

Faculty of Science Department of Physics and Astronomy

Big, Bigger, Biggest

Cosmology from Galaxy Cluster Alignment

Bachelor Thesis Institute for Theoretical Physics

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Abstract

Galaxies are observed to align with the Large Scale Structure. This alignment can be a potential probe for cosmological information. Recently Galaxy Clusters have also been observed to align with the Large Scale Structure. We have looked into Galaxy Cluster Alignment as a potential probe for cosmological information in comparison with Galaxy Alignment. Clusters have a bigger alignment amplitude (A_{IA}) , however galaxies are much more numerous. We investigated where this trade off lies in 2 survey setups. First, we have investigated them in SDSS. Here we use the redMaPPer dataset for clusters. We predict that in this survey they generally have a better signal to noise than galaxies. Second, we have forecasted a cluster sample for LSST. We find that the signal quickly washes out with redshift due to a relatively low number count and a decreasing alignment amplitude $(A_{IA}(z))$ with redshift. Galaxies give a significantly better signal to noise for LSST because of this. However, we also studied the effects of Weak Lensing for clusters. We predict that clusters seem to be less affected by this. Because of this it could be easier to extract the alignment signal from clusters than from galaxies. We have forecasted the CMB lensing - Alignment correlation in LSST as an interesting application of cluster alignments. We find that galaxies have a signal to noise that is about 50 percent better, however we predict clusters to be 10 times less contaminated by Weak Lensing.

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Introduction

Currently, Cosmology is blooming. We have entered an era of precision Cosmology where we have accurate theories which we can compare with multiple probes. For example, the Cosmic Microwave Background and the Large Scale Structure. This will only improve, in 1985 the best galaxy survey catalogued slightly more than a thousand galaxies with 3D positions (Dodelson 2003). However, in 2016 the Sloan Digital Sky Survey (SDSS) has mapped 1.2 million galaxies, but even these numbers look small compared to the number count expected to be observed by Legacy Survey of Space an Time in the Vera C. Rubin Observatory. The prediction is that the Rubin observatory will be able to map almost 2 billion (!) galaxies.

The theoretical model that is currently used most is the ΛCDM model. This is short for Lambda Cold Dark Matter model. Which is interesting, as it is named after two of the biggest question marks of the theory, Lambda and Cold Dark Matter. The Lambda stands for the existence of a 'cosmological constant' in the Einstein field equations. This has always been a reason for much debate, Einstein calling it his biggest mistake to add this constant. The constant nowadays lives under the the name 'dark energy' which is mostly giving our ignorance a name. Dark energy explains the acceleration of the expansion of the Universe, as was observed by looking at distant supernovae (Nobel Prize 2011, Royal Swedish Academy of Sciences 2011). In the ΛCDM parametrisation dark energy accounts for a staggering 68 percent of the energy budget of our Universe. The other part of the name is Cold Dark Matter, a form of matter that only (as far as we know) interacts via the gravitational force. There is a lot of astronomical evidence for the existence of (Cold) Dark Matter, of which the most famous example is the observed rotation curves of galaxies (Rubin et al. 1980). Dark matter accounts for about 27 percent of the energy budget. The remainder of the energy budget is filled with the matter we know and for the most part understand well: baryonic matter and radiation.

 ΛCDM gives a good description of the phases of expansion during the history of our universe. However, to describe the very first moments after the hypothetical start of our Universe we need to extend it with another theory. This is the theory of inflation, it describes the rapid expansion of the universe at the start. Inflation has had a lot of success describing the Universe, but there are still a lot of unanswered questions. We will not use theories of inflation directly, but we will use initial conditions given by inflation. This is useful, since we can look for the fingerprint of these initial conditions as a way of testing inflation theories. For example, many inflation theories induce some sort of non-gaussianity on large scales (Meerburg et al. 2019). We could potentially look for this non-gaussian signature in the position and shapes of galaxies (Schmidt et al. 2015).

To probe the cosmological model we will mainly look at the Large Scale Structure (LSS) of the Universe. This is together with the Cosmic Microwave Background (CMB) our main source of cosmological information. The Large Scale Structure contains all the structure in the Universe bigger than individual galaxies. It turns out that there is a lot of structure in the LSS. You can see voids with barely any galaxies and a filamentary structure forming some sort of cosmic web around these voids. It is interesting to calculate all kind of statistics about the large scale structure, because it is thought that this structure serves as a probe to the underlying density field. The reasoning is that structure forms at places where the density is higher than in other places, places with a so called 'overdensity'. Due to the non-linear nature of gravity, these overdensities grow with time. At some point this overdensity can collapse and a galaxy forms. Comparing the locations of galaxies thus gives us information about these fluctuations. These fluctuations give us information about the Universe. As the speed of light is finite, light from objects far away takes quite some

time to reach us. We get a picture of the object as it was at the moment it emitted the photons, which is thus earlier in the Universe. This has the practical result of looking back in time. We can thus see how the matter fluctuations evolve during the history of the universe. Knowing more about these fluctuations is essential in the understanding of our universe and can be connected to questions about dark energy, dark matter, gravity and inflation (for example via non-gaussianity). Because of this, the Large Scale Structure can be seen as a portal to both the biggest and smallest scales in distance and energy, giving us information about some of the most fundamental questions in physics.

One of the ways we can extract information from these fluctuations is via Intrinsic Alignments. The LSS aligns with the Tidal Field of the Universe (Catelan et al. 2001), because of this galaxies point on average towards overdensities. This is visible in the shape of galaxies as they tend to point towards each other (Singh et al. 2015, Samuroff et al. 2019, Blazek et al. 2011, Johnston et al. 2019, Joachimi et al. 2011). This has been primarily studied as a contaminant to Weak Lensing, but can also be used as a probe in its own right as it holds cosmological information (Chisari and Dvorkin 2013). Not only galaxies align, van Uitert and Joachimi 2017 found that galaxy clusters also align. A galaxy cluster is a group of galaxies ranging from about 20 to up to 1000 galaxies bound together by gravity. Galaxy clusters are embedded in a large dark matter halo and since they consist of galaxies they are much more massive than single galaxies. Van Uitert found the strength of the alignment is bigger for clusters than for galaxies. As it is thought to depend on the mass of the halo, this shows promise for the extraction of cosmological information using galaxy cluster alignments.

In this thesis we will be looking at galaxy cluster alignment as a potential probe for cosmological information. Cluster alignment has not been studied much as a cosmological probe as galaxies are much more numerous in the Universe. However, clusters have a much higher mass, and thus a higher alignment amplitude. We will look at several survey set ups and compare galaxies and clusters. Our goal is to find out if cluster alignment could be useful as a complementary probe for cosmological information. We want to know what the strenghts and weaknesses are of cluster alignments for the extraction of cosmological information. We will do this by comparing the galaxies and clusters in the SDSS¹ survey (York et al. 2000), of which we have the freely available galaxy cluster data (Rykoff et al. 2014), and by forecasting them in the LSST² survey (Ivezić et al. 2019).

This thesis is organised as follows: In section 1 we will describe the Large Scale Structure and several statistical tools used later in this thesis, in section 2 we will go into the modelling of Intrinsic Alignments. In section 3 we will describe several survey setups. In section 4 we will look at the auto-correlation of alignment and the cross-correlation with density, noting the pro's and cons of galaxy clusters. In section 5 we will look into effects of weak lensing on clusters and in section 6 we will apply our framework to CMB-Alignment correlations.

Throughout this thesis we will assume the following flat *Planck* ΛCDM cosmology: $\Omega_b = 0.045$, $\Omega_{CDM} = 0.27$, $\Omega_k = 0$, h = 0.67, $\sigma_8 = 0.83$ and $n_s = 0.96$ (Ade et al. 2016). In a flat Universe, Ω_b , Ω_{CDM} are respectively the baryonic and cold dark matter fractions of the energy budget. h is the dimensionless Hubble constant $H_0/100$, σ_8 is the amplitude of the power spectrum on the scale 8 Mpc/h. n_s is the exponent of the power law that describes the power spectrum during inflation.

¹https://www.sdss.org/

²https://www.lsst.org/

1 Structure, shapes and correlations

As mentioned before, the Large Scale Structure is all structure bigger than individual galaxies. This is a tremendous cosmic web comprised of an enormous amount of galaxies. As this thesis is mainly centered around the Large Scale Structure it is good to know what structures inhabit it. Furthermore we will talk about some of the tools we use to analyse them.

1.1 Galaxies and Galaxy Clusters

First and foremost, The LSS is composed of galaxies. Large structures containing billions of stars embedded in a dark matter halo. A typical galaxy has a weight of about $10^{12} M_{\odot}$. Galaxies can rougly be divided in 2 categories: spirals and ellipticals. Spirals have played an important role in the evidence for Dark Matter and their locations are useful probes for overdensities. In this thesis we will focus on ellipticals, as there is no evidence that spiral galaxies align (Johnston et al. 2019, Samuroff et al. 2019). These are thought to be older galaxies³ as they have lots of older red stars in them.

Galaxy clusters are groups of several galaxies. They mark the most prominent density peaks in the large scale structure, with masses up to $10^{15} M_{\odot}$ (Dodelson 2003). This by itself makes them a valuable probe for Cosmology already (Kravtsov and Borgani 2012). Clusters can be treated as distinct entities. The galaxies in a cluster are trapped in the cluster. They do not expand away from each other as the gravitational pull becomes more important than the Hubble flow (Dodelson 2003). Clusters are considered the most massive virialised structures in the Universe (Kravtsov and Borgani 2012). Since galaxy clusters consist of galaxies and not every galaxy lives in a cluster it is an obvious statement to say that galaxies are much more numerous than galaxy clusters.

We are mainly interested in two properties of clusters, their shape and their mass. As clusters are a distinct entity one can fit a shape to them by looking at the satellite galaxies. It is found that the satellite galaxies trace shape of the dark matter halo of the cluster (Wang et al. 2014, Dong et al. 2014). This shape tends to be elliptical, this will turn out to be a useful property for Intrinsic Alignment measurements. Getting the mass of Galaxy Clusters is tricky business. Most of the mass of a cluster is dark matter (Kravtsov and Borgani 2012), which is hard to measure directly as it does not emit any light. We are however very interested in the mass of the clusters. The reason for this is that a lot of physical mechanisms depend on the mass or give predictions for the mass. Intrinsic Alignments are an example for this, the amplitude of this effect is thought to depend on the mass. Ideally we would like to have a connection between the mass and some easily observed property of clusters. Such relations are known as Mass-Observable relations (MOR). A MOR that is widely used is the relation between the amount of galaxies in a clusters and its mass. The number of galaxies in a cluster is known as its richness λ . For this reason this relation is called the mass-richness relation. Richness is relatively easy to observe and is well correlated with the mass, this makes the mass-richness relation very useful. It is good to note that this is a statistical relation, one uses many clusters to calculate a point in this relation. Mass-richness relations are usually fitted to data with clusters where the mass is determined in some other way. For example by using X-Ray temperatures of gas, the Sunyaev-Zeldovich effect or lensing measurements (Dodelson 2003, Kravtsov and Borgani 2012). Getting a good relation between the mass and an observable (richness for example) is important and subject to quite a bit of research (Murata et al. 2018, Simet et al. 2016, Rykoff et al. 2012).

³Despite being older, for historical reasons ellipticals are also known with the misnomer: Early Type Galaxies

1.2 Scales and Number Densities

To get a better feeling for the scales we will consider an unperturbed Universe (only for this example). The matter density in such a universe is $\rho = \Omega_M \rho_{crit}$. The density in a spherical region is given by:

$$\rho = \frac{M}{4\pi R^3/3} \tag{1.1}$$

We can invert this to:

$$R = 0.951 h^{-1} \operatorname{Mpc} \left(\frac{Mh}{10^{12} \Omega_M M_{\odot}}\right)^{1/3}$$
(1.2)

 Ω_M is the matter fraction, $\Omega_M = \Omega_{CDM} + \Omega_b$. Plugging in our masses for galaxies and clusters gives a result of 1 Mpc for galaxies and about 10 Mpc for clusters.

Actually calculating number counts for galaxies and clusters is much harder. For clusters it is a bit easier as they are less non linear and more sparse. The relevant function is called the Halo Mass functions $\frac{dn}{dM}$, which is a function of mass and redshift. This gives the halo number density per unit mass at a certain redshift. There has been done quite some work on this. This has been done analytically (Press-Schechter theory), but nowadays it is usually done with simulations (for example, Tinker et al. 2010) as it gives significantly better results.

For clusters it is convenient to actually define what the mass is, as it matters at what density difference you draw the line. In this thesis we will use the standard M_{200} mass definitions. This is defined as:

$$M_{200} = \frac{4\pi}{3} R^3 \rho_M \times 200 \tag{1.3}$$

In words, we take the matter of the spherical volume with 200 times the matter overdensity.

For galaxies the number count is harder to predict directly as the halo mass function works primarily well at higher masses. At lower masses non linear effects (that were already important) become even harder to predict. Predictions for number counts observed can be made by extrapolating from previous surveys (for example, Chang et al. 2013).

Once we have number densities, it will be convenient to look at their redshift distribution. This is incorporated in the function dN/dz. The number count per redshift. It can also be useful to use a similar function, $W(\chi) = dN/d\chi$. This is the number count per comoving distance. The relation between these two can be calculated with the expression for the comoving distance (Dodelson 2003):

$$\chi(a) = c \int_{a}^{1} \frac{da'}{a'^{2} H(a')}$$
(1.4)

With *a* the scale factor given by $a = \frac{1}{1+z}$.

1.3 Cosmology with Shapes

We have talked about elliptical galaxies and spiral galaxies. The reason why the elliptical galaxies are used by us is because of the cosmological information hidden in their intrinsic shapes. The shape of galaxies and cluster depend in various ways on the tidal field of the Universe, and this is easiest observed in elliptical structures like early type galaxies and galaxy clusters. We assume the observed shape depends on 3 things:

$$\gamma = \gamma_I + \gamma_L + \gamma_{rnd} \tag{1.5}$$

 γ_I gives the contribution from Intrinsic Alignments. γ_L comes from the structure between our telescopes deflecting the light from the object, which affects the shape. This is called (weak) lensing. The third term accounts for random effects resulting in different shapes. The random shape contribution is by far the biggest, but because we look at shape correlations between lots of galaxies and cluster we assume γ_{rnd} is averaged away and will only produce noise.

Intrinsic Alignments are the focus of this thesis. They depend on the large scale structure surrounding the object. As discussed this contains cosmological information we are interested in (Chisari and Dvorkin 2013). Due to the importance of this subject in this thesis it has its own section (section 2).

The reason why Intrinsic Alignments have gotten attention in the first place is because they are contaminant to weak lensing (Kirk et al. 2012, Croft and Metzler 2000, Krause et al. 2015). Both effects affect the shape, but in different ways. Gravitational lensing is the bending of light trajectories by matter similar to a normal lens. Lensing can be split up in two cases: strong and weak. Strong lensing is the lensing effect that is strong enough to visibly deflect the light of an object. This can produce several images of an object or visibly deform it. The other effect is called weak lensing. This is the statistical variant of the effect. It is not observable on single clusters or galaxies. This is where the statistics come in, by comparing the shapes of lots of galaxies one can see shapes in certain places in the sky being slightly more compressed. This gives the shape contribution γ_L . This is then a probe for the matter in between the object and the observer. As it depends on the trajectory of the photons between the emitting object and the observer its effect increases with redshift. We will discuss cluster lensing more thoroughly in section 5.

1.4 Cosmological statistics: Power spectra and Correlation Functions

Cosmology is mostly based on statistics. This makes sense as we do not know the exact initial conditions of the Universe. Predicting the exact state of the Universe is thus impossible. We can however predict the statistical properties of our universe. For example the average temperature or density. It is then interesting to see how these quantities fluctuate, because we can also predict this. As mentioned in the introduction, these fluctuations are key for the formation of structure and contain a lot of information about the Universe, for example for constraining parameters to better understand the expansion of the universe (Weinberg et al. 2013) or by probing inflation (Meerburg et al. 2019).

To make these predictions quantitative cosmologists usually use correlation functions. As an example we will look at the density density correlation function, for this we first have to define the overdensity δ :

$$\delta(\vec{x}) = \frac{\rho(\vec{x}) - \bar{\rho}}{\bar{\rho}} \tag{1.6}$$

Where $\bar{\rho}$ is the average matter density in the Universe and $\rho(\vec{x})$ the matter density at a point \vec{x} . The correlation function is then defined as:

$$\xi_{\delta}(\vec{x} - \vec{x}') = \langle \delta(\vec{x})\delta(\vec{x}') \rangle \tag{1.7}$$

Physically it means how likely it is that an overdensity at location \vec{x} has another overdensity at a location \vec{x}' . It is convenient to put one overdensity in the center of your coordinate frame and switch to polar coordinates. Because of the isotropy assumption in cosmology only r is relevant. The correlation function then becomes:

$$\xi_{\delta}(r) = \langle \delta(0)\delta(\vec{r}) \rangle \tag{1.8}$$

Another widely used function is the power spectrum. The power spectrum is related to the correlation function by a Fourier transform and thus contains exactly the same information. However it can be more useful to work in Fourier space depending on the situation. The 3D power spectrum is defined in the following way:

$$\langle \delta(\vec{k})\delta(\vec{k}')^* \rangle = (2\pi)^3 \delta^3(\vec{k} - \vec{k}') P_\delta(k) \tag{1.9}$$

This is then simply related to the correlation function by a Fourier transform:

$$\xi_{\delta}(r) = \frac{1}{(2\pi)^3} \int d^3k P_{\delta}(k) e^{-i\vec{k}\cdot\vec{r}}$$
(1.10)

$$= \frac{1}{2\pi^2} \int_0^\infty dk P_{\delta}(k) j_0(kr) k^2$$
 (1.11)

Where the second equality follows from isotropy again. This is however only the tip of the iceberg. There is a whole zoo of correlations functions and thus power spectra. One could correlate most observables, for example temperature and density or galaxy shape and density. All power spectra are usually connected to the 3D density power spectrum. There are analytical formulas for this power spectrum, but we will use more accurate fits from simulations. An important note is that the power spectrum depends on redshift. Lower redshift means a bigger power spectrum as there has been more time for clustering. In this thesis we will mostly use the angular power spectrum.

The angular power spectrum has the benefit of being 2D, 3D surveys need detailed redshift information and this can be expensive and time consuming. Angular power spectra are easier produced from surveys, thus it is good to have theoretical predictions for them. To make predictions for the angular power spectrum we need to know the distribution of galaxies or clusters. We measure galaxies along our line of sight, this is the same as effectively integrating over the distance (χ). For an overdensity $\delta(\vec{x})$ we get the following projection:

$$\delta(\vec{\theta}) = \int d\chi W(\chi) \delta(\vec{x}) \tag{1.12}$$

With $W(\chi)$ normalised to 1, as it is a probability. We can Fourier transform this to $\delta(l)$. The angular matter power spectrum $C_{\delta}(l)$ is then defined by:

$$\langle \delta(\vec{l})\delta^*(\vec{l}')\rangle = (2\pi)^2 \delta^2 (\vec{l} - \vec{l}') C_\delta(l) \tag{1.13}$$

1.5 Bias Function

We have been talking a lot about overdensities in the Universe, but when we are actually going out to measure them we are facing a problem. We cannot directly detect the density field of the universe. We can only observe the density field indirectly via galaxies, clusters and other structures. These reside in dark matter halos which lie in the bottom of potential wells. This gives rise to a bias. This bias is bigger for higher mass structures (Kaiser 1984). The first order Taylor expansion of this relation is given below.

$$\delta_g = b_g(z)\delta\tag{1.14}$$

This proportionality b_g is called the bias function. It can be fitted from data, but there are also some theoretical models for it. The bias function is important in our research as it contributes to the amplitude of the clustering signal. As clusters are more massive, the factor b_g is bigger for clusters than for galaxies.

2 Modelling Intrinsic Alignments

2.1 Shape and the Tidal Field

As mentioned in the introduction, galaxies tend to align with the large scale structure. There are two main reasons for modelling the alignments. The reason they were studied in the first place is that alignments would contaminate weak lensing measurements (Kirk et al. 2012, Croft and Metzler 2000, Krause et al. 2015). To interpret the galaxy shape data properly, the intrinsic alignment mechanism must be well understood. The other reason is the reason we are most interested in, this is the fact that alignments also contain cosmological information (Chisari and Dvorkin 2013). This information could potentially be extracted. Because of the multiple lensing surveys coming up (LSST, Euclid⁴ (Laureijs et al. 2011), Roman Telescope⁵ (Spergel et al. 2015)), we will have access to lots of shape data. Not looking for cosmology in the alignments signal would be a waste of the good data we will have available.

To describe Intrinsic Alignments we will use a simple model known as the Linear Alignment Model, this fits the data well at large scales (Singh et al. 2015, Samuroff et al. 2019, Blazek et al. 2011, Johnston et al. 2019, Joachimi et al. 2011). This model connects the shape of a galaxy to the density field of the Universe. (Catelan et al. 2001, Hirata and Seljak 2004). We will first briefly describe this model and then calculate the angular power spectra (C_l 's) for it.

If an overdensity collapses to become a galaxy in a constant tidal gravitational field, the gravitational pull on one side will be stronger than the pull on the other side. This could both compress it or stretch the halo, depending on the field. This would result in some extra net ellipticity in the shape of the galaxy.

We define the ellipticity as:

$$\epsilon = \frac{1-q^2}{1+q^2} \tag{2.1}$$

Here q is the ratio of the minor axis to the major axis of the best fit ellipse fitted to the galaxy. We decompose this into two components, $\epsilon_{+} = \epsilon \cos(2\theta)$ and $\epsilon_{\times} = \epsilon \sin(2\theta)$. ϵ_{\times} is the 45° rotation from ϵ_{+} . To compare this to the lensing effect on the shape of galaxies this is usually expressed in the distortion γ . The relation between ϵ and the distortion is given by $\gamma = \epsilon/2\mathcal{R}$. \mathcal{R} is the responsivity of the signal, which depends on the measurement.

We do this calculation in a flat sky approximation giving us three coordinates (χ, x, y) where x and y are Cartesian coordinates on the sphere. χ is the comoving distance. In our full calculations we use another derivation that does not include the flat sky approximation.

Using these coordinates the distortion of a galaxy or cluster is connected to the Tidal Field by the following vector (Hirata and Seljak 2004).

$$(\gamma_+, \gamma_{\times}) = -A_{IA} \frac{C_1}{4\pi G} (\partial_x^2 - \partial_y^2, 2\partial_x \partial_y) \phi_p(\vec{x})$$
(2.2)

 A_{IA} gives the amplitude of the signal. C_1 is a pinned constant giving the amplitude of one of the first measurements (Brown et al. 2002). The gravitational primordial potential is given by ϕ_p . We will focus on the + part of the distortion as the × part correlation function with density vanishes. This can be explained by the reasoning that it would mean that there would be some preferential

⁴https://sci.esa.int/web/euclid

⁵https://roman.gsfc.nasa.gov/

rotation axis in the Universe, which goes against the cosmological assumption of isotropy on large scales. In later sections if we are talking about intrinsic alignments, we will be referring to the + component of the alignment distortion.

We Fourier transform the + component to get:

$$\gamma_{+}(\vec{k}) = -A_{IA} \frac{C_{1}}{4\pi G} (k_{x}^{2} - k_{y}^{2}) \phi_{p}(\vec{k})$$
(2.3)

We relate the primordial potential to the density field using the Poisson equation:

$$\phi_p(\vec{k}) = -\frac{4\pi G a^3 \bar{\rho}(z)}{D(z)} \frac{\delta}{k^2}$$
(2.4)

Here *a* is the scale factor, D(z) is the growth function and $\bar{\rho}(z)$ is the mean density of the universe at redshift *z*. The scale factor and growth function are 1 at z = 0. Now we rewrite equation 2.3 with some more convenient cosmological constants. Using $\bar{\rho} = \Omega_M \rho_{crit}/a^3$ we get:

$$\gamma_{+}(\vec{k}) = A_{IA} \frac{C_1 \rho_{crit} \Omega_M}{D(z)} \frac{(k_x^2 - k_y^2)}{k^2} \delta(\vec{k})$$
(2.5)

In our model we then neglect the terms $(k_x^2 - k_y^2)/k^2$, which is a good approximation on large scales. Getting the simple linear expression:

$$\gamma_{+}(\vec{k}) = -A_{IA} \frac{C_1 \rho_{crit} \Omega_M}{D(z)} \delta(\vec{k})$$
(2.6)

It is good to note that for a positive A_{IA} , which has been measured (Blazek et al. 2011, van Uitert and Joachimi 2017, Singh et al. 2015), makes the galaxies stretched towards an overdensity. This is opposite to the effect of Weak Lensing. Weak lensing compresses the shape of a galaxy along the tidal axis.

With our expression for the distortions we can derive the following expressions for 3D power spectra of alignment-alignment and position-alignment correlations:

$$P_{g+}(\vec{k},z) = -b(z)\frac{A_{IA}(z)C_1\rho_{\text{crit}}\ \Omega_M}{D(z)}P_{\delta}(\vec{k},z)$$
(2.7)

$$P_{++}(\vec{k},z) = \left(\frac{A_{IA}(z)C_1\rho_{\text{crit}}\,\Omega_M}{D(z)}\right)^2 P_\delta(\vec{k},z) \tag{2.8}$$

The magnitude of the parameter A_{IA} has been found to depend on luminosity for galaxies (Singh et al. 2015, Joachimi et al. 2011) and richness for clusters (van Uitert and Joachimi 2017). Because of this it is thought that the IA amplitude depends on halo mass. As clusters have a much higher mass, this explains their increased alignment. The previously mentioned measurements found $A_{IA}^{clus} \approx 3A_{IA}^{gal}$ for SDSS.

At small scales ($\approx 1 - 10 \,\text{Mpc/h}$) our approximations break down and it no longer fits the data (Singh et al. 2015), as one now comes to sizes comparable to halo sizes. For this area the model needs to be extended, this is called the halo model (Schneider and Bridle 2010).

2.2 The Angular Power Spectrum

A good way to compare a sample of clusters and a sample of galaxies is by using the angular power spectrum (C(l)). This function is the Fourier transform of the angular correlation function and shows how correlated galaxies are in l space. Below we will give a simplified derivation of the relevant angular power spectra.

We define $\delta(\vec{l})$ as the Fourier transform of $\delta(\vec{\theta})$. We define the same quantities for γ_+ . We can now do a similar calculation as in Dodelson, 2003, chapter 9.

The 2D power spectrum $C_{g,I}$, the angular power spectrum for position-alignment correlations, is defined via the following equation:

$$\langle \delta_g(\vec{l})\gamma_+(\vec{l})^* \rangle = (2\pi)^2 \delta^2(\vec{l}-\vec{l}') C_{g,I}(l)$$
 (2.9)

Integrating over the Dirac deltas gives us:

$$C_{g,I}(l) = \frac{1}{(2\pi)^2} \int d^2 l' \langle \delta(\vec{l}) \gamma(\vec{l'})^* \rangle$$
(2.10)

Fourier transforming and plugging the projection in equation 1.12 results in:

$$C_{g,I}(l) = \frac{1}{(2\pi)^2} \int d^2l' \int d^2\theta e^{i(\vec{l'}\cdot\vec{\theta'}-\vec{l}\cdot\vec{\theta})} \int d\chi W(\chi) \int d\chi' W(\chi') \langle \delta(\vec{x})\gamma(\vec{x'})\rangle$$
(2.11)

Now we write the integral over $\vec{l'}$ as a Dirac delta function over $\vec{\theta'}$ times $(2\pi)^2$ and perform the integral over $\vec{\theta'}$. We also write $\langle \delta(\vec{x})\gamma(\vec{x'})\rangle$ as the inverse Fourier transform of the 3D power spectrum defined in equation 2.7.

$$C_{g,I}(l) = \int d^2\theta e^{-i\vec{l}\cdot\vec{\theta}} \int d\chi W(\chi) \int d\chi' W(\chi') \int \frac{d^3k}{(2\pi)^3} P_{g,I}(k) e^{i\vec{k}[x(\vec{\chi},\vec{\theta}) - \vec{x}(0,\vec{\theta'})]}$$
(2.12)

We can now write the integral over θ as Dirac delta function setting $l_1 = \chi k_1$ and $l_2 = \chi k_2$. We now use these delta functions to perform the integrals over k_1 and k_2 .

$$C_{g,I}(l) = \int d\chi \frac{W(\chi)}{\chi^2} \int d\chi' W(\chi') \int dk_3 P_{g,I}\left(\sqrt{k_3^2 + l^2/\chi^2}\right) e^{ik_3[\chi - \chi']}$$
(2.13)

Now we neglect the factor k_3 since it does not contribute a lot to the integral. This is called the Limber Approximation. With this approximation we write the exponent as a Dirac delta again to get the expression:

$$C_{g,I}(l) = \int d\chi \frac{W^2(\chi)}{\chi^2} P_{g,I}(\frac{l}{\chi})$$
(2.14)

We now make the substitution $k \equiv l/\chi$.

$$C_{g,I}(l) = \frac{1}{l} \int dk W^2(l/k) P_{g,I}(k)$$
(2.15)

To get $C_{I,I}$, the angular power spectrum function of alignment-alignment correlations, we simply need to change $P_{g,I}$ to $P_{I,I}$:

$$C_{I,I}(l) = \frac{1}{l} \int dk W^2(l/k) P_{I,I}(k)$$
(2.16)

We use the latest version (v2.1.0) of the PYCCL library (Chisari et al. 2019) to calculate the angular power spectra in this thesis. This calculates slightly more complex power spectra, making

less approximations. These do not make use of the flat sky approximation and are thus valid on the complete sky. These power spectra are given by:

$$C_{\ell}^{ab} = \frac{2}{\pi} \int_0^\infty k^2 dk P_{\phi}(k, z) \Delta_{\ell}^a(k) \Delta_{\ell}^b(k)$$
(2.17)

CCL uses the non linear power spectrum for P_{ϕ} and transfer function via CAMB (Lewis and Challinor 2011). The Δ 's are the kernels which depend on the power spectrum you are calculating. $P_{\phi}(k, z)$ is the z dependent power spectrum of the potential. For clustering and alignments the kernels are:

$$\Delta_{\ell}^{g}(k) = \int dz \frac{dN}{dz} b_{g}(z) T_{\delta}(k) j_{\ell}(k\chi(z))$$
(2.18)

$$\Delta_{\ell}^{\rm IA}(k) = -\sqrt{\frac{(\ell+2)!}{(\ell-2)!}} \int dz \frac{dN}{dz} A_{IA} \frac{C_1 \rho_{crit} \Omega_M}{D(z)} T_{\delta}(k) \frac{j_{\ell}(k\chi(z))}{[k\chi(z)]^2}$$
(2.19)

 T_{δ} is the matter transfer function. Other differences are the inclusion of the Bessel functions j_l , the angular part in front of the IA-kernel and the use of z instead of χ .

3 Galaxy and Cluster samples

To make good predictions we need to know the sample of galaxies and clusters that will be observed. This depends on the survey. We will investigate two different surveys.

First, we will look at what is possible now. For this we look into the galaxies and clusters observed by the Sloan Digital Sky Survey (SDSS). We know how to accurately model them and have observations for Intrinsic Alignments for them (Singh et al. 2015, van Uitert and Joachimi 2017). Using this we can asses some of their strengths and weaknesses. We are also interested in what we can do in the future. Because of this we forecast a galaxies and a cluster sample for LSST. We extrapolate measurements from SDSS to LSST to make predictions for the Intrinsic Alignments in this sample. Using this sample we can estimate how useful clusters will be in the next generation of telescopes.

3.1 Sloan Digital Sky Survey

The Sloan Digital Sky Survey (SDSS) is a survey done by the Apache Point Observatory in the United States. The project started in 2000 and is currently on its fourth observing run. We will use data from SDSS-DR8 (Aihara et al. 2011). Its results have proven their worth for modern cosmology as it was one of the first large surveys. Its data has been used for years and is mostly freely available. It is currently one of the biggest surveys available, this makes it worth looking into.

3.1.1 Cluster sample

For clusters we have used the coordinates from the clusters in the redMaPPer catalogue (Rykoff et al. 2014), which is freely avaiable⁶. This data is obtained via photometric data from the SDSS-DR8. It contains 26111 clusters in the range $0.08 < z \leq 0.6$. For these clusters we have the redshift (z), the ellipticities (ϵ_1 and ϵ_2), with these we can calculate the total ellipticity $\epsilon = \sqrt{\epsilon_1^2 + \epsilon_2^2}$. We have the richness (λ) which is defined as the number of galaxies in the cluster. These distributions are shown in figure 1 and 2.

In figure 1 we see the effects of a selection bias. At higher redshift lower richness clusters are harder to measure, as mentioned in Rykoff et al., 2014, sec. 10 and 11. Below $z \approx 0.35$ the data is mostly complete with the increasing number count being because of the increasing volume at higher redshift.



Figure 1: The SDSS redMaPPer distribution seen as a 2D histogram of richness and redshift

We rely heavily on the values for A_{IA} and b_g meas-

ured by van Uitert and Joachimi 2017, these values for are shown in table 1 for several cuts. They define their IA amplitude in the following way, including a richness and redshift dependence:

$$A_{IA}(\lambda, z) = A_{IA}^{gen} \left(\frac{1+z}{1+z_0}\right)^{\eta} \left(\frac{\lambda}{\lambda_0}\right)^{\beta}$$
(3.1)

⁶http://risa.stanford.edu/redmapper/



Figure 2: Several distributions of the cluster properties in the SDSS redMaPPer cluster catalogue.

For η and β we use their fitted values: $\eta = -3.2^{+1.31}_{-1.40}$ and $\beta = 0.6^{+0.20}_{-0.27}$. We get that the average standard deviation of the shape per component is $\sigma_{\gamma} = 0.098$, using $\sigma_{\gamma}^2 = \frac{1}{2}(\sigma_{\epsilon_1}^2 + \sigma_{\epsilon_2}^2)$. We note that this is significantly lower than the dispersion for galaxies. The ϵ distribution can be seen in figure 2c. If we use the complete redshift distribution we use the value they measured on their pivot redshift $z_0 = 0.3$ and pivot richness $\lambda_0 = 30$ which they measured as $A_{IA}^{gen} = 12.6^{+1.5}_{-1.2}$ combined with equation 3.1. For the complete distribution we use the bias in the second cut. This is reasonable because the values are all within error bars.

Redshift cut	Richness cut	$N_{clusters}$	b_g	A_{IA}
$0.08 < z \leq 0.16$	$19.8 < \lambda \leq 28$	513		16.2 ± 11.8
$0.08 < z \leq 0.16$	$28 < \lambda \leq 40.5$	323	4.22 ± 0.35	48.0 ± 22.0
$0.08 < z \le 0.16$	$\lambda > 40.5$	216		36.9 ± 11.2
$0.16 < z \le 0.35$	$19.8 < \lambda \leq 28$	4953		10.4 ± 2.6
$0.16 < z \le 0.35$	$28 < \lambda \leq 40.5$	2762	$4.25_{-0.16}^{+0.15}$	15.6 ± 3.0
$0.16 < z \le 0.35$	$\lambda > 40.5$	1699	0.10	19.1 ± 3.2
$0.35 < z \le 0.6$	$19.8 < \lambda \leq 28$	3213		16.2 ± 11.8
$0.35 < z \le 0.6$	$28 < \lambda \leq 40.5$	5652	4.61 ± 0.27	10.9 ± 2.4
$0.35 < z \leq 0.6$	$\lambda > 40.5$	6780		15.1 ± 2.4

Table 1: The properties of the SDSS redMaPPer cluster sample. It gives for every redshift and richness cut the corresponding number of clusters. It gives the bias and the amplitude as measured by van Uitert and Joachimi 2017. All the clusters in this sample have accurate shape measurements. Our number of clusters is slightly different from van Uitert since they apply a small selection over the clusters, but this difference is small.

3.1.2 Galaxy sample

For galaxies we approximate the redshift distribution from the SDSS-III BOSS LOWZ sample. This sample contains mostly luminous red galaxies (LRG), these galaxies give a good alignment signal (Hirata et al. 2007, Joachimi et al. 2011). We approximate it by assuming the number of galaxies per comoving volume is constant. This is a good approximation at low redshifts, since at low redshifts we see most of the galaxies. This is the case for the LOWZ sample. We use the measured constants from Singh et al. 2015. These are shown in table 2. They measured a standard deviation of the

shape to be $\sigma_{\gamma} = 0.2$.

The comoving volume is given by Hogg 2000:

$$dV_c = D_H \frac{(1+z)^2 D_A(z)^2 d\Omega dz}{\sqrt{\Omega_M (1+z)^3 + \Omega_k (1+z)^2 + \Omega_\Lambda}}$$
(3.2)

With, $D_H = c/H_0$ the Hubble distance, $D_A(z)$ the angular diameter distance, in the flat Universe we are assuming Ω_M and Ω_{Λ} are the energy density fractions of matter and dark energy given in the introduction and $\Omega_k = 0$. For the redshift distribution we get:

$$\frac{dN}{dz} \propto \frac{dV_c}{dz d\Omega} = D_H \frac{(1+z)^2 D_A(z)^2}{\sqrt{\Omega_M (1+z)^3 + \Omega_k (1+z)^2 + \Omega_\Lambda}}$$
(3.3)

The proportionality constant is irrelevant since the distribution is normalised to 1.

Redshift cut	$N_{galaxies}$	N_{shape}	b_g	A_{IA}
$0.16 < z \le 0.36$	173855	159621	1.77 ± 0.04	4.6 ± 0.5
$0.16 < z \le 0.26$	67880	63180	1.66 ± 0.07	4.1 ± 0.8
$0.26 < z \leq 0.36$	105975	96441	1.88 ± 0.05	5.1 ± 0.8

Table 2: The properties of the SDDS LOWZ galaxies sample. It gives for the redshift cuts the number of galaxies with accurate shape measurements, the number of galaxies. It gives the bias and the amplitude as measured by Singh et al. 2015.

3.2 Legacy Survey of Space and Time

It is also useful to know the results of a more futuristic survey. In this section we will forecast the cluster and galaxy samples of the Legacy Survey of Space and Time (LSST). This is the survey that will be done by the Rubin observatory. The Rubin Observatory telescope is a next generation telescope that is being built in Chile (Ivezić et al. 2019). One of the goals of LSST is constraining dark energy. As this will be the state of the art for the next years, we want to know if it is likely to give good results for clusters. We use our SDSS data from redMaPPer to compare.

3.2.1 Cluster sample

Number counts:

The expected distribution is given by the following equation (Eifler et al. 2020):

$$\frac{dN}{dzd\Omega} = \frac{dV_c}{dzd\Omega} \int dM \frac{dn}{dM} \int_{\ln\lambda_{\min}}^{\ln\lambda_{\max}} d\ln\lambda \ p(\ln\lambda|M,z)$$
(3.4)

Where $\frac{dn}{dM}$ is the halo mass function, we use the Halo Mass function from Tinker et al. 2010. $\frac{dV_c}{dzd\Omega}$ is the comoving volume element we talked about previously and $p(\ln \lambda | M, z)$ is a lognormal distribution. The integral over the distribution gives the fraction of the halo mass function we observe in our richness bin. The halo mass function gives the number density for a unit mass on a certain redshift. To get the number count per redshift it is integrated over a mass range and multiplied by the comoving volume. The Lognormal distribution was found by Murata et al. 2018 by fitting it to data from SDSS with the mass obtained by lensing.

$$p(\ln\lambda|M,z) = \frac{1}{\sqrt{2\pi}\sigma_{\ln\lambda|M,z}} \exp\left[-\frac{(\ln\lambda - \langle\ln\lambda\rangle(M))^2}{2\sigma_{\ln\lambda|M,z}^2}\right]$$
(3.5)

They derived the mass richness relation to be:

$$\langle \ln \lambda \rangle (M, z | A, B, C) = A + B \ln \left(\frac{M}{M_{piv}}\right) + C \ln(1+z)$$
 (3.6)

Where the pivot mass is defined as $M_{piv} = 3 \times 10^{14} M_{\odot}/h$. We follow the approach of Alonso et al. 2018 and use the scatter from Murata et al. 2018 with q_m set to 0. The reasons for this can be found in Murata's discussion.

$$\sigma_{\ln\lambda|M}\left(M, z | \sigma_0, q_M, q_z\right) = \sigma_0 + q_M \ln\left(\frac{M}{M_{piv}}\right) + q_z \ln(1+z)$$
(3.7)

As both q_m and q_z are 0 this is essentially σ_0 . All the constants are found in table 3. The total number count is given by the integral over z multiplied by the amount of sky covered in steradians:

$$N = \Omega \int_{z_{min}}^{z_{max}} dz \frac{dN}{dz d\Omega}$$
(3.8)

A problem with this calculation is that is does not account for selection effects, i.e. the fact that a survey has a harder time measuring lower λ clusters at higher redshifts. This effect could have several reasons, a Malmquist bias for example (Rykoff et al., 2014, section 10), which can be seen in magnitude limited surveys. Another plausible reason could be that the cluster finding algorithm redMaPPer finds clusters by locating the red galaxies and then proceed to look for groups of galaxies (Rykoff et al. 2014). This works well at low redshifts, as clusters in this range are mostly comprised of red galaxies. For higher redshifts this is not always true, at higher redshifts the fraction of clusters containing mostly blue galaxies increases, giving rise to a selection bias (Rykoff et al., 2014, section 11). We mentioned selection effects for the SDSS cluster sample, as it clearly shows in its redshift and richness distributions (figure 1). In figure 1 the lower richness limit seems to be a linear function of z to first order, so we model it in the following way:

$$\lambda_{\min}(z) = \lambda_{\min}^{0} + a(z - z_0)\Theta(z - z_0)$$
(3.9)

Here is λ_{\min}^0 the lower richness limit without selection effects, *a* is how quickly the selections effect increase. $\Theta(z - z_0)$ is the Heaviside step-function which makes z_0 the *z*-value at which selection effects become important.

Estimating the parameters is not straight forward, as we do not have detailed simulations tailored to LSST. We look for an increase in richness at higher redshift as is discussed in Rykoff et al., 2014, sec. 10 and 11. On top of a regular bias in magnitude limited surveys they found in simulations that their algorithm is less complete in its lower richness bins at higher redshift for reasons specified before. We look at the redMaPPer data from SDSS DR-8 (Aihara et al. 2011) and the preliminary year 1 data from the Dark Energy Survey (DES⁷, Samuroff et al. 2020, McClintock et al. 2018) to make an estimate of our parameters. We expect LSST to be more similar to DES, but better as LSST goes deeper than DES. We can see in figure 1, that for the SDSS data selection effects become important after z = 0.33. If we compare this with Table 1 from Samuroff et al. 2020, we see that for DES selection effects are much less present. They give their number count in three redshift, there would be a steep rise in the percentage of higher richness clusters in the second and third bin. We do not see this. This makes sense as both Samuroff et al. 2020 and McClintock et al. 2018 use a

⁷https://www.darkenergysurvey.org/

volume limited selection from DES, this means that up until their maximum redshift the survey can be considered complete. Their catalogs go up until $z \approx 0.65$ and $z \approx 0.7$. However as the maximum z is a local quantity in DES, as is discussed in Rykoff et al., 2016, sec. 3.4, this does not mean 0.65 and 0.7 are their maximum redshifts in their full survey area. This will also be likely for LSST, so choosing a redshift will only be a first order approximation. With this in mind we set our z_0 to 0.8. This is probably conservative. By comparing with the SDSS clusters (figure 1) and roughly extrapolating to LSST we choose a = 40.

This extrapolation is quite rough. However, the selection effects are relatively unimportant as they kick in at later redshift and the signal turns out to be mostly located at lower redshift. This is shown in figure 12. Because of this we think this suffices as a first order approximation.

We evaluate the integral over $\ln \lambda$ in terms of the error function and call this function S_1 .

$$S_1(M|\lambda, z) \equiv \int_{\ln \lambda_{\min}}^{\ln \lambda_{\max}} d\ln \lambda p(\ln \lambda | M) = -\frac{1}{2} \operatorname{erf} \left(\frac{\langle \ln \lambda \rangle (M, z) - \ln \lambda}{\sqrt{2} \sigma_{\ln \lambda | M, z}} \right) \Big|_{\lambda_{\min}}^{\lambda_{\max}}$$
(3.10)

Our original integral is then given by:

$$\frac{dN}{dzd\Omega} = \frac{dV_c}{dzd\Omega} \int dM \frac{dn}{dM} S_1(M|\lambda, z)$$
(3.11)

The cluster number distribution over mass and redshift are shown in figure 3. These show the predicted number counts per redshift and mass bin. We have overlayed the plots with the SDSS number counts for comparison. It shows that LSST includes the SDSS clusters as to be expected. In the panel on the right we see that in the same redshift range LSST mostly sees more lower mass clusters. This makes sense in the context of figure 1. The selection effects for LSST will kick in later, thus LSST sees the lower mass clusters SDSS does not see. We find the total number of clusters to be 124313. This is about 5 times the SDSS cluster count from redMaPPer.



Figure 3: The predicted LSST cluster count distributions plotted with the SDSS data overlayed. The left panel shows the distribution with redshift, the right panel with mass. For a fair comparison the right panel has LSST only evaluated to z = 0.6.

To investigate the distribution some more we will look at the mass probability distribution, we will do this via the joint probability distribution $P(\ln M, \ln \lambda)$. The joint probability distribution is

proportional to $P(\ln \lambda | M)P(\ln M)$, the proportionality constant is not relevant as we will normalise to one. $P(\ln M)$ is simply proportional to the halo mass function logarithmically binned times the comoving volume:

$$P(\ln M) \propto \int_{z_{\min}}^{z_{\max}} dz \frac{dV_c}{dz d\Omega} \frac{dn}{d\ln M}$$
(3.12)

The mass probability distribution is the mass and richness distribution integrated over λ :

$$P\left(\ln M | \lambda_{\min} \le \lambda \le \lambda_{\max}\right) \propto \int_{\ln \lambda_{\min}}^{\ln \lambda_{\max}} d\ln \lambda P(\ln M, \ln \lambda)$$
(3.13)

$$\propto S_1(M|\lambda_{min} < \lambda < \lambda_{max})P(\ln M) \tag{3.14}$$

The mass distribution for three richness cuts is shown in figure 4. As expected the mass increases with richness.



Figure 4: The predicted mass distribution for the LSST clusters for several richness bins. The distributions are calculated with 0 < z < 2 and normalised such that $\int d \ln \lambda \int d \ln M P(\ln M, \ln \lambda) = 1$.

The bias function is obtained by multiplying $p(\ln \lambda | M, z)$ with the halo bias function b_h .

$$b_g(z) = \frac{\int dM \frac{dn}{dM} b_h(M) S_1(M|\lambda, z)}{\int dM \frac{dn}{dM} S_1(M|\lambda, z)}$$
(3.15)

The mean richness per redshift is calculated in a similar way. We integrate over the distribution with λ multiplied to $p(\ln \lambda | M, z)$. This result is then normalised with the original integral. This gives the following expression:

$$\lambda(z) = \frac{\int \mathrm{d}M \frac{\mathrm{d}n}{\mathrm{d}M} S_2(M|\lambda, z)}{\int \mathrm{d}M \frac{\mathrm{d}n}{\mathrm{d}M} S_1(M|\lambda, z)}$$
(3.16)

Where we have defined S_2 in the following way.

$$S_{2}(M|\lambda, z) = \int_{\ln \lambda_{min}}^{\ln \lambda_{max}} d\ln \lambda \ \lambda p(\ln \lambda | M, z)$$

= $-\frac{1}{2} \exp\left(\langle \ln \lambda \rangle + \frac{\sigma_{\ln \lambda | M}^{2}}{2}\right) \exp\left(\frac{\langle \ln \lambda \rangle - \ln \lambda + \sigma_{\ln \lambda | M}^{2}}{\sqrt{2}\sigma_{\ln \lambda | M}}\right)\Big|_{\lambda_{min}}^{\lambda_{max}}$ (3.17)

Alignment Amplitude

The last piece we need is the alignment amplitude $A_{IA}(z)$. To calculate this we use the linear relation between amplitude and mass found by van Uitert and Joachimi 2017.

$$A_{IA} = B_{IA} \log_{10} \left(\frac{M}{M_{piv2}}\right) + C_{IA} \tag{3.18}$$

Where $B_{IA} = 11.5 \pm 1.1$ and $C_{IA} = 6.3 \pm 0.3$. Combining this with equation 3.6 gives a relation between the A_{IA} and λ , note that we already set C to 0:

$$A_{IA}^{\lambda}(z) = B_{IA} \log_{10} \left[\frac{M_{piv}}{M_{piv2}} \left(\frac{\lambda}{e^A} \right)^{\frac{1}{B}} \right] + C_{IA} = -2.04^{+0.48}_{-0.41} + 5.03^{+0.74}_{-0.68} \ln(\lambda)$$
(3.19)

Where we have propagated the errors of the constants. This can be combined with equation 3.16. To get the actual amplitude A_{IA} we also need to include the z dependence in equation 3.1.

$$A_{IA}(z) = A_{IA}^{\lambda}(z) \left(\frac{1+z}{1.3}\right)^{\eta}$$
(3.20)

For the remaining constants $(\eta, \beta \text{ and } \sigma_{\gamma})$ we use the values used in section 3.1.1. The richness and amplitude are plotted in figure 5.



Figure 5: On the left the richness distribution is shown. On the right the alignment amplitude A_{IA} with three different values of η . The black line uses the measured value by van Uitert and Joachimi 2017, the red lines use the value $\eta \pm \sigma$.

It is good to note that the selection effects we imposed, do not result in an increasing A_{IA} after z = 0.8. The suppressing z-dependence of $A_{IA}(z)$ is stronger than the extra amplitude due to increasing richness. This would indicate a noteworthy difference between galaxies and clusters, as the selection effects for galaxies results in an increasing amplitude at higher redshift (figure 6c). We do stress that our 'selection' effects are only a rough fit, and might not be a good enough prediction.

Constant	Value
А	3.207 ± 0.045
В	0.993 ± 0.045
\mathbf{C}	0.0 ± 0.3
σ_0	0.456 ± 0.045
q_m	0 ± 0.03
q_z	0 ± 0.1

Table 3: Values for the constants used in this section, the uncertainties are gaussian and given at 1 σ . The constants are taken from Murata et al. 2018. They are fitted by to a selection of SDSS redMaPPer clusters with the mass obtained by lensing.

3.2.2 Galaxy sample



Figure 6: The first panel shows the expected distribution of galaxies, both the red and the normal distribution is shown. The normalisation is arbitrary. The second panel shows $\frac{dN_{red}/dz}{dN_G/dz}$, thus the red fraction of the sample. The third panel shows the alignment amplitude A_{IA} expected for this distribution. We explain the sudden rise around z = 1.2 with a Malmquist like bias.

For the LSST Galaxies we use a function from Chang et al. 2013 for the distribution of the total galaxy count:

$$\frac{dN}{dz} \propto z^{\alpha} \exp\left[-\left(\frac{z}{z_0}\right)^{\beta}\right] \tag{3.21}$$

With $\alpha = 1.21$, $\beta = 1.05$ and $z_0 = 0.5$. This distribution has a median redshift $z_m = 0.82$. Again, the proportionality constant is irrelevant due to normalisation. We are however interested in the red fraction of this galaxy distribution. For this we rely on work done by Chisari et al. 2016; Chisari et al. 2015. We use their red fractions as well as their results for amplitude $A_{IA}(z)$. They model the bias A_{IA} by using the correlation with luminosity found by Joachimi et al. 2011, taking into account the Malmquist bias. As they use the correlation between luminosity and alignment amplitude this gives rise to a very steep increase in the alignment amplitude after the Malmquist bias becomes relevant. The red fraction and the bias are plotted in figure 6.

We approximate our bias function as (Alonso et al. 2018):

$$b_g(z) = \frac{0.95}{D(z)}$$
(3.22)

Like Chisari et al. 2016 we assume a shape dispersion $\sigma_{\gamma} = 0.26$ and 2.6 galaxies per square arcminute.

4 Forecasting Alignment and Density Correlations

In this chapter we will be comparing two of the most prominent correlations. We will be comparing the cross-correlation between Alignments and number density (g-IA) and the auto-correlation of alignments (IA-IA). They can be used for the extraction of cosmological information (Chisari and Dvorkin 2013). We will compare the LSST clusters and galaxies and the SDSS clusters and galaxies. We will use information from SDSS-DR8. We will compare the correlations in the form of angular power spectra. We go to $l_{max} = 1000$, we assume this to be conservative in comparison with Krause et al. 2015 going to $l_{max} = 5000$.

4.1 Sloan Digital Sky Survey

We have made the same cuts as van Uitert and Joachimi 2017. As we have good measurements for them, shown in table 1. We plotted the C_l 's and the fractional error $(\delta C_l/C_l)$ for the cluster sample. We assume that all the perturbations are Gaussian, then we can use the following covariance matrix derived by Joachimi and Bridle 2010:

$$\operatorname{Cov}[C_{\alpha\beta}(l), C_{\gamma\delta}(l)] = \frac{C_{\alpha\gamma}(l)C_{\beta\delta}(l) + C_{\alpha\delta}(l)C_{\beta\gamma}(l)}{(2l+1)f_{\rm skv}}$$
(4.1)

The factor f_{sky} is the fraction of the sky the survey has observed. The auto-power spectra also contain noise terms, we assume the noise terms to be white:

$$N^{\gamma} = \frac{\sigma_{\gamma}^2}{n_{\gamma}} \qquad N^n = \frac{1}{n} \tag{4.2}$$

The indices $\{\alpha, \beta, \gamma, \delta\}$ run over the number densities and shapes. σ_{γ} is the average dispersion of the ellipticities per component. The number density n_{γ} is the projected number densities with shape measurement per steradian, n is the same for clustering.



Figure 7: The predicted C_l 's and the fractional uncertainties for the SDSS clusters. The cuts from table 1 are applied.



Figure 8: The C_l 's and their fractional error for the IA - IA and the $g - IA C_l$ signals for the galaxy and cluster full SDSS distributions.

For clusters, most of the signal is at low redshifts. In figure 7 we see that the fractional errors are generally better in the lower bins. However, the number of clusters in the fist cut is small and the standard deviation on A_{IA} large, the signal could be different in reality. The second cut seems like a promising cut, with the fractional error being slightly smaller than the third. This is also in line with the amplitudes in the second cut being slightly larger. The third cut suffers from the decreasing amplitude with z. This effect shows to be stronger than the increased number count. The richness cuts seem to be quite comparable in terms of fractional errors. Only in the first redshift cut is a significant improvement visible. But, again the uncertainties on our constants are high because of the low number count in these bins. For the other cuts the extra signal from a higher A_{IA} due to the higher mass seems to be cancelled by the strongly decreasing number count with richness (see figure 2).

We now turn to the full z distribution. We want to compare the cluster sample to the galaxy sample. Specifically we want to see how the $C_{q,I}$ and the $C_{I,I}$ signals compare.

In figure 8 we have plotted the C_l 's for both distributions. It is clear that the cluster signal is much stronger than the analogous galaxy signal. This in line with the higher amplitude and bias factor. The g - IA signal is also stronger than the IA - IA signal, which is also expected. However we are mostly interested in their fractional error. This will determine the size of their error bars and how well it can be used as a cosmological probe. We see that for the errors of g - IA the clusters seem comparable with the galaxies while the errors for IA - IA seem significantly better for the clusters. It is also worth noting that the fractional errors in the cuts in figure 7 do not necessarily produce errors worse than the full distribution.

Since the fractional error is quite big we also calculated the signal to noise (S/N) to see if it is measurable. We use the following expression:

$$\frac{S}{N} = \sqrt{\sum_{l=l_{\min}}^{l=l_{\max}} \frac{C(l)^2}{\operatorname{Var}[C(l)]}}$$
(4.3)

A S/N of above 3 is usually measurable and above 5 it is considered a good signal. The results are

shown in table 4.

	IA - IA	g - IA
Clusters	1.51	9.74
Galaxies	0.66	8.92

Table 4: Signal to noise for both full SDSS distributions with $2 \le l \le 1000$.

This shows that the result is definitely measurable for g - IA. For IA - IA it is harder to measure, especially the galaxy version, however this has been done by Blazek et al. 2011. This might be due to them using a different method, namely the $w(r_p)$ statistic. This isolates the galaxies close together, this way they get less noise (Faltenbacher et al. 2009). The cluster IA - IA signal should be easier to measure, so a positive measurement could potentially be possible with the SDSS data with the same approach.

To investigate the reasons for the difference between clusters and galaxies we have compared the fractional errors with varying amplitude and shape variance. These are plotted in figure 9.



Figure 9: The fractional errors of the C_l 's for the g - IA and IA - IA for the SDSS cluster distribution. The first row shows g - IA and the second IA - IA. We vary the amplitude A_{IA}^{gen} (first column) and the shape dispersion σ_{γ} (second column). The dotted values show the true predictions for the C_l 's.

The fractional error for both C_l 's is sensitive to changes in the shape variance and amplitude, but the IA - IA fractional error is more sensitive to it. This can be explained by the fact that A_{IA} and σ_{γ}^2 are of quadratic order in the equations for the IA - IA fractional error, but of linear order in the equations for the g-IA fractional error. This also gives a reason for the IA-IA signal to noise for clusters being more than twice the signal for galaxies, since the galaxies both have a bigger shape variance and a smaller amplitude. This has a bigger influence on the IA - IA spectrum than on the g-IA spectrum. The g-IA signal is less sensitive to changes in the amplitude and variance. On the other hand, the g-IA signal is sensitive to the bias factor b_g . The cluster bias factor is also about 2-3 times bigger than the bias factor for galaxies also adding to the total amplitude. In figure 10 the ratio of the fractional errors of clusters to galaxies is shown. This shows that clusters are especially better for the IA - IA correlations, as they are better for every multipole l we have calculated.



Figure 10: The figure shows the relative fractional error of the angular power spectrum for both g - IA and IA - IA. This is the redMaPPer fractional error divided by the fractional error of the LOWZ distribution.

For SDSS the clusters have three advantages over galaxies and one big disadvantage. The advantages are:

- A_{IA} is about 2-3 times bigger.
- b_q is about 2-3 times bigger.
- σ_{γ} is about two times smaller.

The disadvantage is that galaxies are much more numerous in the universe. Despite this disadvantage we have predicted that the fractional errors for $C_{I,I}^{Clus}(l)$ will be about 40/50 percent better than galaxies. We predict the fractional errors for $C_{g,I}^{Clus}(l)$ slightly better than the values for galaxies. We thus conclude that the effects are especially promising in measuring and using IA - IAcorrelations.



4.2 Large Synoptic Survey Telescope

Figure 11: The C_l 's and their fractional error for the IA - IA and the $g - IA C_l$ signals for the galaxy and cluster LSST distributions calculated over the full z-space.

We start with looking at the full z-space distribution. If we compare figures 8 and 11 we see that the amplitude of the C_l 's is smaller, especially for the clusters. We will discuss the reason for this later. For galaxies the fractional errors are quite a bit better, but for clusters the fractional errors are comparable. We have quantified this by calculating the signal to noise for it. This is shown in table 5.

$$\begin{array}{c|c} IA - IA & g - IA \\ \hline \text{Clusters} & 1.36^{+0.26}_{-0.08} & 8.72^{+1.48}_{-1.09} \\ \hline \text{Galaxies} & 1.83 & 27.61 \end{array}$$

Table 5: Signal to noise for both full LSST distributions with $2 \le l \le 1000$. The errorbars for clusters are the 1 σ errorbars on η propagated. Note, that the other uncertainties are not included, however they would contribute to the total uncertainty.

We see increased S/N for galaxies, but decreased S/N for clusters in comparison for SDSS. An explanation for this is in the decreasing amplitude A_{IA} with redshift for clusters. The clusters at higher redshift do not contribute much to the signal. If we calculate the C_l 's over the full z-range they wash out the strong signal of the lower z clusters. This combined with the decreasing number count at higher redshift explains the low S/N. At higher redshift, the S/N is quite bad, this washes out the good signal if one calculates the S/N for the complete z-space.

We have calculated the cluster S/N in several bins. First, we have calculated the S/N in two bins, one ranging from 0 to 0.6 and one from 0.6 to 2. For the g-IA signal the first bin gives S/N = 12.37and for the second bin S/N = 4.52, the number count in these bins is comparable (± 60000). This shows that most of the signal is at lower redshift. We also calculated the S/N for 20 evenly spaced redshift bins (figure 12). One sees that in most lower z bins the S/N is much higher than the S/None gets if evaluating it over the complete redshift range. This means that if only this 0.1 range would be taken into account, with the corresponding number count, the S/N would still be better than the S/N for the complete distribution. Evaluating the full z-space distribution is thus clearly



Figure 12: The signal to noise (S/N) for the IA - IA and $g - IA C_l$'s for the LSST clusters. Calculated in various z bins. The black line shows the S/N calculated for evenly spaced z-bins with a width of $\Delta z = 0.1$. The red line gives the value if the S/N is calculated for one bin with the redshift range [0, 1.5], the green does the same with the range [0, 0.6].

not the best way of handling this, as it yields a lower S/N. This is less of a problem for the other distributions as SDSS does not probe the higher redshifts and the LSST galaxies signal varies less with redshift.

To be able to use the full potential Galaxy Clusters have to offer, one does have to use something more sophisticated than simply using the full z-space without using bins. For example, the $w(r_n)$ function would likely give better results (Faltenbacher et al. 2009). This function only takes contributions in a certain range $[\Pi_{min}, \Pi_{max}]$ into account, where Π is a distance from the cluster along the line of sight. Good binning however requires well constrained coordinates. This is a challenge for observational astronomers, but clusters usually have well constrained zcoordinates. For future surveys the increased signal will probably not surpass that of the galaxies. As clusters are massive objects that need a lot of growth, they reside primarily at lower redshift. At these lower redshifts they give a great signal and would be a good addition to alignment statistics, but



Figure 13: The S/N for the LSST cluster C_l 's calculated per $\Delta \lambda = 10$ bin

future surveys are mostly going deeper. We also show the the signal to noise per richness bin in figure 13. This shows that even though the amplitude rises with mass, the reduced number count at higher mass annihilates this positive effect.

Another interesting feature seems to be the relatively good fractional error on large scales (small l) for the IA - IA signal. It would be to presumptuous to take this to seriously, as the differences are relatively small, but a good signal on lower l's could be useful to constrain non-gaussianity. This is because non-gaussianity would be most visible on a low l-regime (Schmidt et al. 2015).

5 Galaxy Clusters and Weak Lensing

In the previous sections we have not looked at the effect of weak lensing. Weak lensing also affects the shape of galaxies (equation 1.5). For galaxies this effect is known (Dodelson, 2003, chapter 10). It is however important that we know how it affects clusters, to know in what degree they contaminate eachother. Chang and Jain 2014 found that galaxies are typically displaced a bit on the sky due to lensing. This would mean that the shape of clusters, as it is constructed out the locations of galaxies, also changes due to lensing. In this section we will make a simple argument that the effect on galaxy clusters is the same by doing a similar calculation as Dodelson does for galaxies.

We start with using the following expression for the deflection of light by the cosmological potential. (Dodelson, 2003, chapter 10)

$$\theta_S^i - \theta^i = 2 \int_0^{\chi} d\chi' \nabla_i \phi\left(\vec{x}(\chi')\right) \left(1 - \frac{\chi'}{\chi}\right)$$
(5.1)

Where θ_S is the angle at the source and θ the angle we observe. It is however more convenient to write this with the following transformation matrix.

$$A_{ij} = \frac{\partial \theta_S^i}{\partial \theta^j} = \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - \kappa - \gamma_1 \end{pmatrix}$$
(5.2)

Here is κ the convergence of the light, how much the image is magnified. γ_1 and γ_2 are the distortions. We now want to know how this affects the shape of clusters. We use the expression for the quadruple moment of clusters used by van Uitert and Joachimi 2017.

$$Q_{ij} = \frac{\sum_{k} (\theta_{i,k} - \theta_i^{BCG})(\theta_{j,k} - \theta_j^{BCG}) p_{mem,k}}{\sum_{k} p_{mem,k}}, \quad i, j \in \{1, 2\}$$
(5.3)

Where we sum over all clustermembers. BCG is the brightest cluster galaxies which corresponds with the center of the cluster. The chance that a galaxy is part of the cluster is is $p_{mem,k}$. This is determined by p_{mem} (Rykoff et al. 2014):

$$p_{\rm mem} = \frac{\lambda u(R|\lambda)}{\lambda u(R|\lambda) + b(R)}$$
(5.4)

With b(R) being the background noise and $u(R|\lambda)$ given by:

$$u(R) = [2\pi R\Sigma(R)]\phi(m_i)\rho_{\nu}(\chi^2)$$
(5.5)

Where Σ is the cluster density profile, ϕ is the cluster luminosity function, ρ_{ν} is the χ distribution with ν degrees of freedom. m_i is the *i*-band magnitude. The background noise term is given by:

$$b(\theta|z) = 2\pi R \bar{\Sigma}_g \left(m_i, \chi^2 | z \right) \tag{5.6}$$

Where Σ_g is the galaxy density as a function of *i* band magnitude. Physically the noise is given by the galaxies in the background that are not part of the cluster. p_{mem} could potentially be lensed, as multiple term depend on *R*. However, it is unlikely to be a big effect. As a first order approximation we thus assume no background noise, in which case $p_{mem} = 1$. Taking this in mind we get for the quadruple moment:

$$Q_{ij} = \frac{1}{n_{mem}} \sum_{k} \Delta \theta_{i,k} \Delta \theta_{j,k}$$
(5.7)

Where we have defined the angular distance to the galaxy center as $\Delta \theta_{i,k} \equiv \theta_{i,k} - \theta_i^{BCG}$. n_{mem} is the total number of galaxies in the cluster. Now we want to know how this shape changes because of the light deflection due to matter in between the observer and the source. We do this by transforming the angles in equation 5.7 with $\theta_i = (A^{-1})_{ij} \theta_i^S$.

$$Q_{ij}^{obs} = \frac{A_{ii'}^{-1} A_{jj'}^{-1}}{n_{mem}} \sum_{k} \Delta \theta_{i',k} \Delta \theta_{j',k}$$
(5.8)

Now we argue that averaged over all clusters we can assume a spherical symmetry. In which case the summation will be proportional to the Kronecker Delta. This then gives us:

$$Q_{ij}^{obs} \propto A_{ij'}^{-1} A_{ji'}^{-1} \tag{5.9}$$

We want to calculate the following two ellipticities:

$$\epsilon_1 = \frac{Q_{11} - Q_{22}}{Q_{11} + Q_{22}} \qquad \epsilon_2 = \frac{2Q_{12}}{Q_{11} + Q_{22}} \tag{5.10}$$

Since Q appears in both the numerator and the denumerator the proportionality will be divided away. For ϵ_1 we then get:

$$\epsilon_1 = \frac{(A_{11}^{-1})^2 - (A_{22}^{-1})^2}{(A_{11}^{-1})^2 + (A_{22}^{-1})^2 + 2(A_{12}^{-1})^2}$$
(5.11)

The inverse of A while neglecting κ and higher order γ 's (not doing this is possible, however not very enlightening).

$$A_{ij}^{-1} = \begin{pmatrix} 1+\gamma_1 & \gamma_2\\ \gamma_2 & 1-\gamma_1 \end{pmatrix}$$
(5.12)

Plugging this into equation 5.11, neglecting higher order terms and simplifying gives us $\epsilon_1 = 2\gamma_1$. A similar calculation for ϵ_2 gives us $\epsilon_2 = 2\gamma_2$. This is the same result as for galaxies. We can therefore use the formalism for galaxies with the number counts and redshift range of clusters to calculate the contribution to the signal. We calculate the power spectra with equation 2.17. For the lensing we use the following kernel (Schmidt and Jeong, 2012, App. E):

$$\Delta_{\ell}^{\rm L}(k) = -\frac{1}{2} \sqrt{\frac{(\ell+2)!}{(\ell-2)!}} \int \frac{dz}{H(z)} W^{\rm L}(z) T_{\phi+\psi}(k,z) j_{\ell}(k\chi(z))$$
(5.13)

Where W^L is given by:

$$W^{L}(z) \equiv \int_{z}^{\infty} dz' \frac{dN}{dz'} \left(z'\right) \frac{\chi' - \chi}{\chi' \chi}$$
(5.14)

In standard $\Lambda CDM T_{\phi+\psi}$ is related to the matter transfer function by.

$$T_{\delta} = -\frac{k^2}{3H_0^2 \Omega_m} \frac{T_{\phi+\psi}}{1+z}$$
(5.15)

We calculate these functions with CCL (Chisari et al. 2019).

We show the angular power spectra for several distributions in figure 14. For SDSS the Intrinsic Alignments contribute most of the total shape signal. This makes sense, the lensing amplitude



Figure 14: The angular power spectra and their fractional errors are shown in this figure. We show the C_l 's for Weak Lensing (WL), Intrinsic Alignments (IA). We show this for the clusters in SDSS and LSST.

increases with redshift as the light has to travel through more structure to reach us and the alignment amplitude is stronger at lower redshift. For LSST we see a comparable amplitude with alignments being slightly better in most of the regime. The same is true for the fractional errors, alignments seem to be slightly easier to detect in LSST. We have also compared the signal with galaxies. This can be seen in figure 15, we predict that the ratio of alignments to lensing is significantly higher for clusters than for galaxies. This could mean that the IA signal is less contaminated by Weak Lensing for clusters. This might mean that it is easier for clusters to extract the alignment signal than for galaxies, which becomes more important in future surveys which probe higher redshifts. In realistic models the lensing and alignment contribution are usually modelled together, to get a total prediction for the shape. A bigger alignment fraction means that the cosmological fingerprint in alignments could be easier to see, as they are not buried as much by the lensing.



Figure 15: In this figure the ratio of alignment to weak lensing is plotted for both galaxies and clusters. This is thus for IA - IA the fraction of the WL - WL signal and for g - IA the fraction of the g - WL signal. We have plotted this for the LSST distributions.



6 Forecasting CMB Lensing and Alignment Correlations

Figure 16: The C_l 's and their fractional error for the $\kappa_{CMB} - IA$ and the $\kappa_{CMB} - WL C_l$ signals for the galaxy and cluster LSST distributions calculated over the full z-range.

The two main sources of cosmological information are the Cosmic Microwave Background and the Large Scale Structure. However they are not totally separate. The CMB photons travel through the LSS to reach us, and can thus consequently interact with it. This leads to an effect called CMB lensing, where the trajectory of the photons is changed due to the matter field of the LSS.

We look at the convergence field of the CMB lensing (κ_{CMB}), this can be derived from the lensing matrix (equation 5.2). This depends on the matter field (via equation 5.1) between the surface of last scatter for CMB-photons and the observer. As both alignments and weak lensing depend on the matter field it makes sense that a measurement of the $\kappa_{CMB} - shape$ cross-correlation would yield a non-zero result. This has also been measured by Hand et al. 2015. To properly study this effect we need good predictions for the intrinsic alignments and the lensing part of the galaxies and clusters. For galaxies this has been studied by Chisari et al. 2015, Hall and Taylor 2014 and Troxel and Ishak 2014. They found that for certain surveys Intrinsic Alignments are about 10 to 15 percent of the magnitude of the lensing signal.

To calculate the power spectra we use equation 2.17. The relevant kernel for CMB lensing is (Lewis and Challinor 2006, Schmidt and Jeong 2012):

$$\Delta_{\ell}^{\kappa}(k) = -\frac{\ell(\ell+1)}{2} \int_{0}^{\chi} \frac{dz}{H(z)} \frac{\chi_{*} - \chi}{\chi\chi_{*}} T_{\phi+\psi}(k, z)$$
(6.1)

Where χ_* is the comoving distance to the surface of last scatter. We will make predictions for the $C_l^{\kappa_{CMB}, \mathbf{x}}$ spectra with \mathbf{x} being weak lensing or alignments using CCL (Chisari et al. 2019). We will do this for both galaxy and cluster distributions. The noise is estimated using equation 4.1 with Poisson noise for the galaxies and clusters. We get the CMB noise from *Planck* (Aghanim et al. 2018), the noise is publically availabe⁸. We show the signal to noise S/N in figure 17b. The total S/N for the LSST clusters is 4.47 which should measurable.

⁸https://pla.esac.esa.int/



Figure 17: The left panel shows the C_l fraction: $(\kappa_{CMB} - IA)/(\kappa_{CMB} - WL)$. We compare the LSST galaxies with the LSST clusters. The right panel shows the signal to noise S/N for different bins similar to figure 12.

A benefit of the the CMB lensing correlation is that the signal is less washed out in comparison to g - IA and IA - IA. For g - IA and IA - IA the signal was washed out by the tail of the distribution. The CMB is being lensed by all the large scale structure between the surface of last scatter and us, this makes it that the higher redshift signal does not decay that much. We also calculated the S/N for the CMB lensing - IA correlation with the galaxies distribution. This gives us a value of S/N = 6.85. While this is about 50 percent better than the S/N of clusters, this does not mean they are clearly the better choice for the analysis. In section 5 we found that galaxies are more contaminated by lensing than clusters are, for this correlation this is shown in figure 17a. We see that galaxy alignments are about 10 percent of the amplitude of the lensing signal, which agrees nicely with Chisari et al. 2015. Cluster alignments are about as big as the cluster lensing. This could be more useful than having a better signal to noise for reasons specified in the previous section.

7 Conclusion and Discussion

Galaxy Cluster Alignment is a phenomenon worth studying. Galaxy clusters are the biggest virialised structures in the Universe and form on the largest matter overdensities. We have analysed them in the context of Intrinsic Alignments and have evaluated their use as a cosmological probe. We can structure our conclusions in two categories: lower and higher redshift.

For lower redshifts our results are very promising. Clusters have a lower shape dispersion. This has as a consequence that the clusters have relatively less Poisson noise for the shape. This combined with the stronger amplitude, about three times the value galaxies have (van Uitert and Joachimi 2017), gives a better signal to noise than galaxies in lower redshift surveys.

At higher redshifts this is still true. However, the amplitude A_{IA} decreases quickly with redshift (van Uitert and Joachimi 2017). This causes the signal to water down with redshift. There is another reason the signal waters down. We found that the LSST will only find about five times as many clusters as there are in the SDSS redMaPPer sample. This is not a big increase compared with the increase in galaxies we expect to see. This could be because clusters form relatively late in comparison to galaxies as they need a lot of growth. Clusters are usually brighter than single galaxies, this might mean that SDSS already saw a bigger fraction of the total cluster population than it saw galaxies as a fraction of the total galaxy population. The result is that clusters suffer from a relatively low observed number count at higher redshift compared to galaxies. Making good use of this signal at higher redshift requires smart data analysis to extract as much out of the data as possible, as binning the data improves the S/N significantly. A potential advantage of clusters has to do with their susceptibility to lensing. Our calculations indicate that lensing has the same effect on galaxies as on clusters. Because the alignment amplitude of clusters is much bigger than the amplitude for galaxies, the ratio of lensing to alignment is for the LSST clusters 10 to 100 times bigger depending on the statistic (alignment-alignment is more sensitive to the shape). This makes it easier to isolate the alignment signal from the shape. Our calculation however might only be a first order approximation and more work is necessary.

We have studied the correlation between CMB lensing and alignments. This could be a useful application. We predict that the S/N will be about 50 percent better for galaxies. This might be a trade off worth suffering as we predict that the clusters are less contaminated by Weak Lensing.

There are a some things that are good to note. First, our cluster distribution is made with priors from SDSS. This is because at the moment of writing the simulations for LSST are not yet available. Once they are available a more accurate distribution could be made. Our lensing calculation for clusters can also be done without assuming no background noise, this could give more accurate results. Third, we assumed the shape dispersion for clusters in LSST to be the same as SDSS, this does not necessarily have to be true. For galaxies there is a slight difference between SDSS and LSST, this could also be the case for clusters. Fourth, we restricted our analysis to C_l 's. This might not be the best tool, $w(r_p)$ would probably give better results, as binning the signal gives better results (Faltenbacher et al. 2009). Fifth, our predictions use the measured values from van Uitert and Joachimi 2017 and Singh et al. 2015. Some of the constants have significant errors, we do not take this into account in our analysis. All constants are also calibrated at low redshifts, it is yet to be seen whether we can extrapolate to higher redshifts without any issues. New surveys as DES (Abbott et al. 2016), KiDS⁹ (de Jong et al. 2013) and HSC¹⁰ (Aihara et al. 2017) could potentially already shed some light in this. The last two points are both very relevant for the cluster redshift

⁹http://kids.strw.leidenuniv.nl/

¹⁰https://www.naoj.org/Projects/HSC/

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dependence η . The uncertainties on this measured by van Uitert and Joachimi 2017 are significant and this parameter is especially relevant on higher redshift. Related, the redshift dependence can be explored more. Chisari et al. 2017 found in simulations that halos are more aligned at higher redshift. The galaxies within are not perfectly aligned with the halo, but they catch up with time, this would give different redshift dependence for galaxies and their halos. This also suggests halos being more aligned than the galaxies in them, especially at higher redshifts. As the shape of clusters is determined by its halo (Wang et al. 2014, Dong et al. 2014) they would be more aligned at higher redshift. This is not what van Uitert and Joachimi 2017 suggests. Last, something we have not addressed is the advantage of clusters that you do not need accurate galaxy shape measurements to get its shape, the shape can be constructed from the locations of the galaxies living in the cluster. In reality you would need accurate observations for algorithms that find clusters, this trade off could be taken into account.

8 Outlook

Large Scale Structure cosmology is a rapidly evolving field with lots of useful information coming in every day. In the future, the data volume will increase drastically and it will be optimal for mining many correlations that so far have been buried under the noise. First, we theorised that the IA - IA signal for clusters might be measurable in SDSS. Probably not for C(l)'s as the S/N is quite low, but with a more sophisticated approach using the $w(r_p)$ statistic (Faltenbacher et al. 2009) this might be possible as Blazek et al. 2011 had good results with it. This could be investigated. A measurement could further constrain our constants and give us extra information about the mechanism of Intrinsic Alignments at higher mass scales. Research into the z-dependence of clusters and galaxies is also useful, for reasons previously stated based on the work of Chisari et al. 2017. It is important for future surveys as Euclid and LSST to know how aligned they are at higher redshifts. It will however be challenging to separate this from lensing in observations, as the lensing signal increases with redshift. It is also an interesting question how to best handle this low amplitude data at higher redshift, data should be extracted in a smart way. Choosing the right bins to not wash out the signal could significantly improve the results. Having good results also depends on being able to extract the signal from the shape. This needs detailed knowledge of weak lensing of clusters. We have made some progress on this, but this could be improved. For example one could look in simulations how background noise affects the lensing signal.

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