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Black holes and supersymmetry breaking in type IIA string theory

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Abstract

In this thesis, there is a study of four-dimensional black holes in type IIA string theory. The black holes are studied via a dimensional reduction from ten to five-dimensions through Kaluza-Klein reduction and from five to four-dimensions via a Scherk-Schwarz reduction. This reduction can partially break the initial $\mathcal{N} = 8$ supersymmetry to $\mathcal{N} = 6$, 4, 2, and 0. Specifically, there is a study of the vector fields supporting four distinct black holes and an analysis of which black holes survive which Scherk-Schwarz reduction.

Contents

List of Tables

Chapter 1

Introduction

A black hole is formed when a star with large enough mass reaches the end of its lifetime. This is called a progenitor star. To specify, if the mass of the star is bigger than sixteen solar masses, it will become an unstable neutron star. This unstable neutron star will collapse immediately to a black hole. But, even if the mass of the progenitor star is less than sixteen solar masses, it can still end up collapsing to a black hole, even though it will first become a white dwarf or a stable neutron star. For example, if the mass of the progenitor star is less than sixteen but more than eight solar masses, it will collapse to a stable neutron star, which can become unstable after gaining mass (Tolman-Oppenheimer-Volkoff limit [41], [53], [54]). If the mass of the progenitor star is less than eight solar masses, it will collapse to a white dwarf, which in turn may become a stable neutron star, after gaining more than 1.44 solar mass (Chandrasekhar limit [11], [38]). More information can be found in [45], [46].

A theory of quantum gravity has to include black holes, thus black holes can be studied through string theory. In this thesis, black holes will be described using branes, which are extended objects parallel to a number of spatial dimensions. The superstring theory perspective is consistent in a ten-dimensional spacetime, hence several steps are needed to view these ten-dimensional black holes from the perspective of our four-dimensional world. In addition to that, the second aim of this thesis is to partially break supersymmetry from $\mathcal{N} = 8$ to $\mathcal{N} = 6, 4, 2$ and 0.

Only extremal BPS black holes will be studied in this thesis. An extremal electrically charged black hole has the smallest possible mass compatible with its charge. In supersymmetry algebras when the bound between mass and charge is saturated then BPS states are obtained. The entropy of extremal or near-extremal black holes has been studied extensively and some results have even been confirmed microscopically [8], [23], [35], [52].

To compactify ten-dimensional black holes, the branes that are used to describe them will be wrapped around an object on the directions of compactification. There are many ways and many manifolds on which this can be done, but for this thesis, the focus will be on a Kaluza-Klein reduction on a five-torus and a Scherk-Schwarz reduction on a circle. A five-torus is a product of five circle reductions.

To perform the reduction from five- to four-dimensions, a mass matrix will be inserted in the theory. The parameters of said matrix are $m_i, i = 1, 2, 3, 4$, which will become mass parameters in four-dimensions. After the Scherk-Schwarz compactification, some of the vector fields supporting the four-dimensional black hole may gain mass. However, the fields supporting a specific black hole should remain massless after supersymmetry breaking for the black hole to survive the twist. It is of particular interest to see what happens to type IIA black holes compactified from ten to four-dimensions in $\mathcal{N} = 8, 6, 4, 2$, and 0.

There is a lot of separate literature regarding Scherk-Schwarz reduction and black holes. However, not many papers combine both ideas. This is the goal of this thesis. An article studying black holes in type IIB string theory compactified to five-dimensions was recently written [25]. One can also read more on type IIB black holes in the thesis [51].

1.1 Outline

In chapter 2, there is a general introduction of string and superstring theory. There is a discussion of the action of supergravity in ten-dimensions in both Einstein and string frame. An introduction for D-branes and p-branes, is given. Branes are essential in constructing black holes.

Chapter 3 introduces the compactification theories of Kaluza-Klein and Scherk-Schwarz. In addition to that, there is a discussion of dualities between different D-branes.

In chapter 4 there is a general introduction of supersymmetry algebra and multiplets in supergravity.

Chapter 5 depicts how to reduce the field spectrum of type IIA string theory from ten-dimensions on a five-torus and the massless five-dimensional spectrum acquired. There is also a study on the massless and massive multiplets produced from the reduction of five-dimensions to four-dimensions with a Scherk-Schwarz twist. A tool that will prove to be vital when studying the field strengths of black holes is the Hodge duality discussed in section 5.1.

In chapter 6 there is a review of the Schwarzschild and Reissner-Nordström black holes. Four additional black holes are shown: $D_{4a}/D_{4b}/D_{4c}/D_0$, $D_{4a}/D_{4b}/D_{2a}/D_{2b}$, $D_6/D_{2a}/D_{2b}/D_{2c}$, and $D_6/NS_5/D_2/W$, and their corresponding compactification. These reduced black holes are written in both string and Einstein frame. There is a short summary of black hole thermodynamics and a calculation of the macroscopic entropy of all four black holes. There is also a summary of near-horizon geometry of black holes followed by an example.

In chapter 7, there is an analysis on which types of supergravities the previous black holes survive after a Scherk-Schwartz twist from five-dimensions to four-dimensions in $\mathcal{N} = 8, 6, 4, 2$ and 0.

1.2 Conventions

The units used, are set such that $c = k_B = \hbar = 1$ and the ten-dimensional Newton constant is $8\pi G_N^{(10)} = \kappa_{10}^2$. The D-dimensional Newton's constant is defined in terms of the Newton constant in ten-dimensions and the volume of the compactified manifold: $(2\pi)^{10-D} G_N^{(D)} V_{10-D} = G_N^{(10)}$.

The signature for the Minkowski metric is $\eta_{\mu\nu} = \text{diag}(-1, 1, .., 1)$ and $\epsilon_{\mu_1..\mu_n}$ is the Levi-Civita symbol and $\bar{\epsilon}_{\mu_1..\mu_n}$ the Levi-Civita tensor, related by:

$$\bar{\epsilon}_{\mu_1..\mu_n} = \sqrt{|g|}\epsilon_{\mu_1..\mu_n}. \tag{1.1}$$

The Hodge star operator acting on an $(n-p)$-form F:

$$(*F)_{\mu_1..\mu_{n-p}} = \frac{1}{p!}\bar{\epsilon}^{\nu_1..\nu_p}{}_{\mu_1..\mu_{n-p}} F_{\nu_1..\nu_p}. \tag{1.2}$$

The Hodge duality will be discussed in detail in section 5.1.

Chapter 2

String theory

Originally, string theory aimed to explain hadrons and their interactions, but it was later replaced by quantum chromodynamics as a theory for strong interactions. String theory faced some difficulties in its aspiration of being a theory of hadrons, such as containing a massless particle with spin two, which is inconsistent with the hadronic world. This problem ended up giving new perspectives to string theory as a popular candidate for quantum gravity after Scherk and Schwarz suggested identifying this mysterious particle as the graviton. This particle obeys the laws of general relativity at low energies. So far, on-going research in string theory has provided us with remarkable answers for some problems, but others remain unsolved.

Matter according to string theory consists of tiny loops of string instead of point-particles. A string is a one-dimensional object. It propagates through space and time. Polchinski later shed light on D-branes, topological defects on which open strings can end [43], [44]. They are non-perturbative objects in the weakly coupled string theories. Every object in a string theory description is a composition of strings and branes. There are two types of strings, open and closed and their main difference is that the former has two-endpoints. Any open string theory contains both open and closed strings, and the particle identified as graviton can be found in the spectrum of closed strings, thus it is inevitably in all string theories. There are also other massless fields that appear in string spectra, such as the dilaton ϕ, a scalar of particular interest for this thesis. There are five types of string theories in ten-dimensions, which are related to each other through dualities. Some great books and lecture notes on string theory are found in [7], [29], [43], [44], and [55].

The main focus in section 2.1 is the bosonic string theory and in 2.2 the superstring theory. The principal theory examined in this thesis is type IIA string theory which will be introduced in section 2.2 and in 2.2.1 there is a discussion of its action from string to Einstein frame. In section 2.3 one can find a detailed description of branes and specifically the relation between p-branes from supergravity and D-branes from string theory. These branes will later be needed to construct black holes.

2.1 Bosonic string theory

Bosonic string theory was developed in the late 1960s and it was the first version of string theory. The bosonic string theory only includes bosons and it is physically consistent only in twenty-six dimensions. The bosonic string theory falls short in its consistency with the physical world because it does not include fermions and it predicts tachyons, particles with imaginary mass. On the contrary, superstring theory which includes both fermions and bosons does not include tachyons.

The embedding of a string in spacetime is described by the worldsheet. The worldsheet is a two-dimensional manifold parameterized by a timelike and a spacelike coordinate, τ, and σ, respectively. The worldsheet coordinates are $\sigma^\alpha = (\tau, \sigma),\ a = 0, 1$ and the surface sweeped out by the string, defines a map $X^\mu(\sigma, \tau),\ \mu = 0, .., D-1$ from the worldsheet to Minkowski spacetime. In the closed string description, σ is periodic and it takes values from zero to 2π and $X^\mu(\sigma, \tau) = X^\mu(\sigma + 2\pi, \tau)$. The spatial coordinate σ of an open string takes values from zero to π. The string tension is:

$$T = \frac{1}{2\pi l_s^2}, \tag{2.1}$$

where l_s is called the characteristic string length scale and $\alpha' = l_s^2$.

A popular form of the string action is the Polyakov action. Assume strings propagate in a flat background, then the target manifold metric will just be the Minkowski spacetime metric, $\eta_{\mu\nu}$. The Polyakov action, in this case, is:

$$S = -\frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{-g} g^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu}, \tag{2.2}$$

where g is the determinant of the worldsheet metric. The symmetries of the Polyakov action are Poincaré invariance, reparameterization invariance (diffeomorphisms) and Weyl invariance. A Weyl transformation is a local metric rescaling.

2.1.1 Boundary conditions

Boundary conditions explain how the endpoints of open strings move. There are Neumann and Dirichlet boundary conditions. The Neumann boundary condition allows the endpoints of an open string to move freely by imposing:

$$\partial_\sigma X^\mu = 0 \text{ at } \sigma = 0, \pi. \tag{2.3}$$

In contrast, the Dirichlet boundary condition keeps the end of the string at a fixed position in space using the condition:

$$X^\mu = c^\mu, \tag{2.4}$$

which can also be expressed as:

$$\delta X^\mu = 0 \text{ at } \sigma = 0, \pi. \tag{2.5}$$

2.2 Superstring theory

Superstring theory in ten-dimensions includes both bosons and fermions. Different versions of superstring theory emerge from M-theory in eleven-dimensions [55]. Even though the bosonic string theory is unique, there are five perturbative superstring theories. Type II has both left- and right-moving worldsheet fermions. Heterotic theories have superstrings with only right-moving fermions. Type I has both closed strings with left and right-moving fermions and open strings with Neumann boundary conditions [46], [55].

Fermions can either have periodic or antiperiodic boundary conditions. The periodic boundary condition corresponds to the sector R and the anti-periodic, to the sector NS. After combining two open strings into a closed string, one can obtain one of the following combinations: R-R, NS-R, R-NS, NS-NS. These periodic conditions are chosen independently for left- and right-moving superstrings.

The cases which will be more of interest in this thesis are the type II theories. Type IIA is non-chiral and type IIB is chiral. The massless bosonic fields are included in the NS-NS sector (Neveu-Schwarz) and the R-R sector (Ramond). Additionally, the sectors R-NS and NS-R contain fermions.

The NS-NS sector contains a graviton G (also written as $G_{\mu\nu}$), a 2-form field called Kalb-Ramond B_2 (also written as $B_{\mu\nu}$) and a scalar field ϕ, the dilaton. The R-R sector is different for the two type II theories. Type IIA has a 1-form potential, C_1 (C_μ) and a 3-form C_3 ($C_{\mu\nu\rho}$) . Type IIB has a scalar C_0, a 2-form potential C_2 ($C_{\mu\nu}$) and a 4-form C_4 ($C_{\mu\nu\rho\sigma}$) [55].

2.2.1 String and Einstein frame

In this thesis different configurations of black holes will be studied. These black holes should be able to be expressed in both string and Einstein frame. The same is true for type IIA string theory. The action for type IIA in ten-dimensions, in string frame is [7], [13]:

$$\begin{aligned} S_{IIA} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g^{(s)}}[e^{-2\phi}(R^{(s)} + 4|d\phi|^2 - \frac{1}{2}\frac{1}{3!}|H_3|^2) \\ -\frac{1}{2}\frac{1}{2!}|F_2|^2 - \frac{1}{2}\frac{1}{4!}|\tilde{F}_4|^2] + \frac{1}{4\kappa_{10}^2}\int dC_3 \wedge dC_3 \wedge B_2, \end{aligned} \tag{2.6}$$

where $F_2 = dC_1$, $\tilde{F}_4 = dC_3 - dB_2 \wedge C_1$ and $H_3 = dB_2$. The action in the Einstein frame in ten-dimensions can be obtained by substituting the metric, the transformed Ricci scalar and the p-forms into the string frame (2.6).

The metric for the string frame will be denoted by $g^{(s)}_{\mu\nu}$ and the metric for the Einstein frame by $g^{(E)}_{\mu\nu}$. The relation between them is given by:

$$g^{(s)}_{\mu\nu} = g^{(E)}_{\mu\nu} e^{\frac{4}{D-2}\phi_D}, \tag{2.7}$$

where ϕ_D is the D-dimensional dilaton. This relation originates from the Weyl rescaling formula. The following relation between the square root of the determinant of the two metrics is found by first calculating the relation between the two determinants in ten-dimensions and then taking their square root:

$$\sqrt{-g^{(s)}} = \sqrt{-g^{(E)}} e^{\frac{5}{2}\phi_{10}}, \tag{2.8}$$

where ϕ_{10} is the ten-dimensional dilaton.

The notation which will be used for a p-form field strength is:

$$\frac{1}{p!}|F_p|^2 = \frac{1}{p!} F_{\mu_1..\mu_p} \bar{F}_{\nu_1..\nu_p} g^{(s)\mu_1\nu_1}..g^{(s)\mu_p\nu_p}, \tag{2.9}$$

where $\bar{F}$ is the complex conjugate of F. The relation between the fields in Einstein $|F_p|_E$ and string $|F_p|_s$ frame, is obtained when substituting the metric from (2.7) in ten-dimensions:

$$|F_p|_s^2 = (e^{-\frac{1}{2}\phi_{10}})^p |F_p|_E^2 \tag{2.10}$$

The Ricci scalar can be calculated in string frame in terms of the Ricci scalar in Einstein frame using the transformation between the two metrics $g^{(s)} \rightarrow g^{(E)}e^{2h}$, which gives the following relation [9]:

$$R^{(s)} = e^{-2h}(R^{(E)} - 2(D-1)\nabla^2(h) - (D-2)(D-1)\partial_\mu h \partial^\mu h). \tag{2.11}$$

To find the ten-dimensional Ricci scalar, one has to substitute $D = 10$ and $h = 2\frac{\phi_{10}}{D-2}$ in (2.11), to obtain:

$$R_s = e^{-\frac{1}{2}\phi_{10}}(R_E - 9/2\nabla^2\phi - 9/2\partial_\mu\phi\partial^\mu\phi) \tag{2.12}$$

Finally, one should substitute (2.8), (2.10), and (2.12) in (2.6) to obtain the Einstein frame of type IIA ten-dimensional action:

$$\begin{aligned} S_{IIA} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g^{(E)}} [R^{(E)} - \frac{1}{2}|d\phi|^2 - \frac{1}{2}\frac{1}{3!}e^{-\phi}|H_3|^2 \\ -\frac{1}{2}\frac{1}{2!}e^{\frac{3}{2}\phi}|F_2|^2 - \frac{1}{2}\frac{1}{4!}e^{\frac{1}{2}\phi}|\tilde{F}_4|^2] + \frac{1}{4\kappa_{10}^2}\int dC_3 \wedge dC_3 \wedge B_2 \end{aligned} \tag{2.13}$$

2.3 Branes

Branes are hypersurfaces which are fundamental in both supergravity and string theory. In supergravity, one encounters p-branes, which are the generalized version of Dp-branes from string theory. In both cases, p denotes the spatial dimension of the object. Another example of a p-brane is the NS_5 and F_1-branes, which are S-dual to D_5 and D_1, respectively. Dualities will be discussed later in the thesis in section 3.3.

2.3.1 p-branes

The p-branes are solutions to the actions of type IIA and IIB. The p-brane in Minkowski spacetime in D-dimensions causes the Lorentz group decomposition $SO(1, D-1) \rightarrow SO(1, p) \times SO(D-p-1)$, where $(D-p-1)$ are the directions which have rotational symmetry. These directions are perpendicular to the brane. One can use spherical coordinates in these directions. This decomposition allows the coordinate system in ten-dimensions, to be written in terms of time, p-coordinates on which the p-brane lies and $(9-p)$ spherical coordinates [55]. In this thesis only extremal branes will be used.

The extremal p-branes are 1/2 BPS solutions, which means their presence breaks half of supersymmetry. The metric of extremal p-branes in string frame is:

$$\begin{aligned} ds^2 &= H_p^{-1/2}[-dt^2 + dx_1^2 + .. + dx_p^2] + H_p^{1/2}[dr^2 + r^2 d\Omega_{8-p}^2] \\ &= H_p^{-1/2}[-dt^2 + dx_1^2 + .. + dx_p^2] + H_p^{1/2}[dx_{p+1}^2 + .. + dx_9^2] \\ e^\phi &= H_p^{\frac{3-p}{4}} \\ C_{0..p} &= H_p^{-1} - 1. \end{aligned} \tag{2.14}$$

The harmonic function is given by $H_p = 1 + \frac{Q_p}{r^{7-p}}$, where Q_p is the p-brane charge and

$$r^2 = x_{p+1}^2 + .. + x_9^2. \tag{2.15}$$

The charge can also be written as the multiplication of the charge of a single brane with the number of p-branes: $c_p\ N_p$. A $C_{0..p}$ is a $(p+1)$-form potential that belongs in the R-R sector and ϕ is the dilaton. There are restrictions on the value that p can take depending on which type II the theory is. Type IIA only contains even values of p and hence odd potentials and the opposite happens at type IIB. The branes NS_5 and F_1 are exempt from this rule.

The $(p+1)$-potential couples to the p-brane electrically. The field strength of the $(p+1)$-form potential is $F_{p+2} = dC_{p+1}$. The Hodge star operator takes F_{p+2} to F'_{D-p-2}. F' denotes the dualized field strength. By definition $F'_{D-p-2} = dC_{D-p-3}$, which is magnetically coupled to a $(D-p-4)$-brane. The p-brane and $(D-p-4)$-brane are related from this duality [46].

The p-brane has an event horizon in $r = 0$ in the isotropic coordinates. [7].

2.3.2 NS_5-brane

The 2-form $B_{\mu\nu}$ from the Neveu-Schwarz sector couples to F_1 electrically and to NS_5 magnetically. Both of these branes, just like the potential $B_{\mu\nu}$ appear in both type IIA and type IIB string theories. The NS_5-brane breaks half of supersymmetry. The metric of NS_5 brane parallel to the spatial directions $x_1, .., x_5$ in string frame is given by:

$$\begin{aligned} ds^2 &= [-dt^2 + dx_1^2 + .. + dx_5] + H_{ns5}[dx_6^2 + .. + dx_9^2] \\ e^\phi &= H_{ns5}^{1/2} \\ H_{\mu\nu\rho} &= \frac{1}{2}\epsilon_{\mu\nu\rho\lambda}\partial_\lambda H_{ns5}, \mu, \nu, \rho, \lambda = 6, 7, 8, 9[7] \\ &= -\frac{1}{2}\bar{\epsilon}_{\mu\nu\rho}{}^{\lambda} H_{ns5}^{-1}\partial_\lambda H_{ns5}, \mu, \nu, \rho, \lambda = 6, 7, 8, 9[44] \end{aligned} \tag{2.16}$$

The harmonic function is $H_{ns5} = 1 + \frac{Q_{ns5}}{r^2}$, and

$$r^2 = x_6^2 + .. + x_9^2. \tag{2.17}$$

2.3.3 F_1-brane

The metric of a fundamental string F_1, parallel to the spatial direction x_1 in string frame is given by:

$$
\begin{aligned}
ds^2 &= H_1^{-1}[-dt^2 + dx_1^2] + [dx_2^2 + .. + dx_9^2] \\
e^{\phi} &= H_1^{-1/2} \\
H_{\mu 01} &= \partial_\mu H_1^{-1}, [7]
\end{aligned}
\tag{2.18}
$$

where the harmonic function is $H_1 = 1 + \frac{Q_{f1}}{r^6}$, and

$$
r^2 = x_2^2 + .. + x_9^2. \tag{2.19}
$$

2.3.4 D-branes

In string theory when the dynamics of open strings are discussed, then D-branes emerge. Both open and closed strings are described by the Polyakov action, with the difference that the open string has endpoints, thus boundary conditions. Consider the following parametrization for the spatial coordinate of a string:

$$
\sigma \in [0, \pi]. \tag{2.20}
$$

In the following example, the endpoints of an open string lie in a Dp-brane, a $(p+1)$-dimensional hypersurface:

$$
\begin{aligned}
\partial_\sigma X^\alpha &= 0 \text{ for } \alpha = 0, .., p \\
X^I &= c^I \text{ for } I = p+1, .., D-1.
\end{aligned}
\tag{2.21}
$$

A D-brane is a dynamical object, which is infinite in space [28] [29], [30], [55].

Chapter 3

Compactification

Compactification is the reduction of a number of spatial dimensions on a compact manifold. Different manifolds can either preserve all supersymmetry or a fraction of it. For example, an untwisted n-torus, also denoted as T^n, preserves all supersymmetry and a Calabi-Yau manifold can preserve supersymmetry only partially. Kaluza-Klein is such a method of compactifying a theory. An intuitive idea on how to compactify spacetime directions is that they can be wrapped on a circle with a radius so small that it cannot be observed. For example, a two-dimensional plane wrapped in a circle becomes a cylinder. If this cylinder is wrapped again on a circle it will become a torus. An observer from far away will only see a line when looking at the cylinder and a circle when looking at the torus. It appears that the other dimensions are hidden from him.

To calculate the compactification of black holes, one needs to know how arraying branes works. This is discussed in detail in section 3.1. After that, a theoretical background is given for Kaluza-Klein in section 3.2, and Scherk-Schwarz in 3.4. In section 3.3 the dualities between different types of string theories are discussed.

3.1 Arraying branes

A solution of a theory in ten dimensions that obeys periodic boundary conditions, can also be a solution to the compactified theory. Assume that periodicity is obeyed in the y-direction such that $y \sim y + 2\pi nR$, $n \in \mathbb{Z}$. To compactify on a D_p-brane which is pointlike in the y-direction, one must replace this D_p-brane with an array of BPS branes to satisfy the periodicity. The array is placed in the transverse y-direction and the distance of the branes in the array is $2\pi R$. This lattice has a harmonic function:

$$H_p = 1 + \sum_i \frac{Q_p}{|\vec{r} - \vec{r}_i|^{7-p}} \ , \tag{3.1}$$

where the brane labeled as i, has a respective position $\vec{r}_i$.

Branes have a repulsive force from gauge forces and an attractive force from gravitational and dilatonic forces. These forces cancel against each other for BPS branes, bringing an array of BPS branes to a static equilibrium.

As discussed, the radii of a circle on which extra dimensions are wrapped on should be very small. This allows the summation to be replaced with an integral and to generalize, assume that there are more than one transverse directions. Thus the integral will have measure $d^n\vec{y}$, where n are the transverse directions on the D_p-brane. Thus, the harmonic function becomes:

$$H_p = 1 + \frac{Q_p}{r^{7-p-n}}. \tag{3.2}$$

Instead of arraying branes, it is also mentioned in the literature as smearing of branes [6], [21], [51].

3.2 Kaluza-Klein reduction

Assume a $(D+1)$-dimensional theory, with coordinates $x^{\hat{\mu}}$ that decompose as the coordinates x^μ of a D-dimensional Minkowski spacetime and a coordinate y on a circle. Then the higher-dimensional theory coordinates can be written as $x^{\hat{\mu}} = (x^\mu, y)$, where $\mu = 0, .., D-1$. The coordinate on the circle y, has a periodicity $y \sim y + 2\pi n R$, where R is the circle radius. This allows the $(D+1)$-dimensional metric to be rewritten in terms of the D-dimensional metric:

$$d\hat{s}^2 = e^{2\alpha_D \phi} ds^2 + e^{2\beta_D \phi}(dy + \mathcal{A}_\mu dx^\mu)^2, \tag{3.3}$$

where α_D, β_D are constants which depend on the dimension of the compactified theory in Einstein frame and they remain constant in all dimensions in string frame.

3.2.1 Kaluza-Klein tower

The example of a massless scalar can be used to show what a Kaluza-Klein tower is. A massless scalar on a $(D+1)$-dimensional space obeys:

$$\partial^{\hat{\mu}} \partial_{\hat{\mu}} \hat{\phi} = 0 \tag{3.4}$$

and the relation between the $(D+1)$-dimensional to a D-dimensional scalar is:

$$\hat{\phi}(x^\mu, y) = \sum_{z \in \mathbb{Z}} \phi_z(x^\mu) e^{\frac{izy}{R}} \tag{3.5}$$

By substituting (3.5) to (3.4), the following equation is obtained:

$$\partial^\mu \partial_\mu \phi_z - \frac{z^2}{R^2} \phi_z = 0, \tag{3.6}$$

where $z \in \mathbb{Z}$. From the above equation, it can be seen that the lower-dimensional scalars have masses $\frac{|z|}{R}$, and that the scalar corresponding to $z = 0$ is massless. This is called the Kaluza-Klein tower. These masses depend on the radius of the compactified dimensions, which is assumed to be very small leading to very large masses. In fact, these masses are so large that the massive modes can be neglected. Thus one can only consider the case for $z = 0$ corresponding to a massless scalar.

3.2.2 Dimensional reduction of metric

Using the coordinates $x^{\hat{\mu}} = (x^\mu, y)$, the higher dimensional metric $g^{D+1}_{\hat{\mu}\hat{\nu}}$ is reduced to the lower dimensional metric $g^D_{\mu\nu}$, a vector $\mathcal{A}_\mu$ given by $g^D_{\mu y}$ and a scalar g^D_{yy}. This can be shown using the Kaluza-Klein ansatz:

$$g^{D+1}_{\hat{\mu}\hat{\nu}} = \begin{bmatrix} e^{2\alpha_D\phi} g^D_{\mu\nu} + e^{2\beta_D\phi}\mathcal{A}_\mu\mathcal{A}_\nu & e^{2\beta_D\phi}\mathcal{A}_\mu \\ e^{2\beta_D\phi}\mathcal{A}_\nu & e^{2\beta_D\phi} \end{bmatrix}, \tag{3.7}$$

where $g^{D+1}_{\hat{\mu}\hat{\nu}}$ is the $(D+1)$-dimensional metric and $g^D_{\mu\nu}$ is the D-dimensional metric. The parameters α and β are $\beta_D = -(D-2)\alpha_D$ in Eistein frame or $\alpha = 0$ and $\beta = 1$ in string frame.

In the rest of this chapter, all higher dimensional objects will be denoted with a hat, or their dimension will be specified. Using the Kaluza-Klein ansatz (3.7) in Einstein frame to find α_D for each separate circle, one has to consider the relation between the higher and lower dimensional Ricci scalars:

$$\hat{R} = e^{-2\alpha_D\phi} R + .. \tag{3.8}$$

and the relation between the determinant of the higher and lower dimensional metrics:

$$\sqrt{-\hat{g}} = \sqrt{-g}\, e^{\frac{2\beta_D + 2\alpha_D D}{2}\phi} \tag{3.9}$$

The value of α_D in D-dimensions has to satisfy $\sqrt{-\hat{g}}\hat{R} = \sqrt{-g}R$ after substituting the relation between β_D and α_D in D-dimensions. Thus, α_D is [13], [34]:

$$\alpha_D^2 = \frac{1}{2(D-1)(D-2)}. \tag{3.10}$$

3.2.3 Dimensional reduction of the dilaton

Now that some familiarity has been established with the Kaluza-Klein ansatz, the dimensions will be dropped from the metrics g^{D+1}, and g^D and instead denote them as $\hat{g}$ and g, respectively. The dimensions will only be denoted from now on if they play an important part in the rest of the formula. From (3.7), the relation between the determinant of the $(D+1)$-dimensional metric with the D-dimensional metric in string frame is:

$$\sqrt{-\hat{g}} = \sqrt{-g}\, e^{\phi}. \tag{3.11}$$

It is important to establish a relation between the D- and ten-dimensional dilatons. This can be done using the ten-dimensional metric. There is a change in the Newton constant, which can be balanced out by replacing the value for the dilaton, specifically in the case of the ten-dimensional dilaton the following equation becomes:

$$e^{-2\phi_D} = \sqrt{g^{10}_{DD} .. g^{10}_{99}}\, e^{-2\phi_{10}}, \tag{3.12}$$

where ϕ_D is the D-dimensional dilaton, and ϕ_{10} the ten-dimensional dilaton.

3.2.4 Dimensional reduction of a p-form

Potentials in $(D+1)$-dimensions can be written as:

$$\hat{F}_n = d\hat{C}_{n-1}, \tag{3.13}$$

where $\hat{F}_n$ is an antisymmetric tensor field. The y-coordinate cannot occur more than once in the indices of the field potential (at most once), which decomposes the $(D+1)$-dimensional potential into two reduced fields, independent of the coordinate y. The initial $(n-1)$-form potential is written as:

$$\hat{C}_{n-1}(x,y) = C_{n-1}(x) + C_{n-2}(x) \wedge dy. \tag{3.14}$$

Then the fiend strength is written as $\hat{F}_n = dC_{n-1} + dC_{n-2} \wedge dy$. Dimensional reduction gives terms such as tensors coupled to Kaluza-Klein vector fields, thus giving the field strength with a Chern-Simons correction:

$$\hat{F}_n = F_n + F_{n-1} \wedge (dy + \mathcal{A}), \tag{3.15}$$

where $\mathcal{A}$ is the Kaluza-klein potential from the dimensional reduction of the metric and $F_n = dC_{n-1} - dC_{n-2} \wedge \mathcal{A}$, $F_{n-1} = dC_{n-1}$ [50].

The $(D+1)$-dimensional field strength will be reduced to D-dimensional field strengths and when they are inserted in a D-dimensional lagrangian term in Einstein frame, the reduced fields can be expressed as:

$$\begin{aligned} \frac{\sqrt{-\hat{g}}}{n!}\hat{F}_n^2 &= \frac{\sqrt{-g}}{n!}e^{(-2\alpha_D\phi)n}e^{2\alpha_D\phi}F_n^2 - \frac{\sqrt{-g}}{(n-1)!}e^{(-2\alpha_D\phi)(n-1)}e^{2\alpha_D\phi}e^{-2\beta_D\phi}F_{n-1}^2 \\ &= \frac{\sqrt{-g}}{n!}e^{(-2\alpha_D\phi)n}e^{2\alpha_D\phi}F_n^2 - \frac{\sqrt{-g}}{(n-1)!}e^{2\alpha_D(-n+D)\phi}F_{n-1}^2 \end{aligned} \tag{3.16}$$

3.2.5 Dimensional reduction of a pp-wave

A $(D+1)$-dimensional plane-fronted gravitational wave that has parallel rays (pp-wave) with $D > 3$ has the following metric [40]:

$$d\hat{s}^2 = (-1+K)dt^2 + d\vec{x}^2 + (1+K)dy^2 - Kdydt - Kdtdy. \tag{3.17}$$

The metric of the pp-wave in the string frame for the use of this thesis, has the same format as in (3.17), but it is delocalized at the spatial direction which will be compactified on a circle after the Scherk-Schwarz reduction. To find the D-dimensional metric of the pp-wave, one has to use the Kaluza Klein ansatz:

$$\hat{g}_{\mu\nu} = \begin{bmatrix} g_{\mu\nu} + e^{2\phi}\mathcal{A}_\mu\mathcal{A}_\nu & e^{2\phi}\mathcal{A}_\mu \\ e^{2\phi}\mathcal{A}_\nu & e^{2\phi} \end{bmatrix} = \begin{bmatrix} \hat{g}_{00} & \hat{g}_{0y} \\ \hat{g}_{y0} & \hat{g}_{yy} \end{bmatrix} \tag{3.18}$$

The following relations are obtained:

$$\begin{aligned} \hat{g}_{yy} &= 1 + K = e^{2\phi} \\ \hat{g}_{0y} &= -K = e^{2\phi}\mathcal{A}_0 = (1+K)\mathcal{A}_0 \\ \hat{g}_{00} &= -1 + K = g_{00} + e^{2\phi}\mathcal{A}_0\mathcal{A}_0 \\ &= g_{00} + (1+K)(-K/(1+K))^2 \end{aligned} \tag{3.19}$$

Hence $g_{00} = -1/(1+K)$. The D-dimensional metric is:

$$ds^2 = -(1+K)^{-1}dt^2 + d\vec{x}^2, \tag{3.20}$$

where the harmonic function of the pp-wave is:

$$H_K = 1 + K(r) \tag{3.21}$$

3.3 String theory dualities

The different types of string theories mentioned in section 2.2 are related to each other with dualities. These dualities are S-duality, T-duality, and U-duality. Type I is related to type $SO(32)$ heterotic through S-duality, the latter is related to $E_8 \times E_8$ heterotic through T-duality. $E_8 \times E_8$ heterotic and Type IIA are related to M-theory via S-duality and the latter is related to type IIB via T-duality. Lastly, type IIB theory has the property of being related to itself via S-duality, this means that it can be proven that two string theories of type IIB string theory with coupling constant g_s and $1/g_s$ are equivalent [46]. Even though this is a property mostly studied for type IIB string theory, it is also important to keep in mind when studying type IIA as well, since branes such as F_1, NS_5 are present in both theories. This is also true for background fields such as W, which was introduced in 3.2.5 and it is used in the black hole in 6.3.5. Their difference, as mentioned before is that type IIA has only even branes from D_0 to D_8, and Type IIB has only odd branes from D_{-1} to D_9.

S-duality sends D_1 to F_1, D_5 to NS_5, and vice versa. D_3 remains unchanged under this transformation and the case of D_7 is examined in [20] , but it is not needed in this thesis.

T-duality is applied in the coordinates on which a D_p-brane is tangent or transverse, the result will be a D_{p-1}-brane or D_{p+1}-brane, respectively [19]. This duality switches the chirality of right-moving fermions, which allows calculations from type IIA, compactified on a circle of radius R to be T-dual to type IIB theory compactified on a radius α'/R [44]. U-duality is the symmetry group including both S-duality and T-duality [26], [39].

T-duality can be seen in the string mass. Assume a dimensional reduction was applied in coordinate y, resulting in the quantization of the string momentum in the direction of y-coordinate:

$$P_y = \frac{z}{R},\ z \in \mathbb{Z}. \tag{3.22}$$

The compact direction obeys $y \sim y + 2\pi R$, where R is the circle radius on which it was compactified. This means that there is a winding around this direction $y \to y + 2\pi nR$, $n \in \mathbb{Z}$. Both the winding number n and quantized momentum contribute to the string mass which is:

$$M^2 = \alpha'[(\frac{z}{R})^2 + (\frac{nR}{\alpha'})^2] + 2N_L + 2N_R - 4, \tag{3.23}$$

with the restriction that the levels of left- and right-moving oscillations N_L, N_R : $N_R - N_L = zn$. Both the restriction and (3.23) are invariant when interchanging n with z and R with α'/R. Regarding open strings, T-duality can swap Neumann with Dirichlet boundary conditions [4], [28].

3.4 Scherk-Schwarz reduction

Contrary to the Kaluza-Klein reduction, the Scherk-Schwarz reduction [47], [48] makes use of symmetries in the uncompactified theory to give masses in the reduced dimension. In general, the introduction of the masses in the theory comes from the y-dependence of the fields in the higher-dimensional theory. Usually the fields in non-trivial representations of the group G will gain mass.

Consider a theory coupled to gravity with a generic field ψ in $(D+1)$-dimensions, which possesses a global symmetry $g \in G$, and g acts on this generic field as $g(\psi)$. The ansatz for the Scherk-Schwarz reduction is:

$$\psi(x^{\mu}, y) = g(y)\psi(x^{\mu}), \tag{3.24}$$

where $g(y)$ is a symmetry transformation. A Scherk-Schwarz reduction on the scalars leads to a scalar potential.

The group element g is written as an exponential of M and the y-coordinate. M is an element of the lie algebra of group G, and it is proportional to the mass matrix of the lower-dimensional theory:

$$g(y) = \exp(\frac{My}{2\pi R}). \tag{3.25}$$

The goal is, to begin with a generalized field that depends on the compact direction and by using the symmetry of the higher-dimensional theory, to get the equations of motion in the lower-dimensional theory which are independent of this direction. The transformation depending on the compact direction is not periodic but by going around the circle of the compact coordinate, a twist is generated which is a power of the monodromy, $\mathcal{M}$. The monodromy $\mathcal{M}$ is $g(2\pi R)g^{-1}(0)$, for $\mathcal{M} \in G$:

$$\mathcal{M} = \exp(M). \tag{3.26}$$

The matrix M is independent of y [15], [27].

Instead of applying the Scherk-Schwarz reduction on a circle, it is also possible to obtain D-dimensional theory from an initial $(D+E)$-dimensional theory, where the E-dimensional manifold is compact. This is a generalization of what was described above, hence a symmetry from the higher dimensional theory is used again to define the y-dependence of the fields and transformation laws. To be exact, this dependence should be chosen in a way that the fields define continuous fiber bundles on the E-dimensional space. The coordinates of y have to be related to an E-dimensional lie group. This dependence of the coordinate y, as expected from the previous description, gives mass terms in the lower dimensional theory of D-dimensions. [47].

3.4.1 Dimensional reduction of scalar fields

The difference between applying a Kaluza-Klein compactification ($\mathcal{M} = I$) on a circle and then gauging the resulting global symmetry, and doing a Scherk-Schwartz compactification is that in the latter case, a potential appears which can be applied to the theory to give mass to scalar fields. The former case also has a potential introduced to satisfy specific requirements but it does not occur naturally [60].

The $(D+1)$-dimensional scalars take values in the coset G/K, where usually K is a maximal compact subgroup of G and G a non-compact group. G denotes the global symmetry of the theory and K the local symmetry. Then these scalars can be represented by a vielbein $\mathcal{V}(x)$ which transforms as:

$$\mathcal{V} \rightarrow k(x)\mathcal{V}g, \tag{3.27}$$

where k and g are elements of K and G, respectively. Let the vielbein be a real matrix and η a K-invariant metric. Define $\mathcal{H}$ as:

$$\mathcal{H} = \mathcal{V}^T \eta \mathcal{V}, \tag{3.28}$$

and transforms as:

$$\mathcal{H} \rightarrow g^T \mathcal{H} g, \tag{3.29}$$

then $\mathcal{H}$ is K-invariant. The kinetic term of the scalar fields is proportional to:

$$tr[\partial_{\hat{\mu}} \mathcal{H}^{-1} \partial^{\hat{\mu}} \mathcal{H}], \tag{3.30}$$

and the dimensional reduction gives the potential, from the ansatz $\mathcal{H}(\phi(x), y) = \mathcal{M}^T(y)\mathcal{H}(\phi(x))\mathcal{M}(y)$:

$$V = e^{\frac{6}{(D-1)(D-2)}\phi} tr[M^2 + M^T \mathcal{H} M \mathcal{H}^{-1}], \tag{3.31}$$

where ^{T} denotes either the transpose or Hermitian conjugate, depending on the maximal compact subgroup of the theory [15].

Chapter 4

Supergravity

Supergravity combines supersymmetry and general relativity. In supersymmetry theories one encounters gravitinos that have spin 3/2 and they are the supersymmetric fermionic partners of gravitons. In fact, supersymmetry provides a unified description of fermions and bosons in a supermultiplet. Supergravity actions can be viewed as the low-energy effective theories of string theory. In supersymmetry theories, the number of supersymmetries and gravitinos contained in the theory are the same. Extended supersymmetry theories have $\mathcal{N} > 1$.

In section 4.1 there is an overview of the structure of the supersymmetry algebra and in section 4.2 there is a detailed account of massless multiplets which will be needed later on.

4.1 Supersymmetry algebra

An operator Q that generates supersymmetric transformations can transform fermionic to bosonic states and vice-versa. This is an anti-commuting spinor generator $Q_i^A, \bar{Q}_{i'}^A$, $A = 1, .., \mathcal{N}$. In addition the Poincaré generators $P^\mu, M^{\mu\nu}$ are needed to describe the supersymmetry algebra.

Define:

$$\begin{aligned}(\sigma^{\mu\nu})_i^{\ j} &= \frac{i}{4}(\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu)_i^{\ j} \\ (\bar{\sigma}^{\mu\nu})_{i'}^{\ j'} &= \frac{i}{4}(\bar{\sigma}^\mu\sigma^\nu - \bar{\sigma}^\nu\sigma^\mu)^{i'}_{\ j'},\end{aligned} \tag{4.1}$$

where $\sigma^\mu = (I, \vec{\sigma})$, $\bar{\sigma}^\mu = (I, -\vec{\sigma})$, I is the identity matrix and $\vec{\sigma}$ are the Pauli matrices.

4.1.1 $\mathcal{N} = 1$ supersymmetry algebra

The commutation relations from the Poincaré algebra are known [33], [58] and the additional ones needed to describe the supersymmetry algebra are:

$$\begin{aligned} &[Q_i, P^\mu] = [\bar{Q}^{i'}, P^\mu] = 0 \\ &[Q_i, M^{\mu\nu}] = (\sigma^{\mu\nu})_i^{\ j} Q_j \\ &\{Q_i, Q_j\} = 0 \\ &\{Q_i, \bar{Q}_{j'}\} = 2(\sigma^\mu)_{ij'} P_\mu, \end{aligned} \tag{4.2}$$

where $Q_i^\dagger = \bar{Q}_{i'}$ and $(\sigma^\mu \bar{Q})_i^\dagger = (Q\sigma^\mu)_{i'}$. The indices i, j belong in the fundamental representation and i', j' in the conjugate representation. The last commutator to consider is the one for Q_i with the generators of internal symmetry, which is zero except when considering R-symmetry. R-symmetry is the $U(1)$automorphism of the supersymmetry algebra. The commutation relations between them are:

$$\begin{aligned} &[Q_i, R] = Q_i \\ &[\bar{Q}_{i'}, R] = -\bar{Q}_{i'}. \end{aligned} \tag{4.3}$$

Define p^μ as the operator eigenvalues of P^μ.
To find the massless multiplets, let $p_\mu = (E, 0, 0, E)$, thus:

$$\{Q_i, \bar{Q}_{j'}\} = 4E \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}_{ij'}. \tag{4.4}$$

Resulting in $Q_2 = 0$.

For massive supermultiplets, let $p_\mu = (m, 0, 0, 0)$, thus:

$$\{Q_i, \bar{Q}_{j'}\} = 2mI_{ij'}. \tag{4.5}$$

4.1.2 Extended supersymmetry algebra

For extended supersymmetry algebras, the spinor generators also need additional labels $A, B = 1, 2, .., \mathcal{N}$ and the only commutation relations that change are:

$$\begin{aligned} &\{Q_i^A, \bar{Q}_{j'B}\} = 2(\sigma^\mu)_{ij'} P_\mu \delta^A_{\ B} \\ &\{Q_i^A, Q_j^B\} = \epsilon_{ij} Z^{AB}, \end{aligned} \tag{4.6}$$

where ϵ_{ij} is defined at section 1.2 and it will be discussed further in (5.1). Z^{AB} are antisymmetric central charges. They commute with all generators.

To obtain the massless supermultiplets, again let $p_\mu = (E, 0, 0, E)$, thus:

$$\{Q_i^A, \bar{Q}_{j'B}\} = 4E \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}_{ij'} \delta^A_{\ B}. \tag{4.7}$$

That leads to $Q_2^A = 0$ and from (4.6), $Z^{AB} = 0$. The massless multiplets have $2^{\mathcal{N}}$ states.

The same applies for massive multiplets, let $p_\mu = (m, 0, 0, 0)$, thus:

$$\{Q_i^A, \bar{Q}_{j'B}\} = 2m \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{ij'} \delta^A_B. \tag{4.8}$$

There are two distinct cases regarding central charges. They can either be zero, meaning that half generators will simply vanish, $Q_2^A = 0$. The multiplets for this case have $2^{2\mathcal{N}}$ states. In the other case the central charges are non-zero. If Z^{AB} is non-zero, then for even $\mathcal{N}$ it can be expressed as:

$$Z^{AB} = \begin{bmatrix} 0 & q_1 & 0 & 0 & \dots & & \\ -q_1 & 0 & 0 & 0 & \dots & & \\ 0 & 0 & 0 & q_2 & \dots & & \\ 0 & 0 & -q_2 & 0 & \dots & & \\ \vdots & \vdots & \vdots & \vdots & \ddots & & \\ & & & & & 0 & q_{\frac{\mathcal{N}}{2}} \\ & & & & & -q_{\frac{\mathcal{N}}{2}} & 0 \end{bmatrix}. \tag{4.9}$$

In this case, there is a bound for $m \leq \frac{1}{2\mathcal{N}} tr(\sqrt{Z^\dagger Z})$. The states for which, m takes the minimum value are called BPS states. In this case, the multiplets have $2^{\mathcal{N}}$ states [18], [31], [37].

4.2 Massless multiplets

Multiplets are irreducible representations of the supersymmetry algebra and they contain both fermions and bosons. There are equal numbers of bosonic and fermionic degrees of freedom on-shell in any supersymmetry algebra of the following form: $\{Q, Q\} = P$. In the off-shell case, this equality is not always guaranteed. Particles that belong in the same supermultiplet have the same weak isospin, electric charges, and color degrees of freedom since supersymmetry generators commute with the generators of gauge transformations in supersymmetry algebra [37].

There are several types of massless multiplets. Define s as the maximum spin in the representation. In four-dimensions, there is the graviton multiplet($s = 2$) which contains a graviton, $\mathcal{N}$ gravitinos, and in extended supergravity theories, additional fields. The vector (gauge) multiplet ($s = 1$) exists only for supergravities less or equal than four and there are also the chiral multiplets and hypermultiplets ($s = 1/2$). Chiral multiplets exist in $\mathcal{N} = 1$ and hypermultiplets in $\mathcal{N} = 2$, they do not exist for higher supergravities.

Assume P_μ are the momentum operators. The massless representations P_μ satisfy $P^2 = 0$. In $\mathcal{N} = 2$ supergravity, the $s = 2$ multiplet contains a graviton, two gravitinos, and a vector. Additionally, the $s = 3/2$ multiplet contains a gravitino, two vectors, and a spin $1/2$ fermion, the $s = 1$ multiplet contains a vector, two spin $1/2$ fermions, and $1+1$ scalars. Finally, the $s = 1/2$ multiplet contains two spin $1/2$ fermions and $2 + 2$ scalars.

In $\mathcal{N} = 4$ supergravity theory, the $s = 2$ multiplet contains a graviton, four gravitinos, six vectors, four spin $1/2$ fermions and $1 + 1$ scalars. The $s = 3/2$ multiplet contains a gravitino, four vectors, $6 + 1$ spin $1/2$ fermions and $4 + 4$ scalars. The $s = 1$ multiplet contains a vector, four spin $1/2$ fermion and six scalars

In $\mathcal{N} = 6$ supergravity, the $s = 2$ multiplet has a graviton, six gravitinos, $15 + 1$ vectors, $20 + 6$ spin $1/2$ fermions and $15 + 15$ scalars. The $s = 3/2$ multiplet has one gravitino, six vectors, fifteen spin $1/2$ fermions, and twenty scalars.

In $\mathcal{N} = 8$ supergravity, the $s = 2$ multiplet contains one graviton, eight gravitinos, twenty-eight vectors, fifty-six spin $1/2$ fermions and seventy scalars [18].

One can use superalgebra to construct the particle content of supersymmetric theories. In non-supersymmetric particle physics, particles are usually defined to be irreducible representations of the Poincare algebra. Since the Poincaré algebra is a subalgebra of the supersymmetry algebra, any irreducible representation of the supersymmetry algebra is a representation of the Poincaré algebra.

Chapter 5

Dimensional reduction and supersymmetry breaking

In this chapter dimensional reductions will be discussed in further detail. In section 5.2, there will be an analysis of reductions of the field content of type IIA. In section 5.3 the reduction of the five-dimensional theory on a circle using the Scherk-Schwarz mechanism is discussed giving the number of massless and massive fields, which can be grouped in multiplets. Finally, in section 5.4 the mass matrices and their corresponding monodromy are shown.

5.1 Hodge duality

The Levi-Civita symbol is a completely antisymmetric object with the property:

$$\epsilon_{\mu_1..\mu_n} = \begin{cases} +1 \text{ if } \mu_1\mu_2..\mu_n \text{ is an even permutation of } 01..(n-1) \\ \text{-1 if } \mu_1\mu_2..\mu_n \text{ is an odd permutation of } 01..(n-1) \\ \qquad\qquad 0 \text{ otherwise} \end{cases} \quad (5.1)$$

The Levi-Civita symbol $\epsilon_{\mu_1..\mu_n}$ is related to the Levi-Civita tensor $\bar{\epsilon}_{\mu_1..\mu_n}$ by:

$$\bar{\epsilon}_{\mu_1..\mu_n} = \sqrt{|g|}\epsilon_{\mu_1..\mu_n} \quad (5.2)$$

The Levi-Civita symbol is called a symbol because it does not transform as a tensor under coordinate transformations, it only behaves as a tensor in flat spacetime in inertial coordinates.

The Hodge star operator is defined as a mapping from p-forms to $(n-p)$-forms:

$$(*F)_{\mu_1..\mu_{n-p}} = \frac{1}{p!}\bar{\epsilon}^{\nu_1..\nu_p}{}_{\mu_1..\mu_{n-p}}F_{\nu_1..\nu_p}. \quad (5.3)$$

By taking the Hodge dual twice:

$$**F = (-1)^{s+p(n-p)}F, \quad (5.4)$$

where s in the amount of negative eigenvalues of F. Most of the definitions in this paragraph can be found in the book [10].

5.2 Dimensional reduction of type IIA in string theory

As mentioned in chapter 2, type IIA string theory in ten-dimensions has a field spectrum $G_{\mu\nu}$, $B_{\mu\nu}$, ϕ from the NS-NS sector and C_μ and $C_{\mu\nu\rho}$ from the R-R sector. The spacetime coordinates in ten-dimensions are $\mu, \nu, \rho = 0, 1, .., 9$. To compactify n-dimensions of this initial theory, one can use the methods described in chapter 3 and perform n different reductions on a circle $S^1 \times S^1 \times .. \times S^1$ or a reduction on an n-torus. Upon compactification on an n-torus, the lower-dimensional theory obtained is a $(10 - n)$-dimensional theory containing the reduced fields. To find these reduced fields, one ought to keep in mind that the $(D+1)$-dimensional metric reduced on a circle gives a D-dimensional metric, 1-form and scalar. In addition a $(D + 1)$-dimensional p-form produces a D-dimensional p-form and an $(p - 1)$-form and the $(D + 1)$-dimensional scalar reduces to a D-dimensional scalar.

The six-dimensional theory is obtained via a compactification of the ten-dimensional theory on a four-torus and the results of the compactification are shown in table 5.1 analytically. To reduce the graviton one should keep in mind that the graviton also represents the string metric. The reduction of the graviton $G_{\mu\nu}$ produces a metric, four vectors and ten scalars in six-dimensions. The 2-form $B_{\mu\nu}$ decomposes into a six-dimensional 2-form, four vectors and six scalars, and the ten-dimensional scalar ϕ leads to a six-dimensional scalar. The R-R fields C_μ and $C_{\mu\nu\rho}$ reduce to four scalars each, a vector and six vectors, respectively. The 3-form $C_{\mu\nu\rho}$ also reduces to four 2-forms and a 3-form.

Type IIA on T^4	$G_{\mu\nu}$	$B_{\mu\nu}$	ϕ	C_μ	$C_{\mu\nu\rho}$	Total
scalars	10	6	1	4	4	25
1-form	4	4		1	6	15
2-form		1			4	5
metric	1					1
3-form					1	1

Table 5.1: Analytical description of the field content of type IIA reduced on a four-torus

The compactification from the initial ten-dimensional theory on a four-torus leads to a theory in six-dimensions, which is subject to dualities explained in section 5.1. The 3-forms from table 5.1 are dual to 1-forms. The final theory is shown at table 5.2.

Type IIA on T^4	Total
scalars	25
1-form	16
2-form	5
metric	1

Table 5.2: Field content of type IIA reduced on a four-torus

The compactification of a ten-dimensional theory on a five-torus provides a collection of fields in the five-dimensional perspective depicted in table 5.3. The ten-dimensional metric provides the lower-dimensional theory with fifteen scalars, five vectors, and a metric. From the NS-NS sector 2-form there are ten scalars, five vectors and one 2-form in five-dimensions and the ten-dimensional scalar reduces to a five-dimensional scalar. The 1-form becomes five scalars and one 1-form from the five-dimensional perspective and the 3-form in ten-dimensions reduces to ten scalars and vectors, five 2-forms and one 3-form.

Type IIA on T^5	$G_{\mu\nu}$	$B_{\mu\nu}$	ϕ	C_μ	$C_{\mu\nu\rho}$	Total
scalars	15	10	1	5	10	41
1-form	5	5		1	10	21
2-form		1			5	6
metric	1					1
3-form					1	1

Table 5.3: Analytical description of the field content of type IIA reduced on a five-torus

The dualities in the field content of type IIA in five-dimensions are between 2-forms and vectors, and scalars and 3-forms, resulting in forty-two scalars and twenty-seven vectors shown in table 5.4.

Type IIA on T^5	Total
scalars	42
1-form	27
metric	1

Table 5.4: Field content of type IIA reduced on a five-torus

The last field reduction shown in this chapter is on a six-torus, where the higher-dimensional metric reduces to a four-dimensional metric, six vectors and twenty-one scalars. The 2-form reduces to fifteen scalars, six vectors and one 2-form and the ten-dimensional scalar to a four-dimensional scalar. The 1-form from the R-R sector produces six scalars and one 1-form. In addition, the 3-form from the R-R sector decomposes to twenty scalars, fifteen vectors, six 2-forms and one 3-form in four-dimensions. The fields acquired are shown analytically at table 5.5.

Type IIA on T^6	$G_{\mu\nu}$	$B_{\mu\nu}$	ϕ	C_μ	$C_{\mu\nu\rho}$	Total
scalars	21	15	1	6	20	63
1-form	6	6		1	15	28
2-form		1			6	7
metric	1					1
3-form					1	1

Table 5.5: Analytical description of the field content of type IIA reduced on a six-torus

In four-dimensions 2-forms are dual to scalars and the 3-form is the cosmological constant. The final four-dimensional theory in shown at table 5.6.

Type IIA on T^6	Total
scalars	70
1-form	28
metric	1

Table 5.6: Field content of type IIA reduced on a six-torus

The main interest of this thesis lies in the fields acquired from compactification up to a six-torus, but the field content of lower-dimensional theories can be obtained following the same procedure.

5.3 Supersymmetry breaking

After acquiring the five-dimensional field content in section 5.1, the goal is, to begin with an $\mathcal{N}=8$ theory and partially break the supersymmetry to $\mathcal{N}=6,4$, and 2 in four-dimensions. The field content of $\mathcal{N}=0$ will only be mentioned and not listed explicitly. This circle reduction is done using the Scherk–Schwarz mechanism and breaks the supersymmetry partially. Once the four-dimensional fields are obtained, they can be grouped into multiplets for each type of supergravity.

The global symmetry group in five-dimensions is $E_{6(6)}$ and the R-symmetry group is $USp(8)$. To partially break supersymmetry one has to consider a mass matrix in the lie algebra of $USp(8)$. This lie algebra is $\text{Lie}(USp(8)) = usp(8)$.

5.3.1 Massless multiplets

The mass matrix of the theory, as an element of $usp(8)$, depends on the choice of the symplectic metric. Further analysis will be given in section 5.4. For this section, consider the symplectic metric:

$$\Omega = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}, \tag{5.5}$$

which results in the mass matrix:

$$\begin{aligned} M(m_i) &= \sum_{i=1}^{4} m_i \mu_i \\ \mu_i = a \otimes s_{ii},\ a &= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}. \end{aligned} \tag{5.6}$$

The matrix s_{ii} is the 4×4 matrix with 1 in the position of intersection of the ith row with ith column of the matrix. The eigenvalues of $iM(m_i)$ are given by [17]:

Five-dimensional spin	Eigenvalues of $iM(m_k)$
$J = 2$	0
$J = 3/2$	$\pm m_i$
$J = 1$	$3(0), \pm(m_i \pm m_j);\ i < j$
$J = 1/2$	$2(\pm m_i), \pm(m_i \pm m_j \pm m_k);\ i < j < k$
$J = 0$	$2(0), \pm(m_i \pm m_j), \pm(m_1 \pm m_2 \pm m_3 \pm m_4);\ i < j$

Table 5.7: Eigenvalues of mass matrix and corresponding five-dimensional fields [17]

The five-dimensional fields correspond to specific eigenvalues of the mass matrix. This correspondence can be verified using (3.24)-(3.26) and substituting the mass matrix (5.6). The results are shown in table 5.7. Table 5.7 depicts that in five-dimensions, the graviton corresponds to an eigenvalue zero, meaning that it remains massless. The gravitino has spin 3/2, and its corresponding eigenvalue from the mass matrix is $\pm m_i$. There are three zero eigenvalues, and $\pm(m_i \pm m_j)$ for vectors. The spin 1/2 fermion has two $\pm m_i$ and $\pm(m_i \pm m_j \pm m_k)$ eigenvalues, and the scalar has two zero eigenvalues, $\pm(m_i \pm m_j)$ and $\pm(m_1 \pm m_2 \pm m_3 \pm m_4)$, where as stated $i < j < k$ in each case.

The five-dimensional massless sector for $\mathcal{N} = 8, 6, 4$ and 2 is calculated from the eigenvalues that become or were already found to be zero when setting $\mathcal{N}/2$ parameters of the mass matrix to zero. In $\mathcal{N} = 0$, all mass parameters are non-zero. The four-dimensional products of these five-dimensional fields are the actual fields that the Scherk-Schwarz mechanism, will leave massless. The same applies for the five- and four-dimensional massive fields. The five-dimensional fields are used to identify how the Scherk-Schwarz mechanism will act on the four-dimensional field spectrum when it is applied in the initial theory. A list of the number of these fields will be given in the tables below. The rest of the fields will gain masses and will be divided into multiplets, in section 5.3.2. In general, the R-symmetry group in four-dimensions for $\mathcal{N}$ is $U(\mathcal{N})$ and in five-dimensions it is $USp(\mathcal{N})$ [18]. The branching rules between these groups can be found in [49], [59].

The massless sector of $\mathcal{N} = 8$ is obtained by setting $m_1 = m_2 = m_3 = m_4 = 0$ in all eigenvalues of table 5.7. This statement stems from the fact that there are eight massless gravitinos in $\mathcal{N} = 8$, so all masses have to be set to zero to find eight massless eigenvalues corresponding to spin $J = 3/2$ in table 5.7.

Spin	Massless five-dimensional fields	Massless four-dimensional fields
$J = 2$	1	1
$J = 3/2$	8	8
$J = 1$	27	28
$J = 1/2$	48	56
$J = 0$	42	70

Table 5.8: Massless five- and four-dimensional fields in $\mathcal{N} = 8$ supergravity

In $\mathcal{N} = 8$, the massless five-dimensional fields are one graviton as expected, eight gravitinos, twenty-seven vectors, forty-eight spin 1/2 fermions and forty-two scalars.

The massless fields in four-dimensions can be found by reducing the ones in five-dimensions and the result is one graviton, eight gravitinos, twenty-eight vectors, fifty-six spin 1/2 fermions and seventy scalars as shown in table 5.8. The fields in four-dimensions are contained in one graviton multiplet of $\mathcal{N} = 8$ supergravity.

Spin	Massless five-dimensional fields	Massless four-dimensional fields
$J = 2$	1	1
$J = 3/2$	6	6
$J = 1$	15	16
$J = 1/2$	20	26
$J = 0$	14	30

Table 5.9: Massless five- and four-dimensional fields in $\mathcal{N} = 6$ supergravity

The five-dimensional massless fields of $\mathcal{N} = 6$ can be found by setting three arbitrary masses to zero. Without loss of generality after setting $m_1 = m_2 = m_3 = 0$ there is one graviton, six gravitinos, fifteen vectors, twenty spin 1/2 fermions and fourteen scalars. The corresponding fields in four-dimensions are one, six, sixteen, twenty-six and thirty as indicated in table 5.9. The resulting theory has one graviton multiplet in $\mathcal{N} = 6$ in four-dimensions.

Spin	Massless five-dimensional fields	Massless four-dimensional fields
$J = 2$	1	1
$J = 3/2$	4	4
$J = 1$	7	8
$J = 1/2$	8	12
$J = 0$	6	14

Table 5.10: Massless five- and four-dimensional fields in $\mathcal{N} = 4$ supergravity

The massless sector of $\mathcal{N} = 4$ is obtained by setting two masses out of four to be zero. Assume $m_1 = m_2 = 0$. From table 5.7, there is one zero eigenvalue for the five-dimensional graviton, four for gravitinos, seven for vectors, eight for spin 1/2 fermions and six for scalars. In four-dimensions, there is one graviton, four gravitinos, eight vectors, twelve spin 1/2 fermions and fourteen scalars. These fields in $\mathcal{N} = 4$ in four-dimensions are included in one graviton and two vector multiplets.

Spin	Massless five-dimensional fields	Massless four-dimensional fields
$J = 2$	1	1
$J = 3/2$	2	2
$J = 1$	3	4
$J = 1/2$	4	6
$J = 0$	2	6

Table 5.11: Massless five- and four-dimensional fields in $\mathcal{N} = 2$ supergravity

The massless sector of $\mathcal{N} = 2$ is obtained by setting only one mass to zero and keeping the other three non-zero, without loss of generality assume $m_1 = 0$. The massless five-dimensional fields are one graviton, two gravitinos, three vectors, four spin 1/2 fermions, and two scalars and the corresponding fields in four-dimensions are one, two, four, six, and six. In four-dimensions, the fields are contained in one graviton and three vector multiplets in $\mathcal{N} = 2$.

The mass spectrum of the four-dimensional theory is directly given by [14]:

Four-dimensional spin	Eigenvalues of $i\mathcal{M}(m_k)$
$J = 2$	0
$J = 3/2$	$\pm m_k$
$J = 1$	$4(0), \pm(m_i \pm m_j);\ i < j$
$J = 1/2$	$2(\pm m_k), \pm(m_i \pm m_j \pm m_k);\ i < j < k$
$J = 0$	$6(0), \pm(m_i \pm m_j), \pm(m_1 \pm m_2 \pm m_3 \pm m_4);\ i < j$

Table 5.12: Eigenvalues of mass matrix and corresponding four-dimensional fields [14]

To find the massless fields, it should be considered in table 5.12 that the 1/2-spin fermions have one additional $\pm m_k$ eigenvalue from the gravitinos and the scalars have one additional $\pm(m_i \pm m_j)$ eigenvalue from the vectors. The massless fields in $\mathcal{N} = 8$ in four dimensions are the same with the four-dimensional part of table 5.8. For $\mathcal{N} = 6, 4$ and 2 the number of fields obtained from the table 5.12, are the same with numbers in the tables 5.9 - 5.11.

The fields for $\mathcal{N} = 0$, can be obtained easily from both tables 5.7 or 5.12 since all $m_i, i = 1, 2, 3, 4$ are non-zero.

5.3.2 Multiplets

Of particular interest are also the massive multiplets obtained from a dimensional reduction. The massive multiplets can be found from table 5.7.

There are no massive multiplets obtained for $\mathcal{N} = 8$, since all four masses will be set to zero. The massless fields were already calculated in the previous section and they all form one graviton multiplet.

In $\mathcal{N} = 6$ only three arbitrary masses out of a total of four are set to zero. There is also only one massless multiplet consisting of the fields found in table 5.9. In addtion, there are two massive multiplets with mass $|m|$ of one gravitino, six vectors, fourteen spin 1/2 fermions and fourteen scalars.

To obtain the multiplets of $\mathcal{N} = 4$ half of the masses parameters are set to zero. The massless fields found in the previous section are grouped into three multiplets. One massless multiplet consists of a graviton, four gravitinos, six vectors, four spin 1/2 fermions and two scalars and there are also two other multiplets, each one consisting of one vector, four spin 1/2 fermions and six scalars. There are also four massive multiplets with mass $|m_i|$ (two for a fixed i) each containing one gravitino, four vectors, six spin 1/2 fermions and four scalars. The last two massive multiplets have mass $|m_i \pm m_j|$, each having one vector, four spin 1/2 fermions and five scalars.

For the $\mathcal{N} = 2$ only one arbitrary mass has to be set to zero. There are four massless multiplets, the first one has a graviton, two gravitinos, and one vector. The other three contain the same number of fields, each has one vector, two spin 1/2 fermions and two scalars. There are six multiplets (two for fixed i) with mass $|m_i|$ with one gravitino, two vectors and one spin 1/2 fermion. In addition, there are also six more multiplets with mass $|m_i|$ (two for fixed i) with one spin 1/2 fermion and two scalars, each. There are six in total multiplets with mass $|m_i \pm m_j|$ (two for fixed i) each consisting of one vector, two spin 1/2 fermions and a scalar. The last multiplet is obtained two times for mass $|m_i \pm m_j \pm m_k|$ with one spin 1/2 fermion and two scalars.

5.4 The mass matrix

The mass matrix with respect to the symplectic metric (5.5) used in (5.6) is explicitly given by:

$$M = \begin{bmatrix} 0 & 0 & 0 & 0 & m_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_4 \\ -m_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -m_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -m_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -m_4 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{5.7}$$

The monodromy of (5.7) is:

$$\mathcal{M}= \begin{bmatrix} cos(m_1) & 0 & 0 & 0 & sin(m_1) & 0 & 0 & 0 \\ 0 & cos(m_2) & 0 & 0 & 0 & sin(m_2) & 0 & 0 \\ 0 & 0 & cos(m_3) & 0 & 0 & 0 & sin(m_3) & 0 \\ 0 & 0 & 0 & cos(m_4) & 0 & 0 & 0 & sin(m_4) \\ -sin(m_1) & 0 & 0 & 0 & cos(m_1) & 0 & 0 & 0 \\ 0 & -sin(m_2) & 0 & 0 & 0 & cos(m_2) & 0 & 0 \\ 0 & 0 & -sin(m_3) & 0 & 0 & 0 & cos(m_3) & 0 \\ 0 & 0 & 0 & -sin(m_4) & 0 & 0 & 0 & cos(m_4) \end{bmatrix}, \tag{5.8}$$

where $m_i, i = 1, 2, 3, 4$ are real mass parameters.

The mass matrix with respect to the symplectic metric:

$$\Omega = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}, \tag{5.9}$$

is the matrix:

$$M = \begin{bmatrix} 0 & m_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -m_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -m_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & -m_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & -m_4 & 0 \end{bmatrix}. \tag{5.10}$$

Then the exponential of this matrix is:

$$\mathcal{M}= \begin{bmatrix} cos(m_1) & sin(m_1) & 0 & 0 & 0 & 0 & 0 & 0 \\ -sin(m_1) & cos(m_1) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & cos(m_2) & sin(m_2) & 0 & 0 & 0 & 0 \\ 0 & 0 & -sin(m_2) & cos(m_2) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & cos(m_3) & sin(m_3) & 0 & 0 \\ 0 & 0 & 0 & 0 & -sin(m_3) & cos(m_3) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & cos(m_4) & sin(m_4) \\ 0 & 0 & 0 & 0 & 0 & 0 & -sin(m_4) & cos(m_4) \end{bmatrix}, \tag{5.11}$$

The matrices M, and $\mathcal{M}$ are elements of the lie algebra $usp(8)$ and group $USp(8)$, respectively. Both of these matrices will be considered in chapter 7.

Chapter 6

Black holes

String theory is a promising candidate as a theory for quantum gravity, which means string theory should be capable of describing black holes. Simple perturbative string theory is not sufficient to describe them because black holes have a strong coupling. To solve this problem, D-branes are used, whose non-perturbative properties enhance their ability to describe black holes [36]. The initial ten-dimensional black holes in this section are solutions of $\mathcal{N} = 2$ supergravity. In this chapter, all of the previously discussed theoretical tools are applied in black holes. At first, in sections 6.1 and 6.2 some famous examples of black holes are discussed. Later in section 6.3 different black holes are created and their potentials, and metric after the compactification are given, as well as their equivalent Einstein frame. In section 6.4 there will be a brief introduction in black hole thermodynamics and the calculation of the entropy for all examples of black holes. Finally, in section 6.5 the notion of near-horizon geometry for extremal black holes is discussed.

6.1 Schwarzschild metric

The Einstein-Hilbert action in D-dimensions is:

$$S_{EH} = \frac{1}{16\pi G_N^D} \int d^D x \sqrt{-g} R, \tag{6.1}$$

where D is the spacetime dimension, g the determinant of the metric, and R the Ricci scalar. The Schwarzschild metric is the unique, spherical solution of the action (6.1) in a vacuum:

$$ds^2 = -(1 - \frac{2G_N M}{r})dt^2 + (1 - \frac{2G_N M}{r})^{-1} dr^2 + r^2 d\Omega^2, \tag{6.2}$$

where $d\Omega^2 = d\theta^2 + sin^2\theta d\phi^2$, M is the mass of the gravitational object, the black hole, and G_N the gravitational constant.

The Minkowski spacetime with metric $ds^2 = -dt^2 + dr^2 + r^2 d\Omega^2$ is obtained as M approaches zero. Additionally, as the radius of the Schwarzschild metric approaches infinity, then the Minkowski metric is acquired again. This property is called asymptotic flatness.

The horizon of this black hole is found at the radius that satisfies: $g^{rr} = 0$. The event horizon is at $r = 2GM$ distance from the black hole center [10].

6.2 Reissner-Nordström metric

The Reissner-Nordström metric is the exact solution of electrically and magnetically charged black holes, which are spherically symmetric. The metric is:

$$ds^2 = -\Delta dt^2 + \Delta^{-1} dr^2 + r^2 d\Omega^2, \tag{6.3}$$

where Δ is defined in terms of the black hole mass and electric charge:

$$\Delta(r) = 1 - \frac{2G_N M}{r} + \frac{G_N(Q^2 + P^2)}{r^2}, \tag{6.4}$$

and M is the black hole mass, G_N the gravitational constant, Q the total electric charge and P the total magnetic charge. It also has a gauge field with non-zero components:

$$\begin{aligned} F_{rt} &= \frac{Q}{r^2} \\ F_{\theta\phi} &= P sin\theta. \end{aligned} \tag{6.5}$$

The event horizon is located at the values of r that satisfy the equation $\Delta(r) = 0$,

$$r_{\pm} = G_N M \pm \sqrt{G_N^2 M^2 - G_N(Q^2 + P^2)}. \tag{6.6}$$

The square root of (6.6) indicates three separate cases:

$$G_N M^2 < Q^2 + P^2 \tag{6.7}$$

$$G_N M^2 > Q^2 + P^2 \tag{6.8}$$

$$G_N M^2 = Q^2 + P^2. \tag{6.9}$$

The cosmic censorship conjecture, states that singularities should be hidden from an observer by the event horizon, there cannot be naked singularities. The case (6.7) indicates that there is no event horizon. This establishes an argument that this black hole is not a physically possible solution. The case (6.8) has two event horizons and it is expected to be found in nature. The third case, (6.9) is called the extreme solution. Extremal black holes are unstable, with only one event horizon, but they are a very useful theoretical model, since all calculations become much more simple. For example, some symmetries are left unbroken in supersymmetry.

To find the coordinate system in isotropic coordinates of the extremal black hole, one can substitute $\rho = r - G_N M$ in (6.3). This leads to an event horizon located at $\rho = 0$ and the extremal black hole metric in isotropic coordinates is given by:

$$ds^2 = -H(\rho)^{-2} dt^2 + H(\rho)^2 (dr^2 + r^2 d\Omega_2^2), \tag{6.10}$$

where H is a harmonic function given by:

$$H(\rho) = 1 + \frac{G_N M}{\rho}. \tag{6.11}$$

6.3 Black holes, D-branes and a gravitational wave

In this section, the objects used to construct a black hole are only D-branes and the aim is to find extremal four-dimensional black holes via the reduction of six-dimensions. To reduce the corrections needed for the low energy solution, the moduli and dilaton should be finite at the horizon and to be able to manage quantum corrections, and a fraction of the initial supersymmetry has to be preserved. A finite moduli condition means having a finite metric on the torus dimensions. For these conditions to be satisfied, some amount of calculation is needed to find all possible configurations. Even though the explanation and results can already be found in literature [2], the next three paragraphs will serve as an overview of this.

Consider a D_p and a $D_{p'}$-brane. These branes in type IIA string theory, lie in an even number of spatial dimensions, with the exception of branes that couple to the NS-NS sector field $B_{\mu\nu}$ with field strength $H_{\mu\nu\rho}$. The supersymmetry condition concludes that in a configuration of a D_p-brane and a D_p'-brane: $p+p'-2k=0$ mod 4, where k is the number of dimensions the two branes intersect. Additionally, to construct a black hole with non-vanishing and finite entropy, one needs to consider four D-branes.

After applying all these restrictions, one can obtain the three possible black holes. The first black hole (case A) consists of one D_0-brane and three D_4, the second one (case B) with two D_4 and two D_2 and the third (case C) with one D_6 and three D_2. The moduli condition plays a significant role in the spatial dimensions that these branes will be parallel to.

After acquiring a configuration of black holes, one has to address how to combine the D-branes. As it was already mentioned, the metric and dilaton of these D-branes are expressed via their equivalent harmonic functions. The harmonic function rule [56] states that the metric components of the summation of D-branes can be written as the product of the metric components for each different D-brane. For example, in the case of three D_4-branes and one D_0-brane, from (2.14) the corresponding g_{00} components are:

$$H_{4a}^{-1/2}dt^2,\ H_{4b}^{-1/2}dt^2,\ H_{4c}^{-1/2}dt^2 \text{ and } H_0^{-1/2}dt^2, \tag{6.12}$$

thus using the harmonic function rule, the g_{00} component for the whole system of D-branes is:

$$dt^2(H_{4a}^{-1/2}H_{4b}^{-1/2}H_{4c}^{-1/2}H_0^{-1/2}). \tag{6.13}$$

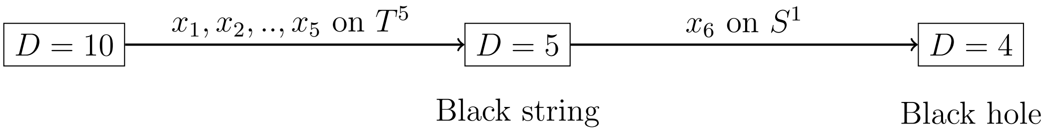
The same rule also applies to the dilaton. The three cases of black holes with only D-branes are studied further below.

The fourth black hole (case D) consists of D_6, D_2, NS_5 and the *pp*-wave introduced in section 3.2.5. This wave adds momentum on the direction it lies and all of the branes in the configuration are parallel in that direction.

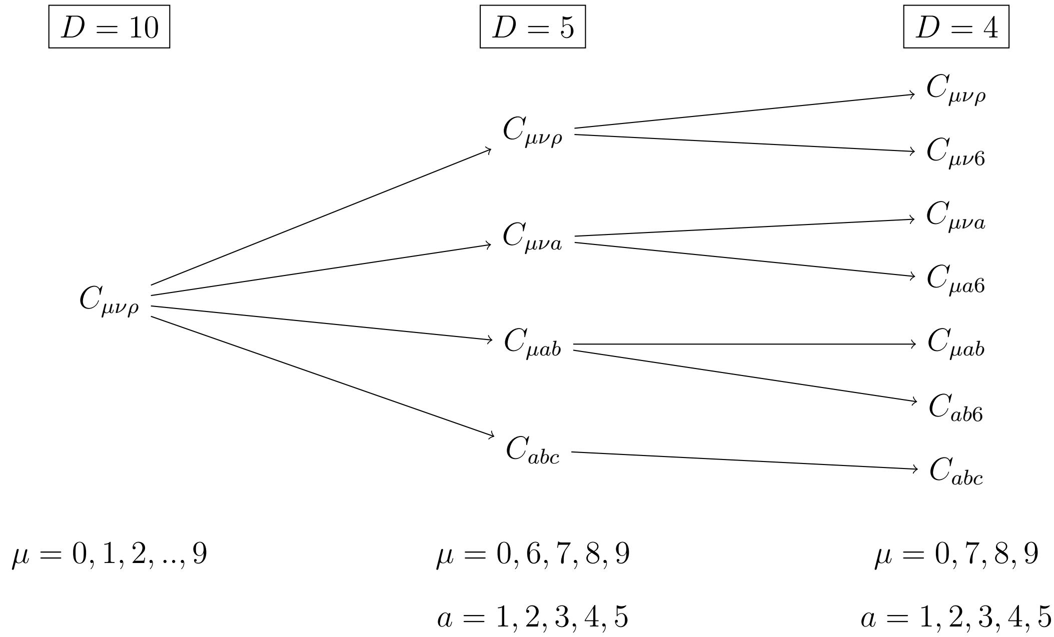
The dimensions marked with $\times$ are the ones where the branes lie on, and the spatial dimensions $x_1, .., x_6$ are the dimensions which will be compactified. The spatial dimensions denoted with a dot, along with time dimension will be the dimensions of the four-dimensional black hole.

6.3.1 Compactification of black holes

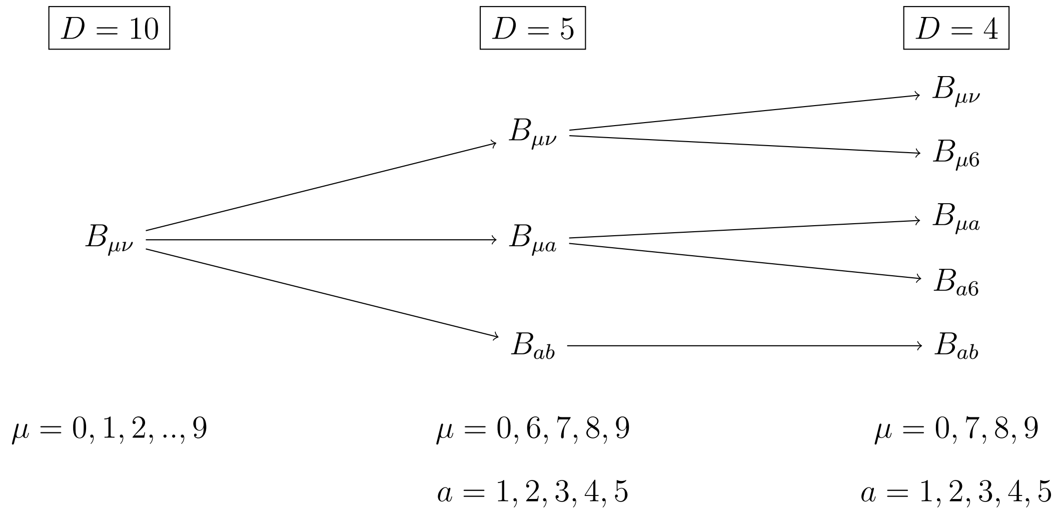
First the ten-dimensional black hole will be reduced on a five-torus resulting on a black string. Finally, a compactification on a circle on x_6 will give a four-dimensional black hole. The three dimensions denoted with a dot, and the time x_0 are the dimensions that will survive the compactification.



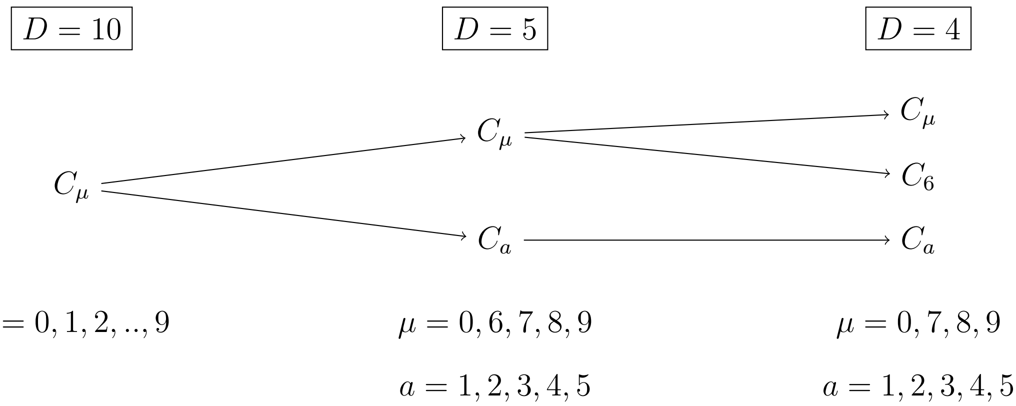
The fields that support a black hole are 1-forms, 3-forms and one 2-form, $B_{\mu\nu}$ from the NS-NS sector. The reduction of the ten-dimensional 3-form is shown in the following diagram.



The reduction of the 2-form is given below.



Finally, the ten-dimensional 1-form and its reduced five- and four-dimensional potentials are given in the following diagram.



Following the above diagrams, one can see how potentials reduce. Keep in mind that some of these five-dimensional fields, when reduced to four-dimensional fields with a Scherk-Schwarz reduction may gain masses.

An example of a reduced potential followed from these diagrams is the potential coupled with a D_2-brane, the ten-dimensional potential $C_{\mu\nu\rho}$, where $\mu = 0, .., 9$, with non-zero entries: $C_{025} = \frac{1}{2}(H_2^{-1} - 1)$. In five-dimensions only the five-vector of type $C_{\mu ab}$ is non-zero and specifically the five-vector $C_{\mu 25}$. The only non-zero entry of this vector is $C_{025} = \frac{1}{2}(H_2^{-1} - 1)$, at $\mu = 0$. In four-dimensions, only the four-vector $C_{\mu 25}$ is non-zero with the same non-zero entry as the five-vector, at $\mu = 0$.

Another interesting example is that of a D_2-brane with ten-dimensional potential $C_{016} = \frac{1}{2}(H_2^{-1} - 1)$. From the diagram, only the 2-form $C_{\mu\nu 1}$ is non-zero in five-dimensions, with non-zero entries at $\mu, \nu = 0, 6,\ \mu \neq \nu$, where $C_{016} = \frac{1}{2}(H_2^{-1} - 1)$. For the purpose of this thesis, the dualized vector of this 2-form will be used. In four-dimensions, only the four-vector with the form $C_{\mu 16}$ is non-zero, with non-zero entry at $\mu = 0$, $C_{016} = \frac{1}{2}(H_2^{-1} - 1)$.

An example will be given on how to reduce a field-strength written as $F'_{\mu\nu\rho\sigma}$ in ten-dimensions, with indices $\mu, \nu, \rho, \sigma = 5, 6, 7, 8, 9$. These field strengths will be encountered often in the rest of the thesis from the dualization of the fields of a black hole. The compactification from ten- to five-dimensions is:

$F'_{\mu\nu\rho\sigma}$ in ten-dimensions $\rightarrow F'_{\mu\nu\rho 5}$ in five-dimensions, for $\mu, \nu, \rho = 6, 7, 8, 9$

All the possible terms of $F'_{\mu\nu\rho 5}$ are:

$$F'_{\mu\nu\rho 5} \qquad \begin{array}{l} \partial_\mu C_{\nu\rho 5} dx^\mu \wedge dx^\nu \wedge dx^\rho \wedge dx^5 \\ \partial_\nu C_{\rho 5 \mu} dx^\nu \wedge dx^\rho \wedge dx^5 \wedge dx^\mu \\ \partial_\rho C_{5\mu\nu} dx^\rho \wedge dx^5 \wedge dx^\mu \wedge dx^\nu \\ \partial_5 C_{\mu\nu\rho} dx^5 \wedge dx^\mu \wedge dx^\nu \wedge dx^\rho \end{array}$$

The solution $\partial_5 C_{\mu\nu\rho} dx^5 \wedge dx^\mu \wedge dx^\nu \wedge dx^\rho$ is zero, since x_5-direction was compactified. Thus the only possible reduction is of the form $C_{\mu\nu a}$.

To find the field in four-dimensions, one should compactify on a circle on direction 6.

$F'_{\mu\nu\rho\sigma}$ in ten-dimensions $\rightarrow F'_{\mu\nu 56}$ in four-dimensions, for $\mu, \nu = 7, 8, 9$

The field strength $F'_{\mu\nu 56}$ can be written in terms of the potential $C_{\mu\nu\rho}$ as:

$$F'_{\mu\nu 56} \qquad \begin{array}{l} \partial_\mu C_{\nu 56} dx^\mu \wedge dx^\nu \wedge dx^5 \wedge dx^6 \\ \partial_\nu C_{56\mu} dx^\nu \wedge dx^5 \wedge dx^6 \wedge dx^\mu \\ \partial_5 C_{6\mu\nu} dx^5 \wedge dx^6 \wedge dx^\mu \wedge dx^\nu \\ \partial_6 C_{\mu\nu 5} dx^6 \wedge dx^\mu \wedge dx^\nu \wedge dx^5 \end{array}$$

With the same logic as the five-dimensional field, some terms are zero. This procedure is done for all reduced fields.

The reduction of a *pp*-wave was shown in section 3.2.5.

6.3.2 Black hole with D_{4a}, D_{4b}, D_{4c} and D_0

The first black hole comprised of three D_4-branes and one D_0-brane, has the following configuration:

Case A	0	1	2	3	4	5	6	7	8	9
D_{4a}	×	×	×	×	×			.	.	.
D_{4b}	×			×	×	×	×	.	.	.
D_{4c}	×	×	×			×	×	.	.	.
D_0	×							.	.	.

To derive the metric of this configuration, the harmonic function rule should be used to combine the metrics of D_{4a}, D_{4b}, D_{4c} and D_0 from (2.14). The metric of this black hole, dilaton and potentials in string frame, are given below.

$$\begin{cases} ds^2_{10} = - dt^2 (H_{4a}^{-1/2} H_{4b}^{-1/2} H_{4c}^{-1/2} H_0^{-1/2}) \\ \qquad + (dx_1^2 + dx_2^2)(H_{4a}^{-1/2} H_{4b}^{1/2} H_{4c}^{-1/2} H_0^{1/2}) \\ \qquad + (dx_3^2 + dx_4^2)(H_{4a}^{-1/2} H_{4b}^{-1/2} H_{4c}^{1/2} H_0^{1/2}) \\ \qquad + (dx_5^2 + dx_6^2)(H_{4a}^{1/2} H_{4b}^{-1/2} H_{4c}^{-1/2} H_0^{1/2}) \\ \qquad + (dx_7^2 + dx_8^2 + dx_9^2)(H_{4a}^{1/2} H_{4b}^{1/2} H_{4c}^{1/2} H_0^{1/2}) \\ e^\phi = H_0^{\frac{3}{4}} H_{4a}^{\frac{-1}{4}} H_{4b}^{\frac{-1}{4}} H_{4c}^{\frac{-1}{4}} \\ C_{01234} = \frac{1}{2}(H_{4a}^{-1} - 1) \\ \tilde{C}_{03456} = \frac{1}{2}(H_{4b}^{-1} - 1) \\ \tilde{\tilde{C}}_{01256} = \frac{1}{2}(H_{4c}^{-1} - 1) \\ \tilde{\tilde{\tilde{C}}}_0 = \frac{1}{2}(H_0^{-1} - 1) \end{cases} \tag{6.14}$$

A tilde will be used to differentiate between potentials corresponding to different branes. The harmonic functions of the branes are given by:

$$\begin{aligned} H_0 =&1 + \frac{Q_0}{r}, \\ H_{4a} =&1 + \frac{Q_{4a}}{r}, \\ H_{4b} =&1 + \frac{Q_{4b}}{r}, \\ H_{4c} =&1 + \frac{Q_{4c}}{r} \end{aligned} \tag{6.15}$$

Where Q_p is the charge of a Dp-brane. The harmonic function of the whole system of D-branes has a rotational symmetry on directions $7, 8$, and 9: $H_p = 1 + \frac{Q_p}{r}$, where $r^2 = x_7^2 + x_8^2 + x_9^2$. This is true for all four black holes in this thesis.

One first has to rewrite the gauge fields in forms compatible with string theory, either 2- and 4-form field strengths, or with their equivalent 1- and 3-form potentials: $F_2 = dC_1$ and $F_4 = dC_3$.

The first three gauge fields has to be written in a dual form. The gauge field:

$$C_5 = C_{01234} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^4 \tag{6.16}$$

can be rewritten as:

$$F_6 = dC_5 = \partial_\mu C_{01234} dx^\mu \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^4 \tag{6.17}$$

and

$$\partial_\mu C_{01234} = -\frac{1}{2} H_{4a}^{-2} \partial_\mu H_{4a}, \tag{6.18}$$

in this case $\mu = 5, 6, 7, 8, 9$, because the rotational symmetry is in the directions that the brane does not lie as discussed in the section of p-branes. As one can see from the table of Case A, each brane has a rotation symmetry in different directions, for example, the brane D_{4b} has rotational symmetry in $\mu = 1, 2, 7, 8, 9$, but after compactification, all branes will posses rotational symmetry only in the directions $\mu = 7, 8, 9$.

Afterwards, one must find the Hodge dual of F_6. All the dualized field strengths will be written as F', which is defined as $*F_6 = F'_4$. F'_4 can be written in terms of F_6, using the levi-civita symbol ϵ and levi-civita tensor $\bar{\epsilon}$ and equations (5.2), (5.3):

$$\begin{aligned} F'_{\mu\nu\rho\sigma} &= \frac{1}{6!} \bar{\epsilon}^{\tau_1..\tau_6}{}_{\mu\nu\rho\sigma} F_{\tau_1..\tau_6} \\ &= \bar{\epsilon}^{\lambda 01234}{}_{\mu\nu\rho\sigma} F_{\lambda 01234}, \text{ where } \lambda, \mu, \nu, \rho, \sigma = 5, 6, 7, 8, 9. \end{aligned} \tag{6.19}$$

Using this calculation for the rest of the potentials that need to be dualized and defining $\epsilon_{01234\lambda\mu\nu\rho\sigma}$ as $\epsilon_{\lambda\mu\nu\rho\sigma}$, one will obtain:

$$\begin{cases} F'_{\mu\nu\rho\sigma} =& -\frac{1}{2} \epsilon_{\lambda\mu\nu\rho\sigma} \partial_\lambda H_{4a}, \;\; \lambda, \mu, \nu, \rho, \sigma = 5, 6, 7, 8, 9 \\ \tilde{F}'_{\mu\nu\rho\sigma} =& -\frac{1}{2} \epsilon_{\lambda\mu\nu\rho\sigma} \partial_\lambda H_{4b}, \;\; \lambda, \mu, \nu, \rho, \sigma = 1, 2, 7, 8, 9 \\ \tilde{\tilde{F}}'_{\mu\nu\rho\sigma} =& -\frac{1}{2} \epsilon_{\lambda\mu\nu\rho\sigma} \partial_\lambda H_{4a}, \;\; \lambda, \mu, \nu, \rho, \sigma = 3, 4, 7, 8, 9 \\ \tilde{\tilde{\tilde{C}}}_0 =& \frac{1}{2}(H_0^{-1} - 1) \end{cases} \tag{6.20}$$

When the metric in ten-dimensions is diagonal one is allowed to just erase the metric components of the compactified dimensions. The compactified metric is obtained by first compactifying in T^5 for $x_1, ..x_5$, this gives out a black string. Afterwards, compactify on S^1.

Then the metric for the black string in case A in five-dimensions is:

$$\begin{aligned} ds_5^2 =& - dt^2(H_{4a}^{-1/2} H_{4b}^{-1/2} H_{4c}^{-1/2} H_0^{-1/2}) \\ &+ (dx_6^2)(H_{4a}^{1/2} H_{4b}^{-1/2} H_{4c}^{-1/2} H_0^{1/2}) \\ &+ (dx_7^2 + dx_8^2 + dx_9^2)(H_{4a}^{1/2} H_{4b}^{1/2} H_{4c}^{1/2} H_0^{1/2}) \end{aligned} \tag{6.21}$$

The metric for the black hole in four-dimensions for case A is:

$$\begin{aligned} ds_4^2 =& - dt^2(H_{4a}^{-1/2} H_{4b}^{-1/2} H_{4c}^{-1/2} H_0^{-1/2}) \\ &+ (dx_7^2 + dx_8^2 + dx_9^2)(H_{4a}^{1/2} H_{4b}^{1/2} H_{4c}^{1/2} H_0^{1/2}) \end{aligned} \tag{6.22}$$

To perform a Scherk-Schwarz reduction and analyze which vector fields remain massless in four dimensions, the five-dimensional fields needed are 1-forms and 2-forms, which can be dualized to 1-forms. This dualization of fields is done after performing the field reduction [34].

Coupled to the D_{4a}-brane the five-dimensional vector fields are the dualized fields from $C_{\mu\nu a}$. In four-dimensions this is $C_{\mu a 6}$, where $a = 5$. For D_{4b}-brane and D_{4c}-branes, the vector field potentials in both five and four-dimensions are $C_{\mu ab}$, where $a, b = 1, 2$ and $a, b = 3, 4$, respectively. For D_0-brane after compactification one obtains a five-vector in five-dimensions and a four-vector in four-dimensions C_μ, with only non-zero element $C_0 = \frac{1}{2}(H_0^{-1} - 1)$.

To find the metric in Einstein frame, one has to first calculate the relation with the metric in string frame using (2.7). In four-dimensions, it becomes:

$$g_{\mu\nu}^{(s)} = g_{\mu\nu}^{(E)} e^{2\phi_4}, \tag{6.23}$$

where ϕ_4 is the four-dimensional dilaton. This formula applies to all three black holes with D-branes. After the compactification on directions $x_1, x_2, .., x_6$:

$$e^{2\phi_4} \sqrt{g_{11}^{(s)} g_{22}^{(s)} .. g_{66}^{(s)}} = e^{2\phi_{10}}. \tag{6.24}$$

The expressions for $e^{2\phi_{10}}$ and $g_{11}^{(s)} g_{22}^{(s)} .. g_{66}^{(s)}$ are taken from (6.14), resulting in $e^{2\phi_4} = 1$. This means that $g_{\mu\nu}^{(s)} = g_{\mu\nu}^{(E)}$, thus the metric in Einstein frame is:

$$\begin{aligned} ds_4^2 =& - dt^2(H_{4a}^{-1/2} H_{4b}^{-1/2} H_{4c}^{-1/2} H_0^{-1/2}) \\ &+ (dx_7^2 + dx_8^2 + dx_9^2)(H_{4a}^{1/2} H_{4b}^{1/2} H_{4c}^{1/2} H_0^{1/2}) \\ e^{\phi_4} =& 1. \end{aligned} \tag{6.25}$$

6.3.3 Black hole with D_{4a}, D_{4b}, D_{2a} and D_{2b}

The second case (case B) of a black hole consists of two D_4-branes and two D_2-branes.

Case B	0	1	2	3	4	5	6	7	8	9
D_{4a}	×	×	×	×	×			.	.	.
D_{4b}	×			×	×	×	×	.	.	.
D_{2a}	×	×					×	.	.	.
D_{2a}	×		×			×		.	.	.

To derive the metric of case B, the harmonic function rule and the metrics of D_{4a}, D_{4b}, D_{2a} and D_{2b} from (2.14) must be used. The metric of the black hole in string frame is:

$$\begin{cases} ds^2_{10} = & - dt^2(H_{4a}^{-1/2}H_{4b}^{-1/2}H_{2a}^{-1/2}H_{2b}^{-1/2}) \\ & + (dx_1^2)(H_{4a}^{-1/2}H_{4b}^{1/2}H_{2a}^{-1/2}H_{2b}^{1/2}) \\ & + (dx_2^2)(H_{4a}^{-1/2}H_{4b}^{1/2}H_{2a}^{1/2}H_{2b}^{-1/2}) \\ & + (dx_3^2+dx_4^2)(H_{4a}^{-1/2}H_{4b}^{-1/2}H_{2a}^{1/2}H_{2b}^{1/2}) \\ & + (dx_5^2)(H_{4a}^{1/2}H_{4b}^{-1/2}H_{2a}^{1/2}H_{2b}^{-1/2}) \\ & + (dx_6^2)(H_{4a}^{1/2}H_{4b}^{-1/2}H_{2a}^{-1/2}H_{2b}^{1/2}) \\ & + (dx_7^2+dx_8^2+dx_9^2)(H_{4a}^{1/2}H_{4b}^{1/2}H_{2a}^{1/2}H_{2b}^{1/2}) \\ e^\phi = & H_{4a}^{\frac{-1}{4}}H_{4b}^{\frac{-1}{4}}H_{2a}^{\frac{1}{4}}H_{2b}^{\frac{1}{4}} \\ C_{01234} = & \frac{1}{2}(H_{4a}^{-1}-1) \\ \tilde{C}_{03456} = & \frac{1}{2}(H_{4b}^{-1}-1) \\ \tilde{\tilde{C}}_{016} = & \frac{1}{2}(H_{2a}^{-1}-1) \\ \tilde{\tilde{\tilde{C}}}_{025} = & \frac{1}{2}(H_{2b}^{-1}-1) \end{cases} \tag{6.26}$$

The harmonic functions of the D_4- and D_2-branes are given by:

$$\begin{aligned} H_{4a} =& 1+\frac{Q_{4a}}{r} \\ H_{4b} =& 1+\frac{Q_{4b}}{r} \\ H_{2a} =& 1+\frac{Q_{2a}}{r} \\ H_{2b} =& 1+\frac{Q_{2b}}{r} \end{aligned} \tag{6.27}$$

Only the gauge fields from the D_4-branes have to dualized. Following the calculation mentioned in (6.16) - (6.19) one obtains:

$$\begin{cases} F'_{\mu\nu\rho\sigma} = & -\frac{1}{2}\epsilon_{\lambda\mu\nu\rho\sigma}\partial_\lambda H_{4a}, \ \ \lambda,\mu,\nu,\rho,\sigma = 5,6,7,8,9 \\ \tilde{F}'_{\mu\nu\rho\sigma} = & -\frac{1}{2}\epsilon_{\lambda\mu\nu\rho\sigma}\partial_\lambda H_{4b}, \ \ \lambda,\mu,\nu,\rho,\sigma = 1,2,7,8,9 \\ \tilde{\tilde{C}}_{016} = & \frac{1}{2}(H_{2a}^{-1}-1) \\ \tilde{\tilde{\tilde{C}}}_{025} = & \frac{1}{2}(H_{2b}^{-1}-1) \end{cases} \tag{6.28}$$

The reduced metric on a five-torus for case B is:

$$\begin{aligned} ds_5^2 =& - dt^2(H_{4a}^{-1/2}H_{4b}^{-1/2}H_{2a}^{-1/2}H_{2b}^{-1/2}) \\ &+ (dx_6^2)(H_{4a}^{1/2}H_{4b}^{-1/2}H_{2a}^{-1/2}H_{2b}^{1/2}) \\ &+ (dx_7^2 + dx_8^2 + dx_9^2)(H_{4a}^{1/2}H_{4b}^{1/2}H_{2a}^{1/2}H_{2b}^{1/2}) \end{aligned} \tag{6.29}$$

A further compactification on a circle gives:

$$\begin{aligned} ds_4^2 =& - dt^2(H_{4a}^{-1/2}H_{4b}^{-1/2}H_{2a}^{-1/2}H_{2b}^{-1/2}) \\ &+ (dx_7^2 + dx_8^2 + dx_9^2)(H_{4a}^{1/2}H_{4b}^{1/2}H_{2a}^{1/2}H_{2b}^{1/2}) \end{aligned} \tag{6.30}$$

In case B, there are two D_4-branes and two D_2-branes. The field $F'_{\mu\nu\rho\sigma}$ of D_{4a}-brane is the potential of the 2-form $C_{\mu\nu a}$ in five-dimensions and $C_{\mu a 6}$ in four-dimensions, where $a = 5$. The D_{4b}-brane is charged by the potential $C_{\mu ab}$, $a = 1, b = 2$ in both four and five-dimensions. The D_{2a}-brane is coupled to the potential $C_{\mu\nu 1}$ in five-dimensions and $C_{\mu 16}$ in four-dimensions. The D_{2b}-brane has potential $C_{\mu ab}, a = 2, b = 5$ in both reduced dimensions, with the difference that it is a five-vector in five-dimensions and a four-vector in four-dimensions. The only non-zero element of the potential for D_{2a}-brane in both reduced dimensions is $C_{016} = \frac{1}{2}(H_{2a}^{-1} - 1)$ and for D_{2b}-brane, it is $C_{025} = \frac{1}{2}(H_{2b}^{-1} - 1)$.

Repeating the calculation of case A, the string and Einstein frame metrics are the same for case B. Hence, in Einstein frame the metric is:

$$\begin{aligned} ds_4^2 =& - dt^2(H_{4a}^{-1/2}H_{4b}^{-1/2}H_{2a}^{-1/2}H_{2b}^{-1/2}) \\ &+ (dx_7^2 + dx_8^2 + dx_9^2)(H_{4a}^{1/2}H_{4b}^{1/2}H_{2a}^{1/2}H_{2b}^{1/2}) \\ e^{\phi_4} =& 1 \end{aligned} \tag{6.31}$$

6.3.4 Black hole with D_6, D_{2a}, D_{2b} and D_{2c}

The last case of a black hole consisting of only D-branes (case C) is that of a D_6-brane and three D_2-branes.

Case C	0	1	2	3	4	5	6	7	8	9
D_6	×	×	×	×	×	×	×	.	.	.
D_{2a}	×	×	×					.	.	.
D_{2b}	×			×	×			.	.	.
D_{2c}	×					×	×	.	.	.

The metric of this configuration in string frame is given using the harmonic function rule and the metric of each of the branes D_6, D_{2a}, D_{2b} and D_{2c} from (2.14):

$$\begin{cases} ds_{10}^2 = & - dt^2(H_6^{-1/2}H_{2a}^{-1/2}H_{2b}^{-1/2}H_{2c}^{-1/2}) \\ & + (dx_1^2 + dx_2^2)(H_6^{-1/2}H_{2a}^{-1/2}H_{2b}^{1/2}H_{2c}^{1/2}) \\ & + (dx_3^2 + dx_4^2)(H_6^{-1/2}H_{2a}^{1/2}H_{2b}^{-1/2}H_{2c}^{1/2}) \\ & + (dx_5^2 + dx_6^2)(H_6^{-1/2}H_{2a}^{1/2}H_{2b}^{1/2}H_{2c}^{-1/2}) \\ & + (dx_7^2 + dx_8^2 + dx_9^2)(H_6^{1/2}H_{2a}^{1/2}H_{2b}^{1/2}H_{2c}^{1/2}) \\ e^{\phi} = & H_6^{\frac{-3}{4}}H_{2a}^{\frac{1}{4}}H_{2b}^{\frac{1}{4}}H_{2c}^{\frac{1}{4}} \\ C_{0123456} = & \frac{1}{2}(H_6^{-1} - 1) \\ \tilde{C}_{012} = & \frac{1}{2}(H_{2a}^{-1} - 1) \\ \tilde{\tilde{C}}_{034} = & \frac{1}{2}(H_{2b}^{-1} - 1) \\ \tilde{\tilde{\tilde{C}}}_{056} = & \frac{1}{2}(H_{2c}^{-1} - 1) \end{cases} \tag{6.32}$$

The corresponding harmonic functions are:

$$\begin{aligned} H_{2a} = & 1 + \frac{Q_{2a}}{r} \\ H_{2b} = & 1 + \frac{Q_{2b}}{r} \\ H_{2c} = & 1 + \frac{Q_{2c}}{r} \\ H_6 = & 1 + \frac{Q_6}{r} \end{aligned} \tag{6.33}$$

The configurations for cases A,B and C are T-dual to each other, and one might say that there is a unique black hole in four-dimensions up to T-duality [2].

The D_6-brane gauge is the only one that has to be dualized:

$$\begin{cases} F'_{\mu\nu} = & -\frac{1}{2}\epsilon_{\lambda\mu\nu}\partial_\lambda H_{4a}, \;\; \lambda, \mu, \nu = 7, 8, 9 \\ \tilde{C}_{012} = & \frac{1}{2}(H_{2a}^{-1} - 1) \\ \tilde{\tilde{C}}_{034} = & \frac{1}{2}(H_{2b}^{-1} - 1) \\ \tilde{\tilde{\tilde{C}}}_{056} = & \frac{1}{2}(H_{2c}^{-1} - 1) \end{cases} \tag{6.34}$$

Finally, the compactification for case C in five-dimensions:

$$\begin{aligned} ds_5^2 = & - dt^2(H_6^{-1/2}H_{2a}^{-1/2}H_{2b}^{-1/2}H_{2c}^{-1/2}) \\ & + (dx_6^2)(H_6^{-1/2}H_{2a}^{1/2}H_{2b}^{1/2}H_{2c}^{-1/2}) \\ & + (dx_7^2 + dx_8^2 + dx_9^2)(H_6^{1/2}H_{2a}^{1/2}H_{2b}^{1/2}H_{2c}^{1/2}) \end{aligned} \tag{6.35}$$

The compactified black hole in four-dimensions is for case C:

$$\begin{aligned} ds_4^2 =& - dt^2(H_6^{-1/2}H_{2a}^{-1/2}H_{2b}^{-1/2}H_{2c}^{-1/2}) \\ &+ (dx_7^2 + dx_8^2 + dx_9^2)(H_6^{1/2}H_{2a}^{1/2}H_{2b}^{1/2}H_{2c}^{1/2}) \end{aligned} \tag{6.36}$$

In the case C, the D_6-brane vector potential is C_μ in both four and five-dimensions. The branes D_{2a} and D_{2b} are coupled to the vector field $C_{\mu ab}$ in both four and five-dimensions, with respective values $a = 1, b = 2$ and $a = 3, b = 4$. Their respective non-zero values are $C_{012} = (H_{2a}^{-1} - 1)$ and $C_{034} = (H_{2b}^{-1} - 1)$. The potential of D_{2a}-brane is $C_{\mu\nu a}$ in five-dimensions and $C_{\mu a6}$ in four-dimensions, $a = 5$. It has non-zero value $C_{056} = \frac{1}{2}(H_{2c}^{-1} - 1)$.

The metric in Einstein frame is:

$$\begin{aligned} ds_4^2 =& - dt^2(H_6^{-1/2}H_{2a}^{-1/2}H_{2b}^{-1/2}H_{2c}^{-1/2}) \\ &+ (dx_7^2 + dx_8^2 + dx_9^2)(H_6^{1/2}H_{2a}^{1/2}H_{2b}^{1/2}H_{2c}^{1/2}) \\ e^{\phi_4} =& 1. \end{aligned} \tag{6.37}$$

6.3.5 Black hole with *D*-branes and a *pp*-wave

In this section, an example with a *pp*-wave, W is studied. This wave was introduced and compactified in section 3.2.5. In this black hole, there is momentum to the direction x_6.

Case D	0	1	2	3	4	5	6	7	8	9
D_6	×	×	×	×	×	×	×	.	.	.
NS_5	×		×	×	×	×	×	.	.	.
D_2	×	×					×	.	.	.
W	×						→	.	.	.

The metric of the above configuration can be found in string frame using the harmonic function rule, the D-brane metric (2.14), the NS_5 metric and the metric of the *pp*-wave (3.17). The metric of the above configuration in string frame [36]:

$$\begin{cases} ds_{10}^2 =& H_2^{-1/2}H_6^{-1/2}[dt^2(K-1) + dx_6^2(K+1) - Kdtdx_6 - Kdx_6dt] \\ &+ dx_1^2(H_6^{-1/2}H_2^{-1/2}H_{ns5}) \\ &+ (dx_2^2 + dx_3^2 + dx_4^2 + dx_5)(H_6^{-1/2}H_2^{1/2}) \\ &+ (dx_7^2 + dx_8^2 + dx_9^2)(H_6^{1/2}H_2^{1/2}H_{ns5}) \\ e^\phi =& H_6^{\frac{-3}{4}}H_2^{\frac{1}{4}}H_{ns5}^{\frac{1}{2}} \\ C_{0123456} =& 1/2(H_6^{-1} - 1) \\ \tilde{H}'_{\mu\nu\rho} =& \frac{1}{2}\epsilon_{\mu\nu\rho\lambda}H_{ns5}^{-1}\partial_\lambda H_{ns5}, \ \ \mu,\nu,\rho,\lambda = 1,7,8,9 \\ \tilde{\tilde{C}}_{016} =& 1/2(H_2^{-1} - 1) \end{cases} \tag{6.38}$$

The harmonic functions of the D-branes and the pp-wave are: The corresponding harmonic functions are:

$$\begin{aligned} H_6 =&1 + \frac{Q_6}{r} \\ H_{ns5} =&1 + \frac{Q_{ns5}}{r} \\ H_2 =&1 + \frac{Q_2}{r} \\ H_K =&1 + K(r) = 1 + \frac{Q_K}{r} \end{aligned} \tag{6.39}$$

The potentials and field strengths of this black hole are [36]:

$$\begin{cases} F'_{\mu\nu} =& -\frac{1}{2}\epsilon_{\lambda\mu\nu}\partial_\lambda H_6, \ \ \lambda,\mu,\nu = 7,8,9 \\ \tilde{H}'_{\mu\nu\rho} =& \frac{1}{2}\epsilon_{\mu\nu\rho\lambda}H_{ns5}^{-1}\partial_\lambda H_{ns5}, \ \ \lambda,\mu,\nu,\sigma = 1,7,8,9 \\ \tilde{\tilde{C}}_{016} =& \frac{1}{2}(H_2^{-1} - 1) \end{cases} \tag{6.40}$$

The compactification on the five-torus is easy since the metric is diagonal in those coordinates. The previous procedure can be used (6.23) - (6.25) to obtain:

$$\begin{aligned} ds_5^2 =& H_2^{-1/2}H_6^{-1/2}[dt^2(K-1) + dx_6^2(K+1) - 2Kdtdx_6] \\ &+ (dx_7^2 + dx_8^2 + dx_8^2)(H_6^{1/2}H_2^{1/2}H_{ns5}) \end{aligned} \tag{6.41}$$

Compactifying again on a circle was calculated in section 3.2.5.:

$$ds_4^2 = -H_2^{-1/2}H_6^{-1/2}(1+K)^{-1}dt^2 + (dx_7^2 + dx_8^2 + dx_9^2)(H_6^{1/2}H_2^{1/2}H_{ns5}) \tag{6.42}$$

An additional Kaluza-Klein field is acquired from the pp-wave. The process for compactifying potentials is the same as before and the results it yields are for D_2-brane are $C_{\mu\nu a}$ in five-dimensions and $C_{\mu a6}$ in four-dimensions, $a = 1$. The NS_5-brane has $C_{\mu a}$ in both five and four-dimensions with $a = 1$. The D_6-brane has C_μ potential in both reduced dimensions.

The metric in Einstein frame is:

$$\begin{aligned} ds_4^2 =& -(H_2H_6(1+K)H_{ns5})^{-1/2}dt^2 \\ &+ (dx_7^2 + dx_8^2 + dx_9^2)(H_6H_2H_{ns5}(1+K))^{1/2} \\ e^{\phi_4} =& (1+K)^{-1/4}H_{ns5}^{1/4} \end{aligned} \tag{6.43}$$

6.4 Black hole thermodynamics

Black holes are thermal systems. They have thermodynamic entropy and obey the laws of thermodynamics. Their thermodynamic entropy is proportional to the area of the event horizon, which increases in classical processes since the area itself is a property of a classical solution [5].

There are four laws, the zeroth law denotes that the surface gravity $\hat{\kappa}$ for a stationary black hole, is constant over the horizon. The first law is expressed as:

$$dM = \hat{\kappa}\frac{dA}{8\pi G} + \omega_H dJ + \Phi_e dQ, \tag{6.44}$$

where M is the black hole mass, A the event horizon area, $\hat{\kappa}$ the surface gravity, ω_H the angular velocity at the horizon, J the angular momentum, Φ the electrostatic potential, and dQ the electric charge. The second law says that the horizon area cannot decrease due to any (classical) process. The assumption for this law is the weak energy condition. The weak energy condition states that the matter density observed for every timelike vector is always non-negative. Finally, the third law states that it is impossible to have a vanishing $\hat{\kappa}$ through a physical process or a physical sequence of operations [42].

The black hole entropy is non-zero for both extremal and non-extremal black holes [3]. The Bekenstein-Hawking entropy has been formulated by Bekenstein up to a proportionality constant [5] and derived exactly by Hawking [22]:

$$S_{BH} = \frac{A}{4G_N}, \tag{6.45}$$

for $k_B = \hbar = c = 1$.

6.4.1 Macroscopic entropy

The formula (6.45) is the same in both ten- and D-dimensional perspectives, since the ten-dimensional Newton's constant is written as: $G_N^{(10)} = (2\pi)^{10-D} G_N^{(D)} V_{10-D}$ and the ten-dimensional area is written in terms of the D-dimensional area as: $A^{(10)} = (2\pi)^{10-D} A^{(D)} V_{10-D}$. It is more convenient then, to use the four-dimensional metric in Einstein frame to calculate the area of the event horizon.

The area of the event horizon for the D_2, D_6, NS_5 and W black hole (6.38), (6.43) is:

$$\begin{aligned} A^{(4)} =& \int_{S^2} \sqrt{g_{77} g_{88}}|_{r=0} \\ =& \sqrt{H_2 H_{ns5} H_6 (1+K)} 4\pi r^2 | r=0 \\ =& 4\pi r^2 \sqrt{(1+\frac{Q_2}{r})(1+\frac{Q_{ns5}}{r})(1+\frac{Q_6}{r})(1+\frac{Q_K}{r})} | r=0 \\ =& 4\pi \sqrt{Q_2 Q_{ns5} Q_6 Q_K} \end{aligned} \tag{6.46}$$

The area of the event horizon for the four-dimensional case A (6.14), (6.25) is [2]:

$$\begin{aligned} A^{(4)} =& \int_{S^2} \sqrt{g_{77} g_{88}}|_{r=0} \\ =& \sqrt{H_{4a} H_{4b} H_{4c} H_0} 4\pi r^2 | r=0 \\ =& 4\pi r^2 \sqrt{(1+\frac{Q_{4a}}{r})(1+\frac{Q_{4b}}{r})(1+\frac{Q_{4c}}{r})(1+\frac{Q_0}{r})} | r=0 \\ =& 4\pi \sqrt{Q_{4a} Q_{4b} Q_{4c} Q_0}. \end{aligned} \tag{6.47}$$

The area of the black hole horizon for the four-dimensional case B (6.26), (6.31) is found with the same calculation [2]:

$$A^{(4)} = 4\pi \sqrt{Q_{4a} Q_{4b} Q_{2a} Q_{2b}}. \tag{6.48}$$

Finally, for the case C (6.32), (6.37) the area of the event horizon is [2]:

$$A^{(4)} = 4\pi\sqrt{Q_6 Q_{2a} Q_{2b} Q_{2c}}. \tag{6.49}$$

As mentioned in section 2.3.1, $Q_p = c_p N_p$, thus the area in (6.46), can be written as:

$$A^{(4)} = 4\pi\sqrt{c_2 N_2 c_{ns5} N_{ns5} c_6 N_6 c_K N_K}, \tag{6.50}$$

where the product of the charges was explicitly calculated in [46]:

$$c_2 c_{ns5} c_6 c_K = 4(G_N^{(4)})^2, \tag{6.51}$$

thus the entropy of this black hole is [36]:

$$S = 2\pi\sqrt{N_2 N_{ns5} N_6 N_K}. \tag{6.52}$$

Similarly, the entropy for the case A is:

$$S = 2\pi\sqrt{N_{4a} N_{4b} N_{4c} N_0}, \tag{6.53}$$

the entropy for case B is:

$$S = 2\pi\sqrt{N_{4a} N_{4b} N_{2a} N_{2b}}, \tag{6.54}$$

and the entropy for case C is:

$$S = 2\pi\sqrt{N_6 N_{2a} N_{2b} N_{2c}}. \tag{6.55}$$

6.5 Near-horizon geometry

The near-horizon geometry is the spacetime geometry that occurs near the event horizon of a black hole. The concept of near-horizon geometry is well defined for extremal black holes. In more than four-dimensions, the near-horizon geometry is important in the classification of black holes. The problem that might arise when one is only working with near-horizon geometries is that their existence does not necessarily provide the corresponding existence of an extremal black hole [32].

6.5.1 Near-horizon geometry of Reissner-Nordström black hole

The near-geometry of the Reissner-Nordstrom Black Hole can be found by taking the limit of the coefficients of the metric in isotropic coordinates to zero, this means, taking the limit at the horizon. Then a simple substitution of this limit will give the form of $AdS_2 \times S^2$ geometry. Consider the metric (6.3), (6.4) of Reissner-Nordström black hole without a magnetic field. The metric at isotropic coordinates is (6.10), (6.11).

The limit of ρ-coordinate at the event horizon of the black hole is:

$$\lim_{\rho\to 0}(1 + \frac{G_N M}{\rho})^{-2} = (\frac{G_N M}{\rho})^{-2}, \text{ and } \lim_{\rho\to 0}(1 + \frac{G_N M}{\rho})^{2} = (\frac{G_N M}{\rho})^{2}. \tag{6.56}$$

After substituting these limits to the metric of the extremal Reissner-Nordström black hole (6.10) the metric becomes:

$$ds^2 = -(\frac{G_N M}{\rho})^{-2} dt^2 + (\frac{G_N M}{\rho})^2 d\rho^2 + (G_N M)^2 d\Omega^2. \tag{6.57}$$

For the near-horizon geometry to emerge in the form needed, one has to substitute $r' = \frac{(G_N M)^2}{\rho}$, $dr' = -\frac{(G_N M)^2}{\rho^2} d\rho$ in (6.57):

$$\begin{aligned} ds^2 &= -(\frac{G_N M}{r'})^2 dt^2 + (\frac{r'}{G_N M})^2 dr'^2 + (G_N M)^2 d\Omega^2 \\ &= (\frac{G_N M}{r'})^2 (-dt^2 + dr'^2) + (G_N M)^2 d\Omega^2 \end{aligned} \tag{6.58}$$

The resulting metric has a geometry of AdS_2 from the first summand and a two-sphere from the second one, this proves that the near-horizon geometry is:

$$AdS_2 \times S^2 \text{ [1]}. \tag{6.59}$$

Chapter 7

Black holes and supersymmetry breaking

This chapter aims to study which of the previously studied black holes will survive a Scherk-Schwarz twist in $\mathcal{N} = 8, 6, 4, 2$ and 0. In section 7.1, the five-dimensional theory is studied, and in section 7.2, the results of the compactification to four-dimensions and their impact is shown on the black holes of chapter 6.

7.1 Five-dimensional theory

The five-dimensional action is obtained from the ten-dimensional action (2.13) compactified on a T^5, or from the eleven-dimensional theory compactified on T^6, using Kaluza-Klein compactification. The eleven-dimensional action is given by:

$$\mathcal{L}_{11} = \frac{1}{2\kappa_{11}^2}\int d^{11}x\sqrt{-g}(R - \frac{1}{48}|\tilde{F}_4|^2) + \frac{1}{12\kappa_{11}^2}\int dC_3 \wedge dC_3 \wedge C_3, \tag{7.1}$$

where the 4-form $\tilde{F}_4 = dC_3 - dB_2 \wedge C_1$, is denoted with a tilde to not be confused with $F_4 = dC_3$.

From (3.10), define i as the reduction step from eleven dimensions to a D-dimensional theory $i = 11 - D$, then:

$$2\alpha_i = \sqrt{\frac{2}{2(10-i)(9-i)}} \tag{7.2}$$

The labels m, n are defined to denote the coordinates of the five-torus from the ten-dimensional theory, $m, n = 1, .., 5$ and i, j as defined, are the six-torus directions from the eleven-dimensional theory.

After compactification of the eleven-dimensional theory once on a circle, the ten-dimensional IIA theory is obtained. The fields $B_{\mu\nu}, C_{\mu\nu\rho}$ and C_μ, from the R-R and NS-NS sector emerge. The reduced fields coming from $C_{\mu\nu\rho}$, will be denoted with $^{(3)}$ and the ones coming from C_μ will be denoted with the label $^{(1)}$. First the lagrangian reduced from the eleven-dimensional theory case will be shown, using the indices i, j, and identifying the ten-dimensional field spectrum among the fields. Then the reduction of the ten-dimensional theory to five-dimensions will be shown.

This process allows one to match the results found in the literature with the desired ones from ten- to five-dimensions since the black holes constructed in chapter 6 are ten-dimensional. The kinetic term of the Lagrangian in Einstein frame is written as [34]:

$$\begin{aligned}\mathcal{L} =& R - \frac{1}{2}|d\vec{\phi}|^2 - \frac{1}{2}\frac{1}{4!}e^{\vec{a}\cdot\vec{\phi}}|\tilde{F}_4|^2 - \frac{1}{2}\frac{1}{3!}e^{\vec{a_1}\cdot\vec{\phi}}|d\tilde{B}_{\mu\nu 1}|^2 - \frac{1}{2}\frac{1}{3!}\sum_{i=2}^{6} e^{\vec{a_i}\cdot\vec{\phi}}|\tilde{F}_{3,i}|^2 \\ &- \frac{1}{2}\frac{1}{2!}\sum_{2\leq j\leq 6} e^{\vec{a}_{1j}\cdot\vec{\phi}}|d\tilde{B}_{\mu 1j}|^2 - \frac{1}{2}\frac{1}{2!}\sum_{2\leq i<j\leq 6} e^{\vec{a}_{ij}\cdot\vec{\phi}}|d\tilde{C^{(3)}}_{\mu ij}|^2 - \frac{1}{2}\frac{1}{2!}e^{\vec{b}_1\cdot\vec{\phi}}|d\tilde{C^{(1)}}_{\mu}|^2 \\ &- \frac{1}{2}\frac{1}{2!}\sum_{i=2}^{6} e^{\vec{b}_i\cdot\vec{\phi}}|d\tilde{g}^i_\mu|^2 - \frac{1}{2}\sum_{2\leq j<k\leq 6} e^{\vec{a}_{1jk}\cdot\vec{\phi}}|d\tilde{B}_{1jk}|^2 - \frac{1}{2}\sum_{2\leq i<j<k\leq 6} e^{\vec{a}_{ijk}\cdot\vec{\phi}}|d\tilde{C}^{(3)}_{ijk}|^2 \\ &- \frac{1}{2}\sum_{2\leq j\leq 6} e^{\vec{b}_{1j}\cdot\vec{\phi}}|d\tilde{C}^{(1)1}_j|^2 - \frac{1}{2}\sum_{2\leq i<j\leq 6} e^{\vec{b}_{ij}\cdot\vec{\phi}}|d\tilde{g}^i_j|^2,\end{aligned} \tag{7.3}$$

where the dilaton vectors considered are given in the appendix A, $\vec{\phi} = (\phi_0, \phi_1, \phi_2, \phi_3, \phi_4, \phi_5)$, $\vec{H} = (H_0, H_1, H_2, H_3, H_4, H_5)$. The p-form field strengths are denoted with a tilde, because they correspond to a big expression which is explicitly given below [34]:

$$\begin{aligned}\tilde{F}_4 =& dC^{(3)}_{\mu\nu\rho} - \gamma^1_j dB_{\mu\nu 1}\wedge D^j_\rho - \gamma^i_j dC^{(3)}_{\mu\nu i}\wedge D^j_\rho + \frac{1}{2}\gamma^1_k\gamma^j_l dB_{\mu 1j}\wedge D^k_\nu \wedge D^j l_\rho \\ &+ \frac{1}{2}\gamma^i_k\gamma^j_l dC^{(3)}_{\mu ij}\wedge D^k_\nu\wedge D^l_\rho - \frac{1}{6}\gamma^1_l\gamma^j_m\gamma^k_n dB_{1jk}\wedge D^l_\mu\wedge D^m_\nu\wedge D^n_\rho \\ &- \frac{1}{6}\gamma^i_l\gamma^j_m\gamma^k_n dC^{(3)}_{ijk}\wedge D^l_\mu\wedge D^m_\nu\wedge D^n_\rho \\ d\tilde{B}_{\mu\nu 1} =& dB_{\mu\nu 1} + \gamma^j_1 dC^{(3)}_{\mu\nu j} + \gamma^k_l dB_{\mu 1k}\wedge D^l_\nu + \gamma^k_l\gamma^l_n dB_{1kl}\wedge D^m_\mu\wedge D^n_\nu \\ \tilde{F}_{3,i} =& \gamma^j_i dC^{(3)}_{\mu\nu j} + \gamma^j_i\gamma^k_l dC^{(3)}_{\mu jk}\wedge D^l_\nu + \gamma^j_i\gamma^k_m\gamma^l_n dC^{(3)}_{jkl}\wedge D^m_\mu\wedge D^n_\nu \\ d\tilde{B}_{\mu 1j} =& \gamma^l_j dB_{\mu 1l} - \gamma^1_i\gamma^l_j\gamma^m_n dB_{1lm}\wedge D^n_\mu \\ d\tilde{C}^{(3)}_{\mu ij} =& \gamma^k_i\gamma^l_j dC^{(3)}_{\mu kl} - \gamma^k_i\gamma^l_j\gamma^m_n dC^{(3)}_{klm}\wedge D^n_\mu \\ d\tilde{C}^{(1)}_\mu =& dC^{(1)}_\mu - \gamma^j_k dC^{(1)}_\mu \wedge D^k_\mu \\ d\tilde{g}^i_\mu =& dg^i_\mu - \gamma^j_k dg^i_j\wedge D^k_\mu \\ d\tilde{B}_{1jk} =& \gamma^1_1\gamma^m_j\gamma^n_k d\tilde{B}_{1mn} \\ d\tilde{C}^{(3)}_{ijk} =& \gamma^l_i\gamma^m_j\gamma^n_k dC^{(3)}_{lmn} \\ d\tilde{C}^{(1)}_\mu =& \gamma^k_j dC^{(1)}_\mu \\ d\tilde{g}^i_j =& \gamma^k_j dg^i_k\end{aligned} \tag{7.4}$$

where:

$$D^i_\rho = \begin{cases} C^{(1)}_\mu, i = 1 \\ g^i_\mu,\ i = 2, .., 6, \end{cases} \tag{7.5}$$

and $\gamma^i_j = (1 + E^{-1})^i_j$, where:

$$E^i_j = \begin{cases} C^{(1)}_j, i = 1 \\ g^i_j,\ i = 2, .., 6. \end{cases} \tag{7.6}$$

The variables E^i_j, are only defined for $i \leq j$.

The second term of (7.1) in five-dimensions becomes [13],[34]:

$$\begin{aligned}
&\frac{1}{12}(d\tilde{B}_{\mu\nu 1} \wedge d\tilde{C}^{(3)}_{\rho jk} C^{(3)}_{lmn})\bar{\epsilon}^{1jklmn} + \frac{1}{12}(d\tilde{C}^{(3)}_{\mu\nu i} \wedge d\tilde{B}_{\rho 1k} C^{(3)}_{lmn})\bar{\epsilon}^{i1klmn} \\
&+ \frac{1}{12}(d\tilde{C}^{(3)}_{\mu\nu i} \wedge d\tilde{C}^{(3)}_{\rho jk} dB_{1mn})\bar{\epsilon}^{ijk1mn} + \frac{1}{48}(d\tilde{B}_{\mu 1j} \wedge d\tilde{C}^{(3)(3)}_{\nu kl\rho mn})\bar{\epsilon}^{1jklmn} \\
&+ \frac{1}{48}(d\tilde{C}^{(3)}_{\mu ij} \wedge d\tilde{B}_{\nu 1l} \wedge C^{(3)}_{\rho mn})\bar{\epsilon}^{ij1lmn} + \frac{1}{48}(d\tilde{C}^{(3)}_{\mu ij} \wedge d\tilde{C}^{(3)}_{\nu kl} \wedge B_{\rho 1n})\bar{\epsilon}^{ijkl1n} \\
&- \frac{1}{72}(d\tilde{B}_{1jk} \wedge d\tilde{C}^{(3)}_{lmn} \wedge dC^{(3)}_{\mu\nu\rho})\bar{\epsilon}^{1jklmn} - \frac{1}{72}(d\tilde{C}^{(3)}_{ijk} \wedge d\tilde{B}_{1lm} \wedge dC^{(3)}_{\mu\nu\rho})\bar{\epsilon}^{ijk1lm}
\end{aligned} \tag{7.7}$$

The ten-dimensional fields are reduced to five-dimensions as mentioned in section 3.2.4. To apply (7.8) in ten-dimensional black holes reduced on a five-torus with torus coordinates $m, n = 1, .., 5$, then the fields should be written in terms of these coordinates. If one ignores interaction terms, with [13] then five-dimensional lagrangian reads:

$$\begin{aligned}
\mathcal{L} =& R - \frac{1}{2}|d\vec{\phi}|^2 - \frac{1}{2}\frac{1}{4!}e^{\vec{a}\cdot\vec{\phi}}|d\tilde{C}^{(3)}_{\mu\nu\rho}|^2 - \frac{1}{2}\frac{1}{3!}e^{\vec{a_1}\cdot\vec{\phi}}|d\tilde{B}_{\mu\nu}|^2 - \frac{1}{2}\frac{1}{3!}\sum_{m=1}^{5} e^{\vec{a}_{(m+1)}\cdot\vec{\phi}}|d\tilde{C}^{(3)}_{\mu\nu m}|^2 \\
&- \frac{1}{2}\frac{1}{2!}\sum_{1\leq m\leq 5} e^{\vec{a}_{1(m+1)}\cdot\vec{\phi}}|d\tilde{B}_{\mu m}|^2 - \frac{1}{2}\frac{1}{2!}\sum_{1\leq m<n\leq 5} e^{\vec{a}_{(m+1)(n+1)}\cdot\vec{\phi}}|d\tilde{C}^{(3)}_{\mu mn}|^2 \\
&- \frac{1}{2}\frac{1}{2!}e^{\vec{b}_1\cdot\vec{\phi}}|d\tilde{C}^{(1)}_{\mu}|^2 - \frac{1}{2}\frac{1}{2!}\sum_{m=1}^{5} e^{\vec{b}_{m+1}\cdot\vec{\phi}}|d\tilde{g}^m_\mu|^2 - \frac{1}{2}\sum_{1\leq m<n\leq 5} e^{\vec{a}_{1(m+1)(n+1)}\cdot\vec{\phi}}|d\tilde{B}_{mn}|^2 \\
&- \frac{1}{2}\sum_{1\leq m<n<r\leq 5} e^{\vec{a}_{(m+1)(n+1)(r+1)}\cdot\vec{\phi}}|d\tilde{C}^{(3)}_{mnr}|^2 - \frac{1}{2}\sum_{1\leq m\leq 5} e^{\vec{b}_{1(m+1)}\cdot\vec{\phi}}|d\tilde{C}^{(1)}_{m}|^2 \\
&- \frac{1}{2}\sum_{1\leq m<n\leq 5} e^{\vec{b}_{(m+1)(n+1)}\cdot\vec{\phi}}|d\tilde{g}^m_n|^2,
\end{aligned} \tag{7.8}$$

where the tilde denotes similar expressions as in (7.4). But for the $E_{6(6)}$ invariance to manifest in this lagrangian, the maximum amount of scalars and vectors should be considered, so the 4- and 3-forms should be dualized to a scalar and vectors, respectively. For example, the first two irreducible representations of $E_{6(6)}$ are **1** and **27**. In this theory there are twenty-one vectors and six 2-forms, but after dualizing the 2-forms there are twenty-seven vectors belonging in the **27** representation of $E_{6(6)}$.

In five-dimensions, the Hodge dual of the 3-form will result in a scalar:

$$*|dC^{(3)}_{\mu\nu\rho}|^2 = -2|db|^2 \tag{7.9}$$

The hodge dual of 2-forms in five-dimensions gives vectors:

$$\begin{aligned}
*|dB_{\mu\nu}|^2 =& 2|\partial_\mu B_\nu|^2 \\
*|F_{3,m}|^2 =& 2[|\partial_\mu C^{(3)}_{\nu m} - \partial_\mu B_\nu \wedge dC^{(1)}_m|^2 + |\partial_\mu C^{(3)}_{\nu m} - \partial_\nu B_m \wedge dC^{(1)}_\mu|^2 \\
&+ |\partial_\mu C^{(3)}_{\nu m} - \partial_\mu B_m \wedge dC^{(1)}_\nu|^2]
\end{aligned} \tag{7.10}$$

To substitute these dualizations correctly in the five-dimensional lagrangian, a Bianchi multiplier imposing the Bianchi identity should also be considered. This was done in [13] in detail, but to find which vectors gain masses it is more important to find where these fields are located in the 27-representation of $E_{6(6)}$ with respect to the vielbein. Their full expression can be absorbed in redefinitions.

The scalars are described in the coset $E_{6(6)}/USp(8)$ and the coset is described by the vielbein [13], [34]:

$$\begin{aligned}\mathcal{V} =& e^{\frac{1}{2}\vec{\phi}\cdot\vec{H}} \prod_{1\leq m\leq 5} e^{C^{(1)}_m E^{m+1}_1} \prod_{1\leq m<n\leq 5} e^{g^m_n E^{n+1}_{m+1}} \\ & e^{\sum_{2\leq m<n\leq 5} B_{mn} E^{1(m+1)(n+1)} + \sum_{1\leq m<n<r\leq 5} C^{(3)}_{mnr} E^{(m+1)(n+1)(r+1)}} e^{bJ}\end{aligned} \tag{7.11}$$

The commutation relations are given by:

$$\boxed{\begin{aligned}[\vec{H}, E^j_i] =& \vec{b}_{ij} E^j_i \\ [\vec{H}, E^{ijk}] =& \vec{a}_{ijk} E^{ijk} \\ [E^j_i, E^l_k] =& \delta^j_k E^l_i - \delta^i_l E^j_k \\ [E^m_l, E^{ijk}] =& -3\delta^{[i}_l E^{|m|jk]} \\ [\vec{H}, J] =& -\vec{\alpha} J \\ [E^j_i, J] =& 0 \\ [E^{ijk}, J] =& 0 \\ [E^{ijk}, E^{lmn}] =& -\epsilon^{ijklmn} J\end{aligned}} \tag{7.12}$$

The ϵ^{ijklmn} is defined at (5.1). The root generators are given in the Appendix A. The kinetic term of the lagrangian for the scalar fields can also be rewritten in terms of [15]:

$$\mathcal{H} = \mathcal{V}^\dagger \eta \mathcal{V}, \tag{7.13}$$

and kinetic term of the scalar fields becomes:

$$\mathcal{L}_{scalars} = \frac{1}{4} Tr[\partial_\mu \mathcal{H}^{-1} \partial^\mu \mathcal{H}]. \tag{7.14}$$

Accordingly, the kinetic term of the lagrangian for the vectors is [14]:

$$\mathcal{L}_{vectors} = -\frac{1}{8} g^{\mu\rho} g^{\nu\sigma} \Omega_{ac} \Omega_{bd} \mathcal{V}^{ab}_{\alpha\beta} \mathcal{V}^{cd}_{\gamma\delta} F^{\alpha\beta}_{\mu\nu} F^{\gamma\delta}_{\rho\sigma}, \tag{7.15}$$

where $\alpha, \beta, .. = 1, .., 8$ are pairs of $E_{6(6)}$ and $a, b, .. = 1, .., 8$ are $USp(8)$ indices. The vectors lie in $A^{\alpha\beta}_\mu$ which belongs in the 27-representationf of $E_{6(6)}$, as seen in Appendix B.

The vectors and scalars gain masses from the matrix M, by substituting $\mathcal{M} = e^{My}$ in the five-dimensional lagrangian [14]:

$$\begin{aligned}A^{\alpha\beta}_\mu(x, y) =& \mathcal{M}^\alpha_\gamma(y) \mathcal{M}^\beta_\delta(y) A^{\gamma\delta}_\mu(x) \\ \mathcal{V}^{ab}_{\alpha\beta}(x, y) =& (\mathcal{M}^{-1}(y))^\gamma_\alpha (\mathcal{M}^{-1}(y))^\delta_\beta \mathcal{V}^{ab}_{\gamma\delta}(x)\end{aligned} \tag{7.16}$$

7.2 Supersymmetry breaking and black holes in four-dimensions

Using (A.13) and (7.8) to match the dilaton vectors from the lagrangian to the Cartan generators, it should be noted that a_{ij} and b_i remain the same, and a_i becomes $-a_i$, due to the 2-forms which were dualized to vectors, it is paired with. The twenty-seven five-dimensional vectors after dualization, can be found in the 27-vector J with the order given below:

$$J = (B_\mu, C^{(3)}_{\mu 1}, C^{(3)}_{\mu 2}, C^{(3)}_{\mu 3}, C^{(3)}_{\mu 45}, C^{(3)}_{\mu 4}, C^{(3)}_{\mu 35}, C^{(3)}_{\mu 5}, C^{(3)}_{\mu 25}, C^{(3)}_{\mu 34}, C^{(3)}_{\mu 15}, \\ C^{(3)}_{\mu 24}, B_{\mu 5}, C^{(3)}_{\mu 14}, C^{(3)}_{\mu 23}, B_{\mu 4}, C^{(3)}_{\mu 13}, B_{\mu 3}, C^{(3)}_{\mu 12}, B_{\mu 2}, C^{(1)}_{\mu}, g^1_\mu, B_{\mu 1}, g^2_\mu, \\ g^3_\mu, g^4_\mu, g^5_\mu) \tag{7.17}$$

The mass spectrum for the mass matrix (5.7), (B.2) - (B.6), and solving for (7.16) is shown in Appendix C, table C.1. The black hole $D_{4a}/D_{4b}/D_{4c}/D_0$ can survive the reduction in $\mathcal{N}=6$ and $\mathcal{N}=4$. To obtain $\mathcal{N}=4$, m_3 and m_4 should be set to zero. The four-dimensional massless vectors $C^{(1)}_\mu$, $C^{(3)}_{\mu 46}$, $C^{(3)}_{\mu 23}$ and $C^{(3)}_{\mu 15}$ correspond to D_0 and D_{4a}, D_{4b} and D_{4c}, respectively. The directions on which these branes lie, however should be changed. The D_{4a} brane corresponding to $C^{(3)}_{\mu 46}$ sits in directions (1235), D_{4b} should sit in (1456) and D_{4c} should be in (2346). This configuration should then be:

Case A'	0	1	2	3	4	5	6	7	8	9
D_{4a}	×	×	×	×		×		.	.	.
D_{4b}	×	×			×	×	×	.	.	.
D_{4c}	×		×	×	×		×	.	.	.
D_0	×							.	.	.

The same fields for $\mathcal{N}=4$, can be used in the black hole for Case C, $D_6/D_{2a}/D_{2b}/D_{2c}$ but again, with a different configuration:

Case C'	0	1	2	3	4	5	6	7	8	9
D_6	×	×	×	×	×	×	×	.	.	.
D_{2a}	×				×		×	.	.	.
D_{2b}	×		×	×				.	.	.
D_{2c}	×	×				×		.	.	.

The black hole in Case B, $D_{4a}/D_{4b}/D_{2a}/D_{2b}$ survives up to $\mathcal{N}=6$, when three mass parameters are set to be zero, $m_1 = m_2 = m_3 = 0$. The four-dimensional fields $C^{(3)}_{\mu 16}$, $C^{(3)}_{\mu 35}$, $C^{(3)}_{\mu 15}$ and $C^{(3)}_{\mu 56}$ corresponding to D_{2a}, D_{2b}, D_{4b} and D_{4a}, resulting to the new configuration:

Case B'	0	1	2	3	4	5	6	7	8	9
D_{4a}	×	×	×	×	×			.	.	.
D_{4b}	×		×	×	×		×	.	.	.
D_{2a}	×	×					×	.	.	.
D_{2b}	×			×		×		.	.	.

Finally, the case D: $D_6/NS_5/D_2/W$ black hole has massless fields up to $\mathcal{N}=6$, with $m_1 = m_3 = m_4 = 0$. The fields $C^{(1)}_\mu$, $C^{(3)}_{\mu 16}$, $B_{\mu 1}$ correspond to D-branes D_6, D_2, NS_5, with the same configuration as (6.38).

The mass matrix from (5.10), (B.8) and (B.9) gives the mass spectrum shown in Appendix C, table C.2. This mass matrix shows better results for case D. The black hole $D_6/NS_5/D_2/W$ survives up to $\mathcal{N} = 4$ for $m_1 = m_4 = 0$ with corresponding fields $C_\mu^{(1)}$, $B_{\mu 5}$ and $C_{\mu 56}^{(3)}$ and the massless field obtained from the reduction g_μ^6. The configuration should also be different than the one in chapter 6, it should be:

Case D'	0	1	2	3	4	5	6	7	8	9
D_6	×	×	×	×	×	×	×	.	.	.
NS_5	×	×	×	×	×		×	.	.	.
D_2	×					×	×	.	.	.
W	×						→	.	.	.

All of these black holes can survive to $\mathcal{N} = 2$, meaning that their reduced vectors can remain massless in $\mathcal{N} = 2$, if the parameters of the mass matrix can be taken to be a linear combination of each other. This can be done using any of the two mass matrices. To be more specific, assume the mass matrix (5.10) and one example will be given for each case. To study cases A and C, set $m_3 = 0$ and $m_1 = m_2 = m_4 = m \neq 0$, leading to $C_\mu^{(1)}$, $C_{\mu 13}^{(3)}$, $C_{\mu 24}^{(3)}$ and $C_{\mu 56}^{(3)}$ are massless, then with these fields, both cases can be re-written as:

Case A"	0	1	2	3	4	5	6	7	8	9
D_{4a}	×		×		×	×	×	.	.	.
D_{4b}	×	×		×		×	×	.	.	.
D_{4c}	×	×	×	×	×			.	.	.
D_0	×							.	.	.

Case C"	0	1	2	3	4	5	6	7	8	9
D_6	×	×	×	×	×	×	×	.	.	.
D_{2a}	×					×	×	.	.	.
D_{2b}	×	×		×				.	.	.
D_{2c}	×		×		×			.	.	.

By setting $m_4 = 0$ and $m_1 = -m_2 = m_3 = m \neq 0$, then $C_{\mu 56}^{(3)}$, $C_{\mu 12}^{(3)}$, $C_{\mu 16}^{(3)}$ and $C_{\mu 25}^{(3)}$ are massless. Thus, the black hole $D_{4a}/D_{4b}/D_{2a}/D_{2b}$ can survive at $\mathcal{N} = 2$, but with the following configuration:

Case B"	0	1	2	3	4	5	6	7	8	9
D_{4a}	×	×	×	×	×			.	.	.
D_{4b}	×			×	×	×	×	.	.	.
D_{2a}	×	×					×	.	.	.
D_{2b}	×		×			×		.	.	.

Let $m_2 = 0$ and $m_1 = m_4 = m \neq 0$, then $C_{\mu 56}^{(3)}$, $B_{\mu 5}$ and $C_\mu^{(1)}$ are massless, then the configuration of case D', can survive up to $\mathcal{N} = 2$.

At $\mathcal{N} = 0$, all masses are non-zero. Setting $m_1 = m_2 = m_3 = m_4 = m \neq 0$ in (5.10), then all four black holes survive, with the configurations of case A, case B"', case C and case D'. In fact, case D' can survive with only the restriction between two mass parameters: $m_1 = m_4 = m \neq 0$. Case B"' is given below:

Case B"'	0	1	2	3	4	5	6	7	8	9
D_{4a}	×	×		×	×	×		.	.	.
D_{4b}	×	×	×		×		×	.	.	.
D_{2a}	×					×	×	.	.	.
D_{2a}	×		×	×				.	.	.

There are other possible configurations that result in the same four-dimensional extremal black hole.

Chapter 8

Conclusion

The objective of this thesis is to study the untwisted reduction of a black hole from type IIA in ten-dimensions to five-dimensions and then to reduce with a twist to four-dimensions. The supergravity is partially broken from five- to four-dimensions from $\mathcal{N} = 8$ to $\mathcal{N} = 6, 4, 2$, and 0. To summarize, the reduction of ten-dimensional type IIA supergravity on $T^5 \times S^1$, was studied with a Scherk-Schwarz twist on S^1 and untwisted T^5.

In section 5.2, the dimensional reduction was studied from IIA in ten-dimensions to six-, five- and four-dimensions. In section 5.3, the reduction with a twist on a circle was presented from five- to four-dimensions. The massive and massless fields were obtained.

In chapter 6, the black holes with D-brane configurations: $D_{4a}/D_{4b}/D_{4c}/D_0$, $D_{4a}/D_{4b}/D_{2a}/D_{2b}$, $D_6/D_{2a}/D_{2b}/D_{2c}$ and $D_6/NS_5/D_2/W$ were studied. Their dimensional reduction was also shown.

In chapter 7, there is an analysis on which fields become massive in four-dimensions using two different mass matrices. The previously studied black holes are shown to survive in $\mathcal{N} = 6$ or up to $\mathcal{N} = 4$, but after changing the directions their branes are parallel to. By writing the non-zero entries of the mass matrix in be a linear combination of each other, these black holes can even be found in $\mathcal{N} = 2$ and $\mathcal{N} = 0$.

8.1 Outlook

Further analysis can be done on the scalars and fermionic fields that remain massless after a Scherk-Schwarz twist from five- to four-dimensions. The outcome of this analysis can be implemented on four-dimensional black holes. In addition, more mass matrices that can be studied.

Another similar research can be done for near-extremal black holes. Are there twists that allow four-dimensional near-extremal black holes to survive up to $\mathcal{N} = 2$ or 0?

In this thesis, the macroscopic entropy of the four-black holes was calculated. Additional analysis can be done on their microscopic entropy. The microscopic entropy can be calculated by counting the brane configurations.

Appendix A

Roots and root generators of five-dimensional lagrangian

The vectors $\vec{g}$, $\vec{f_i}$ are given [13], [34]:

$$\begin{aligned} \vec{g} =& 3(\alpha_1, \alpha_2, .., \alpha_6) \\ \vec{f_i} =& (0, .., 0, (10-i)\alpha_i, \alpha_{i+1}, .., \alpha_6), \end{aligned} \tag{A.1}$$

where there are $(i-1)$ zeroes in $\vec{f_i}$. The values of $\alpha_1, .., \alpha_6$, used in (A.1) can be obtained from (7.2). The vectors $\vec{a}$, $\vec{a}_i$, $\vec{a}_{ij}$, $\vec{a}_{ijk}$, $\vec{b}_i$, $\vec{b}_{ij}$ can be calculated from [13], [34]:

$$\begin{aligned} \vec{a} =& -\vec{g} \\ \vec{a}_i =& \vec{f_i} - \vec{g} \\ \vec{a}_{ij} =& \vec{f_i} + \vec{f_j} - \vec{g} \\ \vec{a}_{ijk} =& \vec{f_i} + \vec{f_j} + \vec{f_k} - \vec{g} \\ \vec{b}_i =& -\vec{f_i} \\ \vec{b}_{ij} =& -\vec{f_i} + \vec{f_j} \end{aligned} \tag{A.2}$$

The dilaton vectors in the five-dimensional lagrangian are obtained after dimensionally reducing via Kaluza-Klein, the initial lagrangian. The dilaton vector associated with the four-form is:

$$\vec{a} = (-\frac{1}{2}, -\frac{3\sqrt{7}}{14}, -\frac{3\sqrt{21}}{21}, -\frac{3\sqrt{15}}{15}, -\frac{3\sqrt{10}}{10}, -\frac{3\sqrt{6}}{6}) \tag{A.3}$$

The dilaton vectors of C_μ and $g_{\mu m}$ are:

$$\begin{aligned}
\vec{b}_1 =& (-\frac{3}{2}, -\frac{\sqrt{7}}{14}, -\frac{\sqrt{21}}{21}, -\frac{\sqrt{15}}{15}, -\frac{\sqrt{10}}{10}, -\frac{\sqrt{6}}{6}) \\
\vec{b}_2 =& (0, -\frac{8\sqrt{7}}{14}, -\frac{\sqrt{21}}{21}, -\frac{\sqrt{15}}{15}, -\frac{\sqrt{10}}{10}, -\frac{\sqrt{6}}{6}) \\
\vec{b}_3 =& (0, 0, -7\frac{\sqrt{21}}{21}, -\frac{\sqrt{15}}{15}, -\frac{\sqrt{10}}{10}, -\frac{\sqrt{6}}{6}) \\
\vec{b}_4 =& (0, 0, 0, -6\frac{\sqrt{15}}{15}, -\frac{\sqrt{10}}{10}, -\frac{\sqrt{6}}{6}) \\
\vec{b}_5 =& (0, 0, 0, 0, -5\frac{\sqrt{10}}{10}, -\frac{\sqrt{6}}{6}) \\
\vec{b}_6 =& (0, 0, 0, 0, 0, -4\frac{\sqrt{6}}{6})
\end{aligned} \tag{A.4}$$

The dilaton vectors of C_m and g_{mn} are the following:

$$\begin{aligned}
\vec{b}_{12} =& (-\frac{3}{2}, \frac{7\sqrt{7}}{14}, 0, 0, 0, 0) \\
\vec{b}_{13} =& (-\frac{3}{2}, -\frac{\sqrt{7}}{14}, \frac{6\sqrt{21}}{21}, 0, 0, 0) \\
\vec{b}_{14} =& (-\frac{3}{2}, -\frac{\sqrt{7}}{14}, -\frac{\sqrt{21}}{21}, \frac{5\sqrt{15}}{15}, 0, 0) \\
\vec{b}_{15} =& (-\frac{3}{2}, -\frac{\sqrt{7}}{14}, -\frac{\sqrt{21}}{21}, -\frac{\sqrt{15}}{15}, \frac{4\sqrt{10}}{10}, 0) \\
\vec{b}_{16} =& (-\frac{3}{2}, -\frac{\sqrt{7}}{14}, -\frac{\sqrt{21}}{21}, -\frac{\sqrt{15}}{15}, -\frac{\sqrt{10}}{10}, \frac{3\sqrt{6}}{6}) \\
\vec{b}_{23} =& (0, -\frac{8\sqrt{7}}{14}, \frac{6\sqrt{21}}{21}, 0, 0, 0) \\
\vec{b}_{24} =& (0, -\frac{8\sqrt{7}}{14}, -\frac{\sqrt{21}}{21}, \frac{5\sqrt{15}}{15}, 0, 0) \\
\vec{b}_{25} =& (0, -\frac{8\sqrt{7}}{14}, -\frac{\sqrt{21}}{21}, -\frac{\sqrt{15}}{15}, \frac{4\sqrt{10}}{10}, 0) \\
\vec{b}_{26} =& (0, -\frac{8\sqrt{7}}{14}, -\frac{\sqrt{21}}{21}, -\frac{\sqrt{15}}{15}, -\frac{\sqrt{10}}{10}, \frac{3\sqrt{6}}{6}) \\
\vec{b}_{34} =& (0, 0, -\frac{7\sqrt{21}}{21}, \frac{5\sqrt{15}}{15}, 0, 0) \\
\vec{b}_{35} =& (0, 0, -\frac{7\sqrt{21}}{21}, -\frac{\sqrt{15}}{15}, \frac{4\sqrt{10}}{10}, 0) \\
\vec{b}_{36} =& (0, 0, -\frac{7\sqrt{21}}{21}, -\frac{\sqrt{15}}{15}, -\frac{\sqrt{10}}{10}, \frac{3\sqrt{6}}{6}) \\
\vec{b}_{45} =& (0, 0, 0, -\frac{6\sqrt{15}}{15}, \frac{4\sqrt{10}}{10}, 0) \\
\vec{b}_{46} =& (0, 0, 0, -\frac{6\sqrt{15}}{15}, -\frac{\sqrt{10}}{10}, \frac{3\sqrt{6}}{6}) \\
\vec{b}_{56} =& (0, 0, 0, 0, -\frac{5\sqrt{10}}{10}, \frac{3\sqrt{6}}{6})
\end{aligned} \tag{A.5}$$

The dilaton vectors of the 2-forms $B_{\mu\nu}$ and $C'_{\mu\nu m}$ are:

$$
\begin{aligned}
\vec{a}_1 =&(1, -\frac{2\sqrt{7}}{14}, -\frac{2\sqrt{21}}{21}, -\frac{2\sqrt{15}}{15}, -\frac{2\sqrt{10}}{10}, -\frac{2\sqrt{6}}{6})\\
\vec{a}_2 =&(-\frac{1}{2}, \frac{5\sqrt{7}}{14}, -\frac{2\sqrt{21}}{21}, -\frac{2\sqrt{15}}{15}, -\frac{2\sqrt{10}}{10}, -\frac{2\sqrt{6}}{6})\\
\vec{a}_3 =&(-\frac{1}{2}, -\frac{3\sqrt{7}}{14}, \frac{4\sqrt{21}}{21}, -\frac{2\sqrt{15}}{15}, -\frac{2\sqrt{10}}{10}, -\frac{2\sqrt{6}}{6})\\
\vec{a}_4 =&(-\frac{1}{2}, -\frac{3\sqrt{7}}{14}, -\frac{3\sqrt{21}}{21}, \frac{3\sqrt{15}}{15}, -\frac{2\sqrt{10}}{10}, -\frac{2\sqrt{6}}{6})\\
\vec{a}_5 =&(-\frac{1}{2}, -\frac{3\sqrt{7}}{14}, -\frac{3\sqrt{21}}{21}, \frac{3\sqrt{15}}{15}, \frac{2\sqrt{10}}{10}, -\frac{2\sqrt{6}}{6})\\
\vec{a}_6 =&(-\frac{1}{2}, -\frac{3\sqrt{7}}{14}, -\frac{3\sqrt{21}}{21}, \frac{3\sqrt{15}}{15}, -\frac{3\sqrt{10}}{10}, \frac{\sqrt{6}}{6})
\end{aligned}
\tag{A.6}
$$

The dilaton vector of $B_{\mu m}$ and $C_{\mu mn}$ are:

$$
\begin{aligned}
\vec{a}_{12} =&(1, \frac{6\sqrt{7}}{14}, -\frac{\sqrt{21}}{21}, -\frac{\sqrt{15}}{15}, -\frac{\sqrt{10}}{10}, -\frac{\sqrt{6}}{6})\\
\vec{a}_{13} =&(1, -\frac{2\sqrt{7}}{14}, \frac{5\sqrt{21}}{21}, -\frac{\sqrt{15}}{15}, -\frac{\sqrt{10}}{10}, -\frac{\sqrt{6}}{6})\\
\vec{a}_{14} =&(1, -\frac{2\sqrt{7}}{14}, -\frac{2\sqrt{21}}{21}, \frac{4\sqrt{15}}{15}, -\frac{\sqrt{10}}{10}, -\frac{\sqrt{6}}{6})\\
\vec{a}_{15} =&(1, -\frac{2\sqrt{7}}{14}, -\frac{2\sqrt{21}}{21}, -\frac{2\sqrt{15}}{15}, \frac{3\sqrt{10}}{10}, -\frac{\sqrt{6}}{6})\\
\vec{a}_{16} =&(1, -\frac{2\sqrt{7}}{14}, -\frac{2\sqrt{21}}{21}, -\frac{2\sqrt{15}}{15}, -\frac{2\sqrt{10}}{10}, \frac{2\sqrt{6}}{6})\\
\vec{a}_{23} =&(-\frac{1}{2}, \frac{5\sqrt{7}}{14}, \frac{5\sqrt{21}}{21}, -\frac{\sqrt{15}}{15}, -\frac{\sqrt{10}}{10}, -\frac{\sqrt{6}}{6})\\
\vec{a}_{24} =&(-\frac{1}{2}, \frac{5\sqrt{7}}{14}, -\frac{2\sqrt{21}}{21}, \frac{4\sqrt{15}}{15}, -\frac{\sqrt{10}}{10}, -\frac{\sqrt{6}}{6})\\
\vec{a}_{25} =&(-\frac{1}{2}, \frac{5\sqrt{7}}{14}, -\frac{2\sqrt{21}}{21}, -\frac{2\sqrt{15}}{15}, \frac{3\sqrt{10}}{10}, -\frac{\sqrt{6}}{6})\\
\vec{a}_{26} =&(-\frac{1}{2}, \frac{5\sqrt{7}}{14}, -\frac{2\sqrt{21}}{21}, -\frac{2\sqrt{15}}{15}, -\frac{2\sqrt{10}}{10}, \frac{2\sqrt{6}}{6})\\
\vec{a}_{34} =&(-\frac{1}{2}, -\frac{3\sqrt{7}}{14}, \frac{4\sqrt{21}}{21}, \frac{4\sqrt{15}}{15}, -\frac{\sqrt{10}}{10}, -\frac{\sqrt{6}}{6})\\
\vec{a}_{35} =&(-\frac{1}{2}, -\frac{3\sqrt{7}}{14}, \frac{4\sqrt{21}}{21}, -\frac{2\sqrt{15}}{15}, \frac{3\sqrt{10}}{10}, -\frac{\sqrt{6}}{6})\\
\vec{a}_{36} =&(-\frac{1}{2}, -\frac{3\sqrt{7}}{14}, \frac{4\sqrt{21}}{21}, -\frac{2\sqrt{15}}{15}, -\frac{2\sqrt{10}}{10}, \frac{2\sqrt{6}}{6})\\
\vec{a}_{45} =&(-\frac{1}{2}, -\frac{3\sqrt{7}}{14}, -\frac{3\sqrt{21}}{21}, \frac{3\sqrt{15}}{15}, \frac{3\sqrt{10}}{10}, -\frac{\sqrt{6}}{6})\\
\vec{a}_{46} =&(-\frac{1}{2}, -\frac{3\sqrt{7}}{14}, -\frac{3\sqrt{21}}{21}, \frac{3\sqrt{15}}{15}, -\frac{2\sqrt{10}}{10}, \frac{2\sqrt{6}}{6})\\
\vec{a}_{56} =&(-\frac{1}{2}, -\frac{3\sqrt{7}}{14}, -\frac{3\sqrt{21}}{21}, -\frac{3\sqrt{15}}{15}, \frac{2\sqrt{10}}{10}, \frac{2\sqrt{6}}{6})
\end{aligned}
\tag{A.7}
$$

The dilaton vectors of the scalars B_{mn} and C'_{mnr} are:

$$\begin{aligned}
\vec{a}_{123} =&(1, \frac{6\sqrt{7}}{14}, \frac{6\sqrt{21}}{21}, 0, 0, 0)\\
\vec{a}_{124} =&(1, \frac{6\sqrt{7}}{14}, -\frac{\sqrt{21}}{21}, \frac{5\sqrt{15}}{15}, 0, 0)\\
\vec{a}_{125} =&(1, \frac{6\sqrt{7}}{14}, -\frac{\sqrt{21}}{21}, -\frac{\sqrt{15}}{15}, \frac{4\sqrt{10}}{10}, 0)\\
\vec{a}_{126} =&(1, \frac{6\sqrt{7}}{14}, -\frac{\sqrt{21}}{21}, -\frac{\sqrt{15}}{15}, -\frac{\sqrt{10}}{10}, \frac{3\sqrt{6}}{6})\\
\vec{a}_{134} =&(1, -\frac{2\sqrt{7}}{14}, \frac{5\sqrt{21}}{21}, \frac{5\sqrt{15}}{15}, 0, 0)\\
\vec{a}_{135} =&(1, -\frac{2\sqrt{7}}{14}, \frac{5\sqrt{21}}{21}, -\frac{\sqrt{15}}{15}, \frac{4\sqrt{10}}{10}, 0)\\
\vec{a}_{136} =&(1, -\frac{2\sqrt{7}}{14}, \frac{5\sqrt{21}}{21}, -\frac{\sqrt{15}}{15}, -\frac{\sqrt{10}}{10}, \frac{3\sqrt{6}}{6})\\
\vec{a}_{145} =&(1, -\frac{2\sqrt{7}}{14}, -\frac{2\sqrt{21}}{21}, \frac{4\sqrt{15}}{15}, \frac{4\sqrt{10}}{10}, 0)\\
\vec{a}_{146} =&(1, -\frac{2\sqrt{7}}{14}, -\frac{2\sqrt{21}}{21}, \frac{4\sqrt{15}}{15}, -\frac{\sqrt{10}}{10}, \frac{4\sqrt{6}}{6})\\
\vec{a}_{156} =&(1, -\frac{2\sqrt{7}}{14}, -\frac{2\sqrt{21}}{21}, -\frac{2\sqrt{15}}{15}, \frac{3\sqrt{10}}{10}, \frac{3\sqrt{6}}{6})
\end{aligned} \tag{A.8}$$

$$\begin{aligned}
\vec{a}_{234} =&(-\frac{1}{2}, \frac{5\sqrt{7}}{14}, \frac{5\sqrt{21}}{21}, \frac{5\sqrt{15}}{15}, 0, 0)\\
\vec{a}_{235} =&(-\frac{1}{2}, \frac{5\sqrt{7}}{14}, \frac{5\sqrt{21}}{21}, -\frac{\sqrt{15}}{15}, \frac{4\sqrt{10}}{10}, 0)\\
\vec{a}_{236} =&(-\frac{1}{2}, \frac{5\sqrt{7}}{14}, \frac{5\sqrt{21}}{21}, -\frac{\sqrt{15}}{15}, -\frac{\sqrt{10}}{10}, \frac{3\sqrt{6}}{6})\\
\vec{a}_{245} =&(-\frac{1}{2}, \frac{5\sqrt{7}}{14}, -\frac{2\sqrt{21}}{21}, \frac{4\sqrt{15}}{15}, \frac{4\sqrt{10}}{10}, 0)\\
\vec{a}_{256} =&(-\frac{1}{2}, \frac{5\sqrt{7}}{14}, -\frac{2\sqrt{21}}{21}, -\frac{2\sqrt{15}}{15}, \frac{3\sqrt{10}}{10}, \frac{3\sqrt{6}}{6})\\
\vec{a}_{345} =&(-\frac{1}{2}, -\frac{3\sqrt{7}}{14}, \frac{4\sqrt{21}}{21}, \frac{4\sqrt{15}}{15}, \frac{4\sqrt{10}}{10}, 0)\\
\vec{a}_{346} =&(-\frac{1}{2}, -\frac{3\sqrt{7}}{14}, \frac{4\sqrt{21}}{21}, \frac{4\sqrt{15}}{15}, -\frac{\sqrt{10}}{10}, \frac{3\sqrt{6}}{6})\\
\vec{a}_{356} =&(-\frac{1}{2}, -\frac{3\sqrt{7}}{14}, \frac{4\sqrt{21}}{21}, -\frac{2\sqrt{15}}{15}, \frac{3\sqrt{10}}{10}, \frac{3\sqrt{6}}{6})\\
\vec{a}_{456} =&(-\frac{1}{2}, -\frac{3\sqrt{7}}{14}, -\frac{3\sqrt{21}}{21}, \frac{3\sqrt{15}}{15}, \frac{3\sqrt{10}}{10}, \frac{3\sqrt{6}}{6})
\end{aligned} \tag{A.9}$$

The root generators for the nodes in the dynkin diagram: $b_{12}, b_{23}, b_{34}, b_{56}$ and a_{123} are found in the adjoint representation with respect to a linear combination of a basis of root generators. To find the rest of them, the roots have to be written as a sum of the roots corresponding in the dynkin diagram [24], [57]:

$$\begin{aligned}
E_1{}^2 &= s_{1,2} + s_{11,13} + s_{14,16} + s_{17,18} + s_{19,20} + s_{21,22} \\
E^{123} &= s_{4,5} + s_{6,7} + s_{8,10} + s_{19,21} + s_{20,22} + s_{23,24} \\
E_2{}^3 &= s_{2,3} + s_{9,11} + s_{12,14} + s_{15,17} + s_{20,23} + s_{22,24} \\
E_3{}^4 &= s_{3,4} + s_{7,9} + s_{10,12} + s_{17,19} + s_{18,20} + s_{24,25} \\
E_4{}^5 &= s_{4,6} + s_{5,7} + s_{12,15} + s_{14,17} + s_{16,18} + s_{25,26} \\
E_5{}^6 &= s_{6,8} + s_{7,10} + s_{9,12} + s_{11,14} + s_{13,16} + s_{26,27}
\end{aligned} \tag{A.10}$$

$$\begin{aligned}
E_1{}^3 &= s_{1,3} + s_{19,23} + s_{21,24} - s_{9,13} - s_{15,18} - s_{12,16} \\
E_1{}^4 &= s_{1,4} + s_{21,25} - s_{15,20} + s_{7,13} + s_{10,16} - s_{17,23} \\
E_1{}^5 &= s_{1,6} + s_{21,26} + s_{10,18} - s_{5,13} + s_{12,20} + s_{14,23} \\
E_1{}^6 &= s_{1,8} + s_{21,27} - s_{5,16} - s_{7,18} - s_{9,20} - s_{11,23} \\
E_2{}^4 &= s_{2,4} + s_{15,19} + s_{22,25} - s_{7,11} - s_{10,14} - s_{18,23} \\
E_2{}^5 &= s_{2,6} + s_{22,26} - s_{10,17} + s_{5,11} - s_{12,19} + s_{16,23} \\
E_2{}^6 &= s_{2,8} + s_{22,27} + s_{5,14} + s_{7,17} + s_{9,19} - s_{13,23} \\
E_3{}^5 &= s_{3,6} + s_{10,15} + s_{24,26} - s_{5,9} - s_{14,19} - s_{16,20} \\
E_3{}^6 &= s_{3,8} + s_{24,27} - s_{5,12} - s_{7,15} + s_{11,19} + s_{13,20} \\
E_4{}^6 &= s_{4,8} + s_{5,10} + s_{25,27} - s_{9,15} - s_{11,17} - s_{13,18} \\
E^{124} &= s_{6,9} + s_{8,12} + s_{23,25} - s_{3,5} - s_{17,21} - s_{18,22} \\
E^{125} &= -s_{4,9} + s_{8,15} + s_{23,26} - s_{3,7} + s_{14,21} + s_{16,22} \\
E^{126} &= -s_{4,12} - s_{6,15} + s_{23,27} - s_{3,10} - s_{11,21} - s_{13,22} \\
E^{134} &= -s_{8,14} + s_{20,25} - s_{6,11} - s_{2,5} - s_{15,21} + s_{18,24} \\
E^{135} &= s_{4,11} - s_{8,17} + s_{20,26} - s_{2,7} + s_{12,21} - s_{16,24} \\
E^{136} &= s_{4,14} + s_{6,17} + s_{20,27} - s_{2,10} - s_{9,21} + s_{13,24} \\
E^{145} &= s_{8,19} + s_{2,9} + s_{18,26} + s_{3,11} + s_{10,21} + s_{16,25} \\
E^{146} &= s_{2,12} + s_{18,27} + s_{3,14} - s_{6,19} - s_{7,21} - s_{13,25} \\
E^{156} &= -s_{2,15} - s_{3,17} + s_{16,27} - s_{4,19} - s_{5,21} + s_{13,26} \\
E^{234} &= s_{6,13} + s_{8,16} + s_{19,25} - s_{1,5} + s_{15,22} + s_{17,24} \\
E^{235} &= -s_{4,13} + s_{8,18} + s_{19,26} - s_{1,7} - s_{12,22} - s_{14,24} \\
E^{236} &= -s_{4,16} - s_{6,18} + s_{19,27} - s_{1,10} + s_{9,22} + s_{11,24} \\
E^{245} &= s_{8,20} + s_{3,13} - s_{17,26} - s_{1,9} + s_{10,22} - s_{14,25} \\
E^{246} &= s_{3,16} - s_{17,27} - s_{1,12} - s_{6,20} - s_{7,22} + s_{11,25} \\
E^{256} &= s_{3,18} - s_{1,15} + s_{11,26} + s_{4,20} + s_{5,22} + s_{14,27} \\
E^{345} &= s_{8,23} - s_{1,11} + s_{15,26} - s_{2,13} + s_{10,24} + s_{12,25} \\
E^{246} &= -s_{1,14} + s_{15,27} - s_{2,16} - s_{6,23} - s_{7,24} - s_{9,25} \\
E^{356} &= -s_{1,17} - s_{2,18} - s_{9,26} + s_{4,23} + s_{5,24} - s_{12,27} \\
E^{456} &= -s_{1,19} - s_{2,20} + s_{5,25} - s_{3,23} + s_{7,26} + s_{10,27} \\
J &= s_{4,25} + s_{6,26} + s_{1,21} + s_{2,22} + s_{3,24} + s_{8,27}
\end{aligned} \tag{A.11}$$

where $s_{a,b}$ is the matrix with 1 in the (a, b) position and zero elsewhere. It is easy to see that the root generators satisfy (7.12). Combining the root generator representations, and the general equation for roots:

$$[H_i, E_j^k] = (\epsilon_j - \epsilon_k)(H_i)E_j^k, \tag{A.12}$$

the Cartan generators are:

$$\begin{aligned} H_i = \mathrm{diag}(& -f_1(i)+g(i), -f_2(i)+g(i), -f_3(i)+g(i), -f_4(i)+g(i), \\ & f_5(i)+f_6(i)-g(i), -f_5(i)+g(i), f_4(i)+f_6(i)-g(i), \\ & -f_6(i)+g(i), f_3(i)+f_6(i)-g(i), f_4(i)+f_5(i)-g(i), \\ & f_2(i)+f_6(i)-g(i), f_3(i)+f_5(i)-g(i), f_1(i)+f_6(i)-g(i), \\ & f_2(i)+f_5(i)-g(i), f_3(i)+f_4(i)-g(i), f_1(i)+f_5(i)-g(i), \\ & f_2(i)+f_4(i)-g(i), f_1(i)+f_4(i)-g(i), f_2(i)+f_3(i)-g(i), \\ & f_1(i)+f_3(i)-g(i), -f_1(i), -f_2(i), f_1(i)+f_2(i)-g(i), -f_3(i), \\ & -f_4(i), -f_5(i), -f_6(i)) \end{aligned} , \tag{A.13}$$

where $i = 1, .., 6$.

Appendix B

The 27-representation of E6

The vectors $A_\mu^{\alpha\beta}$, where [12], [14] $\alpha, \beta = 1, .., 8$ have the properties: $A_\mu^{\alpha\beta} = A^*_{\mu\alpha\beta} = -A_\mu^{\beta\alpha}$ and they are traceless. A map can be constructed with respect to the symplectic metric:

$$\Omega = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix} \tag{B.1}$$

from the real representation of $E_{6(6)}$, $J = (x_1, x_2, .., x_{27})$ to a matrix A of $A_\mu^{\alpha\beta}$, with a suitable choice of basis. This matrix A can be written as [16]:

$$A = \begin{bmatrix} B + iC & S + iD \\ -S + iD & B - iC \end{bmatrix}, \tag{B.2}$$

with the properties $A^*_{\gamma\delta} = (\Omega_{\gamma\alpha} A^{\alpha\beta} \Omega^T_{\beta\delta})^* = A^{\gamma\delta}$ and $A^{\beta\alpha} = -A^{\alpha\beta}$ and $A^\alpha_\alpha = 0$, which result in $B^T = -B, C^T = -C, D^T = -D$ and $S^T = S$ and $tr(S) = 0$. Hence the matrices B, C and D have six independent elements and S has nine. The matrix A can be written as:

$$B = \begin{bmatrix} 0 & x_1 & x_2 & x_3 \\ -x_1 & 0 & x_4 & x_5 \\ -x_2 & -x_4 & 0 & x_6 \\ -x_3 & -x_5 & -x_6 & 0 \end{bmatrix}. \tag{B.3}$$

$$S = \begin{bmatrix} x_7 & x_8 & x_9 & x_{10} \\ x_8 & x_{11} & x_{12} & x_{13} \\ x_9 & x_{12} & x_{14} & x_{15} \\ x_{10} & x_{13} & x_{15} & -x_7 - x_{11} - x_{14} \end{bmatrix}. \tag{B.4}$$

$$C = \begin{bmatrix} 0 & x_{16} & x_{17} & x_{18} \\ -x_{16} & 0 & x_{19} & x_{20} \\ -x_{17} & -x_{19} & 0 & x_{21} \\ -x_{18} & -x_{20} & -x_{21} & 0 \end{bmatrix}. \tag{B.5}$$

and

$$D = \begin{bmatrix} 0 & x_{22} & x_{23} & x_{24} \\ -x_{22} & 0 & x_{25} & x_{26} \\ -x_{23} & -x_{25} & 0 & x_{27} \\ -x_{24} & -x_{26} & -x_{27} & 0 \end{bmatrix}. \tag{B.6}$$

A similar map can be taken for the symplectic matrix:

$$\Omega = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}, \tag{B.7}$$

$A = F + iG$, where:

$$F = \begin{bmatrix} 0 & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ -x_1 & 0 & -x_3 & x_2 & -x_5 & x_4 & -x_7 & x_6 \\ -x_2 & x_3 & 0 & x_8 & x_9 & x_{10} & x_{11} & x_{12} \\ -x_3 & -x_2 & -x_8 & 0 & -x_{10} & x_9 & -x_{12} & x_{11} \\ -x_4 & x_5 & -x_9 & x_{10} & 0 & x_{13} & x_{14} & x_{15} \\ -x_5 & -x_4 & -x_{10} & -x_9 & -x_{13} & 0 & -x_{15} & x_{14} \\ -x_6 & x_7 & -x_{11} & x_{12} & -x_{14} & x_{15} & 0 & -x_1 - x_8 - x_{13} \\ -x_7 & -x_6 & -x_{12} & -x_{11} & -x_{15} & -x_{14} & x_1 + x_8 + x_{13} & 0 \end{bmatrix} \tag{B.8}$$

$$G = \begin{bmatrix} 0 & 0 & x_{16} & x_{17} & x_{18} & x_{19} & x_{20} & x_{21} \\ 0 & 0 & x_{17} & -x_{16} & x_{19} & -x_{18} & x_{21} & -x_{20} \\ -x_{16} & -x_{17} & 0 & 0 & x_{22} & x_{23} & x_{24} & x_{25} \\ -x_{17} & x_{16} & 0 & 0 & x_{23} & -x_{22} & x_{25} & -x_{24} \\ -x_{18} & -x_{19} & -x_{22} & -x_{23} & 0 & 0 & x_{26} & x_{27} \\ -x_{19} & x_{18} & -x_{23} & x_{22} & 0 & 0 & x_{27} & -x_{26} \\ -x_{20} & -x_{21} & -x_{24} & -x_{25} & -x_{26} & -x_{27} & 0 & 0 \\ -x_{21} & x_{20} & -x_{25} & x_{24} & -x_{27} & x_{26} & 0 & 0 \end{bmatrix} \tag{B.9}$$

Appendix C

Vector masses from Scherk-Schwarz reduction

The mass matrix (5.7) results in the following vectors to gain the corresponding masses shown in the table below:

Fields	Mass
$C^{(3)}_{\mu 35}$, $C^{(3)}_{\mu 15}$, $C^{(3)}_{\mu 14}$, g^6_μ	massless
$B_{\mu 6}$, $B_{\mu 4}$	$\lvert m_1 + m_2 \rvert$
$C^{(3)}_{\mu 56}$, g^1_μ	$\lvert m_1 - m_2 \rvert$
$C^{(3)}_{\mu 16}$, $C^{(3)}_{\mu 13}$	$\lvert m_1 + m_3 \rvert$
$C^{(3)}_{\mu 25}$, $B_{\mu 1}$	$\lvert m_1 - m_3 \rvert$
$C^{(3)}_{\mu 26}$, $B_{\mu 3}$	$\lvert m_1 + m_4 \rvert$
$C^{(3)}_{\mu 34}$, g^2_μ	$\lvert m_1 - m_4 \rvert$
$C^{(3)}_{\mu 36}$, $C^{(3)}_{\mu 12}$	$\lvert m_2 + m_3 \rvert$
$C^{(3)}_{\mu 24}$, g^3_μ	$\lvert m_2 - m_3 \rvert$
$C^{(3)}_{\mu 45}$, $B_{\mu 2}$	$\lvert m_2 + m_4 \rvert$
$B_{\mu 5}$, g^4_μ	$\lvert m_2 - m_4 \rvert$
$C^{(3)}_{\mu 46}$, $C^{(1)}_\mu$	$\lvert m_3 + m_4 \rvert$
$C^{(3)}_{\mu 23}$, g^5_μ	$\lvert m_3 - m_4 \rvert$

Table C.1: Four-dimensional vectors with corresponding masses from mass matrix (5.7)

The second mass matrix (5.10) results in the following massive vectors:

Fields	Mass
$B_{\mu 6}$, $C^{(3)}_{\mu 56}$, $B_{\mu 5}$ g^6_μ	massless
$C^{(3)}_{\mu 16}$, $B_{\mu 4}$	$\lvert m_1 + m_2 \rvert$
$C^{(3)}_{\mu 26}$, $C^{(3)}_{\mu 13}$	$\lvert m_1 - m_2 \rvert$
$C^{(3)}_{\mu 36}$, $B_{\mu 3}$	$\lvert m_1 + m_3 \rvert$
$C^{(3)}_{\mu 45}$, $C^{(3)}_{\mu 12}$,	$\lvert m_1 - m_3 \rvert$
$C^{(3)}_{\mu 46}$, $B_{\mu 2}$	$\lvert m_1 + m_4 \rvert$
$C^{(3)}_{\mu 35}$, $C^{(1)}_{\mu}$	$\lvert m_1 - m_4 \rvert$
$C^{(3)}_{\mu 25}$, g^1_μ	$\lvert m_2 + m_3 \rvert$
$C^{(3)}_{\mu 34}$, $B_{\mu 1}$	$\lvert m_2 - m_3 \rvert$
$C^{(3)}_{\mu 15}$, g^2_μ	$\lvert m_2 + m_4 \rvert$
$C^{(3)}_{\mu 24}$, g^3_μ	$\lvert m_2 - m_4 \rvert$
$C^{(3)}_{\mu 14}$, g^4_μ	$\lvert m_3 + m_4 \rvert$
$C^{(3)}_{\mu 23}$, g^5_μ	$\lvert m_3 - m_4 \rvert$

Table C.2: Four-dimensional vectors with corresponding masses from mass matrix (5.10)

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