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A Solution to the Free Choice Permission Paradox with Truthmaker Semantics

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Introduction

The main topic of this thesis is *free choice permission.* Consider the following sentences:

> You may have coffee or tea.
>
> You may have coffee and you may have tea.

In the first sentence, you are clearly given a free choice to have coffee or tea. Intuitively, the second sentence then follows from the first: if you may have coffee or tea, then you may have coffee, and you may have tea. What characterizes the principle of free choice permission is exactly this intuitive reasoning.

Reasoning in general is an important research topic of artificial intelligence. Artificial Intelligence (AI) is devoted to developing agents that display intelligence – ideally, human-like intelligence [23]. Perhaps the most fundamental requirement for an agent to be intelligent is the ability to reason. There is, however, no consensus in AI on the approach towards achieving this ability. Research areas of sub-symbolic AI, such as natural language processing and artificial neural networks, have a connectionist or neurocomputational approach [6], whereas symbolic AI is concerned with formalizing reasoning into logical systems.

Although logic is a discipline of its own, it has many applications within (symbolic) AI: propositional and first-order logic for making logical deductions in problem solving; dynamic logic for multi-agent interaction; fuzzy logic, probability theory, and probabilistic reasoning for dealing with uncertainty and incomplete information; modal logic for modeling belief and knowledge of other agents; and so on [6]. Research in the field of logic is thus essential for the advancement of AI systems.

John McCarthy, one of the founders of AI and the man who coined the term "artificial intelligence," would have probably agreed; he advocated the approach of using logical techniques to formalize reasoning and the associated problems, which AI needs to solve [23]. McCarthy claimed that a reasoning problem must first be understood before its solution can be implemented, and a logical formalization of the problem helps us in this understanding. In line with this philosophy, the first aim of this thesis is to understand and formalize such a reasoning problem, specifically a problem of reasoning with deontic concepts.

Deontic logic is the branch of logic that is concerned with formal reasoning about permission, obligation and other related deontic concepts [20]. Von Wright [26] first developed a symbolic system of deontic logic that dealt with the obligation and permission of *acts*. Later, his work, as well as deontic logic in general, was strongly influenced by the ideas in modal logic. This resulted in the rise of Standard Deontic Logic (SDL), which is still the most studied system of deontic logic [20]. However, the fact that it is the most studied system does not mean it is a perfect system. One of its flaws, as of other deontic systems, is that it cannot deal well with free choice permission.

We have already introduced the principle of free choice permission. Now consider the following line of reasoning:

1. You may have tea.
2. Having tea implies having coffee or tea.
3. You may have coffee or tea.
4. You may have coffee and you may have tea.
5. You may have coffee.

According to the principle of free choice permission we can infer 4 from 3. But because having tea implies having coffee or tea, permission to have tea now entails permission to have coffee, which seems terribly wrong. In deontic logic, this problem is called the *free choice permission paradox.*

The first aim of this thesis is to understand and formalize free choice permission and its associated paradox. The second aim is then to provide a solution to this paradox. Several proposals have already been made over the years. However, no approach has thus far provided a satisfying solution. In this thesis, a solution will be explored based on the logical framework of truthmaker semantics, as proposed by Kit Fine [11]. As opposed to possible worlds semantics, in which statements are either true or false at a world, with truthmaker semantics, statements are verified or falsified by so-called truthmakers. A truthmaker in general is an act or state of affairs that makes a certain statement true, such as an apple's being red which verifies the statement "the apple is red." In Fine's sense [8], truthmakers must be *exact*. This means that truthmakers must be wholly relevant to the statement they verify. In other words, a truthmaker may not contain anything irrelevant to the truth of the statement.

The research goals of this thesis will have been achieved when I, as well as the reader, can answer the following question:

> How can truthmaker semantics provide a solution to the free choice permission paradox?

This main question will be tackled by answering the following sub-questions:

1. What is (free choice) permission?
2. What is the free choice permission paradox?
3. What is truthmaker semantics, and how does it work?
4. How can truthmaker semantics deal with (free choice) permission?
5. How, or in what way, does this solve the free choice permission paradox?
6. What other solutions have been put forward? And (why) is truthmaker semantics a better approach?

The structure of this thesis will closely follow the questions posed above. Chapter 2 gives an introduction to free choice permission. Section 2.1 will begin by outlining deontic logic and concepts of permission. Section 2.2 gives a formal as well as informal account of the principle of free choice permission. The paradox that accompanies it is then discussed in Section 2.3. Chapter 3 explores the framework of truthmaker semantics and its potential to represent permission. The basic framework is introduced in Section 3.1. A semantic clause for permission is defined in section 3.2, after which a solution is provided to the free choice permission paradox. Chapter 4 discusses this solution and its limitations in Section 4.1, and compares it to other solutions in literature in Section 4.2. Improvement and an extension of the proposal is then suggested in Section 4.3. Chapter 5 concludes with a summary, discussion and potential further research.

Free Choice Permission

This chapter opens with section 2.1, providing a short introduction and discussion of linguistic and logical concepts of deontic logic, specifically permission. Sections 2.2 then explains the principle of free choice permission both formally and informally. In section 2.3 it is shown how the formalization of free choice permission leads to disaster in traditional systems of deontic logic, giving rise to the free choice permission paradox.

2.1 Deontic Concepts

Deontic[1] logic is the branch of logic that is concerned with modalities of obligation [26]. Informally, these modalities are normative concepts such as obligation, permission and prohibition. According to Hansson [13, p. 196], "permission statements indicate the absence of norms." He seems to imply that norms can only be obligations or prohibitions. However, although it might seem like norms are either prescriptive, like "keep your dog on leash," or prohibitive, like "it is forbidden to drive without a license," they can also serve as guidelines. Especially social norms, such as "it is customary to shake hands when meeting someone," have a more permissive character than an obligatory one.

To be able to give a formal account of permission, we first have to establish what permission signifies in natural language. We would like the logic to match our intuitions. Hence, we shall first outline what the linguistic concepts of permission are to make a nice transition towards the logical concepts.

2.1.1 Linguistic Concepts of Permission

In ordinary language, granting permission is giving someone the choice to "either perform the action or refrain from performing it" [22, p. 161]. With this notion of permission, one is free to perform the action, that is, one could not be obliged to perform the action. In standard deontic logic, however, permission *is* compatible with obligation. In fact, permission is implied by obligation. For example: You have to be at work at 8:30. Surely if you are obligated, then you are permitted to be there at 8:30. This latter notion of permission is called *unilateral permission*; the former is *bilateral*[2] *permission* [13, p. 199]. In linguistic terms, we would intuitively interpret permission in the bilateral sense. It would be weird if someone was obliged to have coffee if it was allowed. Vice versa would it be weird to say it is 'allowed to stop for a red traffic light', when it is clearly obligated.

In addition to permission being interpreted differently depending on the context (formal or not), it can also be granted in different ways [13]. Maybe the most obvious type of permission is *explicit permission*, in which the permission is explicitly stated: "The teacher allowed us to eat during class," states that we were permitted to eat during class. The second type is *implied permission*, which is implicitly granted within some statement:

[1] comes from the Ancient Greek $\delta\varepsilon o\nu$ (déon), meaning 'that which is proper or binding' [20]

[2] 'having two sides' [5]

"Beyond this gate, dogs are allowed to run free," implies that dogs are permitted beyond the gate. The last type of permission is often not considered as actual permission [13, p. 201]. A so-called tacit permission is a permission for something that is merely not forbidden: "There is no prohibition against skating on the road," allows me to do so. Both implied and tacit permissions are inferred from explicit permissions, though in a different way; an implied permission follows from the presence of some explicit permission, whereas a tacit permission follows from the absence of it. In that sense, we can make a dichotomy between permissions that are created (explicit and implied permissions) and permissions that exist "by default" (tacit permissions) – by default, because no statements prohibit their content. These types of permission are most commonly referred to as "strong" and "weak" permissions, respectively [13], as does Von Wright [27].

Another distinction can be made between statements that create permissions and ones that report their existence. A sentence such as "You are allowed to write your thesis in English," may either report that the permission holds, or indicate that it did not before. According to Hansson [13], it is debatable whether explicit permissions always form an exception to some prohibition. This example suggests they are not; statements with explicit permissions can also repeat pre-existing permissions.

2.1.2 Logical Concepts of Permission

In natural language, permission can be expressed in many different words: allowed, permitted, may, can, and so on. To express logical permission, however, we use a single operator: P. Similarly, the obligation operator is denoted by O. But what do these operators operate on? It is perhaps easiest and most intuitive to think of them as operating on acts. An "act" is broadly defined as "the doing of a thing" [1]. So for example, if we let t be the act of me drinking tea, then Pt would be interpreted as "I am permitted to drink tea." It is clear that this permissive statement has a truth value; it can be true or false that it is permitted to drink tea. An act itself, however, does not have a truth-value. This can pose a problem if we want formulas such as $A \wedge PA$ to be logically valid as well. Fortunately, we can quite simply solve this problem by changing our interpretation of the P operator. Instead of interpreting PA as "it is permitted to A", with A an act, we can interpret it as "it is permitted that A (is true)," with A a proposition. This way $t \wedge Pt$ translates to "I am drinking tea and it is permitted that I am drinking tea."

Several analogies can be made between the concepts of (standard) deontic logic and alethic logic. The first is the mutual definability of the operators. The alethic modalities[3], possibility and necessity, are respectively denoted by the diamond, $\Diamond$, and the box operator, $\Box$. These operators are mutually definable:

$$\Diamond A \leftrightarrow \neg\Box\neg A$$

$$\Box A \leftrightarrow \neg\Diamond\neg A$$

In words, something is possible if and only if its negation is not necessary; and something is necessary if and only if its negation is not possible. Similarly, the operators for the deontic modalities, permission and obligation, are mutually definable:

$$PA \leftrightarrow \neg O\neg A$$

$$OA \leftrightarrow \neg P\neg A$$

In words, something is permitted if and only if its negation is not obligated (i.e., iff it is not forbidden); and something is obligated if and only if its negation is not permitted (i.e., iff its negation is forbidden).

[3]modalities 'of truth' [2]

Sometimes, a third operator, usually F, is added to denote prohibition, as in Hansson [13]. Since we will not be discussing prohibition, and forbidding (prohibiting) can easily be defined as 'not allowing' ($FA \leftrightarrow \neg PA$) or 'obligating not to' ($FA \leftrightarrow O\neg A$), adding the operator is redundant here. The obligation operator might seem redundant as well, because it can be defined in terms of permission (see definition above). However, "not being permitted to not do something" is quite an awkward and unintuitive way to talk about obligation, so it is convenient to use a distinct operator for it.

A second analogy is the unilateral notion of possibility and permission. Like I mentioned before, logical permission is implied by obligation: if something is obligated, then it is permitted. In the same way, possibility is implied by necessity: if something is necessary, then surely it is possible. For now, we will not further concern ourselves with alethic modalities. It might be nice, however, to keep these analogies in mind.

Let us quickly revisit the unilateral notion of permission. In natural language, "it is permitted that A," means $PA \wedge P\neg A$, in agreement with the bilateral notion of permission. Von Wright [26] uses the term "moral indifference"[4], which refers to the situation where an act and its negation are both permitted. In deontic logic, however, permission is defined unilaterally. With this notion, permission is implied by obligation:

$$OA \rightarrow PA$$

In words, if something is obligated, then it is permitted. Note that this notion does not exclude moral indifference. Rather, it becomes a (special) case of unilateral permission. Combined with duality, we get $OA \rightarrow \neg O\neg A$, which 'simply' makes it impossible for both A and its negation to be obligatory, thus excluding inconsistent obligations.

Deontic logic is, in a way, an extension of propositional logic; the deontic operators, P and O, are an addition to the logical connectives of propositional logic[5]. Propositions that include the use of deontic operators will be called deontic propositions, because they state something about what is obligatory or permitted. Sometimes, however, such propositions express logical truths that do not depend on the logical character of these deontic operators [26], for example:

$$PA \wedge (PA \rightarrow PB) \rightarrow PB$$

In words, if A is permitted and permission of A implies permission of B, then B is permitted. This is an application of the *modus ponens*, which is valid for any propositional formula. It should thus be trivial in deontic logic.

More interesting are logical truths that arise from the logical character of the deontic operators themselves, instead of the logical connectives between them, for example:

$$OA \wedge O(A \rightarrow B) \rightarrow OB$$

In words, if A is obligated and it is obligated that A implies B, then B is obligated. This rule of inference is not valid for any propositional formula; it is dependent on the deontic operators, specifically O.

Axioms like this one form the foundation of formal reasoning with deontic concepts. However, carelessly adding axioms to a logical system can lead to problems, even axioms that do not seem to be problematic at first because they match our intuition. Understanding these problems can help us further in the development a new system of deontic logic, in which – preferably – no logical paradoxes arise, and the logical concepts match the linguistic ones.

[4]Hansson [13] finds this a confusing terminology.

[5]negation ($\neg$), conjunction ($\wedge$), disjunction ($\vee$), implication ($\rightarrow$), biconditional ($\leftrightarrow$)

2.2 The Principle of Free Choice Permission

Recently, my mother adopted a dog. To teach him some basic commands, she decided to have weekly obedience training sessions with a trainer. Last week I wanted to go for a short walk with the dog, so my mom told me:

"You may use his collar or his harness." (i)

The most intuitive interpretation of this sentence is that she offered me a free choice; I could decide which one I wanted to use. But then she said:

"You may use his collar or his harness,
but I don't remember which one." (ii)

Apparently, the trainer had told her that one of the two was for short walks and the other for long walks, but she had forgotten which.

Both i and ii can be represented by $P(A \vee B)$, with A for using his collar and B for using his harness. In common language, such a disjunctive permission indicates a free choice. This is what we call *free choice permission*. It satisfies the following axiom[6]:

$$P(A \vee B) \rightarrow PA \wedge PB \tag{FCP}$$

In words, if A or B is permitted, then A is permitted and B is permitted. The first utterance, i, was an example of free choice permission. Because I was permitted to use the collar or the harness, I was both allowed to use his collar and allowed to use his harness – I could choose which one I would take. In ii, however, the free choice axiom does not hold, because I was only allowed to use one of the two. Here, the weaker axiom $P(A \vee B) \rightarrow PA \vee PB$ holds instead.

Though the latter axiom is valid in standard deontic logic, the former is not. We do want this to be the case, so we add the FCP axiom. However, now it could also be inferred from ii that I was both allowed to use the collar and the harness, which I was not. I argue that ii should not be formalized as $P(A \vee B)$ but as $PA \vee PB$ because of the disclaimer my mom gave.

2.3 The Free Choice Permission Paradox

As mentioned above, FCP is not valid in standard deontic logic (SDL). It also never could be, because disaster would follow. First I will explain the free choice permission paradox informally. Later I will formally show some implausible derivations that are associated with the FCP axiom.

Suppose, in the example from 2.2, the trainer had said that the collar was for short walks (and the harness for long walks). Then, using some simple logic, I could still have reasoned that using the harness was allowed as well:

1. I am allowed to use the collar.
2. Using the collar implies using the collar or the harness.
3. I am allowed to use the collar or the harness.
4. I am allowed to use the collar and I am allowed to use the harness.
5. I am allowed to use the harness.

[6] Other definitions have been suggested as well, such as $P(A \vee B) \leftrightarrow PA \wedge PB$ [e.g. 25].

The inference from 3 to 4 in the informal derivation above is an example of FCP, where A would be "I use the collar," and B would be "I use the harness": if it is permitted that I use the collar or the harness, then it is permitted that I use the collar and it is permitted that I use the harness.

To show the problem in formal reasoning, let us suppose FCP was added to SDL. A great number of implausible results could now be derived in combination with other deontic axioms. In this thesis, however, we will only look at the following, which was informally discussed above:

Derivation with SDL: $PA \rightarrow PA \wedge PB$ (2.1)

1.	PA	Assumption
2.	$P(A \vee B)$	1, SDL
3.	$PA \wedge PB$	2, FCP

The theorem used to make the inference from 1 to 2 is a theorem of SDL. Remember that in SDL, the P operator works like the diamond operator from modal logic. Now assume that PA is true, as in 1. By definition of P, there exists an accessible world in which A is true. Then surely at that world $A \vee B$ is also true. Hence, there is an accessible world in which $A \vee B$ is true. So, again by definition of P, $P(A \vee B)$ is true. Since PA was our assumption, $PA \rightarrow P(A \vee B)$ is thus indeed a theorem of SDL[7]. In combination with FCP, $PA \rightarrow PA \wedge PB$ is now also a theorem of SDL. But this seems terribly wrong: since A and B are arbitrary, a permission PA would entail any, and thus every permission PB. Since B can also be $\neg A$, there would be a moral indifference towards any statement A, $PA \wedge P\neg A$. Thus, there can be no obligation, because, with $OA \rightarrow PA$, it would always lead to an inconsistency – something being obligated and its negation permitted, OA and $P\neg A$.

Unfortunately, SDL is not the only system in which the free choice permission paradox manifests itself. In fact, even when only applying FCP and substitution of logical equivalents (SLE)[8] an implausible result can be derived [16]:

Derivation with SLE: $PA \rightarrow P(A \wedge B)$ (2.2)

1.	PA	Assumption
2.	$P((A \wedge B) \vee (A \wedge \neg B))$	1, SLE
3.	$P(A \wedge B) \wedge P(A \wedge \neg B)$	2, FCP
4.	$P(A \wedge B)$	3, PL

The inference from 1 to 2 is made by substituting A for $(A \wedge B) \vee (A \wedge \neg B)$. These two formulas are clearly logically equivalent. The inference from 3 to 4 uses a theorem of propositional logic (PL): $X \wedge Y \rightarrow X$, in which X and Y can be any formula. In this case, X is $P(A \wedge B)$ and Y is $P(A \wedge \neg B)$.

[7] A similar theorem holds for obligation: $OA \rightarrow O(A \vee B)$. This gives rise to another paradox of deontic logic, Ross's paradox [20], which shall not be further discussed in this thesis.

[8] Two formulas can be substituted for each other if they are logically equivalent. For the permission operator this means: if $A \equiv B$, then $PA \equiv PB$. Later, we will see that SLE does not work in truthmaker semantics.

Truthmaker Semantics for Permission

The first section of this chapter, 3.1, will focus on basic truthmaker semantics as proposed by Kit Fine [11]. A formal as well as informal account is given of truthmakers, their properties, and the semantics. Section 3.2 will propose a solution to the free choice permission paradox using this framework of truthmaker semantics. A semantic clause for permission is given, after which several lemmas are proven to show how the paradox is solved exactly.

3.1 Truthmaker Semantics

In general, a truthmaker is some state of affairs or act that verifies, makes true, some statement or proposition [11, p. 1]. For example, an apple's being red is a verifier for the statement "the apple is red". In this thesis, we will not discuss truthmakers in the metaphysical sense, that is, what it is in the world that makes something true. Rather, we will focus on how they serve as a means to provide a semantics for deontic logic. In other words, we are not interested in the relata, the things that *make* true and are *made* true, but in the properties of the truthmaking relation, specifically in the context of permission (and obligation).

Truthmaking comes in three 'flavors' so to speak: exact, inexact and loose [11]. These notions are successively broader, meaning that an exact truthmaker will also be an inexact truthmaker and an inexact truthmaker will also be a loose verifier, but not vice versa. A truthmaker is a loose verifier for a statement if it impossible for the truthmaker to obtain and the statement not be true. For example, the apple's being juicy is a loose verifier for the statement "the apple is red or not," because it will be true regardless the content of the loose verifier. Unlike loose truthmakers, inexact truthmakers need to be relevant to the statement. However, an inexact truthmaker need only be *partially* relevant to the statement, whereas an exact truthmaker need be *wholly* relevant. Wholly relevant, in this sense, means that a truthmaker may not contain something irrelevant to the truth of the statement. For example, the apple's being red is wholly relevant to the statement "the apple is red," and thus an exact verifier, whereas the apple's being red and juicy is only partially relevant, making it an inexact verifier. In this thesis, we are only interested in the notion of exact truthmaking for reasons that shall become clear.

3.1.1 States and State Spaces

To understand how truthmaker semantics works, we will briefly contrast it with possible worlds semantics. The main reason why SDL cannot deal with free choice permission is because it employs an approach with possible worlds semantics. With this semantics, statements are either true or false *at* a possible world. Truthmaker semantics, on the other hand, works with 'states' that *make* statements true or false, but not necessarily either. A 'state' is an abstract concept, and does not specifically refer to any state of being, or some other notion in natural language [11, p. 560]. In the context of truthmakers, states are merely representations of truthmakers that verify and falsify statements, as the apple's being red verifies "the apple is red," and falsifies "the apple is green."

It is important to note that truthmakers are nonfactual, which means even false statements have truthmakers. An example is you being the author of this thesis for the (false) statement "[insert your name] is the author of this thesis." Truthmakers may even be impossible states, unlike *possible* worlds. A truthmaker is thus a state that *would* make a statement true if the state were to obtain, not a state that *does*.

Another important property of truthmakers is that they have a so-called *mereological*[1] structure. This means that states can be part of other states, and they can combine to form more complex states, even a world [11]. For example, an apple's being red is part of the apple's being red and juicy. And the apple's being red and the apple's being juicy combine to the apple's being red and juicy.

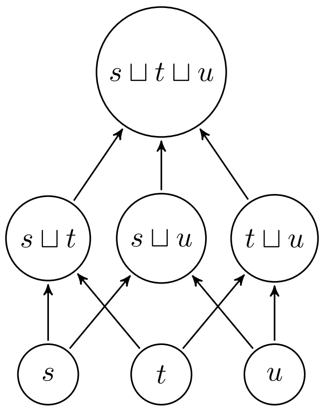
The mereological structure of truthmakers is formalized in a *state space*. A *state space* is an ordered pair $(S, \sqsubseteq)$, where S is a non-empty set of states (truthmakers) and $\sqsubseteq$ is a binary relation on S [11, p. 560]. Specifically, $\sqsubseteq$ is a *partial order*, meaning that for any states s, t, and u of S it is:

(i) reflexive: $s \sqsubseteq s$; a state is part of itself

(ii) anti-symmetric: if $s \sqsubseteq t$ and $t \sqsubseteq s$, then $s = t$; different states cannot be part of each other

(iii) transitive: if $s \sqsubseteq t$ and $t \sqsubseteq u$, then $s \sqsubseteq u$; .

These three properties of the partial relation are the core principles of mereology [24]. State spaces have an additional condition imposed upon them, for which we need some definitions. A state s is an *upper bound* of a subset of states $T \subseteq S$ if it contains each state of T, i.e., if $t \sqsubseteq s$ for each $t \in T$; and s is a *least upper bound* of T if s is an upper bound of T and all upper bounds of T contain s, i.e., if $s \sqsubseteq s'$ for any upper bound s' of T [11, p. 560]. The requirement for state spaces is that every subset $T \subseteq S$ has a least upper bound. This least upper bound will thus always be the *fusion* of (the members of) T, denoted by $\sqcup T$. For example, the least upper bound of $\{r, j\} \subseteq S$ is $r \sqcup j$, the fusion of r and j. If we think of r as the apple's being red and j as the apple's being juicy, then $r \sqcup j$ is the apple's being red and juicy. It is the least upper bound, because there is no state which holds less information yet still contains both r and j.



[1] 'having relations of part to whole and part to part within the whole' [24]

The figure above shows what a state space $(S, \sqsubseteq)$ might look like. The states are represented by the nodes, and the partial order is represented by the arrows going in and coming out of those nodes. In this thesis, state spaces will be represented by Hasse diagrams. Hasse diagrams are used to represent a partially ordered set [14], which a state space effectively is; the set of states S is partially ordered by the relation $\sqsubseteq$. The reflexive and transitive arrows are left out for convenience.

In a Hasse diagram, 'smaller' elements of the set appear lower than 'bigger' elements, and two elements x and y are connected by an upward arrow from x to y iff $x \leq y$ and there is no z such that $x \leq z \leq y$. This gives us a natural way to visualize (least) upper bounds of state spaces; states higher in the diagram are upper bounds of the states lower in the diagram, and the arrows are drawn from every state of a subset $T \subseteq S$ towards the least upper bound of T. So in the figure above, $s \sqcup t \sqcup u$ is an upper bound of $\{s, t, u\}$, and $s \sqcup u$ is the least upper bound of $\{s, u\}$ for example.

3.1.2 Verification and Falsification

Let us now turn to the semantic clauses that state the conditions under which a statement is verified or falsified. These clauses describe the properties of the truthmaking relation. Remember that a verifier is a state that makes a statement true (verifies it), and a falsifier is a state that makes a statement false (falsifies it). The clauses for negation, disjunction and conjunction are as follows [11, p. 562]:

($\neg$) a state verifies $\neg A$ iff it falsifies A;
a state falsifies $\neg A$ iff it verifies A;

($\vee$) a state verifies $A \vee B$ iff it verifies A or it verifies B;
a state falsifies $A \vee B$ iff it is a fusion of states that falsify A and B respectively;

($\wedge$) a state verifies $A \wedge B$ iff it is a fusion of states that verify A and B respectively;
a state falsifies $A \wedge B$ iff it falsifies A or it falsifies B.

Note that with our notion of exact truthmaking, the verifiers of logical equivalent formulas will in general not be the same [10, p. 335], and thus truthmaker semantics does not support SLE. Possible worlds semantics, however, *does*, because equivalent formulas always have the same truth value in a possible world, making it impossible to distinguish them. Let us take the formulas from derivation 2.2 as an example. Suppose A translates to "the apple is red," and B translates to "the apple is juicy." A is true in a possible world where the apple is indeed red. But because the apple also has to be either juicy or not, at least in a possible world, $(A \wedge B) \vee (A \wedge \neg B)$ is true at that world as well.

In truthmaker semantics, the distinction between these equivalent formulas *can* be made, because the formulas are not verified by the same state. The apple's being red is a truthmaker that verifies A, and the apple's being juicy is a truthmaker that verifies B. By the semantic clause for conjunction, the fusion of the two states verifies $A \wedge B$, and therefore it verifies $(A \wedge B) \vee (A \wedge \neg B)$ as well, by the clause for disjunction. $(A \wedge B) \vee (A \wedge \neg B)$ is thus verified by a combination of the verifiers for A and B – the apple's being red and the apple's being juicy – whereas A is only verified by the apple's being red.

We have now informally shown how truthmaker semantics provides a solution to derivation 2.2. In Section 3.2, a formal proof will be given to show how this derivation no longer holds in truthmaker semantics, and how SLE is no longer supported[2]. But before we can do that, we need a more technical account of the semantics of truthmaking.

[2]There are rare exceptions, however. For example, A can not always be distinguished from $A \wedge A$. Suppose some state s verifies A. The fusion of s with itself will just be s. So by the semantic clause for conjunction, s verifies $A \wedge A$ as well.

We define a (*state*) *model* as an ordered triple $(S, \sqsubseteq, |\cdot|)$, where $(S, \sqsubseteq)$ is a state space and $|\cdot|$ is a valuation [11, p. 562]. This valuation is a function that maps each atomic proposition p to a pair (V, F) of subsets of S, in which $|p|^+ = V$ is the set of its verifiers and $|p|^- = F$ is the set of its falsifiers.[3]

Now we can give the formal conditions under which an arbitrary formula X is *verified* by a state s ($s \Vdash X$) or *falsified* by a state s ($s \mathrel{\dashv\!|} X$) [11, p. 563]:

$(\cdot)^+$ $s \Vdash p$ iff $s \in |p|^+$; s verifies p iff s is in the set of verifiers of p;

$(\cdot)^-$ $s \mathrel{\dashv\!|} p$ iff $s \in |p|^-$; s falsifies p iff s is in the set of falsifiers of p;

$(\neg)^+$ $s \Vdash \neg A$ iff $s \mathrel{\dashv\!|} A$; s verifies $\neg A$ iff s falsifies A;

$(\neg)^-$ $s \mathrel{\dashv\!|} \neg A$ iff $s \Vdash A$; s falsifies $\neg A$ iff s verifies A;

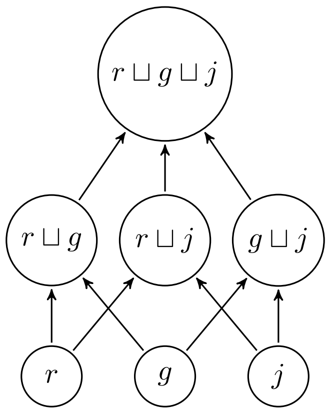
$(\vee)^+$ $s \Vdash A \vee B$ iff $s \Vdash A$ or $s \Vdash B$; s verifies $A \vee B$ iff s verifies A or s verifies B;

$(\vee)^-$ $s \mathrel{\dashv\!|} A \vee B$ iff for some states t and u, $t \mathrel{\dashv\!|} A$, $u \mathrel{\dashv\!|} B$, and $s = t \sqcup u$; s falsifies $A \vee B$ iff s is the fusion of some state t that falsifies A and some state u that falsifies B.

$(\wedge)^+$ $s \Vdash A \wedge B$ iff for some states t and u, $t \Vdash A$, $u \Vdash B$, and $s = t \sqcup u$; s verifies $A \wedge B$ iff s is the fusion of some state t that verifies A and some state u that verifies B;

$(\wedge)^-$ $s \mathrel{\dashv\!|} A \wedge B$ iff $s \mathrel{\dashv\!|} A$ or $s \mathrel{\dashv\!|} B$; s falsifies $A \wedge B$ iff s falsifies A or s falsifies B;

Note that $(\neg)^+$ and $(\neg)^-$ correspond to $(\neg)$ in the previously mentioned informal clauses, $(\vee)^+$ and $(\vee)^-$ correspond to $(\vee)$, and $(\wedge)^+$ and $(\wedge)^-$ to $(\wedge)$.



The figure above shows almost the same state space as before. By adding a valuation, we can turn it into a model $\mathcal{M} = (S, \sqsubseteq, |\cdot|)$. We introduce the variables a, b, c to denote the propositions "the apple is red," "the apple is green," and "the apple is *not* juicy," respectively. If it makes it easier to understand, you could think of r as the apple's being red, g as the apple's being green, and j as the apple's being juicy. Now, r is a verifier of a, $r \Vdash a$, and a falsifier of b, $r \mathrel{\dashv\!|} b$; g is a verifier of b, $g \Vdash b$, and a falsifier of a, $g \mathrel{\dashv\!|} a$; and j is a falsifier of c, $j \mathrel{\dashv\!|} c$. With the semantic clauses we can say, for example, the following:

$r \mathrel{\dashv\!|} \neg a$ because $r \Vdash a$;
$j \Vdash \neg c$ because $j \mathrel{\dashv\!|} c$;
$g \Vdash b \vee c$ because $g \Vdash b$;
$g \sqcup j \Vdash b \wedge \neg c$ because $g \Vdash b$ and $j \Vdash \neg c$;
$r \sqcup j \mathrel{\dashv\!|} b \vee c$ because $r \mathrel{\dashv\!|} b$ and $j \mathrel{\dashv\!|} c$.

[3] The following two plausible conditions can be imposed, if desired:
Exclusivity: no state is both a verifier and a falsifier;
Exhaustivity: any possible state is a verifier or a falsifier.

3.2 Solving the Free Choice Permission Paradox

Now that we have outlined the general set-up of truthmaker semantics, we can focus on finding a semantic clause for permission. The solution to the free choice permission paradox is based on the proposal Fine makes at the end of *Permission and Possible Worlds* [10]. In the first seventeen pages, Fine argues that statements of permission are, in a way, a guide to action, and therefore an approach to their truth-conditions via possible worlds semantics will never work, let alone be satisfying [10, p. 317]. After this argument, he provides a positive proposal of only two pages long, in which he gives the following (informal) semantic clause for the permission operator:

$$PA \text{ is true iff all of the verifiers for } A \text{ are permitted.} \tag{3.1}$$

This indeed seems a promising means to deal with statements of permission in truthmaker semantics. However, Fine does not propose any explicit or formal solution. We still do not know what it means for PA to be true, or when a verifier is permitted or not. In fact, the definition is recursive, but there is no non-recursively defined foundation, making it kind of circular. In what follows, I will work out a (better) solution in detail.

We define a *deontic state space* as an ordered triple $(S, S_{OK}, \sqsubseteq)$, where $(S, \sqsubseteq)$ is a state space and S_{OK} is a (possibly empty) set of okay states of S. Similarly, a *deontic state model* is an ordered quadruple $(S, S_{OK}, \sqsubseteq, |\cdot|)$. Because permission already works as an operator on statements, we cannot use it when talking about states as well. Instead, we talk of states being 'okay', meaning they would be admissible if they were to obtain; intuitively, it is okay for the act to be executed. For example, the state of 'me drinking tea' would be an okay state if it were okay for me to be drinking tea. Since we are only concerned with the semantics, we assume that the set of okay states is always given. In the Hasse diagrams, we will indicate okay states with a light green filling, as opposed to normal states, which will be left blank.

The set of okay states, S_{OK}, is not closed under part, that is, for some states s and t, if $t \sqsubseteq s$ and $s \in S_{OK}$, then not necessarily $t \in S_{OK}$. Informally, this means that parts of okay states do not have to be okay states as well. For example, speeding is generally not okay, but it is if you are driving an ambulance with sirens. Similarly, the fusion of two okay states may not be okay. For example, it is okay to occupy the left seat in a train during rush hour and it is okay to occupy the right seat, but it is not okay to occupy both because someone else might have no seat at all.

We can now formalize the semantic clause for permission in a deontic state model $\mathcal{M} = (S, S_{OK}, \sqsubseteq, |\cdot|)$ as follows:

$$\mathcal{M} \vDash PA \text{ iff for all } s \in S\text{: if } s \Vdash A\text{, then } s \in S_{OK} \tag{3.2}$$

Informally, this clause states that PA is true in a deontic model iff all verifiers for A are okay.[4,5] Notice that this is roughly what Fine proposed (3.1).

[4] One might think that since S_{OK} can be empty, everything can be permitted, in which case, using duality, there would be no obligations. However, S cannot be empty, so with exhaustivity (3) there must be at least some statement verified, which, in case of S_{OK} being empty, would not be permitted.

[5] We can give an apparent counter example for the right-to-left direction, in which PA is true but not all verifiers for A are okay. Suppose A denotes the proposition "the wall is colored," and let PA be true, i.e., it is permitted that the wall is colored. Now let r be the state of the wall being red. It is clearly an exact verifier of A, because the wall being red is wholly relevant to the statement "the wall is colored." But what if was okay for the wall to be any color, except red? Then r would not be an okay state, and therefore PA could not be true. But we assumed PA *was* true, so we have reached an apparent contradiction.

'Apparent' because it actually is not. We have to keep in mind that we are trying to include free choice permission, not exclude it. With such a reading, the permission of "the wall is colored" would imply the conjunction of the permissions of the wall having any color. If it were not okay for the wall to be red, it would have to be explicitly stated; $P(A \wedge \neg B)$, with B as "the wall is not red." Evidently, r would no longer be a verifier. The contradiction is eluded.

Now that we have defined a semantic clause for the permission operator, the logical system can deal with (free choice) permission as desired. Before we can prove this, however, we need to define the following: the syntax of the logic, truth in a model, and entailment.

The syntax basically consists of two parts, because we need a language for inside a deontic model, where we talk about verification and falsification, and a language for outside the model, where we talk about entailment. The inner language $\mathcal{L}$ is a propositional language with the connectives $\neg$ (negation), $\vee$ (disjunction), and $\wedge$ (conjunction),[6] which is defined over a set of propositional variables (e.g. p, q, r). $\mathcal{L}$ is given in Backus-Naur form as:

$$A ::= p \mid \neg A \mid (A \vee A) \mid (A \wedge A) \tag{3.3}$$

The conditions under which formulas of $\mathcal{L}$ are verified or falsified were previously given in 3.1.2.

The outer language $\mathcal{L}_{\mathcal{D}}$ is a (deontic) language of permission, which is given in Backus-Naur form as:

$$B ::= PA \mid \neg B \mid (B \vee B) \mid (B \wedge B), \tag{3.4}$$

with A a formula of $\mathcal{L}$. Note that this is a language of statements which exclusively contain permissions.[7] So for example, $p \wedge q$ is not a well-formed formula of $\mathcal{L}_{\mathcal{D}}$ because it is not a permission or a conjunction of permissions. Also, iterated permission does not occur in the language, i.e., no P occurs in A of any statement PA.

We can now give the truth-conditions for the formulas of $\mathcal{L}_{\mathcal{D}}$. Given a deontic model $\mathcal{M} = (S, S_{OK}, \sqsubseteq, |\cdot|)$, we have the following:

$$\begin{aligned}
&\mathcal{M} \vDash PA \text{ iff for all } s \in S\text{: if } s \Vdash A\text{, then } s \in S_{OK}\\
&\mathcal{M} \vDash \neg B \text{ iff } \mathcal{M} \nvDash B\\
&\mathcal{M} \vDash B \vee C \text{ iff } \mathcal{M} \vDash B \text{ or } \mathcal{M} \vDash C\\
&\mathcal{M} \vDash B \wedge C \text{ iff } \mathcal{M} \vDash B \text{ and } \mathcal{M} \vDash C
\end{aligned}$$

Note that the truth-conditions for non-deontic statements are classical.

Lastly, we define semantic consequence or entailment ($\vDash$) as follows:

$$A \vDash B \text{ iff for all } \mathcal{M}\text{: if } \mathcal{M} \vDash A\text{, then } \mathcal{M} \vDash B$$

Informally, $A \vDash B$ iff if A is true in a model, then B is true in the model.

Now, I will show how our logic of permission can deal with (free choice) permission by proving some lemmas.

Lemma 3.2.1. *$P(A \vee B) \vDash PA$*

Proof. Let $\mathcal{M} = (S, S_{OK}, \sqsubseteq, |\cdot|)$ be a deontic state model. Assume that $\mathcal{M} \vDash P(A \vee B)$ for some arbitrary formulas A and B. By definition of P, for all $s \in S$: if $s \Vdash A \vee B$, then $s \in S_{OK}$. And for all $s \in S$ such that $s \Vdash A$ it is also the case that $s \Vdash A \vee B$ by definition of disjunction. Therefore, for all $s \in S$: if $s \Vdash A$, then $s \in S_{OK}$, because if $s \Vdash A$, then $s \Vdash A \vee B$ and if $s \Vdash A \vee B$, then $s \in S_{OK}$. Thus, by definition of P, $\mathcal{M} \vDash PA$. Since $\mathcal{M}$ is an arbitrary model and A and B are arbitrary formulas, $P(A \vee B) \vDash PA$ holds for any A and B in any model. □

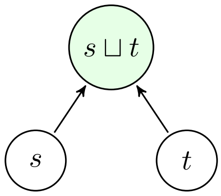
Lemma 3.2.1 shows how the principle of free choice permission is supported in our new system of deontic logic.

[6] Other connectives such as $\rightarrow$ (implication) can be defined in terms of these three.

[7] It is interesting to look at extensions that include other statements, but unfortunately this is outside the scope of this thesis. See Anglberger and Korbmacher [3] for an example of such an extended language.

Lemma 3.2.2. *$P(A \wedge B) \nvDash PA$*

Proof. We construct a counter model $\mathcal{M} = (S, S_{OK}, \sqsubseteq, |\cdot|)$ to show that $P(A \wedge B) \vDash PA$ does not hold for every A and B. We show that $P(p \wedge q)$ is true but Pp is not, for some p and q. Let $S = \{s, t, s \sqcup t\}$ and $S_{OK} = \{s \sqcup t\}$, and let $s \Vdash p$ and $t \Vdash q$. By definition of conjunction, $s \sqcup t \Vdash p \wedge q$ because $s \Vdash p$ and $t \Vdash q$. Since $s \sqcup t$ is the only state that verifies $p \wedge q$ and $s \sqcup t \in S_{OK}$, $\mathcal{M} \vDash P(p \wedge q)$ by definition of P. However, $\mathcal{M} \nvDash Pp$ because $s \Vdash p$ but $s \notin S_{OK}$.

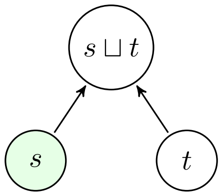


□

Lemma 3.2.2 shows the property of the set of okay states, S_{OK}, that they are not closed under part. Informally, it shows that two statements might only be permitted in combination with each other and not by themselves.

Lemma 3.2.3. *$PA \nvDash P(A \vee B)$*

Proof. We construct a counter model $\mathcal{M} = (S, S_{OK}, \sqsubseteq, |\cdot|)$ to show that $PA \vDash P(A \vee B)$ does not hold for every A and B. We show that Pp is true but $P(p \vee q)$ is not, for some p and q. Let $S = \{s, t, s \sqcup t\}$ and $S_{OK} = \{s\}$, and let $s \Vdash p$ and $t \Vdash q$. By definition of P, $\mathcal{M} \vDash Pp$ because s is the only state that verifies p and $s \in S_{OK}$. Also, $s \Vdash p \vee q$ by definition of disjunction because $s \Vdash p$; and similarly, $t \Vdash p \vee q$ because $t \Vdash q$. Since $t \Vdash p \vee q$ but $t \notin S_{OK}$, $\mathcal{M} \nvDash P(p \vee q)$ by definition of P.

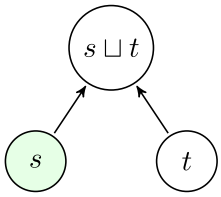


□

In Section 2.3, I showed what trouble adding FCP to standard deontic logic could cause. Lemma 3.2.3 shows how derivation 2.1 is no longer valid in this truthmaker semantics approach to permission. The invalidity is caused by the 'for all states' in the semantic clause for permission. In possible worlds semantics, which is used in SDL, permission has a 'there is a world' definition. If we had used 'there is a state,' we would have again run into the problem that $PA \vDash P(A \vee B)$ is valid. After all, if there exists an okay state that verifies A, then there also exists an okay state that verifies $A \vee B$, namely that same state.

Lemma 3.2.4. *$PA \nvDash P(A \wedge B)$*

Proof. We construct a counter model $\mathcal{M} = (S, S_{OK}, \sqsubseteq, |\cdot|)$ to show that $PA \vDash P(A \wedge B)$ does not hold for every A and B. We show that Pp is true but $P(p \wedge q)$ is not, for some p and q. Let $S = \{s, t, s \sqcup t\}$ and $S_{OK} = \{s\}$, and let $s \Vdash p$ and $t \Vdash q$. By definition of P, $\mathcal{M} \vDash Pp$ because s is the only state that verifies p and $s \in S_{OK}$. Also, $s \sqcup t \Vdash p \wedge q$ by definition of conjunction because $s \Vdash p$ and $t \Vdash q$. Since $s \sqcup t \Vdash p \wedge q$ but $s \sqcup t \notin S_{OK}$, $\mathcal{M} \nvDash P(p \wedge q)$ by definition of P.

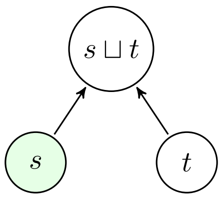


$\square$

In Section 2.3, I showed what trouble was caused when allowing substitution of logical equivalents for the permission operator. Lemma 3.2.4 shows how derivation 2.2 is no longer valid in this truthmaker semantics approach to permission.

Lemma 3.2.5. $PA \nvDash P((A \wedge B) \vee (A \wedge \neg B))$

Proof. We construct a counter model $\mathcal{M} = (S, S_{OK}, \sqsubseteq, |\cdot|)$ to show that $PA \vDash P((A \wedge B) \vee (A \wedge \neg B))$ does not hold for every A and B. We show that for some p and q, Pp is true but $P((p \wedge q) \vee (p \wedge \neg q))$ is not. Let $S = \{s, t, s \sqcup t\}$ and $S_{OK} = \{s\}$, and let $s \Vdash p$ and $t \Vdash q$. By definition of P, $\mathcal{M} \vDash Pp$ because s is the only state that verifies p and $s \in S_{OK}$. Also, $s \sqcup t \Vdash p \wedge q$ by definition of conjunction because $s \Vdash p$ and $t \Vdash q$. Therefore, $s \sqcup t \Vdash (p \wedge q) \vee (p \wedge \neg q)$ by definition of disjunction. Since $(p \wedge q) \vee (p \wedge \neg q)$ but $s \sqcup t \notin S_{OK}$, $\mathcal{M} \nvDash P((p \wedge q) \vee (p \wedge \neg q))$ by definition of P.



$\square$

In Section 2.3, I showed what trouble was caused when allowing substitution of logical equivalents for the permission operator. Lemma 3.2.5 is similar to 3.2.4 in that it shows how derivation 2.2 is no longer valid in this truthmaker semantics approach to permission. However, 3.2.5 specifically shows how substitution of logical equivalent formulas A and $(A \wedge B) \vee (A \wedge \neg B)$ no longer holds. Of course this is just a single example of how SLE is no longer supported in truthmaker semantics. However, it should be easy to see that any two logical equivalent formulas cannot be substituted for each other in truthmaker semantics, as pointed out earlier in 3.1.2.

Discussion

4.1 Statements of Iterated Permission

We have now seen that truthmaker semantics offers a solution to the free choice permission paradox when we give 3.2 as the semantic clause for the permission operator. The proven lemmas correspond to some intuitions we have about (free choice) permission, such as a permitted disjunction entailing the permission of both disjuncts (3.2.1), and permission not being closed under part (3.2.2). The proofs of lemma 3.2.3 and 3.2.4 show the solution truthmaker semantics can provide to the implausible derivations from Section 2.3, respectively to 2.1 and 2.2.

Unfortunately, although 3.2 has solved the free choice permission paradox with one nice and easy definition, it has created a new problem concerning statements of iterated permission. We speak of iterated permission when permitting something is itself permitted. Let us review the example I gave to explain the principle of free choice permission in Section 2.2. In i, my mom gave me permission to use the dog's collar or the harness. Now suppose that the trainer had permitted my mom to permit me to use the collar. Then the trainer had uttered a statement of iterated permission.

To see what problem arises with statements of iterated permission, let us have another look at the clause in question:

$$\mathcal{M} \vDash PA \text{ iff for all } s \in S\text{: if } s \Vdash A\text{, then } s \in S_{OK}$$

Suppose A was itself a statement of permission, PB. Then the clause states that PPB is true in a model iff all verifiers of PB are okay. However, we have never defined what the truthmakers of permission are. So there is no way of saying something about the truth of statements of iterated permission.

Although we could settle for the semantic clause for permission as it is, it is better to find a clause that takes care of non-iterated permission as well as iterated permission. Barcan Marcus [19] already argued for this in 1966. Although she does not specifically talk about statements of permission, she does make an intuitive argument for the inclusion of iterated deontic modalities in general in any system of deontic logic. She claims for example that, "parking on highways ought to be forbidden," intuitively shows why $O(OA \to A)$ is an acceptable deontic axiom [19, p. 580]. We can rewrite "parking on highways ought to be forbidden," as "it is obligatory that if one is obligated to not park on highways, then one does not park on highways." Now, if we let A denote "one does not park on highways," then the sentence indeed translates to $O(OA \to A)$. In a more natural way, the axiom may be read as "it ought to be the case that what ought to be the case is the case" [19, p. 580]. We can construct a similar axiom for permission, $P(PA \to A)$, which could be read as "it is permitted that what is permitted is the case." For example, it is permitted that the permission for me to have tea implies that I do.

4.2 Other Solutions

A clear downside to the semantic clause proposed in this thesis is thus its disability to deal with statements of iterated permission. Let us discuss what other solutions have been proposed to the free choice permission paradox. We can then reflect on the proposal here, and add suggestions for further research if necessary.

Hansson [13] discusses several solutions to the free choice permission paradox, though none of them are satisfying enough. For example, the first solution he discusses is rather an argument to dismiss free choice permission altogether. According to some (e.g. Parks [21]), the paradox arises due to a mistranslation of the disjunction from natural language to logical formula. They say that "you may have tea or coffee" should not be initially translated as $P(p \vee q)$ but immediately as $Pp \wedge Pq$. However, although every permitted disjunction, $P(A \vee B)$, indeed implies a conjunctive permission, $PA \wedge PB$ – the principle of free choice permission – we might not always want to take $PA \wedge PB$ as $P(A \vee B)$. For example, if I have a friend over, I will tell them they may have a cup of tea. Of course they would also be permitted to use the bathroom, so formally speaking this would be a case of $PA \wedge PB$. However, it would be weird if this implied $P(A \vee B)$, because I would never tell them "you may have a cup of tea or use the bathroom." So it seems that free choice permission is not a case of mistranslation, and we should actually distinguish between $P(A \vee B)$ and $PA \wedge PB$, even though the former implies the latter.

Dignum, Meyer and Wieringa [7] have provided a solution to the free choice permission paradox by giving a strong definition for the permission operator, which operates on acts rather than propositions. Although this definition indeed solves the paradox, it also raises some new issues. The most important issue is that the actor is no longer free to choose which act to perform, which goes against the principle of *free* choice permission. The so-called *only* operator restricts the actor to exactly those acts that do not lead to any violation. With this definition of the permission operator, we would always have to make explicit which acts are permitted and which are not. This is very undesirable, because we would like to be able to permit more than one act at the same time without having to explicitly mention all of them, especially permissions that are otherwise unrelated. Besides not dealing well with free choice permission – rather, it works around it – the proposal from Dignum et al. presents no way of dealing with iterated permission. They have defined the permission operator to operate on acts. A permission, however, is not an act. Therefore the definition cannot recursively be applied to permission.

Just as is done in this thesis, Anglberger, Faroldi and Korbmacher [4] approach free choice permission from truthmaker semantics. But instead of proposing a clause that states the truth-conditions for a permission, they give a clause that states the condition under which an exact truthmaker verifies a permission. They also give a clause for obligation. In the paper, they propose that every act (truthmaker) is associated with a set of acts that are admissible as a result of the act being performed, and a set of acts that are required as a result of the act being performed [4, p. 3]. To illustrate this idea, they give the example of John checking in at the airport. This act allows him to proceed to the gate and requires him to keep his luggage with him. Thus, as a result of John checking in, him proceeding to the gate is admissible, and him keeping his luggage with him is required.

The following semantic clauses for permission and obligation are informally given in the paper in slightly other words:

- An act verifies PA iff every verifier of A is admissible as a result of performing the act.
- An act verifies OA iff every verifier of A is required as a result of performing the act.

According to the semantic clause for permission, John checking in is a verifier for his permission to proceed to his gate, because proceeding to his gate is admissible as a result of him checking in.

A nice thing about the proposal is that the FCP axiom need not be added explicitly; it is already a theorem of the logic [4, see p. 10]. Another big plus for the proposal is that they provide more than just a solution to the free choice permission paradox. By defining the exact truthmakers of permission, the semantic clause can also be applied to iterated permissions. Naturally, the same holds for iterated obligations.

4.3 Towards an Improved System of Deontic Logic

Of the solutions discussed so far, the proposal from Anglberger et al. [4] seems the most plausible. However, there is something their semantics cannot deal with, which possible worlds semantics can: the mutual definability of the operators for permission and obligation. According to the SDL definition of these operators (see 2.1.2), something is obligated iff its negation is not permitted. Unfortunately, with the semantic clauses from Anglberger et al. [4], permission and obligation cannot be mutually defined like this. It is clear to see that this is due to the fact that they use different sets of acts; the clause for permission uses a set of admissible acts, and the clause for obligation uses a set of required acts, whatever these sets may be.

The question is, however, whether it is really such a bad thing that permission and obligation cannot be mutually defined. Let us have another look at the duality of permission and obligation:

$$OA \leftrightarrow \neg P \neg A$$

For example, it is obligated to stop at a red light iff it is not permitted to not stop at a red light, i.e., iff it is not permitted to go. This seems to make sense. But if we add a negation on the left, and remove the negation on the right, we get:

$$\neg OA \leftrightarrow P \neg A$$

In words, something is not obligated iff it is permitted to not do it. For example, it is not obligated to stop at an orange light iff it is permitted to not stop at an orange light, i.e., if it is permitted to go. This also seems fine. But what if we do not yet know whether it is permitted to not do something? Imagine someone were to come over for tea at my new apartment. I always take off my shoes inside the house, but my guest is yet to find out; I have not obligated them to take off their shoes nor have I permitted them to keep their shoes on.

Some might argue that the obligation for my guest to take off their shoes already exists, so they might advocate the mutual definability of permission and obligation. Others might argue that the act simply has not been ascribed a ‘deontic label’ yet, and therefore that duality does not (yet) hold. In 2.1.1, we discussed strong and weak permission. Here, we seem to have come across a difference between strong and weak obligation; before I explicitly or implicitly tell my guest it is obligated to take off their shoes, it is only a tacit, and thus weak obligation. A similar example can be given to argue for strong and weak permission.

The idea of differentiating between strong and weak obligation and permission is compatible with the idea of verification and falsification in truthmaker semantics. At least by definition, not all statements have to be either verified or falsified, they can be neither. Similarly, statements might not have to be either strongly permitted or obligated (or forbidden). We will soon continue this discussion. There is another, more important, though related issue we need to address first however.

As mentioned earlier in 2.1.2, logical permission should be interpreted unilaterally, that is, permission is implied by obligation. The solution to the free choice permission paradox proposed by Anglberger et al. [4], nor the one in this thesis, include this unilateral definition of permission, not yet at least. In SDL, however, this unilateral definition even holds as a theorem:

$$OA \to PA \tag{4.1a}$$

In 2.1.2, we mentioned that prohibition, denoted by the F operator (for forbidden), could be defined in terms of obligation as well as permission. So including the mutual definitions of obligation and permission, we now have the following:

$$OA \leftrightarrow \neg P \neg A \tag{4.2a}$$

$$FA \leftrightarrow \neg PA \tag{4.3a}$$

$$OA \leftrightarrow F \neg A \tag{4.4a}$$

In words, something is obligated iff its negation is not permitted; something is forbidden iff it is not permitted; and something is forbidden iff its negation is obligated. The following is now also a theorem of SDL if we combine 4.4a with 4.1a:

$$FA \to P \neg A \tag{4.5a}$$

We would like 4.1a and 4.5a both to be theorems in our new deontic system. First, we extend the previously defined language $\mathcal{L}_{\mathcal{D}}$ (3.4) with the connectives O and F:

$$B ::= PA \mid OA \mid FA \mid \neg B \mid (B \vee B) \mid (B \wedge B),$$

with A a formula of $\mathcal{L}$ (3.3). Now, I propose an extension of the previously defined semantics (see Section 3.2) by adding semantic clauses for obligation and prohibition. Given a deontic model $\mathcal{M} = (S, S_{OK}, \sqsubseteq, |\cdot|)$, we have the following truth-conditions for deontic statements:

$$\begin{aligned}
\mathcal{M} \vDash OA \text{ iff for all } s \in S\text{: if } s \Vdash A, \text{ then } s \in S_{OK};\\
\text{and if } s \dashv\mid A, \text{ then } s \notin S_{OK}\\
\mathcal{M} \vDash PA \text{ iff for all } s \in S\text{: if } s \Vdash A, \text{ then } s \in S_{OK}\\
\mathcal{M} \vDash FA \text{ iff for all } s \in S\text{: if } s \dashv\mid A, \text{ then } s \in S_{OK};\\
\text{and if } s \Vdash A, \text{ then } s \notin S_{OK}
\end{aligned}$$

Informally, in a deontic model:

$$\begin{aligned}
OA \text{ is true iff all verifiers for } A \text{ are okay,}\\
\text{and no falsifier for } A \text{ is okay}\\
PA \text{ is true iff all verifiers for } A \text{ are okay}\\
FA \text{ is true iff all falsifiers for } A \text{ are okay,}\\
\text{and no verifier for } A \text{ is okay}
\end{aligned}$$

The following semantic consequences now hold in our extended system:

$$\vDash OA \to PA \tag{4.1b}$$

$$\vDash FA \to P \neg A \tag{4.5b}$$

$$\vDash OA \leftrightarrow F \neg A \tag{4.4b}$$

From the definitions of obligation and permission, it should be obvious that 4.1b holds. Similarly, 4.5b holds, though a clarification for this might be neat. The clause for permission states that $P\neg A$ is true iff all verifiers for $\neg A$ are okay. By definition of negation, we get: $P\neg A$ is true iff all falsifiers for A are okay. Now it can be seen that FA entails $P\neg A$.

Theorem 4.4b can also be shown in a similar way. The clause for obligation states that OA is true iff all verifiers for A are okay, and no falsifier for A is okay. The clause for prohibition states that $F\neg A$ is true iff all falsifiers for $\neg A$ are okay, and no verifier for $\neg A$ is okay. By definition of negation, we get: $F\neg A$ is true iff all verifiers for A are okay, and no falsifier for A is okay. OA thus entails $F\neg A$.

Note that the following semantic consequences do not hold:[1]

$$\nvDash \neg OA \rightarrow P\neg A \tag{4.2b}$$

$$\nvDash FA \rightarrow \neg PA \tag{4.3b}$$

We now have the semantic 'versions' of 4.1a, 4.5a and 4.4a in our new deontic system, respectively 4.1b, 4.5b and 4.4b. These all correspond to our intuitions of permission, obligation and prohibition, specifically 4.1b corresponds to unilateral permission, and 4.5b corresponds to unilateral negated permission. The last theorem, 4.4b, is intuitive in a natural language way: something is obligated iff its negation is forbidden, and something is forbidden iff its negation is obligated. We can use the traffic light example again: it is obligated to stop at a red light iff it is forbidden to not stop, and it is forbidden to go at a red light iff it is obligated to not go. Note that both actually say the same.

Also, we have excluded 4.2a and 4.3a. This is consistent with strong deontic notions, as discussed at the beginning of this section. The system I have now proposed is thus a deontic system for strong obligation, strong permission and strong prohibition. This also seems to intuitively follow from the proposed clauses; every clause says something about *all* verifiers or *all* falsifiers, giving them a 'strong' deontic notion. With these definitions a 'weak statement' does not have to be either obligated or permitted or forbidden, it can be none of the three.

This extended proposal of the previously defined semantic clause for permission looks promising. It might even be more appealing than the one from Anglberger et al. [4], because they have no means to define permission unilaterally – this extended proposal *does*. However, further research is needed to explore the proposal in more depth. For one, no formal proof has been given for the added clauses. Furthermore, this new system still does not provide a way to deal with iterated permission.

[1] The proofs are left to the reader.

Conclusion

The first aim of this thesis was to understand and formalize free choice permission and its associated paradox. The second aim was to provide a solution to this paradox. Both goals have been achieved. Furthermore, all research questions that were posed in the introduction, 1, have been answered.

To introduce the principle of free choice permission, linguistic and logical concepts of permission were discussed to support the idea that these concepts should correspond. It was shown how adding the axiom for free choice permission leads to implausible results for multiple reasons in existing systems of deontic logic. The problematic derivation in standard deontic logic was caused by the possible worlds semantics it employs, making it possible to derive $P(A \vee B)$ from PA. The second problematic derivation was caused by the acceptance of substitution of logical equivalents.

After understanding and formalizing free choice permission, and the associated paradox, a new approach was explored using Fine's truthmaker semantics [11]. First, a general outline of the semantics was given. Afterwards, Fine's proposal [10] for a semantic clause for permission was discussed, and from that proposal a formalized clause was worked out in detail. It was shown how, with this clause, the free choice permission paradox could be solved in a nice and easy way, without raising any new inconsistencies. The definition of the semantic clause caused the first problematic derivation to fail, because possible worlds semantics uses a notion for permission of 'there is a world,' whereas the proposal here uses a notion of 'in all states,' with which $P(A \vee B)$ could no longer be derived from PA. The second problematic derivation now failed as well. In truthmaker semantics, logical equivalent formulas do not have the same verifiers, making it impossible for them to be verified by the same state, and thus obstructing the substitution of logical equivalents.

The proposed semantic clause did, however, raise some other issues, one of them being that the clause provides no means to deal with statements of iterated permission. After all, we have never defined the conditions to be a truthmaker of permission. With this limitation in mind, we compared the current solution to other solutions to the free choice permission paradox. It turned out that the proposal from Anglberger et al. [4] is the most plausible, because it can solve the free choice permission paradox as well as provide a clause that can deal with iterated permission. Unfortunately though, this proposal cannot account for the unilateral definition of permission.

As a stepping stone towards an improved system of deontic logic, semantic clauses were proposed for obligation and prohibition, additional to the already defined clause for permission. It was informally argued how this new system could account for the unilateral definition of permission, as well as other intuitions we have about obligation, permission and prohibition. As promising as this extended proposal may be, our work is not yet done. Further research is needed to explore this proposal in more depth, and the additional clauses still have to be formally accounted for. Also, this new system can still not deal with iterated permission. In short, this extended system, approached from truthmaker semantics, has both great intuitive and logical power, but its potential has not yet been exploited.

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