MSc History and Philosophy of Science

Master Thesis

Emergent Solitons and the Philosophy of Non-Perturbative Quantum Field Theory

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Abstract

Solitons in quantum field theory are finite-energy solutions to the non-linear equations of motion. In this thesis, I argue that the solitons in two specific models, namely the sine-Gordon model and the Seiberg-Witten model, are emergent from dual models without solitons. Using the framework for emergence introduced by De Haro (2019), I argue that the behaviour in the sine-Gordon model fits the notion of epistemic emergence and that the behaviour in the Seiberg-Witten model fits the notion of ontological emergence. In these cases, a duality or an approximate duality serves as a linkage map between two models that contain some novelty with respect to each other. Contrary to the common relation between emergence and fundamentality in which a less fundamental theory emerges from a more fundamental theory, I argue that in these case studies, the emergent particle can be understood as equally fundamental as the elementary particle in the dual theory. This claim is made on an ontological level for the sine-Gordon model and on an epistemic level for the Seiberg-Witten model. Because soliton solutions are found by non-perturbative methods, an investigation of these models avoids many foundational problems related to perturbative renormalization, the method that is commonly used to treat unphysical infinities that arise in quantum field theory. My claims are further substantiated by the fact that the particle spectra of the case studies can be computed exactly, which grants the models a mathematical rigour that is exceptional compared to other models formulated in Lagrangian quantum field theory.

Contents

1 Introduction

Since its early developments in the 1930s, quantum field theory (QFT) has become the most successful theory to describe nature at the smallest scales. QFT is a mathematical formalism in which the principles of quantum mechanics and special relativity are combined to form a field theory. A popular paradigm for physicists has been to understand QFT in terms of elementary particles, that interact with each other through the exchange of a particular type of particles called gauge particles. The cluster of models describing the physical interactions collectively form the Standard Model of particle physics. In addition to the particles of the Standard Model, many non-elementary particles that have been postulated in the framework of QFT have also been experimentally observed. Of particular interest for this thesis are QFT models in which there exist exact solutions to the classical non-linear equations of motion that are stable and posses particle-like properties. These solutions are called 'solitons'.

In general, exact computations of the physical parameters of particle interactions are very hard to execute. To overcome this problem, physicists apply approximation methods. The most common method is perturbation theory, in which the exact solution is approximated by expanding in the coupling constant of an interaction. While the perturbative approximations in quantum field theory have led to astounding empirical successes, their applicability also has its limits. In some cases, such as in models with soliton solutions, perturbative approximations are not valid or sufficient to extract all the interesting knowledge on the system. Whenever this is the case, the physicist needs to find different, non-perturbative methods to describe the system. Often, these non-perturbative methods are applied to a simplified model, in order to optimally study the aspects of the model that resemble realistic phenomena.

In this thesis, I will discuss two of these models in which soliton solutions exist. The first model is the sine-Gordon model. This is a two dimensional model that has been around since the nineteenth century and that has become a paradigm example of soliton physics in the twentieth century. In 1975, the model was found to be dual to the massive Thirring model, which led physicists to study it in the context of bosonization. The second model that I will discuss is called the Seiberg-Witten model. This is a supersymmetric low energy effective model that was introduced in the 1990s in the context of strong coupling behaviour in gauge theories. It has been studied to shed light on the phenomenon of particle confinement in quantum field theory and is also interesting in the context of electrodynamics, as it contains an electromagnetic type of duality.

From a philosophical perspective, these models are interesting in the context of emergence and fundamentality. The notion of emergence is often associated with some kind of irreducible or inexplicable relative novelty. In case of a duality between a soliton model and another model, such a novelty can be a soliton or an elementary particle. In this thesis, I will examine this idea in more detail in the context of the sine-Gordon model and the Seiberg-Witten model. The central argument is that these soliton models exhibit behaviour that fits the notion of emergence given in De Haro (2019). In this paper, de Haro provides a formal framework for ontological emergence in the physical sciences in terms of novelty and a linkage, where the notion of emergence concerns the entities and properties of systems. Furthermore, I will argue that our common understanding of fundamental particles in quantum field theory is challenged by these models. By means of the dualities, what is considered to be the fundamental elementary particle is exchanged with the less fundamental soliton. An additional aim is to provide a discussion of the Seiberg-Witten model that is accessible to philosophers, which can serve as a groundwork for further investigations. For example, an interesting aspect of this model is the phenomenon of confinement, which has not received much attention from philosophers yet.

In the first part of this thesis, the focus will be on introducing the various mathematical and philosophical concepts that are relevant for a detailed discussion of my case studies. In section 2, I will provide an introduction to the physics of quantum field theory, and focus on the perturbative and non-perturbative methods used in computations of particle interactions. Furthermore, I will explain how solitons can be interpreted as particles in quantum field theory. In sections 3 and 4, my focus will turn towards philosophy. I will discuss how, among other things, the perturbative approximation of interactions has led philosophers to advocate different approaches to QFT in section 3, and I will argue against claims that the type of physics that my case studies belong to is not suitable for philosophical investigation. I will then introduce the notions of fundamentality and emergence in section 4, and relate these concepts to the concept of duality. The last two sections are entirely devoted to a discussion of the two specific soliton models. In section 5, I will provide a discussion of the sine-Gordon model that is largely based on earlier work by Sebastian de Haro and Elena Castellani (2018). In line with Castellani and de Haro, I will argue that the notion of epistemic emergence applies to this physics and I will reformulate and specify their argument in terms of the formal framework that I introduced in the preceding section. This discussion will provide some groundwork for the discussion of the Seiberg-Witten model in section 6. Here I will introduce new work and argue that in this case, the notion of ontological emergence applies.

2 Quantum Field Theory

Quantum field theories can be formulated in terms of Lagrangians. For a theory of interactions, the Lagrangian can be split into a free part and an interacting part,

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I. \tag{1}$$

Let's take the ϕ^4-theory as an example. This theory is well understood, and is commonly used as a heuristic example in QFT textbooks. The field described by this theory is a scalar field ϕ. In high energy physics, the excitation of this field is taken to be a scalar particle (a boson with zero spin). The free Lagrangian of the system is given by

$$\mathcal{L}_0 = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2, \tag{2}$$

where the first term is called the kinetic term and the second term is the mass term. Quantum field theories that describe particles without interactions, 'free' particles, are generally under good mathematical control, meaning that it is often possible to solve the equations of motion exactly.

However, realistic theories also incorporate interactions between particles. For example, the Standard Model, more specifically the theory of quantum electrodynamics (QED), describes interactions between electrons as the exchange of (virtual) photons. In the ϕ^4-theory we are looking at, an interaction that is often taken into account is that of the scalar particle interacting with itself, given by

$$\mathcal{L}_I = -\frac{\lambda}{4!}\phi^4. \tag{3}$$

Here the physical understanding of λ is that it is the coupling constant of this interaction, whose value encodes the strength of the coupling between the fields. In the case of QED, this constant is the electric charge. For a Lagrangian that includes interaction terms like this, the equations of motion are not so easily solved. We can distinguish between two situations in quantum field theory from the perspective of perturbation theory, which I will discuss in the next sections.

2.1 The Perturbative Approximation

In the first situation, the coupling constant of the interaction is small. In this case, the interaction term can be seen as representing a small disturbance to the free system. The fact that the coupling constant is small allows one to approximate the solution of the interacting system, using the exact solution of the free theory. This is called the weak-coupling approximation. In most cases, it is then possible to accurately evaluate and sum the perturbative contributions up to a finite order. The approximate solutions that result from this method can be used to calculate the various physical quantities, such as the cross-section of an interaction. The more perturbative contributions that are taken into account, the better the result will agree with the values of physical quantities found by experiment.

The success of perturbation theory in QFT is exemplified by QED. Since the 1950s, physicists have been able to calculate with high accuracy the values of for example the fine-structure constant and the anomalous magnetic moment using perturbation theory. The theoretical predictions of QED match experimental results up to eleven significant figures, which is "like predicting the measured diameter of the USA to within the width of a human hair"(Butterfield, 2014, p.11).

2.1.1 Treating the Infinities

It is worth going into a bit more detail on the actual computation of physical quantities in the cases where a perturbative expansion can provide reliable results. In order to compute the values of physical quantities such as the cross-section of a certain interaction process, one needs to evaluate certain momentum integrals. Going back to our example of ϕ^4-theory, a perturbative expansion in the coupling constant of this interaction includes momentum integrals of the form

$$\int_0^\infty d^4q \frac{1}{(q^2+m^2)}. \tag{4}$$

As the momentum goes to infinity, this integral diverges and we are left with an unphysical infinity. We call this ultraviolet (UV) divergence. Similarly, there exist interactions for which the integral diverges as the momentum goes to zero. In that case, we speak of infrared (IR) divergence. Besides the many interpretive problems that I will discuss in section 3, there is also the problem that these infinities give rise to unphysical values for physical quantities.

Following Ruetsche (2020), we can identify two parts in the treatment of these infinities. The first part consist of the process of regularization, where the integral is rendered finite by introducing a cutoff to the integral that imposes an upper (in case of UV divergence) or lower (in case of an IR divergence) limit.[1] The consequence is that the outcome of the integral now depends on this cutoff.

The second part is the process of renormalization, where the coupling constant in which the expansion was made is redefined to include so-called counterterms. If a finite number of counterterms is needed to treat the divergent integral, then the theory is called renormalizable. After the introduction of the counterterms, the cutoff can be removed again from the integral. This way, the theory and its predictions will not depend on the choice of the value for the cutoff. If successful, the renormalization procedure leaves us with a finite integral.

I will call the process of regularization plus renormalization 'perturbative renormalization'.[2] The upshot of perturbative renormalization is that the infinities are removed at the cost of a change in the value of coupling constants. Since a cut in the momentum integral corresponds to a cut on the energy scale of the system, one is now able to compute the low energy physics from the new integral, without exact knowledge of what happens at high energies.

2.2 Beyond Perturbation Theory

In the second type of situation the relevant coupling constant is not small. In this case, the perturbative approximation does not provide reliable information and using it to investigate the system might lead one to miss important features. In this case, non-perturbative methods can be invoked in order to make accurate predictions. An example for which this is the case is in computations of the spectrum of hadrons. In the Standard Model of particle physics, hadrons are understood as a collection of two or three quarks. These quarks and the interactions between them are described by the theory of quantum chromodynamics (QCD). At low energies, the value of the coupling constant of the interactions between quarks is large and perturbation theory cannot be used any more to obtain accurate pre-

[1] There are other methods of regularization such as dimensional regularization (see Peskin & Schroeder (1995), p.249-252), but for simplicity I will only discuss this one.

[2] For a more extensive introduction to renormalization in quantum field theory aimed at philosophers, see Butterfield & Bouatta (2015).

dictions. Instead, other methods such as lattice QCD are invoked in order to study the system.

However, this problem of unphysical infinite integrals in QCD dissolves when one looks at the high energy regime. Contrary to QED, the value of the QCD coupling constant decreases as the energy rises. At high enough energies, the coupling strength between the particles is negligible, and the particles behave is if they are free. This behaviour is called *asymptotic freedom*. Because of this characteristic of QCD, perturbation theory will give reliable approximations of the physics at high energies.

Since the introduction of quarks in 1964, it became clear that hadrons such as the nucleons can be understood as composed of quarks. Quarks are never found independently: they cannot exist as individuals, but only as a collection of quarks in the form of a meson or baryon. As such, there is a type of existential dependence between quarks (Tahko, 2018). This phenomenon is known as quark confinement. QCD describes quarks as strongly interacting at accessible energies. Quark confinement is therefore a phenomenon that cannot be approximated by PT.

Similarly, solitons are new objects that only arise at the non-perturbative level. As I will discuss in the next section, solitons are classical solutions to the equations of motion of a (quantum) field theory whose contribution to the physical quantities will not be taken into account when one follows the usual computational methods and performs a perturbative expansion around a single minimum of the theory. I will also introduce solitons as particles in QFT in this section, and discuss their properties. In section 2.2.2, I will provide a more detailed discussion of how non-perturbative phenomena can be approached.

2.2.1 Solitons as Particles

Although many choose to interpret QFT in terms of particles, its name might suggest one should adopt a field interpretation. There are considerable differences between the two interpretations and the objects they focus on. A field interpretation looks at the properties of every spacetime point, whereas a particle interpretation looks at individual entities. In other words, fields have an infinite number of degrees of freedom, whereas that number is finite for particles. Much has been said about which interpretation fits QFT best, but I will not go into this discussion here. Instead, I want to assume that we can adopt a particle interpretation and discuss in more detail what that entails.

Apart from the differences with respect to fields, we can discern several characteristics that are attributed to particles. Firstly, a particle is a localized object in spacetime that is spherically symmetric. A particle is also considered to have an exact trajectory, discreteness and 'primitive thisness' (Teller, 1995).[3] Discreteness means that particles can be counted individually. 'Primitive thisness' can be understood as an additional property that makes it possible to identify which particle is counted and distinguish it from other particles.

These characteristics are relatively straightforward in classical mechanics, but the situation becomes more complicated in the quantum regime. Quantum particles or 'quanta', are still similar to classical, relativistic and non-interacting particles in the sense that both types of particles are aggregable, and that they satisfy the same energy-momentum conditions, such as $E^2 = p^2 + m^2$. However, according to Teller (1995), quantum particles do not have the property of primitive thisness. He provides a thought experiment for this argument. Suppose two people hold the ends of a rope, and give it a shake. That creates two traveling bumps, both moving towards the middle of the rope. Once there, the bumps merge and continue down the rope in opposite directions. The question is: did the bumps pass through each other, or bump off each other? The bumps represent quantum particles, so there is no way to distinguish the particles from each other after they have merged. Quantum particles are therefore not the sorts of things that bear 'primitive thisness', argues Teller.

Besides the property of primitive thisness, the presence of other characteristics such as localization are also still subject of debate for quantum particles. The work of several philosophers appears to show that quantum particles cannot be localized to any extent, let alone have exact trajectories.[4] If localizability is taken as an essential characteristic of particles, one may take these results as strong circumstantial evidence against the possibility of a particle interpretation (Kuhlmann, 2010, ch. 8.3). There is much more to be said about particle interpretations in QFT, but I believe we now know enough to establish in what sense solitons can be understood as particles.

Solitons are found in theories as stationary solutions that are non-dissipative. From these stationary solutions, we can construct "lump-like" solutions by Lorentz transforma-

[3] Lancaster (2019) also considers a longer lifetime of existence to be a characteristic of particles. Without that, excitations of fields "lose their meaning as particles and instead exist as slightly ill-defined parts of a whole ... system." (p. 285)

[4] See for example Malament (1996); Reeh & Schlieder (1961); Fleming & Butterfield (1999); Busch (1999).

tion that move with a velocity smaller than the speed of light (Coleman, 1988, p.192). Provided they satisfy a certain number of other particle-like properties, these solutions are called ‘solitons’.[5] These classical solutions can be quantized so that they can be described using QFT.

One of the particle-like properties of solitons is their scattering behaviour. Two solitons scatter such that the individual waves pass through each other without their velocities and shapes changing. The only difference between before and after the scattering is a phase shift. As such, the ‘primitive thisness’ of the soliton particles is exactly as questionable as Teller describes for quantum particles. Furthermore, solitons have an extended topological structure. In other words, they are extended in spacetime. This is an important characteristic of solitons, because it is associated with their stability. Because soliton solutions are found in non-linear wave equations, the shape of the wave pulses will disperse. However, the topological nature of the solution ensures a solitary wave of constant shape: the wave remains localised in space, and satisfies the energy-momentum relation $E^2 = m^2 + p^2$ that holds for particles in QFT. As such, they can be understood as particles with extended but finite structure whose stability under perturbations is ensured by their topological nature.

Solitons arise independently of elementary particles and are fundamentally different in nature. In contrast to solitons, elementary particles are considered to be point particles. They arise from the quantization of the excitations of a field that is present in the Lagrangian. Furthermore, whereas elementary particles can carry a Noether charge, solitons carry a conserved topological charge.

To see how solitons arise in quantum field theories, it is convenient to shift to a path integral formulation. The general form of a QFT path integral is given by

$$Z = \int \mathcal{D}\phi e^{-\frac{1}{g^2}S[\phi]}, \tag{5}$$

where S is the action[6] that depends on the set of fields of the system ϕ, and g is the coupling

[5]A related phenomenon that also cannot be found by perturbation theory is an instanton. Although instantons are sometimes understood as solitons, they differ from other solitons in that they describe a decay process, and are only localised in Euclidean spacetime. As such, their physical interpretation is less clear. In any case, the properties of instantons differ so much from the solitons that are localised in Minkowsi spacetime, that I will not discuss them in this thesis.

[6]The dominant contributions to the observable quantities are usually easier to recognize in the Euclidean version of the path integral than in the Lorentzian version. If we want to know what contributions there are to the Lorentzian path integral, the Euclidean result may be analytically continued.

constant of the gauge field. To evaluate the integral, a weak-coupling approximation can be used. In this approximation, the path integral is dominated by the vacua that correspond to the lowest action. However, there can also exist other contributions to the path integral that are minima of the action, but that are not taken into account when one evaluates the path integral using the weak-coupling perturbative approximation around the vacuum state. These additional minima correspond to soliton solutions. To find these solutions, we need to *go beyond* perturbative physics.

Soliton solutions can be found in a large number of theories and have been known in mathematics since the nineteenth century. In general, solitons in a theory of one, two or three space dimensions will differ from each other in name and character. Solitons in one space dimension are called 'kinks'. Such soliton solutions arise in the sine-Gordon model. In three space dimensions, several types of solitons appear. A well-known example is the monopole, which arises in the Seiberg-Witten model.

One of the first instances that showed the relevance of solitons in particle physics was a three dimensional pion model that was introduced as a low energy approximation to QCD in a series of papers published between 1958 and 1962 by Tony Skyrme. In his papers, Skyrme argued for two original ideas (Filippov, 2000). Firstly, he took protons to be solitons. These solitons are now called 'Skyrmions'. This went against the prevalent idea at the time that all particles in quantum field theory are point particles, which are without any spatial extension. Because of the stability of the Skyrmions, he was able to provide an explanation for why a proton does not fall apart. The second interesting idea was introduced by Skyrme in the context of the quantized version of his theory. In this theory, the fermionic quantum Skyrmions can be constructed from bosons. This process is called bosonization. To get a better understanding of this process, Skyrme analyzed a simpler version of the theory in two dimensions: the sine-Gordon theory, which will be discussed in section 5. Nowadays, we can understand the Skyrmion model as a low energy approximation to the theory of quantum chromodynamics. It is an example of how solitons have been used to study non-perturbative phenomena. Similar ideas have been used to explain confinement in the Seiberg-Witten model. Unfortunately, the details are beyond the scope of this thesis.

2.2.2 When Perturbative Methods Fail

As was described in section 2, perturbation theory is the common method to describe phenomena in QFT. Phenomena such as quark confinement, where perturbation theory fails due to a large coupling constant, and soliton physics, where perturbation theory around one minimum is not able to find these solutions at all, require a different approach. There are several options. I will focus on two approaches that are relevant for my case studies: simplifications and non-perturbative methods. I will illustrate these approaches with a number of examples, which will all play an important role in my case studies.

The first approach is to use simplified models that exhibit similar phenomenology to that of more realistic QFTs. To be more specific, what I mean by 'simplified' here is that the number of degrees of freedom in simplified models are lower than that of a more realistic model, thereby rendering computations more tractable. For physicists, one of the motivations for looking into these simplified models is the hope is that the simplified model still resembles the realistic model accurately enough to say something about the phenomena that the realistic model exhibits.

One example of a simplification is to lower the number of spacetime dimensions of the model. In less than the observable four dimensions, the spacetime manifold is simpler both conceptually and computationally. In two dimensions (one space dimension and one time dimension) for example, the number of degrees of freedom of each field is smaller compared to four dimensions, because there is no rotation symmetry or angular momentum (Frishman & Sonnenschein, 2010).

A second example of simplification is to increase the symmetry of a system. In particle physics, one way this has been done is by means of supersymmetry, which extends the Lorentzian spacetime symmetries such that each boson has a supersymmetric fermion partner. These partner particles have not been found in any particle experiments yet. However, if supersymmetry were to exist in nature, it could possibly solve several outstanding problems, such as the hierarchy problem.[7] Supersymmetric models have a reduced number of degrees of freedom compared to a system that is not supersymmetric, because the extension of the Lorenz invariance and the particular renormalization conditions that this extension brings constrain the field content. In theories with an 'extended' supersymme-

[7]For more details on why supersymmetry is particularly useful to study strong coupling theories, see the discussion in Bertolini (2021), p.16-18 and chapter 9.4.

try, the system is invariant under more than one supersymmetry transformation. The number of copies of supersymmetries is usually denoted by $\mathcal{N}$. The larger this $\mathcal{N}$, the more constrained the theory is. The restrictions on the theory are made apparent by the fact that supersymmetric theories are holomorphic.[8] I will return to supersymmetry and holomorphy in section 6, where the Seiberg-Witten model will be discussed.

Supersymmetry is particularly suitable for studying non-perturbative physics, because it can constrain the dynamics in the strong coupling regime of a theory to the extent that it is possible to understand it analytically. For example, whereas confinement is still an inaccessible phenomenon for non-supersymmetric theories, there exist supersymmetric accounts that explain it as the result of monopole condensation which have been analytically proven.[9]

The second approach to learn more about the physics in regimes where perturbation theory fails, is to use non-perturbative methods that link these regimes to those where perturbation theory can reliably be used. In terms of degrees of freedom, we can say that in these situations, non-perturbative methods serve as a shift from the degrees of freedom in the non-perturbative regime to those in the perturbative regime.

An example is given by the renormalization group methods. In these methods, the cutoff scale is used to investigate only those degrees of freedom that are present at a chosen energy scale. In section 2.1.1, perturbative renormalization was described as the treatment for unphysical infinities arising in computations on particle interactions. This treatment was completed by taking this cutoff back to infinity. However, we can also adopt a method first introduced by Kenneth Wilson and others in which the cutoff is not taken out. In this case, we can interpret what is left after the treatment as an effective action in which the effects of the high energy degrees of freedom are included in the value of the coupling constants of the fields that are present at low energies. Changing the value of the cutoff leads to a change in the values of the coupling constant, so that the observables stay the same. In terms of the path integral of the system, the effective action S_{eff} is given by

$$e^{iS_{eff}} = \int_{\phi(p),p>\mu} D\phi e^{iS_0[\phi,g]}, \tag{6}$$

where $S_0[\phi, g]$ is the bare action that depends on the fields ϕ and a set of coupling constants denoted by g.

[8]For more details on this, see chapter 5.1 of Bertolini (2021).

[9]See Bertolini (2022) for more details.

One of the benefits of using the path integral formulation of the Wilsonian effective action is that there is no coarse-graining or averaging over quantities in this formulation. In terms of degrees of freedom, the renormalization group equation is a transformation that exchanges the degrees of freedom of the full theory with effective degrees of freedom at low energies. So as Matthew Schwartz states in his introductory book on QFT, we can understand the renormalization group methods as "one of those brilliant ideas that lets you get something for nothing through clever reorganization of things you already know" (Schwartz, 2014, p.417).

An important component of the renormalization group methods is the renormalization group equation, which tells us how the coupling constants behave as the energy scale varies. One particular type of behaviour that the renormalization group methods provide a framework for is the asymptotic behaviour of a theory. Using the renormalization group equation, one can compute the rate of change of the coupling constant with respect to the (logarithm of) the relevant energy scale. As this scale goes to infinity, behaviour such as asymptotic freedom can be identified. For a comprehensive introduction to renormalization group methods aimed at philosophers, see Williams (2021) or Butterfield & Bouatta (2015).

Another non-perturbative method that has become a popular topic in the study of quantum field theory, one that will play an important role in my case studies, is the use of duality transformations. Dualities can be understood as a class of symmetries, where instead of acting on *solutions* to theories (as is the case with standard (gauge) symmetries), dualities act on a space of *theories*.[10] In terms of degrees of freedom, we can say that a duality transformation transforms one set of degrees of freedom to another. As Castellani & Rickles (2016) point out, a duality transformation is more radical than an ordinary symmetry transformation, because on top of a change in theoretical description, it can also bring a change in the interpretation of the physical system.

An important example of a duality in quantum field theory is an S-duality that maps the quantities in a strong coupling regime to those in a weak coupling regime where perturbative methods can provide reliable predictions. I will return to this type of duality and others in section 4.3, where I discuss them in relation to fundamentality and emergence. Dualities can also play an important role in the process of bosonization. In that case, a duality transformation exchanges the degrees of freedom that describe bosons in one theory with the degrees of freedom that describe fermions in the dual theory, or the other way

[10]For more on the relation between dualities and gauge symmetries, see De Haro et al. (2016).

around.

There are many more non-perturbative methods that can form an interesting topic for the philosopher. Take for example the discussion of the 't Hooft limit by Bouatta & Butterfield (2015), where a limit of a large number is taken in order to retrieve knowledge in regimes where perturbation theory fails. However, the discussion above is sufficient to analyze the case studies in sections 5 and 6.

3 Interpreting Quantum Field Theory

When interpreting scientific theories, we ask questions such as "what does the theory say the world is like?" or "what would the world be like if this theory is true?" (Van et al., 1991). Interpretations of physical theories can be subject to various criteria of adequacy, such as logical coherence, adequacy to experimental practice, consistency with empirical data or mathematical rigour. Interpretations can also be subject to extrinsic criteria of adequacy, that make reference to theories besides the one we are looking at. Both type of criteria will appear in this section. I will discuss what approach to interpretation is suitable to QFT, according to different philosophers. Furthermore, I will argue why an interpretive investigation of soliton physics is justified.

3.1 Approaches to QFT

So far, I have implicitly assumed that all physicists use the same approach to quantum field theory. This is the approach that is commonly taught in physics textbooks, and is used by most modern physicists to make experimentally verifiable predictions. It is a Lagrangian formulation of QFT, in which perturbation theory and the renormalization procedure described in section 2.1.1 play an important role. I will call this approach 'Lagrangian quantum field theory' (LQFT). However, there is an alternative approach to quantum field theory that physicists have taken, which differs from LQFT in several important respects. This approach introduces mathematically rigorous axioms to construct models, and is called the 'axiomatic approach to quantum field theory' (AQFT). Examples are the algebraic approach of Haag and Kastler (Haag & Kastler, 1964) and Wightman's axiomatic framework (Wightman, 1956).[11] In AQFT, renormalization techniques are not used to treat infinities, but infinities are avoided altogether in constructing the model.

The difference between the two approaches is highlighted in a discussion by Fleming (2002) of two acclaimed books, one introducing the subject of AQFT and the other of LQFT. The book on AQFT is called *Local Quantum Physics*, and was written by Rudolf Haag in 1992. Fleming characterises this book as containing thorough definitions and rigorous proofs, but as one in which applications are not the primary concern and calculations on experimentally testable quantities for realistic QFTs are absent. Contrary to that, the book on LQFT is focused on the development of attainable calculation methods that can be used to compute transition amplitudes that can be compared with experiment. This book is called *Quantum Field Theory* (1996), and is a famous lecture series given by Steven Weinberg.

These diverging approaches to QFT leave a question for the philosopher: which variant should be subject to interpretative and foundational philosophical investigation? Recall Fleming's discussion on the book on AQFT and LQFT by Haag and Weinberg. He concludes his comparison of these books with the comment that "the mode of presentation of QFT in these two books, and the experience of the student in plowing through them, and the battery of learned material once the study is completed, is so different that one may argue that it is a tenuous claim that both books are about the same QFT."[12] Naturally, which approach one chooses will influence the answers one will find on interpretive questions. In the past two decades, several papers have been published by David Wallace and Doreen Fraser in which the question of which approach to QFT should be subject to philosophical investigation serves as the dominant issue. Generalizing to a certain extent, and highlighting the aspects that are most important for the remainder of my thesis, I will summarize their views as follows.

In Wallace (2006, 2011), Wallace defends LQFT as the right candidate for philosophical investigation. For him, the subject of foundational and interpretive analyses should be the theory that is commonly used by physicists to make verifiable empirical predictions. Because of its extreme success in making these predictions, Wallace argues that LQFT is currently the best theory to look at if we want to understand reality. He argues that

[11] For an overview of the differences between algebraic QFT and Wightman's framework, see Swanson (2017).

[12] Fleming (2002, p.136).

LQFT is not only highly successful in making empirical predictions, but it has also achieved many explanatory successes. Examples of LQFT providing a framework include asymptotic freedom, anomalous symmetry breaking and various aspects of (non-)renormalizability. Therefore, he states that "if the goal of philosophy of physics is to understand the deep structure of reality via our best extant physics, to be lured away from the Standard Model by algebraic quantum fields is sheer madness"(Wallace, 2011, p.124).

So far, AQFT is far less successful in making physical empirical predictions. Because of AQFT's strict requirement of mathematical rigour, the number of physical models that has been set up is limited. Nevertheless, Fraser considers an axiomatic approach to QFT to be more suitable for philosophical interpretation, because the great mathematical rigour provided by well-defined axioms allows philosophers to formulate precise questions and answers (Fraser, 2011). For example, one aspect of AQFT that is celebrated by Fraser is the success of unitarily inequivalent representations in explaining phenomena such as spontaneous symmetry breaking (Fraser, 2009, p.560). Since LQFT employs an energy cutoff, all representations are rendered unitarily equivalent and as such, the approach cannot employ the explanatory utility of inequivalent representations.

According to Fraser, the main problem of the LQFT approach is that it is insufficiently mathematically rigorous for foundational and interpretative analyses. In section 2 we saw that in certain situations, where the coupling is strong, the physicist is forced to work without the help of perturbation theory if he or she wants to make reliable predictions. From an interpretative perspective however, there are good arguments for being careful with the use of perturbation theory, and even to avoid it altogether. Fraser argues that the method of perturbative renormalization, in which energy (or momentum) cut-offs are taken to infinity, makes LQFT an ill-defined framework. Therefore, it is not clear how the philosopher should interpret the mathematics.[13] In line with James D. Fraser (2020), we can call this issue the 'rigour problem'.

James D. Fraser identifies two other worries on perturbative QFT that are raised in philosophical literature: the consistency problem and the justification problem. The consistency problem is the result of a famous theorem proved by Haag in 1955 and generalized by Hall and Wightman in 1957, which is argued to show that standard perturbative calculations in the interaction picture rest on an inconsistent set of assumptions. Because the interaction picture plays an important role in setting up the perturbative approximation,

[13]See for example her discussion on the possibility of a quanta interpretation in Fraser (2009, p.543).

Haag's theorem can be interpreted as implying a fundamental inconsistency in perturbative QFT.

Even if we do not take Haag's theorem into account, the use of perturbative renormalization lacks justification according to J.D. Fraser. Perturbative renormalization is presented in LQFT textbooks as a mathematical trick, and questions such as "why are these infinities there?" and "what justifies the procedure used to remove them?" remain without satisfactory answer (Fraser, 2020, p.393). The idea is then that without satisfactory justification, perturbative renormalization is not suitable for philosophical investigation. J.D. Fraser calls this the justification problem.

Following Fraser (2009, p.2), we can summarize the discussion above in terms of two aims that philosophers have when philosophically analyzing QFT. The first is to "stick as close to actual scientific practice as possible". The second is to "clarify the foundations of theories and to provide interpretations of theories, where necessary." The two aims can be in conflict with each other, and Fraser points out that the latter is "more easily achieved by focusing on cleaner versions of theories which are farther removed from actual applications". We have seen that for philosophers who advocate LQFT, the first aim is prioritized, whereas for those that advocate AQFT, the second aim gets priority. According to Fraser, LQFT sheds light on the empirical content of QFT, whereas AQFT sheds light on its theoretical content.

3.2 Interpreting Soliton Physics

How does soliton physics fall into this discussion? The soliton models that I discuss are formulated within the LQFT approach. Soliton physics is an active area of physics and both the sine-Gordon model and the Seiberg-Witten model are of significant scientific importance. A philosophical investigation of them will therefore be an investigation of what scientists actually study and some of the same criticism towards interpreting LQFT applies to my case studies: as interacting quantum field theories, a philosopher in favour of rigour could point to the consistency problem. However, the other two problems of perturbation theory that were presented above, the justification problem and the rigour problem, are largely avoided or less severe in the soliton models that I will discuss. In this section, I will consider this in more detail. I will argue that the soliton models are mathematically rigorous enough for philosophical investigation, and that as such, these models can provide valuable insights into the theoretical structure of QFT.

As complex case studies, my work fits into what Fraser (2009) has identified as a theme in the philosophy of science where it is believed that "the messy context of application is important for foundational and interpretive questions" (p.2). One advocate of this view is Batterman (2011). He has argued that the divergences that appear in high energy regimes are not always a bad sign for the philosopher. In fact, such mathematical features can be viewed as sources of information and can be used to expand our understanding of the system. He finds support for this view in an article of Roman Jackiw (1996), in which Jackiw argues that the divergences of QFT "must not be viewed as unmitigated defects", because they "convey crucially important information about the physical situation" (p.4). He therefore argues, referring to the physicist Eugene Wigner's famous article on the unreasonable effectiveness of mathematics in physics, that the effectiveness of LQFT as a language to describe physical reality is not unreasonable at all.

Jeremy Butterfield provides a different argument in favour of investigating the Lagrangian approach to QFT, one that also applies to my case studies. He states that the time is ripe for philosophical assessment of what he calls the 'heuristic, informal work' of the quantum field theories that make up the standard model, i.e. (perturbative) Lagrangian QFT with interactions (Butterfield, 2014; Bouatta & Butterfield, 2015). Butterfield argues against the idea that ontological investigation is only suited for fundamental theories and entities that are valid at all energies. The methods of perturbative renormalization presuppose that at some high energy scale, the framework of QFT breaks down altogether. Given this expectation, the question is then if it reasonable to make ontological claims about QFT. If so, one might argue that an effective QFT such as the one I will discuss in section 6 should not be the subject of ontological investigation. However, I agree with Butterfield (2014), who argues that in these cases, we should not beware of all ontological claims, but just those about the ontology at very high energies. He provides three reasons for this.[14] Firstly, he argues that it is reasonable to believe that the physics at higher energies subvene the physics at the lower energies that we can access. Secondly, there are good reasons for believing that at the lower, accessible energies, the physics must be described by a quantum field theory. He provides an argument given by Steven Weinberg in his famous lectures. Weinberg aims to show that 'any theory combining the principles of

[14]Butterfield actually names four reasons. However, because the second and the third reason on his list are intimately related, I will present these reasons as one.

special relativity and quantum mechanics and with a plausible locality property ... must at low energies take the form of a quantum field theory.' (Butterfield, 2014, p.12). According to Butterfield, it is therefore reasonable to believe that QFT will remain a reliable source of knowledge of physics at accessible low energies, no matter what new and possibly more fundamental theories future scientific endeavours might bring. Thirdly and lastly, claims about ontology need not to be restricted to the level of facts that subvene all other facts. In other words, we can talk about the ontology of solitons in low energy theories, without the exact knowledge of the high energy theory.

The idea that LQFT is particularly suitable for investigating the ontology of QFT in the low energy regime is shared by Baker (2015) and Swanson (2017). Baker argues that the controversy between Fraser and Wallace is a question about a domain of application of the two theories. As such, Swanson argues that the two approaches to quantum field theory emphasize different aspects of QFT and leave other aspects out. AQFT will not provide insights into realistic interacting QFTs, whereas LQFT cannot give us insights about the physics at high energy scales beyond the cutoff. Because our current knowledge on QFT is still limited, Swanson urges philosophers to study puzzles, tools and techniques from both the axiomatic and the Lagrangian approach. Similarly, Fraser (2011) argues that there is room for parallel research programs in QFT, because the experimental success of LQFT makes it particularly suited for interpretative questions on the empirical content of QFT, whereas the mathematical rigour of AQFT makes is suitable to shed light on the theoretical content of QFT.

In the same spirit, I argue that there are certain aspects of the theoretical content of QFT that an investigation of soliton physics can shed light on. One can argue that my case case studies will not be suitable to provide knowledge on the empirical content of QFT, since they are necessarily simplified in some respect. The sine-Gordon model is a lower dimensional theory and the Seiberg-Witten model is a supersymmetric theory. As such, the models cannot be verified by experiment and the gap between theory and reality is much bigger than in most models that are set up by the LQFT approach. Therefore these models are arguably less suitable to study the empirical content of QFT.

However, I argue that the sine-Gordon model and the Seiberg-Witten model are particularly suited for interpretive questions on the theoretical content of QFT, because simplified models or toy models such as these are "cleaner" or mathematically more rigorous compared to the bulk of messy models found in scientific practice.

The sine-Gordon model is exceptionally ‘clean’, because it is exactly integrable. In physics, the notion of exact integrability means that the system is exactly solvable in some sense. In this case, the exact integrability of the system allows one to exactly determine its particle spectrum. Bouatta & Butterfield (2015) argue that such mathematical rigour plays an essential role in determining the existence of quantum field theories. According to them, a theory that is rigorously defined is likely to exist, and therefore philosophically investigating the ontology of that theory is worthwhile. For a theory that probably does not exists, one should be careful in making claims about its ontology. For example, in Butterfield (2014), Butterfield argues that the simplifications that the asymptotic freedom of QCD brings (together with its properties of conformal invariance and asymptotic safety[15]) ‘makes it reasonable to hope that [the QFT] will be defined rigorously, i.e. independently of perturbative analyses.’ Similarly, I argue that the mathematical rigour of the sine-Gordon model, namely its integrability, makes a philosophically investigation of the ontology worthwhile.

In the case of the Seiberg-Witten model, the mathematical rigour is provided by its supersymmetry. The supersymmetry allows one to compute exact low-energy results, in the sense that all the quantum corrections are known and calculable and the particle spectrum is exactly determined. Because the sine-Gordon model and the Seiberg-Witten model provide exact results and avoid the use of perturbation theory to a certain extent, I argue that they are mathematically rigorous enough to make claims on emergence and fundamentality.

Additionally, I believe that because these models can shed light on those regimes of QFT that perturbative methods cannot reach, they can be used to study the theoretical content of (the Lagrangian formulation of) QFT. More specifically, the non-perturbative methods used in the sine-Gordon model and the Seiberg-Witten model provide a window into strong coupling regimes. With these methods, one is able to identify new entities such as solitons, which as solutions to the theories are likely to contribute to the physical parameters of the system and which would have been missed if one were to use only perturbative methods.

[15]Conformal invariance reflects the idea of scale independence. Asymptotic safety is a generalized notion of renormalizability, which can allow a QFT to be well defined in the high energy limit without the use of perturbation theory.

4 Framework for Emergence

In this section, I will introduce three concepts in physics and philosophy of physics: fundamentality, emergence and duality. In particular, I will identify different relationships between emergence and fundamentality, and use a formal definition of duality to distinguish between cases where the notion of epistemic emergence or the notion of ontological emergence is applicable. Throughout this section, I will use a definition of emergence that it focused on its physical nature, in terms of entities and properties, rather than a notion of emergence formulated in terms of causality, which is a more common approach in the philosophy of mind.[16]

4.1 Fundamentality

In both physics and philosophy, whether or not something is fundamental is determined in relation to other things. According to Schaffer (2003), fundamentality is the idea that there is a hierarchical picture of nature divided into levels, and the assumption that there is a bottom level that is fundamental. Following Glick (2018), we can be more specific and distinguish between four versions of fundamentality. The first notion of fundamentality is mereological fundamentality, which identifies what is fundamental in terms of parts and wholes. In this case, the whole is less fundamental than the parts that make up the whole. The second notion of fundamentality is grounding fundamentality, which holds that grounds are more fundamental than that which they ground. From this perspective, to be fundamental is to be fully ungrounded. The third notion of fundamentality is dependence fundamentality, which represents the idea that something needs to be independent to be fundamental. In this case, that which depends on something is less fundamental than that which it depends on.

The fourth and final notion that I will discuss is called energy fundamentality. This view of fundamentality is common in particle physics, where higher energy equals shorter distances. Energy fundamentality takes a higher energy theory is to be more fundamental than a lower energy effective theory, where the latter is obtained by the renormalization method described in section 2.1.1. In other words, the theory that is able to describe physics at arbitrary small distance scales is understood to be more fundamental than the theory that describes physics at a longer distances. Glick (2018) argues that this notion

[16]See for example Lowe (2005).

of fundamentality must be accompanied by one of the notions above if we want to make any metaphysical claims of fundamentality, because this notion of fundamentality applies exclusively to theories, whereas the other three notions apply to entities.

In general, the soliton is considered to be a 'composite' particle, and is taken to be less fundamental than an elementary particle because of that characteristic. In string theory for example, the job of determining the fundamentality of solitons is rather straightforward. In this case, elementary particles are excitation modes of strings, while solitons are made up of more than one string (Sen, 1999, p.91). This implies that we can take a mereological stance towards determining fundamentality: the composite particle is made up from other entities, hence elementary particles are more fundamental than solitons. The same stance is generally taken towards solitons in quantum field theory. In this case however, what physicists take to be a 'composite' particle is not necessarily defined by the mereological nature of the particles. In QFT, it is common practice to denote any particle that is not an elementary particle as a composite particle. This includes bound states for example, but also solitons and other topological particles. This use of the notions of composite and elementary reflects the epistemic procedures of physicists, who take the more fundamental elementary particles as the starting point for a theory (Castellani & De Haro, 2018). In general, we should be careful in extending our understanding of the 'composite' label to the ontological domain, since it is not always clear if topological solitons are composed of or can be reduced to any other particles.

4.2 Emergence

In the context of scientific theories, the notions of fundamentality described above all suggest that there is a hierarchical structure of theories, which are ordered such that there is a top theory and a bottom theory. One particular inter-theory relationship in this hierarchical structure of theories on which philosophers have focused extensively in philosophy of mind, biology and physics is that of emergence.

Emergence and fundamentality are usually understood to be related in the sense that a less fundamental theory emerges from a more fundamental theory. Following Butterfield (2011), I will call the emerging theory the top theory, and the theory that the top theory emerges from the bottom theory. Emergent top theories are understood to describe novel and robust phenomena, in the sense that their emergence is relatively insensitive to the

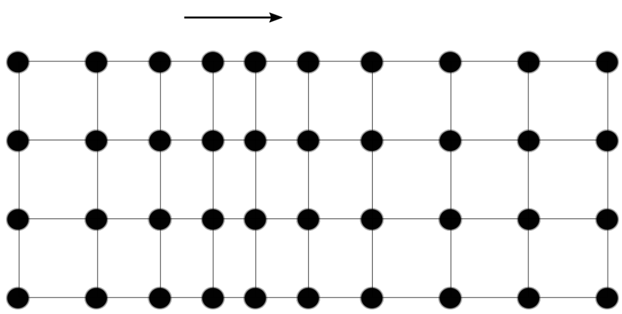

Figure 1: A phonon as a lattice vibration: a collective displacement of the particles inside the lattice.

details of the bottom theory. Other features that philosophers have used to characterize emergent phenomena are irreducibility, inexplicability or unpredictability from the bottom theory. For example, according to Falkenburg & Morrison (2015, p.1), the traditional understanding of emergence in philosophy is that "a phenomenon is emergent if it cannot be reduced to, explained or predicted from its constituent parts." This can be seen as a mereological notion of emergence, where the hierarchical nature of emergence relates constituent parts and wholes.

An example of the relation between fundamentality and emergence can be found in a paper by Jeremy Butterfield on renormalization in QFT (Butterfield, 2014). Butterfield defines emergence as "behaviour that is novel and robust relative to some comparison class"(Butterfield, 2011, p.4). As such, he takes renormalizability as an emergent feature of low energy effective QFT. This corresponds to the notion of energy fundamentality, because full QFTs that describe physics at arbitrary energy scales are taken to be more fundamental than the lower energy effective QFT. Just like in the mereological context, the emergent feature is again found in the less fundamental theory. In section 4.2.1, I will argue that this is not always the case for soliton physics.

A common example of emergence is found in the physics of phonons.[17] Phonons are described in various fields of science, but are perhaps best known from condensed matter physics. This can be defined as the study of properties of the condensed phases of matter, such as liquids and solids. In the context of condensed matter physics, phonons are found as a massless collective excitation of many particles in a solid; see figure 1. The properties

[17]For examples of phonons in philosophical discussions of emergence, see Franklin & Knox (2018), De Haro (2019) or Lancaster (2019).

of a phonon depend on properties of the lattice such as the distance between the particles and the symmetries that are present, but are completely disconnected from the properties of those particles inside the lattice. Furthermore, the phonon cannot exist at the level of the individual atoms, and is therefore novel at the level of the collection of particles. Hence we can say that "more is different", just as Philip Anderson famously did in 1972. As such, phonons exhibit many of the characteristics that philosophers have attributed to emergent phenomena.

For the purpose of this thesis, I will use a more formal framework of emergence that was given in De Haro (2019). In this framework, the notion of emergence is given in terms of properties and entities.[18] De Haro follows Butterfield (2011) in taking a general definition of emergence as novelty and proposes a notion of ontological emergence in terms of ontological novelty. I will come back to ontological emergence and contrast it to epistemic emergence in the next section. In the remainder of this section, I will introduce some important elements of the framework and then discuss the general definition of emergence in more detail.

The first important element in the framework is the contrast between a bare theory and an interpreted theory (De Haro & Butterfield, 2017; De Haro, 2020). A bare theory T is an uninterpreted theory that constitutes states, quantities, dynamics and a set of rules for evaluating physical quantities on the states. One can add an interpretation of the bare theory to get an interpreted theory. Here 'interpretation' means a set of partial maps from the theory to a domain of application that preserves the appropriate structure. The domain of application that a theory describes is part of a physically possible world, and consists of a set of elements and their relations. Often this is an idealised version of the target system that one wants to describe. A top theory and a bottom theory may have different domains of application, even though they provide a description of the same target system.

In this setup, the concept of emergence is defined in terms of two features: a formal feature that consists of a linkage map, and an interpretive feature that consists of novelty. The linkage is an inter-theoretic relationship between bare theories that specifies how the top and the bottom theory *depend* on each other. In general, this is a surjective map that is non-injective, meaning that for every element in the top theory, there is an element in the bottom theory that maps to it. The non-injective nature of the linkage map reflects the

[18]Rather than for example in terms of causality.

idea that there are two physical levels that play a role in emergence. Well-known examples of a linkage in the philosophy of emergence are mathematical limits and approximations. In order for the linkage map to correspond to emergence, there is the additional condition that it must introduce novelty: the top theory must have a referent that is novel compared to the bottom theory. That is, parts of the theories are *independent*.

To summarize, emergence is defined as follows:

"We have emergence iff two bare theories, T_b and T_t, are related by a linkage map, and if in addition the interpreted top theory has novel aspects relative to the interpreted bottom theory." (De Haro, 2019, p.10)

This definition of emergence in terms of novelty is a relatively general account. For example, the feature of 'irreducibility' that was mentioned above as a characteristic of emergence can be understood as a specific version of 'novelty'. The virtue of this generality is that it allows one to identify emergence in a wide variety of systems that are interesting to physicists, and as such is able to find true cases of emergence in the physical world.

To get a better understanding of the framework, let us apply it to the example of phonons. In this case, the bottom theory is the atomic theory and the top theory is the theory that describes the phonons. The existence of the phonons depends on the lattice, so there is a linkage between the top and the bottom theory. However, not all aspects of the top theory depend on the bottom theory: the phonon theory describes the behaviour in the solid in a novel way with respect to the atomic theory.

4.2.1 Epistemic vs. Ontological Emergence

The framework introduced above allows a distinction between epistemic emergence and ontological emergence, where the novelty can be either epistemic or ontological in nature. We can distinguish between two variants of emergence: epistemic emergence and ontological emergence. Here 'epistemic' refers to what can be derived using the theory's formalism and 'ontological' refers to what is physical in the theory. Epistemic emergence deals with theoretical novelty. It occurs when two interpreted theories describe the same domain of application differently. Relative to the bottom theory, an emergent top theory makes novel statements if the interpretive maps are different, but the domain of application is the same.

Following Guay & Sartenaer (2016), I will make a further distinction between weak and strong types of epistemic emergence. Strong epistemic emergence comes down to a lack of derivability in principle. In this case, it is impossible to derive the emergent theory due to an intrinsic limitation of the original theory. Weak epistemic emergence is the lack of derivability in practice. In this case, it is impossible to derive the emergent theory in a given epistemic situation. In practice, we can say that it is as if one is dealing with strong emergence.

Whereas epistemic emergence entails theoretical novelty, the kind of novelty in ontological emergence concerns the referents of the theory. In this case, the two theories have interpretations that refer to different domains of application. Here we need not restrict ourselves to the real world. One can also consider what is physical in possible worlds, which in my case are worlds with lower spacetime dimensions or with extended Lorentzian symmetries. The notion of novel reference can be reformulated as "the linkage map's failure to mesh with the interpretation".[19] The two interpretation maps are different because the theories refer to different domains of application, even though the target system may be the same. Therefore, the order in which the interpretation map and the linkage map are applied matters for the outcome: first linking and then interpreting will give a different result than first interpreting and then linking. This difference indicates novel reference.

To illustrate the difference between epistemic and ontological emergence, let me return to the example of phonons. According to De Haro (2019), the novelty that arises corresponds to a case of epistemic emergence. The top theory describes the phonons as vibration modes of harmonic oscillators. This harmonic behaviour could not have been found in the bottom theory. The novelty here thus lies in the theory: there is a novel description in the phonon theory with respect to the atomic theory. However, the phonon description can be understood as the same theory as the atomic theory, only with a change in variables. Since the domain of application of both the phonon theory and the atomic theory is the same, there is no novel reference and the notion of ontological emergence does not apply here.

4.3 Dualities

The notion of duality reflects the idea that there are two sides that are interdependent at some level. In physics, dualities have played an important role in the development of string theory and topics of quantum field theory such as electromagnetism. I have

[19] More formally, we can say that the linkage map and the interpretation do not commute. See ch. 2 of De Haro (2019).

already introduced dualities as a method to study non-perturbative regimes in section 2.2, where I discussed that for two dual theories, it is possible to transform from one theoretical description to another using a duality transformation. The aim of this section is to examine this duality transformation in more detail and relate it to the notions of emergence and fundamentality.

In order to achieve this, I will make use of a more formal definition of duality. De Haro & Butterfield (2017) define a duality is an *isomorphism* between two models. Here I use 'models' to refer to an instantiation or representation of a theory with a specific mathematical structure. An isomorphism is a bijective map that preserves these structures. In a bijective map, every element in the model is mapped to only one element in the dual model and vice versa: every element in the dual model is mapped to only one element in the model. Defined as such, a duality implies a one-to-one correspondence between the states, quantities and dynamics of the models and reflects the formal equivalence of two theories.

The notion of duality and the notion of emergence seem to be in tension: whereas duality indicates the equivalence of theories and is therefore a symmetric relation, emergence as defined above involves novelty and is therefore a asymmetric relation. However, as I will demonstrate in my case studies, the two need not exclude one another. I argue that a duality can be understood as an intertheory relation, which in the context of emergence serves as a linkage map between two theories. In order to identify a duality as such, we need to generalize our definition of linkage as an non-injective map such that an injective map is allowed. Linkage as injective map excludes the possibility of ontological emergence, but allows the possibility of epistemic emergence. Because the descriptions and interpretations of the models are not required to be identical, theoretical novelty may arise and in that case, there is an asymmetric relation between the top and the bottom theory. Therefore, a duality can serve as a linkage map between epistemically emergent theories. In this context, allowing the linkage map to be injective reflects the intuition that we are dealing with only one domain of application.

In some systems the situation resembles that of a duality, but the duality map is isomorphic only for part of the theories or models. I will call this an *approximate* duality[20], and refer to the duality described above as an 'exact duality'. In contrast to an exact duality, an approximate duality indicates that the notion of ontological emergence can be

[20]The literature on dualities in physics sometimes calls this an 'effective duality'.

applicable. This implies that the correspondence between the physical states, quantities or dynamics is also only approximate, which leaves room for novelty in the domain of application of one theory with respect to the other.[21] If such novelty exists, we again have an asymmetric relation between theories and the notion of ontological emergence is applicable.

As Dieks et al. (2015) point out, whether a duality is exact or approximate matters to how we interpret the theory or model. Dieks et al. have discussed exact and approximate dualities in the context of gravity. They consider the AdS/CFT duality, a duality that has received considerable attention from both physicists and philosophers in the last decades. This duality was introduced by Juan Maldacena in 1998 (Maldacena, 1998). He argued that a gravity theory (string theory) in an anti-deSitter spacetime is dual to a conformal quantum field theory. According to Dieks et al., the question of which of these theories is descriptively more fundamental cannot be answered, because we are dealing with an exact duality for which no external viewpoint is available, so that it is not possible to determine the physical meaning of theoretical quantities from an external perspective.

In case of an approximate duality, Dieks et al. argue that the question of fundamentality reduces to a question of empirical adequacy. In their view, the fundamental theory is the one that agrees best with experiment, and the other theory would be the emergent theory that approximates the more fundamental one. Dieks et al. thus conclude that "the distinction between exact and approximate dualities is important for the question of differences in fundamentality of the two sides of a duality" (p. 208).

In the context of quantum field theory, several philosophers have discussed S-duality in relation to the notions of fundamentality and emergence. S-duality is a type of duality transformation that changes the coupling constants of the theories in a non-trivial way. A paradigm example is the electromagnetic duality. An electromagnetic duality transformation corresponds to an exchange of electric and magnetic fields. Because one of these fields is weakly interacting and the other strongly interacting, this is also called a 'weak-strong duality'. Maxwell's equations for classical electromagnetism already exhibit this duality, in the form of the electromagnetic duality rotation transformation

$$\vec{E} + i\vec{B} \to e^{i\theta}(\vec{E} + i\vec{B}), \tag{7}$$

[21] This distinction between exact and approximate dualities agrees with the view that dualities need not preserve empirical content. See for example Weatherall (2020).

where $\vec{E}$ and $\vec{B}$ are the electric and magnetic fields and θ is an arbitrary angle. This transformation can be extended to matter carrying electric and magnetic charges. In 1931, Paul Dirac introduced a quantization condition that describes how these charges are related

$$eg = 2\pi n, \tag{8}$$

where e is the electric charge, g is the magnetic charge and n is an integer number. An electromagnetic duality transformation will exchange the electric and magnetic charges as

$$\begin{aligned} e &\to g \\ g &\to -e. \end{aligned} \tag{9}$$

The Dirac quantization condition is invariant under this transformation.[22] Furthermore, we can recognize the weak-strong duality in this equation: when the electric charge is small, the magnetic charge is large.

An S-duality is sometimes accompanied by a T-duality, also called a target-space duality. This duality was originally found in string theory, where in the simplest case two string theories are dual under an exchange of $R \to 1/R$. Together, the S-duality and the T-duality can form the 'duality group'. I will return to these types of dualities in section 5, where the existence of an S-duality is related to the epistemic emergence of solitons, and in section 6, where we will see both an S-duality and a T-duality.

5 Sine-Gordon Model

The sine-Gordon model is a paradigmatic example of soliton physics in quantum field theory. The study of this model has a long history that shows its scientific relevance. It was already known to mathematicians in the 19th century, and the first soliton solutions were already found in 1936 (Filippov, 2000). In the 1950s, it was studied by Skyrme as a lower-dimensional analogue to his pion model.[23] Nowadays, this model is important in condensed matter theory, where its ability to produce exact solutions helps to understand phenomena such as superconductivity.

[22] n is arbitrary, so upon a duality transformation a positive n can be mapped to a negative n.
[23] See section 2.2.1.

Much of the philosophy that I will discuss in this chapter is drawn from Castellani & De Haro (2018) and Castellani (2017), reformulated in terms of the language of the framework for emergence introduced in the previous section. As a relatively simple case study, the sine-Gordon model allows me to introduce many aspects of emergence, fundamentality and duality in the context of soliton physics that will also turn out to be important in my discussion of the Seiberg-Witten model. For this reason, I believe that an overview of the topic will be useful. Because the duality in this case study is proved to be exact, it also provides a good first example of the virtues of philosophical analysis of non-perturbative physics.

5.1 Solitons

Let me start by providing the basics of the theory. The quantum sine-Gordon model is a model in one space dimension and one time dimension of which all classical solutions are known. It is described by the Lagrangian[24] (Coleman, 1975)

$$\mathcal{L}_{sG} = \frac{1}{2}(\partial\phi)^2 - \frac{\alpha}{\beta^2}(1 - \cos(\beta\phi)). \tag{10}$$

Here $\sqrt{\alpha}$ plays the role of the mass of elementary particle excitations around $\phi = 0$, and β is an arbitrary real constant. The second term in the Lagrangian represents the potential, which exhibits the symmetry

$$\phi \to \phi + \frac{2\pi}{\beta},$$

and has a degenerate set of zero-energy vacuum solutions given by

$$\phi_n = \frac{2\pi n}{\beta}.$$

In addition to the elementary particle solutions, the equation of motion of this model admits a bosonic soliton and an anti-soliton solution.[25] The time-independent soliton solution is given by

$$\phi_{sG}(x) = \frac{2\pi n}{\beta} + \frac{4}{\beta}\arctan\left(e^{\sqrt{\alpha}x}\right), \tag{11}$$

[24] Given in natural units, so $\hbar = 1$.

[25] The model also admits a 'breather' or 'doublet' solution, which can be understood as a soliton and an anti-soliton oscillating about their common center of mass. Since it can be interpreted as a bound motion of two solitons, I will not discuss this solution here.

where n can be regarded as a topological quantum number. The anti-soliton is described by $\phi = -\phi_{sG}$.

The energy spectrum of the quantum sine-Gordon soliton is believed to be exactly integrable. As I discussed in section 3.2, exact integrability is often interpreted as being exactly solvable in some sense. In this case, it means that an approach called the inverse scattering method can be taken to establish the exact particle spectrum of the theory. Integrability is a vast topic of research, and the details are beyond the scope of this thesis. The important take away is that the integrability of the sine-Gordon model allows one to find the closed form of multi-soliton solutions that describe the scattering of two or more solitons.[26] These multi-soliton solutions are a prime example of the scattering behaviour described in section 2.2.1: the solitons scatter without losing their shape, so that if we are dealing with two identical solitons, there is no way to determine if the particles have bounced off each other or passed through.

5.2 Duality with the Thirring Model

The most interesting aspect of this model in the context of emergence is that when $\beta^2 < 8\pi$, the quantum sine-Gordon model at the quantum level is identical to the massive Thirring model. The Thirring model is a two dimensional theory (one space dimension and one time dimension) with a massive fermion field ψ. Its Lagrangian is given by

$$\mathcal{L}_{Th} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - M_{Th}\bar{\psi}\psi - \frac{1}{2}g\bar{\psi}\gamma^\mu\psi\bar{\psi}\gamma_\mu\psi, \tag{12}$$

where the last term describes the self-interaction of the fermion field. This Lagrangian has a global U(1) symmetry and a corresponding Noether current. The model is well-defined and exactly soluble only for $g > -\pi$.

In 1975, Sidney Coleman showed that the two models are identical if the coupling constant of the fermion fields in the Thirring model is identified with the β parameter of the sine-Gordon model as follows:

$$\frac{\beta^2}{4\pi} = \frac{1}{1 + g/\pi}. \tag{13}$$

When this equality holds, there exists a weak-strong duality between the two models. From these equations, we see that when the sine-Gordon fields weakly couple (small β),

[26] For details, I refer the reader to chapter 5.3 of Manton & Sutcliffe (2004).

the Thirring fermions are strongly coupling (large g). Furthermore, from the duality follows that the soliton solutions of the sine-Gordon model are dual to the particle solutions of the Thirring model. Because the soliton carries a topological charge and the particles of the Thirring model carry a Noether charge (namely an electric charge), the duality transformation exchanges a Noether charge with a topological charge and vice versa.

Castellani (2017) has pointed out that the exchange between Noether charges and topological charges is accompanied by an exchange between which particles are viewed as elementary and which are viewed as composite. More specifically, the particles that appear as elementary particles in one model, appear as composite particles in the other. In the Thirring model, the Thirring fermions appear as elementary particles, and the sine-Gordon soliton appears as a bosonic bound state of the Thirring fermions, and is therefore a light composite particle. In the sine-Gordon model, the sine-Gordon particles appear as elementary particles, whereas the Thirring particles appear as heavy coherent bound states, and thus as composite particles. As such, the particles interchangeably carry Noether charges or topological charges and Castellani concludes that "the different characterizations of the particles of the theory - as elementary particles or as solitons ...- should not be taken too literally" (p.108).

In the same year that Coleman demonstrated its existence, the duality between the sine-Gordon model and the massive Thirring model was proved to be exact by Stanley Mandelstam (1975). In other words, the duality map is an isomorphism and the two theories describe exactly the same states, quantities and dynamics. While Coleman's result was only valid in the perturbative approximation, Mandelstam showed that the duality is an exact duality that is valid for all physically relevant values of the quantities of the theories. In terms of the formal framework introduced in section 4.2, the fact that this duality is exact tells us that the massive Thirring model and the sine-Gordon theory are formally equivalent bare (uninterpreted) theories.

It is remarkable that the duality transformation leads to an exchange of bosons (the sine-Gordon solitons) and fermions (the Thirring particles). So just like in the Skyrmion model that I briefly discussed in section 2.2.1, we are dealing with a case of bosonization here.[27] As Castellani & De Haro (2018, p.209) say, "the fermionic state of the Thirring model *is already there in the sine-Gordon theory*, and vice-versa: a bosonic state of the

[27]For more on bosonization and the sine-Gordon-Thirring duality, see De Haro & Butterfield (2017), section 5.

sine-Gordon theory is already there in the massive Thirring model" (p.209).

5.3 Emergence and Fundamentality

According to Castellani & De Haro (2018), the sine-Gordon model and the massive Thirring model exhibit epistemic emergence. This type of emergence is depicted in figure 2. Because we are dealing with epistemic emergence, the novelty is introduced between the bare theories T_{sG} and T_{Th}. This novelty is the result of the fact that the two models give different descriptions, namely different Langrangians, of the domain of application. Castellani and de Haro argue that the linkage between the two descriptions is like a change of variables, and therefore the emergence is weakly epistemic.

In addition to the theoretical novelty, there is an interpretative novelty that consists in the fact that the particles are interpreted differently in each model, namely as composite in one description and as elementary in the other. Although the interpretations of the two models are different, they describe the same physical system. In other words, the two interpretations map to the same elements within the domain of application. Because there is no novelty in the domain of application, there is no ontological emergence.

Within the formal framework for emergence, the duality transformation takes the place of a linkage map. As such, it is the formal implementation of the comparison of two different models of the same physical theory. Because the duality is an exact duality, it is an isomorphism between equivalent bare theories and the map is bijective. As such, we can identify a kind of epistemic emergence in which a generalized notion of a linkage map is used.[28] Furthermore, the consequence of taking the duality map as a bijective linkage map is that there is emergence in two directions: each theory can emerge from the other, depending on the perspective we take.

What does this mean for our understanding of the fundamentality of the particles? Castellani & De Haro (2018) argue that because the Thirring particle and the sine-Gordon particle are both present in the complete theory, the solitons of the sine-Gordon model are equally fundamental in an ontological sense as the elementary particles of the Thirring model. This goes against the common idea that the less fundamental theory emerges from the more fundamental theory. In this case, the emergent theory emerges from an equally fundamental theory. Epistemically however, one can argue that which particle is

[28]See section 4.2.1.

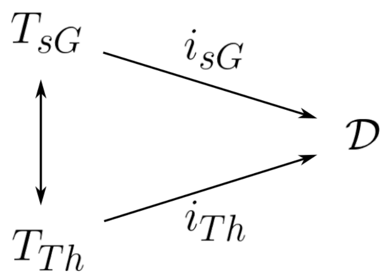

Figure 2: diagram of epistemic emergence in the sine-Gordon model T_{sG} and the Thirring model T_{Th}. The interpretations of the two models map to the same domain of application.

fundamental depends on the theory that is chosen to describe the system with. Even though the physics remains the same upon a duality transformation, the perspective changes: the particles that appear as elementary particles in one model, appear as composite particles in the other. If we follow the epistemic procedures of the physicists and take the elementary particle as the fundamental particle and the composite particle as the less fundamental, then what we identify as the fundamental particle changes too. This type of fundamentality can be understood as epistemic representational (Castellani, 2017, p.108). This agrees with Coleman's ideas, who famously concluded that:
"There is no way of deciding whether the fermion is fundamental and the boson a bound state or the boson is fundamental and the fermion a quantum lump [soliton]. *One is the natural way of putting things if one is describing the massive Thirring model and the other is the natural way of putting things if one is describing the sine-Gordon equation, but these two theories define identical physics. Which you choose to use is purely a matter of taste"* (Coleman, 1988, p.252).

Note that the non-perturbative derivation of the duality establishes it as an exact duality, and as such allows one to confidently rule out the possibility of ontological emergence. In this case, it means that the duality transformation as the linkage map is mathematically rigorous enough to claim that the notion of epistemic emergence is applicable. This claim is further justified by the fact that the sine-Gordon is exactly integrable and therefore its particle spectrum can be identified exactly.

In the next section, a similar approach is taken to investigate emergence and fundamentality of solitons in a more advanced theory: the Seiberg-Witten model. I will argue that in this case, the nature of the duality between the models indicates a case of ontological emergence.

6 Seiberg-Witten Model

In this section, we are looking at a four dimensional $\mathcal{N} = 2$ supersymmetric Yang-Mills (SYM) theory with SU(2) gauge symmetry.[29] In 1994, Nathan Seiberg and Edward Witten published a now famous paper in which they discussed the exact low energy effective model of this theory. Since then, it has had an important impact on the way physicists think about dualities and strong coupling phenomena of gauge theories, such as confinement. Because this model contains multiple duality transformations and it admits soliton solutions, it is interesting to compare it to the sine-Gordon model and investigate emergence and fundamentality.

The following discussion will be formulated in terms of the Wilsonian path integral. As discussed in section 2.2.2, we can interpret the integral after applying the Wilsonian renormalization group methods as a low energy effective action. One of the benefits of using the path integral formulation of the Wilsonian effective action for the philosophical investigation is that there is no coarse-graining or averaging over quantities in this formulation, whereas this would be the case in other formulations.

The supersymmetry of this system extends the Lorentzian spacetime symmetry such that each bosonic field has a fermionic partner field, supersymmetric theories describe multiplet states instead of single particle states. In the $\mathcal{N} = 2$ supersymmetric action of our theory, the complex scalar field ϕ and its supersymmetric partner ψ and the gauge field A_μ and its supersymmetry partner λ_α are combined into a single supersymmetric multiplet Ψ.[30] From the supersymmetry constraints it follows that the Wilsonian effective action of the theory under consideration is restricted to the form

$$S = \frac{1}{16\pi}\text{Im}\int d^4x d^2\theta d^2\bar{\theta}\mathcal{F}(\Psi), \tag{14}$$

[29] A Yang-Mills theory is a quantum field theory with a non-abelian gauge group. It was named after the physicists Chen Ning Yang and Robert Mills, who first introduced such a model in the context of strong interactions between quarks.

[30] This is called the vector multiplet. In the $\mathcal{N} = 2$ supersymmetric action there is also a hypermultiplet, but I will not consider that here.

where θ and $\bar{\theta}$ are coordinates that correspond to the fermionic degrees of freedom and $\mathcal{F}$ is the prepotential of the theory. The idea is that the prepotential includes all the information about the physics in the low energy regime. This prepotential has the important property of being holomorphic, which ensures that it is invariant under the extended $\mathcal{N} = 2$ supersymmetry. This means that it only depends on the superfields and coupling constants and not on their complex conjugates, so $\partial\mathcal{F}(\Psi)/\partial\bar{\Psi} = 0$ and $\partial\mathcal{F}(g)/\partial\bar{g} = 0$. As such, we can understand the holomorphicity condition as a restriction on the form of the prepotential. It is this restriction that makes the supersymmetric theory particularly suitable for the study of the non-perturbative physics of the strong coupling regimes of the theory, because it can simplify the physics to the extent that it is possible to compute its properties analytically. Seiberg and Witten were able to find the full expression for the prepotential.

In the next two subsections, I will introduce the physics of the model. The classical theory, which is computationally more straightforward than the quantum theory, will be discussed in section 6.1. I will then turn to the quantum theory in section 6.2.[31] In section 6.3, I will discuss the soliton solutions in more detail and identify the duality in the model as an approximate duality. In section 6.4, I will turn to emergence and argue that the emergence of solitons in the Seiberg-Witten model is a case of genuine ontological emergence.

6.1 Classical Theory

Before going into the quantum theory, let me start with the classical description. The classical theory admits a family of vacuum solutions. For each solution, a Higgs mechanism spontaneously breaks the SU(2) gauge symmetry to a U(1) symmetry.[32] The Higgs potential is given by

$$V(\phi) = \frac{1}{2}tr[\phi^\dagger, \phi]^2 \geq 0. \tag{15}$$

Unbroken supersymmetry requires that the ground state satisfies $V(\phi_0) = 0$. This means that the fields $\phi_0^\dagger$ and ϕ_0 need to commute, and the family of vacuum states is described by

[31] For a comprehensive introduction to the theory, see the review of Bilal (1996). For a more extensive discussion, see the lecture notes by Bertolini (2021).

[32] For a mathematically rigorous definition of spontaneous symmetry breaking in quantum systems, see Strocchi (2012).

$\phi = \frac{1}{2}a\sigma_3$. We can define a gauge invariant parameter u that labels the possible vacuum solutions in terms of the vacuum expectation value of the Higgs field a,

$$u = \frac{1}{2}a^2 = Tr\phi^2. \tag{16}$$

This complex parameter u is a coordinate of the manifold of vacua M called the moduli space. The understanding of the family of vacua as a manifold is motivated by a deep relationship between supersymmetry and geometry. This relationship is well-illustrated in the non-linear σ-model, discussed for example in chapter 5.1.1 of Bertolini (2021). The moduli space is endowed with a metric that will be the main object of study, because its structure will indicate the validity of the description of the system. This metric is given by the formula

$$(ds)^2 = \text{Im}\tau_{cl}\text{d}a\text{d}\bar{a} = \text{Im}\frac{\partial^2\mathcal{F}(a)}{\partial a^2}\text{d}a\text{d}\bar{a}. \tag{17}$$

By $\mathcal{N} = 2$ supersymmetry constraints, the prepotential $\mathcal{F}$ only depends on a, and not on $\bar{a}$. τ_{cl} is the complexified coupling constant, which is given by

$$\tau_{cl} = \frac{\theta}{2\pi} + \frac{i4\pi}{g^2}, \tag{18}$$

with g the gauge coupling constant. In the classical regime, the physics is independent of the parameter θ, which is known as the instanton or vacuum angle. In the quantum theory that I will discuss in the next section, this parameter can be absorbed in a redefinition of the fields of the system due to an anomalous symmetry, so that it is possible to set $\theta = 0$.

To investigate the particles in this theory, one needs to search for singularities in the moduli space. These singularities can be interpreted as massless particles. An investigation of equation 17 reveals that there is a coordinate singularity on the moduli space at $u = 0$, because at that point $u = \frac{1}{2}a^2$, so $a = 0$. However, it will turn out that this singularity does not belong to the quantum moduli space, so I will leave this point out of the discussion for now. At $u \to \infty$, there is another singularity that corresponds to a W gauge boson. Whenever $u \neq 0$, the SU(2) symmetry is broken into a global U(1) symmetry by a Higgs mechanism that gives the W gauge boson a mass that goes to zero as $a \to 0$. Figure 3 depicts this singularity in the u-plane. As $u \to \infty$, the theory becomes asymptotically free, which means that the fields that were strongly coupled will behave more and more like free fields and we are dealing with a weak coupling theory. In this regime, the physics can be approximated using perturbation theory.

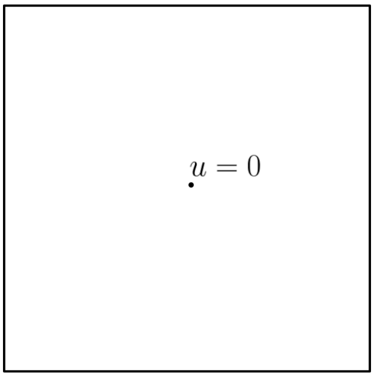

Figure 3: The u-plane of the classical theory, with a singularity at $u = 0$.

6.2 Quantum Theory

The physics described above changes when we extend the theory to the quantum regime. In the classical theory, $u = \infty$ and $u = 0$ are the only singular points in the manifold of vacuum solutions to the theory. In the quantum theory, the prepotential obtains quantum corrections and the coupling matrix τ will be subject to renormalization. Therefore, the metric reveals different singularities in the moduli space. At $u = \infty$, the theory is still weakly coupled and so the same comments apply as in the classical case. However, the classical singularity at $u = 0$ turns out not to be a singularity any more, since equation 16 will obtain quantum corrections. In this section, I will discuss two new singularities in the strong coupling regime. It will become clear that in this strong coupling regime, the metric on the moduli space is not well defined globally any more by the coordinates a and $\bar{a}$. It turns out that different regions in the moduli space have different good coordinates, and therefore each region will be described by a different local theory. These theories are related to each other by dualities.

Before discussing this in more detail, I will describe the physics around the singularity $u \to \infty$. A good coordinate for this region is a. The quantum coupling matrix for this patch in the moduli space, after incorporation of both perturbative and non-perturbative corrections, is given by

$$\tau(a) = \frac{1}{2\pi i} \ln \left(\frac{a}{\lambda}\right)^4 + \frac{a^2}{4\pi i} \sum_k^\infty c_k \left(\frac{\lambda}{a}\right)^{4k}, \tag{19}$$

where the parameter λ is determined by the normalisation of the prepotential. The first term describes the tree- and one-loop perturbative corrections. Higher order corrections are restricted due to the holomorphic properties of the theory.[33] The second term describes non-perturbative instanton corrections, which are summed over with weight c_k.[34] These weight coefficients were determined by Seiberg and Witten (1994b), as well as the way that the prepotential that we can compute from equation 19 transforms to other regions on the moduli space where a is not a good coordinate. This coupling matrix was computed *exactly* by Seiberg and Witten, because the fact that we can understand the family of vacua as a moduli space allowed them to take a geometrical approach towards finding the prepotentials.

Let me turn to the metric on the moduli space again to see which singularities there are. In the quantum theory, the classical coupling constant τ_{cl} is replaced in the metric by equation 19, so that the metric is now given by

$$(ds)^2 = \mathrm{Im}\tau(a)\mathrm{d}a\mathrm{d}\bar{a} = \mathrm{Im}\frac{\partial^2\mathcal{F}(a)}{\partial a^2}\mathrm{d}a\mathrm{d}\bar{a}, \tag{20}$$

Because $\mathcal{F}(a)$ is a holomorphic function of a, it cannot have a global minimum and $\tau(a)$ behaves harmonically.[35] This means that the metric cannot be positive definite everywhere. From equation 18 we know that the complexified coupling $\tau(a)$ depends on the gauge coupling constant g, so a negative metric would correspond to a negative effective gauge coupling. This is not physical, so this cannot be the right way to describe the system. Consequently, the coordinates a and $\bar{a}$ cannot be used for a global description, and the complexified coupling given in equation 19 is only valid locally in the region around $u \to \infty$.

In order to be able to describe the physics in the other regions, one can search for new good coordinates. It turns out that we can find these from the physics around the singularity $u \to \infty$ on the moduli space by making use of an S-duality. Let us define a dual coordinate by the equation

$$a_D = \frac{\partial\mathcal{F}}{\partial a}. \tag{21}$$

Using this new coordinate, the coupling matrix is redefined as

$$\tau = \frac{\partial^2 F}{\partial a^2} = \frac{\partial}{\partial a}\left(\frac{\partial\mathcal{F}}{\partial a}\right) = \frac{\partial a_D}{\partial a}. \tag{22}$$

From this equation, we can see that exchanging $a_D \leftrightarrow a$ leads to the exchange $\tau \leftrightarrow -1/\tau$. With this new coordinate, we now have two ways to describe the low energy physics in

[33]For details, see chapter 9.4 of Bertolini (2022).

[34]For some comments on instantons as solitons, see footnote 5.

[35]In technical terms: the modulus of the holomorphic function cannot exhibit a local minimum or maximum within its domain. For a proof, see Ahlfors (1953, p.241).

this model. One description uses the coordinates a, a_D and τ and contains the bosonic fields ϕ and W. I will call this description the theory T. The other description uses the coordinates $a_D, -a$ and $-1/\tau$ and contains dual bosonic fields ϕ_D and W_D. I will call this description the dual theory T_D.

Seiberg and Witten proposed that there should exist more than one singular point in addition to $u = \infty$. Inspired by the phenomenon of quark confinement in QCD, they assumed there to be exactly two additional singular points in the manifold.[36] Let us set these points to be $u_1 = \Lambda^2$ and $u_2 = -\Lambda^2$, where Λ has a small but non-zero value. To find the corresponding prepotentials describing the physics in the region around these singularities, let us investigate how the coordinates a and a behave as u varies on the moduli space. In the classical theory, a depends on u as described by equation 16. In the quantum theory, this relation is modified by quantum corrections. To find the explicit expression, let us look at the behaviour of the coordinates as u is taken around a closed contour. Because the theory is asymptotically free at $u = \infty$, we expect to get back the classical behaviour described above. From this knowledge, one can find a_D from equation 21 and get

$$a_D(u) = \frac{i}{\pi} a \left(\ln \frac{a^2}{\lambda^2} + 1 \right). \tag{23}$$

Now we take $u \to e^{2\pi i} u$ on the complex u plane; see figure 4. Since equation 16 holds in the asymptotically free regime and $a \to -a$[37], we get

$$a_D = \frac{-i}{\pi} a \left(\ln \frac{e^{2\pi i} a^2}{\lambda^2} + 1 \right) = -a_D + 2a \tag{24}$$

We can formulate this behaviour in terms of a monodromy matrix[38] M_∞,

$$\begin{pmatrix} a_D(u) \\ a(u) \end{pmatrix} = M_\infty \begin{pmatrix} a_D(u) \\ a(u) \end{pmatrix}, \tag{25}$$

[36]One can check this by adding a mass term to the action of the theory that breaks the $\mathcal{N} = 2$ supersymmetry to $\mathcal{N} = 1$. To match the dynamics between the $\mathcal{N} = 2$ and the $\mathcal{N} = 1$ theory, exactly two singularities corresponding to massless solitons should be present on the moduli space, in addition to the singularity at $u \to \infty$. The condensation of these solitons lead to the confinement of the electron-like particles. For details, see chapter 12.3.2 of Bertolini (2021).

[37]This is the result of a $\mathbf{Z}_2$ symmetry acting by $u \leftrightarrow -u$. This symmetry is a subgroup of the U(1) symmetry that was present before its breakdown into SU(2).

[38]'Monodromy' is the name for the study of the behaviour of geometrical objects near singularities.

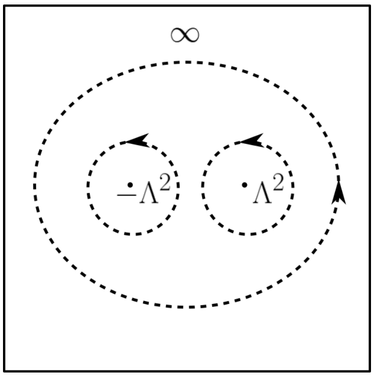

Figure 4: The u-plane of the quantum theory, with three closed contours around the singularities $u = \Lambda^2$, $u = -\Lambda^2$ and $u = \infty$.

where the matrix is given by

$$M_\infty = \begin{pmatrix} -1 & 2 \\ 0 & -1 \end{pmatrix}. \tag{26}$$

To get a theory that describes the full moduli space, the coordinate patches need to be compatible at their overlap. One can use a consistency condition on the monodromy matrices given by $M_\infty = M_{\Lambda^2} M_{-\Lambda^2}$ and depicted in figure 4 to find that the behaviour of the coordinates around $u = \Lambda^2$ and $u = \Lambda^2$ can be written in terms of the matrices

$$M_{\Lambda^2} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \tag{27}$$

and

$$M_{-\Lambda^2} = \begin{pmatrix} -1 & 2 \\ -2 & 3 \end{pmatrix}. \tag{28}$$

Because holomorphic functions are unambiguously determined by the singularities in the moduli space, it is possible to calculate the prepotentials that describe the physics of the patches on the moduli space around the singularities from the monodromy matrices. What we end up with is three low energy effective theories. In the next section, we will see that each of these theories describe the presence of different particles.

6.3 Solitons and Dualities

Let me now focus on the aspects that are interesting in the context of solitons, emergence and fundamentality. So far, I have only discussed the elementary particles in this theory.

We know that in addition to elementary particles, a quantum field theory can also contain soliton solutions. It was already mentioned that singularities in the moduli space can be interpreted as particles. The idea of Seiberg and Witten is that the two singularities at $u_1 = \Lambda^2$ and $u_2 = -\Lambda^2$ in the effective quantum theory correspond to massless solitons, a monopole and a dyon respectively, similar to the massless W boson corresponding to the $u = 0$ singularity in the classical theory. The corresponding patches on the moduli space are described by the low energy effective theories T_D and T'_D. In this section, I will show that these soliton solutions are dual to each other and to the elementary particle solutions of the effective theory T. This will allow us to investigate the duality as a linkage map between emergent theories in section 6.4.

The monopole solutions in this theory were already described in 1974 by Gerard "t Hooft and Alexander Markovich Polyakov, who observed that monopoles naturally occur in Yang-Mills theories with a Higgs mechanism by studying the Georgi-Glashow model. From far away, the 't Hooft-Polyakov solutions look like monopoles that satisfy the Dirac equation. These solutions correspond to a conserved magnetic current that is topological in nature. Like some other theories with topological solitons, the second order field equations of a theory in which the 't Hooft-Polyakov monopole solutions exist can be rewritten as a first order equation whenever the coupling constants take on certain values. The first order field equations of such theories are called 'Bogomolny equations', after Alexander Bogomolny, who published a paper in 1976 in which several examples of this process where discussed. Bogomolny equations are frequently found in supersymmetric theories with soliton solutions, because the values of couplings are the same as the couplings in supersymmetric theories (Hlousek & Spector, 1992). The mass of these solitons has a lower bound called the BPS bound, that for supersymmetric theories is given by

$$M \geq \sqrt{2}|Z|. \tag{29}$$

Here Z is the classical central charge that arises in the supersymmetry algebra of the theory, given by

$$Z = an_e + a_D n_m. \tag{30}$$

n_e and n_m are the electric and magnetic quantum numbers of the state under consideration (Prasad & Sommerfield, 1975; Bogomolny, 1976). The lowest energy solutions to the Bogomolny equations are now known as 'BPS solitons'. The 't Hooft-Polyakov monopole satisfies a Bogomolny equation and is therefore a BPS soliton.

The monopole in this model is described by the dual theory T_D. This theory uses the coordinates $a_D, -a$ to describe the region on the moduli space around the strong coupling singularity $u = \Lambda^2$. This singularity corresponds to a 't Hooft-Polyakov monopole with charge $(n_e, n_m) = (0, 1)$. We can find its mass from the BPS formula, which gives

$$M = \sqrt{2}a_D. \tag{31}$$

We see that the monopole mass is zero at $a_D = 0$. We can also use the BPS equation 29 to check that the monopole has a large mass in the weak coupling regime: for small g, we have small α, thus rendering the central charge Z and the mass M large. Therefore, the monopole and dyon will only contribute to the Wilsonian effective action in the strong coupling regime of the microscopic theory, where they are approximately massless. In the weak coupling regime, their degrees of freedom can be integrated out.

To investigate emergence in this model, it is important to know that the dual theory T_D is interpreted as a $\mathcal{N} = 2$ supersymmetric version of QED, the theory that describes the interactions between electrons in the Standard Model of particle physics. In the previous section, we have seen that the coordinate a_D was obtained from an S-duality transformation from the coordinate a that was used in theory T. This duality is in fact an electromagnetic duality transformation that exchanges magnetically charged particles and electrically charged particles. This duality is similar to the electromagnetic duality in the sine-Gordon model in the sense that the exchange between the electrically charged particles and the magnetically charged particles is an exchange between elementary and composite, and that the duality maps from a weak coupling regime to a strong coupling regime and vice versa. Where in ordinary QED the light matter fields are electrically charged, they are magnetically charged in this SQED variant.[39] This means that the duality transformation maps $S : (n_e, n_m) \to (n_m, n_e)$ and the monopole state becomes an electric state with $(n_e, n_m) = (1, 0)$ with mass $M = \sqrt{2}a$. Furthermore, the dual gauge field W_D, to which the monopoles strongly couple locally, is exchanged under the duality transformation by a gauge field W, to which electron-like particles weakly couple locally.

In addition to the S-duality, the total effective action also describes a T-duality that maps $\tau \to \tau + 1$ and that maps the quantities from the dual theory T_D to the quantities of the dual theory T'_D. Furthermore, a T-duality transformation maps $T : (n_e, n_m) \to$

[39]Because the electrically charged particles are very heavy, they are not part of the low energy effective description of the dual theory T_D .

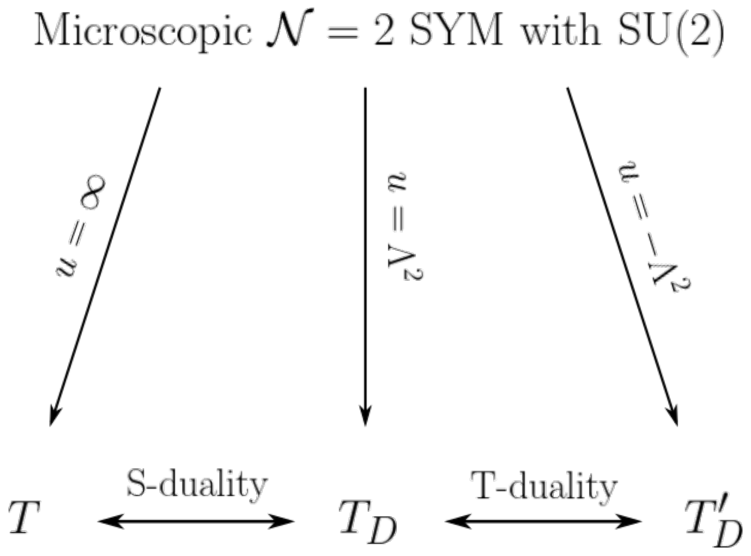

Figure 5: A diagram of the dualities in the Seiberg-Witten model. T, T_D and T'_D represent the low energy effective theories that describe the patches on the moduli space around the singularities $u = \infty$, $u = \Lambda^2$ and $u = \Lambda^2$ respectively.

$(n_e - n_m, n_m)$. Together, the S-duality and T-duality generate the symmetry group SL(2,Z), which is called the duality group.[40] This duality group leaves the metric on the moduli space invariant. By performing a full SL(2,Z) duality transformation on the monopole state, we get the state that corresponds to the second singularity, at $u = -\Lambda^2$. This state describes a dyon, a particle that is both electrically and magnetically charged, $(n_e, n_m) = (-1, 1)$.

Before I discuss the notion of emergence that applies to this theory, I need to make two further comments on the nature of the S-duality described above. Firstly, this duality is much like an electromagnetic duality that was conjectured by Claus Montonen and David Olive in 1977, which was to be a generalization of Dirac's electromagnetic duality.[41] More precisely, it was introduced to investigate the Yang-Mills theory (non-Abelian gauge theories) and the possibility of a theory dual to that. Montonen and Olive claimed the existence of an electric-magnetic *self-duality*: the Yang-Mills theory is theoretically equivalent to the dual description, in which electrically charged particles are exchanged by magnetically charged particles. The duality map only changes the values of the parameters

[40] SL(2,Z)$\simeq$ Sp((2,Z), which is the full electromagnetic duality group.
[41] See section 4.3.

of the theory, but does not map between different theories.

This brings me to my second point. The duality in the Seiberg-Witten model is not exactly the Montonen-Olive duality. If it was, then the spin states of the elementary particle states and the soliton state would also be exchanged.[42] But this is not the case. In the region around $u = \infty$, the super Yang-Mills theory T describes a multiplet with one elementary particle with spin 1, two with spin 1/2 and one with spin 0. However, in the strong coupling regions around $u = \pm\Lambda^2$, the supersymmetric variant of the theory of QED T_D describes a soliton multiplet that contains two particles of spin 1/2, and two with spin 0 (Osborn, 1979, p.326). So the spins of the solitons and the elementary particles that are exchanged by the duality transformation are not the same. We can thus conclude that there is no exact formal equivalence between the dual theories, and we are dealing with two models that are genuinely theoretically distinct.

Nevertheless, the duality can still be understood as an *approximate* electromagnetic duality. In the low energy regime, the duality relates the strong coupling regions of the microscopic theory to the weak coupling region, and exchanges the states and the dynamics of the electron-like particles and the solitons. For a schematic overview of the dualities and the low energy theories, see figure 5.

6.4 Emergence and Fundamentality

So far, we have seen that in the low energy limit of the full microscopic SYM theory with $\mathcal{N} = 2$ supersymmetry and SU(2) gauge symmetry, there are three effective QFTs that each provide a good description for only a region on the moduli space. Suppose we are using one of those theories to describe the target system. Upon a change of the Higgs vacuum expectation value a, a duality transformation needs to be performed in order to preserve the ability to describe the target system. As we perform this duality transformation, we shift between different coordinate patches that ascribe different local properties to the moduli space, thus we shift between different low energy theories. The local properties described by the theories only need to agree where the coordinate patches overlap. By means of the duality transformations, one can obtain one complete effective action from

[42] For the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory, the S-duality is conjectured to be an exact Montonen-Olive duality by (Sen, 1999).

these three local descriptions. The duality therefore has a unifying role here.[43]

In this section, my aim is to determine the intertheory relations between the distinct effective theories that result from taking the low energy limit, in particular between the low energy effective theories T and T_D. Let me start by identifying the candidate for the linkage map. Just like in the sine-Gordon model, the duality in the Seiberg-Witten model can serve as a linkage map between the two bare low energy theories. The S-duality of the sine-Gordon model and the Thirring model is an exact duality: it is an isomorphism. In the case of the Seiberg-Witten model, the duality qualifies as a linkage map, but this map is at most only a partial isomorphism. It is not possible to map all the elements of the theory T to the dual theory T_D, since there is no exact duality that can tell us what spin the particles in T_D should have by looking at the spin of the particles in T. As such, the duality does not indicate a case of formal equivalence of T and T_D, as was the case for the sine-Gordon model. Of course, one might wonder if the duality will turn out to be exact if we extend the theories to take into account higher energies.[44] But since I am only considering emergence in the low energy effective field theories, the physics at higher energies is of no importance to my claim.

In the sine-Gordon-Thirring duality, each dual theory describes a different kind of particle, but the theories are physically exactly equivalent to each other, and so the domain of application can be taken to be the same. Even though it will be computationally far more straightforward to describe the soliton in the sine-Gordon model and the Thirring particle in the Thirring model, the particles are present in the theory regardless of which model we use to describe it with. The situation is different for the Seiberg-Witten model. The different effective theories here are not exactly equivalent, because we are dealing with

[43] This is made more explicit by writing the monodromy matrices for the singularities in terms of the S and T duality matrices. The results are

$$\begin{aligned} M_{A^2} &= ST^2S^{-1} \\ M_{-\Lambda^2} &= (TS)T^2(TS)^{-1} \\ M_\infty &= PT^{-2}, P = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \end{aligned} \tag{32}$$

From this, we get back the consistency relation $M_\infty = M_{\Lambda^2}M_{-\Lambda^2}$ that ensures that the coordinate patches are compatible on their overlap.

[44] I would like to point out that the low energy limits could be taken to suggest another case of emergence, between the full microscopic $\mathcal{N} = 2$ theory and the low energy effective theories.

Low energy effective theory	T	T_D	T'_D
Particles	massless W ϕ electron-like particle	massive W_D ϕ_D monopole	massive W_D ϕ_D dyon
Interpretation	Abelian part of SYM	SQED	SQED
Singularities in moduli space	$u = \infty$	$u = \Lambda^2$	$u = -\Lambda^2$
Good coordinates	a, a_D	$a_D, -a$	$a_D, -a$

Table 1: The effective theories of $\mathcal{N} = 2$ SYM with SU(2) and their properties.

an approximate duality here. Therefore, it is not possible to describe the region on the moduli space around one singularity with the effective theory that was set up to describe the physics in a region around another singularity. The two theories are distinct theories that each provide a description that is only valid locally, even though both describe the same physical system (they result from a low energy limit of the same microscopic theory).

Let me start investigating the possibility of ontological emergence by identifying the domains of application of the theories. Recall that the domain of application was defined as "a set of entities, namely the elements of a set (fluids, particles, molecules, fields charge properties, etc.) and relations between them (distances and correlation lengths, potentials and interaction strengths, etc.)" (De Haro, 2019, p.6). In this case, the domain of application consists of the particles in the BPS spectrum of the theory and the relations between them. From the previous sections, it can be deduced that the domain of application $\mathcal{D}$ of the theory T contains the W gauge boson, the boson ϕ, the electron-like particle and the relations between them, such as the weak coupling between the electron-like particle and the other particles. The domain of application of the theory T_D is $\mathcal{D}_D$ and consists of the monopole, the dual W_D gauge boson, the dual boson ϕ_D and the relations between them, such as the strong coupling between the monopole and the other particles. Because the monopoles couple locally to an effective Lagrangian in terms of the dual field a_D and the dyons couple locally to an effective Lagrangian written in terms of the gauge field $a_D - a$, "there cannot be an effective field theory containing both the monopoles and dyons as elementary fields" (Seiberg & Witten, 1994a, p.32). In other words, both models contain elements in the domain of application that do not map to the other model upon a duality transformation, and so I conclude that $\mathcal{D} \neq \mathcal{D}_D$.

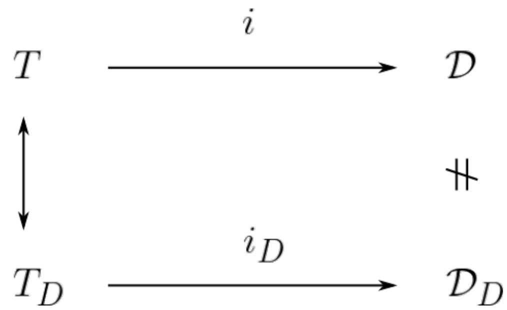

Figure 6: diagram of ontological emergence in the Seiberg-Witten model.

As discussed in section 4, ontological emergence is characterized by novel reference. I therefore need to establish that the range of the interpretation of one effective theory is not the same as the range of interpretation of the other effective theory. In other words, we need different concepts to describe how the theories are to be interpreted. As was discussed in section 6.3, the dual theory T_D is interpreted as a supersymmetric variant of quantum electrodynamics (SQED), coupled to a light hyper multiplet (the soliton). The theory T however is the abelian part of a supersymmetric Yang-Mills theory, which is a gauge theory that describes $U(1)$ gauge bosons (W bosons).[45] First linking the theories by a duality transformation and then interpreting T_D gives SQED, with in the limit $u \to \Lambda^2$ a singularity that can be interpreted as a monopole. This is different from interpreting T and then going to the limit $u \to \Lambda^2$: this point in the moduli space cannot be described by T. There are no concepts found in the interpretation of T that accurately describe this point. So T_D describes properties of the target system that T does not.

What can we say about the fundamentality of the theories in the Seiberg-Witten model? In section 4, ontological emergence is characterized by asymmetry. I want to point out that this is also the case for the emergence in this model. However, the emergence is two-fold, since both theories are ontologically emergent with respect to each other: each domain of application contains novelty with respect to the other theory. Therefore, one can argue that from an ontological point of view, which theory is more fundamental depends on the perspective we take.

As discussed in 3, one might argue that we should be cautious about drawing ontological conclusions in light of the infinities of QFT and the lack of mathematical rigour that their

[45]Photon-like particles which may bear charges, have spin 1 and whose mass is acquired by a Higgs mechanism.

treatment brings. However, I want to reiterate that these objections do not apply in this case. We are dealing with an effective field theory, whose main objects of importance can be exactly calculated. Furthermore, the spectrum of the theory is determined exactly from the supersymmetry algebra. Because these results are exact, we can rule out the possibility that the ontological emergence is merely a case of coarse-graining. If that was to be the case, then the domain of application of one theory should be a subset of the domain of application of the other theory, so that the novel elements that the domain of application of the emergent theory contains are fine-grained elements. The fact that the particle spectrum is determined exactly rules out this possibility. The exactness of the metric ensures us of the fact that there is no global description of the moduli space of vacuum solutions, and that each local description contains different information. Therefore, I believe that it is justified to claim that this is a true case of ontological emergence.

7 Conclusion

Just as investigations of QFT in the algebraic or the Lagrangian approaches both have their benefits for answering a particular type of interpretive question, the preceding sections have shown that soliton physics is particularly well suited to answer questions on the non-perturbative domain of QFT. Because of the mathematical rigour of these models, the non-perturbative methods substantiate interpretative claims on the distinction between what is elementary or composite, what is fundamental and the occurrence of emergence. The usual criticisms against Lagrangian QFT, such as the lack of justification for perturbative renormalization, are weakened or avoided altogether. The non-perturbative methods also allow us to identify entities that are part of the theories' ontology that would not have been found using perturbation theory: solitons.

My discussions of the sine-Gordon model and the Seiberg-Witten model have shown both models exhibit a type of emergence that fits the framework given in De Haro (2019). This framework allows one to identify ontological emergence as relative ontological novelty between two bare theories that are related by a linkage map. In the case of the sine-Gordon model, the duality with the Thirring model serves as a linkage map between two theories that contain relative theoretical novelty and novelty in interpretation. As such, the theories can be identified as epistemically emergent from each other. In the Seiberg-Witten model, taking two low energy limits of the microscopic theory, corresponding to singularities in the moduli space of the theory, results in two effective theories that are linked by an approximate S-duality. This duality can be understood as a linkage map between two theories with ontological novelty relative to each other. The non-perturbative derivation of the dualities and the exact particle spectra make the physics of these models rigorous enough to confidently claim a case of emergence and rule out the possibility of emergence as a consequence of coarse-graining.

The case-studies show that whereas emergence is generally understood to work one way, namely from the bottom theory to the top theory, choosing a duality map to serve as a linkage map between two theories that exhibit emergence allows us to identify a situation where both theories are emergent with respect to each other. Depending on which perspective one takes, the sine-Gordon model is epistemically emergent from the massive Thirring model or vice versa. In the Seiberg-Witten model, the same applies on the level of ontological emergence. Note that the emergence in the Seiberg-Witten model is still an asymmetric relation: the duality is only a partial isomorphism and as such leaves room in the domain of application for ontological novelty.

Furthermore, both models showed that the duality between a theory with elementary particles and a theory with soliton particles provides interesting insights on the fundamentality of particles. As the physicist's tradition is to identify the elementary particles as epistemically fundamental and non-elementary or composite particles as less fundamental, the duality in the sine-Gordon model exchanges which particle is understood to be epistemically fundamental. Similarly, the duality in the Seiberg-Witten model exchanges which particle is understood to be ontologically fundamental. Furthermore, contrary to the common relation between emergence and fundamentality that holds that a less fundamental theory emerges from a more fundamental theory, I have argued that in the sine-Gordon model, the emergent phenomenon can be understood as ontologically equally fundamental to the phenomenon in the theory that it emerges from. This argument transfers to the epistemic domain for the Seiberg-Witten model.

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