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Faculteit Bètawetenschappen

Spin-Cherenkov radiation caused by a magnonic black or white hole

Bachelor Thesis

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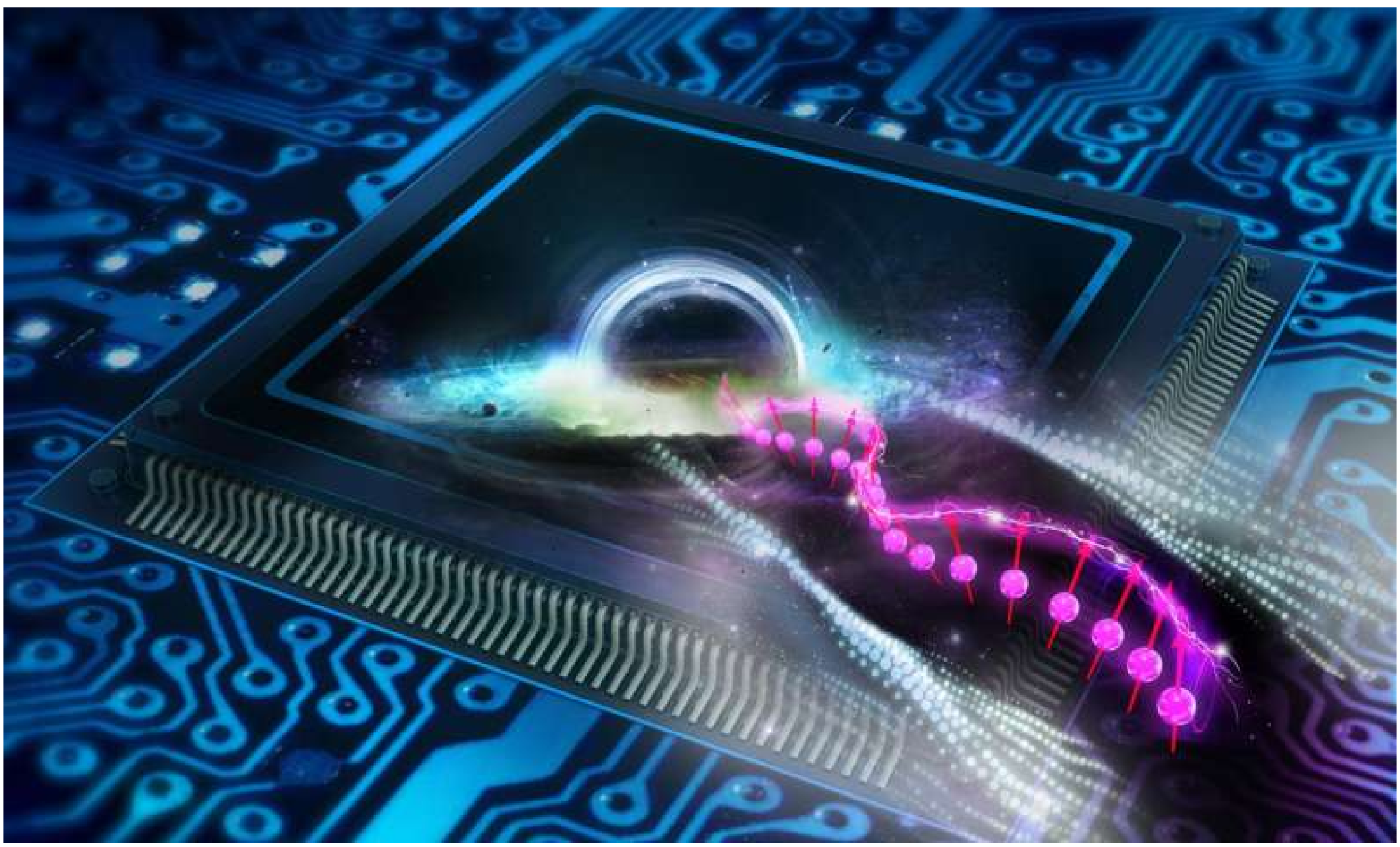
Natuur- en Sterrenkunde



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Abstract

A system of spin waves interacting with a spin-polarized current in a ferromagnetic material has recently been suggested as a black-hole-analogue. When a spin wave encounters such a black or white hole, it is partly reflected and partly transmitted, similar to a quantum mechanical particle encountering a potential step. We calculate the reflection and transmission coefficients of this spin wave and use these coefficients to determine the Green's function that is needed to calculate how spin waves are emitted from imperfections at the horizon Cherenkov processes. This way we find an expression for the wave function describing the Cherenkov radiation on both sides of the black or white hole. We discuss subtleties concerning current conservation in our results, and make suggestions for further research.

Contents

1 Introduction

The death of Stephen Hawking earlier this year (14 March 2018) was naturally followed by obituaries in newspapers around the world, all taking account of his numerous contributions to physics. The most important of these was often said to be his discovery that black holes emit radiation, the so called Hawking radiation. This evaporation of black holes has, ever since its prediction in 1974, been one of the most exciting phenomena in theoretical physics [1]. It has however not been experimentally confirmed in the last fourty-four years, despite physicists best intentions. This is a problem that occurs not only in the case of Hawking radiation, but in a lot more physical theories concerning the horizon of a black hole. While this horizon is an enormously interesting place, questioning multiple of our leading theories, it is almost impossible to subject it to precise measurements, for reasons that are fairly obvious. In 1981, W. G. Unruh proposed a possible solution to this problem [2]. He suggested sending sound waves through a convergent fluid flow. If there is a horizon at a certain position x_0, separating an area with subsonic flow from an area with supersonic flow, one can to large extent imitate the kinematics of a black hole or its time-reversal, the white hole.

With this article a new research field was born: analogue gravity (see also [3]). Besides the system of sounds waves in a fluid flow, there is a growing list of other proposed black-hole-horizon analogues, including systems as flowing Bose-Einstein Condensates (which might already have been successful in finding Hawking radiation, see [4]) and light in dispersive media [5]. The system that is investigated in this thesis is one of the latest additions to the list. A magnonic black hole (visualized by the artist impression on the title page [6]) is created by initiating spin waves in a magnetic material subject to a constant magnetic field, while at the same time letting an electrical spin-polarized current flow through this material. The spin wave interacts with the current via the so called spin transfer torques (STT's), an exchange force between the spin of the itinerant electrons and the spin of the localized electrons. When the current exceeds a critical amperage corresponding to a critical electron drift velocity v_c, these STT's stop the propagation of the wave. A magnonic black or white hole is thus a boundary between a "subsonic" region (with electron velocity $|v| < v_c$) and a "supersonic" region (with $|v| > v_c$) [5].

Although this black-hole-analogue offers several practical and theoretical benefits (see also [5]), there is

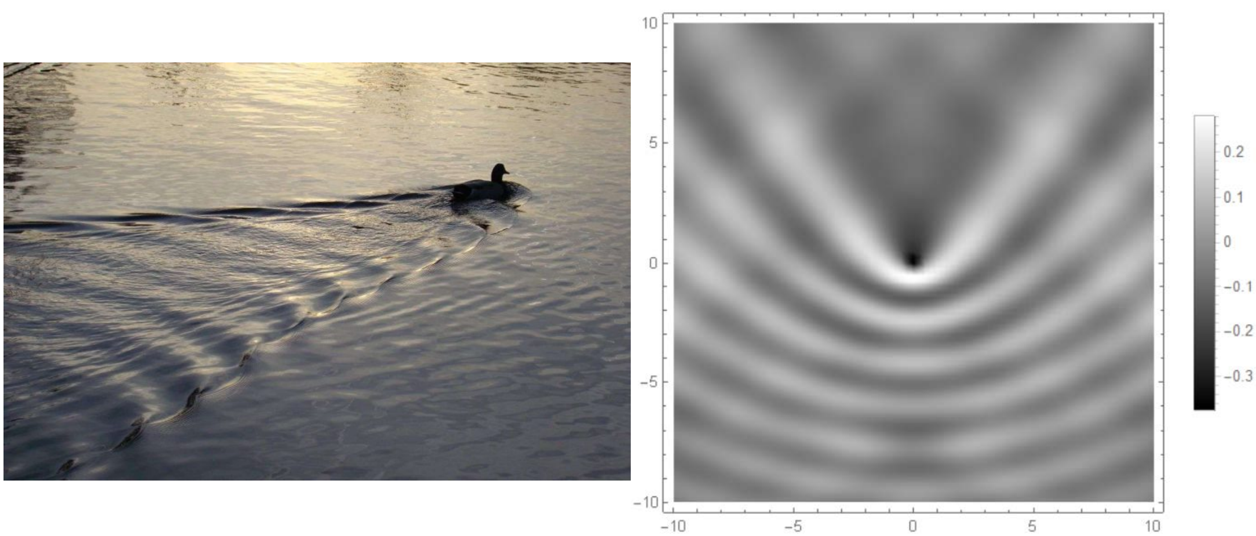

Figure 1: Two examples of Cherenkov-like radiation. On the left photo we see a source of water waves (the duck) moving faster through the medium than the waves themselves, causing a cone-like wave pattern [7]. On the right we see the cone-like pattern of spin-Cherenkov radiation in a square lattice caused by a Gaussian-like source, as calculated by T. Koskamp in his bachelor thesis on the influence of Dzyaloshinskii-Moriya interaction and spin-orbit torques on the current driven spin-Cherenkov effect (see also [8]). The colors indicate the amplitude of the dimensionless magnetization.

still research needed before we can really get useful experimental results. Some aspects of the magnonic black hole are yet to be investigated and described. One of them is the subject of this thesis: the spin-Cherenkov radiation caused by the black or white hole. The Cherenkov effect, in 1934 discovered by the Soviet physicist Pavel Alekseyevich Cherenkov, occurs when a source of light travels faster through a certain medium than the speed of light in that medium, causing radiation [8]. Analogue to this phenomenon there exists a spin-Cherenkov effect, occurring when the source of a spin wave travels faster through the medium than the spin wave itself, or, in our case, when the source is stationary and the medium is flowing faster than the propagation speed of the spin wave. When an initiated spin wave reaches the event horizon (the black or white hole), the sudden change in background current perturbs the wave and causes a part of it to reflect, similar to the reflection and transmission of a quantum mechanical wave at a potential step. The horizon can therefore be seen as a source of spin waves which is partly placed in a medium flowing faster than the speed of the spin waves. While the current driven spin-Cherenkov effect of a random spin wave source has already been described in the bachelor thesis of T. Koskamp [8] and the master thesis of M. de Kruijff [9], the Cherenkov radiation caused by a magnonic black or white hole has never been examined. Because a description of this effect is very important in understanding this black-hole-analogue and essential for obtaining meaningful experimental results, determining this influence is the main goal of this thesis. We focus on spin waves in ferromagnetic materials, although this research could also be done on antiferromagnetic materials or magnetic insulators [5].

We start with the Landau-Lifshitz-Gilbert equation in section 2 to derive the dispersion relation of a spin wave interacting with a current. In section 3 we use this relation to find the critical velocity and we calculate solutions to the scattering problem of the spin wave at the event horizon. We then get reflection and transmission coefficient that depend on the currents on both side of the horizon and the wave frequency. If we plot these coefficients against the frequency, with fixed electron velocities on both sides, we clearly see that the reflection coefficient is almost zero at higher frequencies, but grows fast in the region of low wave frequency. Finally in section 4 we construct a Green's function out of the wave functions found in section 3. With the Green's function, we derive expressions for two new wave functions (one for each side of the event horizon) that give the Cherenkov radiation caused by the horizon.

2 Spin waves

In this chapter we discuss the basics of spin waves and introduce the formulas that are needed to describe the dynamics of spin waves interacting with a current. Hereby we largely follow the discussion of Krüger [10]. For simplicity, a one-dimensional lattice is considered.

2.1 Landau-Lifshitz-Gilbert equation

In 1935, L. D. Landau and E. M. Lifshitz introduced a differential equation that is used to describe the dynamics of the magnetization in a ferromagnetic material [11]:

$$\frac{\partial \mathbf{m}}{\partial t} = -\gamma \mathbf{m} \times \mathbf{H}_{eff}. \tag{1}$$

In this equation $\mathbf{m}$ is the direction of the magnetization, γ is the electron gyromagnetic factor and $\mathbf{H}_{eff}$ is the effective field. In 1955, T. L. Gilbert added an extra damping term to this equation [12],

$$\frac{\partial \mathbf{m}}{\partial t} = -\gamma \mathbf{m} \times \mathbf{H}_{eff} + \alpha \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t}, \tag{2}$$

where α is the damping parameter. The meaning of this latter equation is explained in this subsection.

2.1.1 Magnetization

Every electron has a spin which can have two values: $+\frac{\hbar}{2}$ (which is called spin up) and $-\frac{\hbar}{2}$ (which is called spin down). If an atom has equal amounts of electrons with spin up and electrons with spin down, the total spin of the atom is zero. In the case of unequal amounts of spin ups and spin downs, the atom does have a spin, either up or down. Magnetization occurs when the spins of all the atoms in a certain grid align, due to either the interaction with an external magnetic field or the interaction between the spins of the atoms. This magnetization $\mathbf{M}$ has a certain direction and magnitude. We assume that the temperature of the ferromagnet is way below the Curie temperature, so the magnitude is equal to the saturation magnetization M_s, which allows us to easily obtain a unit vector indicating just the direction of the magnetization:

$$\frac{\mathbf{M}}{M_s} = \mathbf{m}. \tag{3}$$

2.1.2 Effective field

The effective field is calculated by taking the functional derivative of the total energy of the magnetic system with respect to the magnetization:

$$\mathbf{H}_{eff} = -\frac{1}{\mu_0 M_s} \frac{\delta E}{\delta \mathbf{m}}. \tag{4}$$

The energy of the system is given by three components.

Exchange energy: The exchange energy E_E is the result of the Heisenberg interaction between neighbouring spins and is in our case of a one-dimensional lattice given by

$$E_E = \frac{JS^2}{2} \sum_i 2a^2 \left(\frac{\partial \mathbf{m}}{\partial x} \right)^2. \tag{5}$$

In this latter equation, S is the magnitude of the spin, a is the distance between two spins and J is a constant material property, the exchange integral. If J is positive, neighbouring spins tend to align (because then the magnetization is the same at every value of x which minimizes the energy), and the material is ferromagnetic. If J is negative, the material is an antiferromagnet. This means that in a ferromagnet it takes energy to perturb a spin out of its position of alignment with the other spins.

In the case of magnetization changes that are on a far larger scale than that of the distance between different spins, it is useful to turn Eq. (5) into a continuous form, using a spin density of one spin per distance a. Another simplification is the following:

$$\left(\frac{\partial \mathbf{m}}{\partial x}\right)^2 = \frac{\partial}{\partial x}\left(\mathbf{m}\cdot\frac{\partial \mathbf{m}}{\partial x}\right) - \mathbf{m}\cdot\frac{\partial^2}{\partial x^2}\mathbf{m} = \frac{1}{2}\frac{\partial^2}{\partial x^2}(\mathbf{m}\cdot\mathbf{m}) - \mathbf{m}\cdot\frac{\partial^2}{\partial x^2}\mathbf{m} = -\mathbf{m}\cdot\frac{\partial^2}{\partial x^2}\mathbf{m}. \tag{6}$$

Eq. (5) then becomes

$$E_E = -\frac{\mu_0 M_s}{2\gamma} A \int dx\ \ \mathbf{m}\cdot\frac{\partial^2}{\partial x^2}\mathbf{m} \tag{7}$$

where $A = \frac{2\gamma J a S^2}{\mu_0 M_s}$, the exchange constant.

Zeeman interaction: This is the interaction of the spins with an external magnetic field, and is described by

$$E_Z = -\mu_0 M_s \int dx\ \ \mathbf{H}\cdot\mathbf{m}, \tag{8}$$

in which $\mathbf{H}$ is the external magnetic field. We set the magnetic field in the positive $\hat{\mathbf{z}}$-direction, so $\mathbf{H} = H_0\hat{\mathbf{z}}$.

Anisotropy energy: Anisotropy is a property of the material, caused by spin-orbit coupling and dipole-dipole interactions. It makes the spins favour a certain direction, called the easy axis. However, if the magnitude of the external magnetic field is big enough, the spins are forced to align with $\mathbf{H}$, and the anisotropy can be neglected. For simplicity, we assume that this is the case.

Following Eq. (4) we now get

$$\mathbf{H}_{eff} = -\frac{1}{\mu_0 M_s}\left(\frac{\delta E_E}{\delta \mathbf{m}} + \frac{\delta E_Z}{\delta \mathbf{m}}\right). \tag{9}$$

By definition, the functional derivative of the energy is given by

$$\frac{\delta E}{\delta \mathbf{m}} = \lim_{\epsilon\to 0}\frac{1}{\epsilon}\begin{pmatrix} E[\mathbf{m}(x') + \epsilon\hat{\mathbf{x}}\delta(x'-x)] - E[\mathbf{m}(x')] \\ E[\mathbf{m}(x') + \epsilon\hat{\mathbf{y}}\delta(x'-x)] - E[\mathbf{m}(x')] \\ E[\mathbf{m}(x') + \epsilon\hat{\mathbf{z}}\delta(x'-x)] - E[\mathbf{m}(x')] \end{pmatrix}. \tag{10}$$

Then the contribution of the exchange energy to the effective field is the following:

$$\frac{\delta E_E}{\delta \mathbf{m}} = \lim_{\epsilon\to 0}\frac{1}{\epsilon}\begin{pmatrix} -\frac{\mu_0 M_s}{2\gamma} A \int dx'\ \ \left[\big(\mathbf{m} + \epsilon\hat{\mathbf{x}}\delta(x'-x)\big)\cdot\frac{\partial^2}{\partial x'^2}\big(\mathbf{m} + \epsilon\hat{\mathbf{x}}\delta(x'-x)\big) - \mathbf{m}\cdot\frac{\partial^2}{\partial x'^2}\mathbf{m}\right] \\ -\frac{\mu_0 M_s}{2\gamma} A \int dx'\ \ \left[\big(\mathbf{m} + \epsilon\hat{\mathbf{y}}\delta(x'-x)\big)\cdot\frac{\partial^2}{\partial x'^2}\big(\mathbf{m} + \epsilon\hat{\mathbf{y}}\delta(x'-x)\big) - \mathbf{m}\cdot\frac{\partial^2}{\partial x'^2}\mathbf{m}\right] \\ -\frac{\mu_0 M_s}{2\gamma} A \int dx'\ \ \left[\big(\mathbf{m} + \epsilon\hat{\mathbf{z}}\delta(x'-x)\big)\cdot\frac{\partial^2}{\partial x'^2}\big(\mathbf{m} + \epsilon\hat{\mathbf{z}}\delta(x'-x)\big) - \mathbf{m}\cdot\frac{\partial^2}{\partial x'^2}\mathbf{m}\right] \end{pmatrix}. \tag{11}$$

Using integration by parts two times, we get $m_i\frac{\partial^2}{\partial x'^2}\delta(x'-x) = \delta(x'-x)\frac{\partial^2}{\partial x'^2}m_i$, because the boundary terms are 0. Then the latter equation simplifies to

$$\frac{\delta E_E}{\delta \mathbf{m}} = \lim_{\epsilon\to 0}\frac{1}{\epsilon}\begin{pmatrix} -\frac{\mu_0 M_s}{\gamma} A \int dx'\ \ \left(\epsilon\hat{\mathbf{x}}\delta(x'-x)\cdot\frac{\partial^2}{\partial x'^2}\mathbf{m}\right) \\ -\frac{\mu_0 M_s}{\gamma} A \int dx'\ \ \left(\epsilon\hat{\mathbf{y}}\delta(x'-x)\cdot\frac{\partial^2}{\partial x'^2}\mathbf{m}\right) \\ -\frac{\mu_0 M_s}{\gamma} A \int dx'\ \ \left(\epsilon\hat{\mathbf{z}}\delta(x'-x)\cdot\frac{\partial^2}{\partial x'^2}\mathbf{m}\right) \end{pmatrix}, \tag{12}$$

which is easy to solve:

$$\lim_{\epsilon\to 0}\frac{1}{\epsilon}\begin{pmatrix} -\frac{\mu_0 M_s}{\gamma} A \int dx'\ \ \left(\epsilon\hat{\mathbf{x}}\delta(x'-x)\cdot\frac{\partial^2}{\partial x'^2}\mathbf{m}\right) \\ -\frac{\mu_0 M_s}{\gamma} A \int dx'\ \ \left(\epsilon\hat{\mathbf{y}}\delta(x'-x)\cdot\frac{\partial^2}{\partial x'^2}\mathbf{m}\right) \\ -\frac{\mu_0 M_s}{\gamma} A \int dx'\ \ \left(\epsilon\hat{\mathbf{z}}\delta(x'-x)\cdot\frac{\partial^2}{\partial x'^2}\mathbf{m}\right) \end{pmatrix} = -\frac{\mu_0 M_s}{\gamma} A \frac{\partial^2}{\partial x^2}\mathbf{m}. \tag{13}$$

The contribution of the Zeeman energy to the effective field is given by

$$\frac{\delta E_Z}{\delta \mathbf{m}} = \lim_{\epsilon\to 0}\frac{1}{\epsilon}\begin{pmatrix} -\mu_0 M_s \int dx' & \left(\mathbf{H}\cdot[\mathbf{m}+\epsilon\hat{\mathbf{x}}\delta(x'-x)] - \mathbf{H}\cdot\mathbf{m}\right) \\ -\mu_0 M_s \int dx' & \left(\mathbf{H}\cdot[\mathbf{m}+\epsilon\hat{\mathbf{y}}\delta(x'-x)] - \mathbf{H}\cdot\mathbf{m}\right) \\ -\mu_0 M_s \int dx' & \left(\mathbf{H}\cdot[\mathbf{m}+\epsilon\hat{\mathbf{z}}\delta(x'-x)] - \mathbf{H}\cdot\mathbf{m}\right) \end{pmatrix} = -\mu_0 M_s \mathbf{H}. \tag{14}$$

That makes the total effective field

$$\mathbf{H}_{eff} = -\frac{1}{\mu_0 M_s}\left(-\frac{\mu_0 M_s}{\gamma}A\frac{\partial^2}{\partial x^2}\mathbf{m} - \mu_0 M_s \mathbf{H}\right) = \frac{A}{\gamma}\frac{\partial^2}{\partial x^2}\mathbf{m} + \mathbf{H} = \begin{pmatrix} \frac{A}{\gamma}\frac{\partial^2}{\partial x^2} m_x \\ \frac{A}{\gamma}\frac{\partial^2}{\partial x^2} m_y \\ \frac{A}{\gamma}\frac{\partial^2}{\partial x^2} m_z + H_0 \end{pmatrix}. \tag{15}$$

2.1.3 Gilbert damping

Now it is clear what the Landau-Lifshitz equation (Eq. (1)) means. If the spin of an atom in a ferromagnet is disturbed from its equilibrium position (the alignment with the effective field), it will precess around the equilibrium axis. If there is no loss of energy, this movement never stops. However, this is not a realistic situation, because in reality the system always loses energy in the form of heat. To take this into account, Gilbert introduced the damping term in Eq. (2). Because of this damping term, the direction of the magnetization will move towards the equilibrium axis again.

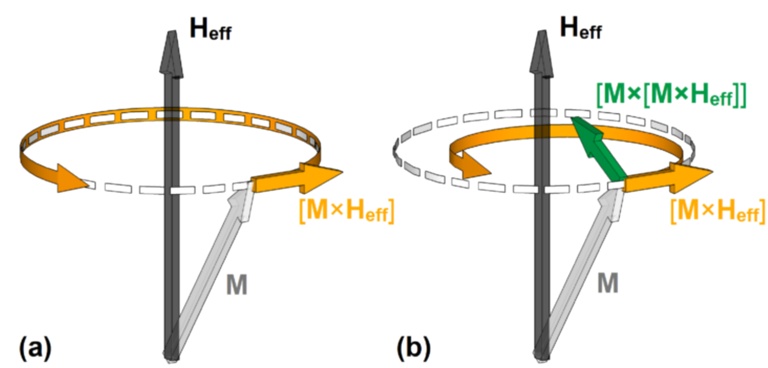

Figure 2: The precession of spin around the equilibrium axis without Gilbert damping (a) and with Gilbert damping (b) [13].

2.2 Spin transfer torques

The existence of the exchange energy described above has an important consequence. If a spin is disturbed from its equilibrium position, it changes the effective field, so the spin of the atom next to it will also start to precess as a result of the exchange energy. This movement is then transferred from spin to spin, or in other words, we have created a spin wave. This spin wave is described by a quasi-particle, the magnon. We would like to get a dispersion relation to describe the spin wave, and the Landau-Lifshitz-Gilbert (LLG) equation is the perfect tool to do that. But in this thesis we are interested in propagation of spin waves interacting with a current, so we have to add extra terms to the LLG equation in order to derive a dispersion relation that is useful to us.

2.2.1 Landau-Lifshitz-Gilbert equation with spin transfer torques

While every atom in the lattice has electrons bound to it, giving it a spin, it is also a characteristic of metals that they contain itinerant electrons, electrons that are not bound to atoms so they can move through the material. These electrons are responsible for the current. Just like there is interaction between the spins of the localized electrons, there is interaction between the localized electrons and the itinerant electrons which

is called sd-interaction: the free electrons induce spin transfer torques (STT's) on the bound electrons. This way a current influences the spin wave.
To take account for these STT's, we have to add two terms to the LLG equation, an adiabatic term [14] and a non-adiabatic term [15]. Then the equation looks like this:

$$\frac{\partial \mathbf{m}}{\partial t} = -\gamma \mathbf{m} \times \mathbf{H}_{eff} + \alpha \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t} - b_j \mathbf{m} \times (\mathbf{m} \times (\mathbf{j} \cdot \nabla)\mathbf{m}) - b_j \beta \mathbf{m} \times (\mathbf{j} \cdot \nabla)\mathbf{m}. \tag{16}$$

In the latter equation $b_j \mathbf{m} \times (\mathbf{m} \times (\mathbf{j} \cdot \nabla)\mathbf{m})$ is the adiabatic term and $b_j \beta \mathbf{m} \times (\mathbf{j} \cdot \nabla)\mathbf{m}$ the non-adiabatic term, with $\mathbf{j}$ the current density. The coupling constant between the current and the magnetization is described by $b_j = \frac{\mu_B P}{e M_s (1+\beta^2)}$, with μ_B the Bohr magneton, e the elementary charge and P the polarization of the current. The strength of the damping is given by $\beta = \frac{\tau_{ex}}{\tau_{sf}}$. Here, τ is the relaxation time of the free spins due to sd-interaction (τ_{ex}) or spin flips (τ_{sf}) [10].

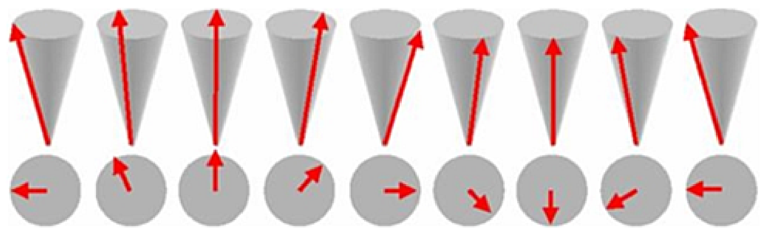

Figure 3: Impression of the propagation of a magnon through a one-dimensional lattice[16].

2.2.2 Dispersion relation

Now we are able to derive a dispersion relation for the spin wave. We linearize the magnetization around the $\hat{\mathbf{z}}$ direction because of the external field. To start the spin wave, we perturb the system in the $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ direction with small deviations:

$$\mathbf{m} = \begin{pmatrix} \delta m_x \\ \delta m_y \\ 1 \end{pmatrix}, \tag{17}$$

where $\delta m_{x,y} \ll 1$, so $\delta m_{x,y}^2 \approx 0$.
Plugging this into Eq. (16) we get the following equations:

$$\begin{bmatrix} \frac{\partial}{\partial t}\delta m_x \\ \frac{\partial}{\partial t}\delta m_y \end{bmatrix} = \begin{bmatrix} \left(-(H - A\frac{\partial^2}{\partial x^2}) - \alpha\frac{\partial}{\partial t} + b_j j \beta \frac{\partial}{\partial x}\right)\delta m_y + b_j j \frac{\partial}{\partial x}\delta m_x \\ \left(H - A\frac{\partial^2}{\partial x^2} + \alpha \frac{\partial}{\partial t} - b_j j \beta \frac{\partial}{\partial x}\right)\delta m_x + b_j j \frac{\partial}{\partial x}\delta m_y \end{bmatrix}, \tag{18}$$

where $H = \gamma H_0$ and we use that $\mathbf{j} = j$, since our current only flows in the $\hat{\mathbf{x}}$ direction.
Now we make an ansatz for the deviations of the following form:

$$\delta m_{x,y} = C_{x,y} e^{i(kx - \omega t)}. \tag{19}$$

Substituting Eq. (19) into Eq. (18) and writing it in matrix form, we get

$$\begin{bmatrix} -i(\omega + k b_j j) & H + Ak^2 - i\alpha\omega - ik b_j j \beta \\ -(H + Ak^2) + i\alpha\omega + ik b_j j\beta & -i(\omega + k b_j j) \end{bmatrix} \begin{bmatrix} C_x \\ C_y \end{bmatrix} = \mathbf{0}. \tag{20}$$

This equation only has solutions for $C_{x,y}$ if the determinant of the left matrix is zero. To solve this matrix for ω, we use that $\alpha, \beta \ll 1$, so $\alpha\beta \approx 0$ and $1 + i\alpha \approx \frac{1}{1 - i\alpha}$. That results in

$$\omega = H + Ak^2 - k b_j j - i\alpha H + Ak^2 + i(\alpha - \beta) k b_j j. \tag{21}$$

Now we have an expression that gives us the frequency ω as a function of the wave number k. This equation has some very important features. The first one is the presence of the term H. This term makes sure that there is a gap in the dispersion relation, or in other words, there is a certain amount of energy needed to initiate a spin wave (see figure 4). It is also the value of the minimum frequency of the spin wave. If we set $H = 1T$, which is a typical magnitude for the magnetic field in spin wave experiments, the minimum frequency is around 30 GHz.

The second feature is the difference between the real and the imaginary part of the dispersion relation. In Eq. (21), $b_j j$ has the dimension of velocity and can be seen as the velocity of the electrons forming the current. This means that if the current is big enough, both parts of the dispersion relation switch sign. If the real part switches sign, there is an energetic instability: the ground state of the system has changed. This is clear if we see the initiation of a spin wave as an excitation of the system that is linearly dependent of the frequency. A frequency with a negative real part than lowers the energy, indicating that the ground state we started with is not the real ground state. If the imaginary part of the dispersion relation switches sign, the system is dynamically unstable. This means that the system will move away from the state that it was in. This becomes clear if we take a look at our ansatz in Eq. (19). Inserting a frequency with a positive imaginary part in this ansatz means that the deviations blow up, so the system is not stable.

The important thing about the dispersion relation is that the critical current v_c at which the energetic instability occurs differs from the current at which the dynamic instability occurs, because of the existence of the β-damping. This means the system can be forced to obtain a new ground state, without moving to this new state. As we will see later in this thesis, this is a crucial condition for the simulation of a black hole and the presence of Cherenkov radiation.

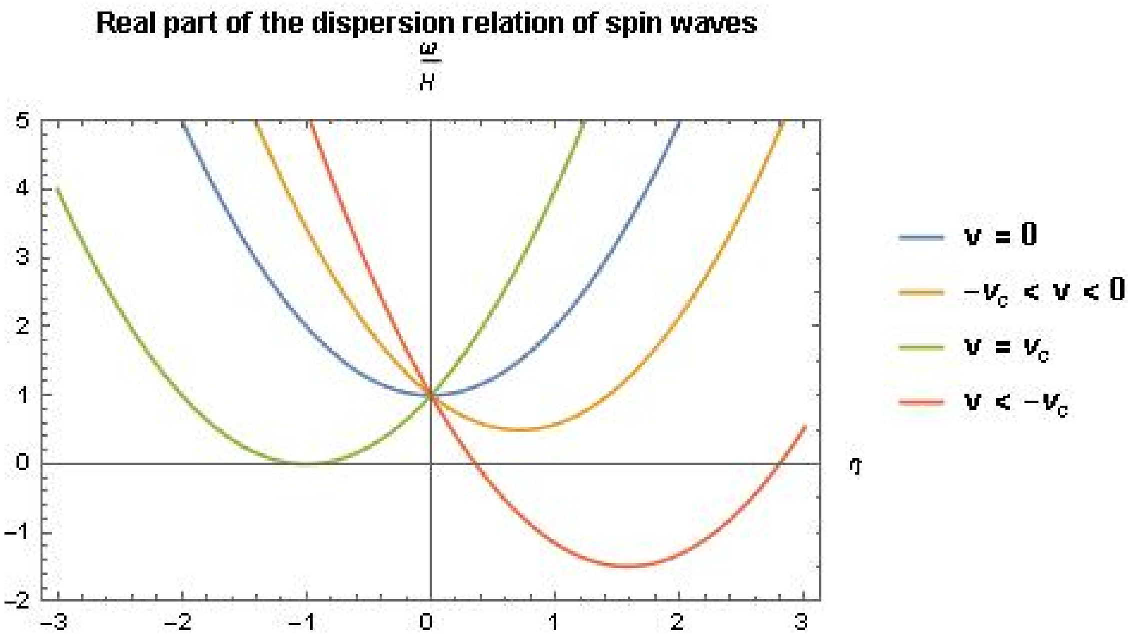

Figure 4: In this figure we plotted the dimensionless frequency $\frac{\omega}{H}$ against the dimensionless wave number η, which is related to k by $\eta = k\sqrt{\frac{A}{H}}$. We can clearly see the shifting of the dispersion relation by the electron velocity $v = -b_j j$. If $|v| > |v_c|$, ω is smaller than zero for certain k meaning the system possesses a different ground state than the one we started with. Also clear in this figure is the energy gap H, which is the minimum energy that is needed to initiate a spin wave at $v = 0$.

3 Reflection and transmission coefficients

Now that we have derived a dispersion relation for a spin wave, and concluded that there is a critical speed v_c at which the system obtains a new ground state without moving towards it, we can start with simulating a black or white hole. First we have to calculate the critical speed, after which we just follow the steps used in calculating the scattering of a quantum mechanical particle at a potential step [17]. In our case there is no potential step, but a sudden change in the velocity of the background electrons from smaller than v_c to larger than v_c or vice versa.

3.1 The critical speed

As said in the previous section, the term $b_j j$ has the dimension of velocity. Because j is the current and we are dealing with electrons (with negative charge), a positive j means a negative electron velocity, so we substitute $b_j j = -v$ in the dispersion relation (Eq. 21):

$$\omega = H + Ak^2 + kv - i\alpha(H + Ak^2) + i(\alpha - \beta)kv. \tag{22}$$

The real part of the latter equation changes sign if v is larger than a certain value v_c, where v_c is the velocity at which the minimum frequency is exactly zero, so when $\mathrm{Re}(\omega) = 0$ and $\frac{d\,\mathrm{Re}(\omega)}{dk} = 0$.

$$\frac{d\,\mathrm{Re}(\omega)}{dk} = 2Ak + v_c = 0, \tag{23}$$

so

$$k = -\frac{v_c}{2A}. \tag{24}$$

Plugging this into

$$H + Ak^2 + kv_c = 0, \tag{25}$$

gives

$$H + A\frac{{v_c}^2}{4A^2} - \frac{{v_c}^2}{2A} = 0. \tag{26}$$

Then we find

$$|v_c| = 2\sqrt{AH}. \tag{27}$$

When we do the same for the imaginary part, we find

$$|v_{c,d}| = |\frac{2\alpha}{\alpha - \beta}|\sqrt{AH}. \tag{28}$$

This means we have a system that is energetically unstable but dynamically stable when $|\frac{2\alpha}{\alpha-\beta}|\sqrt{AH} > |v| > 2\sqrt{AH}$. This is only possible if $0 < \beta < 2\alpha$. Note that when $\beta = \alpha$, the system will always be dynamically stable.

3.2 Scattering of the magnon

The next step in calculating the reflection and transmission coefficients is determining a Schrödinger-like equation for the magnon. Let us define a wave function Ψ so that

$$\mathbf{m} = \begin{pmatrix} \mathrm{Re}(\Psi) \\ -\mathrm{Im}(\Psi) \\ 1 - |\Psi|^2, \end{pmatrix}, \tag{29}$$

which is true for $\Psi = \delta m_x - i\delta m_y$. Then Eq. (18) transforms into

$$i\frac{\partial}{\partial t}\Psi(x,t) = \left(-A\frac{\partial^2}{\partial x^2} + H - iv\frac{\partial}{\partial x}\right)\Psi(x,t), \tag{30}$$

where we assumed the damping parameters α and β to be negligible small, an assumption that is necessary for the scattering calculation, as we shall see later on.
We now set the boundary conditions for the scattering problem. The first boundary condition is

$$\Psi_L(x_0) = \Psi_R(x_0), \tag{31}$$

e.g. the wave has to be continuous at the white or black hole positioned at x_0. For the next boundary condition we integrate both sides of our Schrödinger equation from $x_0 - \epsilon$ to $x_0 + \epsilon$ and then take the limit of ϵ to zero:

$$\lim_{\epsilon \to 0} \int_{x_0-\epsilon}^{x_0+\epsilon} dx\; i\frac{\partial}{\partial t}\Psi(x,t) = \lim_{\epsilon \to 0} \int_{x_0-\epsilon}^{x_0+\epsilon} dx\; \left(-A\frac{\partial^2}{\partial x^2} + H - iv(x)\frac{\partial}{\partial x}\right)\Psi(x,t), \tag{32}$$

which becomes

$$0 = \lim_{\epsilon \to 0}\left(\left[-A\frac{\partial}{\partial x}\Psi(x,t) - iv(x)\Psi(x,t)\right]_{x_0-\epsilon}^{x_0+\epsilon} + i\int_{x_0-\epsilon}^{x_0+\epsilon} dx\; \Psi(x,t)\frac{d}{dx}v(x)\right). \tag{33}$$

We can write $v(x)$ as $v_L + (v_R - v_L)\mathsf{H}(x - x_0)$, with v_L, v_R the electron velocity left and right of x_0, and with $\mathsf{H}(x - x_0)$ the Heaviside step function. The derivative of the Heaviside step function is the Dirac delta function, so we get

$$\left[\frac{\partial}{\partial x}\Psi_L(x,t)\right]_{x=x_0} = \left[\frac{\partial}{\partial x}\Psi_R(x,t)\right]_{x=x_0}. \tag{34}$$

We start solving the problem with the following ansatz:

$$\Psi_L = e^{ik_1x} + Be^{ik_2x}, \tag{35}$$
$$\Psi_R = Ce^{ik_3x}, \tag{36}$$

where B and C are the amplitudes of the reflected and transmitted spin wave respectively. If we set $x_0 = 0$, the two boundary conditions transform to the following equations:

$$1 + B = C, \tag{37}$$
$$k_1 + Bk_2 = Ck_3, \tag{38}$$

which leads to

$$B = \frac{k_3 - k_1}{k_2 - k_3}. \tag{39}$$

From the first boundary condition we now derive

$$C = \frac{k_2 - k_1}{k_2 - k_3}. \tag{40}$$

$k_{1,2,3}$ are determined by using the abc-formula for Eq. (22), with $\alpha, \beta = 0$:

$$k = \frac{-v \pm \sqrt{v^2 - 4A(H - \omega)}}{2A}. \tag{41}$$

This gives us the following k:

$$k_1 = \frac{-v_L + \sqrt{v_L^2 - 4A(H - \omega)}}{2A}, \tag{42}$$
$$k_2 = \frac{-v_L - \sqrt{v_L^2 - 4A(H - \omega)}}{2A}, \tag{43}$$
$$k_3 = \frac{-v_R + \sqrt{v_R^2 - 4A(H - \omega)}}{2A}, \tag{44}$$
$$k_4 = \frac{-v_R - \sqrt{v_R^2 - 4A(H - \omega)}}{2A}, \tag{45}$$

where k_4 is not yet of use to us, but it will be in the following section. Inserting these in Eq. (39) and Eq. (40) leads to

$$B = \frac{v_L - v_R + \sqrt{v_R^2 - 4A(H-\omega)} - \sqrt{v_L^2 - 4A(H-\omega)}}{v_R - v_L - \sqrt{v_L^2 - 4A(H-\omega)} - \sqrt{v_R^2 - 4A(H-\omega)}} \tag{46}$$

$$C = \frac{-2\sqrt{v_L^2 - 4A(H-\omega)}}{v_R - v_L - \sqrt{v_L^2 - 4A(H-\omega)} - \sqrt{v_R^2 - 4A(H-\omega)}}. \tag{47}$$

Having found the reflection and transmission amplitudes, we are now able to define and calculate the reflection and transmission coefficients as the amplitudes times their complex conjugate:

$$|B|^2 = \left(\frac{v_L - v_R + \sqrt{v_R^2 - 4A(H-\omega)} - \sqrt{v_L^2 - 4A(H-\omega)}}{v_R - v_L - \sqrt{v_L^2 - 4A(H-\omega)} - \sqrt{v_R^2 - 4A(H-\omega)}}\right)^2 \tag{48}$$

$$|C|^2 = \frac{4(v_L^2 - 4A(H-\omega))}{\left(v_R - v_L - \sqrt{v_L^2 - 4A(H-\omega)} - \sqrt{v_R^2 - 4A(H-\omega)}\right)^2}. \tag{49}$$

In appendix A, we derive a demand the two coefficients should always meet, using an adjusted version of the continuity equation of the quantum mechanical scattering problem. Curiously enough, the coefficients we just calculated do not match this equation. This can have many reasons, on which we will further elaborate in the discussion (section 6). For now, we assume that our coefficients are right.

Below, in figures 5, 6, 7 and 8, are the two coefficients plotted against the dimensionless frequency $\frac{\omega}{H}$ in the four possible situations for a spin wave encountering the horizon of a black or white hole, where the “supersonic” area has an electron velocity of $|v| = \frac{101}{100}v_c$ and the “subsonic” area an electron velocity of $|v| = \frac{99}{100}v_c$. In the plots it is clearly visible that if the frequency (i.e. the energy of the magnon) is high, there is almost no reflection. Only with low frequencies, approximately $\omega < \frac{H}{2}$, the reflection becomes substantial. Note that this scattering calculation only holds if there is a magnon frequency with $\omega > H - \frac{v^2}{4A}$. For smaller frequencies the wave numbers become imaginary, and we get a different kind of scattering (with exponentially growing or decaying wave functions). Although this is precisely the case we are interesting in for our magnonic black or white hole (because only with lower frequencies we attain energetic instability), we now stick to the scattering calculation with higher frequencies, for reasons that will become clear in the next section.
In the plots, we see similar coefficients for spin waves going into a white or black hole (figure 5 and 6) with a transmission coefficient going to zero and a reflection coefficient going to one for lower frequencies. The coefficients for spin waves going out of a white or black hole (figure 7 and 8) also look similar, with a remarkable high transmission coefficient for low frequencies. Some of the results may seem counter-intuitive (for instance the high reflection coefficient for a wave entering a black hole, or the high transmission coefficient for a wave escaping a black hole), but it is important to realize that our black and white holes are not really black and white holes for the frequencies we plotted, because there is no energetic instability, as we just explained.
The good thing about the reflection and transmission coefficients is that they can be measured. Magnonic scattering experiments have already been conducted in the past, where the cause of the scattering was something different than a black or white hole (for instance an inhomogeneous region in a homogeneous ferromagnetic film [18]). The spin waves can be measured with techniques like ferromagnetic resonance [19].

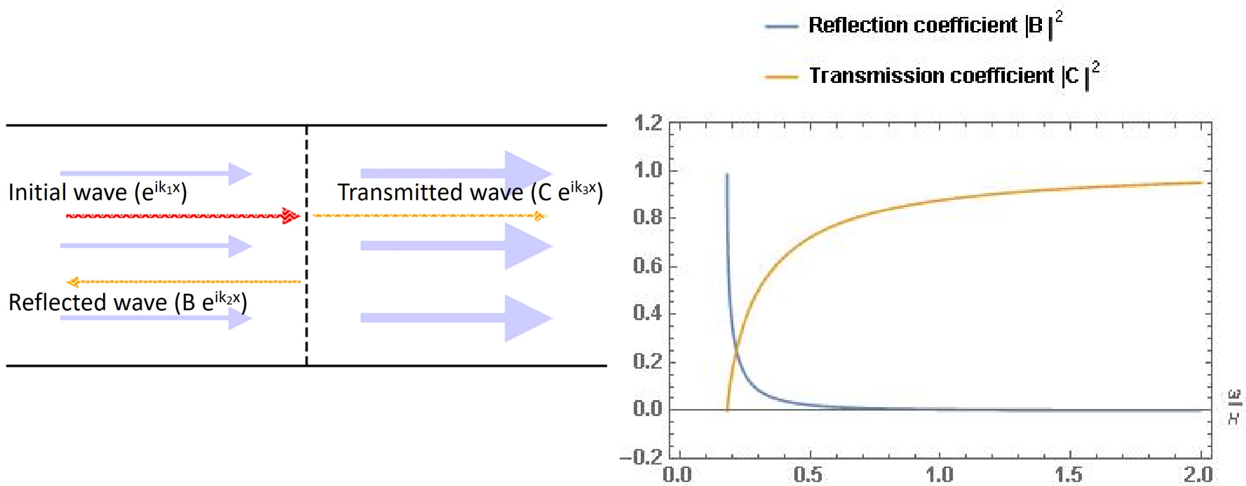

Figure 5: An initial spin wave going into a black hole ($v_L = \frac{99}{100} v_c$, $v_R = \frac{101}{100} v_c$, as indicated by the blue arrows).

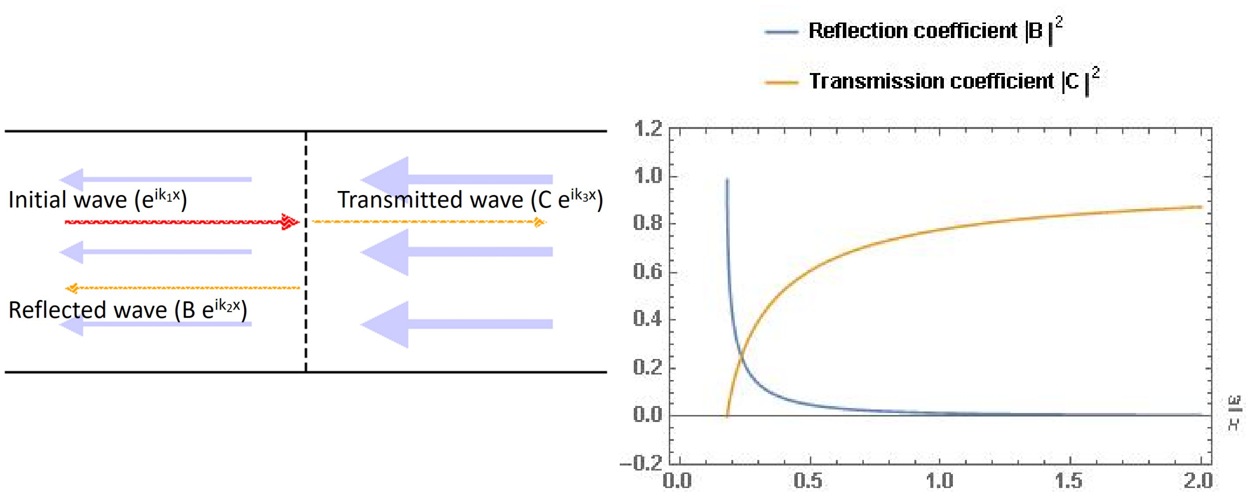

Figure 6: An initial spin wave going into a white hole ($v_L = -\frac{99}{100} v_c$, $v_R = -\frac{101}{100} v_c$, as indicated by the blue arrows).

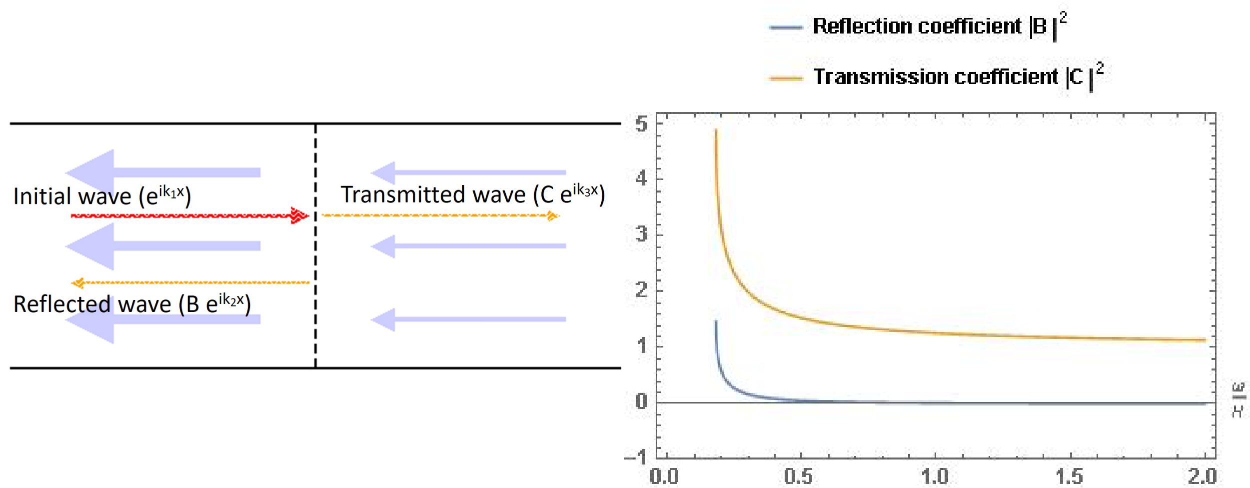

Figure 7: An initial spin wave going out of a black hole ($v_L = -\frac{101}{100}v_c$, $v_R = -\frac{99}{100}v_c$, as indicated by the blue arrows).

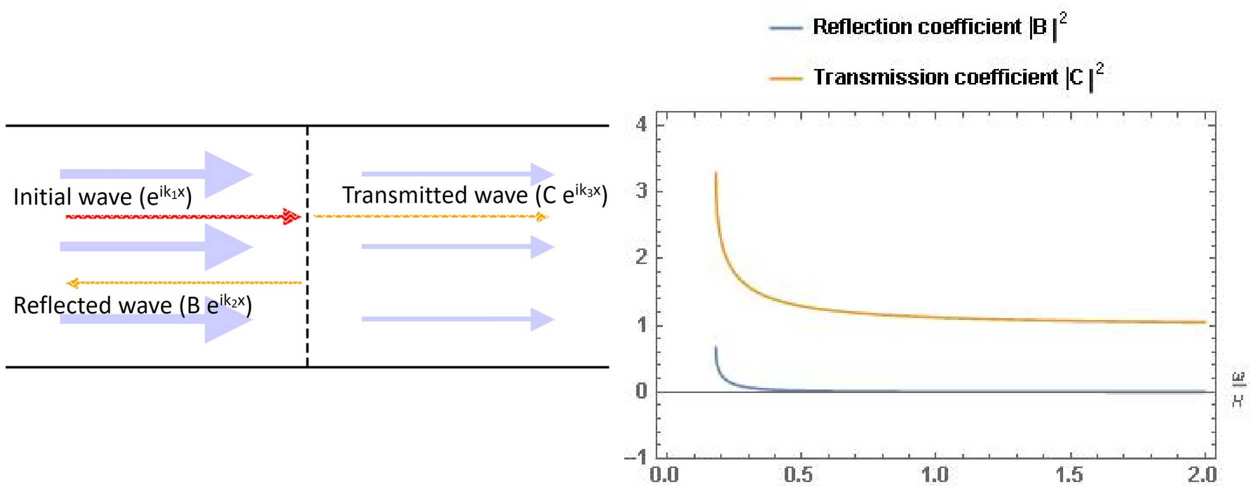

Figure 8: An initial spin wave going out of a white hole ($v_L = \frac{101}{100}v_c$, $v_R = \frac{99}{100}v_c$, as indicated by the blue arrows).

4 The Cherenkov effect

We now continue with the last step of our calculations, having acquired all the necessary mathematical tools to calculate the Cherenkov effect of the magnonic black or white hole.

4.1 The Green's function

A Green's function is normally used to calculate the Cherenkov radiation, as it is the ideal mathematical tool to determine the influence of the moving of a source on the field that source causes. Although our case is different (we are dealing with a stationary source and a moving medium), we still need a Green's function for determining the wave function.
We start with adding a source term $S(x)$ to our wave function:

$$i\frac{\partial}{\partial t}\Psi(x,t) = \left(-A\frac{\partial^2}{\partial x^2} + H - iv\frac{\partial}{\partial x}\right)\Psi(x,t) + S(x). \tag{50}$$

Note that both sides can be multiplied with $\hbar$, and then we associate the left side of the equation as the energy of the magnon E, and the right side minus the source term as $\mathcal{H}_0/\hbar$, a differential operator that gives the energy when acting upon the wave function. Now we get

$$\left[\omega - \frac{\mathcal{H}_0}{\hbar}\right]\Psi(x) = S(x), \tag{51}$$

with $\omega = \frac{E}{\hbar}$ and $\mathcal{H}_0/\hbar = -A\frac{\partial^2}{\partial x^2} + H - iv$. The solution of the differential equation is

$$\Psi(x,t) = \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dt'\, G(x,x',\omega)S(x'), \tag{52}$$

where the Green's function $G(x,x',\omega)$ is a solution to

$$\left[\omega - \frac{\mathcal{H}_0}{\hbar}\right]G(x,x',\omega) = \delta(x-x'). \tag{53}$$

To find the Green's function we redefine the solutions we found for the homogeneous problem (the wave equation without the source term):

$$\Psi_1(x,\omega) = \begin{cases} e^{ik_1(\omega)x} + B_1(\omega)e^{ik_2(\omega)x} & \text{if } x \leq 0; \\ C_1(\omega)e^{ik_3(\omega)x} & \text{if } x \geq 0, \end{cases}$$

$$\Psi_2(x,\omega) = \begin{cases} e^{ik_4(\omega)x} + B_2(\omega)e^{ik_3(\omega)x} & \text{if } x \geq 0; \\ C_2(\omega)e^{ik_2(\omega)x} & \text{if } x \leq 0, \end{cases} \tag{54}$$

with reflection coefficients B_i and transmission coefficients C_i as follows:

$$B_1(\omega) = \frac{v_L - v_R + \sqrt{v_R^2 - 4A(H-\omega)} - \sqrt{v_L^2 - 4A(H-\omega)}}{v_R - v_L - \sqrt{v_L^2 - 4A(H-\omega)} - \sqrt{v_R^2 - 4A(H-\omega)}}, \tag{55}$$

$$C_1(\omega) = \frac{-2\sqrt{v_L^2 - 4A(H-\omega)}}{v_R - v_L - \sqrt{v_L^2 - 4A(H-\omega)} - \sqrt{v_R^2 - 4A(H-\omega)}}, \tag{56}$$

$$B_2(\omega) = \frac{v_R - v_L + \sqrt{v_R^2 - 4A(H-\omega)} - \sqrt{v_L^2 - 4A(H-\omega)}}{v_L - v_R + \sqrt{v_L^2 - 4A(H-\omega)} + \sqrt{v_R^2 - 4A(H-\omega)}}, \tag{57}$$

$$C_2(\omega) = \frac{2\sqrt{v_R^2 - 4A(H-\omega)}}{v_L - v_R + \sqrt{v_L^2 - 4A(H-\omega)} + \sqrt{v_R^2 - 4A(H-\omega)}}, \tag{58}$$

where we calculated B_2 and C_2 the same way as B_1 and C_1 in section 3. In equation 54, Ψ_1 corresponds to an initial wave reaching the horizon from the left side and Ψ_2 corresponds to an initial wave reaching the horizon from the right side.
We know from quantum mechanics that our wave functions are orthonormal, so

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega' \ \Psi_n(x,\omega')\Psi_n^*(x',\omega') = \delta(x-x'). \tag{59}$$

We now construct our Green's function as a linear combination of possible solutions [20]:

$$G(x,x',\omega) = \sum_{n\in\{1,2\}} c_n(x)\frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega' \ \frac{\Psi_n(x,\omega')\Psi_n^*(x',\omega')}{\omega-\omega'(1+i\alpha)}, \tag{60}$$

where we reintroduced an infinitesimal small damping factor α so we integrate over the lower complex plane. This ensures that we have a retarded Green's function, which is what we need in this case [21]. The normalization factors $c_n(x)$ obey $\sum_{n\in\{1,2\}} c_n(x) = 1$.
Substituting the wave functions (Eq. 54) leads to the following:

$$G(x,x',\omega) = \begin{cases} \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega' \ \left[\frac{c_1\left(e^{ik_1x}+B_1e^{ik_2x}\right)\left(e^{-ik_1x'}+B_1^*e^{-ik_2x'}\right)+c_2\left(|C_2|^2e^{ik_2(x-x')}\right)}{\omega-\omega'(1+i\alpha)}\right] & \text{if } x,x'\le 0; \\ \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega' \ \left[\frac{c_1\left(e^{ik_1x}+B_1e^{ik_2x}\right)\left(C_1^*e^{-ik_3x'}\right)+c_2\left(C_2e^{ik_2x}\right)\left(e^{-ik_4x'}+B_2^*e^{-ik_3x'}\right)}{\omega-\omega'(1+i\alpha)}\right] & \text{if } x\le 0\le x'; \\ \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega' \ \left[\frac{c_1\left(C_1e^{ik_3x}\right)\left(e^{-ik_1x'}+B_1^*e^{-ik_2x'}\right)+c_2\left(e^{ik_4x}+B_2e^{ik_3x}\right)\left(C_2^*e^{-ik_2x}\right)}{\omega-\omega'(1+i\alpha)}\right] & \text{if } x'\le 0\le x; \\ \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega' \ \left[\frac{c_1\left(|C_1|^2e^{ik_2(x-x')}\right)+c_2\left(e^{ik_4x}+B_2e^{ik_3x}\right)\left(e^{-ik_4x}+B_2^*e^{-ik_3x}\right)}{\omega-\omega'(1+i\alpha)}\right] & \text{if } x,x'\ge 0. \end{cases} \tag{61}$$

Now we formulate a solution to our nonhomogeneous problem in two parts:

$$\begin{aligned} \Psi_L(x,t) \quad &= \quad \frac{1}{2\pi}\int_{-\infty}^{\infty} dt' \int_{-\infty}^{\infty} \frac{d\omega'}{\omega-\omega'(1+i\alpha)} \\ &\Big[\quad \int_{-\infty}^{0} dx' \ \left(c_1\left(e^{ik_1x}+B_1e^{ik_2x}\right)\left(e^{-ik_1x'}+B_1^*e^{-ik_2x'}\right)+c_2\left(|C_2|^2e^{ik_2(x-x')}\right)\right)S(x') \\ +\int_0^{\infty} \quad dx' \quad &\left(c_1\left(e^{ik_1x}+B_1e^{ik_2x}\right)\left(C_1^*e^{-ik_3x'}\right)+c_2\left(C_2e^{ik_2x}\right)\left(e^{-ik_4x'}+B_2^*e^{-ik_3x'}\right)\right)S(x')\Big], \end{aligned} \tag{62}$$

for $x \le 0$, and

$$\begin{aligned} \Psi_R(x,t) \quad &= \quad \frac{1}{2\pi}\int_{-\infty}^{\infty} dt' \int_{-\infty}^{\infty} \frac{d\omega'}{\omega-\omega'(1+i\alpha)} \\ &\Big[\quad \int_{-\infty}^{0} dx' \ \left(c_1\left(C_1e^{ik_3x}\right)\left(e^{-ik_1x'}+B_1^*e^{-ik_2x'}\right)+c_2\left(e^{ik_4x}+B_2e^{ik_3x}\right)\left(C_2^*e^{-ik_2x'}\right)\right)S(x') \\ &+ \quad \int_0^{\infty} dx' \ \left(c_1\left(|C_1|^2e^{ik_2(x-x')}\right)+c_2\left(e^{ik_4x}+B_2e^{ik_3x}\right)\left(e^{-ik_4x'}+B_2^*e^{-ik_3x'}\right)\right)S(x')\Big], \end{aligned} \tag{63}$$

for $x \ge 0$. These are the expressions we need to find the Cherenkov radiation of a black or white hole.

5 Conclusion

In this thesis we derived expressions that describe the Cherenkov effect caused by a magnonic black or white hole. In section 2, we determined the dispersion relation of a magnon interacting with a spin-polarized current out of the Landau-Lifshitz-Gilbert equation, extended with an adiabatic and a non-adiabatic term to take account for the influence of the electrical current. We assumed that the external magnetic field is so large that all contributions to the effective field apart from the contributions of the exchange energy and the external field can be neglected (for instance the anisotropy). We used this in section 3 to find the critical itinerant electron velocity. With this critical velocity, we considered the black or white hole analogue, with on one side of the horizon an electron velocity higher than critical, and on the other side lower than critical. Following the quantum mechanical scattering calculation at a potential step, we defined wave functions of our magnon on both sides of the horizon by transforming our dispersion relation. We calculated the reflection and transmission coefficients of an initial spin wave propagating out or in of a black or white hole (neglecting the damping terms of the dispersion relation). We found coefficients that are only dependent of the wave frequency and the electron speed on both sides of the horizon.

The next step was substituting these wave functions in a Green's function, so we could derive a new wave function that involved the influence of a source term in a moving medium. We found expressions in section 4 for this wave function that should give the Cherenkov radiation of the black or white hole when evaluated.

6 Discussion and Outlook

The most intriguing point of discussion of this thesis is the contradiction between the reflection and transmission amplitudes found in section 4 and the relation between the coefficients derived out of the continuity equation in appendix A. This contradiction means that we made a mistake in at least one of these derivations. The derivations both look quite straightforward; they both follow from the quantum mechanical scattering calculation, adjusted to our case of a magnonic black or white hole. So the question is: what subtlety did we miss? If we made a mistake in the appendix, which we assumed in section 3, the rest of this thesis still holds. However, if there is something wrong in our derivation of the coefficients, the third and fourth section have to be revised. It is therefore very important that this contradiction will be solved.
One way so solve it that seems to be mathematically correct, is by adding a nonzero constant when solving the integral of equation 68, with value $-v_L|\Psi_L(0)|^2$ for the current on the left side and value $-v_R|\Psi_R(0)|^2$ for the current on the right side of the horizon. Equation 72 then becomes

$$|B|^2 + \frac{v_L - v_R + \sqrt{v_R^2 - 4A(H-\omega)}}{\sqrt{v_L^2 - 4A(H-\omega)}}|C|^2 = 1, \tag{64}$$

which is perfectly correct for the coefficients we found. However, there should be a physical reason to fix the constants of the integral on a certain value (for instance a boundary condition) and up to now, we haven't been able to find a justification for these values. It probably requires a fresh perspective to find this justification, or to find a completely different solution for this problem, so we invite everyone who reads this thesis to take up the challenge.

6.1 Outlook

Unfortunately, there was not enough time in this research project to solve the integrals of the wave function we found in section 4, either numerically or analytically. This should be the first step in following research on magnonic black or white holes, after of course the problem mentioned above is solved. The Cherenkov radiation caused by a magnonic black or white hole is an intriguing phenomenon. Does a black hole causes long-range Cherenkov waves? How does the wave pattern look like in a white hole, where the part of the source term producing Cherenkov radiation can't produce an outgoing wave because of the "supersonic" electron velocity (see figure 9)? These are crucial questions in the research of the magnonic black-hole-analogue, and should be answered before we can really proceed to setting up experiments.

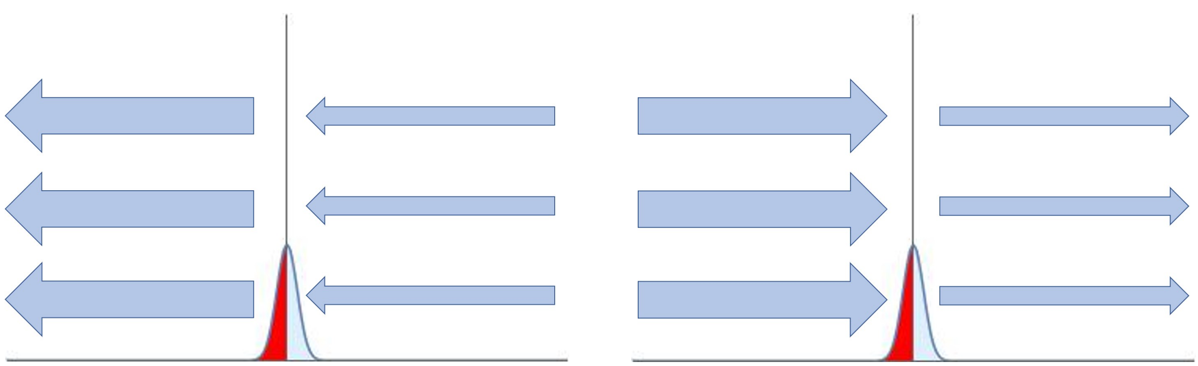

Figure 9: Schematic picture of a magnonic black (left) and white (right) hole, where the blue arrows indicate the electron velocity and the source term is represented by a Gaussian-like curve, centered around the hole. The red part of the source should normally produce Cherenkov radiation, being in the "supersonic" area. At the black hole, this could cause long-range Cherenkov waves into the black hole. However, the spin waves created by the source at the white hole can't propagate away from the source. Calculating the wave pattern of the Cherenkov radiation is therefore far from trivial.

A Calculating the continuity equation for spin waves interacting with a current

In this appendix we use the continuity equation as formulated in hydrodynamics and quantum mechanics to derive the probability current of the spin wave. This offers us a way to check our reflection and transmission coefficients.
The continuity equation in quantum mechanics reads

$$\frac{\partial}{\partial t}|\Psi|^2 = -\frac{\partial}{\partial x}J, \tag{65}$$

with J the probability current [17]. Here J is defined as $J = \Psi^* \frac{\partial}{\partial x}\Psi - \Psi\frac{\partial}{\partial x}\Psi^*$, but our abnormal Schrödinger equation forces us to find a new definition. Knowing that

$$i\frac{\partial}{\partial t}\Psi^*(x,t) = \left(A\frac{\partial^2}{\partial x^2} - H - iv(x)\frac{\partial}{\partial x}\right)\Psi^*(x,t), \tag{66}$$

we obtain the expression

$$\frac{\partial}{\partial t}|\Psi|^2 = \frac{\partial}{\partial x}\left[-iA\left(\Psi\frac{\partial}{\partial x}\Psi^* - \Psi^*\frac{\partial}{\partial x}\Psi\right) - \int^x dx'\ v(x')\frac{\partial}{\partial x'}|\Psi|^2\right]. \tag{67}$$

So the probability current becomes

$$J = iA\left(\Psi(x,t)\frac{\partial}{\partial x}\Psi^*(x,t) - \Psi^*(x,t)\frac{\partial}{\partial x}\Psi(x,t)\right) + \int^x dx'\ v(x')\frac{\partial}{\partial x'}|\Psi(x',t)|^2. \tag{68}$$

Because in our case $\Psi(x,t) = \Psi(x)$, i.e. the wave function is time independent, the probability current is independent of x, so the current is the same at both sides of the event horizon:

$$\begin{aligned} &iA\left(\Psi_L(x)\frac{\partial}{\partial x}\Psi_L^*(x) - \Psi_L^*(x)\frac{\partial}{\partial x}\Psi_L(x)\right) + v_L|\Psi_L(x)|^2 = \\ &iA\left(\Psi_R(x)\frac{\partial}{\partial x}\Psi_R^*(x) - \Psi_R^*(x)\frac{\partial}{\partial x}\Psi_R(x)\right) + v_R|\Psi_R(x)|^2. \end{aligned} \tag{69}$$

Substituting our ansatz (Eq. 35) for Ψ_L and Ψ_R leads to

$$\begin{aligned} &A\left(2k_1 + k_1Be^{i(k_2-k_1)x} + k_2Be^{i(k_2-k_1)x} + k_1B^*e^{i(k_1-k_2)x} + k_2B^*e^{i(k_1-k_2)x} + 2k_2|B|^2\right) + \\ &v_L\left(1 + Be^{i(k_2-k_1)x} + B^*e^{i(k_1-k_2)x} + |B|^2\right) = 2Ak_3|C|^2 + v_R|C|^2. \end{aligned} \tag{70}$$

By substituting our expressions for k (Eq. 42) we obtain

$$\sqrt{v_L^2 - 4A(H-\omega)} - \sqrt{v_L^2 - 4A(H-\omega)}|B|^2 = \sqrt{v_R^2 - 4A(H-\omega)}|C|^2. \tag{71}$$

Now we derived a requirement every reflection coefficient $|B|^2$ and transmission coefficient $|C|^2$ should meet:

$$|B|^2 + \frac{\sqrt{v_R^2 - 4A(H-\omega)}}{\sqrt{v_L^2 - 4A(H-\omega)}}|C|^2 = 1. \tag{72}$$

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