# Transfer of Embodied Experiences in a Tablet Environment Towards a Pen and Paper Task 

Thei J. D. Bongers

Department of Psychology, Utrecht University

201800482: Master Thesis Applied Cognitive Psychology

### 27.5 ECTS

First evaluator: Dr. A. Bakker<br>Second evaluator: Dr. I. Hooge

July 27, 2020


#### Abstract

Transfer is a key concept in learning theories: if someone manages to transfer knowledge to a new situation, it is evidence that this person has learned something beyond the initial experience.

However, how such process takes place is still under debate. Cognitivism stresses the importance of generalized mental schemes, which are created when learning and are used when situations are perceived as similar. Yet, situations that are overlapping are rarely perceived as such. As such, transfer between contexts is known to be challenging. Embodied cognition argues that a lack of transfer can be explained by a lack embodied experience. Because of this, students have difficulty recognizing affordances, which are perceived actions one can do within a situation. Some research has shown learning gained by embodied tasks can be transferred between contexts. In this thesis, we question what is actually transferred according to an embodied view on learning mathematics, in this case proportionality. For this exploratory research we investigate a tablet program meant to embody proportionality and its transfer towards a pen and paper task. From existing data from a previous study, we selected six participants (aged 8 to 10) by maximum variation sampling to show what different information is transferred from a tablet task towards a pen and paper task. We show that many different behaviors are transferred from an embodied task towards another medium. These findings raise questions about how to conceptualize transfer of embodied experiences.


# Transfer of Embodied Experiences in a Tablet Environment Towards a Pen and Paper Task 

According to Lamon (2007) up to $90 \%$ of adults have difficulty with proportional reasoning. This is problematic as proportional reasoning is often used in daily and professional life. To understand proportionality a person must shift from an additive approach towards a multiplicative approach. The struggle in this shift is apparent by the many errors people make when solving proportions such as adding an absolute fixed difference instead of a changing relative difference (example: "What number should X be? 3:5 = 6:X". Participant: "The difference between 3 and 5 is 2, the difference should stay the same. So, X should be 8 , because it is 2 more than 6 "). This difference is also apparent when doing an embodied proportion task; participants are asked to move their hands in an upward motion whilst raising the right hand twice as fast as the left hand. When performing this task people will keep the same distance between their hands (figure 1a through c). It appears that the mental error is grounded in the physical sensori-motor error.


Figure 1. Participant performing an embodied proportion task. The participant is asked to move the right hand twice as fast as the left hand. However, when performing the task, the participant keeps the distance between the hands the same, as indicated by the red line, moving them at the same speed.

In mathematics, students still make many numerical errors. A possible cause for this difficulty is that mathematics is often taught in too formal ways so that students are not involved in meaningmaking (Pouw, van Gog, \& Paas, 2014). In this way, mathematics is but a set of rules and equations, with little relevance to life. To cognitivists, after sufficient practice with the rules of a mathematical concept, students will make a symbolic schema in their mind to retrieve the rules of the mathematical concept with more ease. Transfer of this symbolic schema will only activate if it is perceived as relevant, which requires conscious effort (Tuomi-Gröhn \& Engeström, 2003). As such, transfer only occurs with conscious effort, which often goes wrong due to the relevance of a schema not being accurately perceived. However, in embodied cognition there is evidence that this error of transfer is due to lack of physical experience (Pouw et al. 2014; Shapiro \& Stoltz, 2018), if you overcome the lack of physical experience one will make less numerical errors. This physical experience is important because embodied cognition posits that (mathematical) concepts are grounded in sensori-motor schemas which emerge from the interaction of mind, body, and environment. However, research is still needed about how this interaction affects transfer.

Where cognitivists posit that conceptual knowledge is stored in schemas, embodied cognition posits that schemas are grounded in sensori-motor experiences (Pouw et al., 2014, Tuomi-Gröhn \& Engeström, 2003). In embodied cognition, if transfer does not occur it is because there has not been enough practice with the interaction of body, mind and environment to ground schemas in sensorimotor experiences (Shapiro \& Stolz, 2018). This is a more broader form of transfer, as it predicts that transfer can occur between different contexts without the need of prior conscious abstraction of a schema towards said contexts. However, within embodied cognition not much research has been done that focused on transfer between task. So far, what has been studied is the transition from the proportion 1:2 to $1: 4$ or 3:4 (Van Helden, Alberto \& Bakker, 2019). Instead, embodied cognition has thus far focused on creating learning environments that aid in grounding schemas, which has shown promise in teaching students concepts within the learning environments (Abrahamson, Shayan, Bakker, \& van
der Schaaf, 2015). In order to show that embodied environments can be a useful addition to mathematics education, it needs to show that transfer happens outside of these embodied environments.

For this thesis we focus on what is transferred from an embodied cognition tablet learning task towards a pen and paper task. Since embodied cognition is a relatively new movement within psychology, there has not been a lot of research on transfer of embodied tasks towards different tasks with different modalities. This research is essential, since showing that embodied cognition can transfer information between contexts will show support for embodied cognition. Proportionality is still very difficult to many people and as such requires more research for better ways to teach it. With this thesis we show that the embodied cognition is an addition to current teaching practices because embodied cognition tasks show transfer across media.

## Theoretical background

## Transfer: A brief history

Transfer is a term often used to describe learning beyond the initial experience. As such, when students gain understanding in a classroom, transfer is required to bring this understanding to the outside world. However, the problem with transfer is that it is notoriously difficult, and not fully understood. Transfer is a term that, in psychological terms, has often had its exact meaning changed, and as such requires some explanation on where this thesis stands (Tuomi-Gröhn \& Engeström, 2003).

Classic transfer was first described as the ability to train mental functions such as memory, attention, and judgement (Lobato, 2006; Tuomi-Gröhn \& Engeström, 2003). According to classical transfer, focusing on topics that promote critical thinking would transfer towards other skills. However, this was disproved by Thorndike who showed that students who focused on Latin and geometry, which were considered subjects that improve critical thinking, did not outperform students who focused on other subjects. Transfer is not a skill that can be trained by critical thinking. Thorndike's common
elements posited that transfer occurs to the extent of common elements between the learning situation and the other situation. The more the situations are alike, the more likely one transfers between them. Thorndike's ideas were later used by cognition researchers and reformulated as symbolic representations. When people are learning, they create symbolic representations of what they learn. Transfer occurs when they are in a situation in which symbolic representations are perceived as the same or overlapping with a previous situation. A problem with this is that perceiving overlapping symbolic representations rarely occurs without focusing on perceiving the overlap. Some posit that this poor transfer is due to the lack of grounding when learning (Pouw et al., 2014). Grounding is association information not just to memory, but to the environment and body as well. In terms of learning, we need to learn what the rules are of concepts, but also how they interact with the environment and how this influences or is influenced by the body.

## Embodiment

Where cognitivists believe that we learn with symbolic representation, and therefore transfer with symbolic representation, embodied cognition had questions about representations. Within embodied cognition there is the discussion about how much we rely on representations, ranging from less influence from representations to a more radical no representations at all (Marsh, Johnston, Richardson, \& Schmidt, 2009). Instead, embodied cognition posits that cognition is based on our activity within a richly perceived environment (Wilson \& Golonka, 2013). To influence or navigate this richly perceived environment, affordances are formed (Gibson, 1954). Affordances are the perceived actions one can perform on an object. As an example, a closed door without a handle has the affordance that you have to push to open it, as there is nothing to pull. Additional affordances of this situation are that you can slide to open it, kick it, disassemble it, knock on it, and many more depending on the environment and person. One could posit that the notion of affordance can replace that of transfer; instead of asking if something can be transferred one could also ask "does this task
provide enough opportunities to recognize an affordance? ". However, as previously mentioned, we defined transfer as learning beyond the initial experience. Transfer can still be used as a term to question whether a gained affordance is used in another environment. As an example, one could ask "can an affordance created in a digital environment be transferred towards an analogue environment?"

## Mathematics Image Trainer for Proportions

Inspired by theories of the importance of the human body in learning, Abrahamson and Howison (2010) explored ways of engaging students' bodies in learning about proportion. After many cycles of design research, they created the Mathematics Imagery Trainer for Proportions (MIT-P) in 2010. During the MIT-P, a participant is given two bars on a screen that can be manipulated, with the task: find out the rule that makes the screen green. The height of the devices is measured, and when the device hits a specific proportion, like 1:2, the screen will turn green. The participant will explore with the devices until they find a spot where the screen turns green and is then encouraged to find more spaces where it is green. After finding multiple spots, the participant is asked to find out what the similarities of the spots are and if they can keep the screen green continuously. This exploring and probing is continued until the participant realizes the devices are linked together by their height, after which the participant is encouraged to move the devices in a way the screen will remain green. Later a grid is introduced to help participants construe the rule towards discrete units, which is later followed by adding numbers. The participants are probed for the rule again, and often participants can recite the rule multiplicatively (e.g.: "One device goes twice as fast up as the other device"). So far, the MIT-P has shown a lot of promise guiding participants from an additive approach towards a multiplicative approach, both verbal and motor, and many new iterations of the MIT are being made and researched.

However, with the MIT-P the question remains; "what is learned during the MIT-P". Abrahamson and Howison in later research (2010) included two card sorting tasks as a pre and a post test for the MIT-P. In these card sorting tasks; participants would receive cards with numbers (task \#1)
or balloons on them. They were then asked to put in a sequence that "makes sense". Two sequences were anticipated, one where the cards were sorted by same difference sequence (E.g.: $[1,2][2,3][3,4]$ $[4,5]$ ) or by a proportional sequence (E.g. $[1,2][2,4][3,6][4,8]$ ). The second card sorting task was similar, but instead of numbers had balloons at different heights. The same intended sequences applied. The research showed that, before the MIT-P, the participants were unable to come up with the proportional sequence, whereas after the MIT-P participants were able create the proportional sequence, this applied for both the numbered cards as the balloon cards. It appears that some transfer has occurred, even though the pre and post task were like the MIT-P, the card sorting tasks were visually similar, repeated their movements. However, they received no feedback during the pre and post task, and yet were able to complete the post task. This shows that experiences during the MIT-P can be transferred towards across modalities.

## Transfer in Embodied Cognition

A good example of evidence of transfer in embodied cognition research is from Smith, King, and Hoyte (2014). The children were put in front of a screen and asked to make angles with their arms. The colour of the screen would change colour according to the angle they were making (differentiating between acute, right, obtuse, and straight). The researchers included a pre- and posttest where they measured how good the students were at ordering, estimating, and drawing angles. The pen and paper task had several differences with the screen task; Where the screen task focused on exploration of the rule (when does the screen change colour?), the pen and paper task was more of a test, only asking for results. Furthermore, two parts of the pen and paper task asked for specific angles (E.g.: draw an angle of 110 degrees), whereas the screen task only changed colors on set locations. The Pen and paper task provided no feedback, other than the feedback the participants self-generated (for instance, the drawing they made). The screen task is also rather unconventional, whereas pen and paper are very common in the classroom. Results from the pre- and post-test showed that participants got significantly better at estimating angles, showed an upwards trend for
drawing but showed no change in ordering. When you consider that the estimating and drawing task asked for angles with specific degrees whereas the screen task itself not asked for that, you could say that the participants transferred information about how angles work. So simply a task that shows the difference between acute, right, obtuse, and straight angles transfers information about angles to participants when estimating and drawing angles. So far, there is not much research that focuses on the transfer of embodied instructions, but this example shows that embodied instruction can be beneficial to the understanding of mathematics.

## Transfer and Drawing

For this thesis we will continue upon the work of Boven (2017) to focus on what is transferred from the MIT-P. In the MIT-P of Boven, participants completed a set of tasks on an iPad that uses orthogonal movements to demonstrate proportionality. As an addition, after the iPad tasks the participants completed a pen and paper sketching task which was different from the iPad task in many aspects (see table 1). Though there are many differences between the iPad task and the pen and paper task, both require understanding of proportionality to solve them, and as such can be used to measure transfer. However, within mathematics and embodied cognition, not much research has focused on the effect and usage of drawing, sketching or diagramming tasks, and as such this is a relatively new venture. One example of using a diagramming task to measure transfer is used by de Freitas and Sinclair (2012); students of geometry watched Nicolet films on circles and were asked to describe orally what occurred in the films. After watching the films and describing orally what happened three times, the participants were asked to diagram what they had just seen. From these diagrams the researchers were able ascertain several solution methods and as such were able to see how participants transferred different information. The difference between de Freitas and Sinclair's diagramming task and Boven's pen and paper task is that Boven provides extra aids to help solve the task. However, like in Freitas and Sinclair's diagramming task, Boven's pen and paper task have many possibilities to solve the task, and as such can be used to see what Boven's participants transfer from the iPad task.

## Our Research

We answer the research question; What do students transfer from the tablet task towards the pen and paper task? For this thesis we will expand previous research done by Boven by focusing on what is transferred from the orthogonal MIT-P to the pen and paper task. We use the task of Boven (2017) because there are multiple differences between the MIT-P and the pen and paper task (see table 1), most importantly removing the aids from the MIT-P, forcing the participant to recreate the aids that they need to complete the task. Furthermore, by removing the dynamic feedback from the MIT-P we require participants to use their own feedback (the traces they leave on the paper) This specifically shows what the participants transfer from the MIT-P. We have this focus to assess the MIT-P as a teaching aid, in hopes of increasing knowledge of proportionality. That participants are able to verbalize proportional reasoning within the MIT-P is promising (Abrahamson et al., 2015), however, if participants are unable to continue this approach in other tasks, it might just be that performing the MIT-P makes a participant able to perform the MIT-P better instead of developing a better understanding of proportionality. However, if the participants are able to complete the MIT-P, it is important to see how they solve the end task and see what kind of problem-solving behaviors the MIT-P elicits.

Table 1
Differences Between Tablet and Pen and Paper Task

|  | Tablet | Pen and Paper |
| :--- | :--- | :--- |
| Technology | High tech, uncommon | Low tech, very common |
| Accuracy <br> Feedback | Immediate and dynamic; <br> from technology | Static (feedback only provided when <br> asked for) and self-generated; from <br> person |
| Action history | None, rectangle is lost when <br> fingers lose touch of iPad | Pen leaves a trace on paper that can later <br> be checked |
| Usage of hands | Requires two hands | Requires one hand |
| Aids | X-axis, Y-axis, rectangle <br> that follows fingers. Later in <br> MIT-P, grids, and numeric <br> symbols | X-axis, y-axis, and a printed triangle. <br> Other aides must be added by participant |

## Method

## Participants

For this research we used data collected by Boven (2017). The data was collected during two periods, spring (22-06-16 to 5-07-2016) and fall (20-09-2016 to 2-12-2016). There were 42 participants ( 27 female, 15 male, $M_{\text {age }}=9.50$ years, $S D_{\text {age }}=0.35$ years). Of these participants, 22 were in the full screen and 20 were in the rectangle condition. From the rectangle condition we selected for maximum variation six participants.

## Materials

## Task

The task was a variation of Abrahamson et al. (2015) Mathematical Imagery Trainer for Proportions (MIT-P), designed at Utrecht University. The MIT-P is an interactive technological
device developed for students to create new sensorimotor operatory schemes for mathematical concepts and then aids them to mathematize these schemas.

Boven's research (2017) consisted of two parts. The first part was an embodied interactive task which was a variation of Abrahamson et al. (2015) Mathematical Imagery Trainer for Proportions (MIT-P). The MIT-P was developed for students to create new sensorimotor operatory schemes for mathematical concepts and then aid them to mathematize these schemes using standard frames of references. The idea behind the MIT-P is that using "an embodied-interaction computersupported inquiry activity for learners to discover, rehearse, and thus embody presymbolic dynamics pertaining to the mathematics of proportional transformation" (Abrahamson \& Trninic, 2015, p. 299), or in layman terms; using an interactive program to allow students to use their bodies to learn proportions will embody the mathematical concept.

The MIT-P is conducted on a multitouch tablet (iPad), where the position of the index fingers on the axes influences the color and/or the shape of an element on the screen. The left index finger moves up and down on the left most area of the tablet, whereas the right index finger moves left and right on the lower part of the tablet. The task had two types of feedback: rectangle and full screen. For this thesis, full screen feedback will not be discussed due to time constraints. In rectangle feedback the iPad would create a rectangular shape that would follow the position of the index fingers of the participant (see figure 2a). The rectangle would turn green if it hit the correct proportion of 1:2 (see figure 2b).


Figure 2a. Rectangle feedback. The rectangle follows the position of the index fingers. The black arrows show the direction the fingers can move. The proportion in this figure is $1: 1$, rather than $1: 2$, and thus the rectangle is colored red.

The first task had several phases to aid students to find the goal. The goal was to find out what the "rule" was when the screen or shape turns green, which was a proportion of 1:2. The first phase, exploration, was just the base task as portrayed in figure 2 a and b . The children were encouraged to think aloud, and the interviewer was seated next to them to aid them in their task. The first phase would end when the participant expressed the correct rule or when they were busy for too long (+ 10 minutes). The next phase included a grid that was meant to help them explain the rule better (see figure 3). The goal remained the same; find the rule for when the shape turns green. The participants who discovered the rule were asked to explain the rule with the use of the grid, the participants who could not find the rule were asked to try to find the rule with help of the grid. The grid phase ended when the participant successfully explained the rule, or when the participant could not find the rule for too long ( +5 minutes). After the grid phase the third phase started, the grid + numeral phase. This phase included the same grid as the previous phase, but also added numerals (see figure 4).


Figure 3. Third phase of the task-based interview; grid. A grid is added to aid the participant to find the rule, or aid explaining the rule.


Figure 4. Fourth phase of the task-based interview; Grid + numerals. In addition to the grid, numerals are added in order to aid the participant to discover or explain the rule.

The grid and numeral phase would end after successful explanation of the rule, or when the participant cannot find the rule for too long (>5 minutes). If the participant has not been able to find the rule up to this point, and extra phase was added where the experimenter would guide the participant to find the rule. After successful completion of these phases, the final phase of the tablet task started. In this phase the grid and numbers were removed, and a new rule was introduced; the rectangle would turn green with a proportion of 2:3. This new rule was mentioned to the participants, and their goal was to estimate where the rectangle would be green, and to explain why it would be green there. After five minutes this phase would end, regardless whether the participant was able to correctly explain this phase.

The final task did not utilize the iPad but was instead performed with pen and paper. The final task was added to test what behavior would be transferred to solve a similar proportion problem on a different, static medium. The final task consisted of a graph with a triangle with the proportion of 3:2 without any grids or numerals (see figure 5). There were several consecutive goals; First the participants were asked to create a bigger triangle with the same proportion, then participants were
asked to create an even bigger triangle with the same proportion, and lastly the participant was to create a triangle smaller than the original with the same proportions. In some instances the order of the tasks differed, depending on the participant. During the task, the participants were asked to explain what they were doing. If the participants made a mistake, they were first asked to explain what they were doing and why. If they did not correct their mistake, they were notified of this. If they were still unable to complete the goal, they were aided. The last task would end if the participant successfully completed the task or were unable to do the task even after being aided.


Figure 5. Final pen and paper task. Note that the iPad is turned off, and the task is instead performed on a piece of paper on top of the iPad.

## Video

In total 42 videos were recorded by Boven (2017). For this research we used the videos of participants in the Rectangle condition (20 videos). We did not use the videos of the full screen condition due to time constraints. Two videos had faulty audio, and as such we had 18 usable videos in total. From these videos we inspected the pen and paper task to select participants based on unique ways to attempt to solve this task. This is to show the different ways the MIT-P transfers towards the pen and paper task. From the 18 participants we selected six who showed unique solving techniques.

## Eye-tracker

During the entire experiment participants were eye-tracked using a Tobii X60. During the iPad tasks the data was generally good, as the participants did not move that much the eye tracker did not lose much data. However, when participants had to draw during the pen and paper task, they would often bend over, causing the eye-tracker to lose focus. Since the eye-tracking data in the pen and paper task is very poor it will not be our focus for this thesis.

## Data Analysis

Previous research (Shayan, Abrahamson, Bakker, Duijzer \& van der Schaaf 2015) has shown promise in guiding participants to understand proportions, going from additive to multiplicative approaches, and even teaching more complex mathematical concepts. However, research has not yet focused whether participants can transfer their comprehension of our task towards another task. It might just be that the MIT-P teaches participants to correctly complete the MIT-P, instead of deeper knowledge of proportions. Since mathematical knowledge is usually tested by pen and paper tests, it is important to see whether comprehension of our task can be transferred towards a pen and paper task. We will therefore try to observe what information from the iPad task gets transferred towards the pen and paper task. We will specifically focus on the transfer and emergence of problem-solving techniques.

## Results

## Participant one; Anna

## Tablet task

Our first student, Anna, was very fast when doing the tablet task. She showed capability of reasoning multiplicatively (example; "this (x-axis) needs to be twice as big as this (y-axis)"), as well as being able to explain the different speed of her fingers ("the right finger needs to go twice as fast as the left'). She can also correctly explain that the line between her fingers has the same steepness when
keeping the rectangle green. Throughout the tablet task she prefers to reason with blocks. When making mistakes, she can quickly correct them. She seems to have little difficulty with the tablet task.

## Pen and paper task

For the pen and paper task Anna first took some time to look at the slope of the triangle, after which she asked for a ruler. When no ruler was available, Anna instead asked for a "straight" sheet of paper. She put the paper alongside the slope of the triangle and makes sure the line of the paper matches the line of the triangle (figure 6a). She then slowly pushes the paper away in a parallel direction, effectively making the triangle bigger (figure 6b). The participant stops at a seemingly arbitrary points and asks how big the triangle should be. Interviewer notes that it does not matter how much bigger, just bigger. Participant draws a line alongside the slope of the paper and creates a bigger triangle with the correct proportions (figure 6c).


Note. Orange arrow indicates movement of gaze. Grey is used to show the position of the paper, note that the paper in fact does not have grey edges. Purple arrows indicate movement of paper.

When asked to explain her solutions, the participant explains:

Anna: If you hold the paper like this [puts paper alongside the slope of original triangle], and you move it almost the same with your hands [Participant moves the paper parallel to the
triangle], you can do it like this. And if you hold it like this [points at slope of triangle], then you know how steep it is. And after that, you can draw.

Interviewer: You are correct! But how do you find out that these lines are supposed to be the same?

Anna: If you have this one [puts index fingers at the tops of the original triangle] and you make it bigger with your fingers like this [gestures in a way similar to the tablet task, moving the left finger up, and the right finger to the right], you will need to keep the same line, the same steepness to keep it the same [Anna's eyes follows slope of original triangle while explaining].

When making the triangle smaller, the participant completes this in the same manner as above. On the first try, she moves the paper with two hands, but it goes slightly awry when she pushes a bit more with one hand, making the paper not parallel to the original triangle. Participant notices this however, and remeasures with the paper, and moves the paper with one hand instead of two. She then correctly draws a second triangle. When the interviewer asks to draw an even bigger triangle, the participant swiftly does this in the same manner as previous, without any mistake.

## Corresponding behavior

When we go back to the tablet task, the participant shows three instances where she uses "lines" to explain the rule, which corresponds to her solving the pen and paper task. The interviewer asks her to imagine a line between her fingers and further asks what happens to the line if one makes the triangle bigger. The participant is quick to notice that "The line gets bigger but stays the same" (we assume "the same" refers to the slope). Later, when the grid is added the participant starts explaining by using blocks and calculations. She starts explaining about focusing on the right upper point of the square when making it bigger, which she then explains as seeing it as an oblique line. When trying to show that the line is oblique, she quickly realizes she is wrong and adjusts accordingly. Other than these
instances, the participant does not focus on lines and focuses mostly on grid-reasoning. As such, it is quite surprising this participant chose to focus on the lines of the triangle to solve the task

## Participant two; Bea

Tablet task
Bea starts the task by exploring silently for a long time. For her first explanation of the rule she erroneously states that the $x$-axis is two-and-a-half times as much as the $y$-axis. Throughout the tablet task she often explains with halves. This is an unusual explanation for children her age (8-10 years) as they often struggle with the concept of half numbers. When the grid is added after 8.5 minutes, she can correctly explain that the right finger moves twice as fast as the left finger. She is also capable of explaining that the line between her fingers keeps the same steepness when keeping the rectangle green. When the numbers are added to the grid she starts reasoning with blocks, though she erroneously thinks that the proportion ( $\mathrm{y}: \mathrm{x}$ ) is $1: 1.5$. With the final task of the tablet she also keeps reasoning in halves; she (correctly) explains that the proportion $\mathrm{y}: \mathrm{x}=2: 3$ means adding two halves to the $y$-axis and three halves to the $x$-axis.

## Pen and paper task

For the pen and paper task, Bea starts quickly and erroneously states that the triangle is 4 by 2 . The interviewer does not correct her, and she holds the belief that the original triangle is 4 by 2 for the remainder of the pen and paper task. To make the triangle bigger, she estimates a point on the $x$-axis she believes is twice as big as the original triangle. She overestimates very slightly, realizes this, and adjusts her estimation to be slightly underestimated (figure 7a). For the $y$-axis she measures using her fingers, doubling the height of the $y$-axis (figure 7 b and 7 c ).


Figure 7a. Bea estimates the x-axis when doubled. Bea gets close, first overestimating and later underestimating by a little. Note, the blue grid is not part of the drawing, but an indication where the correct position would


Figure 7a. Bea measures the height of the original rectangle


Figure 7c. Bea adds measurement of the height of the original triangle, almost doubling it.

Bea draws a line between the two points on the y and x axes which results in a roughly correct triangle. When asked to make an even bigger triangle, she tries to make the triangle thrice as big but finds that the graph is too small for that. After a short pause Bea notes that she can also add "two" to the $y$-axis and "one" to the $x$-axis. Though this is incorrect for the task, it is coherent with her belief that the triangle is 4:2. She estimates half of the $x$-axis but hesitates at the $y$-axis. She then measures the addition of the $x$-axis and adds it to the $y$-axis, as she believes it is two (figure 8 a and b ).


Fig. 8a. Bea measures the length of the addition of the base.


Fig. 8b. And adds it to the $y$-axis.

Bea starts drawing an even bigger triangle. She starts correctly, copying the steepness of the other triangles (figure 9a). However, when she notices she will not reach her intended points, she adjusts, making a bit of an untidy triangle that is incorrect, as the base is too small (figure 9b).


When asked why this is correct, the participant explains that she added half of the "blocks" to the triangle, adding two blocks on the $y$-axis and one on the $x$-axis. When asked to make a smaller triangle, she halves the $x$-axis and the $y$-axis, explaining "you do one here ( $x$-axis) and two here ( $y$ axis). Though she does not notice that the proportion is in fact $3: 2$, and not $4: 2$, she makes a triangle by halving the $y$-axis and $x$-axis, making a triangle with the correct proportion.

Since Bea uses blocks, we infer that Bea uses imaginary/estimated grids, multiplicative reasoning, and additive reasoning the solve the task. She misinterprets the triangle as 4:2 throughout the task, as indicated by her explanations. She is not pointed to the fact that her belief is incorrect, and she does not notice this herself. Bea is the only participant who adds half of the original triangle on top of the drawn triangle, an act a lot of participants struggle with. It remains to be seen whether she would be able to add half of the triangle if her belief were 3:2, as participants of this age often struggle with using fractions that are not integers (e.g. 1.5). A peculiarity about this participant is that she only measures the $y$-axis (with her fingers), and always estimates the $x$-axis (visually without support). A particular reason for this behavior is not verbalized and remains unexplained.

## Corresponding behavior

When we look for corresponding behavior, we can clearly see that the participant has a preference in speaking of and working with halves in both tasks. In her first explanation of the tablet
task the participant notices that the $x$-axis is bigger by two-and-a-half (which is incorrect, as it should only be twice as big), which may explain why she started thinking in halves. When doing the tablet task in this way she is quite inaccurate, which could explain why she saw the triangle in the pen and paper task as 4:2 instead of 3:2. More corresponding behavior can be seen in the biggest triangle the participant made (Fig 9 a and b ), as it shows the participant is aware the lines should be the same steepness. It can be argued that the participant was already aware of parallel lines before the tablet task, and as such it is possible that this was not transferred from the tablet task. In the end, the blocks and parallel lines contradicted each other, as parallel lines had a different end coordinates than the block end coordinates, and the participant erroneously decided in favor of the block coordinates.

## Participant three; Caroline

## Tablet task

Caroline struggles with the tablet task. She takes a very long time ( 30 minutes) to state the rule correctly. She gives vague answers and contradicts her own statements. When the grid and the numbers are added (after 20 minutes) she has difficulty making rectangles that completely fills the squares. The interviewer needs to guide the participant to state the correct rule, but once stated the participant has no trouble proving the rule. Since it took her a long time to complete the task, there was not a lot of time to experiment before the interviewer introduced the pen and paper task.

## Pen and paper task

Caroline also struggles greatly with the pen and paper task. She starts the task by gazing a long time, following the slope of the original triangle, and also following the top of the $y$-axis towards the top of the $x$-axis (figure 10). She asks if connecting the tops would be the solution, when asked why she thinks that is the solutions she hesitates in her explanation, indicating confusion:

Interviewer: Why do you think that is the solution?

Caroline: because it is not exactly the same as... [Participant motions along the slope of the triangle] no, yes, because this [gestures imaginary slope of top of $y$-axis to top of $x$-axis] is the same shape as that [gestures at slope of triangle].


Figure 10. Participant gazes from top of $y$-axis to top of $x$-axis. Arrow indicates direction.

When asked to draw, Caroline starts to draw a line at the end of the $x$-axis. The line has roughly the correct steepness but would not reach the $y$-axis before the end of the paper. She stops the line, starts again but at the end of the $y$-axis, and draws a line towards the end of the $x$-axis. The resulting triangle is incorrect, as the line is not steep enough. Caroline admits to being uncertain about the answer and asks for feedback. The interviewer explains the task in more details, gesturing along the $y$ and $x$ axis as one would during the MIT-P.

Caroline is still struggling with the task and asks again what the question was. Caroline gazes at a few hypothetical slopes which would be correct. Caroline starts at a seemingly arbitrary point on the $x$-axis and draws a line towards the $y$-axis but stops. She then starts at the $y$-axis and draws a line towards the $x$-axis, meeting the line she drew earlier. The resulting triangle is steeper than her previous triangle but still too steep for the task. When asked to explain why this is correct, the participant starts to explain, but stops to gaze at the slopes of the original and drawn triangles, sighs of defeat, saying: "I don't know, it is but a guess". Caroline explains that she imagines blocks, and that there are two blocks added to the $y$-axis, and one to the $x$-axis. She struggles to explain further, only saying that she chose two and one because the shapes of the triangles look the same. A reason why she chose two
blocks for the $y$-axis, and one for the $x$-axis is that was the proportion of the tablet task, she might still be focused on that.

Caroline gazes a lot at the task and remains quiet for most of it. She mentions imagining blocks (grid), but she is unable to explain in depth how the blocks relate to the triangle. Even though she is unable to complete the task successfully, there are signs that understand the task. Firstly, her very first attempt of a bigger triangle has roughly the correct steepness. However, she starts it so far in the corner it would not be able to reach the $y$-axis before the sheet ended. Some of her gazes would also be correct triangles, though she is unable to draw them. Other than that, her explanations are very vague, and she admits to guessing, which shows she at least knows that her triangles are not fully correct. Lastly, when making the triangle smaller, she follows the (incorrect) drawn line twice after which she follows an imaginary line that would be correct.

## Corresponding behavior

In the tablet task the participant shows similar behavior to the pen and paper task, she remains mostly quiet and gives vague answers. She appears to see some patterns, but she is unable to verbalize it, e.g. the participant mentions that the green squares look alike, and when the interviewer ask whether the green squares "always look alike", she answers "sometimes". At one point the participant mentions that the line is always the same between her fingers, "if I move this finger, the triangle turns red, so it is not the same line". However, she later contradicts herself: "When I move one finger, the rectangle turns red, and there is still a straight line between my fingers". Adding the grid does not help the participant much, she makes awkward rectangles (e.g. 4.7 blocks by 9.4 blocks), which makes it hard to see on the grid what the rules are. When numbers are added the participant focuses more on making easier rectangles (e.g. 3:6), but shows difficulty predicting correct triangles. After 30 minutes she verbalizes the rule that x must be double of y , which is much longer than most participants. After seeing the rule, she starts predicting accurately where correct triangles will be.

The participant only shows corresponding behavior for basic components of the tasks, such as that the rectangles/triangles need to look alike. She also reverts to a 1:2 grid explanation, which would be incorrect for the pen and paper task but correct for the first three phases of the tablet task. This can be explained because the participant took a long time to verbalize the rule for the iPad task, after which she was quickly guided towards the pen and paper task. Embodied cognition posits that transfer occurs if one has enough experiences with a concept to ground schemas with sensori motor information about said concept. The MIT-P was perhaps too short for this participant to ground sensori motor schemas that can be transferred to a different task.

## Participant four; Dee

## Tablet task

When doing the tablet task Dee was quick to notice that the rectangle turned green when the right hand was twice as far as the left hand. According to the participant, the rectangle turns green when you can fit two identical squares next to each other (correct). The participant was unable to correctly state the speed rule, saying the right-hand moved "slightly" faster. The participant was however able to state that the line between the fingers remained the same steepness when keeping the rectangle green. When the grid is added, the participant was very quick to note that the $x$-axis has twice as many blocks than the $y$-axis when keeping it green. The participant finished the tablet very quickly, and as such the interviewer decided to teach her extra. They took some time to teach Dee about halves, which is a concept students of this age (eight to ten years old) struggle with.

## Pen and paper task

After gazing for a bit, Dee immediately starts to draw a crooked, slightly too steep line (5:4 proportion). The participant explains she added two "blocks" to both the $x$ and $y$-axis but hesitates. After some pause, she a bit more to the $x$-axis, and makes a new triangle, which is an isosceles triangle (figure 11a). The participant crosses out the previous drawn triangle and asks for feedback. She admits
that she has no idea how to solve the task. The interviewer asks for the proportion of the original triangle, and the participant says it is $2: 3$, two blocks at the $y$-axis and three at the $x$-axis, though in reality it is reverse. This mistake might be made because the final task of the tablet task included creating squares with the proportion of 2:3. Dee realized the mistake after the interviewer repeats her explanation questioningly. The interviewer prompts to draw the blocks, which the participant does outside of the triangle. She attempts to make a bigger triangle again by adding two blocks at the $x$-axis and one block at the $y$-axis, which would have made an isosceles triangle. The participant hesitates, and adds another block to the $x$-axis, but realizes that this is faulty (figure 11b). The interviewer explains the task in more detail:

Interviewer: Here we have a triangle that is 3 (points at $y$-axis) by 2 (points at $x$-axis), how could you make this bigger?

Dee: Double?

Interviewer: yes!

Dee: Okay, so you remove this one [Removes excess block on $x$-axis] and make this one a bit larger [Makes a block that is too small a bit larger]. And here ( $y$-axis). [Short pause] You add one [draws block] and another one [draws block].


Figure 11a. Dee her first and second attempt. The first attempt is scribbled out.


Figure 11b. Dee's faulty attempt to make the triangle bigger using blocks.


Figure 11c. Dee's second attempt (scribbled out) and third attempt to make the triangle bigger

Dee then continues to draw a line from the $y$-axis to the $x$-axis, without noticing her intended point on the $x$-axis is one block too far. The interviewer points at the $x$-axis, quietly, and participant corrects her mistake. The result is a correct triangle (figure 11c).

To make the triangle smaller, Dee quickly removes one block from the $x$-axis, stating it needs to be half. She then continues to remove one block from the $y$-axis and hesitates. She continues to remove another half of a block from the $y$-axis and draws a correct smaller triangle.

For the final task Dee must make an even bigger triangle. She does this by adding three blocks to the length of the triangle and adding two blocks to the $x$-axis, and drawing a line between the two points, creating a correct larger triangle. When asked what numbers belong to this triangle, the participant counts the number of blocks aloud on the $y$-axis (nine) and the $x$-axis (six), and states that the proportion is 9:6.

Though Dee was able to successfully complete this task, she did need some guidance from the interviewer to do this. To solve the task the participant used blocks (grid reasoning), 1:2 proportional reasoning, multiplicative and additive reasoning, and halving blocks to make a triangle smaller.

## Corresponding behavior

When we specifically look at corresponding behavior, two examples are very apparent from the tablet task; firstly, the participant tries to make the triangle bigger by adding 1:2 blocks to the triangle, which would be correct in the first three phases of the tablet task, and secondly, the participant refers to the triangle as $2: 3$, which is the proportion of the last phase of the tablet task. The transfer of this information makes it hard for the participant to complete the pen and paper task, but once she overcomes it with a guided question of the interviewer (see earlier quote), she is capable of completing the task quickly.

We can see that all the behavior she showed in the pen and paper task can be found in the tablet task as well. Two types of behaviors are missing in the pen and paper task, as she does not mention speed nor the steepness of the triangle whilst mentioning this in the tablet task. Remarkable of this participant is that she was able to halve her squares to make a smaller rectangle, a skill most other participants lack. The participant did not show this skill during the grid or grid and numerals phase of the tablet task. However, in the last phase (2:3 proportion) of the MIT-P the interviewer took some extra time to explain this to the participant.

Interview: "For every two you add here ( $y$-axis), you add three here ( $x$-axis), so if you add one here ( $y$-axis), how many do you add here ( $x$-axis)?"

Participant: "two? No... here ( $y$-axis), one and then here ( $x$-axis) ... two?"

Interviewer: "Let's try if we can work it out" [interviewer adds grid] "Put your fingers on 2:3, if you add two here ( $y$-axis), you add three here ( $x$-axis), so what happens if you add one here ( $y$-axis), so half. What do you add here ( $x$-axis)?"

Participant: "Also two [participant tries, rectangle turn reddish], no, one [participant adds one, rectangle stays reddish] no, [participant adds one and a half, the rectangle turns green] you add one and a half."

In summary, for this participant there is a lot of corresponding behavior visible, for better (drawing a grid, doubling, halving) or worse (reusing old proportions). This participant shows that some extra explanation about halves in the MIT-P can transfer into a pen and paper task.

## Participant five; Ellen

## Tablet task

Ellen has some difficulty with the tablet task. She spends a lot of time exploring in silence and she gives vague answers. When the grid is added the participant uses blocks to solve the task, but has difficulty in ascertaining the exact proportion, as she mentions that the left finger moves half a block and the right finger a little bit more than that. After adding numbers, the she notices that one needs to "cut" the numbers in half (on the $y$-axis). She has difficulty with the concept of half numbers when asked what happens if you cut an odd number in half but seems to understand it after some explanation by the interviewer.

## Pen and paper task

Though Ellen has no difficulty in recognizing the correct proportion and reasoning her answer according to proportional reasoning, she has difficulty with the accuracy of her drawings. She starts off by pointing (but not sketching) at $1 / 3$ points of the original triangle. She then points at three points on top of the original triangle ( $y$-axis), each with comparably length, doubling it, and sketching a mark on that point. She does the same for the $x$-axis pointing at two placing, but these blocks are much longer then the blocks on the $y$-axis, adding too much. She draws a line between the two points, with a proportion of 6:5. She explains that she added three blocks to the $y$-axis, and two blocks to the $x$ axis, which is a correct way to solve the task. She seems oblivious that the blocks on the $x$-axis are too long. The interviewer asks her to draw the blocks, and she points at a spot on the $x$-axis that would be a block of correct length, but draws a mark between the original and the drawn triangle instead (see
figure 12 a through c) She quickly draws the blocks on the $y$-axis and repeats her explanation that she added three blocks to the $y$-axis and two to the $x$-axis. When asked why, she cannot explain further.


Figure $12 a$ Ellen puts the pencil on this spot...


Figure 12b. But decides to (erroneously)
draw in the midpoint instead


Figure 12c. Result of drawing
blocks. The blocks on the $y$-axis are smaller than the ones on the $x$-axis.

The interviewer asks whether it is on purpose whether she made the blocks on the $x$-axis bigger:

Ellen: Yeah!

Interviewer: Oh, why?

Ellen: Because it is the same here [points at original triangle]
Interviewer: Is that so?

Ellen: (hesitant) Yeah? [she measures the $y$-axis with her hands, and compares it to the $x$-axis] Oh, no, I do not think they are the same. [She measures again and makes a compass movement to compare the $y$-axis] This one ( $y$-axis) is one block bigger.

The interviewer explains that her explanation with blocks is correct, but that the blocks on the $x$-axis should have been the same length. Ellen agrees but remains quiet and does initiate to correct her mistake. The interviewer measures the two blocks and shows Ellen where she should have been on the $x$-axis (figure 13a). Ellen starts a line further than the interviewer indicated and makes a line parallel to her previous erroneous line (figure 13b). When the interviewer asks whether she is guessing, she
responds no, and tries a third time, making a line that starts at the same spot, but ends between her drawn triangles. The interviewer asks her why this is correct:

Ellen: Because you draw a straight line from here [indicates starting point].

Interviewer: A straight line? Okay. [pauses] How do you know it should be a straight line?

Ellen: Because this [indicates slope of triangle] is also a straight line?

Interviewer: Oh, you mean the lines are the same?

Ellen: Yeah!


Figure 13a. Interviewer shows Ellen where the correct point on the $x$-axis would have been.


Figure 13b. Ellen starts further than the interviewer showed and makes a line
parallel to her previous erroneous attempt.


Figure 13c. Ellen's third and fourth attempt, fourth attempt being correct.

The interviewer once again explains that you need to add two blocks to the $x$-axis and shows here where the line should end. Ellen traces this line and creates a correct triangle (figure 13c).

The interviewer asks if she can make a smaller triangle. Ellen starts by diving the original triangle further, making two blocks out of the first block. She draws a line from the middle of the $x$ axis towards the second block of the $y$-axis (making a triangle that is $4: 2$ ) and divides the $y$-axis in smaller blocks. She starts to explain, but notices she made a triangle of 4:2 instead of 3:2 and tries to correct this by removing one block from the $y$-axis, without altering the line drawn. She then explains that the $y$-axis is three blocks, and the $x$-axis is two blocks. Her explanation is coherent with the explanation of 3:2, though her drawing is not (figure 14).


Figure 14. Drawing a smaller triangle. She crossed out the lower block of the $y$-axis but kept the line the same. There are three blocks on the $y$-axis (one twice as big), and two on the $x$-axis.

Ellen now has to make an even bigger triangle. She quickly adds three blocks (smaller than regular) on the $y$-axis and starts drawing a line towards the $x$-axis. It appears the line will intercept on the end of the $x$-axis, but she adjusts, and it results in a rather wobbly line. She then adds two blocks on the $x$-axis and tries to salvage her wobbly line. She adds numbers on the blocks on the $x$-axis, and indicates she is done. Ellen explains she added three to the $y$-axis, and two to the $x$-axis. Once again, her explanation of the task is coherent and correct, but her result is inaccurate.

Ellen shows no difficulty in seeing the proportion and verbalizing how to solve the task. However, the blocks and lines she draws are inaccurate. She is one of the very few who make a triangle smaller by making the blocks within the triangle smaller, however her line is once again inaccurate. Ellen shows difficulty to solve the task in the correct sequence, as she creates blocks on one axis, draws a line towards the other, and then adds blocks on the other axis, whereas it would be easier to add blocks on one axis, then blocks on the other axis, and then add a line between it.

## Corresponding behavior

Though the participant struggled a long time in the tablet task to verbalize the rule, she showed no difficulty in explaining her actions in the pen and paper task. The behavior in the pen and paper task corresponds to her tablet task, as she prefers to use blocks but at the same time has difficulty with accuracy. For instance, when making her first triangle the participant estimates the $x$-axis blocks to be much bigger, which corresponds with the last phase of the tablet task, which introduced the 2:3 proportion. In the end, the participant can correctly verbally explain the tasks and the measures needed to complete it, but she is unable to perform them correctly.

## Participant six; Frank

## Tablet task

From the beginning Frank had difficulty with the execution of the task, as he often moved the fingers in a way that made the tablet remove the rectangle. The interviewer comments many times that the participant moves the fingers in ways that the tablet cannot read the location and tells or shows him how to properly do it. Regardless, throughout the tablet task Frank keeps repeating this error.

When first finding a green rectangle, Frank moves his fingers to an inverted location, going from a green $1: 2$ rectangle to a red $2: 1$ rectangle. He mentions that he cannot make a (green) rectangle "like this", referring to the $2: 1$ rectangle. When exploring further he keeps finding green in the same area and has difficulty in finding the rule until the interviewer asks him to find a small green rectangle. After finding a smaller green rectangle and some nudging from the interviewer he mentions that the width of the rectangle is twice as much as the length to make a green rectangle, which is correct. He uses this explanation for the majority of the task, briefly mentioning the speed of his fingers when nudged by the interviewer. In the final phase of the task the participant has little difficulty finding
green rectangles. He explains a $2: 3$ rectangle as "this one ( $x$-axis) is 1.5 times as big as this one ( $y$ axis)", which is correct.

## Pen and paper task

Frank shows difficulty solving the pen and paper task. He shows hesitation by not finishing many of his sentences, changing tactics often and when one tactic fails, he tries another one or makes changes to the tactic. Despite his difficulty, he is successful in completing the task and does it in a way that is different from other participants.

When the pen and paper task starts, Frank immediately notes that the $x$-axis is half as big as the $y$-axis. When the interviewer indicates he might be wrong, he measures and finds that the $y$-axis is 1.5 times as big as the $x$-axis. To make a bigger triangle Frank measures four blocks to the $y$-axis, and six blocks to the $x$-axis, making an isosceles triangle. When asked to explain, he measures his triangle and finds out the base and the height are the same length. He measures with his fingers and verbalized this with the help of his pencil.

Frank tries again and makes a triangle that is roughly correct. When explaining, he measures with his fingers by making a compass movement but does not notice the length of his fingers decrease whilst doing so, making his measurements inaccurate. He tries again, making the triangle too steep, but when measuring he thinks he is correct, saying the $y$-axis is the $x$-axis times 1.5 , which is a correct, albeit unusual, explanation for the task.

To make a smaller triangle, he creates two lines that are perpendiculars, one towards the $y$-axis and one towards the $x$-axis, differing in length. He then connects the two lines. He tries to explain that one line is one and a half of the other line, but he trails off and starts measuring, concluding that he is incorrect. He tries again as he draws a correct triangle which ends at one of the lines he made earlier. He explains that the length of the perpendicular lines should be the same, which is correct for this task. To further explain this, he draws an even smaller triangle with the same tactic. He also uses this tactic
to explain his earlier bigger triangle, not noticing that the lines are not the same length and that, by his logic, the triangle would be wrong. The interviewer does not point Frank to his flaw in reasoning and ends the task.

Though Frank shows a lot of difficulty with making the task, he still manages to complete it successfully by experimenting. His verbal explanations are correct, but not complete, as he cannot explain why his tactic is correct. He comes to the concluding that, if you draw two same length lines that are perpendicular, you will get a same line, but he is unable to explain that the resulting triangle is parallel.

## Corresponding behavior

In the beginning of the task there are several corresponding behaviors with the tablet task. Frank first mentions that the rectangle is an inverse of the tablet task, but after measuring finds that this is incorrect. Also, when trying to make the triangle bigger he adds four blocks to the $y$-axis and six blocks to the $x$-axis, which would have been correct in the final phase of the tablet task. He uses the same explanation as the final phase from the tablet task, but inverted, the $y$-axis is one and a half times as big as the $x$-axis. However, possibly due to Frank his untidy work, he tries to resort to a different explanation. He discovers that one can solve the task using parallel lines. This is not corresponding with his previous tablet behavior, as he did not mention using lines in the tablet task.

## Conclusion and discussion

The aim of this study was to test whether anything learned from an embodied learning task was transferred to another medium, in our case a task with pen and paper, where a different proportion (3:2 instead of 1:2) and a different shape was used (triangle rather than square). For this thesis we have the research question; What is transferred from the tablet task towards the pen and paper task of the MIT-P? By researching six participants in a clinical task-based interview of about 45 minutes each, selected for maximum variance, we have found that all participants exhibited
behaviors in the pen and paper task that could be traced back to the tablet task. However, the transferred behavior was not always positive, for instance when participants used the initial target proportion of the tablet task (1:2) for the pen and paper task (3:2). Despite this, five out of six participants managed to complete the pen and paper task mostly correctly. It seems likely that the tablet task provides experience that participants are able to transfer across media.

We found that all selected participants showed some behavior that could also be found in the tablet task. The behavior used to solve the pen and paper task included multiplying the triangle, using blocks, and using lines/steepness (see Figures $15 \mathrm{a}, \mathrm{b}, \mathrm{c}$, and d). However, We found one instance where behavior exhibited in the pen and paper task could not be found in the tablet task but rather emerged during the task (see Figure 15e), in this case perpendicular lines were used to create a line parallel to the rectangle. This behavior emerged from a participant who had difficulty verbalizing the task, and as such tried different approaches to find a solution which he could verbalize.


Figure 15a. Participant measures the length of the triangle and multiplies/adds it. Usually, the participant would make a notch


Figure 15b. Participant measures the width of the triangle and multiplies/adds it. Usually, a notch is made and then a line between the notches.


Figure 15c. Participant draws blocks on the paper alongside the triangle. Then the participant adds extra blocks to the length and the width, and then draws a line between the ends.


Figure 15d. Using lines to solve the pen and paper task. The participant either looks at the angle and tries to reproduce it (red line). Alternatively, the participant uses something (example, a ruler) to measure the angle and drags or pushes it towards the intended location.


Figure $15 e$. The participant draws the purple lines first, they are perpendicular and equal in length. Then the participant draws a line between the purple lines (red) which results in a parallel lines to the original. This can be done multiple times.

These behaviors had different outcomes and were not guaranteed a correct or incorrect result. Out of the two participants who used lines to solve the pen and paper task, only one was able to solve the task correctly by measuring the lines, whereas the other solved the task without measuring and instead estimated the steepness of the lines. Moreover, one of the two participants using blocks or grids was able to solve the task correctly, whereas the other made a few mistakes by estimating rather than measuring blocks. The participant who used measuring to solve the task held an incorrect notion of the task; she believed the triangle was $2: 1$ rather than 3:2. As a result, her measurements were inaccurate. The final participant had difficulty verbalizing his actions, and as a way to reinforce his explanations he tried different ways to solve the task, using blocks, measuring, lines and settling on perpendicular lines. His work was untidy, but mostly correct. In summary, the many different behaviors seen during the pen and paper task corresponded with the tablet tasks, with varying results.

Aside from corresponding behavior we also focused on corresponding beliefs which were verbalized during the tasks. Some of the participants initially thought the pen and paper task triangle had a proportion of $2: 1$ or $2: 3$, which would correspond with the tablet task as $2: 1$ is the inverse of the correct proportion of 1:2 in first three phases of the in tablet task, and 2:3 being the correct proportion for the final phase of the tablet task. Usually, these notions were short lived as the interviewer would ask them to explain further and then the participant would see that they are incorrect. In one instance the interviewer did not ask further when a participant verbalize the incorrect proportion (1:2), and the participant did not adjust by herself.

Our research suggests that behavior learned during an embodied tablet task can aid participants with transfer towards a pen and paper task. When we compare affordances and schemas, the difference is that mental schemas need to be similar to the new context, and more importantly, you need to perceive this as overlapping, whereas affordances takes away this overlapping perceiving and replaces it with perceiving possible actions with the environment. It is hard to determine whether this data is in
favor of schemas or affordances. One could argue that there is sufficient similarity between the tablet and pen and paper task for the participants to perceive as such, or one could argue that the tablet provided enough similarity with the pen and paper and as a result strategies emerged.

However, what is transferred differs between our participants, which is unsurprising as we selected on variant behavior during the pen and paper task. Furthermore, this variance can be explained since the experiment was led by an interviewer rather than an operator; the difference being where an interviewer looks for behavior that is emerged and an operator is focused on helping the participant complete the task. For the pen and paper task, the goal of the interviewer was to get a clear explanation from the participants on why they solved the task in their way. The interviewer would only nudge the participants when their explanations were vague or contradictory, giving the participants more freedom to solve the task as they pleased. The benefit of this is that we can see what the participants' initial thought is after the tablet task; what is active in their memory after completing the tablet task. However, the disadvantage is that we do not know if other information is transferred as well. If a participant is able to solve the task using an imaginary grid, would this participant also be able to solve the task by using parallel lines, or vice versa? This should be the next step for research on embodied tasks such as MIT-P, to see whether the behavior is transferred in multiple ways rather than the most prevalent in their memory.

This is exploratory research, and there are many implications for future research. We have shown that transfer across media is possible when using embodied design for learning about proportions, but many questions remained out of the scope of this research. Future research should focus on adding pre- and post-tests to the MIT-P to eliminate the question if the participants simply already know the elements of proportional reasoning targeted here, and in order to quantify transfer and change in behavior. Future research should also focus whether the MIT-P provides experience that is retained over time. Embodied design hopes to ground schemas, does this grounding of
schemas allow it to be remembered over periods of time? Delayed testing, e.g. performing the tablet task and giving the pen and paper task a week later, shows whether the MIT-P is a tool for priming or for learning. For even longer periods of time we should look at longitudinal research to see if the MIT-P can increase adult comprehension of proportions. Finally, this research was done with a one-on-one environment where the participant was guided by an interviewer. The focus here is what an individual gains from the task and how they respond to it. However, this is not how mathematics are taught in the classrooms (yet). Providing the MIT-P in a classroom provides challenges such as distractions, different students experiencing the MIT-P at the same rate, differences between teachers. However, it also poses opportunities, such as students working together (Van Helden et al., 2019). We have concluded that there are many things that can transfer from the MIT-P, and we need to research what effect that will have on the classroom.

## References

Abrahamson, D., \& Howison, M. (2010). Embodied Artifacts: Coordinated Action as an Object-to-Think-With. In Embodied and Enactive Approaches to Instruction: Implications and Innovations (D. L. Holton, Chair, \& J. P. Gee, Discussant). AERA 2010, Denver, May 3, 2010.

Abrahamson, D., Shayan, S., Bakker, A., \& van der Schaaf, M. (2015). Eye-Tracking Piaget: Capturing the Emergence of Attentional Anchors in the Coordination of Proportional Motor Action. Human Development, 58, 218-244.

Abrahamson, D., Trninic, D. (2015). Bringing Forth Mathematical Concepts: Signifying Sensorimotor Enactment in Fields of Promoted Action. ZDM Mathematics Education 47, 295306.

Boven, L. (2017). Coordination of Action and Perception Processes in an Orthogonal Proportion Tablet Task. (Unpublished Master Thesis). Utrecht University, The Netherlands.

Gibson, J.J. (1977). The Theory of Affordances. In R.Shaw and J. Bransford (Eds.) Perceiving, Acting, and Knowing. Toward an Ecological Psychology. Hillsdale: NJ, Lawrence Erlbaum Associates, 67-82

De Freitas, E., \& Sinclair, N. (2012) Diagram, Gesture, Agency: Theorizing Embodiment in the Mathematics Classroom. Educational Studies in Mathematics, 80, 133-152. DOI 10.1007/s10649-011-9364-8

Van Helden, G., Alberto, R., \& Bakker, A. (2019). Verhoudingen Doorgronden. Belichaamd Ontwerp in de Rekenles. Volgens Bartjens, 39(2), 4-7.

Lamon, S. J. (2007). Second Handbook of Research on Mathematics Teaching and Learning (pp. 629668). Charlotte, NC: Information Age Publishing. DOI: 10.1159/000443153

Lobato, J. (2003). How Design Experiments Can Inform a Rethinking of Transfer and Vice Versa. Educational Researcher, 30, 17-20.

Marsh, K.L., Johnston, L., Richardson, M.J., \& Schmidt, R.C. (2009). Towards a Radically Embodied, Embedded Social Psychology. European Journal of Social Psychology. 39(7), 1217-1925.

Pouw, W. T. J. L., van Gog, T., \& Paas, F. (2014). An Embedded and Embodied Cognition Review of Instructional Manipulatives. Education Psychology Review, 26, 51-72. DOI 10.1007/s10648-014-9255-5

Shapiro, L., \& Stolz, S. A. (2018). Embodied cognition and its significance for education. Theory and Research in Education, 17(1), 19-39. https://doi.org/10.1177/1477878518822149

Shayan, S., Abrahamson, D., Bakker, A., Duijzer, A. C. G., \& van der Schaaf, M. F. (2015). The emergence of proportional reasoning from embodied interaction with a tablet application: an eye-tracking study. Proceedings of the 9th International Technology, Education, and Development Conference (INTED 2015), (March), 5732-5741.

Smith, C. P., King, B., \& Hoyte, J. (2014). Learning Angles through movement: Critical Actions for Developing Understanding in an Embodied Activity. Journal of Mathematical Behavior, 36, 95108.

Tuomi-Gröhn, T., \& Engeström, Y. (2007). Advances in Learning and Instruction series. Between School and Work. New Perspectives on Transfer and Boundary-crossing. (pp 19-37). Emerald Publishing

Wilson, A.D., \& Golonka, S. (2013). Embodied Cognition is not What you Think it is. Frontiers in Psychology, 4, 1-13.

