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A Bayesian perspective on the interaction between numerical and temporal perception

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Abstract

This literature research analyzed to what extent Bayesian inference can explain the interaction between numerical and temporal perception. A Theory of Magnitudes (Walsh, 2003) suggests that space, time, and number all interact with each other in a generalized magnitude system. The Bayesian brain hypothesis suggests that integration of sensory input is accomplished by Bayesian inference. Utilizing three potential characteristics of Bayesian perceptual processing, cue integration, adaptation, and the central tendency effect, this research concludes that the interaction between numerical and temporal perception can be understood within a Bayesian framework. The found directionalities of interaction (uni-, bi-, and non-directional) can be explained by optimal cue integration according to the most reliable (and least noisy) cue. These findings suggest that a more liberal interpretation of ATOM could provide a more integrative perspective on the discrepancy in the literature, and in light of Bayesian theories of perception, humans behave and perceive in a statistically optimal manner.

Keywords: Magnitude, numerosity, time, ATOM, Bayes

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Introduction

Numerosity and Time

A number, i.e. numerosity, is an abstraction that can be used to identify and count sets of anything, whether it be apples, humans, seconds, or thoughts.¹ We use numbers oftentimes to measure or estimate quantities as well as for labeling and ranking objects. Millennia ago, we find the Sumerian civilization employing a complex numerical system. However, a more fundamental sense of quantity has been available long before the conceptual formalization and documentation of symbolic numerals. This ‘number sense’ is a perceptual ability found in human adults and infants, as well as in many other animals (Dehaene, 2011). Similarly, most organisms appear to have some form of temporal perception as well (Healy et al., 2013). The perception of time is necessary in order to perceive and act in the dynamics of the external world, and it appears to be linked to both body mass as metabolic rate (Healy et al., 2013).

The ability to discriminate numerosity and duration seem to give an evolutionary advantage (Buetti & Walsh, 2009), the capacity to time action, estimate the size of the pack of hunters or amount of available food all come in good use for survival. Both numerosity and time are fundamental magnitudes used for guiding action in the external world, and interaction between the perception of these magnitudes have long been suspected. The integration of sensory input is in fact necessary, as the world is organized spatiotemporally, and magnitude interference “[...] may provide useful heuristics for things that are statistically true about the physical world: faster things do often get further, [...] bigger things do usually weigh more” (Buetti & Walsh, 2009).

However, it remained to be clarified how and to what extent numerosity and time affect each other. According to Meck and Church (1983), numerical and temporal processing likely use the same underlying accumulation mechanism, a kind of pacemaker or stopwatch, that is able to keep track of both. Following up on their research, a Theory of Magnitudes (ATOM) states that they are intertwined, and likely share a common mechanism and metric (Walsh, 2003). ATOM claims that within this system, all magnitudes (space, time, and number) interact bidirectionally. Indeed, some studies have found a bidirectional interaction (Javadi and Aichelburg, 2012), but various researches

¹ A large philosophical discussion concerning the ontological status of numbers is available. For an overview, see: <https://plato.stanford.edu/archives/spr2019/entries/philosophy-mathematics/>

have found unidirectional results in which time affects number (Martin, Wiener, & van Wassenhove, 2017; Tsouli et al., 2019a; Lambrechts, Walsh, & van Wassenhove, 2013) or the other way around (Alards-Tomalin, Walker, Kravetz, & Leboe-McGowan, 2016), whereas others have found no interaction at all (Agrillo, Ranpura, & Butterwoth, 2010). Evidently, it has yet to be concluded in what manner numerical and temporal perception interact, because these results do not correspond with the literal interpretation of ATOM.

Bayesian Computation

It has been suggested that the integration of these perceptual magnitudes might be achieved by Bayesian inference (Lambrechts et al., 2013; Martin et al., 2017). The 'Bayesian brain hypothesis' has been a popular approach to developing computational models of cognition and perception (Knill & Pouget, 2004). The Bayesian brain is described as a statistical machine that makes sense of the world by use of prior information and integration of sensory data in order to minimize uncertainty. By continuously updating beliefs on the basis of new sensory evidence, it allows for computing a useful representation of the world in order to guide action in a statistically optimal fashion. The Bayesian approach has been proven successful in explaining various sensory phenomena, such as the integration of visual and haptic stimuli (Ernst & Banks, 2002). Bayesian computation might be taking place during temporal and numerical processing as well. Martin et al. (2017) have suggested that Bayesian computation takes place in a generalized magnitude system. Similarly, Lambrechts et al. (2013) suggest that "magnitude representation could be estimated based on the integration of all quantities estimates" (p. 8) and that this integration likely is Bayesian in nature. What role Bayesian inference might exactly play in temporal and numerical perception and their interaction is however to be elucidated further.

Aim and Structure of this Research

This literature research attempted to determine to what extent Bayesian inference can explain the interaction between temporal and numerical perception. With the aim of doing so, firstly a theoretical background is provided, consisting of an overview of Bayesian inference and its role in perception, as well as theories of numerical and temporal perception. Consequently, the interaction between numerical and perception is

analyzed in order to answer whether Bayesian computation can explain said interaction.

Relevance

Gaining new insight into how sensory integration of numerical and temporal data is accomplished might indicate how to approach future research. Moreover, how numerical processing relates to other sensory abilities and higher concepts of mathematics could provide insight into the development of these abilities in humans and animals. These insights can inform the creation of artificially intelligent models as well, as it has often done in the past. For example, the ‘Helmholtz machine’ is a neural network which simulates ‘generative perception’ (Dayan et al., 1995, which is further described in chapter 1.2. Discovering by what means humans (and other animals) interpret the continuously incoming wave of information, could provide a clearer perspective on what artificial reasoning could look like.

Furthermore, whether humans behave rationally has been a topic of research for centuries. Instead of being the most rational animal, humans have shown to be incredibly biased and more often than not appear to make seemingly irrational decisions (Tversky & Kahneman, 1974). However, in the light of the Bayesian theories of cognition, humans might be ‘Bayesian-optimal agents’ (Knill & Pouget, 2004), and not so irrational after all. Bayesian approaches have been already frequently and successfully used in artificial agent modelling. In fact, it is the disciplines of statistics and artificial intelligence that have inspired the use of Bayesian methods in the study of neural computation (Knill & Pouget, 2004). This interplay between disciplines only accentuates the broad application and elegance of Bayes’ theorem.

1 | Bayesian Theories of Perception

The world is an uncertain place. And, as you probably have experienced yourself sometimes with e.g. visual illusions, your senses cannot always be trusted. Efficiently dealing with perceptual uncertainty is an absolute necessity when effectively and safely navigating an ocean of patterns and noise. According to researchers Knill and Pouget (2004), this is exactly what our brain achieves:

Our brains must effectively deal with the resulting uncertainty to generate perceptual representations of the world and to guide our actions. This leads naturally to the idea that perception is a process of unconscious, probabilistic inference. (p. 712)

This probabilistic inference can be described by a Bayesian model of perception.

1.1 | Overview of Bayes' Theorem

Bayes' theorem is a theory originated in mathematics and statistics, when La Place formalized the ideas posthumously published by Thomas Bayes end of the 18th century. Since then it has shown to be useful in many other disciplines such as Artificial Intelligence, where it is used for e.g. the development of classifiers and decision networks. Bayes' theorem allows for the calculation of the conditional probability of an event (the posterior), making use of prior knowledge and updating probabilities when new evidence is observed. In contrast to frequentist statistics, Bayesian statistics is idealist about probabilities - meaning, it rather assigns probabilities to the truth or falsity of the beliefs and hypotheses one has about the world, than to the world itself. Bayes' theorem is mathematically defined as follows:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

The posterior probability, $P(A|B)$ is to be read as 'the probability of B, given that A'. In the case of perception, it stands for the probability of the world, given a certain sensory input. For example, the probability of fire given that smoke is perceived. $P(B|A)$, the likelihood function, reflects the probability of the sense data, given that the world is in that state (the probability of smoke appearing when there is a fire). The prior probability, $P(A)$,

reflects the possibility of a fire occurring. Bayesian inference can be described as updating the hypothesis probabilities based on new information.

1.2 | Bayesian Perceptual Processing

The Bayesian Brain hypothesis has gained popularity over the last few decades, but the initial idea has been around for centuries. It was von Helmholtz who formalized a scientific theory of the brain as a predictive machine, which infers the physical causes of the perceptual input it receives (von Helmholtz, 1866). This reframes perception as the formation of hypotheses about the world (Gregory, 1980) which can be evaluated and (dis)proven when new evidence (perceptual input, neural signals) is collected. This presents us with a radically different model of perception as (Bayesian) inference:

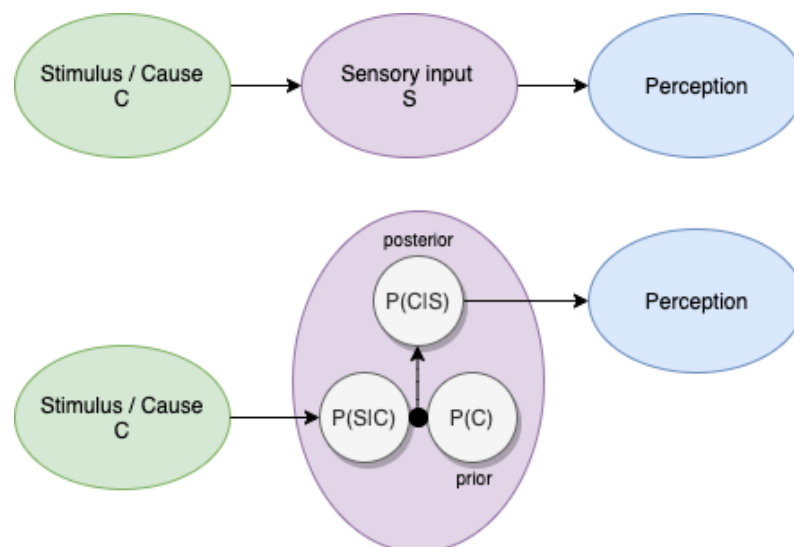


Figure 1: Classical versus generative model of perception. The classical model describes the process of perception as a causal chain between the physical causes, sensory input, and the occurrence of perception (which might lead to a behavioral response). In the generative model, physical causes elicit a (Bayesian) calculation of the likelihood of the sensory input given the physical cause. (Petzschner, Glasauer, & Stephan, 2015) In this model, one infers the causes of the perceived.

There are several perceptual phenomena hypothesized to be characteristic of Bayesian perceptual processing:

The Central Tendency Effect

One way of identifying whether Bayesian processing is occurring is by observation of the 'central tendency effect', also named the 'regression effect' (Petzschner et al., 2015) or 'Vierordt's Law' (Shi, Meck, & Church, 2013). The central tendency effect can be observed in magnitude estimations, which often converge to the mean of the stimuli. For example,

when posed with a successive set of presented numerosities, the subjects' estimates will tend to regress to the mean of these numerosities, meaning that the lower numbers will be overestimated, and the higher numbers will be underestimated (Petzschner et al., 2015; Martin et al., 2017). The central tendency effect might indicate that a central-prior is causing regression to the mean, by biasing the estimates away from the outer points of the distribution (Petzschner et al., 2015). Furthermore, if different magnitudes, for example, numerosity and time, exhibit a similar central tendency effect, there is reason to believe that if it is the case that Bayesian processing is occurring, these magnitudes either share a prior, or a correlation between their priors exists (Martin et al., 2017).

Adaptation

Another potential method to identify Bayesian processing in isolated neural networks is by using adaptation. Adaptation, also called "the psychophysicist's microelectrode" because of its accuracy despite its non-invasiveness, is a reduction in a neuron's response after continuous stimulation (Wolfe et al., 2015). Perceptual adaptation is a powerful method used to isolate neural populations which habituate to a repeated stimulus. When, for example, being exposed to a red image for a while, perceptual adaptation occurs, meaning that the red cone photoreceptors in the eye will be desensitized, causing a green afterimage to be perceived, as signals from the cones responsible for green colour perception are 'overrepresented'. Similarly, adaptation to low numerosity leads to higher estimations, and vice versa (Anobile et al., 2016). Also, adaptation to duration distorts duration perception (Heron et al., 2019; Hayashi et al., 2015). The adaptation to numerosity and duration will be further elaborated upon in the following chapters.

According to Stocker and Simoncelli (2006), and Sato and Aihara (2011), adaptation can be understood as being the result of Bayesian inference. Stocker and Simoncelli (2006) provide a functional explanation of perceptual adaptation by use of a Bayesian framework, in which adaptation does not change the prior probability, but instead causes an improvement of the sensory reliability, which in its turn changes the Bayesian likelihood function. If this is indeed the case, observing perceptual adaptation can be seen as a characteristic of the occurrence of Bayesian processing for perception.

Perceptual Cue Integration

One of the most compelling indications of Bayesian processing is perceptual cue integration (Knill & Pouget, 2004). In order to make sense of the world, the sensory data coming into the different senses need to be matched, because oftentimes different sensory stimuli come from the same source. For example, a loud noise is likely connected to the visual image of an object falling on the ground. These different perceptual cues need to be integrated in a manner that optimizes decision-making and minimizes uncertainty. The integrated estimation will be biased towards the more reliable cue. Remarkably, as one trains one's whole life for optimal cue integration, the brain starts to predict patterns of the incoming sensory data, creating a feedback loop in which the predicted becomes the perceived (Clark, 2015). Compelling examples include the ventriloquist illusion, in which the artist's voice appears to come from elsewhere, rather than his mouth, or the sound-induced flash illusion, in which the hearing of two sounds is accompanied by the perception of two dots flashing, while only one dot is presented (Baysan & Macpherson, 2017).

Observation of the central tendency effect, perceptual adaptation, and cue integration in perceptual studies on numerical and temporal perception and their interaction, might indicate whether the interaction between the two can be understood within a Bayesian model of perception.

2 | Numerical Perception

Humans, as well as many animals, are able to process numerical quantities, i.e. numerosity (Dehaene, 2011). Neural imaging techniques have confirmed that neurons that are tuned to specific numerosities (also dubbed 'numérons') can be found in the parietal cortex in humans (Harvey et al., 2013) and non-human primates (Nieder et al., 2002). A distinction is made between counting numbers up until 4 (Dehaene, 2011), which is called subitizing (from Latin *subitus*, sudden; as it appears to happen instantaneously), the 'Approximate Number System' which represents any number larger than four (Odic & Starr, 2018), and more complex, human, symbolic mathematics (Dehaene, 2011). The focus of this research however is on the more fundamental, non-symbolic number sense, independent of culture and language, which allows for direct, intuitive perception of the numerosity of a set of items as a primary perceptual attribute (Anobile et al., 2015).

2.1 | Theories of Numerical Perception

The number sense has been shown to be present in many animals. For example, the experiments conducted by Mechner (1958), Platt and Johnson (1971) in which a famished rat is conditioned to press a lever n times, have shown that rats are able to produce a Gaussian distribution of responses, with the intended number n as mean. The larger the number, the wider the variability of the distribution became (i.e. the interval between the lowest and highest guess became larger). Optimal results, i.e. a distribution with (the) low(est) variability, were achieved on numbers within the subitizing range. This distribution of estimations might not be as exact as humans are able to count, but it is hypothesized to be a feature of the accumulation system that is in place. Research has suggested that in order to have a basic number sense, a kind of 'accumulator' needs to be present in the brain (Dehaene, 2011; Meck & Church, 1983). This accumulator would be a kind of stopwatch, in which oscillations or pulses accumulate.

It is important to discern numerosity from other magnitudes, such as space (surface, size). It has been suggested that with increased surface or texture density, numerosity estimations increase as well, which indicates that there might not be such a thing as numerosity as a primary perceptual attribute (Durgin, 2008). However, this idea

has been discounted by implementing exact control experimental conditions which control for other perceptual attributes (Ross & Burr, 2010) as well as the finding of 'numerons' in the brain. Meck and Church (1983) followed with the same experimental setup as Mechner, however controlling whether the duration of stimuli would be of influence on numerosity estimation. In summary, they tested whether the rats would perceive the duration or the number of tones. The experiment concludes that rats are indeed able to perceive both numerosity and duration separately, and that perhaps a 'single pacemaker' with separate accumulators and mode switches exists. This will be further discussed in chapter 4, as their experiment is primarily concerned with the interaction between numerical and temporal perception.

Notable phenomena in numerical perception are the distance effect and the size effect. The distance effect shows that when comparing numerosity, the accuracy of estimation correlates with an increase in the ratio between the numerosities. The size effect simply shows a decrease in accuracy when dealing with (relatively) larger numbers, which makes it harder to accurately distinguish numbers that are closer to each other, e.g. 12 and 13, than when they are more distant, e.g. 10 and 20. These effects both follow the Weber-Fechner law (also: 'scalar law'), which states that a logarithmic function describes the psychophysical relation between perceived discriminate stimuli and the ratio of stimulus intensities (Wolfe et al., 2015).

2.2 | Bayesian inference and Numerosity

Estimating numerosity is a task that requires a framework for representing as well as reducing uncertainty. According to Sanborn and Beierholm's (2016) experimental findings, participants' discrete numerical estimations were best fitted by a Bayesian model. Using Bayesian processing, an uncertain representation can be converted into the response that maximizes expected reward (Sanborn & Beierholm, 2016, p. 2), while each component (prior, likelihood and posterior) and their interactions can be identified by the experimenters.

As numerosity estimation shows scalar variability following the Weber-Fechner law, a Bayesian framework might be in place. The center-prior probability, which could also explain the central tendency effect, could account for scalar variability, because the center-prior will have more influence on larger stimuli as the standard deviation of the likelihood increases (Petzschner et al., 2015, p. 287). Furthermore, evidence of

adaptation to numerosity not only confirms the existence of a dedicated neural mechanism (Burr, Anobile, & Turi, 2011), but also indicates the occurrence of Bayesian processing. Not only numbers in the subitizing range, but also larger numerosities (in the ANS range) have shown to produce the adaptation effect (Burr et al., 2011). These findings indicate that Bayesian processing is taking place when estimating numerosity, both in subitizing range and above.

3 | Temporal Perception

The subjective perception of time is a complex phenomenon, which has intimate ties with memory, attention, and other cognitive functions. Just as with illusory tricks and phenomena in the visual system, temporal illusions exist as well. You might yourself have noticed the speeding up and slowing down of perceived time, which is usually dependent on how much one is enjoying the task at hand. Perceiving the passing of time between two intervals, i.e. duration perception, is a skill that is not immediately developed from birth (Droit-Volet, 2012), and a quicker passage of time is experienced when growing of old age (Turgeon, Lustig, & Meck, 2016). Different ranges of timing are hypothesized to be processed by various brain structures: not only on sub-second to second range, second-to-minute level, but also interval timing, and the circadian rhythm. This research will mainly lay focus on the experiments conducted concerning (sub)second to second timing, and the underlying processes that facilitate this function, as this timescale is more relevant for perception of successive states of the world, guiding direct action (Wolfe et al., 2015, p. 474).

3.1 | Theories of Temporal Perception

The traditional view on temporal processing on the (sub)second-to-minute scale, is that a system containing a single ‘pacemaker’ or ‘stopwatch’ emits pulses, which are monitored by a counter or ‘accumulator’, is able to keep track of time (Treisman, 2013; Church, 1984). However, evidence for parallel (but separate) timing processes have been found as well (Buhusi & Meck, 2009; Treisman, 2013). Buhusi and Meck (2009), as well as Treisman (2013), have indeed shown that within the second-to-minute time processing, multiple clocks “[...] that can be independently ran, stopped, or reset by the insertion of a gap into the signal” (p. 5) are present. The proposal of the ‘accumulator’ metaphor in which separate ‘clocks’ reside, remains. The ‘scalar timing theory’ states that interval timing consists of three stages: clock, memory, and decision:

In order to represent a target duration, a pacemaker emits pulses that are passed by a switch into an accumulator. The value in the accumulator is assumed to be normally distributed, which is then compared to the expected time sampled from reference memory. (Shi et al., 2013, p. 558)

On the neural level, Meck and colleagues (2008) have indeed found neurons that detect simultaneous and reoccurring oscillations in the cortex, which gives means to differentiate supra-second durations (Meck, Penney, & Pouthas, 2008).

Similar to numerical perception, temporal perception follows the Weber-Fechner law and shows scalar variability (Jazayeri and Shadlen, 2010). Furthermore, adaptation to duration has been observed (Heron et al., 2019; Hayashi et al., 2015), as well as neural tuning to specific (visual) durations (Heron et al., 2011; Hayashi et al., 2015), indicating that a specific neural mechanism is in place. In fact, multiple channels might be tuned to specific durations (Heron et al., 2011), just as (populations of) neurons can be tuned to e.g. visual orientations (Hubel & Wiesel, 1986) or numerosities (Harvey et al., 2013). The existence of duration-specific channels is in line with the observed scalar variability, and the normally distributed values in the accumulator (Shi et al., 2013).

Other modalities have been shown to modulate time perception, which is not surprising given that the world is organized spatiotemporally, and objects and events are usually connected by causal relations. This indicates the need for an integration of time with other perceptual stimuli.

3.2 | Bayesian inference and Time

The idea that Bayesian processes might guide temporal estimation on the basis of a stored prior notion of duration has gained popularity in recent decades. Turgeon and colleagues (2016) propose a Bayesian framework that accounts for the age-related change of sub-second-to-minute time perception, likely caused by a degeneration of the brain which causes an increase in noise (and thus decrease in accuracy). Furthermore, in a sub-second duration reproduction task, musicians appeared to encode (visual and auditory) temporal information optimally, whereas non-musicians showed a central-tendency effect (Cicchini et al., 2012), which is a characteristic of Bayesian processing. Both observations fit in a Bayesian framework in which non-musicians minimize uncertainty by employing a central-tendency prior and different strategies, but the experts have already maximized their accuracy and therefore do not need to make use of these strategies or a central-tendency prior. Jazayeri and Shadlen (2010) find that a Bayesian model best fit the results of the participants in their experiment, which showed both a

central tendency and scalar variability, indicating the existence of Bayesian priors influencing the likelihood distribution in the sub-second-to-second range.

The importance of contextual effects is emphasized by Shi and colleagues (2013), who illustrate the ways in which context can distort temporal perception and how the brain might accomplish contextual calibration. According to them, the brain is able to incorporate these noisy distortions, using Bayesian inference in order to minimize error. The observation of both the central tendency effect (Shi et al., 2013; Jazayeri & Shadlen, 2010), scalar variability (Jazayeri & Shadlen, 2010), and adaptation (Heron et al., 2011) indicate the possibility of Bayesian processing.

4 | The Interaction of Numerical and Temporal Perception

In the previous chapters, the perceptual abilities concerning numerosity and time have been discussed. It has been hinted at that these are not as separate as presented: the accumulator as described by Meck and Church (1983) comes to mind. The integration, as well as the differentiation of perceptual input according to a correct representation of the statistics of the external world, is of crucial importance for decision-making, and it could be achieved by the use of Bayesian inference. The question remains whether the interaction between the numerical and temporal modalities can indeed be explained within a Bayesian framework, and perhaps whether this framework can give a suggestion on why there is a discrepancy in found directionality of influence.

4.1 | A Theory of Magnitudes

Observing the commonalities between the separate literatures, Walsh proposed an influential theory, 'A Theory of Magnitudes' (ATOM), that integrates the perception of space, time, and quantity (Walsh, 2003; Buetti & Walsh, 2009). Most importantly, ATOM states that the three magnitudes share a common metric and accumulation principles. Walsh suggests that an infant might start off with an undifferentiated magnitude system but will learn to differentiate "according to the statistics of the environment, an environment in which the space, time, speed, size and quantity of events and objects are often highly correlated" (p. 486). Interference and priming between the magnitudes which were once undifferentiated are therefore expected (Buetti & Walsh, 2009). A global prior which reflects the world's statistics could explain the occurrence of interaction between magnitudes (Petzschner et al., 2015).

A crucial detail is that within a literal reading of ATOM, the interaction between the modalities is interpreted as being bidirectional. Indeed, experimental research by Javadi and Aichelburg (2012) has found a bidirectional interaction between numerical and temporal perception. However, various experiments have found different interactions between numerical and temporal perception: Lambrechts et al. (2013) and Martin et al. (2017) found a unidirectional influence of duration on numerosity, Alards-Tomalin et al. (2015) found a unidirectional interaction in which numerosity influenced time, while Agrillo and colleagues (2010) found no interaction at all. Can these findings

be reconciled with each other? It has been suggested that the interaction might be understood and formalized in terms of Bayesian cue integration (Martin et al., 2017; Lambrechts et al., 2013).

4.2 | Bayesian Integration

In order to understand the different findings on the interaction between numerical and temporal perception within a Bayesian framework, we would expect to find the following characteristics of Bayesian inference: cue integration, the central tendency effect, and adaptation.

Central Tendency Effect

Observing a correlation between the central tendency effect of different magnitude estimations could indicate that a prior is shared by those magnitudes (Martin et al., 2017). According to Martin and colleagues (2017), duration and number indeed show a correlated central tendency effect. A correlation between the central tendency of both numerosity and duration estimations might indicate that, in accordance with ATOM, “the representation of these magnitudes is amodally stored” (Martin et al., 2017, p. 2). Important to note is that according to Cicchini et al. (2012), central tendency is dependent on circumstances: “when the temporal judgment is imprecise—such as visual judgments with non-percussionists—then the central-tendency strategy can be beneficial; otherwise, there is no point in sacrificing accuracy” (p. 1060). This makes sense in the context of a Bayesian framework; if there is no need for estimation because one already achieves a high level of accuracy, then using Bayesian processing in order to minimize noise and error is nonsensical.

Adaptation

One of the central arguments for ATOM is that the brain regions associated with time and number overlap in the parietal cortex (Walsh, 2003). Adaptation can be used to detect whether numerosity and time share the same neural mechanism. This has indeed been done by Tsouli and colleagues (2019), who found that cross-adaptation occurs, but only towards one direction: adaptation to visual duration affected numerosity, but there was no evidence either for or against numerosity adaptation affecting duration (Tsouli et al., 2019). If, as Stocker and Simoncelli (2006) proposed, adaptation can be explained within

a Bayesian framework, the finding of cross-adaptation between numerosity and time indicates that the two are integrated.

Cue Integration

Bayesian cue integration could occur and would be dependent on to what extent there is reason to believe that the sensory data is related, as well as the reliability of the cues. These factors could perhaps indicate the reason why different interactions between numerical and temporal perception are observed. According to Lambrechts et al. (2013), number is often the dominant magnitude, as evidence for it is oftentimes presented unambiguously, and therefore numerosity usually influences duration estimation. However, when evidence of numerosity needs to be accumulated, duration will influence numerosity. Thus, they propose that dimensions are integrated in a Bayesian fashion, because the Bayesian model predicts a shift towards the more reliable, or the least noisy, cue. Reviewing the conditions of some notable experiments might give insight into whether their claim is accurate.

Indeed, when inspecting the experimental conditions of the research of Alards-Tomalin and colleagues (2016), who have found a unidirectional effect of numerosity on duration, we see that numerosity can be seen as the dominant magnitude in their experiment. The numerosity of the items is ranged between 1 and 9, following the structural pattern as seen on dice. They were presented unambiguously, as no evidence on the numerosity had to be collected over time. Moreover, the temporal scale the participants had to estimate was between 340 and 460 ms, and can, in this case, be seen as the relatively noisy, non-dominant magnitude, as it usually “appears to be the least reliable dimension i.e. the most susceptible to interference and the least influential on other magnitudes” (Lambrechts et al., 2013, p. 1). Further critique of the experiment is that the presented items had a structural layout similar to dice, which might approach a more symbolic representation that interferes with the numerical perception on itself. Similarly, a group of irregularly distributed items is usually underestimated in comparison to a regularly structured set (Dehaene, 2011). Indeed, the researchers also acknowledge that the structure has influenced some of the participants’ results, whose estimations were biased by the patterns, hypothesized to be the case because the center dot provided a more salient feature than the numerosity itself. Following Lambrechts’ prediction, this could explain why the estimation of the noisy magnitude (time) was

influenced by the more certain magnitude (numerosity). These findings would, therefore, fit in a Bayesian framework for magnitude integration.

Both Lambrechts et al. (2013) and Martin et al. (2017) found a unidirectional influence of duration on numerosity. Their experiments differ from the other studies presented in this chapter, as their experimental conditions allowed for evidence accumulation of all magnitudes, instead of presenting them unambiguously. Within this paradigm, the directionality of interaction is inverted, as duration is not influenced by numerosity, but duration did interfere with numerosity estimations.

Javadi and Aichelburg (2012) observed a bidirectional interaction between numerosity and time, in correspondence with the literal interpretation of ATOM. In their experimental paradigm, a numerosity above subitizing range (28-40) was tested, and short duration (53–106 ms). This forced the participants to use their approximate magnitude system, instead of subitizing, because according to the authors, discrete numerosities would not allow for duration interference, meaning that all of the stimuli were relatively ambiguous and noisy. Important to note is that their main experiment does not control for the occupied surface of the presented items, which impacts the validity of their conclusions, as space (and thus occupied surface) is an explicit and fundamental part of ATOM. A follow-up control experiment has tested the participants on whether they can distinguish occupancy and numerosity of two sets, but this control experiment does not factor in duration and does therefore not support their conclusion that bidirectional interaction between numerosity and time exists. In any case, the bidirectional interaction between two noisy cues can be understood in Bayesian terms if it is the case that they share a prior, because the more uncertainty, the more use of this prior.

In a study that found no interaction, evidence accumulation occurs for both numerosity and time (Agrillo et al., 2010). The numerosity stimuli ranged between 11 and 19, while the duration stimuli ranged between 5 and 13 seconds. However, contrary to the previously mentioned experiments in which visual stimuli were used, this study utilized auditory stimuli. It might be the case that the saliency of information in different sensory modalities varies, which might influence the manner in which they interact (Javadi et al., 2012; Agrillo et al., 2010). Furthermore, the relatively long duration might not be able to be compared with the previously mentioned experiments, as those are rather concerned with sub-second durations.

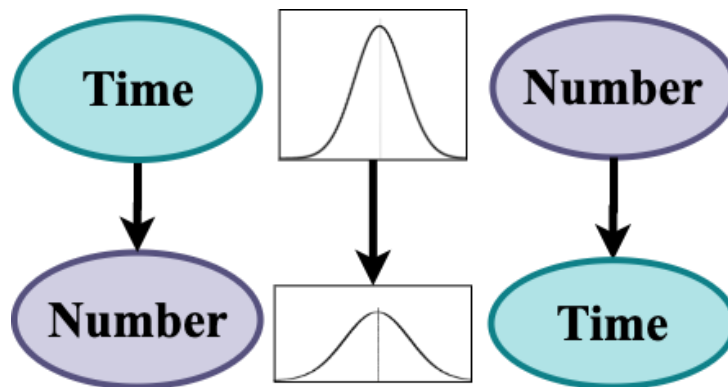


Figure 2: The most reliable cue determines the direction of interaction, according to Bayesian cue integration. If the temporal cue is more certain and less noisy than the numerical cue, the temporal cue will interfere with the numerical estimation, and vice versa. If both cues have a high certainty, no interaction takes place. If both cues have a low certainty, it might be the case that they interact bidirectionally.

The studies discussed above all use varying ranges of time and numerosity. As discussed in previous chapters, there is a difference between subitizing and the Approximate Number System (ANS), as well as sub-second or second-to-minute timing. Hypothesized is that subitizing is not influenced by the other magnitudes at all and that perhaps only the ANS plays a role in ATOM (Agrillo & Piffer, 2012). Furthermore, supra-second durations, which Agrillo et al. (2010) employed in their experiments, allow for explicit counting or cognitive mediation (Agrillo & Piffer, 2012) which might influence the found results because it reduces uncertainty. Future research could further analyze what influence the different scales (subitizing and ANS, sub- and supra-second) have on the found directionality of interaction.

The observations of adaptation, central tendency, and cue integration demonstrate that it is likely that the interaction between numerical and temporal perception is guided by Bayesian means. The different directionalities of the interactions can be reconciled in a Bayesian model, given that the direction is determined by the least noisy cue (see figure 2). A liberal interpretation of ATOM is proposed by Martin et al. (2017), in which multiple priors interact in the context of conflicting sensory cues (p. 11). Instead of a rigid proposal of bidirectional interaction, it is sensible that various interactions are observed because the interaction is determined by the amount of uncertainty in the stimuli and the extent to which the system believes that two magnitudes are related. Cues are not always related, so in order for the system to make sense of the world, a flexible framework needs to be in place that can compensate for the configurations in which the world can be presented.

Discussion

This research investigated whether Bayesian processing could provide an explanation of the interaction between numerical and temporal perception. To do so, three characteristics of Bayesian processing were identified: cue integration, the central tendency effect, and adaptation. Furthermore, scalar variability according to the Weber-Fechner law is hypothesized to be indicative of Bayesian processing as well, as it fits in a framework in which the central-prior could account for both the central tendency effect and scalar variability. All of these characteristics are observable in experiments that examined numerical and temporal perception separately, as well as in experiments that examined the interaction between the two. Notable is that the variation in interactions found (bi-, uni-, and non-directional) could be explained within one Bayesian framework, in which the direction of influence is guided by the least noisy cue. This research concludes that Bayesian inference can to a certain extent explain the interaction between numerical and temporal interaction. The extent is determined by how much uncertainty is present in the estimation task. For example, professional percussionists had no need to integrate noise, as their uncertainty was already minimized (Chicchini et al., 2012). This emphasizes that the utility of Bayesian inference is determined by the extent to which uncertainty is present and noise reduction is needed. These findings are in correspondence with Lambrechts and colleagues' (2013) and Martin et al.'s (2017) claims, which lead to a more liberal interpretation of ATOM, in which various interactions might take place, according to Bayesian cue integration.

A shortcoming of this research, and simultaneously a suggestion for further research, is that more emphasis could be laid on the difference between subitizing and the approximate number system, as some of the mentioned experiments used numerosities ranging between both and did not always differentiate between the two systems. Similarly, the difference between sub- and supra-second duration estimation should be taken into account in order to see the potential influence it has on the found directionality of interaction. Furthermore, future literature research could analyze more studies, such as (but not limited to) Bruno & Cicchini (2016), as this research was limited by time and resources.

ATOM theorizes not only about numerosity and time, but also space. Given the scope of this research, it was not feasible to integrate space in the analysis. However,

space is an important magnitude which might be crucial for understanding the interaction of time with other magnitudes, as the world is spatiotemporally organized.

A Bayesian framework of perception provides an explanation and description at the functional or computational level, but does not give an explanation of the physical, neural implementation of the framework (Marr, 1982). Bayesian explanations are therefore criticized to be ad hoc at times, and integration with mechanistic theories is necessary according to Jones and Love (2011) in order for the Bayesian framework to be theoretically and psychologically valid. Indeed, a Bayesian theory of perception stands or falls with to what extent the brain is able to represent probabilistic information (Knill & Pouget, 2004). The integration of mechanistic and computational theories is therefore crucial for the continuation of the Bayesian research program in psychology and cognitive science.

These results are relevant for the field of artificial intelligence, because they emphasize the embodied nature of knowledge, indicating that learning systems or agents could profit from not merely being fed data, but also discovering patterns in the data by 'doing' and experimenting in the world (be it physical or virtual). Additionally, abolishing the rigid categories that humans have conceptually constrained their research fields and subject with, can inspire the creation of autonomous agents in which these perceptual functions are similar to humans, and not rigidly separated.

Conclusion

This research analyzed to what extent Bayesian inference could account for the interaction between numerical and temporal processing. Several characteristics of Bayesian processing during perception are identified in experiments concerning numerical and temporal perception separately, as well as in their interaction. Bayesian inference provides a framework in which human perception occurs in a statistically optimal manner. The conclusion that Bayesian inference can indeed account for the interaction between numerical and temporal perception is reached, given that the task at hand involves uncertainty, and a statistical integration is necessary in order to maximize performance.

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