Research Project for Master thesis AI

Gradual Acceptability for Structured Argumentation in ASPIC+

by

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Abstract

Argumentation is a reasoning approach in Artificial Intelligence, which is approached by extension-based methods as well as gradual approaches. In the literature one is often vague about the type of argument strength that is studied and it is mostly approached avoiding structured argumentation. Thereby, assumptions are made on abstract level that do not always hold at the structural level.

In this work we answer the question how a semantics for dialectical argument strength in structured approaches to argumentation can be developed and evaluated. To that end two new semantics will be proposed using ASPIC+, one for argumentation frameworks with only attacks, one for argumentation frameworks with only supports. Both of these semantics will be evaluated by the postulates proposed in the literature as well as by postulates proposed in this work. Existing semantics will also be evaluated by the new postulates.

Contents

1. Introduction

Argumentation is a reasoning approach in Artificial Intelligence, which started at the end of the twentieth century with the work of Lin & Shoham (1989), Pollock (1987, 1992), Vreeswijk (1993, 1997) and Simari & Loui (1992). The idea is to use reasoning to construct arguments and evaluate them to say something about validity of an argument.

Most recent work builds on the theory from Dung (1995) about abstract argumentation frameworks (AFs). These AFs can be seen as directed graphs, where the nodes in these graphs represent the arguments and the attack relations are represented by the arrows.

These AFs are used to examine the acceptability of arguments. Methods with this purpose are called argumentation semantics. Dung (1995) proposed four extension-based semantics, namely, complete semantics, grounded semantics, stable semantics and preferred semantics. These semantics are expanded with a lot of different semantics, for example, recursive semantics (Baroni et al., 2005), semi-stable semantics (Caminada, 2006b) and ideal semantics (Dung et al., 2007). A more complete overview can be found in (van der Torre & Vesic, 2017). These extension-based methods give a scientific notion to the principle of "The one who has the last word laughs best".

In extension-based semantics a justified argument kills any argument it attacks. However, is some cases killing is too much and one would only like to weaken an argument. For example, an argument "we should by that car, because our current car is old", should not be killed by the argument "we should not buy that car, because it is expensive," but only weakened (Delobelle, 2017). So, one of the main objections against the extension-based approaches is that acceptability should be a gradual (Delobelle, 2017), while, extension-based semantics are limited by only a small degree of acceptability[1]. Graduality could be achieved in two ways: one could assign a value, which represents the strength, for every argument or one could provide a preorder on the set of arguments i.e. for any argument a and b it is determined if a is ranked higher, lower or equal to b. The first type of semantics are weighting semantics, the second one is called ranking semantics. And trivially each weighting semantics is a ranking semantics.

In the literature different approaches to add graduality are proposed and evaluated against postulates proposed by for example Amgoud & Ben-Naim (2013) and Baroni et al. (2018). Some papers captured graduality using extension based semantics (Cayrol & Lagasquie-Schiex, 2005b; Caminada & Wu, 2010; Bonzon et al., 2018); these are examples of ranking semantics. Other examples are Discussion-based Semantics and Burden-based Semantics (Amgoud & Ben-Naim, 2013), Iterated Graded Defense (Grossi & Modgil, 2015) and Propagation Semantics (Bonzon et al., 2016a). There are many weighting semantics. The general idea of weighting semantics is that they give a value to an argument based on the score of its direct attackers. Examples of weighing semantics are Categoriser-based Semantics (Besnard & Hunter, 2001), Max-based Semantics (Amgoud et al., 2017), Social Semantics (Leite & Martins, 2011), α-burden-based Semantics (Amgoud et al., 2016), Tuples-based Semantics (Cayrol

[1] In extension-based arguments in the extension are ranked higher than arguments outside the extension.

& Lagasquie-Schiex, 2005b), Matt & Toni (Matt & Toni, 2008), Fuzzy Labelling (da Costa Pereira et al., 2011), Counting Semantics (Pu et al., 2015) or Weighted h-categorizer, Weighted Max-based and Weighted Card-based Semantics (Amgoud et al., 2017) and many others.

Besides argumentation frameworks with only attack relations, there are bipolar argumentation frameworks (BAFs). These frameworks can be seen as directional graphs with two types of arrows. As in BAFs the nodes represent the arguments and the arrows represent the attack and support relations between those arguments.

Cohen et al. (2018) defined four notions of such support-relations, namely conclusion support, premise support, intermediate support and sub-argument support. An argument a is conclusion-supported by argument b if argument b has the same conclusion as the conclusion of argument a. An argument a is premise-supported by argument b if argument b has the same conclusion as a premise of argument a. An argument a is intermediate-supported by argument b if the conclusion of argument b is equal to a sub-conclusion of a (which is not the conclusion or a premise). An argument a is sub-argument-supported by argument b if argument b is a sub-argument argument a.

There are various gradual semantics for BAFs. Also for BAFs there are weighting semantics that give a value to each argument and ranking semantics that only rank arguments. Examples of weighting semantics for BAFs are Euler-based restricted semantics (Amgoud & Ben-Naim, 2018) and the quadratic energy model (Potyka, 2018). Examples of ranking semantics for BAFs are the extension-based ranking semantics (Cayrol & Lagasquie-Schiex, 2005b; Bonzon et al., 2018) which are applied on a framework with additional attacks (Cayrol & Lagasquie-Schiex, 2013), for example if a attacks b and b supports c, then an attack from a to c is added.

The above mentioned semantics are all based on abstract argumentation, thereby neglecting the structure of arguments. The only paper about structured argumentation is the paper by Pollock (2001). Recent papers only discuss abstract argumentation. Ignoring the structure of arguments or the nature of their relations may result in odd or undesirable results. For example, for abstract argumentation one makes often assumptions which do not hold for structured argumentation (Prakken, 2020). A second problem with the recent literature is discussed by Prakken (2021). Prakken (2021) distinguishes three aspects of argumentation strength: logic strength, dialectic strength and rhetorical strength. According to Prakken (2021) logical argument strength divides into inferential argument strength (how well do the premises support the conclusion if one only looks at the argument's premises, inferences and conclusion) and contextual argument strength (contextual argument strength is about how well the conclusion of an argument is supported if one looks at the context of all relevant arguments). "Dialectical argument strength looks at how well defended an argument is in the context of an ongoing or terminated critical discussion" (Prakken, 2021) and "Rhetorical argument strength looks at how capable an argument is to persuade other participants in a discussion or an audience". (Prakken, 2021).

Many of the earlier mentioned papers do not mention which kind of argument strength their semantics describes. However, as argued by Prakken (2021), there are three different types of argument strength. Therefore, it is very unlikely that the same set of postulates would fit each of these types of argument strength. For example, unattacked arguments have a very strong logical argument strength, but not necessarily a strong dialectical argument strength. This could lead to problematic results.

In this paper two new semantics will be proposed for structured approaches to argumentation, one for argumentation frameworks with only attacks, one for argumentation frameworks with only supports. Both of these new semantics will be semantics for dialectical argument strength. Both of these semantics will be evaluated by the postulates proposed by respectively Amgoud & Ben-Naim (2013) and Amgoud & Ben-Naim (2016a). Furthermore, we will discuss to what extent each postulates fits for dialectical argument strength. Also additional postulates will be proposed.

In this research project the question we aim to answer is: "How can a weighing semantics for dialectical argument strength in structured approaches to argumentation be developed and evaluated?"

To answer this question the following three research questions will be answered:

- R1 What properties of ASPIC+ can be utilized to create new semantics for dialectical argument strength?
- R2 What postulates defined on an abstract level hold for structured argumentation i.e. when instantiated in ASPIC+?
- R3 What are good postulates for dialectical strength?
- R4 What postulates are satisfied by different semantics?

The paper is organised as follows. Section 2 gives the background of argumentation frameworks, introduces (gradual) semantics formally and consists of a description for structured argumentation using ASPIC+. In section 4 intuitions for a new semantics are discussed and a new semantics is proposed for argumentation frameworks with only attacks. Subsequently, in section 5 this new semantics will be evaluated by existing postulates as well as new postulates, which will be proposed in that section. Then, section 6 contains intuitions and a definition of a semantics for argumentation frameworks with only supports. These semantics will be evaluated on existing postulates as well as postulates proposed in section 7. The final section concludes.

2. Preliminaries

This section starts by recalling the definition of Dung's argumentation frameworks and adaptations of this framework and the extension-based semantics (Dung, 1995). Then, (weighted) bipolar argumentation frameworks will be introduced. Finally, ASPIC+ will be recalled.

2.1. **Argumentation Frameworks.**

As mentioned before, most recent work builds on the theory from Dung (1995) about abstract argumentation frameworks (AFs). These AFs can be seen as directed graphs, where the nodes in these graphs represent the arguments and the attack relations are represented by the arrows, see Definition 1.

Definition 1 (Abstract Argumentation Frameworks)**.** *An abstract argumentation framework (or graph) is a tuple* $(\mathcal{A}, \mathcal{R})$*, where* $\mathcal{A}$ *is a set of arguments and* $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ *a binary attack relation on* $\mathcal{A}$*. For arguments* $a, b \in \mathcal{A}$*,* $(a, b) \in \mathcal{R}$

or $a\mathcal{R}b$ means that a attacks b. Furthermore, $\mathcal{R}^-(a)$ denotes the set of attackers of a and $R^+(a)$ denotes the set of arguments that are attacked by a.

In other papers adaptations of these argumentation frameworks are used. This are the semi-weighted abstract argumentation frameworks (resp. weighted abstract argumentation frameworks), where nodes (resp. nodes and edges) have an initial weight.

Definition 2 (Semi-weighted Abstract Argumentation Frameworks)**.** *(Amgoud et al., 2017) A semi-weighted abstract argumentation framework (or graph) is a tuple $(\mathcal{A}, \omega, \mathcal{R})$, where $\mathcal{A}$ is a set of arguments, $\omega : \mathcal{A} \to [0, 1]$ is a function which maps each argument to its initial weight and $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ a binary attack relation on $\mathcal{A}$.*

Definition 3 (Weighted Abstract Argumentation Frameworks)**.** *(Amgoud & Doder, 2019) A weighted abstract argumentation framework (or graph) is a tuple $(\mathcal{A}, \omega, \mathcal{R}, \pi)$, where $\mathcal{A}$ is a set of arguments, $\omega : \mathcal{A} \to [0, 1]$ is a function which maps each argument to its initial weight, $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ a binary attack relation on $\mathcal{A}$, $\pi : \mathcal{A} \to [0, 1]$ is a function which maps attack to its weight.*

2.1.1. *Extension-based Semantics.*

These AFs are used to examine the acceptability of arguments. Methods with this purpose are called argumentation semantics. Dung (1995) describes four extension-based semantics. In these extension-based methods arguments are skeptically acceptable, credulously acceptable or not acceptable[2]. He defines acceptability of arguments in terms of conflict-free sets and in terms of defending. Each extension represents a different set of arguments that are acceptable.

Definition 4 (Conflict-free)**.** *A set S of arguments is said to be conflict-free if $\forall A \in S\ \nexists B \in S : A$ attacks B.*

Definition 5 (Defends)**.** *Let $AF = (\mathcal{A}, \mathcal{R})$ be an argumentation framework. A set $S \subseteq \mathcal{A}$ of arguments is said to defend argument $a \in \mathcal{A}$ iff $\forall b \in \mathcal{A}\ b\mathcal{R}a$ implies that $\exists c \in S : c\mathcal{R}b$.*

Definition 6. *(Dung, 1995) Let $AF = (\mathcal{A}, \mathcal{R})$ be an argumentation framework and $S \subseteq A$ a conflict-free set. Then:*

- *S is a complete extension iff it defends all its elements and contains any argument it defends.*
- *S is a grounded extension iff it is a minimal (w.r.t. set inclusion) complete extension.*
- *S is a preferred extension iff it is a maximal (w.r.t. set inclusion) complete extension.*
- *S is a stable extension iff it attacks any argument in $\mathcal{A} \backslash S$.*

Definition 7. *For each semantics an argument is skeptically acceptable if it belongs to all extensions; an argument is credulously acceptable if it belongs to at least one extension, but not to all extensions; an argument is not acceptable if it belongs to no extension.*

[2]Other paper sometimes use the terms 'acceptable', 'undecided' and 'not acceptable' or in recursive semantics they use the terms 'undefeated', 'provisionally defeated' and 'defeated'

2.2. **Bipolar Argumentation Frameworks and Corresponding Semantics.**

Besides argumentation frameworks with only attack relations, there are bipolar argumentation frameworks. These frameworks can be seen as directional graphs with two types of arrows. As in BAFs the nodes represent the arguments. Moreover, the arrows represent the attack and support relation between those arguments. There are also semantics that use weighted bipolar abstract argumentation frameworks.

Definition 8 (Bipolar Argumentation Frameworks)**.** *(Cayrol & Lagasquie-Schiex, 2005a) A bipolar abstract argumentation framework is a triple* $(\mathcal{A}, \mathcal{R}, \mathcal{S})$*, where* $\mathcal{A}$ *is a set of arguments and* $\mathcal{R}$ *(resp.* $\mathcal{S}$*) is a binary attack (resp. support) relation on* $\mathcal{A}$*.*

For $a, b \in \mathcal{R}$ *(resp.* $\mathcal{S}$*)* $a\mathcal{R}b$ *(resp.* $a\mathcal{S}b$*) means that* a *attacks (resp. supports)* b*. Furthermore,* $R-(a)$ *(resp.* $\mathcal{S}^-(a)$*) denotes the set of attackers (resp. supporters) of* a*.*

Definition 9 (Weighted Bipolar Abstract Argumentation Frameworks)**.** *(Amgoud & Ben-Naim, 2018) A weighted bipolar abstract argumentation framework (or graph) is a tuple* $(\mathcal{A}, \omega, \mathcal{R}, \mathcal{S})$*, where* $\mathcal{A}$ *is a set of arguments,* $\omega : \mathcal{A} \to [0, 1]$ *is a function which maps each argument to its initial weight and* $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ *(resp.* $\mathcal{S} \subseteq \mathcal{A} \times \mathcal{A}$*) a binary attack (resp. support) relation on* $\mathcal{A}$*. For arguments* $a, b \in \mathcal{A}$*,* $(a, b) \in \mathcal{R}$ *or* $a\mathcal{R}b$ *means that* a *attacks* b *and* $(a, b) \in \mathcal{S}$ *or* $a\mathcal{S}b$ *means that* a *supports* b*. We write* $\omega \equiv x$ *if* $w(a) = x$ *for all arguments* $a \in \mathcal{A}$*.*

2.2.1. *Extension-based Semantics.*

In the literature different interpretations of support i.e. semantics for BAFs are presented. General support (Cayrol & Lagasquie-Schiex, 2005c) adds attack-relations in terms of supported attack (i.e. if a supports b and b attacks c, then a support attacks c) and indirect attack (i.e. if a supports b and c attacks a, then c indirect attacks b). Also set-support (A set-supports b if there is a sequence of arguments $(a_1, \ldots, a_n), a_1 \in A, a_n = b$, where a_i supports a_{i+1}) as addition to defending is added. Boella et al. (2010) add mediated attacks (i.e. if a supports b and c attacks b, then c mediated attacks a). More details for the interpretations can be found in (Cohen et al., 2014) as well as a more complete survey. On the basis of these 'added' attack relations extension-based semantics can be used in terms of conflict-freeness and defending (set-support included as defending)(Cayrol & Lagasquie-Schiex, 2013).

2.3. **ASPIC+.**

ASPIC+ is a framework for structured argumentation that generates abstract argumentation frameworks in the sense of Dung (1995) as described before, which defines the relations between arguments as well as the structure of arguments. The ASPIC+ framework (Prakken, 2010; Modgil & Prakken, 2014) is a formal argumentation framework with attack relations, where there is a distinction between strict (or deductive) inference rules and defeasible inference rules. Strict rules are rules that are deductively valid and so are unattackable. Defeasible rules are rules that have exceptions and so are attackable. The rules, combined with a logical language $\mathcal{L}$ form an argumentation system.

Definition 10 (Argumentation Systems)**.** *An argumentation system is a tuple* $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$ *where:*

- *$\mathcal{L}$ is a logical language with negation symbol $\neg$.*
- *$\mathcal{R} = \mathcal{R}_s \cup \mathcal{R}_d$ is a set of strict ($\mathcal{R}_s$) and defeasible ($\mathcal{R}_d$) inference rules of the form $\varphi_1, \ldots, \varphi_n \rightarrow \varphi$ and $\varphi_1, \ldots, \varphi_n \Rightarrow \varphi$ respectively (where φ_i, φ are meta-variables ranging over well-formed formula in $\mathcal{L}$), and $\mathcal{R}_s \cap \mathcal{R}_d = \emptyset$.*
- *n is a partial function such that $n : \mathcal{R}_s \rightarrow \mathcal{L}$,*
- *$\leq$ is a partial preorder on $\mathcal{R}_d$.*

We write $\varphi = -\psi$ just in case $\psi = \neg\varphi$ or $\varphi = \neg\psi$.

These argumentation systems can be combined with knowledge bases consisting of axiom premises and ordinary premises, to build arguments. Axiom premises are unattackable, ordinary premises are attackable. An argumentation system supplemented with a knowledge base form a argumentation theory.

Definition 11 (Knowledge Bases)**.** *A knowledge base in an Argumentation System $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$ is a set $\mathcal{K} \subseteq \mathcal{L}$ consisting of two disjoint subsets $\mathcal{K}_n$ (the axioms) and $\mathcal{K}_p$ (the ordinary premises).*

Definition 12 (Argumentation Theory)**.** *An argumentation theory is a tuple $AT = (AS, \mathcal{K})$ where AS is an argumentation system and $\mathcal{K}$ is a knowledge base in AS.*

In ASPIC+ an argument is either an element of the knowledge base $\mathcal{K}$, or a subset of the knowledge base, combined with a sequence of rule applications that leads to a conclusion.

Definition 13 (Argument)**.** *An argument A on the basis of an argumentation theory with a knowledge base $\mathcal{K}$ and an argumentation system $(\mathcal{L}, \mathcal{R}, n, \leq)$ is any structure obtainable by applying one or more of the following steps finitely many times:*

(1) *φ is an argument if $\varphi \in \mathcal{K}$ with $\mathrm{Prem}(A) = \{\varphi\}, \mathrm{Conc}(A) = \varphi, \mathrm{Sub}(A) = \{\varphi\}, \mathrm{DefRules}(A) = \emptyset, \mathrm{TopRule}(A) =$ undefined.*

(2) *$A_1, \ldots, A_n \rightarrow \psi$ is an argument if $A_1, \ldots, A_n$ are arguments such that there exists a strict rule $\mathrm{Conc}(A_1), \ldots, \mathrm{Conc}(A_n) \rightarrow \psi$ in $\mathcal{R}_s$.*
$\mathrm{Prem}(A) = \mathrm{Prem}(A_1) \cup \cdots \cup \mathrm{Prem}(A_n)$,
$\mathrm{Conc}(A) = \psi$,
$\mathrm{Sub}(A) = \mathrm{Sub}(A_1) \cup \cdots \cup \mathrm{Sub}(_n) \cup \{A\}$,
$\mathrm{DefRules}(A) = \mathrm{DefRules}(A_1) \cup \cdots \cup \mathrm{DefRules}(A_n)$,
$\mathrm{TopRule}(A) = \mathrm{Conc}(A_1), \ldots, \mathrm{Conc}(A_n) \rightarrow \psi$

(3) *$A_1, \ldots, A_n \Rightarrow \psi$ is an argument if $A_1, \ldots, A_n$ are arguments such that there exists a defeasible rule $\mathrm{Conc}(A_1), \ldots, \mathrm{Conc}(A_n) \Rightarrow \psi$ in $\mathcal{R}_d$.*
$\mathrm{Prem}(A) = \mathrm{Prem}(A_1) \cup \cdots \cup \mathrm{Prem}(A_n)$,
$\mathrm{Conc}(A) = \psi$,
$\mathrm{Sub}(A) = \mathrm{Sub}(A_1) \cup \cdots \cup \mathrm{Sub}(A_n) \cup \{A\}$,
$\mathrm{DefRules}(A) = \mathrm{DefRules}(A_1) \cup \cdots \cup \mathrm{DefRules}(A_n) \cup \mathrm{Conc}(A_1), \ldots, \mathrm{Conc}(A_n) \Rightarrow \psi$,
$\mathrm{TopRule}(A) = \mathrm{Conc}(A_1), \ldots, \mathrm{Conc}(A_n) \Rightarrow \psi$

Where Prem (resp. Conc, TopRule, Sub) returns all the premises (resp. the conclusion, the last rule in the argument, all the sub-arguments of a given argument). We write $\mathcal{A}(AT)$ to denote the set of arguments from argumentation theory AT.

An argument in ASPIC+ is strict and firm if it all its premises are axiom premises and it has no defeasible inference rules.

Definition 14 (Argument Properties)**.** *An argument a is*

- *strict if* $\text{DefRules}(a) = \emptyset$
- *defeasible if* $\text{DefRules}(a) \neq \emptyset$
- *firm if* $\text{Prem}(a) \subseteq \mathcal{K}_n$
- *plausible if* $\text{Prem}(a) \not\subseteq \mathcal{K}_n$

In ASPIC+ there are three kind of attacks, to know rebuttal attack, undercutting attack and undermining attack. Rebuttal attack is an attack on the conclusion of an argument, undercutting attack is an attack on a defeasible rule of an argument and an undermining attack is an attack on an ordinary premise of an argument. Arguments with a strict TopRule can not be rebutted, only arguments with a defeasible TopRule can be rebutted.

Definition 15 (Attacks)**.** *A attacks B iff A undercuts, rebuts or undermines B, where:*

- *A undercuts argument B (on B') iff* $\text{Conc}(A) = -n(r)$ *for some* $B' \in \text{Sub}(B)$ *such that B''s top rule r is defeasible.*
- *A rebuts argument B (on B') iff* $\text{Conc}(A) = -\varphi$ *for some* $B' \in \text{Sub}(B)$ *of the form* $B''_1, \ldots, B''_n \Rightarrow \varphi$.
- *A undermines B (on φ) iff* $\text{Conc}(A) = -\varphi$ *for an ordinary premise φ of B.*

Premises can have a ranking and also defeasible rules can have an ranking in an argumentation system. The ranking of arguments in structured argumentation frameworks can be defined by these rankings.

Definition 16 (Structured Argumentation Frameworks)**.** *Let $AT = (AS, \mathcal{K})$ be an argumentation theory. A Structured Argumentation Framework (SAF) is a triple $(\mathcal{A}, \mathcal{R}_{att}, \preceq)$, where $\mathcal{A}$ is the set of all finite arguments constructed from $\mathcal{K}$ in AS, $\mathcal{R}_{att}$ is the attack relation ($(a, b) \in \mathcal{R}_{att}$ iff a attacks b), and $\preceq$ is a preference ordering on $\mathcal{A}$. We write $a \prec b$ iff $a \preceq b$, but $b \npreceq a$.*

Definition 17. *A defeats B iff A undercuts B, or if A rebuts/undermines B on B' and $A \nprec B'$.*

This notion of defeat is similar to the notion of attack-relation from Argumentation Frameworks of definition 1, more precisely, the defeat-relation is a subset of the attack-relation.

2.3.1. *Instantiations.*
ASPIC+ is used to instantiate Dung's abstract approach with a general account of the structure of arguments and the nature of the defeat relation (Prakken, 2010) as well as BAFs (Cohen et al., 2018). For argumentation frameworks this can be done by defining an argumentation framework in terms of a SAF (see Definition 18). There is a ordering for the ordinary premises, there is in ASPIC+ also the possibility to raise these to orderings for the defeasible inference rules.

Definition 18 (Argumentation frameworks)**.** *(Modgil & Prakken, 2014) An argumentation framework corresponding to a $SAF = (\mathcal{A}, \mathcal{R}_{att}, \preceq)$ is a pair $(\mathcal{A}, \mathcal{R}_{def})$ such that $\mathcal{R}_{def}$ is the defeat relation on $\mathcal{A}$ defined by $(\mathcal{A}, \mathcal{R}_{att}, \preceq)$.*

3. Graduality

In this section we will look at different kind of gradual semantics. For some of the semantics examples are provided to increase understanding of semantics and increase understanding of the difference between different semantics. Firstly, the semantics for frameworks with only attacks will be discussed. Then, the semantics for frameworks with both attacks and supports will be discussed.

Graduality could be achieved in two ways, one could assign a value, which represents the strength, for every argument or one could provide a preorder on the set of arguments i.e. for each argument $a, b \in \mathcal{A}$ is determined if a is ranked higher, lower or equal to b. The first type of semantics are weighting semantics, the second one is called ranking semantics. And trivially each weighting semantic is a ranking semantic. All of the semantics discussed below are displayed in Table 2.

3.1. **Graduality in Extension-based Semantics.**

One could argue, as Cayrol & Lagasquie-Schiex (2005b), that arguments could be uni-acceptable (skeptically acceptable), cleanly-acceptable, exi-acceptable (credulously acceptable) and not-acceptable according to the terminology of Pinkas & Loui (1992) and Cayrol & Lagasquie-Schiex (2005b). Uni-accepted means belonging to each extension; cleanly accepted means belonging to at least one extension but not all and all attackers do not belong to any extension; exi-excepted means belonging to some extension but not all; not-accepted means not belonging to any extension.

Another attempt to capture graduality in the extension-based methods is from the work of Bonzon et al. (2018) elaborating on (Caminada & Wu, 2010). Using the labeling-approach from Caminada (2006a), to label arguments with labels 'in', 'out' or 'undec' where an argument is labelled 'in' iff the argument is accepted for the labelling, it is labelled 'out' iff the argument is rejected, and it is labelled 'undec' iff the argument is undecided. Using the complete labelling, arguments could have multiple labels. On the basis of these labellings, the arguments could be ranked, where $\{\text{in}\} \succeq \{\text{in}, \text{undec}\} \simeq \{\text{in}, \text{out}\} \simeq \{\text{undec}\}, \simeq \{\text{in}, \text{out}, \text{undec}\} \succeq \{\text{out}, \text{undec}\} \succeq \{\text{out}\}$.

Both of the above semantics are ranking semantics.

3.2. **Weighting Semantics for AFs.**

Weighting semantics are semantics that assign a value, which represents the strength, from a fixed ordered scale for every argument in the graph (Amgoud, 2019). The strength of an argument a is dependent on its direct attackers $b_1, \ldots, b_n$. First the strengths of the direct attackers is aggregated by a function g. Which then represents how strongly a is attacked. Then an influence function f takes into account the strength of the attacks (Amgoud, 2019). For acyclic argumentation frameworks is it straight forward how to apply this. For cyclic argumentation frameworks more sophisticated methods are needed, for example fixed-point methods in (Amgoud et al., 2016) or other solutions like the one presented in (Potyka, 2018).

The strength is for argumentation framework $AF = (\mathcal{A}, \mathcal{R})$ and semantics Π and can be defined by $Deg^{\Pi}_{AF} : \mathcal{A} \to [0, \infty)$:

$$Deg^{\Pi}_{AF}(a) = f(g(Deg^{\Pi}_{AF}(b_1), \ldots, Deg^{\Pi}_{AF}(b_n))). \tag{1}$$

Examples of such weighting semantics are categoriser-based semantics (Besnard & Hunter, 2001), maxed-based semantics (Amgoud et al., 2017), social semantics (Leite & Martins, 2011), α-burden-based semantics (Amgoud et al., 2016) tuples-based semantics (Cayrol & Lagasquie-Schiex, 2005b), Matt & Toni (Matt & Toni, 2008), Fuzzy labelling (da Costa Pereira et al., 2011), Counting semantics (Pu et al., 2015) or weighted h-categorizer, weighted max-based and weighted card-based semantics (Amgoud et al., 2017) and many others.

There are many weighting semantics. The general idea of weighting semantics is that they give a value to an argument based on the score of its direct attackers $x_1, \ldots, x_i$, which are on their turn are based on the score of their direct attackers. Categoriser-based semantics (Besnard & Hunter, 2001) is as in Equation 1 with $f(x) = \frac{1}{1+x}$ and $g(x_1, \ldots, x_n) = \sum_{i=1}^{n} Deg_{AF}^{\pi}(x_i)$. So the higher the score of the direct attackers of a, the lower the score of argument a and vice-versa. If the attackers of argument b are strict subset of the attackers of argument a, then argument b's score is strictly higher then a's. In contrast we have the max-based semantics (Amgoud et al., 2017) which as underlying function for Equation 1 has $f(x) = \frac{1}{1+x}$ and $g(x_1, \ldots, x_n) = \max\{Deg_{AF}^{\pi}(x_i) | i \in [1, \ldots, n]\}$. This semantics differs from the previous one in the sense that addition of attackers to argument a does not necessarily change the score of argument a.

Categoriser-based semantics was initially for acyclic argumentation frameworks. Later is shown that also cyclic argument frameworks have categoriser-based rankings. These are obtainable using a system of non-linear equations (Cayrol & Lagasquie-Schiex, 2005b). Pu et al. (2014) show existence and uniqueness of a solution of the categoriser equations and show how to find them using a fixed-point technique. Max-based semantics is also usable for cyclic argumentation frameworks using a fixed-point method (Amgoud et al., 2017).

Example 1. *Consider the argumentation framework* $AF = (\mathcal{A}, \mathcal{R})$, *where* $\mathcal{A} = \{a, b, c, d, e, f, g, h, i\}$ *and* $\mathcal{R} = \{(c, a), (d, b), (e, b), (f, b), (g, d), (h, e), (i, f)\}$. *Then for categoriser-based semantics* $Deg_{AF}^{Cat}(a) = \frac{1}{2}$ *and* $Deg_{AF}^{Cat}(b) = \frac{2}{5}$. *For maxed-based semantics* $Deg_{AF}^{Max}(a) = \frac{1}{2}$ *and* $Deg_{AF}^{Max}(b) = \frac{2}{3}$.

So in categoriser-based semantics the value of a is higher than the value of b and with max-based semantics it is the contrary. So categoriser-based semantics favours the number of attacks, while max-based semantics favours the quality of the attacks. One could choose for quantity or one could choose for quality or even something in between. Then the question arises how many weak arguments are equally strong as one strong argument? Amgoud et al. (2016) present α-burden-based semantics, with $f(x) = 1 + x$ and $g(x_1, \ldots, x_n) = (\sum_{i=1}^{n} (\frac{1}{Deg_{AF}^{\pi}(x_i)})^{\alpha})^{1/\alpha}$. This is a theory to decide on the number of how many weak arguments can compensate one strong argument and based on that calculate a weighting by choosing a value for α in the function g, using a fixed-point method to calculate the values for cyclic argumentation frameworks.

Example 2. *Consider the argumentation framework* $AF = (\mathcal{A}, \mathcal{R})$, *with* $\mathcal{A} = \{a, b, c, d, e\}$ *and* $\mathcal{R} = \{(a, d), (b, a), (b, c), (c, d), (d, e), (e, a)\}$ *as illustrated in Figure 1. If* $\alpha = 2$, *then* $Deg_{AF}^{\alpha=2}(a) \approx 2.18, Deg_{AF}^{\alpha=2}(b) = 1, Deg_{AF}^{\alpha=2}(c) = 2, Deg_{AF}^{\alpha=2}(d) \approx 1.68$ *and* $Deg_{AF}^{\alpha=2}(e) \approx 1.60$. *Depending on the value of* α *the outcome differs, for different values of* α *there are different rankings. There are tipping points at* $\alpha_1 \approx 0.7879$ *and* $\alpha_2 \approx 3.0258$.

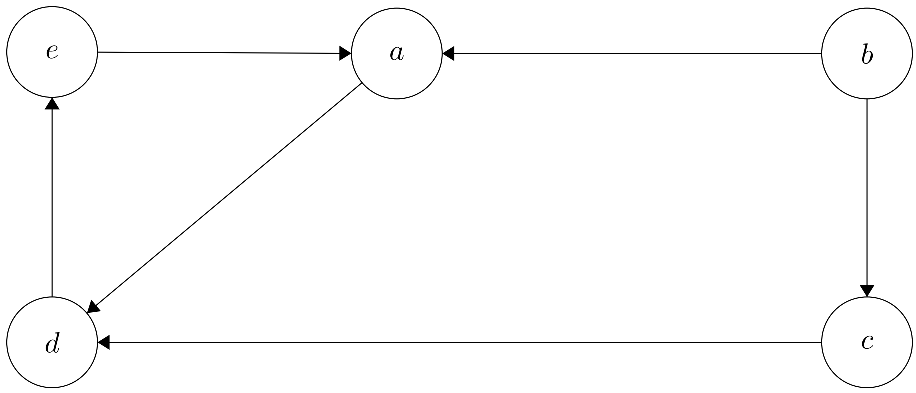

FIGURE 1. Attack relations for Example 2. After the example of Bonzon et al. (2016b).

Another weighting semantics is counting semantics (Pu et al., 2015), with $f(x) = 1 - x$ and $g(x_1, \ldots, x_n) = \sum_{i=1}^{n} \frac{\alpha}{N} Deg_{AF}^{\pi}(x_i)$, where N denotes the number of attacks of the most attacked argument. Again the value of α could affect the weighting and ranking of arguments, where the argument framework is fold out like a tree per node. For illustration see Figure 2.

A next step in complexity is that some weighting semantics use semi-weighted argumentation frameworks in which arguments have an initial strength ω. One semantics using this weighted argumentation frameworks is weighted max-based semantics (Amgoud et al., 2017). This is almost equal to the previously seen max-based semantics, it is this value multiplied by the initial weight of the argument.

Example 3. *Consider the semi-weighted argumentation framework* $AF = (\mathcal{A}, \omega, \mathcal{R})$, *with* $\mathcal{A} = \{a, b, c\}$, $\omega(a) = 0.5$, $\omega(b) = \omega(c) = 1$ *and* $\mathcal{R} = \{(a, b), (b, c)\}$. *Then* $Deg_{AF}^{\Pi}(a) = 0.5$, $Deg_{AF}^{\Pi}(b) = \frac{2}{3}$ *and* $Deg_{AF}^{\Pi}(d) = \frac{3}{5}$. *And thus* $b \succeq c \succeq a$.

One step further there are also semantics that account for varied-strength of attacks (Amgoud & Doder, 2019), with use of weighted argumentation frameworks. First they propose a family S^* of gradual semantics and then show how to change existing semantics for semi-weighted argumentation frameworks into semantics. For example, weighted-max-based semantics for weighted argumentation frameworks, would be defined by the function:

$$Deg_{AF}^{\Pi}(a) = \frac{\omega(a)}{1 + \max_{a \in \mathcal{R}^-(a)} \pi(b, a) Deg_{AF}^{\Pi}(b)}.$$

This varied-strength attacks accounts for the fact that some argument a could be a strong attack on argument b and the same argument could be a weaker attack on argument c.

Example 4. *Consider the weighted argumentation framework* $AF = (\mathcal{A}, \omega, \mathcal{R}, \pi)$, *with* $\mathcal{A} = \{a, b, c, d\}$, $\omega \equiv 1$, $\mathcal{R} = \{(a, b), (a, d), (c, d)\}$ *and* $\pi(a, b) = 1$, $\pi(a, d) = 0.8$, $\pi(c, d) = 0.6$. *Then* $Deg_{AF}^{\Pi}(a) = Deg_{AF}^{\Pi}(c) = 1$, $Deg_{AF}^{\Pi}(b) = \frac{1}{2}$ *and* $Deg_{AF}^{\Pi}(d) = \frac{5}{9}$. *And thus* $a \simeq c \succeq d \succeq a$.

3.3. **Ranking Semantics for AFs.**
There are also ranking semantics. Ranking semantics do not necessarily give a

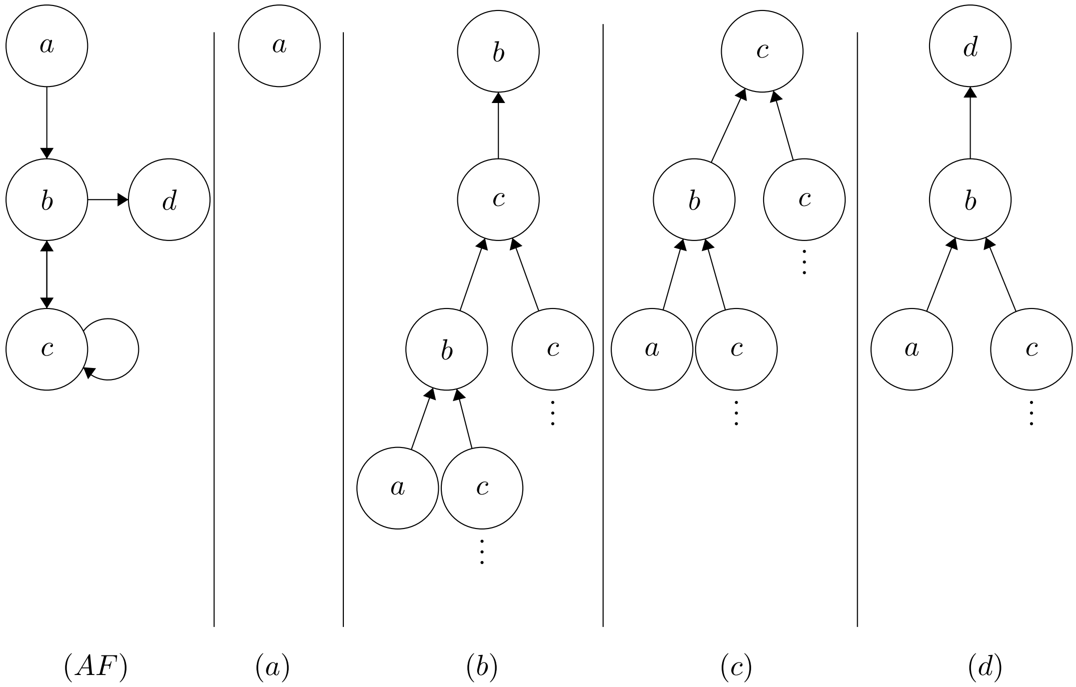

FIGURE 2. Illustration of unfolding the argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, with $\mathcal{A} = \{a, b, c, d\}$ and $\mathcal{R} = \{(a, b), (b, c), (b, d), (c, b), (c, c)\}$ (Pu et al., 2015). In this picture on the left AF, shows the argumentation framework, (a), shows the unfolding from node a, (b) shows the unfolding from node b etc. The dots under the node c mean that the tree continues as after each c, splitting into a b and a c etc. The vertical lines are borders to make clearer separation between the trees.

value to each argument, but return a total preorder on the set of arguments i.e. for each pair of arguments $a, b \in \mathcal{A}$ is determined if a is ranked higher, lower or equal to b. Examples of ranking semantics are the previously seen extension-based methods of Cayrol & Lagasquie-Schiex (2005b) and Bonzon et al. (2018) or discussion-based semantics and burden-based semantics (Amgoud & Ben-Naim, 2013), Iterated graded defense (Grossi & Modgil, 2015) and propagation semantics (Bonzon et al., 2016a). It is trivial that all weighting semantics can be used as ranking semantics, after all, the assigned values can be used to rank arguments.

The discussion-based semantics and burden-based semantic (Amgoud & Ben-Naim, 2013) are rankings based on the discussion counts and burden numbers. These are both methods that compare arguments step-wise. With discussion counts the argumentation framework or graph is converted to trees for each argument as illustrated in Figure 2[3], these trees have infinitely long branches if the argumentation framework is cyclic. A linear discussion is an argument game where each next node attacks the previous one. The different numbers of nodes at each level determine the ranking between to arguments. The burden

[3]this is similar for counting semantics(Pu et al., 2015).

number of an argument is the score of that argument. These scores are updated recursively and used to compare arguments until all arguments are ranked. In cyclic graphs some arguments may have an equal number of nodes on each level of the tree, these are thus ranked equally.

Definition 19 (Linear discussion)**.** *Let $AF = (\mathcal{A}, \mathcal{R})$ an argumentation framework and $a \in \mathcal{A}$. A linear discussion for a in $\mathcal{A}$ is a sequence $s = (a_1, \dots, a_n)$ of elements of $\mathcal{A}$ (where n is a positive integer) such that $a_1 = a$ and $\forall i \in \{2, \dots, n\}$ $a_i \mathcal{R} a_{i-1}$. The length of s is n. We say that: s is won iff n is odd; s is lost iff n is even.*

Definition 20 (Discussion counts)**.** *Let $AF = (\mathcal{A}, \mathcal{R})$ be an argumentation framework, $a \in \mathcal{A}$, i a positive integer and let N be the number of linear discussions for a of length i. We define that:*

$$\mathrm{Dis}_{A_i}(a) = \begin{cases} -N & \text{if } i \text{ is odd;} \\ N & \text{if } i \text{ is even.} \end{cases}$$

Definition 21 (Burden numbers)**.** *Let $AF = (\mathcal{A}, \mathcal{R})$ be an argumentation framework. Then* $\mathrm{Bur}_i(a) = \begin{cases} 1 & \text{if } i = 0; \\ 1 + \sum_{b \in R^-(b)} \frac{1}{Bur_{i-1}(b)} & \text{otherwise.} \end{cases}$

Example 5. *Let $F = (\mathcal{A}, \mathcal{R})$ be an argumentation framework with $\mathcal{A} = \{a, b, c, d, e\}$ and $\mathcal{R} = \{(a, e), (b, a), (b, c), (c, e), (d, a), (e, d)\}$ as illustrated in Figure 1.*

Linear discussions for argument a with length ≤ 3 are (a), (a, b), (a, e) and (a, e, d), the only linear discussion for argument b is the one of length 1: (b), etc.

For an overview see Table 1.

i	a	b	c	d	e
1	-1	-1	-1	-1	-1
2	2	0	1	2	1
3	-1	0	0	-3	-2

i	a	b	c	d	e
0	1	1	1	1	1
1	3	1	2	3	2
2	2.5	1	2	$\frac{11}{6}$	$\frac{4}{3}$

TABLE 1. On the left discussion counts (Dis_i) and on the right burden numbers (Bur_i) for Example 5.

For step 0 all arguments have $\mathrm{Bur}_0 = 1$. *Argument a has two defeaters, namely so* $\mathrm{Bur}_1(a) = 3$. *Argument b has no defeaters so,* $\mathrm{Bur}_1(a) = 1$ *etc., the calculation give the result as displayed in Table 1.*

On the basis of these counts we can conclude, using Definition 43 and 44 that we can rank the arguments for both semantics as follows

$$b \succeq e \succeq c \succeq d \succeq a.$$

One of the main consequences of both of these semantics is that arguments that have more attacks, are ranked lower than arguments that have fewer attacks. This on itself looks likes a reasonable property, but that is not always desirable. After all, one argument a, could have only 1 attacker, but a strong unattacked one, and argument b could have multiple attackers, which are all attacked by one or more unattacked arguments (as for example in Figure 3) as also briefly discussed with α-burden-based semantics.

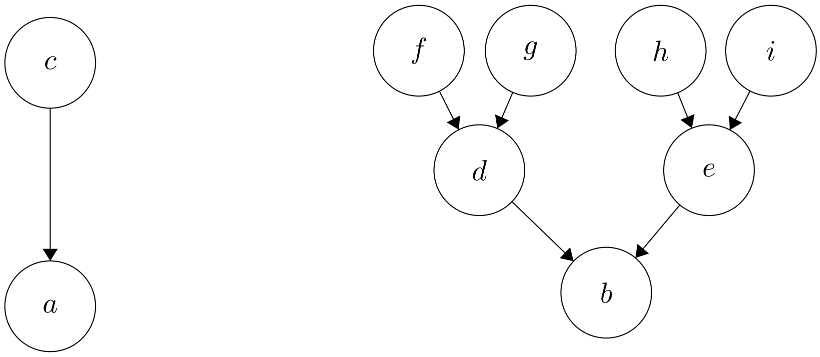

FIGURE 3. Example of one strong attack vs all defence, where c attacks a and d and e attack b, but b is defended by f, g, h and i.

3.4. **Weighting and Ranking Semantics for BAFs.**
There are various semantics for BAFs. Also for BAFs there are weighting semantics that give a value to each argument and ranking semantics that only rank arguments. Examples of weighting semantics for BAFs are Euler-based restricted semantics (Amgoud & Ben-Naim, 2018) and the quadratic energy model (Potyka, 2018). Examples of ranking semantics for BAFs are the extension-based ranking semantics (Cayrol & Lagasquie-Schiex, 2005b; Bonzon et al., 2018) as described in Section 3.1 combined with the semantics for BAFs (Cayrol & Lagasquie-Schiex, 2013) as described in section 2.2.1.

Euler-based semantics (Ebs) is a weighting semantics for acyclic non-maximal (i.e. not every node is connected) argumentation frameworks based on the definition of weighted bipolar argumentation frameworks. These are bipolar argumentation frameworks with initial weighting for the arguments.

For any acyclic non-maximal weighted bipolar argumentation framework $BAF = (\mathcal{A}, \omega, \mathcal{R}, \mathcal{S})$, the Euler-based semantic defines the strength of an argument $a \in \mathcal{A}$ by

$$Deg^{Ebs}_{BAF}(a) = 1 - \frac{1 - \omega(a)^2}{1 + \omega(a)e^E}, \text{where } E = \sum_{x \in \mathcal{S}^-(a)} Deg^{Ebs}_{BAF}(x) - \sum_{x \in R^-(a)} Deg^{Ebs}_{BAF}(x).$$

Both attacks and supports are taken into account in an exponent of e. This ensures that the more and the stronger the attackers the the higher the exponent and therefore the lower strength. The opposite holds for stronger and more supporters, then the exponent is lower and therefore the strength is higher.

Example 6. *Let* $BAF = (\mathcal{A}, \omega, \mathcal{R}, \mathcal{S})$ *be a weighted bipolar argumentation framework with* $\mathcal{A} = \{a, b, c, d\}$, $\omega(a) = \omega(b) = \omega(c) = 0.5$ *and* $\omega(d) = 0.6$, $\mathcal{R} = \{(b, a), (d, c)\}$ *and* $\mathcal{S} = \{(b, c)\}$ *as illustrated in Figure 4.*

Applying the Euler-based semantics on this framework results in the ranking $d \succeq^{Ebs}_{BAF} b \succeq^{Ebs}_{BAF} c \succeq^{Ebs}_{BAF} a$. The attack of d is stronger than the support from b for argument c, because of the higher initial value of d compared to b. However the support partly compensates for the attack. Therefore c is ranked higher than a. Changing the initial values could give a totally different ranking.

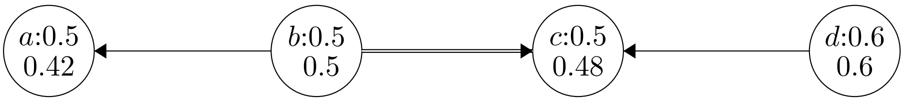

FIGURE 4. The weighted bipolar argumentation framework from Example 6. For any node $x \in \mathcal{A}$ the first line tells the name of the node 'x', followed by a colon, followed by the initial value ω(x), the second line shows $Deg^{Ebs}_{BAF}(x)$.

3.5. **Defeasible Reasoning with Variable Degrees of Justification.**

One of the first contributions on gradual acceptability is from Pollock (2001). Although his research is from far before most recent work, it is almost neglected. Pollock discusses gradual acceptability for structured argumentation. He describes the graduality of acceptability in term of probabilities and argues for example for the previously discussed Weakest Link Principle.

3.6. **Wrap-up.**

A lot of semantics have been discussed. All ranking or weighting semantics discussed above, are displayed in 2. In some of the lines, the semantic is not included because the naming of the proposed semantics was unknown to me.

Paper	Semantic	AF	W/R	IS	WA
(Pollock, 2001)		AF	weighting	No	No
(Besnard & Hunter, 2001)	Categoriser-based	AF	Weighting	No	No
(Cayrol & Lagasquie-Schiex, 2005b)	Tuples-based	AF	Ranking	No	No
(Matt & Toni, 2008)	Matt & Toni	AF	Weighting	No	No
(Caminada & Wu, 2010)		AF	Ranking	No	No
(da Costa Pereira et al., 2011)	Fuzzy Labeling	AF	Weighting	No	No
(Amgoud & Ben-Naim, 2013)	Burden-based	AF	Ranking	No	No
(Amgoud & Ben-Naim, 2013)	Discussion-based	AF	Ranking	No	No
(Cayrol & Lagasquie-Schiex, 2013)		BAF	Ranking	No	No
(Grossi & Modgil, 2015)	Iterated Graded Defense	AF	Ranking	No	No
(Pu et al., 2015)	Counting	AF	Weighting	No	No
(Amgoud et al., 2016)	α-burden-based	AF	Weighting	No	No
(Bonzon et al., 2016a)	Propagation	AF	Weighting	Yes	No
(Amgoud et al., 2017)	Max-based	AF	Weighting	No	No
(Amgoud et al., 2017)	Weighted h-categoriser	AF	Weighting	Yes	No
(Amgoud et al., 2017)	Weighted max-based	AF	Weighting	Yes	No
(Amgoud et al., 2017)	Weighted card-based	AF	Weighting	Yes	No
(Amgoud & Ben-Naim, 2018)	Euler-based	BAF	Ranking	No	No
(Bonzon et al., 2018)		AF	Ranking	No	No
(Potyka, 2018)	Quadratic Energy Model	BAF	Weighing	Yes	No
(Amgoud & Doder, 2019)		AF	Weighting	Yes	Yes

TABLE 2. An overview of the discussed semantics in this paper. In the first column (Paper) the reference. In the second column (Semantic) the name of the semantic. All papers talk about abstract argumentation except for Pollock (2001), which is about structured argumentation. In the third column (AF) if it regards AFs of BAFs. In the fourth column (W/R) if it is a weighting or a ranking semantic. In the fifth column (IS) if the semantic uses initial strengths. In the last column (WA) if the method uses weighted attacks or supports.

4. Dialectical Semantics

As discussed above, many semantics have been proposed over the years. Most of the proposed semantics discuss abstract argumentation, thereby neglecting the structure of arguments. Ignoring the structure of arguments or the nature of their relations may result in odd or undesirable results. A second problem with the recent literature is discussed by Prakken (2021). Prakken (2021) distinguishes three aspects of argumentation strength: logic strength, dialectic strength and rhetorical strength. Authors do not explicitly state which kind of argumentation strength is modelled.

In this section the intuitions of dialectical argument strength will be described and a definition of dialectical argument strength will be provided by proposing a new semantic for dialectical argument strength.

4.1. **Intuitions.**

Zenker et al. (2020) describes dialectical argument strength in terms of move space: "argument strength can be operationalized as the (un)availability of participant moves that constrain further interlocutor moves." The more attacking moves an argument allows to a possible opponent in a discussion, the lower the dialectical argument strength. Zenker et al. (2020) also claims that "it cannot be assumed that material that goes unchallenged is accepted." This suggests that the number of possible attacking points of an argument is a good indicator of the dialectical argument strength, i.e. arguments that have fewer attacking points should have a higher degree of acceptability in a dialectical context.

In the same line of argumentation, to the idea of Prakken (2021), if an argument has refuted some doubts, then it is stronger than if the argument has not refuted some doubts. This suggests that besides the number of attacking points, the number of survived attacks is also important , i.e., arguments that survived more attacks, should have a higher degree of acceptability in a dialectical context.

4.2. **Definition.**

In this subsection dialectical argument strength will be defined.

As Prakken (2021) describes, dialectical strength has both static and dynamic aspects. The static aspect is given by the outcome of a critical discussion and is about how well-defended an argument is in the discussion? First of all this means that accepting an argument in a discussion is better than rejecting it. Furthermore, this means if an argument argument survives more attacks, it will increase the degree of acceptability. The dynamical aspect is given by the opponent's possibilities to attack. If an argument has more attacking points, then this will decrease the degree of acceptability.

4.3. **Possible New Semantics.**

Below a possible new gradual semantics will be presented. To that end, an alternative for Definition 7 will be provided and attacking points of an argument and the number of survived attacks of an argument in a specific argumentation framework will be defined as well as the counting of these.

The difference between skeptically acceptable, credulously acceptable and not acceptable will be extended. Firstly, the skeptically acceptable arguments, will be divided into two groups: the undoubtedly acceptable arguments (those that are strict and firm) and the provisionally acceptable arguments (those that in the future can change status). Secondly, the not acceptable arguments will be divided into two groups: the provisionally unacceptable arguments (those that in the future can change status) and the undoubtedly unacceptable arguments (those that can never change status).

Definition 22. *Let* $AF = (\mathcal{A}, \mathcal{R})$ *be an argumentation framework. An argument* $a \in \mathcal{A}$ *is*

- *Undoubtedly acceptable iff it is strict and firm.*
- *Provisionally acceptable iff it is skeptically acceptable, but is not strict and firm.*
- *Probably acceptable iff it is credulously acceptable.*
- *Provisionally unacceptable iff it is not acceptable and non of its attackers is strict and firm.*
- *Undoubtedly unacceptable iff one of its attackers is strict and firm.*

Note that these statuses are complementary, e.g. if argument a is not undoubtedly acceptable, provisionally acceptable, probably acceptable or provisionally unacceptable, then it is undoubtedly unacceptable. This follows right from the definition.

There are three kinds of attacks that an argument can receive, undermining of the premises, undercutting of inference rules and rebutting of sub-arguments. Undermining is possible on exactly those premises that are in the set of ordinary premises of a. Undercutting is possible on all defeasible inference rules of a. Rebutting is possible on each conclusion of a sub-argument that has a defeasible toprule.

Definition 23 (Attacking Point)**.** *For any argumentation theory* $AT = (AS, \mathcal{K})$*, with* $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$ *an argumentation system, for any argument* $a \in \mathcal{A}$*, the attacking points of a are the points where a can be undermined, undercut and rebut. These are defined by the sets*

- *Undermining:* $Um(a) = \{x \in \mathcal{K}_p \cap \text{Prem}(a)\}$.
- *Undercutting:* $Uc(a) = \{\text{DefRules}(a)\}$.
- *Rebutting:* $Rb(a) = \{\text{Conc}(b) | b \in \text{Sub}(a) \textit{ and } \text{TopRule}(b) \in \text{DefRules}(a)\}$.

The set of attacking points is defined by $AP(a) = Um(a) \cup Uc(a) \cup Rb(a)$

Definition 24 (Number of Attacking Points)**.** *For any argumentation theory* $AT = (AS, \mathcal{K})$*, with* $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$ *an argumentation system, with* $AF = (\mathcal{A}, \mathcal{R})$ *the corresponding argumentation framework, for any argument* $a \in \mathcal{A}$ *the number of attacking points (the number of places where an attack can happen) on an argument is defined by* $|AP(a)|$.

Example 7. *Consider argumentation system* $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$. *with* $\mathcal{L} = \{a_i, r_1, \neg a_i, \neg r_1\}$ *and* $\mathcal{R} = \{r_1 : a_1, a_2 \Rightarrow a_4; a_3, a_4 \rightarrow a_5\}$. *Let* $AT = (AS, \mathcal{K})$ *be an argumentation theory, where* $\mathcal{K} = \mathcal{K}_n \cup \mathcal{K}_p$*, with* $\mathcal{K}_n = \{a_1\}$ *and* $\mathcal{K}_p = \{a_2, b_3\}$. *This leads to the argumentation framework* $AF = (\mathcal{A}, \mathcal{R})$*, with* $\mathcal{A} = \{A\}$ *and* $\mathcal{R} = \emptyset$. *This is displayed in Figure 5, where dashed lines are defeasible inference rules, solid lines are deductive inference rules, dotted boxes display ordinary premises and solid boxes display axiom premises. Then, this argument can be attacked by arguments that have conclusion* $\neg a_2, \neg a_3, \neg r_1$ *or* $\neg a_4$. *In this example* $Um(A) = \{a_2, a_3\}$*,* $Uc(A) = \{r_1\}$ *and* $Rb(A) = \{a_4\}$.

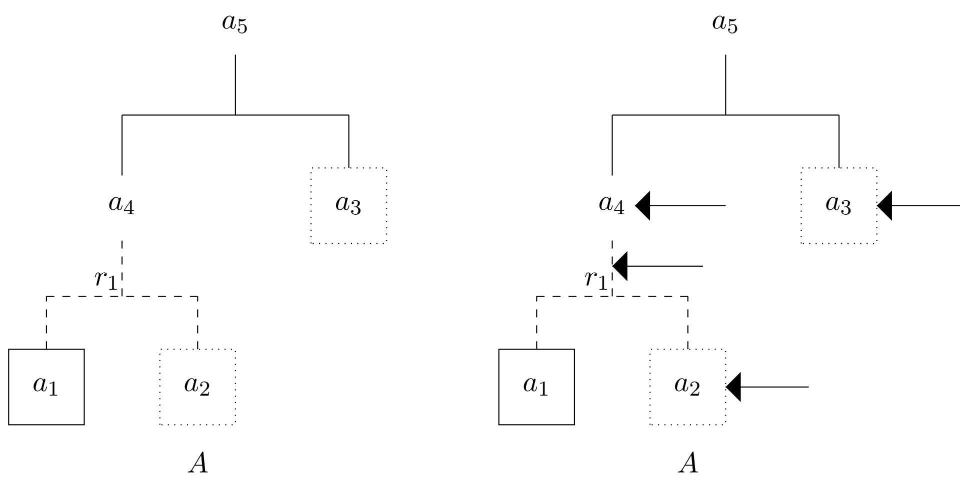

FIGURE 5. ASPIC+ argumentation explaining the attacking points of argument A. To the left the argument A, to the right the argument A with arrows pointing to the attacking points.

The number of attacking points can also be defined by counting the number of ordinary premises (undermining) and two times the number of defeasible inference rules. It is two times the number of defeasible inference rules, because the rules can be attacked (undercutting) and the conclusion of the defeasible inference rule can be rebutted.

Proposition 1. *For any argumentation theory $AT = (AS, \mathcal{K})$, with $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$ an argumentation system, with $AF = (\mathcal{A}, \mathcal{R})$ the corresponding argumentation framework, for any argument $a \in \mathcal{A}$ it holds that $AP(a) = |\mathcal{K}_p \cap \text{Prem}(a)| + 2 \cdot |\text{DefRules}(a)|$.*

Proof.

$$\begin{aligned} AP(a) &= |Um(a)| + |Uc(a)| + |Rb(a)| \\ &= |Um(a)| + |Uc(a)| + |\{\text{Conc}(b) | b \in \text{Sub}(a) \text{ and } \text{TopRule}(b) \in \text{DefRules}(a)\}| \\ &= |Um(a)| + |Uc(a)| + |\{\text{DefRules}(a)| \\ &= |\mathcal{K}_p \cap \text{Prem}(a)| + 2 \cdot |\text{DefRules}(a)| \end{aligned}$$

□

A survived attack is an attack that is defended, i.e., it is an attack that is counter-attacked, such that the attacker is provisionally unacceptable or undoubtedly unacceptable. We define the survived attacks of argument a as the attackers of a that are provisionally unacceptable or undoubtedly unacceptable. The survived attacking points are all attacking points that are only attacked by survived attacks. So, if an argument is attacked on one attacking point by an provisionally unacceptable argument and by a provisionally acceptable argument, then this attacking point is no survived attacking point. The intuition behind this is as follows. Suppose there are two arguments, one weak argument A and one strong argument B with the same conclusion x, which attack argument C with conclusion $\neg x$. It would be odd to say that the weak argumentation of A would bring down the impact of B on C. Otherwise adding weak attacks would increase the degree of acceptability of unacceptable arguments. Therefore when multiple attacks happen at the same attacking point, only the strongest attacker is used. Example 8 illustrates this. The number of survived attacking points is determined by counting the survived attacking points.

It only takes a unique set of conclusions, to avoid counting attacks on one attacking point double. $SA(\cdot)$ is the set of defended attackers. $SAP(\cdot)$ ensures that attacking points that encounter multiple attacks, but where not are not all attacks are defended, are not counted and in addition ensures that attacking points are not counted multiple times.

Definition 25 (Survived Attacks)**.** *For any argumentation theory $AT = (AS, \mathcal{K})$, with $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$ an argumentation system, with corresponding argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, for any argument $a \in \mathcal{A}$ the survived attacks (attacks that are successfully counter-attacked) are defined by $SA(a) = \{b \in R^{-}(a) | b \text{ is unacceptable}\}$.*

Definition 26 (Survived Attacking Point)**.** *For any argumentation theory $AT = (AS, \mathcal{K})$, with $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$ an argumentation system, with corresponding argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, for any argument $a \in \mathcal{A}$, for any attacking point $x \in AP(a)$, x is a survived attacking point iff $\forall b \in R^{-}(a)$ it holds that if $\mathrm{Conc}(b) = \neg x$, then $b \in SA(a)$.*

Definition 27 (Survived Attacking Points)**.** *For any argumentation theory $AT = (AS, \mathcal{K})$, with $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$ an argumentation system, with corresponding argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, for any argument $a \in \mathcal{A}$, the attacking points are defined by the set $SAP(a) = \{x \in AP(a) | x \text{ is a survived attacking point}\}$.*

Definition 28 (Number of Survived Attacking Points)**.** *For any argumentation theory $AT = (AS, \mathcal{K})$, with $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$ an argumentation system, for any argument $a \in \mathcal{A}$ the number of survived attacking points (the number of places where attacks happened that are successfully counter-attacked) of an argument is defined by $|SAP(a)|$.*

Example 8. *The following example illustrates the 'disappearance' of a survived attacker (AT_1), when two attacks happen at the same point versus the situation in which they happen on different attacking points (AT_3). Furthermore, we will see that there is no difference in strength of arguments C in AT_1 and AT_4.*

Consider argumentation system $AS_1 = (\mathcal{L}_1, \mathcal{R}_1, n_1, \leq)$. with $\mathcal{L}_1 = \{u_1, \ldots, y_1, r_1, \neg u_1, \ldots, \neg y_1, \neg r_1\}$ and $\mathcal{R}_1 = \{r_1 : \neg x_1 \Rightarrow w_1, u_1 \rightarrow \neg v_1, v_1 \rightarrow x_1, y_1 \rightarrow x_1\}$. Let $AT_1 = (AS_1, \mathcal{K}_1)$ be an argumentation theory, where $\mathcal{K}_1 = \mathcal{K}_{1p} = \{u_1, v_1, \neg x_1, y_1\}$. This leads to the argumentation framework $AF_1 = (\mathcal{A}_1, \mathcal{R}_1)$, with $\mathcal{A}_1 = \{A_1, B_1, C_1, D_1\}$ and $\mathcal{R}_1 = \{(A_1, C_1), (B_1, C_1), (D_1, A_1)\}$.

Consider argumentation system $AS_3 = (\mathcal{L}_3, \mathcal{R}_3, n_3, \leq)$. with $\mathcal{L}_3 = \{u_3, \ldots, y_3, r_3, \neg u_3, \ldots, \neg y_3, \neg r_3\}$ and $\mathcal{R}_3 = \{r_3 : \neg x_3 \Rightarrow w_3, u_3 \rightarrow \neg v_3, v_3 \rightarrow x_3, y_3 \rightarrow \neg w_3\}$. Let $AT_3 = (AS_3, \mathcal{K}_3)$ be an argumentation theory, where $\mathcal{K}_3 = \mathcal{K}_{3p} = \{u_3, v_3, \neg x_3, y_3\}$. This leads to the argumentation framework $AF_3 = (\mathcal{A}_3, \mathcal{R}_3)$, with $\mathcal{A}_3 = \{A_3, B_3, C_3, D_3\}$ and $\mathcal{R}_3 = \{(A_3, C_3), (B_3, C_3), (D_3, A_3)\}$.

Consider argumentation system $AS_4 = (\mathcal{L}_4, \mathcal{R}_4, n_4, \leq)$. with $\mathcal{L}_4 = \{w_4, x_4, y_4, r_4, \neg w_4, \neg x_4, \neg y_4, \neg r_4\}$ and $\mathcal{R}_4 = \{r_4 : \neg x_4 \Rightarrow w_4, y_4 \rightarrow x_4\}$. Let $AT_4 = (AS_4, \mathcal{K}_4)$ be an argumentation theory, where $\mathcal{K}_4 = \mathcal{K}_{4p} = \{\neg x_4, y_4\}$. This leads to the argumentation framework $AF_4 = (\mathcal{A}_4, \mathcal{R}_4)$, with $\mathcal{A}_4 = \{B_4, C_4\}$ and $\mathcal{R}_4 = \{(B_4, C_4)\}$. All are displayed in Figure 6, where arrows represent attacks.

The number of attacking points for each argument is $|AP(A_i)| = |AP(B_i)| = |AP(D_i)| = 1$ *and* $|AP(C_i)| = 3$ *for* $i = 1, 3$ *or* 4. *The survived attacks are* $SA(A_i) = SA(B_i) = SA(D_i) = \emptyset$, $SA(C_1) = \{A_1\}$, $SA(C_3) = \{A_3\}$ *and* $SA(C_4) = \emptyset$. *However,* $SAP(C_1) = \emptyset$, *because there is a non-defended attack,* B, *on* $\neg x_1$. *Furthermore,* $SAP(C_3) = \{\neg x_3\}$ *and* $SAP(C_4) = \emptyset$.

We see that $Deg^{S_1}_{AF_1}(C_1) = Deg^{S_1}_{AF_4}(C_4) = \frac{1}{5}$. *So, the presence of an attacker with the same conclusion (although survived) does not matter.*

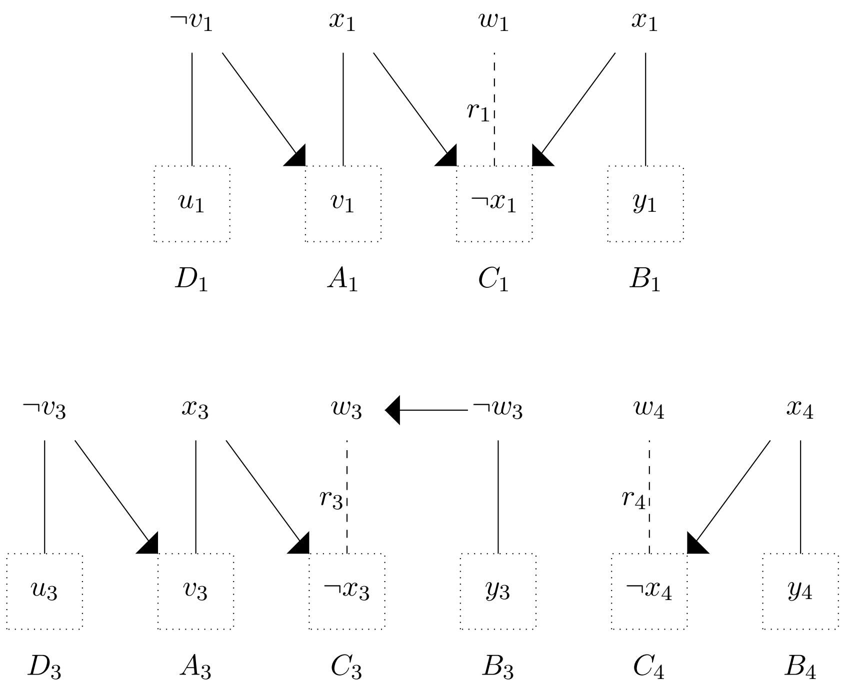

Figure 6. ASPIC+ argumentation with argumentation frameworks AF_1, AF_3 and AF_4 comparing double undermining with undermining and rebutting.

The semantics that will be defined, makes a further distinction between arguments in grounded semantics. The first thing that is ensured is that undoubtedly unacceptable arguments are always ranked lower provisionally unacceptable argument, which are always ranked lower than credulously acceptable arguments, which are always ranked lower than provisionally acceptable arguments, which are always ranked lower than undoubtedly acceptable arguments. This is done by mapping each of these to a separate interval between 0 and 1. In this case it is arbitrarily chosen to map the undoubtedly unacceptable arguments in the interval $(0, \frac{1}{4})$, the provisionally unacceptable arguments in the interval $(\frac{1}{4}, \frac{1}{2})$, the credulously acceptable arguments in $(\frac{1}{2}, \frac{3}{4})$ and the provisionally acceptable arguments in $(\frac{3}{4}, 1)$. Because these values are not re-used to calculate the degree of acceptability of other arguments, these intervals can be chosen arbitrarily. There is chosen to map all undoubtedly acceptable arguments to 1.

Then the percentage of attacking points that are unsuccessfully attacked is calculated. The number of attackers that are unacceptable increased by 1 is divided by the number of attacking points increased by 2. This increment with 1 ensures that there is a difference in unattacked and undefended argument based on the number of attacking points, the increment with 2 ensures two things, i) there is never a division by 0 and ii) arguments that are not strict and firm never reach value 1.

There is a separate case for undoubtedly acceptable arguments; these have a degree of acceptability of 1. So, if an argument is strict and firm, it has a degree of 1. Otherwise, when an argument is provisionally acceptable and unsuccessfully attacked on all its attacking points, then it has degree slightly smaller than 1. All is based on the dialectical principles i) the more attacks an argument survives, the more acceptable it is and ii) the more attacking points an argument has, the weaker it is.

Definition 29. *We define the following semantics S_1, called Grounded Dialectical Semantics for every argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, for every $x \in \mathcal{A}$:*

$$Deg^{S_1}_{AF}(x) = \begin{cases} 1 & \textit{if } x \textit{ is undoubtedly acceptable,} \\ \frac{3}{4} + \frac{1}{4} \cdot \frac{1+|SAP(x)|}{2+|AP(x)|} & \textit{if } x \textit{ is provisionally acceptable,} \\ \frac{1}{2} + \frac{1}{4} \cdot \frac{1+|SAP(x)|}{2+|AP(x)|} & \textit{if } x \textit{ is credulously acceptable,} \\ \frac{1}{4} + \frac{1}{4} \cdot \frac{1+|SAP(x)|}{2+|AP(x)|} & \textit{if } x \textit{ is provisionally unacceptable,} \\ \frac{1}{4} \cdot \frac{1+|SAP(x)|}{2+|AP(x)|} & \textit{if } x \textit{ is undoubtedly unacceptable,} \end{cases}$$

5. Postulates for Dialectical Strength

For gradual notions of argument strength several sets of postulates have been proposed, postulates that good semantics should satisfy. Other terms used for postulates are properties or principles. In this section we will look at some of postulates proposed in the literature, more specifically in (Amgoud & Ben-Naim, 2013). Firstly, we will provide the definitions and postulates from Amgoud & Ben-Naim (2013) accompanied by the intuitions of each postulate and we will discuss the postulates and critically review them in the context of ASPIC+ (i.e. see if they still are desirable in structured argumentation). Subsequently, we will investigate whether Grounded Dialectical Semantics S_1 satisfies the postulates. Then, we will propose some new postulates that should hold for semantics that describe dialectical strength and show that S_1 satisfies these new postulates.

Theorem 1. *Semantics S_1 satisfies postulates (In), (APS), (SAP) and (SP) and does not satisfy postulates (Ab), (VP), (CP), (QP), (DP), (DDP), (CT), (SCT) and (TSAP).*

Proof. The proof consist of all proofs of Claim 1 till Claim 13. □

5.1. **Definitions and postulates.**

Definition 30 (Ranking)**.** *(Amgoud & Ben-Naim, 2013) A ranking on a set $\mathcal{A}$ is a binary relation $\preceq$ on $\mathcal{A}$ such that: $\preceq$ is total and transitive.*

Definition 31 (Ranking-based Semantics)**.** *(Amgoud & Ben-Naim, 2013) A ranking-based semantics is a function S that transforms any argumentation framework $AF = (\mathcal{A}, \mathcal{R})$ into a ranking on $\mathcal{A}$, $(a, b) \in S(AF)$, means $b \preceq a$.*

5.1.1. *Abstraction.*
The intuition behind abstraction is that two equivalent argumentation frameworks should have equivalent rankings. This is defined as follows.

Definition 32 (Isomorphism)**.** *(Amgoud & Ben-Naim, 2013) Let $AF = (\mathcal{A}, \mathcal{R})$ and $AF' = (\mathcal{A}', \mathcal{R}')$ be two argumentation frameworks. An isomorphism from A to A' is a bijective function $f : \mathcal{A} \to \mathcal{A}'$ such that $\forall a, b \in \mathcal{A}$, $a\mathcal{R}b$ iff $f(a)\mathcal{R}'f(b)$.*

Postulate 1 (Abstraction)**.** *(Amgoud & Ben-Naim, 2013) A ranking-based semantics S satisfies abstraction (Ab) iff for any two frameworks $AF = (\mathcal{A}, \mathcal{R})$ and $AF' = (\mathcal{A}', \mathcal{R}')$, for any isomorphism $f : \mathcal{A} \to \mathcal{A}'$, we have that $\forall a, b \in \mathcal{A}, (a, b) \in S(AF)$ iff $(f(a), f(b)) \in S(AF')$.*

If abstraction holds, this implies that any two unattacked arguments are always ranked equally high. This seems undesirable for some notions of argument strength. For dialectical strength, as well as for inferential argument strength as described in (Prakken, 2021), the attackability of an argument plays a role, after all, there is a difference in strict and defeasible inference rules and a difference in axiom and ordinary premises. Thus for these notions of strength this postulate would be a problem. Notice that semantics S_1 does not satisfy abstraction.

Claim 1. *Abstraction does not hold for semantics S_1.*

Proof. Consider argumentation system $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$, with $\mathcal{L} = \{a, b, \neg a, \neg b\}$ and $\mathcal{R} = \emptyset$. Let $AT = (AS, \mathcal{K})$ be an argumentation theory, where $\mathcal{K} = \mathcal{K}_n \cup \mathcal{K}_p$, with $\mathcal{K}_n = \{a\}$ and $\mathcal{K}_p = \{b\}$. This leads to the argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, with $\mathcal{A} = \{A, B\}$ and $\mathcal{R} = \emptyset$. Then, $Deg^{S_1}_{AF}(A) = \frac{5}{6}$ and $Deg^{S_1}_{AF}(B) = 1$. So, S_1 does not satisfy abstraction. □

5.1.2. *Independence.*
The intuition behind independence is that the question whether an argument a is at least as acceptable as an argument b should be independent of any argument c that is neither connected to a nor to b (Amgoud & Ben-Naim, 2013). This is defined as follows.

Definition 33 (Weak Connected Component)**.** *(Amgoud & Ben-Naim, 2013) A weak connected component of an argumentation framework A is a maximal subgraph of A in which any two vertices are connected to each other by a path (ignoring the direction of the edges). We denote by $Com(A)$ the set of every argumentation framework B such that B is a weak connected component of A or the graph union of several weak connected components of A.*

Postulate 2 (Independence)**.** *(Amgoud & Ben-Naim, 2013) A ranking-based semantics S satisfies independence (In) iff for every argumentation framework AF, $\forall B \in Com(AF)$, $\forall a, b \in Arg(B)$, $(a, b) \in S(A)$ iff $(a, b) \in S(B)$.*

There is no reason why independence should not hold for dialectical argument strength. Notice that semantics S_1 satisfies independence.

Claim 2. *Independence holds for semantics S_1.*

Proof. Let $AF = (\mathcal{A}, \mathcal{R})$ be an arbitrary argumentation framework, let $B \in Com(AF)$ arbitrary. Then for every $a \in B$ holds that $Deg^{S_1}_{B}(a) = Deg^{S_1}_{AF}(a)$. After all, $AP(\cdot)$ only depends on the number of ordinary premises and defeasible inference rules. Further more $SAP(\cdot)$ only depends on its attackers, and these do not differ in both graphs. Which is the same reason why arguments do not change in status, i.e., undoubtedly acceptable, provisionally acceptable, credulously acceptable, provisionally unacceptable and undoubtedly unacceptable. So, $(a,b) \in S(AF)$ iff $(a,b) \in S(B)$. So, S_1 satisfies independence. □

5.1.3. *Void Precedence and Cardinality Precedence.*
The intuition behind void precedence is that attacks harm the degree of acceptability regardless of the acceptability of the attacker. So, non-attacked arguments are always ranked higher than attacked arguments. The intuition behind cardinality precedence is that more attackers is always worse than having fewer attackers regardless of the acceptability of the attackers[4]. This is defined as follows.

Postulate 3 (Void Precedence)**.** *(Amgoud & Ben-Naim, 2013) A ranking-based semantics S satisfies void precedence (VP) iff for every argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, $\forall a, b \in \mathcal{A}$, if $R^-(b) \neq R^-(a) = \emptyset$, then $(b,a) \notin S(AF)$.*

Postulate 4 (Cardinality Precedence)**.** *(Amgoud & Ben-Naim, 2013) A ranking-based semantics S satisfies cardinality precedence (CP) iff for every argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, $\forall a, b \in \mathcal{A}$, if $|R^-(b)| > |R^-(a)|$, then $(b,a) \notin S(AF)$.*

For an argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, where $\mathcal{A} = \{a, b, c, d\}$ and $\mathcal{R} = \{(b,a),(c,b)\}$. VP states that argument a is ranked lower than argument d. However, if d consists of ordinary premises and defeasible inference-rules and c consists of axioms and strict inference rules, this is not undoubtful. In terms of dialectical strength, argument b has overcome one of its attackers, so one could argue that argument b should be ranked higher than argument a.

Another argument against Void Precedence is that arguments that survived an attack, i.e. arguments that are provisionally acceptable despite having at least one attacker, have covered possible weaknesses. On the other hand arguments without any attacks could be vulnerable for future attacks.

Notice that semantics S_1 does not satisfy void precedence and cardinality precedence.

Claim 3. *Void precedence does not hold for semantics S_1.*

Proof. Consider argumentation system $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$, with $\mathcal{L} = \{a, b, c, d \neg a, \neg b, \neg c, \neg d\}$ and $\mathcal{R} = \{\neg b \rightarrow \neg a; c \rightarrow b\}$. Let $AT = (AS, \mathcal{K})$ be an argumentation theory, where $\mathcal{K} = \mathcal{K}_p = \{a, \neg b, c, d\}$. This leads to the argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, with $\mathcal{A} = \{A, B, C, D\}$ and $\mathcal{R} = \{(C,B),(B,A)\}$ as displayed in Figure 7. Then, $Deg^{S_1}_{AF}(a) = \frac{11}{12}$ and $Deg^{S_1}_{AF}(d) = \frac{5}{6}$. So, S_1 does not satisfy void precedence. □

Claim 4. *Cardinality precedence does not hold for semantics S_1.*

Proof. Since CP implies VP and VP does not hold, also CP does not hold. □

[4]Cardinality precedence is not a necessary postulate, one gives precedence to either cardinality or quality.

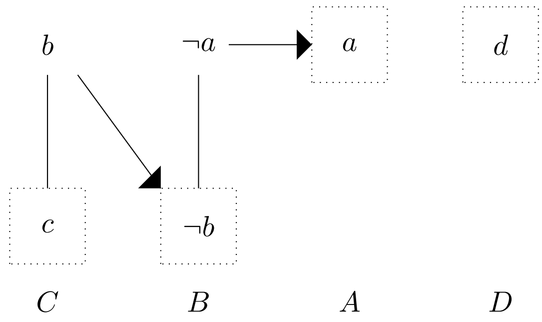

FIGURE 7. ASPIC+ argumentation as counterexample against void precedence.

5.1.4. *Quality Precedence.*
The intuition behind quality precedence is that if an argument a has an attacker with a higher degree of acceptability than all attackers of argument b, then b should be ranked higher than a. This is defined as follows.

Postulate 5 (Quality Precedence)**.** *A ranking-based semantics S satisfies quality precedence (QP) iff for every argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, $\forall a, b \in \mathcal{A}$, if $\exists c \in R^-(b)$ such that $\forall d \in R^-(a)$ holds that $(d, c) \notin S(AF)$, then $(b, a) \notin S(AF)$.*

Quality precedence is not a postulate that should hold for every type of argument strength. For example, it should not hold for dialectical argument strength. Consider two arguments a and b, with both one attacker respectively c and d, where c is ranked slightly higher than d. According to QP b should be ranked higher, but if a has only 1 attacking point and b has a lot, then QP is at least questionable in terms of dialectical argument strength. Notice that semantics S_1 does not satisfy quality precedence.

Claim 5. *Quality precedence does not hold for semantics S_1.*

Proof. Consider argumentation system $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$, with $\mathcal{L} = \{a_i, r_1, r_2, \neg a_i, \neg r_1, \neg r_2\}$ and $\mathcal{R} = \{r_1 : a_1 \Rightarrow a_2; r_2 : \neg a_3 \Rightarrow a_4; a_2 \rightarrow a_3; a_5 \rightarrow a_6\}$. Let $AT = (AS, \mathcal{K})$ be an argumentation theory, where $\mathcal{K} = \mathcal{K}_p = \{a_1, \neg a_3, a_5, \neg a_6\}$. This leads to the argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, with $\mathcal{A} = \{A, B, C, D\}$ and $\mathcal{R} = \{(A, B), (C, D)\}$ as displayed in Figure 8. Then, $Deg^{S_1}_{AF}(A) = \frac{4}{5}$, $Deg^{S_1}_{AF}(C) = \frac{5}{6}$, $Deg^{S_1}_{AF}(B) = \frac{3}{10}$ and $Deg^{S_1}_{AF}(D) = \frac{1}{3}$. So, $\exists c \in R^-(B)$, namely A, such that $\forall d \in R^-(D)$, namely C holds that $(A, C) \notin S(AF)$ and $(D, B) \in S(AF)$. So, S_1 does not satisfy quality precedence. □

5.1.5. *Defense Precedence.*
The intuition behind defence precedence is that being defended is better than not being defended (assuming the number of attackers is the same) (Amgoud & Ben-Naim, 2013). This is defined as follows.

Definition 34. *Let $AF = (\mathcal{A}, \mathcal{R})$ be an argumentation framework. For any argument $a \in \mathcal{A}$ the set of defenders is defined as $Def(a) = \cup_{b \in R^-(a)} \mathcal{R}^-(b)$.*

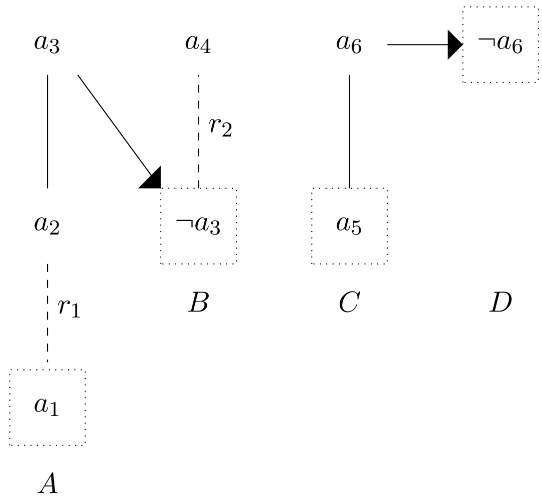

FIGURE 8. ASPIC+ argumentation as counterexample against Quality Precedence.

Postulate 6 (Defence Precedence)**.** *(Amgoud & Ben-Naim, 2013) A ranking-based semantics S satisfies defense precedence (DP) iff for every argumentation framework* $AF = (\mathcal{A}, \mathcal{R})$, $\forall a, b, \in \mathcal{A}$, *if* $|R^{-}(a)| = |R^{-}(b)|$, $Def(a) \neq Def(b) = \emptyset$, *then* $(b, a) \notin S(A)$.

Defence precedence is not a postulate that should hold for semantics for dialectical argument strength. Consider two arguments B and F such that argument F has one provisionally unacceptable attacker, but is itself undoubtedly unacceptable, because of one undoubtedly acceptable attacker and argument B has two provisionally unacceptable attackers. According to Defence precedence argument F should be ranked higher. However, F is rebutted by a strict and firm argument G and so F is for sure not true, while B could be true, only not the right arguments were used. Argument F has fewer possibilities to move, since F can never be provisionally acceptable and B can (by attacking arguments A and C. So, the dialectical argument strength of F should be ranked lower than B. For a sketch of the ASPIC+ argumentation, see Figure 9. Notice that semantics S_1 does not satisfy defence precedence.

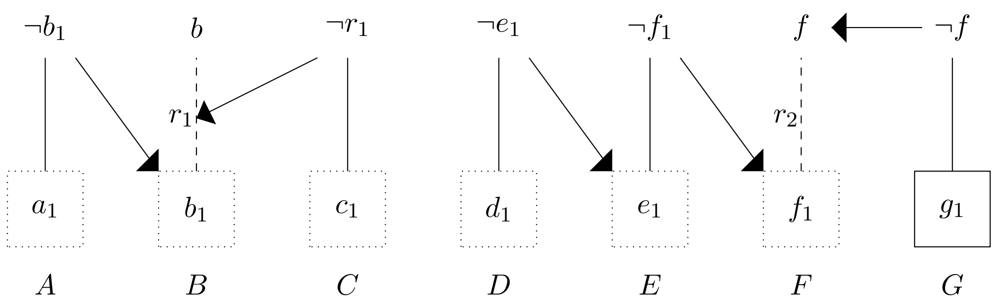

FIGURE 9. ASPIC+ argumentation as counterexample of defence precedence.

Claim 6. *Defence Precedence does not hold for semantics S_1.*

Proof. Consider argumentation system $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$. with $\mathcal{L} = \{a_1, \dots g_1, r_1, b, f, \neg a_1, \dots \neg g_1, \neg r_1, \neg b, \neg f\}$ and $\mathcal{R} = \{r_1 : b_1 \Rightarrow b; r_2 : f_1 \Rightarrow f; a_1 \rightarrow \neg b_1; c_1 \rightarrow \neg r_1; d_1 \rightarrow \neg e_1; e_1 \rightarrow \neg f_1; g_1 \rightarrow \neg f\}$. Let $AT = (AS, \mathcal{K})$ be an argumentation theory, where $\mathcal{K} = \mathcal{K}_n \cup \mathcal{K}_p$, with $\mathcal{K}_n = \{g_1\}$ and $\mathcal{K}_p = \{a_1, b_1, c_1, d_1, e_1, f_1\}$. This leads to the argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, with $\mathcal{A} = \{A, B, C, D, E, F, G\}$ and $\mathcal{R} = \{(A, B), (C, B), (D, E), (E, F), (G, F)\}$ as displayed in Figure 9. Then, $Deg^{S_1}_{AF}(B) = \frac{3}{10}$ and $Deg^{S_1}_{AF}(F) = \frac{1}{10}$. Furthermore, $|R^-(F)| = |R^-(B)|$ and $Def(F) \neq Def(B) = \emptyset$, but $(B, F) \in S(AF)$. So, S_1 does not satisfy Defence Precedence. □

5.1.6. *Distributed Defense Precedence.*
The intuition behind distributed defence precedence is, when comparing two arguments with the same number of attackers and defenders, where each defender attacks exactly one attacker, then the argument where each defender attacks a distinct attacker is the best(Amgoud & Ben-Naim, 2013). This is defined as follows.

Definition 35. *(Amgoud & Ben-Naim, 2013) Let $AF = (\mathcal{A}, \mathcal{R})$ be an argumentation framework. The defense of $a \in A$ is simple iff every defender of a attacks exactly one attacker of a.*

Definition 36. *(Amgoud & Ben-Naim, 2013) Let $AF = (\mathcal{A}, \mathcal{R})$ be an argumentation framework. The defense of $a \in A$ is distributed iff every attacker of a is attacked by at most one argument.*

Postulate 7 (Distributed Defense Precedence)**.** *(Amgoud & Ben-Naim, 2013) A ranking-based semantics S satisfies the postulate distributed defense precedence iff for every argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, $\forall a, b \in \mathcal{A}$ such that $|\mathcal{R}^-(a)| = |\mathcal{R}^-(b)|$ and $|Def(a)| = |Def(b)|$, if the defense of a is simple and distributed and the defense of b is simple but not distributed, then $(b, a) \notin S(AF)$.*

Figure 10 shows the intuitions of DDP. According to DDP argument a should be ranked higher than argument b, because all of a's attacks are defended and only one of b's attacks is defended. However, the ranking of a and b is not clear for dialectical argument strength. If i, j and k are strict and firm and f and g are defeasible and plausible then b should be ranked higher. After all, it is impossible to make argument a provisionally acceptable, while b with a counter argument for f is acceptable. Moreover, when g and f are strict and firm and i, j and k are defeasible and plausible, then a should be ranked higher. After all, argument b is undoubtedly unacceptable and argument a is only provisionally unacceptable and is provisionally acceptable if one finds a counter argument for k. Notice that semantics S_1 does not satisfy distributed defence precedence.

Claim 7. *Distributed Defence Precedence does not hold for semantics S_1.*

Proof. Consider argumentation system $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$, with $\mathcal{L} = \{a_i, \neg a_i\}$ and $\mathcal{R} = \{a_1 \rightarrow \neg a_2; a_2 \rightarrow \neg a_3; a_4 \rightarrow \neg a_3; a_5 \rightarrow \neg a_4; a_6 \rightarrow \neg a_5; a_7 \rightarrow \neg a_9; a_8 \rightarrow \neg a_9; a_9 \rightarrow \neg a_{10}; a_{11} \rightarrow \neg a_{10}\}$. Let $AT = (AS, \mathcal{K})$ be an argumentation theory, where $\mathcal{K} = \mathcal{K}_p = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}\}$. This leads to the argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, with $\mathcal{A} = \{A, B, C, D, E, F, G, H, I, J, K\}$ and $\mathcal{R} = \{(A, B), (B, C), (D, C), (E, D), (F, E), (G, I), (H, I), (I, J), (K, J)\}$ as

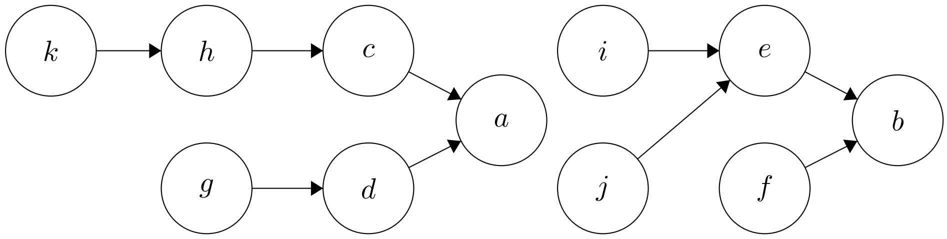

FIGURE 10. Argumentation framework describing distributed defense precedence (Amgoud & Ben-Naim, 2013).

displayed in Figure 11. Then, $Deg^{S_1}_{AF}(C) = \frac{1}{3} = Deg^{S_1}_{AF}(J)$. Furthermore, $|R^-(C)| = |R^-(J)|$ and $Def(C) = Def(J)$, the defense of C is simple and distributed, the defense of J is simple and not distributed, but $(J, C) \in S(AF)$. So, S_1 does not satisfy Distributed Defence Precedence. □

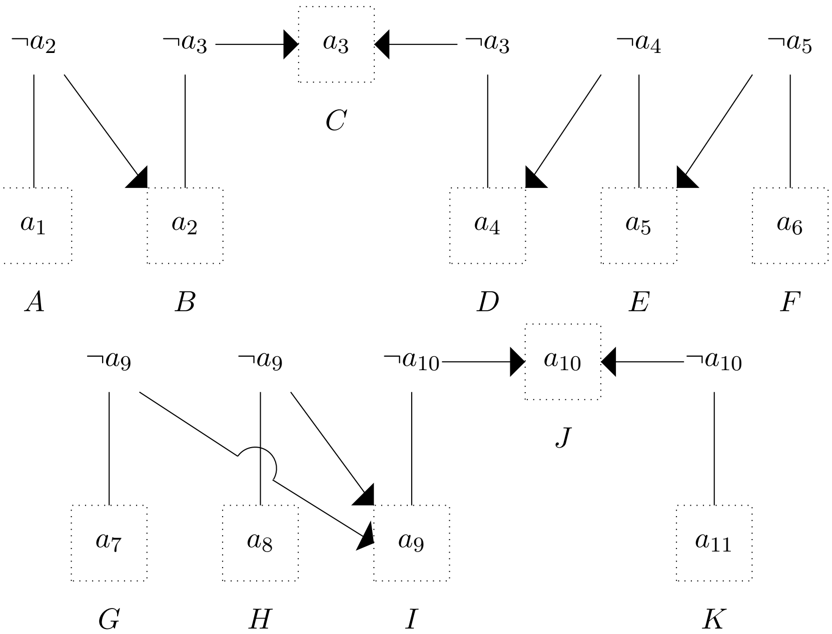

FIGURE 11. ASPIC+ argumentation as counterexample of Distributed Defence Precedence.

5.1.7. *Counter-Transitivity.*
The intuition behind (strict) counter-transitivity is that the more numerous and acceptable the attackers of an argument a, the less the degree of acceptability of a (Amgoud & Ben-Naim, 2013). This is defined as follows.

Definition 37 (Group Comparison). *(Amgoud & Ben-Naim, 2013) Let* $\preceq$ *be a ranking on a set* $\mathcal{A}$ *of arguments. For all* $A, B \subseteq \mathcal{A}$, $(A, B) \in Gr(\preceq)$ *iff there exists an injective function* f *from* B *to* A *such that* $\forall a \in B$, $f(a) \preceq a$.

Definition 38 (Strict Group Comparison). *(Amgoud & Ben-Naim, 2013) Let* $\preceq$ *be a ranking on a set* $\mathcal{A}$ *of arguments. For all* $A, B \subseteq \mathcal{A}$, $(A, B) \in Sgr(\preceq)$ *iff* *there exists an injective function f from B to A such that i) $\forall a \in B$, $f(a) \preceq a$ and ii) $|B| < |A|$ or $\exists a \in B$ such that $a \not\preceq f(a)$.*

Postulate 8 ((Strict) Counter-Transitivity)**.** *(Amgoud & Ben-Naim, 2013) A ranking-based semantics S satisfies the postulate counter-transitivity (CT) iff for every argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, $\forall a, b \in \mathcal{A}$, if $(R^-(b), R^-(a)) \in Gr[S(AF)]$, then $(a, b) \in S(AF)$. Semantics S satisfies the postulate strict counter-transitivity (SCT) iff for every argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, $\forall a, b \in \mathcal{A}$, if $(R^-(b), R^-(a)) \in Sgr[S(AF)]$, then $(b, a) \notin S(AF)$.*

Counter-transitivity is also a postulate that not necessarily applies for dialectical strength. For example, consider the argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, with $\mathcal{A} = \{a, b, c, d\}$ and $\mathcal{R} = \{(b, a), (c, b)\}$. According to counter-transitivity argument d is always ranked higher than argument a. However, when arguments a and d are equally attackable, i.e., they have the same number of attacking points, then, argument a should be ranked higher, after all, argument a has overcome one of its attackers.

Since strict counter-transitivity implies VP (Bonzon et al., 2016b) (which is not satisfied for dialectical argument strength), this means that also SCT is not satisfied for dialectical argument strength. Notice that semantics S_1 does not satisfy counter-transitivity and strict counter-transitivity.

Claim 8. *Counter-transitivity does not hold for semantics S_1.*

Proof. Consider argumentation system $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$, with $\mathcal{L} = \{a, b, c, d\neg a, \neg b, \neg c, \neg d\}$ and $\mathcal{R} = \{\neg b \to \neg a; c \to b\}$. Let $AT = (AS, \mathcal{K})$ be an argumentation theory, where $\mathcal{K} = \mathcal{K}_p = \{a, \neg b, c, d\}$. This leads to the argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, with $\mathcal{A} = \{A, B, C, D\}$ and $\mathcal{R} = \{(C, B), (B, A)\}$ as displayed in Figure 7. Then, $Deg^{S_1}_{AF}(A) = \frac{11}{12}$ and $Deg^{S_1}_{AF}(D) = \frac{5}{6}$. So, $(R^-(A), R^-(D)) \in Gr[S(AF)]$ and $(A, D) \in S(AF)$. So, S_1 does not satisfy counter-transitivity. □

Claim 9. *Strict counter-transitivity does not hold for semantics S_1.*

Proof. Since SCT implies VP and VP does not hold, also CP does not hold. □

5.2. **New Postulates.**

The first postulate for dialectical strength that we propose, is attacking point sensitivity, which intuitively ensures that an argument is ranked lower if it has more attacking points. This is defined below and involves ASPIC+ properties Prem(·) and DefRules(·) (see Definition 13). It says that if an argument has the same attackers, at least as many ordinary premises, at least as many defeasible inference rules and the total number of ordinary premises plus defeasible inference rules is strictly larger, then it should be ranked higher.

Postulate 9 (Attacking Point Sensitivity)**.** *A semantics S satisfies attacking point sensitivity (APS) iff, for all argumentation frameworks $AF = (\mathcal{A}, \mathcal{R})$ and $\forall a, b \in \mathcal{A}$ it holds that if i) $\exists f : R^-(a) \to R^-(b)$, such that f is bijective and $Deg(f(a)) = Deg(a)$, ii) $|\text{Prem}(a) \cap \mathcal{K}_p| \geq |\text{Prem}(b) \cap \mathcal{K}_p|$, iii) $|\text{DefRules}(a)| \geq |\text{DefRules}(b)|$ and iv) $|\text{Prem}(a) \cap \mathcal{K}_p| + |\text{DefRules}(a)| > |\text{Prem}(b) \cap \mathcal{K}_p| + |\text{DefRules}(b)|$, then $Deg^S_{AF}(a) < Deg^S_{AF}(b)$.*

The first condition, i.e. $R^-(a) = R^-(b)$, is a very restrictive condition, but also a necessary condition. Only the number of ordinary premises and the number of defeasible inference rules is not enough to compare arguments. Take the argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, with $\mathcal{A} = \{a, b, c\}$ and $\mathcal{R} = \{(c, b)\}$. Suppose a and b have the same number of ordinary premises and the same number of defeasible inference rules. According to condition ii) and iii) only, arguments a and b should be ranked equally high. However, if argument c is strict and firm, then argument b should have strictly lower degree of acceptability than argument a, since a is undoubtedly unacceptable or provisionally unacceptable and b is provisionally acceptable.

Another way of defining postulates for dialectical strength, is by using the concept of the more attacks an argument survived the better. This have we formalised in the following postulates, without using ASPIC+-properties.

Then the following postulates intuitively state that if an argument has at most the same number of attackers that are skeptically acceptable according to the grounded semantics and at least the same number of attackers that are not acceptable according to the grounded semantics then it is ranked higher. The difference between the Postulate 10 and Postulate 11 is the handling of credulously acceptable arguments. Postulate 10 considers a credulously acceptable attacker as a flaw, Postulate 11 considers a it as a victory. Both postulates can be satisfied together, but they are designed as a choice, where one of these two postulates should hold.

Definition 39 (Uncontested Attackers)**.** *Let $AF = (\mathcal{A}, \mathcal{R})$ be an argumentation framework. For any argument $a \in \mathcal{A}$ the set of uncontested attackers of a is defined as*

$$Uncon(a) = \{b \in R^-(a) | b \textit{ is in the grounded extension}\}.$$

Definition 40 (Questionable Attackers)**.** *Let $AF = (\mathcal{A}, \mathcal{R})$ be an argumentation framework. For any argument $a \in \mathcal{A}$ the set of questionable attackers of a is defined as*

$$Quest(a) = \{b \in R^-(a) | b \textit{ is credulously acceptable in grounded semantics}\}.$$

Definition 41 (Defeated Attackers)**.** *Let $AF = (\mathcal{A}, \mathcal{R})$ be an argumentation framework. For any argument $a \in \mathcal{A}$ the set of defeated attackers of a is defined as*

$$Defeat(a) = \{b \in R^-(a) | b \textit{ is unacceptable in grounded semantics}\}.$$

Postulate 10 (Survived Attacks Precedence)**.** *A semantics S satisfies survived attacks precedence (SAP) iff, for all argumentation frameworks $AF = (\mathcal{A}, \mathcal{R})$ and $\forall a, b \in \mathcal{A}$ it holds that if i) $Uncon(b) \subseteq Uncon(a)$, ii) $Quest(b) \subseteq Quest(a)$, iii) $Defeat(a) \subseteq Defeat(b)$, iv) $|\text{Prem}(a) \cap \mathcal{K}_p| = |\text{Prem}(b) \cap \mathcal{K}_p|$ and $|\text{DefRules}(a)| = |\text{DefRules}(b)|$, then $Deg^S_{AF}(a) \leq Deg^S_{AF}(b)$.*

Postulate 11 (Totally Survived Attacks Precedence)**.** *A semantics S satisfies totally survived attacks precedence (TSAP) iff, for all argumentation frameworks $AF = (\mathcal{A}, \mathcal{R})$ and $\forall a, b \in \mathcal{A}$ it holds that if i) $Uncon(b) \subseteq Uncon(a)$, ii) $Quest(a) \subseteq Quest(b)$, iii) $Defeat(a) \subseteq Defeat(b)$, iv) $|\text{Prem}(a) \cap \mathcal{K}_p| = |\text{Prem}(b) \cap \mathcal{K}_p|$ and $|\text{DefRules}(a)| = |\text{DefRules}(b)|$, then $Deg^S_{AF}(a) \leq Deg^S_{AF}(b)$.*

The next postulate uses the status of an argument using grounded and preferred extensions, as defined in Definition 6 and Definition 7. The postulate demands that not acceptable arguments are never ranked higher than credulously acceptable arguments, which are never ranked higher than skeptically acceptable arguments.

Postulate 12 (Status Precedence)**.** *A semantics S satisfies status precedence (SP) iff, for all argumentation frameworks $AF = (\mathcal{A}, \mathcal{R})$ and $\forall a, b \in \mathcal{A}$ it holds that*

- *if argument a is skeptically acceptable according to both grounded and preferred semantics, and b is credulously acceptable or unacceptable according to both grounded and preferred semantics, then a is ranked higher than b.*
- *if argument a is credulously acceptable according to both grounded and preferred semantics, and b is unacceptable according to both grounded and preferred semantics, then a is ranked higher than b.*

Notice that semantics S_1 satisfies all new postulates.

5.2.1. *Attacking Point Sensitivity.*

Claim 10. *Attacking point sensitivity holds for semantics S_1.*

Proof. Let $AT = (AS, \mathcal{K})$ be an arbitrary argumentation theory for arbitrary argumentation system $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$. Let $AF = (\mathcal{A}, \mathcal{R})$ be the corresponding argumentation framework. Let $a, b \in \mathcal{A}$ arbitrarily. If $\exists f : R^-(a) \to R^-(b)$, such that f is bijective and $Deg^{S_1}_{AF}(f(a)) = Deg^{S_1}_{AF}(a)$, $|\text{Prem}(a) \cap \mathcal{K}_p| \geq |\text{Prem}(b) \cap \mathcal{K}_p|$ and $|\text{DefRules}(a)| \geq |\text{DefRules}(b)|$ and $|\text{Prem}(a) \cap \mathcal{K}_p| + |\text{DefRules}(a)| > |\text{Prem}(b) \cap \mathcal{K}_p| + |\text{DefRules}(b)|$, then arguments a and b have the same status, i.e., both are provisionally acceptable, credulously acceptable, provisionally unacceptable or undoubtedly unacceptable, except if argument b is undoubtedly acceptable and argument a is provisionally acceptable. Furthermore, $SAP(a) = SAP(b)$ and $AP(a) > AP(b)$. If b is undoubtedly acceptable, then $Deg^{S_1}_{AF}(a) < 1 = Deg^{S_1}_{AF}(b)$. Else $Deg^{S_1}_{AF}(a) = x + \frac{1}{4} \cdot \frac{1+SAP(a)}{2+AP(a)} \leq x + \frac{1}{3} \cdot \frac{1+SAP(a)}{2+AP(b)} = Deg^{S_1}_{AF}(b)$, with $x = \frac{3}{4}$ if the arguments are provisionally acceptable, $x = \frac{1}{2}$ if the arguments are credulously acceptable, $x = \frac{1}{4}$ if the arguments are provisionally unacceptable and $x = 0$ if the arguments are undoubtedly unacceptable. So, S_1 satisfies attacking point sensitivity. □

5.2.2. *(Totally) Survived Attacks Precedence.*

Claim 11. *Survived attacks precedence holds for semantics S_1.*

Proof. Let $AT = (AS, \mathcal{K})$ be an arbitrary argumentation theory for arbitrary argumentation system $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$. Let $AF = (\mathcal{A}, \mathcal{R})$ be the corresponding argumentation framework. Let $a, b \in \mathcal{A}$ arbitrarily. If $|\text{Prem}(a) \cap \mathcal{K}_p| = |\text{Prem}(b) \cap \mathcal{K}_p|$ and $|\text{DefRules}(a)| = |\text{DefRules}(b)|$, then arguments $AP(a) = AP(b)$. If $Defeat(a) \subseteq Defeat(b)$, then $SAP(a) \leq SAP(b)$. If also $Uncon(b) \subseteq Uncon(a)$ and $Quest(b) \subseteq Quest(a)$, then there are 7 different scenario's:

(1) Argument b is undoubtedly acceptable, or
(2) Argument a and b are both provisionally acceptable, or
(3) Argument b is provisionally acceptable and argument a is credulously acceptable or provisionally unacceptable, or
(4) Argument a and b are both credulously acceptable, or
(5) Argument b is credulously acceptable and argument a is provisionally unacceptable, or
(6) Argument a and b are both provisionally unacceptable, or
(7) Argument a is undoubtedly unacceptable.

In case of (1), then

$$Deg^{S_1}_{AF}(a) \leq Deg^{S_1}_{AF}(b) = 1.$$

In case of (2), (4) and (6), then

$$Deg^{S_1}_{AF}(a) = x + \frac{1}{3} \cdot \frac{1 + SAP(a)}{2 + AP(a)} \leq x + \frac{1}{3} \cdot \frac{1 + SAP(b)}{2 + AP(b)} = Deg^{S_1}_{AF}(b),$$

with x respectively $\frac{2}{3}$, $\frac{1}{3}$ and 0. In case of (3) and (5), then

$$Deg^{S_1}_{AF}(a) < x \leq x + \frac{1}{3} \cdot \frac{1 + SAP(b)}{2 + AP(b)} = Deg^{S_1}_{AF}(b),$$

with x is respectively $\frac{2}{3}$ and $\frac{1}{3}$. In case of (12), then

$$Deg^{S_1}_{AF}(a) = Deg^{S_1}_{AF}(b) = 0.$$

So, $Deg^{S_1}_{AF}(a) < Deg^{S_1}_{AF}(b)$. So, S_1 satisfies survived attacks precedence. □

Claim 12. *Totally Survived Attacks Precedence does not hold for semantics S_1.*

Proof. Consider argumentation system $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$, with $\mathcal{L} = \{a_i, \neg a_i\}$ and $\mathcal{R} = \{a_1 \rightarrow \neg a_3; a_2 \rightarrow \neg a_1; a_3 \rightarrow \neg a_2\}$. Let $AT = (AS, \mathcal{K})$ be an argumentation theory, where $\mathcal{K} = \mathcal{K}_p = \{a_1, a_2, a_3, a_4\}$. This leads to the argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, with $\mathcal{A} = \{A, B, C, D\}$ and $\mathcal{R} = \{(B, A), (C, B), (A, C)\}$ as displayed in Figure 12. Then, $Deg^{S_1}_{AF}(A) = \frac{4}{9} < \frac{7}{9} = Deg^{S_1}_{AF}(D)$. Furthermore, $Uncon(D) = Uncon(A) = \emptyset$, $Quest(A) = \{B\} \subseteq \emptyset = Quest(D)$ and $Defeat(A) = Defeat(D)$. So, S_1 does not satisfy Totally Survived Attacks Precedence. □

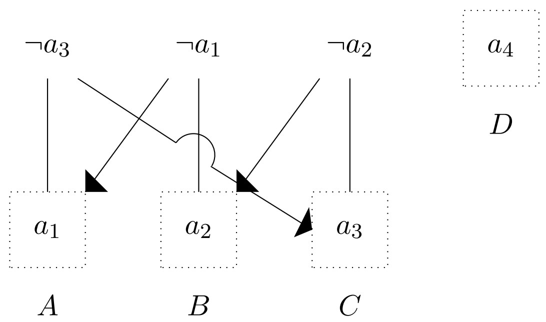

FIGURE 12. ASPIC+ argumentation as counterexample of Totally Survived Attacks Precedence.

5.2.3. *Status Precedence.*

Claim 13. *Status precedence holds for semantics S_1.*

Proof. Let $AT = (AS, \mathcal{K})$ be an arbitrary argumentation theory for arbitrary argumentation system $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$. Let $AF = (\mathcal{A}, \mathcal{R})$ be the the corresponding argumentation framework. Let $a, b \in \mathcal{A}$ arbitrarily. If argument a is skeptically acceptable according to both grounded and preferred semantics and argument b is credulously acceptable or unacceptable according to both grounded and preferred semantics. Then, $Deg_{AF}^{S_1}(a) \geq \frac{2}{3} > Deg_{AF}^{S_1}(b)$. If argument a is credulously acceptable according to both grounded and preferred semantics and argument b is unacceptable according to both grounded and preferred semantics. Then, $Deg_{AF}^{S_1}(a) \geq \frac{1}{3} > Deg_{AF}^{S_1}(b)$. So, S_1 satisfies status precedence. □

5.3. **Existing Semantics.**

In this section Max-based Semantics, Categoriser-based Semantics, Discussion-based Semantics, Burden-based Semantics and grounded semantics will be compared with Grounded Dialectical Semantics.

Table 3 gives an overview of the postulates satisfied by respectively Max-based Semantics, Categoriser-based Semantics, Discussion-based Semantics, Burden-based Semantics, grounded semantics and our proposed semantics.

A cross × means that the postulate is not satisfied, a checkmark ✓means that the postulate is satisfied. Cells highlighted in grey are the results already proven in the literature. Cells highlighted in red are proven below, non-highlighted cells are proven in section 5.1 and 5.2.

Properties	Max	Cat	Dbs	Bbs	Grounded	S_1
Ab	✓	✓	✓	✓	✓	×
In	✓	✓	✓	✓	✓	✓
VP	✓	✓	✓	✓	×	×
CP	×	×	✓	✓	×	×
QP	✓	×	×	×	✓	×
DP	×	✓	✓	✓	×	×
DDP	×	×	×	✓	×	×
CT	✓	✓	✓	✓	✓	×
SCT	×	✓	✓	✓	×	×
APS	×	×	×	×	×	✓
SAP	×	×	×	×	×	✓
TSAP	×	×	×	×	×	×
SP	✓	×	×	×	✓	✓

Table 3. Postulates satisfied by the semantics in the literature and the new semantics.

5.3.1. *Max-based Semantics.*

Max-based Semantics (Amgoud & Ben-Naim, 2018) is for argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, for any $a \in A$, defined as in Equation 2.

$$Deg_{AF}^{Max}(a) = \frac{1}{1 + \max_{b \in R^-(a)}(Deg_{AF}^{max}(b))} \tag{2}$$

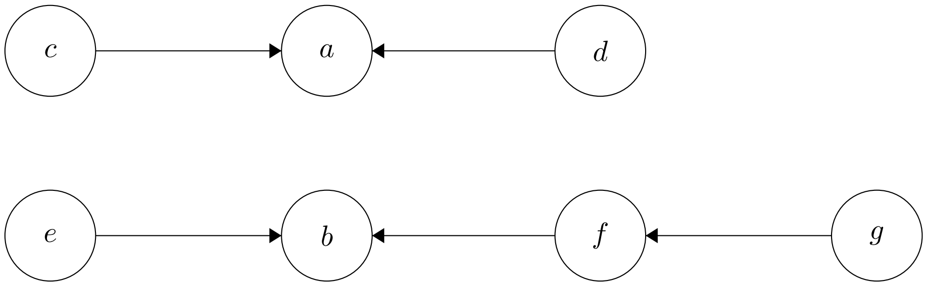

FIGURE 13. Argumentation Framework as example against defence precedence for Max-based Semantics.

Claim 14. *Abstraction holds for Max-based Semantics.*

Proof. Consider arbitrary argumentation frameworks $AF = (\mathcal{A}, \mathcal{R})$ and $AF' = (\mathcal{A}', \mathcal{R}')$ and let $f : \mathcal{A} \to A'$ be an isomorphism. Then, $\forall a \in \mathcal{A}$, $Deg_{[}^{Max}AF](a) = Deg_{[}^{Max}AF'](f(a))$. After all, the strength of any argument only depends on the attacker with the highest degree of acceptability. Therefore, $(f(a), f(b)) \in S(AF')$. So, Max-based Semantics satisfies abstraction. $\square$

Claim 15. *Independence holds for Max-based Semantics.*

Proof. Let $AF = (\mathcal{A}, \mathcal{R})$ be an arbitrary argumentation framework, let $B \in Com(AF)$ arbitrary. Then for every $a \in B$ holds that $Deg_{B}^{Max}(a) = Deg_{AF}^{Max}(a)$. After all, the strength of an argument only depends on the degree of acceptability of its attackers. So, Max-based Semantics satisfies independence. $\square$

Claim 16. *Void precedence holds for Max-based Semantics.*

Proof. Consider arbitrary argumentation framework $AF = (\mathcal{A}, \mathcal{R})$ and let $a, b \in \mathcal{A}$ arbitrary. If $R^{-}(a) \neq R^{-}(b) = \emptyset$, then $Deg_{AF}^{Max}(a) = \frac{1}{1+\max_{c \in R^{-}(a)}(Deg_{AF}^{max}(c))} < \frac{1}{1} = Deg_{AF}^{Max}(b)$. So, Max-based Semantics satisfies void precedence. $\square$

Claim 17. *Cardinality precedence does not hold for Max-based Semantics.*

Proof. Consider argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, with $\mathcal{A} = \{a, b, c, d, e\}$ and $\mathcal{R} = \{(c, a), (d, a), (e, b)\}$. Then $Deg_{AF}^{Max}(a) = \frac{1}{2} = Deg_{AF}^{Max}(b)$. So, Max-based Semantics does not satisfy cardinality precedence. $\square$

Claim 18. *Quality precedence does hold for Max-based Semantics.*

Proof. Consider arbitrary argumentation framework $AF = (\mathcal{A}, \mathcal{R})$ and let $a, b \in \mathcal{A}$ be arbitrary. If $\exists c \in R^{-}(b)$ such that $\forall d \in R^{-}(a)$ holds that $(d, c) \notin S(AF)$, then, by definition, $Deg_{AF}^{Max}(a) > Deg_{AF}^{Max}(b)$. So Max-based Semantics satisfies quality precedence. $\square$

Claim 19. *Defence precedence does not hold for Max-based Semantics.*

Proof. Consider argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, with $\mathcal{A} = \{a, b, c, d, e, f, g\}$ and $\mathcal{R} = \{(c, a), (d, a), (e, b), (f, b), (g, f)\}$, as displayed in Figure 13. Then $|R^{-}(a)| = |R^{-}(b)|$ and $Def(b) \neq Def(a) = \emptyset$, but $Deg_{AF}^{Max}(a) = Deg_{AF}^{Max}(b)$. So, Max-based Semantics does not satisfy defence precedence. $\square$

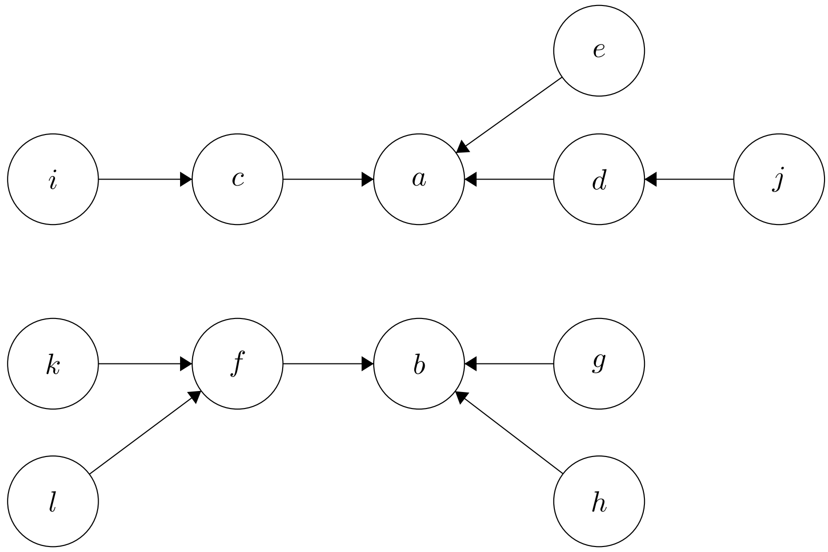

Figure 14. Argumentation Framework as example against distributed defence precedence for Max-based Semantics.

Claim 20. *Distributed defence precedence does not hold for Max-based Semantics.*

Proof. Consider argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, with $\mathcal{A} = \{a, b, c, d, e, f, g, h, i, j, k, l\}$ and $\mathcal{R} = \{(c, a), (d, a), (e, a), (f, b), (g, b), (h, b), (i, c), (j, d), (k, f), (l, f)\}$, as displayed in Figure 14. Then $|R^-(a)| = |R^-(b)|$ and $|Def(a)| = |Def(b)|$ and the defense of a is simple and distributed and the defence of b is simple and not distributed, but $Deg_{AF}^{Max}(a) = Deg_{AF}^{Max}(b)$. So, Max-based Semantics does not satisfy distributed defence precedence. □

Claim 21. *Counter-transitivity holds for Max-based Semantics.*

Proof. Consider arbitrary argumentation framework $AF = (\mathcal{A}, \mathcal{R})$ and let $a, b \in \mathcal{A}$ be arbitrary. If $(R^-(a), R^-(b)) \in Gr[S(AF)]$, then

$$\begin{aligned} Deg_{AF}^{Max}(b) &= \frac{1}{1 + \max_{c \in \mathcal{R}^-(b)} Deg_{AF}^{Max}(c)} \\ &\geq \frac{1}{1 + \max_{c \in \mathcal{R}^-(b)} Deg_{AF}^{Max}(f(c))} \geq Deg_{AF}^{Max}(a). \end{aligned}$$

So, Max-based Semantics satisfies counter-transitivity. □

Claim 22. *Strict counter-transitivity does not for Max-based Semantics.*

Proof. Consider argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, with $\mathcal{A} = \{a, b, c, d, e\}$ and $\mathcal{R}\{(c, a), (d, a), (e, b)\}$. Then $(R^-(a), R^-(b)) \in Sgr[S(AF)]$, but $Deg_{AF}^{Max}(a) = Deg_{AF}^{Max}(b)$. So, Max-based Semantics does not satisfy strict counter-transitivity. □

Claim 23. *Attacking point sensitivity does not hold for Max-based Semantics.*

Proof. Consider argumentation system $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$, with $\mathcal{L} = \{a, b, c\}$ and $\mathcal{R} = \{b \to c\}$. Then for argumentation theory $AT = (AS, \mathcal{K})$, with

$\mathcal{K} = \mathcal{K}_n \cup \mathcal{K}_p$, where $\mathcal{K}_n = \{a\}$ and $\mathcal{K}_p = \{b\}$, this leads to argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, where there are two argument a (A) and $b \to c$ (B). There are no attacks, so the first condition of the postulate is satisfied trivially. Furthermore, the three other conditions are satisfied, after all, $|\text{Prem}(A) \cap \mathcal{K}_p| + |\text{DefRules}(A)| = |\text{Prem}(A) \cap \mathcal{K}_p| = 1 > 0 = |\text{Prem}(B) \cap \mathcal{K}_p| + |\text{DefRules}(b)|$ and $Deg^{Max}_{AF}(a) = Deg^{Max}_{AF}(b)$. So, Max-based Semantics does not satisfy attacking point sensitivity. □

Claim 24. *Totally survived attacks precedence nor survived attacks precedence holds for Max-based Semantics.*

Proof. Consider argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, with $\mathcal{A} = \{a, b, c, d\}$ and $\mathcal{R} = (c, b), (d, c)$. Then, even if $|\text{Prem}(a) \cap \mathcal{K}_p| = |\text{Prem}(b) \cap \mathcal{K}_p|$ and $|\text{DefRules}(a)| = |\text{DefRules}(b)|$, $Deg^{Max}_{AF}(a) > Deg^{Max}_{AF}(b)$. So, Max-based Semantics does not satisfy (totally) survived attacks precedence. □

Claim 25. *Status precedence holds for Max-based Semantics.*

Proposition 2. *For the sequence defined by* $a_0 = 1$ *and* $a_n = \frac{1}{1+\frac{1}{1+a_{n-1}}}$ *holds* $a_n > \frac{1}{2}\sqrt{5} - \frac{1}{2}$.

Proof. We will proof by induction. As base step we see that $1 > \frac{1}{2}\sqrt{5} - \frac{1}{2}$. Suppose $a_n > \frac{1}{2}\sqrt{5} - \frac{1}{2}$. Then,

$$\begin{aligned} a_{n+1} - \frac{1}{2}\sqrt{5} - \frac{1}{2} &= \frac{1}{1 + \frac{1}{1+a_n}} - \frac{1}{2}\sqrt{5} - \frac{1}{2} \\ &= \frac{a_n + 1}{a_n + 2} - \frac{1}{2}\sqrt{5} - \frac{1}{2} \\ &= 1 - \frac{1}{a_n + 2} - \frac{1}{2}\sqrt{5} - \frac{1}{2} \\ &> 1 - \frac{1}{\frac{1}{2}\sqrt{5} - \frac{1}{2} + 2} - \frac{1}{2}\sqrt{5} - \frac{1}{2} = 0. \end{aligned}$$

By the Principle of Mathematical Induction holds that $a_n > \frac{1}{2}\sqrt{5} - \frac{1}{2}$. □

Proposition 3. *For the sequence defined by* $a_0 = \frac{1}{2}$ *and* $a_n = \frac{1}{1+\frac{1}{1+a_{n-1}}}$ *holds* $a_n < \frac{1}{2}\sqrt{5} - \frac{1}{2}$.

Proof. We will proof by induction. As base step we see that $\frac{1}{2} < \frac{1}{2}\sqrt{5} - \frac{1}{2}$. Suppose $a_n < \frac{1}{2}\sqrt{5} - \frac{1}{2}$. Then,

$$\begin{aligned} a_{n+1} - \frac{1}{2}\sqrt{5} - \frac{1}{2} &= \frac{1}{1 + \frac{1}{1+a_n}} - \frac{1}{2}\sqrt{5} - \frac{1}{2} \\ &= \frac{a_n + 1}{a_n + 1} - \frac{1}{2}\sqrt{5} - \frac{1}{2} \\ &= 1 - \frac{1}{a_n + 2} - \frac{1}{2}\sqrt{5} - \frac{1}{2} \\ &< 1 - \frac{1}{\frac{1}{2}\sqrt{5} - \frac{1}{2} + 2} - \frac{1}{2}\sqrt{5} - \frac{1}{2} = 0. \end{aligned}$$

By the Principle of Mathematical Induction holds that $a_n < \frac{1}{2}\sqrt{5} - \frac{1}{2}$. □

Proof. Let $AF = (\mathcal{A}, \mathcal{R})$ be an arbitrary argumentation framework. Since every skeptically acceptable argument can be is either not-attacked or attacked by one or more not acceptable attackers, the degree of acceptability of all these arguments can be described by the sequence $a_0 = 1$ and $a_n = \frac{1}{1+\frac{1}{1+a_{n-1}}}$ and the not acceptable arguments can be described by the sequence $b_0 = \frac{1}{2}$ and $b_n = \frac{1}{1+\frac{1}{1+b_{n-1}}}$. After all, for $n > 0$, $Deg^{Max}_{AF}(a_n) = \frac{1}{1+Deg^{Max}_{AF}(b_{n-1})}$, with $Deg^{Max}_{AF}(b_{n-1}) = \frac{1}{1+Deg^{Max}_{AF}(a_{n-1})}$. Furthermore $a_n > \frac{1}{2}\sqrt{5} - \frac{1}{2}$, see Proposition 2 and $b_n > \frac{1}{2}\sqrt{5} - \frac{1}{2}$, see Proposition 3.

Credulously acceptable argument a the degree of acceptability is $Deg^{Max}_{AF}(a) = \frac{1}{1+Deg^{Max}_{AF}(a)}$. So, $Deg^{Max}_{AF}(a) = \frac{1}{2}\sqrt{5} - \frac{1}{2}$.

So, skeptically acceptable arguments always get a degree of acceptability in the interval $(\frac{1}{2}\sqrt{5} - \frac{1}{2}, 1]$. Credulously acceptable arguments always get a degree of acceptability of $\frac{1}{2}\sqrt{5} - \frac{1}{2}$ and not acceptable argument always get a degree of acceptability in the interval $[\frac{1}{2}, \frac{1}{2}\sqrt{5} - \frac{1}{2})$. So Max-based Semantics satisfies status precedence. □

5.3.2. *Categoriser-based Semantics.*

Categoriser-based Semantics (Besnard & Hunter, 2001) is for argumentation framework AF as in Equation 3, where $b_1, \ldots, b_n$ are attackers of a. Categoriser-based Semantics is only defined for a-cyclic argumentation frameworks. However, for cyclic argumentation framework solving a system of linear equations could provide a solution.

$$Deg^{Cat}_{AF}(a) = \frac{1}{1 + \sum_{i=1}^{n} Deg^{Cat}_{AF}(b_i)} \tag{3}$$

Claim 26. *Attacking point sensitivity does not hold for Categoriser-based Semantics.*

Proof. Consider argumentation system $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$, with $\mathcal{L} = \{a, b, c\}$ and $\mathcal{R} = \{b \rightarrow c\}$. Then for argumentation theory $AT = (AS, \mathcal{K})$, with $\mathcal{K} = \mathcal{K}_n \cup \mathcal{K}_p$, where $\mathcal{K}_n = \{a\}$ and $\mathcal{K}_p = \{b\}$, this leads to argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, where there are two argument a (A) and $b \rightarrow c$ (B). There are no attacks, so the first condition of the postulate is satisfied trivially. Furthermore, the three other conditions are satisfied, after all, $|\text{Prem}(A) \cap \mathcal{K}_p| + |\text{DefRules}(A)| = |\text{Prem}(A) \cap \mathcal{K}_p| = 1 > 0 = |\text{Prem}(B) \cap \mathcal{K}_p| + |\text{DefRules}(b)|$ and $Deg^{Cat}_{AF}(a) = Deg^{Cat}_{AF}(b)$. So, Categoriser-based Semantics does not satisfy attacking point sensitivity. □

Claim 27. *Totally survived attacks precedence nor survived attacks precedence holds for Categoriser-based Semantics.*

Proof. Consider argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, with $\mathcal{A} = \{a, b, c, d\}$ and $\mathcal{R} = \{(c, b), (d, c)\}$. Then, even if $|\text{Prem}(a) \cap \mathcal{K}_p| = |\text{Prem}(b) \cap \mathcal{K}_p|$ and $|\text{DefRules}(a)| = |\text{DefRules}(b)|$, $Deg^{Cat}_{AF}(a) > Deg^{Cat}_{AF}(b)$. So, Categoriser-based Semantics does not satisfy (totally) survived attacks precedence. □

Claim 28. *Status precedence does not hold for Categoriser-based Semantics.*

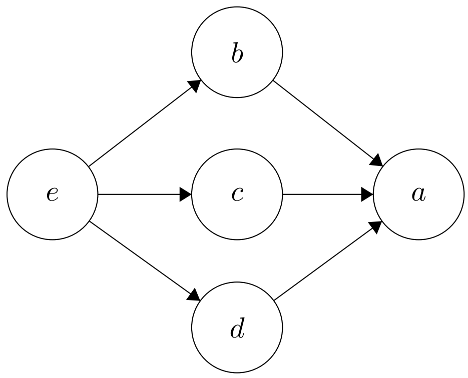

FIGURE 15. Argumentation Framework as example against status precedence for Categoriser-based Semantics.

Proof. Consider argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, with $\mathcal{A} = \{a, b, c, d, e\}$, with $\mathcal{R} = \{(b, a), (c, a), (d, a), (e, b), (e, c), (e, d)\}$, as in Figure 15. Then, argument a is skeptically acceptable and argument b is not acceptable. However, $Deg_{AF}^{Cat}(a) = \frac{2}{5} < \frac{1}{2} = Deg_{AF}^{Cat}(b)$. So, Categoriser-based Semantics does not satisfy status precedence. □

5.3.3. *Discussion-based Semantics.*
The Discussion-based Semantics(Amgoud & Ben-Naim, 2013) is a ranking based on the discussion counts. These compare arguments step-wise and count the number of linear discussion that end with these argument. Two arguments can be compared lexicographically based on their discussion counts, i.e., $a \preceq_{AF}^{Dbs} b$, if $Dis(b) \preceq_{lex} Dis(a)$.

Definition 42 (Linear discussion)**.** *Let $AF = (\mathcal{A}, \mathcal{R})$ an argumentation framework and $a \in \mathcal{A}$. A linear discussion for a in $\mathcal{A}$ is a sequence $s = (a_1, \ldots, a_n)$ of elements of $\mathcal{A}$ (where n is a positive integer) such that $a_1 = a$ and $\forall i \in \{2, \ldots, n\}$ $a_i \mathcal{R} a_{i-1}$. The length of s is n. We say that: s is won iff n is odd; s is lost iff n is even.*

Definition 43 (Discussion counts)**.** *Let $AF = (\mathcal{A}, \mathcal{R})$ be an argumentation framework, $a \in \mathcal{A}$, i a positive integer and let N be the number of linear discussions for a of length i. We define that:*

$$\mathrm{Dis}_{A_i}(a) = \begin{cases} -N & \textit{if } i \textit{ is odd;} \\ N & \textit{if } i \textit{ is even.} \end{cases}$$

Claim 29. *Attacking point sensitivity does not hold for Discussion-based Semantics.*

Proof. Consider argumentation system $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$, with $\mathcal{L} = \{a, b, c\}$ and $\mathcal{R} = \{b \to c\}$. Then for argumentation theory $AT = (AS, \mathcal{K})$, with $\mathcal{K} = \mathcal{K}_n \cup \mathcal{K}_p$, where $\mathcal{K}_n = \{a\}$ and $\mathcal{K}_p = \{b\}$, this leads to argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, where there are two argument a (A) and $b \to c$ (B). There are no attacks, so the first condition of the postulate is satisfied trivially. Furthermore, the three other conditions are satisfied, after all, $|\mathrm{Prem}(A) \cap \mathcal{K}_p| + |\mathrm{DefRules}(A)| =$

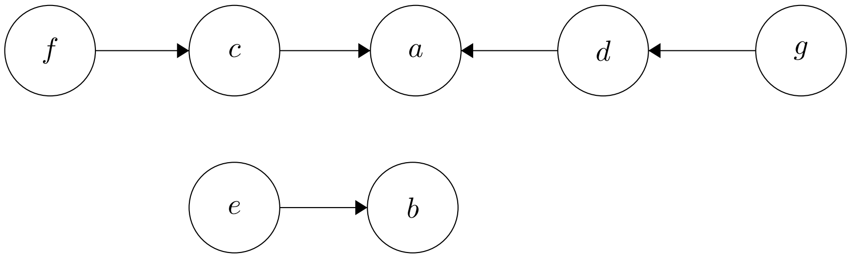

FIGURE 16. Argumentation Framework as example against status precedence for Discussion-based Semantics.

$|\text{Prem}(A) \cap \mathcal{K}_p| = 1 > 0 = |\text{Prem}(B) \cap \mathcal{K}_p| + |\text{DefRules}(b)|$ and $a \simeq^{Dbs}_{AF} b$. So, Discussion-based Semantics does not satisfy attacking point sensitivity. □

Claim 30. *Totally survived attacks precedence nor survived attacks precedence holds for Discussion-based Semantics.*

Proof. Consider argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, with $\mathcal{A} = \{a, b, c, d\}$ and $\mathcal{R} = (c, b), (d, c)$. Then, even if $|\text{Prem}(a) \cap \mathcal{K}_p| = |\text{Prem}(b) \cap \mathcal{K}_p|$ and $|\text{DefRules}(a)| = |\text{DefRules}(b)|$, $b \prec^{Dbs}_{AF} a$. So, Discussion-based Semantics does not satisfy (totally) survived attacks precedence. □

Claim 31. *Status precedence does not hold for Discussion-based Semantics.*

Proof. Consider argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, with $\mathcal{A} = \{a, b, c, d, e, f, g\}$ and $\mathcal{R} = \{(c, a), (d, a), (e, b), (f, c), (g, d)\}$, as displayed in Figure 16. Then, argument a is skeptically acceptable and argument b is not acceptable. However, $a \prec^{Dbs}_{AF} b$. So, Discussion-based Semantics does not satisfy status precedence. □

5.3.4. *Burden-based Semantics.*
Burden-based Semantics also compares arguments step-wise and assigns a Burden number to each argument at each step i. Two arguments a and b can be compared lexicographically based on their Burden numbers, i.e., $a \preceq^{Bbs}_{AF} b$, if $Bur(b) \preceq_{lex} Bur(a)$.

Definition 44 (Burden numbers)**.** *Let $AF = (\mathcal{A}, \mathcal{R})$ be an argumentation framework. Then* $\text{Bur}_i(a) = \begin{cases} 1 & \text{if } i = 0; \\ 1 + \sum_{b \in R^-(b)} \frac{1}{Bur_{i-1}(b)} & \text{otherwise.} \end{cases}$

Claim 32. *Attacking point sensitivity does not hold for Burden-based Semantics.*

Proof. Consider argumentation system $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$, with $\mathcal{L} = \{a, b, c\}$ and $\mathcal{R} = \{b \to c\}$. Then for argumentation theory $AT = (AS, \mathcal{K})$, with $\mathcal{K} = \mathcal{K}_n \cup \mathcal{K}_p$, where $\mathcal{K}_n = \{a\}$ and $\mathcal{K}_p = \{b\}$, this leads to argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, where there are two argument a (A) and $b \to c$ (B). There are no attacks, so the first condition of the postulate is satisfied trivially. Furthermore, the three other conditions are satisfied, after all, $|\text{Prem}(A) \cap \mathcal{K}_p| + |\text{DefRules}(A)| = |\text{Prem}(A) \cap \mathcal{K}_p| = 1 > 0 = |\text{Prem}(B) \cap \mathcal{K}_p| + |\text{DefRules}(b)|$ and $a \simeq^{Bbs}_{AF} b$. So, Burden-based Semantics does not satisfy attacking point sensitivity. □

Claim 33. *Totally survived attacks precedence nor survived attacks precedence holds for Burden-based Semantics.*

Proof. Consider argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, with $\mathcal{A} = \{a, b, c, d\}$ and $\mathcal{R} = (c, b), (d, c)$. Then, even if $|\text{Prem}(a) \cap \mathcal{K}_p| = |\text{Prem}(b) \cap \mathcal{K}_p|$ and $|\text{DefRules}(a)| = |\text{DefRules}(b)|$, $b \prec^{Bbs}_{AF} a$. So, Burden-based Semantics does not satisfy (totally) survived attacks precedence. □

Claim 34. *Status precedence does not hold for Burden-based Semantics.*

Proof. Consider argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, with $\mathcal{A} = \{a, b, c, d, e, f, g\}$ and $\mathcal{R} = \{(c, a), (d, a), (e, b), (f, c), (g, d)\}$, as displayed in Figure 16. Then, argument a is skeptically acceptable and argument b is not acceptable. However, $a \simeq^{Bbs}_{AF} b$. So, Burden-based Semantics does not satisfy status precedence. □

5.3.5. *Grounded Semantics.*

Claim 35. *Attacking point sensitivity does not hold for grounded semantics.*

Proof. Consider argumentation system $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$, with $\mathcal{L} = \{a, b, c\}$ and $\mathcal{R} = \{b \to c\}$. Then for argumentation theory $AT = (AS, \mathcal{K})$, with $\mathcal{K} = \mathcal{K}_n \cup \mathcal{K}_p$, where $\mathcal{K}_n = \{a\}$ and $\mathcal{K}_p = \{b\}$, this leads to argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, where there are two argument a (A) and $b \to c$ (B). There are no attacks, so the first condition of the postulate is satisfied trivially. Furthermore, the three other conditions are satisfied, after all, $|\text{Prem}(A) \cap \mathcal{K}_p| + |\text{DefRules}(A)| = |\text{Prem}(A) \cap \mathcal{K}_p| = 1 > 0 = |\text{Prem}(B) \cap \mathcal{K}_p| + |\text{DefRules}(b)|$ and both argument are in the grounded extension. So, grounded semantics does not satisfy attacking point sensitivity. □

Claim 36. *Totally survived attacks precedence nor survived attacks precedence holds for grounded semantics.*

Proof. Consider argumentation framework $AF = (\mathcal{A}, \mathcal{R})$, with $\mathcal{A} = \{a, b, c, d\}$ and $\mathcal{R} = (c, b), (d, c)$. Then, even if $|\text{Prem}(a) \cap \mathcal{K}_p| = |\text{Prem}(b) \cap \mathcal{K}_p|$ and $|\text{DefRules}(a)| = |\text{DefRules}(b)|$, argument a and b are both in the grounded extension. So, grounded semantics does not satisfy (totally) survived attacks precedence. □

6. Dialectical Support Semantics

In this section support argumentation frameworks are discussed. Like Amgoud & Ben-Naim (2016b) we will discuss argumentation frameworks with supports only, i.e., $AF = (\mathcal{A}, \mathcal{S})$. Firstly, we will provide the definitions of premise support and support argumentation frameworks. Then we will provide the intuitions for a semantics that will be defined in the last part of this section.

6.1. **Definitions.**

Recall that an argument a is premise-supported by argument b if argument b has the same conclusion as a premise of argument a, see Definition 45.

Definition 45 (Premise Support)**.** *(Cohen et al., 2018) Let* $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$ *be an argumentation system and let* $AT = (AS, \mathcal{K})$ *be an argumentation theory and* $A_1, A_2 \in \mathcal{A}(AT)$*. Argument* A_1 *provides premise support for* A_2 *iff* $A_1 \neq A_2$ *and* $\text{Conc}(A1) \in \text{Prem}(A_2)$*.*

A support argumentation framework is just like an abstract argumentation framework, except there are no attackers but only supporters. For simplicity we will say 'argumentation framework' instead of 'support argumentation framework'.

Definition 46 (Support Argumentation Frameworks)**.** *A support argumentation framework is a tuple* $(\mathcal{A}, \mathcal{S})$*, where* $\mathcal{A}$ *is a set of arguments and* $\mathcal{S} \subseteq \mathcal{A} \times \mathcal{A}$ *a binary support relation on* $\mathcal{A}$*. For arguments* $a, b \in \mathcal{A}$*,* $(a, b) \in \mathcal{S}$ *or* $a\mathcal{S}b$ *means that a premise supports* b*. Furthermore,* $\mathcal{S}^{-}(a)$ *denotes the set of supporters of* a*.*

6.2. **Intuitions.**
We will discuss dialectical argument strength for premise support. An argument consists of premises and inference rules (see Definition 13). For now we will restrict ourselves to premise support. Therefore it is only relevant to look at the differences between premises of arguments.

As described earlier, Zenker et al. (2020) described dialectical argument strength in terms of the opponent's availability to move. In terms of premise support possible moves are questioning why a certain premise holds. So, the fewer ordinary premises an unsupported argument has, the better. As a baseline, each non-supported argument a has as degree of acceptability $\frac{1}{1+|Um(a)|}$.

Premise support in dialectical context is a good feature. Supporters should not decrease the acceptability. Otherwise, the opponent can devalue an argument by adding a supporter. So, supporters of argument a do not break down the argument strength of a, but supporters do not necessarily contribute to the acceptability of argument a. What we mean by the latter is illustrated by Example 9. Notice that more supporters is not necessarily better. A supporter is only contributing when it is firm. After all, arguments with 1 or more ordinary premises expand the number of possibilities for the opponent to move. Otherwise, with a firm argument, the possibilities to move are decreased by 1. If in Example 9 argument a was firm, i.e., $a \in \mathcal{K}_n$, then the opponent would have no possibility to move.

Example 9. *Consider argumentation system* $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$*, where* $\mathcal{L} = \{a, b\}$*,* $\mathcal{R} = \{a \to b\}$ *and consider argumentation theory* $AT = (AS, \mathcal{K})$*, with* $\mathcal{K} = \mathcal{K}_p = \{a, b\}$*. Then argument* b *is supported by argument* $a \to b$*. However the opponent than still has an option to question the argument, after all,* a *is only an ordinary premises. In case of no support the opponent could ask "why* b*?" However, in the case of this example the opponent could ask "why* a*?". So, the number possibilities to move are equal.*

6.3. **Possible New Semantics.**
One could involve the difference between strict and plausible inference rules and argue that arguments with strict rules are better than argument with plausible rules, if the number of ordinary premises is the same. However, this would make the semantics more complicated than necessary. Moreover, there is no possibility in ASPIC+ to support plausible inference rules. Therefore this will be ignored in the following semantics.

Below we will present a possible new gradual semantics, which exactly meets the intuitions above.

Definition 47. *For argumentation framework* $AF = (\mathcal{A}, \mathcal{S})$*. The degree of acceptability of an argument* $a \in \mathcal{A}$*, is one over one plus the number of weakly* *or unsupported premises, i.e.,*

$$Deg^{S_2}_{AF}(a) = \frac{1}{1 + \sum_{b \in Um(a)} \min_{c \in \mathcal{S}^-(a)}(f_b(c))},$$

where $f_b : \mathcal{A} \to (0,1]$ *is defined by*

$$f_b(x) = \begin{cases} 0 & \text{, if } \mathrm{Conc}(x) = b \text{ and } x \text{ is firm,} \\ 1 & \text{, otherwise.} \end{cases}$$

If an argument a has no supports, i.e. $\mathcal{S}^-(a) = \emptyset$, then this semantics is equivalent to $\frac{1}{1+|Um(a)|}$. After all, then

$$\sum_{b \in Um(a)} \min_{c \in \mathcal{S}^-(a)} (f_b(c)) = \sum_{b \in Um(a)} 1 = |Um(a)|.$$

So, the more ordinary premises an unsupported argument has, the lower its degree of acceptability. Notice, this is also the minimal value.

Proposition 4. *For any argumentation framework* $AF = (\mathcal{A}, \mathcal{S})$*, for any argument* $a \in \mathcal{A}$ *always* $Deg^F_{AF}(a) \geq \frac{1}{1+|Um(a)|}$.

Proof. Consider arbitrary argumentation framework $AF = (\mathcal{A}, \mathcal{S})$, then $\forall a \in \mathcal{A}$,

$$\begin{aligned} Deg^{S_2}_{AF}(a) &= \frac{1}{1 + \sum_{b \in Um(a)} \min_{c \in \mathcal{S}^-(a)}(f_b(c))} \\ &\geq \frac{1}{1 + \sum_{b \in Um(a)} 1} = \frac{1}{1 + |Um(a)|} \end{aligned}$$

□

The function f_b ensures that only firm supporters contribute to the degree of acceptability and that supporters only contribute to the degree of acceptability for the premises they support.

Example 10. *Consider argumentation system* $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$ *, with* $\mathcal{L} = \{a_i, \neg a_i\}$ *and* $\mathcal{R} = \{a_2 \to a_1; a_1 \to a_3; a_5, a_6 \to a_4; a_4 \to a_7\}$*. Let* $AT = (AS, \mathcal{K})$ *be an argumentation theory, where* $\mathcal{K} = \mathcal{K}_n \cup \mathcal{K}_p$*, with* $\mathcal{K}_n = \{a_2\}$ *and* $\mathcal{K}_p = \{a_1, a_4, a_5, a_6\}$*. This leads to the argumentation framework* $AF = (\mathcal{A}, \mathcal{S})$*, with* $\mathcal{A} = \{A, B, C, D\}$ *and* $\mathcal{S} = \{(A, B), (C, D)\}$*, see Figure 17, where double-lined arrows represent supports. Then,* $Deg^{S_2}_{AF}(A) = 1$ *and*

$$\begin{aligned} Deg^{S_2}_{AF}(B) &= \frac{1}{1 + \sum_{b \in Um(a)} \min_{c \in \mathcal{S}^-(a)}(f_b(c))} \\ &= \frac{1}{1 + f_{a_1}(A)} = \frac{1}{1} = 1. \end{aligned}$$

Furthermore, $Deg^{S_2}_{AF}(C) = \frac{1}{3}$ *and* $Deg^{S_2}_{AF}(D) = \frac{1}{2}$.

Notice, one can also construct an arguments $E : (a_2 \to a_1) \to a_3$ *and* $F : (a_5, a_6 \to a_4) \to a_6$*. Argument* F *would have* $Deg^{S_2}_{AF}(F) = \frac{1}{3}$*, which is lower than argument* D*. This is a desirable outcome, because the argumentation is weaker than* D*. Argument* E *would have* $Deg^{S_2}_{AF}(E) = 1$*, which is equal to* $Deg^{S_2}_{AF}(B)$.

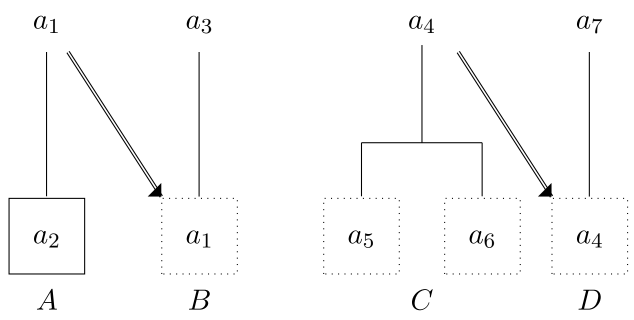

FIGURE 17. ASPIC+ example where support by a firm argument is compared to support by a plausible argument.

7. POSTULATES FOR SUPPORT SEMANTICS

In this section postulates from Amgoud & Ben-Naim (2016b) will be covered as well as new postulates, which we will propose.

Amgoud & Ben-Naim (2016b) have proposed postulates for semi-weighted support argumentation frameworks. These are defined as a regular support argumentation framework, except with an extra initial weight, see Definition 48. For simplicity, we will call them argumentation frameworks.

Definition 48 (Semi-weighted Support Argumentation Frameworks)**.** *A semi-weighted support argumentation framework is a tuple* $(\mathcal{A}, \omega, \mathcal{S})$*, where* $\mathcal{A}$ *is a set of arguments,* $\omega : \mathcal{A} \to [0,1]$ *is a function that maps each argument to its initial weight and* $\mathcal{S} \subseteq \mathcal{A} \times \mathcal{A}$ *a binary support relation on* $\mathcal{A}$*. For arguments* $a, b \in \mathcal{A}$*,* $(a,b) \in \mathcal{S}$ *or* $a\mathcal{S}b$ *means that a premise supports b. Furthermore,* $\mathcal{S}^-(a)$ *denotes the set of supporters of a.*

7.1. **Existing Postulates.**

Amgoud & Ben-Naim (2016b) propose several postulates for semi-weighted support argumentation frameworks. Since we do not use initial weights explicitly, we will work with for $a \in \mathcal{A}$ with $\omega(a) = \frac{1}{1+|um(a)|}$. This seems justified, since this is the value of each argument when $\mathcal{S} = \emptyset$. By this assumption the postulate Minimality (see Postulate 13) is exactly satisfied. The formulation of the postulates might slightly differ from (Amgoud & Ben-Naim, 2016b).

Postulate 13 (Minimality)**.** *(Amgoud & Ben-Naim, 2016b) A semantics F satisfies Minimality iff, for any argumentation framework* $AF = (\mathcal{A}, \omega, \mathcal{S})$*,* $\forall a \in \mathcal{A}$ *if* $S^-(a) = \emptyset$*, then* $Deg^F_{AF}(a) = \omega(a)$*.*

Theorem 2. *Semantics* S_2 *satisfies postulates support independence, non-dilution, dummy, monotony, coherence, strengthening soundness, boundedness, strict monotony, Ind and firm counting and does not satisfy postulates anonymity, equivalence, strengthening, counting, reinforcement, imperfection, cardinality precedence, quality precedence and compensation.*

Proof. Since claim 37 up to and including Claim 55 are true, Theorem 2 is true. □

7.1.1. *Anonymity.*
The intuition of the first postulate (anonymity) is that equivalent arguments are ranked equivalently.

Definition 49 (Isomorphism)**.** *(Amgoud & Ben-Naim, 2016b) Let* $AF = (\mathcal{A}, \omega, \mathcal{S})$ *and* $AF' = (\mathcal{A}', \omega', \mathcal{S}')$ *be two argumentation frameworks. An isomorphism from* AF *to* AF' *is a bijective function* f *from* $\mathcal{A}$ *to* $\mathcal{A}'$ *such that i)* $\forall a \in \mathcal{A}$ *holds that* $\omega(a) = \omega(f(a))$ *and ii)* $\forall a, b \in \mathcal{A}$ *holds* $a\mathcal{S}b$ *iff* $f(a)\mathcal{S}f(b)$*.*

Postulate 14 (Anonymity)**.** *A semantics* F *satisfies anonymity iff, for any two argumentation frameworks* $AF = (\mathcal{A}, \omega, \mathcal{S})$ *and* $AF' = (\mathcal{A}', \omega', \mathcal{S}')$ *for any isomorphism* f *from* AF *to* AF'*, the following property holds:* $\forall a \in \mathcal{A}, Deg^F_{AF}(a) = Deg^F_{AF}(f(a))$*.*

Anonymity is a debatable postulate for dialectical argument strength. It simplifies to much, it does not account for multiple supporters with the same conclusion. Notice that semantics S_2 does not satisfy anonymity.

Claim 37. *Anonymity does not hold for semantics* S_2*.*

Proof. Consider argumentation systems $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$ and $AS' = (\mathcal{L}', \mathcal{R}', n, \leq)$, with $\mathcal{L} = \{a_i\}$, $\mathcal{L}' = \{b_i\}$ and $\mathcal{R} = \{a_1, a_2 \to a_3; a_4 \to a_1; a_5 \to a_2\}$ and $\mathcal{R}' = \{b_1, b_2 \to b_3; b_4 \to b_1; b_5 \to b_1\}$. Let $AT = (AS, \mathcal{K})$ and $AT' = (AS', \mathcal{K}')$ be argumentation theories, where $\mathcal{K} = \mathcal{K}_n \cup \mathcal{K}_p$, $\mathcal{K}_n = \{a_4, a_5\}$, $\mathcal{K}_p = \{a_1, a_2\}$ and $\mathcal{K}' = \mathcal{K}'_n \cup \mathcal{K}'_p$, $\mathcal{K}_n = \{b_4, b_5\}$ and $\mathcal{K}_p = \{b_1, b_2\}$. This leads to the argumentation frameworks $AF = (\mathcal{A}, \mathcal{S})$ and $AF' = (\mathcal{A}', \mathcal{S}')$, with $\mathcal{A} = \{A_1, A_2, A_3\}$, $\mathcal{A}' = \{B_1, B_2, B_3\}$ and $\mathcal{S} = \{(A_1, A_3), (A_2, A_3)\}$ and $\mathcal{S}' = \{(B_1, B_3), (B_2, B_3)\}$, see Figure 18. Take isomorphism $f : \mathcal{A} \to \mathcal{A}'$, such that $f(A_i) = B_i$. However, $Deg^{S_2}_{AF}(A_3) = 1 \neq \frac{1}{2} = Deg^{S_2}_{AF}(B_3)$. So, S_2 does not satisfy anonymity. □

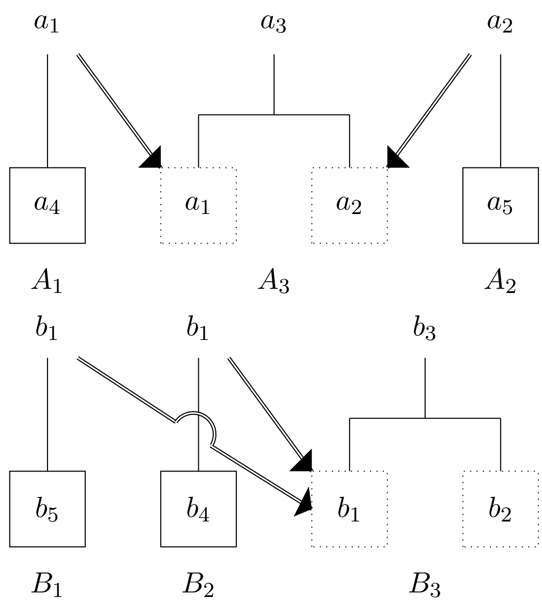

FIGURE 18. ASPIC+ argumentation as counterexample of anonymity.

7.1.2. *Support Independence.*
The intuition of the next postulate (support independence) is that the degree of acceptability of an argument a should be independent of any argument that is not connected to a(Amgoud & Ben-Naim, 2016b).

Definition 50. *For any two argumentation frameworks $AF = (\mathcal{A}, \omega, \mathcal{S})$ and $AF' = (\mathcal{A}', \omega', \mathcal{S}')$, $AF \oplus AF'$ is defined as the argumentation framework $AF'' = (\mathcal{A} \cup \mathcal{A}', \mathcal{S} \cup \mathcal{S}')$, where for $a \in \mathcal{A}$ (respectively $a \in \mathcal{A}'$) $\omega''(a) = \omega(a)$ (respectively $\omega''(a) = \omega'(a)$)*

Postulate 15 (Support Independence)**.** *A semantics F satisfies support independence iff, for any two argumentation frameworks $AF = (\mathcal{A}, \omega, \mathcal{S})$ and $AF' = (\mathcal{A}', \omega', \mathcal{S}')$ such that $\mathcal{A} \cap \mathcal{A}' = \emptyset$, the following property holds: $\forall a \in \mathcal{A}, Deg^F_{AF}(a) = Deg^F_{AF \oplus AF'}(a)$.*

The intuitions of support independence are not debatable, but the definition above is, most importantly because the definition of the $\oplus$-operator seems incomplete. What we mean by that is, when $AF = (\{a\}, \emptyset)$ and $AF' = (\{b\}, \emptyset)$, with Conc$(a) \in$ Prem(b), then $(a, b) \in \mathcal{S}''$ should hold according to independence, but it does not. Notice that semantics S_2 satisfies support independence.

Claim 38. *Support independence holds for semantics S_2.*

Proof. Consider arbitrary argumentation frameworks $AF = (\mathcal{A}, \omega, \mathcal{S})$ and $AF' = (\mathcal{A}', \omega', \mathcal{S}')$, such that $\mathcal{A} \cap \mathcal{A}' = \emptyset$. Then, $\forall a \in \mathcal{A}$,

$$\begin{aligned} Deg^{S_2}_{AF}(a) &= \frac{1}{1 + \sum_{b \in Um(a)} \min_{c \in \mathcal{S}^-(a)}(f_b(c))} \\ &= \frac{1}{1 + \sum_{b \in Um(a)} \min_{c \in \mathcal{S}'^-(a)}(f_b(c))} = Deg^{S_2}_{AF'}(a). \end{aligned}$$

So, S_2 satisfies support independence. □

7.1.3. *Non-dilution.*
The intuition of the next postulate (non-dilution) is that supporting another argument does not affect its own strength (Amgoud & Ben-Naim, 2016b).

Postulate 16 (Non-dilution)**.** *A semantics F satisfies non-dilution iff, for any two argumentation frameworks $AF = (\mathcal{A}, \omega, \mathcal{S})$ and $AF' = (\mathcal{A}, \omega', \mathcal{S}')$, such that $\mathcal{S}' = \mathcal{S} \cup \{(a, b)\}$ and $S^+(b) = \emptyset$, the following property holds: $\forall x \in \mathcal{A} \backslash \{b\}, Deg^F_{AF}(x) = Deg^F_{AF'}(x)$.*

Non-dilution is one of the key intuitions for support relation in dialectical argument strength and therefore a good postulate. Notice that semantics S_2 satisfies non-dilution.

Claim 39. *Non-dilution holds for semantics S_2.*

Proof. Consider two arbitrary argumentation frameworks $AF = (\mathcal{A}, \omega, \mathcal{S})$ and $AF = (\mathcal{A}, \omega', \mathcal{S}'\})$, such that $\mathcal{S}' = \mathcal{S} \cup \{(a, b)\}$ and $S^+(b) = \emptyset$. Then

$$Deg^{S_2}_{AF}(a) = \frac{1}{1 + \sum_{d \in Um(a)} \min_{c \in \mathcal{S}^-(a)}(f_d(c))} = Deg^{S_2}_{AF'}(a).$$

So, S_2 satisfies non-dilution. □

7.1.4. *Dummy.*
The intuition of the next postulate (dummy) is that zero-valued arguments do not affect the acceptability of arguments by supporting another arguments.

Postulate 17 (Dummy)**.** *(Amgoud & Ben-Naim, 2016b) A semantics F satisfies dummy iff, for any argumentation framework* $AF = (\mathcal{A}, \omega, \mathcal{S})$*,* $\forall a, b \in \mathcal{A}$ *if i)* $\omega(a) = \omega(b)$ *and ii)* $S^{-}(a) = \mathcal{S}^{-}(b) \cup \{x\}$*, such that* $Deg^{F}_{AF}(x) = 0$*, then* $Deg^{F}_{AF}(a) = Deg^{F}_{AF}(b)$*.*

Dummy seems a reasonable postulate. For semantics S_2 arguments never have a degree of acceptability of 0. So, this postulate is satisfied trivially.

Claim 40. *Dummy holds for semantics* S_2*.*

Proof. Consider arbitrary argumentation framework $AF = (\mathcal{A}, \omega, \mathcal{S})$. Then for any $a \in \mathcal{A}$, $\exists n \in \mathbb{N}$ such that $Deg^{S_2}_{AF}(a) > \frac{1}{n} > 0$. So, S_2 satisfies dummy. □

7.1.5. *Monotony.*
The intuition of the next postulate (monotony) is that the more supporters an argument has, the higher its degree of acceptability.

Postulate 18 (Monotony)**.** *(Amgoud & Ben-Naim, 2016b) A semantics F satisfies monotony iff, for any argumentation framework* $AF = (\mathcal{A}, \omega, \mathcal{S})$*,* $\forall a, b \in \mathcal{A}$ *if i)* $\omega(a) = \omega(b)$ *and ii)* $S^{-}(a) \subseteq \mathcal{S}^{-}(b)$*, then* $Deg^{F}_{AF}(a) \leq Deg^{F}_{AF}(b)$*.*

Proposition 5. *For any argumentation framework* $AF = (\mathcal{A}, \omega, \mathcal{S})$*, for any* $a \in \mathcal{A}$*,*

$$Deg^{S_2}_{AF}(a) = \frac{1}{1 + |\{b \in Um(a) | \forall x \in \mathcal{S}^{-}(a), \mathrm{Conc}(x) \neq b \vee x \text{ is not firm}\}|}.$$

Proof. Let $AF = (\mathcal{A}, \omega, \mathcal{S})$ be an arbitrary argumentation framework. Then,

$$\begin{aligned}
& \sum_{b \in Um(a)} \min_{c \in \mathcal{S}^{-}(a)} (f_b(c)) \\
= & \sum_{b \in Um(a)} \min_{c \in \mathcal{S}^{-}(a)} (\begin{cases} 0 & \text{, if } \mathrm{Conc}(c) = b \text{ and } c \text{ is firm,} \\ 1 & \text{, otherwise.} \end{cases}) \\
= & \sum_{b \in Um(a)} \begin{cases} 0 & \text{, if } \exists c \in \mathcal{S}^{-}(a), \mathrm{Conc}(c) = b \text{ and } c \text{ is firm,} \\ 1 & \text{, otherwise.} \end{cases} \\
= & \sum_{b \in Um(a)} \begin{cases} 1 & \text{, if } \forall c \in \mathcal{S}^{-}(a), \mathrm{Conc}(c) \neq b \text{ and } c \text{ is not firm,} \\ 0 & \text{, otherwise.} \end{cases} \\
= & |\{b \in Um(a) | \forall c \in \mathcal{S}^{-}(a), \mathrm{Conc}(c) \neq b \text{ and } c \text{ is not firm}\}|.
\end{aligned}$$

So,

$$Deg^{S_2}_{AF}(a) = \frac{1}{1 + |\{b \in Um(a) | \forall x \in \mathcal{S}^{-}(a), \mathrm{Conc}(x) \neq b \vee x \text{ is not firm}\}|}.$$

□

Claim 41. *Monotony holds for semantics* S_2*.*

Proof. Consider arbitrary argumentation framework $AF = (\mathcal{A}, \omega, \mathcal{S})$. Then for arbitrary $a, b \in \mathcal{A}$ if i) $\omega(a) = \omega(b)$ and ii) $S^{-}(a) \subseteq \mathcal{S}^{-}(b)$. Since $S^{-}(a) \subseteq \mathcal{S}^{-}(b)$, $|\{c \in Um(a) | \forall x \in \mathcal{S}^{-}(a), \mathrm{Conc}(x) \neq c \vee x \text{ is not firm}\}| \geq |\{c \in Um(b) | \forall x \in \mathcal{S}^{-}(b), \mathrm{Conc}(x) \neq c \vee x \text{ is not firm}\}|$. So, $Deg^{F}_{AF}(a) \leq Deg^{F}_{AF}(b)$. So, S_2 satisfies monotony. □

7.1.6. *Equivalence.*
The intuition of the next postulate (equivalence) is that the degree of acceptability of an argument depends on the acceptability of its direct supporters and its initial strength (Amgoud & Ben-Naim, 2016b).

Postulate 19 (Equivalence)**.** *(Amgoud & Ben-Naim, 2016b) A semantics F satisfies Equivalence iff, for any argumentation framework* $AF = (\mathcal{A}, \omega, \mathcal{S})$, $\forall a, b \in \mathcal{A}$ *if i)* $\omega(a) = \omega(b)$ *and ii)* $\exists f : S^{-}(a) \to \mathcal{S}^{-}(b)$*, a bijective function, such that* $\forall x \in \mathcal{S}^{-}(a), Deg^{F}_{AF}(x) = Deg^{F}_{AF}(f(x))$*, then* $Deg^{F}_{AF}(a) = Deg^{F}_{AF}(b)$*.*

Equivalence is not a good postulate for dialectical argument strength. It does not include the place of the support. If an argument has one premise that is supported twice by firm arguments, then it should be ranked lower than an argument that has two firm supporters on different premises. Notice that semantics S_2 does not satisfy equivalence.

Claim 42. *Equivalence does not hold for semantics* S_2*.*

Proof. Consider argumentation systems $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$, with $\mathcal{L} = \{a_i, b_i\}$ and $\mathcal{R} = \{a_1, a_2 \to a_3; a_4 \to a_1; a_5 \to a_2; b_1, b_2 \to b_3; b_4 \to b_1; b_5 \to b_1\}$. Let $AT = (AS, \mathcal{K})$ be an argumentation theory, where $\mathcal{K} = \mathcal{K}_n \cup \mathcal{K}_p$, $\mathcal{K}_n = \{a_4, a_5, b_4, b_5\}$ and $\mathcal{K}_p = \{a_1, a_2, b_1, b_2\}$ This leads to the argumentation framework $AF = (\mathcal{A}, \mathcal{S})$, with $\mathcal{A} = \{A_1, A_2, A_3, B_1, B_2, B_3\}$ and $\mathcal{S} = \{(A_1, A_3), (A_2, A_3), (B_1, B_3), (B_2, B_3)\}$, see Figure 18. Then, $Deg^{S_2}_{AF}(A_3) = 1 \neq \frac{1}{2} = Deg^{S_2}_{AF}(B_3)$. So, S_2 does not satisfy equivalence. □

7.1.7. *Coherence.*
The intuition of the next postulate (coherence) is that the impact of support is proportional to the initial strength of its target (Amgoud & Ben-Naim, 2016b).

Postulate 20 (Coherence)**.** *(Amgoud & Ben-Naim, 2016b) A semantics F satisfies Coherence iff, for any argumentation framework* $AF = (\mathcal{A}, \omega, \mathcal{S})$, $\forall a, b \in \mathcal{A}$ *if i)* $\omega(a) > \omega(b)$*, ii)* $Deg^{F}_{AF}(b) < 1$ *and iii)* $S^{-}(a) = \mathcal{S}^{-}(b)$*, then* $Deg^{F}_{AF}(a) > Deg^{F}_{AF}(b)$*.*

Coherence seems a reasonable argument for dialectical argument strength. After all, when an argument a has more weaknesses, i.e., ordinary premises, than argument b and exactly the same supporters, then some of the weaknesses of a are not supported. So, the number of unsupported weaknesses of a is greater than the number of unsupported weaknesses of b. Notice, semantics S_2 satisfies coherence.

Claim 43. *Coherence holds for semantics* S_2*.*

Proof. Consider arbitrary argumentation framework $AF = (\mathcal{A}, \omega, \mathcal{S})$. Then for any $a \in \mathcal{A}$ if $\omega(a) > \omega(b)$, $Deg^{F}_{AF}(b) < 1$ and $S^{-}(a) = \mathcal{S}^{-}(b)$. $\omega(a) > \omega(b)$, so

$\exists n > 0, |Um(a)| + n = |Um(b)|$. Then,

$$\begin{aligned} Deg^{S_2}_{AF}(a) &= \frac{1}{1 + |\{c \in Um(a) | \forall x \in \mathcal{S}^-(a), \mathrm{Conc}(x) \neq c \vee x \text{ is not firm}\}|} \\ &> \frac{1}{1 + |\{c \in Um(a) | \forall x \in \mathcal{S}^-(a), \mathrm{Conc}(x) \neq c \vee x \text{ is not firm}\}| + n} \\ &= \frac{1}{1 + |\{c \in Um(b) | \forall x \in \mathcal{S}^-(b), \mathrm{Conc}(x) \neq c \vee x \text{ is not firm}\}|} \\ &= Deg^{S_2}_{AF}(b) \end{aligned}$$

So, S_2 satisfies coherence. □

7.1.8. *Strengthening.*
The intuition of the next postulate (strengthening) is that supporters strengthen the arguments they support.

Postulate 21 (Strengthening)**.** *(Amgoud & Ben-Naim, 2016b) A semantics F satisfies strengthening iff, for any argumentation framework* $AF = (\mathcal{A}, \omega, \mathcal{S})$*,* $\forall a \in \mathcal{A}$ *if i)* $\omega(a) < 1$ *and ii)* $\exists b \in \mathcal{S}^-(a)$*, such that* $Deg^F_{AF}(b) > 0$*, then* $Deg^F_{AF}(a) > \omega(a)$*.*

Strengthening is not a good postulate for dialectical strength. After all, only support by firm arguments strengthens the acceptability for dialectical argument strength. Notice that semantics S_2 does not satisfy strengthening.

Claim 44. *Strengthening does not hold for semantics* S_2*.*

Proof. Consider argumentation systems $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$, with $\mathcal{L} = \{a_i\}$ and $\mathcal{R} = \{a_1 \to a_2; a_2 \to a_3\}$. Let $AT = (AS, \mathcal{K})$ be an argumentation theory, where $\mathcal{K} = \mathcal{K}_p = \{a_1, a_2\}$ This leads to the argumentation framework $AF = (\mathcal{A}, \mathcal{S})$, with $\mathcal{A} = \{A, B\}$ and $\mathcal{S} = \{(A, B)\}$, see Figure 19. Then, $Deg^{S_2}_{AF}(B) = \frac{1}{2} = \omega(B)$. So, S_2 does not satisfy strengthening. □

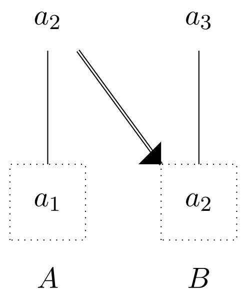

FIGURE 19. ASPIC+ argumentation as counterexample of strengthening.

7.1.9. *Strengthening Soundness.*
The intuition of the next postulate (strengthening soundness) is that arguments only gain strength by supporting the argument with an acceptable one (Amgoud & Ben-Naim, 2016b).

Postulate 22 (Strengthening Soundness)**.** *(Amgoud & Ben-Naim, 2016b) A semantics F satisfies strengthening soundness iff, for any argumentation framework* $AF = (\mathcal{A}, \omega, \mathcal{S})$*,* $\forall a \in \mathcal{A}$ *if* $Deg^F_{AF}(a) > \omega(a)$*, then* $\exists b \in \mathcal{S}^-(a)$*, such that* $Deg^F_{AF}(b) > 0$*.*

Claim 45. *Strengthening soundness holds for semantics* S_2*.*

Proof. We will use a proof by contraposition. Consider arbitrary argumentation framework $AF = (\mathcal{A}, \omega, \mathcal{S})$. Notice, $\forall b \in \mathcal{A}, Deg^F_{AF}(b) > 0$. Furthermore, if $S^-(a) = \emptyset$, then $Deg^F_{AF}(a) = \omega(a)$. So, S_2 satisfies strengthening soundness. □

7.1.10. *Counting.*
The intuition of the next postulate (counting) is that the more supporters an argument has, the stronger the argument. (Amgoud & Ben-Naim, 2016b).

Postulate 23 (Counting)**.** *(Amgoud & Ben-Naim, 2016b) A semantics F satisfies counting iff, for any argumentation framework* $AF = (\mathcal{A}, \omega, \mathcal{S})$*,* $\forall a, b \in \mathcal{A}$ *if i)* $\omega(a) = \omega(b)$ *and ii)* $S^-(a) = \mathcal{S}^-(b) \cup \{y\}$*, with* $Deg^F_{AF}(y) > 0$*, then* $Deg^F_{AF}(a) > Deg^F_{AF}(b)$*.*

Counting is not a good postulate for dialectical strength, because supporters with the same conclusion do not necessarily have an effect on the degree of acceptability. Notice that semantics S_2 does not satisfy counting.

Claim 46. *Counting does not hold for semantics* S_2*.*

Proof. Consider argumentation systems $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$, with $\mathcal{L} = \{a_i\}$ and $\mathcal{R} = \{a_1 \to a_2; a_2, a_3 \to a_4; a_5 \to a_3; a_3, a_6 \to a_7\}$. Let $AT = (AS, \mathcal{K})$ be an argumentation theory, where $\mathcal{K} = \mathcal{K}_n \cup \mathcal{K}_p$, $\mathcal{K}_n = \{a_5\}$ and $\mathcal{K}_p = \{a_1, a_2, a_3, a_6\}$. This leads to the argumentation framework $AF = (\mathcal{A}, \mathcal{S})$, with $\mathcal{A} = \{A, B, C, D\}$ and $\mathcal{S} = \{(A, B), (C, B), (C, D)\}$, see Figure 20. Then, $\omega(B) = \frac{1}{3} = \omega(D)$ and $S^-(B) = \mathcal{S}^-(D) \cup \{A\}$, with $Deg^F_{AF}(A) = \frac{1}{2} > 0$. Furthermore, $Deg^{S_2}_{AF}(B) = \frac{1}{2} = Deg^{S_2}_{AF}(D)$. So, S_2 does not satisfy counting. Notice, this is due to the fact that argument A is not firm. □

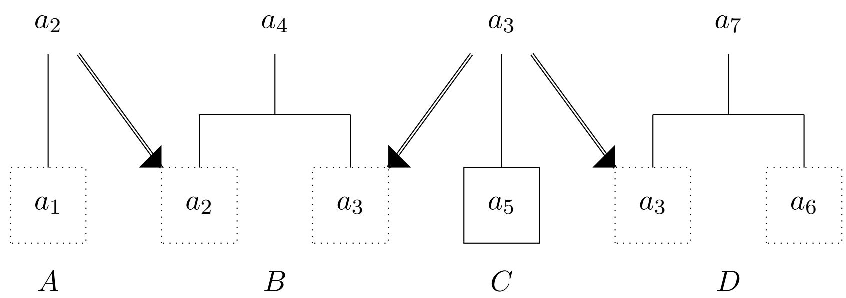

FIGURE 20. ASPIC+ argumentation as counterexample of counting.

7.1.11. *Reinforcement.*
The intuition of the next postulate (reinforcement) is that if argument a has one different supporter from argument b, which is more acceptable, then argument a has a higher degree of acceptability.

Postulate 24 (Reinforcement)**.** *(Amgoud & Ben-Naim, 2016b) A semantics F satisfies reinforcement iff, for any argumentation framework* $AF = (\mathcal{A}, \omega, \mathcal{S})$, $\forall a, b \in \mathcal{A}$ *if i)* $\omega(a) = \omega(b)$, *ii)* $Deg^F_{AF}(b) < 1$, *iii)* $S^-(a) \backslash \mathcal{S}^-(b) = \{x\}$, *iv)* $S^-(b) \backslash \mathcal{S}^-(a) = \{y\}$ *and* $Deg^F_{AF}(x) > Deg^F_{AF}(y) > 0$, *then* $Deg^F_{AF}(a) > Deg^F_{AF}(b)$*.*

Reinforcement does not match the intuition that arguments only contribute as a supporter when they are firm. The impact of arguments with different strength could all be nothing. Therefore semantics S_2 does not satisfy reinforcement.

Claim 47. *Reinforcement does not hold for semantics* S_2*.*

Proof. Consider argumentation systems $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$, with $\mathcal{L} = \{a_i\}$ and $\mathcal{R} = \{a_1, a_2 \to a_3; a_3 \to a_4; a_5 \to a_6; a_6 \to a_7\}$. Let $AT = (AS, \mathcal{K})$ be an argumentation theory, where $\mathcal{K} = \mathcal{K}_p = \{a_1, a_2, a_3, a_5, a_6\}$. This leads to the argumentation framework $AF = (\mathcal{A}, \mathcal{S})$, with $\mathcal{A} = \{A, B, C, D\}$ and $\mathcal{S} = \{(A, B), (C, D)\}$, see Figure 21. Then, $\omega(B) = \frac{1}{2} = \omega(D)$, $Deg^{S_2}_{AF}(B) = \frac{1}{2} < 1$, $S^-(B) \backslash \mathcal{S}^-(D) = \{A\}$, $S^-(D) \backslash \mathcal{S}^-(B) = \{A\}$ and $Deg^{S_2}_{AF}(B) = \frac{1}{2} > \frac{1}{3} = Deg^{S_2}_{AF}(A) > 0$. However, $Deg^{S_2}_{AF}(B) = Deg^{S_2}_{AF}(D)$. So, S_2 does not satisfy reinforcement. □

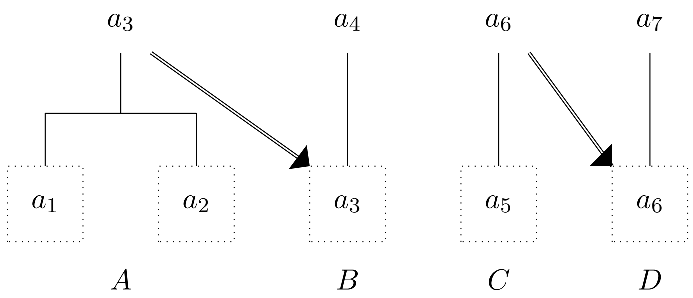

FIGURE 21. ASPIC+ argumentation as counterexample of reinforcement.

7.1.12. *Boundedness.*
The intuition of the next postulate (boundedness) is that an argument which has a maximal degree keeps the same degree if one of its supporters is strengthened (Amgoud & Ben-Naim, 2016b).

Postulate 25 (Boundedness)**.** *(Amgoud & Ben-Naim, 2016b) A semantics F satisfies boundedness iff, for any argumentation framework* $AF = (\mathcal{A}, \omega, \mathcal{S})$, $\forall a, b \in \mathcal{A}$ *if i)* $\omega(a) = \omega(b)$, *ii)* $S^-(a) \backslash \mathcal{S}^-(b) = \{x\}$, *iii)* $S^-(b) \backslash \mathcal{S}^-(a) = \{y\}$, $Deg^F_{AF}(x) > Deg^F_{AF}(y)$ *and* $Deg^F_{AF}(b) = 1$, *then* $Deg^F_{AF}(a) = 1$*.*

Claim 48. *Boundedness holds for semantics* S_2*.*

Proof. Consider arbitrary argumentation framework $AF = (\mathcal{A}, \omega, \mathcal{S})$. Then for any $a, b \in \mathcal{A}$ suppose $\omega(a) = \omega(b)$, $S^-(a)\backslash\mathcal{S}^-(b) = \{x\}$, $S^-(b)\backslash\mathcal{S}^-(a) = \{y\}$, $Deg^{S_2}_{AF}(x) > Deg^{S_2}_{AF}(y)$ and $Deg^{S_2}_{AF}(b) = 1$. Since, $Deg^{S_2}_{AF}(x) > Deg^{S_2}_{AF}(y)$, $Deg^{S_2}_{AF}(y) < 1$. Then for $AF' = (\mathcal{A}\backslash\{x, y\}, \mathcal{S}\backslash\{(u, v)|u\mathcal{S}v \wedge (u = x \vee u = y \vee v = x \vee v = y)\}$, holds that $Deg^{S_2}_{AF}(b) = 1$. Since, $\omega(a) = \omega(b)$, each ordinary premise of b has at least one firm supporter and all supporters of b also support ordinary premises of a it holds that $Deg^{S_2}_{AF}(b) \geq Deg^{S_2}_{AF'}(b) = 1$. So, S_2 satisfies boundedness. □

7.1.13. *Imperfection.*
The next four postulates are optional postulates according to Amgoud & Ben-Naim (2016b), unlike the previous postulates, these were mandatory. The intuition of the first optional postulate (imperfection) is that plausible arguments never have the maximal value.

Postulate 26 (Imperfection)**.** *(Amgoud & Ben-Naim, 2016b) A semantics F satisfies imperfection iff, for any argumentation framework* $AF = (\mathcal{A}, \omega, \mathcal{S})$, $\forall a \in \mathcal{A}$ *if* $\omega(a) < 1$, *then* $Deg^F_{AF}(a) < 1$.

Imperfection is an optional postulate. For dialectical argument strength it is not preferable that imperfection holds, since this postulate ignores the fact that arguments where each ordinary premise is supported by a firm argument are unquestionable. Notice that semantics S_2 does not satisfy imperfection.

Claim 49. *Imperfection does not hold for semantics* S_2.

Proof. Consider argumentation systems $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$, with $\mathcal{L} = \{a_i\}$ and $\mathcal{R} = \{a_1 \to a_2; a_2 \to a_3\}$. Let $AT = (AS, \mathcal{K})$ be an argumentation theory, where $\mathcal{K} = \mathcal{K}_n \cup \mathcal{K}_p$, with $\mathcal{K}_n = \{a_1\}$ and $\mathcal{K}_p = \{a_2\}$. This leads to the argumentation framework $AF = (\mathcal{A}, \mathcal{S})$, with $\mathcal{A} = \{A, B\}$ and $\mathcal{S} = \{(A, B)\}$, see Figure 22. Then, $\omega(B) = \frac{1}{2} < 1$ and $Deg^{S_2}_{AF}(B) = 1$. So, S_2 does not satisfy imperfection. □

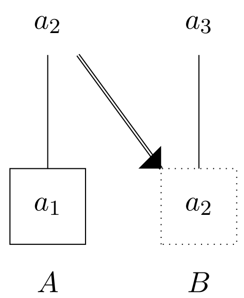

FIGURE 22. ASPIC+ argumentation as counterexample of imperfection.

7.1.14. *Cardinality Precedence.*
The intuition of the next postulate (cardinality precedence) is that an argument a is stronger than an argument b if the acceptable supporters of a are more numerous than those of b (Amgoud & Ben-Naim, 2016b).

Postulate 27 (Cardinality Precedence)**.** *(Amgoud & Ben-Naim, 2016b) A semantics F satisfies cardinality precedence iff, for any argumentation framework $AF = (\mathcal{A}, \omega, \mathcal{S})$, $\forall a, b \in \mathcal{A}$ if i) $\omega(a) = \omega(b)$, ii) $Deg^F_{AF}(b) < 1$, iii) $0 < |\{x \in \mathcal{S}^-(b) | Deg^F_{AF}(x) > 0\}| < |\{y \in \mathcal{S}^-(a) | Deg^F_{AF}(y) > 0\}|$ and iv) $\exists x \in \mathcal{S}^-(b) \forall y \in \mathcal{S}^-(a) Deg^F_{AF}(x) > Deg^F_{AF}(y)$, then $Deg^F_{AF}(a) > Deg^F_{AF}(b)$.*

Cardinality precedence is an optional postulate. For dialectical argument strength it is not preferable that cardinality precedence holds, since this postulate ignores the fact that arguments that are not firm, should not contribute. Furthermore, it ignores the fact that a second supporter for the same premise does not have an effect. Notice that semantics S_2 does not satisfy cardinality precedence.

Claim 50. *Cardinality precedence does not hold for semantics S_2.*

Proof. Consider argumentation systems $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$, with $\mathcal{L} = \{a_i\}$ and $\mathcal{R} = \{a_1, a_2 \to a_3; a_3 \to a_4; a_5, a_6 \to a_3; a_7 \to a_8; a_8 \to a_9\}$. Let $AT = (AS, \mathcal{K})$ be an argumentation theory, where $\mathcal{K} = \mathcal{K}_p = \{a_1, a_2, a_3, a_5, a_6, a_7, a_8\}$. This leads to the argumentation framework $AF = (\mathcal{A}, \mathcal{S})$, with $\mathcal{A} = \{A, B, C, D, E\}$ and $\mathcal{S} = \{(A, B), (C, B), (D, E)\}$, see Figure 23. Then, $\omega(B) = \frac{1}{2} = \omega(E)$, $Deg^{S_2}_{AF}(E) = \frac{1}{2} < 1$, $0 < 1 = |\{x \in \mathcal{S}^-(E) | Deg^F_{AF}(x) > 0\}| < 2 = |\{y \in \mathcal{S}^-(B) | Deg^F_{AF}(y) > 0\}|$ and $\exists x \in \mathcal{S}^-(E)$, namely D, such that $\forall y \in \mathcal{S}^-(B)$, $Deg^{S_2}_{AF}(D) = \frac{1}{2} > Deg^F_{AF}(y)$. However, $Deg^{S_2}_{AF}(B) = \frac{1}{2} = Deg^{S_2}_{AF}(E)$. So, S_2 does not satisfy cardinality precedence. □

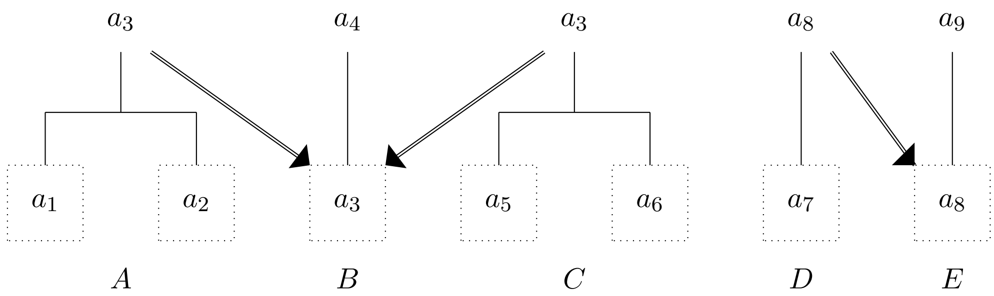

FIGURE 23. ASPIC+ argumentation as counterexample of cardinality precedence.

7.1.15. *Quality Precedence.*
The intuition of the next postulate (quality precedence) is that an argument a is stronger than an argument b, if some supporter of a is stronger than any supporter of b (Amgoud & Ben-Naim, 2016b).

Postulate 28 (Quality Precedence)**.** *(Amgoud & Ben-Naim, 2016b) A semantics F satisfies quality precedence iff, for any argumentation framework $AF = (\mathcal{A}, \omega, \mathcal{S})$, $\forall a, b \in \mathcal{A}$ if i) $\omega(a) = \omega(b)$, ii) $Deg^F_{AF}(b) < 1$, iii) $0 < |\{x \in \mathcal{S}^-(b) | Deg^F_{AF}(x) > 0\}| < |\{y \in \mathcal{S}^-(a) | Deg^F_{AF}(y) > 0\}|$ and iv) $\exists x \in \mathcal{S}^-(b) \forall y \in \mathcal{S}^-(a) Deg^F_{AF}(x) > Deg^F_{AF}(y)$, then $Deg^F_{AF}(a) > Deg^F_{AF}(b)$.*

Quality Precedence is an optional postulate. For dialectical argument strength it is not preferable that quality precedence holds, since this postulate ignores the fact that a second supporter for the same premise does not have an effect. Notice that semantics S_2 does not satisfy quality precedence.

Claim 51. *Quality precedence does not hold for semantics S_2.*

Proof. Consider argumentation framework $AF = (\mathcal{A}, \omega, \mathcal{S})$, with $\mathcal{A} = \{A, B, C, D, E\}$ and $\mathcal{S} = \{(A, B), (C, B), (D, E)\}$ as in the previous proof and in Figure 23. Then, $\omega(B) = \frac{1}{2} = \omega(E)$, $Deg^{S_2}_{AF}(B) = \frac{1}{2} < 1$, $0 < 1 = |\{x \in \mathcal{S}^-(E)| Deg^F_{AF}(x) > 0\}| < 2 = |\{y \in \mathcal{S}^-(B)| Deg^F_{AF}(y) > 0\}|$ and $\exists x \in \mathcal{S}^-(E)$, namely D, such that $\forall y \in \mathcal{S}^-(B) Deg^{S_2}_{AF}(D) = \frac{1}{2} > Deg^F_{AF}(y)$. However, $Deg^{S_2}_{AF}(B) = \frac{1}{2} = Deg^{S_2}_{AF}(E)$. So, S_2 does not satisfy quality precedence. □

7.1.16. *Compensation.*
The intuition of the next postulate (compensation) is that a small number of strong supporters compensates a greater number of weak supporters (Amgoud & Ben-Naim, 2016b).

Postulate 29 (Compensation)**.** *(Amgoud & Ben-Naim, 2016b) A semantics F satisfies Compensation iff, for any argumentation framework $AF = (\mathcal{A}, \omega, \mathcal{S})$, $\forall a, b \in \mathcal{A}$ if i) $\omega(a) = \omega(b)$, ii) $|S^-(a)| < |S^-(b)|$, iii) $\forall x \in \mathcal{S}^-(a) Deg^F_{AF}(x) = d$, iv) $\forall y \in \mathcal{S}^-(b) Deg^F_{AF}(y) = d'$ and iv) $0 < d' < d$, then $Deg^F_{AF}(a) = Deg^F_{AF}(b)$.*

Compensation is an optional postulate. For dialectical argument strength it is not preferable that compensation holds, since this postulate ignores the fact that arguments that are not firm, should not contribute. Furthermore, it ignores the fact that a second supporter for the same premise does not have an effect. Notice that semantics S_2 does not satisfy compensation.

Claim 52. *Compensation does not hold for semantics S_2.*

Proof. Consider argumentation systems $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$, with $\mathcal{L} = \{a_i\}$ and $\mathcal{R} = \{a_1 \to a_2; a_2 \to a_3; a_4 \to a_2; a_5 \to a_6; a_6 \to a_7\}$. Let $AT = (AS, \mathcal{K})$ be an argumentation theory, where $\mathcal{K} = \mathcal{K}_n \cup \mathcal{K}_p$, with $\mathcal{K}_n = \{a_5\}$ and $\mathcal{K}_p = \{a_1, a_2, a_4, a_6\}$. This leads to the argumentation framework $AF = (\mathcal{A}, \mathcal{S})$, with $\mathcal{A} = \{A, B, C, D, E\}$ and $\mathcal{S} = \{(A, B), (C, B), (D, E)\}$, see Figure 24. Then, $\omega(B) = \frac{1}{2} = \omega(E)$, $|S^-(E)| = 1 < 2 = |S^-(B)|$, $\forall x \in \mathcal{S}^-(E) Deg^{S_2}_{AF}(x) = d = 1$, $\forall y \in \mathcal{S}^-(B) Deg^{S_2}_{AF}(y) = d' = \frac{1}{2}$, so $d > d' > 0$. However, $Deg^{S_2}_{AF}(B) = \frac{1}{2} \neq 1 = Deg^{S_2}_{AF}(E)$. So, S_2 does not satisfy compensation. □

7.2. **New Postulates.**
Additional to those postulates that are proposed by (Amgoud & Ben-Naim, 2016b), we will propose three postulates for dialectical argument strength. The first one is strict monotony, with the intuition that if argument a has a smaller initial strength than argument b and b has a strict superset of supporters, then b should be ranked strictly higher than a.

Postulate 30 (Strict Monotony)**.** *A semantics F satisfies strict monotony iff, for any argumentation framework $AF = (\mathcal{A}, \omega, \mathcal{S})$, $\forall a, b \in \mathcal{A}$ if i) $\omega(a) < \omega(b)$ and ii) $S^-(a) \subset \mathcal{S}^-(b)$, then $Deg^F_{AF}(a) < Deg^F_{AF}(b)$.*

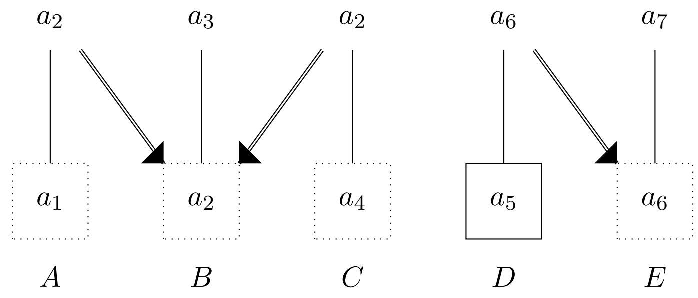

Figure 24. ASPIC+ argumentation as counterexample of compensation.

The second postulate we would like to propose is independence as in Postulate 2, which does avoid the trouble with the $\oplus$-operator. Although this postulate is a postulate for argumentation frameworks with attacks, this postulate also fits argumentation frameworks with only supports[5] and can be defined equivalently.

Postulate 31 (Independence)**.** *(Amgoud & Ben-Naim, 2013) A ranking-based semantics S satisfies independence (Ind) iff for every argumentation framework AF, $\forall B \in Com(AF)$, $\forall a, b \in Arg(B)$, $(a, b) \in S(A)$ iff $(a, b) \in S(B)$.*

The third and last postulate we would like to propose is based on the following intuition. When two arguments are equally weak, but one has more premises that are supported by firm supporters, then that one has a higher degree of acceptability.

Postulate 32 (Firm Counting)**.** *A semantics F satisfies firm counting iff, for any argumentation framework $AF = (\mathcal{A}, \omega, \mathcal{S})$, $\forall a, b \in \mathcal{A}$ if i) $\omega(a) = \omega(b)$ and ii) $|\{\text{Conc}(x) | x \in S^-(a) \text{ and } x \text{ is firm}\}| > |\{\text{Conc}(x) | x \in S^-(b) \text{ and } x \text{ is firm}\}|$, then $Deg^F_{AF}(a) > Deg^F_{AF}(b)$.*

Notice that semantics S_2 satisfies all these three postulates.

Claim 53. *Strict monotony holds for semantics S_2.*

Proof. Consider arbitrary argumentation framework $AF = (\mathcal{A}, \omega, \mathcal{S})$. Then for arbitrary $a, b \in \mathcal{A}$ if i) $\omega(a) < \omega(b)$ and ii) $S^-(a) \subset \mathcal{S}^-(b)$. Since $\omega(a) < \omega(b)$ and $S^-(a) \subset \mathcal{S}^-(b)$, $|\{c \in Um(a) | \forall x \in \mathcal{S}^-(a), \text{Conc}(x) \neq c \vee x \text{ is not firm}\}| > |\{c \in Um(b) | \forall x \in \mathcal{S}^-(b), \text{Conc}(x) \neq c \vee x \text{ is not firm}\}|$. So, $Deg^F_{AF}(a) < Deg^F_{AF}(b)$. So, S_2 satisfies strict monotony. $\square$

Claim 54. *Independence holds for semantics S_2.*

Proof. Let $AF = (\mathcal{A}, \mathcal{S})$ be an arbitrary argumentation framework, let $B \in Com(AF)$ arbitrary. Then for every $a \in B$ holds that $Deg^{S_2}_B(a) = Deg^{S_2}_{AF}(a)$. After all, $|Um(a)|$ only depends on the number of ordinary premises. Furthermore, $\forall x \in S^-(a)$ holds that $x \in B$. So, S_2 satisfies independence. $\square$

Claim 55. *Firm counting holds for semantics S_2.*

[5]this is because the weakly connected component is defined in terms of edges and not specifically in terms of attacks or supports.

Proof. Consider arbitrary argumentation framework $AF = (\mathcal{A}, \omega, \mathcal{S})$, an arbitrary $\forall a, b \in A$. If $\omega(a) = \omega(b)$ and $|\{\text{Conc}(x)|x \in S^-(a)x \text{ is firm}\}| > |\{\text{Conc}(x)|x \in S^-(a)x \text{ is firm}\}|$. Notice $|\{\text{Conc}(x)|x \in S^-(a)x \text{ is firm}\}| \geq |Um(a)| = \frac{1}{\omega(a)} - 1$. So,

$$\begin{aligned} Deg_{AF}^{S_2}(a) &= \frac{1}{\frac{1}{\omega(a)} - |\{\text{Conc}(x)|x \in S^-(a)x \text{ is firm}\}|} \\ &> \frac{1}{\frac{1}{\omega(b)} - |\{\text{Conc}(x)|x \in S^-(b)x \text{ is firm}\}|} = Deg_{AF}^{S_2}(b). \end{aligned}$$

So, S_2 satisfies firm counting. □

7.3. **Existing Semantics.**

In this section Top-based Semantics, Reward-based Semantics and Aggregation-based Semantics will be compared with semantics S_2. For each of the proofs below we will use for any a as initial weight $\omega(a) = \frac{1}{1+|Um(a)|}$.

Table 4 gives an overview of the postulates satisfied by respectively Top-based Semantics, Reward-based Semantics, Aggregation-based Semantics and our proposed semantics S_2.

A cross $\times$ means that the postulate is not satisfied, a checkmark ✓means that the postulate is satisfied, a questionmark means that it is uncertain if this postulate is satisfied. Cells highlighted in grey are the results already proven in (Amgoud & Ben-Naim, 2016a). Cells highlighted in red are proven below, non-highlighted cells are proven in section and .

7.3.1. *Top-based Semantics.*

The first semantics we will evaluate is Top-based Semantics (Amgoud & Ben-Naim, 2016b). This semantics is semantics based on quality precedence. Each argument has an initial strength and the strongest supporter determines the final strength. Top-based Semantics uses a multiple steps scoring function, but is proven to be equivalent to Equation 4, for argumentation framework $AF = (\mathcal{A}, \omega, \mathcal{S})$, for any $a \in \mathcal{A}$.

$$Deg_{AF}^{Tbs}(a) = \omega(a) + (1 - \omega(a)) \cdot \max_{b \in S^-(a)} Deg_{AF}^{Tbs}(b). \tag{4}$$

Claim 56. *Strict monotony does not hold for Top-based Semantics.*

Proof. Consider argumentation system $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$, with $\mathcal{L} = \{a_i\}$ and $\mathcal{R} = \{a_1, a_2, a_3 \to a_4; a_5 \to a_3; a_3, a_6 \to a_7, a_8 \to a_6\}$. Let $AT = (AS, \mathcal{K})$ be an argumentation theory, where $\mathcal{K} = \mathcal{K}_n \cup \mathcal{K}_p$, $\mathcal{K}_n = \{a_5, a_8\}$, $\mathcal{K}_p = \{a_1, a_2, a_3, a_6\}$. This leads to the argumentation framework $AF = (\mathcal{A}, \mathcal{S})$, with $\mathcal{A} = \{A_1, A_2, A_3, A_4\}$ and $\mathcal{S} = \{(A_2, A_1), (A_2, A_3), (A_4, A_3)\}$, see Figure 25. Then, $\omega(A_1) = \frac{1}{4} < \frac{1}{3} = \omega(A_3)$ and $S^-(A_1) \subset S^-(A_3)$. However, $Deg_{AF}^{Tbs}(A_1) = 1 = Deg_{AF}^{Tbs}(A_3)$. So, Top-based Semantics does not satisfy strict monotony. □

Claim 57. *Independence holds for Top-based Semantics.*

Proof. Let $AF = (\mathcal{A}, \omega, \mathcal{S})$ be an arbitrary argumentation framework, let $B \in Com(AF)$ arbitrary. Then for every $a \in B$ holds that $Deg_B^{Tbs}(a) = Deg_{AF}^{Tbs}(a)$. After all, the strength of an argument only depends on the initial strength and on

Properties	Tbs	Rbs	Gbs	S_2
Anonymity	✓	✓	✓	×
Support Independence	✓	✓	✓	✓
Non-Dilution	✓	✓	✓	✓
Monotony	✓	✓	✓	✓
Equivalence	✓	✓	✓	×
Dummy	✓	✓	✓	✓
Minimality	✓	✓	✓	✓
Strengthening	✓	✓	✓	×
Strengthening Soundness	✓	✓	✓	✓
Coherence	✓	✓	✓	✓
Counting	×	✓	✓	×
Boundedness	✓	✓	✓	✓
Reinforcement	×	✓	✓	×
Imperfection	×	✓	✓	×
Cardinality Precedence	×	✓	×	×
Quality Precedence	✓	×	×	×
Compensation	×	×	✓	×
Strict Monotony	×	✓	✓	✓
Independence	✓	✓	✓	✓
Firm Counting	×	×	×	✓

TABLE 4. Postulates satisfied by the support semantics in the literature and the new support semantics.

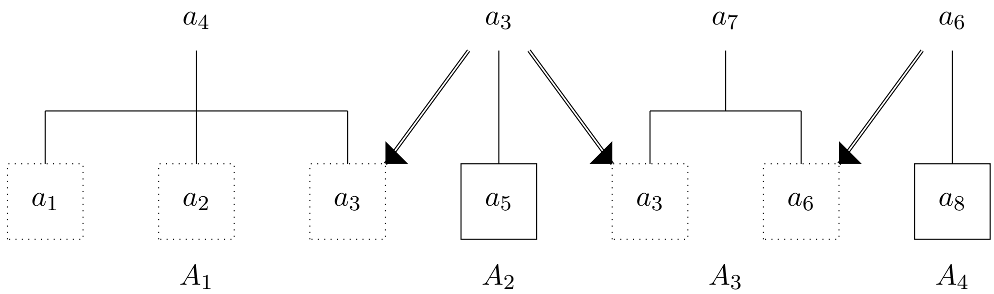

FIGURE 25. ASPIC+ argumentation as counterexample of strict monotony.

the degree of acceptability of its supporters. So, Top-based Semantics satisfies independence. □

Claim 58. *Firm counting does not hold for Top-based Semantics.*

Proof. Consider argumentation system $AS = (\mathcal{L}, \mathcal{R}, n, \leq)$, with $\mathcal{L} = \{a_i\}$ and $\mathcal{R} = \{a_1, a_2 \to a_3; a_4 \to a_1; a_5 \to a_2; b_1, b_2 \to b_3; b_4 \to b_1; b_5 \to b_1\}$. Let $AT = (AS, \mathcal{K})$ an be argumentation theory, where $\mathcal{K} = \mathcal{K}_n \cup \mathcal{K}_p$, $\mathcal{K}_n = \{a_4, a_5, b_4, b_5\}$ and $\mathcal{K}_p = \{a_1, a_2, b_1, b_2\}$, as displayed in Figure 18. Then, $Deg^{Tbs}_{AF}(A_3) = 1 = Deg^{Tbs}_{AF}(B_3)$. So, Top-based Semantics does not satisfy firm counting. □

7.3.2. *Reward-based Semantics.*
The second semantics we will evaluate is Reward-based Semantics (Amgoud & Ben-Naim, 2016b). The intuition is that an argument receives a reward for each of its supporters. Reward-based Semantics uses a multiple steps scoring function, but is proven to be equivalent, for argumentation framework $AF = (\mathcal{A}, \omega, \mathcal{S})$, for any $a \in \mathcal{A}$, to Equation 5, where $n = |\{b \in S^-(a) | Deg^{Rbs}_{AF}(b) \neq 0\}|$ and $m = \frac{\sum_{b \in S^-(a)} Deg^{Rbs}_{AF}(b)}{n}$.

$$Deg^{Rbs}_{AF}(a) = \omega(a) + (1 - \omega(a)) \cdot (\sum_{j=1}^{n-1} \frac{1}{2^j} + \frac{m}{2^n}), \tag{5}$$

Claim 59. *Strict monotony holds for Reward-based Semantics.*

Proof. Let $AF = (\mathcal{A}, \omega, \mathcal{S})$ be an arbitrary argumentation framework and suppose $\omega(a) < \omega(b)$ and $S^-(a) \subset S^-(b)$. Then, $n_a < n_b$ and $\sum_{j=1}^{n_a-1} \frac{1}{2^j} + \frac{m_a}{2^{n_a}} < \sum_{j=1}^{n_b-1} \frac{1}{2^j} + \frac{m_b}{2^{n_b}}$. After all,

$$\begin{aligned} & \sum_{j=1}^{n_a-1} \frac{1}{2^j} + \frac{m_a}{2^{n_a}} - \sum_{j=1}^{n_b-1} \frac{1}{2^j} + \frac{m_b}{2^{n_b}} \\ = \ & \frac{m_a}{2^{n_a}} - \frac{m_b}{2^{n_b}} - \sum_{j=n_a}^{n_b-1} \frac{1}{2^j} \\ \leq \ & \frac{1}{2^{n_a}} - \frac{m_b}{2^{n_b}} - \frac{1}{2^{n_a}} \\ < \ & \frac{1}{2^{n_a}} - \frac{1}{2^{n_a}} = 0. \end{aligned}$$

Call $x := \sum_{j=1}^{n_a-1} \frac{1}{2^j} + \frac{m_a}{2^{n_a}}$ and $y := \sum_{j=1}^{n_b-1} \frac{1}{2^j} + \frac{m_b}{2^{n_b}}$. Notice, x and y are always smaller than 1. After all, $\forall n < \infty$,

$$\sum_{j=1}^{n-1} \frac{1}{2^j} + \frac{m}{2^n} < \sum_{j=1}^{n-1} \frac{1}{2^j} + \frac{1}{2^n} < \sum_{j=1}^{n} \frac{1}{2^j} < 1.$$

Then, $Deg^{Rbs}_{AF}(a) < Deg^{Rbs}_{AF}(b)$. After all,

$$\begin{aligned} & Deg^{Rbs}_{AF}(a) - Deg^{Rbs}_{AF}(b) \\ = \ & \omega(a) + (1 - \omega(a)) \cdot x - \omega(b) - (1 - \omega(b)) \cdot y \\ < \ & \omega(a) - \omega(b) + y \cdot (\omega(b) - \omega(a)) \leq 0. \end{aligned}$$

So, Reward-based Semantics satisfies strict monotony. □

Claim 60. *Independence holds for Reward-based Semantics.*

Proof. Let $AF = (\mathcal{A}, \omega, \mathcal{S})$ be an arbitrary argumentation framework, let $B \in Com(AF)$ arbitrary. Then for every $a \in B$ holds that $Deg^{Rbs}_{B}(a) = Deg^{Rbs}_{AF}(a)$. After all, the strength of an argument only depends on the initial strength and on the degree of acceptability of its supporters. So, Reward-based Semantics satisfies independence. □

Claim 61. *Firm counting does not hold for Reward-based Semantics.*

Proof. Consider the argumentation theory from the proof claim 58, displayed in Figure 18. Then, $Deg^{Rbs}_{AF}(A_3) = \frac{5}{6} = Deg^{Rbs}_{AF}(B_3)$. So, Reward-based Semantics does not satisfy firm counting. □

7.3.3. *Aggregation-based Semantics.*
The third semantics we will evaluate is Aggregation-based Semantics (Amgoud & Ben-Naim, 2016b). The intuition is that a small number of strong supporters can compensate a large number of weaker supporters. Aggregation-based Semantics uses a multiple steps scoring function, but is proven to be equivalent, for argumentation framework $AF = (\mathcal{A}, \omega, \mathcal{S})$, for any $a \in \mathcal{A}$, to Equation 6, where $k = \sum_{b \in S^-(a)} Deg^{Gbs}_{AF}(b)$.

$$Deg^{Gbs}_{AF}(a) = \omega(a) + (1 - \omega(a)) \cdot \frac{k}{1+k}, \tag{6}$$

Claim 62. *Strict monotony holds for Aggregation-based Semantics.*

Proof. Let $AF = (\mathcal{A}, \omega, \mathcal{S})$ be an arbitrary argumentation framework and suppose $\omega(a) < \omega(b)$ and $S^-(a) \subset S^-(b)$. Then, $k_a < k_b$, after all,

$$k_a < k_b \Leftrightarrow k_a + k_a \cdot k_b < k_b + k_a \cdot k_b \Leftrightarrow \frac{k_a}{1+k_a} < \frac{k_b}{1+k_b}.$$

Call $x := \frac{k_a}{1+k_a}$ and $y := \frac{k_b}{1+k_b}$. Notice, x and y are always smaller than 1. Then, $Deg^{Gbs}_{AF}(a) < Deg^{Gbs}_{AF}(b)$. After all,

$$\begin{aligned} & Deg^{Gbs}_{AF}(a) - Deg^{Gbs}_{AF}(b) \\ = \; & \omega(a) + (1 - \omega(a)) \cdot x - \omega(b) - (1 - \omega(b)) \cdot y \\ < \; & \omega(a) - \omega(b) + y \cdot (\omega(b) - \omega(a)) \leq 0. \end{aligned}$$

So, Aggregation-based Semantics satisfies strict monotony. □

Claim 63. *Independence holds for Aggregation-based Semantics.*

Proof. Let $AF = (\mathcal{A}, \omega, \mathcal{S})$ be an arbitrary argumentation framework, let $B \in Com(AF)$ arbitrary. Then for every $a \in B$ holds that $Deg^{Gbs}_{B}(a) = Deg^{Gbs}_{AF}(a)$. After all, the strength of an argument only depends on the initial strength and on the degree of acceptability of its supporters. So, Aggregation-based Semantics satisfies independence. □

Claim 64. *Firm counting does not hold for Aggregation-based Semantics.*

Proof. Consider the argumentation theory from the proof claim 58, displayed in Figure 18. Then, $Deg^{Gbs}_{AF}(A_3) = \frac{7}{9} = Deg^{Gbs}_{AF}(B_3)$. So, Aggregation-based Semantics does not satisfy firm counting. □

Thus we see that Independence is satisfied by all these semantics, that strict monotony is satisfied by Reward-based Semantics and Aggregation-based Semantics and that none of these semantics satisfy Firm Counting. This advocates that these semantics are not good semantics for dialectical argument strength.

8. Conclusion and Discussion

In our work we investigated dialectical argument strength, with the purpose to select and create postulates that should be satisfied by semantics describing dialectical argument strength for structured argumentation in ASPIC+. To that end, we investigated the intuitions behind dialectical argument strength for both attacks and support and proposed two dialectical semantics. We evaluated these semantics using postulates proposed by Amgoud & Ben-Naim (2013) and Amgoud & Ben-Naim (2016a). Furthermore we created some postulates ourselves. We also evaluated other semantics. The main question we answered is: How can a weighing semantics for dialectical argument strength in structured approaches to argumentation be developed and evaluated?

8.1. Answering The Research Questions.

Our work is divided into two parts, one about argumentation frameworks with only attacks and one about argumentation frameworks with only supports. Our work started, in section 4, by proposing the Grounded Dialectical Semantics, as a semantics for dialectical argument strength. This semantics utilizes different properties of ASPIC+ to account for attacking points and survived attacks. The properties that are utilized are about the structure of the argument, to use the place of an attack. Furthermore, we used the fact if an argument is strict or defeasible, and firm or plausible. This answers research question R1. This was followed, in section 5, by checking postulates that have been proposed in the literature. Although our proposed semantics only satisfies the postulate independence (of these postulates from the literature), this was not a problem. This has not weakened the position of the Grounded Dialectical Semantics, since we argued that many of these postulates are not suitable postulates for semantics describing dialectical argument strength, since these postulates do not meet the intuitions for dialectical argument strength. Grounded Dialectical Semantics satisfies three postulates we proposed, which meet the intuitions of dialectical argument strength. Table 3 provided an overview of the postulates satisfied by respectively Max-based Semantics, Categoriser-based Semantics, Discussion-based Semantics, Burden-based Semantics, grounded semantics and our proposed semantics.

To answer research question R2, a semantics for dialectical argument strength should satisfy postulates (In), (APS),(SP) and either (SAP) or (TSAP) and should not satisfy the other postulates discussed in this paper. Thus we have seen that these semantics from the literature are not suitable semantics for dialectical argument strength. After all, these semantics violate dialectical principles from the proposed postulates, but also satisfy postulates that follow intuitions that are in contradiction with intuitions about dialectical argument strength. For example, Max-based, Categoriser-based, Discussion-based and Burden-based Semantics satisfy void precedence, which should not apply for dialectical argument strength or the postulate abstraction, that holds for each of the reviewed semantics from literature, but neglects the structure of arguments.

In the second part of our work, starting in section 6, we proposed a simple semantics for dialectical argument strength in support argumentation frameworks. This simplified to frameworks where arguments only can be supported, where the supports were only premise supports. This utilizes ASPIC+, to see whether a support is a premise support. Furthermore, our semantics utilizes ASPIC+ properties to check whether an argument is firm or plausible. This answers research question R3. Once again our semantics was evaluated by postulates proposed in the literature. In section 7 we have seen that some of these postulates hold and some of them do not hold. This is not a problem at all. Some of the postulates simplified too much, some were poorly formulated and some were not suitable postulates for semantics for dialectical argument strength. We proposed new postulates that meet the intuitions for dialectical argument strength. Table 4 gives an overview of the postulates satisfied by respectively Top-based Semantics, Reward-based Semantics, Aggregation-based Semantics and our proposed semantics S_2. The existing semantics satisfy postulates that should not hold for dialectical argument strength. When the existing semantics do not satisfy a postulate, that do not match the intuitions for dialectical argument strength, for example equivalence and counting, then this is because they simplify. They ignore the structure of arguments, which can be problematic. To answer research question R4, a support semantics for dialectical argument strength should satisfy postulates Non-dilution, Monotony, Dummy, Strengthening Soundness, Coherence, Boundedness, Strict Monotony, Independence and Firm Counting and should not satisfy anonymity, equivalence and counting. Other postulates covered in this paper are not mandatory, but also do not violate the intuitions of dialectical argument strength.

8.2. **Further Research.**

A subject that was not studied in this work is the dynamics of a discussion. What we mean by that, is that adding information to the knowledge base might change the status of arguments. Baumann (2012) presents a pseudo-metric which gives a value to the number of permutations needed to enforce a specific set in the extension. This could be promising for a dialectical semantics and might also expose weaknesses of the Grounded Dialectical Semantics.

Although the semantics S_2 meets the intuitions for dialectical argument strength, it could be improved by taking weaknesses in inference rules into account. One of the challenging factors is how to value weaknesses in inference rules relative to weaknesses in premises.

Another logical following step would be to combine both proposed semantics into a semantics for bipolar argumentation frameworks. In doing so, one should be careful, because we made the assumption that in the weighted support argumentation framework only firm arguments contribute. However, when combined with attacks, it is not so clear that this is a good assumption.

In this work we came up with some postulates for dialectical argument strength, but there was no systematic way of creating postulates. Therefore, there might be other postulates that should hold for dialectical argument strength. For example, another type of independence one could consider, is formula independence, where two arguments should be independent if they have no well-formed formulas in common.

We briefly discussed different types of argument strength, logical, dialectical and rhetorical argument strength. There are no semantics specifically for rhetorical argument strength. It would be interesting to have a set of postulates to evaluate if an semantics is a proper semantics for each type of argument strength.

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