The accumulated regret of trip chaining

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Abstract

Past research on commuting behaviour has tried to assemble the factors that effectuate mode choice. Thereby, trip chaining was found to be an important influence. The aim of this thesis is to apply the effect of trip chaining in a random regret minimization (RRM) model. In this thesis, a revealed preference data set will be used where individuals have registered their travel behaviour for one day. The attributes of work-related trip chains have been summed up such that mode choice relies on all trips instead of merely the work-related trip. The main question of this thesis is to what extent a regret-based model with trip chaining data would improve model fit. This thesis not only contributes to existing literature by testing a new methodology to process trip chaining data, but also by applying it to the RRM model, where it has never been incorporated before. Model estimation was based on the cost and time for each trip, and precipitation data from the Royal Dutch Meteorological Institute (KNMI). The results showed that aggregating trip chain data did not improve model fit. Furthermore, it was found that combining software- and human estimated travel times and distances led to faulty parameter estimates. On top of that, the random regret minimization model showed signs of low robustness; convergence strongly depended on the starting values of the parameters. Nevertheless, there is indication that private modes of transport are preferred for trip chains. Future research is encouraged to further explore this effect of trip chaining in regret-based models. In addition, it is recommended to first test new trip chaining methods on stated preference data to control for the travel time and distance on which the decision-maker bases its decision.

Keywords— Discrete choice models, mode of transport, commuting, prospect theory, regret minimization, trip chaining.

Contents

1 Introduction

Discrete choices are made every day: choosing a product in the supermarket, selecting a task from a list of chores, or deciding whether to go to work by car or train. These decisions are made on a daily basis and despite how small they may seem to the individual, on a global and even local level they produce quite complex behaviour. The field of discrete choice theory studies human decision-making in settings where a fixed number of alternatives is available and exactly one alternative must be chosen (or none of the alternatives). In particular, mode of transport choice has proven to be a challenge, since it is sensitive to individual preferences, context (e.g. weather), moods and habits, as well as affected by long-term decisions such as residential and employment locations (Garcia-Sierra, van den Bergh, & Miralles-Guasch, 2015). With the growing concerns about the environment, the burden on transport models has also increased. Modelling transport behaviour aides effective policy making and the ability to detect bottlenecks in the traffic flow. Disentangling this complex network into individual behaviour requires a thorough understanding of what constitutes a choice.

Often, mode choice for commuting is approached in a singular fashion. That is, if a person dropped his children off at school before going to work, then the mode choice for the trip "going to work" is evaluated independently from the trip "dropping off the children". However, since this person would go directly from school to work, he cannot switch modes. Therefore, the mode choice for the first trip does influence the mode choice for the second trip and vice versa. This is just one example of a trip that is interwoven with commuting. Other examples are doing groceries on the way home, a quick stop at the dentist in-between lunch, or visiting grandmother before going to work.

The focus of this thesis is which mode commuters choose under uncertainty when they have multiple destinations planned in their trip. The process of linking trips with multiple purposes together is called *trip chaining* and it is not a new phenomenon (e.g. Adler & Ben-Akiva, 1979). Still, trip chaining can be seen as one of the underdogs in discrete choice literature: in the article of De Witte, Hollevoet, Dobruszkes, Hubert, and Macharis (2013), the authors examined 76 papers to find out which attributes were used to define mode choice and thereby, define a common definition for mode choice. They summarized how often a particular attribute was chosen and how often it was found to be a significant contributor to mode choice. The factor trip chaining was only tested in 18% of the papers they reviewed, but still significant in 80% of the cases. Thus, a model could benefit from taking into consideration that the trip to or from work is just a link of a larger chain.

1.1 Problem definition

Previous literature that studied the causes of trip chaining (e.g. Hensher & Reyes, 2000) has used discrete choice models as a tool to study these effects, in particular the baseline model: random utility maximization (henceforth RUM) model (McFadden et al., 1973). In these models each individual is assumed to have a finite number of alternatives available to him, denoted as the choice set, and a selection of attributes (price, travel time, etc.) on which the individual bases his decision. Each alternative is evaluated in isolation of the other alternatives based on these traits. Due to the extensive number of variations of the model, its easy interpretation and convenient simulation methods (Train, 2009), the researcher has a lot of flexibility in defining the model and therefore, it allows him to better represent the various choice situations in the world.

Despite this flexibility, this type of discrete choice models has one major limitation: it is unable to capture the *decoy effect* (Guevara & Fukushi, 2016). When faced with two extreme options (e.g. a long waiting time and low costs versus no waiting time, but high costs), the preference orderings for the alternatives can change when a third "medium" option is introduced. For instance, if a supermarket that sells Coca Cola and Fanta, also decides to sell Pepsi Cola, then this most likely would influence the Coca Cola purchase more than Fanta. RUM models are constructed to ignore this part of the choice context; alternatives are evaluated independent of each other[1]. Therefore, researchers have searched for context-dependent variations of the RUM model.

This led to the development of alternative discrete choice models. One of them is the random regret minimization (henceforth RRM) model (C. G. Chorus, Arentze, & Timmermans, 2008; C. G. Chorus, 2010). There is an increasing amount of research (Jing, Zhao, He, & Chen, 2018) that supports the use of this model for analyzing route and mode choice behaviour. Regret is said to arise when a competing alternative performs better than the chosen alternative. The more uncertainty there is, the more people express regret-evasive behaviour (Kahneman & Tversky, 1979). The RRM model evaluates each alternative by comparing its performance (e.g. time, price) to the other alternatives [2]. Thus, it measures the possible regret that may arise when choosing an alternative i. Since the regret for each alternative depends on the other alternatives in the choice set, the introduction of a new alternative (e.g. Pepsi Cola) can change the preference ordering of the other alternatives (Coca Cola and Fanta). The main assumption in this model is that individuals choose the alternative that is expected to lead to the smallest amount of regret. With this approach, the golden mean is selected, instead of choosing the alternative with the highest satisfaction.

This thesis aims to build a stronger foundation for trip chaining as a determinant for mode choice using the RRM model rather than RUM. Instead of determining the regret for a single purpose trip, the regret model aggregates the anticipated regret for all the trips per mode. Thereby, this multi-purpose approach overcomes the limitations of previous work,

[1] In literature known as the *independence of irrelevant alternatives* (IIA) property. See Train (2009) or Guevara and Fukushi (2016) for more information on this.

[2] See Guevara and Fukushi (2016) for a thorough explanation why the IIA property does not apply to RRM and how this is related to the decoy effect.

where the mode choice for commuting is evaluated as a singular event, independent of linked trips. The main question is

to what extent can the influence of trip chaining on mode choice for commuters be implemented in RRM models by aggregating the attributes?

The attributes will be aggregated by summing up the attribute values for each alternative. It is expected that by accumulating the attribute values, the differences between alternatives increase, thereby making it easier for the RRM model to distinguish the golden mean alternative.

1.2 Data

Four different modes of transport are considered: car, train, bicycle and btm (bus, tram or metro). The data is a revealed preference data set obtained from the Central Bureau of Statistics (CBS). Each year, a survey about travel behaviour, denoted as OViN[3] (Centraal Bureau voor de Statistiek (CBS), 2017), is conducted among approximately 0.25% of the Dutch population. The participants receive a date where they must keep a record of all the trips they make. The survey also informs about their socio-demographic characteristics, such as household composition, age and income.

To obtain full information on all the alternatives that were available to the respondents of OViN, the software tool OpenTripPlanner (McHugh, 2011) was used to determine the choice set available to the individual and the time and cost for each alternative in this choice set. Furthermore, the data was enriched with precipitation data. Multiple studies (e.g. Kashfi, Lee, & Bunker, 2013; Liu, Susilo, & Karlström, 2015) have already showed the significance of rain data to mode choice. In the literature review of Jonkeren (2020), the weather was found to influence bicycle use the most. Furthermore, he found that precipitation has a positive effect on car usage, whereas the results for public transport and walking were ambiguous.

1.3 Contribution and overview

The contribution of this research to literature, and in particular the field of Artificial Intelligence, is to study the effect of trip chaining in a regret-based model, in which it has never been implemented before. The claim is not that the RRM model is generally better than the RUM model, but rather that it is more suitable in this choice environment. The RRM model is better equipped to handle the uncertainty (e.g. weather, traffic jams) that arises in the complex network of human decision-making. This different approach may lay bare new insights on the role of trip chaining in mode choice.

[3] Onderzoek Verplaatsingen in Nederland

The remainder of this thesis is structured as follows. The next section discusses the relevant literature on trip chaining and previously used methodologies to study this factor. Section 3 provides a brief summary of the theoretical background of the RUM and RRM model. Section 4 contains a thorough description of the data sets that were combined for testing the models. Section 5 will unveil the mathematical particularities of the RRM model, as well as the altercations that were made to account for trip chaining. In section 6 the results are discussed. Section 7 contains the discussion and implications for further research. The conclusion can be found in section 8.

2 Literature review

Trip chaining is defined as a sequence of trips that begin at home and end at home (Shiftan, 1998). Former research on trip chaining has mainly focused on the influence on public transport and car usage. The articles, accumulated by De Witte et al. (2013), generally showed that the length of the trip chain is correlated positively with car use and negatively with public transport. That is, the more complex a trip chain becomes (i.e., a higher number of side activities), the more likely it is that the car is used. This stresses the unavoidable downside of public transport: passengers have to rely on the efficiency and frequency of the network. They are not in control and this leads to uncertainty. The car gives a sense of control and moreover, brings you door-to-door. To fully comprehend why trip chaining stimulates car use, researchers first had to identify what caused these complex chains.

In literature, two socio-demographic characteristics were found related to complex trip chains. First of all, multiple researchers (e.g. O'Fallon, Sullivan, & Hensher, 2004; Currie & Delbosc, 2011; Hensher & Reyes, 2000) have found that the presence of children increased the chance of taking the car. Families generally have a larger number of trips to make (daycare, school, extracurricular activities) and as a result, have to construct more complex trip chains.

Second of all, Hensher and Reyes (2000) found that age had a positive impact on non-work related trip chains. They suspected that this was due to the increasing number of service trips. More complex chains have to be constructed to combine doctor visits, family responsibilities and other services. Contrary to this, Hensher and Reyes (2000) also found that larger families had shorter trip chains. An explanation for this is that the available modes have to be shared across the family members and similarly the household responsibilities are shared. For instance, the oldest child picks up the youngest child from school, while the mother does the groceries on her way home. Moreover, in a study of Krygsman, Arentze, and Timmermans (2007) it was found that women mostly were trip chaining as main caretaker of the household and children.

Alternatively, a recent study on trip chaining attempted to find which factors convinced Chinese commuters to switch from car to public transport (Sun, Li, & Wu, 2018). Their results showed that activities, which took place after work and were limited by time constraints, positively influenced the use of public transport. The risk of being delayed by congestion was too high. In addition, the uncertainty of arrival time and parking made car owners give up driving and resort to public transport.

Finally, one paper has focused on determining what part of the trip chain is essential for mode of transport choice (Habib, Day, & Miller, 2009). They found that mode choice is influenced by all the trips in the chain. The only exception were chains where the first trip is a work-related trip. In that case, this trip was the main determinant of mode choice.

3 Discrete choice models

Discrete choices are choices with a limited number of alternatives to select from. For example, answering a multiple choice test, deciding which card to play in a card game or, of course, which transportation mode to take to work. Sometimes, discrete choices are a sequence of decisions over time, such as deciding each year if you want to move house or not. The decision-maker only selects one option or opts out (choose none of the alternatives).

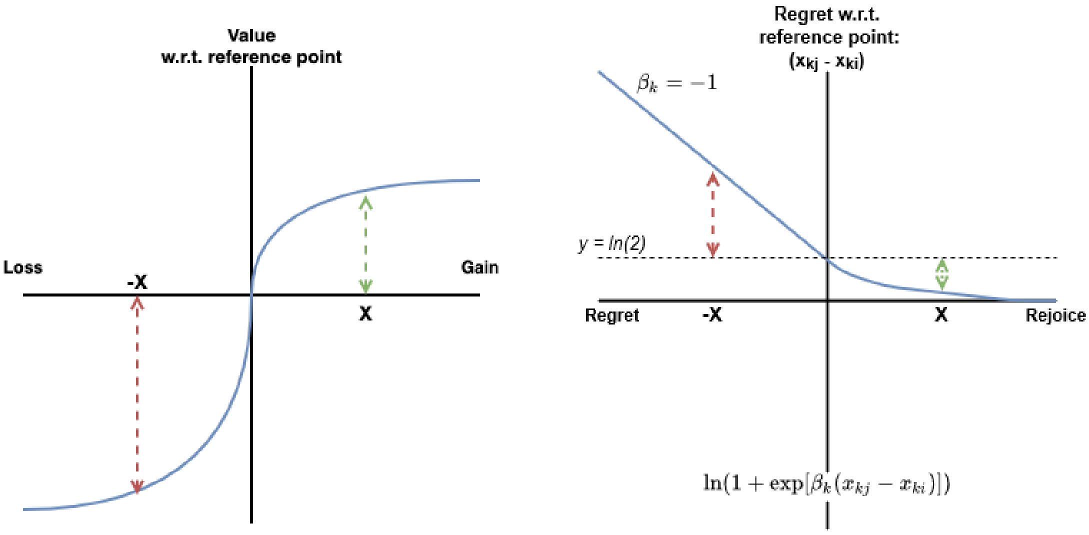
The purpose of discrete choice models is to help us understand the behavioural process that leads to the choice of the decision-maker. There are (internal or external) factors that collectively determine this choice. However, it is not always possible for a researcher to observe or measure these factors. Therefore, the field of discrete choice models distinguishes these factors as either *observed* or *unobserved factors*, denoted respectively as x and ϵ. To model the choice of a decision-maker, a function $f(x, \epsilon)$ is constructed. Let us use the vector y to refer to the available alternatives for the decision-maker. Then $y = f(x, \epsilon)$ is the function that models the behavioural process. Note, that it is a causal approach: if we know x and ϵ, then we can determine the choice y. Unfortunately, ϵ is always unknown as it encompasses all the unobserved variables. Therefore, it is common practice in discrete choice theory to model ϵ as stochastic with an unknown distribution, such that $f(x, \epsilon)$ can be used to calculate the probability of choosing some alternative in y. It is left to the researcher to specify a distribution for ϵ. The distribution determines what type of discrete choice model is exactly used to model the problem. When this distribution is chosen, the researcher obtains the probability for any particular outcome (i.e. all the alternatives y), given the function $f(x, \epsilon)$. The alternative with the highest probability, is assumed to be chosen by the decision-maker.

3.1 Utility-based models

The most commonly used models in discrete choice theory, define $f(x, \epsilon)$ as a *utility function*. The utility of an alternative is defined by summing over its characteristics (e.g. price, quality, etc.). Positive traits increase the utility, while negative traits decrease the utility. Essentially, the utility function is a scoring function. The higher the utility, the higher the chance that this option is being chosen by the decision-maker.

The baseline model that uses utility functions, is the *random utility maximization* model (McFadden et al., 1973). In this thesis, I will use the abbreviation RUM. In RUM models, decision-makers are assumed to make decisions under rational conditions. That is, it is assumed that they maximize a utility function with unlimited computational capabilities and full knowledge. In order to do this, they must know all the options that are available to them.

However, the assumption of unlimited computational abilities and the isolated analysis



(a) Loss aversion

(b) Regret function

Figure 2: Illustration of loss aversion. Left: loss aversion as explained by Kahneman and Tversky (1979). Right: loss aversion in the random regret minimization model of C. G. Chorus (2010).

of the alternatives is not in line with the concept of *bounded rationality* (Simon, 1955), which states that decision-makers are limited in their choice by their cognitive abilities, the information available to them and the time to make a choice. In the area of mode of transport choice, decisions are made with a high level of uncertainty. For instance, the time loss someone faces when there is risk of congestion or the uncertainty that comes with switching modes in public transport. Mode of transport is not the only domain where the outcome of choices is unpredictable (Tversky & Kahneman, 1974). When the field of economics started accepting the psychological theory of bounded rationality, it lead to the development of a large variety of models that tried to incorporate new decision rules describing human behaviour (e.g. Collins et al., 2012).

3.2 Regret-based models

One of these models was the *random regret minimization model* (C. G. Chorus et al., 2008; C. G. Chorus, 2010). In this model, the behavioural process $f(x, \epsilon)$ is defined as a *regret function*. The random regret minimization model (henceforth: RRM) rests on two theoretical foundations: *regret theory* (Bell, 1982; Loomes & Sugden, 1982; Quiggin,

1994) and *prospect theory* (Kahneman & Tversky, 1979). In regret theory, individuals are assumed to minimize anticipate regret that arises when a chosen option performs worse than another. While regret theory deals with single-attribute (monetary) choice situations, the RRM model extends this view to multi-attribute decision-making. For each alternative, a regret function is constructed that compares the attribute values of this reference alternative to the attribute values of the other alternatives.

When constructing this function, Chorus involved the second foundation: prospect theory. This theory makes a distinction between people's attitudes towards gains and losses. One of its principles is loss aversion, which states that people on the one hand show risk seeking behaviour when they face gains, but on the other hand, rather play it safe when they risk loosing something. Figure 2a illustrates this outcome. The shape of the function in the gain area is concave and flattens fast, indicating that each additional gain has less additional value. The shape of the function in the loss area is convex and does not flatten as in the gain area. As a result, each additional loss has a larger negative value than the positive value of an equivalent gain. In other words, the negative value that comes from loosing X units (e.g. X euros) is larger than the positive value of gaining the same amount of X, unless X is close to zero. Prospect theory requires that the decision-maker has a reference point on which he bases whether he is facing a loss or gain.

In the RRM model, loss aversion is incorporated as follows. One alternative is taken as a reference point. This alternative is compared to all the other alternatives; what are the price differences? Are the other modes of transport faster or slower? After the reference point is compared to all the alternatives, the total amount of regret for the reference point is calculated (the formula for this is given in section 5). Then another alternative is selected as the reference point and the comparison process starts again. In the RRM model, it is possible that losses have a higher value than gains. Consequently, regret has a larger impact on a decision than rejoice. Instead of maximizing a utility function, the decision-maker is expected to minimize regret. That is, the alternative that is expected to result in the least amount of regret is chosen.

Note that the RRM model does not deal with the cognitive limitations of the brain regarding decision-making. In this respect, it is still bounded by the same limitations as RUM. Instead, the focus of RRM is to incorporate the uncertainty that drives the decision-makers' behaviour in a model by modelling it as regret minimization behaviour.

Figure 2b shows the shape of an arbitrary regret function (blue line). The formula of this regret function will be explained in section 5. For now, the focus is on the theoretical foundation, i.e. the concept of loss aversion. The black dotted line indicates the baseline. The point where the regret function crosses this line, is the point where the alternatives have an equal performance on an attribute, e.g. the same travel time. The distance from the regret function until this line indicates the regret or rejoice from choosing the reference alternative i over another alternative j. If alternative j is expected to perform better than reference alternative i, e.g. the travel time of alternative j (tt_j) is lower than the travel time of the reference alternative i (tt_i), then $tt_j - tt_i < 0$ and we speak of regret. When $tt_j - tt_i > 0$, we speak of rejoice. As can be seen in figure 2b, the distance between the regret function and the black-dotted line for a regret of $-X$ is much larger than the distance for an equal amount of rejoice X.

3.3 Modelling details

In this thesis, assumptions were made for the behavioural process of the decision-makers to keep the computational problem tractable. First, the decision-maker is assumed to minimize regret when selecting which transportation mode will take him to work as previous literature has shown that this is a better approximation of mode choice behaviour (Jing et al., 2018).

Second, it is assumed that the decision-maker must go to work, i.e. there is no opt-out alternative. Working from home is normally planned beforehand and not every profession has this flexibility.

Third, the decision-maker is assumed to be aware of all the alternatives available to him. The assumption emanates from the limitations of the data set; the revealed preference data set only shows which decision has been made, but not the alternatives that were considered. Previous literature has studied methods to vary choice set size (Ben-Akiva & Boccara, 1995). However, the model used in this thesis is not suitable to handle different choice set sizes (van Cranenburgh & Prato, 2016) and therefore, it was assumed that the decision-maker was fully aware of all alternatives.

Finally, it should be noted that, over the years, multiple definitions of the RRM model have been given, each new version slightly adapted and improved, giving rise to a whole "family" of regret models: RRM2010 (C. G. Chorus, 2010), G-RRM (C. G. Chorus, 2014), and P-RRM and μRRM (van Cranenburgh, Guevara, & Chorus, 2015)). The version used in this thesis, is the most recent version, μRRM, that was introduced by van Cranenburgh, Guevara, and Chorus (2015). As the name suggests, this version contains an additional factor μ, which was added because previous RRM models were not scale-invariant. Section 5.1 will elaborate on this.

Henceforth, to avoid confusion in literature, I will refer to the regret model that is used in this thesis as μRRM.

4 Materials

The main focus of this thesis is to model the behavioural process of mode choice selection for commuters as part of a trip chain. The behavioural process is modelled as a regret function. This function compares alternatives with each other based on their attributes. This means that the data set must contain information on all the alternatives that were available to the decision-maker.

In order to do this, two data sources were combined. The first data set, **OViN**, contains information on individual travel behaviour of the Dutch population (Centraal Bureau voor de Statistiek (CBS), 2017). It is a yearly survey of the Dutch Central Bureau of Statistics to investigate the day-to-day mobility of the Dutch population. The survey only contains information regarding which mode(s) the decision-makers used for their trip, but not the alternatives that were available to them. To obtain the full choice set for each individual, **OpenTripPlanner** was used (McHugh, 2011). This open software tool estimates travel routes for means of transport such as transit, walking, bicycle and car. For each route, it can provide the travel time, cost (of public transport modes only), distance and number of mode changes.

On top of that, previous research has shown that rain has a strong influence on mode of transport choice (Hagenauer & Helbich, 2017; Jonkeren, 2020). Therefore, the data of OViN and OpenTripPlanner is supplemented by precipitation data obtained from the **KNMI** (Royal Dutch Meteorological Institute). All in all, these data sets are used to acquire the information on the three attributes that will be used in this thesis: travel time, cost (based on travel distance) and precipitation. The following sections provide more in-depth information on the processing of the OViN, OpenTripPlanner and KNMI data.

4.1 OViN

OViN is a revealed preference data set, containing the transit behaviour of the Dutch population since 2010. For this thesis, the years 2013-2017 were accumulated, since the structure of the survey has changed after 2012 and therefore, it was less compatible.

The survey is conducted as follows: each year about 40 thousand respondents (roughly 0.25% of the population) fill out a survey that registers their travel behaviour for one specific day[4]. Their transit behaviour is registered as trips, where each trip consists of a label and a number of legs. Each leg stands for a specific mode that was used. The label indicates the main mode of transportation during that trip. Since multiple modes can be used during the same trip, the CBS defined a preference ordering to determine the label:

1. Train

[4]People in assisted living facilities and institutions were excluded from the sampling, because their mobility is limited.

2. Bus/tram/metro
3. Car driver
4. Car passenger
5. Moped
6. Bicycle
7. Walking

For instance, consider an individual that has made a trip to work by first walking 500 meter, then travelling by train for 15 km and then another 2 km in the bus and 250 m walking to the destination. This trip consists of four legs (walking, train, bus, walking). The label for this trip would be the train, because it has the highest ranking. Even if someone would drive 20 km by car to a P+R and then travelled just 2 km by train, the main label of the trip would still be *train*.

Apart from travel behaviour, the survey also includes a wide range of socio-demographics of the participant. They can be divided into three main categories:

- structure of the household (single, married, single parent with children, etc.);
- transportation modes available to the participant;
- other socio-demographic traits of the individual such as age, gender, ethnicity and profession.

Respondents have been proportionally sampled by CBS, based on the number of people in their municipality. For each trip the purpose (e.g. commuting, groceries, sport, ...) of the individual was reported.

4.1.1 Reduction and cleaning of the data set

This study focuses on commuting behaviour on weekdays. To reduce computational load, the model was only applied to the MRDH region ("Metropoolregio Rotterdam Den Haag"), which already provided ample data (51,879 trips from 16,307 individuals). This region covers big cities such as Rotterdam and The Hague (figure 3), which are part of the *Randstad*, an urban area in the Netherlands. High congestion during rush hours is not uncommon in this area. This region also has an extensive subway and tram network.

Trips that went abroad were excluded, because the data set only included information until the border and thus, was incomplete. Freight traffic was also not taken into account.

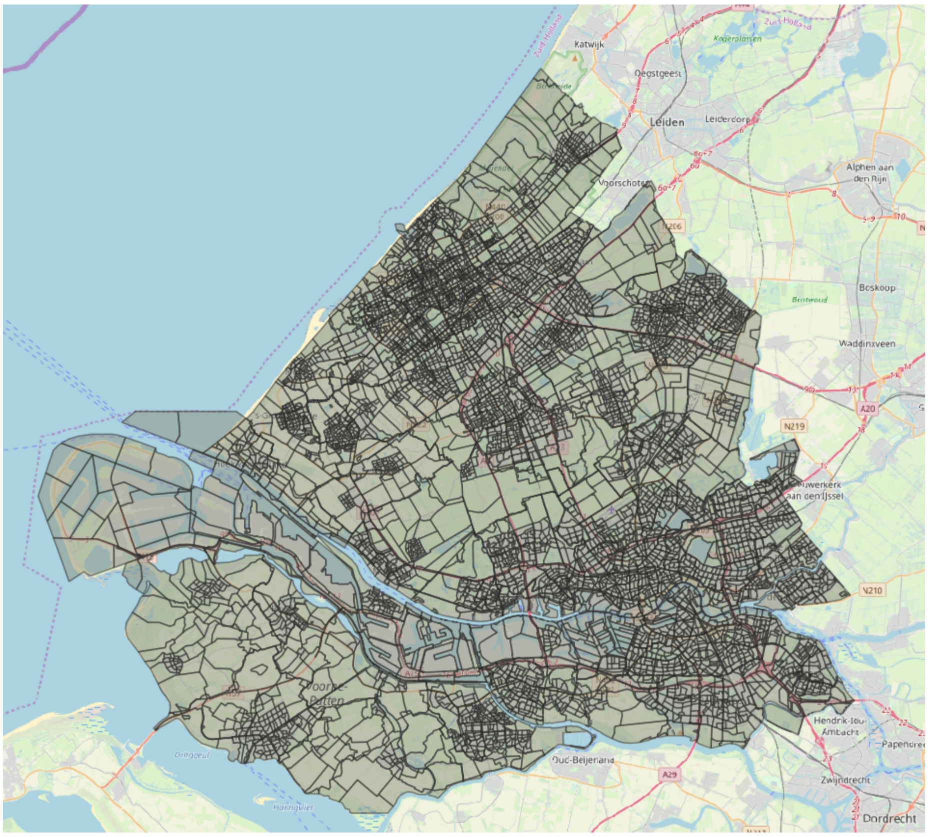


Figure 3: The MRDH area; obtained from TNO. Constructed with data obtained from OpenStreetMap contributors (2017) and V-MRDH data from TNO.

Furthermore, two age requirements were made. Participants younger than 18 were omitted, since they are not allowed to drive a car (which restricts their choice set) and most likely rely on their parents resources and schedule. Participants older than 75 were also taken out. At this age, a medical check is required to renew a driver's license and physical health might also restrict the choice set, such as excluding bicycle or public transport as transportation modes.

Finally, the data set included some peculiarities. Five trips where participants reported a severe physical performance (e.g. cycling more than 2 hours to work), were removed from the data set, because it was questionable whether they correctly filled in the survey. The same was done for 65 observations where respondents claimed that their trip to or from work took over 4 hours. Considering that, in this time frame, one could even travel from Maastricht to Groningen by car or train, it is very likely that these participants have misunderstood the

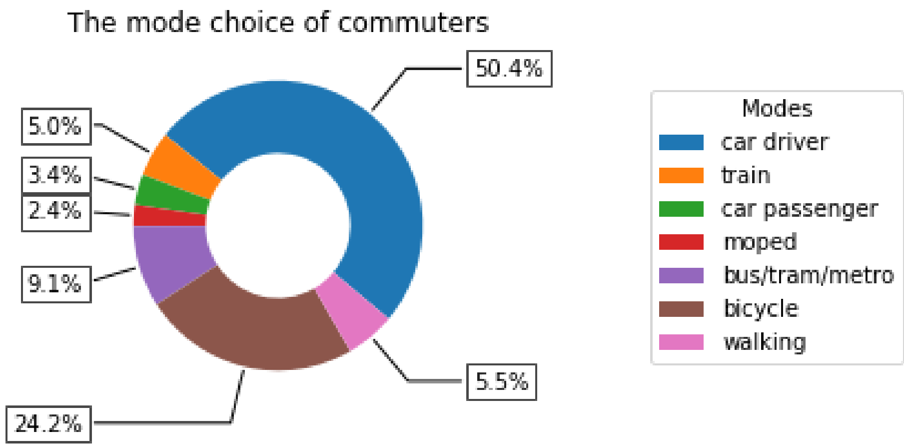


Figure 4: Donut chart with distribution of mode choices as registered in OViN

survey. The same goes for five participants who claimed to have taken the car to work as a driver, yet did not possess a driver's license. Finally, another 51 participants that failed to register there income were also taken out of the data set, because this information was needed for the cost variable (further discussed in section 5.1.2). Eventually, the 51,879 trips were narrowed down to a total amount of 6,816 trips (from 3,195 individuals) for model estimation.

4.1.2 Choice set

Respondents could choose among 8 different main classes that best described their main mode of transportation for the trip: car driver, train, car passenger, moped, bus/tram/subway, bicycle, walking, or the residual class. None of the participants had selected the residual class for their commuting trips. The mode choice division is given in figure 4. The largest share consists of car users, followed by the bicycle, which is not an odd outcome in the Netherlands.

The size of this initial choice set provided a challenge: the large number of options created a high computational cost. This effect was twofold.

Firstly, the μRRM model compares each alternative binary wise. For example, if there are three alternatives $\{A, B, C\}$ and one attribute x, then six variables have to be calculated:

$$(x_A - x_B),\ (x_A - x_C),\ (x_B - x_A),\ (x_B - x_C),\ (x_C - x_A) \text{ and } (x_C - x_B).$$

For two attributes this would already become 12 variables. Hence, if all seven alternatives were used, then $7 \times k \times n$ values have to be calculated, where k denotes the number of attributes and n the number of observations in the data set.

Secondly, the parameters of each attribute were chosen to be alternative specific in this thesis. Therefore, each attribute would have seven different parameters: one for each alternative in the choice set. For merely two attributes, the model would already have to estimate $7 \times 2 = 14$ parameters. Thus, the larger the choice set, the higher the computational cost.

Therefore, the choice set had to be reduced. The following alternatives were taken out:

- *Car passenger.* The information on this mode choice was incomplete. For instance, it was impossible to determine for each individual if they could arrange a driver: this depends on coworkers that live nearby, spouses that work in the same area and the willingness to share. Furthermore, there was no data on the costs (e.g. whether the passenger contributed to the gas price).

- *Pedestrian.* This option was only chosen 5.5% of the time. People often live too far away from their work to walk, let alone walk an entire trip chain.

- *Moped.* Contributing to only 2.1% of the observations, the model would have a small amount of data to estimate its parameters. On top of that, OpenTripPlanner (discussed in subsection 4.3) was not able to estimate routes with these modes of transportation.

Observations where these modes were chosen, were removed from the data set. Thus, the final choice set consisted of the four alternatives: car (driver), train, bicycle and bus/tram/metro.

4.1.3 Trip chaining

Trip chaining refers to combining multiple destinations in one route without switching modes. It was one of the two variables of De Witte et al. (2013) that was found to be often significant (almost 80% of the time), but not often studied (only 18% of the time). The other variable was *habits*, and is left for future research. The structure of the data set is not appropriate to include this attribute, since each individual registers his behaviour for only one day.

In a trip chain, each route starts at home and ends at home. The three types of simple trip chains for commuters are illustrated in figure 5. An individual either combines an activity before going to work, after leaving work or both. These examples consist of only one or two activities, but in complex trip chains multiple activities can take place in-between the routes home to work or work to home. In OViN, individuals with multiple work-related trips were present. In this thesis, this is also considered as a trip chain.

Based on results from earlier research on trip chaining (De Witte et al., 2013), it is expected that individuals who made multiple trips during their commute are inclined to take the car.

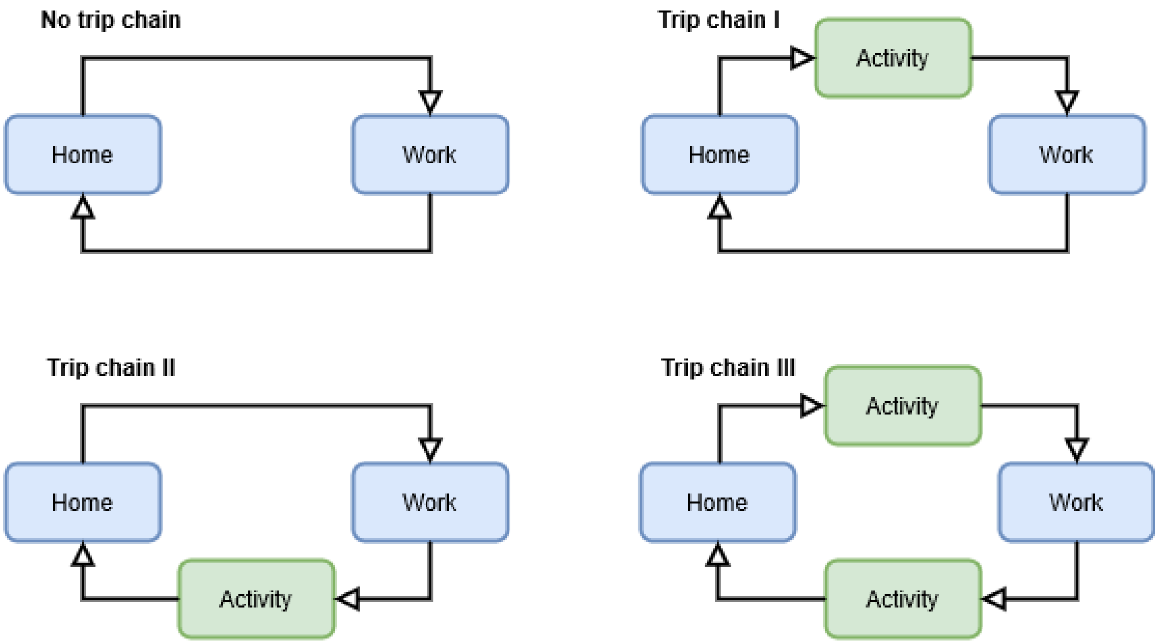


Figure 5: Examples of trip chaining

4.2 KNMI

According to the site of KNMI (*KNMI: Zware neerslag*, n.d.), the average precipitation rate is 800 millimeter per year in the Netherlands and due to climate change they predict that this will only become more. The literature review of Jonkeren (2020) found that cycling becomes less attractive when it is pouring, while the chance of taking the car increases. Thus, when investigating mode choice in the Netherlands, one cannot avoid rain. Therefore, the OViN data is combined with weather information obtained from KNMI.

A dummy variable was constructed, which was set to 1 if it was raining on departure time, otherwise 0. The precipitation on arrival time was disregarded in this thesis for two reasons. Firstly, because the arrival time depends upon the mode of transport used. For example, when taking the car the arrival time might be ten minutes sooner than by public transport. Secondly, the expected arrival time of the participant might not necessarily match the estimation of the real arrival time. In particular, previous research has found that car is perceived as more punctual and reliable than public transport (Garcia-Sierra et al., 2015), even leading to confirmation seeking behaviour that promotes this "car bias" (Innocenti, Lattarulo, & Pazienza, 2013). Hence, the expected arrival time of individuals is not necessarily the same as the actual arrival time that has been registered in OViN.

Therefore, only the precipitation data on departure time was required. This data has been gathered and processed by TNO. The next section describes their process in detail.

4.2.1 Retrieving precipitation data

To obtain the precipitation data, the time and location of each trip were extracted from OViN. The precipitation data was recorded in 5-minute intervals. In this thesis, it has been assumed that individuals based their decision on the weather conditions right before and during their departure. This was approximated by a 15-minute time interval. First, this 15-minute time interval had to be determined for each trip. The departure time was rounded down to the closest 5 minute instance, because this provides the best indication of the weather right before departure. The interval consisted of 10 minutes before this departure time and the first 5 minutes after. For example, if a person left at 13.18, then this was rounded down to 13.15 and the data from the intervals 13.05-13.10, 13.10-13.15 and 13.15-13.20 were extracted from the KNMI database. After this, the location data was used to extract the corresponding files, which gave the precipitation value of each 5 minute-time interval in mm/5min. These values were summed to obtain the mm/15min precipitation value.

Matching the KNMI data with the OViN locations demanded some additional steps: OViN data consists of PC4 (postal code) zones, whereas KNMI data is stored on a 1x1 km grid. To restore this discrepancy, centroids are computed for each PC4 zone and KNMI grid cell (see figure 6). Each KNMI centroid is then linked to the nearest PC4 centroid. Since this is still a rough estimation, the KNMI data from the 8 adjacent zones are also taken into account. The average precipitation is then calculated for these 9 cells. This is the final precipitation value. Later this was converted to the binary variable described before, because it was expected that the amount of precipitation would not provide different behaviour than the presence of rain. The exact value of precipitation is superfluous for the choice task.

4.3 OpenTripPlanner

As mentioned before, the OViN data set only provides us information about which decision has been made by the decision-maker, but not which alternatives were available to him. To obtain these alternative travel routes, OpenTripPlanner was used. It is an open source software that, given a starting point A and end point B of a trip, combines transit, pedestrian, bicycle and car segments to provide all possible routes from A to B. The following (combinations of) mode choices were possible:

- bicycle

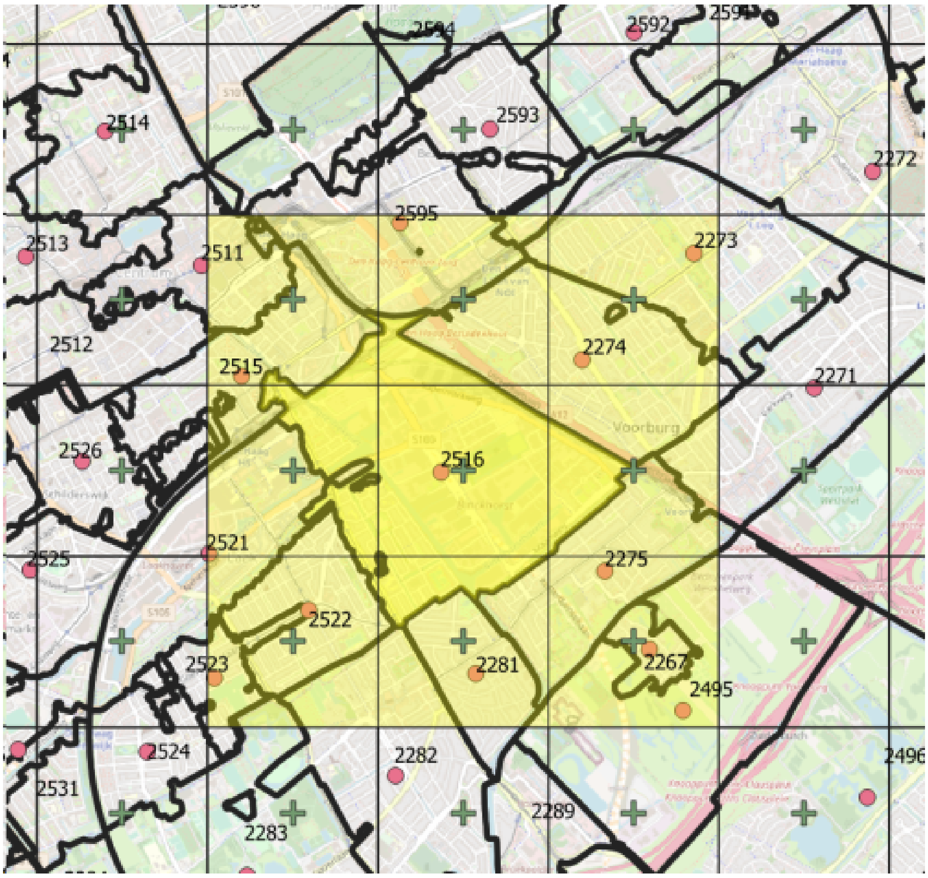


Figure 6: Centroids of PC4 zones (red dots) and KNMI grid cells (blue crosses). Map obtained from OpenStreetMap contributors (2017) and PC4 zone data from Central Bureau of Statistics. Data combined by TNO, including the 1x1 km grid.

- bus, tram, subway, walk
- car
- train, walk
- transit (train and bus/tram/metro), walk

If multiple routes were possible for the same mode of transportation, then the fastest route was reported by OpenTripPlanner. Some routes can be banned (e.g. toll routes), others can be marked as preferred (e.g. highways with little congestion). Furthermore, OpenTripPlanner is able to eliminate alternatives such as walking if the traveler has a handicap and avoid walking more than X kilometers, where X remains to be specified by the researcher.

Again, this data had already been collected and processed by TNO, including the fusion of the data sources. Therefore, this thesis adheres to the restrictions that were chosen during this process. The only assumption that was made by TNO for this process was a maximum walking distance of 2 kilometers.

4.3.1 Estimating costs and rush hour delays

OpenTripPlanner is able to provide four attributes for a route: cost, time, distance and number of mode changes. The only exception are the cost for the car alternative and the 837 cases where the cost of public transport could not be estimated. These costs were estimated by using the average cost per kilometer obtained from Snelder et al. (2019). For the car alternative this is €0.17 per km, for the train €0.17 and for bus/tram/metro €0.10. Furthermore, for public transport fixed costs were also taken into account (train: €2.20, bus/tram/metro: €0.78), also obtained from Snelder et al. (2019). The car cost is solely determined by the average gas price (per km), since fixed costs such as regular technical inspections and road taxes are often separated from gas prices in the mental model of an individual (e.g. Garcia-Sierra et al., 2015). Thus, people tend to disregard the fixed costs of the car when they compare it to the costs of other alternatives.

If no route could be found for a certain transportation mode, OpenTripPlanner would return a missing value. Section 5.1.4 contains information on how the regret model will handle this information.

The most complicated limitation of OpenTripPlanner is its inability to take into account traffic jams and public transport delays in the calculation of travel times. These setbacks determine the travel time variability and therefore, cause a lot of uncertainty. The literature accumulated by Garcia-Sierra et al. (2015) showed that travellers dislike travel time uncertainty more than long travel times.

Thus, in this thesis adjustments had to be made to the estimated travel times to incorporate this variability. A binary variable was used, that was 1 if the departure time was during rush hour (6:30-9:30 and 15:30-19:30) and 0 otherwise. With this variable, the estimated travel time of the car alternative was increased with a time penalty if the departure time of the trip was during rush hour. In section 6.1.2 the exact estimation method will be elaborated.

4.3.2 Knowledge of the choice set

The alternative routes given by OpenTripPlanner are assumed to be known by the decision-maker. Of course, this is questionable: taking into account that commuting might be habitual behaviour, it raises the possibility that some alternatives were never considered by an individual. The exact structure of the choice set for each individual is unfortunately not available for this thesis, since the respondents were not asked about this. Still, one restriction on the choice set was made. When an individual did not possess a driver's license, the car alternative was marked as unavailable in the choice set.

5 Methods

In this thesis, trip chaining is incorporated in the *random regret minimization* model (μRRM) to study its effect on mode choice in commuting. Section 5.1 provides the mathematical description of the model and the adjustments that were made for this thesis. In particular, sections 5.1.2-5.1.4 explain which attributes have been used and how to handle different choice set compositions and missing values in the data. The final subsection is devoted to the adjustments that were made to account for trip chaining in μRRM. All models in this thesis, were implemented using Biogeme 3.2.5 (Bierlaire, 2018). This software was developed to easily build and evaluate all types of discrete choice models.

5.1 General notations

Suppose a decision-maker n faces a set of alternatives, denoted by J. Each alternative has k attributes. The values of these attributes are known and are stored in a $(k \times J)$ matrix X. They are called the *explanatory variables.* These variables come in two types: *general attributes*, such as price and travel time, and *socio-demographics* (e.g. age, gender, household structure). Note that the latter are user-related, whereas the former are attribute-related. They are called explanatory, because the researcher can extract them from the data set.

Now, regret is said to arise when an alternative performs better than the alternative chosen by the decision maker. Suppose that decision-maker n considers choosing alternative i. To compute the total regret that decision-maker n anticipates for an alternative i, the alternative is compared to each alternative $j \neq i$ on their k attributes $([x_{kjn} - x_{kin}])$. From the RRM model it follows that the regret that a decision-maker n anticipates when comparing alternative i to an alternative j is:

$$R_{ijn} = \sum_k \mu \cdot \ln\Big(1 + \exp\big(\frac{\beta_{ki}}{\mu}[x_{kjn} - x_{kin}]\big)\Big), \quad \text{with } \mu > 0. \tag{1}$$

Equation 1 is repeated for all the alternatives in the choice set $J \setminus \{i\}$. Thus, the total regret that decision-maker n will anticipate when choosing an alternative i is given by:

$$RR_{in} = ASC_i + \sum_{j \in J \setminus \{i\}} R_{ijn} + \epsilon_{in}, \quad \text{where } \epsilon_i \sim \text{i.i.d. EV}(0, \mu)^5. \tag{2}$$

Let us go over this equation step-by-step. The next three paragraphs explain respectively: alternative specific constant (ASC), β parameters, μ parameter and the error term ϵ.

[5] Independent and identically distributed random variables with mean 0 and variance $(\epsilon_{in}) = (\pi^2/6) \cdot \mu$.

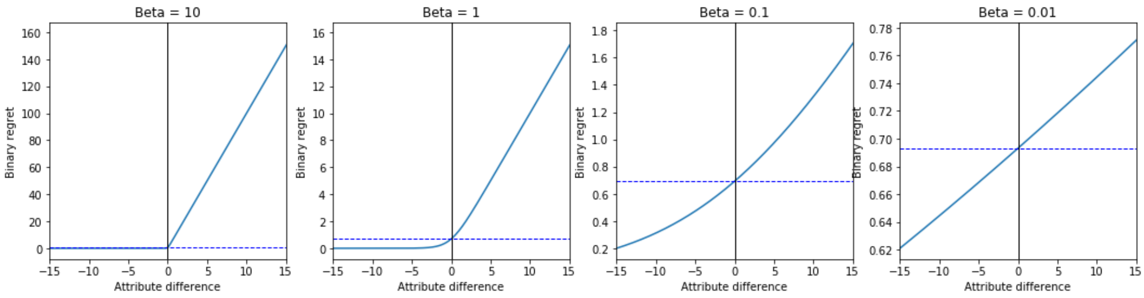
ASC The first part of equation 2 is the alternative specific constant (ASC). It is included to capture the average impact of unobserved variables that are affecting the regret function. One of the ASCs must be normalized to zero. For instance, if ASC_{car} is normalized to zero and ASC_{train} is positive, then with all attributes constant there generally is a higher regret for the train alternative than the car alternative. In this thesis, the ASC of the car is normalized to zero, because there generally is a bias for this alternative (Innocenti et al., 2013).

β **parameters** The second part of equation 2 is the calculation of regret of reference alternative i determined by the binary comparison of all attributes k. The β parameter measures the impact of the difference in performance on attribute k. In other words, how important regret is for this attribute. Note that the β parameter is alternative specific in this thesis, because it is expected that cost and time do not induce the same amount of regret across alternatives. For example, the time parameter of the bicycle is expected to be more sensitive to changes in travel time than the car, because the car provides comfort and no physical effort. This altercation was derived from C. G. Chorus (2012, p.36), where it was applied to one of the first versions of the RRM model (C. G. Chorus, 2010).

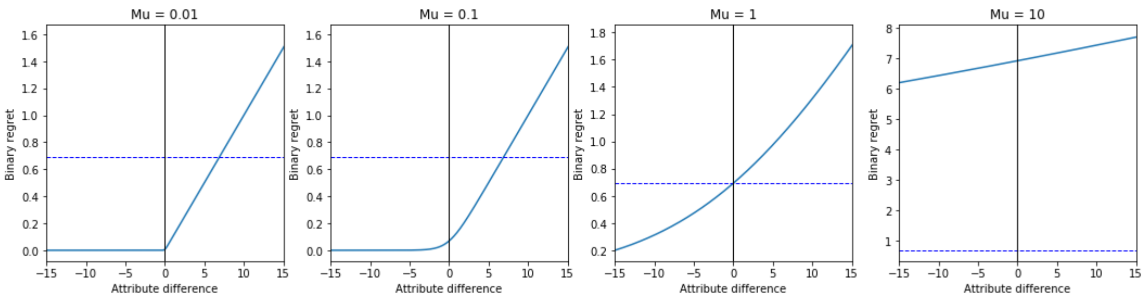
The sign of β is quite intuitive. If an increase of attribute k for the reference alternative is desired (e.g. more comfort), then the sign is positive such that regret will decrease. Vice versa, the parameter is estimated as negative when an increase of the corresponding attribute of the reference alternative increases regret (e.g. higher costs). For more information on the interpretation of the beta-parameters, see appendix B. Similar to linear regression, the goal of the model is to estimate the value of the β parameters. This is more thoroughly explained in the next subsection.

μ **parameter** The μ factor has only been recently (2015) added to the regret model and is referred to as the scaling factor. However, this parameter is not the exact scale, but rather is confounded by the scale. Without this factor, the RRM model is not scale invariant. That is, by changing the scale, the properties of the model also alter. As a result, the regret minimization behaviour that is imposed by the model changes.

To illustrate this, figure 7a has been constructed. Without μ, the value of the β parameter is influenced by 1) the anticipated regret and 2) the *profundity of regret* (see appendix A, or go directly to the source: van Cranenburgh, Guevara, & Chorus, 2015). The anticipated regret refers to how important an attribute is for the total amount of regret that is anticipated when choosing an alternative i. For instance, if a businessman is more concentrated on minimizing travel time than cost, then the absolute β value for the attribute *travel time* will be higher than the absolute β value for the attribute *cost*. Thus, an alternative that is five minutes quicker than alternative i will induce more regret than an alternative that is five euros cheaper than i. The profundity of regret, however, refers to the amount of loss aversion that is present in the data. Figure 7a shows what happens if μ is not considered in



(a) Different β values.



(b) Different μ values, β fixed at 0.1.

Figure 7: Regret function with and without scaling parameter μ. The dotted line indicates the regret if two alternatives have equal performance on an attribute (i.e. their difference is zero). Adjusted from van Cranenburgh, Guevara, and Chorus (2015).

the regret function. For high values of β, there is a strong element of loss aversion (note that the positive values indicate regret, because $x_{kj} - x_{ki} > 0$ when an alternative j performs better than alternative i on attribute k). When β is lowered, we are essentially zooming in on the function or in other words, the scale changes, and the loss aversion curve is disappearing. For low values of β, the function is nearly linear, which means that losses weigh equally compared to gains. In the μRRM model, μ intercepts the profundity of regret, such that β only measure the influence of anticipated regret. In figure 7b, the regret functions for different values of μ take the same shape as the functions in figure 7a. Note that the scale of 7b is different, because the regret is also multiplied by μ in the function. The value of μ can be used to determine the profundity of regret that is present per attribute. Both the β parameters and μ parameter are estimated in the μRRM model. For more information on μ and the profundity of regret, the reader is referred to appendix A and (van Cranenburgh, Guevara, & Chorus, 2015).

Error term ϵ The error term ϵ is standard practice in discrete choice models. It is there to capture the regret of the variables which the researcher cannot observe. Since these variables are unknown, the researcher must assume a distribution for ϵ. The chosen distribution determines what type of discrete choice model is applied (Train, 2009). In the μRRM model, the error term has the same distribution as the random utility maximization model (McFadden et al., 1973): Gumbel distributed[6] with mean 0 and variance $(\pi^2/6)\cdot\mu^2$. (Note that the scale parameter μ is linked to the estimated variance of the error term. If $\mu = 1$, then we obtain the RUM model.) This means that for two alternatives $i, j \in J$:

- $\text{var}(\epsilon_i) = \text{var}(\epsilon_j)$,
- $\text{corr}(\epsilon_i, \epsilon_j) = 0$.

In essence, the error terms are assumed to be independent across alternatives and have the same variance: $\pi^2/6 \cdot \mu^2$. One major advantage of this approach is that the solution of the regret function is of closed form and thus, no simulation is required. Nevertheless, the reader should be aware that these assumptions also pose a great restriction on the model: the researcher must be certain that the known variables X are sufficiently specified, such that no correlation may exist between the remaining, unknown variables (ϵ). If this is uncertain, a different distribution for ϵ should be considered.

A full explanation of the different types of discrete choice models (that are brought forth by different assumptions on ϵ) and their mathematical derivations is beyond the scope of this research. Interested readers are referred to Train (2009) to satisfy their curiosity.

5.1.1 Estimation and probabilities

As mentioned before, the purpose of the regret model is to estimate the β parameters and the μ parameter. In the μRRM model, it is assumed that the decision-maker consistently chooses the alternative that is expected to lead to the lowest regret. This means that if, for example, it is known that alternative i was chosen by decision-maker n, the weights of each attribute (β) are estimated such that:

$$P_{in} = P(-RR_{in} > -RR_{jn}, \forall j \neq i) \tag{3}$$

Since a Gumble distribution is assumed for ϵ, this probability can be written as:

$$P_{in} = \frac{e^{-RR_{in}}}{\sum_{j=1}^{J} e^{-RR_{jn}}}. \tag{4}$$

For the full derivation of this probability, the reader is again referred to Train (2009). The alternative with the highest probability is expected to produce the lowest amount of regret

[6] Also referred to in literature as type I extreme value or i.i.d. extreme value.

and is therefore most likely chosen. Since the data contains information on what modes where chosen and the values of their attributes, the μRRM model tries to find the β values such that the chosen mode always has the highest probability of being chosen.

5.1.2 Attributes

The attributes that were used in the μRRM model of this thesis are cost, travel time and rain. Since travel costs are expected to have a larger impact on low-income individuals than high-income individuals and individuals were divided among income classes in OViN, the costs will be divided by the average income of the individuals' income class to see if this improves model fit. If multiple passengers were in the car, the travel cost was divided by the number of people in the car. The bicycle alternative was assumed not to have any costs and thus, no cost parameter was estimated for this alternative.

The dummy variable for rain was implemented as in a RUM model (that is: $\beta_{rain}x_{rain}$), because at least three levels are necessary to distinguish the difference between RUM and μRRM decision behaviour. There can be no compromise effect on a two-level scale (C. G. Chorus, 2010). The variable x_{rain} was 1 if and only if it was raining during departure time (as explained in section 4.2), otherwise 0. The implementation is as follows:

$$RR_{in} = ASC_i + \beta_{rain}x_{rain} + \sum_{j \neq i} \sum_{k} \mu \cdot \ln\left(1 + \exp(\frac{\beta_{ki}}{\mu}[x_{kjn} - x_{kin}])\right) \tag{5}$$

Thus, if $\beta_{rain} > 0$, then regret increases when it rains. When regret increases, the probability of choosing i decreases.

5.1.3 Availability of choice set

If an individual did not possess a driver's license, the car alternative was omitted from his/her choice set. The availability of an alternative i is denoted by the dummy variable α_i, which was 1 if an alternative was available and 0 otherwise. It is incorporated in the probability function of 3:

$$P_{in} = \frac{\alpha_i \cdot e^{-RR_{in}}}{\sum_{j=1}^{J} \alpha_j \cdot e^{-RR_{jn}}}, \tag{6}$$

such that the probability of choosing this alternative is 0.

On top of that, the total amount of regret is higher for alternatives with a larger choice set, because these alternatives are compared to four options instead of three. Therefore, van Cranenburgh, Prato, and Chorus (2015) advice to include a choice set correction factor. The total regret for an alternative is multiplied by this correction factor (c_f) to account for the size of the choice set where the participant had to choose from: $RR_{in} \cdot c_f$. For relatively large choice sets (10 or more alternatives) a constant factor can be used (which is further explained in van Cranenburgh, Prato, & Chorus, 2015), but for smaller choice sets the correction factor has to be estimated. Similar to the method used in appendix B of van Cranenburgh, Prato, and Chorus (2015), the correction factor is 1 if the choice set consists of three alternatives (i.e. the participant does not possess a driver's license) and in the case of four alternatives, the correction factor has to be estimated. Thus, the final probability of choosing an alternative i becomes:

$$P_{in} = \frac{\alpha_i \cdot e^{-RR_{in} \cdot c_f}}{\sum_{j=1}^{J} \alpha_j \cdot e^{-RR_{jn} \cdot c_f}}, \tag{7}$$

where c_f is the correction factor for the choice set size of individual n.

5.1.4 Missing values

For certain observations OpenTripPlanner was not able to estimate the time and cost of all the alternatives. For instance, due to the short distance of the trip (such that public transport is not an option) or the remote location of the destination. The empty cells were filled with a missing value (999). In random utility maximization models, it is standard practice to set these values to an arbitrarily large number. As a result, their utility spikes to minus infinity (if the corresponding β is negative) and the probability that this alternative is chosen goes to zero. This is called a *disutility.*

The same approach can be used for regret-based models, despite that the regret of each alternative depends on the attribute values of the other alternatives. To see why this solution also works for a minimization problem, a simplified example is used.

Suppose that the four alternatives of this thesis are all available in the choice set. For simplicity we compare these alternatives on only one attribute: travel time. Furthermore, let us assume that OpenTripPlanner was not able to estimate the time attribute for train and bus/tram/metro. Therefore, $x_{time,train} = x_{time,btm} = 999$ minutes. The estimated travel time of the bicycle is estimated as 15 minutes and 5 minutes for the car. Finally, we assume that the β value of the time attribute is negative, since an increase in travel time is expected to increase regret and thus, decrease the probability of choosing this alternative. In this example, we assume that $\frac{\beta}{\mu} = -1$ for simplicity (remember that always $\mu > 0$ and so $\beta < 0$).

There are three cases for the function $\ln\left(1 + \exp(\frac{\beta}{\mu} \cdot [x_j - x_i])\right)$ when missing values are involved:

1. Only x_j is missing.

2. Only x_i is missing.

3. Both x_j and x_i are missing.

In the first case, $x_j = 999$ and the function $\exp(-1 \cdot [999 - x_i])$ will go to zero as long as $x_i \ll 999$. This gives us the lowest amount of regret possible: $\ln(1+0) = 0$. Vice versa, the second case will result in the highest amount of regret, $\ln(\infty)$, because $\exp(-1 \cdot [x_j - 999])$ goes to infinity. Finally, in the third case, the difference between the time attribute is zero and thus, this provides a regret value of $\ln(2)$.

Only considering the time attribute, it is expected that the car will result in the least amount of regret. When we compute the regret of the attributes, we get (the first regret function has been written out in detail, the others are left for the reader):

$$\begin{aligned}
R_{car} &= R_{train \leftrightarrow car} + R_{bike \leftrightarrow car} + R_{btm \leftrightarrow car}, \\
&= \ln\big(1 + \exp(\tfrac{\beta}{\mu} \cdot [x_{train} - x_{car}])\big) + \ln\big(1 + \exp(\tfrac{\beta}{\mu} \cdot [x_{bike} - x_{car}])\big) + \ln\big(1 + \exp(\tfrac{\beta}{\mu} \cdot [x_{btm} - x_{car}])\big), \\
&= \ln\big(1 + \exp(-1 \cdot [999 - 5])\big) + \ln\big(1 + \exp(-1 \cdot [999 - 5])\big) + \ln\big(1 + \exp(-1 \cdot [15 - 5])\big), \\
&= \ln\big(1 + \exp(-994)\big) + \ln\big(1 + \exp(-994)\big) + \ln\big(1 + \exp(-10)\big), \\
&= \ln(1) + \ln(1) + \ln\big(1 + exp(-10)\big) \approx 0.000045. \\
R_{bike} &= R_{car \leftrightarrow bike} + R_{train \leftrightarrow bike} + R_{btm \leftrightarrow bike}, \\
&= \ln(1) + \ln(1) + \ln\big(1 + exp(10)\big) \approx 10.0. \\
R_{train} &= R_{car \leftrightarrow train} + R_{bike \leftrightarrow train} + R_{btm \leftrightarrow train}, \\
&= \ln\big(1 + \exp(994)\big) + \ln\big(1 + \exp(984)\big) + \ln(2) \to \infty. \\
R_{btm} &= R_{train}.
\end{aligned}$$

As expected, the lowest regret is found for the car alternative. The regret functions that go to infinity will not cause problems as long as they do not belong to the chosen alternative. To ensure that this situation would not occur, the reported travel time and distance (on which the cost was based) in OViN were filled in for the chosen mode per observation. Thus, for each trip there was always information available on the attributes of the chosen mode.

5.2 Trip chaining in μRRM

In OViN each participant registered all the trips that he or she made on a certain day. An individual made a trip chain if he

- traveled a sequence of at least three trips
- starting at home and ending at home
- without stopping at home in the mean-time
- and without switching modes.

Three adjustments for the μRRM model were considered in this thesis: trip chaining as panel data, aggregating data of trip chains, and accumulating regret of each trip in the chain. Due to time constraints the third option has not been implemented, but it is described in appendix D. The other adjustments are discussed in detail in the next sections.

5.2.1 Panel data

The registered trips of the OViN data used in this thesis were ordered by individual as a list of sequential choices. Due to this structure, it was considered to process the trip chaining data as panel data. In this type of data, each individual makes a sequence of decisions over time. The advantage of this approach is that correlations between the decisions can be captured by the explanatory variables.

However, this method was rejected due to theoretical objections. The advantage of panel data is a disadvantage for OViN: correlations among decision must be captured by the explanatory variables, such that the unobserved factors (ϵ) are independent over time. In other words, $corr(\epsilon_{nit}, \epsilon_{ni(t-1)}) = 0$, for two sequential choices in time periods t and $t-1$. This is an essential assumption of logit (Train, 2009), which is the foundation of the utility and regret models.

For trip chaining this assumption is problematic, since each choice in the chain is influenced by the previous decision. As a solution, Train (2009) suggests adding a lagged dependent variable (denoted by z). A lagged variable is a reference to the previous choice. It can refer to the number of times that an alternative has been chosen so far, the similarity of the attribute of the previous chosen alternative to the attributes of the current alternative or to the last chosen alternative. This approach is often used in stated preference data sets to capture unobserved preferences or strategies of individuals (e.g. always stick to the first choice). This implies that the choice set for the second choice is always equal to the choice set in the first choice. Since OViN is a revealed preference data set, this assumption does not hold: if an individual first went to work by car, then his choice set for the second trip he will make is limited, because for instance, his bicycle is still at home. Once the trip chain has started, the choice set of the individual becomes limited based on the mode that he chose for the first trip. Hence, unlike the stated preference data sets, the choice set of an individual has not been consistent in a revealed preference data set. Therefore, the lagged variable is useless for a revealed preference data set and has not been implemented.

5.2.2 Aggregating data

One straightforward way to implement trip chaining is by aggregating the attribute data for all the trips in the chain. Consider the trip chain:

$$home \rightarrow work \rightarrow groceries \rightarrow home.$$

Then the attributes (time, cost and rain) for these trips would be aggregated. Thus, the time-attribute for a decision-maker n becomes:

$$x_{n,time} = x_{n,home-work} + x_{n,work-groc} + x_{n,groc-home}$$

and idem dito for cost. The functions of 2 and 7 do not change, but rather the data input does.

It is possible that the label of the trip chain (i.e. the mode choice) is not consistent for all trips in the chain. For example, one might go to work by bicycle, take the bus to shop in the city centre, take the bus back to work and then cycle back home. In these cases the label of the trip chain is determined in the same way as the mode choices in OViN, namely by a preference ordering (see 4.1).

The advantage of this approach is that it is quite easy to implement and it might increase the differences between the attribute values of the alternatives. The latter is valuable for cases where the differences between the commuting routes are similar, but the differences for the activity-related trips are not. For instance, consider an individual who can either take the bus or the car to work and arrive in 15 minutes for approximately the same cost. Suppose that this person wants to make a short stop at the grocery store before heading back home. If this means a detour of 10 minutes for the bus, but only 5 minutes for the car, then the probability of taking the car will most likely increase.

Nevertheless, the disadvantages of this approach have to be recognized. By aggregating data certain information is lost. For instance, the combination of multiple modes during a trip chain. Furthermore, the aggregation blurs out which part of the trip chain is crucial for mode choice. Finally, it can not be excluded that the summation of attribute values actually decreases the differences between alternatives. Consider again the previous example of the car and the bus, but this time the bus arrives 5 minutes sooner at work. Now the total travel time of both the bus and the car is 20 minutes. In this case, it is essential to know whether the work-related trip or the detour is determining for mode choice.

Another difficulty of this approach is how to model the binary variable for rain. This variable is 1 if it was raining on departure time. However, in the example above, person i now has three departure times. There are three options to model this:

1. Create a new dummy for rain that is one if it was raining on at least one of the departure times.
2. Model x_{rain} as a categorical value by summing over the dummy values (e.g. if it was only raining when person n left for work, then $x_{rain} = 1 + 0 + 0 = 1$.

3. Model x_{rain} as a continuous variable, i.e. the precipitation rate measured by the KNMI.

The second option was deemed the best, because the multiple precipitation observations hopefully better reflect the forecast of that day. This assumes that people have checked on the weather forecast before deciding their mode of transport for that day. The more rainfall on a day, the higher the regret for the bicycle becomes and hence, the lower the probability of taking this mode.

To summarize, the aim of this thesis is to investigate whether the accumulated regret of trips will improve the estimation of the parameters of μRRM. The first regret model takes a singular approach. It merely looks at the trips with commuting as purpose and tries to estimate the parameters of cost and time, given these observations. The contribution of this thesis is to implement the effect of trip chaining in regret models. That is, the mode that was used for commuting can also be determined by other trips in the chain. This lead to the construction of a μRRM model where the attributes of the trips in trip chains are summed up. The trips will be aggregated by a unique trip chain identification number that has been assigned to the them.

6 Results

Two regret minimization models will be compared: μRRM with aggregated data of trip chains and a basic μRRM model for reference. In each model, alternative specific constants (henceforth: ASC) have been included to capture inherent preferences that could not be captured by the explanatory variables. The ASC of the car alternative has been normalized to zero. The researcher is allowed to specify a minimum and a maximum bound for the beta parameters. For this thesis, a minimum of -10 and maximum of 10 was set for each parameter. Model performance will be based on the following three criteria:

Log likelihood is the logarithm of all the probabilities that the model estimated for the chosen modes:

$$LL^* = \sum_n \sum_i y_{ni} \ln(P_{ni}),$$

where the binary variable y_{ni} is 1 if and only if individual n chose alternative i. The log likelihood value is maximized by the model.

Likelihood ratio index (ρ^2) measures the goodness-of-fit. The log likelihood of the estimated model is compared to the initial likelihood, that is, the likelihood of a model that predicts choices y with the default starting values for the parameters. The ratio lies between 0 and 1. The closer ρ^2 is to 1, the better the goodness-of-fit. The formula is:

$$\rho^2 = 1 - \frac{LL^*}{LL^i},$$

where LL^* is the final log likelihood of the estimated model and LL^i is the initial log likelihood. Note that always $0 \leq \frac{LL_i}{LL^*} \leq 1$, because the model fit of the final estimated model is always an improvement of the initial model.

The AIC score indicates the relative distance between the fitted log likelihood function of the model and the unknown "true" log likelihood function of data. The AIC score is used as an indicator for overfitting. Complex models with too many parameters might start to focus on noise in the data, instead of capturing general behaviour. Therefore, with AIC, each additional parameter, k, in the model is punished by a penalty of 2:

$$AIC = 2k - 2LL^*.$$

The lower this AIC score is, the closer this model is to the "true" model. When comparing two models with the same data set, the model with the lower AIC score is considered to be better. Since the sizes of the data sets in this thesis differed, the AIC score was divided by the sample size (N) to compare the models on equal terms.

In addition to the three measures of model fit, the **profundity of regret** will be estimated for each regret-based model in this thesis. This measure was introduced, together with the μRRM model, in the paper of van Cranenburgh, Guevara, and Chorus (2015). The exact formulation of this parameter can be found in appendix A. The profundity of regret indicates the amount of regret minimization behaviour that is present in the model. If this value is close to 1, then the results imply that a high amount of regret minimization is imposed by the models' estimation of choice behaviour regarding this attribute. However, if it is close to 0, then almost no regret is present, but instead utility maximization behaviour seems to be the best explanation of the choice behaviour. Time and cost have a separate profundity of regret. Thus, it is possible that one attribute is best processed with regret minimization behaviour and the other by utility maximization.

6.1 Results reference model

6.1.1 Tuning the parameters

First, the regret minimization model, described in sections 5.1-5.1.4, was estimated as a reference model. In total, 13 parameters had to be estimated (three alternative specific constants, four time parameters, three cost parameters, one rain parameter and the correction factor of section 5.1.3). Since there is a lot of freedom in defining the reference model, three versions of the reference model were estimated to find out which version provided the best model fit:

- **Model M1:** combines the data from OViN and OpenTripPlanner. That is, for each observation the distance and time of the chosen mode are retrieved from OViN, while OpenTripPlanner estimates the distance and time for the other modes. No adjustments were made to the cost or time attributes.
- **Model M2:** the cost parameter is divided by income to measure regret relative to income. Apart from that, it uses the same data as M1.
- **Model M3:** only uses the distance and time estimates of OpenTripPlanner. No adjustments to cost or time were made.

Each model has been estimated ten times with different starting values for the parameters, because the μRRM model frequently (2 out of 10 estimations) stranded in a local minimum during estimation. Out of these ten models, the one with the highest log likelihood was selected for comparison. In addition, a RUM model has also been estimated (on the data of M1) to compare its performance to regret-based models. Noteworthy, the RUM model was less volatile than the regret-based models: the outcome of the ten RUM models was the same, despite the different starting values.

Table 1 contains the performance of the selected models (expressed in log likelihood, ρ^2 and AIC score) and table 2 the corresponding profundities of regret. Note that the sample size of M3 is higher than the others, since the estimations of OpenTripPlanner were more

Model	Nr of parameters	Log Likelihood	ρ^2	Sample size (N)	AIC	$\frac{\text{AIC}}{\text{N}}$
RUM	11	-4965.2	0.472	6816	9952.4	1.460
M1	13	-4702.6	0.497	6816	9431.1	1.384
M2	13	-4759.6	0.491	6816	9545.3	1.400
M3	13	-5556.7	0.406	6830	11139.4	1.630
μRRM_{ref}	13	-3691.1	0.518	5587	7408.1	1.326
μRRM_{TC}	13	-2260.4	0.500	3286	4546.7	1.391
μRRM_{work}	13	-2133.4	0.524	3267	4292.8	1.314

Table 1: Overview of estimation results per model. The first three models are the precursors of the reference model and are introduced in section 6.1.1. Model μRRM_{ref} is the final reference model introduced in section 6.1.2, model μRRM_{TC} is the trip chaining model introduced in section 6.2 and μRRM_{work} is the reference model with only work-related trips and is also introduced in section 6.2.

often within reasonable limits (see section 4.1.1) than the travel times or distances reported in OViN. Hence, less observations had to be discarded. The AIC score divided by sample size is also provided for a fair comparison between models.

The results in table 1 show that model M1 has the best model fit compared to M2 and M3, and also exceeds the RUM model. It has the highest ρ^2 value (0.497) and the lowest AIC score per observation (1.384). In particular, the difference between model fit of M1 and M3 is remarkable: the model where OViN and OpenTripPlanner data was combined (M1), outperformed the model relying solely on OpenTripPlanner (M3). This shows that the anticipated travel times of individuals differ from free-flow estimations. Moreover, the profundity of regret for time is merely 0.09 for model M3 (table 2), which indicates that barely any regret minimization behaviour is imposed by this attribute. This is contradicted by the profundity of regret measured in M1 and M2 (both: 0.51). Thus, the more uncertainty in travel time (due to congestion or other hindrances), the higher the attribute differences and the higher the profundity of regret.

These results implied that the estimations of OpenTripPlanner are not as reliable as has been initially expected. Further analysis lead to a cesspool of anomalies in the data of this thesis (see appendix C for the thorough examination). The variance of the differences (between the reported time and distance in OViN and estimated time and distance of OpenTripPlanner) was larger than anticipated.

There are three causes. The first one was known: time estimations of OpenTripPlanner

	Profundity of regret	
Model	Cost	Time
M1	0.57	0.51
M2	0.62	0.51
M3	0.60	0.09
μRRM_{ref}	0.61	0.52
μRRM_{TC}	0.57	0.36
μRRM_{work}	0.62	0.53

Table 2: Profundity of regret for the regret-based models.

did not include rush hour[7] delays and therefore, would be underestimated. The suggested compensation method in section 4.3 did not improve model fit. Several rush hour penalties (increase travel time by 10%, 20%, 30%, 40% or 50%) were applied to the travel time (for all modes except the bicycle), but none of them resulted in a better model fit (all $\rho^2 \leq 0.497$). A randomized time penalty (uniform distribution, between 0 and 50%) even decreased model performance to $\rho^2 = 0.495$. Thus, it was decided to use the estimated values of OpenTripPlanner as they were for M1-M3.

The second cause posed an even bigger problem: for privacy reasons, OpenTripPlanner had to estimate alternative routes based on postal codes (for both arrival and destination location). In OViN, the respondents had to fill in the time and distance themselves, which naturally is more precise. Therefore, the attribute values between OViN and OpenTripPlanner deviated in varying degrees. This in turn affected the outcome of the model, because it estimates parameters based on differences between alternatives.

Finally, the data exploration showed that there were still outliers in OViN. They are possibly a result of human error. Even though the CBS has applied imputations, the results of appendix C implied that not all the anomalies have been accounted for, leading to larger attribute differences in the data.

6.1.2 Final reference model

Considering that the model M1, which combined OViN and OpenTripPlanner data, resulted in the best model fit so far despite the estimation differences, it was attempted to improve this model by excluding severe outliers in the data. A *tolerance value* was used to filter the data. That is, if the estimated distance or time did not deviate more than a certain percentage (the tolerance value), then the observation remained in the data set, otherwise, it was discarded. Several tolerance values (10%, 20%, 30%, 40% and 50%) were tested

[7] Between 6.30-9.30 or 15.30-19.00

% deviation allowed	% removed Car	Train	BTM
10	74.3	31.9	80.8
20	57.8	24.2	66.9
30	42.3	14.1	51.3
40	28.7	9.6	37.9
50	19.4	6.3	28.1
Total nr of observations	4155	426	729

Table 3: Percentage of overestimated values with different tolerance boundaries

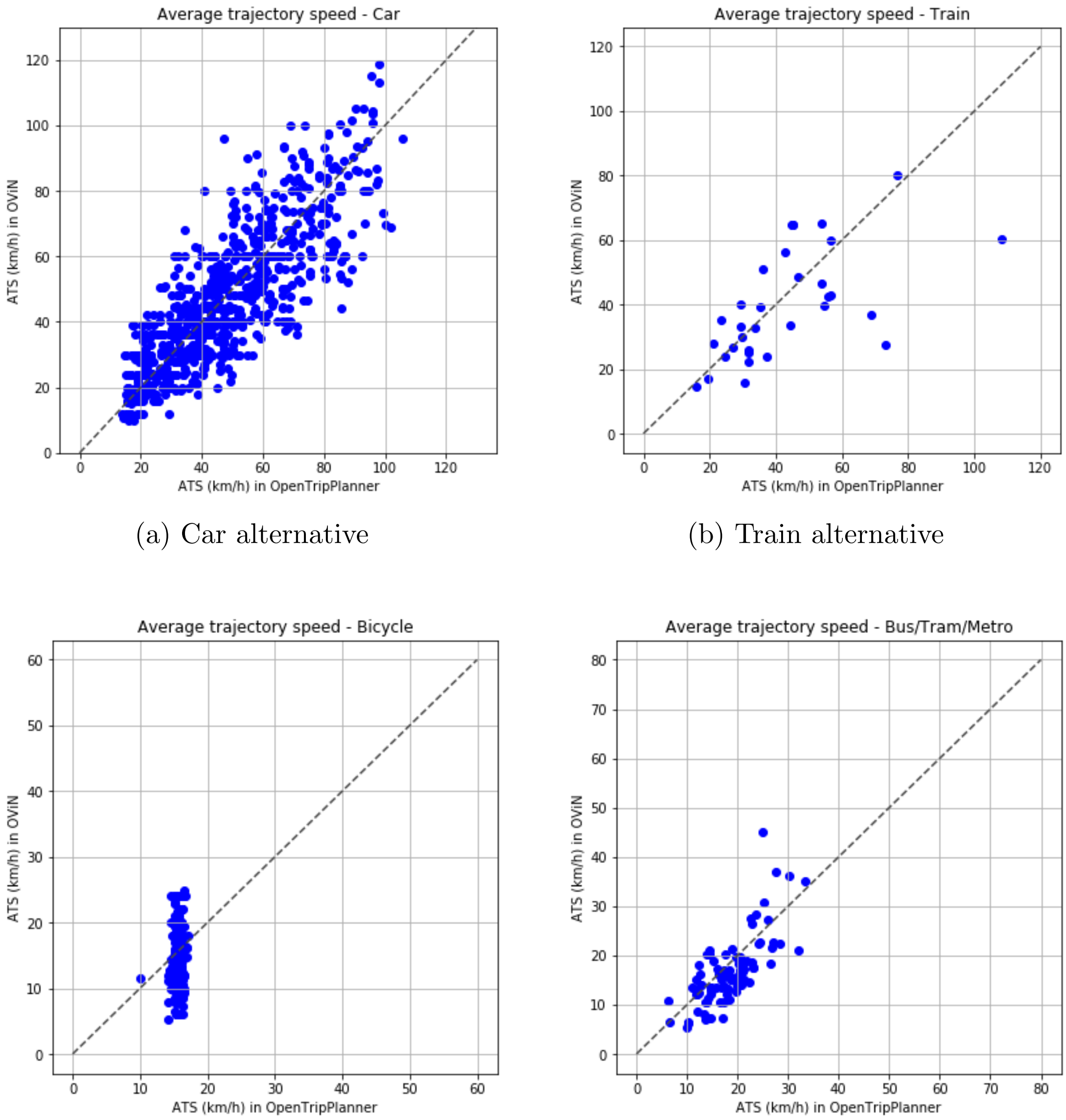
for the car and public transport alternatives[8]. The goal was on the one hand to remove extreme outliers in the data, but on the other hand to make sure enough data would remain for estimation. This was accomplished with a tolerance of 50% (see table 3). With these restrictions, 5,587 observations remained, a reduction of 18.0%. The scatter plots of figure 8 show the estimated (OpenTripPlanner) and measured (OViN) average trajectory speed of the observations in the data set that remained.

With this curtailed data set, M1 was estimated again. The results of this model are denoted as μRRM_{ref} in table 1. The model fit ($\rho^2 = 0.518, AIC/N = 1.326$) improved on all conditions compared to M1. The trip chaining model will be compared to this model.

6.2 Results trip chaining model

The second trip chaining model of section 5.2 was estimated. The data of the reference model was enriched by aggregating the travel time and cost of each trip in the trip chain. A trip chain is defined as a sequence of trips that started at home and ended at home, and where at least one trip was work-related and at least two activities were in-between. In total, 608 trip chains were found matching this definition. Another 2,117 individuals in the data set reported going to work and straight back home, without combining activities. Though, they are strictly not a trip chain, they were also aggregated, but this will not affect estimation since the differences between the alternatives are all multiplied by a factor of two. The remaining 561 trips were singular observations. This is not because only one trip

[8]Since the bicycle covers smaller distances and different travel speeds were recorded by OViN (appendix C), it was restricted in a different (more intuitive) way. Observations where the average trajectory speed was above 25 km/h or below 5 km/h were removed and also, the observations where the distance was overestimated with more than 100%. By this restriction 2.8% of the observations were removed.



(a) Car alternative

(b) Train alternative

(c) Bicycle alternative

(d) Bus/tram/metro alternative

Figure 8: Scatter plots of average trajectory speeds for the chosen mode according to OpenTripPlanner and OViN. Outliers in data have been removed. R-squared values for car, train, bicycle and bus/tram/metro are now respectively: 0.52, 0.30, 0.02 and 0.42.

was made, but rather because the other trips of the same individual had been removed. For instance, because they were completed with one of the modes that have been removed in the data processing stage (see section 4.1), or because it was one of the outliers in the data set. Eventually, 3,286 observations were used for estimating the trip chaining model.

6.2.1 Statistics on trip chaining

In the data set 608 trip chains were found. The majority of these chains was executed by car and bicycle (see table 4). Only 24 trip chains were executed by the bus/tram/metro alternative and merely 13 by train. In addition, the 55 trip chains with high complexity (3 or more activities combined with commuting) were mostly executed by car (69.1%). Surprisingly, 25.5% of these trip chains have been carried out by bicycle, which shows that private transportation modes in general are preferred for trip chains compared to public transport. This is in line with previous research (De Witte et al., 2013) where the probability of taking public transport decreased when the length of the trip chain increased.

Mode	**Total nr of observations**	**Total nr of trip chains**	**Percentage of trip chains**
Car	1964	389	19.8%
Train	100	13	13.0%
Bicycle	925	182	19.7%
Bus/tram/metro	297	24	8.1%

Table 4: Percentage of trip chains in μRRM_{TC} model per mode.

Previous research (Krygsman et al., 2007) also found that trip chains were mostly performed by women, possibly due to their role as main caretakers of household and children. This effect was not found in this study; only a slight majority (51.6%) of the trip chains was undertaken by women.

Trip chaining lengths vary between 3 and 8 activities. The complexity of the trip chains remained stable over household size: the average size of the trip chains per number of people in the household varied between 3.0 and 3.7. Figure 9 shows that trip chains with increased complexity (trip chain length 5 or more) are mostly carried out by households of size 2-4, while the complexity of trip chains is downsized for larger households. This finding is in line with previous research, where it was found that the complexity of the trip chains decreased with household size (Hensher & Reyes, 2000).

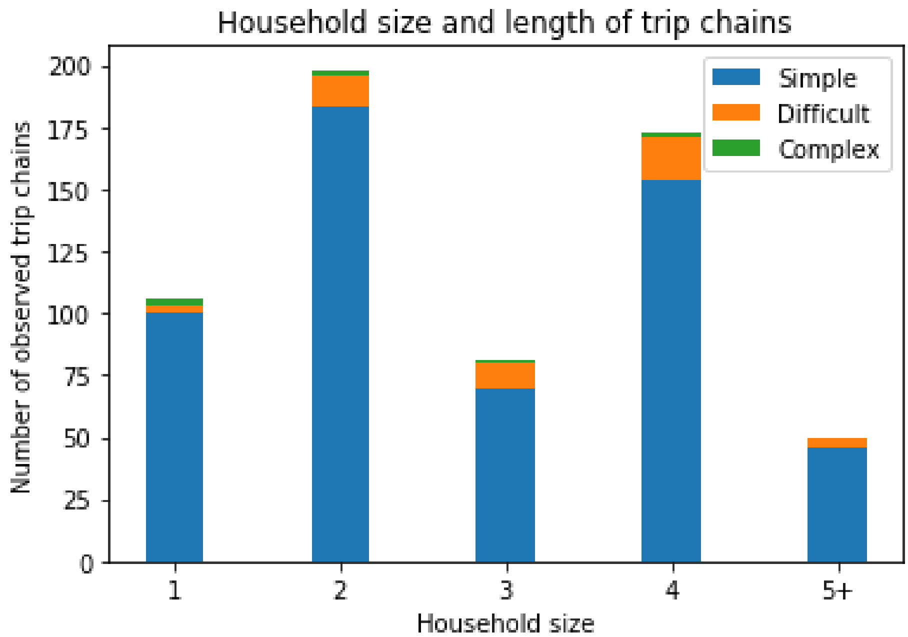


Figure 9: Household size and the simple, difficult and complex trip chains of respectively length 3-4, 5-6 and 7-8.

The main purposes of the side activities in the trip chain were 1) shopping (165 times), 2) dropping off or picking up a person (116 times) and 3) business-related visits (72 times). It could not be tested if the presence of children increased the chance of taking the car, because the survey did not include information on which person was dropped off or picked up. However, it was found that approximately half of the trip chains were performed by someone in a household with children (52.5%) and the number of advanced trip chains for households of 3 or 4 members (figure 9) indicates that the presence of one or two children may cause more (complex) trip chains. Still, this information is not specific enough to draw steadfast conclusions.

	Reference model			**Trip chaining model**		
Parameter	**Value**	**Rob. t-test**	**Bound reached?**	**Value**	**Rob. t-test**	**Bound reached?**
ascBTM	1.75	24.7**	No	1.99	24.1**	No
ascBike	1.7	27.9**	No	1.86	15.5**	No
ascTrain	1.19	12.6**	No	1.26	10.1**	No
β_{class}	10	12.6**	Yes	1.17	13.1**	No
$\beta_{cost-btm}$	-10	-4.09**	Yes	-0.915	-2.94**	No
$\beta_{cost-car}$	7.26	2.84**	No	10	1.47	Yes
$\beta_{cost-train}$	-0.328	-2.24*	No	-1.46	-3.18*	No
β_{rain}	0.151	1.49	No	-0.0477	-0.574	No
$\beta_{time-bike}$	-10	-4.99**	Yes	-10	-1.63	Yes
$\beta_{time-btm}$	-2.37	-7.16**	No	-0.943	-5.59**	No
$\beta_{time-car}$	-0.892	-12.9**	No	-1.73	-10.5**	No
$\beta_{time-train}$	-3.99	-21.9**	No	-0.611	-3.11**	No
μ	0.13	10.2**	No	0.824	6.12**	No
* = p-value < 0.05						
** = p-value < 0.01						

Table 5: The estimated parameters of the reference model and the aggregated trip chaining model. The ASC of the car has been normalized.

6.2.2 Estimation results

As was done with the reference models, the trip chaining model was also estimated ten times with the same starting values as the previous models. Out of the 10 models that were estimated, 4 converged to the same log likelihood. Given their equal performance, one of these four models was selected at random and compared to the reference model. The corresponding parameters are in table 5.

In both models, the rain variable is not significant, which might be explained by its rare occurrence (approximately 10% of the observations in μRRM_{ref} and approximately 20% in μRRM_{TC}). The values of the alternative specific constants are also similar in both models. Their positive values indicate that the regret for these alternatives is generally higher compared to the car alternative (which had been normalized). This shows that there is a bias towards the car alternative.

There are also some distinctions between the models. The sign of the cost and time parameters is expected to be negative (see appendix B), because a higher cost or travel time has a negative effect on the probability of being chosen. Nevertheless, the cost parameter of the car is estimated as positive in the reference model. One possible explanation for this is that car costs are underestimated due to an unfounded preference for the car, i.e. a car bias (Garcia-Sierra et al., 2015). The cost parameter of the car is not significant in the trip chaining model. Furthermore, the results show that in both models two or three parameters reach their bounds (-10 or 10), but for different parameters. This indicates that the structure of both data sets is troublesome in general for regret-based models.

The model fit results of the trip chaining model can be found in table 1 (denoted as μRRM_{TC}). Overall, model fit decreased compared to the reference model. Considering the smaller data set, a ρ^2 of 0.5 does not seem like a terrible performance, but the normalized AIC clearly shows a higher AIC per observation. To exclude the possibility that the size of the data set is of influence, the reference model was estimated with only the work-related trips, which consists of 3,267 observations. This version of the reference model (denoted as μRRM_{work} in table 1) has less observations than μRRM_{TC}, but still has a better model fit ($\rho_{work} = 0.524 > \rho_{TC} = 0.468$). Thus, model fit of the trip chaining model declines, regardless of the sample size.

6.2.3 Profundity of regret

As a final analysis, the profundity of regret was calculated for the aforementioned models (see the three models at the bottom of table 2). The results for the reference models (μRRM_{ref} and μRRM_{work}) are similar, but for the trip chaining model the profundity of regret is lower for both attributes. In particular, the profundity of regret for time has decreased. This means that the model imposes only mild differences between the regret incited by a loss and the rejoice incited by an equivalent gain.

To understand why this is the case, histograms of the attribute differences have been created (see figure 10 on page 44). The two upper figures show the attribute differences of the aggregated trip chaining data. The bottom two figures display the attribute differences of only work-related trips (the data of μRRM_{work}, chosen because they have a nearly equal amount of observations). Furthermore, observations where missing values were presents have not been taken into account.

The graphs of the cost differences show that the attribute differences of the trip chaining data has a remarkable higher number of differences close to zero (almost 400 more). This explains why the profundity of regret for the cost attribute is slightly lower in the trip chaining model (work-related data: 0.62, trip chain data: 0.57). Of the same model, the profundity of regret for time suffered the most (work-related data: 0.53, trip chain data: 0.36). The upper-right graph shows that the distribution of the trip chaining data is slightly more spread out for time, but it is still closely centered around zero. Thus, the distribution of the differences in the trip chaining model is less suitable for the random regret minimization model, because it only imposes weaker loss aversion on the attributes than the data that has not been aggregated.

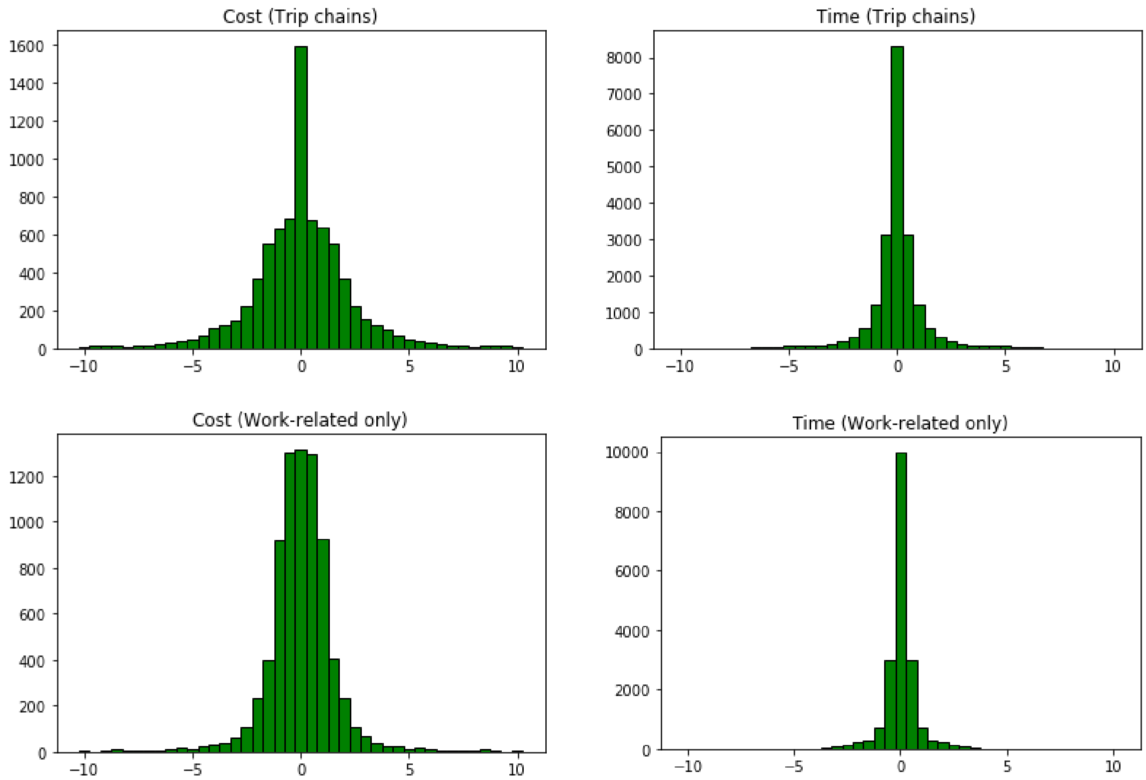


Figure 10: Histograms of attribute differences, bin size 0.5. The upper two graphs: the aggregated trip chaining data with profundities of regret 0.57 (cost) and 0.36 (time). The lower two graphs: only work-related trips with profundities of regret 0.62 (cost) and 0.53 (time).

7 Discussion

The aim of this thesis was to study the effect of trip chains in regret-based models predicting mode choice for commuting. In order to do this, a random regret minimization model (μRRM of van Cranenburgh, Guevara, & Chorus, 2015) was estimated on a data set where the attribute values of trips were added up to one single value. To the author's knowledge, this implementation of trip chaining has never been applied before in regret-based models or any discrete choice model for that matter. Considering that regret-based models were found to have equal or even improved model fit compared to utility-based models for estimating mode choice (Jing et al., 2018), applying trip chaining in the former might lead to new insights on how this factor influences this type of choice behaviour.

The trip chaining model was compared to two reference μRRM models, distinguishable by their data input (one data set with all commuting trips and one with only work-related trips). The data of these models was obtained by combining a revealed preference data set (OViN) with estimation data for alternative routes (OpenTripPlanner) and precipitation data (KNMI). The main findings of this thesis can be summarized in four points.

First, the trip chaining effects found in the data of this thesis only partially overlapped with what has been found in literature. In previous research generally two results were consistently found:

- the presence of children increases the average length of the trip chains (e.g. O'Fallon et al., 2004),
- the car is preferred over public transport for complex trip chains (e.g. Currie & Delbosc, 2011).

In this thesis, the first statement could not be replicated, because it was not always possible to establish if children (or their needs) were involved in the trip chain. It was found that households with children had executed half of the trip chains, but it remained uncertain if the needs of the children were the cause of these chains. For instance, respondents could choose "picking up or dropping off a person" as their the purpose of the trip, but no further information was provided about this passenger. Nevertheless, it was found that trip chains with over 4 trips hardly occurred for households with 5 or more members. This is in accordance with Hensher and Reyes (2000), whom explained this result by the shared responsibility of the family members to fulfill household services and demands, thereby reducing the complexity of the trip chain.

The car was found to be involved in a large majority of the trip chains, supporting the second statement. In addition, it was found that not only the car, but also the bicycle was preferred over public transport methods for complex trip chains. This strengthens the results of Currie and Delbosc (2011), where it was indicated that private modes of transport are more flexible and therefore, preferred for complex trip chains. The more activities that are planned in the trip chain, the higher the time constraints on the individual. Future research could focus on the usage of multiple types of autonomous vehicles, such as moped and motorcycles, in particular during rush hour times, since these travel modes are less affected by heavy traffic.

Second, the μRRM model was less stable than the classic RUM model. Each μRRM model converged for either 2 or 4 estimations out of 10 attempts with different starting values. However, for the RUM model convergence was realized for all the ten cases and the resulting log likelihoods were consistent. Thus, this model is more robust. This could be due to the complexity of μRRM compared to the simple definition of RUM, but another possible cause is misspecification of the model (van Cranenburgh & Prato, 2016). RRM models in general are not well-suited to handle choice set variation.

Third, the profundities of regret found in this thesis were lower for the μRRM with aggregated trip chains than the reference μRRM models (see table 2). The distribution of the differences (figure 10) shows that by aggregation the distribution not only spreads out (i.e. larger differences are created), but also becomes more centered around zero (i.e. attribute differences become nil). Thus, the aggregated trip chains actually made the data less suitable for random regret minimization.

Fourth, the estimation results of this thesis do not support the new approach to apply trip chaining, but neither do they support the use of RRM models for this data set. In previous literature, random regret minimization models for mode choice have been successfully estimated on both stated preference and revealed preference data sets (Jing et al., 2018 provides an overview on this). The profundities of regret found in this thesis (table 2) also indicated that regret minimization behaviour is imposed by the model. Nevertheless, the parameter estimates depicted in table 5 indicate that even the reference model has its limitations. For instance, the sign of the cost parameter of the car is positive, whereas it is expected to be negative since higher costs normally decrease the probability of choosing this alternative. On top of that, parameters such as the time parameter of the bicycle reached their bounds (-10 or 10), indicating that the model has trouble estimating them.

These results laid bare the limitations of the combined data set. As was shown in appendix C, the main cause of the aforementioned seems to be the combination of the revealed preference data set OViN and the estimated data of OpenTripPlanner in this thesis. On the one hand, OpenTripPlanner underestimated travel time, because it had no information on congestion. On the other hand, distance was both over- and underestimated, because the exact starting and end location have been generalized in OViN, forcing OpenTripPlanner to use centroids of postal codes for its estimations. Finally, this thesis could not exclude the possibility that human error was still present in OViN data, although most outliers have been removed (section 6.1.2).

7.1 Future work

Future research can focus on exploring the different methodologies to implement trip chaining. The main purpose of this thesis was to implement the effect of trip chaining - that previous literature found in (variations of) RUM models - in μRRM models, but under the current limitations of the data set this method cannot be fully rejected, nor confirmed. Therefore, future research is encouraged to implement other trip chaining method in regret-based models on more consistent data. Stated preference data, for instance, might be more suitable to test this method, since it is easier to control the choice set of the participants. For instance, by letting them estimate their travel time(s). Another angle to study trip chaining is to use GPS data, as was done by Huang and Levinson (2017).

The profundities of regret found in this thesis do indicate that data aggregation is not a suitable method, because it not only enlarges attribute differences, but also nullifies them. Hence, we either obtain very large or very small differences. Therefore, future research can also make use of alternative methods such as structural equation modelling (Hadiuzzaman, Farazi, Hossain, Barua, & Rahman, 2019) or the co-evolutionary approach (Krygsman et al., 2007) that have been used so far to study trip chaining in utility-based models. Since μRRM and RUM have similar estimation methods, the aforementioned techniques are also applicable to the former. Another approach is combining the Dense Neural Network approach of Sifringer, Lurkin, and Alahi (2018) with regret-based modelling. So far, this method has only been combined to a simple RUM model to estimate the parameters of a preference data set with three alternatives. The strength of this approach is that the neural network is able to process a large amount of variables that have not been included in the RUM model. However, since the results of the network are used in the RUM model, the interpretation of the results is more straightforward than in a neural network alone. This approach allows for a wide variety of factors that influence trip chaining to be studied.

Another subject that has been neglected in this thesis, yet was marked as important by De Witte et al. (2013), is the influence of habit formation on mode choice. The static nature of the current data set was unsuitable for such a dynamic analysis of habit formation. It is left to future research to study under which conditions individuals will break from their habitual behaviour.

8 Conclusion

This thesis was not able to implement the effect of trip chaining on mode choice in μRRM models. It was found that the estimation differences for time and distance in the data set led to faulty parameter estimates, even in the RUM and reference μRRM models. However, the analysis of the data set provided three insights. First, commuters take into account congestion and other hindrances when estimating their travel time. Second, by aggregating trip chaining data profundities of regret decrease, because summing up attribute values also makes more differences between alternatives nihil. Third, apart from the car, the bicycle is also preferred for complex trip chains. Future research could focus on the discrepancy between private transportation modes and shared transportation modes, instead of merely examining the different effect(s) of the car and public transport for trip chaining.

Furthermore, stated preference data sets seem to be a more logical first step to understand whether commuters base their decision on past experiences, habits or time-constraints of the trip chain. In these type of data sets the information provided to the participant can be controlled. If this is established, then the step to revealed preference data can be made, to escape the inevitable pitfall of stated preference data sets: the gap between what individuals claim to choose and what they actually choose (the behaviour they want to accomplish and the behaviour that is actually observed). Future research is also encouraged to make use of the wide range of unexplored methods to implement trip chaining that thus far, have only been applied in or combined with utility-based models.

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A Profundity of regret

In this thesis, the *profundity of regret* has been used as one of the measures to compare the regret-based models. It measured the amount of regret minimization behaviour that is imposed by the μRRM model for the attributes time and cost. This appendix further clarifies its formulation and interpretation.

With the introduction of the μRRM model in the paper of van Cranenburgh, Guevara, and Chorus (2015), the notion profundity of regret was also defined. This value is attribute specific and indicates whether the corresponding attribute is best processed as regret minimization or utility maximization behaviour. The closer this value is to 1, the higher the influence of regret compared to rejoice, i.e. a stronger aversion to loss. However, if this value is found to be close to zero, then losses are (de)valued just as much as gains. In this case, the decision behaviour in the data set is better reflected by processing this attribute in a random utility maximization manner. Essentially, the profundity of regret measures the quantity of loss aversion that is imposed by the model for a specific attribute.

The equation of the profundity of regret for an attribute k is given by:

$$\gamma_k = \frac{1}{|A_k|} \sum_{A_k} \left(\left| \frac{\exp(\beta_k/\mu \cdot [x_{kjn} - x_{kin}]) - 1}{\exp(\beta_k/\mu \cdot [x_{kjn} - x_{kin}]) + 1} \right| \right), \quad A_k = \{x_{kjn} - x_{kin} | x_{kjn} - x_{kin} \neq 0\}.,$$

where k denotes the attribute,
$|A_k|$ denotes the cardinality of the set of attribute differences ($\neq 0$),
i, j denote the alternatives that are compared,
n denotes the decision-maker.

Binary comparisons where the difference between attributes is zero are excluded, because this contains no information on the extent to which the regret generated by a loss has more or equal influence on a decision than the rejoice of an equivalent gain.

Since all the parameters in this thesis are alternative specific, the aforementioned equation can be generalized to:

$$\gamma_k = \frac{1}{|A_k|} \sum_{A_k} \left(\left| \frac{\exp(1/\mu \cdot [\beta_{kj} x_{kjn} - \beta_{ki} x_{kin}]) - 1}{\exp(1/\mu \cdot [\beta_{kj} x_{kjn} - \beta_{ki} x_{kin}]) + 1} \right| \right), \quad A_k = \{x_{kjn} - x_{kin} | \beta_{kj} x_{kjn} - \beta_{ki} x_{kin} \neq 0\}.$$

The set A now only consists of differences where the $\beta_{kj} x_{kjn} \neq \beta_{ki} x_{kin}$. Semantically, the inclusion of the β parameters means that only if there is a sensation of loss or gain, then the binary comparison is taken into account for the profundity of regret.

To intuitively understand this function, remember the shapes of the regret function for different values of μ (see figure 7b). For large values of μ, attribute differences between alternatives must be high otherwise they are scaled down to zero. If the exponential function approaches $\exp(0) = 1$, then the numerator goes to zero and as a result, $\gamma_k \to 0$. Hence, when attribute differences are low and μ is high, less regret minimization behaviour is imposed by the model for this attribute. In this case, anticipated regret is just as important as anticipated rejoice. Contrariwise, when $\mu \approx 0$, then attribute differences (high or low) are barely scaled down, meaning even small differences in regret or rejoice will increase the value of γ_k. Thus, strong regret minimization behaviour is imposed by the model.

The authors van Cranenburgh, Guevara, and Chorus (2015) proposed the profundity of regret to gain more insight in the regret minimization behaviour that is imposed by the model. In the paper the profundity of regret is even described as a crucial analysis for researchers to correctly interpret the results of their regret minimization models. Previous literature had shown that a forerunner of μRRM, namely the RRM model of C. G. Chorus (2010), often reached a nearly similar model fit as the linear-additive RUM model (C. Chorus, van Cranenburgh, & Dekker, 2014). According to van Cranenburgh, Guevara, and Chorus (2015) this might be due to impartial regret minimization behaviour across the attributes. Despite that the μRRM model is able to mimic RUM behaviour by adjusting its scale value μ (see figure 7b), this value is not attribute specific. Therefore, the profundity of regret measures the presence of regret minimization behaviour for each attribute separately.

In short, the importance of the profundity of regret is that it quantifies the amount of loss aversion that is imposed by the model *for each attribute separately.* Values close to 1 imply high regret-averse behaviour, whereas values close to 0 indicate that the loss of an anticipated regret is not more important than the rejoice of an equivalent gain. Low profundities of regret for attributes are an indication that the classic RRM model (without the scaling factor μ) will have a similar performance to the RUM model. Hence, the profundity of regret is an essential tool for the researcher to analyze the results of a regret-based model.

B Interpretation of the β parameters in μRRM models

In this appendix, the commonalities and differences between interpreting β parameters in μRRM and RUM models will be highlighted. When validating the results of a discrete choice model, the researcher should be aware which interpretation applies in the model. In RUM models, the interpretation of the β value and its sign is straightforward and intuitive: assuming that all attribute values are positive, attributes with a positive sign, positively influence utility and vice versa. More specifically, a positive sign indicates that a one unit increase of this attribute, will increase utility and thus, raise the chance of choosing the corresponding alternative. A negative sign means that each unit increase (e.g. in costs or travel time) will decrease utility with factor β and therefore, decrease the probability of choosing this alternative.

Generally, this interpretation of the sign holds for μRRM models: for instance, if the β value of an attribute has a negative sign, then an increase of this attribute increases regret and thus, decreases the probability of choosing the corresponding alternative. However, there are three complications.

First of all, due to the binary comparison of alternatives, the increase of an attribute x_i not only influences the probability of choosing alternative i, but also the probability of the other alternatives. For example, suppose that we have two alternatives i and j which we compare on cost with the traditional RRM function: $\ln(1+\exp(\beta_{cost}[x_j - x_i])), \text{with} \beta_{cost} < 0$. If the cost of alternative i increases, then the difference $x_j - x_i$ will be lower. Since the corresponding β parameter is negative, this decreasing difference will increase the regret for choosing alternative i. Thus, the probability of choosing i decreases. Reversely, the difference $x_i - x_j$ increases and therefore, the regret of choosing j decreases. Hence, the probability that this alternative is chosen increases. Thus, when the attribute value of merely one alternative changes, then the regret of the other alternatives is also influenced by this.

Second of all, in the μRRM model, the value of β does not refer to the influence of one unit increase of the attribute, but rather the influence of attribute differences between alternatives. This is due to the loss aversion principle that RRM encompasses. The higher β, the more likely that a marginal increase (or decrease) of the attribute increments (or diminishes) regret.

Third of all, in this thesis, each attribute has a unique β parameter for each alternative in the choice set instead of just one for each attribute. Most explanations on the sign of β parameters in μRRM models do not take this into account. Even though, this difference does not change the interpretation, in this appendix, I will include this distinction.

The following abstract example illustrates that the sign of β parameters in the μRRM model of this thesis has the same interpretation as in the RUM model. That is,

if an increase of an attribute x_i is expected to negatively influence the probability of choosing alternative i, then the sign of the corresponding β parameter will also be negative and vice versa when x_i decreases.

For simplicity, the regret function will be written as $\ln(1 + \exp(\beta_j x_j - \beta_i x_i))$, without the μ-scaling factor, since it is always positive and therefore, has no influence on the sign of β. The subscript referring to the attribute is also left out, because this simplified regret function only compares one attribute.

This informal proof will only explicate the cases where the sign of β_i is negative. The same reasoning applies for the positive case and are left to the reader.

Case $\boldsymbol{\beta_i < 0}$. Consider the regret function of two alternatives, i and j, for an attribute x: $\ln(1 + \exp(\beta_j x_j - \beta_i x_i))$. Assume that $x_i, x_j > 0$ and $\beta_i < 0$[9]. It will be shown that if $\beta_i < 0$ and the attribute value x_i increases, then the probability of choosing i will decrease, regardless of the sign of β_j.

Suppose that x_i is increasing and x_j remains fixed. Since $\beta_i < 0$, the expression $\beta_j x_j - \beta_i x_i$ is increasing. Noting that the function $\exp(z)$ is increasing when z is increasing, this shows us that the limit

$$\lim_{x_i \to \infty} \exp(\beta_j x_j - \beta_i x_i)$$

goes to infinity and therefore,

$$\lim_{x_i \to \infty} \ln(1 + \exp(\beta_j x_j - \beta_i x_i))$$

also goes to infinity. Thus, if the attribute x_i increases, the regret function $\ln(1+\exp(\beta_j x_j - \beta_i x_i))$ will always increase. In other words, choosing alternative i is expected to result in more regret. Since the goal of the μRRM model is to minimize regret, a higher regret implies that the likelihood of choosing alternative i decreases.

In short, there are two take-away messages. First, the value of β has a different meaning in a μRRM model than in a RUM model. In the latter, β indicates the influence of one unit increase of the attribute. In the former, β indicates the effect of higher or lower attribute differences on the probability of choosing each alternative. Thus, it measures the marginal influence on regret. Second, the interpretation of the sign is the same for the

[9] A critical note: this reasoning does not hold for when an attribute x_i increases from a negative value to a positive value. However, this is not possible for the attributes used in this thesis for the μRRM model (time and cost). The sign of these attributes is consistent.

μRRM and RUM model. If the increase of an attribute, such as cost and travel time, is expected to increase regret, then the sign of the estimated parameter must be negative. This interpretation does not change when each attribute has a unique β parameter for each alternative.

C Comparison OpenTripPlanner and OViN

In this thesis, the software OpenTripPlanner was used to estimate the distance and time for each alternative route that the decision-maker could have taken. This information is crucial for discrete choice models, otherwise alternatives can not be compared on their attributes. Given a departure and arrival location (extracted from OViN), OpenTripPlanner would calculate the possible alternative routes as explained in section 4.3 and report information such as the distance and time. However, when this data was compared to the registered travel time and distance in OViN, discrepancies were found in varying magnitudes. This appendix explores the causes of this variance (section C.1) and the magnitude of these discrepancies (section C.2).

C.1 Causes of discrepancies

Initially, the estimation method of OpenTripPlanner and data collection process of OViN was analyzed more thoroughly. The process is illustrated in figure 11. With this information, three possible causes were found.

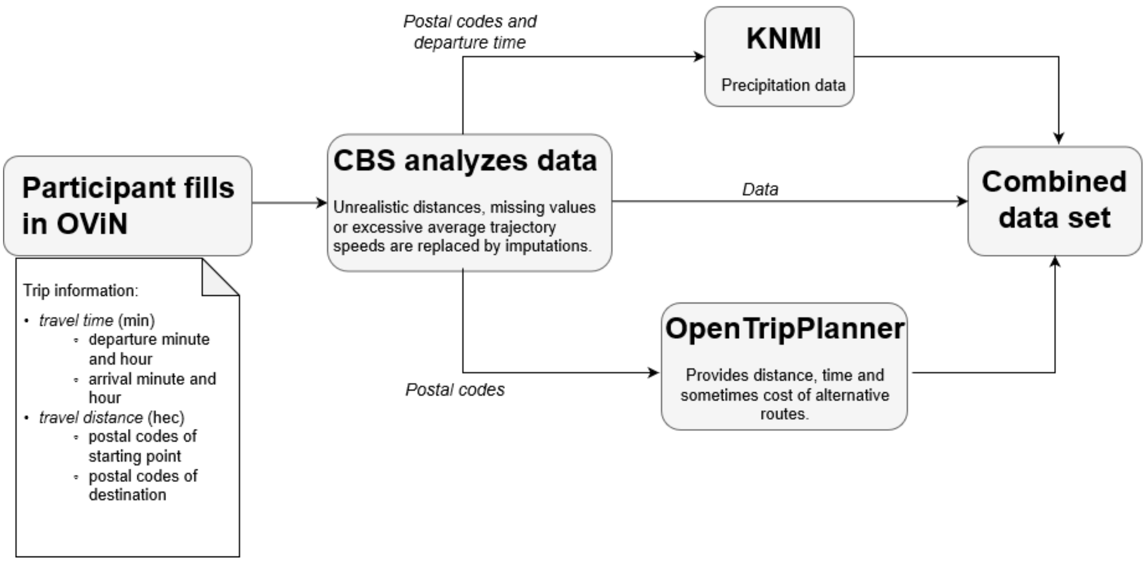


Figure 11: The process of data collection and preparation

First of all, OpenTripPlanner was limited by privacy warranty: only the postal codes of the departure and arrival location were registered in OViN. Respondents did register their indication of the travel time and distance for each trip. However, OpenTripPlanner was only given the centre of the aforementioned postal codes as the start and end point for each trip to estimate the alternative routes. As can be see in figure 6 on page 20, the size of some of these postal code areas is more than 1x1 km and the shapes are irregular. Therefore, the exact departure and arrival location of the individual would matter greatly for the distance covered, especially if an individual started or ended the trip at the edge of a postal code. The distance, in turn, determines the estimated travel time and cost.

Second of all, OpenTripPlanner was also limited by missing information on traffic congestion or other disruptions. Each route was estimated in free flow (i.e. travelling without adverse conditions). Thus, travel times were most likely underestimated by OpenTripPlanner, especially during rush hour.

The third and final possible cause is human error. The respondents of OViN were asked to fill in *their estimation* of the travel time and distance. Incorrect units, one zero too many, misinterpreting the question; human error is not uncommon in surveys. Therefore, the data has been reviewed by CBS. Entries where large distances were covered or high average trajectory speeds were reported, have been replaced by imputations (see the research description of Centraal Bureau voor de Statistiek (CBS), 2017). In the raw data set of the MRDH region alone, the CBS has applied data imputation to 7,118 out of the 51,879 reported distances (13.7%). Yet, as will be seen in the next section, the credibility of some entries is still questionable.

C.2 Magnitude of discrepancies

It was to be expected that the estimations from OpenTripPlanner were not perfect, but the extent to which it influenced the estimation of the model was unforeseen. OpenTripPlanner was missing two important factors: the exact departure and arrival point of the respondents in OViN and the lack of information on congestion or other delays. The latter meant that the data it provided was rather the *expected* travel time and distance in free flow. Meanwhile, OViN contained after-the-fact information: *experienced* travel times and distances.

To illustrate the magnitude of the difference between the expected and experienced travel times and distances, the average trajectory speeds (km/h) have been mapped. Figure 12 shows the average trajectory speed (henceforth: ATS) for each mode. On the x-axis we have the ATS calculated with OpenTripPlanner estimations and on the y-axis the ATS computed with the distance and time from OViN. Each point on the diagonal is a perfect estimation. If a point is on the left of the diagonal, then the ATS has been underestimated by OpenTripPlanner. On the right of the diagonal are all the observations that have been overestimated. Note that for each observation in the data set of this thesis, one mode could be compared, since OViN only has the distance and travel time of the chosen mode. Furthermore, observations of which OpenTripPlanner was not able to estimate the distance or time, were excluded for this comparison. Therefore, 1,388 out of 7,166 observations remained unobserved.

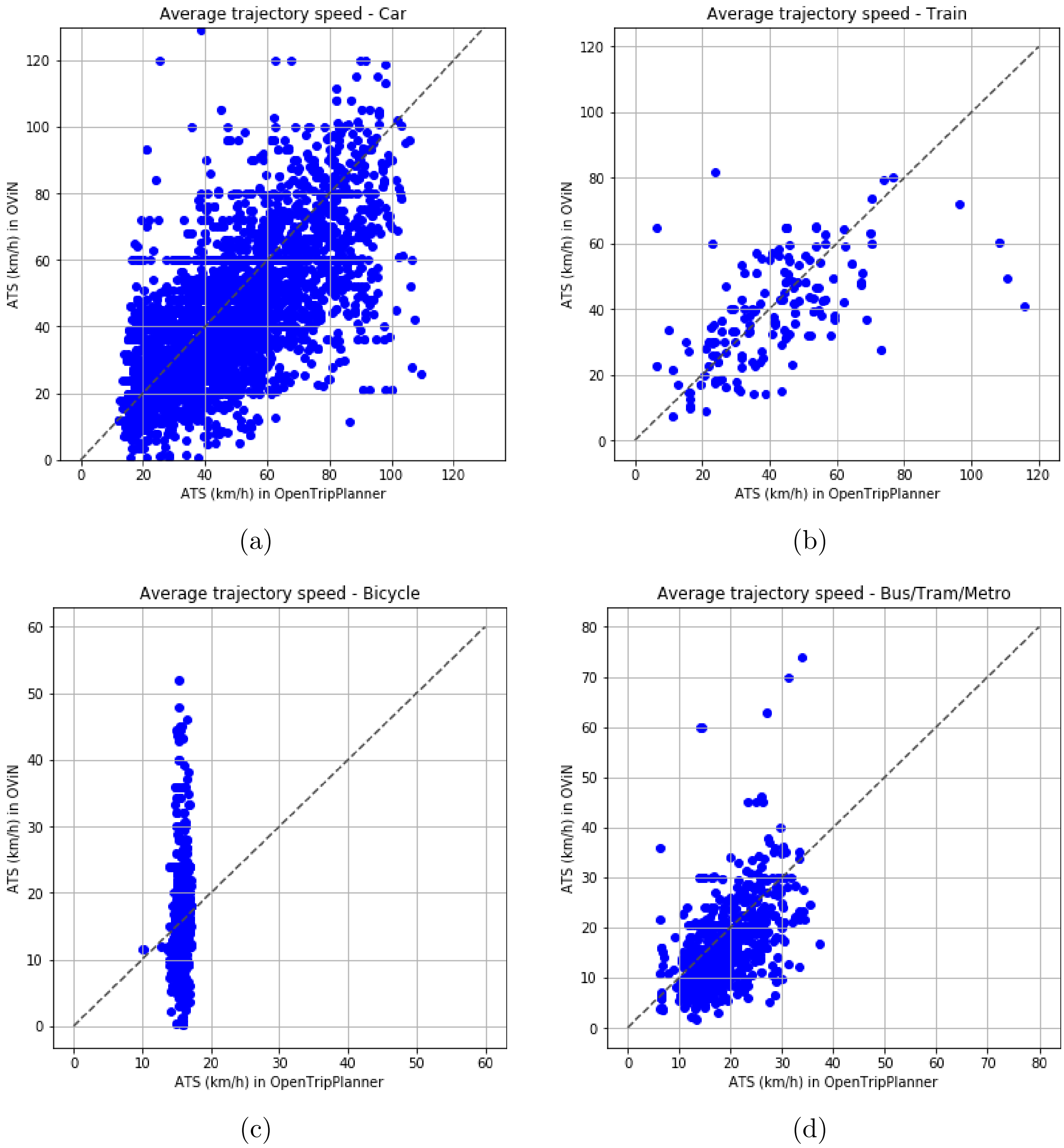


Figure 12: Comparison of OpenTripPlanner and OViN average trajectory speeds (ATS). R-squared values for car, train, bicycle and bus/tram/metro respectively: 0.24, -0.07, 0.02 and 0.20.

The R-squared values for the car, train, bicycle and bus/tram/metro alternatives are respectively: 0.24, -0.07, 0.02 and 0.20. The scatter plots show that OpenTripPlanner both over- and underestimates the ATS of each mode. Nevertheless, this data does not only expose shortcomings of OpenTripPlanner. The vertical shape of the bicycle scatter plot reveals that the ATS in OpenTripPlanner is consistent, whereas the reported ATSs in OViN run off in all directions. Considering that no participant reported using an electrical bicycle, ATSs above 25 km/h were deemed very unlikely. The same goes for speeds below 5 km/h. A further analysis into the correlation of age and the ATS in OViN was fruitless; figure 13 clearly shows that it is impossible to use age as an approximation for ATS.

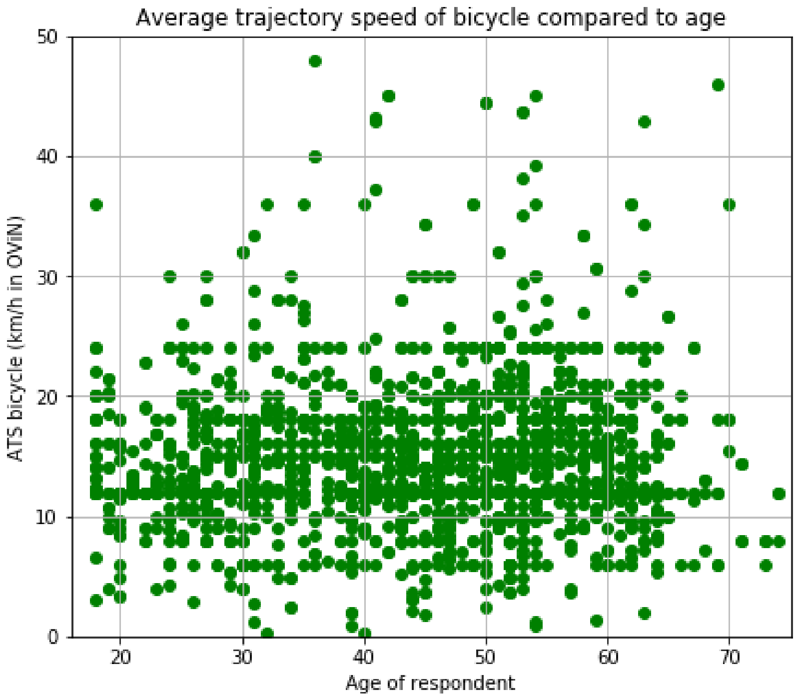


Figure 13: The average trajectory speeds as recorded in OViN in comparison with age.

Apart from the bicycle speeds, the other modes also show remarkable ATSs obtained with OViN data. For instance, for one car observation the ATS would have been 120 km/h according to OViN, whereas the ATS based on estimations of OpenTripPlanner is barely 35 km/h. Entries like this imply that not only OpenTripPlanner, but also OViN still contains unrealistic observations, despite the imputations of CBS.

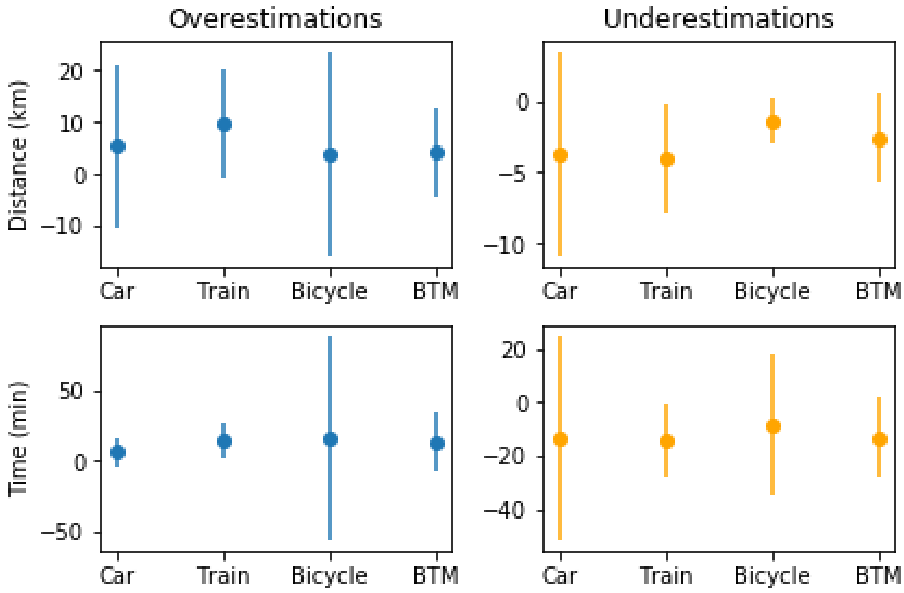


Figure 14: The mean and standard deviation of the over- and underestimations per mode

C.2.1 Distance and time

The next step is to establish whether time, distance or both disturb the data of the model, and whether they are mostly over- or underestimated. First, the estimations of OpenTripPlanner were split up in over- and underestimations per mode. After this, the mean and standard deviation were calculated. The results are in figure 14. Note that time is now in *minutes*.

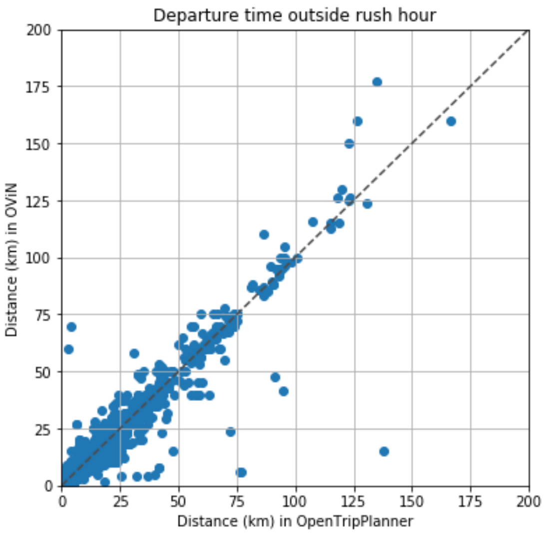
Let us start with the distance. The two upper figures show that the overestimations (in blue) are generally larger and more volatile than the underestimations (in orange). In particular, the overestimations of the bicycle alternative are quite dispersed (SD = 19.8), most likely because it is more affected by the generalization problem of the postal codes. For the underestimations, the largest variance was found for the car alternative (SD = 7.2).

For the time attribute, the standard deviations of the overestimations are between 10 and 22 minutes (SD for car, train and bus/tram/metro resp.: 10.0, 12.5 and 21.3), with an extraordinary exception for the bicycle alternative (SD = 72.9). This is due to the different cycling speeds that are difficult to predict. In terms of underestimation, the car is most affected (SD = 38.3). To see whether this might be caused by the rush hour information that is missing in OpenTripPlanner, the scatter plot of figure 12a was split up in four categories (see figure 15 on page 63):

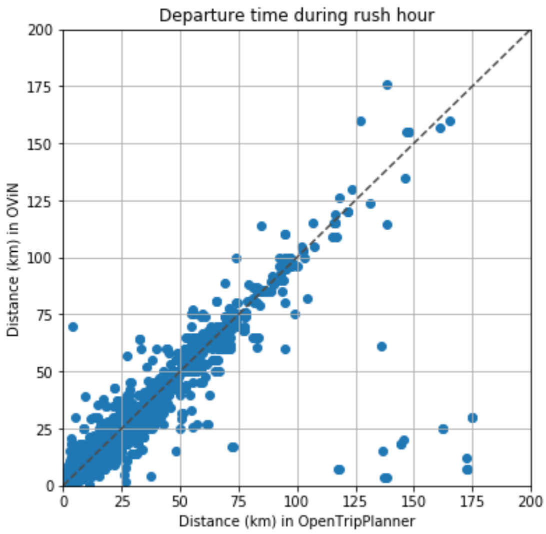
1. the measured and estimated time outside rush hour (figure 15a);
2. the measured and estimated time during rush hour (figure 15b);
3. the measured and estimated distance outside rush hour (figure 15c);
4. the measured and estimated distance during rush hour (figure 15d).

The scatter plots of the distances cluster around the diagonal, which means that the estimations of OpenTripPlanner are comparable to the measured distances in OViN. Contrarily, the scatter plots of time spread out to the left of the diagonal, indicating OpenTripPlanner is underestimating. Moreover, the underestimations become more severe during rush hour: in figure 15c the recorded time in OViN is mostly below 1.5 hours, in contrast to figure 15d. Thus, it seems that the lack of congestion information takes its toll.

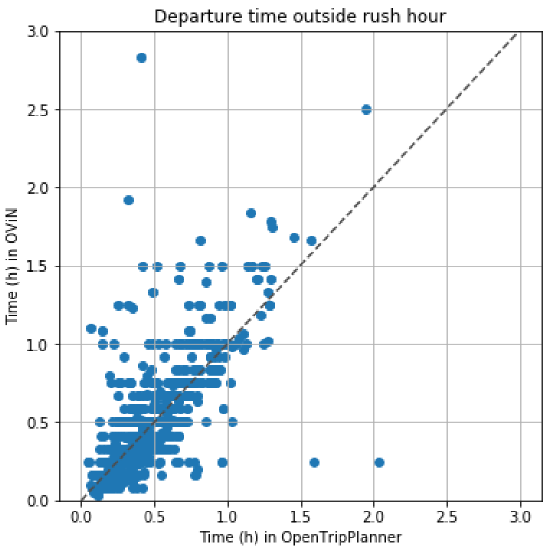
All in all, these statistics illustrate that both distance and time play their part in over- and underestimating ATSs, but with different causes. Distance was mostly overestimated. The main cause, affecting all modes, is the difference between the location of the centroid and the actual starting/end point of the trip. Other possible causes are poor distance estimations by respondents, or short-cuts that were taken by the respondent to avoid congestion (although these routes are mostly longer), but from the data it can not be retrieved whether and how often these abnormalities occur. Time was mostly underestimated by OpenTripPlanner, especially for the car alternative, since it was only able to estimate routes in free-flow. The results illustrate that the combination of OViN and OpenTripPlanner data is an ill-fitting puzzle and the researcher has to be critical when specifying the data sample for the model.



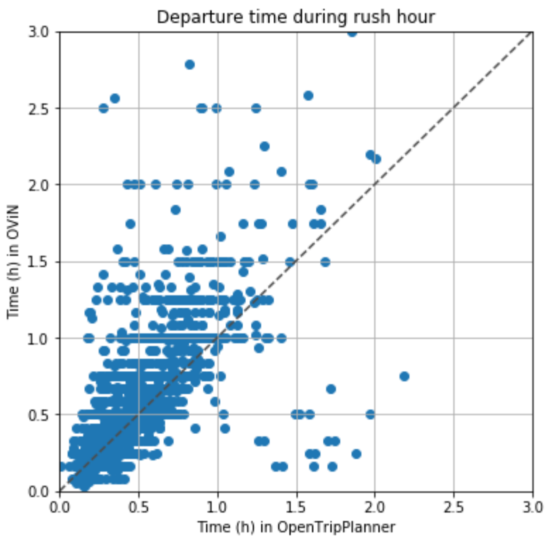
(a) Distance (km) of the car alternative - No rush hour



(b) Distance (km) of the car alternative - Rush hour



(c) Time (h) of the car alternative - No rush hour



(d) Time (h) of the car alternative - Rush hour

Figure 15: Comparison of OpenTripPlanner and OViN estimated values for the *car alternative* based on distance and time.

D Adjustment: class division

A second option to implement trip chaining was considered in this thesis, but has not been implemented due to time constraints. This method implements trip chaining by taking the chain apart, rather than aggregating it. To see the relevance of this, consider the trip chain:

$$home \rightarrow work \rightarrow groceries \rightarrow home,$$

which can be split up in three trips (home-work, work-groceries and groceries-home). Suppose that we know that these trips were executed by car. How do we know which "subtrip" was most important for the decision to take the car? Perhaps this person did not want to carry the heavy groceries or perhaps the office is not easy to reach by public transport. In the previous model this data stays hidden, because the values of the attributes are aggregated together. Therefore, with this approach, the regret of the three different trips are calculated separately and then their contribution to the total amount of regret is estimated. In formula this is as follows:

$$R_{n,i} = ASC_i + \sigma_{work} R_{n,i,work} + \sigma_{groc} R_{n,i,groc} + \sigma_{home} R_{n,i,home}, \tag{8}$$

where the σ's are estimated in a similar fashion as the β parameters.

The estimated value of σ_c for a class c will show how much influence the regret of this class has on the total amount of anticipated regret. The calculation of $R_{n,i,c}$ is the same as equation 1 (page 23). The only difference between $R_{n,i,c1}$ and $R_{n,i,c2}$ is the value of the attributes. For clarification, let's write out the last part of 8:

$$R_{i,n,home} = \sum_{j \neq i} \sum_{k} \mu \cdot \ln \left(1 + \exp \left(\frac{1}{\mu} [\beta_{k,j} x_{j,n,home} - \beta_{k,i} x_{i,n,home}] \right) \right). \tag{9}$$

Note that the $\beta_{k,i}$ only depends on the attribute k and alternative i, **not** the class c. Otherwise, the number of parameters to be estimated would explode and the model would never be able to converge.

D.1 Critical notes

Applying this method in Biogeme requires that the data is split up in multiple subsets. For example, one data set with all the work related trips, one for grocery/shopping trips, etc. Then the regret functions have to be calculated for each data set. Moreover, the more trip purposes that are registered in a data set, the more regret models have to be separately estimated. In OViN, respondents could select from 13 different trip purposes. This places a heavy burden on the estimation of the model and naturally, future research should first explore whether this approach is feasible before applying it in Biogeme. Given the experienced difficulty with the convergence of the μRRM models in this thesis, it is highly unlikely that this complex version of multiple combined μRRM would have converged.

E Code of trip chaining model

```
    # -*- coding: utf-8 -*-
"""
Created on Thu Apr 23 10:19:10 2020

@author: polwrvd

muRRM model with aggregated trip chains
"""

import pandas as pd
import numpy as np
import profundityOfRegret as por

# import data
rawdata = pd.read_csv(r"...\ovin_13_17_rain_7alts.csv")
missing_value = 999
rawdata = rawdata.replace(9999, missing_value) # lower missing value

# TRIP CHAINING
print("Start processing for trip chaining")

ids = list(set(rawdata.opid)) # list of individuals
tc = [] # list of trip chains indices
tc_id = [] # list of trip chain ids
for i in ids:
    # subset of data on one individual
    op_data = rawdata.loc[rawdata.opid == i]

    # check if there is a work-related trip
    if 2 in op_data.doel.tolist():
        # find indices of trips going home
        indices = op_data.loc[op_data.doel == 1].index.tolist()

        # find trip chain(s) if it is present
        index0 = op_data.index[0]
        while indices:
            j = indices.pop(0)
            if 2 in rawdata.doel.tolist()[index0:(j+1)]:
```

```
                tc.extend(range(index0,(j+1)))
                # add a unique trip chain ID which is linked to the first index
                tc_id.extend([index0] * (j+1 - index0))
            index0 = j+1

print("End process trip chaining")

# Define data set with trip chaining and add the ID's
data = rawdata.loc[tc]
data['tripchain_id'] = tc_id

# note that trips going to work and straight back home also share the same ID
#   even though, this is officially not a trip chain.
# Aggegrating this data won't change the results, because all the alternatives
#   are multiplied by factor 2.

###################################
####### FILTER OBSERVATIONS #######
###################################
# drop observations in weekend
weekend = data.loc[data.weekday == 0].index
data.drop(weekend, inplace = True)

# And remove participants with age under 18 or above 75+
age_filter = data.loc[data.age == 0].index
data.drop(age_filter, inplace = True)

# remove modes
# 1 = car, 2 = train, 3 = bike, 4 = walk, 5 = btm, 6 = car pass. and 7 = snor/bromfiets
# remove unknown modal choices
data.drop(data.loc[data.modal_choice == 0].index, inplace = True)
# remove walking
data.drop(data.loc[data.modal_choice == 4].index, inplace = True)
# remove car passengers
data.drop(data.loc[data.modal_choice == 6].index, inplace = True)
# remove snorfiets/bromfiets alternative
data.drop(data.loc[data.modal_choice == 7].index, inplace = True)

# Add btm as a fourth option
data.loc[data.modal_choice == 5, 'modal_choice'] = 4
```

```
# Now: 1 = car, 2 = train, 3 = bike, 4 = btm

# drop observations where income is unknown
data.drop(data[data.hhgestink == 7].index, inplace = True)

# collect a list of commuting and trip chains with modes choices and purposes
trip_chains = []
for i in list(set(data.opid)):
    trip_chains.append([data.loc[data.opid == i].doel.tolist(),
        data.loc[data.opid == i].modal_choice.tolist()])

# determine nr of trip chains (length of chain > 2)
xtc = 0
for j in trip_chains:
    if len(j[0]) > 2:
        xtc += 1
# reports 981 trip chains

###################################
########### METRICS ###############
###################################
# Define distance per km -- OVIN
data.loc[:,'dist_km'] = pd.Series(data.afstv/10, index = data.index)

# Distance from meters to kilometers, except for missing values -- OTP
mask_dcar = (data.d_car != missing_value)
mask_dtrain = (data.d_train != missing_value)
mask_dbike = (data.d_bike != missing_value)
mask_dbtm = (data.d_btm != missing_value)
data.loc[mask_dcar, 'd_car'] = data.loc[mask_dcar, 'd_car']/1000
data.loc[mask_dtrain, 'd_train'] = data.loc[mask_dtrain, 'd_train']/1000
data.loc[mask_dbike, 'd_bike'] = data.loc[mask_dbike, 'd_bike']/1000
data.loc[mask_dbtm, 'd_btm'] = data.loc[mask_dbtm, 'd_btm']/1000

###################################
############# TIME ################
###################################
# Reisduur from minutes to hours -- OVIN
data.reisduur /= 60
```

```
# convert time (sec) to hours -- OTP
mask_car = (data.t_car != missing_value)
mask_train = (data.t_train != missing_value)
mask_bike = (data.t_bike != missing_value)
mask_btm = (data.t_btm != missing_value)
data.loc[mask_car, 't_car'] = data.loc[mask_car, 't_car']/3600
data.loc[mask_train, 't_train'] = data.loc[mask_train, 't_train']/3600
data.loc[mask_bike, 't_bike'] = data.loc[mask_bike, 't_bike']/3600
data.loc[mask_btm, 't_btm'] = data.loc[mask_btm, 't_btm']/3600


#############################################
########### TIME DISCREPANCIES ##############
#############################################


# replace the travel time registered in OViN, depending on the chosen mode of transport
# this functions returns indices of outliers in the data
def adjust_travel_times(df):
    indices = []
    for index,row in df.iterrows():
        if row['modal_choice'] == 1:
            ratio = abs(row.t_car - row.reisduur) / row.reisduur
            if row.t_car != missing_value and ratio > 0.5:
                indices.append(index)
            df.at[index,'t_car'] = row.reisduur
        elif row['modal_choice'] == 2:
            ratio = abs(row.t_train - row.reisduur) / row.reisduur
            if row.t_train != missing_value and ratio > 0.5:
                indices.append(index)
            df.at[index,'t_train'] = row.reisduur
        elif row['modal_choice'] == 3:
            ats = row.dist_km/row.reisduur
            if row.t_bike != missing_value and (ats < 5 or ats > 25):
                indices.append(index)
            df.at[index,'t_bike'] = row.reisduur
        elif row['modal_choice'] == 4:
            ratio = abs(row.t_btm - row.reisduur) / row.reisduur
            if row.t_btm != missing_value and ratio > 0.5:
                indices.append(index)
```

```
            df.at[index,'t_btm'] = row.reisduur
        else:
            print('Unknown input at index', index)
    return indices

# Replace reisduur (in hours) at the right places and remove outliers in the data
indexlist = adjust_travel_times(data)
data.drop(indexlist, inplace = True)

# The same function, but now for distance
# this functions returns indices of outliers in the data
def adjust_travel_distances(df):
    indices = []
    for index,row in df.iterrows():
        if row['modal_choice'] == 1:
            ratio = abs(row.d_car-row.dist_km) / row.dist_km
            if row.d_car != missing_value and ratio > 0.5:
                indices.append(index)
            df.at[index,'d_car'] = row.dist_km
        elif row['modal_choice'] == 2:
            ratio = abs(row.d_train-row.dist_km) / row.dist_km
            if row.d_train != missing_value and ratio > 0.5:
                indices.append(index)
            df.at[index,'d_train'] = row.dist_km
        elif row['modal_choice'] == 3:
            ratio = abs(row.d_bike-row.dist_km) / row.dist_km
            if row.d_bike != missing_value and ratio > 1:
                indices.append(index)
            df.at[index,'d_bike'] = row.dist_km
        elif row['modal_choice'] == 4:
            ratio = abs(row.d_btm-row.dist_km) / row.dist_km
            if row.d_btm != missing_value and ratio > 0.5:
                indices.append(index)
            df.at[index,'d_btm'] = row.dist_km
        else:
            print('Unknown input at index', index)
    return indices

# Replace distance (in km) at the right places and remove outliers in the data
indexlist = adjust_travel_distances(data)
```

```
data.drop(indexlist, inplace = True)

###################################
############# COSTS ###############
###################################
# Define costs for each mode of transport (prices obtained from M. Snelder article)
# - car costs always distance * 0.17
# - replace cells where train cost is -1 with variable and fixed costs of train
# - same for bus/tram/metro
# - student OV is free travel
data['c_car'] = 0.17 * data['dist_km']
data.loc[(data['c_train'] == -1), 'c_train'] = 0.17 *
    data.loc[(data['c_train'] == -1), 'dist_km'] + 2.20
data.loc[data['student_week'] == 1, 'c_train'] = 0
data.loc[(data['c_btm'] == -1), 'c_btm'] = 0.10 *
    data.loc[(data['c_btm'] == -1), 'dist_km'] + 0.78
data.loc[data['student_week'] == 1, 'c_btm'] = 0

# In case of carpooling: divide by the number of people in the car (incl. driver)
carpool = (data.bessamen == 2)
data.loc[carpool,'c_car'] = data.loc[carpool,'c_car'] / (data.loc[carpool,'besaantin'])

################################
########## AVAILABILITY ########
################################
# availability of alternatives; car depends on drivers license
data['car_av'] = pd.Series((data.driving_license == 1) * 1, index = data.index)
data['train_av'] = pd.Series(1, index = data.index)
data['bike_av']  = pd.Series(1, index = data.index)
data['btm_av'] = pd.Series(1, index = data.index)

######################################
#### REMOVE ABNORMAL OBSERVATIONS ####
######################################
total_removed = 0

# if OP drove the car to work, but says not to have a driver's license
abnormalities = data.index[(data.driving_license==0) & (data.modal_choice==1)].tolist()
data.drop(abnormalities, inplace = True)
```

```
# Also exclude the observations of people who cycled for over 2 hours
abnormalities2 = data.index[(data.modal_choice == 3) & (data.reisduur > 2)].tolist()
data.drop(abnormalities2, inplace = True)

# Remove observations where people travelled by car for over 4 hours to work
# In that time you can ride from Maastricht to Groningen
abnormalities4 = data.index[(data.t_car != missing_value) & (data.t_car > 4)].tolist()
data.drop(abnormalities4, inplace = True)

# Drop public transport options where the travel time is longer than 4 hours
abnormalities5 = data.index[(data.t_train!=missing_value) & (data.t_train>4)].tolist()
data.drop(abnormalities5, inplace = True)

abnormalities6 = data.index[(data.t_btm != missing_value) & (data.t_btm > 4)].tolist()
data.drop(abnormalities6, inplace = True)

# reset index
data.reset_index(drop = True, inplace = True)

###################################
####### MISSING VALUES ############
###################################

# Remove observations where the cost of the chosen alternative is unknown
def modal_choice_check(database, miss):
    useless_indices = []
    for index,row in database.iterrows():
        if (row['c_train'] == miss) and (row['modal_choice'] == 2):
                useless_indices.append(index)
        if (row['c_btm'] == miss) and (row['modal_choice'] == 4):
                useless_indices.append(index)
    new_df = database.drop(useless_indices)
    return new_df, len(useless_indices)

# remove observation if time is a missing value for the chosen alternative
data, missing = modal_choice_check(data, miss = missing_value)

###################################
######### TRIP CHAINS #############
###################################
```

```
# collect a list of commuting and trip chains with 1) purposes and 2) modes choices
trip_chains = []
for i in list(set(data.opid)):
    trip_chains.append([data.loc[data.opid == i].doel.tolist(),
        data.loc[data.opid == i].modal_choice.tolist()])

# determine nr of trip chains
lengths = []
modes = []

xtc = 0
for j in trip_chains:
    if len(j[0]) > 2:
        lengths.append(len(j[0]))
        if 2 in j[1]:
            modes.append(2)
        elif 4 in j[1]:
            modes.append(4)
        elif 1 in j[1]:
            modes.append(1)
        else: modes.append(3)
        xtc += 1

# now reports 608 trip chains
print(" Number of trip chains: ", xtc)

###################################
####### SELECTION #################
###################################
# Select the columns you need for clarity
selection = data.loc[:, ['op','opid', 'maand', 'jaar', 'kleeft',
                         'dist_km', 'passenger','driving_license',
                         'departure_rain', 'd_hhchildren',
                         'c_car','c_train','c_btm','t_car','t_train','t_bike','t_btm',
                         'car_av','train_av','bike_av','btm_av',
                         'modal_choice','tripchain_id']]

###################################
####### AGGREGATE TRIPS ###########
###################################
```

```
# First define dummy variable for rain on departure time
selection['rain'] = (data.departure_rain != 0) * 1

# aggregate trips based on unique trip chain ID number
agg = selection.groupby('tripchain_id').agg({'c_car': ['sum'], 'c_train': ['sum'],
    'c_btm': ['sum'], 't_car':['sum'], 't_train':['sum'],
    't_bike':['sum'], 't_btm':['sum'], 'rain':['sum'],
    'car_av':['min'],'train_av':['min'],
    'bike_av':['min'],'btm_av':['min']})
# maintain column names (without "sum" and "min")
agg.columns = agg.columns.droplevel(1)

# aggregate mode choice
# Sometimes multiple modes are chosen, so we define the label by the preference ordering:
# train > btm > car > bike
mc = []
for id in agg.index:
    mode_choices = selection.loc[selection.tripchain_id == id, 'modal_choice'].tolist()
    if 2 in mode_choices:
        mc.append(2)
    elif 4 in mode_choices:
        mc.append(4)
    elif 1 in mode_choices:
        mc.append(1)
    else:
        mc.append(3)

agg['modal_choice'] = mc

######################################################
###################### BIOGEME #####################
######################################################
import biogeme.database as db
import biogeme.biogeme as bio
import biogeme.models as mod

# import data to biogeme
ovin = db.Database('ovin', agg)

from biogeme.expressions import Beta, DefineVariable, log, exp
```

```
# headers as variables
globals().update(ovin.variables)

# define a few difference variables to check if the scaling is right
t21 = DefineVariable('t21', t_train - t_car, ovin)
c21 = DefineVariable('c21', c_train - c_car, ovin)
t31 = DefineVariable('t31', t_bike - t_car, ovin)

# Check if scaling is necessary
ovin.suggestScaling()


# scaling
st = 0.001
sc = 0.001
ovin.scaleColumn('t_car', st)
ovin.scaleColumn('t_train', st)
ovin.scaleColumn('t_bike', st)
ovin.scaleColumn('t_btm', st)

ovin.scaleColumn('c_car', sc)
ovin.scaleColumn('c_train', sc)
ovin.scaleColumn('c_btm', sc)


# Alternative specific constants (ASC), one is set to zero as reference
ascCar = Beta('ascCar', 0, None, None, 1)
ascTrain = Beta('ascTrain', 0, None, None, 0)
ascBike = Beta('ascBike', 0, None, None, 0)
ascBTM = Beta('ascBTM', 0, None, None, 0)




### Very likely that model gets stuck in local minimum, so try multiple starting values
# Number of random sets of Starting Values
R = 10
# Number of parameters to be estimated (except correction factor and mu)
B = 8
```

```
# Set seed
np.random.seed(0)

# Generate random starting values (SV) from an uniform distribution
minimum = -0.1
maximum = 0.1
sv = np.random.uniform(minimum,maximum,(R,B))

# Generate random SV for correction factor between 0 and 1
cf = np.random.rand(R,1)

# Generate random SV for mu parameters between 0 and mumax (mu itself lies between 0-10)
mumax = 0.5
muset = np.random.uniform(0,mumax,R)

# Store profundity of regret for each model
profs = []

for r in range(R):
    # Seperate beta values
    beta_time_car = Beta('beta_time_car', sv[r,0], -10, 10, 0)
    beta_time_tr = Beta('beta_time_tr', sv[r,1], -10, 10, 0)
    beta_time_bi = Beta('beta_time_bi', sv[r,2], -10, 10, 0)
    beta_time_btm = Beta('beta_time_btm', sv[r,3], -10, 10, 0)

    beta_cost_car =  Beta('beta_cost_car', sv[r,4], -10, 10, 0)
    beta_cost_tr = Beta('beta_cost_tr', sv[r,5], -10, 10, 0)
    beta_cost_btm = Beta('beta_cost_btm', sv[r,6], -10, 10, 0)

    beta_rain = Beta('beta_rain',sv[r,7],-10,10,0)
    # beta class is the correction factor
    beta_class = Beta('beta_class',cf[r,0],-10,10,0)
    mu = Beta('mu', muset[r], 0, 10, 0)


    # regret functions; choice set size = 3 (no drivers license, so no car)
    Rtr = mu * (log(1 + exp((beta_time_bi/mu)*t_bike - (beta_time_tr/mu)*t_train)) + \
                log(1 + exp((beta_time_btm/mu)*t_btm - (beta_time_tr/mu)*t_train)) + \
                log(1 + exp((beta_cost_btm/mu)*c_btm - (beta_cost_tr/mu)*c_train)))
    Rbi = mu * (log(1 + exp((beta_time_tr/mu)*t_train - (beta_time_bi/mu)*t_bike)) + \
```

```
            log(1 + exp((beta_time_btm/mu)*t_btm - (beta_time_bi/mu)*t_bike)))
Rbtm = mu * (log(1 + exp((beta_time_tr/mu)*t_train - (beta_time_btm/mu)*t_btm)) + \
            log(1 + exp((beta_time_bi/mu)*t_bike - (beta_time_btm/mu)*t_btm)) + \
            log(1 + exp((beta_cost_tr/mu)*c_train - (beta_cost_btm/mu)*c_btm)))

# regret functions; choice set size = 4
R4car = beta_class *
    mu * (log(1 + exp((beta_time_tr/mu)*t_train - (beta_time_car/mu)*t_car)) + \
            log(1 + exp((beta_time_bi/mu)*t_bike - (beta_time_car/mu)*t_car)) + \
            log(1 + exp((beta_time_btm/mu)*t_btm - (beta_time_car/mu)*t_car)) + \
            log(1 + exp((beta_cost_tr/mu)*c_train - (beta_cost_car/mu)*c_car)) + \
            log(1 + exp((beta_cost_btm/mu)*c_btm - (beta_cost_car/mu)*c_car)))
R4tr = beta_class *
    mu * (log(1 + exp((beta_time_car/mu)*t_car - (beta_time_tr/mu)*t_train)) + \
            log(1 + exp((beta_time_bi/mu)*t_bike - (beta_time_tr/mu)*t_train)) + \
            log(1 + exp((beta_time_btm/mu)*t_btm - (beta_time_tr/mu)*t_train)) + \
            log(1 + exp((beta_cost_car/mu)*c_car - (beta_cost_tr/mu)*c_train))+ \
            log(1 + exp((beta_cost_btm/mu)*c_btm - (beta_cost_tr/mu)*c_train)))
R4bi = beta_class *
    mu * (log(1 + exp((beta_time_car/mu)*t_car - (beta_time_bi/mu)*t_bike)) + \
            log(1 + exp((beta_time_tr/mu)*t_train - (beta_time_bi/mu)*t_bike)) + \
            log(1 + exp((beta_time_btm/mu)*t_btm - (beta_time_bi/mu)*t_bike)))
R4btm = beta_class *
    mu * (log(1 + exp((beta_time_car/mu)*t_car - (beta_time_btm/mu)*t_btm)) + \
            log(1 + exp((beta_time_tr/mu)*t_train - (beta_time_btm/mu)*t_btm)) + \
            log(1 + exp((beta_time_bi/mu)*t_bike - (beta_time_btm/mu)*t_btm)) + \
            log(1 + exp((beta_cost_car/mu)*c_car - (beta_cost_btm/mu)*c_btm))+ \
            log(1 + exp((beta_cost_tr/mu)*c_train - (beta_cost_btm/mu)*c_btm)))

# regret functions
RRcar = car_av * -(ascCar + R4car)
RRtr = (1-car_av) * -(ascTrain + Rtr) + car_av * -(ascTrain + R4tr)
RRbi = (1-car_av) * -(ascBike + Rbi + beta_rain * rain) +
    car_av * -(ascBike + R4bi + beta_rain * rain)
RRbtm = (1-car_av) * -(ascBTM + Rbtm) + car_av * -(ascBTM + R4btm)

V = {1: RRcar,
     2: RRtr,
     3: RRbi,
     4: RRbtm}
```

```
    av = {1: car_av, 2: train_av, 3: bike_av, 4: btm_av}

    # Define the likelihood function for the estimation
    biogeme = bio.BIOGEME(ovin,mod.loglogit(V,av,modal_choice))

    # Name biogeme object to identify each repetition
    biogeme.modelName = "muRRM_aggr_rep_" + str(r+1)

    # Estimate!
    # It is possible that starting values are infeasible.
    # The following condition allows to skip errors.
    try:
        results = biogeme.estimate()
        print("Rep " + str(r+1) + "/ LogLik: " +
            str(results.getGeneralStatistics()['Final log likelihood'][0]))
        # profundity of regret
        param = results.getBetaValues()
        profs.append([por.profCost2(agg,param),por.profTime2(agg,param)])
    except:
        print("Rep " + str(r+1) + "/ Convergence Error")

# Calculate profundity of regret
print("Profundities of regret: (cost and time):", profs)
print("Starting values:", sv)
```