Utrecht University

Master's Thesis Project

The Effect of Structured Interactions on the Development of Fairness in the Ultimatum Game

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November 20, 2020

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Abstract

Software agents are increasingly employed to negotiate on behalf of human users as a solution for human disadvantages. Agents are however less adequate at addressing social dilemmas in which cooperation as best outcome is complicated due to conflicting interests. Human behaviour in such situations is mediated by considerations of fairness. An often used model for social dilemmas that emphasises bilateral negotiation, is the Ultimatum Game. Previous attempts to accurately model human negotiation behaviour in the Ultimatum Game have proposed extensions that are exploitable or not realistically applicable to real negotiation situations. An extension endogenous to such situations is structure from complex spatial networks, which was shown in other social dilemma models to provide fairer solutions. Implementations in the Ultimatum Game are ho(wever limited. In this project, complex network structure is implemented as an extension of the Ultimatum Game. We study how clustering and degree-heterogeneity as network characteristics influence the evolution of fair negotiation strategies and allocation outcomes. To accomplish this, a multi-agent model is designed to simulate the Ultimatum Game played in a population with differing interaction structure. We find that clustering and degree-heterogeneity are favourable to the evolution of fair negotiation strategies. Additionally, for both network characteristics we find an increase in homogeneity for utility per interaction. Our findings show that fair negotiation behaviour emerges despite self-interest in structured interactions. They further suggest that an automated negotiation system embedded in a spatial interaction network is more appropriate for social dilemmas.

Acknowledgements

This project came about in collaboration with the Centrum voor Wiskunde en Informatica (CWI). I would like to thank my first supervisor Tim Baarslag and daily supervisor Daan Bloembergen for their support and guidance and members of the Intelligent and Autonomous Systems department at CWI for their warm welcome and inspiring and motivating atmosphere.

Contents

Chapter 1

Introduction

1.1 Negotiation

With interdependence in outcomes but independence in decisions, a negotiation process is a self-contradicting aspect of social interactions. Nevertheless, the process of joint decision-making appears in all levels of society, from diplomacy between nations and lobbying in politics to the division of household chores. Negotiation is crucial in forming alliances, reaching agreements and resolving conflict, explaining its ubiquity in life (Baarslag, 2016).

Human performance in negotiations varies with the context in which it occurs and the needs and preferences of parties involved. Among human shortcomings in complicated negotiation processes are that humans are influenced by social cues that may be irrelevant to the negotiation, are limited in the amount of information they can account for, are too slow for some implementations of negotiation, and may mediate between their desired outcome and behaviour out of fear for social conflicts resulting from the negotiation process (Bala & Chishti, 2017; Filzmoser, 2010). Such limitations of human performance gave rise to a growing interest in automated negotiation.

In automated negotiation, humans are represented by software agents that negotiate based on a preference model of their user. The use of software agents for negotiations yields lower transaction costs and higher transaction volumes due to higher information processing speed. Software agents are argued to find better negotiation outcomes due to agents being superior in dealing with complex problems. Furthermore, use of software agents can relieve cognitive effort and stress on part of the user (Baarslag, 2016; Baarslag, Kaisers, Gerding, Jonker, & Gratch, 2017; Filzmoser, 2010).

Automated negotiation addresses a need for flexible and efficient trading and distribution solutions. Such need is typically found in application domains that become increasingly decentralised in negotiations and heterogeneous in negotiators. This includes e-commerce which sees increases in independent sellers and buyers (e.g. Chen, Chao, Godwin, & Soo, 2004; Padovan, Sackmann, Eymann,

& Pippow, 2002). Apart from competitive markets, automated negotiation approaches have also been applied to address societal challenges, such as the distribution of renewable energy through smart energy grids (e.g. Chakraborty, Baarslag, & Kaisers, 2018; Camarinha-Matos, 2016; Etukudor et al., 2020) and reducing urban congestion through automating traffic flow (e.g. Gaciarz, Aknine, & Bhouri, 2015; Takahashi, Kanamori, & Ito, 2013).

It is argued that software agents find better negotiation outcomes due to agents being superior in dealing with complex problems. This superiority may however not be found for non-competitive problems such as social dilemmas, in which an outcome is based on the actions of all parties, with self-interested choices being the most attractive. The difficulty here lies in pure self-interest on part of all parties leading to sub-optimal solutions for all involved (De Jong, Uyttendaele, & Tuyls, 2008). The distribution of public resources is a typical real situation that features this social dilemma (Dawes & Messick, 2000).

Preference models for software agents are usually designed according to classical game theory, which posits humans to be individually rational and self-interested. Numerous findings however contest this position by showing that humans do consider others and are naturally inclined to cooperate, and expect others to act similarly (e.g. Bowles, Boyd, Fehr, & Gintis, 1997; Fehr & Schmidt, 1999; De Jong, Tuyls, Verbeeck, & Roos, 2005). In steering away from individually rational and self-interested negotiation behaviour, humans are found to more adequately address social dilemmas (De Jong et al., 2008). With increasing need for automated negotiation not only in competitive environments but also as a solution to societal problems, models are needed that more accurately reflect human considerations and human negotiation behaviour.

1.2 Ultimatum Game

A popular game-theoretic method for studying preference models in negotiation, is the Ultimatum Game. In the basic setup of this game, two individuals are involved in a special form of negotiation where one proposes an allocation of some divisible good. The other player is then left to respond by either accepting the proposed allocation or rejecting it, leaving both players with nothing. The Ultimatum Game showcases how human behaviour deviates from the classical game-theoretical model (Güth, Schmittberger, & Schwarze, 1982). The deviation from the expected behaviour is explained as that human behaviour is mediated by considerations of fairness. From subsequent behavioural studies these considerations are shown to be mostly driven by inequity aversion (e.g. Kahneman, Knetsch, & Thaler, 1986; Bone & Raihani, 2015; Fehr & Schmidt, 1999).

In the evolutionary counterpart of the Ultimatum Game, various computational models have been proposed that describe the emergence of fairness in a population, or elicit fairness through implementations that are external to the agent. The issue with such approaches is that though they contribute to the body of research into human negotiation behaviour in the Ultimatum Game, they may not be applicable to the context of non-competitive automated negotiation. This can be because applied mechanisms are not realistically applicable in a real scenario or because such external mechanisms require agents to abide by such mechanisms. The latter cannot be ensured as real implementations must consider the possibility of agents that are consciously designed to be self-interested and aim exploit a system (de Jong & Tuyls, 2011).

1.3 Structured Interactions

A solution to the exploitability of a system is to ground a system in mechanisms that constitute general human interaction. As complex networks, social networks are characterised by natural clustering as well as degree-heterogeneity. Both these substructures are shown to fortify cooperative strategies in evolutionary games. Regarding clustering, cooperative behaviour is shown to become feasible through recurring interactions resulting from group formation (Axelrod & Hamilton, 1981). Group formation results from the introduction of spatiality in evolutionary games: with interactions occurring locally rather than globally, cooperative strategies become more resilient to invasion by forming clusters (Ellison, 1993a; Nowak & May, 1992). With regards to hub forming, heterogeneity in neighbourhood size is shown to contribute to the spread and resilience of strategies (Lieberman, Hauert, & Nowak, 2005). Outside of the Ultimatum Game, heterogeneity favours cooperative strategies due to the presence of highly connected and thus influential agents that are resilient against strategic invasions and are more selective to beneficial strategies (F. C. Santos, Pacheco, & Lenaerts, 2006).

For the Ultimatum Game, the introduction of spatiality is found to favour fair strategies (Page, Nowak, & Sigmund, 2000; Iranzo, Román, & Sánchez, 2011). Regarding complex, social networks however, only a limited amount of literature exists that has studied the Ultimatum Game in such settings. Those that have do point towards benefit for the implementation of social network structure to the Ultimatum Game, but extend the model further to the context of e.g. the Multiplayer Ultimatum Game (for clustering and degree-heterogeneity, respectively: F. P. Santos, Santos, & Pacheco, 2018; Bo & Yang, 2010).

1.4 Problem Definition

An increasing need in flexible and efficient trading and distribution solutions have caused automated negotiation to rise in popularity. Current models are generally based on classical game-theoretical models of human negotiation behaviour, which are shown not to correspond with behavioural experimental results. With automated negotiation also providing a solution to non-competitive societal challenges, cooperative models must be developed to adequately approach social dilemmas.

Current models predominantly see theoretical benefit in explaining how fairness considerations are formed or incorporate mechanisms that are either not fit for our scenario, or require the deliberate adherence of agents present in the system. Therefore the study and development of mechanisms endogenous to society and human interaction are needed.

Such a mechanism is the presence of structure in social networks. Social structure in populations is shown to benefit cooperative strategies in various evolutionary models of social dilemmas. Though many approaches to spatiality and graphs in the Evolutionary Ultimatum Game exist, research on social structure however is scarce. This has led us to define the following research questions and hypotheses:

Research Question 1 *How does clustering in the topological structure of a population influence the evolution of fair negotiation behaviour in the Evolutionary Ultimatum Game?*

In literature we have found that having repeated interactions with the same individuals is beneficial for fair strategies. This benefit is further extended to indirect interactions, providing resilience to clusters against invasive strategies. Therefore we define our first hypothesis as follows:

Hypothesis 1: For population structures with high Clustering Coefficient, we expect to find population convergence to fairer negotiation strategies in comparison with population structures that demonstrate a lower degree of clustering.

Research Question 2 *How does degree-heterogeneity in a population affect the evolution of fair negotiation behaviour in the Evolutionary Ultimatum Game?*

In literature we found that populations with high heterogeneity in the amount of neighbours are driven to higher offers and acceptance thresholds due to the presence of nodes with disproportionally high node degree. This effect is attributed to these hub nodes being more resilient against strategic invasions and being more selective in adopting strategies. Therefore we define our second hypothesis as follows:

Hypothesis 2: For population structures heterogeneous in degree distribution we expect to find population convergence to fairer negotiation strategies in comparison with degree-homogeneous population structures.

Goal of the research

The goal of this project is to study the influence of structured interactions on the evolution of negotiation behaviour in the Evolutionary Ultimatum Game.

In this, we hope to find that population structure benefits the emergence of fair strategies. For this, we develop a model that emulates the Ultimatum Game dilemma played in a population that is structured according to real social networks.

Chapter 2

Literature Review

This chapter discusses relevant literature on fairness in the Ultimatum Game. The aim of this section is to familiarise the reader with past research on the Ultimatum Game, evolutionary games and games played in graph structures. It commences with an overview of findings of fairness in human Ultimatum Game experiments. Next, we discuss the Ultimatum Game in the context of evolutionary games along with important findings from other relevant evolutionary games. Lastly, we discuss games on graphs and the Ultimatum Game in graph structures.

2.1 The Ultimatum Game

The original Ultimatum Game was first analysed by Güth et al. (1982) to study in detail aspects of negotiation behaviour. In this experiment, two players are involved with each other in a bilateral negotiation scenario in which one is assigned the role of proposer and the other that of responder. The proposer is tasked with allocating a common good such that their offer $p \in [0,1]$ resembles the amount of the common good they are willing to share, keeping the remainder $1-p$ for themselves. The responder can only state whether they accept, receiving the offered amount p, or reject the allocation, leaving both players with an income of 0. The tendency for the responder to accept is typically explained by a minimum threshold from which they are or should be willing to accept, $q \in [0,1]$. Below q, an offer is deemed undesirable for a responder.

The main aspect of the Ultimatum Game receiving continued interest, is the discrepancy between expected and observed negotiation behaviour. The expected behaviour for proposers and responders is for the former to propose the lowest divisible amount ϵ and for the latter to have threshold q set to or below this value. This behaviour is deemed individually rational for each player in this one-shot scenario: the responder prefers receiving ϵ over 0. The proposer in turn prefers receiving as much as possible by offering ϵ, assuming that the responder is likely to accept. This state with the lowest possible settings for p

and q is regarded as the Nash equilibrium for the original Ultimatum Game. Rather than offering as low as possible, Güth et al. found that the modal offer p was near a 50% share of the money to be allocated and the average p was 0.35. Of all offers that were as low as possible, most were rejected by the responder. A second run with the same participants showed a decrease in 50% splits. However, the average p (0.31) was still far from ϵ. The explanation that Güth et al. offer for the observed deviation from the expected rational behaviour is that for the responder, p should be an appropriate amount deemed fair or justified. If the offer is deemed lower than a hypothetical amount q they would be willing to accept, the responder is willing to pay this price as to punish the proposer. For the proposer, offering an amount higher than ϵ is motivated by either considerations of fairness or by the threat of the responder possibly rejecting $p < q$.

The analysis from Güth et al. (1982) contradicts the modelling of human negotiation behaviour according to the *homo economicus*, the profile of a rational agent that acts optimally out of self-interest. In the study, human behaviour is closer to the *homo reciprocans*-model, as cooperators that retaliate against non-cooperative, opportunistic agents. Subsequent studies have attempted to further explain the incongruence between individually rational negotiation behaviour and observed behaviour with the pervading question herein being how fairness considerations mediate in negotiations. In the remainder of this section we will discuss fairness definitions in the setting of bilateral negotiations and distribution and elaborate on further Ultimatum Game extensions for their findings and explanations for the incongruence between expected and observed behaviour.

2.1.1 Fairness in distribution mechanisms

Research investigating the processes behind fairness judgments in distribution mechanisms generally focuses on two aspects. *Procedural justice* is the extent to which the procedure of allocation in some form of organisation is deemed a fair process. *Distributive justice* refers to how individuals relate to each other in their outcome of the allocation process (Yaari & Bar-Hillel, 1984). When the outcome of an allocation process is known, individuals will base their judgments of fairness on how their revenue relates to comparable others. If this is not known however, due to only knowing your own revenue and having a small chance for repeated interactions, individuals will emphasise the procedures leading to the allocation in their judgments. This stems from the assumption that if the procedure is fair, the final outcome must be as well (van den Bos, Lind, & Wilke, 2001).

Fair distribution may be regarded as allocating utilities to all individuals based on what configuration maximises the sum or mean of each their utility. This approach leads to an equal distribution of utilities with the assumptions that individuals' utility functions are similar. On the other hand, one can assign unequal proportions of goods to individuals based on their needs such that individuals that have a higher need or a lower valuation of parts of the goods, receive more such that their total outcome utility is equal (van den Bos et al., 2001; de Jong & Tuyls, 2011). These two approaches show a challenge for fair allocations: whereas assigning equal amounts of goods to each individual regardless of their own revenue may be deemed fair for one, assigning unequal quantities of goods to individuals as to minimise inequity in a group may be deemed fairer for the other.

2.1.2 Fairness in the Ultimatum Game

The original Ultimatum Game has seen several extensions in an attempt to further describe fairness considerations and investigate the incongruence between the expected rational behaviour and behaviour as observed. Following is an overview of important findings on fairness and fair behaviour that extensions have offered.

Sequential Bargaining with shrinking common good The sequential Ultimatum Game is an extension in which the influence of threats and punishment is diminished until the final round such that rational behaviour is more appealing for both proposer and responder (Ochs & Roth, 1989). In one game, rejection of an offer leads to a reversal of roles and game progression to the next round until the final round is reached. Rejection in the final round renders payoff for both players 0. With each progression to a next round, the common good to distribute is discounted by a discount rate δ per each new period.

Ochs and Roth found that responders' behaviour still reflected a preference for near-equal splits. Though responders were aware of the total divisible amount being discounted for each progression, a substantial amount of rejections for unequal first-round division was observed. Along with these rejections, near-equal counter-proposals were made in the following round. This behaviour held even for when gained utility from near-equal counter-proposals was known to be less than the original offer. Furthermore, first-round proposers with low offers on average showed the least earnings over the span of multiple games. The tendency for participants to prefer fair distributions over utility strengthens the idea of humans acting not out of a purely monetary interest. Rather, humans are inclined to divide fairly and react to unfair allocations.

Multiplayer Negotiation With the inclusion of multiple players per role, within-role competitiveness mediates in players' negotiation behaviour. In playing the Ultimatum Game with multiple proposers and a single responder, the two proposers both make an offer p_1, p_2 to which the responder will react. This leads to a competition among proposers to offer the highest share. This is because the responder rationally picks $\max(p_1, p_2)$. With multiple negotiators and a single proposer, this dynamic is turned around as responders now compete with their thresholds since when $(q_1, q_2) \leq p$, $\min(q_1, q_2)$ will receive p. In both scenarios, utility was not shared further among group members (Guth, Huck, & Ockenfels, 1996; Güth & Kocher, 2014).

Incomplete or Imperfect Information Multiplayer negotiation has seen further extensions which include alternative player types by ranging the amount of information responders have of the common good or the allocations made. In the Ultimatum Game with reduced information, information was either incomplete in the amount of a good to be allocated (Guth et al., 1996) or imperfect in how it was divided among three players (Güth & Van Damme, 1998).

In the incomplete scenario, the Ultimatum Game was played with three players X, Y and Z such that player X proposes an amount $p_x \in [0, 1]$ to player Y, after which Y responds and subsequently proposes a non-zero division $p_y \in [0, (p_x - 2\epsilon)]$ of the received amount for themselves and player Z. Player Z acts as a regular responder. The total amount to be allocated was varied between a high and low setting. All players were aware of the two possible settings, however only player X had knowledge of the actual amount to be allocated in the concerning round. In the low amount setting, Y is uncertain of whether the allocation behaviour of X truly represents fair behaviour or whether X conceals opportunistic behaviour, consequently influencing the behaviour of X.

In the imperfect information scenario, player X divides a common good among all three players including themselves, being the only proposer. Player Y receives information on how the good is allocated and is able to accept or reject: player Z has no direct influence at all and only receives the amount allocated, should Y accept. The amount of information Y has, is varied: information is either complete (Y is aware of allocations for all players), incomplete but relevant (Y is aware only of the amount they will receive themselves) or incomplete and irrelevant (Y is unaware of their own share; only aware of that of Z). In this scenario, the behaviour of X is expected to be influenced by the inferences Y can draw on the allocation based on the amount of information received.

Both studies concluded that when some possibility of pretending to be fair arises, proposers become more opportunistic in their demands while fearing exposure of their unfair allocations. Though the proposers may not be intrinsically interested in fair allocations, these observations do show that proposer behaviour is mediated by fairness considerations. Guth et al. (1996) however did find that there was a strong positive relationship between higher x and y propositions, suggesting a kind of non-bilateral reciprocity.

Punishment and Justice Rejection behaviour in the Ultimatum Game may in part be motivated by a form of punishment on the account of the responder. Such punishment is motivated by the responder deeming an allocation unfair or driving opportunistic players to change their future behaviour. In the study of Kahneman et al. (1986) most subjects preferred to split equally over proposing a $90 - 10\%$ division (proposer-responder). When in a later stage responders were asked to split equally an amount C with an opportunistic player or $C - \epsilon$ with a fair player, nearly all chose the latter. Cooperative players were thus favoured over non-cooperative ones, though the resulting utility would be less.

In the further debate of whether punishment occurs due to negativity stemming from breaking cooperative norms or to simply reduce inequality in total payoff, Raihani and McAuliffe (2012) found that humans are more receptive to inequity than to experiencing losses. Cheating was punished less when it was justified by the proposer having less total utility than the responder. Bone and Raihani (2015) further found that explicit punishment was motivated by revenge as well as inequity aversion. When individuals could however choose the severity of punishment as a discount on the other player's utility, the severity was motivated predominantly by disadvantageous inequity for the punisher. Punishment even occurred in the setting where this would lead to a lower total payoff than when the player would have continued playing without ineffective punishment. Given possible explanations for ineffective punishment are that though such punishment is ineffective for the player, it does reduce the standard deviation of total utilities for the whole group and may change the future behaviour of the other player.

From the three studies, rejection behaviour and punishment are shown to result in part from unfair allocations and a disliking for inequity. The negative affect to inequity and willingness to sacrifice utility provide support in the direction of humans considering the fairness of distributions, further deviating from the *homo economicus*-model. Furthermore, the act of punishment is shown to have a collective benefit in that though the punisher may not see individual benefit, punishment contributes in the reduction of inequity within the whole group.

Summary In this selection of the large amount of extensions that exist for the Ultimatum Game, we can draw two main conclusions regarding fairness. First, we find that negotiation behaviour is influenced by fairness considerations and that such considerations rely on inequity aversion between partners as well as overall. Second, under extensions of the original Ultimatum Game, fair negotiation can become a more profitable strategy regardless of whether an individual is concerned with fairness in allocations.

Regarding the influence of fairness considerations, monetary payoffs are shown to be insufficient in explaining behaviour and do not capture subjects' utility function (Güth et al., 1982; Ochs & Roth, 1989). The behaviour of both proposers and responders is influenced by considerations of fairness, though this does not ensure cooperative behaviour or fair play. Cooperative behaviour does however elicit reciprocity, not only directly but also indirectly (Kahneman et al., 1986; Guth et al., 1996). Lastly, fairness judgments in the Ultimatum Game were based on how equal utility divisions were between players and how equal the final utility distribution was in the population (Raihani & McAuliffe, 2012; Bone & Raihani, 2015).

Extending the Ultimatum game provides mechanisms that promote or demote fairness. With less information on either allocations or the total amount to allocate, proposals become greedier. Also with more recurring interaction, opportunistic play becomes less profitable. Furthermore, increasing the number of players per role can strengthen their positions or elicit competition in making or accepting offers. Lastly, providing punishment mechanisms or options to choose partners may further promote cooperative behaviour through the possibility of reducing inequity between players and within the whole group, and enforcing cooperation on behalf of opportunistic proposers.

2.2 Evolutionary Ultimatum Game

Behavioural experiments on the Ultimatum Game have presented findings on the degree to which fair allocations are preferred, contesting the idea that human negotiation behaviour is led by a self-interested notion of rationality. Experimental approaches to human behaviour in the Ultimatum Game suggest that some of the actions observed serve little benefit individually, but do collectively. Some extensions were further found to promote fair negotiation behaviour by making fair offering more appealing or allowing for corrective measures on part of responders.

From behavioural experiments we learn what is seen as fair as well as how external mechanisms elicit fairness in negotiations. With computational models however we can simulate the Ultimatum Game and study why and when fairness emerges, by modelling internal constructs of environments or human properties. We can thus find settings under which fair negotiation behaviour is less a deliberate choice and more the most optimal due to environmental circumstances.

The Evolutionary Ultimatum Game is a computational framework of the Ultimatum Game that draws from evolutionary game theory and population dynamics. The evolutionary counterpart results from situating agents in a population in which interactions between agents consist of playing the Ultimatum Game. The game is played for a set amount of rounds, with mechanisms in place for agents to change their strategy based on the expected utility. With agents adapting strategies to what better suits against their opponents, over time a population converges to a single or set of strategies which best suit that composition of settings and initial strategies.

In this section we will expand on population dynamic approaches to evolutionary games, among which the Evolutionary Ultimatum Game. In evolutionary games, a game denotes the payoff model and possible actions for interactions between agents. Apart from the game, dynamics are influenced by the further design of interactions, the composition of a population, how the progression of the game is further defined and other exogenous constraints put on the dynamics in a population. Such components that are exterior to the basic game itself determine how a population evolves and to what extent population dynamics are guided by the underlying game. For the reason that evolutionary implementations of serious games overlap in their environment as a population that evolves, we also include findings from other evolutionary games that prove helpful for our implementation.

2.2.1 Emergence of Fairness

In discussing how cooperative behaviour can evolve in the Evolutionary Prisoner's Dilemma, Axelrod and Hamilton (1981) explain that fairness in general may emerge through three mechanisms: kin selection, group selection and reciprocity. Important for any of these strategies to develop is that the environment permits cooperative behaviour to exist and improve inclusive fitness: the act of cooperation must be beneficial among other members of the population that cooperate and overrule the benefit of opportunistic play. As strategies in the environment progress and the cooperative sub-population grows, recurring interaction among similar individuals becomes the strongest force behind the further evolution of cooperation. These mechanisms can however only exist when a population is small enough such that the probability of recurring interaction is sufficiently large and agents have a way of pushing consequences to opportunistic agents, e.g. when agents can punish opportunistic players, when agents have memory of other agents' past moves or when interaction is restricted to each agent's direct surroundings.

Work from Traulsen, Hauert, De Silva, Nowak, and Sigmund (2009) in the Public Goods game suggested that mutation rate (μ) configurations might influence the interaction between agents: when mutation rate $\mu \to 0$, interaction becomes less important and dominance of a strategy relies more on its relative abundance in the population. With larger μ more strategies are found in the population at a given time because agents mutate and thus explore more with a benefit for cooperation. Furthermore, Traulsen et al. state that a higher rate of exploration deemed more fitting for the simulation of cultural evolution as humans are more prone to exploring their strategic options and being inconsistent in their behaviour.

Rand, Tarnita, Ohtsuki, and Nowak (2013) confirmed and further studied the influence of mutation rate on the Evolutionary Ultimatum Game. In the regular one-shot Evolutionary Ultimatum Game, natural selection is found to support convergence to the rational solution of the classical Ultimatum Game setting. When the natural selection rate is weakened however, average q and in turn p increase. With weakened selection and larger mutation rate μ, average p increases but q seems relatively unaffected, as the best strategy is the one that maximises its expected absolute payoff in a heterogeneous population. With smaller μ, it is however the expected relative payoff that must be maximised as strategic sub-populations must be able to resist the intrusion of a newly introduced strategy. The best strategy to then arise is one with higher p and q that are close together.

Page and Nowak (2001) found that restricting agents' acceptance thresholds q to be similar to their offers p, steers a population to the level of equal splits whereas otherwise would converge to the classical, rational solution. Agents that offer and accept low, may receive payoff from cases in which they act as responders but will be rejected as proposers. It will thus be more profitable to raise offers and acceptance thresholds such that the agent will receive payoff both as proposer and responder. The empathic strategy ($p = q$) resembles the strategy a population converges to under weakened selection and small mutation rate from Rand et al. (2013). Page and Nowak (2002) found that the effects of this form of empathy held even when only a small proportion of agents was constrained to empathic play. The issue however is that the independent evolution of p and q is favoured: when the amount of empathy-constrained agents is allowed to evolve with the population due to higher settings for μ, the population again converges to the Nash equilibrium strategy from the original Ultimatum Game.

In above mechanisms the evolution of fairness is stimulated by certain configurations of the Ultimatum Game. They are based on the case of the Ultimatum Game where the game is played among agents in the population paired at random and the expected absolute payoff of a strategy matters more due to a global nature of interaction. Models with global interaction are fitting for theoretical analysis. It is however hard to generalise these findings to settings of social behaviour and interaction. Social creatures typically form structured social networks where interaction is more frequent among direct neighbours in the network (Nowak, Tarnita, & Antal, 2010b). Culturally shared behaviours arise through a high level of information exchange, interaction, common experiences and common beliefs among a cluster of individuals. But also infrastructure and city planning have a sense of spatiality in the Euclidean distance between objects and the frequency of travel on shorter or faster roads between them. For this reason, research has also focused on the influence of spatial interactions and population structure on the evolution of cooperation and fairness.

2.2.2 Structured Populations

The presence of structure in populations that are involved in evolutionary games is found to influence the evolution of population behaviour. Nowak and May (1992) adapted the work of Axelrod and Hamilton (1981) to introduce spatiality in a simplified version of the Prisoner's Dilemma with only the two pure strategies. Having interaction occur only between neighbouring agents can by itself yield complex and chaotic patterns of interplay between agents. More importantly, in the spatial game chaotically shifting balances were found in which agents were quick to change strategies without however affecting the proportion of cooperators in the population, whereas without spatiality the environment converges to defection. In discussing the success of Tit-for-Tat in their original Prisoner's Dilemma, Axelrod and Hamilton (1981) state that the success of Tit-for-Tat stems from its formation of clusters in which interaction with similar agents occurs, which evolutionarily stabilises the group. Ellison (1993b) further found for the Coordination Game that a crucial determinant for the speed and configuration of population convergence was whether interactions were of a global or local nature, as with spatial interactions. Ellison also concluded that fair strategies benefit from local interaction: interaction among neighbours that use cooperative strategies evolutionarily stabilises said cooperative behaviour.

Page et al. (2000) applied evolutionary game theory to the Ultimatum Game to examine the effects of spatiality on fairness. In each round, all agents interact with each other after which agents leave offspring in numbers proportional to their fitness. Offspring agents take their parents' mutated strategy, with mutation implemented as noise sampled uniformly at random from an interval α applied to the p and q. In the absence of mutation in the non-spatial case, natural selection favoured the classical rational solution. The average p however climbed toward equal splits when rate of mutation increased. This effect was similar but weaker for q such that $p > q$. Spatiality was introduced by arranging agents on a one-dimensional ring with two, and on a two-dimensional lattice (fixed boundaries) with four neighbours. Agents now play only against their neighbours and again leave offspring, however proportional to their neighbourhood instead of the whole population. In the one-dimensional version, selection initially globally favoured domination of neighbourhoods with $p \approx q$ when average p and q were spread far apart. After development of average $p \approx q$, the effect from clustering arises with the whole population slowly converging to the fair strategy of equal splits. In the two-dimensional case with four neighbours, total grid increase (corresponding to the size of the population) caused convergence to fairer average values for p and q. For every grid size, fairness could only rise due to the formation of 3×3 clusters of fair strategies.

In the one-dimensional case, the development of $p \approx q$ preceded the convergence to fair average p and q. The importance of empathic strategies to the evolution of fairness was subsequently clarified by Page and Nowak (2001), discussed earlier. Iranzo et al. (2011) revisited the issue of spatiality and empathy in expanding both these works. With a population on a 2D lattice with four neighbours, two settings were defined: the empathic setting in which $p = q$, and the independent setting where p and q evolve independently. Each round the Ultimatum Game is played twice between agents such that each agent once focally plays the Ultimatum Game against all its neighbours. Role assignment was either non-random such that each agent plays as proposer and responder against each neighbour or random with the possibility of playing as either role twice. Furthermore, strategy updating was either through imitation of the best in the neighbourhood or imitating a random neighbour with higher payoff with probability relative to their payoff difference. Mutation was implemented similarly with noise centered around parents' parameters.

Findings for the empathic setting indicated a general convergence to $(p, q) = \frac{1}{2}$. These final values decreased when adapting the neighbourhood size from four to eight. In the independent setting with some stochasticity due to proportional imitation or random role assignment, the population mostly converged to quasiempathic strategies ($p, q \gg 0$ and $p \approx q$). This was mostly observed in large (40×40) networks. Interesting was that for both settings, the absence of stochasticity enabled multiple strategies to coexist in the final population. Authors explain that this is possible due to stable spatial configurations and can thus not be observed on random networks. Furthermore, whereas mutation caused values p and q to climb in the non-spatial setting (as seen in Page et al., 2000), the introduction of mutational noise in otherwise deterministic settings caused the population to converge to the classical Nash equilibrium strategy due to emphasis on individual play rather than playing as clusters because neighbours are too dissimilar to oneself. This effect is averted by other introductions of stochasticity. Lastly, larger populations seem to elicit a fairer population: average p and q were generally higher for larger grid sizes (Page et al., 2000; Iranzo et al., 2011).

2.2.3 The Ultimatum Game on graphs

The previous studies on spatiality addressed population structure as neighbourhood interaction on a 1D ring and a 2D lattice in which all agents have the same number of neighbours, k (apart from the edges of the grid). It is however unlikely to find a population in which all members have the same amount of connections or influence, whether it be e.g. individual agents in bee colonies, packs of wolves, companies or in social groups.

Lieberman et al. (2005) addressed the primal effects of differences in the connectivity of populations on the spread of strategies outside of the Ultimatum Game. The topology of a population here is defined by a graph or network in which agents are represented by vertices, with edges between the vertices denoting whether agents are connected and thus can interact. The application of graphs on evolutionary processes stems from the important finding that when all nodes are equal in their connections or accompanied weights, Moran-processes 2.1 are also indicative of dynamics on graphs.

Small spatial populations are more subject to the introduction of new strategies due to random processes such as mutation. For large populations, such influences make less of a difference whereas natural selection becomes more dominant. This is illustrated with the fixation probability ρ of a newly introduced mutant, defined by Lieberman et al. as:

$$\rho = \frac{1 - 1/r}{1 - 1/r^N} \ , \tag{2.1}$$

where N denotes the size of a population and r denotes the relative fitness of the mutant compared to the population. The fitness of the population here is taken as 1. The probability that a mutant takes over a whole population is inversely proportional to population size. With heterogeneity in connectivity for network nodes, network substructures can emerge that reduce or amplify the spread of a single mutant strategy that is more advantageous than its neighbours. This is due to the difference in connectivity of vertices, meaning the amount of edges vertices have and thus the amount of vertices that they can influence and get influenced by. A simple example is that of a star configuration as can be seen in figure 2.1. The probability for a mutant strategy to take over a whole structure now becomes

$$\rho = \frac{1 - 1/r^2}{1/r^{2N}} \ , \tag{2.2}$$

amplifying selective difference r in equation 2.1 to r^2. Advantageous mutant strategies are thus amplified whereas for disadvantageous mutant strategies, the fixation probability is reduced. The work of Lieberman et al. (2005) is

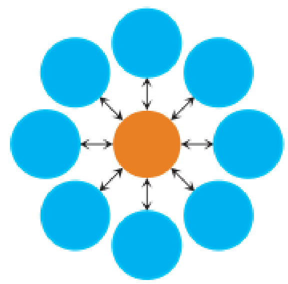

Figure 2.1: A network in which vertices are configured in a star-like formation. Central vertex has 8 neighbours and thus is heavily subject to change as opposed to outer vertices with each having only the central vertex as neighbour. An advantageous strategy adopted by an outer agent is imitated by the central agent, which quickly spreads to other agents given that their strategies are disadvantageous to the central agent's strategy.
Source: Lieberman et al. (2005)

an approximation to evolutionary dynamics for population structure, based on evolutionary graph theory.

In further studying scale-free structures, F. C. Santos et al. (2006) found degree heterogeneity to be beneficial to the emergence of cooperative behaviour. This effect is attributed to the presence of hub nodes that have a disproportionately high amount of connections relative to the rest of the population. Hub nodes have a higher number of interactions from which revenue can be gained, as well as a greater sample of strategies to imitate from. With a higher number of interactions and thus greater probability of receiving non-zero utility, a hub node is less likely to perform worse than a regular node. If the hub node does perform worse than one of its neighbours, it will adopt the strategy and remain until a better performing neighbour surfaces. In turn, neighbouring agents are likely to adopt well performing strategies of a hub node. Hub nodes thus serve as strategic amplifiers and filter out bad performing strategies due to their high selectivity on which strategies to adopt.

Bo and Yang (2010) studied scale-free structures in the Evolutionary Ultimatum Game with the addition of incomplete information in other agents' strategies. Instead of receiving a neighbours strategy when imitating, agents were to learn the neighbour's strategy from their interactions. The results were in contrast to the results from F. C. Santos et al. (2006), indicating that scale-free network structures were of moderate influence on the development of fair negotiation behaviour. This result was found when agent degree was accounted for in comparing fitness levels. In the case of normalised payoff values, populations with scale-free network structures showed even less effect on the development of fair negotiation behaviour. Their differences in performance is attributed to the difference in game setting. In the implementation of Bo and

Yang (2010) however, the establishment of fair strategies is hindered by the addition of incomplete information due to slowing down the spread of a strategy in a neighbourhood. Furthermore, strategies present in the population show p and q being further apart under incomplete information as compared to quasiempathy under complete information. Greater distance between strategy values p and q benefits opportunistic strategies (Page & Nowak, 2001).

A different approximation to dynamics with population structure is that of Nowak, Tarnita, and Antal (2010a). The rationale in their study is that humans not only look at other individuals to imitate their behaviour but also take into account their own and others' social group memberships. Only having individual imitation is deemed as being insufficient in explaining cultural evolutionary processes. With evolutionary set theory Nowak et al. (2010a) propose a framework in which the population network is dynamic rather than static. Agents play the Prisoner's Dilemma with only strategies C and D. A population of N individuals is distributed over M sets, within which individuals interact. Sets are defined as a group of vertices which are all interconnected by edges. A visual representation is displayed in figure 2.2. Individuals can take part in multiple

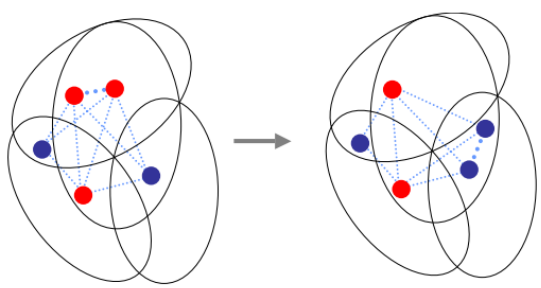

Figure 2.2: A social group is defined as a set of vertices that are all connected to each other. Sets are depicted as closed curves. The transition from the left to right image depicts the change of strategy and social group of the top right agent in the left image.
Source: Nowak et al. (2010a)

sets. At every time step agents are picked at random to imitate. The imitator at random selects another agent proportional to its fitness and copies its strategy s and set inclusions $K \subseteq M$ with probability $1 - \mu_s$ and $1 - \mu_K$ respectively: with probability μ_s the agent adopts a new randomly chosen strategy and with μ_K adopts a random sample of new sets to be included by. With too small values for μ_K, individuals mostly belong to the same sets. With too high μ_K however, individuals change sets too fast for the group level interactions to have effect on evolutionary dynamics. An optimal set mutation rate given by the authors is $\sqrt{\frac{M}{K}}$. A further implementation in the article is that cooperators only cooperate with agents they have a minimum amount of set overlap with, L. An agent cooperates i times with another agent if $i \geq L$. For lower i, the agent defects. With $L = 1$, it is most beneficial for an agent to have $K = 1$.

$L > 1$ favours the evolution of cooperation among agents for any given number of K.

A measure to evaluate approximations to evolutionary dynamics was introduced by Nowak et al. (2010b). The measure relies on the structure coefficient σ, that is affected by population size, population structure, the update rule used and the mutation rate but does not depend on the payoff matrix of a given game. The influence of σ can be written as a linear inequality in discrete payoff values:

$$\sigma a + b > c + \sigma d \tag{2.3}$$

Large well-mixed populations have $\sigma = 1$. Structured populations however generally have $\sigma > 1$. With values for σ greater than 1, payoffs on the main diagonal of the payoff matrix become more important, favouring interaction among agents with similar strategies. σ thus denotes how competing strategies are favoured as a result of population structure, size, update rule and mutation rate. σ is most predictive in cases where natural selection rate is low. A strategy s is favoured over other strategies s' if

$$\sum_{s'=1}^{n} \sigma a_{(s,s)} + a_{(s,s')} - a_{(s',s)} - a_{(s',s')} > 0, \tag{2.4}$$

with $a_{(s,s')}$ here denoting the payoff an agent with strategy s receives from an agent with strategy s'. σ is calculated differently for evolutionary graph theory (i) and evolutionary set theory (ii) approaches:

$$\sigma = (k+1)/(k-1) \qquad \text{(i)} \tag{2.5}$$

$$\sigma = \frac{M(2\nu+3) + K\nu(\nu+2)}{M + K\nu(\nu+2)} \cdot \frac{\nu+1}{\nu+3} \quad \text{(ii)} \tag{2.6}$$

Here, a large population size and low mutation rate are assumed and $\nu = 2N\mu_K$ for (ii). In the Prisoner's Dilemma it is found that selection benefits cooperators when $b/c > k$ for graphs. For evolutionary set theory, this is $b/c > 1 + 2\sqrt{K/M}$ for the optimum μ_K. For graphs, cooperation is stimulated with lower connectivity. For social sets, this holds when the amount of sets that agents belong to is small enough to cluster in social groups effectively.

F. P. Santos, Pacheco, Paiva, and Santos (2017) studied the proceedings of the Evolutionary Ultimatum Game on a Small-World network, a complex network type that employs the clustering characteristic of real social networks. Their version of the Evolutionary Ultimatum Game constituted a multiplayer-approach to the negotiation dynamics in which one proposer allocates among a group of agents. With their research, a structural measure of the complexity of population topology was introduced, named Structural Power (SP). SP measures the relative influence that one agent has over other agents through the average overlap the agent has with others. It was found that with increased SP, individuals have more relative influence as their proposal and acceptance strategies reach a larger number of individuals. In turn, better performing strategies can spread faster within and between groups.

The work from F. P. Santos et al. (2017) strikes resemblance to that of Nowak et al. (2010a) in the emphasis on interaction among social groups. From both studies it can be taken that population structure and the resulting interaction among clusters of agents with similar strategies supports the development of fairness in a population. Main differences are however that, apart from implementing their models to different games, Nowak et al. (2010a) propose a model in which networks are dynamic in cluster composition as well as the number of clusters present in the network. Their approach utilises different rules for e.g mutation and strategy updating while noting that these are of effect for the structural measure that is discussed in Nowak et al. (2010b). In the work of Lieberman et al. (2005) and F. C. Santos et al. (2006) it was seen that structures in which a subset of agents has a highly disproportionate amount of connections compared to the rest of the population, serve as highly selective amplifiers for strategies. Such selective amplifiers are shown to be beneficial for cooperation for a range of evolutionary games, by providing central selective hubs. SP is predictive of the benefit of clusters by denoting the relative influence on other agents through neighbourhoofd overlap. With high relative influence on neighbours, selectivity in which strategies to adopt and an ensured high overlap in neighbourhoods with substantial part of a population, we expect SP to be predictive as well for hubs and their propagation of fair strategies.

Chapter 3

Background

This chapter offers information on theories relevant to the subject of this thesis as to provide less familiar readers with a sufficient background on the matter. It commences with an explanation of the Ultimatum Game and provides introductory information to Game Theory and Network Theory. The Ultimatum Game is further explained with terminology used throughout this paper.

3.1 Game Theory

Game theory is the study of human strategic behaviour in the attempt of approximating rationality in mathematical models. It has seen increased use as a simplification of real-life scenarios. Possible behaviours are quantified to a set of actions such that human behaviour in a situation and its outcome can be summarised by how players discriminate between actions given the utility they expect to receive from their chosen actions. A game is defined as a formal interaction with a finite set of N players and a finite set of actions $A = \{A_1, A_2, ..., A_n\}$ such that A_i denotes the set of actions player i can take. An action profile is then a vector of actions $a = \{a_1, a_2, ..., a_n\}$ taken by n players. The set $U = \{u_1, u_2, ..., u_n\}$ denotes sets of utility functions of which u_i maps for player i the set of actions to a real valued utility. With multiple interactions, a player can choose to play only one or vary between multiple actions. To use one same action for each interaction is to have a pure strategy; in playing with a set of actions, a player is said to have a mixed strategy. We can thus define a strategy $s_i \in S_i$ as the action or set of actions that a player i chooses to take. S_i here denotes the set of all possible strategies for player i. Finally, the set of strategies taken by n players is defined as a strategy profile $s = \{s_1, s_2, ..., s_n\}$.

A player that is rational aims to maximise its expected utility, $\mathrm{E}(u_i)$. Should the actions or strategies of other players be known, a rational player would then pick the action or strategy that produces the most favourable outcome with regards to that action- or strategy profile. Figure 3.1 represents a payoff matrix

	C	D
C	2, 2	0, 3
D	3, 0	1, 1

Figure 3.1: An $n = 2$ normal-form game depicting actions for row- and column-player. For actions C (cooperate) and D (defect) available to each player, possible action profiles are: CC, CD, DC, DD. The first action and the utility in each cell depict those of the row-player.

for the *Prisoner's Dilemma*. In playing the strategy 'always D', DC and DD are the possible action profiles with outcomes that both are more favourable for the row player than playing 'always C' with action profiles CC and CD. Because the row player prefers $u_{row}(D, C) = 3$ over $u_{row}(C, C) = 2$ and $u_{row}(D, D) = 1$ over $u_{row}(C, D) = 0$, strategy 'always D' is said to strictly dominate 'always C' because there is no action profile for which playing C would yield an equal or higher payoff in a single round (Wooldridge, 2009). Let us denote s_i as a strategy for player i and $s_{-i} = \{s_1, ..., s_{i-1}, s_{i+1}, ..., s_n\}$ as the strategy profile of all involved players minus player i. Strict and weak domination are formally defined as follows:

- A strategy s_i *strictly* dominates an alternative strategy s'_i when for all $s_{-i} \in S_{-i}$, $u_i(s_i, s_{-i}) > u_i(s'_i, s_{-i})$
- A strategy s_i *weakly* dominates an alternative strategy s'_i when for all $s_{-i} \in S_{-i}$, $u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i})$ (Shoham & Leyton-Brown, 2008, p.78).

3.1.1 Nash equilibria

An action or strategy that maximises $\mathrm{E}(u_i)$ with regards to the actions of others is also defined as a best response. A game may enter a stable state for consecutive rounds in which no player will diverge from their current strategy as no other strategy will yield higher $\mathrm{E}(u_i)$. Such a stable state from which players are unlikely to depart, is a *Nash equilibrium*. In figure 3.1 a Nash equilibrium is found in action profile DD as playing action D for both players is a strictly dominating strategy over playing action C. The concept of Nash equilibria can explain convergence of systems to solutions that are optimal for a given state of the game but may not be the globally optimal solution. For the Prisoner's Dilemma as depicted, both players would receive a higher payoff when both cooperate to attain a payoff from action profile CC rather than defect and receive the payoff from action profile DD. Playing D for an individual player however yields a higher payoff for any action that the opponent chooses to take. Therefore, utility maximising players are tempted at any action profile in the game to play and remain playing D.

A Nash equilibrium is either strict or weak depending on whether the equilibrium strategy is strictly or weakly dominating. More formally,

- A strategy profile s is a *strict Nash equilibrium* if, for all agents i and for all strategies $s'_i \neq s_i,\ u_i(s_i, s_{-i}) > u_i(s'_i, s_{-i})$
- A strategy profile s is a *weak Nash equilibrium* if, for all agents i and for all strategies $s'_i \neq s_i,\ u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i})$, and s is not a strict Nash equilibrium,

with $s_{-i} = \{s_1, ..., s_{i-1}, s_{i+1}, ..., s_n\}$ again being a strategy profile s excluding agent i's strategy (Shoham & Leyton-Brown, 2008, p.62).

Consider again the Prisoner's Dilemma from figure 3.1. The Nash equilibrium DD is strict as strategy 'always D' is strictly dominating over 'always C'. Suppose that action profile CC now yields $(3, 3)$. Due to a given player now being indifferent between actions C and D when the opponent plays C, the strategy 'always C' is only weakly dominated by 'always D': the given player is no longer ensured that the latter will perform strictly better regardless of the opponent's strategy. Action profile DD is however still a *strict* Nash equilibrium because regarding an opponent playing 'always D', the best course of action is to follow suit. Suppose instead we substitute payoffs for DC and CD with $(3, 1)$ and $(1, 3)$ respectively. Action profile DD is now rendered a *weak* Nash equilibrium because when faced with an 'always D'-playing opponent, diverting one's strategy yields no improvement or decline.

Subgame-Perfect Nash Equilibria

In sequential games with complete information on opponent utility, a refinement of Nash equilibria can be found. Due to the nature of such games where actions are taken sequentially rather than simultaneously, subgames can be identified in which one of the players acts. In figure 3.2, a total of two subgames can be found (one on either side of the dashed line).

For player A, to always play U may seem a strictly dominating strategy as $u_A(UL) > u_A(DL)$ and $u_A(UR) > u_A(DR)$. Since this is a sequential game however, player A relies on the consecutive action of player B, knowing that B will aim to maximise their utility. Because $u_B(UL) > u_B(UR)$, player A will only receive 2. By backward induction, it can be seen that player B will prefer action profile UL over UR and DR over DL. Taking UL and DR into account, player A prefers DR over UL. Therefore DR is the subgame-perfect Nash equilibrium in figure 3.2.

Evolutionarily Stable Strategies

A similar sense of stability is found in the presence of strategies when a game is played in a population of players for an indefinite number of rounds. In such

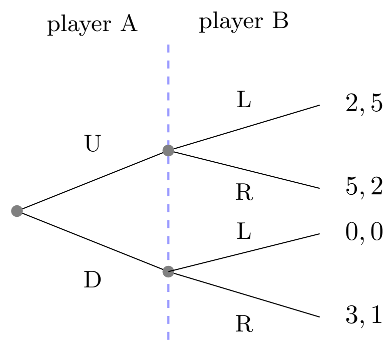

Figure 3.2: An $n = 2$ extensive-form game depicting actions for players A and B. Possible actions for player A are U (up) and D (down); for player B, these are L (left) and R (right). Possible action profiles are: UL, UR, DL, DR. The first action is taken by player A. For corresponding payoffs, the first value is the utility for player A.

a setting, a game is played each round between members of the population who act according to their strategy. Over the span of multiple rounds, players adapt their strategies based on the strategies' average performance in the population. Through this, a population evolves in its composition of adopted strategies.

Suppose now that the one-versus-one Prisoner's Dilemma is played among this evolving population. Though 'always D' still is the equilibrium strategy for one-to-one interactions, $u(D, D) = (1, 1)$ is sub-optimal to $u(C, C) = (2, 2)$ in accumulating payoff. Therefore it may be beneficial to continually cooperate (C) with other cooperating opponents rather than defecting (D), which would subsequently tempt the opponent to defect as well. Because players' income is no longer following from a single round interaction but from an iterated game against multiple players, it can be rational to choose to cooperate continually even though some opponents choose to defect. Important however is that the strategy to always cooperate is rewarding enough for the player, meaning that the amount of cooperating opponents is big enough and that the temptation of diverting to defection is low enough.

In the above-mentioned scenario, to defect is an *evolutionarily stable strategy*, meaning that strategies in a population will converge to defection. As defecting against cooperating opponents yields a higher payoff than cooperating, an individual may divert to defection. Subsequently, to defect yields a higher payoff against defecting opponents than to remain cooperating. A homogeneous, cooperating population may therefore converge to one in which only defection remains due to the introduction of defecting individuals. Formally, a strategy s is a strict ESS and cannot be invaded by another strategy s' when these conditions hold, as defined by Smith and Price (1973):

$$\begin{aligned} &1. \quad E(u_i(s, s)) > E(u_i(s', s)), \text{ or} \\ &2. \quad \text{if } E(u_i(s, s)) = E(u_i(s', s)), \\ &\quad \text{then } E(u_i(s, s')) > E(u_i(s', s')) \end{aligned} \tag{3.1}$$

We can regard an ESS as another refinement of the Nash equilibrium. A strategy must be a best response to itself for it to be a strict ESS, similar to a strict Nash equilibrium. If s is not a best response against itself however due to s' faring equally well against s, s must have a benefit against s' over s' against itself and must thus at the minimum weakly dominate s' (Shoham & Leyton-Brown, 2008).

3.1.2 Evolutionary Dynamics

Mathematical principles such as Nash equilibria and ESSs are used to study the evolution of players' behaviour in a population. Evolutionary dynamics can be generalised to the evolution of species in a biotope or the evolution of human culture. Populations may alter though the selection of individuals that fare better relative to others in the population or random mutations that diversify the population. As shown above, a game theoretical approach is to regard an environment as a population that is composed of agents utilising different strategies. With a game specifying how these agents interact, strategies used by agents can converge to what must be either a local or global optimum for a given population composition by evolutionary mechanisms such as selection and mutation.

Such dynamics are simulated within a Multi-Agent System (MAS) in which agents represent genomes or individuals and are limited in their behaviour to the strategy they represent. In their interaction with others and through the implementation of earlier mentioned dynamical mechanisms, the agent population converges to the set(s) of best performing agents based on the strategy they maintain. In applying game theory to the context of an evolving population, we can study recurring interactions between individuals, identify the characteristics of dynamics between strategies that may be acute or long-term in their onset and consider the evolution of a strategy in specific environments. Axelrod and Hamilton (1981) proposed three metrics to determine the success of a given strategy. The *robustness* of a strategy refers to how well it can thrive and remain standing in an environment that can contain any composition of diverse strategies, the *stability* of a strategy tells us how easily a strategy can be invaded by others under given circumstances after it has already formed a cluster and the *initial viability* of a strategy denotes whether a strategy can fixate itself initially in a given environment without going extinct directly.

3.1.3 The Ultimatum Game

The Ultimatum Game is a form of bilateral negotiation where one player receives a certain amount of a common good and must make an offer $p \in [0,1]$ after which the responder must react by either accepting the division. Each gets their share of the common good as per the division if the responder accepts. If the responder chooses conflict and rejects the division, neither of the players receive anything. The willingness to accept an offer is often thought of as following from some (in human experiments often unspecified) acceptance threshold $q \in [0,1]$ such that

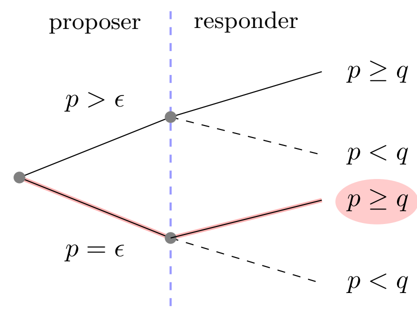

Figure 3.3: The Ultimatum Game presented in extensive form. Each branch represents an action taken by the proposer and responder respectively. Dashed gray lines for the responder indicate refusal of the offer, leading to payoff of zero. The subgame-perfect Nash equilibrium outcome is marked red.

a responder is prompted to accept an offer when $p \geq q$. The payoff for proposer (u_{pro}) and responder (u_{res}) is as follows:

$$u_{pro} = \begin{cases} 1 - p, & \text{if } p \geq q \\ 0, & \text{if } p < q \end{cases}$$

$$u_{res} = \begin{cases} p, & \text{if } p \geq q \\ 0, & \text{if } p < q \end{cases} \tag{3.2}$$

The players are asymmetric in the actions they can take and move sequentially, such that one player proposes a distribution and the other is left with the ultimatum of accepting the distribution or rejecting so that both player will receive nothing. The game is a positive-sum example of a non-cooperative game as individuals can act out of self-interest at the expense of the other. Both players at any time have perfect and complete information on the other's actions in the sense that they are directly informed of the opponent's previous choice and are not restricted in their information on what actions the other can take.

The Ultimatum Game in its simple form was proposed by Güth et al. (1982) who were interested in finite ultimatum bargaining games in which players, due to the sequential nature of the game, are involved in one-player subgames such that their actions should be determined only by the best action they can take in that subgame. A graphical representation can be found in figure 3.3.

The Ultimatum Game has numerous Nash equilibria if we consider the full game, which take equal values for p and q. Regarding the game as two separate subgames however, we can identify a subgame-perfect Nash equilibrium: reasoning backwards, a rational responder must accept any proposition p as otherwise they will receive 0 due to disapproval. With this in mind, the proposer may suggest any allocation $p > 0$ as no non-zero offer is expected to be declined and thus offer the smallest distributable amount. These two actions are the best actions each player can take in a one round game. Therefore this combination of proposing the least possible and accepting (since the least possible is still non-zero) constitutes a subgame-perfect Nash equilibrium for the Ultimatum Game. For both players, this dictates the expected behaviour of a rational agent that aims to maximise its total payoff. This holds even for an iterated game setting in which players alternate roles where defection is the best option in the last round, making greedy play also the best action in the prior-to-last and each preceding round until the first.

3.2 Complex Systems

As a system grows in its number of, the heterogeneity of, the set of actions for, and the interactions between its components, the system becomes more and more complex to the point where that system cannot solely be described as the sum of its parts. Simple behaviour on the component level may result in complex emergent behaviour on the system level, from the interactions between those simple constituents (Siegfried, 2014a). A few examples of systems that we can call complex are governments (as they consist of different branches with each their own functions, responsibilities and hierarchical substructure), the human brain (in a neurological sense; as it consists of a large set of neurons that can be subdivided in regions that each are attributed with a specific purpose to our processing of certain impulses), and social networks (as individuals are heterogeneous in the amount of others they are related to, how these relations are defined, what their beliefs are and which status or permissions they have in that social network, Bar-Yam, 1998).

Siegfried (2014a) identifies these (non-exhaustive) four typical characteristics of complex systems:

- state space complexity (complexity in the number of possible states of the system)
- structural complexity (complexity in the connections between system components)
- behavioural and algorithmic complexity (complexity in behaviour and interactions of system components)
- temporal complexity (complexity in time- and state-dependent behaviour of a system)

A way of representing such complex systems in an abstract, concise manner is through wiring diagrams, or networks. Expressing complex systems as networks can clarify the individual components present and the relations that can be found between them.

3.2.1 Network Concepts

A network is defined as a graph $G = (V, E)$ where V denotes the set of vertices or nodes in a network (representing the components) and E denotes the set of edges or links between those nodes (representing component interactions). The properties of the network's structure depend on what it represents. In literature, terminology from graph theory and network science are often used interchangeably. A subtle difference in usage is that graph-theoretical terminology is more often used in the context of discussing mathematical principles whereas terms from network science are found regularly in the context of real-world applications, such as a network underlying to groups of friends or a network representing corporate connections within a field or city.

Network Science		**Graph Theory**
Network	$\equiv$	Graph
Node ($n \in N$)	$\equiv$	Vertex ($v \in V$)
Link ($l \in L$)	$\equiv$	Edge ($e \in E$)

These terms are commonly used synonymously and they will be used as such throughout this project.

Undirected and Directed Graphs A network may be depicted as a directed graph in which edges are arrows directed from a node to another or as an undirected graph. In an undirected network, an edge e_{xy} between nodes x and y is equivalent to the pair $(e_{xy},\ e_{yx})$, denoting a symmetric relationship between x and y (ignoring any further notion of edge weights for now). How the edges relate to each other depends on whether relations are hierarchical/ordinal or not: if x is the father of y, the reverse cannot be true. However, if x is family of y, the reverse must be true as well, and can thus be expressed through two reciprocal, directed edges or a single undirected edge.

Multigraph and Weighted Graph Some relations may also require additional measures to be represented, such as an acquaintance network in which the degree of interaction is relevant or a network in which not only the presence of public transit lines between cities matter, but also the amount of lines that are present. Such networks can be expressed by attaching weights to edges to express a quantity of that relationship (amount of interaction) resulting in a weighted graph and/or have multiple edges (public transit lines), thereby making it a multigraph (Barabási et al., 2016).

Clusters and Hubs Within a network certain sub-structures can be defined based on how nodes within these structures are connected. Two structures relevant for the focus of this project are clusters and hubs.

A set of nodes in a network is defined as a cluster when these nodes show a high number of connections amongst themselves. This is analogous to the set of friends for an individual in which most are also friends of each other. Due to this high density of connections within the set, information possessed by a cluster member is likely to reach another cluster node faster than an arbitrary node in the network.

In the case of multiple nodes being connected to a same node with an absence or low amount of connections amongst themselves, we define the single node as a hub. To draw a parallel with the previous analogy, most friends of the given individual would not regard other members as their friends. Another analogy for a hub is an individual enjoying a high degree of popularity or status, such as an individual with a large social media following or a superior in a hierarchical system. When being passed by hub nodes, information has a larger reach compared to nodes that have lower degree.

3.2.2 Structural Properties

Real-world networks are subject to some topology stemming from the characteristics of the system it represents and its components. Models exist for the generation of synthetic networks used for studying such systems. These synthetic networks differ in their structural properties. Below, an explanation of important structural properties is given followed by an introduction of three influential network generation models that are used in this project.

The **Average Path Length** (APL) or Average Shortest Path Length is a measure for how clustered or connected a network is and thus how efficiently information can be transferred between any potential pair of nodes. It is the average of the shortest path lengths between all possible pairs of nodes,

$$APL = \frac{1}{N(N-1)} \sum_{i \neq j}^{N} s(v_i, v_j), \tag{3.3}$$

with N being the total amount of nodes in a network G and $s(v_i, v_j)$ being the separation between arbitrary nodes i and j, calculated as the least amount of edges between them (Pasta, 2019).

The average **Clustering Coefficient** (CC) denotes the tendency of nodes in a graph to be clustered together rather than spread apart with less connections. The more connections between the neighbouring nodes of a focal node, the higher its individual clustering coefficient will be. Social networks tend to be strongly clustered. The average clustering coefficient for a network is calculated as:

$$CC = \frac{1}{N(N-1)} \sum_{i=1}^{N} \frac{2d_{-i}}{d_i(d_i-1)}, \tag{3.4}$$

with d_i denoting the amount of neighbours for node i and d_{-i} denoting the amount of edges between neighbours of node i (Pasta, 2019).

The **Degree Distribution** (DD) denotes to what extent a network is distributed or centralised. The degree d of a node is the amount of neighbours it has. With all degrees in a network known, a frequency distribution can be made displaying the proportion of all N nodes for which the degree is d (Pasta, 2019).

The **Structural Power** of a network denotes the amount of cluster overlap that nodes have. Structural power can be calculated for a single relationship (SP_{ij}), the average for a single node (SP_i) or the overall average for a network (SP_{graph}). Whereas the average clustering coefficient of a network represents the fraction of triads (e.g. two friends that have another friend in common) over all possible triples in that network, structural power represents the amount of influence a node i has over another node j through their mutual connections, relative to the total neighbourhood size of node j (F. P. Santos et al., 2017). The structural power of node i over node j is calculated as:

$$SP_{i,j} = \frac{2d_{i,j} + \sum\limits_{x \in N} d_{i,x} \times d_{x,j}}{\sum\limits_{x \in N} d_{x,j} + 1} , \tag{3.5}$$

with $d_{i,j}$ being 1 if there is an edge between nodes i and j and $d_{i,x} \times d_{x,j}$ referring to the amount of neighbours that both i and j are connected to. The average SP of an agent is:

$$SP_i = |R_i|^{-1} \sum_{j \in R_i} SP_{i,j} , \tag{3.6}$$

with R_i denoting the agents that are reached by agent i directly or indirectly through a common neighbour. In other words, R_i is the combined set of agents in the neighbourhood of agent i and the agents in the neighbourhoods agent i participates in.

The average SP of a network G can then be defined as follows:

$$SP_{graph} = \frac{\sum\limits_{i=1}^{N} SP_i}{N} \tag{3.7}$$

When speaking of SP_i in general without specifying a specific node, we will denote this as SP_{node}. For $SP_{i,j}$ we will use SP_{pair}.

3.2.3 Network Models

Erdős–Rényi (Random Graph)

The random graph model formally introduced by Pál Erdős and Alfréd Rényi preluded more sophisticated complex network generation models that better simulated the stochasticity of connections found in real-world networks. In the Erdős–Rényi model, N isolated nodes are linked randomly pairwise with a probability p. This is done for every possible pair of nodes in the graph (Erdős & Rényi, 1960). Figure 3.4 shows three generated ER graphs with varying p.

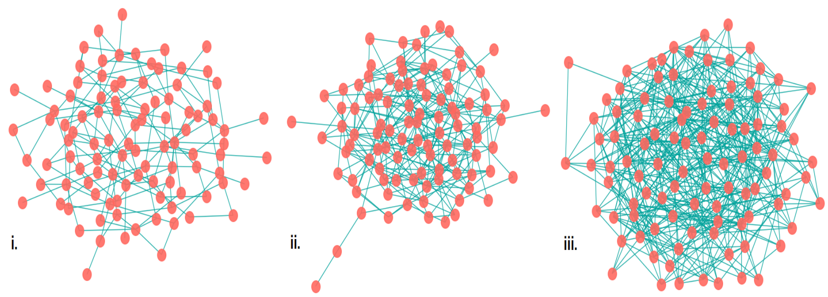

Figure 3.4: Erdős–Rényi random graphs generated with $N = 100$ and $p = .03$ (i), $p = .06$ (ii) and $p = .10$ (iii).

ER graphs are characterised by low average path length and a low clustering coefficient. This is because the probability of an edge being wired between a pair of nodes is equal for all possible pairs. Therefore, ER graphs have a low standard deviation in the amount of edges d that each node has. Statistics for the ER graphs are shown in table 3.1.

p	APL	CC	SP_G
.03	3.759	.0197	.288
.06	2.877	.0737	.202
.10	2.261	.1083	.163

Table 3.1: Structural properties for the generated Erdős–Rényi random graphs depicted in figure 3.4.

Contrary to real networks, ER graphs show little to no triadic closure or clustering due to the independent, equal probability of connection between nodes. Real networks also exhibit the formation of larger hubs where a small subset of nodes exhibits a degree orders of magnitudes higher than others, whereas ER graphs have a degree distribution that converges to Poisson distribution (for $N \gg \overline{d}$; when $N > \overline{d}$ but $N \not\gg \overline{d}$, converges to a binomial distribution) (Barabási & Albert, 1999; Barabási et al., 2016; Watts & Strogatz, 1998). Degree distributions can be found in figure 3.5 (i - iii).

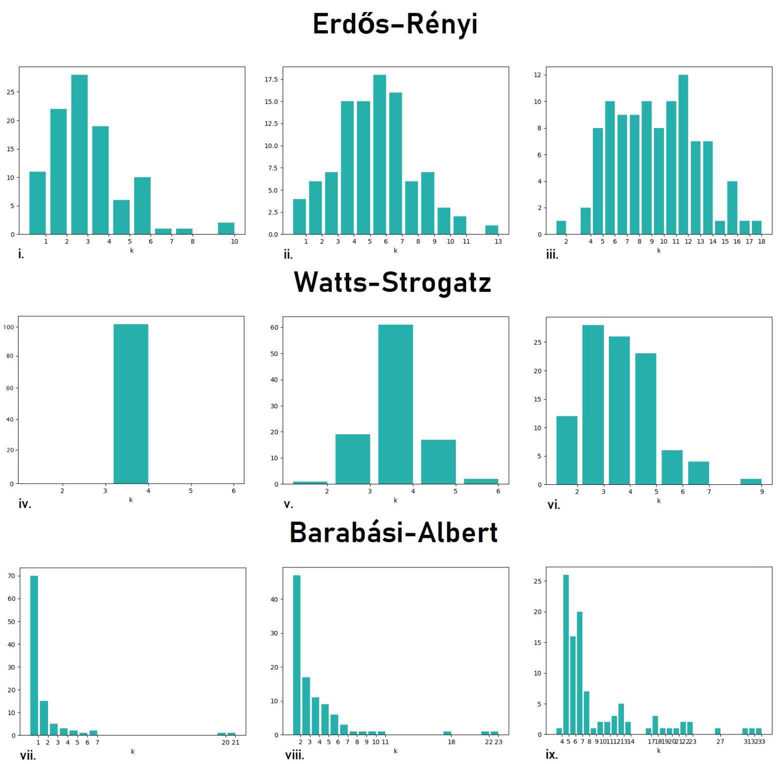

Figure 3.5: Degree distributions for generated graphs. For Erdős–Rényi: $n = 100$, $p = .03$ (i), $p = .06$ (ii), $p = .1$ (iii). For Watts-Strogatz: $n = 100$, $K = 4$, $p = 0$ (iv), $p = .10$ (v), $p = .80$ (vi). For Barabási-Albert: $n = 100$, $m = 1$ (vii), $m = 2$ (viii), $m = 5$ (ix).

Watts-Strogatz (Small-World Network)

The Watts-Strogatz network generator creates small-world networks that exhibit the first of two general properties of real networks, the formation of clusters through triadic closures. Watts-Strogatz small-world networks exhibit a low average path length as found in random graphs and high clustering as found in lattice graphs. Small-world networks are generated by starting with nodes oriented on a regular ring lattice with d neighbours from which all edges are rewired uniformly at random with some probability p_{rewire}. With high values for p_{rewire}, generated networks resemble Erdős–Rényi random graphs. Figure 3.6 shows three small-world networks generated with varying probabilities of rewiring.

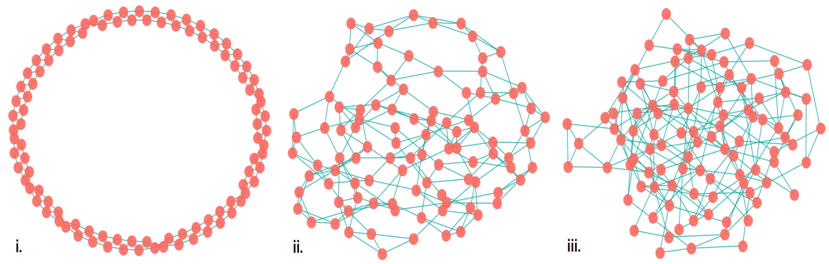

Figure 3.6: Watts-Strogatz small-world networks generated with $N = 100$, $\overline{d} = 4$ and $p_{rewire} = 0$ (i), $p_{rewire} = .10$ (ii) and $p_{rewire} = .80$ (iii).

Different from random graphs, nodes in the small-world network that have common neighbours have a higher probability of being connected as well. Due to the larger overlap in neighbours between nodes, small-world networks also exhibit a higher average structural power than random graphs. Structural properties for the small-world networks displayed can be found in table 3.2.

p_{rewire}	APL	CC	SP_G
.0	12.89	.5	.5
.10	4.591	.3317	.3804
.80	3.462	.0391	.2594

Table 3.2: Structural properties for the generated Watts-Strogatz small-world networks depicted in figure 3.6

To what degree a network exhibits small-world properties is measured by its small-worldness. A measure for small-worldness is ω. For ω, the average path length of a network (APL) is compared with that of an equivalent random network (APL_r) and its clustering coefficient(CC) with that of an equivalent regular lattice (CC_l). Equivalent here refers to having the same amount of nodes and edges per node. Small-worldness ω is then calculated as:

$$\omega = \frac{APL_r}{APL} - \frac{CC}{CC_l} \tag{3.8}$$

Possible outcomes for ω are found in $[-1, 1]$. For values of ω close to -1, a network exhibits the APL and CC of a lattice whereas for values close to 1, its properties resemble those of a random graph. For ω close to 0, a network features small-world characteristics (Telesford, Joyce, Hayasaka, Burdette, & Laurienti, 2011).

Small-world networks follow real networks in that there is a higher amount of clustering and triadic closures, along with network-spanning edges that reduce the average distance between two arbitrary nodes. However, they do not display a power relation in their degree distribution and thus do not exhibit degree heterogeneity as found in real networks.

Barabási-Albert (Scale-Free Network)

Barabási-Albert scale-free networks do exhibit the presence of hubs in real networks. The scale-free networks are generated by starting with m initial nodes to which in each time-step a new node is attached to m nodes already present in the graph. Node selection displays preferential attachment, meaning that higher degree for nodes results in a higher probability of subsequent attachment by new nodes. This is done until the amount of nodes N is met. The preferential attachment resembles the preference of individuals to associate themselves with others who have a large amount of connections or the growth of the World Wide Web through hyperlinking popular websites. Figure 3.7 shows three generated scale-free networks with varying m.

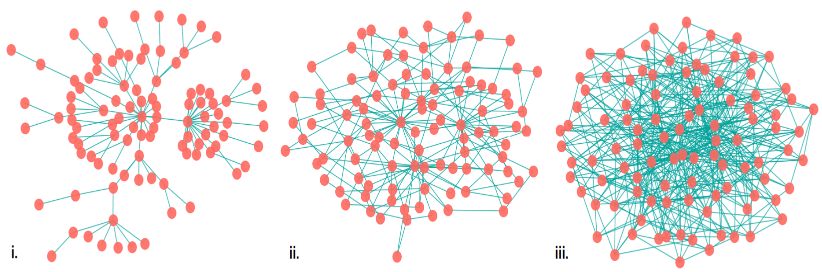

Figure 3.7: Barabási-Albert scale-free networks generated with $N = 100$ and $m = 1$ (i), $m = 2$ (ii) and $m = 4$ (iii).

The APL and CC for Barabási-Albert scale-free networks are heavily reliant on the settings for m. For $m = 1$, nearly all paths cross the small subset of nodes with high degree without any faster shortcut from one hub to another, necessitating longer paths between nodes with low degree. And since nodes are attached preferentially, triadic closures are sparse. The average structural power is higher for lower m however, caused by the subset of nodes with higher degree.

m	APL	CC	SP_G
1	4.052	.0	.4042
2	3.079	.1149	.2568
4	2.229	.2040	.1898

Table 3.3: Structural properties for the generated Barabàsi-Albert scale-free networks depicted in figure 3.7

Contrary to ER graphs and small-world networks, Barabási-Albert scale-free networks do not consider all nodes to be present at the time of creation. Therefore Barabási-Albert scale-free networks can also simulate network growth. Barabási-Albert scale-free networks do model hub formation through preferential attachment as can be found in some real networks, but fail to include the formation of clusters for low settings for m. Scale-free network generators that do account for the formation of clusters with extreme hub formations however do exist (e.g. Holme & Kim, 2002; Klemm & Eguíluz, 2001).

3.2.4 Agent-Based Models

Agent-based modelling is a simulation modelling technique with which complex systems can be represented. An Agent-Based Model (ABM) is built with a focus on the individual components that comprise the system, rather than the system itself. Whereas network models can represent the topology or hierarchy of units constituent to a complex system, ABMs can represent the behaviour of and interactions between such units. This allows for the modelling of macro dynamics on the level of the system, as an outcome of micro dynamics on the level of components.

ABMs are able to:

- *capture emergent phenomena*, in modelling a system through its smaller constituents,
- *provide a natural description of a system*, by focusing on causation behind emergent behaviour as to recreate it indirectly rather than directly model said behaviour,
- *allow for flexibility in modelling*, as properties underlying the system can be adjusted easily (Bonabeau, 2002).

Agents

The smaller constituents that serve as building blocks for ABMs are agents. An ABM situates agents in a common simulation environment in and with which they interact (Siegfried, 2014b). An agent is a software object that perceives its surroundings through its sensors and has ways to act on it through its effectors (Russell & Norvig, 2002). There is no single form of agent as their design is heavily reliant on their purpose. Generally, agents possess the following characteristics as defined by Wooldridge and Jennings (1995, p. 116). An agent is:

- *autonomous* in that it has control over its actions and internal state and operates individually,
- *social* in its capability of interaction with other agents,
- *reactive* in perceiving and reacting to changes in their environment, and
- *proactive* or *goal-oriented* in changing their environment to pursue a desired goal or state.

Important further characteristics for an agent are that it is *rational* and *situated* (Regli et al., 2009; Russell & Norvig, 2002). The rationality of an agent refers to having a measure of performance based on its desired state or goals for which it aims to improve its performance. Situatedness refers to the agent being embedded in some environment, whether it be real or virtual, that the agent can sense and affect.

The internal structure that makes up the agent is referred to as the agent's architecture. An agent consists of *sensor* - and *effector* components, and may also include a *reasoner* component depending on the complexity of the agent's behaviour. Sensor components enable an agent to perceive its environment; effector components enable an agent to interact with it. Reasoner components refer to modules that enable additional processing on recent or past sensor information.

Multi-Agent Systems

A Multi-Agent System (MAS) is a system consisting of multiple autonomous agents that interact with each other and their environment to achieve an individual or joint objective. MAS tackle complex problems in a distributed manner: agents contribute to a global solution by individually solving a global or local problem (Serugendo, Gleizes, & Karageorgos, 2005). Due to limited or no central management, a typical MAS is self-organising: global, system-level behaviour is predominantly the aggregate of local, agent-level actions.

Much like an agent, a MAS can be embedded in the real world, integrated in a physical system (e.g. in building management systems, Priyadarshana et al., 2017) or fully existing in a virtual environment. Regardless of the implementation, behaviour found in MAS can be characterised by the following:

- there is an *absence of explicit external control*: a MAS is itself autonomous with system reorganisations originating from internal decisions,
- control in MAS is *decentralised*: the system itself is mostly composed from its constituent parts with little to no global control on the level of the system,
- *dynamic operation*: MAS evolves independently over time without external control, implying continuity of agent behaviour (Serugendo et al., 2005).

MAS vary in complexity depending on the size of the system, the complexity of local dynamics and the heterogeneity of those computations. Following these dimensions, (Regli et al., 2009) define these terms to describe typical MAS:

- *monolithic system*: system consisting of a single, highly specialised agent with complex internal computation with a focus on autonomy, proactivity and continuity,
- *median system*: system consisting of a small number of agents that are heterogeneous in their architecture and display moderate computational complexity with a focus on coordination, cooperation and resource sharing,

- *swarm system*: system consisting of a high number of agents that are predominantly homogeneous in their architecture and of low complexity, with a focus on emergent behaviour as the aggregate of local dynamics.

It serves the reader right to provide a disambiguation between the definitions for ABM and MAS. Both techniques imply systems that are defined mostly by the behaviour of agents as their constituent parts, allowing for emergent behaviour. Additionally, both can also represent complex systems. Multiple definitions exist for both ABMs and MAS as both can refer to specific techniques as well as a framework or mindset that encompass programming approaches. In this project, Agent-Based Modelling is approached as constructing a model so that a complex system as found in the real world is represented with the use of agents. A MAS is approached as a form of decentralised computing on the basis of agents. Whereas for an ABM the purpose is the resemblance of a real complex system with emphasis on simulation, a MAS aims to offer a global solution by convergence of local computations. Both approaches can work in accordance with each other, with their intersection being a MAS implemented to provide a global solution to a real world problem that is expressed as a model.

Chapter 4

Methodology

This chapter provides the methodology for this thesis project. The methodology is structured as follows. First, our definition of fairness that will be used for the experiments will be further specified, along with a further specification of our hypotheses. Then we will discuss our developed Multi-Agent Model in detail according to ODD protocol.

4.1 Specifications

4.1.1 Fairness

In specifying our criteria on when a result is deemed fair, we will use two definitions of fairness for our experiment to reflect procedural fairness as well as distributional fairness. For our first definition we focus on players' negotiation behaviour in that players exhibit fairness in their strategies. We therefore define fairness as population convergence to an average strategy $\overline{s} = (\overline{p}, \overline{q})$ that splits equally and is empathetic:

$$\overline{s} = (0.5, 0.5)$$

Our second definition considers the distribution of utility in a population in the final round. We define a fair distribution of utility to be one in which $u_i = u_j$ for each possible pair of agents, meaning that for the standard deviation of utilities σ_u within a population in the last round we have:

$$\sigma_u = 0$$

These criteria denote what we would ultimately see as perfectly fair strategies and distributions. For our experiments, we focus however on the relative improvement in fairness of our model under different settings of network characteristics. These criteria are thus used as reference points by which we evaluate whether the relative difference in strategies and utility distribution between different population structures is in favour of or against fairness.

4.1.2 Hypotheses

Having formalised our definition of fairness, we now include this formalisation in our hypotheses. With $a \to b$ we denote that variable a converges in the direction of value b.

Hypothesis 1a: For population structures with high Clustering Coefficient (CC), we expect to find $\overline{s} = (\overline{p}, \overline{q}) \to (0.5, 0.5)$, with $(\overline{p}, \overline{q})$ for high CC populations being greater than $(\overline{p}, \overline{q})$ for low CC populations

Hypothesis 1b: For population structures with high CC, we expect to find $\sigma_u \to 0$, with σ_u for high CC populations being less than σ_u for low CC populations.

Hypothesis 2a: For population structures high in degree-heterogeneity we expect to find $\overline{s} = (\overline{p}, \overline{q}) \to (0.5, 0.5)$, with $(\overline{p}, \overline{q})$ for high degree-heterogeneic populations being greater than $(\overline{p}, \overline{q})$ for less degree-heterogeneic populations.

Hypothesis 2b: For population structures heterogeneous in degree distribution we expect to find $\sigma_u \to 0$, with σ_u for high degree-heterogeneic populations being less than σ_u for low degree-heterogeneic populations.

4.2 Multi-Agent Model

In the following section we will provide a thorough description of the model designed for our research aim. We first discuss the model on a general level, providing the model's purpose (4.2.1), its entities and variables (4.2.2) and a process overview (4.2.3). Secondly, we discuss design concepts and model attributes (4.2.4). Lastly, we discuss the initialisation (4.2.5) and provide detailed descriptions along with pseudo-algorithms for submodels (4.2.7). The model description follows the ODD (Overview, Design concepts, Details) protocol (Grimm et al., 2006, 2010; Grimm, Polhill, & Touza, 2017).

4.2.1 Purpose

The purpose of this model is to capture the influence that social network structure has on the evolution of fair negotiation behaviour in bilateral negotiations. The model simulates a population of individuals that interact with each of their neighbours by playing the one-versus-one Ultimatum Game. The population is structured according to the properties of small-worldness and scale-freeness, as found in real social networks. Individuals continually and simultaneously interact with their neighbours for a set amount of rounds. Through social comparison with their neighbours and creatively trying new strategies, individuals attempt to improve their own negotiation tactics. The idea is that self-interested negotiation tactics are individually rational and optimal when others can be exploited.

However, individuals have repeated interactions only with their neighbours and are a source of influence for their neighbours' negotiation behaviour. When adopted by neighbours, self-interested negotiation tactics will therefore lead to sub-optimal results for an individual.

4.2.2 Entities, state variables, scales

This model contains two types of entities, being the individuals and the population. Individuals are represented by agents (and will further be referred to as such) that inhabit a node in a network. The population is the set of agents, coupled to a network. The network determines the social structure within the population, i.e. which agents are connected and can thus interact. Since agents and nodes are paired bijectively, variables concerning nodes are included in the state variables for agents.

Agents

Agents represent individuals in a population that are involved in bilateral negotiations with their neighbours. Each agent has an identifier that is unique within the population and inhabits a node in the social structure network. Agents interact with their set of neighbours by playing the Ultimatum Game, for which each agent has its own strategy. The resulting payoff from the interactions is stored as an agent's revenue, with the agent's fitness denoting its performance relative to the size of its neighbourhood and thus the resulting amount of interactions.

There are no discrete types that distinguish sets of agents from others. Each agent has a unique id and assigned node. Agents are assigned their own neighbourhood and strategy, with similarities with other agents being possible but unintentional.

Following the classification of Sloman (1999, p. 4-6), our agent can be seen as a *reactive* agent. Reactive agents receive information through their sensors and act on that using their actuators without much processing before taking action or determining which action to take. The agents are however slightly more complex than regular reactive agents. Though actions during game interactions are simple, agents employ a slight sense of reasoning in the comparison of their performance with that of their neighbours and calculating a probability with which they exploit neighbours' strategies. They do not however reason on a history of previous states in determining consecutive actions, which Sloman (1999) appoints as the core difference between his class of reactive agents and his subsequent, and slightly more complex, deliberative agents.

Population

The population is the set of all agents present, along with a social structure network. The population entity is not described by many state variables. Rather, the population entity is used to address all agents and interaction couples and in

State variables	Brief description
id	Agent identifier
node	Social structure network node associated with the agent
neighbourhood	List of agents that inhabit nodes adjacent to *node*
strategy	Tuple $s = (p, q)$ with $p, q \in [0, 1]$ denoting offer value p and acceptance threshold q for the agent
revenue	Sum of all payoffs received in the current round
fitness	Performance measure calculated from the revenue gained and the amount of interactions per round
data	List with per round a tuple (p, q, u) containing offer strategy p, acceptance threshold q and round revenue u
exemplar	A neighbour selected for social comparison in the updating phase

Table 4.1: Table listing all state variables of the agent entity with a brief description of each variable.

coupling agents to the social structure network. The population entity executes actions that occur globally, and is by itself the environment for the agents. The population entity is a central component for model in addressing each agent to interact with their neighbours by playing the Ultimatum Game, and handling the update sequence each round. One time step then denotes a single round in which all interactions have occurred and all agents have updated their strategy.

state variables	brief description
agents	List of all agents present in the population
graph	Generated social structure object that is passed to the population
edge list	List of all connections present between agents in the network

Table 4.2: Table listing all state variables of the population entity with a brief description of each variable.

The conjunction between population and the social structure network forms the complete environment for the agents. Agents are paired bijectively to nodes of a new social structure network for each simulation. This generated network dictates the structure of the population and thus which agents can and cannot interact. Nodes have access to the agents on its neighbouring nodes. Figure 4.1 offers a schematic representation of the agent-to-network coupling.

The environment that all agents are situated in and are part of is described following the environment properties from Russell and Norvig (2002, p. 46) in table 4.3.

Note that this takes account of the perceived environment from the perspec-

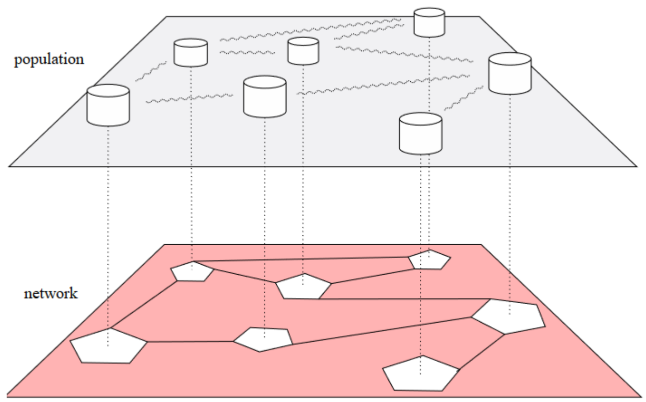

Figure 4.1: Schematic representation of the coupling of the agent population to a network. Pentagrams in the lower layer represent nodes, with edges between them. Cylinders in the upper layer represent individual agents, with dotted phase lines between them indicating the possibility of interacting. These interaction lines correspond to the edges in the lower layer.

Property	Description
inaccessible	An agent is not aware of other players' strategy values when acting
nondeterministic	The agent's environment is subject to stochasticity beyond the agent's control or knowledge
nonepisodic	Actions in earlier rounds will affect negotiations in later rounds, through mutual influence on strategies between agent and neighbours
static	The environment does not change for an agent during deliberation before taking action
continuous	Strategies for the agents are continuous. Therefor the strategies of an agent's neighbours are unlimited in their configuration.

Table 4.3: Properties of the model's environment following the classifications of Sloman (1999).

tive of an arbitrary agent rather than the complete environment.

4.2.3 Process Overview and Scheduling

A flow diagram for the round process of the model is shown in figure 4.2. Each time step consists of two parts. In the first part, each agent plays the Ultimatum game with each of their neighbours, storing their resulting payoffs after each interaction. Agents interact twice per neighbour in a single round; this will be further explained under *interaction* in the design concepts (4.2.4). After all agents have interacted, fitness for each agent is calculated and stored. All agents store round performance in their data variable and select an exemplar neighbour from their neighbourhood.

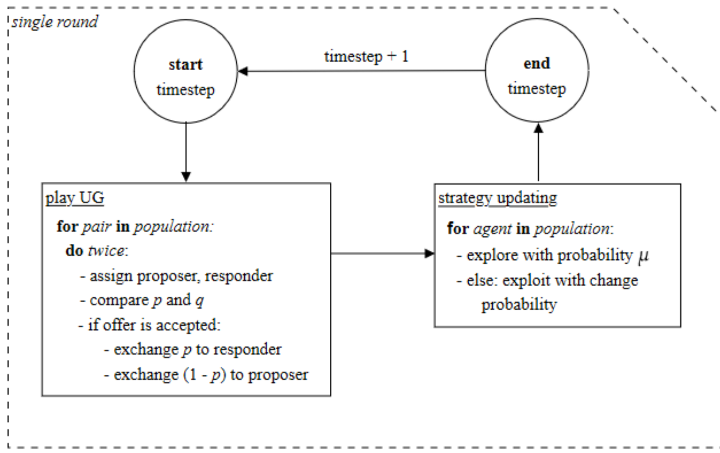

Figure 4.2: Flowchart showing the process of a single iteration. p denotes the proposer's offer strategy; q denotes the responder's acceptance threshold.

In the second part, agents enter an update process in which each has a chance of imitating their exemplar neighbour's strategy or exploring the strategy space by adopting a newly randomised strategy. This update process is finished when all selected agents have stored their adapted strategy. The final action in the process is all revenues being reset to zero before commencing with the next round.

4.2.4 Design Concepts

Basic Principles The theoretical underpinning for our model stems from game theory, evolutionary dynamics, evolutionary graph theory and network science. Our model is designed according to the principles of Agent-Based Modelling (ABM). Further implementations to our model stem from earlier approaches to the Evolutionary Ultimatum Game, as are reflected in our update rules which will be explained under submodels (4.2.7).

Game theory contributes in our use of the Ultimatum Game as a game-theoretic model of bilateral negotiations. It further contributes in our explanation of behaviour on the local level in our model, being the considerations an individual has in the UG-model of bilateral negotiations as based on the Nash equilibrium in the Ultimatum Game.

Following evolutionary game theory and evolutionary dynamics, we extend this bilateral negotiation model to a population in which the Ultimatum Game is the mode of interaction. Population members interact for a given amount of rounds over which they adapt strategies according to fitness and random exploration. The core principle taken from these theories is that, given enough time, the average population strategy will converge toward a single or multiple attractor points in the strategy space. These attractor points are Evolutionarily Stable Strategies, from which we deduce that these strategies are actually the most viable under given settings.

The introduction of a topology in the Evolutionary Ultimatum Game stems from the principles of Evolutionary Graph Theory. This posits that topological structure has a strong influence on the evolutionary dynamics occurring within a population, as shown by e.g. Lieberman et al. (2005). We expand this introduction of topological structure by regarding structures that describe social networks specifically. With this we include network science, which is, much like graph theory, the study of graphs/networks. As explained earlier however, graph theory more so refers to the mathematical approach to graphs/networks whereas network science refers to the study and description of found networks in the real world.

Our model is designed as an ABM. Though mathematical modelling is common in evolutionary game theoretical studies, we believe that ABMs better allow for a natural description of our intended phenomenon. Furthermore, with use of an ABM studying our model in different networks requires only the tweaking of hyperparameters whereas otherwise this would constitute a different mathematical model per structure.

Emergence The explicit behaviour of our model is the local interaction, being the bilateral negotiation. Slightly less explicit is the exchange of information (being: strategies) between neighbouring agents, as influenced by population structure. The most implicit, and emergent property of behaviour in our model is the aggregate exchange of information throughout the whole population, and the collective individual convergence to fairer strategies. This behaviour is not ascribed to individual agents or a central authority. Rather, it emerges from local, utility-maximising behaviour from agents given their position in the network.

A further emergent property that shows up in the model is selection pressure versus neutral drift as characteristic of the global dynamics. In short, this is the degree to which global dynamics are actually controlled by the performance of a strategy rather than being controlled by stochastic evolutionary mechanisms such as exploration, respectively.

Adaptation Agents adapt only in their strategies. Adaptation in our Evolutionary Ultimatum Game model is designed according to the evolutionary game-theoretic principles of exploration and exploitation. An update sequence is present at the end of each round up until the last. In this update sequence, both exploration and exploitation occur with a probability.

On the level of a single agent, the exploration probability is exogenous in the sense of being set as hyperparameter before initialisation and thus being fixed. The exploitation probability however is calculated for each agent and depends on the relative performance difference between an agent and its chosen exemplar neighbour. Important to note is that exploitation occurs only when an agent is not selected to explore, and thus when the exploration probability is not met.

Objectives The prime objective for agents to adapt is to maximise their performance, meaning the utility they gain in each round. The more specific objectives of exploration and exploitation differ but ultimately reflect utility-maximisation. For exploration, the underlying objective is to explore the strategy space in search of a strategy that might outperform the former strategy in the agent's neighbourhood. For exploitation, the objective is to emulate a neighbour that may be performing better, in the hopes of achieving similar success. Depending on the update rule, different interpretations for exploitation can be ascribed. An update rule that places non-zero exploit probability on worse performing strategies can still be regarded as exploration within one's own field of view, being the neighbourhood.

Learning No form of learning has been applied on the level of individual agents. As a dynamical system it can be said however that the population as an entity undergoes a learning process, as found in swarm intelligence. Though this is our intended use for the model, it is an emergent property and not explicitly incorporated.

Prediction Agents do not apply any form of prediction in the model.

Sensing Agents can sense their direct surroundings in the sense of having access to which agents are in their neighbourhood. Agents cannot however sense any state variable of other agents, apart from exemplar agents' strategy and fitness when explicitly given in the update sequence. Furthermore, agents are only aware of the outcome of an interaction. This will further be explained under *Interaction*.

Regarding agents only being aware of the outcome of an interaction due to the current implementation, we like to address that knowledge of an offer or acceptance threshold is not necessary for the scenario that the model intends to simulate. Agents only act based on their current strategy and do not anticipate on the actions of others; therefore strategic information on neighbours before acting is also irrelevant and therefore not stored or further accounted for.

Interaction An interaction between agents constitutes playing the one-versus-one Ultimatum Game. It is implemented as being conducted by a 'central authority', meaning interacting agents only share their relevant strategy value (offer or acceptance threshold) with the authority and are returned a payoff depending on the turnout of the interaction. This has been a optimisation choice rather than explicitly and meaningfully designed.

Furthermore, an interaction constitutes of two bilateral negotiations: in one round, against one neighbour, an agent is involved in the Ultimatum Game twice. Based on whether role-determination is random or alternating, agents can either alternate between roles or play as one role twice per round per neighbour.

The choice for two bilateral negotiations follows the implementation by Iranzo et al. (2011). There are two motivations for this design choice. Firstly, having two negotiations per round per neighbour emulates the idea of a focal agent interacting with each of its neighbours before a consecutive agent interacts with its neighbours, allowing for each agent to play once being focal agent and once being a neighbour of a focal agent.

Secondly, having a higher amount of interactions in a round before the updating sequence increases the probability of an agent using both its offer and acceptance threshold. Therefore, round revenue is likely to be more representative of both an agent's offer and acceptance threshold, meaning an agent has more interactions it can test its strategy in.

Further interaction between agents occurs in the updating sequence, where an agent receives the fitness and strategy values of its exemplar neighbour.

Stochasticity Stochasticity is present in exploration and exploitation. For exploration, as well as initialisation of a simulation, an agent receives a new strategy composed of $(p, q) \in [0, 1]$, sampled uniformly at random.

For exploitation, stochasticity is present in the selection of an exemplar neighbour, in the probability of adopting the neighbour's strategy and in an amount of noise applied to the strategy values of the neighbour, to encourage local exploration.

Collectives No explicit agent collectives are present in the model. Implicitly, collectives can be present in the form of substructures in the social structure network. The social structure network is however in no way dynamic; agent connections do not change during a simulation and such collectives are in no way separated from the rest of the population.

Furthermore, the neighbourhood of a given agent can be seen as a collective, with all interacting indirectly through said agent unless connections also exist between neighbours.

Observation Data is stored per agent as a tuple (p, q, u) being the offer value (p), acceptance threshold value (q) and the utility (u) gained in that round. Main analysis will only cover data from the last round to reflect the converged population state and final strategies and income for the agents.

In studying the data, we will look at average population p and q as well as distribution of utility among a population.

Networks This design concept is supplementary and not part of the standard ODD protocol. We add a small discussion of the social structure networks to provide some detail before discussing the initialisation of a simulation. Pseudocode algorithms for the generation of networks will be provided in the submodels subsection (4.2.7), along with a more in-depth discussion of the network models.

The social structure of the population of agents in our model is determined by networks that are generated before commencing the simulation. These networks are created with the amount of agents (and thus nodes) that will be present in the population, as well as the average degree of each agent, meaning the amount of edges that an agent's node has. The networks are underlying to the model and have no further influence on the population apart from being used to initialise agents and determine their neighbours at the start.

4.2.5 Initialisation

At the start of a simulation a generated network is passed to the empty population entity, after which the population generates an agent per node. At creation, an agent is assigned an ID and random strategy and is linked to the corresponding network node. After all agents are generated, each agent stores its neighbouring agents in its neighbourhood list variable. A pseudocode overview for initialisation is provided in algorithms 1 for the population and algorithm 2 for the agent.

Algorithm 1: Population initialisation

```
input  : Generated network object G
output: Generated agent objects within population object
def populate(G):
    agents ⟵ ∅
    agentID ⟵ 0
    for node in G:
        agent ⟵ create Agent(node, agentID)
        add agent to agents
        agentID += 1
    for agent i in agents:
        agent do store neighbouring nodes in list K_i
    return agents
```

4.2.6 Input Data

Our model makes no use of external data for initialisation or processes during simulations.

4.2.7 Submodels

All model parameters and hyperparameters are listed in the following table.

Algorithm 2: Agent initialisation

input : *node*, *agentID*
output: Generated agent object

```
def create Agent(node, agentID):
    agent ⟵ Agent Object
    agent id ⟵ agentID
    agent node ⟵ node
    agent strategy ⟵ random strategy()
    agent revenue ⟵ 0
    agent fitness ⟵ 0
    agent K_agent ⟵ ∅
    agent data ⟵ ∅
    return agent
```

Parameter	Description
N	The total number of agents in the population
n_{rounds}	The number of rounds for a simulation
n_{sim}	The number of simulations per setting
$p_i \in [0, 1]$	The offer that a given proposer i will make
$q_i \in [0, 1]$	The acceptance threshold for a given responder i
$s_i \in S$	A strategy as a tuple (p, q) for a given agent i. $S = \{s_1, s_2, ..., s_n\}$ denotes the set of all strategies present in the population
$u_i \in U$	The utility received by a given agent i. $U = \{u_1, u_2, ..., u_n\}$ denotes the set of utilities for all agents
μ	The probability of exploring the strategy space by randomising strategy values p and q of an agent. Synonymous to the mutation rate in Evolutionary Game Theory literature
α	The amount of noise present in exploiting the strategy of another agent
π_{ij}	The probability of agent i exploiting the strategy of agent j
f_i	The fitness for a given agent i
$d_i \in D$	The degree (amount of connections) for agent i. $D = \{d_1, d_2, ..., d_n\}$ denotes the set of all degrees present in the population
$k \in K_i$	A neighbour from the set of neighbours K_i for agent i, such that $K_i = \{k_1, k_2, ..., k_{d_i}\}$
p_{rewire}	The probability of rewiring an edge for the Watts-Strogatz small-world network
γ	The value to which counts are raised to model preferential attachment for the scale-free network generator

Run of the program

Flowchart 4.3 shows a schematic representation of the occurrences during the complete program. When the program is run, a set of specified networks is generated according to algorithms 6 and 7. The number of networks generated is equal to the network settings times the amount of simulations set. All networks per network setting are individually generated but are however the same in hyperparameter settings.

After networks are generated, the subset of the networks corresponding to a network setting is passed to the simulation handler that initialises the population. At the end of each simulation, data of the Ultimatum Game is returned to the simulation handler. This data is stored in a DataFrame after which a new simulation commences with a new initialisation of the population, with no data or parameter settings present in the game process from the previous simulation. When all simulations for a given network setting have passed, simulations are run with subsequent network setting subsets of the generated networks.

After simulations for all network settings have ended, data is added together in a final DataFrame that is further used for analysis.

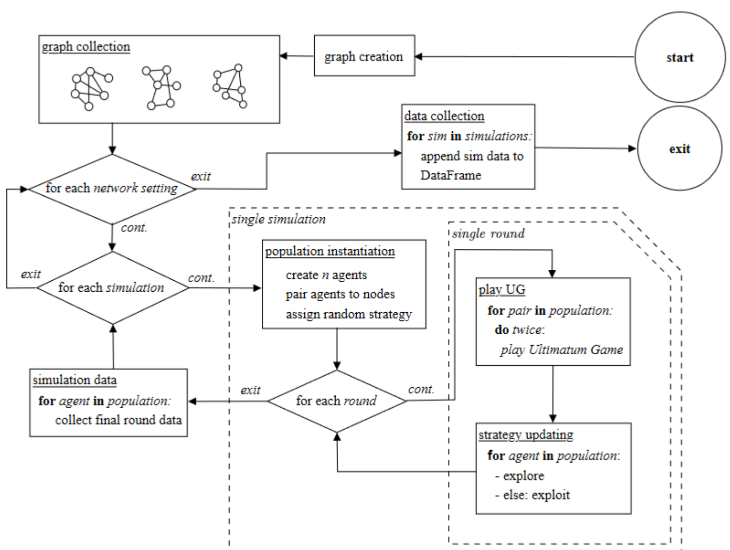

Figure 4.3: Flowchart showing the process of the total program. Single round description is summarised; more extensive description of the single round can be found in figure 4.2 and algorithms 3 and 5.

Ultimatum Game

The Ultimatum Game is an interaction between two neighbouring agents. Algorithm 3 shows the implementation of the actual game progression. In a single interaction, the proposing agent i shares their offer p_i after which agent j will either accept or reject the offer, dependent on whether $p_i \geq q_j$. If not, both agents receive nothing. More formally, the utility functions for the proposer u_{ij} and responder u_{ji} are defined as:

$$u_{ij} = \begin{cases} 1 - p_i, & \text{if } p_i \geq q_j \\ 0, & \text{if } p_i < q_j \end{cases}$$
$$u_{ji} = \begin{cases} p_i, & \text{if } p_i \geq q_j \\ 0, & \text{if } p_i < q_j \end{cases} \tag{4.1}$$

Strategies

A strategy for agent i is defined as a tuple $s_i = (p, q)$ with $(p, q) \in [0, 1]$. Values for p and q are selected uniformly at random.

Algorithm 4: Random Strategy Generation

```
def random strategy():
    p ⟵ value ∈ [0,1] sampled uniformly at random
    q ⟵ value ∈ [0,1] sampled uniformly at random
    strategy ⟵ (p, q)
    return strategy
```

Update Sequence

Algorithm 5 shows the code for the update sequence that is referenced in algorithm 3. The update sequence submodel is developed such that concurrence is emulated in updating: it is ensured that no agent updates its variable values before all other agents have calculated their new values.

Update Rule

In the updating sequence, agents that do not explore have a chance of exploiting their exemplar neighbour's strategy. The chance of exploiting is calculated with the fitness of the agent and its exemplar neighbour according to the Fermi equation for pairwise comparison. The Fermi imitation rule allows for more local exploration in the strategy space around existing strategies. The Fermi imitation rule places non-zero probability on any strategy an exemplar may have. Probabilities do reflect how much better or worse an exemplar's fitness is compared to an agent's own. For an agent i, s_i becomes s_j with probability

Algorithm 3: Evolutionary Ultimatum Game

```
input  : population, edgelist, n_rounds
def play(population, edgelist, n_rounds):
    for n in range(n_rounds):
        for pair in edge list:
            /* Ultimatum Game is played twice per pair.  Here
               depicted in the randomRoles-configuration.    */
            for i in range(2):
                /* Assign player roles at random.             */
                proposer ⟵ random choice from pair
                responder ⟵ (pair − proposer)
                if proposer p ≥ responder q:
                    u_proposer += 1 − p
                    u_responder += p
    /* Agent degree duplified due to double interactions per
       neighbour.                                              */
    for agent i in population:
        income ⟵ u_i
        f_i ⟵ income / (2 ∗ d_i)
        agent's exemplar j ⟵ random choice from K_i
        agent data ⟵ (s_i, income)
        if n_rounds ≠ final round:
            update sequence(population)
    /* When the final round has ended                          */
    dataset ⟵ ∅
    for agent in population:
        add agent data to dataset
    return dataset
```

$$\pi_{ij} = \frac{1}{1 + e^{-\beta(f_j - f_i)}}$$

Note that if $f_j = f_i$, the probability of imitating that neighbour π_{ij} is .5.

The β in the Fermi equation denotes the selection pressure. For higher values for β, pressure to exploit strategies with higher fitness becomes increasingly deterministic. For lower values, the probability of imitating neighbours becomes less influenced by the relative difference between agents' fitness values.

Performance Measures

The utility for an agent i is defined as the sum of its payoffs resulting from playing the Ultimatum Game with all its neighbours,

Algorithm 5: Update Sequence

```
input  : population
def update sequence(population):
    updateList ⟵ ∅
    for agent i in population:
        if random value ∈ [0,1] < μ:
            add agent to updateList
            new strategy for agent ⟵ new random strategy
        else:
            π_ij ⟵ update rule(agent i, agent's exemplar j)
            if random value ∈ [0,1] < π_ij:
                add agent to updateList
                p, q ⟵ strategy of agent's exemplar
                new strategy for agent ⟵ (p ± α,  q ± α)
    /* Agent strategies are updated only after all agents
       have calculated their new strategy.                */
    for agent in updateList:
        agent strategy ⟵ new strategy for agent
```

$$u_i = \sum_{j=1}^{K_i} u_{ij}. \tag{4.2}$$

The fitness for an agent i is then defined as the payoff the agent has received per interaction, weighed by its amount of neighbours:

$$f_i = \frac{u_i}{2d_i} \tag{4.3}$$

Note that the amount of neighbours is doubled, since the agent has two interactions with each neighbour. The fitness of an agent is reset to zero before the following round commences.

An alternative definition for fitness used in our second experiment takes into account the amount of neighbours of the exemplar. Instead of weighing the utility of agent i by its own degree, u_i is weighed by the maximum of agent i's and its exemplar j's degree d_j:

$$f_i = \frac{u_i}{2\max(d_i, d_j)} \tag{4.4}$$

This is done to increase the weight of degree-heterogeneity between neighbours in exploiting strategies.

Network generation

The networks that determine the population structure are intended to resemble real networks as can be found in social groups. As explained in the background section, small-world networks and scale-free networks resemble such real social networks in their clustering and hub-forming, respectively. The display of these social network characteristics underlies our focus on these two network types in this project. Networks are generated using the Networkx package for the creation, manipulation, and study of the structure, dynamics, and functions of complex networks.

Watts-Strogatz (Small-World) Networks The small-world networks generated according to the methods of Watts and Strogatz have been chosen for their clustering property which can be varied from regular ring lattices with high APL and high CC, through small-world networks with low APL and high CC, towards random networks with low APL and low CC. The algorithm for small-world network generation is shown in algorithm 6.

Algorithm 6: Watts-Strogatz Network Generator

input : N, d, p_{rewire}
output: Small-World Network

```
def watts_strogatz_graph(N, d, p_rewire):
    G ⟵ networkx.Graph()
    /* Create a ring by connecting all nodes                    */
    nodes ⟵ list(range(N))
    /* Then let each node add its d/2 neighbours s.t.
       finally each node has d neighbours                      */
    for j in range(1, (d/2 + 1)):
        targets ⟵ list(j, j+1, ..., N, 0, ..., j-1)
        add edges between bijective pairs from (nodes, targets) to G
    /* Then rewire edges with probability p_rewire              */
    for j in range(1, (d/2 + 1)):
        targets ⟵list(j, j+1, ..., N, 0, ..., j-1)
        for node, target from bijective pairs in (nodes, targets):
            if random value < p_rewire:
                new neighbour ⟵ random choice(nodes)
                while new neighbour == node or edge already exists:
                    new neighbour ⟵ random choice(nodes)
                else:
                    remove edge from G
                    add new edge to G
    return G
```

Barabàsi-Albert (Scale-Free) Networks Barabàsi-Albert scale-free networks are generated with the concept of preferential attachment. d edges are introduced to a graph structure after which sequentially a new node is added and attached preferentially to the existing nodes, with nodes with higher degree having higher probability of receiving an edge.

Algorithm 7: Scale-Free Network Generator

```
input  : N, d, γ
output: Scale-Free Network
def scale_free_graph(N, d, γ):
    G ⟵ networkx.Graph()
    /* Create a ring by connecting all nodes                    */
    nodes ⟵ list(range(N))
    /* Then let each node add its d/2 neighbours s.t.
       finally each node has d neighbours                       */
    for j in range(1, (d/2 + 1)):
        targets ⟵ list(j, j+1, ..., N, 0, ..., j-1)
        add edges between bijective pairs from (nodes, targets) to G
    /* Then rewire all edges                                    */
    for j in range(1, (d/2 + 1)):
        targets ⟵list(j, j+1, ..., N, 0, ..., j-1)
        for node, target from bijective pairs in (nodes, targets):
            new neighbour ⟵ preferentialChoice(nodes)
            while new neighbour == node or edge already exists:
                new neighbour ⟵ preferentialChoice(nodes)
                if degree for node ≥ n-1:
                    break out of loop
            else:
                remove edge from G
                add new edge to G
    return G
```

The original Barabàsi-Albert scale-free network generation algorithm was not fitting for our research as it did not allow for a smooth transition from low to no preferential attachment, to a desired amount. Furthermore, with the original algorithm no setting could be created that is completely random or shows no further influence from preferential attachment. Therefore we have composed a new algorithm fitting to our intended use.

For the generation of our desired scale-free networks we have adapted the small-world network generator due to its ease in adaptation and the possibility of creating a structurally simpler graph that is equivalent in nodes and number of edges, which can then be rewired. Instead of picking a new node uniformly at random, a probability distribution is calculated based on nodes' degree at that time. Algorithm 7 shows the main algorithm for the scale-free network generator

Algorithm 8: Preferential Attachment

```
input  : N, G, γ
output: Target For New Neighbour
def preferentialChoice(nodes, G, γ):
    /* Take degree of nodes from G                                  */
    counts ⟵ list(node degrees from G)
    /* Raise all counts by γ and afterwards divide by sum of
       counts to calculate preferential probability                 */
    counts ⟵ counts^γ
    counts ⟵ counts/sum(counts)
    /* pick random node weighed by its preferential
       probability                                                  */
    target ⟵ random choice(nodes, probability distribution = counts)
    return target
```

which differs only slightly from the algorithm for small-world networks (6), with the most important distinction being the rewiring of all edges. Algorithm 8 shows the preferential selection of new target nodes.

Our implementation of a smoother preferential attachment centres around γ, the exponent to which counts are raised to emphasise degree differences in the calculation of attachment preference. With the current implementation, for $\gamma = 0$ a regular random graph is returned. For $\gamma = 1$, our network behaves as the original algorithm from Barabàsi and Albert.

Standard deviation equations

A further formulation necessary before discussing our experiments is that of standard deviations within- and between populations. The first formula denotes the mean of standard deviations for strategy offer values p. This indicates the average spread of strategies within each population over the given amount of simulations and thus indicates how homogeneous populations were on average for a given setting. On this we reason whether population members still showed discordance in their strategies and utilities in the final round.

$$\overline{\sigma_p} = n_{sim}^{-1} \sum_{i=1}^{n_{sim}} \sqrt{N^{-1} \sum_{j=1}^{N} (p_j - \overline{p})} \tag{4.5}$$

The second formula pertains to the standard deviation of average p-values that are found for each population within that setting. This thus focuses on how strong results for given simulations coincide with each other. With this we evaluate to what degree a final result is actually predictive of the experimental settings due to similar results occurring in consecutive simulations or whether the final average strategy values and utility are not representative because outcomes are less conclusive due to broader spread. Π refers to the set of all p-values across simulations, whereas $\overline{p}$ represents the average offer strategy value of a single simulation.

$$\sigma_{\overline{p}} = \sqrt{N^{-1} \sum_{i=1}^{N} (\overline{p_i} - \overline{\Pi})} \tag{4.6}$$

Chapter 5

Results

In this chapter we will present our findings for the Evolutionary Ultimatum Game played in structured populations. First, preliminary studies on hyperparameter setting and implementation details are presented with their influence on population convergence. After, we will explore the influence of population structure on the evolution of fair negotiation behaviour in the Evolutionary Ultimatum Game.

5.1 Experiment 1: Clustering in populations

For our first experiment we set out to answer the following research question:

Research Question 1 *How does clustering in the topological structure of a population influence the evolution of fair negotiation behaviour in the Evolutionary Ultimatum Game?*

First part of our hypothesis is that for graphs with high Clustering Coefficient (CC) we will find that average population strategy values $\overline{p}$ and $\overline{q}$ converge toward $(0.5, 0.5)$ and will be higher than for graphs with lower CC. The second part of our hypothesis is that for graphs with high CC we will find that utility standard deviation σ_u will approach zero and will be lower than for graphs with lower CC.

Convergence to higher population average strategy values $\overline{p}$ and $\overline{q}$ is expected due to established strategies in clusters profiting from utility gained from neighbours with similar strategies. Additionally, due to recurring interactions p and q values can further consolidate due to noise in strategy imitations. Lastly, in being played by multiple agents, more agents must be converted to an invasive strategy before an established strategy disappears from the population. With similar strategies in one's neighbourhood, an agent is more likely to revert back to the established strategy after being converted, making clusters serve as memory for established strategies and a buffer against invasions.

Our expectation of lower standard deviation in final utilities is based on the (expected) presence of higher average $\overline{p}$ and $\overline{q}$, resulting in fairer distributions of utilities.

5.1.1 Experimental Setup

For our first experiment, the Evolutionary Ultimatum Game is played with a population of $N = 60$ agents with average degree $\overline{d} = 4$. The game is played over 10.000 rounds, and we average over 100 simulations for each value of p_{rewire}. For each simulation, a small-world network is generated according to algorithm 6 with associated p_{rewire} so as to take into account stochasticity in network generation. The exploration rate μ is set to 0.001 to ensure the introduction of new strategies with low enough probability to allow existing strategies to further develop. For exploitation, agents update strategies according to the Fermi pairwise comparison rule with selection intensity $\beta = 10$ to ensure that strategies with lower performance can also be adopted. For subtle exploration surrounding existing strategies through noise, we set $\alpha = 0.01$.

The main variation in this setup is the value for the rewiring probability p_{rewire} with which we generate our networks using the Watts-Strogatz small-world network generator. Through the adjustment of values for p_{rewire}, we find graphs generated with differing values for APL, CC and SP_{graph}. We allow for appropriate amounts of stochasticity as to provide enough room for agents to explore the strategy space without losing the possibility for introduced strategies to get a foothold in the population.

For the experimental setup we performed two preliminary studies on the effects of average node degree and rewiring probability on structural properties and population strategy convergence for the current context. Before stating the results, we will briefly discuss the findings from these studies to further motivate the choices made for the setup.

Preliminary Studies

Average Degree For lower average degree, populations were found to converge to fairer strategies whereas with higher average degree, populations averted to the subgame-perfect Nash solution of the classical Ultimatum Game, $(p, q) \rightarrow 0$ (Page et al., 2000; Iranzo et al., 2011). Though this was found for the Ultimatum Game played on one- and two-dimensional lattices with two, four and eight neighbours, these findings suggest that node degree has a strong influence on population convergence regardless of topology. Therefore we conducted a preliminary study on the effect of average node degree on average population strategy. Since average degree is of importance to the generation of Watts-Strogatz small-world networks, we report as well the influence of average degree on structural network properties.

From the top plot on the influence of average degree (figure 5.1) a strong initial decline in converged population strategy values is visible for average neighbourhood sizes lower than 8% of total population size N. This follows the

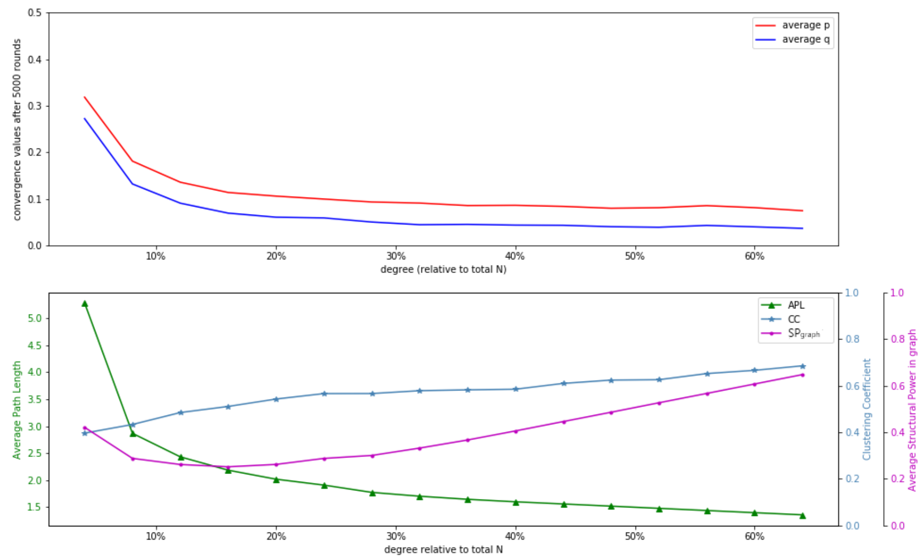

Figure 5.1: Plot showing influence of average degree $\bar{d}$. Top plot shows convergence of average population (p, q) for realisations with different degrees. Bottom plot shows the progression of APL, CC and SP_{graph}. The Evolutionary Ultimatum Game was run with $N = 100$ agents and exploration probability $\mu = 0.005$ on a small-world network following $p_{rewire} = .1$.

findings from Iranzo et al. (2011) who report a strong decline in strategy values in changing the neighbourhood size from 4 to 8 in a population of $N = 100$. In the remainder of the plot the values for p and q continue to decline in a less drastic manner.

The bottom plot from figure 5.1 shows the influence of average degree on Average Path Length (APL), Clustering Coefficient (CC) and average Structural Power for nodes in the graph (SP_{graph}). For APL we find an effect similar to the strategy values, with a strong decline for values until 8% of total N. A similar but weaker effect is found for SP_{graph}, with values increasing from 16% onward. Regarding CC, we see a slow increase along the values for average degree.

The importance for APL, CC and SP_{graph} stem from the amount of adoptions needed for a strategy to spread through a population and the amount of strategic overlap agents have. For a fair strategy to establish, multiple agents must adopt the strategy such that the utility gained from playing with each other can outweigh the utility gained from playing intrusive strategies. For fair strategies to consolidate to quasiempathetic strategies, it is further needed that interplay with similar strategies is prolonged. For high APL, on average more adoptions are needed for a newly introduced strategy to reach any other agent in the population. With less local competition, a fair strategy is less likely to be intruded by an opportunistic and rapidly adopted strategy. For high CC and SP_{graph}, the overlap in neighbourhoods for agents is larger. A single strategy is therefore likely to be played by a larger set of agents that improve the strategy through local exploration. A strategy being played by a larger set of neighbours also has a lower likelihood of disappearing since more agents need to adopt a new strategy for the old to disappear.

Initial increase in average degree in a network shows a strong influence on APL and average p and q. It is undesirable to have additional influences on values for p and q, because such influences can lead to the over- or underestimation of the effect of population structure on strategy convergence. Therefore we select $\overline{d}$ such that repressive effects of high $\overline{d}$ on $\overline{p}, \overline{q}$ are avoided, while providing enough stability for differences in d that may occur between agents in a population. With this reasoning we have chosen to use average degree values around 8% of the total population size.

Rewiring Probability: For our first experiment we generated different graphs created with the Watts-Strogatz small-world network generator. The network generator attains small-world phenomena through placing agents on a regular ring and then rewiring each edge with a certain probability. For low probabilities, we are likely to end with a network structure close to regular rings. For high probabilities, networks increasingly resemble random graphs. Both graph types are defined by different structural properties.

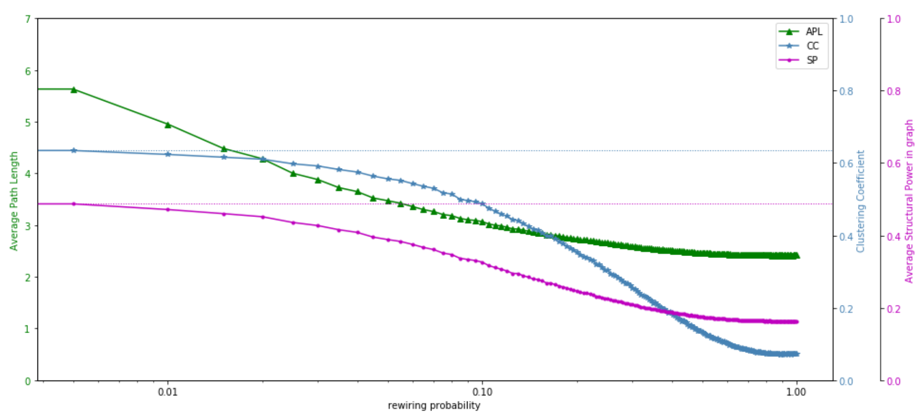

Figure 5.2: Plot of the influence of rewiring probability on structural properties for networks generated with the Watts-Strogatz small-world network generation algorithm. 200 networks were generated in the range [0, 1] with steps of 0.005, $N = 100$ and $\overline{d} = 8$. The horizontal axis is displayed in logarithmic scale.

To decide on an appropriate set of rewiring probabilities, we generated and studied networks for different values of p_{rewire}. Results are depicted in figure 5.2. APL shows a decline for higher values of the rewiring probability, with reduced effect for higher probability values. The increase in rewiring probability has an effect similar on both CC and SP_{graph} until $p_{rewire} = 0.1$. From there on, CC is more drastically affected by the rewiring of edges. This difference is explained by CC being computed as the amount of neighbourhood overlap for direct neighbouring nodes, whereas SP_{graph} takes into account indirect neighbours as well. A rewired edge therefore has less immediate effect on the set of nodes SP_{graph} is calculated on, in comparison with the set for CC.

Our set of probabilities used for the experiment consists of 21 p_{rewire} values from the range [0.0, 0.915]. These values are picked such that all values are equally spaced along found CC values. With the importance of clustering in mind, the values are based on CC so that we can compare population convergence in equal increases for clustering.

5.1.2 Results

We will now discuss our first experiment and results for the first research question following the experimental setup above. The results displayed in figure 5.3 and table 5.1 show higher final average population strategy values for low rewiring probability. A measure for small-worldness, ω, is included in the plot. For $\omega < 0$ we stray from small-world networks toward regular lattices; for $\omega > 0$ we stray toward random graphs. Our main result is that for rewiring probabilities in the range where $\omega < 0$, we see higher converged population $(\overline{p}, \overline{q})$ than in the range $\omega > 0$.

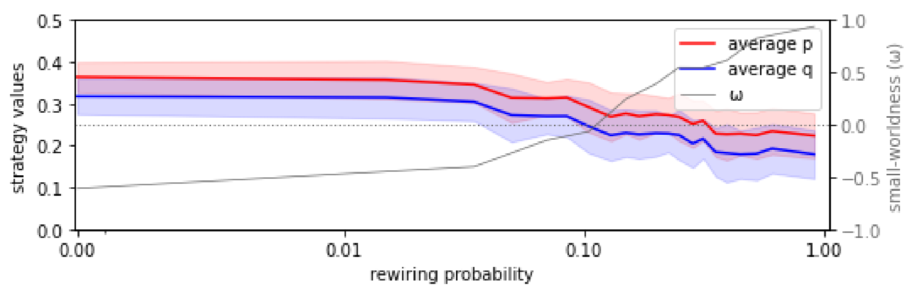

Figure 5.3: Plot for converged population strategy values p and q over small-world network structures with increasing rewiring probability. Shaded areas represent standard deviations across simulations. The horizontal axis is displayed in logarithmic scale. Gray line shows ω-measure for small-worldness, with $\omega = 0$ depicted as gray dotted line.

For each setting of p_{rewire}, population strategy values have converged such that $\overline{p} > \overline{q}$ and $\overline{q} > 0$. Furthermore, results show little difference in standard deviations for $\overline{p}$ and $\overline{q}$ between different rewiring probabilities.

The relation between APL, CC, SP_{graph} and $\overline{p}, \overline{q}$ is displayed in figure 5.4. Values for $\overline{p}$ and $\overline{q}$ increase for population structures with higher average path length, clustering and average structural power. This positive relationship emphasises the influence of population structure, and specifically clustering, on fair negotiation. Note that an increase for these structural properties equals a decrease for p_{rewire}. The influence for CC and SP_{graph} is relatively similar, with the main difference being the earlier rise of $\overline{p}, \overline{q}$ for SP_{graph}. As explained earlier, SP_{graph} and CC are closely related as both measure the overlap in

p_{rewire}	$\overline{p}$	$\sigma_{\overline{p}}*$	$\overline{\sigma_p}*$	$\overline{q}$	$\sigma_{\overline{q}}*$	$\overline{\sigma_q}*$
0.0	.3633	3.575	1.957	.3172	4.207	2.779
0.085	.3144	4.511	1.464	.2701	4.951	2.310
0.105	.2916	5.959	1.601	.2459	6.354	2.175
0.915	.2236	5.366	1.581	.1791	5.782	1.806

*: values $\times 10^{-2}$

Table 5.1: Highlight of simulation results for indicated values of p_{rewire}. $\sigma_{\overline{p}}$ denotes the standard deviation of the average values for p that are found for each population within that setting, indicating the dissimilarity between simulations. $\overline{\sigma_p}$ denotes the average of the standard deviations that are found within each population, indicating on average how dissimilar agents' p-values are within a population. A mathematical formulation for both $\overline{\sigma_p}$ and $\sigma_{\overline{p}}$ is given at the end of our methodological section, submodel 6.

neighbourhood size, with CC having the restriction of only measuring overlap for direct neighbours between agents.

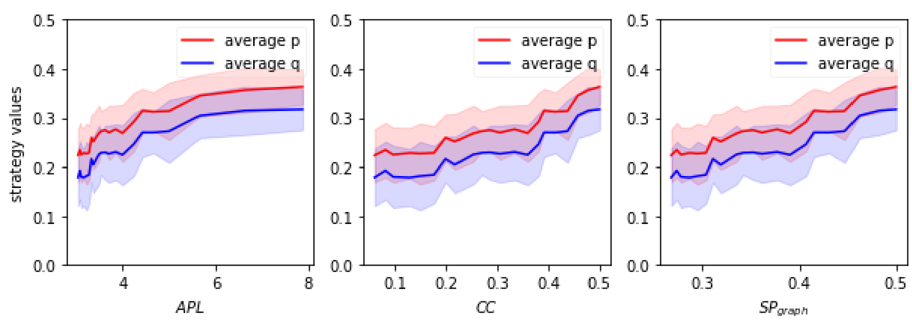

Figure 5.4: Plot for converged population strategy values p and q over increasing values for average path length (APL, left), clustering coefficient (CC, centre) and average structural power (SP_{graph}, right). Shaded areas represent standard deviations.

Figure 5.5 shows a plot for average income and average standard deviation of income in populations. The maximum utility agent i can receive with $d_i = 4$ in one round is 8, since the Ultimatum Game is played twice per neighbour. Regarding the average income of agents in the Evolutionary Ultimatum Game, rewiring probability has little effect on convergence values for $\overline{u}$, with only a slight increase for higher p_{rewire}. Values for $\overline{u}$ differ little between p_{rewire} settings, however within a setting do fluctuate between simulations as shown by the shaded area. The green line shows progression of the standard deviation of utilities within populations. We find for for lower values for p_{rewire}, utilities within populations differ less than for higher rewiring probabilities, meaning that clustered populations show a more egalitarian division of utility.

Four settings are selected for local analysis of the effects of clustering on fairness. These settings are the extrema for p_{rewire} to study fairness in populations for the lowest and highest p_{rewire}-values, and the two values closest to $\omega = 0$ before and after the tipping point of small-worldness coefficient ω. Data for the

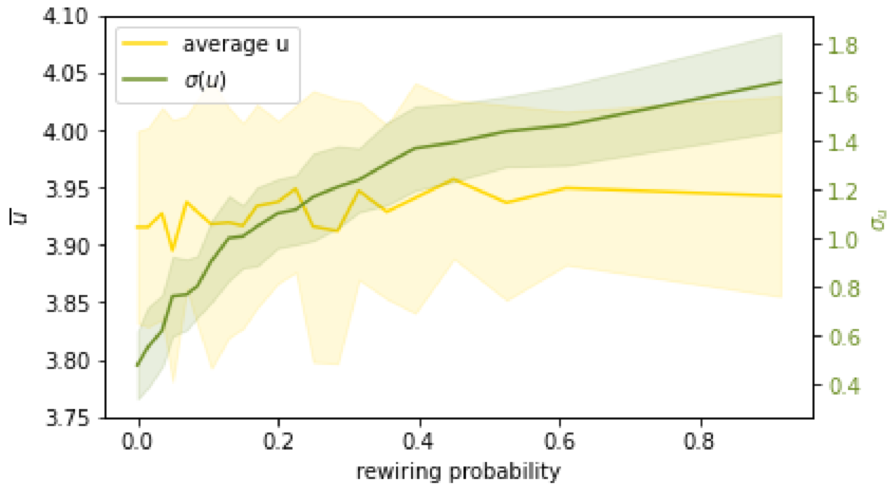

Figure 5.5: Plot for found average utility $\overline{u}$ and utility standard deviation within populations for increasing rewiring probability. Shaded areas represent standard deviations across simulations. Scale for σ_u is according to the right y-axis.

local analysis is shown in table 5.1 and figures 5.6 and 5.7. In the histograms from figure 5.6 we find the spread of all observed final values of p and q for agents in the given setting, for all simulations. The heatmaps display strategies (p, q) on a two-dimensional plane with lighter colours indicating a higher frequency of strategy observations within that bin for the given setting.

First discussing the histograms, we see the set of p and q-values widen and shift to lower values in the span of $p_{rewire} = .0$ toward $p_{rewire} = .915$. For the two top settings agents' p and q show low spread, suggesting that most values found are near $\overline{p}, \overline{q}$ for the given settings as indicated in table 5.1. For the bottom settings observed p and q are spread out more. From the standard deviations for p and q within and between simulations found in table 5.1, we derive that the spread for observed values is explained most by spread in values between simulations. This indicates that though final states differ between simulations, the final states of simulations themselves do show homogeneity in strategies and thus convergence to a strategy for a population.

In the heatmaps we find that for each setting of p_{rewire} the spread of agent strategies is along and above the main diagonal formed by $p = q$. This indicates that the modal strategy has $p > q$, as we have already seen in figure 5.3. For the top settings shown, we see smaller spread of values along with limited bright yellow coloured bins, indicating higher homogeneity of values for p and q. In the bottom heatmaps, strategies show wider spreads that are in total closer to the attractor $(p, q) \to (0, 0)$, the subgame perfect Nash equilibrium from the original Ultimatum Game.

For the four settings figure 5.7 offers a scatter plot showing further all strategy profiles found in the final state over all simulations. Different from the

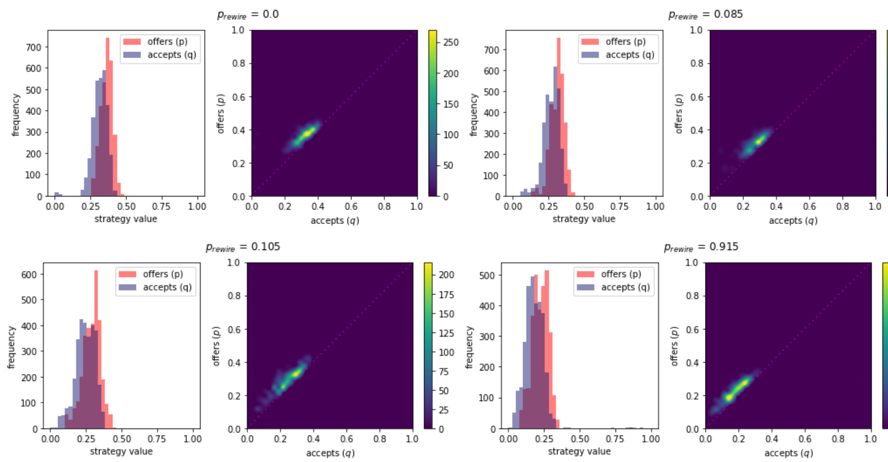

Figure 5.6: Histogram and heatmap showing distribution of p- and q-values for all simulations per highlighted p_{rewire}. The diagonal line is the identity line $p = q$. Bin size for the heatmaps is 2.5×10^{-2}

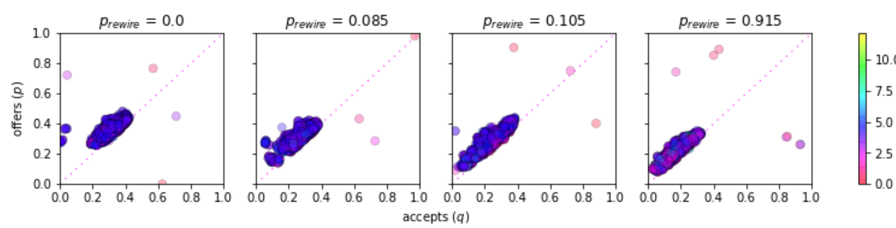

Figure 5.7: Scatterplot showing distribution of p- and q-values for all simulations per highlighted p_{rewire}. The diagonal line is the identity line $p = q$. The colour of a scatter point displays the utility for the agent it represents, values indicated by the colour bar on the right.

heatmap, the scatter plots show raw values for p and q. Agents' utility in the final round is mapped to marker colour. In the scatter plots, we see an increase in the deviation of agents' utility values as p_{rewire} increases. Furthermore, we see with increase in p_{rewire} that agents' strategies converge towards the rational strategy from the classic Ultimatum Game, $(p, q) \to 0$. As we have also discussed regarding the deviation between simulations versus within a simulation, we see in the scatter plots that for $p_{rewire} = .0$ and $p_{rewire} = .085$ agents' strategy values are more consistently present in the same area than for the two settings past ω.

5.1.3 Summary

With this experiment we set out to find whether clustering in population structure results in convergence to fairer play for the Evolutionary Ultimatum Game. Fairness was regarded here as values in offer (p) and acceptance threshold (q)

moving towards equal splits $(0.5, 0.5)$ as well as spread of utility in the end state of a simulation. With the use of the Watts-Strogatz small-world generator we found that in increasing the rewiring probability, leading to decreasing amounts of clustering (CC, SP) and average path length (APL), values for p and q decreased. This indicates that for populations in which agents are less associated with the neighbours of their own neighbours (CC), and agents also have less overlap in their common neighbours with each other (SP), offers and acceptance thresholds decrease. This is also to be said of a decrease in APL, indicating that a faster spread of information from one to another point in the population leads to agents resorting to more opportunistic strategies, as characterised by $(\overline{p}, \overline{q}) \rightarrow 0$. Regarding egalitarian notions of fairness, we further did not find an effect for the average amount of utility that agents received from their neighbours. We did however find that for low p_{rewire} agents in a population are more homogeneous in their income.

The effect of clustering on fair negotiation behaviour is explained by agents having a high number of neighbourhood overlap with their own direct neighbours (CC) and high overlap in their neighbourhood with indirectly connected agents (SP_{graph}). With shared neighbours, agents are likely to maintain similar strategies due to a shared influence on strategy selection and fitness, increasing homogeneity for subsets of the population. The formation of clusters furthermore makes it possible for cooperative strategies to consolidate, by p and q growing together due to local exploration. Lastly, sharing neighbours and having a higher degree of homogeneity within a cluster provides resilience to invasive opportunistic strategies. In line with the definition of an evolutionarily stable strategy, a fair strategy may not provide an agent with high utility when placed against more opportunistic strategies. If the utility gained from interactions with similar strategies is however higher than what an opportunistic strategy would yield against itself, the fair strategy is more likely to remain in the population. In the event of a cluster agent adopting an opportunistic strategy, the agent is still probable to revert to the former cluster strategy. This is due to its frequency in the cluster, the imitation model selection being a random process and worse performing strategies having a non-zero probability of being adopted. Due to the hardship for opportunistic strategies to spread, a cluster thus also functions as a filter that is less likely to fully adopt strategies if they are not truly more profitable.

For population structures determined by low p_{rewire}, we found further that observed values for p and q are more homogeneous than for population structures with high p_{rewire}. We explain this effect by the invasion of new strategies being delayed due to having to overcome different clusters before being able to take over a whole population. This conclusion is further strengthened by the fact that for findings in the region $\omega < 0$, values for $\overline{p}$ and $\overline{q}$ decrease only slightly though regular ring networks $(p_{rewire} = 0)$ and small-world networks $(p_{rewire} \approx 0.1)$ strongly differ in their APL. With lower APL, the total distance over a population an invasive strategy has to traverse is smaller, thus being quicker in being adopted across different parts of the population. Though APL for small-world networks is low, we still see relatively high values for $\overline{p}, \overline{q}$ that are close to values found in regular ring topologies. We believe that this reduced effect for low APL on $\overline{p}, \overline{q}$ is due to the high degree of clustering still present, which helps the obstruction of invasions and the retainment of opportunistic strategies.

5.2 Experiment 2: Degree heterogeneity

For our second experiment we set out to answer the following question:

Research Question 2 *How does degree-heterogeneity in a population affect the evolution of fair negotiation behaviour in the Evolutionary Ultimatum Game?*

We hypothesised that for networks that show heterogeneity in the degree distribution, such as scale-free graphs, we will find that populations converge to higher strategy values p and q. Furthermore, we expect a lower standard deviation of utilities σ_u when corrected for degree-heterogeneity. Values for $\overline{p}$ and $\overline{q}$ are expected to rise due to the presence of hub nodes that have a disproportionately high node degree compared to the rest of the population, allowing such hub nodes to be selective in the strategies they exploit. σ_u is expected to be smaller as a result of higher average population strategy.

5.2.1 Experimental Setup

To study the effect of heterogeneity in node degree, and thus the amount of connections agents have, the Evolutionary Ultimatum Game is played by populations for which the structure is defined by graphs that differ in their degree distribution. Our scale-free networks are generated according to algorithms 7 and 8 such that arbitrary node edges are rewired preferentially to nodes with higher degree. This preference for rewiring nodes is managed by an exponent, γ, for which we increase the value per setting. We will vary the rate from 0 to 2 with increments of 0.1 for a gradual progression from a random to scale-free degree distribution. Figure 5.8 shows 6 of total 21 degree distribution settings as an illustration of the increase in degree heterogeneity for used networks.

The game is played in a population of $N = 60$ agents, with each agent having degree $d = 4$ before edges are rewired. Small increases in initial degree settings are of little effect on results for the Evolutionary Ultimatum Game on scale-free graphs (Bo & Yang, 2010). Therefore we retain the same starting degree, and thus the amount of edges and interactions, as we used for our first experiment. The game is played over 10.000 rounds, with each setting consisting of 100 simulations. For each of these simulations, a network with given γ is generated so that we can take into account stochasticity in the process of generating the networks. The exploration rate is set to $\mu = 0.001$ so that the strategy space can be explored while again allowing existing strategies to further develop. The selection intensity β is set to 10 with the trade-off between

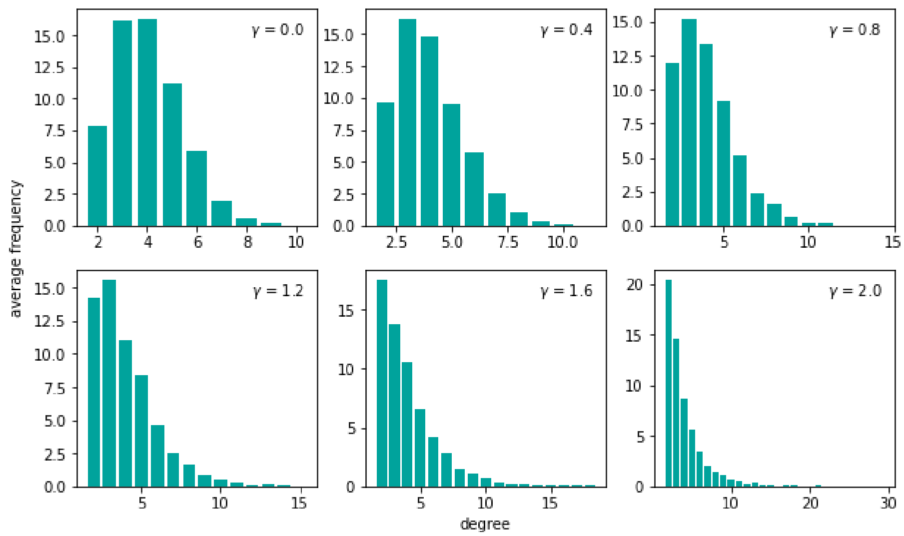

Figure 5.8: Degree histograms for a subset of graphs showing increase in positive skew of degree distributions as γ increases. Distributions displayed are created from average frequencies for the set of simulations for given settings.

deterministic imitation of better performing neighbours versus reduced influence of relative fitness differences in mind. Finally, for noise we again have $\alpha = 10$.

The aim for this experiment is to study the influence of node degree heterogeneity on fair negotiation behaviour in the Evolutionary Ultimatum Game. Therefore we vary the extent to which, during the rewiring of edges, nodes with higher degree are preferred over nodes with lower degree. This produces networks with increasing right-tailed degree distributions. The network generator is adapted such that for $\gamma = 0$ we have a random network, allowing the comparison of population strategy convergence between random and scale-free networks. For $\gamma = 1$, we find our network generator to behave as the original Barabási-Albert scale-free networks generator.

We distinguish between two cases for our results: the case in which fitness for an agent is calculated with its own degree, and the case in which this is calculated with the highest degree of the two agents that are involved in a comparison for exploitation. This choice is motivated by Bo and Yang (2010) who found drastic differences in results when degree-heterogeneity was not accounted for. Without accounting for degree-heterogeneity, hub agents have less security in being ensured of high payoffs. The strategy of an agent with only two neighbours can take over the hub node after a single lucky round. The hub node however has a large neighbourhood of agents that are likely to play similar strategies and thus return a previous hub node strategy in a consecutive round, serving as a form of memory. In order to reason on the influence of implicit versus explicit involvement of degree-heterogeneity on the hypothesised benefits of hubs we will run our experiment in both settings.

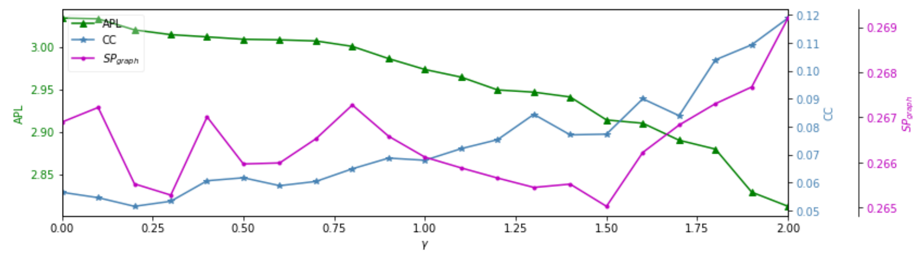

Figure 5.9: Plot showing the progression of average path length (APL), clustering coefficient (CC) and average structural power in the graph (SP_{graph}) for increase in γ.

Structural Properties for generated graphs Before discussing the results for our second experiment, we first elaborate on the structural properties of the generated networks. The progression for APL, CC and SP_{graph} is shown in figure 5.9. For APL and CC we can see a clear effect of γ. Note however that though a decrease for APL is visible, the effect is limited as the decrease is only small compared to the differences we have seen for small-world networks. The same can be said for CC. For SP_{graph}, the effect of γ is limited with a steady increase visible only for $\gamma \geq 1.5$.

Both random networks and scale-free networks in general show low APL, however for different reasons. In random networks, APL is low due to nodes being connected at random, with equal probability for nodes that are close and nodes that are further away. Consequently, a high number of population-spanning edges can be found. The low APL for scale-free networks however does not result from population-traversing edges. Rather, APL for scale-free networks is low due to most nodes being connected to a small set of hub nodes with high degree.

This difference in origins for low APL has implications for the spread of strategies through a network. Paths spanning a network for scale-free networks are more likely to overlap than for random networks. In scale-free networks it is therefore important for a strategy to be adopted by a hub node whereas in random networks, strategies can spread through different paths with less importance for adoption by specific nodes.

Regarding CC and SP_{graph}, these measures are both computed locally and averaged over all nodes to provide a global representation. Whereas nodes in random networks are likely to show similar local values, nodes in scale-free networks are highly heterogeneous in degree and therefore also in their neighbourhood size and neighbourhood overlap. This results in averaged values that are heavily reduced by the high number of nodes that show low local clustering and SP whereas we can witness high values for hub nodes. A clear example of the predictability for SP_{node} for degree heterogeneity can be seen in figure 5.15.

5.2.2 Results

Now we will discuss our second experiment and results for our second research question in the experimental setting we have discussed above. As stated in the experimental setup, the results will be split in two parts to shortly discuss the influence of implicit versus explicit incorporation of degree-heterogeneity in exploitation.

Fitness only weighed by agent's own degree

In the case of fitness being determined by an agent's own node degree, we found limited effect of heterogeneity in the amount of neighbours on the evolution of fair negotiation behaviour in the Evolutionary Ultimatum Game. Results for our experiment are displayed in figure 5.10. Values for converged population strategy values $\overline{p}$ and $\overline{q}$ are seen to be roughly equal for all settings, with $\overline{p} > \overline{q}$ and $\overline{q} > 0$. $\overline{p}$ maintains an equal distance above $\overline{q}$ across settings. Furthermore, standard deviations for simulations do not vary noticeably between settings.

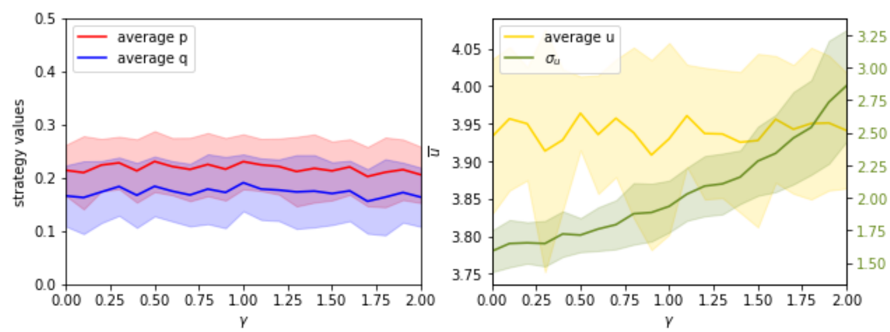

Figure 5.10: Plot for converged population strategy values p and q (left-hand side), and progression of average utility $\overline{u}$ and standard deviation σ_u (right-hand side) over different values for γ.

The second plot in figure 5.10 shows the progression of average utility and within-population difference in utility. Regarding the distribution of utility between settings for γ, we see no noticeable difference in the average utility that is found within a population across settings, apart from a more stable progression starting at $\gamma = 1.1$. The average utility is found to vary less between simulations for higher γ settings. We do find however that the standard deviation of utility increases for higher values for γ, indicating a less egalitarian population with regards to the income of agents.

Fitness weighed with exemplar's degree taken into account

For the case where agents take into account the node degree of their exemplar, we find a slight increase in converged population strategy values $\overline{p}$ and $\overline{q}$ for higher values of γ, as can be seen in the left-side plot of figure 5.11. For all settings, $\overline{p}$ is higher than $\overline{q}$ and $\overline{q} > 0$. Moreover, an increase in standard deviation can be seen as γ increases, leading us to believe that populations with heterogeneous degree distributions become more susceptible to advantageous mutations. Additionally, $\overline{p}$ and $\overline{q}$ diverge for $\gamma \geq 1.3$.

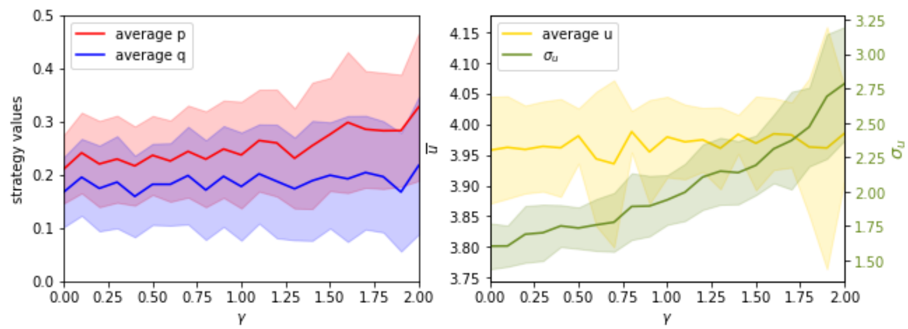

Figure 5.11: Plot for average strategy values $\overline{p}$ and $\overline{q}$ on the left-hand side, and for average utility $\overline{u}$ and standard deviation σ_u on the right-hand side, over different values for γ.

On the right side in figure 5.11 we see again the distribution of utilities and the within-population difference in utility. Noticeable is that found average utility values show less perturbance than those found for the setting in which node degree is not explicitly taken into account. Regarding the distribution of utility within a population, we do not find a remarkable difference, with inequality again increasing with an increase in degree heterogeneity.

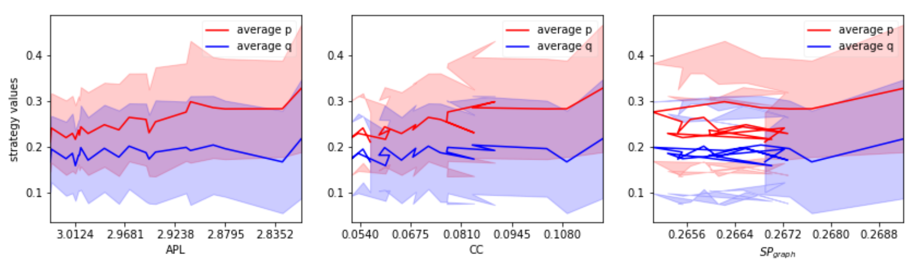

Figure 5.12: Plot for converged population strategy values $\overline{p}$ and $\overline{q}$ over increasing values for average path length (APL, left), clustering coefficient (CC, centre) and average structural power (SP_{graph}, right). Shaded areas represent standard deviations. Note that for APL, the horizontal axis is inverted such that progressions follow increase in γ. Middle and right plots seem distorted due to strategy values as well as SP_{graph} developing non-monotonically for initial settings.

The development of $\overline{p}, \overline{q}$ for APL, CC and SP_{graph} is shown in figure 5.12. Converged population values for $\overline{p}$ and $\overline{q}$ rise for decrease in APL. This may seem remarkable since though low APL is beneficial for opportunistic strategies, average population strategy is highest for $\gamma = 2.0$ with lowest APL. However, in discussing structural properties before the results we already explained that the difference in APL between random and scale-free networks is small in absolute sense, and low APL for both network types is of different origin. Regarding the influence of CC on $\overline{p}$ and $\overline{q}$, we observe a modest but turbulent rise with increase in clustering. This disturbed progression is caused by initial non-monotone progressions in population strategies and CC for different settings of γ. For SP_{graph}, little conclusions can be drawn on connections with the progression of $\overline{p}$ and $\overline{q}$. As we have seen in figure 5.9, SP_{graph} shows little relation with γ for generated networks.

The adjustment of γ for this experiment was used to generate networks that show increasing amounts of heterogeneity in its degree distributions, following scale-free networks. The actual heterogeneity in degrees for generated networks is calculated by the standard deviation of degrees found within a population. Figure 5.13 displays average strategy values $\overline{p}, \overline{q}$ plotted against found values for heterogeneity. For the left-side plot, we can see better the increase in $\overline{p}$ from $\sigma_d > 1.793$ (corresponding to $\gamma = 1.0$) and the divergence of $\overline{p}$ from $\overline{q}$ with both values showing a stronger standard deviation between simulations. In the right-side plot we notice that heterogeneity in income increases linearly with heterogeneity in degrees, however when divided by degree for each agent we find that the population becomes more homogeneous in their revenue from each interaction.

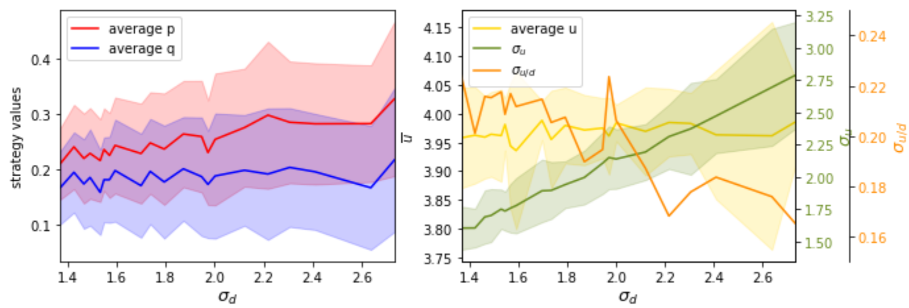

Figure 5.13: Plot showing converged $\overline{p}$ and $\overline{q}$ values on the left-hand side, and average utility $\overline{u}$, utility standard deviation σ_u and relative utility standard deviation $\sigma_{u/d}$ on the right-hand side, for average degree standard deviation.

Three settings are highlighted in figure 5.14 and table 5.2 that we will further analyse. All found strategies for the given settings are included in the plot. For all three settings we see that apart from a small set of outliers all strategies are converged s.t. at least $p \geq q$.

For $\gamma = 0.0$, found strategy values show moderate spread along the identity line with only slight deviation in u compared to $\gamma = 1.0$ and $\gamma = 2.0$. The spread for strategy values is explained mostly by differences between simulations, as can be deduced from table 5.2. The markers are of roughly equal size, meaning agents show little difference in their degree. This makes sense as $\gamma = 0.0$ is the setting for which the degree distribution is most homogeneous.

For $\gamma = 1.0$, we see a broader spread in strategy values along the identity line,

γ	$\overline{p}$	$\sigma_{\overline{p}}$*	$\overline{\sigma_p}$*	$\overline{q}$	$\sigma_{\overline{q}}$*	$\overline{\sigma_q}$*
0.0	.2106	6.433	1.209	.1675	6.582	1.536
1.0	.2369	10.01	1.129	.1778	10.14	1.495
2.0	.3271	13.86	1.770	.2173	12.91	1.573

*: values $\times 10^{-2}$

Table 5.2: Overview of simulation results for indicated values of γ. $\sigma_{\overline{p}}$ denotes the standard deviation of the average values for p that are found for each population within that setting, indicating the dissimilarity between simulations. $\overline{\sigma_p}$ denotes the average of the standard deviations that are found within each population, indicating on average how dissimilar agents' p-values are within a population.

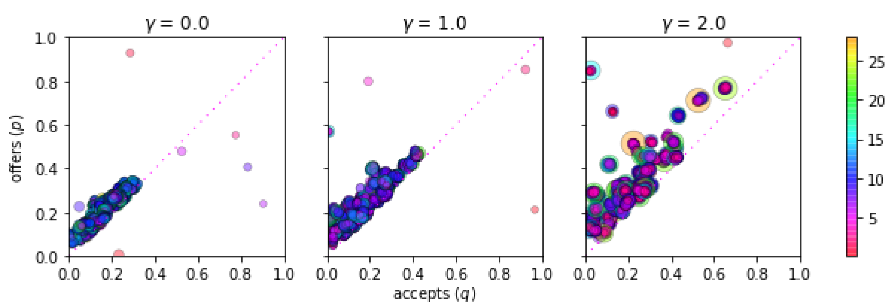

Figure 5.14: Scatter plot for the spread of values for p and q found in all simulations for given setting. Marker colour indicates utility for the represented agent with colour mapping according to colour bar on the right. Marker size represents agent degree. The diagonal line represents the identity line $p = q$.

however still with a high amount of overlap. From the table we can again deduce that the spread of strategies is on account of differences between simulations rather than within. Apart from a few outliers, the spread in strategies does seem to be in unison with the set of found values extending towards $p, q \to 0.5$. Noticeable is that strategies also spread out in perpendicular direction from the identity line towards the p-axis, indicating that values for p are diverging from q, compared to values in setting $\gamma = 0.0$. Additionally, we can see that markers differ slightly in size, indicating that degrees are more heterogeneously distributed compared to $\gamma = 0.0$. This is what we expected based on the degree distribution plots in figure 5.8.

For $\gamma = 2.0$ strategies are most spread out for all three settings. Values are found along most of the identity line, with a subset of strategies being found slightly below $p, q = (0.5, 0.5)$. Most strategies are however still concentrated within the area of the past two settings. For this setting, strategies show the least overlap as is also evident from $\sigma_{\overline{p}}$ and $\sigma_{\overline{q}}$ from table 5.2, again indicating that strategy values differ more between simulations than within.

The scatter plot shows a broader range of differences in colour for $\gamma = 2.0$ than for other settings, with brighter coloured markers also being larger in size. This correspondence between size and utility is to be expected since the maximum utility to be received is tied to the number of interactions and thus the degree an agent has. Something important to notice further is that each larger marker, representing a hub node, is accompanied by smaller markers sharing the exact same strategy. We reason that these smaller markers are hub node neighbours, for which the hub node is of substantial influence on their strategy and income. This explains why these smaller markers are positioned in the near-exact same location.

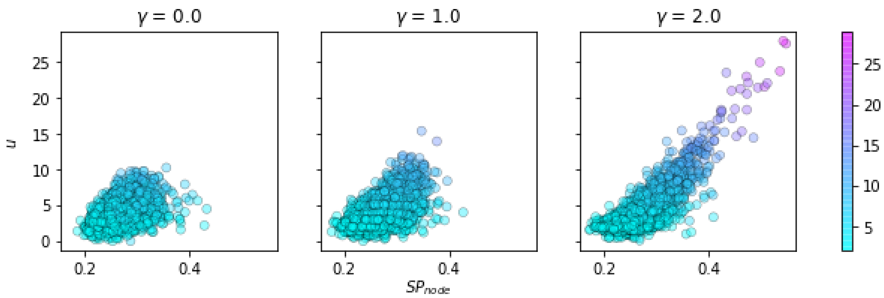

Figure 5.15: Scatter plot showing utility for individual structural power SP_{node}. Colour for markers indicates the amount of neighbours, with colours mapped according to colour bar on the right.

As indicated earlier, the average structural power found in a graph offers little information for further analysis as it is not discriminant between settings for γ. Figure 5.15 presents a scatter plot for all values found within indicated settings for γ. In this scatter plot, the individual structural power for nodes (SP_{node}) is plotted against the utility gained in the last round. For $\gamma = 0.0$, we see only limited differences in u as well as SP_{node} between agents. Discernible differences in SP_{node} are explained only by random degree differences between agents and coincidental differences in neighbourhood overlap. For degree we see no meaningful and structural difference in colour for agents. Differences in u are the result of both random degree differences and the high presence of opportunistic strategies.

For $\gamma = 1.0$, we can see a slight influence of degree heterogeneity on the spread of values. Though we still find similar values for SP_{node} as in $\gamma = 0.0$, agents spread out towards higher values for u. The increase for u for similar values for SP_{node} is explained by SP_node no longer being the result of coincidental overlap in neighbourhoods, but meaningful overlap resulting from the starting increase in degree for a subset of agents. With higher SP_{node}, this subset of agents has a greater influence on the average population strategy. Furthermore, interactions are assumed to centre more around this subset of agents than for agents with equally high SP_{node} in $\gamma = 0.0$. Involving the spread of strategies for $\gamma = 1.0$ in figure 5.14, we explain higher u to be due to the slight presence of hub nodes that are less prone to adapt their strategy and more likely to have higher values for (p, q).

For $\gamma = 2.0$, a strong increase is found for u as well as SP_{node} and degree.

The strong increase in SP_{node} is explained by a strong increase in degree found for agents and less overlap between agents with low degree. Furthermore, we see a straighter distribution of agents between SP_{node} and u, from which we infer that income for agents is determined more by actual differences in degree. Along with the strategy distribution from figure 5.14, we furthermore explain the narrower distribution as resulting from a greater part of interactions being defined by hub nodes and the lower presence of opportunistic strategies in the population.

5.2.3 Summary

In this experiment we set out to find whether heterogeneity in node degree in a population causes populations to converge to fairer bargaining behaviour in the Evolutionary Ultimatum Game. To answer this, we produced networks with increasingly heterogeneous degree distributions on which the Evolutionary Ultimatum Game was played. We again regarded fairness as offer- and acceptance threshold values $p, q \to 0.5$ and fair allocations as reduced deviation in the amount of utility each agent has at the end of the last round. Our reasoning for the expectancy of fairer strategy values, as a result of increasing degree heterogeneity, was in two parts. First, we expected that the disproportionately large amount of interactions for the hub node would result in the acceptance and development of fair strategies, and rejection of opportunistic strategies. Second, we expected average population strategy values to rise due to the increased influence of hub nodes on the rest of the population. This increased influence is due to hub nodes' neighbourhood size and further helped by their centrality in a network with low average path length, in turn causing a fast spread of fair strategies. There were then two cases surrounding the origin of selectivity for hub nodes. One assumes that selectivity arises implicitly due to neighbours serving as memory which allows hub nodes to revert to old strategies. The other implements selectivity by tying the calculation of imitation probabilities to neighbourhood size.

Selectivity arising implicitly A first finding was that for neighbourhood size to have effect on strategy convergence, it must be explicitly accounted for. The first case in the results, where an agent's fitness is only relative to its own neighbourhood size, showed no distinguishable effect of nodes with disproportionately higher neighbourhood sizes on population strategies. Fitness for hub nodes here was expressed as the average income from their interactions.

Though it was foreseen that hub nodes are equally likely to be influenced by better performing neighbours, we reasoned that the amount of interactions that are involved in weighing the agent's fitness caused hub nodes to be less affected by less fortunate interactions. This in our reasoning should have resulted in hub nodes remaining with a strategy longer, whilst bringing together values for p and q to reduce the possible difference between offers given and received. Also, neighbouring nodes were expected to serve as an additional measure for fair strategies to remain and evolve by functioning as a form of memory for the hub node.

Whereas we expected hub nodes to profit from interactions with a larger amount of neighbours, thus allowing for hub nodes to play more risky, fair strategies, hub nodes did not show influence from their higher degree apart from the utility that they received. We reason that the absence of influence is due to the tendency of hub node neighbours to quickly exploit others' strategies because of their low degree. In the case where the hub node changes strategies, hub node neighbours lose a substantial amount of their utility if the new strategy is incompatible with theirs. Rather than serving as memory, hub neighbours are therefore more likely to exploit strategies. In the case of remaining with said strategy, utility is reduced drastically such that the probability to be imitated by the hub node has shrunk severely. Changes in strategy for the hub node are thus definitive in the sense of having a low probability of returning to previous strategies. Without an explicit implementation for selectivity for nodes with higher degree, we thus do not find beneficial effects of degree heterogeneity on the evolution of fair behaviour.

Selectivity by explicitly accounting for neighbourhood size When we do account for neighbourhood sizes in calculating imitation probabilities, we find that population strategy values $\overline{p}$ and $\overline{q}$ converge toward 0.5 as degrees are more heterogeneously distributed. This leads us to the conclusion that a stricter selection procedure for hub nodes on which strategies to exploit, allows for the exploitation and development of fairer strategies.

As degree heterogeneity increased, $\overline{p}$ was shown to deviate more from $\overline{q}$. We explain this difference between values as being beneficial especially for the hub agent. This is in two parts. Firstly, with its neighbourhood size a hub agent is encountered with invasive strategies more often. A low acceptance threshold q allows a hub agent to gain revenue from such invasive strategies when playing as responder. Secondly, due to neighbourhood sizes of other agents being included in imitation probability calculation, agents are more likely to imitate the hub agent than the reverse. This reduces the time and severity for a hub agent to have intrusive, opportunistic strategies in their neighbourhood that yield less utility for the hub node than for the intruder. Without a foothold for opportunistic strategies to further develop, the costs of accepting and ratifying lower offers become less than the costs of receiving 0 instead.

Another finding for the influence of degree heterogeneity on fair negotiation behaviour was that heterogeneity in utility within a population is positively linearly related to degree heterogeneity. This finding is to be expected when for every agent i its strategy $s_i \approx \overline{s} = (\overline{p}, \overline{q})$, with $\overline{p} > \overline{q}$, such that income relative to agents' neighbours is equal and absolute differences are determined by how many interactions agents have. When corrected for degree however, we find that the standard deviation for utilities within populations becomes smaller with increasing degree heterogeneity. We believe this to be an effect of fairer offers being made. An important reminder here is that player roles are assigned uniformly at random: for each neighbour, an agent has a 25% probability of playing as responder twice. In a setting with $(\overline{p}, \overline{q}) \to 0$, the agent receives a minimal amount from the interactions with a neighbour. From these findings we conclude that relative to agents' degrees, degree heterogeneity in a population leads to more egalitarian, and thus fairer allocations.

Chapter 6

Discussion

In this project we set out to study the influence of population structure on the evolution of fair negotiation behaviour in the Evolutionary Ultimatum Game. For this, we sought to find how strategy convergence for populations is affected by two defining characteristics for real networks. One characteristic is clustering, the presence of overlap in connections between two acquaintances. The other characteristic is hub formation, due to heterogeneity in the amount of connections and thus the amount of influence people have. In this chapter we will discuss the findings on our experiments and relate our findings to the reviewed literature. We will discuss our two experiments separately before unifying these results in a general conclusion and suggesting future directions.

6.1 Clustering

Our first research question pertains to how clustering in population structure influences the evolution of fair negotiation behaviour in the Evolutionary Ultimatum Game. In our experiment we sought to prove that populations that show a higher degree of clustering in their structure will show convergence to values close to 0.5 for $(\overline{p}, \overline{q})$ as well as a more egalitarian distribution of utility. We found that for higher values for Clustering Coefficient (CC) and average Structural Power in the graph (SP_{graph}), populations converge to higher average offers ($\overline{p}$) and acceptance thresholds ($\overline{q}$) than for populations that show low CC and SP_{graph}. Though we have seen in our preliminary studies that low average path length (APL) is beneficial for the fast spread of opportunistic strategies, small-world populations, defined by low values for APL but high values for CC and SP_{graph}, showed only minimal decrease in $\overline{p}, \overline{q}$ compared to regular rings that possess higher APL. Furthermore, we found that for high CC and SP_{graph} agent strategy values were more homogeneous than for low CC and SP_{graph}.

Regarding utility gained in the Evolutionary Ultimatum Game, we found only little effect for the average utility for agents in a population, meaning that the amount of successful interactions in the final round was slightly less than for settings with lower CC and SP_{graph}. For populations with higher CC and SP_{graph} utility was slightly less than for less clustered populations. For the distribution of utility however we found lower standard deviation of utility for populations with higher CC and SP_{graph}.

We attribute the results of our first experiment to the implications of clustering for the spread of strategies. With more overlap in neighbourhoods between direct neighbours, as well as indirect neighbours, the spread of a strategy is more frequent on a local scale. This increases the frequency with which a strategy is played, causing strategy values to grow closer due to local exploration. Such consolidation of strategy values is needed for fair strategies to survive, along with the benefit of utility gained from interactions with similar strategies. This last necessity follows from the measures for Evolutionarily Stable Strategies (ESS). Here it is important to mention however that no strategy in this implementation can become a strict ESS, such that the only way it can be invaded is due to mutation. The strict ESS is defined in the context of evolutionary games with deterministic exploitation of strategies. Exploitation for our experiments is implemented as proportional imitation with imitation probability calculated following the Fermi equation. This places non-zero probability on worse performing strategies, thus making it possible for fair strategies to destabilise.

Regarding similar studies within the Evolutionary Ultimatum Game, our findings are in line with research from F. P. Santos et al. (2017) in that populations defined by higher CC and SP_{graph} show convergence in the direction of $(\overline{p}, \overline{q})$. A distinction to make however is that their research features a multi-responder approach in which values for $\overline{p}$ and $\overline{q}$ denote the share to be divided among all involved in an interactions, lowering the comparability of results. Furthermore, in their setting exploitation features two randomly selected agents per round that imitate with probability whereas in this implementation, all agents have a probability of exploiting as well as a probability of exploring. Further regarding the findings of Page et al. (2000) and Iranzo et al. (2011), we did not find our population to actually converge to quasi-empathic strategies ($\overline{p} \equiv \overline{q}$) before rising $\overline{p}$ and $\overline{q}$. In their studies, quasi-empathy is hypothesised to be required for a population to converge to stable, fair strategies, and to occur only in spatial societies. Our results however show that $\overline{p}$ and $\overline{q}$ are spread far apart, meaning converged population strategies are still sensitive to opportunistic strategies that have p_i such that $\overline{p} > p_i > \overline{q}$. This is possibly due to more sources of stochasticity in our implementation due to which which the further consolidation toward $\overline{p} \equiv \overline{q}$ is less profitable.

6.2 Degree-heterogeneity

Our second research question concerns how heterogeneity in degree distribution within a population affects the evolution of fair negotiation behaviour in the Evolutionary Ultimatum Game. We aimed to find whether higher heterogeneity in node degree causes populations to converge to population strategies close to fair splits as well as a more egalitarian distribution of utility. We motivated our expectation with two benefits of high node degree. Firstly, in having a high number of neighbours, the probability for an agent to be successful in part of its interactions is higher. Therefore higher selectivity on which strategies to adopt was expected. Secondly, due to its disproportionately high amount of connections an agent has more influence on the average population strategy at a given time. In combining the selectivity with the increase in influence, hub nodes were expected to guide the population towards fairer strategies.

A first finding was that though we assumed that high degree for hub nodes would be sufficient to elicit selectivity in which strategies to adopt, degree heterogeneity showed nearly no effect without explicitly including node degree in imitation probability calculation. The only effect we did find was an increase in inequality for the distribution of utility in a population. With an explicit implementation of selectivity however, we witnessed values for converged population strategy values $\overline{p}$ and $\overline{q}$ converge closer toward $(05, 0.5)$ compared to regular networks with no structural heterogeneity in degree. Converged population strategy values however did grow further apart as degree heterogeneity decreased. Regarding the structural properties for our generated networks, only APL and CC were somewhat predictive for the direction of $\overline{p}$ and $\overline{q}$. Our generated graphs did not show a consistent increase in SP_{graph}; therefore it did not show a strong relationship with the rise for converged population strategies. Regarding the distribution of utility in the explicit selectivity-case, we found little effect for degree heterogeneity on the average utility found in a population. This means that degree heterogeneity does not lead to higher payoffs or a higher ratio of successful interactions. Furthermore, heterogeneity in utility within populations was positively linear in relation to degree heterogeneity, suggesting that as populations become more heterogeneous, the distribution of utility does as well. However when controlled for agent degree, agents were shown to become more homogeneous in their utilities with increase in degree heterogeneity.

An explanation for why the implicit account of degree in selectivity did not show effects is that hub node neighbours were more likely than expected to exploit other agents' strategies. This was assumed to be due to their low degree, causing them to lose substantial part of their revenue if it is not successful in an interaction. In the case of the hub agent changing strategies, hub neighbours thus do not serve as a sense of memory for the hub agent, making strategy adaptions for the hub agents more definitive than expected. For the explicit case, fitness was determined by weighing utility with the maximum amount of neighbours between the two involved agents, rendering the hub node with the benefit of being less likely to adapt strategies that perform significantly worse. We further found $\overline{p}$ and $\overline{q}$ to increase for increases in degree heterogeneity. We explain this with the selectivity for the hub agent, the fast spread of hub agent strategies due to low APL with most paths running through the hub and the difficulty for opportunistic strategies to spread through the population due to most paths running through the hub. Furthermore, with most of the interactions taking place with a hub agent, the probability for hub strategies to dominate the overall set of strategies is high. Though models for imitation are selected at random, hub neighbours are highly likely to continually imitate the hub agent due to the weight placed on neighbourhood size in the calculation of imitation probabilities. Therefore an established strategy played by the hub agent is more likely to spread throughout the population than to disappear.

In comparing the effect of hubs on the spread of strategies, we find that star structures indeed increase the spread of strategies as shown by Lieberman et al. (2005). For our setting of the Evolutionary Ultimatum Game however increased selection from neighbours did not result in selection for fairer strategies. Rather, the selection of opportunistic strategies was favoured. This can be explained by the time necessary for a fair strategy to establish and to consolidate. Strategies with p and q far apart are easily beaten by opportunistic strategies with lower p that still allows for successful interaction. With a fair strategy with p and q close together, surrounding strategies are needed that play a similar strategy as the small distance between p and q does not allow for high utility otherwise. For this reason, the presence of star formations without an explicit implementation for selectivity is not likely to support the convergence of populations to fair strategies.

Comparing our results with Evolutionary Ultimatum Game findings from Bo and Yang (2010), we too find a weaker effect for degree-heterogeneity on fair negotiation behaviour than for clustering. Furthermore, fluctuations discussed by Bo and Yang (2010) for their scale-free network setting is possibly more descriptive of scale-free network processes than we thought. In their study, the fluctuations eventually lessen. It is probable that this disappears due to them having implemented a form of learning on the level of exploitation, in which agents may learn to cope with initial strategic turbulence. However, final distributions of $\overline{p}$ and $\overline{q}$ values still show spread and are far apart much like in the results of our experiments. Beyond that, we however find little agreement. Bo and Yang (2010) report that the presence of a small amount of nodes with disproportionately high degree in fact obstructs the evolution of fair negotiation behaviour, a conclusion that we do not draw from our results. There is however no strong expectancy of overlap in results. Firstly, the Scale-Free Network generator for this paper operates differently from the Barabàsi-Albert generator used in their research and comparable settings for them depend strongly on the amount of agents and the amount of nodes an agent preferentially attaches to. Second, Bo and Yang (2010)'s study employs a strictly larger amount of agents. Third, their research centres around an Evolutionary Ultimatum Game implementation where agents have incomplete information of the strategies of their neighbours. Such an extension fundamentally impacts agents' strategy development since imitation of neighbours' strategies requires learning what neighbours' strategies are.

6.3 Conclusion

In this project we examined the evolution of fairness in the Evolutionary Ultimatum Game under social network structures. Two research questions were formulated to address each of the two defining characteristics for social networks. Our first research question considers the influence of clustering in population structure on the development of fair negotiation behaviour. Our second research question considers the influence that degree-heterogeneity in population structure has on the development of fair bargaining behaviour. We designed two experiments to address each research question separately. Our experiments showed that both clustering and degree-heterogeneity led to higher offers and acceptance thresholds and a more egalitarian distribution of utility. These structural characteristics however did not lead to equal splits and an egalitarian distribution in the absolute sense. Still, the finding of fairer negotiation strategies and less inequality in outcomes, for both clustering and degree-heterogeneity, lead us to conclude that social structure in a population benefits the evolution of fair negotiation behaviour.

In the Evolutionary Ultimatum Game, only limited studies were performed to evaluate the influence of social structure on fair negotiation behaviour. Our results contribute to Evolutionary Ultimatum Game literature by introducing a model of bilateral negotiation in populations with social structure without further extensions. Moreover, this project confirms the direction of earlier findings in evolutionary games in that fairness and cooperation are favoured by population structure according to social networks. Our results further suggest that fairness considerations in human negotiation may not need to be accounted for explicitly in preference models. Rather, fairness can occur naturally in networks with dense social groups and networks in which highly influential individuals show selectivity in who they let themselves be influenced by.

The beneficiality of social structure for fair negotiation behaviour has implications for the design of automated negotiation systems that confront non-competitive distribution problems. Rather than extending preference models to reflect human users, fair and egalitarian allocations may be achieved by situating interactions in networks similar to human networks. In promoting fairness with mechanisms that are out of reach for an agent, fair negotiation behaviour becomes a rational choice rather than imposed. As a result automated negotiation may approximate human performance in social dilemmas not through the modelling of human preferences, but through the modelling of human constraints.

6.4 Future Work

There are few ways in which this work can be extended. In discussing the benefits of heterogeneity in degree, F. C. Santos et al. (2006) hint at the presence of interaction effects when small-world networks and scale-free networks are combined. Whereas low Average Path Length is generally seen as detrimental for the fixation of fair strategies, and random- and small-world networks are characterised by the presence of such population-spanning connections, hub nodes are said to ameliorate such problems. With the presence of clusters around hubs however, hubs may see a decrease in their dominating structural power over other nodes. With reduced influence on surrounding agents and thus reduced selectivity, hub nodes will be introduced to foreign strategies more often, being more prone to adopting self-interested strategies. Hub nodes can thus experience a reduction in their fairness-promoting characteristics. In such an event we could see hub nodes functioning more as amplifiers of strategies and less as selective strategy filters. This in turn contests the buffering property of agent clusters and the time for a strategy to be introduced throughout clusters. With clustering and degree-heterogeneity being the two important characteristics that describe structure in social networks, research into the concatenation of the two and the degrees to which these counteract one another is necessary for any system to be implemented according to social structure.

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