Universiteit Utrecht

Graduate School of Natural Sciences

Dissipative Antiferromagnetic Dynamics

Thesis

submitted in partial fulfillment of the
requirements for the degree of

Master of Science
in
Theoretical Physics

Daan van Seters

July 10, 2022

First supervisor Prof. Dr. R. Duine
Second supervisor Prof. Dr. C. Morais Smith
Daily supervisors Dr. T. Ludwig
Dr. H. Y. Yuan

Abstract

No consensus exists on the equations that govern the dissipative dynamics of antiferromagnets. In particular, from phenomenological considerations (H. Y. Yuan, Liu, et al. 2019) one expects inter-sublattice damping terms in the sublattice dynamics. These terms are not present in the Landau-Lifshitz-Gilbert equations that are commonly used to describe individual sublattice dynamics. In this thesis, we present a Caldeira-Leggett approach within the Keldysh path integral formalism to base the dissipative equations for two-sublattice antiferromagnets on a more microscopic description. From this, we argue that the dissipative dynamics are determined by the specific form of system-bath structure through which the system dissipates. We recover phenomenological equations in the quasi-classical limit and find that the presence of inter-sublattice damping terms originate from the thermal coupling between the sublattices. Remarkably, we find that in a special case the Néel order is a dissipation-free mode. Subsequently, we argue that the existence of dissipation-free modes is a more general phenomenon and will occur in systems composed of two subsystems where the bath is shared between the two subsystems.

Acknowledgements

I would like to thank my supervisors Rembert Duine, Tim Ludwig and Huaiyang Yuan for their invaluable guidance, support and contributions to the research presented in this thesis. Furthermore, I would like to thank all other members of the spintronics research group at Utrecht University for their helpful suggestions, questions and discussions. Lastly, thanks for all the support of my friends and family during this year of master research and thesis writing.

Contents

1 Introduction

While the world today thrives on data acquisition and computation power, reduction of energy consumption becomes more vital than ever. Spintronics is a research field focusing on a new territory of (spin-)electronic devices that increase memory and processing capacity, while decreasing demand of power.

Whereas conventional electronic devices utilise the property of electric charge of electrons or other charge carriers, spintronics exploits the spin degree of freedom of these particles through spin-polarised currents (currents in which the spins have a preferred direction). For instance, it is possible to create a magnetic memory from ferromagnets, which are materials in which all spins align, see figure 1a. Labelling a ferromagnet where all spins point in a given 'up'-direction as '1' and a ferromagnet where all spins pointing in the opposite direction as '0', one can create a magnetic binary code. Then, a digit can be changed by sending a spin-polarised current through the ferromagnet, upon which its direction is switched, hence the code is changed. Such a magnetic memory does not suffer from information loss through ionising radiation, as is the case for conventional charge-based memory. However, by perturbation of external magnetic fields, even this ferromagnetic based memory can lose its information.

For antiferromagnets, this problem is much less relevant. Antiferromagnets are materials that have an intrinsic magnetic ordering: neighbouring spins in a lattice energetically favour to arrange themselves in a way that the total magnetisation is vanishingly small, see for a simple example figure 1b. Whereas for ferromagnets the magnetic field of all spins add up to an significant effective total magnetisation, in antiferromagnets the magnetic moment of each lattice cell is cancelled by the opposing spin directions and consequently, the net magnetisation field of an antiferromagnet is zero. On the one hand, it is hard to experimentally manipulate such materials due to their lack of net magnetisation. For this reason, Louis Néel, the originator of antiferromagnetics, famously declared that antiferromagnets are 'interesting but useless' (Néel 1970). On the other hand, this means the material is largely resistant to the influence of external magnetic fields for the same reason.

Therefore it is of no surprise that since the discovery of spin transfer torque and anisotropic magnetoresistance (Xu, Wang, and Xia 2008; Núñez et al. 2006; Wei et al. 2007; Haney and MacDonald 2008), there has been enormous interest in antiferromagnetic spintronics. Continuing on the above example of a magnetic memory, it is possible to switch the antiferromagnetic ordering by sending a current through the antiferro-

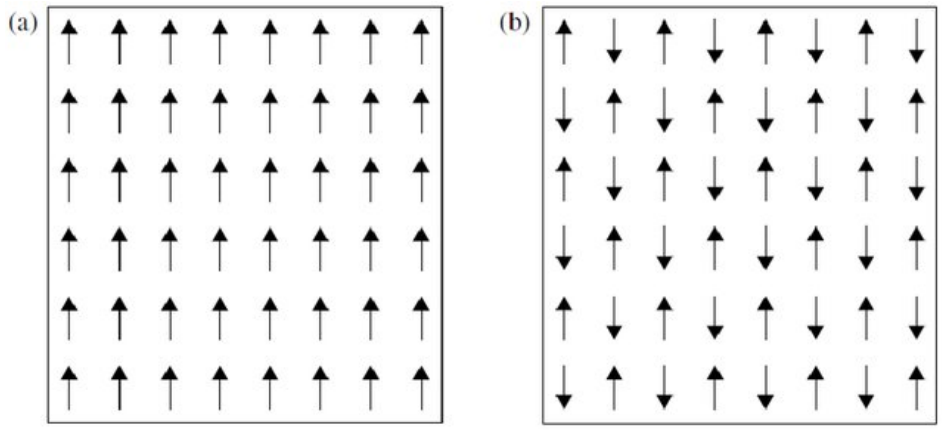

Figure 1: Simple two-dimensional examples of ferromagnetic ordering (a) and antiferromagnetic ordering (b). Whereas in ferromagnets spins will align, they will antialign to their neighbouring spins in an antiferromagnet, leading to systematic alternation between up- and down-spins.

magnet whose field acquires, due to the spin-orbit coupling, the same periodicity as the antiferromagnet lattice (Wadley et al. 2016). This allows for electric writing and readout of the magnetic storage memory.

Other notable discoveries and development within antiferromagnetic spintronic over recent years include for example spin pumping (Cheng, J. Xiao, et al. 2014) which is the generation of spin-polarised currents, ultra-fast control over spin states up to terahertz regime (Kampfrath et al. 2011; Cheng, D. Xiao, and Brataas 2016) and the research into skyrmions in antiferromagnets (Barker and Tretiakov 2016).

This short overview is not meant to be exhaustive, but may serve to convey one message: due to its unique properties, antiferromagnetic spintronics may pave the way for key technology of the future.

However, it is in this light peculiar that there is no consensus on the fundamental equations that describe antiferromagnetic dynamics. In particular, no universal agreement exist on the exact form for dissipation. To investigate this matter, it will be useful to first understand the dynamics of the ferromagnet. On a macroscopic level, the dynamics of a ferromagnet can be described by a single parameter; its magnetisation $\mathbf{M}$. This is the collective field of all the spins in the material, that in (in the simplest case) point in the same direction. The magnetisation dynamics of a (macro)spin with magnetisation $\mathbf{M}$ is described by the Landau-Lifshitz-Gilbert (LLG) equation (Gilbert 2004),

$$\dot{\mathbf{M}} = \mathbf{M} \times \mathbf{H} - \alpha_0 \mathbf{M} \times \dot{\mathbf{M}} \,, \tag{1.1}$$

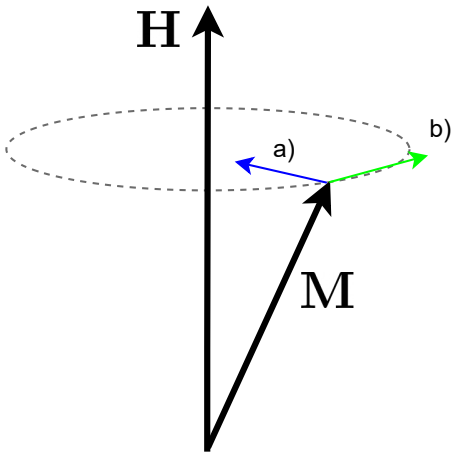

Figure 2: Diagrammatic representation of the LLG-equation: a magnetisation **M** will align to an external field **H** (a) while rotating around it (b).

where **H** is an effective magnetic field that includes the external field, the exchange field and the anisotropy field and α_0 is the damping parameter. The first term describes the precession of the spin around the external field, while the second term describes the alignment to the external field, see figure 2. For an antiferromagnet, there is no universally accepted counterpart to the LLG equation. Nevertheless it is clear that, because there are multiple spin-directions, there are more order parameters in such a system. A convenient way to organise the description of the antiferromagnet is to divide it into sublattices: each sublattice consist of all spins pointing in one particular direction. In this thesis, we focus on two-sublattice antiferromagnets — the antiferromagnets with a structure like in figure 1b. Hence we have two order parameters, which are the macrospins $\mathbf{S}_1$ and $\mathbf{S}_2$. They are the collective field of all spins pointing in the same direction.[1] However, we can equivalently formulate the system in an average magnetisation $\mathbf{M} = \frac{1}{2}(\mathbf{S}_1 + \mathbf{S}_2)$ and the *Néel vector* $\mathbf{N} = \mathbf{S}_1 - \mathbf{S}_2$. As the spins tend to anti-align in an antiferromagnet, usually $|\mathbf{N}| \gg |\mathbf{M}|$ and the Néel vector is the crucial order parameter that is physically loosely interpreted as the 'amount of antiferromagnetism'.

By considering a Rayleigh dissipation functional in the Lagrangian of a general N-sublattice antiferromagnet, it has been shown that in general the antiferromagnetic

[1] In this thesis, the quantities $\mathbf{S}_1$ and $\mathbf{S}_2$ generally refer to the macrospins of the sublattices. Nevertheless, the formalism presented is not only valid in the large-spin limit, but is quite general; the result can, at least in principle, also be applied to atomic and subatomic bosonic particles.

dynamics is governed by N equations of the form (H. Y. Yuan, Liu, et al. 2019)

$$\dot{\mathbf{S}}_i = \mathbf{S}_i \times \mathbf{H}_i - \mathbf{S}_i \times \left(\sum_{j=1}^{N} R_{ij} \dot{\mathbf{S}}_j \right) , \tag{1.2}$$

where R_{ij} are, due to the second law of thermodynamics, strictly non-negative elements of a real dissipation matrix $\mathbf{R}$. Note that each sublattice has its distinct effective field $\mathbf{H}_i$. For the two-sublattice antiferromagnet, the dissipation matrix is given by

$$\mathbf{R} = \begin{pmatrix} \alpha & \alpha_c \\ \alpha_c & \alpha \end{pmatrix} . \tag{1.3}$$

The diagonal elements are identical due to the sublattice permutation symmetry that is assumed. That is, we assume the interchange between $\mathbf{S}_1$ and $\mathbf{S}_2$ should not affect the dissipative dynamics of the system. For the same reason the matrix is symmetric ($R_{ij} = R_{ji}$). The equations of motion for the two-sublattice antiferromagnet are

$$\dot{\mathbf{S}}_1 = \mathbf{S_1} \times \mathbf{H}_1 - \alpha \mathbf{S}_1 \times \dot{\mathbf{S}}_1 - \alpha_c \mathbf{S}_1 \times \dot{\mathbf{S}}_2 \tag{1.4a}$$

$$\dot{\mathbf{S}}_2 = \mathbf{S_2} \times \mathbf{H}_2 - \alpha \mathbf{S}_2 \times \dot{\mathbf{S}}_2 - \alpha_c \mathbf{S}_2 \times \dot{\mathbf{S}}_1 . \tag{1.4b}$$

The significance of this result is the appearance of *inter-sublattice damping terms*. These terms have anti-damping characteristics, which means they increase the lifetime of excitations in the antiferromagnet, see figure 3. This can be intuitively understood by recognising that for an antiferromagnetic interaction, the velocity $\dot{\mathbf{S}}_1$ is approximately opposite to the velocity of the other spin $\dot{\mathbf{S}}_2$. Hence the inter-sublattice term reduces the damping effect of the (intra-sublattice) Gilbert damping term. Written in terms of magnetisation and Néel order, the dynamical equations are given by

$$\dot{\mathbf{M}} = 2\mathbf{M} \times (\mathbf{H}_m - \alpha_m \dot{\mathbf{M}}) + \frac{1}{2} \mathbf{N} \times (\mathbf{H}_n - \alpha_n \dot{\mathbf{N}}) , \tag{1.5a}$$

$$\dot{\mathbf{N}} = \mathbf{M} \times (\mathbf{H}_n - \alpha_n \dot{\mathbf{N}}) + \mathbf{N} \times (\mathbf{H}_m - \alpha_m \dot{\mathbf{M}}) . \tag{1.5b}$$

Here we defined the effective field as subject to the magnetisation as $\mathbf{H}_m = \frac{1}{2}(\mathbf{H}_1 + \mathbf{H}_2)$ and the effective field for the Néel vector as $\mathbf{H}_n = \mathbf{H}_1 - \mathbf{H}_2$. Moreover, the damping parameters for the magnetisation and the Néel order are given by $\alpha_m = (\alpha + \alpha_c)/2$ and $\alpha_n = (\alpha - \alpha_c)/2$ respectively.[2]

[2]Note the difference to the original paper due to the difference in magnetisation definition.

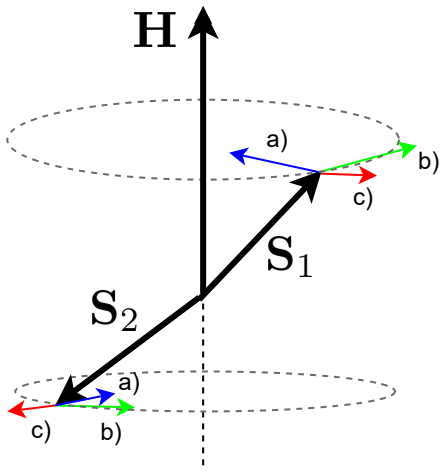

Figure 3: Diagrammatic representation of the phenomenological dynamics for an antiferromagnet: spins $\mathbf{S}_1$ and $\mathbf{S}_2$ align to an external field $\mathbf{H}$ through the intra-sublattice Gilbert damping term (a) while precessing around the external field (b). Moreover, the inter-sublattice Gilbert damping terms (c) are opposed to damping and cause an anti-damping force to occur. Note that for simplicity of illustration, in this diagram it is assumed $\mathbf{H} = \mathbf{H}_1 = \mathbf{H}_2$: only an external field contributes to the effective field of both spins.

The dissipation matrix eq. (1.3) is especially useful to calculate the energy dissipation rate of the system; it is given by (H. Y. Yuan, Liu, et al. 2019)

$$\dot{E} = -\sum_{i,j} R_{ij}\dot{\mathbf{S}}_i \cdot \dot{\mathbf{S}}_j = \alpha(\dot{\mathbf{S}}_1^2 + \dot{\mathbf{S}}_2^2) + 2\alpha_c \dot{\mathbf{S}}_1 \cdot \dot{\mathbf{S}}_2 = 2(\alpha + \alpha_c)\dot{\mathbf{M}}^2 + \frac{\alpha - \alpha_c}{2}\dot{\mathbf{N}}^2 \,, \quad (1.6)$$

where the energy dissipation is considered in terms of sublattices as well as in terms of the magnetisation and Néel order.

The phenomenological equations of motion are a very general account of antiferromagnetic dynamics; many previous approaches are special cases by choosing the damping parameters to have specific properties. For example, some (Takei et al. 2016) use $\alpha_m = 0$, while others use $\alpha_m = \alpha_n$ (Cheng, J. Xiao, et al. 2014). From the perspective of the sublattices, also different approaches are used: one may choose $\alpha_c = 0$ with α finite (Kittel 1951), or $\alpha = \alpha_c$ (Baryakhtar and Danilevich 2015), amongst other possibilities.

The general dynamics as described above are results of a quasi-classical phenomenological approach: they are derived by assuming a general dissipation functional in a classical Lagrangian. In other words, the approach does not specify the physical

process that explains the dissipative behaviour of antiferromagnets. Consequently, such a model is not able to explain why the choice for specific values is suitable in some systems, while other models benefit from the choice of what appears to be very different assumptions. Moreover, from general scattering theory and first-principle calculations, it has been shown that at least in certain antiferromagnets, $\alpha_m \gg \alpha_n$, which is contrary to many assumptions listed above (Liu et al. 2017).

The goal of this thesis is to understand the more fundamental physical process that underlies the dissipative behaviour of two-sublattice antiferromagnets, by analysis of the thermal structure that is necessary for dissipation. First and foremost, we will argue that the dissipative behaviour of two-sublattice antiferromagnets is determined by the system-bath structure that gives rise to this dissipation. Not only will such a fundamental understanding provide a theoretical foundation to the phenomenological equations; moreover, it can explain the origin of the different phenomenological assumptions to the dissipative dynamics. Lastly the results may give physical explanation to the above-mentioned first-principle calculations for the damping of the antiferromagnetic modes.

This research includes several aspects: first, a quantum theory for dissipative spin systems needs to be developed. It is important that this theory is able to re-derive the phenomenological equations of motion. The construction of such a theory will be the subject of chapter 2. After this, dissipative antiferromagnets can be modelled in this theory. However, as will become clear, the construction of a model is not trivial: based on different physical assumptions, different models arise. Of course, this is exactly the argument in this thesis: the goal is to argue that the damping behaviour depends on the dissipative structure of the system, hence on the physical assumptions underlying the model. Therefore, in chapter 3 we will develop these models and compare them to the phenomenological equations. Remarkably, eqs. (1.4) are recovered in the model where both spins couple to the same thermal bath, but we find that there are no inter-sublattice damping terms when the spins couple to separate thermal baths. These findings indicate that the inter-sublattice damping, and therefore the magnon lifetime, is dependent on the thermal separation between interacting antiferromagnetic layers.

Moreover, it will turn out that under specified circumstances, our theory predicts that the Néel vector will be a dissipation-free mode of the system. In chapter 4, this result is generalised by arguing that this phenomenon is not unique to antiferromagnets, but will arise in many systems with an analogous dissipative structure. Lastly, the thesis ends with a summary of the results and an outlook on further research in chapter 5.

2 Theory for Dissipation and Spin Dynamics

In this thesis, we are interested in the dynamics of dissipative interacting (macro-)spins from a microscopic perspective. The description and behaviour of such systems requires a special physical theory: first, it needs to be a theory that is able to include dissipation of quantum systems, and second, this theory should include the possibility to describe the dynamics of spin systems. Lastly, since the quasi-classical behaviour of the spin systems plays an essential role in our research, we wish to construct a theory that is able to restore the quasi-classical limit quite straightforwardly.

To meet these requirements we use the Caldeira-Leggett model, which accounts for dissipation in quantum systems by considering the composite of the system and its environment (Caldeira and Leggett 1983). In this thesis we consider this model formulated in terms of the closed-time contour or *Keldysh* formalism, which is a general theory for non-equilibrium quantum systems. The exposition by Kamenev is being used as principal guide (Kamenev 2011). A major benefit of the Keldysh approach is that it is expressible as a path integral formulation. Therefore, quasi-classical dynamics can be derived from the action with techniques analogous to Lagrangian mechanics.

Nevertheless, the Keldysh formalism in its standard form does not include a treatment of spin dynamics. A theory for the dynamics of spin systems is non-trivial, as even the classical dynamics of spin cannot be treated in standard Hamiltonian mechanics. Yet, a path integral formalism for spin systems can be formulated by the definition of *spin coherent states*. In this thesis we follow the derivation by (Altland and Simons 2010) and extend it to the Keldysh formalism.

In section 2.1, we will first derive the spin path integral and define the key concepts. After that, the dynamics of two interacting spins is considered in section 2.2. This illustrates the general methods of application of the spin path integral in this research and serves as a basis for all forthcoming models. Subsequently in section 2.3 dissipation is included into the theory. We introduce the Keldysh formalism through the derivation of the dissipative dynamics of a single spin.

2.1 Construction of the path integral

We will derive the path integral formulation for a system that depends on two spins $\hat{\mathbf{S}}_i$ $(i = 1, 2)$, which in general may interact with each other, described by the system's

Hamiltonian $\hat{H} = \hat{H}(\hat{\mathbf{S}}_1, \hat{\mathbf{S}}_2)$.[3] The Hilbert space of spin states can be constructed by a basis of *spin coherent states* $\{|\tilde{g}_i\rangle\}$ $(i = 1, 2)$, where $\{|\tilde{g}_1\rangle\}$ is the basis of the Hilbert space of $\mathbf{S}_1$, and similarly, $\{|\tilde{g}_2\rangle\}$ is the basis of the Hilbert space of $\mathbf{S}_2$. Conceptually, the state $|\tilde{g}_i\rangle = \tilde{g}_i\,|\uparrow\rangle$ is a rotation of the maximum weight state $|\uparrow\rangle$ by an element $\tilde{g}_i \in SU(2)$. Hence the state $|\tilde{g}_i\rangle$ is interpreted as living on a sphere. The $|\uparrow\rangle$-states of the two spins are different states in principle; however, since this state is defined by the choice of quantisation, we can choose them to be in the same direction. Moreover, external fields will be introduced later on that will be the same for all spins in the system. Consequently, these external fields suggest a simple choice the quantisation direction.

Any spin state on the sphere can be represented by a rotation of the Euler angles ϕ_i, θ_i and ψ_i of the $|\uparrow\rangle$-state, see figure 4. In other words, we can write any state $|\tilde{g}_i\rangle$ in terms of the Euler angles as (Altland and Simons 2010, section 3.3)

$$|\tilde{g}_i\rangle = e^{-i\phi_i \hat{S}_i^z} e^{-i\theta_i \hat{S}_i^y} e^{-i\hat{S}_i^z \psi_i}\,|\uparrow\rangle \ , \tag{2.1}$$

where by definition we have $\hat{S}_i^z\,|\uparrow\rangle = S_i\,|\uparrow\rangle$; S_i is the maximal eigenvalue of the $\hat{S}_i^z$. Loosely speaking, this eigenvalue is the 'magnitude of the total spin' of the system under consideration, and in our model it is the radius of the sphere.

The basic intuition one must keep in mind is that the dynamics of spin describe a massless particle moving on the unit sphere. This movement is described by the angles ϕ_i and θ_i. The Euler angle ψ_i will only rotate the spin vector around its own axis, which has no consequence on the dynamics and no physical meaning. In other words, ψ_i is a gauge freedom of the system. This means we can define a simpler representation of the spin coherent state by

$$|g_i\rangle = e^{-i\phi_i \hat{S}_i^z} e^{-i\theta_i \hat{S}_i^y}\,|\uparrow\rangle \ \ (i = 1, 2) \ , \tag{2.2}$$

which is related to the first representation by $|\tilde{g}_i\rangle = |g\rangle\, e^{-iS_i\psi_2}$.

Now let us look at the properties of the spin coherent states. Each set of spin coherent states form a complete basis of their respective Hilbert space:

$$C \int \mathrm{d}g_i\,|g_i\rangle\,\langle g_i| = \mathbb{1} \ (i = 1, 2) \ . \tag{2.3}$$

[3] Although not of interest in this research, the number of spins may be extended to an arbitrary number M. The derivation in that case is analogous to the one presented here.

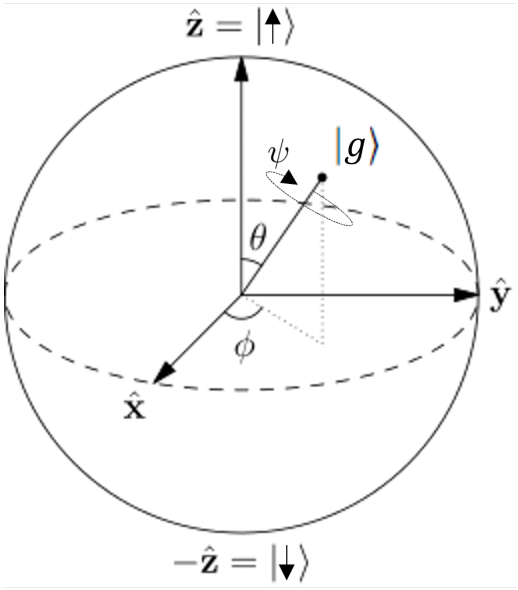

Figure 4: Visualization of the spin coherent state. The Euler angles ϕ_i and θ_i rotate the vector around the sphere, while ψ_i is a gauge parameter, since it only rotates the vector around its own axis.

Although the details do not concern us here, note that this is actually a rather special integral; it assigns a volume measure to the group $SU(2)$. For our purposes, the constant C can be set to unity without affecting the physical results.

Since all spins are initially defined within their own Hilbert space, the combined system is described by the tensor product of these spaces. Consequently the following resolution of unity also holds:

$$\int \mathrm{d}g_1 \, \mathrm{d}g_2 \;\; |g_1\rangle \otimes |g_2\rangle \, \langle g_1| \otimes \langle g_2| \;\; = \mathbb{1} \; . \tag{2.4}$$

From now on, the tensor product symbol $\otimes$ will be omitted for brevity, although it is important to keep in mind that operators related to $\hat{\mathbf{S}}_i$ will only act on states $|g_j\rangle$ if $i = j$.

Now we will construct the path integral for a general Hamiltonian $\hat{H} = \hat{H}(\hat{\mathbf{S}}_1, \hat{\mathbf{S}}_2)$. In order to achieve this, we consider the transition amplitude $\mathcal{Z} = \mathcal{Z}[\mathbf{S}_1, \mathbf{S}_2]$

$$\mathcal{Z} = \langle g_{1,N}| \, \langle g_{2,N}| \, e^{-i\hat{H}t} \, |g_{1,0}\rangle \, |g_{2,0}\rangle \equiv \langle g_N| \, e^{-i\hat{H}t} \, |g_0\rangle \;\; , \tag{2.5}$$

where $|g_0\rangle \equiv |g_{1,0}\rangle \, |g_{2,0}\rangle$ is the shorthand notation for the initial states of the two

subsystems and $|g_N\rangle \equiv |g_{1,N}\rangle\,|g_{2,N}\rangle$ for all final states. Then this transition amplitude is split into N time intervals of $\Delta t = \frac{t}{N}$, where N limits to infinity:

$$\mathcal{Z} = \lim_{N\to\infty} \langle g_N| \underbrace{e^{-i\hat{H}\Delta t} e^{-i\hat{H}\Delta t} \dots e^{-i\hat{H}\Delta t}}_{N\text{times}} |g_0\rangle \ . \tag{2.6}$$

Subsequently, between each exponential a resolution of unity is inserted:

$$\mathcal{Z}[\{\hat{\mathbf{S}}_i\}] = \lim_{N\to\infty} \int \prod_{j=0}^{N} \left(\mathrm{d}g_j\right) \langle g_N|\, e^{-i\hat{H}\Delta t}\, |g_{N-1}\rangle \langle g_{N-1}|\, e^{-i\hat{H}\Delta t}\, |g_{N-2}\rangle \dots \langle g_1|\, e^{-i\hat{H}\Delta t}\, |g_0\rangle \ . \tag{2.7}$$

This allows us to consider transition amplitudes of infinitesimal time individually:

$$\begin{aligned} \mathcal{Z}_j &= \langle g_{j+1}|\, e^{-i\hat{H}\Delta t}\, |g_j\rangle \\ &\simeq \langle g_{j+1}|\, 1 - i\hat{H}\Delta t\, |g_j\rangle \\ &= \langle g_{j+1}|g_j\rangle - i\Delta t\, \langle g_{j+1}|\, \hat{H}\, |g_j\rangle + 1 - \langle g_j|g_j\rangle \ , \end{aligned} \tag{2.8}$$

where the added terms in the last line cancel each other. However, the presence of the 1 allows us to re-exponentiate. This results in

$$\mathcal{Z}_j \simeq \exp\left\{ \Delta t \left(\frac{\langle g_{j+1}|g_j\rangle - \langle g_j|g_j\rangle}{\Delta t} - i\, \langle g_{j+1}|\, \hat{H}\, |g_j\rangle \right) \right\} \ , \tag{2.9}$$

where we deliberately rewrote the expression, such that the appeared fraction is reminiscent of a discrete derivative. Yet, we must not be misguided by our notation. If we would disregard our shorthand notation, the numerator is written as

$$\begin{aligned} \langle g_{j+1}|g_j\rangle - \langle g_j|g_j\rangle &= \langle g_{1,j+1}|\, \langle g_{2,j+1}|g_{1,j}\rangle\, |g_{2,j}\rangle - \langle g_{1,j}|\, \langle g_{2,j}|g_{1,j}\rangle\, |g_{2,j}\rangle \\ &= \langle \Delta g_{1,j}|g_{1,j}\rangle + \langle \Delta g_{2,j}|g_{2,j}\rangle + \langle \Delta g_{1,j}|g_{1,j}\rangle\, \langle \Delta g_{2,j}|g_{2,j}\rangle \ , \end{aligned} \tag{2.10}$$

where in the second line, we introduced the notation $\langle \Delta g_i| \equiv \langle g_{i+1}| - \langle g_i|$. Moreover, we reordered the terms in order to recover the independent infinitesimal change of each state. As a consequence, an additional correlation term appears. This means that the nominator of the infinitesimal transition amplitude eq. (2.9) actually considers all orders of change in the spin coherent states simultaneously.

However, when we will perform the limit in eq. (2.7) explicitly, $\Delta t = \frac{t}{N}$ goes to zero. In this limit, only first order Δ-terms in the denominator do not vanish. Hence the higher order correlations can be disregarded and we exactly get the required discrete

derivatives. As a consequence, the final expression of the path integral in the continuum limit is given by

$$\mathcal{Z}[\{\hat{\mathbf{S}}_i\}] = \int \mathcal{D}g \exp\{i\mathcal{S}\} \ , \tag{2.11}$$

where $\mathcal{D}g = \lim_{N\to\infty} \int \prod_{i=0}^{M} \prod_{j=0}^{N} \left(\mathrm{d}g_{i,j}\right)$, and the *action* $\mathcal{S}$ is given by

$$\mathcal{S} = \int \mathrm{d}t \left(-i \left\langle \dot{g}_1 | g_1 \right\rangle - i \left\langle \dot{g}_2 | g_2 \right\rangle - \left\langle g_1 \right| \left\langle g_2 \right| \hat{H} \left| g_1 \right\rangle \left| g_2 \right\rangle \right) \ , \tag{2.12}$$

where $\langle \dot{g}_j |$ denotes the time derivative of the state $\langle g_j |$. The first two terms in the action are *Berry phase terms* or *Berry connections*, which are a result of the geometrical trajectory of each spin state inside its Hilbert space (Berry 1984). As will become clear what follows, these terms play the role of kinetic energy inside the action. The third term is the Hamiltonian, evaluated in the eigenbases of the spins, that describes the interaction between the two spins.

The spin path integral that we derived, and the action in particular, is the starting point of this research. Although the path integral allows for calculations of quantum mechanical transition amplitudes in principle, we are primarily interested in the quasi-classical behaviour of spin systems. These can be derived from the action by taking the variation with respect to the dynamical variables, which is equivalent to calculating the Euler-Lagrange equations. However, it is crucial to realise what quantities are actually the dynamical variables in our action: not the spin vectors themselves, but the coordinates that parameterise the spin coherent states are the relevant variables. These are the Euler angles ϕ_1, θ_1 and ϕ_2, θ_2 of the two spins respectively.

We can find an explicit form for the Berry connections (see appendix A), which is given by

$$\langle \dot{g} | g \rangle = -iS\dot{\phi}(1 - \cos\theta) \ , \tag{2.13}$$

where S is the maximal eigenvalue of $\hat{S}^z$.

The expectation value of a spin operator is defined by

$$S_i \equiv \langle \tilde{g} | \hat{S}_i | \tilde{g} \rangle \ . \tag{2.14}$$

and from the definition of the spin coherent states, it follows that the spin vector $\mathbf{S}_i$ is

the vector on a sphere with radius S_i: (see also appendix A)

$$\mathbf{S}_i = S_i \begin{pmatrix} \sin\theta_i \cos\phi_i \\ \sin\theta_i \sin\phi_i \\ \cos\theta_i \end{pmatrix} , \tag{2.15}$$

i.e. $\mathbf{S}_i$ is the product of S_i and a vector on the unit sphere. This result is consistent with the intuition of spin being a rotating vector of fixed length.

2.2 Dynamics of two interacting spins

In this section we calculate the dynamics of two antiferromagnetically interacting spins in the spin coherent state formulation. This serves two ends: first, this is the basis of all our following models, and second, it illustrates the practice and details of the spin coherent states. Starting from the interaction defining Hamiltonian, we will derive the quasi-classical equations of motion.

The antiferromagnetic coupling of spins $\hat{\mathbf{S}}_\mathbf{1}$ and $\hat{\mathbf{S}}_\mathbf{1}$ is described by the Heisenberg Hamiltonian

$$\hat{H}_S = +J\hat{\mathbf{S}}_\mathbf{1} \cdot \hat{\mathbf{S}}_\mathbf{2} , \tag{2.16}$$

where J is a positive coupling constant. Using the main result of the previous section, the action is of the form

$$S = \int \mathrm{d}t \, \left(-i \langle \dot{g}_1 | g_1 \rangle - i \langle \dot{g}_2 | g_2 \rangle - J \langle g_1 | \hat{\mathbf{S}}_\mathbf{1} | g_1 \rangle \cdot \langle g_2 | \hat{\mathbf{S}}_\mathbf{2} | g_2 \rangle \right) . \tag{2.17}$$

Nevertheless, to continue we have to derive the explicit form of the action in terms of the spherical coordinates θ_1, ϕ_1 of $\mathbf{S}_1$ and θ_2, ϕ_2 of $\mathbf{S}_2$. The Berry connections are given by eq. (2.13). Using eq. (2.15) and the substraction formula for cosine, the interaction is in spherical coordinates written as

$$\langle g_1 | \hat{\mathbf{S}}_\mathbf{1} | g_1 \rangle \cdot \langle g_2 | \hat{\mathbf{S}}_\mathbf{2} | g_2 \rangle = S_1 S_2 (\sin\theta_1 \sin\theta_2 \cos(\phi_1 - \phi_2) + \cos\theta_1 \cos\theta_2) . \tag{2.18}$$

In total, the action in the Euler angle representation is given by

$$\begin{aligned} S = -\int \mathrm{d}t \, \Big[& S_1 \dot{\phi}_1 (1 - \cos\theta_1) + S_2 \dot{\phi}_2 (1 - \cos\theta_2) \\ & + J S_1 S_2 \left(\sin\theta_1 \sin\theta_2 \cos(\phi_1 - \phi_2) + \cos\theta_1 \cos\theta_2 \right) \Big] . \end{aligned} \tag{2.19}$$

We are interested the quasi-classical equation of motion. This corresponds to finding the path that is weighted the heaviest in the path integral, which is the solution that extremises the action. Hence we can take the variation of the action, or equivalently, we calculate the Euler-Lagrange equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = 0 \tag{2.20}$$

for any generalized coordinate q. In our case we get four equations of motion, from $\theta_1, \phi_1, \theta_2$ and ϕ_2:

$$\theta_1 : \qquad S_1\dot{\phi_1}\sin\theta_1 = +JS_1S_2(\sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2\cos(\phi_1-\phi_2)) \tag{2.21a}$$

$$\theta_2 : \qquad S_2\dot{\phi_2}\sin\theta_2 = +JS_1S_2(\cos\theta_1\sin\theta_2 - \sin\theta_1\cos\theta_2\cos(\phi_1-\phi_2)) \tag{2.21b}$$

$$\phi_1 : \qquad S_1\dot{\theta}_1 = -JS_1S_2(\sin\theta_2\sin(\phi_1-\phi_2)) \tag{2.21c}$$

$$\phi_2 : \qquad S_2\dot{\theta}_2 = +JS_1S_2(\sin\theta_1\sin(\phi_1-\phi_2))\ . \tag{2.21d}$$

From these equations, we want to restore vectorial equations of motion for $\mathbf{S}_1$ and $\mathbf{S}_2$. To acquire these, we calculate the time derivative of the spin vectors in terms of the Euler angles and decompose it into a simpler vector and a rotation ϕ around the z-axis:

$$\dot{\mathbf{S}}_i = S_i\begin{pmatrix}\dot{\theta}_i\cos\theta_i\cos\phi_i - \dot{\phi}_i\sin\theta_i\sin\phi_i\\ \dot{\theta}_i\cos\theta_i\sin\phi_i + \dot{\phi}_i\sin\theta_i\cos\phi_i\\ -\dot{\theta}\sin\theta_i\end{pmatrix} = S_1\begin{pmatrix}\cos\phi_1 & -\sin\phi_1 & 0\\ \sin\phi_1 & \cos\phi_1 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\dot{\theta}_1\cos\theta_1\\ \dot{\phi}_1\sin\theta_1\\ \dot{\theta}_1\sin\theta_1\end{pmatrix}\ . \tag{2.22}$$

The equations of motion for θ_1, eq. (2.21a), and ϕ_1, eq. (2.21c), can be inserted in the vector, to get for $\dot{\mathbf{S}}_1$:

$$\dot{\mathbf{S}}_1 = -JS_1S_2\begin{pmatrix}\sin\theta_1\cos\theta_2\sin\phi_1 - \cos\theta_1\sin\theta_2\sin\phi_2\\ \cos\theta_1\sin\theta_2\cos\phi_2 - \sin\theta_1\cos\theta_2\cos\phi_1\\ -\sin\theta_1\sin\theta_2\sin(\phi_1-\phi_2)\end{pmatrix}\ . \tag{2.23}$$

The cross product of the two spins is in terms of the components is given by

straightforward computation as

$$\mathbf{S_1} \times \mathbf{S_2} = S_1 S_2 \begin{pmatrix} \sin\theta_1 \cos\theta_2 \sin\phi_1 - \cos\theta_1 \sin\theta_2 \sin\phi_2 \\ \cos\theta_1 \sin\theta_2 \cos\phi_2 - \sin\theta_1 \cos\theta_2 \cos\phi_1 \\ -\sin\theta_1 \sin\theta_2 \sin(\phi_1 - \phi_2) \end{pmatrix} . \tag{2.24}$$

Comparing, it is clear that the time derivative, up to the prefactor, exactly corresponds to the cross-product. The components of $\dot{\mathbf{S}}_2$ are exactly the same but opposite in sign, which can be seen by simply swapping the indices $(1 \leftrightarrow 2)$ in above equation. Therefore we end up with the compact equations of motion:

$$\dot{\mathbf{S}}_1 = -\dot{\mathbf{S}}_2 = -J(\mathbf{S_1} \times \mathbf{S_2}) . \tag{2.25}$$

We find the expected result: two antiferromagnetically coupled spins act on each other as magnetic fields and move anti-parallel to each other. In other words, the two spins will rotate around their common axis.

2.3 Dissipation

Having established the formalism and techniques for the modelling of spin dynamics, in this section we will develop the Keldysh formalism to describe energy dissipation in quantum field theory, and for spin systems in particular.

In short, the Keldysh formalism, in this thesis used in its functional form, is a general framework for non-equilibrium field theory. Its main feature that makes treatment of non-equilibrium systems possible is the *closed-time contour*. Where standard field theory evaluates the evolution of a system from infinite past to infinite future,

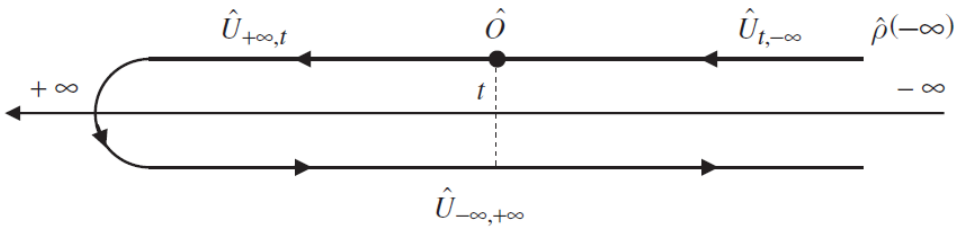

Figure 5: Illustration of the Keldysh contour: starting in initial state $\hat{\rho}(-\infty)$ at $t = -\infty$, the system is evolved with evolution operator $\hat{U}$ to time t, where observable $\hat{O}$ is measured. Then the system evolves to $t = \infty$, and subsequently it evolves backward to the initial state at $t = -\infty$. *(figure from Kamenev 2011.)*

the closed-time contour additionally considers the backwards evolution from future to past, creating a closed 'time contour', see figure 5. This involves doubling all degrees of freedom in the system under consideration, as one splits the contour at $t = +\infty$ into a forward and backward evolution. The integral over this closed-time contour is denoted as $\oint_K \mathrm{d}t$.

Although counter-intuitive, this backwards evolution is required for all field theories, in principle. However, in standard equilibrium quantum field theory, all perturbations are adiabatically switched on from the infinite past to time t, and in turn are adiabatically turned off from time t to the infinite future. One assumes that the net effect is that the system in the infinite future is back in its ground state; it has acquired a phase factor at most. The backward evolution then is equivalent to accounting for this phase factor, which is the same as removing the vacuum bubbles in the diagrammatic approach (Kamenev 2011, p. 3).

Contrarily, in non-equilibrium quantum field theory, the perturbations are not guaranteed to be adiabatic. As a consequence, even when the disturbance is turned off, the system may end up in another state than the ground state at $t = \infty$. In other words, the backward evolution must be included.

Dissipation of quantum systems can be treated in the Keldysh formalism by making use of the Caldeira-Leggett approach. This is a method of treating quantum dissipation, where the system is coupled to a reservoir of harmonic oscillators. Subsequently, these oscillators are integrated out to obtain the effective dissipative dynamics of the system in question (Caldeira and Leggett 1983). We will follow this approach for a macro-spin $\mathbf{S}$ in a magnetic field.

First, we will derive the dissipative action for one spin $\mathbf{S}$ coupled to a bath of harmonic oscillators. The significance of this example lies not so much in the physical interest; rather, the techniques to be developed and objects to be defined will be used extensively throughout the rest of this thesis.

The dynamics of each harmonic oscillator in the bath is governed by the familiar Hamiltonian

$$\hat{H}_\alpha = \frac{\hat{\mathbf{p}}_\alpha^2}{2m_\alpha} + \frac{m_\alpha \omega_\alpha^2}{2} \hat{\mathbf{x}}_\alpha^2 \,. \tag{2.26}$$

Here, m_α en ω_α are the mass and oscillation frequency of the oscillator α which has momentum $\hat{\mathbf{p}}$ and position $\hat{\mathbf{x}}$. Energy dissipation of the spin $\hat{\mathbf{S}}$ is described by minimal

coupling of the spin to each of these bath particles:

$$\hat{H}_c = \sum_\alpha \gamma_\alpha \hat{\mathbf{S}} \cdot \hat{\mathbf{x}}_\alpha \ , \tag{2.27}$$

where γ_α is the coupling strength between oscillator α and the spin. Moreover, we have to assume that there is coupling to some external field $\mathbf{H}$, which in general can be time-dependent. If there was not such a field, there would be no preferred (lowest energy) direction of the spin and therefore no energy dissipation. For this purpose, it is convenient to chose $\mathbf{H} = (0, 0, H)^T$ with H positive. Note that this choice does not lead to loss of generality; actually this external field fixes the z-direction and hence the coordinate system. The total Hamiltonian then is given by:

$$\hat{H} = -\hat{\mathbf{S}} \cdot \mathbf{H} + \sum_\alpha \gamma_\alpha \hat{\mathbf{S}} \cdot \hat{\mathbf{x}}_\alpha + \sum_\alpha \left(\frac{\hat{\mathbf{p}}_\alpha^2}{2m_\alpha} + \frac{m_\alpha \omega_\alpha^2}{2} \hat{\mathbf{x}}_\alpha^2 \right) \ . \tag{2.28}$$

Following the path integral procedure of previous section, we arrive at the Keldysh partition function

$$\mathcal{Z}[\mathbf{S}, \mathbf{x}_\alpha, \mathbf{p}_\alpha] = \int \mathrm{d}g \left(\prod_\alpha \int \mathrm{d}\mathbf{x}_\alpha \mathrm{d}\mathbf{p}_\alpha \right) e^{i\mathcal{S}} \ . \tag{2.29}$$

Note that the spin integral of previous section has to be combined with the standard Feynman path integral over the phase space of all harmonic oscillators. The action is

$$\mathcal{S} = \oint_K \mathrm{d}t \left[-i \langle \dot{g} | g \rangle + \mathbf{S} \cdot \mathbf{H} - \sum_\alpha \gamma_\alpha \mathbf{S} \cdot \mathbf{x}_\alpha - \sum_\alpha \left(\frac{\mathbf{p}_\alpha^2}{2m_\alpha} + \frac{m_\alpha \omega_\alpha^2}{2} \mathbf{x}_\alpha^2 - \mathbf{p}_\alpha \dot{\mathbf{x}}_\alpha \right) \right] \ , \tag{2.30}$$

where the contour integral K refers to the Keldysh contour. As explained above, this contour goes from the infinite past to the infinite future, and backwards to the infinite past. Note that as a consequence of the path integral procedure, all operators turn into their expectation value within the action. Given that the dependence on the momenta is quadratic in the action, we recognise that its dependence in the path integral is Gaussian and we can integrate them out, using the identity

$$\int \frac{\mathrm{d}\mathbf{x}}{\sqrt{2\pi}} e^{-\frac{1}{2}\mathbf{x}^T A \mathbf{x} + \mathbf{x}^T J} = \frac{e^{\frac{1}{2} J^T A^{-1} J}}{\mathrm{Det}(A)} \ , \tag{2.31}$$

where J is a vector and A a matrix with non-zero eigenvalues that have a non-negative

real part. First, we perform the Gaussian integration over momenta to get

$$\mathcal{S} = \oint_K \mathrm{d}t \left[-i \langle \dot{g}|g\rangle + \mathbf{S}\cdot\mathbf{H} - \sum_\alpha \gamma_\alpha \mathbf{S}\cdot\mathbf{x}_\alpha + \sum_\alpha \frac{m_\alpha}{2}\left(\dot{\mathbf{x}}_\alpha^2 - \omega_\alpha^2 \mathbf{x}_\alpha^2\right)\right] . \tag{2.32}$$

By partial integration, the dynamics of the oscillator α can be written as

$$\frac{m_\alpha}{2}\oint_K \mathrm{d}t \left(\dot{\mathbf{x}}_\alpha^2 - \omega_\alpha^2\mathbf{x}_\alpha^2\right) = \frac{m_\alpha}{2}\oint_K \mathrm{d}t \mathbf{x}_\alpha \left(-\partial_t^2 - \omega_\alpha^2\right)\mathbf{x}_\alpha \, . \tag{2.33}$$

Hence the dynamics of the oscillators can collectively be written in one *propagator*, that we define as:

$$G_\alpha^{-1}(t) \equiv \frac{m_\alpha}{2}((i\partial_t)^2 - \omega_\alpha^2) \, . \tag{2.34}$$

Inserting the propagator, the action acquires the form

$$\mathcal{S} = \oint_K \mathrm{d}t \left[-i \langle \dot{g}|g\rangle + \mathbf{S}\cdot\mathbf{H} + \sum_\alpha \left[-\gamma_\alpha \mathbf{S}\cdot\mathbf{x}_\alpha + \mathbf{x}_\alpha^T G_\alpha^{-1}(t)\mathbf{x}_\alpha\right]\right] . \tag{2.35}$$

In this form, it is apparent that the position-dependence of the harmonic oscillators in the action is also quadratic. As a consequence, these degrees of freedom can also be eliminated by a second Gaussian integration. This results in the *dissipative action*:

$$\mathcal{S} = \oint_K \mathrm{d}t \left[-i \langle \dot{g}|g\rangle + \mathbf{S}\cdot\mathbf{H} + \oint_K \mathrm{d}t' \mathbf{S}(t)\alpha(t-t')\mathbf{S}(t')\right] . \tag{2.36}$$

In principle, this action describes the dissipative dynamics of the spin $\mathbf{S}$. However, we must investigate into the structure of the newly defined *dissipation propagator* $\alpha(t-t')$ to learn how energy dissipation affects the system exactly. First, the dissipation propagator in eq. (2.36) is defined as

$$\alpha(t-t') = -\frac{1}{4}\sum_\alpha G_\alpha(t-t') \, , \tag{2.37}$$

where $G_\alpha(t-t')$ is an operator such that $G_\alpha^{-1}(t)$ is its inverse. To find an explicit form for the dissipation operator, we have to use the defining characteristic of the Keldysh formalism; we have to analyse the behaviour of the propagator on the closed-time contour.

First, we split the Keldysh contour into its positive and negative time part, such that we have

$$\mathcal{S} = \oint_K \mathrm{d}t L = \int_{-\infty}^{+\infty} \mathrm{d}t L^+ + \int_{+\infty}^{-\infty} \mathrm{d}t L^- = \int_{-\infty}^{+\infty} \mathrm{d}t [L^+ - L^-] \,, \tag{2.38}$$

where L is the total Lagrangian. This is the above-mentioned doubling of degrees of freedom: now for every dynamical quantity, for example the spin $\mathbf{S}$, there appear two quantities in the action: the forward quantity $\mathbf{S}^+$ and the backward quantity $\mathbf{S}^-$. Now the crucial step to gain physical insight into this matter is to perform a *Keldysh rotation* on all dynamical fields, again illustrated by the spin $\mathbf{S}$:

$$\begin{aligned} \mathbf{S}^{cl} &= \frac{1}{2}(\mathbf{S}^+ + \mathbf{S}^-) \,, \\ \mathbf{S}^{q} &= \frac{1}{2}(\mathbf{S}^+ - \mathbf{S}^-) \,. \end{aligned} \tag{2.39}$$

The first, $\mathbf{S}^{cl}$, is called the *classical field*, while the second, $\mathbf{S}^q$, is refered to as the *quantum field*. Moreover we redefine $\mathbf{S} \equiv (\mathbf{S}^{cl}, \mathbf{S}^q)^T$ and the same for all $\mathbf{x}_\alpha$ and $\mathbf{H}$. In other words, all quantities are defined in $(cl, q)-$space now. After this split, rotation, and introduction of $(cl, q)-$space, the inverse propagator G_α^{-1} has turned into a matrix in $(cl, q)-$space:

$$G_\alpha^{-1} = \begin{pmatrix} 0 & (G_\alpha^{-1})^A \\ (G_\alpha^{-1})^R & (G_\alpha^{-1})^K \end{pmatrix} \,, \tag{2.40}$$

where $(G_\alpha^{-1})^K$ is the *Keldysh part*, which is a regularisation, whose specifics are not relevant in our discussion at this stage. The advanced [retarded] part are given by $(G_\alpha^{-1})^{R[A]} = \frac{1}{2} m_\alpha ((i\partial_t \pm i\eta)^2 - \omega_\alpha^2)$. Here and in the following, the expressions are read in upper signs $(+)$ for the retarded part and lower signs $(-)$ for the advanced part. Furthermore, $\eta = (2\tau)^{-1}$ is the inverse mean lifetime of the bath modes and is assumed to be infinitely small: it makes sure the Green's functions evaluate the retarded and advanced part of the trajectory. This infinitesimality is a consequence of the fact that the bath modes are assumed to have infinite lifetime τ.

Implementing the Keldysh rotation, the action looks like

$$\mathcal{S} = 2 \int_{-\infty}^{\infty} \mathrm{d}t \left[- i \langle \dot{g} | g \rangle + \mathbf{S}^T \sigma_1 \mathbf{H} + \gamma^2 \int_{-\infty}^{\infty} \mathrm{d}t' \mathbf{S}^T(t) \alpha(t - t') \mathbf{S}(t') \right] \,, \tag{2.41}$$

where $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ is the first Pauli matrix, which is just a result of the Keldysh rotation and the fact that we write all quantities in $(cl, q)-$space. In practice, this means the dot product of two vector quantities multiply the quantum component of the first with the classical component of the second and vice versa.

The object under investigation, the dissipation propagator, also has turned into a matrix in $(cl, q)-$space and has the form

$$\alpha(t-t') = \begin{pmatrix} 0 & \alpha^A(t-t') \\ \alpha^R(t-t') & \alpha^K(t-t') \end{pmatrix} . \tag{2.42}$$

We are primarily interested in the explicit form of the retarded and advanced dissipation propagator, as these parts contain the dissipative dynamics. (The Keldysh dissipation propagator contains the fluctuation, that are necessarily present due to the dissipation-fluctuation theorem.) Therefore, by eq. (2.37), we first have to find the explicit form of $G_\alpha(t-t')$. The Fourier transform of $(G_\alpha^{-1})^{R[A]}(t)$ is given by

$$(G_\alpha^{-1})^{R[A]}(\omega) = \frac{m_\alpha}{2} \int \mathrm{d}t \left((i\partial_t \pm i\eta)^2 - \omega_\alpha^2 \right) e^{-i\omega t} = \frac{m_\alpha}{2} ((\omega \pm i\eta)^2 - \omega_\alpha^2) \tag{2.43}$$

Using eq. (2.37), the dissipation propagator $\alpha^{R[A]}(t-t')$ in Fourier space then is

$$\alpha^{R[A]}(\omega) = -\sum_\alpha \frac{1}{2m_\alpha} \frac{1}{(\omega \pm i\eta)^2 - \omega_\alpha^2} . \tag{2.44}$$

This is the explicit form we were looking for. We see that the propagator has a familiar Green's function structure and contains simple poles that depend on the oscillation frequency of the bath particle. However, we are not so much interested in the specifics of each particle on the spin system individually, as on the dissipative physics that the bath creates as a whole. To this end, we introduce the spectral density function, defined by

$$J_\alpha(\epsilon) = \pi \sum_\alpha \frac{\gamma_\alpha^2}{\omega_\alpha m_\alpha} \delta(\epsilon - \omega_\alpha) . \tag{2.45}$$

This quantity can be understood as specifying 'how much states there are at energy ϵ, and how easy it is for the system to transfer energy to these states'. Making use of the shifting property of the Dirac delta function, we can rewrite the dissipation propagator

as an integral over energy:

$$\alpha^{R[A]}(\omega) = -\int_0^\infty \frac{\mathrm{d}\epsilon}{2\pi} \frac{\epsilon J_\alpha(\epsilon)}{(\omega \pm i0)^2 - \epsilon^2} \ . \tag{2.46}$$

The spectral density function specifies the type of bath we are considering. Given the form of a certain spectral density, we can (depending on how nice the specific form is) explicitly calculate the integral to find an closed-form expression for the dissipation propagator.

In this thesis, we will model our bath as an Ohmic bath. This means the bath has a linear spectral density, and in general will lead to linear dissipation terms in the quasi-classical equations of motion. Hence it can be viewed as a first order approximation to the bath behaviour. The spectral density function for an Ohmic bath is explicitly given by

$$J_\alpha(\epsilon) = \alpha_0 \epsilon \Theta(\epsilon) \ , \tag{2.47}$$

where α_0 is a proportionality constant and $\Theta(\epsilon)$ is the Heaviside step-function, indicating that we are only considering positive ϵ. Plugging this in to the equation of the propagator, we get

$$\alpha^{R[A]}(\omega) = -\alpha_0 \int_0^\infty \frac{\mathrm{d}\epsilon}{2\pi} \frac{\epsilon^2}{(\omega \pm i0)^2 - \epsilon^2} \ . \tag{2.48}$$

This integral can be evaluated by complex integration, see appendix B. The result is

$$\alpha^{R[A]}(\omega) = \pm\frac{1}{2} i\alpha_0 \omega + \alpha^{R[A]}(\omega = 0) \ . \tag{2.49}$$

The last term can be ignored: it is an ω-independent constant that does not contribute to the dynamics (again, see appendix B). To find an explicit time-dependent solution, we perform an inverse Fourier transform, giving

$$\alpha^{R[A]}(t - t') = \mathcal{F}^{-1}(\alpha^{R[A]}(\omega)) = \mp\frac{\alpha_0}{2} \delta(t - t')\partial_t \ . \tag{2.50}$$

This form of the dissipation propagator finally allows for evaluation inside the action. For now, we ignore the contributions from the Keldysh part of the propagator, because we are interested in the noiseless dynamics only. Alternatively, this disregard of fluctuations can be understood as the first order approximation of the dynamics, as the Keldysh part depends on the square of the quantum components (Kamenev 2011, p.34). Note however that noiseless dynamics is not equal to the classical trajectory: not only

quantum fluctuations are ignored; moreover, classical fluctuations as dictated by the fluctuation-dissipation theorem also vanish (one should take the limit $\hbar \to 0$ according to (Kamenev 2011, section 4.1) to find the classical dynamics including fluctuations).

Inserting the resulting expression for the dissipation propagator gives us the action

$$\mathcal{S} = -2\int_{-\infty}^{\infty} \mathrm{d}t \left[i \langle \dot{g}|g\rangle - \mathbf{S}\sigma_1 \mathbf{H} - \frac{\alpha_0}{2}\mathbf{S}\chi\dot{\mathbf{S}}\right] . \tag{2.51}$$

Here and in the following, $\chi = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, where the minus sign comes from the sign difference between $\alpha^R(t-t')$ and $\alpha^A(t-t')$. This matrix will always appear when these propagators are evaluated. This matrix does not have a special physical value; it only means the quantum component of $\mathbf{S}$ are multiplied with the classical component of $\dot{\mathbf{S}}$ and minus vice versa.

Let us rewrite the action now such that we can find the quasi-classical equations of motion. First note that

$$\int_{-\infty}^{\infty} \mathrm{d}t \left(\mathbf{S}\chi\dot{\mathbf{S}}\right) = \int_{-\infty}^{\infty} \mathrm{d}t \Big(\mathbf{S}^{cl}\dot{\mathbf{S}}^{q} - \mathbf{S}^{q}\dot{\mathbf{S}}^{cl}\Big) = \int_{-\infty}^{\infty} \mathrm{d}t \frac{\mathrm{d}}{\mathrm{d}t}\Big(\mathbf{S}^{cl}\mathbf{S}^{q}\Big) - 2\int_{-\infty}^{\infty} \mathrm{d}t \mathbf{S}^{q}\dot{\mathbf{S}}^{cl} , \tag{2.52}$$

where the first integral on the right hand side vanishes as it is a total derivative. Furthermore, the Berry phase term is given by:

$$\langle \dot{g}|g\rangle = \left\langle \dot{g}^{+}\middle|g^{+}\right\rangle - \left\langle \dot{g}^{-}\middle|g^{-}\right\rangle = -iS_i\dot{\phi}^{+}(1-\cos\theta^{+}) + iS_i\dot{\phi}^{-}(1-\cos\theta^{-}) . \tag{2.53}$$

Then rotating to Keldysh space gives

$$\langle \dot{g}|g\rangle = -2iS\left[\dot{\phi}^{cl}\sin\theta^{cl}\sin\theta^{q} + \dot{\phi}^{q}(1-\cos\theta^{cl}\cos\theta^{q})\right] . \tag{2.54}$$

Hence the action is given by

$$\mathcal{S} = -2\int_{-\infty}^{\infty} \mathrm{d}t \left[i \langle \dot{g}|g\rangle - \mathbf{S}^{q}\mathbf{H} + \alpha_0 \mathbf{S}^{q}\dot{\mathbf{S}}^{cl}\right] . \tag{2.55}$$

Note that by eq. (2.39) we have that $\mathbf{H}^{q} = 0$ while $\mathbf{H}^{cl} = \mathbf{H}$. Straightforward calcula-

tion gives the explicit forms of the classical and quantum fields:

$$\mathbf{S}^{cl} = \frac{1}{2}(\mathbf{S}^{+} + \mathbf{S}^{-}) = \begin{pmatrix} \sin\theta^{cl}\cos\theta^{q}\cos\phi^{cl}\cos\phi^{q} - \cos\theta^{cl}\sin\theta^{q}\sin\phi^{cl}\sin\phi^{q} \\ \cos\theta^{cl}\sin\theta^{q}\cos\phi^{cl}\sin\phi^{q} + \sin\theta^{cl}\cos\theta^{q}\sin\phi^{cl}\cos\phi^{q} \\ \cos\theta^{cl}\cos\theta^{q} \end{pmatrix} \; ; \quad (2.56)$$

$$\mathbf{S}^{q} = \frac{1}{2}(\mathbf{S}^{+} - \mathbf{S}^{-}) = \begin{pmatrix} \cos\theta^{cl}\sin\theta^{q}\cos\phi^{cl}\cos\phi^{q} - \sin\theta^{cl}\cos\theta^{q}\sin\phi^{cl}\sin\phi^{q} \\ \sin\theta^{cl}\cos\theta^{q}\cos\phi^{cl}\sin\phi^{q} + \cos\theta^{cl}\sin\theta^{q}\sin\phi^{cl}\cos\phi^{q} \\ -\sin\theta^{cl}\sin\theta^{q} \end{pmatrix} \; . \quad (2.57)$$

The action explicitly written in Euler angle coordinates is given by:

$$\begin{aligned} \mathcal{S} = -2\int_{-\infty}^{\infty} \mathrm{d}t \Bigg[& - S\left[\dot{\phi}^{cl}\sin\theta^{cl}\sin\theta^{q} + \dot{\phi}^{q}(1-\cos\theta^{cl}\cos\theta^{q})\right] - SH\sin\theta^{cl}\sin\theta^{q} \\ & + \alpha_0 S^2\Big[\sin\phi^{q}[\dot{\theta}^{q}\sin 2\theta^{cl}\sin\phi^{q} + \dot{\phi}^{cl}\cos\phi^{q}(\cos 2\theta^{q} - \cos 2\theta^{cl})] \\ & + \dot{\theta}^{cl}\cos^2 2\phi^{q}\sin 2\theta^{q}\Big]\Bigg] \end{aligned} \quad . \quad (2.58)$$

To derive the quasi-classical equations of motions, we can calculate the Euler-Lagrange equations. However, in this formalism, when we wish to compute the equations of motion for the classical coordinates (as opposed to the quantum coordinates), we should vary the action with respect to the quantum coordinates and regard the quantum fluctuations as small, i.e. setting the quantum coordinates to zero. This means the quasi-classical equations of motion are given by calculating

$$0 = \left(\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{\phi}^{q}} - \frac{\partial L}{\partial \phi^{q}}\right)\Bigg|_{\phi^q=\dot{\phi}^q=\theta^q=\dot{\theta}^q=0} \quad (2.59a)$$

$$0 = \left(\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{\theta}^{q}} - \frac{\partial L}{\partial \theta^{q}}\right)\Bigg|_{\phi^q=\dot{\phi}^q=\theta^q=\dot{\theta}^q=0} \quad (2.59b)$$

Computation gives

$$\theta^{q}: \qquad S\dot{\phi}\sin\theta = -SH\sin\theta + \alpha_0 S^2\dot{\theta} \quad (2.60a)$$

$$\phi^{q}: \qquad S\dot{\theta} = -S^2\alpha_0\sin\theta\dot{\phi} \quad (2.60b)$$

These are the quasi-classical equations of motion for the spins, where we dropped the

cl superscripts on the coordinates. To find a solution for $\dot{\mathbf{S}}$, we can write it again according to eq. (2.22) and substitute the solutions to the Euler-Lagrange equations in the vector:

$$\begin{aligned}\dot{\mathbf{S}} &= S \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \dot{\theta}\cos\theta \\ \dot{\phi}\sin\theta \\ \dot{\theta}\sin\theta \end{pmatrix} \\ &= SH \begin{pmatrix} \sin\theta\sin\phi \\ \sin\theta\cos\phi \\ 0 \end{pmatrix} - S^2 \begin{pmatrix} -\dot{\theta}\sin\phi - \dot{\phi}\sin\theta\cos\theta\cos\phi \\ \dot{\theta}\cos\phi - \dot{\phi}\sin\phi\sin\theta\cos\theta \\ \dot{\phi}\sin^2\theta \end{pmatrix}\end{aligned} \tag{2.61}$$

We see that the first term is the cross product with the external field $\mathbf{H} = (0, 0, H)$, and the second term is exactly the form of the cross product of $\mathbf{S}$ with its time derivative. Therefore the resulting equation of motion is

$$\dot{\mathbf{S}} = \mathbf{S} \times \mathbf{H} - \alpha_0 \mathbf{S} \times \dot{\mathbf{S}} \tag{2.62}$$

This is precisely what we would expect from a spin that dissipates energy: the first term describes the rotation around the external field, while the second term describes the alignment to the external field. In other words, we recovered the LLG-equation as described in chapter 1, see eq. (1.1). However, this is only the quasi-classical limit of our microscopic approach that in principle is able to describe the quantum dynamics of the system; first, from the path integral, one can in principle derive quantum correlation functions. Secondly, in the derivation of the quasi-classical equation of motion we ignored the quantum *and* classical fluctuations in the system. Including these would let us obtain a Heisenberg-Langevin equation.

From eqs. (2.25) and (2.62), we may see a pattern emerging: all quantities of which the classical component multiplies the quantum component of the spin in the action (see eq. (2.55)), turn into a cross product in the quasi-classical equation of motion. Indeed, this is the case in general, the proof of which is included in appendix C. This simplifies our derivation significantly: from now on, we may directly read off the quasi-classical equations of motion from the action, without intervention of the Euler coordinates.

2.4 Summary

In this chapter, we have constructed the mathematical formalism to describe dissipative spin dynamics. First, in section 2.1 we derived the spin path integral for two spins, eq. (2.12), by the use of spin coherent states. In section 2.2, we calculated the path integral for the Heisenberg Hamiltonian specifically, which in the quasi-classical limit indeed reproduces the expected equations of motion for two antiferromagnetically coupled spins. Then in section 2.3, we included dissipation to the formalism by considering the spin path integral within the Keldysh formalism, in order to couple the system to its environment according to the Caldeira-Leggett approach. By integrating out the degrees of freedom of the bath, we obtained a dissipative action from which we recovered the Gilbert damping term in the quasi-classical limit. Here, we made some important assumptions: first, we have modelled the bath by a collection of non-interacting harmonic oscillators. Second, we assumed that the bath obtains a Ohmic energy spectral density in the continuum limit. Third and lastly, we have ignored the Keldysh part of the bath propagator, which includes the quantum and classical fluctuations.

With this formalism, we can model the dissipative dynamics of interacting spin systems. In the next chapter, we will use this to model two-sublattice antiferromagnets and to describe their dissipative dynamics.

3 Dissipative Spin Systems

In the previous chapter, we considered a system of two interacting spins. In this section, we will submerge this system into a thermal environment; in other words, we will model the dissipation of such a system. Dissipation is introduced into the system by the coupling to a surrounding thermal bath of harmonic oscillators, according to the Caldeira-Leggett method, which was also developed in the previous section.

However, it is not immediately clear how this coupling should be constructed in the case of two interacting spins. One of the possible approach es is to view the two spins as independent dissipative systems. This means each spin has its own thermal surrounding to which energy is dissipated. Therefore, energy dissipation of the total system would be modelled as the dissipation of the first spin in one bath, and of the second spin in another bath. Hence this approach requires the presence of two different baths. This scenario is potentially applicable to synthetic antiferromagnets, where often the sublattices are separated by a magnetically neutral metal.(Duine et al. 2018) Depending on the properties of this barrier, the environments of the two spins may be different to certain degrees in such systems. Furthermore, this model may be applicable to bilayer Van der Waals magnets, where each magnetic layer has its own thermal environment.

Another approach would be to couple the entire system to a single thermal bath. That implies that the spins will be in the same environment, since they are coupled to the same particles. For instance, this could be a model to approximate the system of crystalline two-sublattice antiferromagnet, where one material (the antiferromagnet) is modelled as two interpenetrating sublattices. Hence, the assumption of a common thermal bath can be justified by the (close to) identical spatial and thermodynamical properties of the two sublattices.

In the following subsections, we will consider both possibilities. In section 3.1, we will derive the quasi-classical equations of motions for a system where each spin is coupled to a separate environment, while in section 3.2, we calculate the equations of motion for the system where both spins are coupled to the same bath.

3.1 Separated thermal baths

We consider the two-spin system of section 2.1 where each spin is coupled to a thermal bath and these baths are separate; this means we assume there is no connection between the environment of $\mathbf{S}_1$ and $\mathbf{S}_2$. In other words, the Hamiltonian of the baths will consists

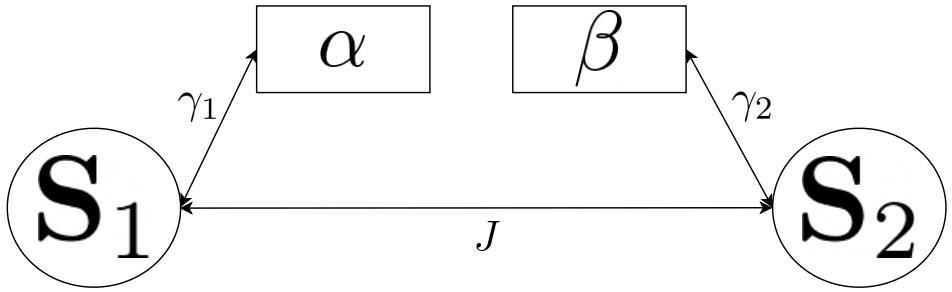

Figure 6: Diagrammatic representation of the separated bath model. Spins S_1 and S_2 dissipate energy into their own bath, denoted by α and β respectively. The constants γ_1 and γ_2 are the coupling strengths to the baths.

of two collections of oscillators:

$$\hat{H}_{HO} = \sum_{\alpha} \left(\frac{\hat{\mathbf{x}}_\alpha^2}{2m_\alpha} + \frac{m_\alpha \omega_\alpha^2}{2} \hat{\mathbf{x}}_\alpha^2 \right) + \sum_{\beta} \left(\frac{\hat{\mathbf{p}}_\beta^2}{2m_\beta} + \frac{m_\beta \omega_\beta^2}{2} \hat{\mathbf{x}}_\beta^2 \right) \ , \tag{3.1}$$

where the oscillators labelled by α will couple to $\hat{\mathbf{S}}_\mathbf{1}$ and the β-oscillators will couple to $\hat{\mathbf{S}}_\mathbf{2}$. In turn, the coupling between the spins and the bath is given by

$$\hat{H}_c = \sum_{\alpha} \gamma_{1,\alpha} \hat{\mathbf{S}}_1 \cdot \hat{\mathbf{x}}_\alpha + \sum_{\beta} \gamma_{2,\beta} \hat{\mathbf{S}}_2 \cdot \hat{\mathbf{x}}_\beta \ . \tag{3.2}$$

Note that the two spins are able to have different coupling constants with respect to their bath. In the Hamiltonian and action, we will suppress the presence of an external field $\mathbf{H}$ for notational ease. Although in general it will be there, from the previous section we already know the dependence of the (quasi-classical) dynamics of the spins on such fields, and dissipation does not affect this. Hence, we can add a term $\mathbf{S}_i \times \mathbf{H}$ in the equation of motion of spin i in the end without calculation. Furthermore, the same reasoning is true for $\hat{H}_S$, the spin interaction Hamiltonian, which is given by eq. (2.16), of which we know it will contribute a term $\pm \mathbf{S}_1 \times \mathbf{S}_2$ in the dynamical equations. Consequently, the remaining relevant part of the Hamiltonian for calculating the dissipative properties of the separate bath model is given by $\hat{H} = \hat{H}_{HO} + \hat{H}_c$.

Substituting the total Hamiltonian in the action (see eq. (2.12)), we find that the

action is given by

$$\mathcal{S} = \oint_K \mathrm{d}t \Bigg[- i \langle \dot{g}_1 | g_1 \rangle - i \langle \dot{g}_2 | g_2 \rangle - \sum_\alpha \gamma_{1,\alpha} \mathbf{S}_1 \cdot \mathbf{x}_\alpha - \sum_\beta \gamma_{2,\beta} \mathbf{S}_2 \cdot \mathbf{x}_\beta + \sum_\alpha \left(\mathbf{x}_\alpha^T G_\alpha^{-1}(t) \mathbf{x}_\alpha \right) + \sum_\beta \left(\mathbf{x}_\beta^T G_\beta^{-1}(t) \mathbf{x}_\beta \right) \Bigg] . \tag{3.3}$$

The action now contains the Berry connection of each spin, the spin-spin coupling, the two spin-bath couplings, and finally the two ensembles of harmonic oscillators. Note that the momentum dependence is already integrated out in the exact same way as in the previous section. This system is diagrammatically illustrated in figure 6. Just like in section 2.3, we can integrate out all degrees of freedom of the baths. Since the baths are not connected, we can just perform this procedure twice: once for each bath. This results in the action

$$\mathcal{S} = 2 \int_{-\infty}^{\infty} \mathrm{d}t \Bigg[- i \langle \dot{g}_1 | g_1 \rangle - i \langle \dot{g}_2 | g_2 \rangle + \int_{-\infty}^{\infty} \mathrm{d}t' \mathbf{S}_1(t) \alpha(t - t') \mathbf{S}_1(t') + \int_{-\infty}^{\infty} \mathrm{d}t' \mathbf{S}_2(t) \beta(t - t') \mathbf{S}_2(t') \Bigg] , \tag{3.4}$$

where $\alpha(t-t')$ is defined in eq. (2.37) and $\beta(t-t')$ is analogous with index replacements $\alpha \to \beta$. Due to the difference in coupling constants between $\gamma_{1,\alpha}$ and $\gamma_{2,\beta}$, α and β are in general not the same. As in section 2.3, we assume that both baths have an Ohmic spectral density and that again the quantum corrections are small. Hence we can immediately write down the action:

$$\mathcal{S} = -2 \int_{-\infty}^{\infty} \mathrm{d}t \left[i \langle \dot{g}_1 | g_1 \rangle + i \langle \dot{g}_2 | g_2 \rangle + \alpha_0 \mathbf{S}_1^q \dot{\mathbf{S}}_1^{cl} + \beta_0 \mathbf{S}_2^q \dot{\mathbf{S}}_2^{cl} \right] . \tag{3.5}$$

Here, we already performed the integration by parts according to eq. (2.52). Making use of appendix C, we can write down the equations of motion directly:

$$\dot{\mathbf{S}}_1 = \mathbf{S_1} \times \mathbf{H} - J \mathbf{S_1} \times \mathbf{S_2} - \alpha_0 \mathbf{S}_1 \times \dot{\mathbf{S}}_1 , \tag{3.6a}$$

$$\dot{\mathbf{S}}_2 = \mathbf{S_2} \times \mathbf{H} - J \mathbf{S_2} \times \mathbf{S_1} - \beta_0 \mathbf{S}_2 \times \dot{\mathbf{S}}_2 . \tag{3.6b}$$

Note that we recovered the presence of the external field $\mathbf{H}$ and the spin interaction term that we suppressed earlier for convenience. We find two LLG-equations where the spins act upon each other as external field, although with a minus sign; this is the anti-ferromagnetic interaction. The interaction terms are not affected by the presence of dissipation in the system. Due to the fact that each spin is coupled to a different bath, the proportionality constants of their Gilbert damping term are in general not equal to each other.

3.2 Shared thermal bath

Now we will derive the quasi-classical equations of motion for the case that the subsystems $\mathbf{S}_1$ and $\mathbf{S}_2$ are coupled to one single bath, that is, their thermal environment is completely identical. This system is diagrammatically presented in figure 7.

As in the previous section, we will suppress the coupling to the external field and the spin interaction in the Hamiltonian, to only recover these terms in the dynamical equations. Hence, the relevant Hamiltonian of this system consists of the dynamics of the harmonic oscillators and the coupling between the spins and the bath: $\hat{H} = \hat{H}_{HO} + \hat{H}_c$. The harmonic oscillator Hamiltonian is now given by:

$$\hat{H}_{HO} = \sum_{\alpha} \left(\frac{\mathbf{p}_\alpha^2}{2m_\alpha} + \frac{m_\alpha \omega_\alpha^2}{2} \mathbf{x}_\alpha^2 \right) , \tag{3.7}$$

The crucial difference with the previous approach is that there is only a single bath now; the α-oscillators couple to both $\hat{\mathbf{S}}_\mathbf{1}$ and $\hat{\mathbf{S}}_\mathbf{2}$ equally. Hence the coupling between the spins and the bath is given by

$$\hat{H}_c = \sum_{\alpha} \gamma_\alpha \left(\hat{\mathbf{S}}_1 \cdot \mathbf{x}_\alpha + \hat{\mathbf{S}}_2 \cdot \mathbf{x}_\alpha \right) . \tag{3.8}$$

We now want to compute the action for our total Hamiltonian. This is carried out analogously to the case for separate baths, by substituting the Hamiltonian in the action:

$$\mathcal{S} = \oint_K \mathrm{d}t \left[-i \langle \dot{g}_1 | g_1 \rangle - i \langle \dot{g}_2 | g_2 \rangle - \sum_{\alpha} \left[\gamma_\alpha \left(\hat{\mathbf{S}}_1 + \hat{\mathbf{S}}_2 \right) \cdot \mathbf{x}_\alpha - \mathbf{x}_\alpha G_\alpha^{-1}(t) \mathbf{x}_\alpha \right] \right] . \tag{3.9}$$

When we will perform a Gaussian integration over the oscillator positions now, we see we have to treat the sum of the two spins as one object now. By Gaussian integration,

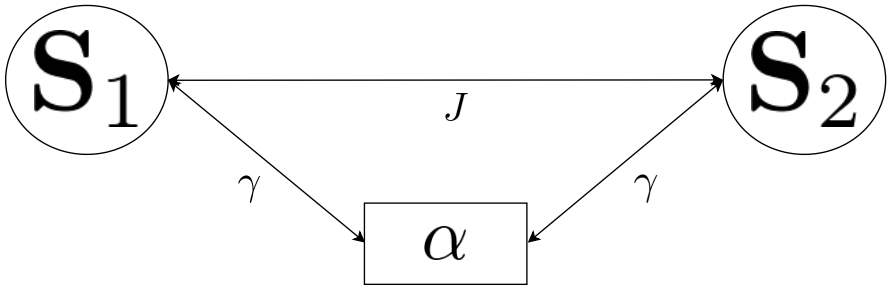

Figure 7: Diagrammatic representation of the shared bath model. Spins S_1 and S_2 dissipate energy into their shared bath, denoted by α. The constant γ is the coupling strength to the baths.

we get:

$$\begin{aligned}\mathcal{S} = &-2\int_{-\infty}^{\infty} \mathrm{d}t\; i\,\langle \dot{g}_1|g_1\rangle + i\,\langle \dot{g}_2|g_2\rangle \\ &+2\int_{-\infty}^{\infty}\mathrm{d}t\int_{-\infty}^{\infty}\mathrm{d}t'\,(\mathbf{S}_1(t)+\mathbf{S}_2(t))\,\alpha(t-t')\,(\mathbf{S}_1(t')+\mathbf{S}_2(t'))\ ,\end{aligned} \tag{3.10}$$

where we also split the Keldysh contour and rotated to (q, cl)-space. This means the propagator $\alpha(t-t')$ now couples the spins dissipatively to themselves, but also to each other. Figure 7 illustrates the dissipative structure of this action: from the figure as well as from the action it is evident that $\mathbf{S}_1$ and $\mathbf{S}_2$ are coupled to the bath in an identical matter. Assuming the bath is Ohmic, and using the results of the previous chapter, we arrive at the action

$$\mathcal{S} = -2\int_{-\infty}^{\infty}\mathrm{d}t\left[i\,\langle \dot{g}_1|g_1\rangle + i\,\langle \dot{g}_2|g_2\rangle + \alpha_0\Big(\mathbf{S}_1^q\dot{\mathbf{S}}_1^{cl}+\mathbf{S}_2^q\dot{\mathbf{S}}_2^{cl}+\mathbf{S}_1^q\dot{\mathbf{S}}_2^{cl}+\mathbf{S}_2^q\dot{\mathbf{S}}_1^{cl}\Big)\right]. \tag{3.11}$$

Again we can directly read off the equations of motion by employing the result of appendix C. The full equations of motion for two spins coupled to the same environment, including the spin interaction and coupling to the external field, are

$$\dot{\mathbf{S}}_1 = \mathbf{S_1}\times\mathbf{H} - J\mathbf{S_1}\times\mathbf{S_2} - \alpha_0\mathbf{S}_1\times\dot{\mathbf{S}}_1 - \alpha_0\mathbf{S}_1\times\dot{\mathbf{S}}_2 \tag{3.12a}$$

$$\dot{\mathbf{S}}_2 = \mathbf{S_2}\times\mathbf{H} - J\mathbf{S_2}\times\mathbf{S_1} - \alpha_0\mathbf{S}_2\times\dot{\mathbf{S}}_2 - \alpha_0\mathbf{S}_2\times\dot{\mathbf{S}}_1 \tag{3.12b}$$

Comparing to the result of previous section, we see that sharing the same bath actually makes a truly significant difference: although the first two terms are identical to the case for seperate baths (when $\alpha_0 = \beta_0$), now additional *inter-sublattice damping terms*

have appeared in the equations of motion.

3.3 Bath comparison and analysis

In the previous two sections, we calculated the dynamical equations for two different systems: first for the system where two interacting spins are each coupled to their own bath, and second for the system where two interacting spins are both coupled to the same single bath. The most apparent and significant result is the difference in presence of the inter-sublattice damping terms; only for the shared bath they occur. In this section we will further analyse this significant structural difference. First, we will take a different perspective by translating the problem into terms of the magnetisation and the Néel order. Then, we will consider the consequences of the combination of both models, which will naturally lead to the third point, which is the comparison to the phenomenological approach, discussed in section 1. Lastly, we will shortly discuss how the bath model may be advanced into a more realistic description of the environment.

3.3.1 Magnetisation and Néel order

More insight in the physical properties of both systems can be obtained when we rewrite the solutions into the average magnetisation $\mathbf{M} = (\mathbf{S}_1 + \mathbf{S}_2)/2$ and the Néel order $\mathbf{N} = \mathbf{S}_1 - \mathbf{S}_2$.

For the separate bath, the quasi-classical equations of motion in terms of the magnetisation and the Néel order are given by

$$\dot{\mathbf{M}} = -\frac{\alpha_0 + \beta_0}{2}(\mathbf{M} \times \dot{\mathbf{M}} + \frac{1}{4}\mathbf{N} \times \dot{\mathbf{N}}) + \frac{\alpha_0 - \beta_0}{4}(\mathbf{M} \times \dot{\mathbf{N}} - \mathbf{N} \times \dot{\mathbf{M}}) + \mathbf{M} \times \mathbf{H} \tag{3.13a}$$

$$\begin{aligned}\dot{\mathbf{N}} = &-(\alpha_0 - \beta_0)(\mathbf{M} \times \dot{\mathbf{M}} + \frac{1}{4}\mathbf{N} \times \dot{\mathbf{N}}) + \frac{\alpha_0 + \beta_0}{2}(\mathbf{M} \times \dot{\mathbf{N}} - \mathbf{N} \times \dot{\mathbf{M}}) \,, \\ &+ \mathbf{N} \times \mathbf{H} - 2J\mathbf{N} \times \mathbf{M} \,.\end{aligned} \tag{3.13b}$$

It seemed we have worsened the situation: it is hard to derive a picture of the general dynamics of the system from these equations. In order to compare the separate bath model to the shared bath model, and to achieve more insight in these equations of motion, let us assume that the energy spectral density of the baths α and β are the same, that is $\alpha_0 = \beta_0$. This assumption corresponds to the assumption of symmetry under sublattice permutation, that is, the assumption that the system behaves equivalent

when the sublattices are interchanged.

Alternatively this assumption may be understood as follows: the terms remaining in the equations after assuming $\alpha_0 = \beta_0$ are the strongest contributions to the general dynamical equations (3.13) in the case that the environment of the sublattices is *similar*, although not *identical*. In other words, we assume that the spectral density $J(\epsilon)$ of the two baths are (close to) equal.

Under the assumption $\alpha_0 = \beta_0$, the equations of motion reduce to

$$\dot{\mathbf{M}} = -\alpha_0(\mathbf{M} \times \dot{\mathbf{M}} + \frac{1}{4}\mathbf{N} \times \dot{\mathbf{N}}) + \mathbf{M} \times \mathbf{H} \,, \tag{3.14a}$$

$$\dot{\mathbf{N}} = -\alpha_0(\mathbf{M} \times \dot{\mathbf{N}} + \mathbf{N} \times \dot{\mathbf{M}}) - 2J\mathbf{N} \times \mathbf{M} + \mathbf{N} \times \mathbf{H} \,. \tag{3.14b}$$

From this form of the equations, the dynamics become apparent. The magnetisation evolves according to a Gilbert damping term, a Néel vector Gilbert damping term and the precession around the external field. The Néel Gilbert damping term will generally be an anti-damping term, since it points roughly in the opposite direction of the magnetisation Gilbert damping. The Néel order on the other hand does not obtain a Gilbert damping term, but terms that depend on both the magnetisation and the Néel vector, and their time derivatives. Moreover, the Néel order does not only precess around the external field, but additionally around the magnetisation through the Heisenberg coupling.

Straightforward calculation gives the equations of motion for the magnetisation and Néel vector for the shared bath model:

$$\dot{\mathbf{M}} = -2\alpha_0\mathbf{M} \times \dot{\mathbf{M}} + \mathbf{M} \times \mathbf{H} \,, \tag{3.15a}$$

$$\dot{\mathbf{N}} = -2\alpha_0\mathbf{N} \times \dot{\mathbf{M}} - 2J\mathbf{N} \times \mathbf{M} + \mathbf{N} \times \mathbf{H} \,. \tag{3.15b}$$

Comparing to the quasi-classical eqs. (3.14 for the separate bath model, we see the consequence of the presence of inter-sublattice damping terms in the shared bath model: the dynamics of the average magnetisation do not depend on the Néel vector at all now. The only contributions to the dynamics are the rotation around the external field and a Gilbert damping term, that aligns the magnetisation with an external field. In short, the dissipative dynamics of the magnetisation obey the LLG-equation. Contrarily, the Néel vector does not obtain a Gilbert damping term. Together with the precession around the external field and the magnetisation vector, there is a term $-2\alpha_0\mathbf{N} \times \dot{\mathbf{M}}$,

that is clearly induced by the bath. However, this is not a dissipative term in the sense that it pushes the Néel vector to equilibrium. One way to see this, is to insert the dynamic equation for the magnetisation into the bath-induced term. Using the triple product expansion, we find that

$$\begin{aligned}\mathbf{N}\times\dot{\mathbf{M}} &= \mathbf{N}\times\big(-2\alpha_0\mathbf{M}\times\dot{\mathbf{M}}+\mathbf{M}\times\mathbf{H}\big)\\ &= -2\alpha_0\big((\mathbf{N}\cdot\dot{\mathbf{M}})\mathbf{M}-(\mathbf{N}\cdot\mathbf{M})\dot{\mathbf{M}}\big)+\big((\mathbf{N}\cdot\mathbf{H})\mathbf{M}-(\mathbf{N}\cdot\mathbf{M})\mathbf{H}\big)\\ &= -2\alpha_0(\mathbf{N}\cdot\dot{\mathbf{M}})\mathbf{M}+(\mathbf{N}\cdot\mathbf{H})\mathbf{M}\,,\end{aligned} \tag{3.16}$$

where in the last line, two terms vanished by the fact that $\mathbf{M}$ and $\mathbf{N}$ are perpendicular when $|\mathbf{S}_1| = |\mathbf{S}_2|$, which is the case for antiferromagnets. As a consequence, the bath-induced term in the Néel vector dynamics always moves the Néel vector according to the direction of the magnetisation. Effectively this amounts to a deviation of the Néel vector, where the magnitude of deviation (from its 'dissipation-free position'; the position without the bath-induced term) is determined by α_0 and the strength and direction of the external field $\mathbf{H}$. This type of movement is also known as *nutation*: where precession is the change of the Euler angle ϕ (see figure 4 for reference), nutation is the change of the angle θ, which results in a typical 'wobbly' motion, see figure 8. However, these additional dynamics do not have a net effect of bringing the Néel vector closer to equilibrium. In other words, the bath-induced term is *not* a dissipative term.

Furthermore, upon relaxation, the magnitude of the nutation will reduce to zero. This can be seen from the equations of motion (3.15): the magnetisation relaxes to its equilibrium state (alignment with the external field) and hence it will stop precessing. As a consequence $\dot{\mathbf{M}}$ goes to zero and in turn the bath-induced term in eq. (3.15b) becomes zero; the nutation fades away. Because the Néel vector does not have a dissipative term, it will stay in precession around the external field and the magnetisation vector.

Remarkably, the Néel vector has turned out to be a dissipation-free mode in the shared bath model. For the separate bath model, there is no such dissipation-free mode; even when the two baths are similar ($\alpha_0 = \beta_0$), the extra terms in eqs. (3.14) still lead to dissipation of the Néel vector. This interesting result will be the subject of next chapter, in which we will generalise it.

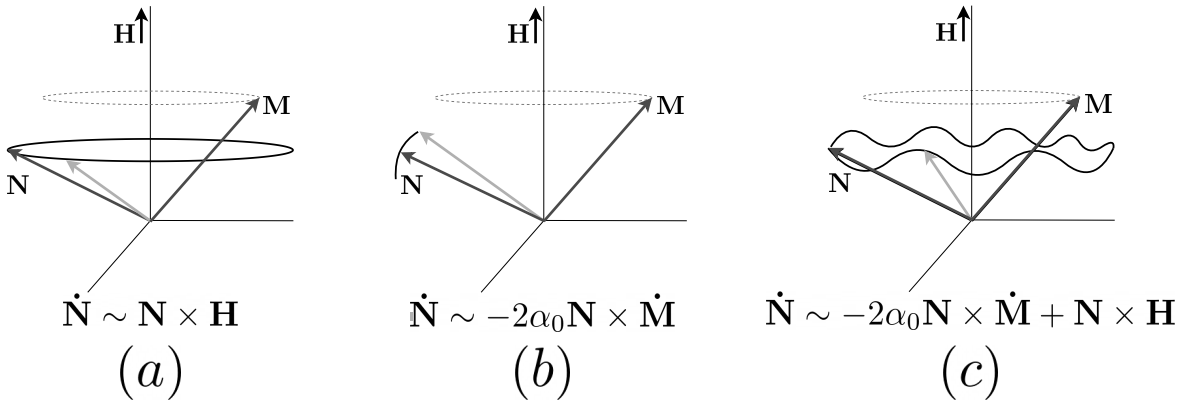

Figure 8: The effect of dissipation on the Néel vector in the shared bath model: subfigure (a) illustrates the precessing motion around **H** as a consequence of the last term in eq. (3.15b). Subfigure (b) shows the effect of the bath-induced term, which is, according to eq. (3.16) to always change the Néel vector according to the magnetisation. Combining these effects, subfigure (c) gives the resulting nutation-like behaviour. Note that for simplicity of the figure, we omitted the dynamics given by the second term in eq. (3.15b), which is the preciesion of the Néel vector around **M**. Quantities are not to scale.

3.3.2 Three baths

As we discussed above, both the shared bath model and the separate bath model are ideal cases: in real systems the sublattices probably will have *some* thermal contact, but do also not have a *completely* shared bath. Therefore, a more general model can be constructed by the combination of above models. In other words, we consider a system where each spin has its own thermal bath, denoted by α and β for $\mathbf{S}_1$ and $\mathbf{S}_2$ respectively, and a shared bath, now denoted by κ; see figure 9. The calculation of the action follows quite trivially from the composition of the previous approaches. Ignoring the non-dissipative terms for the moment, the dissipative action for this system is

$$\begin{aligned}\mathcal{S}_{diss} = 2\int_{-\infty}^{\infty} \mathrm{d}t \int_{-\infty}^{\infty} \mathrm{d}t' \Bigg[&(\mathbf{S}_1(t) + \mathbf{S}_2(t))\, \kappa(t-t')\, (\mathbf{S}_1(t') + \mathbf{S}_2(t')) \\ &+ \mathbf{S}_1(t)\alpha(t-t')\mathbf{S}_1(t') + \mathbf{S}_2(t)\beta(t-t')\mathbf{S}_2(t') \Bigg] .\end{aligned} \tag{3.17}$$

Here, the separated baths are denoted by α and β in accordance to section 3.1. The shared bath propagator is denoted by κ and likewise the proportionality constant of the energy spectral density is κ_0.

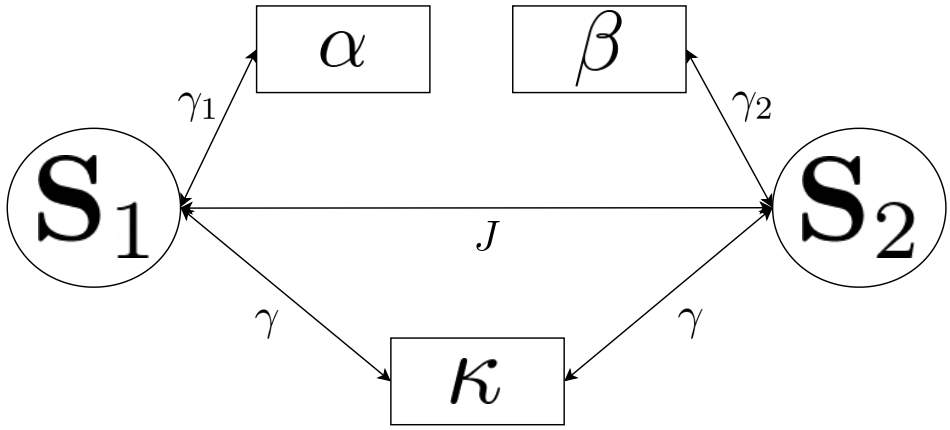

Figure 9: Diagrammatic representation of the three-baths model. Spins S_1 and S_2 dissipate energy into their own bath, which are denoted by α and β respectively. In addition, they share the third bath denoted by κ. The constants γ, γ_1 and γ_2 are the coupling strengths to the baths.

The equations of motion for this system are a straightforward combination of the equations of motion for the separated bath and the shared bath:

$$\dot{\mathbf{S}}_1 = \mathbf{S_1} \times \mathbf{H} - J\mathbf{S_1} \times \mathbf{S_2} - (\kappa_0 + \alpha_0)\mathbf{S}_1 \times \dot{\mathbf{S}}_1 - \kappa_0 \mathbf{S}_1 \times \dot{\mathbf{S}}_2 \tag{3.18a}$$

$$\dot{\mathbf{S}}_2 = \mathbf{S_2} \times \mathbf{H} - J\mathbf{S_2} \times \mathbf{S_1} - (\kappa_0 + \beta_0)\mathbf{S}_2 \times \dot{\mathbf{S}}_2 - \kappa_0 \mathbf{S}_2 \times \dot{\mathbf{S}}_1 \tag{3.18b}$$

The structure of these equations is as expected from the previous two sections: while the separate baths are only contributing to the Gilbert damping, the shared bath also contributes the inter-sublattice damping terms. Stated differently, the three bath model is a generalisation of the previous two models to which the latter are special cases. To illuminate on this point, we will assume sublattice symmetry again, such that $\alpha_0 = \beta_0$. Then, we define the *bath parameter*

$$\xi \equiv \frac{\kappa_0}{\alpha_0 + \kappa_0} \,. \tag{3.19}$$

This parameter is interpreted as a measure for how much of the environment is shared between the sublattices and it ranges from 0 to 1. When $\kappa_0 = 0$, $\xi = 0$ and the model describes fully separated baths. On the contrary, the model describes a shared bath model when $\alpha_0 = 0$, which means $\xi = 1$.

3.3.3 Comparison to the phenomenological approach

Using the definitions established above, we may rewrite the dynamical equations into

$$\dot{\mathbf{S}}_1 = -J\mathbf{S_1} \times \mathbf{S_2} - \alpha \left(\mathbf{S}_1 \times \dot{\mathbf{S}}_1 + \xi \mathbf{S}_1 \times \dot{\mathbf{S}}_2 \right) , \tag{3.20a}$$

$$\dot{\mathbf{S}}_2 = J\mathbf{S_1} \times \mathbf{S_2} - \alpha \left(\mathbf{S}_2 \times \dot{\mathbf{S}}_2 + \xi \mathbf{S}_2 \times \dot{\mathbf{S}}_1 \right) , \tag{3.20b}$$

where $\alpha = \kappa_0 + \alpha_0$ is the phenomenological damping parameter that quantifies all damping terms and $\xi = \alpha_c/\alpha$ is the inter-sublattice damping parameter.

Comparing to the phenomenological equations (1.4), it may be noted that $\xi = \alpha_c/\alpha$ and hence the bath parameter is equivalently interpreted as the system's amount of inter-sublattice damping, relative to the amount of Gilbert damping. As ξ is also the measure for how much the bath is shared, this parameter eminently illustrates that in the current microscopic approach, the inter-sublattice damping terms in antiferromagnetic dissipative dynamics is due to the bath structure of the system.

Quantitatively, the model predicts that in general, the inter-sublattice damping will never be greater than the Gilbert damping terms. This stems from the fact that in the shared bath model, these contributions are equal, and the separate bath model does only contribute to the Gilbert damping.Both models are not only the limits of the three-baths model, but also are special cases of the phenomenological equations as presented in section 1. The shared bath is the special case that $\alpha = \alpha_c$ (as for example used by (Baryakhtar and Danilevich 2015)), while the separate bath model is the special case that $\alpha_c = 0$ and α finite, as used by (Kittel 1951).

The equations of motion for the average magnetisation and Néel vector are given by

$$\dot{\mathbf{M}} = -\alpha(1+\xi)\mathbf{M} \times \dot{\mathbf{M}} - \frac{\alpha(1-\xi)}{4}\mathbf{N} \times \dot{\mathbf{N}} + \mathbf{M} \times \mathbf{H} \; ; , \tag{3.21a}$$

$$\dot{\mathbf{N}} = -\frac{\alpha(1-\xi)}{2}\mathbf{M} \times \dot{\mathbf{N}} - \frac{\alpha(1+\xi)}{2}\mathbf{N} \times \dot{\mathbf{M}} - 2J\mathbf{N} \times \mathbf{M} + \mathbf{N} \times \mathbf{H} \, , \tag{3.21b}$$

where we used the relation $\alpha_m = (\alpha + \alpha_c)/2 = \alpha(1+\xi)/2$ and $\alpha_n = (\alpha - \alpha_c)/2 = \alpha(1-\xi)/2$.

From first principle calculations it was shown that in several crystalline antiferromagnets the magnetisation damping is 1 to 3 orders of magnitude bigger that the Néel damping (H. Y. Yuan, Z. Yuan, et al. 2021). In terms of the bath parameter, this

damping ratio is in our model given by

$$\frac{\alpha_m}{\alpha_n} = \frac{1+\xi}{1-\xi} \,. \tag{3.22}$$

Hence, we see that indeed the damping ratio is determined by the bath structure, and that its value increases when ξ goes to 1, that is, when the bath is increasingly shared. Even more, we may invert this expression to find the bath parameter when the damping ratio of a particular antiferromagnet is known:

$$\xi = \frac{\frac{\alpha_m}{\alpha_n} - 1}{\frac{\alpha_m}{\alpha_n} + 1} = \frac{\alpha_m - \alpha_n}{\alpha_m + \alpha_n} \approx 1 - 2\frac{\alpha_n}{\alpha_m} \,, \tag{3.23}$$

where the approximation is valid when $\alpha_m/\alpha_n \gg 1$. This shows that the bath structure asymptotically approaches a shared bath when the damping ratio increases.

In our model, we can calculate the energy dissipation rate by making use of the phenomenological theory presented in chapter 1. For the three-baths model, the energy dissipation rate is given as function of the bath parameter by

$$\dot{E}(\xi) = 2\alpha(1+\xi)\dot{\mathbf{M}}^2 + \alpha\frac{1-\xi}{2}\dot{\mathbf{N}}^2 \,. \tag{3.24}$$

Again it becomes clear that in the shared bath model ($\xi = 1$) no energy dissipation rate is due to the Néel vector. This will be further discussed in the next chapter.

3.3.4 Advanced bath models

The three-baths model presented above accounts, by its ability to describe any combination of the shared and separate baths, for a very wide range of phenomenological dynamics. It is also possible to include some time dependence in the bath parameter $\xi(t)$ to account for baths for which the coupling to the system is time-dependent. Nevertheless, the fact remains that this model is a basic approximation to the behaviour of thermal baths in real systems.

One of the simplifying assumptions that are implicitly made in the derivation of the shared bath model as well as the separate bath model is the 'ideal' behaviour of the baths; we assume there is no interaction between the bath oscillators themselves. However, in real systems, the bath is not composed of abstract harmonic oscillators, but it consists for instance of electrons or phonons, or any other (quasi-)particles. These

baths will in general have some self-interaction; moreover they are themselves coupled to other modes in the system.

Of course the goal of our model is not to describe all the interactions of all excitations in the system; here, we are mainly interested in the dissipation of the antiferromagnetic parameters. However, taking into account further interactions in the system, it may be the case that in the separate bath model, inter-sublattice damping terms still will appear due to the coupling of the baths themselves. In other words, this possibility can be explored in first approximation by coupling the baths in the separate model. Although an extensive account of these considerations does not serve the goal of this thesis, an introduction into this model (and its problems) is given in appendix D.

3.4 Summary

In this section, we have investigated into a microscopic model for the dissipation of two-sublattice antiferromagnets. The first important result is that in our model, the phenomenologically predicted inter-sublattice damping terms have their physical origin in the presence of a shared thermal environment between the sublattices. Secondly, the most general model we considered is presented in section 3.3.2 and consists of the system coupled to three baths: both sublattices are coupled to a shared bath, while each sublattice is also coupled to its own bath. Within this model, we have defined the bath parameter ξ that indicates how much the environment is shared between the sublattices. Furthermore, this parameter was then related to the damping ratio within real antiferromagnets in eq. (3.23), which quantitatively states the hypothesis we established: the dissipative behaviour of the modes within the two-sublattice antiferromagnet is determined by the system-bath structure.

4 Dissipation-Free Modes

The results in this chapter were partly published in Seters et al. 2022.

In section 3.3.1 we recognised that in the shared bath model, the Néel vector actually does not contribute to the dissipation of the system. This result is also clear from eq. (3.24), where $\dot{E}(1)$ does not depend on the Néel order. It is important to notice that this result is unique for $\xi = 1$; in other words, the dissipation-free Néel order does not occur in any other system than the shared bath model. Reversely, it may be argued that the occurrence of such a dissipation-free mode is not a unique property of two-sublattice anti-ferromagnets, but that it is a consequence of the shared bath structure itself, regardless of the specific system. In this section, we will explore this hypothesis and argue that *there will be a dissipation-free mode when identical subsystems couple according to the shared bath model*. Additional conditions that must be satisfied will be specified in the course of the argument.

To demonstrate that the presence of dissipation-free modes is a property peculiar to the shared bath model, we will consider the dynamics of two dissipating interacting (bosonic) particles in the Keldysh formalism. From this, we will recover a dissipation-free mode of the system. Afterwards, we illustrate why one can (in hindsight) expect a dissipation-free mode in the shared bath model, and lastly, we discuss how the results in this chapter may be applied in experimental situations to engineer low-dissipation modes.

4.1 Dissipation-free modes for two coupled particles

First, we will show that in a system of two coupled quantum particles that dissipate energy into a single shared bath, a dissipation-free mode like the Néel vector for the antiferromagnet will exist. For this purpose, we consider the Hamiltonian

$$\hat{H}_o = \frac{\hat{p}_1^2}{2m} + \frac{\hat{p}_2^2}{2m} + V(\hat{q}_1, \hat{q}_2) \,, \tag{4.1}$$

where $\hat{q}_1, \hat{p}_1$ and $\hat{q}_2, \hat{p}_2$ are the positions and momenta of the two particles, respectively. For now, the potential $V(\hat{q}_1, \hat{q}_2)$ is very general; it may include any position-dependent one-particle potentials and any interaction potential. Just like for the antiferromagnetic systems in previous sections, dissipation of this composite system will be modelled by

the coupling to an environment, described as a collection of harmonic oscillators. This coupling consists of the subsystems each coupling to their shared thermal bath, since this is the bath structure in which we expect dissipation-free modes to arise. The bath Hamiltonian is given by

$$\hat{H}_b = \sum_i \left(\frac{\hat{p}_i^2}{2m_i} + \frac{m_i \omega_i^2}{2} \hat{x}_i^2 \right) , \tag{4.2}$$

and the coupling of the system to the bath by

$$\hat{H}_c = \sum_i \gamma_i \, \hat{x}_i \hat{q}_1 + \sum_i \gamma_i \, \hat{x}_i \hat{q}_2 \, . \tag{4.3}$$

In analogy to previous sections, we will construct a path integral within the Keldysh formalism. In this case however, we deal with the more familiar Feynman path integral over positions of the particles, $\mathcal{Z} = \int Dq_1 Dq_2 \exp[i\mathcal{S}]$, where the Keldysh action is given by

$$\mathcal{S} = \oint_K \mathrm{d}t \left(\frac{m}{2}(\dot{q}_1^2 + \dot{q}_2^2) - V(q_1, q_2) \right) + \oint_K \mathrm{d}t \oint_K \mathrm{d}t' \, \big(q_1(t) + q_2(t)\big) \alpha(t - t') \big(q_1(t') + q_2(t')\big) \, . \tag{4.4}$$

The first part of this action is recognised as the 'non-dissipative Lagrangian' $L = T - V$; it is simply the kinetic energy minus the potential energy of the particles. The second part of the action we will refer to as the *dissipative part* of the action, where we already integrated out all bath degrees of freedom according to section 2.3, see the derivation from eq. (2.31) to eq. (2.36). The propagator $\alpha(t - t')$ here is exactly the same dissipation propagator as defined in eq. (2.37). This is no surprise, since we have integrated out the same type of bath as in section section 2.3 and the coupling of the subsystems to the bath is analogous.

From the action, we can already guess the form of the dissipation-free mode that will appear: the dissipative part of the action only depends on the linear combination of coordinates $q_1 + q_2$. Therefore, we define the change in coordinates to the *centre-of-mass coordinate* Q and the *relative coordinate* q as

$$\begin{aligned} Q &= \frac{q_1 + q_2}{2} \, , \\ q &= q_1 - q_2 \, . \end{aligned} \tag{4.5}$$

Note that mass m does not appear in these definitions, since we assume the mass of both particles are equal. We call these coordinates the *eigenmodes* of the system.

Rewriting the action in the newly defined coordinates, we get

$$\mathcal{S} = \oint_K \mathrm{d}t \left(\frac{m}{2}(2\dot{Q}^2 + \frac{1}{2}\dot{q}^2) - \bar{V}(Q,q) \right) + \oint_K \mathrm{d}t \oint_K \mathrm{d}t'\, Q(t)\alpha(t-t')Q(t') \;, \tag{4.6}$$

where $\bar{V}(Q,q) = V(Q+q/2, Q-q/2)$. Clearly, the dissipative part of the action only depends on the centre-of-mass coordinate of the system. To derive the quasi-classical equations of motion, we first rewrite the action according to the Keldysh rotation into classical and quantum components:

$$\begin{aligned}\mathcal{S} = 2\int_{-\infty}^{\infty} \mathrm{d}t \left(\frac{m}{2}(2\dot{Q}^{cl}\dot{Q}^{q} + \frac{1}{2}\dot{q}^{cl}\dot{q}^{q}) - \frac{1}{2}\big(\bar{V}(Q^{cl}+Q^{q}, q^{cl}+q^{q}) - \bar{V}(Q^{cl}-Q^{q}, q^{cl}-q^{q})\big) \right) \\ +2\int_{-\infty}^{\infty}\mathrm{d}t \int_{-\infty}^{\infty}\mathrm{d}t' \big(Q^{cl}(t)\alpha^{A}(t-t')Q^{q}(t') + Q^{q}(t)\alpha^{R}(t-t')Q^{cl}(t') \big) \;,\end{aligned} \tag{4.7}$$

where we used the fact that in the Keldysh rotation, the dissipation propagator obtains the form given by eq. (2.42). Here we disregarded the effect of the Keldysh part of the dissipation propagator. As specified above, the (cl)-coordinates specify the quasi-classical dynamics, which are found by taking the variation of the action with respect to the quantum components, and setting the quantum components to zero afterwards. This procedure results in the following quasi-classical equations of motion:

$$m\ddot{Q}(t) = -\frac{1}{2}\frac{\partial}{\partial Q}\bar{V}(Q,q) - \int_{-\infty}^{\infty}\mathrm{d}t'\alpha^{R}(t-t')Q(t') \;; \tag{4.8a}$$

$$m\ddot{q}(t) = -2\frac{\partial}{\partial q}\bar{V}(Q,q) \;. \tag{4.8b}$$

Indeed, the effect of dissipation is only present in the quasi-classical equation for $Q(t)$; we recognise Newtons second law $\mathbf{F} = m\mathbf{a}$ in both equations, with an additional bath-induced term in the first equation.

If we would choose the bath to be Ohmic ($J(\epsilon) = \alpha_0\epsilon$), as we did in the anti-ferromagnetic case, we would find a linear dissipation term $-\alpha_0\dot{Q}(t)$ that for example characterises the dissipation associated with Brownian motion (Caldeira and Leggett 1983). However, as is clear from the structure of the equations, the fact that only the centre-of-mass coordinate is affected by the dissipation within the system is not at all

dependent on the choice of bath.

Nevertheless, on closer inspection it is not ensured that the relative coordinate q is indeed entirely unaffected by the dissipation of the system. Since the potential $\bar{V}(Q,q)$ is allowed to include interaction terms, it may be possible that the term $\partial\bar{V}(Q,q)/\partial q(t)$ depends on Q (the simplest example being $\bar{V} \sim qQ$). Therefore, we find a restriction on the space of possible systems that can obtain a dissipation-free mode: the potential must be separable in its eigenmodes according to

$$\bar{V}(Q,q) \stackrel{!}{=} \bar{V}(Q) + \bar{V}(q) = V((q_1+q_2)/2) + V(q_1-q_2)\ . \tag{4.9}$$

In other words, the relative coordinate will still obtain indirect dissipation when it is coupled to the centre-of-mass coordinate through the potential. This requirement of separability on the potential can be viewed as the requirement that the system's dynamics can be decomposed in an 'extrinsic motion' (centre-of-mass) and an 'intrinsic motion' (relative).

In multiple systems this is indeed the case. For example, one may consider two identical linearly-coupled harmonic oscillators. Their potential is given by $V(q_1,q_2) = m\omega_0^2 q_1^2 + \gamma q_1 q_2 + m\omega_0^2 q_2^2$, and one can rewrite this into the centre-of-mass coordinate and the relative coordinate as $\bar{V}(Q,q) = (2m\omega_0^2+\gamma)Q^2 + (m\omega_0^2/2 - \gamma/4)q^2$. Furthermore, if systems are coupled through a spring-like potential $V(q_1,q_2) = k(q_1-q_2)^2$ this can be written in terms of the relative coordinate as $V(q) = kq^2$. Also inverse distance potentials, such as the gravitational and Coulomb potential are trivially rewritten into a potential of the relative coordinate only.[4]

Hence, the main concern is to write the non-interacting dynamics in terms of the eigenmodes; the terms that describe the one-particle dynamics must be expressible in the composite eigenmodes Q and q nevertheless. From above discussion it is already clear that, for the kinetic energy to be separable, we require that both subsystems have identical mass. For the one-particle potential of each subsystem, more nuanced conditions hold, see appendix E.

[4]It is in this context worth mentioning that the Heisenberg Hamiltonian we used in the case of antiferromagnets also satisfies eq. (4.9): $J\mathbf{S}_1 \cdot \mathbf{S}_1 = J(\mathbf{M}\cdot\mathbf{M} - \frac{1}{4}\mathbf{N}\cdot\mathbf{N})$.

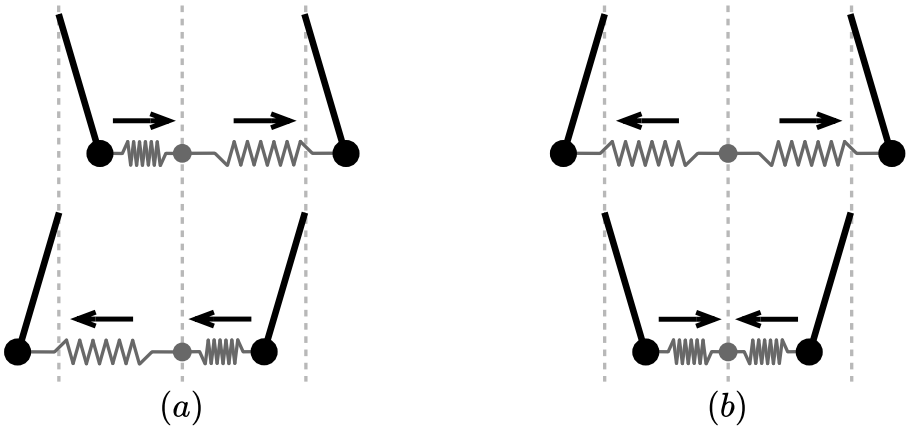

Figure 10: Illustration of dissipative and dissipation-free modes for two coupled oscillators (illustrated as pendulums). The bath degrees of freedom are illustrated by the grey ball and the linear oscillator-to-bath coupling is illustrated by springs. The oscillation is between upper and lower figures. Oscillations of the centre-of-mass coordinate (a) exert a force (black arrows) onto the bath degrees of freedom and, in turn, dissipates energy by exciting bath modes. In contrast, in relative coordinate oscillations (b) the forces on the bath modes balance each other and, in turn, the bath modes are not excited and no energy is dissipated. (Figure adopted from Seters et al. 2022)

4.2 Physical intuition on the dissipation-free modes

Both in section 3.2 and section 4.1, we found a dissipation-free mode when dissipation is modelled by the coupling to a thermal bath that is shared by both subsystems. From the second of these, one may develop a physical intuition on the nature of these dissipation-free modes. After all, we have proved that under certain (ideal) conditions, these dissipation-free modes exist, but the physical understanding remains to be shown.

In order to do so, it is useful to interpret the particles of previous subsection as being coupled (classical) pendulums. The calculation of the dynamics of two (non-dissipative) coupled pendulums is a popular problem and often involves the introduction of an in-phase coordinate and an out-of-phase coordinate to decouple the dynamical equations. Not coincidentally do these coordinates correspond to the centre-of-mass coordinate and the relative coordinate introduced here, respectively.

Interpreting the centre-of-mass dynamics as in-phase motion, it becomes clear why this is the dissipative mode: both pendulums move in unison, exerting always an equally directed force onto each bath oscillator to which they are both coupled (i.e. all the oscillators in the shared bath). As a consequence, this mode loses energy to the bath. The out-of phase motion on the other hand corresponds to the oppositely directed

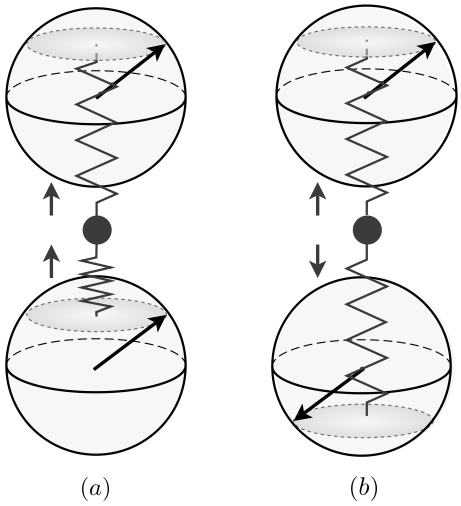

Figure 11: Illustration of dissipative and dissipation-free modes for two coupled spins; for z-components only. The bath degrees of freedom are illustrated by the grey ball and the linear spin-to-bath coupling is illustrated by springs. The motion of the average magnetization (a) exerts a force (grey arrows) onto the bath degrees of freedom and, in turn, dissipates energy by exciting bath modes. In contrast, the motion of the Néel vector (b) does not excite the bath, as forces exerted onto the bath degrees of freedom always balance each other; in turn, the Néel vector is a dissipation-free mode. (Figure adopted from Seters et al. 2022)

movement of the pendulums. This amounts to a net force of zero onto each bath oscillator; the forces are always equal but opposite in direction. In other words, the out-of-phase motion cannot lose any energy to a bath oscillator to which both pendulums are coupled in equal strength (again: the shared bath). This phenomenon is illustrated in figure 10.

In other words, the relative coordinate is a dissipation-free mode because the coupling to the bath is always orthogonal to its motion (in some abstract (Q, q)-space). Actually, this is already clear from the form of the coupling Hamiltonian, eq. (4.3). Rewriting this into the system's eigenmodes, we obtain $\hat{H}_c = \sum_i \gamma_i \hat{x}_i \hat{Q}_i$, from which it is directly clear the relative coordinate simply does not couple to the bath.

Recalling the antiferromagnet in the shared bath, we see that in a similar way only the magnetisation couples to the bath in eq. (3.8). Comparing to the system of coupled pendulums, we may now also understand from a microscopic perspective how a dissipation-free mode arises in the antiferromagnet. Since the relation of the magnetisation and the Néel vector to the sublattice spins $\mathbf{S}_1$ and $\mathbf{S}_2$ is identical to the

relation of the centre-of-mass and relative mode to the individual pendulums, the magnetisation and Néel vector can identically be interpreted as some in-phase motion and out-of-phase motion. As this concerns precessing vector dynamics in three-dimensional space now, this analogy is harder to visualise. However, as is clear from figure 11, the effect of the z-component of the spins on the bath degrees of freedom can for instance be illustrated to show the analogy to the pendulum system.

Although eq. (3.24) already showed that the presence of a dissipation-free mode was unique to the shared bath model, from this discussion one may also understand on a physical level why this is the case. If there are additional baths that are not fully shared, the relative coordinate [or Néel vector] dissipates by the fact that q_1 and q_2 [or $\mathbf{S}_1$ and $\mathbf{S}_2$] are coupled to different bath oscillators. Hence the force exerted on such an oscillator cannot be cancelled out. On the level of the Hamiltonian this amounts to the fact that a separate bath coupling (see for example eq. (3.2)) cannot be rewritten in terms of Q [or $\mathbf{M}$] only. Also the coupling strength of the subsystems to the bath should be equal; otherwise, one encounters a similar problem.

4.3 Applications

The results discussed in this section make clear that the presence of dissipation-free modes is unique to systems where subsystems couple to a shared bath only. When the subsystems also couple to some separate baths, all eigenmodes are dissipative. Moreover, we showed that the dissipation-free mode can turn into a indirectly dissipating mode, when it is coupled through the potential $V(Q, q)$ to the dissipative eigenmode.

Contrariwise, one may revert these statements to apply them in the design of physical low-dissipation modes. Although in real physical systems, the bath between subsystems is never completely shared, our results indicate that one can reduce the dissipation rate of one of the modes by designing the system such that the common bath of the subsystems increases, and the separate dissipation channels are reduced. This can for example be achieved by reducing the physical distance between the subsystems or by increasing the mean free path of excitations within the total system-bath configuration.

Furthermore, the potential of, and coupling between the subsystems must be designed in such a way that one can decompose the coupling between the systems in the eigenmodes of the composite system according to eq. (4.9), insofar that is possible. In other words, terms in the potential that are not separable in the eigenmodes must be as much suppressed as possible.

In crystalline antiferromagnets, both conditions are already full-filled to a large extent: as the sublattices are physically inter-located, it is plausible to assume that they are mainly coupled to a shared bath. Furthermore, we already noted that the Heisenberg Hamiltonian is indeed separable in the eigenmodes. As a consequence, our results may explain why the damping of the magnetisation mode is found to be much higher than the damping of the Néel order in scattering theory and first-principle calculations (Liu et al. 2017).

In real systems, one may also encounter the problem that the subsystems are not completely identical; they do not have the same mass for instance. In that case, one may define a new set of eigenmodes by introducing the possibility that the subsystems are differently coupled to the bath. For example, in the case of two interacting non-identical particles, the Hamiltonian of the particles and the system-bath coupling is given by (the Hamiltonian is written in terms of velocities rather than momenta for clarity of the argument)

$$\hat{H} = \frac{1}{2} m_1 \left(\hat{\dot{q}}_1^2 + \frac{m_2}{m_1} \hat{\dot{q}}_2^2 \right) + V(\hat{q}_1, \hat{q}_2) + \sum_i \gamma_{1,i} x_i \left(\hat{q}_1 + \frac{\gamma_{2,i}}{\gamma_{1,i}} \hat{q}_2 \right) . \tag{4.10}$$

Then, by designing the coupling of the subsystems to the bath such that approximately $\gamma_{2,i}/\gamma_{1,i} = \sqrt{m_2/m_1}$ holds, the Hamiltonian may be rewritten in the eigenmodes $Q = \hat{q}_1 + \sqrt{m_2/m_1}\hat{q}_2$ and $q = \hat{q}_1 - \sqrt{m_2/m_1}\hat{q}_2$:

$$\hat{H} = \frac{1}{4} m_1 \left(\hat{\dot{Q}}^2 + \hat{\dot{q}}^2 \right) + \bar{V}(\hat{Q}, \hat{q}) + \sum_i \gamma_{1,i} x_i \hat{Q} . \tag{4.11}$$

In other words, the non-identicality of the subsystems may be artificially compensated by differing the couplings to the bath. However, it must be noted that this has also consequences for the potential; the mass ratio m_2/m_1 will necessarily appear in the potential of q_2 and in all interaction terms between the subsystems.

Lastly, given the results for two-particle systems, one may wonder whether dissipation-free modes can also exist in dissipative N-particle systems. While this is possible in principle, these dissipation-free modes will often not correspond to physical normal modes of the system, as the form of the potential gets more restricted in more-particle systems. Nevertheless, like the two-particle system, one may use the results for N-particle systems to engineer low-dissipation modes. An overview of this generalisation is given in appendix F.

4.4 Summary

In the formulation of the dissipative dynamics of antiferromagnets, we discovered the presence of a dissipation-free mode in the system under the condition that the bath of the sublattice is completely shared. In this section, we have generalised this result, by arguing that it is not so much a peculiar property of the antiferromagnet, as it is a result of the system-bath structure in composite systems. The presence of dissipation-free modes can be traced to the linear coupling of each subsystem to the same set of bath particles. Furthermore, for a truly dissipation-free mode, it is required that the potential is separable in the eigenmodes of the system according to eq. (4.9). By illustration of two coupled pendulums, we gained physical insight into the occurrence of dissipation-free modes, and why it is a property unique to the shared bath model for dissipation. Lastly, we discussed how these observations can be used to experimentally design low-dissipation modes, and how the theory can be slightly adjusted to account for non-identical subsystems.

5 Summary and Outlook

Understanding of the dynamics of antiferromagnets is crucial in the development of antiferromagnetic spintronic devices, such as digital memory and information processing tools. In this thesis, we have tried to understand the dissipative dynamics of two-sublattice antiferromagnets from a microscopic perspective, by modelling dissipation according to the Caldeira-Leggett approach. In order to do so, we used the path integral for spin-coherent states within the Keldysh formalism, as was presented in chapter 2. The strength of the combination of these formalisms is that we can use all the advantages of the functional formulation of quantum field theory, while at the same time we are able to derive the quasi-classical dynamics from the action. Especially this last property was important in this thesis, since it allowed the comparison between our current approach and phenomenological considerations from (H. Y. Yuan, Liu, et al. 2019).

In chapter 3 multiple models for the dissipation of the two-sublattice antiferromagnet were considered: while the separate bath model describes the dissipation of each sublattice as the coupling to their own separate bath, the shared bath model considers the coupling of both sublattices to the same bath. Remarkably, the inter-sublattice damping terms, as predicted in the phenomenological approach, are only present in the shared bath model. Hence, our results indicate that the inter-sublattice damping terms are due to an environment that is shared between the two sublattices.

Furthermore, we considered the slightly more general case where there is a shared bath, as well as two separate baths. In this three-baths model, the bath parameter ξ indicates how much of the environment between the two sublattices is shared, ranging from completely separate ($\xi = 0$) to completely shared ($\xi = 1$). The resulting dynamical equations (3.20) allow direct comparison to the phenomenological equations (1.4), and we see that $\xi = \alpha_c/\alpha$: the bath parameter indicates how much inter-sublattice damping the system obtains, relative to the (intra-sublattice) Gilbert damping.

As we expect that the physically inter-located sublattices in a crystalline antiferromagnet mainly have a shared bath, our three-baths model may explain by eq. (3.23) why the damping parameter of the magnetisation mode is found to be several orders of magnitude higher than that of the Néel vector (Liu et al. 2017).

Moreover, in the shared bath model the Néel order turned out to be a dissipation-free mode of the system. As this arises from the linear coupling of the two subsystems to a shared bath, in chapter 4 we argued that the presence of dissipation-free modes

is not confined to antiferromagnets, but holds for more general composite systems in which two identical subsystems are linearly coupled to a shared bath. From these considerations, we were able to deduce the conditions necessary for a dissipation-free mode to exist. As a consequence, our results indicate how to engineer low-dissipation modes in real systems.

According to our goal, we managed to provide a more fundamental understanding of the dissipative dynamics of two-sublattice antiferromagnets: our results indicate physical reasons for the variety of values for phenomenological damping parameters that are chosen in different systems, and we have established a quantitative connection between the bath parameter of the system and the damping parameters.

However, it has already been mentioned in 3.3.4 that the harmonic oscillator bath is a first approximation to the behaviour of environments in real systems. Therefore, the presented models may be advanced by replacing the oscillator bath by an electron bath or a phonon bath, for a more accurate description of metallic or insulating antiferromagnets, respectively. Furthermore, an interesting direction to explore is the generalisation to the so-called interacting bath models. An interesting approach is to include an interaction between the baths in the separate bath model. The motivation for these models is this: the current results imply that the inter-sublattice damping terms are a consequence of the thermal connection between the two sublattices. Hence, one expect that similar terms will arise when there is some indirect thermal coupling, that is, the coupling of the two baths. An introduction to this topic is presented in appendix D.

Another extension to the bath model is to consider the effects of non-Ohmic baths. The Ohmic bath is a linear approximation to the bath behaviour that recovers linear dissipation. However, it is clear that it is not physical; the energy spectral density $J(\epsilon) = \alpha_0 \epsilon$ diverges at high energy. Hence, in real systems, there is necessarily a cut-off at high frequencies. Recently, it has been shown how the high-frequency behaviour of the bath leads to the presence of bath-induced spin inertia, that result in nutation effects in the dynamics of ferromagnets (Quarenta 2022). In a similar way, one may find these effects in antiferromagnets. The basic concepts and mathematical methods are exposed in appendix G. In short, the high-frequency behaviour of the bath gives rise to new dynamics. Even more, within this method, it is also possible to include different types of low-frequency behaviour by changing the Ohmic spectral density to sub-Ohmic or super-Ohmic densities ($J(\epsilon) = \alpha_0 \epsilon^p$ with $p < 1$ or $p > 1$ respectively).

Lastly, in the entire thesis, we ignored the fluctuations that according to the

dissipation-fluctuation theorem necessarily occur in dissipative systems. However, it has been shown that multiplicative noise can induce a change in the expected trajectory (Volpe and Wehr 2016). In other words, the noiseless dynamics we have considered are not necessarily equivalent to the classical dynamics in the case that this *Stratonovich drift* is present in the system. Hence it is worthwhile to consider the inclusion of noise in our results. This can either be done by considering classical fluctuations only (by taking the limit $\hbar \to 0$ in the action, see (Kamenev 2011, chapter 4)) or both classical and quantum fluctuations, by considering the full contribution of the Keldysh part of the bath propagator $\alpha(t - t')$.

We have argued that the existence of dissipation-free modes is a property of the shared bath model; when identical subsystems are coupled to a shared bath, a dissipation-free mode will arise. In section 3.3 and chapter 4, and similarly in (Seters et al. 2022), we demonstrated the presence of dissipation-free modes in antiferromagnets and composite systems of identical (bosonic) particles. In other words, we established that our hypothesis holds in the case of spins coupling to the bath by minimal spin-position coupling and in the case of particles coupling to the bath by minimal position-position coupling. Interestingly, a similar result was already found for qubits that are coupled to the same environment (Duan and Guo 1998). However, we have not proved a general theorem that applies to *all* composite systems and types of coupling (and it is hard to imagine there could be such a theorem). Consequently, it is up to further research to theoretically test this hypothesis for other physical systems. For instance, one might consider systems of fermions and velocity-dependent coupling to the bath.

A Berry Connections

In this appendix, we derive the form of the Berry connections that appear in the spin path integral in terms of the Euler angles that parameterise the underlying Hilbert space. In order to do so, we write the spin coherent states in their Euler angle representation explicitly:

$$\begin{aligned}\langle \dot{g}|g\rangle &= \langle\uparrow| \partial_t(e^{i\theta S^y}e^{i\phi S^z})e^{-i\phi S^z}e^{i\theta S^y}|\uparrow\rangle \\ &= i\langle\uparrow|\left(e^{i\theta S^y}\dot{\phi}\hat{S}^z + \dot{\theta}\hat{S}^y e^{i\theta S^y}\right)e^{i\phi S^z}e^{-i\phi S^z}e^{-i\theta S^y}|\uparrow\rangle \\ &= i\dot{\phi}\langle\uparrow|e^{i\theta S^y}\hat{S}^z e^{-i\theta S^y}|\uparrow\rangle + i\dot{\theta}\langle\uparrow|\hat{S}^y|\uparrow\rangle \\ &= i\dot{\phi}\langle\uparrow|e^{i\theta S^y}\hat{S}^z e^{-i\theta S^y}|\uparrow\rangle \ .\end{aligned} \tag{A.1}$$

In the last line we used the fact that $S^y|g\rangle = 0$. Using the identity (Altland and Simons 2010, p. 138)

$$e^{-i\phi S^i}S^j e^{i\phi S^i} = S^j\cos\phi + \epsilon_{ijk}S^k\sin\phi \quad (i\neq j)\ , \tag{A.2}$$

we get

$$\langle\dot{g}|g\rangle = i\dot{\phi}\cos\theta\langle\uparrow|\hat{S}^z|\uparrow\rangle - i\sin\theta\langle\uparrow|\hat{S}^x|\uparrow\rangle = i\dot{\phi}S\cos\theta, \tag{A.3}$$

where $\hat{S}^x|g\rangle = 0$ and S is the eigenvalue of $\hat{S}^z$. We could do the same calculation for $\langle\dot{\tilde{g}}|\tilde{g}\rangle$ which, due to the extra gauge term $e^{-i\psi\hat{S}_3}$ would result in

$$\langle\dot{\tilde{g}}|\tilde{g}\rangle = iS(\dot{\psi}+\dot{\phi}\cos\theta)\ . \tag{A.4}$$

Choosing the gauge $\dot{\psi} = 0$ gives back the previous result. However, the gauge fixing condition $\dot{\psi} = -\dot{\phi}$ is much more usual, such that we get

$$\langle\dot{\tilde{g}}|\tilde{g}\rangle = -iS\dot{\phi}(1-\cos\theta)\ . \tag{A.5}$$

Of course this does not affect the equations of motion, as this term merely adds a total derivative to the action.

The expectation value of a spin operator is defined by

$$S_i \equiv \langle\tilde{g}|\hat{S}_i|\tilde{g}\rangle \tag{A.6}$$

and it follows from eq. (A.2) and writing $|g\rangle$ in terms of the Euler angles, that

$$\mathbf{S}_i = S_i \begin{pmatrix} \sin\theta_i \cos\phi_i \\ \sin\theta_i \sin\phi_i \\ \cos\theta_i \end{pmatrix} . \tag{A.7}$$

B Ohmic Dissipation Propagator

The dissipation propagator for an Ohmic spectral density $J(\epsilon) = \alpha_0 \epsilon \Theta(\epsilon)$ is given by

$$\alpha^{R[A]}(\omega) = \alpha_0 \int_0^\infty \frac{\mathrm{d}\epsilon}{2\pi} \frac{\epsilon^2}{\epsilon^2 - (\omega \pm i\eta)^2} \,. \tag{B.1}$$

Since the integrand is symmetric with respect to ϵ, we can equivalently calculate half of the integral over the entire real domain:

$$\alpha^{R[A]}(\omega) = \frac{\alpha_0}{2} \int_{-\infty}^\infty \frac{\mathrm{d}\epsilon}{2\pi} \frac{\epsilon^2}{\epsilon^2 - (\omega \pm i\eta)^2} \,. \tag{B.2}$$

Moreover, extending the domain of the integrand to the complex plane, we recognise that it exhibits two simple poles; one at $\epsilon = \omega \pm i\eta$ and on at $\epsilon = -\omega \mp i\eta$. Hence we can make use of the residue theorem to calculate this integral. The contour is given by the union of the arc integral with a radius limiting to infinity, and the symmetric integral over the real line with bounds equally limiting to infinity:

$$\alpha^{R[A]}(\omega) = \frac{\alpha_0}{2} \left(\oint - \int_\Gamma \right) \frac{\mathrm{d}\epsilon}{2\pi} \frac{\epsilon^2}{\epsilon^2 - (\omega \pm i\eta)^2} \,. \tag{B.3}$$

The contour integral only encloses one of the two poles; the result is given by

$$\oint \frac{\mathrm{d}\epsilon}{2\pi} \frac{\epsilon^2}{(\epsilon - \omega \mp i\eta)(\epsilon + \omega \pm i\eta)} = i\mathrm{Res}(\epsilon = \omega \pm i\eta) = \pm \frac{i}{2}(\omega \pm i\eta) \,. \tag{B.4}$$

Note that the advanced propagator picks up an extra minus sign because the contour integral for the negative imaginary pole is oppositely directed to the contour of the positive imaginary pole, see figure 12. The arc integral is calculated as:

$$\int_0^\pi \frac{\mathrm{d}\theta}{2\pi} \frac{i|\epsilon| e^{3i\theta}}{e^{2i\theta} - \frac{\omega \pm i\eta}{|\epsilon|}^2} = \frac{|\epsilon|}{2\pi} \left(e^{i\theta} - \frac{\omega \pm i\eta}{|\epsilon|} \tanh^{-1}\left(\frac{|\epsilon| e^{i\theta}}{\omega \pm i\eta} \right) \right) \Bigg|_0^\pi \tag{B.5}$$

$$= -\frac{|\epsilon|}{\pi} + \frac{\omega \pm i\eta}{\pi} \tanh^{-1}\left(\frac{|\epsilon|}{\omega \pm i\eta} \right) \,. \tag{B.6}$$

The arc-integral is then found by taking the radius $|\epsilon|$ within this expression to infinity:

$$\lim_{|\epsilon| \to \infty} \left\{ -\frac{|\epsilon|}{\pi} + \frac{\omega \pm i\eta}{\pi} \tanh^{-1}\left(\frac{|\epsilon|}{\omega \pm i\eta} \right) \right\} = -\frac{1}{\pi} \lim_{|\epsilon| \to \infty} |\epsilon| \mp \frac{i}{2}(\omega \pm i\eta) \,, \tag{B.7}$$

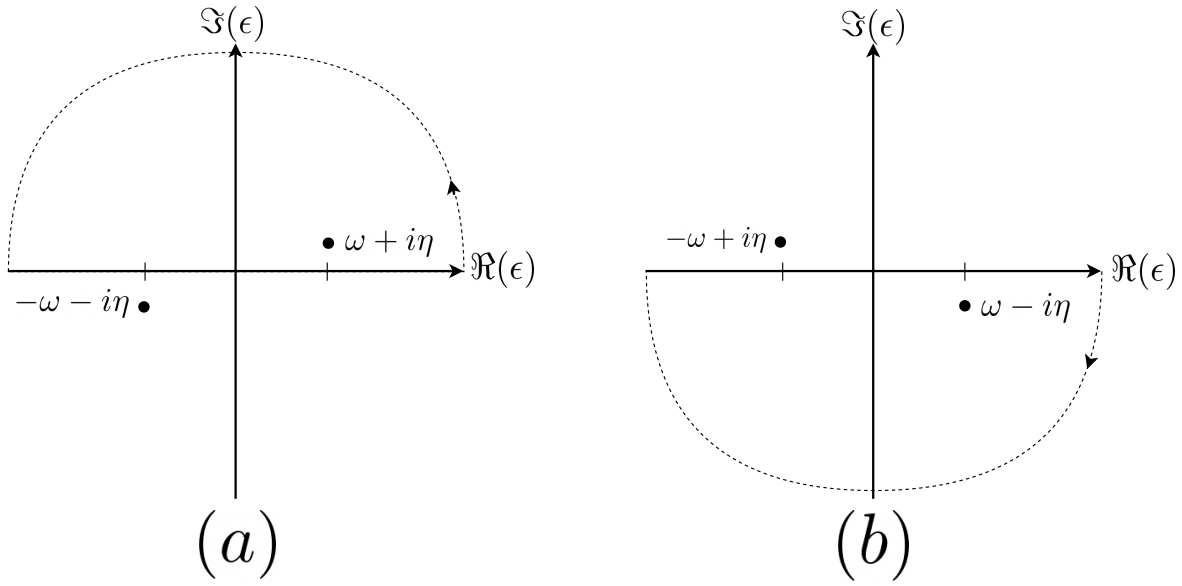

Figure 12: The contour integral over the complex plane for the retarded propagator (a) and the advanced propagator (b). Both pick up the contribution of only one pole and of the arc at infinity.

where we used that $\lim_{|\epsilon|\to\infty} \tanh^{-1}\left(\frac{|\epsilon|}{\omega \pm i\eta}\right) = \mp\frac{\pi}{2}$. Therefore the propagator results in

$$\alpha^{R[A]}(\omega) = \pm\frac{i\alpha_0}{2}(\omega \pm i\eta) + \alpha^{R[A]}(\omega = 0) \ , \tag{B.8}$$

where we defined $\alpha^{R[A]}(\omega = 0) = \frac{\alpha_0}{2\pi}\lim_{\epsilon\to\infty}\epsilon$. Although this term is infinite, there is no ω-dependence, and as a consequence, it does not affect the dynamics of the system in question. From eq. (2.41), it is clear that this term contributes a term proportional to $\mathbf{S}(t)\times\mathbf{S}(t)$, which is evidently zero by the nature of the cross-product. When considering the coupling of two spins, the term proportional to $\mathbf{S}_1(t)\times\mathbf{S}_2(t)$ that the infinite constant would produce can be absorbed by redefinition of the Heisenberg coupling constant J.

Moreover, in section G.1 we introduce a renormalised coupling to the bath, that produces the same dynamics. This renormalisation means that the infinite term does not appear at all. Hence, we are justified to ignore the infinite contribution here.

C Shortcut to the Equations of Motion

From the derivations in section 2.2 and 2.3 it becomes apparent that all inner products of $\mathbf{S}^q$ with any classical field $\mathbf{A}^{cl}$ in the action will correspond to a cross product of $\mathbf{S}^{cl}$ with $\mathbf{A}^{cl}$ in the quasi-classical equations of motion. In this appendix, we prove that this relation is true in general. This result is very useful, since it allows us to directly read off the equations of motion from the action instead of doing increasingly complicated calculations in terms of the Euler coordinates. For this proof we consider the action $S = \oint_K \mathrm{d}t L$, where the Lagrangian after the Keldysh rotation is given in terms of the classical and quantum fields by

$$L = i\langle \dot{g}|g\rangle + \mathbf{S}\sigma_1\mathbf{A} = -S\left[\dot{\phi}^{cl}\sin\theta^{cl}\sin\theta^q + \dot{\phi}^q(1-\cos\theta^{cl}\cos\theta^q)\right] + \mathbf{S}^q\cdot\mathbf{A}^{cl} + \mathbf{S}^{cl}\cdot\mathbf{A}^q\ . \tag{C.1}$$

To find the most general result, we will assume that the vector $\mathbf{A} = (\mathbf{A}^{cl}, \mathbf{A}^q)$ can be any vector quantity or sum of vector quantities, including $\mathbf{S}$ itself or any order derivative of $\mathbf{S}$. First note that

$$\mathbf{A}^{[cl/q]}(x^{cl}, x^q) = \frac{1}{2}(\mathbf{A}^+(x^+) \pm \mathbf{A}^-(x^-))\ . \tag{C.2}$$

Therefore, when we calculate the partial derivative to any quantum coordinate x, we find that

$$\left.\frac{\partial\mathbf{A}^{[cl/q]}}{\partial x^q}\right|_{x^q=0} = \frac{1}{2}\left(\frac{\partial\mathbf{A}^+}{\partial x^+}\frac{\partial x^+}{\partial x^q} \pm \frac{\partial\mathbf{A}^-}{\partial x^-}\frac{\partial x^-}{\partial x^q}\right)\Bigg|_{x^q=0} = \frac{1}{2}\left(\frac{\partial\mathbf{A}^+}{\partial x^+} \mp \frac{\partial\mathbf{A}^-}{\partial x^-}\right)\Bigg|_{x^q=0}\ , \tag{C.3}$$

where $x^{[+/-]} = x^{cl} \pm x^q$. Hence, we see that the operation of setting the quantum component to zero at the end of the calculation is equivalent to relabelling all $+$-fields and $-$-fields to classical fields. Explicitly, this means

$$\left.\frac{\partial\mathbf{A}^{cl}}{\partial x^q}\right|_{x^q=0} = 0\ , \qquad\qquad \left.\frac{\partial\mathbf{A}^q}{\partial x^q}\right|_{x^q=0} = \frac{\partial\mathbf{A}^{cl}|_{x^q=0}}{\partial x^{cl}} = \frac{\partial\mathbf{A}}{\partial x}\ . \tag{C.4}$$

Therefore we see we can ignore any derivative of a classical field and treat the derivative of a quantum field as 'normal' derivatives. Moreover,

$$\mathbf{A}^{cl}(x^{cl}, x^q)\Big|_{q=0} = \frac{1}{2}(\mathbf{A}^+(x^{cl}) + \mathbf{A}^-(x^{cl})) = \mathbf{A}(x)\ . \tag{C.5}$$

Going back to our problem, we will split the Lagrangian and rewrite the Euler-Lagrange equations as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial i\,\langle \dot{g}|g\rangle}{\partial \dot{\phi}^q}\bigg|_{q=0} = \bigg(\frac{\partial \mathbf{S}^q}{\partial \phi^q}\cdot\mathbf{A}^{cl} + \frac{\partial \mathbf{S}^{cl}}{\partial \phi^q}\cdot\mathbf{A}^q + \mathbf{S}^q\cdot\left[\frac{\partial \mathbf{A}^{cl}}{\partial \phi^q} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial \mathbf{A}^{cl}}{\partial \dot{\phi}^q}\right] + \mathbf{S}^{cl}\cdot\left[\frac{\partial \mathbf{A}^q}{\partial \phi^q} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial \mathbf{A}^q}{\partial \dot{\phi}^q}\right]\bigg)\bigg|_{q=0}\,, \tag{C.6}$$

$$-\frac{\partial i\,\langle \dot{g}|g\rangle}{\partial \theta^q}\bigg|_{q=0} = \bigg(\frac{\partial \mathbf{S}^q}{\partial \theta^q}\cdot\mathbf{A}^{cl} + \frac{\partial \mathbf{S}^{cl}}{\partial \theta^q}\cdot\mathbf{A}^q + \mathbf{S}^q\cdot\left[\frac{\partial \mathbf{A}^{cl}}{\partial \theta^q} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial \mathbf{A}^{cl}}{\partial \dot{\theta}^q}\right] + \mathbf{S}^{cl}\cdot\left[\frac{\partial \mathbf{A}^q}{\partial \theta^q} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial \mathbf{A}^q}{\partial \dot{\theta}^q}\right]\bigg)\bigg|_{q=0}\,, \tag{C.7}$$

where $'q'$ refers to all quantum coordinates. Immediately we see the last two terms of both equations are zero, as they are the Euler-Lagrange equations that describe the independent dynamics of the general vector $\mathbf{A}$ with respect to the coordinate ϕ. Moreover, applying the rules we found above and calculating the left hand side, the Euler-Lagrange equations reduce to

$$-S\dot{\theta}\sin\theta = \frac{\partial \mathbf{S}}{\partial \phi}\cdot\mathbf{A}\,, \tag{C.8}$$

$$S\dot{\phi}\sin\theta = \frac{\partial \mathbf{S}}{\partial \theta}\cdot\mathbf{A}\,. \tag{C.9}$$

The derivatives of $\mathbf{S}$ are given by

$$\frac{\partial \mathbf{S}}{\partial \phi} = \begin{pmatrix} -\sin\theta\sin\phi \\ \sin\theta\cos\phi \\ 0 \end{pmatrix}\,, \qquad \frac{\partial \mathbf{S}}{\partial \theta} = \begin{pmatrix} \cos\theta\cos\phi \\ \cos\theta\sin\phi \\ -\sin\theta\,. \end{pmatrix} \tag{C.10}$$

Like before, we want to derive an expression for $\dot{\mathbf{S}}$, so according to eq. (2.22) we rewrite

$$\dot{\mathbf{S}} = S\begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \dot{\theta}\cos\theta \\ \dot{\phi}\sin\theta \\ \dot{\theta}\sin\theta \end{pmatrix}$$

$$
\begin{aligned}
&= S \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta\sin\phi A^x - \cos\theta\cos\phi A^y \\ \cos\theta\cos\phi A^x + \cos\theta\sin\phi A^y - \sin\theta A^z \\ \sin\theta\sin\phi A^x - \sin\theta\cos\phi A^y \end{pmatrix} \\
&= -\mathbf{S} \times \mathbf{A} \,,
\end{aligned}
\tag{C.11}
$$

where in the second line we inserted the Euler-Lagrange equations and the last line follows from straightforward computation. So, indeed we have verified that the inner product with a vector $\mathbf{A}$ in the action corresponds to a cross product in the quasi-classical equations of motion. The vector $\mathbf{A}$ can be any vector, including $\mathbf{S}$ itself or any order derivative, since the independent dynamics of $\mathbf{A}$ are cancelled by its own dynamic equation. Concluding, we have proved that we can directly read of the equations of motion for $\mathbf{S}$ when we can write the action as the Berry connection plus a series of inner products with $\mathbf{S}$.

D Interacting Baths

In section 3.3.2, we modelled the dissipation of an antiferromagnet by the coupling to three baths; each sublattice coupled to its own bath and both coupled to a shared bath, where the bath parameter ξ is indicative for how much the sublattices share their environment. Although this model is fairly general, one may also consider a more advanced model, where each sublattice dissipates into their separate bath, but these bath are in turn interacting. Physically, one may for example imagine that the spins couple to a phonon or electron modes, which are themselves interacting within the material. In this section, we will present a first attempt for such a model. However, in its current form, the model may be considered unsatisfactory, as tension arises between different assumptions within the model. After the initial exposition to the model, these questions will be discussed.

D.1 The interacting bath model

In this section we will explicate the interacting bath model as described above. The basic idea is illustrated in figure 13; each spin is coupled to a bath, and moreover these baths are coupled to each other. Like previous sections, the bath is modelled as a collection of harmonic oscillators and coupling of the spins to the bath is described by minimal coupling. In terms of a Keldysh action, this system is described by

$$\begin{aligned}\mathcal{S}_{diss} = \oint_K \mathrm{d}t \Bigg[&\sum_\alpha \left[-\gamma_1 \mathbf{S}_1 \cdot \mathbf{x}_\alpha + \mathbf{x}_\alpha (G_1^{-1})_\alpha(t) \mathbf{x}_\alpha \right] \\ &+ \sum_\beta \left[-\gamma_2 \mathbf{S}_2 \cdot \mathbf{x}_\beta + \mathbf{x}_\beta (G_2^{-1})_\beta(t) \mathbf{x}_\beta \right] - \sum_{\alpha,\beta} \left(\mathbf{x}_\alpha^T \gamma_{\alpha\beta} \mathbf{x}_\beta \right) \Bigg] .\end{aligned} \tag{D.1}$$

Here, the first sum over α describe the bath dynamics of first bath and the coupling to $\mathbf{S}_1$ with coupling constant γ_1. The sum over β includes the similar bath dynamics of the second bath and the coupling to $\mathbf{S}_2$ with coupling constant γ_2. The propagators $(G_1^{-1})_\alpha(t)$ and $(G_2^{-1})_\beta(t)$ are defined by eq. (2.34). Then the two baths are minimally coupled to each other by the last sum over α and β, where the coupling constants $\gamma_{\alpha\beta}$ indicate the coupling between a particle α and a particle β. Note that we did not include the Heisenberg interaction and the coupling to effective fields, as we are only interested in the dissipative structure of the action in this appendix.

Now we would like to perform a Gaussian integration over all bath variables. In

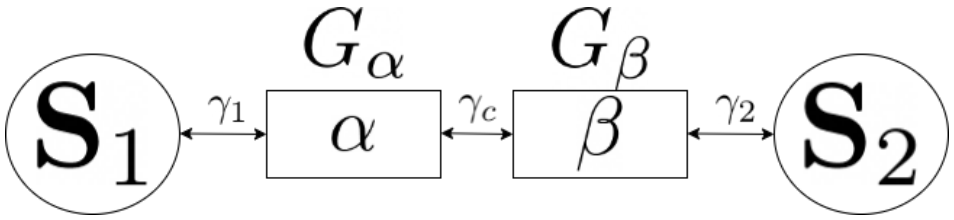

Figure 13: Diagrammatic representation of the interacting bath model. Spins S_1 and S_2 dissipate energy into their own bath, denoted by α and β respectively. The constants γ_1 and γ_2 are the coupling strengths to the baths. Furthermore, the baths are minimally coupled with coupling constant γ_c.

order to integrate both baths simultaneously, we rewrite the action into a vector form with vectors of dimension 2N with the first N positions refer to the particles in the first bath and the rest correspond to particles in the second bath. This gives us the action

$$\mathcal{S}_{diss} = \oint_K \mathrm{d}t \left[- \begin{pmatrix} \mathbf{X}_\alpha \\ \mathbf{X}_\beta \end{pmatrix}^T \begin{pmatrix} \gamma_1 \mathbf{1} \mathbf{S}_1 \\ \gamma_2 \mathbf{1} \mathbf{S}_2 \end{pmatrix} + \begin{pmatrix} \mathbf{X}_\alpha \\ \mathbf{X}_\beta \end{pmatrix}^T \begin{pmatrix} \mathbf{G}_1^{-1} & \mathbf{\Gamma} \\ 0 & \mathbf{G}_2^{-1} \end{pmatrix} \begin{pmatrix} \mathbf{X}_\alpha \\ \mathbf{X}_\beta \end{pmatrix} \right], \tag{D.2}$$

where $\mathbf{X}_\alpha = (\mathbf{x}_{\alpha=1}, \ldots, \mathbf{x}_{\alpha=N})^T$ and $\mathbf{X}_\beta = (\mathbf{x}_{\beta=1}, \ldots, \mathbf{x}_{\beta=N})^T$ are vectors of dimension N, containing all degrees of freedom of the bath. Furthermore, with a some abuse of notation, $\mathbf{1}$ is an N-dimensional vector with 1 at each entry, to ensure the right matrix multiplication with the spin vectors. The matrices $\mathbf{G}_1^{-1} = \mathrm{Diag}(\{(G_1^{-1})_\alpha\})$ and $\mathbf{G}_2^{-1} = \mathrm{Diag}(\{(G_2^{-1})_\beta\})$ are matrices with the propagators for all bath oscillators on the diagonal. Lastly, the *interaction matrix* $\mathbf{\Gamma}$ is defined by:

$$-\sum_{\alpha,\beta} \left(\mathbf{x}_\alpha^T \gamma_{\alpha\beta} \sigma_1 \mathbf{x}_\beta \right) = \mathbf{X}_\alpha^T \mathbf{\Gamma} \mathbf{X}_\beta \, . \tag{D.3}$$

In words, it is the block matrix with all interactions between the baths. Integrating out the harmonic oscillators by Gaussian integration (see eq. 2.31)) will give us the action

$$\mathcal{S}_{diss} = 2 \int_{-\infty}^{+\infty} \mathrm{d}t \left[- \frac{1}{4} \begin{pmatrix} \gamma_1 \mathbf{1} \sigma_1 \mathbf{S}_1 \\ \gamma_2 \mathbf{1} \sigma_1 \mathbf{S}_2 \end{pmatrix}^T \begin{pmatrix} \mathbf{G}_1 & -\mathbf{G}_1 \mathbf{\Gamma} \mathbf{G}_2 \\ 0 & \mathbf{G}_2 \end{pmatrix} \begin{pmatrix} \gamma_1 \mathbf{1} \sigma_1 \mathbf{S}_1 \\ \gamma_2 \mathbf{1} \sigma_1 \mathbf{S}_2 \end{pmatrix} \right], \tag{D.4}$$

where we used the fact that

$$\begin{pmatrix} \mathbf{G}_1^{-1} & \mathbf{\Gamma} \\ 0 & \mathbf{G}_2^{-1} \end{pmatrix}^{-1} = \begin{pmatrix} \mathbf{G}_1 & -\mathbf{G}_1 \mathbf{\Gamma} \mathbf{G}_2 \\ 0 & \mathbf{G}_2 \end{pmatrix} . \tag{D.5}$$

In the action, the diagonal matrix entries will just result in the dissipative self-coupling of $\mathbf{S}_1$ and $\mathbf{S}_2$ respectively, such that under the assumption of Ohmic baths, we recover Gilbert damping terms in the quasi-classical equations. Hence, only the mixed term is new and of interest; this signifies that the two spins are coupled. Since this coupling depends on the interaction matrix $\mathbf{\Gamma}$, this coupling derives from the interaction of the baths. We can define the *interacting bath propagator*:

$$\Delta = \frac{1}{4}\sum_{i,j} \sigma_1 (\mathbf{G}_1)_{ii} \mathbf{\Gamma}_{ij} (\mathbf{G}_2)_{jj} \sigma_1 \ . \tag{D.6}$$

Using the Fourier transform of the Green's functions, as calculated in eq. (2.43), this quantity is explicitly given by

$$\Delta^{R[A]}(\omega) = -\sum_{i,j} \frac{\gamma_{ij}}{m_{\alpha_i} m_{\beta_j}} \frac{1}{(\omega \pm i\eta)^2 - \omega_{\alpha_i}^2} \frac{1}{(\omega \pm i\eta)^2 - \omega_{\beta_j}^2} \ . \tag{D.7}$$

Due to the unknown coupling constants γ_{ij} between the two baths in this expression, it is hard to obtain a simpler form of this equation. As a consequence, we will need to make a simplificating assumption on the interaction of the baths. The most obvious and basic assumption is that of equal coupling; if all the particles in the baths are evenly strongly coupled to each other, that is, $\gamma_{ij} \equiv \gamma_c$ is constant, this equation is simply the product of the two bath propagators. Introducing the bath spectral densities $J_\alpha(\epsilon)$ and $J_\beta(\epsilon)$ of the first and second bath respectively allows as to rewrite the interacting bath propagator as

$$\Delta^{R[A]}(\omega) = -4\gamma_c \left(\int_0^\infty \frac{\mathrm{d}\epsilon}{2\pi} \frac{\epsilon J_\alpha(\epsilon)}{(\omega \pm i\eta)^2 - \epsilon^2} \right) \left(\int_0^\infty \frac{\mathrm{d}\epsilon}{2\pi} \frac{\epsilon J_\beta(\epsilon)}{(\omega \pm i\eta)^2 - \epsilon^2} \right) \ . \tag{D.8}$$

Focusing on the case of Ohmic baths, from appendix B we know the dissipation propagator is of the form:

$$\alpha^{R[A]}(\omega) = \pm \frac{i\alpha_0}{2}(\omega \pm i\eta) + \frac{\alpha_0}{2\pi} \lim_{\epsilon \to \infty} \epsilon \ . \tag{D.9}$$

Then the interacting bath propagator becomes

$$\begin{aligned}\Delta^{R[A]}(\omega) &= -\gamma_c\alpha_0\beta_0\left(\pm i\omega - \eta + \frac{1}{\pi}\lim_{\epsilon\to\infty}\epsilon\right)\left(\pm i\omega - \eta + \frac{1}{\pi}\lim_{\epsilon\to\infty}\epsilon\right) \\ &= -\gamma_c\alpha_0\beta_0\left(-\omega^2 \pm 2i\omega\left(-\eta + \lim_{\epsilon\to\infty}\frac{\epsilon}{\pi}\right)\right) + \dots\ ,\end{aligned} \tag{D.10}$$

where the ω-independent terms are disregarded, and we assumed the mean lifetimes of the excitations in both baths are the same ($\eta_1 = \eta_2 = \eta$). From this equation, we see a problem arising; whereas in non-interacting bath, the infinite term was neglected because it does not contribute to the dynamics, in the interacting bath this is not the case. The infinite term multiplies the dynamics the contributions of the other bath. For this result to make physical sense, the infinite term in the linear ω-term has to be cancelled. This means we require

$$\eta = -C + \lim_{\epsilon\to\infty}\frac{\epsilon}{\pi}\ , \tag{D.11}$$

such that

$$\Delta^{R[A]}(\omega) = -\gamma_c\alpha_0\beta_0\left(-\omega^2 \pm 2i\omega C\right) + \dots\ . \tag{D.12}$$

Here, C is the (positive) finite remainder that is the proportionality constant to the inter-sublattice Gilbert damping. However, as is clear from eq. (D.11), now η is infinitely big. This means our initial contour as calculated in appendix B is not correct; there we assumed η to be small, or at least to be finite. In other words, we run into an inconsistency in our approach: on the one hand, η must be infinite to prevent the Gilbert damping terms to have infinite magnitude, which on the other hand contradicts the assumption that η is finite in the calculation of the bath propagator.

Moreover, even without this paradox, it is already in the first place remarkable that in this model, the presence of inter-sublattice damping terms seems to depend on the lifetime of the bath, while this plays no role in the shared bath (where inter-sublattice damping terms also arise).

One possibility to circumvent these difficulties is to formulate the theory in terms of renormalised couplings, as is introduced in appendix G. In that case, the infinite terms would disappear, but in return one obtains a far more complex coupling between all the components of the system.

E Conditions on a Dissipation-Free Potential

In chapter 4, we derived that for a dissipation-free mode in a two-particle system to exist, the potential must satisfy the condition of separability, eq. (4.9). In this appendix, we will derive the consequences of this condition for the one particle potentials $V_1(q_1)$ and $V_2(q_2)$ for the first and second particle, respectively. More precisely, we will assume that the potential can be written as a sum of two power series (for instance, the Taylor expansions around the minima of the eigenmodes).

The general form of the potential satisfying the condition of separability is given by

$$V(q_1, q_2) = V(q_1 + q_2) + V(q_1 - q_2) \; . \tag{E.1}$$

Note that we dropped the $\frac{1}{2}$-factor for the centre-of-mass coordinate for notational convenience; this does not affect the result in a significant way. Now, let us expand both terms in the potential:

$$V(q_1, q_2) = \sum_n a_n (q_1 + q_2)^n + b_n (q_1 - q_2)^n \; . \tag{E.2}$$

In principle, this sum can range from $n = 0$ to $n = \infty$, for example when it expresses the Taylor series of an analytic potential. However, as in practice one usually only considers to first few orders, this sum may only have finite terms. For each term, we employ the binomial theorem to expand them as

$$\begin{aligned} V(q_1, q_2) &= \sum_n \sum_{k=1}^{n} \binom{n}{k} \left(a_n q_1^{n-k} q_2^k + (-1)^k b_n q_1^{n-k} q_2^k \right) \\ &= \sum_n \sum_{k=1}^{n} \binom{n}{k} \left(a_n + (-1)^k b_n \right) q_1^{n-k} q_2^k \; . \end{aligned} \tag{E.3}$$

From the current form, we see that for every n, possible contributions to the one particle potentials $V_1(q_1)$ and $V_2(q_2)$ come from the $k = 0$ and the $k = n$ terms, respectively. In other words, we can separate the potential by splitting the sum into two one-particle potentials (which come from the $k = 0$ and $k = n$ terms of the binomial sum) and an interaction potential $V_{int}(q_1, q_2)$ (which consists of the remaining $n - 2$ terms of the

binomial sum:

$$V(q_1, q_2) = \sum_n (a_n + b_n) q_1^n + (a_n + (-1)^n b_n) q_2^n \tag{E.4}$$

$$+ \sum_n \sum_{k=2}^{n-1} \binom{n}{k} \left(a_n q_1^{n-k} q_2^k + (-1)^k b_n q_1^{n-k} q_2^k \right) \tag{E.5}$$

$$\equiv \sum_n V_1^n(q_1) + V_2^n(q_2) + V_{int}^n(q_1, q_2) \ .$$

When n is even, the prefactors of both one-particle potentials are equal, since $(-1)^n = 1$ in that case. On the other hand, for n odd, the n^{th} term for particle one is determined by the prefactor $a_n + b_n$, while the same term for the second particle has prefactor $a_n - b_n$. In other words, when the condition of separability holds, *the one-particle potential for both particles are equal in their even powers, while their odd-powered terms may in general be different.*

Moreover, from this expression it becomes clear that in general, the interaction potential and the one-particle potentials are fully dependent on each other, since both are determined by the set of constants $\{a_n, b_n\}$. However, in special cases, there is still freedom to choose the interaction potential independently. For example, when we only consider $n = 2$ contributions to the potential, the potential is given by

$$V(q_1, q_2) = (a_2 + b_2)(q_1^2 + q_2^2) + 2(a_2 - b_2) q_1 q_2 \ , \tag{E.6}$$

where we recognise that the one particle potentials and the interaction potential are fully independent, as they are determined by orthogonal linear combinations of a_2 and b_2. Importantly, the potential for two coupled identical harmonic oscillators may be recovered from this form by choosing $a_2 + b_2 = m\omega_0^2$ and $a_2 - b_2 = \gamma/2$.

F Dissipation-free Modes in N-particle Systems

In chapter 4, we found that in dissipative system composed of two identical subsystems, one expects a dissipation-free mode when both subsystems are coupled to the same thermal bath. Furthermore, for a completely dissipation-free mode, the potential should be separable in the eigenmodes of the system.

The presence of dissipation-free modes is not limited to 2-particle systems. In this appendix, we will show that regardless of the amount of particles, one can find dissipation-free modes when all subsystems couple to the same thermal bath and the potential is separable in the eigenmodes. However, this comes with a cost; the potential becomes increasingly restricted when we require that the dissipative eigenmodes correspond to some physical normal modes of the system's dynamics.[5]

First, we derive the eigenmodes of the system from the dissipative action in an analogous way as chapter 4. We consider a system composed of N particles with position q_i, $(i = 1, 2, \ldots, N)$ and velocity $\dot{q}_i$. Furthermore, we assume that all particles have identical mass and that the potential is given by $V(\{q_i\}) \equiv V(q_1, q_2, \ldots, q_N)$. For notational convenience, we introduce the vector notation $\mathbf{q} = (q_1, q_2, \ldots, q_N)^T$. Lastly, all particles are coupled to the same bath of harmonic oscillators with equal coupling strength. As a consequence, the dissipative Keldysh action of this system is the N-particle generalisation of eq. (4.4), and is given by

$$\mathcal{S} = \oint_K \mathrm{d}t \left(\frac{m}{2} \dot{\mathbf{q}}^2 - V(\{q_i\}) \right) + \oint_K \mathrm{d}t \oint_K \mathrm{d}t' \, \mathbf{q}^T(t) R \alpha(t - t') \mathbf{q}(t') \, . \tag{F.1}$$

Here, the bath degrees of freedom are already integrated out. The dissipation propagator is given by eq. (2.37). Furthermore, we have introduced the dissipation matrix R into the action (see eq. (1.3)), which due to equal coupling of all particles is a $N \times N$ all-ones matrix. Of course, from the action it is readily clear that the dissipative eigenmode will be the mode that is proportional to $q_1 + q_2 + \cdots + q_N$. However, let us be more careful in finding the dissipation-free modes.

The eigenmodes of the dissipative system correspond to the eigenvectors of the dissipation matrix. The eigenvectors of this dissipation matrix are easy to find; first, since an all-ones matrix is equivalently written as $R = \mathbf{1}\mathbf{1}^T$ (with $\mathbf{1}$ being a N-dimensional

[5] In this appendix, we will refer to the modes of the dissipative part of the action as the (dissipative) *eigenmodes*, which are not necessarily equivalent to the system's *normal modes*, that we understand as the modes of the total action (including kinetic and potential) that describe fully decoupled dynamics.

vector of all ones), the first eigenvector is $\mathbf{x}_1 = \mathbf{1}/\sqrt{N}$ with eigenvalue $\sqrt{N}$:

$$R\mathbf{x}_1 = \frac{1}{\sqrt{N}}\mathbf{1}(\mathbf{1}^T\mathbf{1}) = \sqrt{N}\mathbf{1} \ . \tag{F.2}$$

This is the eigenvector that will correspond to a dissipative mode. The reason for the normalisation factor $\sqrt{N}$ will be made clear below.

Furthermore, since all rows of R are identical, $\mathrm{Rank}(R) = 1$ and consequently all other eigenvalues of R are 0, such that the remaining eigenvectors are $N-1$ linearly independent solutions of the equation

$$\mathbf{0} = R\mathbf{v} = \mathbf{1}\mathbf{1}^T\mathbf{v} = \mathbf{1}(\mathbf{1}\cdot\mathbf{v}) \ \ \Rightarrow \sum_i v_i = 0 \ , \tag{F.3}$$

where $\mathbf{v}$ is an eigenvector and v_i its i^{th} entry. Clearly, there more than $N-1$ possible solutions to this equation. However, we should choose the eigenvectors such, that they form a orthonormal basis. In that case, we can diagonalise the dissipation matrix R into a diagonal matrix D and an orthogonal matrix A (such that $A^{-1} = A^T$) according to $R = ADA^T$. The entries of the diagonal matrix are given by the eigenvalues of the dissipation matrix, $D = \mathrm{diag}(\sqrt{N}, 0, \ldots, 0)$. The matrix A is composed of the corresponding eigenvectors in its columns.

Now we introduce the change of coordinates to the eigenmodes of the system, of which we claim they correspond to the eigenvectors of the dissipation matrix. We define the eigenmodes as

$$\mathbf{x} = A^T\mathbf{q} \ , \tag{F.4}$$

where $\mathbf{x}$ is given by $(x_1, x_2, \ldots, x_n)^T$; it is the vector composed of the n eigenmodes of the system. Using this coordinate change, we can write the action in terms of the dissipative eigenmodes as

$$\mathcal{S} = \oint_K \mathrm{d}t \left(\frac{m}{2}\dot{\mathbf{x}}^2 - \bar{V}(\{x_i\})\right) + \oint_K \mathrm{d}t \oint_K \mathrm{d}t' \, \mathbf{x}^T(t) D\alpha(t-t')\mathbf{x}(t') \ . \tag{F.5}$$

Here we used the orthonormal property in the kinetic part; $\dot{\mathbf{q}}^T\dot{\mathbf{q}} = \dot{\mathbf{x}}^T A^T A\dot{\mathbf{x}} = \dot{\mathbf{x}}^T\dot{\mathbf{x}}$. Furthermore, the potential $\bar{V}(\{x_i\})$ is shorthand notation for $V(\{\sum_j A^T_{ij}x_j\})$. Lastly, in the dissipative part, we inserted the diagonalisation of R and the coordinate change to eigenmodes simultaneously, such that only the diagonal matrix remains.

Since D_{11} is the only non-zero entry of the diagonalisation matrix, it is manifest

that x_1 is the only dissipative mode in the system. However, just like for the two-particle case, when we require that the other modes cannot indirectly dissipate through coupling to x_1, the potential should be separable into dissipative and dissipation-free eigenmodes:

$$\bar{V}(\{x_i\}) = \bar{V}(x_1) + \bar{V}(x_2, x_3, \ldots, x_N) \; . \tag{F.6}$$

Note that we do not require that no eigenmodes are coupled, but only that dissipation-free modes are not coupled to dissipative modes. The quasi-classical equations of motion for the eigenmodes are then given by

$$m\ddot{x}_1(t) = -\frac{\partial}{\partial x_1}\bar{V}(x_1) - \int_{-\infty}^{\infty} \mathrm{d}t' \alpha^R(t-t') x_1(t') \; ; \tag{F.7a}$$

$$m\ddot{x}_i(t) = -\frac{\partial}{\partial x_i}\bar{V}(x_2, x_3, \ldots, x_N) \;\; (i = 2, 3, \ldots, N) \; . \tag{F.7b}$$

In the derivation, we required that the dissipation matrix R is diagonalised by an orthogonal matrix. Actually, such a diagonalisation is unique (up to permutation of the columns) and the orthogonal matrix A is given by

$$A = \begin{pmatrix} \frac{1}{\sqrt{N}} & \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} & \cdots & \frac{-1}{\sqrt{N(N-1)}} \\ \frac{1}{\sqrt{N}} & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} & \cdots & \frac{-1}{\sqrt{N(N-1)}} \\ \frac{1}{\sqrt{N}} & 0 & \frac{2}{\sqrt{6}} & \cdots & \frac{-1}{\sqrt{N(N-1)}} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{1}{\sqrt{N}} & 0 & 0 & \cdots & \frac{N-1}{\sqrt{N(N-1)}} \end{pmatrix} \; , \tag{F.8}$$

such that $R = ADA^T$. Note that except for the first, all columns add to zero, according to eq. (F.3).

By the uniqueness of A, we find that irrespective of the potential, the eigenmodes $x_2, x_3, \ldots, x_N$ must always be the same. However, some arbitrary potential is probably not separable into these modes; in other words, often the dissipative eigenmodes *do not* correspond to the physical normal modes of the total system.

We could have chosen instead to diagonalise the dissipation matrix by a non-orthogonal, but still invertible matrix P such that $R = PDP^{-1}$. Then, we would have much more freedom in the choice of eigenvectors, such that in some cases, we can choose them to be also eigenmodes of the potential.

However, as $\dot{\mathbf{q}}^T\dot{\mathbf{q}} = \dot{\mathbf{x}}^T P^T P\dot{\mathbf{x}}$, velocity-dependent coupling between the dissipation-

free modes will necessarily arise when $P^T P \neq I$. As a consequence, when one wishes to find dissipative eigenmodes that are the (decoupled) physical normal modes of the system, one *must* use the orthogonal diagonalisation. As the dissipative eigenmodes often do not correspond to these physical normal modes, it is often *not* possible to find decoupled eigenmodes in a N-particle dissipative system. Only in the cases where the dissipative eigenmodes defined by eq. (F.8) and eq. (F.4) correspond *exactly* to the normal modes that decouple the entire dynamics, the dissipation-free coordinates are true eigenmodes of the system, in the sense that they define completely decoupled dynamics of the system.

In conclusion, we find that in a dissipative system where N particles are coupled to a shared thermal bath, there is one dissipative mode, while there are $N-1$ dissipation-free modes. However, as the potential of many N-particle systems cannot be written in terms of these modes, they generally do not correspond to the eigenmodes of the system. In that sense the two-particle case is a special case; there, the dissipative and dissipation-free mode have a physical interpretation as centre-of-mass and relative coordinate.

G Renormalised Propagators and Non-Ohmic Baths

In this appendix, we consider renormalised coupling between the bath and the spins, as is described in (Weiss 2012). In our discussion, we ignore the non-dissipative dynamics of the spin system, since these are unaltered by an alternative approach to the dissipative coupling. First, we consider the renormalised coupling for a two-spin system, consisting of spins $\mathbf{S}_1$ and $\mathbf{S}_2$ as defined in previous sections, coupled to one and the same thermal bath. After that, we will show that this approach can be used to find the dynamics that result from the high-frequency behaviour of the bath. This result is fully adopted from (Quarenta 2022); here, we only apply it to the case of antiferromagnets. Lastly, the results for the shared bath system are easily translated into the dynamics for the separate bath system.

G.1 Renormalised bath propagator

The action that defines the bath dynamics and the coupling of the spins to the bath is given by

$$\mathcal{S} = -\oint_K \mathrm{d}t \sum_\alpha \frac{m_\alpha}{2}\left(\dot{\mathbf{x}}_\alpha^2 - \omega_\alpha^2\left(\mathbf{x}_\alpha - \frac{\gamma_\alpha}{m_\alpha\omega_\alpha^2}(\mathbf{S}_1 + \mathbf{S_2})\right)^2\right) , \tag{G.1}$$

where $\mathbf{x}_\alpha, \dot{\mathbf{x}}_\alpha, m_\alpha, \omega_\alpha$ are the position, velocity, mass and angular frequency of the particle α respectively, and γ_α is its coupling strength to the spins.

As usual, the bath dynamics are integrated out by introducing the propagator $G_\alpha^{-1}(\omega)$, which is defined by its Fourier transform $G_\alpha^{-1}(t-t') = \frac{m_\alpha}{2}((i\partial_t)^2 - \omega_\alpha^2)$. However, because of this new coupling, new terms appear, and the action reads

$$\mathcal{S} = -\oint_K \mathrm{d}t\mathrm{d}t' \sum_\alpha (\mathbf{S}_1 + \mathbf{S_2})\left(\frac{\gamma_\alpha^2}{4}G_\alpha(t-t') - \frac{\gamma_\alpha^2}{2m_\alpha\omega_\alpha^2}\right)(\mathbf{S}_1 + \mathbf{S_2}) . \tag{G.2}$$

Therefore, we define a new propagator $\alpha'^{[R/A]}(\omega)$ as

$$\alpha'^{[R/A]}(\omega) = -\sum_\alpha \frac{\gamma_\alpha^2}{2m_\alpha}\left(\frac{1}{(w \pm i\eta)^2 - \omega_\alpha^2} + \frac{1}{\omega_\alpha^2}\right) = -\sum_\alpha \frac{\gamma_\alpha^2}{2m_\alpha\omega_\alpha^2}\left(\frac{(w \pm i\eta)^2}{(w \pm i\eta)^2 - \omega_\alpha^2}\right) . \tag{G.3}$$

The action in terms of the new propagator is given by

$$\mathcal{S} = \oint_K \mathrm{d}t\mathrm{d}t'(\mathbf{S}_1 + \mathbf{S_2})\alpha'(t-t')(\mathbf{S}_1 + \mathbf{S_2}) . \tag{G.4}$$

The explicit form of the propagator in time domain is found, as is usual at this point, by introducing the energy spectral density $J(\epsilon)$ of the bath and Fourier transforming by complex integration. First, by definition $J(\epsilon) = \pi \sum_\alpha \frac{\gamma_\alpha^2}{m_\alpha \omega_\alpha} \delta(\epsilon - \omega_\alpha)$, such that the propagator reads

$$\alpha'^{[R/A]}(\omega) = -\int_0^\infty \frac{\mathrm{d}\epsilon}{2\pi} \frac{J(\epsilon)}{\epsilon} \frac{(\omega \pm i\eta)^2}{(\omega \pm i\eta)^2 - \epsilon^2} \tag{G.5}$$

Hence it becomes apparent why this type of coupling accounts for renormalisation: it adds an inverse power of ϵ to the integral, which in the Ohmic case makes the integral converge on the arc at infinity in the complex plane. Explicitly, choosing the Ohmic bath $J(\epsilon) = \alpha_0 \epsilon$ gives

$$\alpha'^{[R/A]}(t - t') = -\alpha_0 (\omega \pm i\eta)^2 \int_0^\infty \frac{\mathrm{d}\epsilon}{2\pi} \frac{1}{(\omega \pm i\eta)^2 - \epsilon^2} \tag{G.6}$$

Since this integral is symmetric, it is equal to half of its value integrated from minus infinity to infinity, and one can apply the reside theorem by considering integration in the complex plane. The important difference with the non-renormalised propagator $\alpha(t-t')$ is that the arc does not contribute an infinite value. In this sense this propagator is renormalised. Evaluation of the integral then gives the final result:

$$\alpha'^{[R/A]}(t - t') = \pm \frac{i\alpha_0}{2} (\omega \pm i\eta) \ . \tag{G.7}$$

Hence, we find the same result as in appendix B without the infinite constant. In other words, this calculation justifies the fact that earlier, we ignored this constant.

G.2 Bath-induced spin inertia

On the other hand, we may choose a bath that is more general than the Ohmic bath, and contains higher powers in ϵ. In other words, we consider a spectral density

$$J(\epsilon) = J_O(\epsilon) + J_{hf}(\epsilon), \tag{G.8}$$

where $J_O(\epsilon)$ is the Ohmic contribution we calculated above, and $J_{hf}(\epsilon) = J(\epsilon) - J_O(\epsilon)$ is the remaining part of the spectral density. The contribution to the propagator of this remaining part can be calculated:

$$\alpha_{hf}'^{[R/A]}(t - t') = (\omega \pm i\eta)^2 \int_0^\infty \frac{\mathrm{d}\epsilon}{2\pi} \frac{J_{hf}(\epsilon)}{\epsilon} \frac{1}{(\omega \pm i\eta)^2 - \epsilon^2} \tag{G.9}$$

Since the Ohmic part is the first order of the energy spectral density, the remaining part represents the behaviour at higher energies. Therefore, in the high frequency limit, that is $\epsilon \gg \omega$, we can approximate the integral as:

$$\alpha_{hf}'^{[R/A]}(t-t') \approx -(\omega \pm i\eta)^2 \int_0^\infty \frac{\mathrm{d}\epsilon}{2\pi} \frac{J_{hf}(\epsilon)}{\epsilon^3} = -\omega^2 M \ . \tag{G.10}$$

Here we defined the *bath-induced spin inertia* M, which is defined as the value of above integral. This constant characterises the higher order energy behaviour of the bath, including its cut-off structure. Furthermore, the infinitesimal contributions due to η are ignored, as they play no relevant role below. With this renormalised propagator that includes higher orders in energy structure, the quasi-classical equations of motion of the system become

$$\dot{\mathbf{S}}_1 = -J\mathbf{S_1} \times \mathbf{S_2} - \alpha_0 \mathbf{S}_1 \times (\dot{\mathbf{S}}_1 + \dot{\mathbf{S}}_2) + M\mathbf{S}_1 \times (\ddot{\mathbf{S}}_1 + \ddot{\mathbf{S}}_2) \ , \tag{G.11a}$$

$$\dot{\mathbf{S}}_2 = -J\mathbf{S_2} \times \mathbf{S_1} - \alpha_0 \mathbf{S}_2 \times (\dot{\mathbf{S}}_1 + \dot{\mathbf{S}}_2) + M\mathbf{S}_2 \times (\ddot{\mathbf{S}}_1 + \ddot{\mathbf{S}}_2) \ , \tag{G.11b}$$

where the Heisenberg interaction terms are included into the dyanmics. Changing variables to $\mathbf{M} = (\mathbf{S}_1 + \mathbf{S}_2)/2$ and $\mathbf{N} = \mathbf{S}_1 - \mathbf{S}_2$ gives

$$\dot{\mathbf{M}} = -2\alpha_0 \mathbf{M} \times \dot{\mathbf{M}} + 2M\mathbf{M} \times \ddot{\mathbf{M}} \ , \tag{G.12a}$$

$$\dot{\mathbf{N}} = -2\alpha_0 \mathbf{N} \times \dot{\mathbf{M}} + 2M\mathbf{N} \times \ddot{\mathbf{M}} - 2J\mathbf{N} \times \mathbf{M} \ . \tag{G.12b}$$

The quasi-classical equations of motion for separated baths (as described in section 3.1) including the bath-induced spin inertia terms are easily inferred from the above result:

$$\dot{\mathbf{S}}_1 = -J\mathbf{S_1} \times \mathbf{S_2} - \alpha_0 \mathbf{S}_1 \times \dot{\mathbf{S}}_1 + M\mathbf{S}_1 \times \ddot{\mathbf{S}}_1 \ , \tag{G.13a}$$

$$\dot{\mathbf{S}}_2 = -J\mathbf{S_2} \times \mathbf{S_1} - \alpha_0 \mathbf{S}_2 \times \dot{\mathbf{S}}_2 + M\mathbf{S}_2 \times \ddot{\mathbf{S}}_2 \ . \tag{G.13b}$$

This is simply do to the fact that there is no dissipative inter-sublattice connection in the separate bath model. In terms of the magnetisation and Néel order, the quasi-classical dynamics are given by

$$\dot{\mathbf{M}} = -\alpha_0(\mathbf{M} \times \dot{\mathbf{M}} + \frac{1}{2}\mathbf{N} \times \dot{\mathbf{N}}) + 2M(\mathbf{M} \times \ddot{\mathbf{M}} + \frac{1}{2}\mathbf{N} \times \ddot{\mathbf{N}}) \ , \tag{G.14a}$$

$$\dot{\mathbf{N}} = -\alpha_0(\mathbf{N} \times \dot{\mathbf{M}} + \mathbf{M} \times \dot{\mathbf{N}}) + 2M(\mathbf{N} \times \ddot{\mathbf{M}} + \mathbf{M} \times \ddot{\mathbf{N}}) - 2J\mathbf{N} \times \mathbf{M} \ . \tag{G.14b}$$

G.3 Sub-Ohmic and super-Ohmic baths

In previous section, eq. (G.8) defines a general decomposition into the low-frequency behaviour and the high-frequency behaviour. The high-frequency part gives rise to the bath-induced spin intertia terms in the dynamical equations. By assuming the low-frequency part to be Ohmic, we find the Gilbert damping and inter-sublattice damping terms. However, in this decomposition it is not necessary to assume the bath is Ohmic at low frequency: one may also consider the cases of *sub-Ohmic* and *super-Ohmic* baths, where the bath spectral density shows power law behaviour $J(\epsilon) \sim \alpha_0 \epsilon^p$ at low frequency, where $p < 1$ for sub-Ohmic baths and $p > 1$ for super-Ohmic baths. However, as is clear from eq. (G.5), the acquisition of a closed form for the bath propagator in these cases depends on the specific value of p and requires closer consideration.

References

Yuan, H. Y., Qian Liu, et al. (July 2019). "Proper dissipative torques in antiferromagnetic dynamics". In: *EPL (Europhysics Letters)* 126.6. Publisher: IOP Publishing, p. 67006. ISSN: 0295-5075. DOI: 10.1209/0295-5075/126/67006.

Néel, Louis (1970). *Nobel Lecture*. URL: https://www.nobelprize.org/prizes/physics/1970/neel/lecture/.

Xu, Yuan, Shuai Wang, and Ke Xia (June 2008). "Spin-Transfer Torques in Antiferromagnetic Metals from First Principles". In: *Physical Review Letters* 100.22. Publisher: American Physical Society, p. 226602.

Núñez, A. S. et al. (June 2006). "Theory of spin torques and giant magnetoresistance in antiferromagnetic metals". In: *Physical Review B* 73.21. Publisher: American Physical Society, p. 214426.

Wei, Z. et al. (Mar. 2007). "Changing Exchange Bias in Spin Valves with an Electric Current". In: *Physical Review Letters* 98.11. Publisher: American Physical Society, p. 116603.

Haney, Paul M. and A. H. MacDonald (May 2008). "Current-Induced Torques Due to Compensated Antiferromagnets". In: *Physical Review Letters* 100.19. Publisher: American Physical Society, p. 196801.

Wadley, P. et al. (Feb. 2016). "Electrical switching of an antiferromagnet". In: *Science* 351.6273. Publisher: American Association for the Advancement of Science, pp. 587–590.

Cheng, Ran, Jiang Xiao, et al. (July 2014). "Spin Pumping and Spin-Transfer Torques in Antiferromagnets". In: *Physical Review Letters* 113.5. Publisher: American Physical Society, p. 057601.

Kampfrath, Tobias et al. (Jan. 2011). "Coherent terahertz control of antiferromagnetic spin waves". In: *Nature Photonics* 5.1. Number: 1 Publisher: Nature Publishing Group, pp. 31–34. ISSN: 1749-4893.

Cheng, Ran, Di Xiao, and Arne Brataas (May 2016). "Terahertz Antiferromagnetic Spin Hall Nano-Oscillator". In: *Physical Review Letters* 116.20. Publisher: American Physical Society, p. 207603.

Barker, Joseph and Oleg A. Tretiakov (Apr. 2016). "Static and Dynamical Properties of Antiferromagnetic Skyrmions in the Presence of Applied Current and Temperature". In: *Physical Review Letters* 116.14. Publisher: American Physical Society, p. 147203.

Gilbert, T.L. (Nov. 2004). "A phenomenological theory of damping in ferromagnetic materials". In: *IEEE Transactions on Magnetics* 40.6. Conference Name: IEEE Transactions on Magnetics, pp. 3443–3449. ISSN: 1941-0069.

Takei, So et al. (May 2016). "Spin Superfluidity in the v=0 Quantum Hall State of Graphene". In: *Physical Review Letters* 116.21. Publisher: American Physical Society, p. 216801.

Kittel, C. (May 1951). "Theory of Antiferromagnetic Resonance". In: *Physical Review* 82.4. Publisher: American Physical Society, pp. 565–565.

Baryakhtar, V. G. and A. G. Danilevich (Oct. 2015). "Dissipative function of a ferromagnet and the theory of Onsager's kinetic equations". In: *Low Temperature Physics* 41.10. Publisher: American Institute of Physics, pp. 778–780.

Liu, Qian et al. (Nov. 2017). "Mode-dependent damping in metallic antiferromagnets due to intersublattice spin pumping". In: *Physical Review Materials* 1.6. Publisher: American Physical Society, p. 061401.

Caldeira, A. O and A. J Leggett (1983). "Quantum tunnelling in a dissipative system". In: *Annals of Physics* 149.2, pp. 374–456. ISSN: 0003-4916.

Kamenev, Alex (Sept. 2011). *Field Theory of Non-Equilibrium Systems*. Cambridge University Press. ISBN: 978-1-139-50029-6.

Altland, Alexander and Ben D. Simons (Mar. 2010). *Condensed Matter Field Theory*. Cambridge University Press.

Berry, M. V. (1984). "Quantal Phase Factors Accompanying Adiabatic Changes". In: *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences* 392.1802, pp. 45–57. ISSN: 0080-4630.

Duine, R. A. et al. (Mar. 2018). "Synthetic antiferromagnetic spintronics". In: *Nature Physics* 14.3. Number: 3 Publisher: Nature Publishing Group, pp. 217–219.

Yuan, H. Y., Zhe Yuan, et al. (Jan. 2021). "Recent progress in antiferromagnetic dynamics". In: *EPL (Europhysics Letters)* 132.5. arXiv:2010.14940 [cond-mat], p. 57001.

Seters, Daan van et al. (June 2022). *Dissipation-free modes in dissipative systems*. DOI: 10.48550/arXiv.2206.07471.

Quarenta, Mario G. (2022). "Bath-Induced Spin Inertia". MA thesis. Utrecht University.

Volpe, Giovanni and Jan Wehr (Apr. 2016). "Effective drifts in dynamical systems with multiplicative noise: a review of recent progress". In: *Reports on Progress in Physics* 79.5. Publisher: IOP Publishing, p. 053901. ISSN: 0034-4885.

Duan, Lu-Ming and Guang-Can Guo (1998). "Reducing decoherence in quantum computer memory with all quantum bits coupling to the same environment". In: *Phys. Rev. A* 57, p. 737.

Weiss, Ulrich (2012). *Quantum dissipative systems*. World Scientific.