Inquiry-Based Mathematics Teaching at Secondary level in Lesson Study

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Abstract

Purpose – Investigate the development of IBL practices in LS and foster the sustainability

Design/methodology/approach – Qualitative research based on reflection reports, questionnaires, and interviews

Findings – Teachers' Understanding of IBL and students doing inquiries have the potential to improve in LS.

Practical implications – Inquiry-based teaching in Lesson study is essential for teachers to own the teaching method and sustain it in their class after the project end.

Social implications – Inquiry-based mathematics teaching is important to develop the 21st-century skills of students and prepare them for the requirement of today's knowledge-based economies.

Originality/value – LS practices have a specific teaching method in focus.

Keywords Lesson study, Inquiry-based learning

Paper type Research paper

Intended journal: IJLLS (The word limit is exceeded in this paper)

1. Introduction

Globalization and modernization are changing the world, which has also led to a requirement to change in students' skills. Those skills, referred to as 21st-century skills, are not all new but have gained importance in our current society (Silva, 2009). Problem-solving and critical thinking skills, vital to surviving, have become popular due to the demands in today's knowledge-based economies (Levy and Murnane 2004; Rotherham and Willingham 2009). Inquiry-Based Learning (IBL) can be used to promote and improve these 21st-century skills (Chu, Reynolds, Tavares, et al., 2021). For that reason, teachers are supported by projects or professional development programs to improve their teaching methods and adapt to IBL (e.g., Maass & Doorman, 2013). However, teachers are prone to return to their daily practices after the project they participated in for professional development ends (Maass & Artigue, 2013). In many projects or studies, teachers experience the teaching method as learners but do not know how to practice it fully in their classroom since they are not owners of the method (Liljedahl, 2016). Lesson Study as a continuous professional development practice starting from teachers' practice-oriented questions can help them integrate the new practices they have learned from the project into their daily teaching.

Lesson study, as a teaching method, originated in Japan and has been in practice for more than a century (Isoda, 2007). It comes from the Japanese word *jugyou kenkyuu* (授業研究.) which is translated as 'research live lesson' or 'lesson study' (Stigler & Hiebert, 1999; Yoshida, 2000). It is a collaborative and cyclic process where teachers study teaching materials and curriculum, plan lessons, observe, and reflect on them to improve students' learning (Stigler & Hilbert,1999). Teachers' teaching practices are investigated by themselves in lesson study. In this way, teachers with a common goal have the chance to observe each other's classes and improve their lessons and practices as owners.

The Teachers' Inquiry in Mathematics Education (TIME) project, which has 14 partners from four countries, has a goal to explore how teaching practices, mainly IBL practices, can be improved by a group of math teachers working together in Lesson Study projects with university professors (Time,2021). This format, which is an integral part of the professional lives of Japanese teachers at the lower and secondary levels, seems promising for creating ownership and developing innovative teaching practices (Cobb et al., 2017). However, the application and continuum of such a promising format in countries where it is not a part of the school culture is a challenge that has only recently been identified in the Netherlands.

There are several challenges found so far by studies that affect the continuum of lesson study. Firstly, due to the differences in culture, organizational and governmental structure, it is not easy to implement lesson study in a different context than in Japan. There is national and district funding for LS-designated research schools, and the national curriculum changes are expected to be investigated via lesson study at a system level (Lewis & Takahashi, 2013). Secondly, school organizations can cause some obstacles to sustaining lesson study. For example, part-time working teachers in the Netherlands are not always available for team discussions and are less flexible in observing each other's lessons (Wolthuis et al., 2020). Also, teachers' understanding of lesson study affects how they adapt lesson study to their school environment and its continuation (Wolthuis et al., 2021). For example, teachers who see lesson study as research and work according to the lesson study procedure successfully adapt lesson study in their school system and keep using it. In this study, we will focus on teachers' understanding and adoption of specific teaching methods, namely IBL, to improve their teaching practices, how they do that in lesson study projects, and how that affects the continuation of lesson study for continuous professional development.

This paper explores supporting factors to implement and sustain the Lesson Study method in IBL practices to promote IBL as a part of the teachers' teaching processes. Several dimensions can affect the continuation of such a method which may include factors mentioned in previous studies. In this study, we focus on how the support needed by teachers and their understanding of IBL affect the continuation of lesson study from the teachers' perspectives. Teachers' reflection reports and semi-structured interviews are the main data source. We employed a mixed-methods approach containing qualitative and quantitative data.

2. Theoretical Background

2.1. Inquiry-Based Learning

Inquiry-based learning (IBL) has become popular in science and mathematics education. It can be defined as constructing knowledge, with the learner formulating hypotheses and testing them by conducting experiments and/or making observations (Pedaste, Mäeots, Leijen, & Sarapuu, 2012). Active participation and the learner's responsibility for discovering new knowledge are required in IBL lessons (Swan et al., 2013). IBL aims to promote student engagement, making it possible by letting students take responsibility for their learning (Savelsbergh et al., 2016). Inquiry-based learning in mathematics is similar to inquiry-based learning in science. Maass and Dorier (2020) define Inquiry-Based Mathematics education as

> *Inquiry-based mathematics education (IBME) refers to a student-centered paradigm of teaching mathematics and science. Students are invited to work in ways similar to how mathematicians and scientists work. This means they have to observe phenomena, ask questions, and look for mathematical and scientific ways of how to answer these questions (like carrying out experiments, systematically controlling variables, drawing diagrams, calculating, looking for patterns and relationships, and making conjectures and generalizations), interpret and evaluate their solutions, and communicate and discuss their solutions effectively.* (p.1)

Maass and Dorier combined science and mathematics in their definition. Thus, the list developed to measure elements of inquiry in science teachers' teaching practices by Capps and Crawford will be used to measure elements of inquiry in mathematics teachers' teaching practice with a little adaptation in this study. The list of understanding of inquiry, abilities to do inquiry, and essential features of inquiry are based on three elements of inquiry. The first element is an understanding inquiry as a way to develop scientific concepts and skills. The second element is based on students' actions during the lesson, such as asking questions and creating a hypothesis. The third element focuses on teachers' teaching practice by using table 1. In this study, we will use the list to decide the presence or absence of doing inquiry and understanding of inquiry in the teachers' reflection reports.

Understanding about inquiry (U)	Doing Inquiry(D) Derived from abilities to do inquiry (A) and essential features (EF) of inquiry
U1: Different kinds of questions suggest a different kinds of scientific investigations U2: Current scientific knowledge and understanding guide scientific investigations U3: Mathematics is important in all aspects of inquiry U4: Technology used to gather data enhances accuracy and allow scientist to analyze and quantify the results of the investigation U5: Scientific explanations emphasize evidence, have logically consistent arguments, and use scientific principles, models, and theories. U6: Science advances through legitimate skepticism U7:Scientific investigations sometimes result in new ideas and phenomena for study, generate new methods or procedures for an investigation, or develop new technologies to improve data collection.	D1(EF1/A1): Involved in sci-oriented problems D2(A2): Design and conduct investigation. D3(E2): Priority to evidence in resp. to a problem: observe, describe, record, graph D4(EF3/ A4): Uses evidence to develop an explanation (e.g., cause for effect, establish relationship based on evidence) D5(EF4/A5, A6): Connect explanation to scientific knowledge: does evidence support explanation? Evaluate explain in light of alternative exp., account for anomalies D6(EF5/A7): Communicates and justifies D7(A3): Use of tools and techniques to gather, analyze and interpret data D8(A8): Use of mathematics in all aspects of inquiry

Table 1: List of understanding about inquiry, ability to do inquiry, and essential features of inquiry (Capps & Crawford, 2013, p.500)

2.2 Lesson Study

Lesson study is a cyclic process consisting of preparation, actual class, and class review sessions. Figure 1 includes steps given by Stigler and Hiebert. Teachers study curriculum material and find resources related to the lessons to align those findings with the actual needs of students. After the planned curriculum is turned into a curriculum that can be implemented, the lesson is observed by many teachers, and sometimes professors also join. After the lesson, observers share their findings and observations to improve the lesson plan. In the TIME project, teachers will prepare research lessons, implement and observe lessons with teachers from other countries to improve the lesson plan, and finally share their results through reflection reports, which will be the data source for this study.

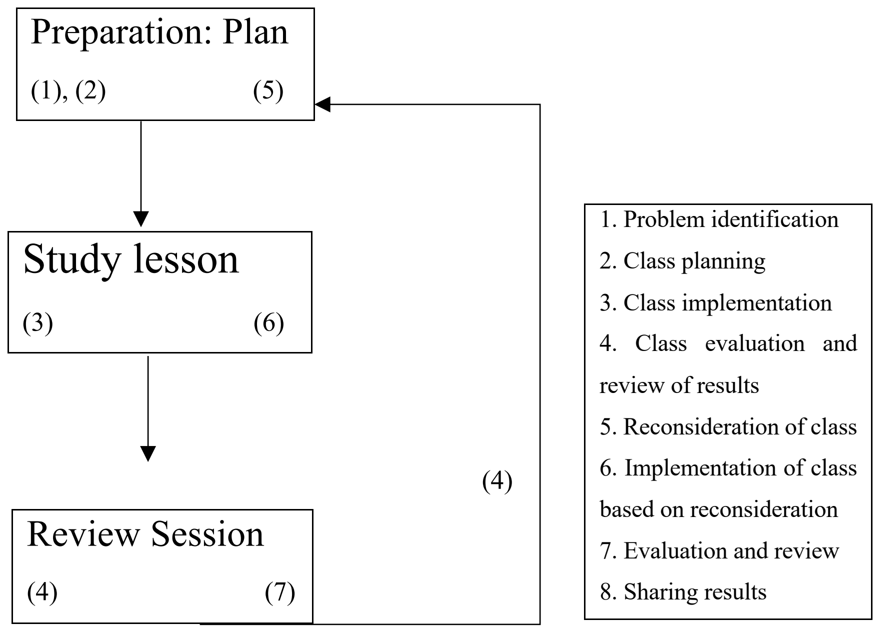


Figure 1: Lesson study cycle (Stigler & Hiebert, 1999)

Research shows that teachers' knowledge can be improved (Lewis et.al, 2009), teachers' growth can be promoted (Lewis et al., 2009; Lewis et al., 2006; Lewis et al., 2011), and sustaining professional learning communities are possible with LS practices(Moss et al., 2012).

2.2.1 The popularity of lesson study in Japan

Lesson study has now become a popular method of teaching even in some western countries such as the US, the UK as well as the Netherlands. It can be traced back to the 1990s in Japan. It started as a government initiative after WWII and became immensely popular within the regular school culture. Almost all elementary schools in Japan actively apply the lesson study method and have made it an integral part of the teachers' teaching practice and school culture. Despite the government's interest and support, it remained a voluntary act. According to Fernandez and Yoshida (2004), there are two main reasons for this: government support and teachers' opinions about the usefulness of lesson study.

Lesson study remained voluntary, but many schools consider it quasi-required (Kitayama & Yamada, 1992). They are doing it because other schools do. There is almost no school where lesson study is not a part of the school culture. Additionally, the government has financial support for an external commentators, university professors, and teachers visiting other schools. There is, however, no support for the teachers.

Another reason for the popularity of the lesson study method in Japan is that the teachers find it helpful (Inagaki, Terasaki, & Matsudaira, 1988). Even though lesson study is time-consuming, teachers say they learn their weaknesses and strengths with this practice (Inagaki, Terasaki, & Matsudaira, 1989). Participation in lesson study is seen as the main reason for many Japanese educators' success in changing teaching practices (Lewis et al., 2006; Murata and Takahashi, 2002; Shimizu et al., 2005). This shows that teachers' expectations and perceptions regarding the positive impact on their teaching experience can be a crucial element in the quest for finding ways to sustain the lesson study practice.

2.2.2 Obstacles to sustain Lesson Study Practice

Several elements such as adaptation of LS, organizational structure, and teachers' motivation and understanding are found to impact the repetition of lesson study practice (e.g., Wolthuis et al., 2020, 2021). For this reason, support needed to overcome obstacles mentioned in the studies to sustain lesson study practices will be researched at the national, school, and personal levels.

National level

Lesson study requires a group of teachers, a facilitator who can be one of the teachers with experience in lesson study, and an external commentator, who is generally a university professor or an education expert, to be involved in the practice. Professors as external commentators or facilitators can support teachers with the new educational practices and content knowledge. Fernandez (2004) says lesson study supports teachers' knowledge. However, content knowledge is a prerequisite to having a better experience in lesson study practice. It plays a significant role in lesson study practice for a rich teacher learning experience (e.g., Ball 1990; Borko et al. 1992). This shows that teachers' content knowledge should be improved to have a better experience. The presence and feedback of an external commentator are helpful to ensure this. However, outside of Japan, university professors cannot get involved formally due to the lack of this structure.

In Japan, university professors and schools have financial support from the government. However, teachers do not receive any additional salary from joining lesson study. A similar kind of application exists in the Netherlands. Even though there is no government support, schools can hire an expert from Lesson Study Professional Learning Network (LSPLN) to help them apply the lesson study method and improve their teaching practices. It is gradually becoming popular but is still not as common as in the Japanese school system. Also, due to no government support for specific professional development practices, the school administration has to take the initiative to decide where to spend money for professional development. Several studies mention financial issues as one of the reasons why schools cannot continue lesson study practice in the Netherlands (Wolthuis et al., 2021).

On the other hand, the possibility of preserving all the elements of lesson study while adapting it to an international context is discussed by several researchers (e.g., Akiba, 2016; Takahashi & McDougal, 2016). In the study conducted by Takahashi and Mc Dougal (2016), it has been found that seeking guidance from a facilitator who is experienced in lesson study and input from an expert are not core elements but are optional. So, the absence of university professors might actually not be an obstacle to sustaining lesson study practice if teachers are well supported in the project.

Another obstacle might be creating schedules to meet with university professors. Teachers and professors can create time during the weekends or holidays for these meetings in Japan (Fernandez & Yoshida, 2004). Even though initiatives of schools and professors make the time for meetings, the government support probably affects creating time for it.

School-level

The organizational structure of schools has an impact on sustaining innovative professional development practices that are taking place in their context. Since innovative professional development initiatives are not easy to sustain (Hargreaves & Goodson, 2006; Hubers et al.,2017), schools need to reconsider their organizational structure and adapt accordingly to support these initiatives (Akiba & Wilkinson, 2016). Wolthuis (2021) found that part-time working teachers, the turnover of people acquainted with LS, policy on school improvement, and scheduling are the common obstacles related to school organizations and reasons why schools do not continue lesson study practices in the Netherlands. There is no time to plan lesson study since part-time teachers are not available for longer hours during the week and obviously have fewer hours for professional development practices than full-timers. The turnover of people acquainted with lesson study is another reason people lose their enthusiasm to continue the practice. Having different initiatives to follow due to policy on school improvement creates a high workload for teachers, so following the lesson study becomes even more difficult. Lastly, instruction time has priority over lesson study meetings. This makes collaborative work of lesson study harder.

Personal level

Studies show that in lesson study, three main domains are in interaction. These include the teachers' knowledge, commitment, community, and learning resources (Murata et al., 2004; Lewis et al., 2006; Lewis et al., 2007). Since distinct types of knowledge are integrated into the lesson study cycle, Fernandez (2004) suggests that lesson study improves teachers' pedagogical knowledge. Mass and Dorier claim (2010) that secondary school teachers have weaknesses in pedagogical content knowledge. Although studies suggest that teachers' knowledge is developing with lesson study(Lewis et al., 2009), teachers' awareness of what they have learned from lesson study can affect their commitment to lesson study practice. Lesson study is collaborative work between teachers. After they applied the research lesson, they come together and discuss their observations and ideas. This leads to developing meaningful learning communities which strengthen teachers' commitments to their profession and their motivation to improve their teaching practices (Grossman et al. l, 2009). The lack of meaningful learning communities within the school and district level affects teachers' motivation to continue the lesson study practice. Thus, the lack of teachers' motivation becomes an obstacle in sustaining lesson study.

Interpretation of lesson study varies from teacher to teacher and develops with their experience. Teachers with a lesson study experience focus more on student learning, while ones without experience focus more on the observing phase (Bocala, 2015). Also, some reports show that some teachers confused lesson study with lesson planning rather than focusing on teacher research (Fujii, 2014; Lewis et al., 2006; Yoshida, 2012). A recent study shows that teachers' interpretation affects the sustaining lesson study practice after learning it through external experts. Teachers perceive lesson study as researching students' learning and improving pedagogical content knowledge and are more willing to embed lesson study practice into their teaching routine (Wolthuis et al., 2019). Teachers' understanding of lesson study is another element that can become an obstacle in continuing the lesson study practice.

These main topics derived from literature on personal, school, and national levels will be used in creating the questionaries and semi-structured interviews for the study. The results will be categorized under personal, school, and national level obstacles.

2.3. IBL and Lesson Study

Research lessons where teachers focus on observing students' responses and collecting student learning data are rather prominent in lesson study (Yoshida, 2012). These student-centered lessons provide opportunities for students to do mathematical inquiry and investigate with LS. How the investigation by students can be supported and how the outcomes of such inquiring activities can further feed into a lesson are studied in these research lessons. The external commentator, generally a university professor, helps and guides teachers during this process. He moderates the discussions after the observation phase.

Different lesson study models emerge after adaptation to different countries. According to Ding, Jones, and Pepin (2013), an external commentator has critical importance in the Chinese model. Even in the Japanese model, there are different types of lesson study such as school, local, and national levels. The Japanese lesson study model has been taken as a focus in the Time project. TIME project aims to help teachers to develop their IBL teaching practices through lesson study. It connects a specific teaching method, IBL, with lesson study practice. During the project, teachers will be supported by a facilitator and an external commentator from university partners. Also, teachers are expected to follow the main phases adapted from Stigler and Hilbert (figure 3) for this project.

Adapted phases in TIME projects correspond to the number in figure 3:

1. Teaching problem identification and study
2. Lesson planning
3. Lesson implementation
4. Lesson evaluation and review of the results
5. Reconsideration of the lesson
6. Implementation of a lesson based on reconsiderations

7. Evaluation and review
8. Sharing of the results

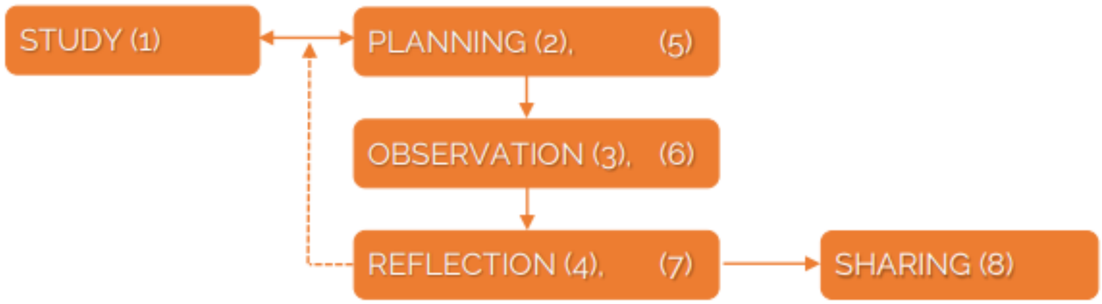


Figure 3: Main phases of the TIME project

In this study, teachers are expected to follow an adapted version of the main phases to support lesson study practice. After completing all steps, a reflection report can be created to share findings and learnings from an LS experience with the international community of the TIME project.

The teacher teams are required to identify a research theme rather than just agree on a topic that is hard to teach. According to the research theme, finding an appropriate mathematical context to teach and subsequently deciding the learning goals are the following steps to be taken. After the lesson plan is completed with the help of a facilitator, one of the teachers applies the lesson plan while the other team members observe and take notes. During the reflection phase, all teachers who attend the lesson share their ideas. An external commentator is also present, who summarizes the discussion by emphasizing the important points made and providing some additional comments. Teams in the project then reflect on their learnings and struggles as a teacher in reflection reports. The team goes back to the lesson plan, attempts to improve it using their newly acquired knowledge from the previous phases, and implements it again. This cycle continues until the teachers are satisfied with the result. During these cycles, it is expected that teachers develop their understanding of the potential of IBL and of ways to implement IBL in their own practice. Thus, the support needed by a teacher and how their understanding of IBL is chancing in lesson study will be studied with the first and second research questions. The relation between teachers' understanding of IBL and sustainability from teachers' perspectives will be derived from the results.

In the Time project, teachers are supported by university professors not only by teaching lesson study practices and feedback to research lessons but workshops on the design principles to design teaching materials. This aims to decrease the need for external commentators and equip teachers with designing educational material on their own.

Research questions

This study will address these research questions:

(1) How does teachers' understanding of IBL develop in lesson study?

(2) What kinds of support do teachers need to be able to continue lesson study practices?

Hypotheses

Teachers can focus on their own development of teaching practice rather than grasping the notion of IBL. Student inquiry is expected to be the focus of lesson plans, and teacher guidance can be adapted to the lesson study experience. We expect improvement in students doing inquiry and understanding of inquiry by teachers.

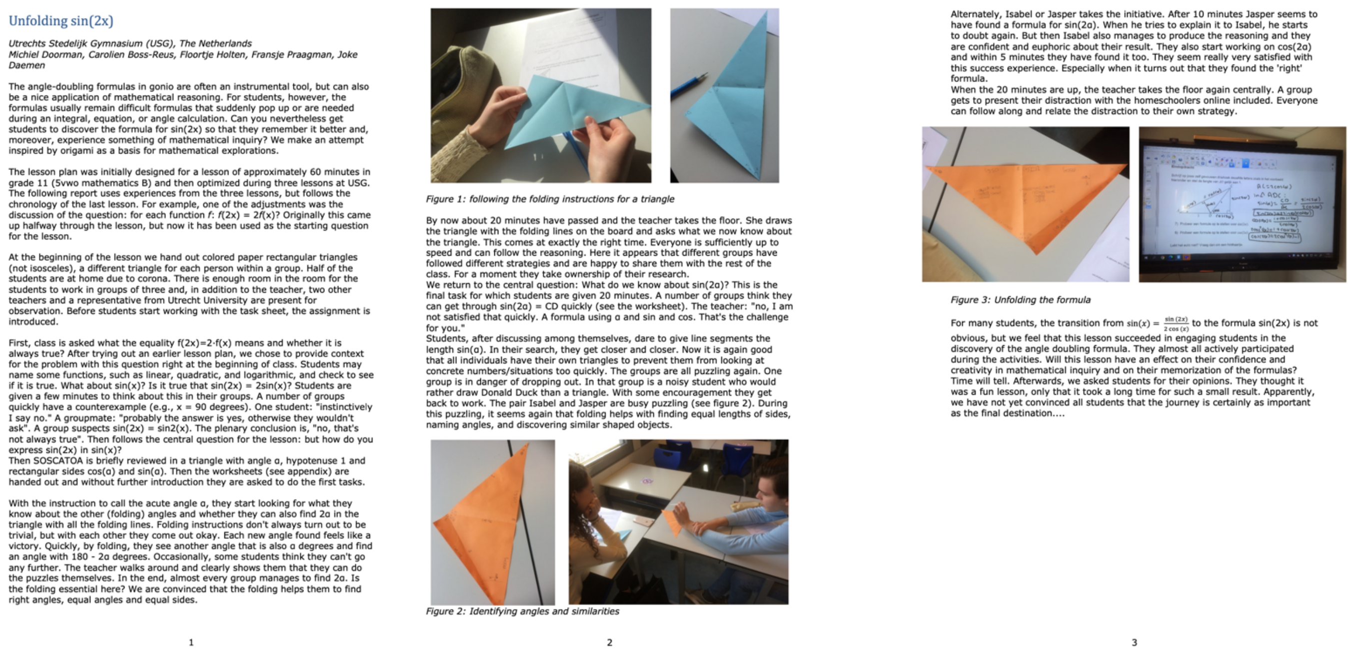
Teachers can need financial and organizational support on the school and national levels. However, any changes at the national level are not expected. In the TIME project, teachers are supported with workshops and design principles to design teaching materials based on their lesson and research goals. In this way, the project aims to sustain lesson study practice by decreasing the need for a facilitator and an external commentator.

3. Methods

3.1.Context and Participants

This study is performed within the European Time Project with participants and researchers from four countries: Croatia, Slovenia, Denmark, and The Netherlands (TIME,2021). Each country includes at least a university and two secondary-level schools participating in the project. Each country has two different schools, which means two different teams. Mathematics teachers will be the main participants of this study, with thirty-seven total. More specifically, nineteen teachers are from Croatia, eight of them from Slovenia, seven from Denmark, and three teachers participating from the Netherlands. All the teachers share a similar interest in applying IBL to improve their teaching practice. Most of them collaborated before in the Mathematics Engaging Realistic Interesting and Applicable (MERIA,2019) project where IBL teaching methods for mathematics were developed and explored. However, most of the participating teachers do not have any experience in lesson study.

The Time project brought a new perspective to lesson study application in these two areas, integrating specific teaching methods, IBL, and supporting teachers on design principles. During the project, participants are supported by workshops introducing lesson study and the design principles for integrating IBL into their lessons, namely Realistic Mathematic Education(RME) and Theory of Didactic Situation (TDS). While designing the research lesson, they receive support from a researcher since participating schools have a collaboration agreement with universities, making an external commentator available to all participants. During the observation phase, different countries joined classes from other countries to give feedback on the research lesson in the discussion phase which is moderated by an external commentator. Based on the feedback team makes changes to the lesson plan and implements it again. This cyclic process continues as long as the teachers want. In the end, findings are shared with the international community through reflection reports in English (figure 4) which are prepared according to a template provided for teachers (Basic, 2021) in the project. These reflection reports include the explanation of task and intention, observation, and reflection on changes made based on observation of students' learning process.



Unfolding sin(2x)

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The angle-doubling formulas in gonio are often an instrumental tool, but can also be a nice application of mathematical reasoning. For students, however, the formulas usually remain difficult formulas that suddenly pop up or are needed during an integral, equation, or angle calculation. Can you nevertheless get students to discover the formula for sin(2x) so that they remember it better and, moreover, experience something of mathematical inquiry? We make an attempt inspired by origami as a basis for mathematical explorations.

The lesson plan was initially designed for a lesson of approximately 60 minutes in grade 11 (5vwo mathematics B) and then optimized during three lessons at USG. The following report uses experiences from the three lessons, but follows the chronology of the last lesson. For example, one of the adjustments was the discussion of the question: for each function f: $f(2x) = 2f(x)$? Originally this came up halfway through the lesson, but now it has been used as the starting question for the lesson.

At the beginning of the lesson we hand out colored paper rectangular triangles (not isosceles), a different triangle for each person within a group. Half of the students are at home due to corona. There is enough room in the room for the students to work in groups of three and, in addition to the teacher, two other teachers and a representative from Utrecht University are present for observation. Before students start working with the task sheet, the assignment is introduced.

First, class is asked what the equality f(2x)=2·f(x) means and whether it is always true? After trying out an earlier lesson plan, we chose to provide context for the problem with this question right at the beginning of class. Students may name some functions, such as linear, quadratic, and logarithmic, and check to see if it is true. What about sin(x)? Is it true that sin(2x) = 2sin(x)? Students are given a few minutes to think about this in their groups. A number of groups quickly have a counterexample (e.g., x = 90 degrees). One student: "instinctively I say no." A groupmate: "probably the answer is yes, otherwise they wouldn't ask". A group suspects sin(2x) = sin2(x). The plenary conclusion is, "no, that's not always true". Then follows the central question for the lesson: but how do you express sin(2x) in sin(x)?
Then SOSCATOA is briefly reviewed in a triangle with angle α, hypotenuse 1 and rectangular sides cos(α) and sin(α). Then the worksheets (see appendix) are handed out and without further introduction they are asked to do the first tasks.

With the instruction to call the acute angle α, they start looking for what they know about the other (folding) angles and whether they can also find 2α in the triangle with all the folding lines. Folding instructions don't always turn out to be trivial, but with each other they come out okay. Each new angle found feels like a victory. Quickly, by folding, they see another angle that is also α degrees and find an angle with 180 - 2α degrees. Occasionally, some students think they can't go any further. The teacher walks around and clearly shows them that they can do the puzzles themselves. In the end, almost every group manages to find 2α. Is the folding essential here? We are convinced that the folding helps them to find right angles, equal angles and equal sides.

1

Figure 1: following the folding instructions for a triangle

By now about 20 minutes have passed and the teacher takes the floor. She draws the triangle with the folding lines on the board and asks what we now know about the triangle. This comes at exactly the right time. Everyone is sufficiently up to speed and can follow the reasoning. Here it appears that different groups have followed different strategies and are happy to share them with the rest of the class. For a moment they take ownership of their research.
We return to the central question: What do we know about sin(2α)? This is the final task for which students are given 20 minutes. A number of groups think they can get through sin(2α) = CD quickly (see the worksheet). The teacher: "no, I am not satisfied that quickly. A formula using α and sin and cos. That's the challenge for you."
Students, after discussing among themselves, dare to give line segments the length sin(α). In their search, they get closer and closer. Now it is again good that all individuals have their own triangles to prevent them from looking at concrete numbers/situations too quickly. The groups are all puzzling again. One group is in danger of dropping out. In that group is a noisy student who would rather draw Donald Duck than a triangle. With some encouragement they get back to work. The pair Isabel and Jasper are busy puzzling (see figure 2). During this puzzling, it seems again that folding helps with finding equal lengths of sides, naming angles, and discovering similar shaped objects.

Figure 2: Identifying angles and similarities

2

Alternately, Isabel or Jasper takes the initiative. After 10 minutes Jasper seems to have found a formula for sin(2α). When he tries to explain it to Isabel, he starts to doubt again. But then Isabel also manages to produce the reasoning and they are confident and euphoric about their result. They also start working on cos(2α) and within 5 minutes they have found it too. They seem really very satisfied with this success experience. Especially when it turns out that they found the 'right' formula.
When the 20 minutes are up, the teacher takes the floor again centrally. A group gets to present their distraction with the homeschoolers online included. Everyone can follow along and relate the distraction to their own strategy.

Figure 3: Unfolding the formula

For many students, the transition from $\sin(x) = \frac{\sin(2x)}{2\cos(x)}$ to the formula sin(2x) is not obvious, but we feel that this lesson succeeded in engaging students in the discovery of the angle doubling formula. They almost all actively participated during the activities. Will this lesson have an effect on their confidence and creativity in mathematical inquiry and on their memorization of the formulas? Time will tell. Afterwards, we asked students for their opinions. They thought it was a fun lesson, only that it took a long time for such a small result. Apparently, we have not yet convinced all students that the journey is certainly as important as the final destination....

3

Figure 4: Example Reflection Report from teachers

The main aim of this paper is to explore how lesson study in IBL practice can become a sustainable way of working. To reach this aim, we will investigate the support teachers need and how their understanding of IBL develops in the TIME project by analyzing reflection reports and questioning the teachers.

3.2 Data collection instruments

A questionnaire to study obstacles encountered by teachers is designed to find an answer to the second research question. It has eight questions in total. The first question aims to understand the motivation behind participating in inquiry-based teaching in lesson study. The second question checks the extent to which teachers understand lesson study. Questions three to five aim to draw insight into obstacles teachers encountered and their views on having an external commentator. Questions six to eight are prepared to have an understanding of the willingness of teachers to continue lesson study practice after the TIME project ends (Table 2).

Sub Questions to answer 2nd Research question	The aim
What are the obstacles that teachers encountered during the lesson study (personal, school and national level)?	Motivation of teachers
	Teachers' Understanding of lesson study
	Views of teacher on external commentator
	Obstacles during the lesson study practices
What is the perspective of teachers on continuing lesson study practice after the project ends?	Support needed to continue of lesson study
	Views on continuation of lesson study practices after the TIME project

Table 2: Overview of questionnaire items

The items are developed based on the obstacles found in the literature, such as financial support, motivation, and scheduling. After questions are formed, the survey is sent to two experts. As a result, some questions are changed not to be biased. Instead of asking which ones are important to continue the lesson study, the question is formed objectively by asking which ones are important to continue or not to continue LS. Also, teachers are allowed to add more options to the answers.

The interview scheme is created based on the literature review and findings in reflection reports and questionnaires. Two main questions with two sub-questions are prepared. One of the main questions is related to IBL, and the other is to LS. For example, the sub-questions on IBL are about where teachers struggle the most applying IBL to their teaching, and another one is about how helpful the support they got from the project was. The interview questions are discussed with an expert and a pilot interview is conducted. After the pilot interview, the question on LS is changed to make it more clear for teachers.

3.3 Data collection

Reflection reports of teachers, questionnaires, and semi-structured interviews are the main instruments for answering the first research question (Table 3). To answer the second question, a questionnaire is used to investigate supports and obstacles that teachers encountered during the TIME project. These instruments are followed up with a semi-structured interview with four teachers to gather more insight on both research questions. These four teachers represented the participating countries. Four participants from each country for the interview are selected based on the questionnaire analysis and the result of reflection reports. Unfortunately, an interview couldn't be conducted with a Slovenian teacher due to limitations in English proficiency. Therefore, the fourth interview is conducted with another participant from the Netherlands. Interview questions were sent before the scheduled interview day to give teachers time to discuss the questions with their teams.

Instrument	Quantity	
Reflection reports (Data collected During LS)	15	10 from first, 5 from second cycle
Questionnaire (Data collected After LS)	19	From 37 teachers
Semi-structured interview (Data collected After Questionnaire)	4	1 Croatian, 1 Danish and 2 Dutch

Table 3: Type of Instruments used in the research

Teams of teachers from each school write a reflection report after a lesson study cycle. Teachers write down their observations from the class and explain the changes that have been made in the lesson study. The project has fifteen reflection reports, ten from the first cycle and five from the second one. The first cycle reports include the first report of each team, and the second cycle consists of the second trial of the same research theme. The Slovenian team used the same teaching activity for the first and second cycle while other teams changed theirs. The Dutch team has no reports on the second cycle.

A questionnaire is used to create a scheme for the obstacles that teachers encounter during the lesson study process. It is sent to all teacher participants, and nineteen answers are collected. In addition to the questionnaire, we developed an instrument for semi-structured interviews to dive into the findings emerging from the analyses of the questionnaires and the practice reports.

3.4 Data analysis

Reflection reports

The development of understanding IBL is analyzed by using reflection reports created during the Time project by teams of teachers. These reflection reports are coded according to table 2 adapted version of essential features of inquiry and phases of inquiry in science(Table1). Since mathematics is already an essential element and has to be used in the inquiry process, elements U3 and D8 are deducted from table 1. One full sentence is used as a unit for coding. The actions of the teachers form the base for the understanding of inquiry, while students' actions are the base for doing the inquiry part. "*...For example, one of the adjustments was the discussion of the question: for each function f: f(2x) = 2f(x)? Originally this came up halfway through the lesson, but now it has been used as the starting question for the lesson...*" is an example of an action of teachers while the sentence "*…Quickly, by folding, they see another angle that is also α degrees and find an angle with 180 – 2α degrees…*" shows the inquiry being done by students. Based on the category the sentence belongs to a code is attained from table 2. In the previous example sentences, the act of the teachers is coded as U1, and the act of the students is coded as D3. The elements of IBL (understanding of inquiry and doing inquiry) are found in reflection reports to provide an answer to the first research question.

The reliability of the coding scheme is checked by coding the same reports by a second coder. The usage of the coding scheme is explained to the second coder. Two reports with a different number of codes density are used among the reflection reports. The coding was found to have an 88% match. The second coder chose to code the planning part, where teachers mentioned the expected behavior of students. This part was not coded by the researcher mainly because it is impossible to get any insights into students doing inquiry. When the coding related to the planning part is taken out, there is a match of 91%. However, it is possible to get insights into teachers' understanding of inquiry; based on their lesson plan, the reports are reviewed and coded for the planning part. Moreover, the code using tools and techniques to gather, analyze, and interpret data(D7) is taken out since tools are given by teachers and using techniques is already checked in recording and collecting data (D3).

Understanding about inquiry (U)	Explanations Teachers' acts based on their reflections reports	Doing Inquiry(D)	Explanations Students' actions based on teachers' reflection reports
U1: Different kinds of questions suggest a different kinds of scientific investigations	U1: Teachers asking specific questions, using specific context or delaying attention for misconceptions to help students to continue inquiry. Ex: The teacher does not give any further instructions purposefully.	D1: Involved in sci-oriented problems	D1: Students are presented a problem/question which is brought by a teacher and start asking questions about it Ex: Each group is given an A3 worksheet leading them to define a term on their own based on real life example from immediate surroundings
U2: Current scientific knowledge and understanding guide scientific investigations	U2: Teachers using students current knowledge to guide them in inquiry stage Ex: "Ultimately, the teacher sends the discussion to the correct solution, which is then shown, see Figure 3".	D2: Design and conduct investigation.	D2: Students start asking questions, creating models, making estimations. Ex: They were moving the chairs, opening and closing window panes and their textbooks, looking at the projector lens.
U4: Technology used to gather data enhances accuracy and allow scientist to analyze quantify results of the investigation	U4: Teachers designing lesson material which allows students to gather data, create cases, have a systematic approach to gather data.	D3: Priority to evidence in resp. to a problem: observe, describe, record, graph	D3: Students gathering data to find an answer to their problem. Ex: After the first devolution students in groups start drawing a coordinate system full of confidence the way they usually do it paper is turned landscape, the origin is in the middle of the paper, axes are named x and y, unit is the same on both axes, and the scale is linear.

U5: Scientific explanations emphasize evidence, have logically consistent arguments, and use scientific principles, models, and theories. \	U5: Teachers prepare a discussion environment for students to improve their arguments based on their models and mathematical concepts. Ex: In the classroom hangs a clothesline on which the students hang their worksheet when they are finished.	D4: Uses evidence to develop an explanation (e.g., cause for effect, establish relationship based on evidence)	D4: Based on their observation develop/ change their explanation or solution. Ex: They notice that all the values are positive, so they only need the first quadrant and place the origin in one corner of the paper.
U6: Science advances through legitimate skepticism	U6: Teachers creating an environment where students question their findings.	D5: Connect explanation to scientific knowledge: does evidence support explanation? Evaluate explain in light of alternative exp., account for anomalies	D5: Compare the explanations with formal/ generalized definitions. (?) Ex: After the discussion the teacher hangs a formal definition on the board so everybody can see it and comments whether those two groups definitions were accurate or had some deficiencies and need to be improved.
U7:Scientific investigations sometimes results in new ideas and phenomena for study, generate new methods or procedures for an investigation, or develop new technologies to improve the collection of data.	U7: Teachers create a design where some students reach the general formula where students can use in different context.	D6: Communicates and justifies	D6: Discuss in teams or class about findings to justify their solution or explanation. Ex: After the second devolution students eagerly discuss the questions on the handout.

Table 4: Reflection report coding scheme

Questionnaire

The data analysis from the questionnaire is conducted using descriptive statistics and thematic analysis (Table 5). Common schemes are created based on personal, school, and national(international) level obstacles from the literature that teachers encountered.

Sub Questions to answer 2nd Research question	The aim	The analysis	Question of the survey
What are the obstacles that teachers encountered during the lesson study (personal , school and national level)?	Motivation of teachers	Descriptive statistics	Question 1,6 and 7
	Teachers' Understanding of lesson study	Inductive Coding	Question 2
	Views of teacher on external commentator	Descriptive statistics and thematic analysis	Question 4 and 5
	Obstacles during the lesson study practices	Descriptive statistics	Question 3
What is the perspective of teachers on continuing lesson study practice after the project ends?	Support needed to continue of lesson study	Descriptive statistics	Question 6,7
	Views on continuation of lesson study practices after the TIME project	Thematic analysis	Question 8

Table 5: Analysis Scheme of Questionnaire

Semi-structured Interviews

Semi-structured interviews with the selected participant teachers are analyzed to delve deeper into understanding the findings from the reflection reports and questionnaires to answer both research questions. However, it is important to note that while the outcome cannot be generalized, it provides insights into differences from one nation to the other. The interview is recorded in an online environment by using Zoom or Teams. Interviews and questionaries are analyzed by looking for the shared themes on personal, school, and national levels as mentioned in the literature that teachers mentioned negatively or positively. After the analysis, a new level which is international, occurred due to the TIME project being a European project with international participants. Obstacles are considered as problems or negatively mentioned themes such as time-consuming, while the themes that they mentioned positively or as a request from the school are considered as support. For instance, needing help with the schedule to continue lesson study. The last question of the interview measures the willingness of teachers to continue lesson study practice by simply asking their thoughts and plans on lesson study as a team.

The inter reliability of the coding scheme for interviews is checked by using a second coder. The coding scheme is explained and ten percent of a random selection of sentences are given. Categorization of obstacles and support is discussed. The workload is taken on a personal level and not at the school level since it changes from person to person.

3.5 Procedure

Firstly, reflection reports are collected during the project when teachers are done with their LS cycle. They have shared a reflection report at least for two complete LS cycles except for the Netherland team. These reflection reports are analyzed according to the coding scheme in Table 2. The reliability check is done by a second coder. Considering the number of IBL elements in the reflection reports, the first research question is answered.

The second step in this study is conducting the questionnaire with all teacher participants of the TIME project. The questionnaire results collect participants' views on lesson study practice, obstacles they have encountered, and continuing to LS provide answers regarding the support teachers need on a personal, school, national, and international level. To clarify and have more insight into the support and IBL understanding, a questionnaire is followed by a semi-structured interview. The pilot interview is conducted with a teacher from the project. Based on the results, the interview is improved. After the improvement, it is conducted with other teachers through Zoom and Teams online environment. The meeting is recorded and the transcript of the interview is written. The answers will be collected under the common schemes. Based on the result, the second and third research questions are answered.

4. Results

4.1 Reflection Reports

Most of the teams of teachers were involved in two successive LS cycles. The first cycle includes ten reflection reports in total, and the second cycle has five reports. IBL elements in these reports were analyzed in two categories: teachers' understanding and preparing of inquiry and accounts of students doing inquiry. In the reports, statements for teachers' understanding inquiry were assigned to six different elements of IBL. The number of reports having these codes assigned to teachers' understanding inquiry elements is shown in figure 5. Understanding of different questions leading to different inquiries was found in most of the reports, followed by the understanding of discussing models with scientific explanation and creating a discussion environment for inquiry (figure 5). On the other hand, the understanding of inquiry leading to a general formula or explanation is only found in two reports.

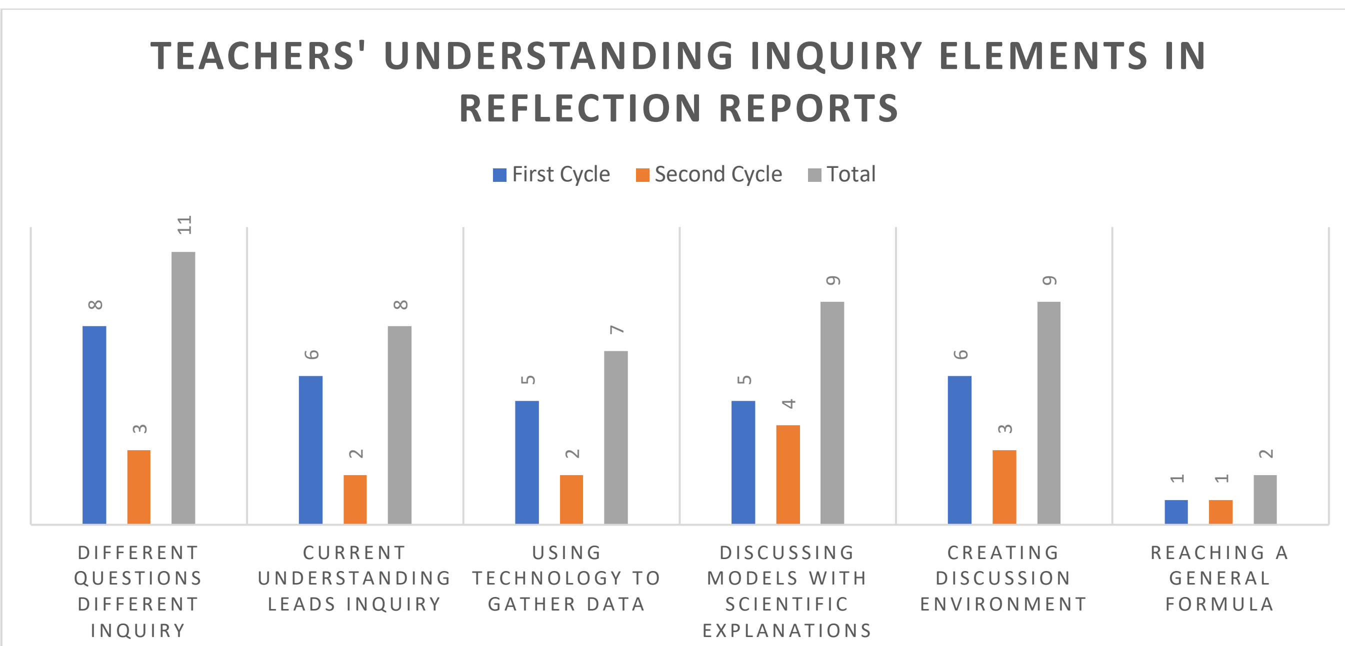

Figure 5: IBL elements in Reflection Report- Understanding of Inquiry

Students doing inquiry is also assigned six different IBL elements. The number of reports with these codes is shown in figure 6. Start questioning through a mathematical problem and design/investigation are the most frequent elements in the category of students doing inquiry (figure 6). Creating models, justification, and communications are found in slightly more than half of the reports. Connecting findings with a mathematical argument is rarely found in the reports.

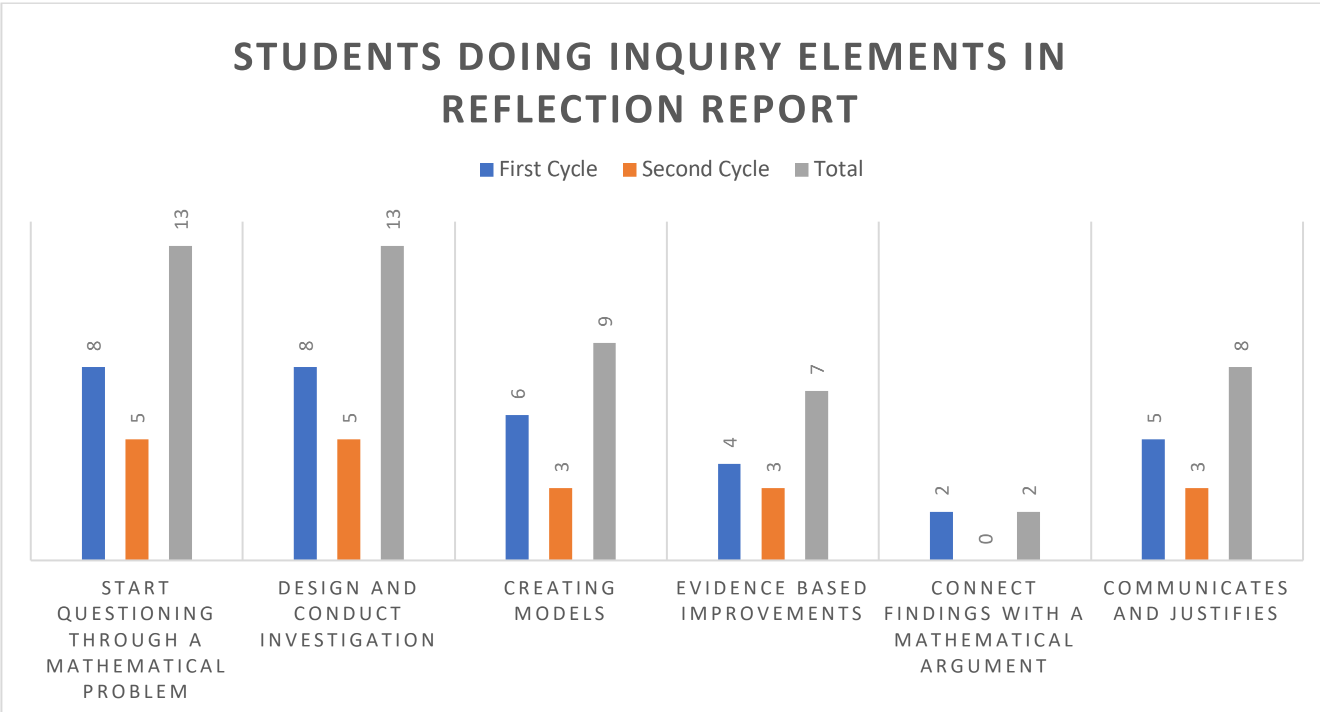

Figure 6: IBL elements in the reports- Students doing inquiry

We also compared the change of inquiry-related codes in the reports for each of the (country) teams (table 6). When looking at the reports from different teams, the highest percentage of the existing codes in the reports is the one from the team of Netherlands with 84% in the first cycle, followed by the Croatian and the Danish teams with 77% in the second cycle (Table 6). One of the reports includes all understanding of inquiry codes, while one of them includes none. One of the reports from the Croatian team has all the codes for doing inquiry, followed by the reports from the Dutch and the Danish, and another report from the Croatian team with 83% existing codes. Overall, the Croatian and the Slovenian reports have higher percentages of codes concerning inquiry done by students than those concerning the teachers' understanding of inquiry. This trend is reversed for the Dutch and the Danish teams. The reports from these teams have more codes on teachers' understanding of inquiry than students' doing inquiry (Table 6).

The Slovenian team used the same activities on both cycles while the Danish teachers changed the activities. The number of teachers' understanding of inquiry codes dropped for the Slovenian team while it stayed the same for the Danish team. Different understanding code creating a discussion environment (U5) is found in the second cycle in the Slovenian team. However, different questions lead to different inquiry(U1), and current understanding leads the inquiry (U2) are disappear in the second cycle(Table 6). Similarly, for the codes, students' doing inquiry is lowered since the creating models (D3) is not found for the second cycle reports (Table 3). Overall, total number of the codes found in the reports is dropped for The Slovenian team from five codes (U1,2, D1,2,3,4) to the four (U5, D1,2,4). The Denmark team's number of codes has remained similar for the teachers' understanding. However, the codes for students' understanding have increased. Similarly, the total number of codes in the reports has increased for Croatian and Denmark (figure 6).

	Teams	**Existing Codes ono Understanding of Inquiry**	**Existing Codes on Doing Inquiry**
First Cycle	Croatia	U1,5 and 6	D1,2,3,4 and 6
		U1,4,5 and 6	D1,2,3,4,5 and 6
		U2	
		None	None
	Denmark	U1,2,4 and 6	D1,2,3 and 6
		U1,2,4,5 and 6	D1,2
	Netherlands	U 1,2,4,5 and 6	D1,2,4,5 and 6
		U 1,2,4,5,6 and 7	D1,2,3 and 6
	Slovenia	U1,2	D1,2 and 3
		U1	D1,2,3 and 4
	Croatia	U1,2,4 and 6	D1,2,3 and 6
		U1,2,5 and 6	D1,2,3 and 6

Second Cycle	Denmark	U1,4,5,6 and 7	D1,2,3,4 and 6
		U5	D1,2 and 4
	Slovenia	U5	D1,2 and 4

Table 6 :Existing codes in the reports

The average of existing codes in a report is calculated by dividing the total codes by the number of reports excluding the Netherlands (Figure 7). Since no data was obtained from the Netherlands for the second cycle, the first one is not included in calculating the average number of codes. Based on the result, the average number of existing codes on teachers' understanding of inquiry and students doing inquiry has increased from the first to the second cycle.

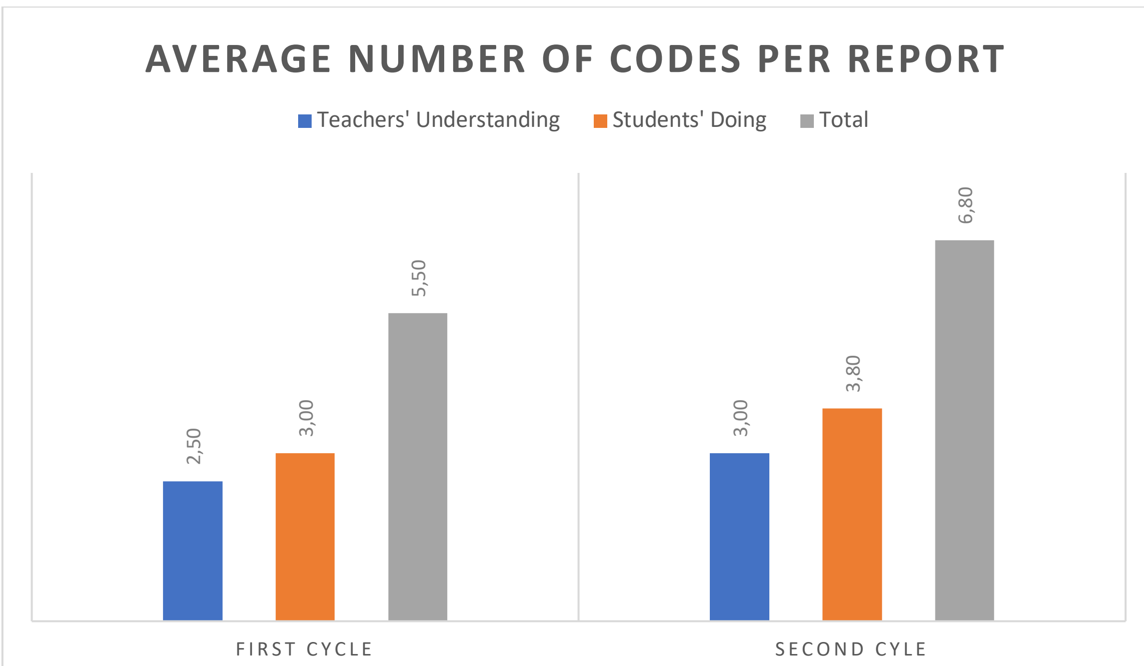

Figure 7: Average Number of Codes in a Reflection Report

4.2.Questionnaire

The number of teachers who participated in the questionnaire is nineteen out of thirty-seven.(Table 7)

Country	Number of Teachers
Croatia	7
Denmark	4
Slovenia	5
The Netherlands	3

Table 7: Number of Participating Teachers in Questionnaire

The overall analysis of motivation to join LS shows that learning new teaching approaches, inquiry-based teaching, and working as a team are the important elements from teachers' perspectives (Figure 8).

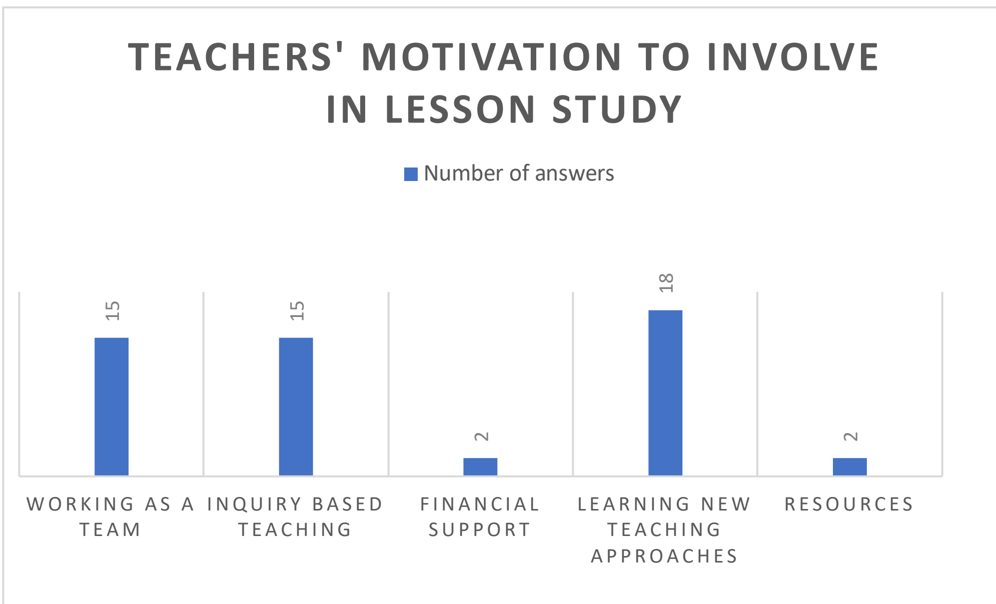

Figure 8: Teachers' Motivation to Involve in Lesson study

The elements that are found to be effective in continuing lesson study are motivation, team, and schedule by more than half of the teachers (figure 9). These elements are followed by having an external commentator, students' development, and workload. Moreover, financial support is important for two teachers in the continuum of lesson study. Most of the teachers, namely 73 percent, said they feel comfortable designing lesson material. Slightly more than half of the teachers mentioned that their principal also wants to continue LS practices after the TIME project ends. At the same time, one of them said the principal supports continuing LS in an action-based learning framework. Moreover, almost half the teachers said their school will have a university partner even if the TIME project ends.

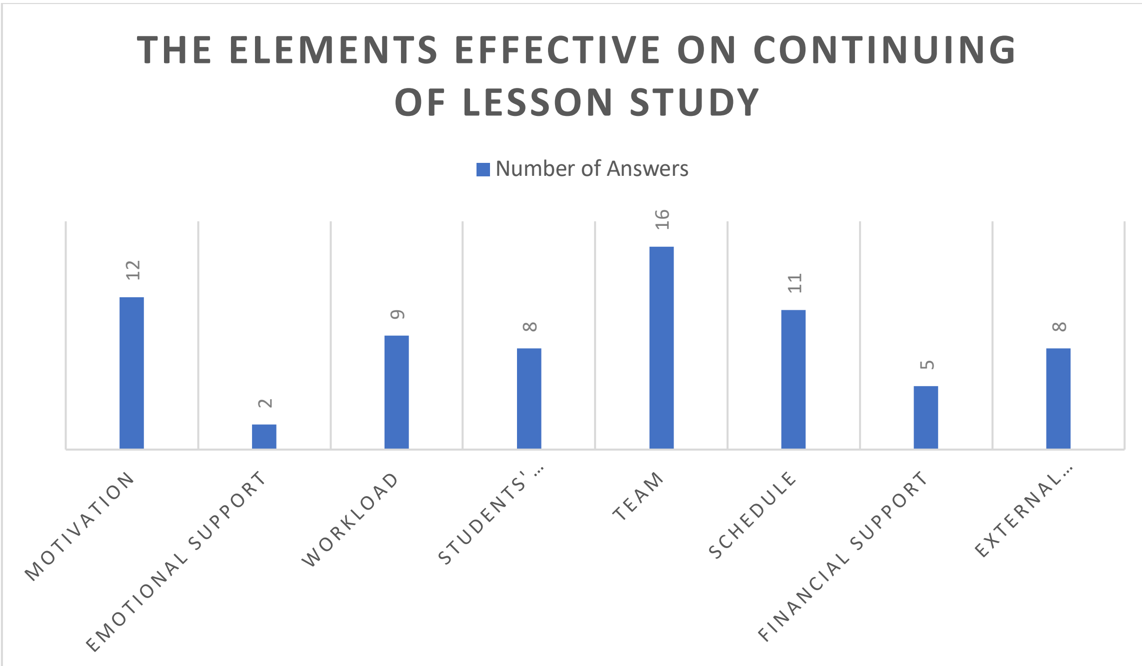

Figure 9: The elements effective in the continuing lesson study

The elements like motivation, workload, and development of students are considered at the personal level; schedule and team are regarded at the school level. The team is considered school-level since the principal is responsible for hiring teachers. The literature considers financial support and external commentators at the national level. However, in this project, those are provided at the project level. Thus, we took them to the international level. Questionnaire results show that the teachers need more support at the personal and school levels to sustain LS practices(Figure 10).

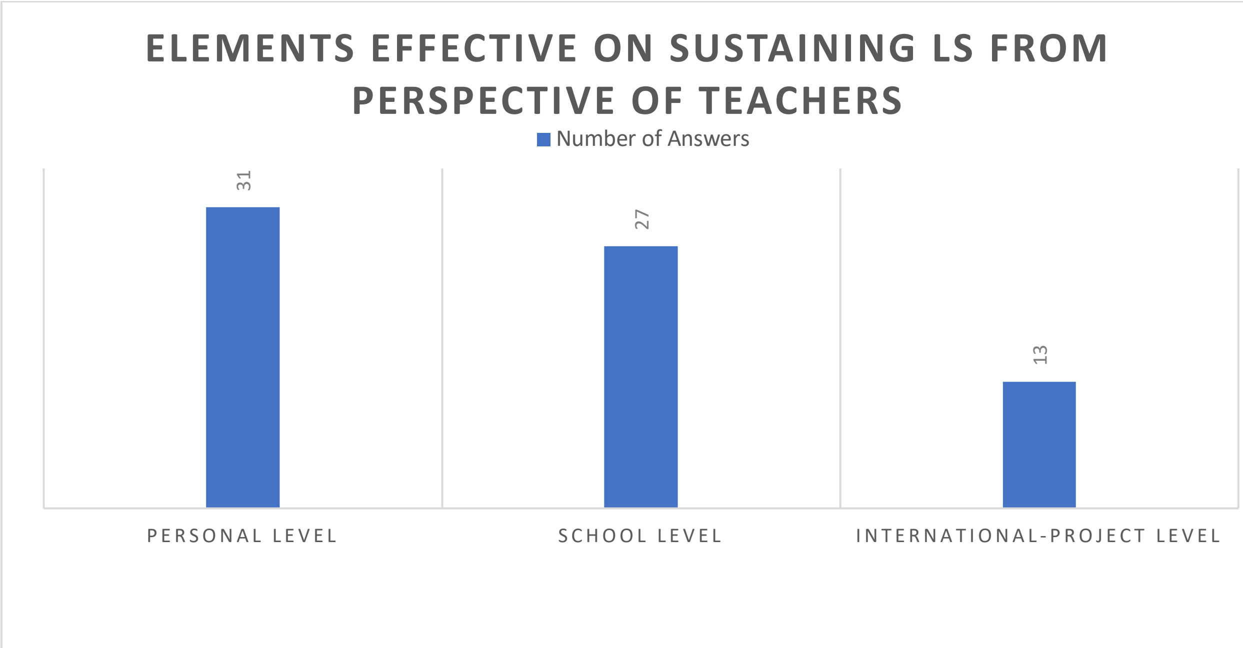

Figure10: Personal, school, national and international effects

The percentage of the teachers who found having an external commentator very important and important is 79 percent, while only 10 percent found that it is not needed (Figure 11). The remaining participants who stayed neutral stated that having an external commentator is not necessary. Teachers who found having an external commentator important state that an external commentator provides constructive feedback and helps them look at the lesson plan and lesson from a different perspective. Also, one teacher mentioned the motivational effect of the external commentator.

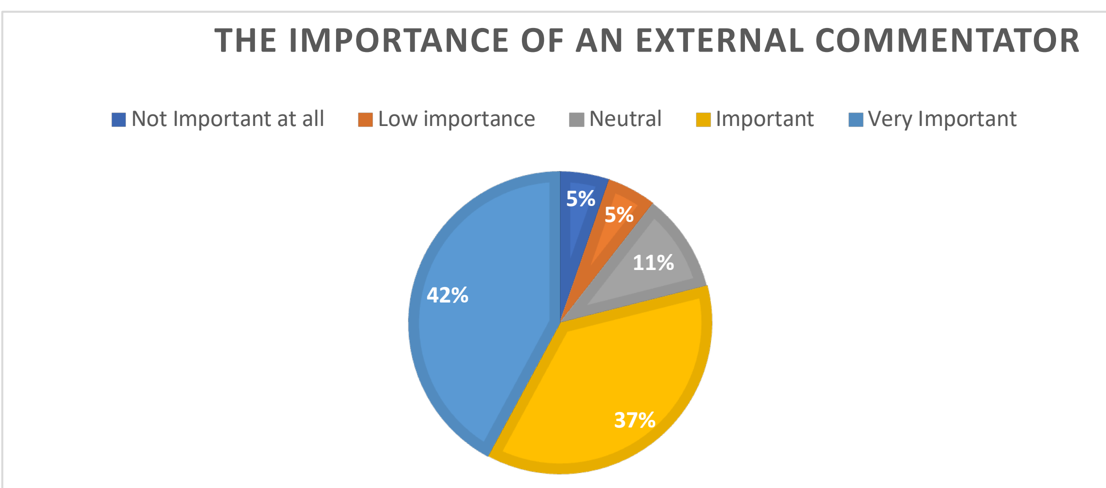

Figure 11: Importance of An External Commentator

4.3.Interview

Interviews are conducted with four female teachers with experience in IBL, varying from five to fifteen years. They are all secondary-level teachers with experience ranging from fifteen to twenty-five years. The group of students teachers responsible for differs in the Netherlands. The secondary school has four grades in all countries except the Netherlands, where the secondary level has six grades. Thus, teachers in the Netherlands are also responsible for lower grades compared to other countries.

Participants

Male/Female	0/4
Teaching Experience	15 to 25 years
Class-Student Group	13 to 18 years
Country	Croatia (1), Denmark (1), The Netherlands (2)

Table 8: Background information

Replies of teachers to the IBL-related questions are coded according to the understanding of IBL codes in table 2. Based on the result, teachers only focused on different questions leading to different inquiries, how current knowledge affects the inquiry of students, and the use of science to improve through legitimate skepticism. None of the teachers explicitly mentioned the need for creating a discussion environment for students. However, three of them did mention that students tend to be more active than in a traditional classroom.

Regarding obstacles, four levels of obstacles are mentioned, while there are only three levels for support. On a personal level, all teachers found that lesson study requires too much time. One of the teachers mentioned that being motivated and finding lesson study useful for their personal goals is important and can affect a team's productivity based on a case during the project.

	Obstacles	Support
Personal	Time, Motivation, Workload	Lesson material, Professional Development
School	Schedule	Time, Schedule
National	Curriculum	
International	Language,Curriculum, Understanding of IBL	External Commentator, Time

Table 9: Obstacles-Supports

At the school level, only one teacher mentioned the schedule as an obstacle. Teachers have done the project duties in their personal time. Being a part-timer also made it hard to find a common spot for each teacher. At the national level, the curriculum is only mentioned by one team '…*I think that we are old-fashioned, and our teachers are very afraid of innovation. Because they are insecure and sometimes, they think that we don't have time because we have a lot of content...*'. On the international level, language has been seen as a problem. At the observation stage, they mentioned that they could not understand the discussion between groups, which provides valuable evidence of students' learning process.

Similarly, the language problem occurred when teachers were discussing with teachers from different countries. One of them mentioned that having a different view on IBL teaching prevented fruitful discussion by saying, '*If they didn't think there should be some inquiry, then we are talking not the same language, and that's been a struggle, I think.'* Considering support teachers expect from school is having more flexibility regarding time and schedule.

Overall, all teachers mentioned that time and scheduling a meeting was an obstacle. All of them do find the external commentator useful in the beginning. Two teachers do not think an external commentator is necessary to continue lesson study practice. However, all the teachers think having an external commentator gives more structure and value to the process in terms of creating time, such as *'It makes our work serious for the head of the school, for the principal*.' Three teachers mentioned that the practice would fade away without an external commentator. Three teachers mentioned their willingness to continue IBL practices in LS after the project ended.

However, one of the teachers mentioned that they do not want to continue LS practices due to personal level obstacles, namely time.

5. Discussion

This study researched how teachers' understanding of IBL develops and explores the support needed by teachers. It is outlined that LS practices have the potential to improve teaching practices on IBL. Studies claim that LS practices can improve teachers' knowledge (Lewis,2009) and promote their growth (Lewis 2013,2011,2006). The increase in the average number of IBL elements found per report supports this claim. Teachers' understanding of inquiry elements in the reports has increased from the first cycle to the second one. Besides that, the increase in students doing inquiry combined with teachers' understanding indicates improvement in teaching practices.

Regarding sustainability from teachers' perspective, more personal level obstacles are mentioned than the others. Time and schedule are the most important factors mentioned by teachers as an effect on continuing LS practices. As was found, an external commentator is helpful to support and guide teachers in the process (reference) but more importantly, they structure the practice. The relationship between university professors and schools is important to sustain lesson study practice and support teachers to improve their teaching practice. Thus, schools can prioritize having those relations with the universities.

The results from this study cannot be generalized. However, they give valuable insight into lesson study practice at a high-school level in the European context. In a limited scope, IBL practices in LS have the potential to improve teaching practices by giving teachers a chance to apply in their own classrooms. The study can be repeated in a broader scope, and lesson observation forms can be used rather than just reflection reports. The presence of specific codes might also be impacted by the topic or the task of the LS. The Slovenian team continued with the same tasks while other teams changed to a new one for the second cycle. The different trend in the Slovenian team might also be created by these different choices of the country teams. Time and support for scheduling LS meetings are needed to sustain LS practices from teachers' perspectives. An external commentator is also useful in structuring meetings and creating time for the LS. In the long term, teachers can be contacted again to study how they sustained the IBL in lesson study or not. A continuation of this process of designing lessons and reflecting on findings with colleagues seems to be fundamental for implementing IBL in the daily educational practice of mathematics teachers.

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