment and understanding of mathematics and how they may feel when having to do mathematical problems. One of the ways in which the student suggests that understanding and enjoyment could be present was through the use of context, in this case the arts.

There was integument to whether the context of the arts to teach math was the reason for students' enjoyment. Thus, a follow up question was asked to each student, namely: "Learning through context and application seems to be a factor in understanding and enjoying mathematics and this course. Was it the context of the arts in particular that fostered these two aspects?"

These were the responses attained:
"Context is very important for understanding math. You can use equation and know how to use it, but I would not understand at all why I would use it this way (without context)."

"Learning and teaching through application is important. It doesn't matter what kind of application it is."

"If they don't make the situation too complicated, any context to explain math can help to understand math better."

Thus, we can see these responses as recommendations/suggestions of what can be done to aid students' understanding the subject, as well as enjoy it within mathematics education.

Students also reflected and gave course specific recommendations, and mentioned the following:

"I didn't really see like a line or a link between everything taught, so it was like if this, then this, then this. There were too many different aspects."

"Explain the mathematical issue in what we've seen in the art and after that let us practice immediately with it, I think it would have been more effective."

"If using art, stick to that to make people interested so we don't lose attention, especially as non-science students."

This can all be seen as areas of improvement for the future of the course.

Conclusion

The aim of this study was to investigate whether the inclusion of the arts within higher level mathematics education has the potential to enrich students' perceptions of mathematics. We will first answer the sub questions in accordance with the following perceptions:

- Mathematics in relation to everyday life
- Mathematics as dull and inflexible
- Mathematics taught through the context of art
- Mathematics and Creativity

The Extent to which Perceptions Change

The first sub-question was concerned in the change in perceptions of students based on the agreement to statements given in pre-/posttest. By using the pre-, posttest, it was found that perceptions were enriched as there was an increase (or decrease in accordance with a less favorable statement) in the percentage of agreement between the pretest and posttest results.

Less than half of the respondents of the pretest and posttest agreed with the statement "Mathematics helps me to understand life in general". However, we also saw a small percentage increase in the number of students that agree with this statement between the pretest and posttest (5.8 %). But the question is what caused students to change this view. Is it the based on the medium in which this course was taught?

The pretest-posttest comparison showed a big increase in students' agreement that using the arts to teach mathematics fosters mathematical creativity (see Figure 5). Thus, experiencing math lessons through this context changed their perception on what this intervention can do, namely encourage students' mathematical creativity.

Then we see that after also taking the course more students also ascribed to the idea that the subject of mathematics stimulates creative thinking when solving questions. The level of agreement increasing my approximately 8%. We also observed that more students perceived the subject of mathematics to be exciting and interesting between the pretest and posttest, the percentage of agreement increased by approximately 24, where in the pretest around 33% of students agreed, to over half of the students agreeing with the statement after they completed the course, namely to around 57% agreement. So, we can infer that after taking part of this course students had a more positive outlook on the subject, as more students found the subject more exciting, seeing the link of the subject to everyday life, as well as appreciating the context in which the course was taught.

What Students Report

The second sub-question focused on what students report to gain insight on what they truly think of the subject of mathematics, and their opinions on the new intervention being used. This intervention being: using the arts as the context to teach mathematics at higher level education.

Students at first reported that they either didn't see the use of mathematics in their everyday lives or seeing mathematics as a means for practical things such as counting money, measuring quantities, or to be learnt if you'd like to further your studies in the field. Where math is described to have very little usage in the respondents' lives or as one student said seeing the subject as a bit useless outside of education. However, this view gradually changed as they then say that math can be used for many things outside of the practical aspects. Additionally, students realizing that math is used indirectly to solve a lot of problems that arise in everyday like. It was interesting to see how students expanded their view, where one participant mentioned that in math classes, we adopt a structured yet creative way of thinking that is we use essentially everyday.

However, this aspect of creativity was not something students previously reported. We saw that although some report to still see math as passive, rigid and so on, they now also saw math as an exploration tool. They expressed more excitement to do math as the course progressed, but only for particular questions namely the questions that made use of the art/creative context.

Thus, the question arose, whether the context of the arts was the reason for students' enrichment of perceptions or their increased enjoyment and curiosity. Two of the three students reported that the inclusion of the arts as it relates to mathematical concepts aided their enjoyment and understanding of math. Additionally, this inclusion allowed them to realize that math can be a facilitator within artwork, visual or verbal. However, they did believe that any context can be used once taught properly and it's not too difficult to comprehend. Whereas our first participant was very keen of this context of the art being the reason for his change in perception of seeing math as having a dual characteristic, namely structured and evoking creativity. It was explicitly reported that the context of the arts is what worked for them because art is visual, accessible and has fluid bounds.

Discussion

It was not uncommon to see that most students thought math was boring at the beginning of the course namely 55.6% on the pretest, as Hagan, (2020) proposed that mathematics tends to be of little interest to some. Sam & Ernest, (2000) expound saying that some students also perceive the subject to be solely about computation. We saw this was indeed the case at the start of this study, as interview participants reported believing that math was solely about methods or solving practical things and disagreed that there was any elements of creativity or avenues of freedom when doing math. Where they deemed the concept of mathematics to be a digestive process rather than a creative one (Dreyfus & Eisenberg, 1996).

Silver, (1997) argued that for most students, mathematics is associated with being one of the subjects least associated with creativity, and this was indeed true for these participants. Paksu, (2008) suggests that this might be due to the fact that some teachers focus on teaching procedures to get mathematical solutions, instead of promoting creative applications within mathematics. This reinforces student's conception of mathematics as solely an application of mathematical techniques, overemphasizing rules, algorithms, and convergent thinking to produce a single right answer (Mann, 2006). Thus, it was hypothesized that the context of the art in mathematics would be a good starting point for challenging students' negative perceptions of math. Arts was supposed to be a context that can heighten creativity within mathematics education (Burnard et al., 2016). Through this study we regard this hypothesis to be true. There are various reasons for this. Firstly, posttest results showed that students started to think more positively about the mathematics subject and the only difference between pre and posttests was the fact that students partook in a course that incorporated the arts to teach math. Secondly, a higher percentage of students agreed that this context stimulated mathematical creativity in the posttest as well. Thirdly, one interviewee stated the inclusion of art within the course worked well for them. Additionally, all three interview participants mentioned their appreciation to the application of art within the math coursework.

According to English et al., (2008), creative activities such as art support students in recognizing that doing “real” mathematics is, in actuality, the process of creative thinking. Thus, it’s this element of creativity that makes the arts so appealing as well as a good context to be used within mathematics education. This was apparent also through students’ responses as they expanded their view on the function of math being both stimulating creativity and structure. They also reported enjoying the creative and more artistic questions than the didactical questions. Some also made mention of seeing the ‘real’ aspects of things they knew were intriguing, for example seeing math in the structure of a snowflake or well-known paintings like the Monalisa.

The findings within Figure 1 was also of importance as it shows that after the course more students could identify that they have the ability to be creative in mathematics as well as having the autonomy to discover and explore new things within the subject. The fact that more students felt able to explore new things also indicates the beginning of exploration within the structure of mathematics, where students can begin to experience the beauty of the domain and develop a realistic sense of mathematics (Mann, 2006). Also, creative thinking in mathematics implies that you do your own mathematics, thus thinking and solving questions in your own way (English et al., 2008). Figures 4 indicates the same, where it is shown that more students perceive mathematics to be a subject that stimulates creative thinking, and the true essence of mathematics is indeed to think creatively (Dreyfus & Eisenberg, 1966; Ginsburg, 1996).

There were also clear indications through the interview statements, that a good way for students to adopt more positive perceptions of mathematics is through the addition of context. Thus, implying that a way to solve the issue Hagan, (2020) presented, specifically that students find it difficult to acquire mathematical skills needed to comprehend the subject is to give students a context to link mathematics to. As the use of contexts disrupts students’ perception of mathematics as a remote body of knowledge and isolated disciple (Boaler, 1993). Additionally, contexts provide students with strategies needed to solve mathematical problems (Van den Heuvel-Panhuizen, 1996), enabling them to see the usefulness of mathematics within their daily lives (De Lange, 1987). This find is akin to what if hypothesized by Haylock, (1987) and Plucker & Zabelina, (2009), namely that integrating mathematics and arts education stimulates the integration of different conceptual systems from both disciplines and enables students to create something new and attach greater meaning to mathematics and art.

Based on all the findings within this study it can therefore be concluded that this course and the context used to teach mathematics was indeed an advantageous starting point for enriching students' perceptions about mathematics. More specifically, the incorporation of the arts when teaching higher level mathematics education does enrich the non-science students' perceptions of the subject.

Limitations

The findings of this study cannot be generalized without considering general limitations of the methods. Firstly, the participants used in this study was a very specific group, namely mainly non-science students studying in the field of humanities. Thus, if the research was done on a different target group (e.g., Mathematics' students, different level of education, etc.), results may have been different. Secondly, the number of students enrolled in this course was also generally small, thus there was a limited number of students partaking to come up with results for this study. There was also a big contrast in the number of students that answered the pretest and posttest. So, there was a limited number of responses acquired for the posttest. In addition to this, the pretests were given to the entire class before individual interviews took place, thus there could have been social desirability bias as students were somewhat aware of the goal of this research. Lastly, the artistic elements of this course were also limited as there were only references to music, poetry, and visual arts. This was of course due to curriculum requirements; however, research findings may vary if other forms of art were used to explain mathematical content.

As a result, in order to properly generalize these findings, this study should be carried out with a different sample group of students (in field or academic level) and/or a larger group of students, and potentially fostering to more artistic mathematical contexts.

Implications and future research

This study shows that the inclusion of the arts can enrich perceptions of mathematics for non-science Bachelor level students. Additionally, that this context of the arts can be used as a way to reintroduce the element of creativity in mathematics education as well as stimulating students' creative thinking and increasing their autonomy in the subject. This in turn causes students to have fewer negative perceptions of mathematics.

Future research could focus on introducing arts as context in mathematics education at lower levels of education. For example, students at high school as this may yield different results. However, when doing so, ensure that the mathematical theory is linked to the context in an understandable and approachable way. It may also be interesting to investigate, the result of what may take place when a context like art is introduced to university mathematics students.

A major aspect of this research was the fostering of creativity as a means to reduce negative perceptions. For example, promoting that mathematics is indeed linked to everyday life, it may be interesting to investigate if other contexts can invoke elements of creativity and enrich students' perceptions. Such studies also help to better understand this relationship between mathematics and creativity.

Overall, this study has shown the advantages of using contexts in mathematics education, but also the use of a context that ignites creativity, namely art. Therefore, this reinforces ideas of art (context) being a way to improve students' perceptions. Thus, this specific inclusion of the arts has the potential to change negative perceptions of mathematics.

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Appendixes

Appendix I: Interview Questions

Round 1 (April 26th/May 3rd)

1. What level of mathematics education did you have prior to this course?
   - What level of high school education did you do?
   - What kind of mathematics did you specialize in? eg. Wiskunde A,B,C,D
2. What are you currently studying here at university?
3. In your own words, what is mathematics all about?
   - What are your first thoughts when I say the word "mathematics"?
   - What are your thoughts on the mathematics subject?
   - What image do you have when you think of math?
4. How do you feel when knowing mathematics exercises need to be done?
   - Learning math is like __________________.
5. What was your experience in mathematics classrooms prior to this course?
   - In terms of instruction and teacher/student interaction?
6. As a non Beta student, you view mathematics as a tool to ___________.
   - Do you think you have freedom to explore within mathematics? (within education)
   - What kind of skills do you think one can adopt and use from mathematics? (within other domains)
7. What in your opinion is the purpose of mathematics education?
8. What are you expecting to attain from this course?
9. This course is entitled Mathematics for Poets, Thinkers and Doers. Do you think there is a link in both of these avenues?

Round 2 (May 31st)

1. What are your thoughts on the course thus far?
2. Has anything caused you to change your initial thoughts of mathematics?
   - What are these factors that stimulated change?
3. The mathematics taught in this course is like __________________.
4. What is your current experience in the classroom?
   - In terms of instruction and teacher(TA)/student interaction?
5. Do you think you have freedom to explore within mathematics information or exercises given in this course?
6. Is there any relationship between mathematics and other domains that you have now come to realize?
7. As a non Beta student, you view mathematics as a tool to ___________.
   - Do you think you have freedom to explore within mathematics? (within education)
   - What kind of skills do you think one can adopt and use from mathematics? (within other domains)

Round 3 (June 21st)

1. Now that the course is completed, what would you describe mathematics to be all about?
    - What image do you have when you think of math ________.
2. Has anything caused you to change your initial thoughts of mathematics?
3. What are these factors that stimulated change?
    - The mathematics taught in this course is like __________________.
4. Do you think you have freedom to explore within mathematics information or exercises given in this course?
5. Is there any relationship between mathematics and art that you have now come to realize?
6. Would you argue that mathematics can foster creativity and different avenues for solving problems?
7. How do you feel when knowing mathematics exercises need to be done?
    - Learning math is like __________________.
8. As a non Beta student, you view mathematics as a tool to ___________.
    - Do you think you have freedom to explore within mathematics? (within education)
    - What kind of skills do you think one can adopt and use from mathematics? (within other domains)
9. What in your opinion is the purpose of mathematics education?

Appendix II: Pre/Posttest Interview

Explorations of Students' Mathematical Beliefs and Behavior	Totally Agree	Agree	Neutral	Disagree	Completely Disagree
The math that I learnt in school is ..					
1. Mostly facts and procedures that have to be memorized					
2. Thought provoking					
3. Just a way of thinking about space, numbers, and problems					
Ideas					
4. Some people are good at math, and some just aren't					
5. In mathematics something is either right or it's wrong					
6. Everything important about mathematics is already known by mathematicians					
7. In mathematics you can be creative and discover things by yourself					
8. Math problems can only be done correctly in only one way					

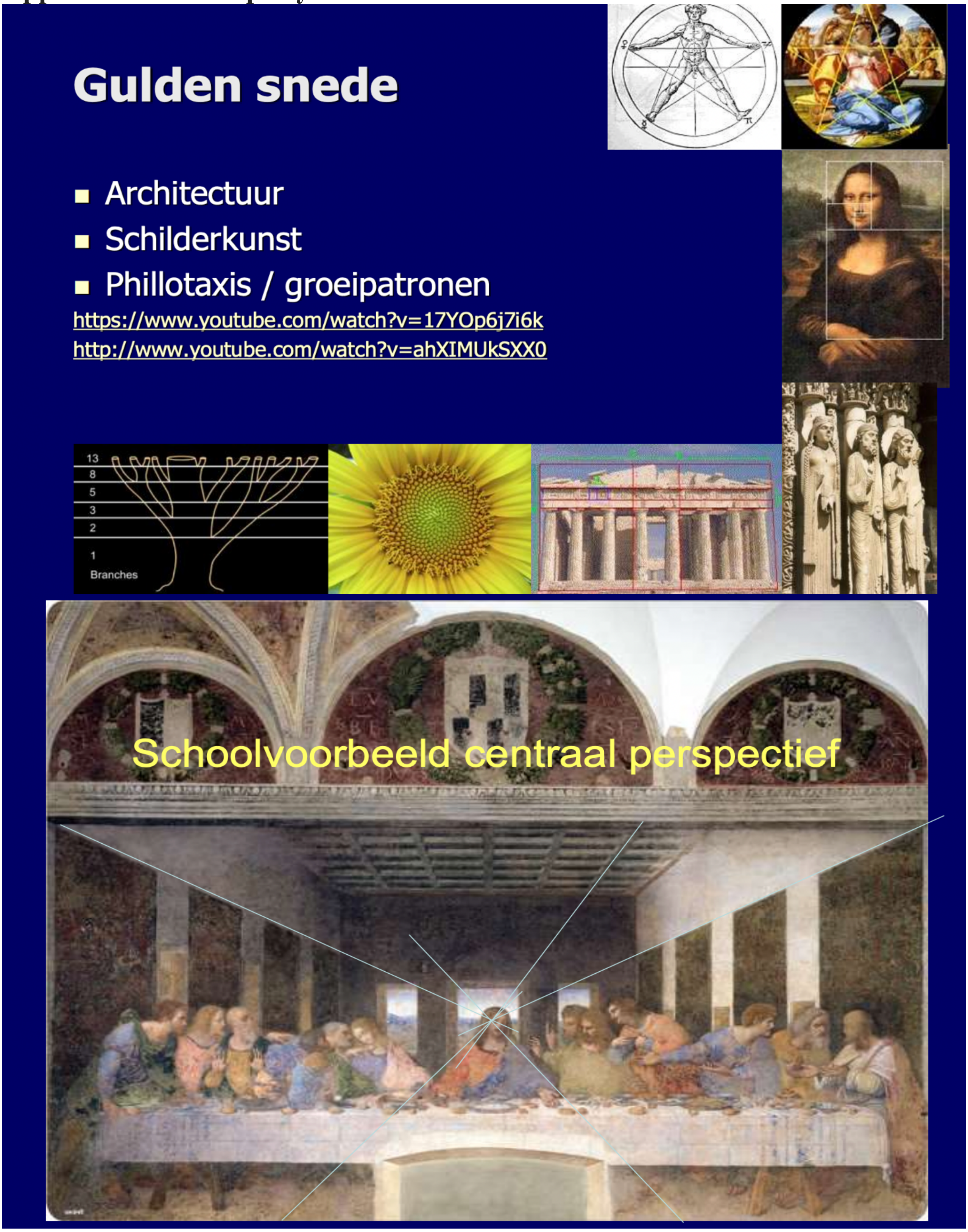
(Schoenfeld, 1989)

Students' Mathematics-Related Beliefs Questionnaire (MRBQ)	Totally Agree	Agree	Neutral	Disagree	Completely Disagree
Mathematics as a social activity					
9. Mathematics is exciting and interesting.					
10. I never get tired of doing mathematics.					
11. I like to do and think about mathematics also out of school.					
12. Mathematics helps me to understand life in general.					
13. Mathematics helps those who make important decisions.					
14. Mathematics is boring.					
15. Good mathematical knowledge makes it easier to learn other subjects.					

(Op 't Eynde & de Corte, 2003)

Additional Statements	Totally Agree	Agree	Neutral	Disagree	Completely Disagree
Mathematics and the arts/creativity					
16. Mathematics gives students the freedom to be creative in solving questions					
17. There is a creative side in doing mathematics, not everything is strategic/rigid					
18. Mathematical creativity is encouraged within a mathematics classroom					
19. There is mathematics within the arts					
20. There is relationship between math and the arts					

Appendix III: Exemplary Lecture Slides



Appendix IV: Exemplary Tutorial Questions

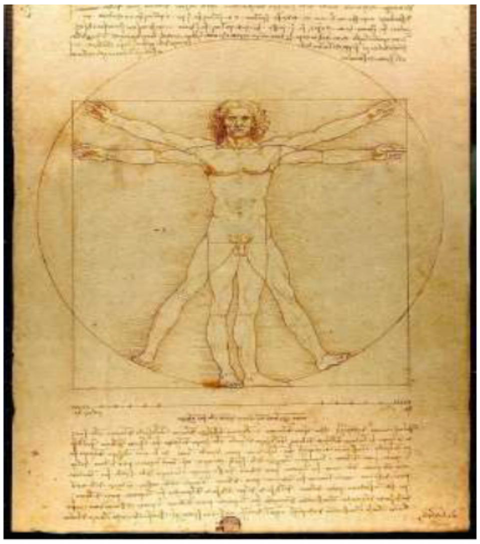
Example 1.

De verhouding tussen opeenvolgende getallen van Fibonacci naderen op den duur de gulden snede. Er wordt vaak gesteld dat Leonardo Da Vinci in zijn tekening van de Mens van Vitruvius gebruikmaakte van de gulden snede als de verhouding tussen de hoogte van het vierkant en de straal van de cirkel in de tekening.

a) Meet deze verhouding op in de (verhoudingsgetrouwe) kopie van de tekening en vergelijk met de gulden snede. Klopt de veronderstelling?

Je zou de gulden snede als verhouding ook op diverse plekken bij je eigen lichaam moeten kunnen vinden. We gaan dit testen.

b) Meet de afstand van je navel tot de grond, recht naar beneden en vergelijk deze met de afstand van je navel tot de bovenkant van je hoofd. Vind je hier de gulden snede? En als je de afstand van je navel tot de grond



Example 2.

In de wijk Wittevrouwen in Utrecht is een grijs kastje beschilderd (zie hieronder). Dit is gedaan door ieder huis in de wijk te vervangen door een punt en vervolgens een gebiedsindeling te maken met een Voronoi diagram. Hoe je zo'n diagram maakt kun je op internet vinden. Het basisidee is dat een grenslijn tussen twee punten bepaald wordt door hun middelloodlijn. Voronoi diagrammen kennen vele toepassingen in de biologie, aardwetenschappen en informatica: https://nl.wikipedia.org/wiki/Voronoi-diagram

In deze opdracht ga je zelf zo'n diagram maken.

Figuur 5: Een Voronoi-diagram.

a. In de figuur hierboven zie je de kaart van Nederland met alle spoorlijnen. Markeer in deze kaart alle universiteitssteden van Nederland: Amsterdam, Nijmegen, Maastricht, Utrecht, Rotterdam, Tilburg, Groningen, Delft, Enschede, Wageningen en Eindhoven.
Maak met behulp van die punten een Voronoi-diagram (gebruik de kaart in de bijlage).

Opmerking: teken met liniaal (en liefst geodriehoek) en potlood (zodat je kunt gummen), of teken op de computer (maar zorg dan dat je goed middens van lijnstukken kunt bepalen). Je mag je resultaat kleuren.

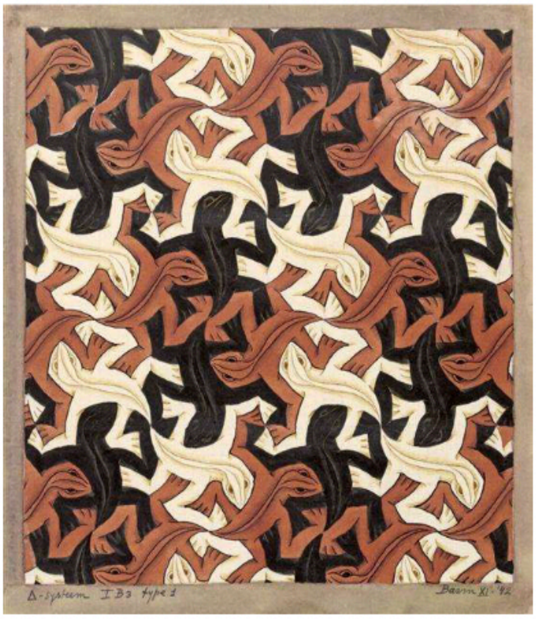
b. Wat kun je met je diagram concluderen over de ligging van de universiteitssteden?

Example 3.

b. Volg het stroomschema om de patronen uit Figuur 7 te classificeren. Leg uit hoe je door het stroomschema bent gegaan. Ondersteun je uitleg door in de figuren spiegelassen en rotatiecentra te tekenen (indien mogelijk).

Figuur 7: twee Islamitische patronen.

De wiskundige tekenaar M.C. Escher is ook geïnspireerd door dit soort behangpatronen. Je kent misschien zijn levendige vlakvullingen zoals die van Figuur 9.



Figuur 9: “Hagedis” van Escher.

c. Herhaal opdracht b voor Figuur 9. Zou je antwoord verschillen als de hagedissen niet gekleurd maar allemaal zwart-wit zouden zijn zodat ze niet van elkaar te onderscheiden zijn? Leg uit?

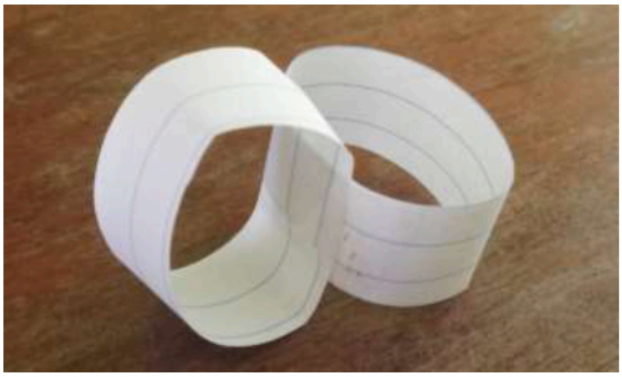
Example 4.

In het figuur hieronder is een Möbiusband te zien. Dit is eenvoudig zelf te maken. Je pakt een strookje papier. Je brengt de twee uiteinden bij elkaar maar je draait één van de uiteinden een halve slag voordat je de uiteinden op elkaar plakt. Möbiusbanden hebben verschillende interessante eigenschappen. Een voorbeeld hiervan is het resultaat na het knippen van een Möbiusband in de lengte op ½ van de breedte.

Figuur 1: Möbiusband.

Nu gaan we onderzoeken wat deze eigenschappen zijn als we Möbiusbanden op verschillende plekken doorknippen.

a. Knip twee Möbiusbanden in de lengte doormidden: één op de helft van de breedte, de andere op een derde. Geef na het onderzoeken van de resultaten voor deze twee gevallen een vermoeden over hoe de resultaten zullen voortzetten bij starten met knippen op een vierde, of een vijfde van het midden. Ondersteun je antwoord eventueel met schetsen of foto's bij het inleveren van wat je hebt waargenomen.



Figuur 2: geplakte niet-gedraaide banden.

Naast alle interessante eigenschappen die Möbiusbanden hebben, heeft de niet gedraaide band ook interessante eigenschappen als je deze boven op een andere niet gedraaide band plakt. Zoals de afbeelding hierboven, maar dan met 2 banden in plaats van meerdere.

b. Voorspel wat er zou kunnen gebeuren als je deze 2 banden knipt bij de helft van de breedte.
c. Wat is het resultaat van het doorknippen? Schets of maak een foto.
d. Doe de vorige opdracht maar dan met twee enkel gedraaide Möbiusbanden in plaats van niet gedraaide. Wat is het resultaat? Maak een schets of foto.

Example 5.

Een kettingbreuk is een uitdrukking van een getal die er als volgt uit ziet:

$$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cdots + \cfrac{1}{a_n}}}$$

We kunnen deze kettingbreuk ook simpeler opschrijven met de volgende notatie: $\langle a_0, a_1, a_2, \ldots, a_n \rangle$. Een voorbeeld is:

$$1 + \cfrac{1}{2 + \cfrac{1}{3}}$$

Ofwel $\langle 1,2,3 \rangle$. We kunnen deze breuk als volgt versimpelen:

$$1 + \cfrac{1}{2 + \cfrac{1}{3}} = 1 + \cfrac{1}{\cfrac{7}{3}} = 1 + \frac{3}{7} = \frac{10}{7}$$

a) Versimpel de kettingbreuk $\langle 1,3,5 \rangle$.

Je ziet misschien wel in dat je elke kettingbreuk op deze manier kunt versimpelen tot een normale breuk. Andersom blijk je ook elke normale breuk $\frac{p}{q}$, met p en q gehele getallen, als kettingbreuk te kunnen schrijven. Kijk bijvoorbeeld naar de breuk $\frac{18}{5}$:

$$\frac{18}{5} = 3 + \frac{3}{5} = 3 + \cfrac{1}{\cfrac{5}{3}} = 3 + \cfrac{1}{1 + \cfrac{2}{3}} = 3 + \cfrac{1}{1 + \cfrac{1}{\cfrac{3}{2}}} = 3 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2}}}$$

Ofwel $\frac{18}{5} = \langle 3,1,1,2 \rangle$.

b) Schrijf $\frac{25}{7}$ als kettingbreuk.

Kettingbreuken kunnen ook oneindig lang zijn. In onze simpele notatie zien ze er dan uit als $\langle a_0, a_1, a_2, \ldots \rangle$. Oneindige kettingbreuken horen bij irrationale getallen. In het dictaat kun je hier iets over lezen op pagina 188 en 189. Daar staat de oneindige kettingbreuk van $\sqrt{2}$, die er in onze notatie uitziet als:

$$\sqrt{2} = \langle 1,2,2,2, \ldots . \rangle$$

We schrijven ook wel $\langle 1, \overline{2} \rangle$, wat betekent dat 2 zich blijft herhalen in de kettingbreuk. Hoe kunnen we nu zien dat deze kettingbreuk bij $\sqrt{2}$ hoort? In het boek staat een handige manier beschreven, maar het kan ook anders m.b.v. het merkwaardige product uit opgave 1.2:

$$\sqrt{2} = 1 + \left(\sqrt{2} - 1\right) = 1 + \cfrac{1}{\cfrac{1}{(\sqrt{2} - 1)}} = 1 + \cfrac{1}{2 + \left(\cfrac{1}{(\sqrt{2} - 1)} - 2\right)} = 1 + \cfrac{1}{2 + \left(\cfrac{\sqrt{2} + 1}{(\sqrt{2} - 1)(\sqrt{2} + 1)} - 2\right)}$$

$$= 1 + \cfrac{1}{2 + (\sqrt{2} - 1)} = 1 + \cfrac{1}{2 + \cfrac{1}{2 + \left(\cfrac{1}{(\sqrt{2} - 1)} - 2\right)}} = 1 + \cfrac{1}{2 + \cfrac{1}{2 + (\sqrt{2} - 1)}} = \cdots$$

En dit proces blijft zich herhalen. We noemen dit ook wel een algoritme, en het werkt voor elk irrationaal getal. Als x een reëel getal is, dan bedoelen we met $\lfloor x \rfloor$ het grootste gehele getal dat kleiner is dan x. Stel nu dat we de kettingbreuk van x uit willen rekenen. Dan doen we dat als volgt: Zeg eerst even dat $x_0 = x$. Dan is

$$a_0 = \lfloor x_0 \rfloor, \qquad x_1 = \frac{1}{x_0 - a_0}$$