Mathematical explanations

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Summary

Do we need to be realists about mathematical entities? The indispensability argument states that we ought to have ontological commitment towards the entities used in our best scientific explanations. In order to argue in favour of mathematical realism using the indispensability argument, one needs to show that mathematical explanations exist in science. Therefore the question 'Does mathematics play an explanatory role in science?' is the main question this paper addresses. Because explanations tell us something about the true causes of some phenomenon, explanations are usually taken to be solely causal. However, in the first chapter we provide an indispensable mathematical explanation of the existence of a physical phenomenon, namely the Kirkwood gaps between the rings of Saturn, and we argue that a causal explanation does not provide the whole story. Supposedly, a mathematical analysis gives us both the existence and the location of these gaps.[1] With the use of this explanation we take a look at the difference between program and process explanations. In the second chapter arguments against the realism of mathematics are given, most notably the argument that the indispensability of mathematical explanations does not provide ontological commitment towards the content of these explanations, and the argument that mathematics only describes, and that the explanation emerges from a correct physical interpretation of the mathematics.[2] In the last chapter we evaluate these arguments and come to the conclusion that we have no reason to doubt the existence of mathematical explanations, and that we therefore ought to be ontologically committed towards mathematical entities used in indispensable mathematical explanations.

[1] Colyvan (2012) P 92
[2] Bueno (2012)

1 Introduction

1.1 Explanations

An important part of science is to provide explanations of physical phenomenon, and on the basis of whether a theory provides explanations, it is determined whether a theory scientific or not. As we will see throughout this paper, explanations often answer why questions.

Explanations differ from descriptions. Explanations often tell us something about the true causes of a phenomenon, while a description of a phenomenon provides less information. Because explanations reveal us something about the true causes of a phenomenon, explanations are often thought to be causal. Colyvan, an author discussed in the second chapter, even claims that most philosophers think that causal explanations is all there is.[3]

Explanations consist of two parts, something that is in need of explaining, and something that does the explaining work. The first is called the explanandum, the second the explanans. We can, for example, have the explanandum 'The yellow Billiard ball is moving' and have the explanans 'Because the white billiard ball hit it' form an explanation of a physical phenomenon. It is difficult to give a full definition of what an explanation truly is. There are many different theories of explanation, each one tries to give a definition of what an explanation is, but none is without problems. Later in this paper we will introduce a popular theory of explanation, difference-making, and see whether it is able to cover non-causal explanations.

This paper will focus on the question 'Does mathematics play an explanatory role in science?' This question is an important one, because as we will see in the next section, the mathematical indispensability argument claims that if mathematics does play an indispensable role in science, we ought to be realists about mathematics.

1.2 Mathematical indispensability argument

The Quine and Putnam indispensability argument[4] states that we ought to rationally believe in the existence of entities that are indispensable to our best scientific theories. And because the mathematical indispensability argument claims mathematics plays an indispensable part in our scientific theories, we ought to be realists about mathematical entities. However, it is much debated whether mathematics plays an indispensable part in science, and in what way the mathematics must be indispensable to be able to claim ontological commitment towards the mathematics. As we will see in the next section, mathematical realists, also called Platonists, and anti-realists about mathematics, also called nominalists, agree that this is not enough to claim ontological commitment towards the mathematics used in our best scientific theories. The mathematics needs to be indispensable in an explanatory way.

Baker notes that Colyvan (taking the Platonist side) and Melia (taking the nominalist side) have come up with a revisionary account of this argument. They found it necessary to come up with this revised indispensability argument because they agreed that the fact that mathematics is indispensable for science was not enough to establish Platonism. It is not enough to show that mathematics play an

[3] Baron & Colyyvan (2016) P 83

[4] The Quine and Putnam indispensability argument is an influential argument used to argue in favour of scientific realism. In general the basic argument goes as follows: P1: We ought rationally to believe in the existence of any entity indispensable in our best scientific theories. P2: Some entity X plays an indispensable role in a certain scientific theory, which is the best scientific theory to explain a certain phenomenon. C: we ought to rationally believe in the existence of entity X.

indispensable role for science, 'it has to be indispensable in the right kind of way'.[5] What is necessary to be shown is that reference to mathematical object sometimes plays an *explanatory* role in science. Both sides of the argument agree that inserting the word 'explanatory' in the first premise makes it more plausible because 'it restricts attention to cases where we can posit the existence of a given entity by inference to the best explanation.'[6] In other words, based on inference to the best explanation, we can rationally be allowed to believe in the existence of an entity referred to in this explanation. It is, for example, not enough to claim ontological commitment towards the mathematics if it is only indispensable in the description of a scientific theory.

The mathematical indispensability argument makes use of inference to the best explanation. Inference to the best explanation is a way of choosing between different competing hypotheses which are all empirically adequate given certain data. Inference to the best explanation states that if all hypotheses predict the data equally well, the hypothesis that best explains the data must be chosen.

We ought to rationally believe in the content of our best scientific theories. And we determine what our best scientific theories are on the basis of inference to the best explanation. Because we determine what is the best theory on the basis of the explanation, and not for example on basis of the description, of a phenomenon, we need the mathematics to be indispensable in the explanation provided by this theory, and not for example the description. So to have ontological believe in the mathematics involved in our best scientific explanations, the mathematics must play an explanatory role in the theory.

Baker states an 'enhanced' Indispensability argument as follows:

> Premise 1: We ought rationally to believe in the existence of any entity that plays an indispensable explanatory role in our best scientific theories.
> Premise 2: Mathematical objects play an indispensable explanatory role in science.
> Conclusion: Hence, we ought to believe in the existence of mathematical objects.

The mathematical realist uses this argument to argue that we ought to believe in the existence of mathematical entities. Because if it can be shown that mathematics plays an indispensable explanatory role in science, then it would show that we ought to have ontological commitment to the mathematics involved in this explanation. So mathematical realists argue that there are mathematical explanations of non-mathematical facts. And nominalists, anti-realists about mathematics, refute the idea that we ought to be realists about mathematics by refuting the second premise, thus claiming mathematical explanations do not exist. On top of that they can argue we ought not to have ontological commitment to mathematical entities even if there are mathematical indispensable explanations for physical phenomena, and thus refuting the whole indispensability argument for mathematics.

1.3 Explainable AI

Within the field of Artificial Intelligence, the importance of explanations has gain increasing interest. As we have seen, it is difficult to formulate a definition of what an explanation is. In their paper, Doran et al. note that the variety of ways 'explanations' are currently handled within the field of explainable AI is well summarized by Lipton when he states "*the term [explanation] interpretability holds no agreed upon meaning, and yet machine learning conferences frequently publish paper which wield the term in a quasi-mathematical way*"'[7].

[5] Baker (2009) p 613
[6] Baker (2009) p 613
[7] Doran, Schulz & Besold (2017) P 2

To overcome the dangerous practise of blindly accepting the outcome of an AI, it is prudent for an AI to provide not only an output, but also a human understandable explanation that expresses the rationale of the machine. In other words, complete trustworthiness and an evaluation of the ethical and moral standards of a machine seem to be able to be achieved only by a detailed 'explanation' of AI decisions. However, Doran et al. ask themselves whether present systems that claim to make 'explainable' decisions really do provide explanations.[8]

They claim that those who argue that present systems really do provide an explanation, point either to (a) Machine Learning algorithms that produce rules about data features to establish classification decisions, or point to (b) the fact that rich visualizations or text that is supplied along with a decision, offer sufficient information to draw an explanation of why a particular decision was reached by an AI. However, neither of these 'explanations' are explanations in a strict sense.[9]

On the one hand, while explanations typically answer why questions, rules about data features do not seem to answer any 'why' questions, only 'how' questions. On the other hand, 'rich visualizations' or 'text' supplied along with the output from a program is at best something from which a subject can obtain an explanation, not an explanation in itself, and at worst is not an explanation at all.

Doran et al. give three notions of explainable AI based on various corpora between different fields relating to machine learning. We will not go into detail regarding these notions of explainable AI, we will only note that at best there is a notion of explainable AI where the user of the program cannot only see, but also study and understand how inputs are mathematically mapped to outputs. But as we will see at a later point in this paper, seeing and understanding how mathematics helps map inputs to outputs is not nearly enough to be called an explanation.[10]

Since there is no agreed upon meaning of explanation in the field of machine learning, since we have seen that the best current notion of explainable AI does not come close to providing an explanation, and since we have seen that the existence of mathematical explanations is being debated, this paper will be useful in that it will shed light on what a supposedly mathematical explanation is, and whether mathematics can in fact play an explanatory role in science. And as we have seen, explanations are mostly taken to be causal. This provides a problem for explainable AI, because in machine learning it seems difficult to provide a causal explanation of the output. In this paper we will see whether causal explanations truly are all there is, and whether mathematics can only play a descriptive role in science. If mathematics can only play a descriptive role in science, this poses a problem to explainable AI, making it increasingly difficult to argue machine learning can provide true explanations, and not merely descriptions of how the output came to be.

In the following two sections we will take a closer look at how Colyvan et al.[11] use the indispensability argument to argue in favour of the existence of mathematical entities and the further implications of their argument. After that we will consider Bueno's[12] nominalist notion, and how he claims that even if mathematics turns out to provide indispensable explanations for scientific phenomena, we still ought not to have ontological commitment in mathematical entities. In the last section, arguments will be

[8] Doran, Schulz & Besold (2017) P 1
[9] Doran, Schulz & Besold (2017) P 2
[10] Doran, Schulz & Besold (2017) P 4
[11] Colyvan (2012). Baron & Colyvan (2016). Baron, Colyvan & Ripley (2017). I refer to all of these as Colyvan's arguments in favour of mathematical realism because Colyvan (2012) presents the main indispensable mathematical explanation we will be concerned with.
[12] Bueno (2012)

pitted against each other and we will come to the conclusion that we think mathematical explanations do exist, and that the counterarguments brought forward by Bueno are not convincing.

2 An example of a mathematical explanation and 'causal explanations are all there is'

Colyvan et al. are realists about mathematical entities. They believe in the existence of indispensable mathematical explanations of physical phenomena. Colyvan uses the indispensability argument to argue in favour of mathematical realism. He gives an example of an indispensable mathematical explanation and thereby claims we ought to be ontologically committed towards the content of this explanation. We will first take a look at an example given by Colyvan, of an indispensable mathematical explanation of the Kirkwood gaps, next we will take a look at the difference between process and program explanation. In the last part of this chapter we will take a look at a popular theory of explanation, and see if we can extend this theory to cover mathematical explanations.

2.1 Kirkwood gaps, an example of an indispensable mathematical explanation

Let us first look at an example of what Colyvan calls a mathematical explanation of a physical phenomenon, the Kirkwood gaps and it's eigenvalues. The Kirkwood gaps are gaps between the asteroid belts of Saturn in which there are no asteroids. According to Colyvan these gaps and their exact location can be explained 'in terms of the eigenvalue of the local region of the solar system.'[13] Colyvan notes the following:

> The basic Idea is that the system has certain resonances and as a consequence some orbits are unstable. Any object initially heading into such an orbit will be dragged off to an orbit on one side or other of its initial orbit as a result of regular close encounters with other bodies.[14]

Colyvan thus argues the explanation of these gaps is mathematical because a mathematical analysis gives us both the existence of these gaps and their location.

Colyvan also notes that we can seek a causal explanation of each individual asteroid telling us why it does not orbit in the Kirkwood gaps. However, these causal explanations do not provide the whole story. The singular causal explanations do not explain why no asteroid is able to maintain a stable orbit in the Kirkwood gaps. The explanandum is thus: No asteroids are able to maintain a stable orbit in the Krikwood gaps. And the explanans uses the mathematical eigenvalues to explain this phenomenon, and thus provides a mathematical explanation. Even though a causal explanation is available, this does not provide us with the whole story, Colyvan argues. Colyvan notes that '[t]he explanation of this important astronomical fact is provided by the mathematics of eigenvalues.'[15]

> We thus have scientific statements involving mathematical entities (the eigenvalues of the system) explaining physical phenomena (the relative absence of asteroids in the Kirkwood gaps)[16]

Colyvan calls the causal explanation a process explanation, and the higher, mathematical, explanation a program explanation. In the next section we will take a closer look program and process explanations, their differences and the different explananda program and process try to explain.

2.2 Program and process explanations

Colyvan notes the following about process and program explanations:

> A process explanation is an account of the actual causes that culminated in a particular explanandum. A program explanation, by contrast, is an explanation that appeals to some entity

[13] Colyvan (2012) P 92
[14] Colyvan (2012) P 92
[15] Colyvan (2012) P 92
[16] Colyvan (2012) P92

> or property that is not itself causally efficacious but, rather, ensures the existence of whatever it is that causes the explanandum. (…) A program explanation can explain why a particular explanandum must be the case (…) as opposed to why it is the case *de facto*.[17]

In their paper *Time enough for explanation*, Colyvan et al. note that there are two different explanations regarding the same phenomenon. Both explanations explain something different. On the one hand we have process explanations, which is a causal explanation of why a specific physical phenomenon occurs; on the other hand we have program explanations, explaining why, in some system, a specific physical phenomenon must occur.

The process explanation 'provides details of the particular causal process that led to a particular failure.'[18] So for example in the Kirkwood gaps case, the process explanation explains why, in the case of a particular asteroid, it does not orbit at the Kirkwood gaps' location. It explains why it was pulled out of the location of the Kirkwood gaps, and thus gives a causal explanation. It does not explain why it has to be the case that there are no asteroid orbiting at the location of the Kirkwood gaps.

The program explanation appeals to 'certain properties that are not causally efficacious', namely the eigenvalues explaining why there are no asteroids orbiting at the location of the Kirkwood gaps. In contrast with the process explanation, the program explanation does not provide details of a particular asteroid and why it was pulled out of the unstable orbit. Program explanations abstract away from the particular causal details. Colyvan notes that the program explanation of the Kirkwood gaps states that '[w]ere a particle to be orbiting in one of the gaps, its orbital period would be such that it would pull it out of the original orbit and into another.'[19]

Colyvan notes that 'the distinction between process explanations and program explanations appears intuitive and offers a useful way to distinguish two different kinds of explanation.'[20] He also notes that one way of elucidating the importance of program explanations, is via the notion of structural constraints. 'Roughly speaking,' Colyvan writes, 'structural constraints on a system are features that constrain the manner in which they operate.'[21] It is the structural constraint on the Kirkwood gaps system that facilitate the program explanation for why there cannot be any of asteroids in the unstable orbits.

A important difference between program and process explanations can be found in their explananda. Colyvan notes that '[w]hen considering the distinction between a program and a process explanation, there is an ambiguity in the explanandum.'[22] For instance, in the Kirkwood gaps case, there are really two explananda, 'one that is best explained by appealing to the process and one that is best explained by appealing to programming properties.'[23] These two explananda correspond to the following 'why' questions:

(A) Why is a particular asteroid, as a matter of fact, unable to orbit in the Kirkwood gaps?
(B) Why can't asteroids ever orbit in the Kirkwood gaps?

The process explanation appears to answer (A), giving us a causal explanation of why a particular asteroid is unable to orbit in the Kirkwood gaps. For example giving us the explanandum 'because it was pulled out of the orbit by physical object X.' The program explanation appears to answer (B), giving

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[17] Baron & Colyvan (2016) P 64
[18] Baron & Colyvan (2016) P 65
[19] Baron & Colyvan (2016) P 67
[20] Baron & Colyvan (2016) P 65
[21] Baron & Colyvan (2016) P 65
[22] Baron & Colyvan (2016) P 66
[23] Baron & Colyvan (2016) P 66

us a non-causal explanation of why no asteroids are able to orbit in the location of the Kirkwood gaps. Giving us an explanans based on the eigenvalues of the system. Colyvan notes the following:

> Each particle in the system will have its own peculiar causal story of how it got to be where it is and why it is not in a gap region. But the eigenanalysis guarantees that there will be very few particles in the gap region and why.[24]

Colyvan does not deny that there is a causal stories that try to explain the fact particles can only orbit in stable orbits. However, he notes that 'in the context of explaining the gaps in the rings of Saturn, these causal stories are at best only part of the story and at worst, misleading.'[25] Colyvan notes that it is misleading because, given only the causal explanations of every particle and why it does not orbit in the Kirkwood gaps, it would look like the gaps come into existence by accident. But the existence, location and width of the gaps is not an accident, it had to be the way they are. Colyvan's point is that '[t]his important modal element is absent from the causal story.'[26]

We have seen that Colyvan argues there is an indispensable mathematical explanation, and that this explanation provides something the causal explanation cannot provide, namely that the existence, location and width of the Kirkwood gaps are no coincidence.

In the last section of this chapter, we discuss the possibility of including mathematical explanations in popular theories of explanation. As we have seen in the introduction, explanations are mostly taken to be causal. Colyvan even claims that most philosophers think causal explanations are all there is.[27] The last part of this chapter makes clear come distinctions between causal and non-causal explanations, it shows us that when philosophers talk about explanations, they mostly take them to be causal, and it shows us the difficulty of having a theory of explanation cover non-causal explanation

2.3 Difference making and 'causality is all there is'

In the end of his paper *Time enough for explanation*, Colyvan arrives at what an explanation is, and if his proposed mathematical explanations fit in with theories of explanation. He notes that there is an implicit idea in the field of explanatory theories that 'causal explanation is, in some sense, where the actions is.'[28] This places Colyvan with a problem, because 'a theory of causal explanation has trouble with the [non-causal] cases we have considered here.'[29] The question now is, do causal theories of explanation cover the higher level explanations mathematics often provides**?**

Colyvan starts with explaining a popular theory of explanation called 'difference-making' which usually covers causal explanations. Difference making is to be understood via classes of counterfactual dependencies. In short difference-making states that 'X explains Y', means that 'if ¬X then ¬Y'. In other words, if X has not happened, then Y would also not have happened. Colyvan calls this a more liberal conception of causation, and he is sure he can extend this view far enough to have it cover program explanations, and thereby also mathematical explanations.[30] To extend the theory of difference making to cover these higher level explanations, Colyvan has to solve one difficulty. The problem Colyvan is faced with is the problem of impossible antecedents. While difference-making seems to hold the laws of the universe fixed, this is a difficulty for mathematical counterfactuals.

[24] Baron & Colyvan (2016) P 67
[25] Baron & Colyvan (2016) P 68
[26] Baron & Colyvan (2016) P68
[27] Baron & Colyvan (2016) p83
[28] Baron & Colyvan (2016) p83
[29] Baron & Colyvan (2016) p83
[30] Baron & Colyvan (2016) p84

2.4 Modal information

Program explanations, such as the explanation of the Kirkwood gaps using the eigenvalues, provide an important modal component.[31] As we have seen above, there is a causal story for every single asteroid explaining why it was pulled out of the Kirkwood gaps, but these causal explanations do not provide the whole story. For example, when all the causal explanations for each asteroid are taken together, it would look like the existence of the Kirkwood gaps, their location and their width are all accidental. However, it is not accidental, the Kirkwood gaps have to exist on the exact location with the exact width they have. This modal component is missed by the causal process explanation.

Colyvan notes that '[d]ifference-making accounts do deal in the modality of explanation, up to a point. Because these accounts rely on counterfactuals, explanations modelled in this way typically possess some modal force.'[32] However, Colyvan also notes that counterfactual accounts, at least as they are standardly developed, do not imbue a modal force strong enough to cover the mathematical cases considered here

The problem with mathematical counterfactual, Colyvan notes, is that, assuming mathematical truths are necessary truths, we are trying to asses a counterfactual with an impossible antecedent. This is a problem. The normal notion of difference making is focussed on causality, it holds the laws of the universe fixed, and asks the question of whether some event would have happened, had some other event not happened. Let me illustrate this with a small example: let's say one billiard ball hit another billiard ball, we might explain the movement of the second ball using the counterfactual

> If the first ball had not hid the second ball, the second ball would not have moved.

In this counterfactual, we can see that to be able to evaluate this counterfactual, we must hold the laws of the universe, in this case physical laws, fixed.

However, in the case of a mathematical explanation (assuming the mathematical truths are necessary truths) , we do not hold the laws of the universe fixed. A mathematical explanation in counterfactual terms would look something like

> If certain mathematics had not been true, a certain physical event would not have happened.

However, in the case of counterfactuals of a mathematical explanation of a physical event, we do not hold the laws of the universe fixed. In the antecedent we change these laws. Colyvan notes this might provide a problem for trying to asses counterfactuals of mathematical explanations.

2.5 Impossible antecedents

The way in which Colyvan tries to solve the problem of impossible antecedents is by allowing for difference-making of the relevant kind across both possible and impossible worlds. And as we have seen, once difference-making has been extended to include counterfactuals with impossible antecedents, it is too far removed from what we usually mean by 'causation' to count as an analysis of causal explanation. This is because we do not hold the laws of the universe fixed, and causality is typically underpinned by the actual laws of nature. The kind of cases we are considering are not constrained by the laws of nature in the right way to be able to deploy the same nomic similarities counterfactuals usually makes use of. Colyvan notes:

[31] Baron & Colyvan (2016) P 84
[32] Baron & Colyvan (2016) P 84

> The upshot, then, is this: while it may be an option to extend a difference-making conception of causal explanation to cover the high-level mathematical and logical explanations we have considered here, doing so severs the conceptual connection with causation.[33]

What Colyvan tries to show is that there are explanations which are not causal, and that therefore there are explanations, extra-mathematical ones, which have a higher modality than causality has. A correct theory of explanation needs be able to cover these cases, and therefore cannot be limited to only causal explanations. Theories of explanation need to cover higher modalities, not only causal modalities. Colyvan argues this point by extending the notion of difference-making with counterfactuals with impossible antecedents. He argues that because difference-making in a non-classical scenario does not line up with causation, the modal character of high-level explanations (which are possible in our tweaked notion of difference-making) of the kind considered here is much higher than the modal character of causation, and therefore cannot be easily subsumed under a theory of causal explanation.

In the last part of this chapter we have seen that Colyvan tries to make difference-making cover explanations with a higher modality, for example mathematical explanations. In the beginning of 2.3 we have seen that there is an implicit idea among the philosophers of explanation that causality is all there is. We have now seen that this idea is not only implicit among philosophers, but that it is also the basis of a popular theory of explanation, difference-making, and that the current notion of difference-making does not include non-causal explanations. In the next chapter we will consider some counterarguments, provided by a nominalist. And in the chapter after that, we will try to evaluate the different arguments.

[33] Baron & Colyvan (2016) P 86

3 Nominalism

In his paper *An Easy Road to Nominalism*, Bueno argues against mathematical realism. He challenges Colyvan's claim that 'there are genuine mathematical explanations of physical phenomena and that easy-road nominalists are unable to make room for them.'[34] His paper is mainly focussed on a supposedly indispensable mathematical explanation given by Colyvan, namely the mathematical explanation of the Kirkwood gaps we have seen in the previous chapter. In contrast with Colyvan, What Bueno considers to be an explanation, is strictly limited to causal explanations. He argues that the mathematical explanation given by Colyvan is not really an explanation, but that the real explanation is provided by identifying the physical interpretation of the mathematical formalism, the mathematics only describes the phenomenon. On top of that, he challenges the mathematical indispensability argument, arguing that the fact that mathematics is indispensable is not enough to claim ontological commitment towards the content of the mathematics.

Bueno states that an easy road approach to mathematics can be articulated and that an easy road approach questions the possibility of genuine mathematical explanations.

Colyvan challenges easy road nominalism by arguing in favour of extra-mathematical explanations Bueno claims there are two moves the nominalist is able to make to refute the challenge against easy road nominalism: The first is to deny that mathematical explanations are genuine explanations; Bueno has set up four criteria for genuine explanations, and will show that mathematical explanations do not live up to these criteria. The second being that even if there were to be alleged mathematical explanations that do meet these four criteria, 'we would not be justified in assigning any ontological significance to such mathematical explanations.'[35]

3.1 The four criteria

What is needed to show mathematics play an indispensable role in science are genuine mathematical explanations. 'The use of mathematics (…) needs to be ultimately responsible for the explanation in question.'[36] For this to be the case, a number of requirements need to be met:

- *Indispensability*: Mathematics needs to play an indispensable role in the explanation.
- *Explanation versus description*: Mathematics needs to not only *describe* the phenomena, but also *explain* them.
- *Understanding*: Mathematics should offer understanding of the phenomena concerned.
- *Epistemic significance*: mathematics involved in a genuine explanation may receive epistemic significance.[37]

A main concern Bueno has is that extra-mathematical explanations may turn out to be descriptions of the phenomena instead. On top of that, Bueno does not think an extra-mathematical explanation adds to the understanding of the phenomena, a point elaborated on in a later part of his paper.

Let us now see how Bueno argues in favour of easy road nominalism.

3.2 Mathematics and physical interpretations

[34] Bueno (2012) p 967
[35] Bueno (2012) p 971
[36] Bueno (2012) P 968
[37] Bueno (2012) P 968-969

Bueno begins with arguing that mathematics does not explain a physical phenomenon, but only describes it, and that the actual work in these cases are done by 'identifying and defending the relevant physical interpretations.'[38] Bueno claims we often think mathematics does the explaining, because the relevant physical interpretations, that which does the actual explaining, 'are often not carefully distinguished from the mathematical formalism.'[39] The explanation comes from an adequate physical interpretation, 'which identifies the relevant physical processes responsible for the production of the relevant phenomenon.'[40]

Bueno gives a few examples, we will take a closer look at two of them to clarify how Bueno argues in favour of this claim.

First he sketches a scenario in which a rock is thrown, and where we have a mathematical equation that describes the stone's motion. The mathematical equation would be an equation in which we enter a time, and get an number which represents the upward or downward force of the stone. He asks whether the fact that the equation equals zero explains why the stone is at rest, and answers that it surely does not, it only provides a mathematical description of the fact that the stone is at rest. The fact that an equation has value zero is by itself not an explanation of a physical phenomenon, the number zero has no explanatory value towards any physical phenomenon, it is just a number resulting from an equation. What Bueno says is needed to get an explanation from the value which results from an equation, is a physical interpretation of this value. The number zero resulting from the equation with a certain time has to be interpreted as the fact that the stone has no upward or downward kinetical energy, so at this exact moment the stone does not move. Nothing about the value zero is about the physical phenomenon the equation describes. The mathematical equation does not provide an explanation, what is required to provide an acceptable explanation is '[a]n adequate physical interpretation, which identifies the relevant physical processes responsible for the production of the relevant phenomenon.'[41]

Second, In the case of the Kirkwood gaps, he asks whether the eigenvalues plays an explanatory role. Bueno states that surely it does not, the explanatory work is done by determining a suitable physical event, which in this case is the gravitational force exerted by Jupiter. Mathematics only provides a mathematical description. The eigenvalues do not explain the system's behaviour, '[r]ather such values emerge from the particular physical interactions among the objects that characterize the system, as long as the mathematics used to describe the system is interpreted in a suitable way.'[42]

We have seen that Bueno states that mathematical expressions by themselves are not about physical phenomena, the mathematical formalism needs to be interpreted before it can become relevant to the description of a certain physical phenomenon. [43] Only the physical interpretations of a mathematical equations states something about the world.

3.3 Indispensability and why it does not matter

After having made clear that mathematics does not explain physical phenomena, but only describes them, and that the actual work in these cases is done by 'identifying and defending the relevant physical interpretations.'[44] Bueno goes on to examine whether 'the use of mathematics in the

[38] Bueno (2012) P 967
[39] Bueno (2012) P 967
[40] Bueno (2012) P 972
[41] Bueno (2012) P 972
[42] Bueno (2012) P 973
[43] Bueno (2012) P 973
[44] Bueno (2012) P 967

explanation of the Kirkwood gaps [exemplifies] the four requirements on the mathematical explanation discussed above'[45]

- *Indispensability:* Bueno notes that proving indispensability is a difficult task to accomplish. What needs to be shown is that 'without the use of the particular sort of mathematics that was in fact invoked', no explanation could be obtained.

He does not claim, however, that no such thing is possible or has not been shown. What he does claim is that the claim about the indispensability of mathematics is not one to be worried about for the nominalist, for even if mathematics is shown to be indispensable, it does not justify ontological commitment in mathematical entities. Three reasons are noted for this claim.

First, 'the quantifiers used in the relevant explanation need not be ontologically committing'[46]. Bueno claims the fact that we make use of indispensable mathematics does not provide ontological commitment towards the content of the mathematics used. He thus claims that the mathematical indispensability argument is not a valid argument. To argue for this claim he notes the two distinct roles of the existential quantifier, first that some part of the domain is quantified over, and second the indication of existence of some object in the domain. According to Bueno 'Once this point is recognized, the possibility emerges of quantifying over mathematical objects, (…) without thereby being ontologically committed to their existence.'[47] Bueno notes one example in which we quantify over some object – we say there is some object – without ontological commitment towards this object. The example sentence is 'there is a fictional detective who does not exist', which we can logically formulated as $\exists x(Fx \& \neg Ex)$ in which F means 'is fictional' and E means 'does exist'. Bueno notes that we can clearly see we can state that there is an object x which does not exist, and thus the use of some entity does not imply ontological commitment towards this entity. Because we have seen that using the existential quantifier does not have to result in ontological commitment, 'the nominalist is free to quantify over mathematical objects and to refer to them in an explanatory context while denying that such quantification is ontologically committing.'[48] Even if there are genuine mathematical explanations of physical phenomena, this is not enough to claim ontological commitment.

The second argument is that physical interpretations of mathematical formalism is what is ultimately responsible for the explanatory work. One can allow mathematics to be indispensable for the description of the relevant physical objects, but the explanation of a phenomena 'emerges from the identification of the processes that produce the relevant phenomena'.[49] The mathematical formalisms can be descriptively useful or even descriptively indispensable, but it is the identification of the relevant physical process that explains why the phenomena occurs. The fact that mathematics may be indispensable in a descriptive way does not imply we ought to be realists about mathematics.

The last argument is that 'explanations can be taken as pragmatic rather than epistemic features.'[50] Mathematical explanations may be valued because they provide useful understanding, and even though mathematics maybe a useful way to explain a phenomenon, it does not have to be taken as true.

3.4 The other three criteria

[45] Bueno (2012) P 975
[46] Bueno (2012) P 976
[47] Bueno (2012) P 976
[48] Bueno (2012) P 976
[49] Bueno (2012) P 977
[50] Bueno (2012) P 977

Bueno continues with the other three criteria of mathematical explanation:

- *Explanation versus description:* As we have seen, Bueno has elaborately stated that mathematics does not explain a physical phenomenon, but only describes it, and that the actual work in these cases is done by 'identifying and defending the relevant physical interpretations.'[51] Thus, in the case of the Kirkwood gaps, it is not clear that the mathematics plays an explanatory role instead of merely a descriptive role.
  Bueno also notes that since it is a physical phenomenon, 'it is reasonable to expect that the relevant considerations that explain its emergence be of a physical nature'**.** And that 'clearly the mathematics lacks the appropriate ontological import to bear such a burden.'[52] Mathematics alone cannot deliver an explanation of a physical fact.
- *Understanding:* Mathematics does not provide understanding. The understanding emerges from elsewhere, while mathematics allows the expression of certain relations among the objects in question. As we have seen, mathematics needs to be interpreted to allow for understanding.
- *Epistemic significance:* Since only the entities doing the explaining work, and not mathematics but suitable physical descriptions of relevant phenomena do the explaining work, what should 'receive epistemic significance is not the mathematics, but the proper specification of the relevant physical process.'[53]

However, Bueno notes that even if all four conditions for mathematical explanations were met by a particular explanation, this needs not imply ontological commitment to the mathematical entities. '[T]here are perfectly good explanations (including scientific ones) that are not true, nor are they based on true scientific theories.'[54]

In this chapter we have seen Bueno's arguments to argue against mathematical realism. In the next chapter we will evaluate these arguments, and see whether we find them convincing.

[51] Bueno (2012) P 967
[52] Bueno (2012) P 978
[53] Bueno (2012) P 979
[54] Bueno (2012) P 979

4 Discussion

In this chapter we will take a look at some of Bueno's arguments, evaluate them and see whether we think they are convincing or not. Let us start with shortly taking a broader look at Bueno's standpoint.

Bueno is a nominalist, he does not believe we ought to be realists about mathematical entities. Bueno denies ontological commitment towards mathematical entities by denying that the Kirkwood gaps explanation Colyvan gives is an indispensable mathematical explanation of a physical fact. However, on top of making use of the mathematical indispensability argument and denying it's second premise – that indispensable mathematical explanations exist in science – Bueno also denies the truth of the indispensability argument for mathematics by itself. As we have seen, he notes that having an indispensable mathematical explanation is not enough to have ontological commitment towards mathematical entities. In this chapter we will see whether we find his argumentation convincing, and thus whether we think we ought to have ontological commitment towards mathematical entities. This chapter consists of mainly two parts. First we will have a look at what Bueno says about the indispensability of mathematics in scientific explanations. Last we will take a look at Bueno's argument that mathematics does not explain, but merely describes.

4.1 Indispensability

In his paper *An Easy Road to Nominalism*, Bueno starts with giving four criteria which are necessary to be met by a genuine and indispensable mathematical explanation. These four criteria are: Indispensability, explanation versus description, understanding and epistemic significance. Let us evaluate what Bueno says about indispensability.

Bueno claims that the nominalist need not be worried about the indispensability of mathematics used in explanations of a physical phenomenon, even if the mathematics is indispensable, this does not justify ontological commitment. Bueno gives three reasons for the claim, let us evaluate the strength of these three arguments:

- First, Bueno claims the use of the existential quantifier by itself does not imply we ought to have ontological commitment towards the entity quantified over. Something else is needed. But what that something else is, Bueno does not specify. Bueno gives an example of an entity over which we use the existential quantifier, but about which we do not have ontological commitment. In the sentence 'there is a fictional detective who does not exist' we quantify over a detective, without having existential commitment towards this detective.
  I do not think this claim is a very strong one. Firstly, Bueno implies something else is needed to have ontological commitment towards some entity than only the use of the existential quantifier. However, he does not specify what this something is.
  On top of that I believe the example of a fictional detective to not be very convincing. First, 'fictive' already implies that we have no ontological commitment toward this entity, why would this also be the case with 'mathematical'? When we compare a 'fictive explanation' and a 'mathematical explanation' we automatically know, without it being specified or explained, we do not have to hold ontological commitment towards the fictive explanation. For mathematics however, I do not think we intuitively know we do not need to have ontological commitment towards the explanation. Bueno however, claims we do not have to be ontologically committed towards mathematical entities when we make use of a mathematical explanation, without further specification.
  On top of that, one would not look towards fictive entities to have some speaking power about the world, but we do approach mathematical entities in a way that implies they have at least some speaking power about the physical world.

- The second point Bueno introduces is that mathematics might have some indispensable descriptive power, but the explanation of a phenomena 'emerges from the identification of the processes that produce the relevant phenomena'.[55]
  As we have seen in the introduction, Platonists will agree it is not enough for mathematics to be descriptively indispensable. Baker notes that both sides of the argument agree that mathematics needs to be indispensable in the right kind of way. Mathematics needs to be indispensable in an explanatory sense because then 'it restricts attention to cases where we can posit the existence of a given entity by inference to the best explanation.'[56] Since we determine what theories are thought of as scientific by using inference to the best explanation, and since we ought to be realists about our best scientific theories, we can claim ontological commitment towards the content of the explanation of our best scientific theories.
  So Platonists agree it is not enough for mathematics to be indispensable in a descriptive way, but Platonists and nominalists disagree whether mathematics is in fact indispensable in an explanatory way. At a later point in this chapter we will look at the claim that the explanation emerges from the identification of the correct physical process.
- The last argument Bueno gives to argue that even when we make use of indispensable mathematical explanations in science, we do not need to have ontological commitment towards the mathematics used in the explanation, is that 'explanations can be taken as pragmatic rather than epistemic features.'[57] On this point, Bueno fundamentally disagrees with Platonists about what an explanation is. Bueno holds that explanations need not be true, and can provide an understanding of the world without being true. Platonists will disagree and say a criteria of an explanations is that we have believe they provide a truth about the world, explanations explain in what way the world works and do this on a factive basis. An explanation that does not cover the data correctly, will not be taken as a good explanation. I tend to side with the Platonists. The difference between how Platonists approach explanations and how Bueno approaches explanations becomes perfectly clear when Bueno says 'there are perfectly good explanations (including scientific ones) that are not true, nor are they based on true scientific theories.'[58]

These are the three arguments used to argue against the existence of mathematical entities based on the fact that when we have an indispensable mathematical explanation, this does not imply ontological commitment to the content of this explanation. As we have seen, I do not believe these arguments suffice to argue against the validity of the mathematical indispensability argument.

4.2 Mathematics is an explanation versus a description

Let us next take a look at Bueno's argument that mathematics does not explain but only describes physical phenomena, and that the explanation comes from the correct physical interpretation of the mathematical formalism. About the Kirkwood gaps explanation Bueno notes that it is unclear the mathematics provides an explanation of the physical phenomenon. Let us have a look at an example in which Bueno notes the mathematics only describes.

Bueno gives the following example. We have a physical phenomenon of a stone thrown into the air, and a mathematical equation that describes the stone's motion. At a certain point in time, the equation equals zero, and the stone has no motion, it hangs in the air. Bueno then asks '[d]oes the fact that the equation has such a value explain why the stone is at rest, or does is merely provide a mathematical

[55] Bueno (2012) P 977
[56] Baker (2009) p 613
[57] Bueno (2012) P 977
[58] Bueno (2012) P 979

description of the relevant phenomenon?'[59] Let us break down this case. First, Bueno notes that he takes a physical phenomenon and a mathematical equation that describes the phenomenon. No wonder the mathematics in this case provides a description, and might not provide an explanation. However, I do think Bueno has a point that the simple fact that an equation equals some value, does not provide an explanation. Nothing about the number zero refers to the fact the stone is at rest, the mathematics – the fact that some equation equals zero – needs to be interpreted to obtain an explanation. This argument might also be applicable to the Kirkwood gaps case, but I do not believe it applies to all (supposedly) mathematical explanations. I will give a mathematical explanation, and I will argue the mathematics in this case is not fallible to this critique of Bueno.

4.3 Cicada's

Cicada's have a long life under ground, living as ant-like creatures. After many years, the cicada's begin their brief adult life as winged creatures, having only a couple of weeks to mate. There are two north American species of cicadas who have a life cycle of 13 and 17 years. Biologist have sought an explanation of why the life cycles are exactly this length, and not for example a length of 14 or 16 years. Colyvan et al. give an explanation in which mathematics plays an indispensable role. Their explanation consists of four components:

(i) Ecological constraints on the life cycle of cicada's that restrict their life cycle within the range of 12 to 18 years;
(ii) The assumed presence of predators, who themselves have periodical life cycles;
(iii) Facts about the numbers 13 and 17, most notably that they are both prime numbers; and
(iv) The mathematical facts about primes, namely that they have the fewest common multiples.[60]

Lifecycles that are prime numbered, reduce the chance of overlapping with lifecycles of predators (or competitors). Because a cicada species has a life cycle of 13 years, it will only overlap with predators who's life cycle is 1 or 13 years, and in the case of 13 years, it will only overlap with lifecycles that are synchronised with the cicada's. Colyvan states that '[t]he model tells us that the optimal way for an organism with periodical life cycle to avoid predators with periodic life cycles is for that organism to possess a prime-numbered life cycle'[61] The explanation of cicada's life cycle makes use of a mathematical element, the prime numbers 13 and 17, and the fact that prime numbers in general have the least common multiples with other numbers. The mathematics indispensable for this explanation is the fact that primes have the fewest common multiples.

I believe this explanation is a mathematical explanation of a physical or biological fact. Colyvan surely agrees, but Bueno would try to argue otherwise. I think Bueno would respond with the claim that the mathematics does not play a role in the explanation, the correct physical interpretation of the mathematical formalism is what provides the explanation. It is thus not the facts about the number 13 and 17 that provide an explanation, but the explanation comes from the entity these numbers refer to.

I would argue this is not true. The mathematics indispensable in this explanation is the property of primes, namely the fact that primes have the fewest common multiples with any other number (that is equal or lower then themselves). The mathematics in this explanation does not refer to a physical object or a physical process. Bueno might claim the mathematics, that is the numbers 13 and 17, refers

[59] Bueno (2012) P 971
[60] Baron, Colyvan & Ripley (2017) P 6
[61] Baron, Colyvan & Ripley (2017) P 6

to the physical/biological entities of 13 and 17 years, and thus a physical/biological interpretation provides the explanation. However, even if these numbers refer to years, the explanation remains a mathematical one. As said, the explanation comes from the mathematical fact about primes, namely that they have the fewest common multiples with other numbers. This mathematical fact remains when instead of taking a prime number, we take a prime length of years. In this case the explanation comes from the mathematical fact about a prime length of years, namely that they have the fewest common multiples with other length of years. To reiterate, The mathematical explanation does not come from the number 13 and 17 themselves, but it comes from the mathematical fact about primes. I do not think this mathematical fact about primes can physically interpreted.

4.4 What makes the cicada explanation not fallible to Bueno's argument?

We have given a mathematical explanation that does not seem fallible to Bueno's claim that the mathematical formalism does not explain, but that the physical interpretation does. Because of this explanation we can refute Bueno's argument. The question arises about what exactly makes the cicada explanation not fallible to Bueno's argument. It looks like this explanation does not seem to be a program explanation, but instead looks like a process explanation. However I do not think the question of whether explanations are program or process explanation has anything to do with it. Intuition tells me the difference is made in the fact that this explanation does not make use of an equation or numbers. When a supposedly mathematical explanation makes use of numbers or equations, nothing about the number itself refers to the phenomenon it is trying to explain. *We* have to refer these numbers to the physical phenomenon.

We have for example seen this with the equation Bueno gives to describe the motion of thrown stone. The number produced by the equation is what is supposed to do the explaining work, but the result of an equation in itself does not refer to a specific physical phenomenon, it is just a number placed behind some values and an '=' sign. In the case where an equation describes the motion of a rock, the equation produces some number, but this number by itself does not mean anything, the number by itself does not hold any relation to the phenomenon of the thrown rock. The number always has to be interpreted.

However in the case of the cicada explanation, we do not make use of an equation, or a number referring to some physical phenomenon in any other way. Instead the explanation is provided by a mathematical fact about primes. The use of the number 13 and 17 itself does not provide the explanation, the explanation is provided by the fact that primes have the fewest common multiples. We do not even have to refer to prime numbers, we can make use of a prime length of years, the mathematical fact about primes remains the same. We can thus just as well say that the indispensable mathematical part to this explanation is the mathematical fact about prime lengths of years, namely that they have the fewest common multiples with other lengths of years.

Another question which arises whether Bueno's argument holds against all program explanations, or whether it is a coincidence that the mathematical explanation we have given to counter Bueno's argumentation is a process explanation. Bueno's argument was not only aimed at program explanations, but at all mathematical explanations. We have no reason to believe Bueno's arguments does hold against all program explanations, but elaboration of this argumentation line must be subject to follow-up research and extends the limits of this paper.

In this chapter we have seen that we do not think Bueno's arguments suffice to support the claim that no matter whether a mathematical explanation is indispensable, we cannot claim ontological commitment towards mathematical entities. In the latter part of this chapter we have seen that not all mathematical explanations are fallible to Bueno's claim that mathematics only describes, and that a physical interpretation of the mathematical formalism is what provides an explanation. In the next chapter we will take a look back at the arguments provided in this paper, and we will take a broader look at the importance of the conclusion we reached.

5 Conclusion

In this paper we have considered whether mathematics plays an explanatory role in science. Explanations generally tell us something about true causes, and are therefore mostly taken to be causal. This poses a problem for the Platonist. To argue in favour of mathematical realism one can make use of the mathematical indispensability argument, and argue that because we have indispensable mathematical explanations, we ought to be ontologically committed towards the content of the mathematics involved. We have seen one such argument, Colyvan et al. argue in favour of mathematical realism by providing the Kirkwood gaps explanation. We have also seen that causal explanations of this phenomenon are possible, in the form of a process explanation, but that Colyvan argues this doesn't provide the full story. I agree with this point, because when only the causal explanations of every single particle and why it does not orbit in the Kirkwood gaps, the existence, location and width of the Kirkwood gaps seems to be an accident. However, the existence, location and width are not accidents, they had to be the way they are.

On the other hand, we have considered Bueno's counterarguments, arguing against mathematical realism, making use of broadly two claims. First, Bueno claims mathematics does not explain, but only describes physical phenomena, he argues the explanation comes from a physical interpretation of the mathematical formalism. And second, even if mathematics is indispensable, Bueno argues, this does not imply we ought to have ontological commitment towards the mathematics.

As we have seen in the last chapter, we do not think Bueno's arguments are convincing. We have argued the arguments used in favour of the argument 'even if mathematics is indispensable, this does not imply ontological commitment towards the mathematics' are not satisfying. And we have argued that there is a mathematical explanation in which the explanation does not come from a physical interpretation, but from a mathematical fact about primes. Therefor we conclude that at least one mathematical explanations exist, and therefor that mathematics does play an indispensable explanatory role in science.

The fact that we have concluded that non-causal explanations are possible, and that we have seen at least one explicit example of an indispensable mathematical explanation in science, offers hope for the field of explainable AI. In the introduction we noted that there are several difficulties for explainable AI, for example the fact that there is no agreed upon meaning of explanation in the field of machine learning, that the existence of mathematical explanations is being debated, and even the fact that the existence of non-causal explanations was not an agreed upon fact by philosophers. Because we have come to the conclusion that mathematical explanations do exists, we can confront some of these difficulties. As noted the existence of non-causal explanations offers hope that explanations in the field of explainable AI are possible, and since explanations in the field of explainable AI are often used in some quasi-mathematical way, now that we have established that mathematical explanations exist, the possibility of explainable AI becomes more and more likely.

However, we are far from concluding that actual explainable AI exists at this point in time, and we are not definitely not certain that it ever will. A lot of questions are still open; for example, how mathematical are explanations provided by machine learning?; can we provide an indispensable algorithmic explanation?; to what extend do explanations in the field of explainable AI only describe, and to what extend are explanations in the field of explainable AI fallible to Bueno's critique that mathematics only describes? The fact that we came to the conclusion that this argumentation by Bueno does not cover all indispensable mathematical explanations does not necessarily mean it is not a good argument against explanations provided by machine learning. On top of that, there is another difference between explanations provided by AI and mathematical explanations: mathematical explanations provide explanations about physical phenomenon, or phenomena in other sciences, while explainable AI only provides explanations within the field of machine learning itself. Further research must determine whether the arguments used to argue in favour of indispensable mathematical explanations can also be used to argue explanations in machine learning exist.

On top of that there are other questions left unanswered, and this paper definitely does not cover all current material on the topic of mathematical explanations and mathematical realism. Because of the time limit we were only able to look at a select number of papers and arguments. Further research can for example include a more detailed view on Bueno's arguments, include different writings from Bueno, and include argument brought forward by other nominalists. Further research must also determine how common indispensable mathematical explanations are. In this paper we have only seen one example that is definitely not fallible to Bueno's argument that mathematics only describes, and therefore we can, at most, claim ontological commitment towards the content of this mathematical explanations.

At last we come to theory of explanation. In this paper we have seen one popular theory of explanation, difference-making, and we have seen that this theory covers only causal explanations. We have seen that most philosophers think causality is all there is. But because we came to the conclusion that indispensable mathematical explanations do exist, theories of explanation must include non-causal explanations. There needs to be a major shift in the field of theories of explanation to include non-causal explanations.

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