Master Thesis U.S.E.

The natural interest rate, its drivers, and the future outlook.

An empirical research over the main factors that are driving the natural interest rate of the Euro Area.

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Abstract

This thesis provides a critical analysis of the factors driving the natural interest rate (also referred to as "r*") in the Euro Area from 1995 to 2017. The research shows how the increased level of savings, shrinking globalization growth, and declining potential output growth are placing downward pressure on the r* of the countries of the Euro Area. The analysis suggests that the Great Recession and the European Sovereign Debt Crisis structurally changed the underlying dynamics of the drivers of the natural interest rate, showing a much stronger effect of the factors for the period following the financial crises. Finally, the stress tests conducted with the OLS model show very little flexibility of the natural interest rate under three different scenarios, namely severely adverse, adverse, and positive/stable scenario. This result raises questions about the effectiveness of the suggested factors in explaining the movements of r*, indicating that the underlying dynamics might be regulated by other factors such as the financial cycle, as proposed by another strand of literature. Concluding, the findings suggest that ECB, policymakers, and investors should keep a close eye on the future evolution of r* as the zero-lower bound appears to be here to stay, at least in the foreseeable future.

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1. Introduction

This paper aims to provide investors and financial institutions with a relatively simple model that can accurately explain the movements of the natural interest rate (referred to as “r*”) of the Euro Area. The model is then used as a stress-testing tool for assessing the sensibility of the natural interest rate to the factors driving it, and forecasting what the trend of r* could look like under a severely adverse, adverse, and positive/stable scenario over three years.

Understanding the trend of the natural interest rate is of primary importance for many stakeholders of the financial system, including - but not limited to - the ECB, policy makers, academics, and investors. Following the economic instability experienced by many countries in the 1990s, scholars and practitioners paid growing attention to assessing the financial system’s sensitivity (Crockett, 1997). Traditionally, stress tests are used to evaluate the vulnerabilities of asset portfolios. Still, recent times have been marked by a broader application of such procedures on banks, banking systems, and financial systems (Čihák, 2007).

Interest rates are the most powerful monetary policy instruments, allowing the governing entity to either stimulate or slow down the economic activity by reducing or raising the interest rates, with the final purpose of having control over inflation. However, due to the decline in interest rates faced by the major global economies over the last two decades, the effectiveness of this tool (therefore, of monetary policies) has been limited by the zero-lower bound, i.e., the theoretical constraint that prevents interest rates from going below zero.

The ECB already experienced the problem of the zero-lower bound during the Great Recession (followed by the Sovereign Debt Crisis), when the central banl deployed unconventional monetary policies such as Forward Guidance and Quantitative Easing. As valuable as these tools revealed to be, if the zero-lower bound is here to stay, it will require central banks to come up with further unconventional tools in order to not fall in the so-called “deflation trap” (IMF, 2003). In June 2014, the Governing Council of the ECB introduced the Negative Interest Rate Policy (NIRP), and it is planning on keeping the rates negative for the next four years. Claeys (2021) showed that negative rates can be sustained in the short term and might even lead to an improvement in output, employment, and inflation; however, it is still unknown whether the central bank can hold the same policy in place for a longer period.

Knut Wicksell (1898) described the natural interest rate as the rate which is neutral to commodity prices, i.e., the real interest rate that places neither upward nor downward pressure on inflation. The average r* is alsolosely related to the average long-term real interest rates (Brand, Bielecki, & Penalver, 2018). Taking both perspectives into consideration, a negative neutral interest rate is a sign that the economy will require a negative interest rate in the long run to support its total productivity. Natural interest rates are not only crucial for monitoring the equilibrium of economies; they also determine the expected return for fixed income portfolios. These portfolios are diversified by taking a global scope, exposing investors around the World to the negative interest rates of the Euro Area and other economies, such as Sweden, Denmark, and Japan. The price of fixed-income assets, such as bonds, move in the opposite direction of interest rates, meaning that a decreasing r* will push up prices and decrease yields. For this reason, understanding the trend of r* and how this could evolve in the future is of primary importance for fixed income investors.

Many researchers have already tried estimating the natural interest rate for the Euro Area (as reported in the literature review), and the different estimated models all come to the same conclusion that r* has been falling globally over the last 25 years. Furthermore, a wide of range of studies (for reference, see: Bomfim (1997), Laubach and Williams (2003), Congressional Budget Office (2014), International Monetary Fund (2014), Council of Economic Advisers (2015), Hamilton et al. (2015), and Pescatori and Turunen (2015)) provide evidence of the many factors influencing the natural interest rate, but the results seem to differ among different economies and periods analyzed. The researchers, however, find consensus that the main drivers are of both macroeconomic and demographic nature.

The model developed in this paper is a time-series OLS regression with demographic and macroeconomic variables on the natural interest rate estimated through the Holston, Laubach & Williams (HLW) model. The HLW model is a procedure applied by the Federal Reserve (referred to as "FED") that provides estimates of the unobserved natural interest rate as a time-varying process. The model uses the Kalman filter to estimate the natural interest rate from GDP growth, inflation, and interest rates. Further explanation and implications of such measures are reported in the section '4.1 Data Collection and Description'.

By providing evidence to answer the research question, the analysis covers the following sub-questions:

1. Which factors have driven the natural interest rate of the Euro Area over the last 20 years? (covered in section 5.1)
2. How has the impact of these drivers changed after the Great Recession of 2008-2009? (covered in section 5.2)
3. How will the natural interest behave under stable / adverse / severely adverse stress test scenarios? (covered in section 5.3)

Within the sub-topics, the paper also provides findings that help to either confirm or contradict the following hypotheses:

HP1. The literature review suggested that the underlying drivers of r* for the Euro Area are a mix of demographic and macroeconomic variables. It is expected to see a high explanatory power of the selected variables for the natural interest rate (r* estimated through the HLW model, an estimation procedure that is purely based on macroeconomic components). The explanatory power of the model will be assessed through the adjusted R-squared measure, "detrended" as illustrated in section 4.1 and illustrated in appendix 8.8 of this paper.

HP2. The IMF (2019) suggests that productivity is not the real driver of the natural interest rate for the Euro Area, indicated by the loss in significance for this variable when a proxy for globalization is included in the model. However, this strongly contradicts the paper of Papetti (2019), thus, requiring further investigation.

HP3. Lane (2012), Arteta & Hale (2008), and Achraya et. al (2015) suggested that the financial crises faced by the Euro Area in the last two decades have imposed a real challenge on the system and that the underlying dynamics are deeply scarred. To assess whether this is true also for the natural interest rate, the regression was run on the database split in the period of time prior and after the Great recession. A strong difference in the explanatory power and in the coefficients of the independent variables would confirm the hypothesis.

HP4. Risk aversion has been defined as one of the strongest drivers for the natural interest rate of the Euro Area by the ECB, assessing that its impact has become much more significant in the latest years, placing a minor significance on the effects of productivity growth (Brand, Bielecki, & Penalver, 2018). The split of the regression database into two separate periods will show whether risk aversion is becoming increasingly significant.

The following sections in this paper will first explore the existing scientific literature on the topic. The aim is to define what is already known about r*, the factors driving it on which the literature agrees and those on which it disagrees, and how they are driving it, providing evidence for the variables included in the regression model. The data and methodology sections will then illustrate what data was gathered, how they were gathered, and how they were applied in the regression model and stress test scenarios. Finally, the paper reports the discussion of the regression models, the stress-tests, conclusions, and limitations of the research.

2. Literature Review

2.1 R*, definitions and estimation procedures

According to Laubach & Williams (2015), the natural interest rate is the real short-term interest rate that allows the economy to operate at its full potential. However, the interpretation of the natural rate can take a longer perspective too, defining the outlook of the real interest rate for the next five to ten years. Other theories provide slightly different definitions; however, they all agree on the fact that this rate can not be seen directly, therefore it has to be estimated, and the calculation is based upon data that is not available in real-time, meaning that the estimation has to be continuously updated.

Several researchers have devised different ways of estimating the natural interest rate. Del Negro et al. (2017) and Johanssen et al. (2016) have estimated r* through time-series models, allowing to estimate the non-observable variable, r*, through observable ones. These models include the inflation rate and the business cycle. Other researchers have made use of long-term relationships between the trend variables to impose economic structures (for reference, see: Harvey (1990) and Durbin and Koopman (2012)), while Lubik and Matthes (2015) made use of a Vector Autoregressive Model. Another estimation procedure is the one through General equilibrium models (GEMs). These rely on the assumption that households have rational expectations about the current and to-be state of the economy; therefore, it is possible to estimate the unobservable natural interest rate through their consumption and investment patterns.

Another way of estimating r* is through semi-structural models. These models are built on theoretical structures, but they still offer flexibility for a better fit with the data. Of the semi-structural model, the Holston, Laubach, & Williams (HLW) is the most recognized by academics, both due to its flexibility and to the replicability of the results. This model was developed on top of the Laubach & Williams (LW) model. The theory underlying these two models was born in 1928, when Ramsey (1928) created a growth model that could express equilibrium conditions in the form of equation, assuming that representative households choose optimal saving rates. From there, the LW model was developed in 2015 to estimate the natural rate for the United States (Laubach & Williams, 2015). In 2017, the HLW model was published; this was a “simplified” version of the previous model, which can be applied to any advanced economy to derive the unobservable natural interest rate.

2.2 Underlying dynamics of R*

The models proposed by previous researchers use different estimation procedures and explanatory variables to estimate the natural interest rate. However, they agree on the downward trend that r* has been facing since the 1980s in all major economies around the World. Proof of this is reported in the graph in Appendix 8.1. The lines in the graph represent natural interest rates for the Euro Area estimated through different models, clearly showing that despite the various procedures and explanatory variables, all models conclude that the rate is trending downwards and that some significant macroeconomic developments are driving it.

Demographics, Savings and Risk Aversion

Rachel and Smith (2017) showcased that the natural interest rate is indeed falling globally, which is strongly attributable to a decline in the forecasted global growth. Explanation of this is given by the underlying dynamics impacting the growth of the most advanced economies, which primarily relate to the shift in investments and savings derived from demographic evolution.

Aging population and reduction in population growth are seen as the two most impactful drivers; the first one strongly increases the level of savings (at the expense of investments), while the second one harms the return on capital due to fewer workers. Hong & Shell (2019) also showed that the change in demographics experienced by the developed economies is the primary driver of r*. Longer lives and an aging population are imposing a challenge to the Social Security system, paying more than what it receives from contributors. The potential output decrease is also attributable to the aging population, where the working class is always smaller and the class to be maintained is always greater.

As highlighted by Teulings & Lu (2016), the introduction of the contraceptive pill in the Euro Area has created an imbalance between age cohorts, with the older cohort constantly rising. Furthermore, they argue that the overall aging population will strongly undermine the level of investments. According to the researchers, such a low level of investments, which can be referred to as a low risk appetite, will not allow for a raise in the real interest rate for about 10 to 15 years. The current elderly dependency ratio (calculated as the ratio between individuals older than 65 over those aged between 15 and 64) is estimated at about 25 to 100, however, ECB forecasts show that the ratio might reach 50% by 2050, with a population growth estimated to decline from 0.45% to -0.40% (Brand, Bielecki, & Penalver, 2018).

The ECB developed an 'overlapping generation model' (or OLG model), used for forecasting the impact of demographics on the natural interest rate. The estimates of the model suggest a change ranging between -1.7% and 0.4% in natural interest rate from 1990 to 2030 attributable to demographics only. The paper outlines how this effect is split into two different *'channels'*, namely: decreased effective labour input, and increased savings due to longer lives expectancy (Papetti, 2019). The range of papers developed around OLG models find agreement on the following three channels through which demographic transition affects r*:

1. Downward impact from lower labor input: other things equal, a lower input determines a decrease in production.
2. Downward impact from higher life expectancy: lower mortality and longer lives push up the level of savings, foreseeing a longer retirement phase.
3. Upward impact from the rising percentage of dissavers (individuals that spend more than they earn).

The OLG models applied by the researchers Bielecki et al. (2018), Papetti (2018), and Auerbach and Kotlikoff (1987), show that the decline in the natural interest rate of the Euro Area is mainly attributable to an aging population, with an estimated average effect of about 0.9%. Furthermore, given the forecasted demographic trend, demographic evolution is estimated to keep depressing the interest rate by another 0.5% by 2030. These estimates are aligned with the multi-country OLG model developed by Krueger and Ludwig (2007). They found that the worldwide natural interest rate is estimated to decrease by 0.9% within 2030 due to demographic trends only.

Teulings & Lu (2016) highlight the correlation between aging population and an increased level of savings and increased risk aversion. The impact of risk-aversion on the equilibrium interest rate has been studied since the early 90s'. In 1989, Chang showed that as the temporal risk aversion increases, the natural interest rate faces a downward pressure (Chang, 1989). The ECB also found that risk aversion (proxied as the spread between long and short-term interest rates) has been one of the strongest determinants for the downsloping r* of the Euro Area since the 1980s, placing much lower importance on the drop in productivity growth (Brand, Bielecki, & Penalver, 2018). The result is robust among different periods; however, the increasing share of older individuals has become more impactful over the last decade, with an even more significant effect than risk aversion (Brand, Bielecki, & Penalver, 2018).

Labor Productivity, Potental output, Financial Crises, and Globalization

The results of General Equilibrium Models attribute the trend of r* to the intertemporal substitution elasticity of households, which determines the impact of interest rate changes on their consumption. These models also seem to support the relevance of shocks that affect the savings decisions of households, such as aggregate demand (AD) shocks or changes in labor productivity growth (Woodford, 2003; Galí, 2008). This is in line with the theoretical perspective that a positive AD shock has an impact on the equilibrium point of an economy, and it will necessarily require a higher r* for the economy to go back to equilibrium. On the other hand, productivity growth also plays an essential role in the longer term, where a foreseeable high growth might lead households to save less and consume more, again having a positive impact on r* (and the same is applicable for the opposite case). The multi-country model developed by Hledik and Vlcek (2018) shows that the natural interest rate follows changes in potential GDP quarter by quarter. An estimation of the output gap, the difference between real GDP and potential GDP, is provided by the HLW model. The results show an increasing gap starting from the 1980s, the same period during which natural interest rates began falling globally.

Adverse AD shocks, such as financial crises, have multiple channels through which they can impact the natural interest rates. A well-documented correlation is the one between financial crises and risk-aversion, showing that the second one can be used as part of indicators for forecasting the former one (Coudert & Gex, 2008). Furthermore, financial crises tend to require longer recovery periods compared to standard recessions, with even more persistent effects on potential GDP and long-term growth, which appears to be especially true for the Southern European countries (Guiso, 2014). The research of Guiso argues that financial crises tend to divert individuals from risky investment opportunities with high returns to less-risky assets with lower returns, determining an overall increase in risk-aversion. In turn, risk-aversion slows down economic growth, explaining how financial crises take a longer time to revert their effects.

Apart from risk-aversion, financial crises have other effects on macroeconomic and demographic variables that are important to consider. For example, during the Great Recession, the US and the Euro Area experienced a substantial decline in output, investments, and consumption, followed by a drop in labor force participation and per capital employment (Christiano, Eichenbaum, & Trabandt, 2015). Moreover, further evidence shows that the Great Recession has had a stronger impact on the youth labor market for the OECD countries, highlighting a much stronger constraint in youth labor supply compared to adult labor supply, increasing the pre-existing gap between the two (Bell & Blanchflower, 2011).

Following the Great Recession, the Euro Area experienced the European Sovereign Debt Crisis, which lasted from 2009 to 2012. The start and propagation of such crises has been attributed to the original (and *"flawed"*) design of the euro, marked with a poor understanding of the difficulties that a crisis would have imposed under a monetary union setting (Lane, 2012). A study conducted before the European Sovereign Debt Crisis found that the level of foreign credit to domestic private firms drops significantly during such crises, strongly undermining investments and potential output (Arteta & Hale, 2008). Achraya et. Al (2015) investigated the direct impact of the European Sovereign Debt Crisis on newly syndicated loans to firms, finding that the risk-shifting conduct of banks during this period caused a shrink in the probability of newly syndicated loans by 53%, severely impacting investments and output (similarly to the Great Recession).

It is not clear whether the Great Recession and the European Sovereign Debt Crisis have had a direct impact on the natural interest rate; however, it is evident that these crises had both short and long-term effects on the economic structure and the investment and saving behavior of individuals.

The Eurostat estimates for labor productivity show a decline over the last 15 years (Eurostat, 2022). But the drop goes further back in time, up to the 60s. Since then, productivity has continuously declined from 7% to about 1% in the 2000s. The factors influencing a decrease in productivity are several. They relate both to global and country-specific dynamics, including events that might have caused deep scars in the economy, like the global financial crisis of 2008/2009.

Figure 1. shows the r* estimates reported in the paper of Laubach & Williams (2017) for the Euro Area.

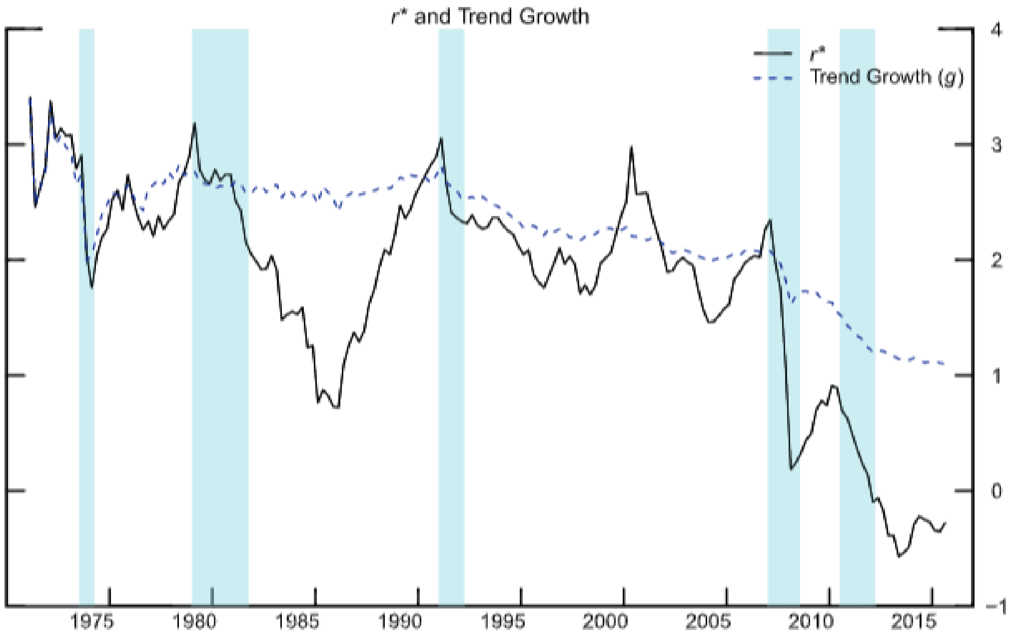

Figure 1. Natural interest rate estimates from HLW model, retrieved from Holston, Laubach, & Williams (2017)

Figure 1. shows how the trend of r* over the last 40 years for the Euro Area has been negative, with solid downward pressures during the periods of economic stress (shaded blue areas in the graph). As reported by the paper, the most substantial drop occurred during the global financial crisis, followed by the European sovereign debt crisis. These two events had such a strong negative impact that r* fell below zero for the first time.

According to a report disclosed in 2015 by the OECD (2015), the median loss in output in 2014 amounted to 5.5% for 19 countries in the Euro Area. A loss that is greater than what it is estimated for any previous crisis. The paper also outlines how the most significant impact of the crisis was on productivity, which resulted from the combined lower factor productivity and capital per worker.

Finally, previous researchers have highlighted the impact of globalization on the downward trending r*. According to the ECB, globalization can influence the natural interest rate through different channels: a higher degree of globalization can increase productivity growth through new investment opportunities, enhance export, and improve the allocation of resources (ECB, 2021). As evidence, globalization is the cause of shifts in the level of savings and investments abroad, which have a strong influence on the domestic interest rate (Obstfeld, 2020). All of this help stimulate the economy and its aggregate demand and in turn, increase long-term interest rates. The International Monetary Fund (IMF) shows that natural interest rates have been rising up to the 1970s, when globalization was also at its peak. Since then, globalization and r* experienced a strong decline, suggesting a robust correlation between the two (IMF, 2019). The paper defines "globalization premium" as the positive contribution of globalization to the natural interest rate level during the 1980s. The model suggested by the IMF finds that total factor productivity growth is not as significant when globalization is included in the analysis; however, the same model indicates that this "globalization premium" has almost completely vanished as globalization growth has slowed down, partially explaining the drop in the natural rates that started around mid-1980s. The IMF believes that globalization is so determinant that it will be impossible for the trend of r* to reverse in the absence of significant changes in the level of global competition (IMF, 2019).

3. Theoretical Framework

This research follows a series of papers that aim at studying the macroeconomic and demographic variables that are driving down the natural interest rate of the Euro Area. The research has the purpose of incorporating the findings of previous research into one model that can explain and predict the trend of R* based on demographic and macroeconomic factors. The following framework was extrapolated from the literature analyzed, and it consolidates elements that were found to have a significant impact on r* by previous research. The framework is reported in Figure 2.

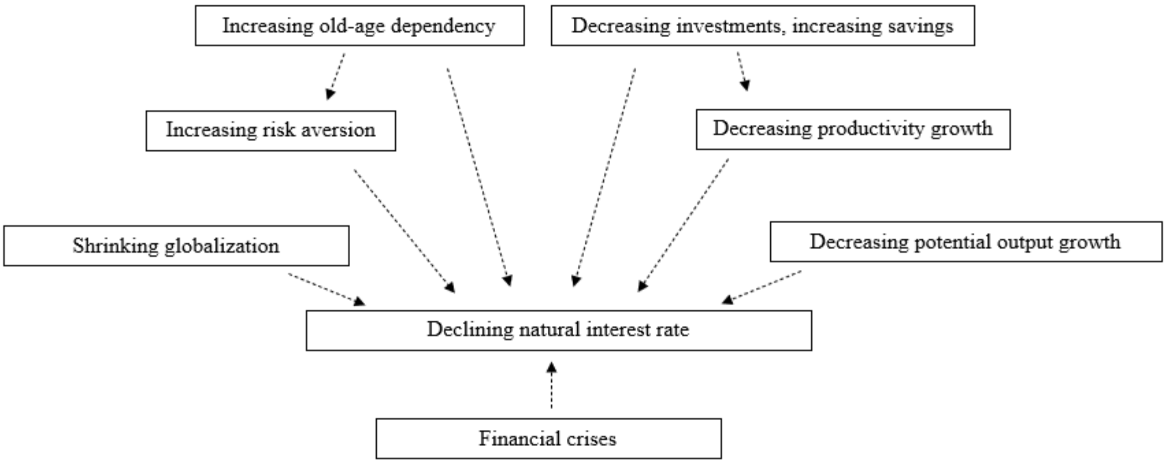

Figure 2. The theoretical framework

As shown in Figure 2., the factors proposed by previous papers strongly relate to the aging population trend experienced by all advanced economies. The decrease in fertility rate and longer life expectancy are slowing down the generational change that would be required to ensure appropriate investments and savings levels for the economy to continue growing; this is also one of the determinants of a decreased productivity growth and potential output growth, along with the shrinking globalization that is negatively impacting the growth estimates of countries whose economies heavily rely on imports/exports. Even though the different nature of the factors reported in the framework, it is clearly visible that these are strongly intertwined, putting pressure on one another and eventually, driving down r*. Finally, the literature suggested that the latest financial crises (namely the Great Recession and the European Sovereign Debt Crisis) have created structural changes in the economy, as well as shifts in the investment and saving behavior of individuals, which appears to have placed further pressure on r*.

4. Empirical Strategy

4.1 Data Collection and Description

The following section covers the description and rationale behind the dependent and the independent variables used for the time-series regression analysis included in this research.

Dependent variable

Natural interest rate: the natural interest rate used in this research is estimated through the HLW model. This model aims to derive the natural interest rate as a time-varying process, making use of the Kalman filter to estimate r*. Developed by Kalman in the 1960, this filter is an estimation methodology of unknown variables assessed through inputs, which are measured over time and re-fit into the model. The algorithm can produce (relatively) accurate estimates through a recursive procedure, in which a higher weight is given to the more precise estimate (providing a lower uncertainty for the state variables). The equation used to derive the natural interest rate comes from the New Keynesian framework, which uses the Phillips curve relationship and the intertemporal Investments-Savings (IS) equation to explain the dynamics affecting output (GDP) gap and inflation as a function of the real rate gap. Considering representative households with Constant Elasticity of Substitution (CES) preferences, the model entails changes in the natural rate of interest when the growth rate of output changes, as well as when shifts in preferences occur. The natural interest rate is estimated with the following formula $r * = \frac{1}{\sigma} g_c + \Theta$, where sigma represents the intertemporal elasticity of substitution in consumption and theta is the rate of time preference (Holston, Laubach, & Williams, 2017).

The estimates provided by this model, as reported by the three authors, do have a relatively high degree of uncertainty, also given by the downward sloping IS curve condition, which seems to fail in the modern economy; conversely, it is important to mention that the estimates of the natural interest rate of the Euro Area computed with different models (see appendix 8.1) show a relatively high degree of correlation in the estimation results, agreeing on the downward trend that the rate has been experiencing over the last decades. The HLW model was chosen for the estimation of the natural interest rate of the Euro Area because of its computation procedure. Demographic models (like OLGs) and other time-series models used by previous researchers base their r* estimates on demographic and macroeconomic factors; it appears to be counterintuitive to re-run a regression with the same factors that were used for the estimation of the dependent variable. On the other hand, the natural interest rate estimated through the Kalman filter purely relies on the relationship between real GDP, short-term interest rates, inflation, and expected inflation, meaning that demographic factors were not included in the estimation procedure. Creating a model that can explain the movements of r* estimated through the HLW model would not only close the gap between the different estimation procedures, but would also provide an empiric confirmation of the underlying correlation among these variables.

A replication code along with the r* estimates are provided on the website of the Federal Reserve Bank of New York. The natural interest rate has been estimated quarterly from the period starting in 1995 Q1 and ending in 2019 Q3, for a total of 99 quarters through 25 years.

Explanatory variables

Productivity: productivity is measured in different ways and for various purposes. Aligning with Fiorentini et al. (2018) and Generali & Neri (2017), productivity is specified here as labor | productivity. Quarterly changes in labor productivity per person for the Euro Area ranging from 1995 Q1 up to 2019 Q3 were retrieved from the Eurostat database.

Potential output growth: potential output growth is the period-in-period change in the potential output that could be produced under full economic activity. The HLW model provides estimates of the output gap for the Euro Area; this measure was used to adjust the real GDP estimates to derive the potential GDP. Quarterly real GDP estimates for the Euro Area ranging from 1995 Q1 up to 2019 Q3 were retrieved from the Eurostat database.

Globalization: this research followed the same proxy procedure often adopted by previous papers, where globalization is expressed as the ratio between the sum of imports and exports over GDP. Quarterly data for Imports, Exports, and GDP for the Euro Area ranging from 1995 Q1 up to 2019 Q3 were retrieved from the Eurostat database.

Savings: gross household saving rate was used for proxying this variable. Quarterly data for the Euro Area ranging from 1995 Q1 up to 2019 Q3 were retrieved from the Eurostat database.

Risk-aversion: following the study of Brand, Bielecki, & Penalver (2018), the spread between long-term and short-term interest rates was used as an imperfect proxy of the term premium to express the risk-aversion of investors. The spread between 6-months and 1-month EURIBOR was calculated from the rates retrieved for the Euro Area ranging from 1995 Q1 up to 2019 Q3 from the Statistical Data Warehouse database. Supplementary, and for robustness check, the variable was proxied also as the spread between 2Y and 10Y Euro Area Benchmark bond yield. Quarterly data are provided by the Statistical Data Warehouse.

Old-age-dependency ratio: aging population is proxied through the old-age-dependency ratio (or "OAD"). This measure represents the ratio between the population older than 64 and the population with ages between 16 and 64. Quarterly data for OAD for the Euro Area ranging from 1995 Q1 up to 2019 Q3 were retrieved from The World Bank database.

Control variables

Time: due to the non-stationary nature of most of the explanatory variables(see appendix 8.4), a time control variable was included in order to avoid spurious regression results that could arise from trending variables.

Great Recession: the OCED Stats database reports the GDP estimates for most economies worldwide. Looking at the data for the G20-zone, the Great Recession can be seen at a global level in the period from Q3 2008 to Q1 2009 (OECD, 2022). The control (dummy) variable takes the value of 1 for this time period.

European Sovereign Debt Crisis: this crisis was faced by the European Union in the period ranging from Q2 2009 up to Q3 2010 (OECD, 2022). The control (dummy) variable takes the value of 1 for this time period.

Summary statistics

Here below are reported the main summary statistics of the variables included in the regression models:

Summary statistics					
Variable	**Obs**	**Mean**	**Std. Dev.**	**Min**	**Max**
Dependent					
Natural Interest Rate	99	0.014373	0.007688	0.001372	0.027987
Independent					
Productivity	99	0.174748	0.432209	-2.3	1
Savings	99	12.95402	0.59543	11.8	14.26
Globalization	99	0.746102	0.116262	0.526999	0.937496
Old-age-dependency ratio	99	0.272536	0.02989	0.225771	0.32932
Potential Output Growth	99	2204020	471922.5	1372721	3168754
Risk Aversion 1	99	0.180552	0.209498	-0.1986	0.6943
Risk Aversion 2	99	0.184753	0.212169	-0.1986	0.6943
Control					
Great Recession	99	0.030303	0.172292	0	1
Sovereign Debt Crisis	99	0.060606	0.239821	0	1

Table 1. Summary statistics of the variables in the database.

The statistics show a total amount of 99 observations for each variable, ensuring model representativity. Furthermore, given the percentage form of the dependent variable, the variables Potential Output Growth and Globalization were transformed in natural logarithms, which allows for an easier and more meaningful interpretation by expressing the results in terms of percentage changes (Wooldridge, 2015). Productivity, Savings and Old-age-dependency are already presented in the form of percentages (or ratios), and therefore, no further action is required for these variables. The variables already present some interesting results, as showcased by the doubling level of globalization from its minimum to mixum value, and with a similar result for the old-age-dependency ratio and potential output growth.

4.2 Methodology

Regression models

Following the steps of Del Negro et al. (2017) and Johanssen et al. (2016), this thesis aims at explaining the movements of r* for the Euro Area through a time-series OLS regression. The sub-questions of this paper are answered through the following models:

Regression models	
SQ 1:	$r_t^* = \beta_0 + \beta_1 Productivity_t + \beta_2 Savings_t + \beta_3 Ln\ Globalization_t + \beta_4 Ln\ Potential\ Output\ Growth_t$ $+ \beta_5 Risk\ Aversion_t + \beta_6 Great\ Recession_t + \beta_7 Sovereign\ Debt\ Crisis_t + \beta_8 Time\ Control_t$
SQ 2:	$r_t^* = \beta_0 + \beta_1 Savings_t + \beta_2 LnGlobalization_t + \beta_3 LnPotential\ Output\ Growth_t + \beta_4 Time\ Control_t$ *Model 1: t = from Q2-1995 up to Q4-2007, Model 2: t = from Q1-2008 up to Q4-2019*
SQ 3:	$r_t^* = \beta_0 + \beta_1 Savings_t + \beta_2 LnGlobalization_t + \beta_3 LnPotential\ Output\ Growth_t + \beta_4 Time\ Control_t$

Time-series OLS regressions rely on a few fundamental assumptions that must be accounted for to define the OLS regression as consistent and unbiased (Poole & O'Farrell, 1971). The assumptions are (1) linear parameters, (2) no perfect (multi)collinearity, (3) zero conditional mean, (4) homoskedasticity, and (5) no serial correlation. We include a further assumption (6) normality of residuals to validate the use of t statistics and F statistics. Assumption (1) is easily met by specifying all variables linearly. Assumption (3) is non-testable but also easily verifiable: this assumption requires the error term (at time t) to be uncorrelated with any explanatory variable, and the condition must hold for every period $(E(u_t|X) = 0, t = 1,2, \dots, n)$. The explanatory variables are then defined as "strictly exogenous". Two leading causes to fail this condition are omitted variables and measurement error in the regressors. Given the in-depth theoretical analysis that has preceded the variables selection, it can be assumed that no relevant variable is omitted from the model and that adequate specification is adopted for each of the explanatory variables.

Assumption (2) of no perfect (multi)collinearity imposes a limit on the correlation among the variables of the regression model, as this can be source of inaccurate or incorrect coefficient estimates (Poole & O'Farrell, 1971). This assumption is tested through a correlation matrix (shown in appendix 8.2). The matrix shows no perfect (multi)collinearity among the explanatory variables; however, near multicollinearity is noticeable between OAD and the time control variable (corr. = 0.9941). As opposed to the findings of Rachel & Smith (2017) Hong & Shell (2019), the coefficient of OAD shows a positive correlation with r*, indicating that the aging population is placing upward pressure on the natural interest rate. The unexpected result appears to be due to the high correlation between the control variable and OAD, invalidating the estimation and interpretation of the coefficient. The availability of quarterly demographic data on the Euro Area is very limited, and neither Eurostat nor The Worldbank seem to provide alternative quarterly proxies. The non-stationarity nature of most of the explanatory variables requires the time control to be included in the model; therefore, the near multicollinearity problem could only be solved by excluding the old-age-dependency ratio from the model.

Assumption (4) requires the variance of the error term to be the same for all time periods, independently of X $(Var(u_t|X) = Var(u_t) = \sigma^2, t = 1,2, \dots, n)$ (Poole & O'Farrell, 1971). The

Breusch-Pagan test is used for this purpose. The test, disclosed in appendix 8.5, shows evidence that the error term has a constant variance, thus, indicating no sign of heteroskedasticity.

Assumption (5) specifies that error terms in two different periods cannot be correlated ($Corr(u_t, u_s)|X) = 0, for\ all\ t \neq s$), if this assumption does not hold, the error terms are said to be autocorrelated across time (Poole & O'Farrell, 1971). The results of the Breusch-Godfrey test shows no sign of autocorrelation, up to 4 periods of lag of the error term (see appendix 8.5). The test is conducted up to four lags due to the quarterly nature of the data.

Finally, assumption (6) requires the residuals to be independent of X, and to be independently and identically distributed as Normal $(0, \sigma^2)$ (Poole & O'Farrell, 1971). This assumption was tested and confirmed by plotting the predicted residuals of the regression model on a histogram (see appendix 8.3).

As previously mentioned, it is also essential to re-estimate R-squared for testing HP.1 by *"detrending"* it. The non-stationary nature of the model variables requires adjusting R^2 by excluding the spurious correlation that arises from trending variables (Brooks, 2014). The detrending process requires regressing y_t (the natural interest rate) on t ($y_t = t$), and finally regress all the explanatory variables and t on the estimated residuals of the first regression ($\ddot{y}_t = X_{t1}, X_{t2} \dots X_{tn}$). The adjusted R^2 that is derived from the computation is then in the form of $R^2 = 1 - \frac{SSR}{\sum_{t=1}^{n} \ddot{y}_t^2}$, as opposed to $R^2 = 1 - (\hat{\sigma}_u^2 / \hat{\sigma}_y^2)$, netting out any effect caused by trending variables (Wooldridge, 2015).

Stress-testing

The banking and the financial systems are not new to stress-testing, and its application has become widely known and recognized over the last two decades. The process entails the simulation of "shock scenarios" on different variables to showcase the impact that these would have on the studied parameter (the natural interest rate in this analysis). In this analysis, three case scenarios, namely a (1) severely adverse, (2) adverse, and (3) positive/stable scenarios are covered.

To conduct a reliable and representative test, Bunn et al. (2005) propose a sequence of steps: (1) defining the shock, (2) determining the size of the impact on the selected (macro)variables, (3) applying the simulated scenario to the model (the paper suggests few other steps that relate to the analysis of portfolios; these steps are ignored in this paper due to the lack of relevance for the analysis of the natural interest rate).

The first critical step of the analysis is to define shocks that represent extreme - but still plausible - events. The test is set to show the impact of a severe event, but this should still be reasonable and within the possibilities of the dynamics of the system. Drehmann et al. (2009) made a similar statement, asserting that given the minimal probability that very extreme scenarios will present themselves, they should not be accounted for in the analysis. Another way to simulate shocks, as proposed by Bunn et al. (2005), is to make use of historical events as a benchmark for replicating similar future scenarios, suggesting that extreme events could be replicated by looking at the end tails of the distribution of changes in the parameters. This approach is strongly aligned with the procedure of Value at Risk (or "VaR") analyses. The VaR is a quantile-based methodology to evaluate the loss that could arise from an extreme case scenario, where the end-tail of the probability distribution is used to quantify the loss. By making use of the probability distribution, it is possible to derive the maximum variance with a certain % of probability, assuming that this will not change in the future.

Kupiec (2002) explains the process of stress testing within a VaR framework, asserting that historical volatility and data correlation are often utilized for this assessment; nevertheless, it is possible to assume levels of volatility that are outside of the historical values. This procedure might have little to no statistical evidence supporting the adjustments but it still provides an overview of unprecedented events that might occur in the future, therefore, should not be excluded entirely from the analysis. This statement is also aligned with the one of Drehmann et al. (2009), suggesting that the shock size should be about two to three standard deviations away from the mean.

The scenarios were simulated as it follows:

(1) Severely adverse: this scenario simulates an unprecedented event that will have a more significant impact than the Great Recession. Given the unparalleled volatility that is expected to be found under such a scenario, and following the points raised by Drehmann et al. (2009), the shock caused by the event was estimated by adding (or subtracting) 5 standard deviations from the mean. Using three standard deviations, as suggested by Drehmann et al. (2009), provides similar variances to those experienced by the Euro Area during the crisis of 2008/2009; thus, five standard deviations are used in this paper for the unprecedented (severely adverse) scenario.

(2) Adverse scenario: this simulation replicates the highest variances experienced by the Euro Area over the last two decades. The shock is estimated by taking the 1st percentile of the distribution as suggested by Bunn et al. (2005).

(3) Stable/Positive scenario: this case scenario aims at studying the natural interest rate under a favorable economic phase. For this reason, the VaR methodology is applied to find the 90th percentile, replicating periods of strong economic recovery and expansion.

The entire disclosure of the relevant percentiles, median, and mean of the quarterly changes in the analyzed variables are provided in appendix 8.6.

5. Results and Interpretation

5.1 Real drivers of r*

Here the regression model is discussed, assessing its feasibility for explaining the movements of r* for the Euro Area, covering the period from 1995 to mid-end 2019. The analysis covers the significance of the variables within the model and the alignment of the results with the findings of the papers reported in the "Literature Review" section. The interpretation of the coefficients is written only for the model specified with the variables that resulted being statistically significant from the first model specification. In this section (under "Model 2"), the hypotheses HP1 and HP2 are tested in this section under Model 2.

Model 1: All variables

$$r_t^* = \beta_0 + \beta_1 Productivity_t + \beta_2 Savings_t + \beta_3 Ln\ Globalization_t + \beta_4 Ln\ Potential\ Output\ Growth_t + \beta_5 Risk\ Aversion_t + \beta_6 Great\ Recession_t + \beta_7 Sovereign\ Debt\ Crisis_t + \beta_8 Time\ Control_t$$

Dependent variable: Natural interest rate (estimated with the HLW model)	
Productivity	-0.0000269 (0.0005)
Savings	0.001386 (0.0005)***
lnGlob	0.0276908 (0.0057)***
lnOutput	0.048864 (0.0044)***
RA	-0.0024567 (0.0013)*
GR	-0.0026444 (0.0016)
DC	0.0000286 (0.0011)
Time	-0.0007302 (0.0000)***
_cons	-0.5683716 (0.0609)
Adj r2	0.9411
Observations	99

Notes

**p<.10 ; **p<.05 ; ***p<0.01*
Standard errors reported in the parenthesis next to coefficients
Data frequency: quarterly

Variables
Productivity: Quarterly change in labour productivity per person (Eurostat Database).
Savings: Gross household saving rate (Eurostat Database).
lnGlob: Natural logarithm of globalization, proxied as [(imports + exports) / GDP] (Eurostat Database).
lnOutput: Natural logarithm of potential output growth, proxied as real GDP + output gap (OG estimated through HLW model).
RA: Risk aversion, proxied as the spread between 6-months and 1-month EURIBOR (Statistical Data Warehouse).
GR: Dummy variable for Great Recession.
DC: Dummy variable for Sovereign Debt Crisis.
Time: Control for trending variables
_cons: Intercept of the regression model

Table 2. Regression results, all variables.

The results show a substantial significance for most of the explanatory variables used in the regression model. The table in appendix 8.7 shows the regression results including the fourth lag for the independent variables, analyzing the 1-year impact of the factors on the natural interest rate. None of the lagged variables result significant, therefore, no lagged independent variables were included in the model specification.

The model's results show an adjusted R-squared value of 0.9411, indicating that the variables included in the model are highly accurate at explaining the movements of r*. In economic terms, savings, globalization, and potential output growth explain about 94% of the variation in the estimated natural interest rate. It has to be remarked that R^2 might result fictitiously high in time-series regressions due to the trending nature of the variables. The "detrended" R^2 is reported for Model 2 (with significant variables only), where the explanatory power of the model is assessed in more detail.

Overall, the theoretical links from the literature review seem to be partially confirmed by the coefficients of the regression results. Households' savings, globalization, and potential output growth are statistically significant, showing a degree of correlation with the natural interest rate independent of the period. The model suggests that savings, globalization, and potential output growth positively correlate with the natural interest rate. This finding appears to confirm the points raised by Obstfeld (Obstfeld, 2020) and the IMF (2019), stating that the shrink in globalization growth and potential output are placing downward pressure on r*. On the other hand, the data used for proxying productivity, risk-aversion (proxied as the spread between 2Y and 10Y bond yields and as the spread between 6-months and 1-month EURIBOR), and the dummy variables representing the two major economic recessions of the Euro Area, result as not significant. Risk-aversion and financial crises were defined as the most impactful drivers of r*. The first one has been described as the most prominent factor in recent times by Brand et al. (2018), and complementary studies concluded that financial crises have deeply scarred the economy through different channels, ultimately affecting the natural interest rate. With this regard, the findings reported in section 5.2 show a different result from the one presented in Table 2., indicating that the financial crises experienced by the Euro Area during the period 2008/2009 did change the underlying dynamics of the drivers of r*.

Model 2: Significant variables only

$$r_t^* = \beta_0 + \beta_1 Savings_t + \beta_2 LnGlobalization_t + \beta_3 LnPotential\ Output\ Growth_t + \beta_4 Time\ Control_t$$

Dependent variable: Natural interest rate (estimated with the HLW model)	
Savings	0.001369 (0.0004661)***
lnGlob	0.030471 (0.0054887)***
lnOutput	0.045806 (0.0046501)***
Time	-0.000728 (0.0000326)***
_cons	-0.523633 (0.0627593)
Adj r2	0.9341
Observations	99

Notes
**p<.10 ; **p<.05 ; ***p<0.01*

Standard errors reported in the parenthesis next to coefficients

Data frequency: quarterly

Variables

Savings: Gross household saving rate (Eurostat Database).

lnGlob: Natural logarithm of globalization, proxied as [(imports + exports) / GDP] (Eurostat Database).

lnOutput: Natural logarithm of potential output growth, proxied as real GDP + output gap (OG estimated through HLW model).

Time: Control for trending variables

_cons: Intercept of the regression model

Table 3. Regression results, significant variables only.

The first hypothesis (HP1) raised in this paper aims at understanding the explanatory power of the model, assessing the strength of the underlying correlation between demographic and macroeconomic variables with the natural interest rate. The adjusted R^2 of this model shows a very high value of 0.93, indicating the high accuracy of the selected variables in explaining the variance of r*. However, following the discussion in section 4.1, the adjusted R-squared of this regression model needs to be re-adjusted before using it as a tool to define the explanatory power of the model. The process and results of the detrending procedure are shown in appendix 8.8. The results suggest a re-estimated R^2 of about 0.72; as expected, this value is lower than the adjusted R^2 obtained from the original regression, but it still shows a relatively high explanatory power, confirming that most of the variance of the natural interest rate is explained by movements in the level of individuals' savings, in the level of globalization, and the potential output growth.

Teulings & Lu (2016) and Rachel & Smith (2017) showed that the aging population and longer life expectancy are placing upward pressure on the level of households' savings, and this appeared to be strongly correlated with the fall of the natural interest rate. Most of the OLG models developed within previous papers came to the same conclusion (Papetti, 2019), but also showing that so-called "dissavers" place an upward pressure on the natural interest rate, spending more than what they are earning. However, the regression results reported here show a positive correlation between savings and r*, suggesting that an increase by 1% of households' savings (as a percentage of disposable income) would increase the natural interest rate by 0.0014 percentage points approximately. This result clearly disagrees with the generally accepted inversed relationship between natural interest rate and savings, indicating a possible (omitted variable) bias of the model. For robustness check, the model was re-run including the variable OAD, but the results show no effect on the sign of the coefficient (as visible from appendix 8.9). It is not clear whether the model is suffering from omitted variable bias or whether it is showing an actual positive correlation between savings and r*; however, the value of the coefficient indicates a very limited impact on r*, especially when this estimate is compared to the size of the effect of globalization and potential output growth (as showcased by their high coefficients).

The regression results are aligned with the paper of Hledil and Vlcek (2018) with regard to the connection between potential output growth and r*. The coefficient shows that an increase in potential output growth places upward pressure on the natural interest rate, confirming the hypothesis that the decrease in potential output growth experienced by the Euro Area over the last two decades is a determinant driver for r*. The model suggests that an increase in output growth of 1% would raise the natural interest rate by 0.04 percentage points, ceteris paribus. The sign of the coefficient is also aligned with the underlying dynamic between interest rates and productivity growth, according to which an increase in potential productivity growth will require higher interest rates for the economy to stay at equilibrium.

The results are partially aligned also with the theories of Woodford (2003) and Gali (2008), which highlighted a correlation between AD shocks and r*. The change in potential output growth can be associated to AD shocks; however, adverse AD shocks, proxied by the Great Recession and the European Sovereign Debt Crisis result not significant within this regression model. The model takes into account the direct impact of the explanatory variables on the movements of r*. Still, it fails to capture long-term structural (economic) changes that arise from financial crises. Guido (2014) suggested that financial crises tend to require a longer period of recovery for the economies to go back to their initial state compared to other crises. Furthermore, the level of foreign credit to domestic firms dropped significantly during these periods, impacting the overall investment level and productivity growth (Arteta & Hale, 2008). The long-term impact that the Great Recession and the Sovereign Debt Crisis had on output growth and on the natural interest rate appear to go beyond the statistical capabilities of this model, explaining why the findings seem to disagree with previous research.

The coefficient for the variable "globalization" is coherent with the findings of previous analyses. Obstfeld (2020) explained that globalization is cause of shifts in savings and international investments, strongly influencing the domestic interest rate. The results of the regression model suggest that a growth in globalization by 1% (proxied as the sum of exports and imports, divided by GDP) determines an increase in the natural interest rate by 0.03 percentage points, ceteris paribus. The value and sign of the coefficient confirm the correlation proposed by the IMF (2019), which showed that the natural interest rate and globalization growth have gone "hand-in-hand", peaking around 1970s, to then gradually falling, up to the latest data available today.

Finally, the regression model was used to test the second hypothesis (HP2) of this paper. The IMF (2019) found that productivity is not a real driver of r*, and that this variable loses its statistical significance when a proxy for the level of globalization is included in the regression model. The regression models and results used for testing this assumption are reported in appendix 8.10. The two regression models (one including "globalization" and excluding "globalization") show that productivity is statistically insignificant in both model specifications. Excluding the variable for globalization from the model, productivity reaches a slightly higher t-value (t-value: -0.46, against t-value: -0.05), showing that the variable proxying globalization captures part of the explanatory power of productivity, thus, partially aligning with the finding of the IMF.

5.2 The evolution of the drivers over the last two decades

To answer sub-question (2), the regression model (including only significant variables) was re-run on the same data, having the database split between two time periods: from Q2 1995 up to Q4 2007, and from Q1 2008 up to Q4 2019. These periods were selected because they mark two distinctive phases for the economy of the Euro Area, prior to and after the Great Recession and the Sovereign Debt Crisis, thus, exposing any change that these might have imposed on the drivers of r*. In this section, HP3 and HP4 are tested.

Dependent variable: Natural interest rate (estimated with the HLW model)		
	1995-2007	**2008-2019**
Productivity	-0.000718 (0.0008922)	0.000961 (0.0005077)
Savings	0.001127 (0.0005995)	0.002032 (0.0008302)
lnGlob	0.026838 (0.0061981)	0.043322 (0.0100357)
lnOutput	0.033955 (0.0094905)	0.066522 (0.0079643)
RA	-0.001402 (0.0018356)	0.001347 (0.0024577)
Time	-0.00058 (0.0001079)	-0.000842 (0.0000621)
_cons	-0.375384 (0.1209456)	-0.811693 (0.1077466)
Adj r2	0.4258	0.8581
Observations	51	48

Notes

**p<.10 ; **p<.05 ; ***p<0.01*
Standard errors reported in the parenthesis next to coefficients
Data frequency: quarterly

Variables

Savings: Gross household saving rate (Eurostat Database).
lnGlob: Natural logarithm of globalization, proxied as [(imports + exports) / GDP] (Eurostat Database).
lnOutput: Natural logarithm of potential output growth, proxied as real GDP + output gap (OG estimated through HLW model).
Time: Control for trending variables
_cons: Intercept of the regression model

Table 4. Regression results, for the periods 1995-2007 and 2008-2019.

The coefficients of the regressions reported in Table 4. show some interesting results. Apart from a drop in significance of savings for the first period, all the variables that result significant for the period 1995-2019 are also statistically significant when the period is split, as illustrated in Table 4.

The results reported in the regression model in Table 3. are robust enough to support changes in time, showing valid correlations between the variables included in the model and the natural interest rates, independent of the period analyzed. However, there is a noticeable difference in the explanatory power, as indicated by R^2. This finding helps validate HP3, which aims at assessing whether the financial crises have determined a change in the underlying dynamics of the natural interest rate. The adjusted R^2 measures for the two time periods reveal a much different explanatory power of the independent variables in the models: the adjusted R-squared for the period 2008-2019 is about double the measurement reported for the time period 1995-2007. Furthermore, the variables lnOutput and Time gain a greater explanatory power than in the first period, while little to no change is noticeable from globalization. On the other hand, the coefficients of all the explanatory variables show a greater degree of impact on r* for the second period, indicating a much stronger correlation compared to the years ranging from 1995 to 2007.

As stated in the previous section, the results of the regression model conducted over the whole period are not capable of identifying the impact of the financial crises on r*; however, the difference in explanatory power and the effect of each of the explanatory variables (illustrated by the coefficients in the Table 4.), show that the two financial crises did affect the structural dynamics that underlies the economy and the movements of r*. Given the accurate division of the periods around the two financial crises, it is possible to attribute (at least partially) the change in explanatory power to the Great Recession and the Sovereign Debt Crisis, supporting the conclusions of Christiano et al. (2015), Acharya et al. (2015), and Arteta & Hale (2008).

The variables proxying productivity and risk aversion result as not significant, showing a poor degree of correlation with r* for both time periods analyzed. This finding agrees with the findings of Table 3., which shows a statistically insignificant effect of these variables for the period ranging from 1995 to 2019; on the other hand this result is partially misaligned with the analysis of Brand et al. (2018) and contradicting HP4. Brand et al. found that risk aversion has played a much more relevant role since the 1980s compared to the drop in productivity. The results of the regression model reported in Table 4. find that neither productivity nor risk aversion were real drivers of r* for the last two decades, instead, the variable proxying savings gained significance in the last decade, contradicting the insignificant result reported for the previous decade. Oppositely from the previous point, this conclusion was also reached by Brand et al. (2018), showing that the older cohort that is driving up the level of savings is becoming much more impactful compared to the level of risk aversion itself, explained by the introduction of the contraceptive pill as showcased by Teulings & Lu (2016).

The variable proxying globalization appears to be strongly significant for every time period analyzed, both when the period is split and when analyzed altogether. The findings show that its impact has almost doubled over the last decade, indicating that an increase in globalization by 1% would raise r* by 0.04%, ceteris paribus. The result aligns with the conclusion of Borio et al. (2019), finding that globalization is one of the two variables (along with cost of capital) that is correlated with the long-term real interest rate of the Euro Area both in the period pre-WWII and post-WWII. Borio et al. (2019) also found that savings are significant only during the post-WWII period, indicating a coincidental relationship; however, the results of the regression models reported here stand for the significance of such variable, with an even more significant impact reported for the period 2008-2019. Given the robustness of the results for the periods analyzed, this variable was included in the final specification of the model.

5.3 Stress-test scenarios

It now follows the analysis of the path of the natural interest rate of the Euro Area under three different simulated scenarios: (1) severely adverse, (2) adverse, (3) positive/stable scenario.

It is interesting to remark that the first percentile for each variable belongs to either Q4 2008 or Q1 2009, right at the outbreak of the Great Recession in the Euro Area. This was among the most impactful events the Euro Area experienced over the last two decades, confirming its fit as a benchmark for the adverse case scenario. Furthermore, Drehmann et al. (2009) suggested that a severely adverse scenario would be represented by going three standard deviations away from the mean. The calculations, however, revealed that three standard deviations were not enough to go beyond the historical variance experienced during the Great Recession, thus, requiring to account for five standard deviations to simulate an unprecedented event. This scenario simulates a severe global recession, with a severe drop in globalization, potential output growth, and a rise in savings levels. Similarly to the definition of severely adverse scenario of the FED, this scenario represents an economic downturn prolonged by remote work, leading to a drop in commercial real estate, and negatively affecting corporates and investors' sentiment (2022).

The gradual shift to economic "normality" (either positive in case of adverse scenario, and negative for the case of positive scenario) is simulated following the trend of GDP growth as disclosed by the FED (FED 2022 Stress Test); similar percentage changes calculated from the GDP growth are applied to the explanatory variables to derive the gradual economic recovery or stability over a time span of three years (full disclosure of the parameters is reported in appendix 8.11).

The chart in Figure 3. provides an overview of the movements of r* estimated under the three simulated scenarios.

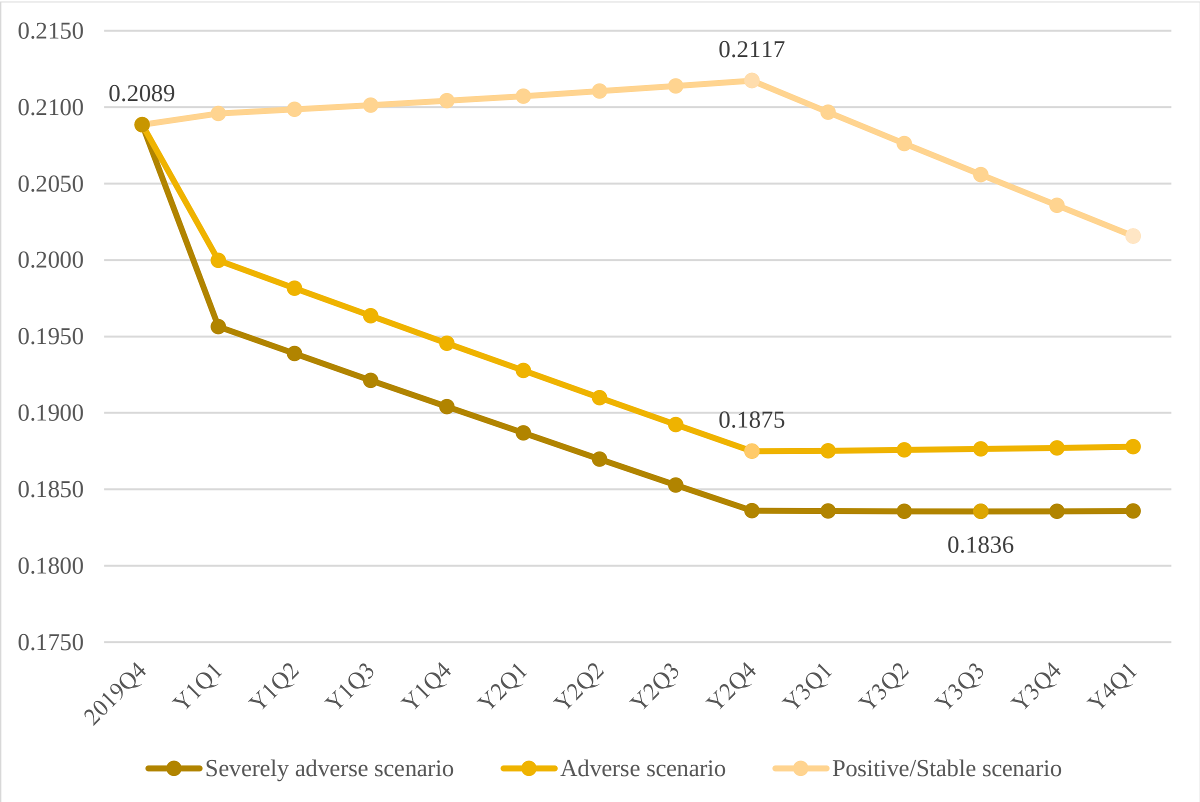

Figure 3. Overview of the three stress-test scenarios.

According to the model, the re-estimated r* for the latest quarter of data available (Q4 2019) lies at about 0.21%, against a natural interest rate of 0.47% (estimated through the HLW model). The R^2 measure reported section 5.1 provided evidence of the good fit of the model with the data, however, the re-estimated r* seem to differ from the r* used to develop the model. It is important to remark that, due to multicollinearity and in the absence of alternative proxies, the variable representing old-age-dependency ratio was excluded from the model. The coefficient of the variable OAD showed a positive sign (see appendix 8.7), contradicting the inverted correlation that should underly the two variables (and suggesting a plausible omitted variable bias). However, if OAD was included in the model, the re-estimated r* would have resulted in a higher estimation, aligning with the r* estimated through the HLW model. Because of the statistical limitations exposed in the section 4.2, the model was not modified further; however, the model might suffer from a negative bias, which will be accounted for in the conclusion and final considerations.

As visible from graph Figure 3., the three paths of r* estimated under the three scenarios are not far away from each other. Specifically, even when accounting for five standard deviations away from the mean (as done for the severely adverse scenario), the natural interest rate drops slightly lower than the estimated r* that would derive from the worst scenarios experienced in the past (represented by the adverse scenario). Conversely, under the positive/stable scenario the natural interest rate still struggles to move away from the zero-lower bound. The model does not account for the structural economic changes that arise from the variance in the variables included in the model, explaining why the natural interest rates seem to converge by the end of the period analysed. The main finding of this stress testing lies in the (relatively) little impact that the variables seem to have on the path of the natural interest rate. Even though the high R^2, the estimated r* under the three scenarios suggest that severe changes (either positive or negative) in savings, potential output growth, and globalization are not enough to neither depress the natural interest rate below zero, nor to move it far away from the zero-lower bound.

The IMF affirmed that a significant change in globalization would be required to reverse the trend of r* (2019). The results of the stress-test partially confirm this statement; however, the enhanced globalization, along with a lower level of savings and higher potential output, do not have the required impact that is necessary to move the natural interest rate away from the zero lower bound.

It is worth noting that there is another group of literature that does not entirely agree with the papers presented in support of this research. An increasing number of researchers are raising doubts on whether the current way of estimating r* is correct, as well as debating whether or not r* should be used for monitoring the macroeconomy. The findings of a research conducted by the World Bank Group in 2021 show that only the relative price of capital seems to be robust to the inclusion of time-fixed effects, suggesting that other influencing factors might be the shift to inflation targeting and the credibility gained during the transition (Ruch, 2021). The Bank for International Settlements (BIS) disclosed a similar analysis, inferring that the current underlying forces that explain the trend of r* are incomplete and that a greater focus should be placed on monetary policies and their impact to the financial cycle (Borio, Disyatat, & Rungcharoenkitkul, 2019).

6. Conclusion and further research

The research aimed at discovering the main drivers of r*, how these have developed over the last two decades, and how their impact would shift the trend of r* under three different scenarios used for stress-testing applying the VaR framework.

Among the variables proposed by the literature review (for reference, see Bomfim (1997), Laubach and Williams (2003), Congressional Budget Office (2014), International Monetary Fund (2014), Council of Economic Advisers (2015), Hamilton et al. (2015), and Pescatori and Turunen (2015)), only savings, globalization and potential output growth appear to be the real drivers of the natural interest rate for the Euro Area over the last two decades, aligning with the researches of Obstfeld (2020), Hledil and Vlcek (2018), Woodford (2003), Gali (2008) and (Papetti, 2019). The resulting model suggests that these variables together explain about 72% of the variance of r*, indicating a high explanatory power for the selected variables; nevertheless, the positive coefficient of the variable proxying savings raised some doubts about the unbiasedness of the model, suggesting an apparent omitted variable bias. Finally, the model revealed that globalization takes part of the explanatory power of productivity, which becomes statistically insignificant when a proxy for globalization is included in the model, aligning with the findings of IMF (2019).

A remarkable finding of this paper is that the impact of all has considerably changed after the period 2008-2009. All the variables that result statistically significant for the whole period (1995-2019) also appear to be significant for the periods 1995-2007 and 2008-2019, but with noticeable differences in their effect on the natural interest rate. The R^2 measure reported for the period 2008-2019 is about double the one measured for the period 1995-2007. The finding seems to suggest that the Great Recession and the European Sovereign Debt Crisis did have a strong structural impact on the drivers of r*, which the model failed to capture in the first analysis reported in this paper, and aligning with the conclusions reached by Arteta & Hale (2008) and Achraya et. Al (2015).

Finally, the stress-tests showed how the variables included in the model have a relatively little impact on r*, with peaks and dips of the natural interest rate at about 0.21%, 0.19%, and 0.18% for the positive/stable, adverse, and severely adverse case scenarios. The analysis reveals that, even though the high explanatory power of the model, the trend of r* will not turn around effortlessly, even under a favorable future scenario where savings decrease, globalization enhances, and potential output growth starts rising again.

Concluding, the points raised in the research and the rigidity showed by r* under different scenarios suggest the following conclusions: (1) the zero-lower bound is here to stay, and (2) the list of drivers of r* might be incomplete. As suggested by Borio et al. (2019), researches over the natural interest rate often focus on demographic and macroeconomic factors, and frequently reach different estimation results. Opposively, greater attention should be given to the impact of inflationary and monetary policies on the financial cycle, requiring a more complicated model from the one derived in this research.

The conclusions of this paper are of highly relevance for the policymakers of the Euro Area and for fixed income investors. Firstly, given the rigidity of the natural interest rate to the factors presented in this paper, the ECB and the complementary governing parties should keep on developing alternative monetary policy tools that could substitute the use of interest rates as inflation regulator. The NIRP has been defined as effective for the short-run, but doubts are still pending on the impact of such policy for the long run economic equilibrium. Finally, the forecasted low natural interest rate expected in the future will keep on depressing the yields of fixed income assets, depreciating this asset class at the expense of overpricing other financial assets.

Even though all the major statistical considerations were followed to ensure the validity and replicability of the study, the paper presented some limitations that could be addressed in further research. The limitations are the followings: (1) old-age-dependency ratio was excluded from the model due to near multicollinearity with the time control (and in the absence of valid alternative quarterly proxies); further research could investigate alternative ways to account for trending variables without compromising the components of the model.; (2) the model failed to capture the structural changes derived from the Great Recession of 2008/2009 in the first instance. This linear-regression model has a very simplistic form and might fail to find appropriate correlations; further research could specify the financial crisis in the model in a different way so as to account for the structural changes of the underlying dynamics that derive from it; (3) the linearity of the model might be another limit to its application; small shocks can be captured by linear relationships; however, extreme shocks might require model non-linearity, suggesting a more complex model specification to capture the impact of highly impacting AD shocks.

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8. Appendices

Appendix 8.1

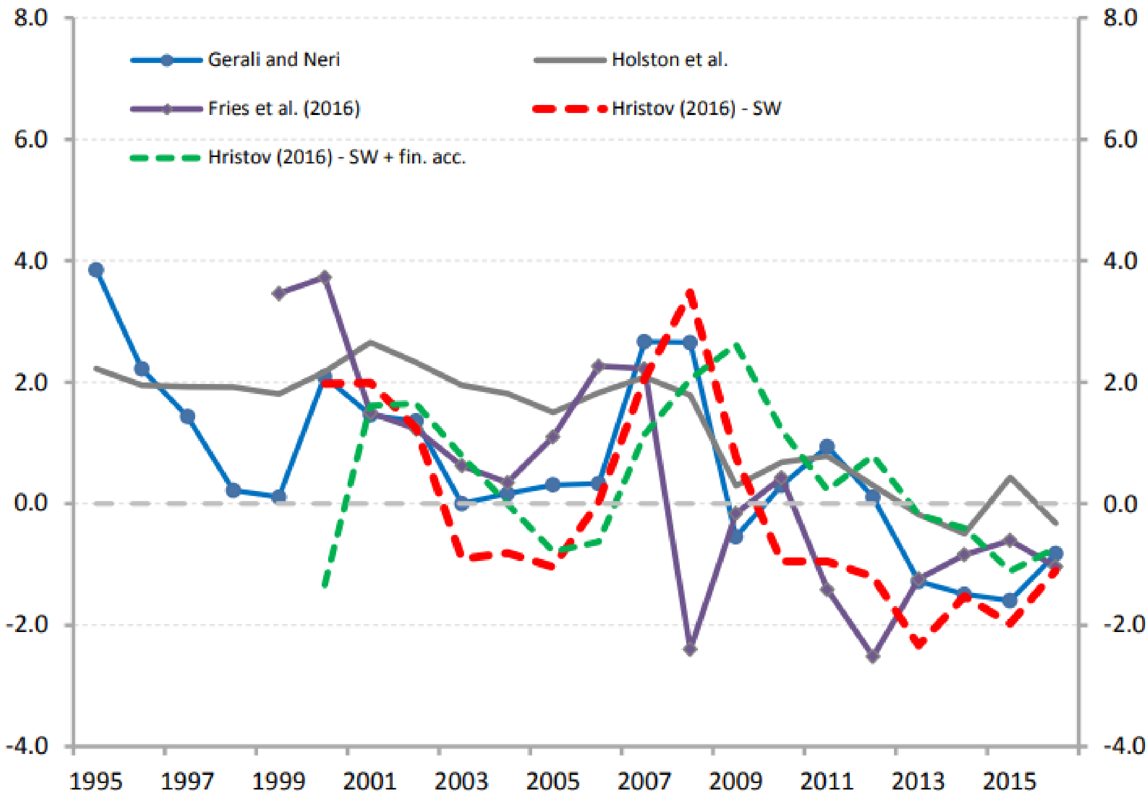

Figure 4. The natural interest rate estimated through different estimation models.

Appendix 8.2

	R	Produc~y	Savings	OAD	RA	GR	DC	qdate	lnGlob	lnGDP
R	1									
Productivity	0.1055	1								
Savings	0.5038	-0.0084	1							
OAD	-0.8779	-0.1254	-0.5355	1						
RA	-0.2911	-0.0289	-0.272	0.2398	1					
GR	-0.1067	-0.4829	0.2634	0.0049	0.2533	1				
DC	-0.1412	0.261	0.2034	0.0458	0.4477	-0.0449	1			
qdate	-0.8726	-0.1359	-0.5286	0.9941	0.3101	0.0371	0.0933	1		
lnGlob	-0.7528	-0.1285	-0.609	0.9441	0.336	-0.055	0.007	0.956	1	
lnGDP	-0.7602	-0.1544	-0.4906	0.9572	0.3327	0.052	0.0808	0.9738	0.9592	1

Table 5. Correlation matrix of selected variables – Assumption (2).

Appendix 8.3

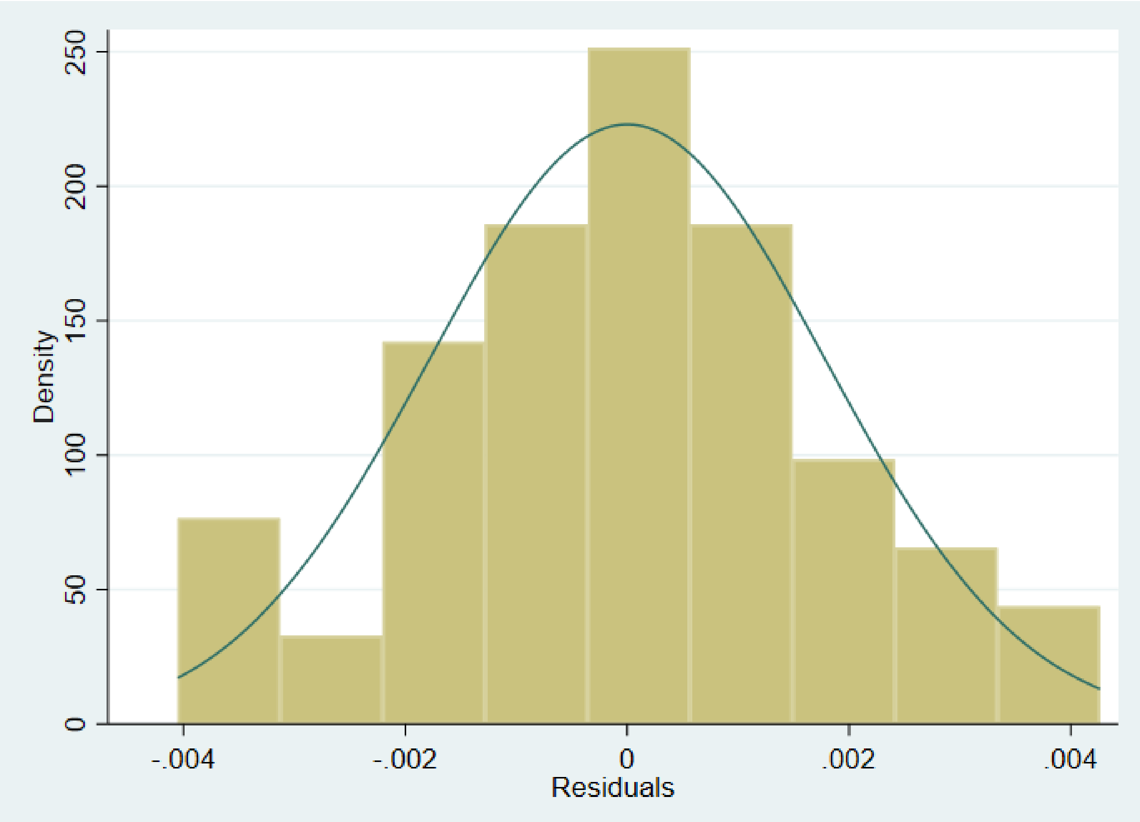

Figure 5. Histogram of regression residuals – Assumption (6).

Appendix 8.4

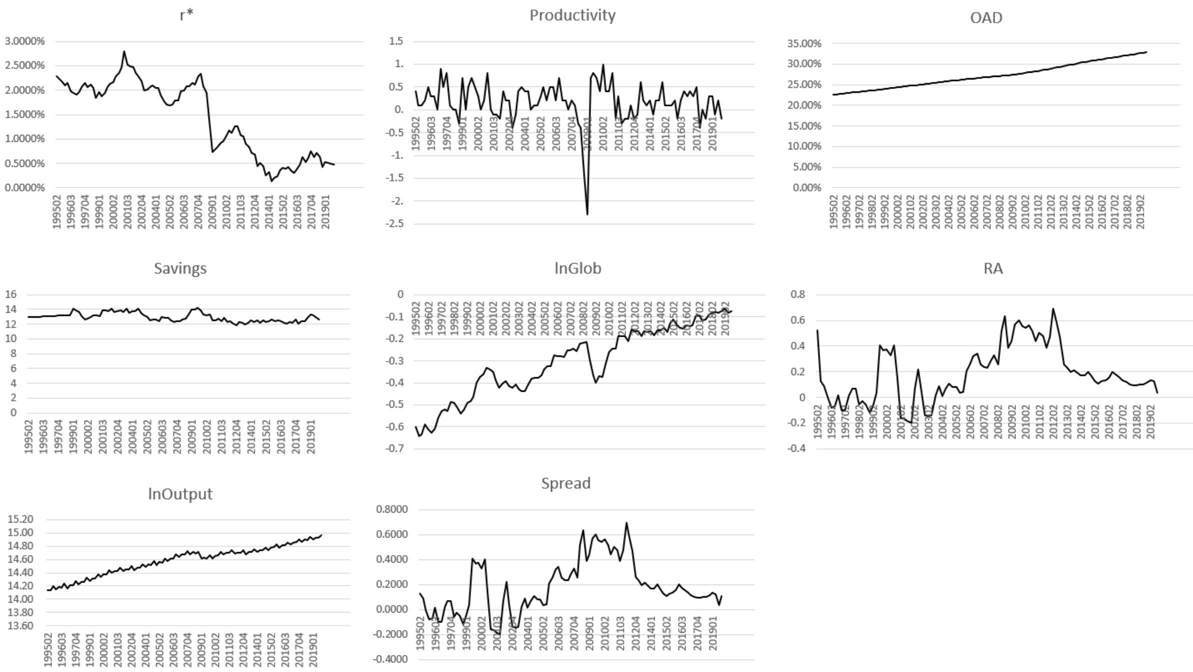

Figure 6. Trend lines of the selected variables.

Appendix 8.5

Breusch-Godfrey LM test for autocorrelation (1 lag)			
lags(p)	chi2	df	Prob > chi2
1	28.412	1	0
H0: no serial correlation			
Breusch-Godfrey LM test for autocorrelation (4 lags)			
lags(p)	chi2	df	Prob > chi2
4	48.882	4	0
H0: no serial correlation			
Breusch-Pagan / Cook-Weisberg test for heteroskedasticity			
Ho: Constant variance			
Variables: fitted values of R			
chi2(1) = 1.20			
Prob > chi2 = 0.2742			

Table 6. Breusch-Godfrey & Breusch-Pagan tests, Assumption (4) & Assumption (5)

Appendix 8.6

Percentile	**Productivity**	**Savings**	**Globalization**	**OAD**	**RA**	**Potential Output growth**	**Spread**
1st percentile	-1.0	0.797	-0.0667	0.0015	0.3702	-232,110	0.3702
10th percentile	-0.6	0.43	-0.0191	0.0015	0.1202	-112,556	0.1202
Median	0.0	0.0238	0.0019	0.001	-0.0001	13,162	0.0007
75th percentile	0.2	-0.19	0.016	0.0009	-0.0409	92,385	-0.0403
90th percentile	0.5	-0.47	0.0284	0.0008	-0.1161	115,518	-0.1133
Mean	-0.0061	-0.0028	0.0039	0.0011	-0.0049	18,327	-0.0002
Std. dev.	0.5011	0.3124	0.0186	0.0002	0.1081	85,752	0.1008

Table 7. Main percentiles, median, mean and standard dev.

Appendix 8.7

Dependent variable: Natural interest rate (estimated with the HLW model)	RA lagged	lnGDP lagged	lnGlob lagged	Savings lagged	Productivity lagged
Productivity					
L0	0.00019 (0.0005)	0.00011 (0.0005)	0.00024 (0.0005)	0.00017 (0.0005)	0.00019 (0.0005)
L4					-0.00006 (0.0004)
Savings					
L0	0.00166 (0.0005)***	0.00179 (0.0005)***	0.00157 (0.0005)***	0.00158 (0.0004)***	0.00164 (0.0004)***
L4				0.00031 (0.0004	
lnGlob					
L0	0.03622 (0.0060)***	0.03328 (0.0065)***	0.03512 (0.0063)***	0.03634 (0.0060)***	0.03642 (0.0062)***
L4			0.00261 (0.0049)		
lnOutput					
L0	0.05105 (0.0043)***	0.06240 (0.0108)***	0.05082 (0.0043)***	0.05036 (0.0043)***	0.05108 (0.0043)***
L4		-0.01244 (0.0109)			
RA					
L0	-0.00424 (0.0014)***	-0.00363 (0.0014)***	-0.00419 (0.0013)***	-0.00428 (0.0013)***	-0.00427 (0.0014)***
L4	0.00005 (0.0011)				
GR	-0.00126 (0.0015)	-0.00068 (0.0014)	-0.00137 (0.0015)	-0.00105 (0.0016)	-0.00124 (0.0015)
DC	0.00100 (0.0012)	0.00099 (0.0016)	0.00115 (0.0011)	0.00079 (0.0011)	0.00102 (0.0011)
Time	-0.00077 (0.0000)***	-0.00075 (0.0011)***	-0.00078 (0.0000)***	-0.00076 (0.0000)***	-0.00077 (0.0000)***
_cons	-0.59257 (0.0588)	-0.58469 (0.0000)	-0.58598 (0.0597)	-0.58706 (0.0589)	-0.59234 (0.0586)
Adj. R^2	0.9471	0.9479	0.9473	0.9474	0.9471
Observations	95	95	95	95	95

Notes
**p<.10 ; **p<.05 ; ***p<0.01*
Standard errors reported in the parenthesis next to coefficients
L0: no lags
L4: 4 lags

Variables
Data frequency: quarterly

Productivity: Quarterly change in labour productivity per person (Eurostat Database).

Savings: Gross household saving rate (Eurostat Database).

lnGlob: Natural logarithm of globalization, proxied as [(imports + exports) / GDP] (Eurostat Database).

lnOutput: Natural logarithm of potential output growth, proxied as real GDP + output gap (OG estimated through HLW model).

RA: Risk aversion, proxied as the spread between 6-months and 1-month EURIBOR (Statistical Data Warehouse).

GR: Dummy variable for Great Recession.

DC: Dummy variable for Sovereign Debt Crisis.

Time: Control for trending variables

_cons: Intercept of the regression model

Table 8. Models including lagged variables, regression results.

Appendix 8.8

Regression 1	
Dependent variable: Natural interest rate	
Time	-0.0002336 (0.0000)***
_cons	0.0587505 (0.0025)
Regression 2	
Dependent variable: Residuals of Regression 1	
Savings	0.0013691 (0.0004)***
lnGlob	0.0304712 (0.0054)***
lnOutput	0.0458061 (0.0046)***
Time	-0.0004947 (0.0000)***
_cons	-0.5823838 (0.0627)
Adj. R^2	0.7239
Observations	99

Notes

**p<.10 ; **p<.05 ; ***p<0.01*
Standard errors reported in the parenthesis next to coefficients

Variables
Data frequency: quarterly

Savings: Gross household saving rate (Eurostat Database).

lnGlob: Natural logarithm of globalization, proxied as [(imports + exports) / GDP] (Eurostat Database).

lnOutput: Natural logarithm of potential output growth, proxied as real GDP + output gap (OG estimated through HLW model).

Time: Control for trending variables

_cons: Intercept of the regression model

Table 9. Detrending R-squared.

Appendix 8.9

Dependent variable: Natural interest rate (estimated with the HLW model)		
	OAD included	OAD excluded
Savings	0.0015029 (0.0004)***	0.0013691 (0.0004)***
OAD	0.2512846 (0.0642)***	-
lnGlob	0.0314977 (0.0051)***	0.0304712 (0.0054)***
lnGDP	0.0523064 (0.0046)***	0.0458061 (0.0046)***
Time	-0.0010414 (0.0000)***	-0.0007282 (0.0000)***
_cons	-0.6288117 (0.0643)	-0.5236333 (0.0627)
Adj. R^2	0.9428	0.9341
Observations	99	99

Notes

**p<.10 ; **p<.05 ; ***p<0.01*
Standard errors reported in the parenthesis next to coefficients

Variables
Data frequency: quarterly

Savings: Gross household saving rate (Eurostat Database)

OAD: Old-age-dependency ratio, proxied as the ratio between population older than 64 over population between 16 and 64 (The World Bank Database)

lnGlob: Natural logarithm of globalization, proxied as [(imports + exports) / GDP] (Eurostat Database).

lnOutput: Natural logarithm of potential output growth, proxied as real GDP + output gap (OG estimated through HLW model).

Time: Control for trending variables

_cons: Intercept of the regression model

Table 10. Regression results, with/without old-age-dependency ratio.

Appendix 8.10

Dependent variable: Natural interest rate (estimated with the HLW model)		
	Globalization excluded	Globalization included
Productivity	-0.0002688 (0.0005)	-0.0000269 (0.0005)
Savings	0.0008821 (0.0005)	0.001386 (0.0005)***
lnGlob	-	0.0276908 (0.0057)***
lnOutput	0.0593486 (0.0043)***	0.048864 (0.0044)***
RA	-0.0004361 (0.0014)	-0.0024567 (0.0013)**
GR	-0.0057198 (0.0016)***	-0.0026444 (0.0016)
DC	-0.0018313 (0.0012)	0.0000286 (0.0011)
Time	-0.0006704 (0.0000)***	-0.0007302 (0.0000)
_cons	-0.7346723 (0.0557)	-0.5683716 (0.0609)***
Adj. R2	0.9269	0.9411
Observations	98	98

Notes

**p<.10 ; **p<.05 ; ***p<0.01*

Standard errors reported in the parenthesis next to coefficients.

Variables

Data frequency: quarterly

Productivity: Quarterly change in labour productivity per person (Eurostat Database).

Savings: Gross household saving rate (Eurostat Database).

OAD: Old-age-dependency ratio, proxied as the ratio between population older than 64 over population between 16 and 64 (The World Bank Database)

lnGlob: Natural logarithm of globalization, proxied as [(imports + exports) / GDP] (Eurostat Database).

lnOutput: Natural logarithm of potential output growth, proxied as real GDP + output gap (OG estimated through HLW model).

RA: Risk aversion, proxied as the spread between 6-months and 1-month EURIBOR (Statistical Data Warehouse).

GR: Dummy variable for Great Recession.

DC: Dummy variable for Sovereign Debt Crisis.

Time: Control for trending variables

_cons: Intercept of the regression model

Table 10. Regression results, with/without globalization.

Appendix 8.11

	Severely adverse scenario			Adverse scenario			Positive/Stable scenario		
Quarters ahead	**Savings**	**Globalization**	**Potential GDP**	**Savings**	**Globalization**	**Potential GDP**	**Savings**	**Globalization**	**Potential GDP**
Q1	14.22	0.84	2,758,319	13.46	0.86	2,936,643	12.19	0.92	3,284,272
Q2	14.36	0.83	2,730,736	13.59	0.85	2,907,277	12.07	0.93	3,317,115
Q3	14.51	0.82	2,703,428	13.73	0.84	2,878,204	11.95	0.94	3,350,286
Q4	14.65	0.81	2,676,394	13.86	0.84	2,849,422	11.83	0.95	3,383,789
Q5	14.80	0.81	2,649,630	14.00	0.83	2,820,928	11.71	0.96	3,417,627
Q6	14.94	0.80	2,623,134	14.14	0.82	2,792,719	11.59	0.97	3,451,803
Q7	15.09	0.79	2,596,903	14.28	0.81	2,764,792	11.48	0.98	3,486,321
Q8	15.25	0.78	2,570,934	14.43	0.80	2,737,144	11.36	0.99	3,521,184
Q9	15.09	0.79	2,596,643	14.28	0.81	2,764,515	11.48	0.98	3,485,973
Q10	14.94	0.80	2,622,609	14.14	0.82	2,792,160	11.59	0.97	3,451,113
Q11	14.79	0.81	2,648,835	14.00	0.83	2,820,082	11.71	0.96	3,416,602
Q12	14.64	0.81	2,675,324	13.86	0.84	2,848,283	11.82	0.95	3,382,436
Q13	14.50	0.82	2,702,077	13.72	0.84	2,876,765	11.94	0.94	3,348,611

Table 11. Estimated factors under severely adverse - adverse – positive/stable scenario.