Utrecht University

Department of Theoretical Physics

Effect of media on elections in an Ising-like voter mode

A thesis submitted in fulfillment of the requirements
for the degree of Master of Science
in the

Complex Systems Studies
Institute for Theoretical Physics

Supervisors:
Prof. dr. ir. Henk Dijkstra
Prof. dr. ir. H.T.C. Henk Stoof
Dr. Claudia Wieners

Candidate:
Andrea Di Benedetto
Student number:
6903657

Academic Year 2020/2021

Acknowledgements

I would like to thank all the people who contributed to the completion of this project and therefore to my entire university experience in Utrecht. First of all I would like to thank my girlfriend Laura who has always been close to me, both in the happy and sad moments and my family who has never stopped believing in me always allowing me to pursue my dreams. I would also like to thank my supervisor Dr. Claudia Wieners without whom it would never have been possible to complete this project. Her enthusiasm and her extraordinary scientific intuitiveness have been the backbone of this experience all along. Obviously, an immense thanking goes to my supervisors Prof. Dr. Henk Dijkstra and Prof. Dr. Henk Stoof, who despite the immense difficulties due to a global pandemic have given me the opportunity to carry out the thesis on a unique and extremely interesting topic, guiding me admirably in the many daily challenges that only such an interdisciplinary topic can bring. It has been a pleasure and an honor to deal with you and you have the merit of having revived my passion for scientific research. I would never have been able to complete this adventure without Suzanne's availability and affection, who managed to make me feel at home despite the 2000 km distance. Finally, I would like to thank the support of my long-time friends Andrea, Carmine, Francesco, Gino, Matteo, with whom I concluded yet another step of my life and the most recent ones who accompanied me on this journey in the Netherlands Ciro, Corey, Lorenzo, Mart and Rodolfo. A final special mention goes to Vishnu, an impeccable friend and workmate, with whom I shared a thousand adventures in just two years.

Abstract

In this project we develop a bottom-up model to study the influence of media on election dynamics in a two-party system (say, the US). We start from a Potts model where nodes can be 3 opinion states: Democratic voter, Republican voter, or non-voter. Voters can interact to influence each other's opinion, depending on transition probabilities and individual affiliations to parties. Some nodes represent media sources. These are highly-connected and influential nodes, which are randomly located in the network and have the role of spreading external influence (e.g. information on the state of the economy) throughout the population. By varying the number of these media in the network, we observed a phase transition between the configuration in which the population is completely uninformed and ordered and one continuously influenced by the many sources, thus extremely disordered. Finally, several experiments were done by adding feedbacks between the media and people, to assess under what conditions deep partians divisions, hence polarisation, can be observed. Therefore, we witnessed how important it is to maintain a certain variety in the media's opinion to prevent the formation of the so-called "echo chambers" and therefore of polarisation as an emergent behavior.

Contents

1 Introduction **4**

2 Preliminaries **4**
2.1 Networks . . . 4
2.1.1 Random networks . . . 5
2.1.2 Scale free networks . . . 5
2.1.3 Small world networks . . . 6
2.2 Phase transitions . . . 6
2.3 The Ising Model . . . 7
2.4 The Potts Model . . . 8
2.5 Voter models . . . 9
2.6 The Fair Model . . . 10
2.7 Autocorrelation functions . . . 12

3 Historical data **13**

4 The Model **17**
4.1 Network properties . . . 17
4.2 Opinion-switch mechanism . . . 18
4.3 The external influence . . . 19
4.3.1 Media nodes . . . 19
4.3.2 The external influence . . . 20

5 Results and discussion **22**
5.1 Baseline . . . 22
5.2 Sensitivity experiments: number of media . . . 24
5.3 Sensitivity experiments: the role of affiliations . . . 26
5.3.1 No affiliations case . . . 26
5.3.2 Everyone affiliated to the same party . . . 27
5.4 Feedback 1 : People can leave media . . . 29
5.5 Feedback 2 : Media can decide to change opinion . . . 32
5.5.1 Case 1 . . . 32
5.5.2 Case 2 . . . 35
5.6 Varying affiliations . . . 37

6 Summary and conclusion **39**

7 Discussion and further work **40**

1 Introduction

Polarisation is defined as the division into two sharply contrasting groups or sets of opinions or beliefs. There is agreement in the literature [5] that from the 1970s polarisation increased in the US and Europe such that, for example, parents affiliated with one US political party are loathe to have their children wed someone affiliated with the other major party [11]. Another example is the contentious debates and votes on health care reform and the debt ceiling in the 2010-2012 Congress that was extremely divided into two contrasting groups. It has become such an important part of our society that for some commentors a decrease in polarization could have negative effects on society, as apparently did in the US in the mid-century (APSA, 1950). But where does polarisation originates from? Many hypothesis have been raised and confronted, blaming the political institutions, big societal impacts such as inequality and immigration patterns or even globalisation [3]. Hand in hand with polarization, mass media have acquired more power of influence over people and there is growing evidence ([7] ,[10]) that they influence the individual opinion of citizens and also their political behavior, thus having a role in the polarization phenomenon. Despite the valuable empirical work, to our best knowledge, no attempt has been made to actually model the processes leading to polarisation. The goal of this project, is to help unravel the governing features of social polarization, by constructing a model for a population of voters influenced by different media outlets. While most of the voter focus on the processes behind decision making ([4], [6], [12]), the role of external influences such as media outlets will be the main feature of the framework that will be presented. We start from a Potts model where nodes can be 3 opinion states: Democratic voter, Republican voter, or non-voter. Voters can interact to influence each other's opinion, depending on transition probabilities and individual affiliations to parties. Some nodes represent media sources. These are highly-connected and influential nodes, which are randomly located in the network and have the role of spreading external influence (e.g. information on the state of the economy) throughout the population. By characterizing the decision-making processes of individuals and the role of external influences in a completely new way, we want to reveal the mechanisms that underlie polarisation as an emergent behavior from simple micro interactions. On the other hand, the results of our model should not be seen as a prediction of who will win the elections in the near future but simply an acceptable simulation of the reality in which he performs experiments dedicated to understanding the role of the media in people's political opinion.

2 Preliminaries

2.1 Networks

Graphs are mathematical structures used to describe pairwise relations between agents. They are formed by nodes (or vertices) which are connected by edges (or links) and can be undirected (if edges connect nodes simmetrically) and directed (if edges connect nodes a-simmetrically). Network theory is the study of graphs whose edges store attributes and has applications in a in a huge number of disciplines, from social systems to particle physics. The first example of proof in Networks theory is Euler's solution of the Seven Bridges of Königsberg problem, dating back to 1736.

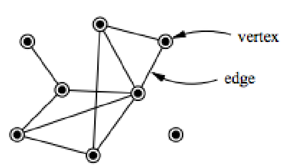


Figure 1: A network object [2]

The network we are interested in in this project is a social network made up of people who move on a train platform. In social networks, not all nodes have the same number of edges and a fundamental characteristic of the macro object is precisely the distribution function $P(k)$, that represents the probability that a randomly selected node has k edges. Depending on this probability distribution, we can have different types of social networks and the three most common cases will be discussed in this section. Networks can be classified according to two independent structural features: the clustering coefficient C and the average node-to-node distance (also called average shortest path length) L. Given a node i, his clustering coefficient is designed as,

$$C_i = \frac{2E_i}{k_i(k_i-1)}. \tag{1}$$

The coefficient of the entire network is given by the average of all the single C_i,

$$C = \sum_i^N \frac{C_i}{N}. \tag{2}$$

On the other hand, the average node-to-node distance is the average distance L between two randomly chosen nodes. Given a network G whose vertices belong to the set V. Let $d(v_1, v_2)$ be the distance between nodes v_1 and v_2 in V and that $d(v_i, v_j) = 0$ if the node j can not be reached from i. Thus, the average node-to-node distance is given by,

$$l_G = \frac{1}{n(n-1)} \sum_{i \neq j} d(v_i, v_j), \tag{3}$$

where n is the number of the nodes of G.

2.1.1 Random networks

The simplest and oldest network model is the random model, also known as Erdős-Rényi model. It can be obtained by starting with a set of isolated vertices and subsequently connecting them with an independent probability. Equivalently, the probability for generating each graph that has n nodes and M edges is

$$P = p^M (1-p)^{\binom{n}{2}-M}. \tag{4}$$

where the average number of edges per node is given by $< k_i >= \binom{n}{2}p$. For enough values of n, this distribution is Poissonian with a peak at $P(< k >)$. In this case, since the edges are distributed randomly, the clustering coefficient is simply given by:

$$C = p = \frac{< k >}{N} \tag{5}$$

Real networks can be described by observing how the number of edges and nodes behave or how the density changes during a day. Density is described as

$$d = \frac{2m}{n(n-1)} \tag{6}$$

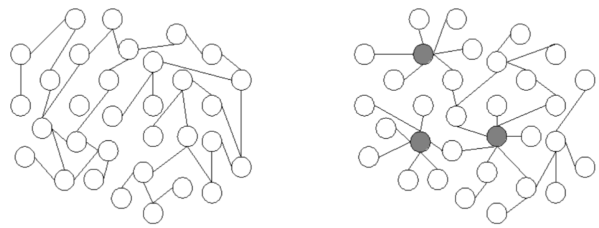
where m is the number of nodes and n the number of edges. Recent empirical results showed that most of real social networks like the World-wide Web or the the airport networks, do not follow this degree distribution but the so called "scale-free" and "small world" distributions.

2.1.2 Scale free networks

If the degree distribution of a network follows

$$P(k) \sim k^{-\gamma} \tag{7}$$

where γ is a parameter in the range $2 < \gamma < 3$ This particular type of network gained interest in 1999 when Barabasi discovered that the network formed by the world wide web consists of particular nodes called hubs that have a very high number of connections compared to the average. Many real-world networks are thought to be scale-free and the major elements that explain the emergence of the scale-free property in a complex networks are two: the growth and the preferential attachment. In this case, the clustering coefficient decreases as as the node degree increases, by following a power law. Similarly to small world networks, the average distance between two vertices in a network is very small compared to ordered graphs and the diameter of a growing scale-free network can even be considered as constant.



(a) Random network (b) Scale-free network

Figure 2: A scale-free network object [14]

2.1.3 Small world networks

A small-world network refers to a group of networks in which the average node-to-node distance between nodes increases sufficiently slowly as a function of the number of nodes in the network. This term is mostly used to refer the specific "Watt-Strogatz" toy network. In this case, the average node-to-node distance between two randomly chosen nodes grows proportionally to the logarithm of the number of nodes n in the network,

$$L \sim \log(n) \tag{8}$$

Furthermore, the clustering coefficient should be large. Empirical example of this network structure can be gene Networks or the underlying architecture of the Internet.

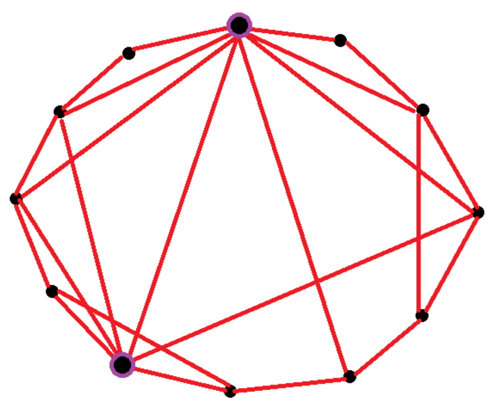


Figure 3: A small world network object with clustering coefficient = 0.522 [15]

2.2 Phase transitions

Given a system (often made of many subsystems) can undergo strong qualitative changes in its macroscopic properties if a suitable observable defined as **order parameter** is adequately tuned and that close to these critical points some sudden changes in the behavior of nonlinear systems as a key characteristic constants (the so-called critical exponents) consequence of a continuous change in a given parameter are are the same for very different systems [13]. This description was developed to describe the transitions between states of matter but it can be generalized to any type of complex system, such as the set of many interacting voters subjected to the influence of an external term. By changing the order parameters, depending on how this macroscopic change happens, we can define two different kinds of phase transitions:

- **First order phase transitions**: they are those in which at the transition there is a drastic change that marks a point of discontinuity. As example we can think of the melting of ice or the boiling of water that happen abruptly at a specific temperature.
- **Second order phase transitions**: They are those in which the order parameter grows continuously from zero. An example can be the glass transition.

2.3 The Ising Model

The Ising model [2] is a model used in statistical physics to describe magnets. It is based on the assumption that the magnetism of a bulk material arises from the local interaction of many atomic spins, for simplicity located in a D-dimensional lattice (in our case D=2 but in solid state physics usually D=3). The dipoles or "spins" defined as S_i can be equal to +1 or -1, respectively corresponding to either "up" or "down" dipole states. The simplest case one dimensional case, is the case in which all the interaction have the same strength J and interactions occur only between nearest neighbors on the lattice. An external magnetic field can be applied to the system, so that the Hamiltonian is given by:

$$H = -J \sum_{<i,j>} s_i s_j - B \sum_{i}^{N} s_i \tag{9}$$

where $< i, j >$ represents the sum over nearest neighbors and N is the total number of sites on the lattice. The configuration where all the spins are aligned is defined as ferromagnetic state, on the other hand the one where all the spins are ant-aligned anti-ferromagnetic state. This system can be analytically solved and the partition function is given by

$$Z = \frac{\sum_{s_i} e^{-\beta H}}{Z} \tag{10}$$

where T is the temperature, k_B the Boltzmann constant and $\beta = 1/K_B T$. The index s_i means that the sum is performed over all spins and for all values of spins (+1 and -1). This model has two notorious distinct phases: paramagnetic phase, where the fluctuations in temperature lead the system to a completely disordered configuration, and ferromagnetic phase, where all spins are aligned. Between these two phases, at some critical temperature $T = T_c$ a second order transition occurs with the magnetization:

$$M = \frac{1}{N} \sum_{i=1}^{N} < s_i > \tag{11}$$

considered as order parameter. In fact, the paramagnetic phase corresponds to $M = 0$ and the ferromagnetic phase to $M \neq 0$.

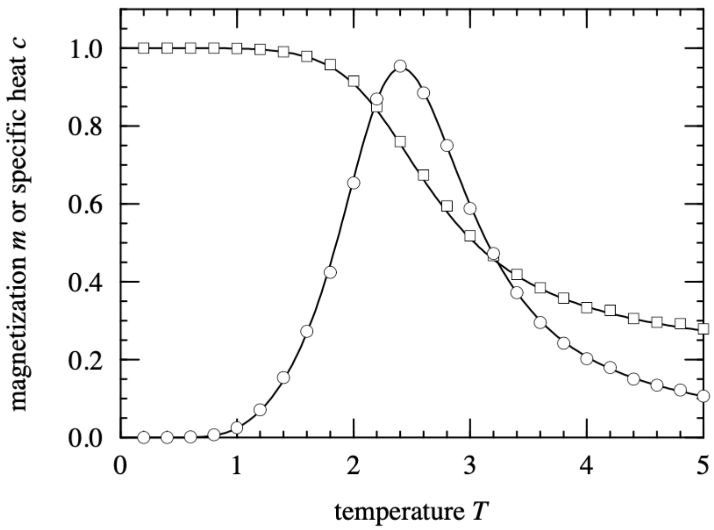


Figure 4: Phase transition of the Ising model. The specific heat is a measure of the fluctuations of the magnetisation, that peak at the critical temperature [13]

2.4 The Potts Model

The Potts model [16] is a generalization of the Ising model (Ising, 1925) to more than two components, and has been a subject of a lot of research in recent years, with application to many different fields such as biology or social sciences. The first Potts model was developed by Askin and Teller (1943) with 4 different states but 30 years later DOmb [Domb (1943)] proposed a work with a general number q of states. Originally, Domb proposed an Ising model with a system of spins confined in a plane, with each spin pointing to one of the q defined by the angles

$$\theta_n = 2\pi n/q, n = 0, 1, 2...., q-1. \tag{12}$$

If considering the interaction only between nearest neighbors, the Hamiltonian depends only on the relative angle between the vectors, thus given by

$$H = -\sum_{<ij>} J(\Theta_{ij}), \tag{13}$$

where the function $J(\Theta) = J(\Theta + 2\pi)$ and $\Theta_{ij} = \Theta_{n,i} - \Theta_{n,j}$ is the angle between two neighbors spins at positions i and j. This is known as the $Z(q)$ model. The standard Potts model is then obtained by choosing

$$J(\Theta_{ij}) = \epsilon_2 \delta_{Kr}(n_i, n_j). \tag{14}$$

This model is ferromagnetic if $\epsilon > 0$ and antiferromagnetic if $\epsilon < 0$. If rewriting the Kronecker delta function,

$$\delta_{Kr} = \frac{1}{q}[1 + (q-1)e^{\alpha} e^{\beta}] \tag{15}$$

where $\alpha = 0, 1, 2..., q-1$ and $\beta = 0, 1, 2..., q-1$ we can reflect the full symmetry in a $q-1$ dimensional space, where e^{α} are q unit vectors pointing in the q symmetric directions of a hypertetrahedron in $q-1$ dimensions.

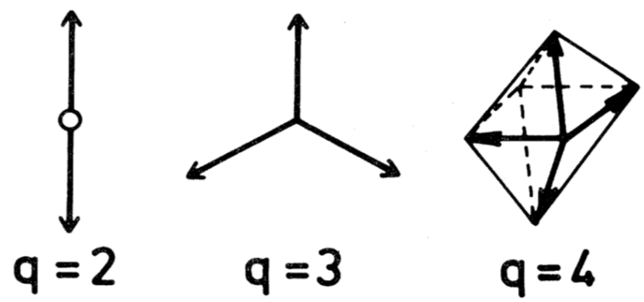


Figure 5: The q unit vectors pointing in the q symmetric directions of a hypertetrahedron in q —1 dimensions [16]

In the last fifty years of research, physicists studied the characteristics of the phase transition of the Potts model for different sizes and number of colors and we can resume them with the following conjecture.

Let $d \geq 2$ (number of dimensions) and $q \geq 2$. The phase transition of the nearest neighbor ferromagnetic q-state Potts model is continuous (2nd order) if $q \leq q_c(d)$ and discontinuous (1st order) if $q > q_c(d)$, where

$$q_c(d) = \begin{cases} 4 & \text{if } d = 2 \\ 2 & \text{if } d \leq 3. \end{cases} \tag{16}$$

The model that is built in this project is based on a Potts model with $d = 2$ and $q = 3$.

2.5 Voter models

Voter models (VM) can defined as an idealized description for the evolution of opinions in a population [21]. Usually, every voters can assume two states and resides on a node of an arbitrary network. At every time step, a random node is selected and he adopts a new state, depending on his neighbors. This particular interaction, which can be local or completely random, mainly defines the type of voter. In the most simple type, in the "classic voter model" (VM) the voter adopts the state a random neighbor.

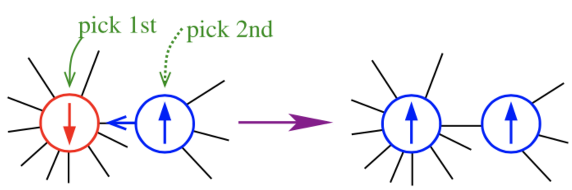


Figure 6: VM rule: the picked voter adopts the opinion of one random neighbor [21]

There are many more ways of deciding how a group of voters would change opinions and the most used example is the majority rule. This can be implemented in two different ways:one is that a voter adopts the state of the majority of its local neighborhood and the second one is when a group of voters is selected and all voters in this group adopt the local majority opinion. Finally, another option can be adopting non linear update rules such as the "vacillating voter model" (VVM) [23] and the non conserved voter model (NVM) [22]. the main difficulty encountered in these theoretical models is that of avoiding that all elements of the network reach the same opinion, that is, a state of consensus. The prevention of it often leads to the addition of unrealistic and difficult to justify characteristics but in our case this will be avoided by applying external influences through particular elements called media nodes.

2.6 The Fair Model

Do economic events affect voting behaviour? If so, how? This is one of the most important questions in political economy and since the 70's the entire scientific community tried to find an answer. The first one to actually explore this problem was Kramer (1971), concluding with his work that the economic fluctuations have an important influence on the congressional elections. As will happen with all subsequent studies, other scholars reached the opposite conclusions, as happened with Stigler (1973) who concluded that there is no correlation between economics and popular vote. The main debates concerning this field are about the scientific validity of the statistical tools used, penalized by the small amount of historical data, given that the history of modern democracies is not so old. Several theoretical models have been developed to actually describe how economic events can affect election results and one of them has been continuously used and updated to describe the US presidential election, namely the Fair (1978) model [24]. Yale economist Ray Fair is one of the pioneers of modern-day election forecasting (Fair, 1978). The previous updates of this equation are in Fair (1982, 1988, 1990, 1996a, 1998, 2002a, 2006, 2010, 2014). The specification of the equation has not been changed since changes following the 1992 election. His presidential vote equation, which was first used to predict the 1980 election, is still applied today, although, not surprisingly, it underwent several revisions in the nearly 40 years since its first publication.

The model can be summarized with 4 principles:

- Incumbent presidents running have an advantage.
- Voters like change. Therefore, parties in office for two or more consecutive terms have a disadvantage.
- There is a slight but persistent bias favoring the Republican Party.
- The state of the economy affects the incumbent party vote.

This translates into the equation:

$$V_d = 48.06 + 0.673(G*I) - 0.721(P*I) + 0.792(Z*I) + 2.25(DPER) - 3.76(DUR) + 0.21(I) + 3.25(WAR) \quad (17)$$

Parameter	Description
G	Growth rate of real per capita GDP in the first three quarters of the on-term election year (annual rate)
P	absolute value of the growth rate of the GDP deflator in the first 15 quarters of the administration (annual rate) except for 1920, 1944, and 1948, where the values are zero.
Z	Number of quarters in the first 15 quarters of the administration in which the growth rate of real per capita GDP is greater than 3.2 percent at an annual rate except for 1920, 1944, and 1948, where the values are zero
I	1 if there is a Democratic presidential incumbent at the time of the election and -1 if there is a Republican presidential incumbent
$DPER$	1 if a Democratic presidential incumbent is running again, -1 if a Republican presidential incumbent is running again, and 0 otherwise
DUR	0 if either party has been in the White House for one term, 1 [-1] if the Democratic [Republican] party has been in the White House for two consecutive terms, 1.25 [-1.25] if the Democratic [Republican] party has been in the White House for three consecutive terms, 1.50 [-1.50] if the Democratic [Republican] party has been in the White House for four consecutive terms, and so on
WAR	1 for the elections of 1918, 1920, 1942, 1944, 1946, and 1948, and 0 otherwise
V_d	Democratic share of the two-party presidential vote

Table 1: Table with all the parameters of Fair's equation [24].

Result	Forecast date	Days to election	Democratic vote	Absolute error
50.3	January 30, 1999	647	45.7	4.6
50.3	May 1, 1999	556	47.9	2.4
50.3	November 3, 1999	370	44.7	5.6
50.3	November 5, 1999	368	47.1	3.2
50.3	January 29, 2000	283	47.9	2.4
50.3	April 28, 2000	193	50.8	0.5
50.3	July 31, 2000	99	50.8	0.5
50.3	October 27, 2000	11	50.8	0.5

Table 2: Table with real polling results (Democratic vote) and predicted value with Fair's equation (Result) of the 2000 US presidential elections.

2.7 Autocorrelation functions

The autocorrelation function is one of the mathematical tools used to find the influence of temporal patterns on data-series. Specifically it measures the correlation between two measures of the same variable, separated by an interval of various time lags. Given measurements, $y_1, y_2, ..., y_N$ at time $x_1, x_2, ..., x_N$ and the lag k, the autocorrelation function is defined as

$$r_k = \frac{\sum_{i=1}^{N-k}(y_i - < y >)(y_{i+k} - < y >)}{\sum_{i=1}^{N}(y_i - < y >)^2}. \tag{18}$$

The quantity we want to calculate in our case is the time interval needed to obtain two completely unrelated measures of the same variable. This is the case when, given two measures of the auto correlation $r_{k,1}$ and $r_{k,2}$, they satisfy

$$r_{k,2} - r_{k,2} \leq 1/e. \tag{19}$$

3 Historical data

Presidential elections in the USA occur quadrennially and indirectly, in which citizens who are registered to vote in their own state cast ballots for members of the Electoral College. These members have then the task of directly vote president and vice-president. The candidate who receives an absolute majority of electoral votes is elected. Historically, from the 1848 where the Whig party won, only two parties have succeeded in power, the Democratic one and the Republican one. This makes the American system an excellent candidate for simulations of elections and dissemination of information, since the system can be represented as a Potts model where the spin state represents political opinion ($S = -1$ for democrat voters, $S = +1$ and $S = 0$ for independents/non voters).

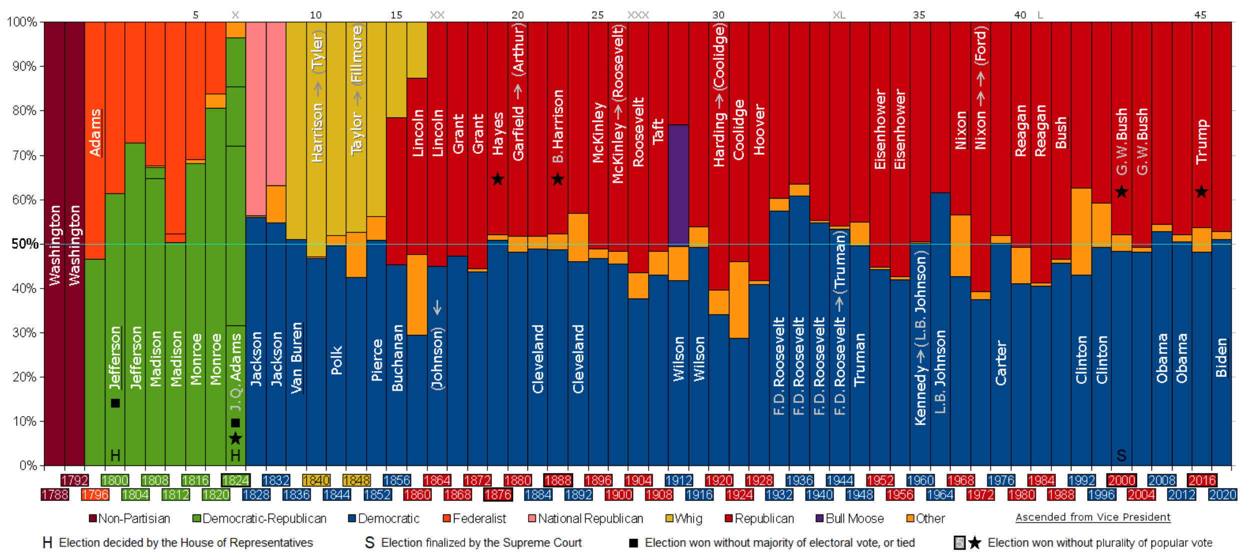


Figure 7: Historical series of candidate presidents and popular votes [17]

Of the US voters, Around a third of registered voters (34%) identify as independents, while 33% identify as Democrats and 29% identify as Republicans [7].
An important point is that even those who define themselves as independents most of the time have a tendency towards a party. In fact, When taking independents' partisan leanings into account, 49% of all registered voters either identify or lean to the Democratic party, while 44% identify or lean to the Republican Party. Being registered with a party seems like something very static in the American system, with virtually no radical changes in portions over the past 25 years. It is also important to underline that being affiliated with a party certainly does not mean voting for this same party but it is possible that you do not go to vote at all or that you can even decide to vote for the opposite party, which happened in 2016 where 5% of Democrats and Democratic leaners reported voting for Trump, and 4% of Republicans and GOP leaners reported voting for Hillary Clinton.

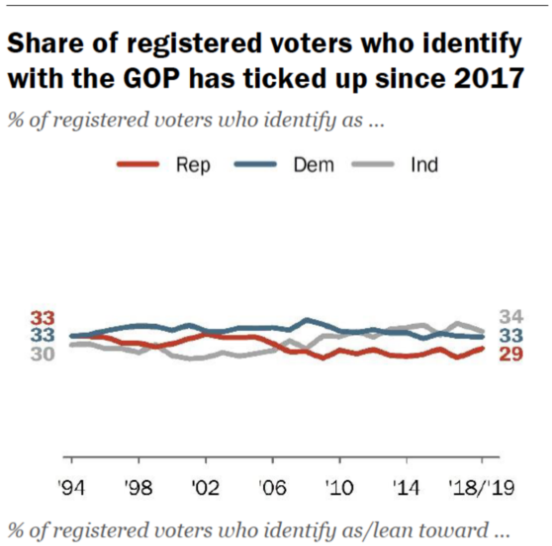
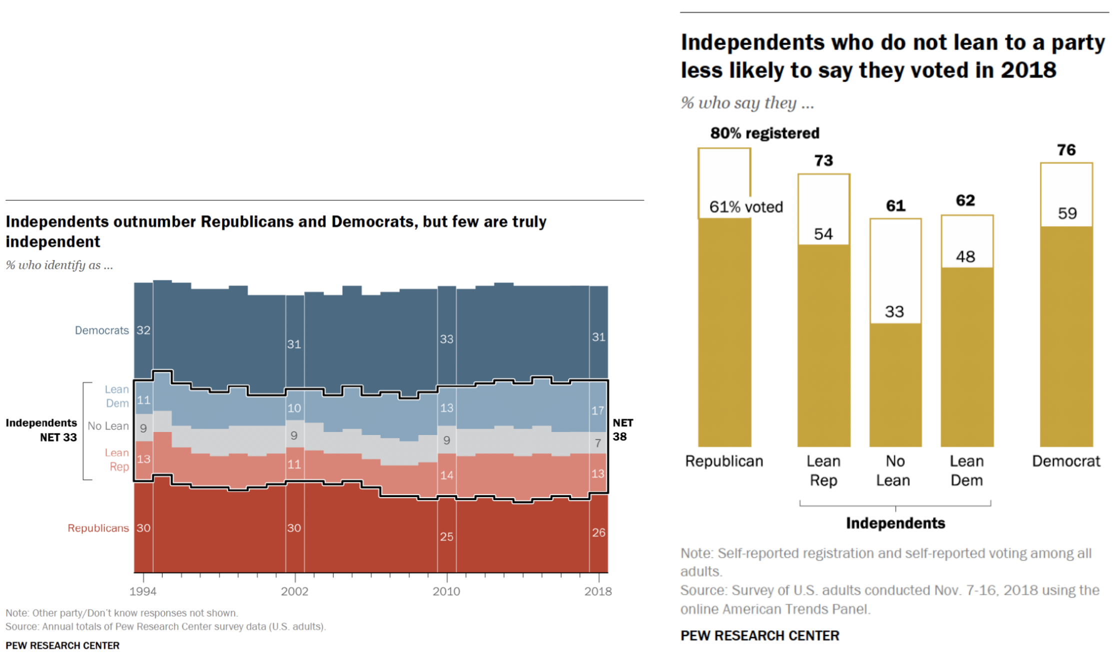


Figure 8: Shares of affiliations in the last 20 years [9]

Thus, the vote in the US depends not just on how many people support which party, but also a lot on how many people (supporting a party) go out and vote. Party affiliates are most likely to vote, leaners intermediate, and true independents least likely. Consequently, to best reproduce these observed phenomena, to each voter will be assigned two variables, i.e. voting intention and affiliation. Each one can be Republican, Democrat or neutral (non voter/non affiliated) and in principle all combinations between two variables will be possible even including Democrat-affiliated and Rep-voter or viceversa.



(a) (b)

Figure 9: Graphs about US independents [8]

From these data, another fundamental factor for this project is to understand how long it takes on average two elections to be unrelated. This can be analyzed by using autocorrelation functions. At the same time we want to exclude any type of increasing/decreasing behaviour on a long time scale, thus we "detrended" the dataset. This has been done by using the function detrend() of the library Scipy of Python. Since the amount of data points of historical elections in US is not enough to obtain an acceptable result for the autocorrelation time, we decided to use poll results to observe how political views change over time.

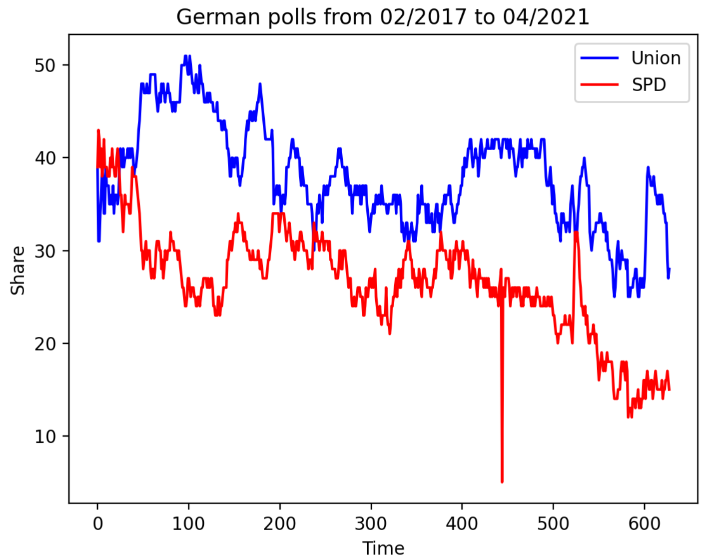
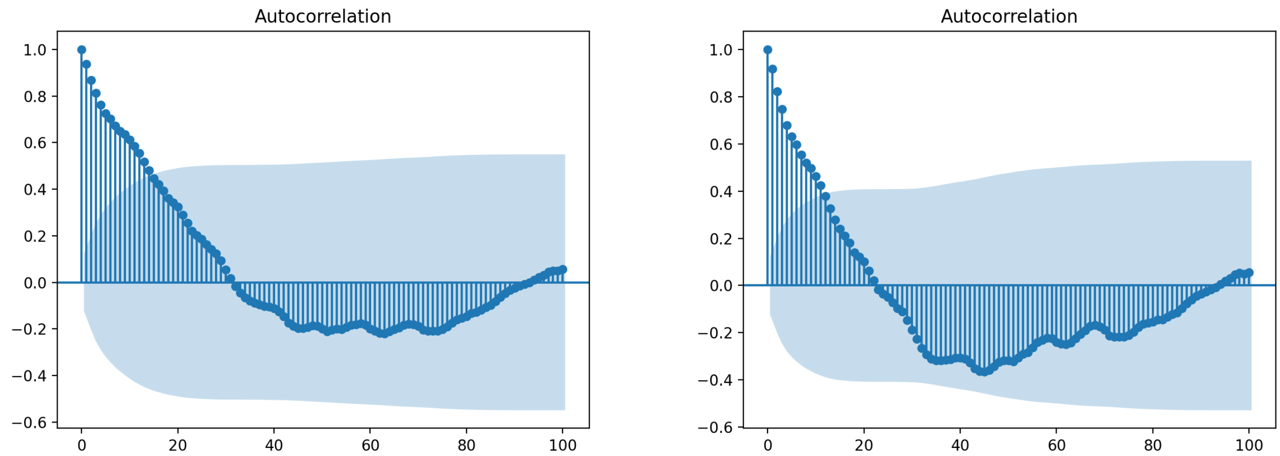


Figure 10: Monthly average share of the two main parties in the political German system

In addition, since it was also not possible to find a dataset based on surveys made on the US population, the analysis on autocorrelation was performed on the German series of polls made by Infratest dimap from the 2000 to 2020 (16). To do this it was necessary to approximate the German multi-party system to the American bipolar one. This was based on the hypothesis that the time interval necessary to observe two unrelated measures does not depend on the number of parties involved which in the German case are 8 but of which 6 in the last twenty years have always obtained significantly lower results than "Union " and "SPD".



(a) Before detrending

(b) After detrending

Figure 11: Autocorrelation of the share of the Union party vs lags in months. The blue shaded region represents the region not statistically significantly different from zero.

4 The Model

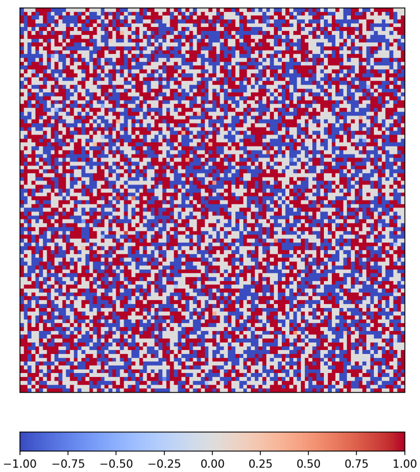
The idea of building this model, intended to reproduce the fundamental mechanisms underlying the US elections, was born after numerous difficulties in applying the regular 2-state Ising model. In fact, the major problems encountered with the first framework are two: the population is easily able to reach consensus and there are not enough parameters in the system to modify besides the temperature, which is not exactly attributable to a real variable such as the number of media outlets per person. So after a period of difficulty in achieving realistic results, based on the physical and social science literature, we added one ingredient at a time such as non-voter status, affiliations and the media nodes. Let us now focus on the specific features of the model.

4.1 Network properties

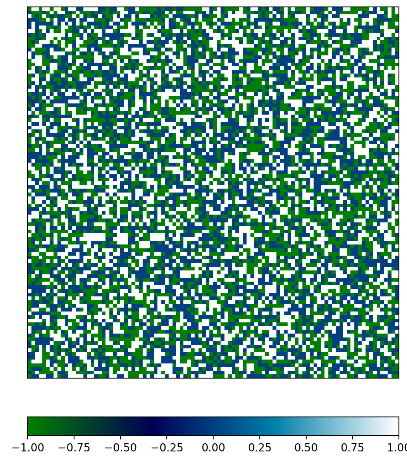
The population is described as a 2-D network of $N = LxL$ nodes. Each node is identified by a coordinate (i, j) in the network that remains fixed and it is connected to his neighbors by edges. In order to make connections more heterogeneous, interpersonal interactions follow a hierarchial structure, with two different levels. The way how the network is created is now explained: first, from the unconnected network, connections are formed over all the nodes until his connectivity k_{ij} is equal to the maximum connectivity K_{ij} or until the network is saturated. First level connections are created by iterating over all nodes S_{ij} and his possible neighbors S_{mn} depending on the distance, by following the distance probability rule given by

$$P(l) \sim \frac{1}{1 + \exp\left[(l-a)\right]/b} + 0.001\frac{L-1}{L} \tag{20}$$

where $n = i \pm l_1$, $m = \pm l_2$, $l = \sqrt{(i-n)^2 + (j-m)^2}$ is the distance between the nodes (i, j) and (m, n) and l_1, l_2 are two independent random variables and the sign is generated with probability 0.5. This represents the first generation of connection, strongly depending on the position in the network. The population is then divided into local groups of $N_G = L_G x L_G$ where $a = L_G$ and $b = L_G/4$. Subsequently, after the formation of a connection between two nodes (i, j) and (m, n), second level connections are formed between (m, n) and all the nodes of (i, j), with probability p_c. The number of edges to a node will be always in between the interval (k_{min}, k_{max}) in order to have the desired degree distribution. The degree distribution of the network has been chosen as scale free, where the probability of having k individuals has the form $P(K) \sim k^{\gamma}$, with $k \in (k_{min}, k_{max})$. It has been decided to leave out the calculation of network size dependence and to focus more on the mechanisms underlying the model. For this reason, by following [1], we considered a total number of nodes equal to $100x100 = 10,000$, local groups of 20 elements, and $(k_{min}, k_{max}) = (8, 24)$ for all the different simulations. Each performed simulation has a duration of 2,920,000 steps, 730 months , in which 20 elections will be made.



(a) Initial configuration of opinions. Every state is indicated with a color (Republican=red, Democrat=blue, non voter=white)



(b) Initial configuration of affiliations. Every state is indicated with a color (Republican affiliated=white, Democrat affiliated green=blue, non voter=blue

Figure 12: Network initial conditions

4.2 Opinion-switch mechanism

This model is based on a Potts 3-states model. The opinion of a node is modeled with three different states ($Sij = -1, 0, +1$) and they represent their choice of political party (+1 means republican, -1 means democrat) and their intention to vote ($Sij = 0$ means that (i,j) is not going to vote). Initially individuals are uniformly and randomly assigned one of three states such that the initial opinion is null ($< S >= 0$). The update of a node (i, j) depends on the average opinion h_{ij} of his neighbors k_{ij} by following different thresholds. This average opinion is defined as

$$h_{ij}(t+1) = \frac{1}{k_{ij}}(\sum_{n=1}^{kij} A_{l(n)m(n)}S_{l(n)m(n)}(t)). \tag{21}$$

The value A_{ij} describes the authority of of the node i, j and it has been fixed to his minimum value 0.1 for people and 1 for media. This was the case because it corresponds to the social importance of an individual's opinion. Another important aspect of the update is the affiliation of the node. Every node, in addition to having the opinion S_{ij}, it has a status of affiliation which can be the same (+1, -1, 0), meaning republican affiliated, democratic affiliated and non affiliated. This second state influences the probability of changing opinion, that is based on a series of thresholds. For example, a republican (democrat) affiliated node, even with a majority of democratic (republican) neighbors, may choose to not change his opinion. These transition probabilities are given in the following table where the conditions for opinion switches from an old state (row) to a new state (column) are given in the following table; / means "transition not possible".

		Rep vote	No vote	Dem vote
Rep affil	Rep vote	$h > -h_{aq}$	$h < -h_{aq}$	/
	No vote	$h > h_{ar}$	else	$h < -h_{ad}$
	Dem vote	/	$h > h_{ab}$	$h < h_{ab}$
not affil	Rep vote	$h > h_{nq}$	$h < h_{nq}$	/
	No vote	$h > h_{nj}$	else	$h < -h_{nj}$
	Dem vote	/	$h > -h_{nq}$	$h < -h_{nq}$
Dem affil	Rep vote	$h > -h_{ab}$	$h < -h_{ab}$	/
	No vote	$h > h_{ad}$	else	$h < -h_{ar}$
	Dem vote	/	$h > h_{aq}$	$h < h_{aq}$

with $h_{nq} = 0$ (non-affiliate quit), $h_{nj} = 0.5$ (non-affiliate join), $h_{aq} = 0.5$ (affiliate quit), $h_{ar} = 0.25$ (affiliate rejoin), $h_{ad} = 0.75$ (affiliate defect), $h_{ab} = 0.0$ (affiliate back-out). In this project, these transition probabilities will be considered as constant but they may be considered depending on time or other features of the system. This may be part of the future developments of the model.

4.3 The external influence

4.3.1 Media nodes

The main difference of this experiment with others is that the external influence is not directly acting as an external influence on the equation 21 but indirectly added through high authority and super connected nodes defined as media. Another important point is that, in order to have more freedom in experiments, their opinion is described as a float in a fixed interval, whose limits (S_{min}, S_{max}) are randomly assigned during the generation of the network. One third of media is generated with limits such that $S_{min} \in (-1, 0)$ and $S_{max} \in (0, 0.5)$, one third with $S_{min} \in (-0.5, 0)$ and $S_{max} \in (0, 1)$ and the final third with $S_{min} \in (-1, 0)$ and $S_{max} \in (0, 1)$. By following the recent experiments on the US population, [[7]], it has been considered that realistically, we expect that a person receives information from a number between 10 and 15 medias. Starting with a configuration of 100x100 network, it has been then fixed that each media node is, on average, connected to 2000 other nodes.

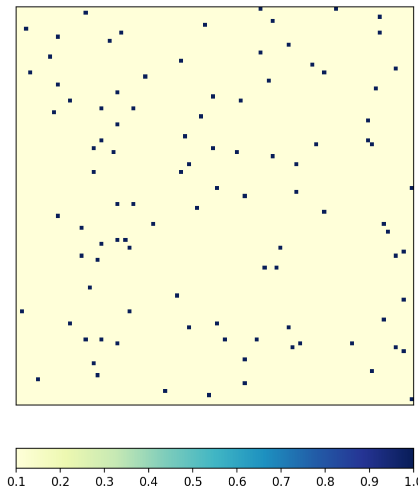


Figure 13: Initial configuration where the color of each square represents the individual authority. All voters have the authority fixed to 0.1 while media's authority is fixed to 1 (ten times more authoritative). Population of $N = 10,000$ elements in which 100 are defined as media.

4.3.2 The external influence

In the original Fair model, this principles are translated in a set of parameters that will influence the share of of the two parties (see 17). In this project, considering that the external influence acts through high authority-super connected media, we have used the same weights of the model on the influence that changes the opinion S_{ij} of a media. The main differences with the Fair's Model are two: first, there is no asymmetry between two parties since we want a setup that initially appears symmetrical and secondly, there are no terms regarding wars. In our case the external influence is then represented by 3 main terms: DUR , G and $DPER$. The G term contains all the fluctuations on the weekly scale caused by economics or other random events such as political scandals. It is a random value between -0.22 and 0.22, incorporating the contributions of G, P and Z terms of the Fair equation (17). Thus every week, the opinion of every media is updated according to

$$S_{ij} = S_{ij} + I * G. \tag{22}$$

with G random value sampled from a uniform distribution in the interval $(-0.22, +0.22)$. On the other hand second term DUR represents the "boredom" of people against the ruler party. Indeed, in literature there is a wide agreement on the fact that, especially in the last decade, the more a party is in power, the more likely it is that a big change is about to happen. Analogously as in the Fair equation, the term DUR will be 0 if either party has been in the White House for one term, 1 [-1] if the Democratic [Re-publican] party has been in the White House for two consecutive terms, 1.25 [-1.25] if the Democratic [Republican] party has been in the White House for three consecutive terms, 1.50[-1.50] if the Democratic [Republican] party has been in the White House for four consecutive terms, and so on. The term follows then,

$$S_{ij} = S_{ij} + 0.376 * (DUR) \tag{23}$$

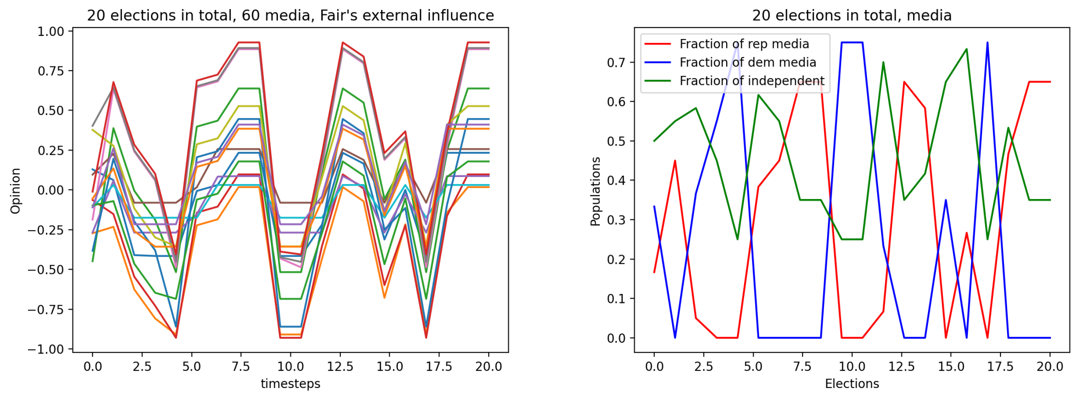
Finally, the second political influence term is defined as $DPER$ and it is equal to -1 if a Democratic presidential incumbent is running again, +1 if a Republican presidential incumbent is running again, and 0 otherwise. Analogously to the previous ones

$$S_{ij} = S_{ij} + 0.24 * DPER \tag{24}$$

Thus, the scheme of external influence follows:

- Every week add a random value G in the interval $(-0.22, +0.22)$ to the opinion of media
- After each election, add a new term depending on DUR and DPER (evaluated following the model description in the table 1).

Of course, every time a new term is added to a media's opinion, the new value is always contained in the corresponding interval (S_{min}, S_{max}) defined initially. If the new S_{ij} results bigger (smaller) than $S_{max}(S_{min})$, then $S_{ij} = S_{max}$ $(S_{ij} = S_{min})$.



(a) Individual opinion of 15 random media nodes. Each media is represented with a different color.

(b) Portions of media divided per political opinion. A media with opinion $S(i,j)$ is defined as republican if $S(i,j) \geq 0.25$, democrat if $S(i,j) \leq 0.25$ and independent otherwise.

Figure 14: Measures on a population of $N = 10,000$ elements in which 60 are media nodes. The number of elections per a simulation is 20. Baseline.

Let us finally resume all the main steps of the model:

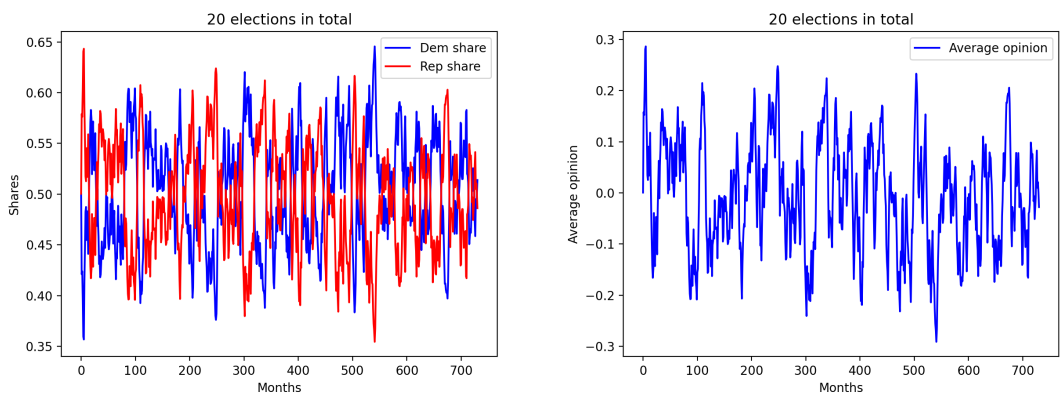
Algorithm 1 Main steps of the algorithm

For each timestep, generate a random position (i,j):
if the node $(i,j) \neq media$ **then**
    Update the opinion depending on his neighbors.
if a week is passed **then**
    add a random term (economy) to medias' opinion
if 4 years are passed **then**:
    measure all the other observables
    define the ruling party
    update the boredom term
    update the incumbency term

5 Results and discussion

5.1 Baseline

The main goal of our model is to reproduces qualitatively, the best we can, the historical data of section 2. This means that our baseline should have an autocorrelation of around 10 months, low likelihood of shares bigger than the 60% and a tendency for ruling parties to be in power for 2-4 electoral terms. The decision of making this model was largely moved by the freedom in the parameters that we can use in order to actually have realistic outputs. The entire set of simulations is typically characterized by an initial equilibration interval that lasts for approximately the first 2.5 elections (initial 100 months). Any results obtained in this range were not considered reliable. In Figure 15, we can observe the behaviour of the shares that like in the historical data oscillate in the correct interval. This results came out by considering the combination of 60 media and 2000 initial "customers", where, on average, an each voter receives information by 11 sources.

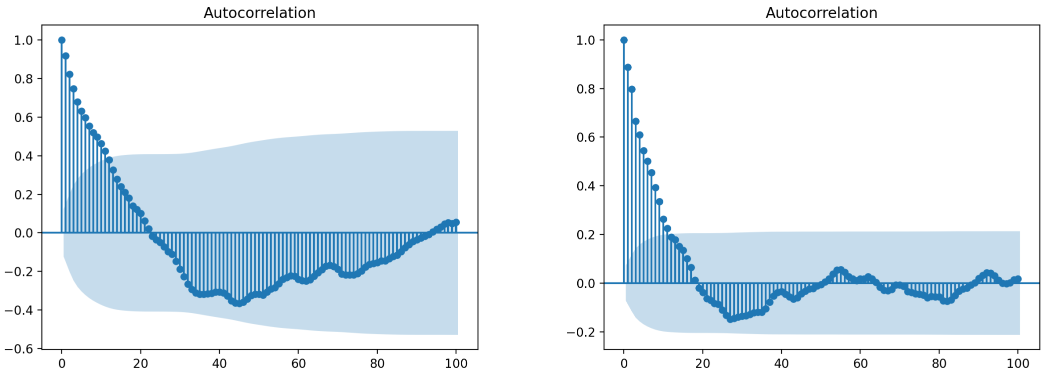


(a) Shares of both parties in time.

(b) Average opinion in time.

Figure 15: Measures on political opinion in the baseline case.

Another key ingredient in the tuning of the baseline is the autocorrelation. As already mentioned in the historical data section, because of the difficulty in finding a proper dataset of polls it has been used the German case. The typical autocorrelation then, is given by a value of between 15 and 20 months, achieved by the model without the need of fix more parameters. In figure 16 we can observe the two autocorrelation functions (real German and the simulation baseline) after being linearly detrended.

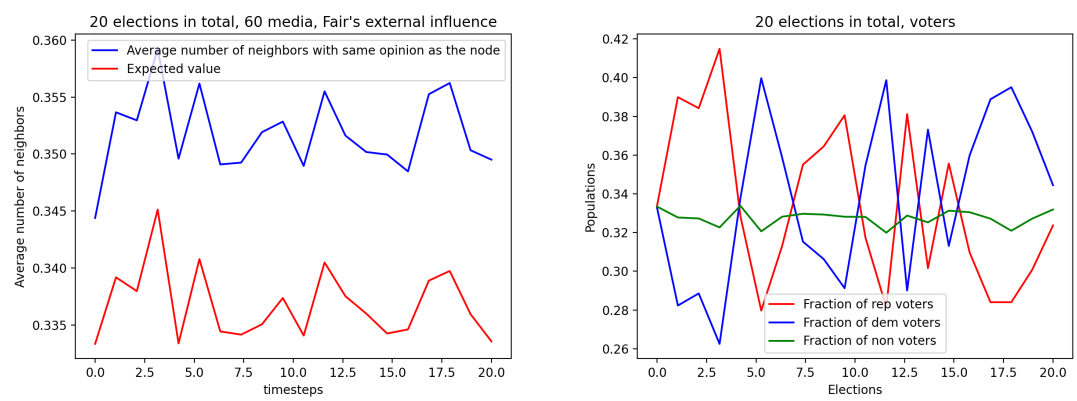


(a) Real autocorrelation in German elections

(b) Typical autocorrelation in the model

Figure 16: Autocorrelation

In the right image of figure 17 we can observe the behaviour of the different portion of populations in the model. In this setup, non-voters seem extremely reluctant to vote and the two opposing populations (D and R) exchange different fractions of elements over the months. Considering that one of the main objectives of this model is to study the role of media in polarization, it was necessary to define a variable that can describe segregation (or clustering), that is, how effectively people tend to be surrounded by like-minded people, and the polarization, that is, how much the system finds itself stuck in a scenario where there is no longer a dialogue between the two challenging groups of opinion. To measure segregation, the average number of elements with the same opinion as a person in the network was considered. This same variable has a considerable role in the description of the influence of media, which for now only disseminate information but which later in the discussion will be subject to different experiments.



(a) Average number of neighbors with same opinion as a random node vs time.
Expected number of neighbors with same opinion as a media node (red line).

(b) Portions of voters vs time.

Figure 17: Measures on a population of $N = 10,000$ elements in which 60 are media nodes. The number of elections per a simulation is 20. Baseline.

On the other hand, to represent how much the system is divided into two sharply contrasting groups, we considered the probability of changing the opinion in the time interval of an election. Basically, the probability is given by the equation

$$P = \sum_{ij} \sum_{k}^{T} \frac{p_k^{ij}}{T}, \tag{25}$$

where $T = 4$ years and p_k^{ij} is equal to one if the voter node located at (i, j) has changed his opinion in timestep k and zero otherwise. In social sciences polarisation is defined as the division into two sharply contrasting groups or sets of opinions or beliefs. In our case, to quantify the level of polarization we will therefore use the probability that a random person in the network has to change their opinion. If this probability is zero, our population can be defined as polarized.

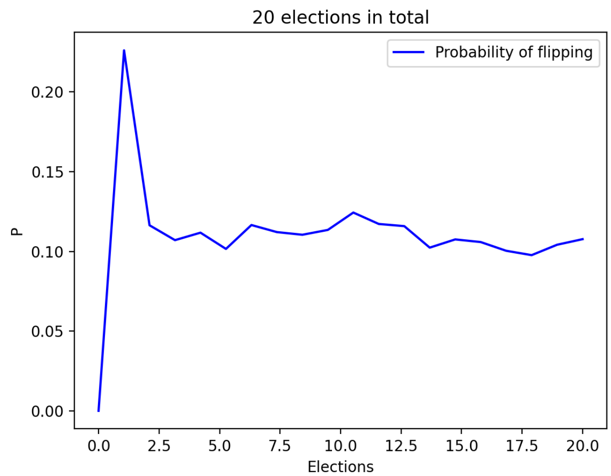
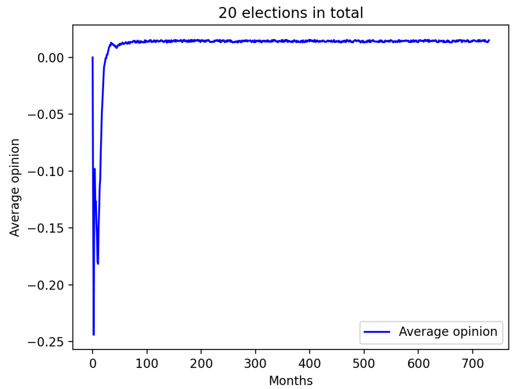


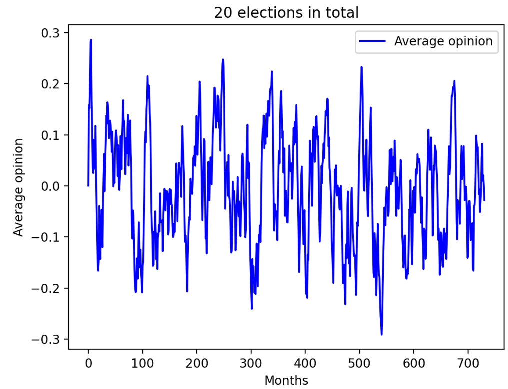
Figure 18: Probability that during the time interval between two elections, a random node in the network changes flips his opinion.

5.2 Sensitivity experiments: number of media

Analogously to the Ising model, in this model it is possible to see two distinct phases. In fact, by increasing the number of the media nodes, we observe the transition between an inert (not informed population) to a volatile (extremely informed population).



(a) Average opinion vs time.



(b) Average opinion in a network of N=10,000 nodes of which 60 are media sources.

Figure 19: Average opinion of a population of $N = 10,000$ for a different number of media nodes. The number of elections per simulation is 20.

To obtain this transition, we observed the standard deviation of the average opinion defined as

$$\sigma_S = \sqrt{\frac{\sum_{t=1}^{t=T} (S(t) - < S >)}{T - 1}} \tag{26}$$

where $S(t)$ is the average opinion at time t and T is the total number of time steps of the simulation.

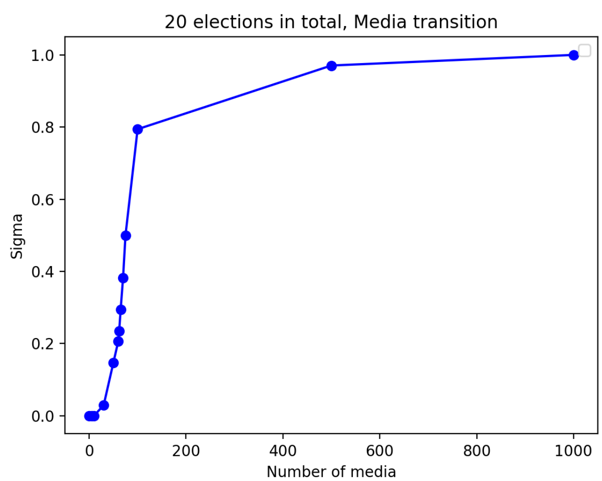


Figure 20: Normalized standard deviation for different amount of media nodes in a network of N=10,000 nodes

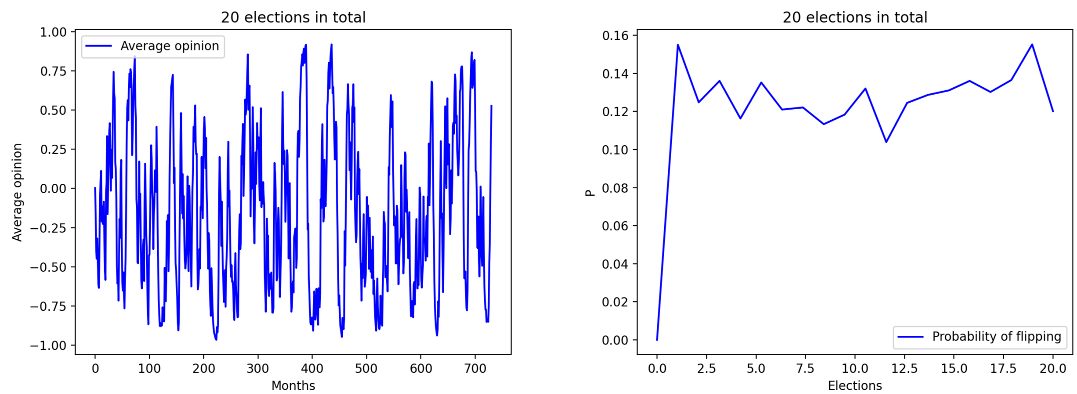
Thus, given a population of $N = 10,000$ nodes, if the number of media nodes is smaller than a certain threshold that is between 50 and 100 media, the network will not respond to external influence. By increasing it we witness a continuous phase transition with the fluctuations in the average opinion continuously increasing and emerging disorder.

5.3 Sensitivity experiments: the role of affiliations

In this subsection we will observe the behavior of the model as the initial condition of the affiliations changes (all affiliated to a party or none) and the opinion of the media.

5.3.1 No affiliations case

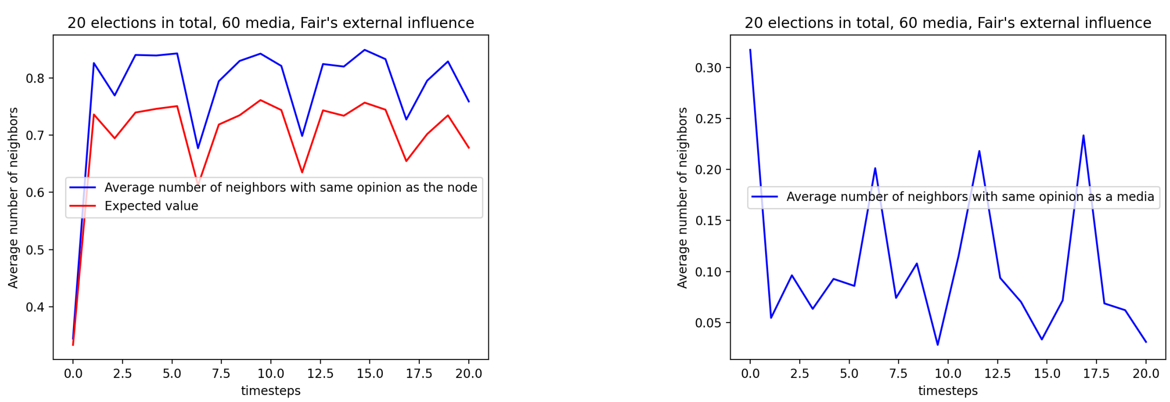
In this case the system is initialized with a population where none has an affiliations in order to discover what is the role of this ingredient in the simulations.



(a) Average opinion vs time.

(b) Probability that during the time interval between two elections, a random node in the network changes flips his opinion.

Figure 21: Measures on a population of $N = 10,000$ elements in which 60 are media nodes. The number of elections per a simulation is 20. All voters are not affiliated.



(a) Average number of neighbors with same opinion as a voter node vs time.
Expected number of neighbors with same opinion as a media node (red line).

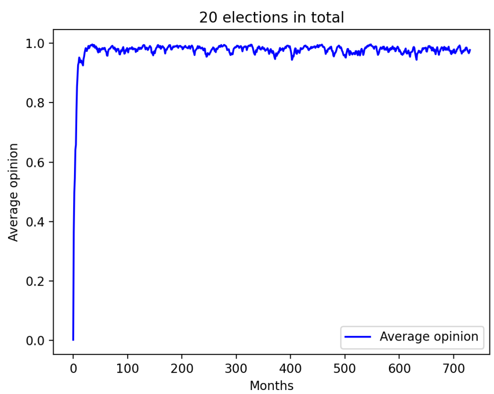
(b) Average number of neighbors with same opinion as a media node vs time

Figure 22: Measures on a population of $N = 10,000$ elements in which 60 are media nodes. The number of elections per a simulation is 20. All voters are not affiliated.

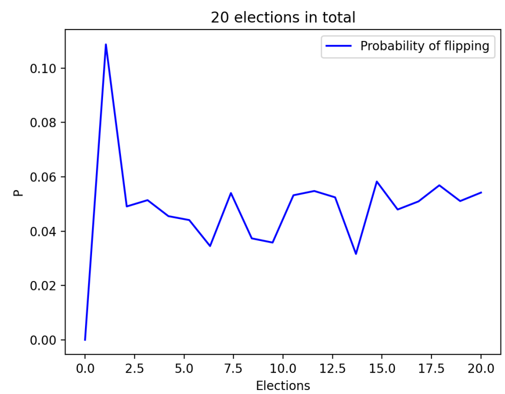
It is clear that the lack of affiliation allows each voter of the network to change their idea several times in a single simulation, raising the probability of filliping to the 14% (the baseline has 10%). This leads the average opinion to oscillate quickly between -1 and +1 (21), implying that in a few moments, consensus has been reached and all elements of the network think the same way. At the same time, media nodes still play a crucial role in fact it is not always the same party that remains in command , even if the system is really "segregated", meaning that on average the portion of neighbors with the same opinion as a random voter node oscillates between the 70% and the 80% 22.

5.3.2 Everyone affiliated to the same party

In this section, we simulate 20 elections analogously as the baseline but with a initial configuration where all the voters are affiliated to the republican party, thus the initial matrix of affiliations is the unity matrix with one at every matrix position.



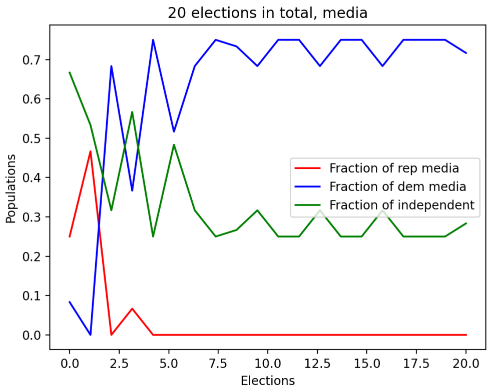
(a) Average opinion vs time



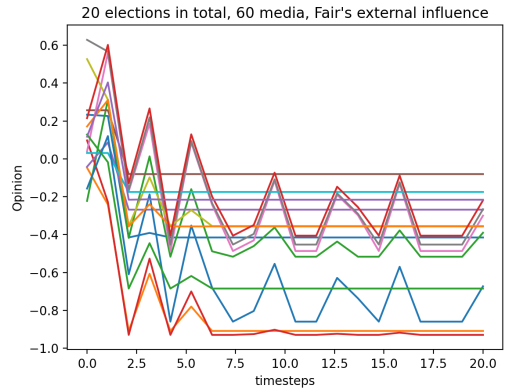
(b) Probability that during the time interval between two elections, a random node in the network changes flips his opinion.

Figure 23: Measures on a population of $N = 10,000$ elements in which 60 are media nodes. The number of elections per a simulation is 20. All voters are affiliated to the republican party.

As we can see in the Image a) of Figure 23, all the population converges to consensus. This was expected since, everyone started with the same affiliations and all the transition probabilities depend on this. From image b) of 23, we also witness that the probability of flipping doesn't reach zero. This means that, even if the vast majority joins the same party, there are some fluctuations, mostly between non voters and the republican party.



(a) Portions of media divided per political opinion. A media with opinion $S(i, j)$ is defined as republican if $S(i, j) \geq 0.25$, democrat if $S(i, j) \leq 0.25$ and independent otherwise.



(b) Individual opinion of 15 random media nodes. Each media is represented with a different color

Figure 24: Measures on a population of $N = 10,000$ elements in which 60 are media nodes. The number of elections per a simulation is 20. All voters are affiliated to the republican party.

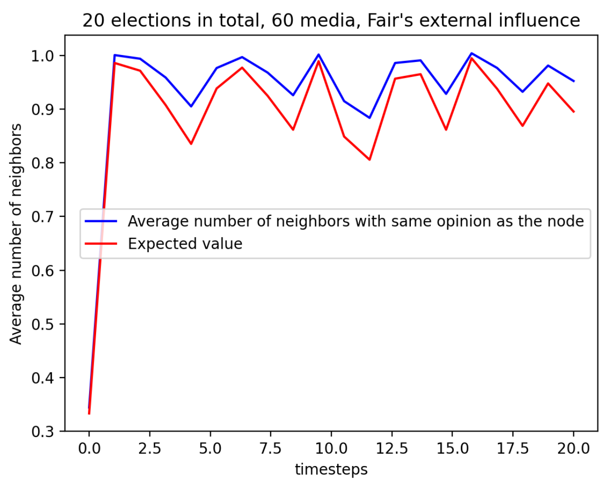
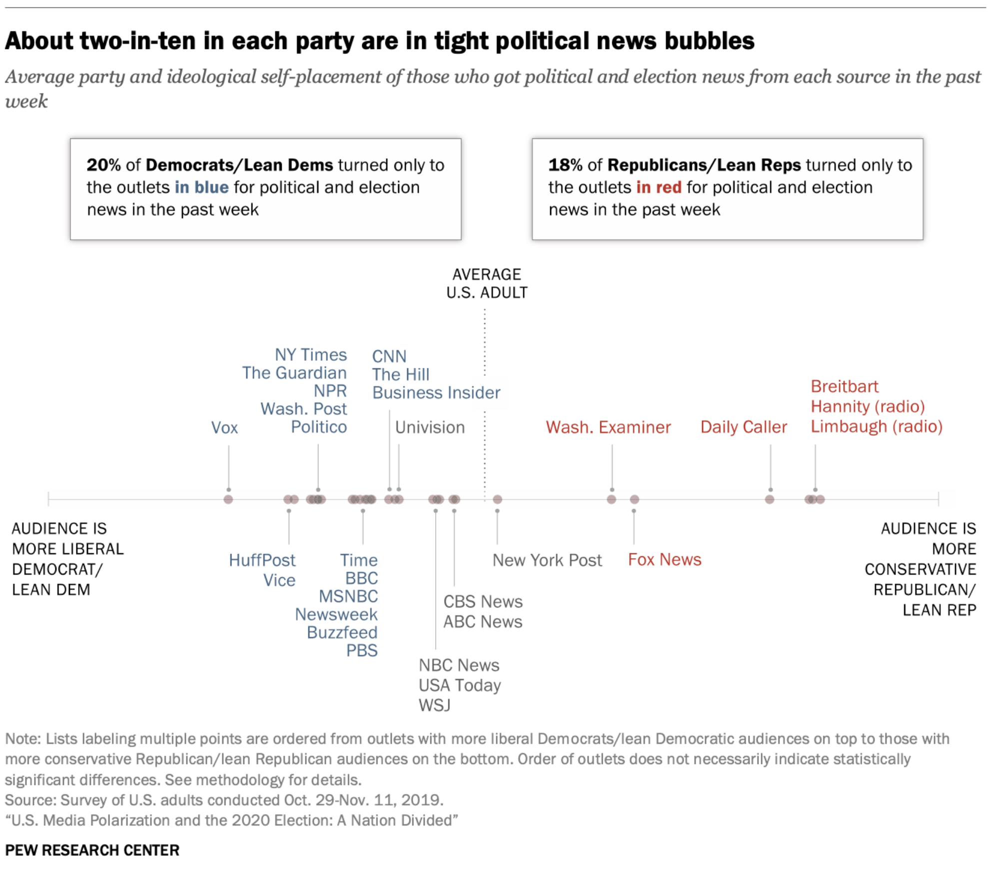


Figure 25: Average number of neighbors with same opinion as a voter node vs time.
Expected number of neighbors with same opinion as a media node (red line). All voters are affiliated to the republican party.

We can see the same result in 25, where the curve has a periodic behaviour, oscillating near one, meaning that the boredom term has still some influence on the systems, even if consensus is reached. Indeed, the initial equilibration, all media decide to oppose the ruling party, shifting their opinion to a negative value. The results are analogous to those obtainable considering an initial configuration in which everyone is affiliated with the democratic party (obviously considering that in that case, the consensus will be reached by the other party).

5.4 Feedback 1 : People can leave media

According to recent social sciences studies([32], [33], [34], [35]) people are increasingly surrounding themselves with media that have a similar opinion to their own one, thus causing the formation of echo chambers. A correct definition for echo chambers can be environments in which the opinion, political leaning, or belief of users about a topic gets reinforced due to repeated interactions with peers or sources having similar tendencies and attitudes [31]. For example, it has been estimated that around the 20% of voters leaning to a party in the US, before the 2020 elections turned only to media sources of the same political view [7]. In order to reproduce this, we added to the simulations a positive feedback intended to facilitate a voter to surround himself with media that have a similar political opinion.



By also considering this new feedback, the entire algorithm becomes:
The feedback works as follow:

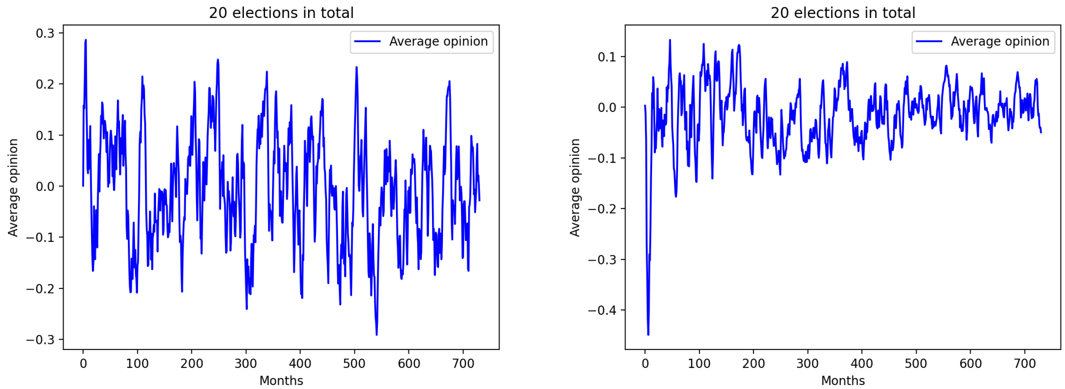
Algorithm 2 Main steps of the algorithm

For each timestep, generate a random position (i,j):
if $thenode(i, j) \neq media$ **then**
  Substitute medias with opinion too different (Feedback 1).
  Update the opinion depending on his neighbors.
if the node $(i, j) = media$ **then**
  Update his interval of opinion depending on the number of customers (Feedback 2).
if a week is passed **then**
  add a random term (ECONOMY) to medias' opinion
if 4 years are passed **then**:
  measure all the other observables
  define the ruling party
  update the boredom term
  update the incumbency term

Algorithm 3 Feedback 1

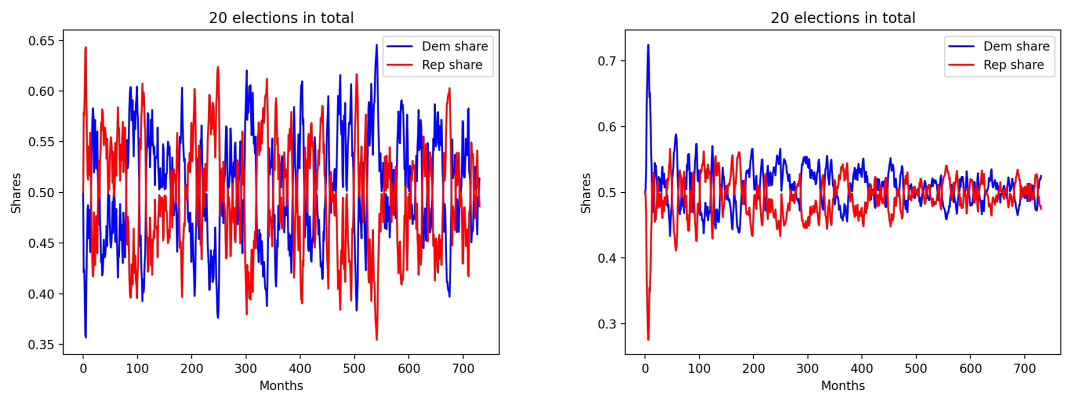
if $(i, j) \neq$ media and $S(i, j) < -0.25$ **then**
  for (x, y) as media neighbors of (i, j) **do** ▷ Democrat quitting overly republican media
    if $S(x, y) > 0$ and $S(x, y) < 0.5$ **then** $P = S(i, j)$
      if $S(i, j) > 0.5$ **then** $P = 0.5$
      else $P = 0$
      if $p < P$ **then**
        Remove the edge between (i, j) and (x, y)
        Find a new medium (v, w) with $S(v, w) \leq 0$
        Create edge between (i, j) and (v, w)

if $(i, j) \neq$ media and $S(i, j) > 0.25$ **then**
  for (x, y) as media neighbors of (i, j) **do** ▷ Republican quitting overly democrat media
    if $S(x, y) < 0$ and $S(x, y) > -0.5$ **then** $P = -S(i, j)$
      if $S(i, j) > -0.5$ **then** $P = 0.5$
      else $P = 0$
      if $p < P$ **then**
        Remove the edge between (i, j) and (x, y)
        Find a new medium (v, w) with $S(v, w) \geq 0$
        Create edge between (i, j) and (v, w)



(a) Average opinion without feedbacks vs time (b) Average opinion vs time with feedback 1 activated.

Figure 26: Measures on a population of $N = 10,000$ elements in which 60 are media nodes. The number of elections per a simulation is 20. Average opinion for different amounts of media



(a) Shares of the two parties vs time with no feedback. (b) Shares of the two parties vs time with feedback 1 activated.

Figure 27: Influence of feedback 1 on a population of $N = 10,000$ elements in which 60 are media nodes. The number of elections per a simulation is 20. Less media imply smaller amplitudes in average opinion.

The results show that, if the feedback is activated there is still an alternation of parties in power, thus no consensus is reached. On the other hand, the amplitudes of oscillations become smaller and smaller 27, meaning that the the portion of the population that on average changes its mind between elections decreases with the passage of time. This happens since voters can choose the media with the closest possible opinion to theirs, some media are completely ignored by the population which is therefore less subject to external influence. In any case, external influence (especially boredom term) is still able to ensure that there is a realistic alternation between parties.

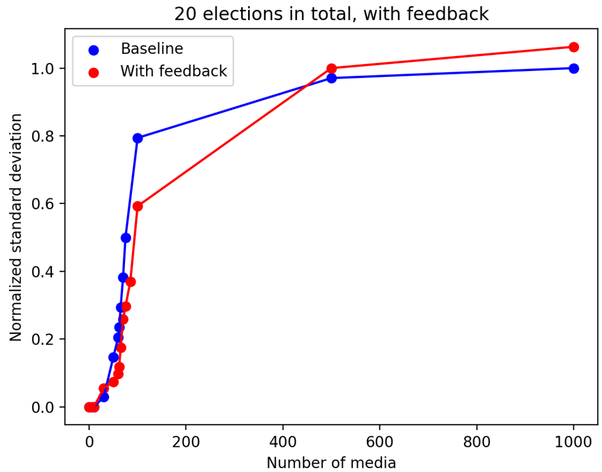


Figure 28: Average normalized standard deviation vs number of media nodes in a population of $N = 10,000$ elements.

In 28, we can see how the activation of this internal feedback influences the phase transition discussed in the previous paragraph. The behaviour of the normalized standard deviation is the same but the number of media needed to observe the transition is larger. This means that the system in this configuration is less responsive to the presence of media outlets than the baseline.

5.5 Feedback 2 : Media can decide to change opinion

In this experiment we add another feedback, this time relative to media nodes. Because of the presence of Feedback 1, now media can start with a given number of customers but loose a part of them during the simulation. This may happen because they are too extreme or mainstream in their opinion and not in agreement with the population of voters. In order to represent the fact that media in reality adapt in such a way as to satisfy the tastes of their customers, in this experiment, depending on the number of customers, they also can change their interval of opinion. Two options are possible, media may decide to become more "extreme" in their opinion or try to change completely their idea, becoming more neutral and mainstream. First we will discuss the first scenario, then the second one. Let us finally resume all the main steps of the model:

5.5.1 Case 1

Considering that the initial number of customers per media is 2000, if the loss of customers reaches a value between the 20% and the 40% of the initial number, it will shift his interval of opinion $[S_{min,ij}, S_{max,ij}]$ such that

$$\begin{aligned} S_{max,ij} &\rightarrow S_{max,ij} + 0.25 * |S_{max,ij}| \\ S_{min,ij} &\rightarrow S_{min,ij} + 0.25 * |S_{min,ij}| \end{aligned} \tag{27}$$

if $S_{ij} > 0.25$ and

$$\begin{aligned} S_{max,ij} &\rightarrow S_{max,ij} - 0.25 * |S_{max,ij}| \\ S_{min,ij} &\rightarrow S_{min,ij} - 0.25 * |S_{min,ij}| \end{aligned} \tag{28}$$

if if $S_{ij} < -0.25$. On the other hand, if the loss of customers is bigger than the 40%, the shift will become

$$\begin{aligned} S_{max,ij} &\rightarrow S_{max,ij} + 0.50 * |S_{max,ij}| \\ S_{min,ij} &\rightarrow S_{min,ij} + 0.50 * |S_{min,ij}| \end{aligned} \tag{29}$$

Algorithm 4 Main steps of the algorithm

For each timestep, generate a random position (i,j):
if $thenode(i, j) \neq media$ **then**
    Substitute medias with opinion too different (Feedback 1).
    Update the opinion depending on his neighbors.
    Substitute media with opinion too different (Feedback 1)
    Look for media with similar opinion (Feedback 1)
if a week is passed **then**
    add a random term (ECONOMY) to medias' opinion
if 4 years are passed **then**:
    measure all the other observables
    define the ruling party
    update the boredom term
    update the incumbency term
=0

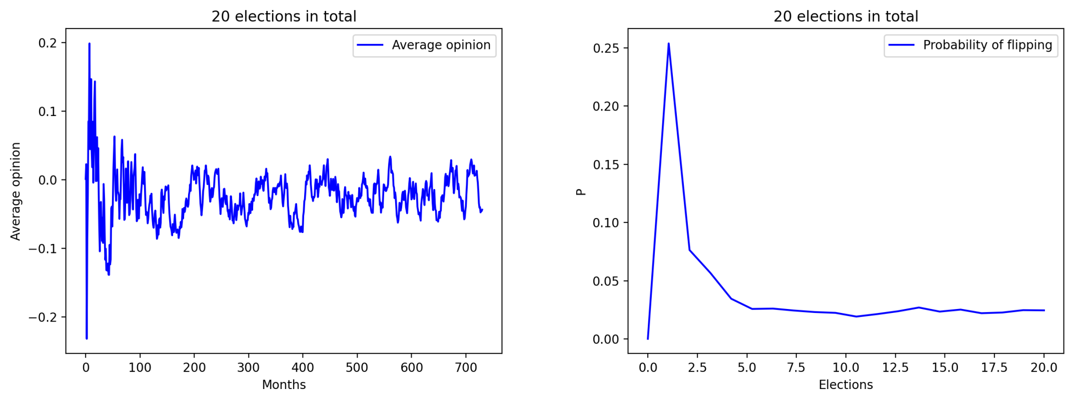
Algorithm 5 Main steps of the algorithm

For each time step, generate a random position (i,j):
if the node $(i, j) \neq media$ **then**
    Update the opinion depending on his neighbors.
    Substitute media with opinion too different (Feedback 1)
    Look for media with similar opinion (Feedback 1)
if the node $(i, j) = media$ **then**
    Update his interval of opinion depending on the number of customers (Feedback 2).
if a week is passed **then**
    add a random term (economy) to the opinion of media
if 4 years are passed **then**:
    measure all the other observables
    define the ruling party
    update the boredom term
    update the incumbency term

if $S_{ij} > 0.25$ and

$$\begin{aligned} S_{max,ij} &\rightarrow S_{max,ij} - 0.50 * |S_{max,ij}| \\ S_{min,ij} &\rightarrow S_{min,ij} - 0.50 * |S_{min,ij}| \end{aligned} \tag{30}$$

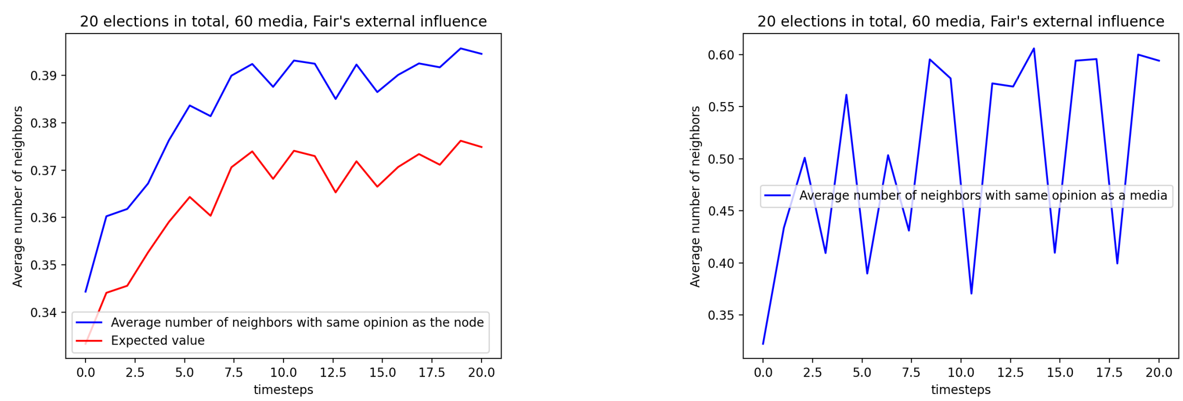
if $S_{ij} < -0.25$. If the media is a neutral one, hence $S_{ij} \in [-0.25, +0.25]$, there will be a 50% probability to go closer increase his opinion and 50% of decreasing it, with the same amounts as the other cases (25% for 20% of loss and 50% for 50% of loss).



(a) Average opinion vs time.

(b) Probability that during the time interval between two elections, a random node in the network changes flips his opinion.

Figure 29: Measures on a population of $N = 10,000$ elements in which 60 are media nodes. The number of elections per simulation is 20. Feedback 1 and Feedback 2 (case 1) activated.

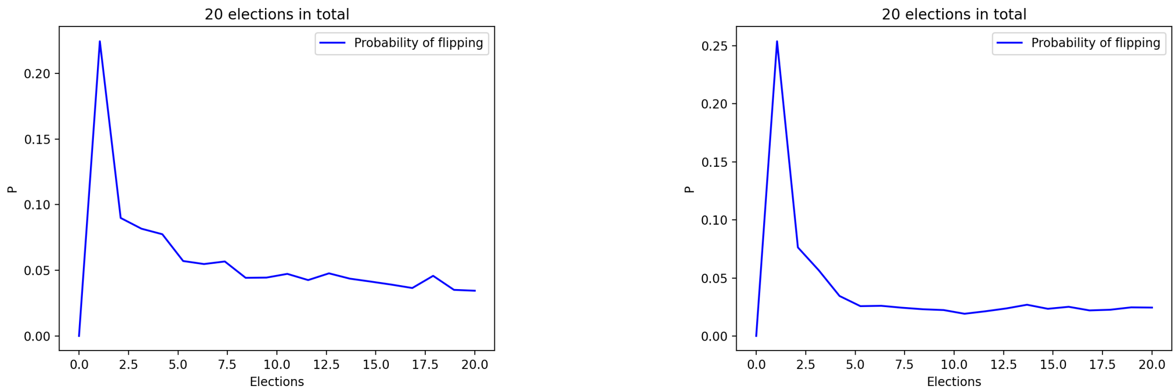


(a) Average number of neighbors with same opinion as a voter node.

(b) Observed average number of neighbors with same opinion as a media node (blue line).
Expected number of neighbors with same opinion as a media node (red line).

Figure 30: Measures on a population of $N = 10,000$ elements in which 60 are media nodes. The number of elections per simulation is 20. Feedback 1 and Feedback 2 (case 1) activated.

In this case we witness that the feedback brings the system to a configuration with smaller amplitudes but not exactly a complete polarised population (29). In fact, the probability of flipping relaxes to a value near the 2%.



(a) Probability that during the time interval between two elections, a random node in the network changes flips his opinion. Feedback 1 activated.

(b) Probability that during the time interval between two elections, a random node in the network changes flips his opinion. Feedback 1 and feedback 2 case 2 activated.

Figure 31: Measures of polarisation with the different feedbacks on a population of $N = 10,000$ elements in which 60 are media nodes. The number of elections per simulation is 20.

This was slightly unexpected, it would be more logical to see total polarization with such an extreme feedback. This is because although groups with the same opinion are emerging in the network (31), the media still have on average half of the customers with a different political idea from theirs. This therefore causes the alternation between the parties but with lower oscillation amplitudes.

5.5.2 Case 2

Let us now discuss the other scenario where

$$\begin{aligned} S_{max,ij} &\rightarrow S_{max,ij} - 0.25 * |S_{max,ij}| \\ S_{min,ij} &\rightarrow S_{min,ij} - 0.25 * |S_{min,ij}| \end{aligned} \tag{31}$$

if $S_{ij} > 0.25$ and

$$\begin{aligned} S_{max,ij} &\rightarrow S_{max,ij} + 0.25 * |S_{max,ij}| \\ S_{min,ij} &\rightarrow S_{min,ij} + 0.25 * |S_{min,ij}| \end{aligned} \tag{32}$$

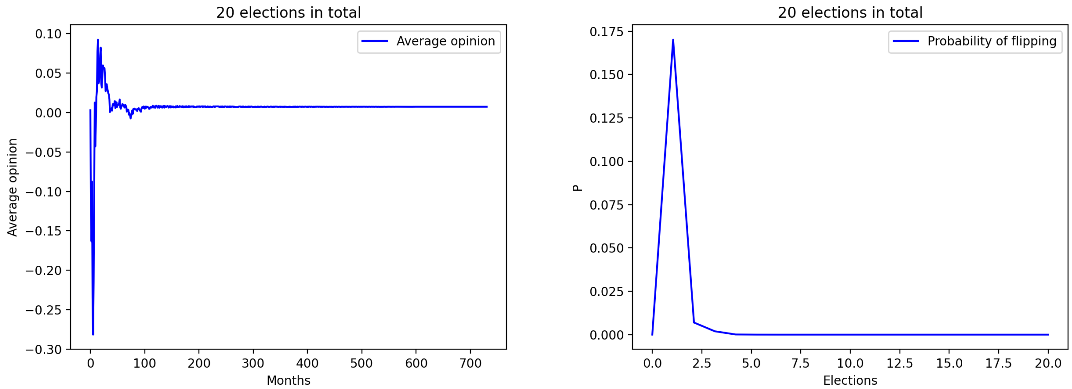
if if $S_{ij} < -0.25$. On the other hand, if the loss of customers is bigger than the 40%, the shift will become

$$\begin{aligned} S_{max,ij} &\rightarrow S_{max,ij} - 0.50 * |S_{max,ij}| \\ S_{min,ij} &\rightarrow S_{min,ij} - 0.50 * |S_{min,ij}| \end{aligned} \tag{33}$$

if $S_{ij} > 0.25$ and

$$\begin{aligned} S_{max,ij} &\rightarrow S_{max,ij} + 0.50 * |S_{max,ij}| \\ S_{min,ij} &\rightarrow S_{min,ij} + 0.50 * |S_{min,ij}| \end{aligned} \tag{34}$$

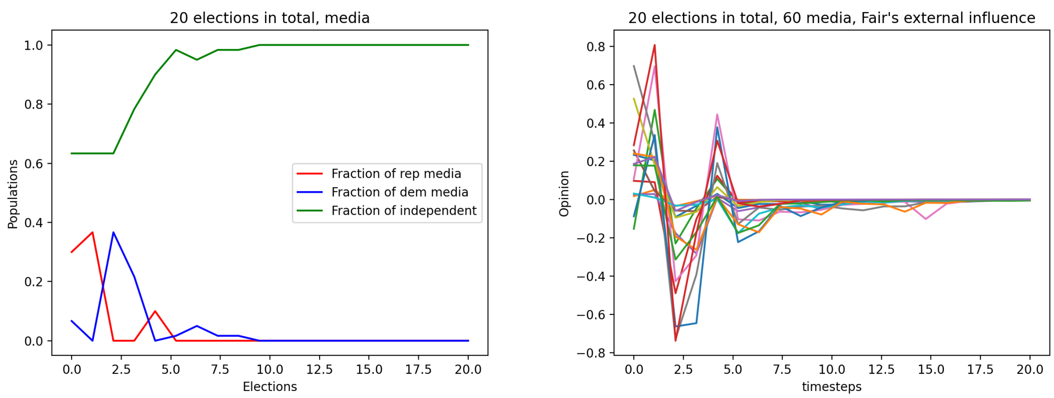
As already mentioned before, depending on how much the number of customers decreased respect than the initial value, media become more mainstream, shifting his interval of opinion to the neutral zone.



(a) Average opinion

(b) Probability that during the time interval between two elections, a random node in the network changes flips his opinion.

Figure 32: Measures on a population of $N = 10,000$ elements in which 60 are media nodes. The number of elections per simulation is 20. Feedback 2 (case 2) activated.



(a) Portions of media divided per political opinion. A media with opinion $S(i,j)$ is defined as republican if $S(i,j) \geq 0.25$, democrat if $S(i,j) \leq 0.25$ and independent otherwise.

(b) Individual opinion of 15 random media nodes. Each media is represented with a different color.

Figure 33: Measures on a population of $N = 10,000$ elements in which 60 are media nodes. The number of elections per simulation is 20. Feedback 2 (case 2) activated.

Contrary to what was expected, if the media tend to become mainstream, the population very quickly reaches a state of total polarization (32). This happens because all the media tend to become neutral for their policies (the interval of opinion tends to zero) and therefore lose the power to influence the voters who therefore remain stuck with the political idea of the first months of the simulation. This is visible from the plot of opinions of 15 randomly taken media, which after around 100 months converge to zero (33).

5.6 Varying affiliations

In this section we will discuss the case in which affiliations, considered constant for all past experiments, may change at the end of each election. In this way, the whole algorithm takes the form:

Algorithm 6 Main steps of the algorithm

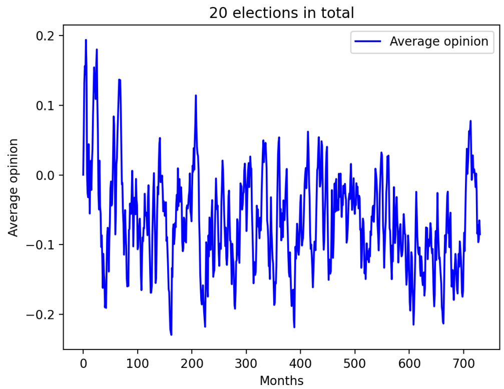
Generate a random position (i,j):
if the node $(i, j) \neq media$ **then**
    Update the opinion depending on his neighbors.
if a week is passed **then**
    add a random term (ECONOMY) to medias' opinion
if 4 years are passed **then**:
    measure all the other observables
    define the ruler party
    update the BOREDOM term
    update the INCUMBENCY term
    Update the affiliations depending on the previous vote

where the specific update for the affiliations follows:

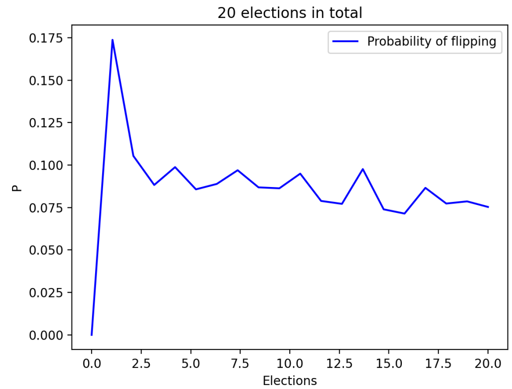
Algorithm 7 Affiliations update

if $(i, j) \neq media$ **then**
    Check for who voted in the election just ended.
    if (i, j) was R-affiliated but voted for the opposite party **then**.
        (i, j) has a 10% of probability to become unaffiliated
    if (i, j) was D-affiliated but voted for the opposite party **then**.
        (i, j) has a 10% of probability to become unaffiliated
    if (i, j) was not affiliated but voted for the D-party **then**.
        (i, j) has a 10% of probability to join the D-party
    if (i, j) was not affiliated but voted for the R-party **then**.
        (i, j) has a 10% of probability to join the R-party.
if a week is passed **then**
    add a random term (ECONOMY) to medias' opinion
if 4 years are passed **then**:
    measure all the other observables
    define the ruling PARTY
    update the boredom term

By considering this setup, we obtain a similar situation to the baseline in terms of average opinion. Indeed, affiliations do not influence amplitudes but only the alternancy of parties. In this case, we observe in the image a) of 34 that the portion of democrats is larger for almost the entire simulation.



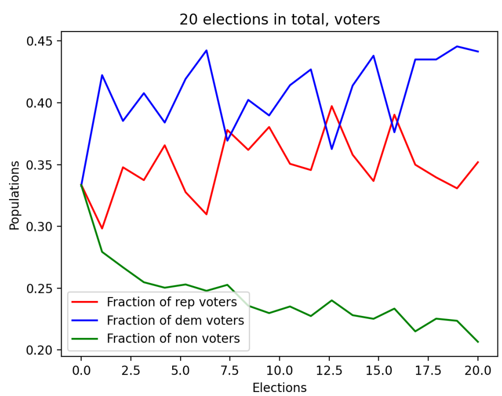
(a) Average opinion vs time.



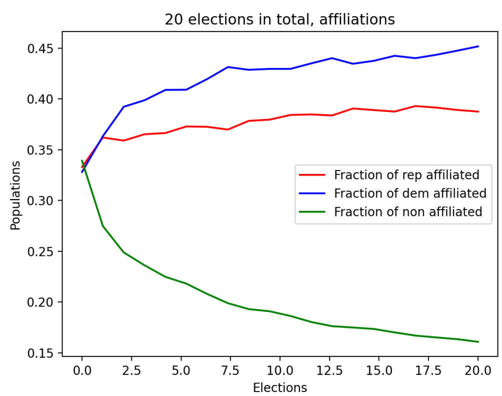
(b) Probability that during the time interval between two elections, a random node in the network changes flips his opinion

Figure 34: Measures on a population of $N = 10,000$ elements in which 60 are media nodes. The number of elections per simulation is 20. Affiliation states have a probability to change after every election.

Furthermore, the main weakness of the model with this setup is visible in 35, where the fraction of non voters tend non realistically to decrease along with the fraction of non affiliated. Contrary to this, in reality in the past 5 years it has been observed that affiliations had a fairly constant behavior. Despite numerous attempts to tune the transition probabilities to a realistic result, a right combination could not be found and another internal feedback will likely be required to make this configuration work realistically.



(a) Portion of voters vs time



(b) Portions of affiliations vs time

Figure 35: Measures on a population of $N = 10,000$ elements in which 60 are media nodes. The number of elections per simulation is 20. Affiliation states have a probability to change after every election.

6 Summary and conclusion

In this project we explored the role of media in the evolution of the political opinion of a population that can vote for two different parties. To properly describe voters and media it was necessary to put on the table different mathematical tools and disciplines such as economics, sociology and physics. For example by using the Fair model [24], we set all the parameters of the equation that regulate the influence of the economy on the elections, whose frequency was defined by observing historical data. The model developed is able to give realistic outputs and reproduce the main characteristics of the elections in the USA, from 1800 until today. One of the strengths of the model is precisely the way in which the media have been constructed, different from what has recently been proposed in the literature. The presence of numerous characteristics such as the portion of non-voters, affiliation, media, external influence, make the model optimal for carrying out experiments on phenomena such as polarization and the emergence of echo chambers from local micro interactions between the agents. We observed that, as the number of media changes, a phase transition between two critical scenarios in which the population is constantly indecisive and chaotic regarding the elections and one in which it is completely static with no source of information. In a population of $N = 10,000$ individuals, this occurred around 60 media, corresponding to around 11 media sources per voter. Furthermore, through the application of feedbacks such as the possibility of abandoning some media if too distant from a voter's opinion (a phenomenon that has been extremely studied in the social sciences lately), we witnessed the formation of groups of individuals with the same opinion, also known as "echo chambers" and polarisation. This social phenomena has recently been the subject of much debate in social sciences literature ([31], [32], [33], [34], [35]) and our experimental results seem to agree with most of them. By also adding the possibility that the media change their opinion based on its customers, the system becomes much less responsive to external influences and polarized. Overall, this agent based model showed that by starting from a simple model framework and local interactions it is possible to simulate the dynamics behind the spread of opinion and political elections. We can therefore conclude that if voters can switch away from media sources they don't agree with, then echo chambers arise, i.e. voters surround themselves by media they agree with and isolates themselves from other opinions. Consequently the probability of a voter to switch opinion drops by half.

7 Discussion and further work

The new framework was able to produce realistic results and interesting information on the dynamics of news diffusion between media and voters. This does not mean that numerous changes can be made to study different scenarios. One above all is the situation in which affiliations can change during the simulation. As can be seen in the numerous plots, the average number of neighbors with the same opinion of a random node never exceeds 50%. This is due to the fact that there are some elements affiliated with a particular party who resist changing their minds even in the most convinced neighborhoods of the opposite political opinion. In the situation where even affiliation can change after a certain interaction, one expects to see the phenomenon of segregation as an emerging behavior as well. Another possible application of this model is the coupling with an integrated assessment model (IAM) [36], that is a model designed to "integrate" society, economy, biosphere, atmosphere into one modeling framework. The IAM, adequately normalized, would replace what until now has been the task of the fair model, simulating the effects of politics on the economy and the environment, and thus influencing the elections.

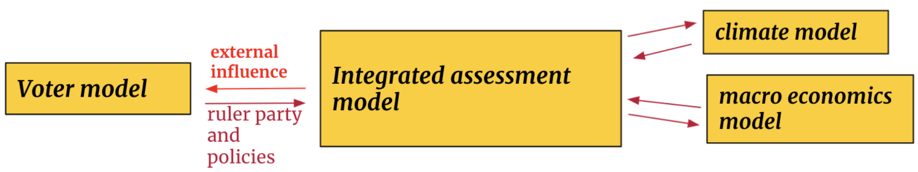


Figure 36

References

[1] Bruno Rodrigues [*"Homeostatic behaviour in democratic societies under climate change".]* Master Thesis, Utrecht University

[2] Ernst Ising. [*"Contribution to the Theory of Ferromagnetism"*]. In: Z. Phys. 31 (1925), pp. 253–258.

[3] McCarty, Nolan, Keith T. Poole and Howard Rosenthal *Polarized America: The Dance of Ideology and Unequal Riches.* Cambridge, MA: MIT Press. (2006)

[4] C. Castellano et al. *["Incomplete ordering of the voter model on small-world networks]"* EPL 63 153 (2003)

[5] Campante F, Hojman D. Media and Polarization *["Evidence from the Introduction of Broadcast TV in the US."]* Journal of Public Economics. 2013;100 :79-92.

[6] Krzysztof Suchecki et al. *["Voter model dynamics in complex networks: Role of dimensionality, disorder, and degree distribution"]* Phys. Rev. E 72, 036132 – Published 30 September 2005

[7] Pew Research Center, *["U.S. Media Polarization and the 2020 Election: A Nation Divided"]*, January, 2020

[8] Pew Research Center, March 2019, *["Political independents, who they are, what they think"].*

[9] Pew Research Center, October 2020, *["What the 2020 electors looks like by party, race, ethnicity, age, education and age"]*

[10] C. A. Bail et al, *["Exposure to opposing views on social media can increase political polarization"]*, 9216–9221 — PNAS — September 11, 2018 — vol. 115 — no. 37

[11] Shanto Iyengar et al. *["The Home as a Political Fortress: Family Agreement in an Era of Polarization"]* J. Politics 80, 1326 (2018).

[12] V. Sood and S. Redner *["Voter Model on Heterogeneous Graphs"]* Phys. Rev. Lett. 94, 178701 – Published 4 May 2005

[13] M. E. J. Newman and G. T. Barkema. [*["Monte Carlo methods in statistical physics."]*] Clarendon Press, 1999.

[14] David Condrey, CC BY-SA 3.0 ¡https://creativecommons.org/licenses/by-sa/3.0¿, via Wikimedia Commons

[15] Schulllz, CC BY-SA 3.0 ¡https://creativecommons.org/licenses/by-sa/3.0¿, via Wikimedia Commons

[16] F. Y. Wu [*The Potts model*] Rev. Mod. Phys. 54, 235 – Published 1 January 1982 DOI:https://doi.org/10.1103/RevModPhys.54.235

[17] ChrisnHouston, CC0, via Wikimedia Commons

[18] Andrzej Grabowski and R.A. Kosincki. [*"Ising-based model of opinion formation in a complex network of interpersonal interactions"*]. In: Physica A: Statistical Mechanics and its Applications 361 (Mar. 2006), 651–664.

[19] Mirta Galesic a, D.L. Stein. [*"Statistical physics models of belief dynamics: Theory and empirical tests"*]. In: Physica A: Statistical Mechanics and its Applications 519 (2019) 275–294

[20] Dan Braha, Marcus A. M. de Aguiar. [*"Voting contagion: Modeling and analysis of a century of U.S. presidential elections"*]. PLoS ONE 12(5): e0177970. https://doi.org/10.1371/journal. pone.0177970

[21] Sidney Redner [*"Reality-inspired voter models: A mini-review"*] In: C. R. Physique 20 (2019) 275–292 https://doi.org/10.1016/j.crhy.2019.05.004

[22] R. Lambiotte, S. Redner, *["Dynamics of non-conservative voters"]*, Europhys. Lett. 82 (2008) 18007.

[23] R. Lambiotte, S. Redner, *Dynamics of vacillating voters*, J. Stat. Mech. L10001 (2007).

[24] Ray C. Fair [*The Effect of Economic Events on Votes for President*] The Review of Economics and Statistics , Apr., 1978, Vol. 60, No. 2 (Apr., 1978), pp. 159-173; The MIT Press; https://www.jstor.org/stable/1924969

[25] Michael S. Lewis-Beck *["Election Forecasting: Principles and Practice"]*, BJPIR: 2005 VOL 7, 145–164

[26] David Sabin-Miller, Daniel M. Abrams. [*["When pull turns to shove: A continuous-time model for opinion dynamics"]*, PHYSICAL REVIEW RESEARCH 2, 043001 (2020). DOI: 10.1103/PhysRevResearch.2.043001

[27] Deepak Bhat and S. Redner [*"Nonuniversal opinion dynamics driven by opposing external influences"*] PHYSICAL REVIEW E 100, 050301(R) (2019) DOI: 10.1103/PhysRevE.100.050301

[28] N. Khalil, M. San Miguel, and R. Toral, *["Zealots in the mean-field noisy voter model"]* Phys. Rev. E 97, 012310 (2018).

[29] V. V. Vasconcelos, S. A. Levin, and F. L. Pinheiro, *["Consensus and polarization in competing complex contagion processes"*] J. R. Soc., Interface 16, 20190196 (2019).

[30] David Sabin-Miller and Daniel M. Abrams [*"When pull turns to shove: A continuous-time model for opinion dynamics"*] Phys. Rev. Research 2, 043001 – Published 1 October 2020

[31] Matteo Cinelli et al, *["The echo chamber effect on social media"]* PNAS 2021 Vol. 118 No. 9 e2023301118 https://doi.org/10.1073/pnas.2023301118

[32] M. Del Vicario et al., *["The spreading of misinformation online"]*. Proc. Natl. Acad. Sci. U.S.A. 113, 554–559 (2016).

[33] R. K. Garrett, *["Echo chambers online?: Politically motivated selective exposure among Internet news users."]* J. Comput. Mediated Commun. 14, 265–285 (2009).

[34] K. Garimella, G. De Francisci Morales, A. Gionis, M. Mathioudakis, *["Political discourse on social media: Echo chambers, gatekeepers, and the price of bipartisanship" in Proceedings of the 2018 World Wide Web Conference]*, WWW '18, P. A. Champin, F. Gandon, L. Medini, Eds. (International ´ World Wide Web Conferences Steering Committee, Geneva, Switzerland, 2018), pp. 913–922.

[35] W. Cota, S. C. Ferreira, R. Pastor-Satorras, M. Starnini, *["Quantifying echo chamber effects in information spreading over political communication networks."]* EPJ Data Sci. 8, 35 (2019).

[36] Hadi Dowlatabadi *["Integrated assessment models of climate change"]* Energy' Policy, VoL 23, No. 4/5, pp. 289-296, 1995