# The influence of regret on moral decision making

## A formal analysis of the relation between moral luck and regret theory in an extension of probabilistic XSTIT logic

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**Abstract**

This interdisciplinary thesis combines topics from philosophy, economy, computer science, and logics. Moral luck is the philosophical problem in which agents can be morally judged for factors beyond their control. The feeling of regret when things turn out the wrong way is an important effect in cases of moral bad luck. In decision theory, regret can play a role as well. This is incorporated in regret theory, which explains human behaviour better than maximum expected utility does. Decisions can be modelled in STIT logic to reason about what actions an agent can perform and what action it actually performs, as well as the decision making process before performing the action of choice. This thesis researches the relation between moral luck and regret theory. This will be done by extending probabilistic XSTIT logic with both maximum expected utility and regret theory. The resulting framework will then be used to formalise two moral luck examples and relate them to regret theory.

*Keywords:* stit logic, moral luck, regret theory, maximum expected utility

# Contents

# 1 Introduction

In daily life, we make all kinds of decisions and we make a lot of decisions. Morality is involved in a significant amount of these decisions. In the decision making process, we consider many different factors. These include the possible outcome and our belief of the chance of success the chosen action has. Someone can avoid risky situations, or maybe attracted to them. One person may opt for a more rational strategy, while another person might follow his gut feeling. A number of axioms and theories have been proposed to model decision making behaviour.

Maximising expected utility is considered to be a rational strategy for making decisions under risk. Empirical experiments have shown that humans do not necessarily follow this rational strategy, but rather seem to act according to regret theory. This alternative strategy considers our feeling of regret after choosing a particular action in the decision making process. In moral decision making, regret can be felt as well. This seems to relate to the philosophical problem of moral luck. When someone makes a morally bad decision, he can be lucky if the outcome turns out to be positive. The ideas of moral luck and regret theory both have something to do with the feeling of regret. In this thesis, it will be investigated what else they have in common.

## 1.1 Relevancy for the field of AI

Artificial intelligence is a young, interdisciplinary field of increasing popularity. Understanding decision making processes better and finding ways to formalise them is part of this field. It is not only relevant to find an optimal, rational decision making strategy, but understanding human decision making behaviour is important as well. An AI should be able to communicate with humans, and should therefore have an idea of what we would do in a certain context in order to adapt to the situation in a appropriate manner. Many scientific fields are involved in artificial intelligence. For this thesis, insights from philosophy, psychology, computer science, and logics will be combined.

The philosophical problem of moral luck will be introduced with the help of two running examples. An important aspect of moral (bad) luck is the feeling of regret. Psychological experiments will provide us with data about human decision making behaviour, or more specifically, their preferences in certain choice situations involving lotteries. Both computer science and economics are interested in game theory and attempt to formally define strategies that are optimal or capture human behaviour. Decision theory can be seen as a subset of game theory, there is only one agent involved in the decision making process and outcomes are usually determined by chance processes.

A well-known strategy is maximum expected utility in which the pay-off of outcomes and the probabilities of those outcomes are involved. This is considered to be a rational, optimal strategy for decision making when probabilities are known. Humans do not always follow this rational behaviour and empirical evidence has shown that humans do not maximise their expected utilities. Therefore, regret theory has been proposed, which not only considers the actual outcome, but the counterfactual outcome as well. Preference of one action over another does not only involve the chosen action and its possible outcomes, but rejecting the other option is part of the decision making process as well. Reasoning about counterfactuals, about what could have happened if one had chosen otherwise, is a key aspect in experiencing the feeling of regret. Reasoning about actions and counterfactuals in a decision making process can be modelled in STIT logic. The acronym STIT stands for "sees to it that" in which an agent (or group of agents) sees to it that some proposition will be true. This thesis uses the probabilistic XSTIT logic as a basis to formally analyse moral luck examples in order to gain a better understanding.

## 1.2 Research Questions

Both the philosophical problem of moral luck and the decision theoretic concept of regret theory involve the feeling of regret in a decision making context. This raises the question how these two are related to each other. Therefore, in this thesis the following two research questions will be answered:

1. How can maximum expected utility and regret theory be incorporated into probabilistic XSTIT logic?

2. How do regret theory and moral luck relate to each other?

To discover how moral luck and regret theory relate, an extension of probabilistic XSTIT logic will be proposed. This extension consists of incorporating maximum expected utility and regret theory into probabilistic XSTIT. Maximum expected utility has been added to a different STIT logic before (Bartha, 2014), but not to probabilistic XSTIT so far. We did not find any publications that incorporate regret theory into STIT logic. In order to contribute to the area of STIT logic and to formulate the extension, we will need to explore decision theory, regret theory, and probabilistic XSTIT. A way to represent games in the logical framework will be presented. This includes the addition of certain concepts used in regret theory.

Examples of moral luck will be formalised in the resulting framework. With these models, moral luck and regret theory will be compared to reveal their relationship. We will analyse the differences and similarities of the two concepts within our logical framework. One obvious similarity between these two topics, is the feeling of regret. As far as we know, the relation between moral luck and regret theory has not been researched before. This means that there are no answers to our research questions yet.

## 1.3 Related work

Moral luck is a widely discussed topic in philosophy. The origin of this phenomenon lies with Williams and Nagel (1976). Some logical analyses have been done, such as by Meder (2018) and Farjami, Meder, Parent and Benzmüller (2018) in input/output logic, and by Horty (2001, pp. 117-121) in STIT logic. An analysis of moral luck and legal luck in probabilistic XSTIT is presented by Broersen (2014), where he argues that moral luck does not exist because it is confused with legal luck. Probabilistic XSTIT is extended with grades of responsibility by Doriot and Broersen (2016). This logic will be used as a basis for the current extension with maximum expected utility and regret theory.

An extension of Horty's (2001) utilitarian STIT logic with maximum expected utility is given by Bartha (2014) and game theory with mixed strategies is considered briefly as well. Bentzen (2010) proves that Horty's (2001) utilitarian STIT can be reduced to a single time step. Kooi and Tamminga (2006, 2008) use STIT logic for game theory, their examples are reduced to a single time step.

Regret as an emotion has been studied in several logics. Some examples of this are the KARO framework (Meyer, 2006), and the combination of this emotion with fuzzy logic (El-Nasr, Yen & Ioerger, 2000). A STIT logic extended with emotions such as regret is proposed by Lorini and Schwarzentruber (2011). It is important to note that these authors approach regret as an emotion, whereas the current work is based directly on regret theory (Bell, 1982; Loomes & Sugden, 1982). The difference is that we are not concerned with the experience of this feeling itself, but rather how regret influences the decision making process. As far as we know, there are no publications that add regret theory to STIT logic.

## 1.4 Structure

This thesis is structured as follows. First, the two main topics, moral luck and regret theory, are introduced as well as the logic that will be used. Chapter 2 introduces moral luck, Chapter 3 will give an outline of regret theory, and in Chapter 4 the basics of probabilistic XSTIT will be given. Subsequently, a comparison between these different topics will be made and we will study how they relate to each other. More specifically, in Chapter 5, we will research how regret theory can be integrated into probabilistic XSTIT. Next, this extension of probabilistic XSTIT will be used to discover the relation between regret theory and moral luck in Chapter 6. This will be followed by a discussion and a conclusion in Chapter 7.

# 2 Resultant Moral Luck

Moral luck is a subject of philosophical debate and can be traced back to Bernard Williams and Thomas Nagel (1976). According to them, if an agent can be punished or praised morally and some of the factors contributing to the outcome were not under its control, then the situation can be seen as a case of moral luck. Here the word *luck* is used to describe both good and bad luck.

Nagel (1979) describes four different kinds of moral luck: constitutive, circumstantial, causal, and resultant moral luck. Constitutive moral luck is based on the person you are, on your traits, capacities, and temperament. This is based on your genes and upbringing, on which you have had little or no influence. If someone usually is very selfish and does something good for another person, it is possible that he still gets morally judged for being selfish. Circumstantial moral luck can be the result of the situations one encounters. The opportunities one encounters determine the possible actions. Someone is judged only for actions he performed in an encountered situation, not for what he would have done in hypothetical situations. Causal moral luck can be seen as the combination of constitutive and circumstantial moral luck. It is "luck in how one is determined by antecedent circumstances" (Nagel, 1979, p. 28). According to him, it is essentially the problem of free will[1]. Resultant moral luck has to do with the consequences of actions performed by an agent. This term was not used by Nagel in 1979 but it is how it is known in the literature nowadays (Nelkin, 2013). There are two ways that can lead to resultant moral luck. One is when two agents have the same intentions and perform the same actions, but the way things turn out is different for both agents due to negligence. The other one involves decisions made under uncertainty, in which the outcome of an action cannot be foreseen.

In this thesis, we will research the relation between moral luck and regret theory, on the basis of the feeling of regret. The feeling of regret is mostly present in the case of resultant moral luck. If an agent made a particular decision and things turn out the wrong way, then the agent will experience regret. This emotion

[1]Nagel (1979) does not go into this category any further, since it was beyond the scope of his essay. He has been criticized for including this category, because it is already covered by constitutive and circumstantial moral luck (Nelkin, 2013).

is described by Williams (1981, p. 27) as follows:

> The constitutive thought of regret in general is something like 'how much better if it had been otherwise', and the feeling can in principle apply to anything of which one can form some conception of how it might have been otherwise, together with consciousness of how things would then have been better.

This reasoning about counterfactuals is an important aspect of resultant moral luck. This thesis will therefore focus on resultant moral luck. For conciseness, we will often use 'moral luck' instead of 'resultant moral luck' in the following chapters.

## 2.1 Examples

Resultant moral luck will be illustrated by the following two examples: the lorry driver example 2.1 (Nagel, 1979) and the Gauguin example 2.2 (Williams, 1981). These two examples will be used for the formalisation in STIT logic in Chapter 6 as well.

**Example 2.1** (Lorry drivers)**.** Suppose there are two lorry drivers, $\alpha$ and $\beta$, each driving their own lorry. Both drivers have been negligent and did not check their brakes for quite some time. The brakes of both lorries are unsafe and should be repaired. While driving, lorry driver $\alpha$ fatally hits a child because he could not brake in time. The other lorry driver, $\beta$, does not cause an accident. Afterwards, driver $\alpha$ will be sent to jail because he killed the child. Although that is not his only punishment. Agent $\alpha$ feels terrible about the death of the child, regrets having been negligent about checking his brakes, and will blame himself for the death of the child.

**Example 2.2** (Gauguin)**.** An artist we will call Gauguin[2] thinks it is a good idea to leave his family behind and move to Tahiti because he believes it will be beneficial for his artwork. Even though he moves to Tahiti, he feels that he neglects his wife and five children. The outcome of his action is uncertain, because he does not know yet if his paintings will be more successful when he is in Tahiti instead of France. If he fails, he will regret his decision of making the morally wrong choice. On the other hand, if he succeeds, he will have a justification for his choice of

[2]The real Paul Gauguin, a French post-impressionist artist, moved to Tahiti permanently in 1895 to improve his artistic success (Thomson, 1987). This was only after he and his wife got divorced. The first year, he did not produce any paintings. Nowadays, Gauguin is famous for his paintings of Tahitian women. He knew his decisions were often judged as morally wrong, but he only cared about getting the attention (Thomson, 1987).

abandoning his family. Gauguin moves to Tahiti, hoping that he will be morally lucky, hoping that his immoral decision leads to a positive outcome anyway.

When Gauguin leaves his family behind and becomes a great painter, he thinks his morally bad decision will be justified by his successful career (Nagel, 1979). These two examples illustrate cases of resultant moral luck because they involve a certain kind of moral decision, including not performing a particular action, and there are factors that are beyond the agent's control which affect the outcomes of their actions. Despite these factors, the agent can be judged morally on the way things turn out. When presented with an action choice the outcomes of those actions are not always certain. This means that the agent can be lucky or unlucky depending on the outcome of the performed action.

## 2.2 Problem

The problem with these cases of resultant moral luck, is that we judge an agent on the basis of the outcome, even though it was due to factors beyond their control. The *control principle* states that we should only be morally judged for outcomes that depend on factors under our control (Nelkin, 2013). In example 2.1, both lorry drivers were negligent about checking their brakes and performed the same voluntary actions. They made the same moral choices, but due to factors beyond their control, the consequences were different for the two agents. We still tend to morally judge agent $\alpha$ in a different way than agent $\beta$. Agent $\alpha$ was morally wrong and will feel this regret himself. In example 2.2, there was one agent that made a moral choice, but the consequences of his action were uncertain. This uncertainty of the outcome was again beyond the control of the agent. In this case, we blame or praise the agent according to the outcome as well. These two examples show the problem of resultant moral luck and how it conflicts with the control principle: that an agent should not be held morally responsible for actions or outcomes due to factors beyond its control (Broersen, 2014).

According to Nelkin (2013), there are three solutions[3] to this problem with moral luck:

> (i) to deny that there is moral luck despite appearances, (ii) to accept the existence of moral luck while rejecting or restricting the Control Principle, or (iii) to argue that it is simply incoherent to accept or deny the existence of some type(s) of moral luck.

[3] We will not go into the philosophical discussion on the existence of (resultant) moral luck, since it is beyond the scope of this thesis. We think it is easy to mistake moral luck for legal luck and the existence of moral luck relies heavily on how certain concepts are defined precisely.

One of the responses to the problem of moral luck is denial. This denial can express itself in different ways, for example by arguing that moral luck is mistaken for legal luck. In example 2.1, the lorry driver that runs over the child will be convicted for involuntary manslaughter, which is a legal punishment. It can be argued that "our views about the moral assessment of actions are influenced and obscured by our legal views on the matter" (Broersen, 2014) and that moral luck does not exist in the lorry driver example 2.1, instead it is a case of *legal* bad luck. To avoid the influence of legal judgement, the lorry driver example can be adapted. Suppose a dog runs onto the road instead of a child. Suddenly, the lorry driver that kills the dog does not get sentenced to jail, but can still be morally judged.

Resultant moral luck includes the uncertainty of outcomes after a moral decision making process. Another important factor is that when things turn out the wrong way, the agent will experience regret and is considered to have moral bad luck. This feeling of regret is the central emotion in regret theory as well. This decision making strategy will be explained in the next chapter.

# 3 Regret Theory

In 1982, both Bell, and Loomes and Sugden independently came up with the notion of *regret* in decision theory. Decision theory is a form of game theory in which we consider just one agent that has to make some decision. Note that this is not necessarily about moral decision making, such as in the moral luck examples, but rather decision making in general. There are many strategies for making decisions; maximin, maximax, and expected utility to name a few (Loomes & Sugden, 1982; Bartha, 2014). Maximum expected utility "has been generally accepted as a normative model of rational choice" (Kahneman & Tversky, 1979, p. 263). Both Bell (1982) and Loomes and Sugden (1982) claim that these choices are made *under uncertainty* although others would not agree with this term (Horty, 2001). Usually, the term *under uncertainty* would imply that there are no probabilities known for the possible outcomes. Both Bell (1982) and Loomes and Sugden (1982) use examples that do have known probabilities and would therefore lead to a different strategy than in a case with unknown probabilities. In this thesis, the term *risk* is used when probabilities are known, whereas *uncertainty* will be used when probabilities are absent.

Kahneman and Tversky (1979) conducted a number of experiments (see Table 3.1) and showed that people generally do not follow expected utility to make decisions. The subjects were asked which one of two options they would prefer. In Problem 3 for example:

1. 80% chance to win $4000 and
   20% chance to win nothing

2. $3000 for sure

These problems or scenarios are formulated with the following notation. A lottery $X_i$ is a combination of pay-offs $x_1, ..., x_n$ and probabilities $p_1, ..., p_n$ where $p_1 + ... + p_n = 1$ and this lottery will be written as $x_1, p_1; ...; x_n, p_n$. Pay-offs of $0 will be omitted such that the lottery $(x, p; 0, 1 - p)$ is written as $(x, p)$. There is a finite number $n$ of states of the world, each state $j$ has a probability $p_j$. These states of the world can be seen as choices for a special agent called *nature*. A decision making agent is presented with a problem where it has to choose between actions. For simplicity, it is assumed that a choice or action $A$ corresponds to a lottery $X$. An important aspect of regret theory is reasoning about counterfactuals or foregone outcomes. Counterfactuals are not what the actual outcome is, but rather what the outcome could have been if one had chosen otherwise (Starr, 2019). The preference relation of lotteries (and therefore actions) for an agent will be denoted with $\succ, \succcurlyeq, \sim$ for strict preference, weak preference, and indifference respectively. Where $X_i \succ X_k$ ought not be read as 'lottery $X_i$ is strictly preferred over lottery $X_k$', but rather that the agent strictly prefers 'choosing lottery $X_i$ and rejecting lottery $X_k$' over 'choosing lottery $X_k$ and rejecting lottery $X_i$' (Loomes & Sugden, 1982).

| Problem number | Lotteries offered | Measured preference | % of subjects with measured preference | Expected utility preference |
|---|---|---|---|---|
| 1 | $X_1 = (2500, 0.33; 2400, 0.66)$ | $X_1 \prec X_2$ | 82 | $X_1 \succ X_2$ |
| | $X_2 = (2400, 1.00)$ | | | |
| 2 | $X_3 = (2500, 0.33)$ | $X_3 \succ X_4$ | 83 | $X_3 \succ X_4$ |
| | $X_4 = (2400, 0.34)$ | | | |
| 3 | $X_5 = (4000, 0.80)$ | $X_5 \prec X_6$ | 80 | $X_5 \succ X_6$ |
| | $X_6 = (3000, 1.00)$ | | | |
| $3'$ | $X_7 = (-4000, 0.80)$ | $X_7 \succ X_8$ | 92 | $X_7 \prec X_8$ |
| | $X_8 = (-3000, 1.00)$ | | | |
| 4 | $X_9 = (4000, 0.20)$ | $X_9 \succ X_{10}$ | 65 | $X_9 \succ X_{10}$ |
| | $X_{10} = (3000, 0.25)$ | | | |
| $4'$ | $X_{11} = (-4000, 0.20)$ | $X_{11} \prec X_{12}$ | 58 | $X_{11} \prec X_{12}$ |
| | $X_{12} = (-3000, 0.25)$ | | | |
| 10 | $X_{17} = (X_5, 0.25)$ | $X_{17} \prec X_{18}$ | 78 | $X_{17} \succ X_{18}$ |
| | $X_{18} = (X_6, 0.25)$ | | | |

Table 3.1: Human behaviour in a decision making experiment compared to maximising expected utility. Adapted[a] from (Loomes & Sugden, 1982, p. 806).

[a]In the original experiment, Israeli pounds were used as currency. "To appreciate the significance of the amounts involved, note that the median net monthly income for a family is about 3,000 Israeli pounds" (Kahneman & Tversky, 1979).

In the experiment, the responses of the subjects were compared to what maximum expected utility would prescribe. In the example of Problem 3, the expected utility would be $u(4000) \times 0.8 + u(0) \times 0.2 = u(3200)$ for gambling and only $u(3000)$ for the sure prize, maximising it would lead to the choice of gambling. The utility function $u(x)$ here means that the monetary value of the pay-off is transformed into some subjective value, such that "as an agent gets richer, every successive dollar (or gold watch, or apple) is less valuable to her than the last" (Briggs, 2017). When comparing two similar situations, for example Problems 3 and 4 (see Table 3.1), the subjects would choose differently. In Problem 4, the subjects do maximise expected utility, while in Problem 3 they do not. This can be seen as contradictory and irrational. Kahneman and Tversky (1979) came up with *prospect theory* to explain this behaviour. In contrast to prospect theory, regret theory is a much simpler explanation for the same behaviour (Loomes & Sugden, 1982).

The key problem that regret theory solves, is that people generally do not maximise expected utility, even though it is usually seen as a rational strategy. When comparing two similar decision making situations, people maximise their expected utility in one situation, but not in the other. In expected utility, such preferences are contradictory and can therefore be seen as irrational. Loomes and Sugden (1982) argue that this behaviour is not irrational, and propose regret theory as a solution. In regret theory, such differences between the preferences of choices in similar situations are not contradictory. Regret theory considers counterfactual outcomes, since they influence the decision making process in humans. Maximising expected utility can still be considered to be the optimal strategy for agents that do not feel any regret or rejoice (the positive counterpart of regret) (Loomes & Sugden, 1982).

## 3.1 Allais Paradox

Allais Paradox (1953) is one of the many examples that show that maximum expected utility is not in line with the experimental data of human behaviour.

**Example 3.1** (Allais Paradox)**.** There are two scenarios:

1. An agent is given a choice between a sure prize of \$1 million or a lottery with a 10% chance at \$5 million, a 89% chance at \$1 million, and a 1% chance of getting nothing. More formally, the agent can choose between $X_1 = (1, 1.00)$ and $X_2 = (5, 0.10; 1, 0.89)$ with the pay-off in millions of dollars.

2. The agent is offered a choice between two lotteries. The first lottery has a 10% chance of \$5 million and a 90% chance of winning nothing. The second lottery has an 11% chance of winning \$1 million and an 89% chance

at nothing. This can be written as a choice between $X_3 = (5, 0.10)$ and $X_4 = (1, 0.11)$, again with pay-offs in millions of dollars.

Allais (1953) showed empirically that most people select the sure prize of \$1 million in the first scenario ($X_1 \succ X_2$) and choose the gamble with 10% chance of \$5 million in the second scenario ($X_3 \succ X_4$). This would mean that

$$u(1) > 0.10u(5) + 0.89u(1) + 0.01u(0)$$

and

$$0.10u(5) + 0.90u(0) > 0.11u(1) + 0.89u(0)$$

where $u(x)$ is the utility of $x$ and the prizes in millions of dollars (Bell, 1982). The utility of a certain prize can be different from the monetary value, it can be seen as the psychological value that belongs to the monetary value (Allais, 1953). The first inequality can be rewritten as

$$\begin{aligned} u(1) &> 0.10 \times u(5) + 0.89 \times u(1) + 0.01 \times u(0) \\ 0.01 \times u(1) + 0.10 \times u(1) + 0.89 \times u(1) &> 0.01 \times u(0) + 0.10 \times u(5) + 0.89 \times u(1) \\ 0.01 \times u(1) + 0.10 \times u(1) &> 0.10 \times u(5) \\ 0.11 \times u(1) &> 0.10 \times u(5) \end{aligned}$$

and the second equation will be

$$\begin{aligned} 0.10 \times u(5) + 0.90 \times u(0) &> 0.11 \times u(1) + 0.89 \times u(0) \\ 0.01 \times u(0) + 0.10 \times u(5) + 0.89 \times u(0) &> 0.01 \times u(1) + 0.10 \times u(1) + 0.89 \times u(0) \\ 0.10 \times u(5) &> 0.01 \times u(1) + 0.10 \times u(1) \\ 0.10 \times u(5) &> 0.11 \times u(1) \end{aligned}$$

which shows that these two formulas are contradictory, since $0.10 \times u(5)$ cannot be both strictly smaller and strictly larger than $0.11 \times u(1)$ at the same time. The inequality $>$ shows the preference of one choice over the other on the basis of expected utilities. The first inequality shows that people would violate maximum expected utility. Whereas in the second situation, they would make a rational decision. The combination of Problems 1 and 2 (Table 3.1) is a variation on the original Allais Paradox leading to the same observations in human behaviour (Kahneman & Tversky, 1979).

## 3.2 The influence of regret on decision making

The emotion of regret plays a significant role in decision making (Bell, 1982; Loomes & Sugden, 1982). Suppose you chose not to participate in the Dutch

Postcode Loterij[1] and it turns out that your postcode has won the jackpot. You will probably regret your earlier decision not to buy a ticket, because you could have won lots of money. The feeling of regret in the Dutch Postcode Loterij has been researched by Zeelenberg and Pieters (2004). It is this emotion that plays a role in decision making. Bell (1982) explicitly assumes that all outcomes will be fully resolved. This means that the subject will hear the actual outcome for all choice options as in the example of the Dutch Postcode Loterij. This information is important for the decision maker and affects his strategy. These simple situations are the subject of the current work, more complex situations are addressed by Bell (1983). The notion of regret depends on the prize won, the prize that could have been won, the probabilities involved, and personal preferences.

Bell (1982) and Loomes and Sugden (1982) both independently came up with the idea of regret theory in the same year. The idea is the same in both articles: human behaviour does not follow expected utility. The experience of regret is an important aspect in the human decision making process. The positive counterpart of regret is called *rejoice*, this involves the feeling of pleasure when the actual outcome turned out to be better than the counterfactual (Loomes & Sugden, 1982; Lorini & Schwarzentruber, 2011).

## 3.3 Violations of expected utility theory

A number of violations of expected utility have been identified in human behaviour, of which Loomes and Sugden (1982) mention the following three main ones:

1. The certainty effect or common ratio effect
2. The common consequences effect
3. The isolation effect

The certainty effect or common ratio effect can be seen in situations where choices between a sure prize and a gamble with high chance are compared to choices between the same prizes with lower probabilities, where the ratio of the probabilities is kept the same. An example is the comparison of Problems 3 and 4 (see

[1]The Dutch Postcode Loterij is a charity lottery in which postcodes are drawn to determine the winners (Nationale Postcode Loterij, 2019). A large jackpot (54.7 million euro in January 2020) called the Eindejaarskanjer is awarded each year, in addition, there are smaller prizes throughout the year. The Eindejaarskanjer is divided between all people that have a lottery ticket for the winning postcode. Approximately 25 addresses (houses, offices, etc.) share the same postcode (PostNL, n.d.). The organisation of the Postcode Loterij encourages you to buy a lottery ticket by pointing out that if you do not participate, all of your neighbours will win lots of money and you will be the only one left out.

Table 3.1), where subjects do not follow maximum expected utility in Problem 3, but do when they are presented with Problem 4. There is a reverse common ratio effect as well, here a negative pay-off can be the result of the offered gambles. This can be seen in comparing Problems 3′ and 4′. Loomes and Sugden (1982) formally describe this as follows:

**Definition 3.1** (The common ratio effect)**.** Let $X_i = (x_1, \lambda p)$ and $X_k = (x_2, p)$ be independent lotteries, where $0 < p \leq 1$ and $0 < \lambda < 1$. If there exists some probability $\bar{p}$ such that $X_i \sim X_k$ when $p = \bar{p}$, then *(i) (the common ratio effect)* if $0 < x_2 < x_1$, then $p < \bar{p} \Rightarrow X_i \succ X_k$ and $p > \bar{p} \Rightarrow X_i \prec X_k$ and *(ii) (the reverse common ratio effect)* if $x_1 < x_2 < 0$, then $p < \bar{p} \Rightarrow X_i \prec X_k$ and $p > \bar{p} \Rightarrow X_i \succ X_k$.

Here two situations can be compared when $x_1$, $x_2$, and $\lambda$ are the same in both situations, but $p$ is different. For Problems 3 and 4, this means that $x_1 = 4000$, $x_2 = 3000$, and $\lambda = 0.8$. Then $p = 1.0$ in Problem 3 and $p = 0.25$ in Problem 4. The common ratio effect occurs when some probability $\bar{p}$ can be found, this is the case when $0.25 < \bar{p} < 1.0$ in comparing Problems 3 and 4.

The common consequences effect occurs in Allais Paradox (Example 3.1 and Problems 1 and 2 in Table 3.1). Prizes and probabilities are the same in the two situations, but some probability modifier $\alpha$ varies between both situations. The following prediction (Loomes & Sugden, 1982) describes this formally:

**Definition 3.2** (The common consequences effect)**.** Let $X_i = (x_1, p_1; x_2, \alpha)$ and $X_k = (x_2, p_2 + \alpha)$ be independent lotteries where $0 < p_1 < p_2 \leq 1$ and $0 \leq \alpha \leq (1 - p_2)$. If there exists some probability $\bar{\alpha}$ such that $X_i \sim X_k$ when $\alpha = \bar{\alpha}$, then *(i) (the common consequences effect)* if $0 < x_2 < x_1$, then $\alpha < \bar{\alpha} \Rightarrow X_i \succ X_k$ and $\alpha > \bar{\alpha} \Rightarrow X_i \prec X_k$ and *(ii) (the reverse common consequences effect)* if $x_1 < x_2 < 0$ then $\alpha < \bar{\alpha} \Rightarrow X_i \prec X_k$ and $\alpha > \bar{\alpha} \Rightarrow X_i \succ X_k$.

Two situations can be compared when $x_1$, $x_2$, $p_1$, and $p_2$ are the same in both cases, but $\alpha$ is different. For example in comparing Problems 1 and 2, we have $x_1 = 2500$, $x_2 = 2400$, $p_1 = 0.33$, and $p_2 = 0.34$. Then in Problem 1 $\alpha = 0.66$, while in Problem 2 $\alpha = 0$. This means that we can find some probability $\bar{\alpha}$, namely when $0 < \bar{\alpha} < 0.66$, thus comparing Problems 1 and 2 show the common consequences effect.

The isolation effect happens in two-stage gambles. The subject chooses between two gambles where one leads to a different gamble and the other to a sure prize. Two-stage gambles (Problem 10) lead to different decisions than one-stage gambles with the same prizes and results (Problem 4). One might think that the isolation effect exists because people generally find it difficult to compute probabilities and a two-stage gamble makes this even harder. Instead, it can be argued that the outcomes of $X_{17}$ and $X_{18}$ are not statistically independent (Loomes & Sugden, 1982). This still leads to the same analysis in expected utility in comparison to Problem 4, but regret theory gives different results, which are in line with the observed human behaviour.

After identifying these main violations of expected utility that are observed in human behaviour, the alternative strategy can be formulated. Regret theory allows these three violations and is in line with the experimental data.

## 3.4 How to calculate regret

To model human behaviour in decision making, Loomes and Sugden (1982) propose a framework that is based on expected utility but incorporates regret and reasoning about counterfactuals. The expected utility of an action $A_i$ can be calculated with

$$E_i = \sum_{j=1}^{n} p_j u(x_{ij})$$

where the state of the world $j$ has a probability $p_j$ which is multiplied by the utility $u(x)$ of pay-off $x$. This formula only considers the outcome of the chosen action, not of rejected actions. In regret theory, the counterfactuals play an important role in the feeling of regret. Loomes and Sugden (1982) transform the utility function $u(x)$ into a choiceless utility function $C(x) = c$, where $c$ is the value the agent would assign to getting pay-off $x$ without having chosen it. Bell (1982) uses a value function $v(x)$ with the same purpose. A difference between \$1000 won and \$2000 that you could have won feels larger than the difference between \$1,000,000 won and \$1,001,000 that you could have won even though the monetary difference is \$1000 in both cases. An increasing concave function is often used as a value function (Bernoulli, 1954). We will use the notation of the choiceless utility $c$ instead of the value function $v(x)$ here. All following definitions in this section are from Loomes and Sugden (1982) unless stated otherwise. To include counterfactuals and values, modified utility is defined as

$$m_{ij}^k = M(c_{ij}, c_{kj})$$

where $m_{ij}^k$ is the modified utility of getting $c_{ij}$ while having rejected the possibility of getting $c_{kj}$. Expected modified utility can then be calculated as follows

$$E_i^k = \sum_{j=1}^{n} p_j m_{ij}^k$$

Next, the regret-rejoice function $R$ is defined as the difference between 'what is' and 'what could have been' and is part of the modified utility function

$$m_{ij}^k = c_{ij} + R(c_{ij} - c_{kj})$$

It is assumed that $R(0) = 0$ and that $R(x)$ is non-decreasing and three times differential. An agent will have a weak preference $A_i \succcurlyeq A_k$ if and only if

$$\sum_{j=1}^{n} p_j[c_{ij} - c_{kj} + R(c_{ij} - c_{kj}) - R(c_{kj} - c_{ij})] \geq 0$$

Next, a function $Q(x)$ is introduced for convenience

$$Q(\xi) = \xi + R(\xi) - R(-\xi)$$

where $\xi$ is the difference $c_{ij} - c_{kj}$ between the pay-offs of two choices. So the preference $A_i \succcurlyeq A_k$ holds if and only if

$$\sum_{j=1}^{n} p_j[Q(c_{ij} - c_{kj})] \geq 0$$

This $Q(x)$ is an increasing function with the symmetry $Q(\xi) = -Q(-\xi)$. After defining these formulas, Loomes and Sugden (1982) argue that $Q(\xi)$ must be concave for all positive values of $\xi$ to be consistent with the empirical data. Unfortunately, there is no formula given in which you can plug in some numbers and then gives the answer. The function $Q$ is only described as having a certain shape.

One of the implications of regret theory, is that multiple pairwise choices can be intransitive. This violates the transitivity axiom of von Neumann and Morgenstern (1947). It can be argued that intransitivity should not be considered to be irrational (Loomes & Sugden, 1982). One objection against intransitivity is that if three pairwise compared lotteries have intransitive preferences, it is not possible to choose a lottery from the set of all three options. Another objection is that one might "get locked into an endless chain of choices" (Loomes & Sugden, 1982, p. 821). Both these arguments are supposedly based on false assumptions and further research on *action weights* is proposed by Loomes and Sugden (1982).

# 4 STIT Logic

For the formalisation of the moral luck examples, probabilistic XSTIT will be used. This is one of the many variants of STIT logic, which belong to the class of logic of action. This class includes dynamic logic as well (Segerberg, Meyer & Kracht, 2016). STIT logic has a robust conceptual and philosophical basis and allows reasoning about what agents *actually do*, instead of being able to express only what agents *can do* such as in alternating-time temporal logic and coalition logic (Lorini & Schwarzentruber, 2011). The acronym STIT stands for “sees to it that” where an agent or a group of agents *see to it that* some proposition will be true. The foundations of this logic are captured by Belnap, Perloff and Xu (2001). An axiomatisation was given by Xu (1998). Later, one of the most important books on STIT logic was written by Horty (2001). Here we use the operator $[\alpha \ \texttt{xstit}^{\geq c}]\varphi$ where agent $\alpha$ sees to it that proposition $\varphi$ will be true in the next time step with chance of success $c$. Other possible STIT operators include Chellas’s STIT, deliberative STIT, and achievement STIT (Xu, 2015). This thesis uses the probabilistic XSTIT logic developed by Broersen (2008), since the probabilities will later be necessary in our extension with regret theory.

## 4.1 Probabilistic XSTIT

The following definitions are taken from Broersen (2014) and Doriot and Broersen (2016). Only the relevant definitions are repeated here. The syntax of the logic is defined by a language.

**Definition 4.1** (Language)**.** Given a countable set of propositions $P$, a finite set $\mathcal{A}$ of agent names and a set of real numbers $C \subseteq [0, 1]$, the formal language $\mathcal{L}$ is:

$$\varphi := p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \Box\varphi \mid X\varphi \mid [\alpha \ \texttt{xstit}^{\geq c}]\varphi$$

where $p \in P$, $\alpha \in \mathcal{A}$ and $c \in C$. The propositional connectives are as usual. The modal operator $\Box\varphi$ expresses historical necessity and $\neg\Box\neg\varphi$ is abbreviated by $\Diamond\varphi$. $X\varphi$ is interpreted as the transition to a next moment. The operator

$[\alpha \; \texttt{xstit}^{\geq c}]\varphi$ means that 'agent $\alpha$ sees to it that $\varphi$ in the next moment with a probability of at least $c$'. The branching time structure is made up of moments, and histories that pass through these moments. Multiple moments can be part of the same time step, or belong to different time steps. When multiple moments have the same predecessor, they belong to the same time step.

**Definition 4.2** (History)**.** Given $M$ a non-empty set of moments, $H$ is a non-empty set of histories. A history $h \in H$ is a sequence $..., m, m', m'', ...$ of mutually different elements from $M$. $H_m = \{h \in H \mid m \in h\}$ is the set of histories through $m$. We define two functions *succ* and *prec* by: $m' = succ(m, h)$ iff $m'$ succeeds $m$ on the history $h$, and $m = prec(m', h)$ iff $m$ precedes $m'$ on the history $h$. We have the following constraints on the set $H$ of histories:
**(One successor)** $\forall m, h$, if $m \in h$ then $\exists! m' = succ(m, h)$
**(Definition predecessor)** $m = prec(m', h)$ if and only if $m' = succ(m, h)$
**(One predecessor)** $\forall m, h$, if $m \in h$ then $\exists! m' = prec(m, h)$
**(One past)** $\forall m, \forall h, h' \in H_m$, $prec(m, h) = prec(m, h')$

Histories are sequences of moments. Along a history $h$, a moment $m$ can have only one successor $succ(m, h)$ and one predecessor $prec(m, h)$. A moment $m$ along different histories $h, h'$ has only one predecessor, which is expressed by 'one past'.

**Definition 4.3** (Choices)**.** $Choices\colon M \times \mathcal{A} \to \wp(\wp(H))$ is the choices function yielding for an agent $\alpha$ in a moment $m$ the set $Choices(m, \alpha)$ of subsets of $H_m$ containing the agent's choices. We have the following constraints on $Choices(m, \alpha)$:
**(No empty choice)** $\forall K \in Choices(m, \alpha),\; K \neq \emptyset$
**(No absence of choice)** $\forall h \in H_m,\; \exists K \in Choices(m, \alpha)$ such that $h \in K$
**(No choice between undivided histories)** $\forall h, h'$, if $succ(m, h) = succ(m, h')$ then $\forall K \in Choices(m, \alpha)$, if $h \in K$ then $h' \in K$
**(Independence of agency)** $\forall \alpha_i \in \mathcal{A}, \forall K_i \in Choices(m, \alpha_i), \bigcap_{\alpha_i \in \mathcal{A}} K_i \neq \emptyset$
The choice made by agent $\alpha$ in a moment $m$ relative to the history $h$ is given by:
$Choice(m, \alpha, h) = \{\bigcup_i K_i \mid K_i \in Choices(m, \alpha) \text{ and } h \in K_i\}$

The set of all choices $Choices(m, \alpha)$ available to agent $\alpha$ at moment $m$ contains at least one choice $K$, defined as a set of histories. 'No choice between undivided histories' and 'independence of agency' are the well-known STIT conditions as usual. The definition for choices of individual agents can be extended to choices for groups of agents.

**Definition 4.4** (Group choices)**.** $Choices_G\colon M \times \wp(\mathcal{A}) \to \wp(\wp(H))$ is the group choice function yielding for a group of agents $A$ in a moment $m$ the set of subsets of $H_m$ containing the group's combined choices: $K \in Choices_G(m, A)$ iff $\exists(K_{\alpha_1}, ..., K_{\alpha_k}) \in \times_{\alpha_i \in A} Choices(m, \alpha_i)$ where $K = \cap K_{\alpha_i}$. The choice made by the group of agents $A$ in a moment $m$ along the history $h$ is given by:
$Choice_G(m, A, h) = \{\bigcup_i K_i \mid K_i \in Choices_G(m, A) \text{ and } h \in K_i\}$

These choices by groups of agents will be used to define the state $S$ of an agent $\alpha$ at moment $m$. These are the choices for all agents other than $\alpha$.

**Definition 4.5** (States)**.** The states of an agent $\alpha$ at a moment $m$ is defined by: $States(m, \alpha) = Choices_G(m, \mathcal{A} \setminus \{\alpha\})$

We will use $K$ for a choice of agent $\alpha$ and $S$ for a state of agent $\alpha$, where states are the combined choices of all the other agents. The expectation function defines a probability distribution over states. An agent $\alpha$ at moment $m$ assigns probabilities to the choices of other agents (states).

**Definition 4.6** (Expectation)**.** The expectation function $B\colon M \times \mathcal{A} \times \wp(H) \to C$ is a subjective probability function such that $B(m, \alpha, S)$ expresses agent $\alpha$'s belief that he will be in state $S$ at moment $m$. We apply the following constraints:

1. $B(m, \alpha, S) \geq 0$ if $S \in States(m, \alpha)$
2. $B(m, \alpha, S) = 0$ otherwise
3. $\sum_{S \in States(m,\alpha)} B(m, \alpha, S) = 1$

Subjective probabilities are in the range $[0, 1]$ and non-zero probabilities can only be assigned to states. The probabilities of all states for an agent $\alpha$ at a moment $m$ should add up to 1. The above definitions can then be used to define a probabilistic XSTIT-frame and -model.

**Definition 4.7** (Probabilistic XSTIT-frame)**.** A probabilistic XSTIT-frame is a tuple $\mathcal{F} = \langle M, H, Choices, B \rangle$ such that:

1. $M$ is a non-empty set of moments
2. $H$ is a non-empty set of histories
3. $Choices\colon M \times \mathcal{A} \to \wp(\wp(H))$ is a choice function
4. $B\colon M \times \mathcal{A} \times \wp(H) \to C$ is an expectation function

**Definition 4.8** (Probabilistic XSTIT-model)**.** A probabilistic XSTIT-frame is extended to a model $\mathcal{M} = \langle M, H, Choices, B, V \rangle$ by adding a valuation $V$ of atomic propositions $V\colon P \to \wp(M)$ assigning to each atomic proposition the set of moments relative to which they are true.

An example of a probabilistic XSTIT-model can be seen in Figure 4.1. The squares represent moments, columns can be seen as choices of agent $\alpha$ and rows as choices of agent $\beta$ (or states of agent $\alpha$). The numbers along the rows are the probabilities agent $\alpha$ assigned to these states. Symmetrically, the numbers along the columns represent the probabilities that agent $\beta$ assigns to the choices of agent $\alpha$. The dashed lines going through the moments are histories grouped in bundles. Propositions such as $\varphi$ can be true in a moment and are therefore written in the square. Moments from different time steps are grouped into rectangles where the lower rectangles are earlier in time than the ones above them.

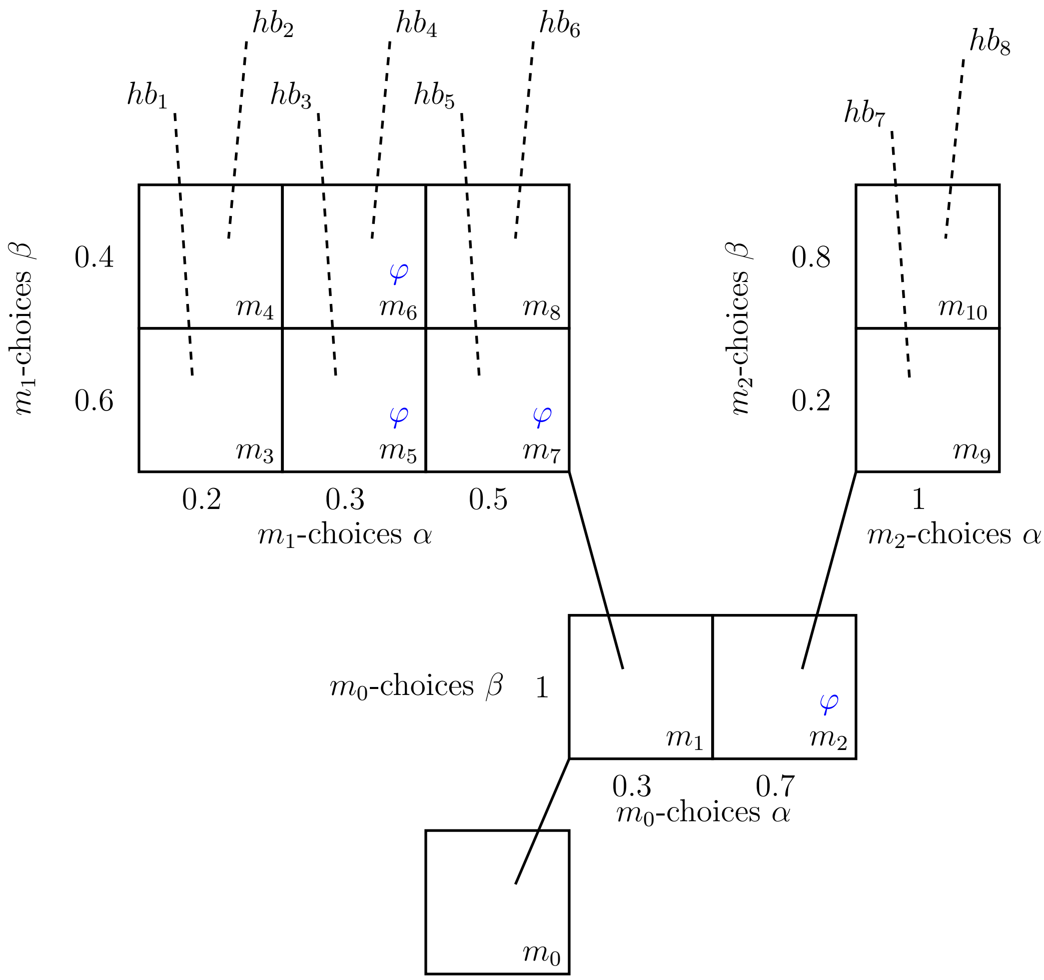


Figure 4.1: Visualisation of a partial probabilistic XSTIT-model with two agents.

**Definition 4.9** (Possible next $\varphi$-states)**.** The possible next $\varphi$-states function $PosX\colon M \times H \times \mathcal{A} \times \mathcal{L}$ which for a moment $m$, a history $h$, an agent $\alpha$, and a formula $\varphi$ gives the possible next states obeying $\varphi$ given the agent's current choice determined by $h$, and is defined by: $PosX(m, h, \alpha, \varphi) = \{S \in States(m, \alpha) \mid \forall h' \in S \cap Choice(m, \alpha, h), \langle succ(m, h'), h' \rangle \models \varphi\}$

Given an agent $\alpha$, a history $h$ indicating the agent's choice, a moment $m$, and a proposition $\varphi$, the function $PosX$ gives the states in the next time step in which

$\varphi$ holds. This can then be used to define the chance of success $CoS$, which is the sum of the probabilities of all possible next $\varphi$-states.

**Definition 4.10** (Chance of success)**.** The chance of success function $CoS\colon M \times H \times \mathcal{A} \times \mathcal{L} \to C$ gives the chance that the agent's choice in moment $m$ relative to $h$ is an action resulting in $\varphi$. This chance of success is defined by:

$$CoS(m, h, \alpha, \varphi) = \sum_{S \in PosX(m,h,\alpha,\varphi)} B(m, \alpha, S)$$

The above definitions can then be used to formulate evaluation rules for the truth values of formulas.

**Definition 4.11.** Relative to a model $\mathcal{M} = \langle M, H, Choices, B, V\rangle$, truth of a formula in a dynamic state $\langle m, h\rangle$, with $m \in h$, is defined as:

$$\begin{array}{lll}
\langle m,h\rangle \models p & \text{iff} & m \in V(p) \\
\langle m,h\rangle \models \neg\varphi & \text{iff} & \text{not } \langle m,h\rangle \models \varphi \\
\langle m,h\rangle \models \varphi \wedge \psi & \text{iff} & \langle m,h\rangle \models \varphi \text{ and } \langle m,h\rangle \models \psi \\
\langle m,h\rangle \models \Box\varphi & \text{iff} & \forall h' \in H_m, \langle m,h'\rangle \models \varphi \\
\langle m,h\rangle \models X\varphi & \text{iff} & \langle succ(m,h), h\rangle \models \varphi \\
\langle m,h\rangle \models [\alpha\ \texttt{xstit}^{\geq c}]\varphi & \text{iff} & CoS(m,h,\alpha,\varphi) \geq c
\end{array}$$

With the help of the example in Figure 4.1, we can evaluate some formulas. Relative to moment $m_1$, agent $\alpha$ has three choices, while agent $\beta$ has two choices. Agent $\alpha$ assigns a probability of 0.4 to the top state, and a probability of 0.6 to the bottom state. Agent $\beta$ thinks that agent $\alpha$ will pick the left choice with probability 0.2, the middle one with probability 0.3, and the right one with probability 0.5. Relative to moment $m_1$ and history $h_3$, agent $\alpha$ ensures that $\varphi$ holds in the next moment, since $\varphi$ is true in both moment $m_5$ and moment $m_6$. Therefore, we can say that $\langle m_1, h_3\rangle \models [\alpha\ \texttt{xstit}^{\geq 1}]\varphi$. In contrast, relative to moment $m_1$ and history $h_3$, agent $\beta$ does not ensure that $\varphi$ holds in the next moment, because it is not true in moment $m_3$. Agent $\beta$ believes that this left state has a chance of 0.2 of happening. The chance of success for the bottom choice is $0.3 + 0.5 = 0.8$. It is then the case that $\langle m_1, h_3\rangle \models [\beta\ \texttt{xstit}^{\geq 0.8}]\varphi$.

The logic described in this chapter will be extended with maximum expected utility and regret theory in the following chapter.

# 5 Regret Theory and STIT Logic

Now that regret theory and probabilistic XSTIT logic have been introduced, we can attempt to combine them. Recall the first research question of this thesis: How can maximum expected utility and regret theory be incorporated into probabilistic XSTIT logic? The reason that we want this, is because STIT logics are sometimes used for modelling decisions to perform an action and not just the action itself (Bartha, 2014). STIT logic has not been extended with regret theory yet, but there have been extensions with maximum expected utility (Bartha, 2014), which was not done in the probabilistic XSTIT we use here. Bartha (2014, p. 31) says that he is "concerned not with a probabilistic version of *stit* but with the importance of probabilities for norms of choice." Therefore, new definitions for adding maximum expected utility to probabilistic XSTIT will be proposed. Since maximum expected utility is not in line with actual human behaviour, it is interesting to add regret to STIT logic. The goal of this extension is to be able to better model human decision making behaviour in STIT logic. To actually add regret theory to probabilistic XSTIT, we first need to consider how we can model the examples from regret theory in this logic.

## 5.1 Representation of Games

### 5.1.1 Pay-off and Value

The examples from regret theory are related to lotteries where the probabilities and outcomes are known. Probabilities from the regret examples can easily be included in probabilistic XSTIT. Next, we need a way to add the jackpot of a lottery to the logic as well. Horty (2001) uses so-called values that are associated to histories, but "It says nothing about what is ultimately taken as a measure of an individual agent's utility pleasure, mental states of intrinsic worth, happiness, money, an index of basic goods" (Horty, 2001, p. 38). The prize of a lottery may have a different psychological value to the agent than the monetary value, as we have seen in Section 3.2. Thus, a pay-off of \$3000 will have a value of $Value(3000)$.

This can be seen as a function that makes mental states, such as happiness, and monetary values comparable in a numerical way. In this extension of probabilistic XSTIT, the pay-off and value will both be associated to moments, rather than histories. This choice is made, because the pay-off is the result in a certain time step and not of an entire history. This is done by Kooi and Tamminga (2006) as well. Events in the future or in the past may change how much money the agent actually has. Here, only the outcomes of the current decision will be considered to compare the local choices of an agent.

**Definition 5.1** (Pay-off)**.** The payoff function *Pay-off*: $M \to \mathbb{R}$ assigns real numbers to moments in $M$, such that *Pay-off(m)* gives the pay-off in moment $m$.

**Definition 5.2** (Value)**.** The value function $Value\colon \mathbb{R} \to \mathbb{R}$ transforms the pay-off, such that $Value(\textit{Pay-off}(m))$ gives the value of the pay-off in moment $m$.

This is in contrast to Horty (2001) and Bartha (2014), who consider the global "goodness" of choices, the preference of certain histories instead of moments. We will slightly abuse the notation and write $Value(m)$ instead of $Value(\textit{Pay-off}(m))$ to keep the notation simple. The $Value(m)$ function here is similar to the value function $v(x)$ (Bell, 1982) and the choiceless utility function $C(x) = c$ (Loomes & Sugden, 1982), where $x$ is the pay-off in both cases. In some situations, it is possible that there is no money involved and the pay-off cannot be expressed as a specific amount of dollars. It is then possible to give a numerical value to a moment, such that it expresses how good or bad that moment is to the decision making agent. Intuitively, it might seem that value should not be associated to a specific moment only, since money can lose its value over time. For simplicity, we will assume that this is incorporated in the value function itself. This means that the value of the same pay-off can be different for different moments in time. It can be assumed that all moments have some value, but values will only be specified when relevant.

### 5.1.2 Propositions to Describe Choices

Propositions will be used to describe moments in the next time step (Broersen, 2008), this again slightly deviates from Horty (2001), where propositions depend on moment-history pairs and the effect of an action is instantaneous. To describe the choices made by the agent and by *nature*, propositions such as $Win$, $Gamble$, and $Sure$ can be used. Here, we will only use propositions to describe choices of the decision making agent, and not for the choices of *nature*. In decision theory, there is only one decision making agent which is not able to influence the choice by *nature*. Therefore, it is not necessary to describe *nature*'s choice with propositions as well. These propositions describing the agent's choice will later be used in the definition of two ought operators: one for maximum expected utility and one for regret theory.

### 5.1.3 Game Forms

In the experiment conducted by Kahneman and Tversky (1979), the subjects were asked to fill in a questionnaire where they had to answer which of two gambles (or a gamble and a sure prize) they preferred. The outcomes of the gambles were not presented to the subjects. Bell (1982) assumes that these problems all have fully resolved outcomes, this information influences the decision making process beforehand. For now, assume that an agent looks at a computer screen where the problem is displayed, as soon as he presses the button that indicates his choice of preference, the outcomes of all involved lotteries appear on the screen. This can be seen as a simultaneous choice of the agent and *nature*. In game theory, these simultaneous choices are formalised in a normal form game such as in Table 5.1, which shows the normal form game of Problem 3 (Loomes & Sugden, 1982).

| Nature's choice | | Agent's choice: Gamble | Agent's choice: Sure |
|---|---|---|---|
| | Win 0.8 | 4000 | 3000 |
| | Lose 0.2 | 0 | 3000 |

Table 5.1: Problem 3 in normal form.

Table 5.1 shows "Gamble" as lottery $X_5$ and "Sure" as lottery $X_6$, "Win" means that lottery $X_5$ leads to the highest pay-off, while "Lose" means that lottery $X_5$ leads to the lowest pay-off. A normal form game can easily be translated into a STIT-model, which will be done in Section 5.4. However, in real life examples such as the Dutch Postcode Loterij, the outcome of the lottery will be known some time after the agent has made its decision to buy a lottery ticket or not. Since the decision of the agent and the decision of *nature* are not made at the exact same time here, this could be formalised with an extensive form game. In this thesis, we will normalise extensive form games to normal form games for simplicity and to ensure the possibility of reasoning about counterfactuals. This is only possible when the choices for the involved agents (including *nature*) are independent of each other. For example, the outcome of the Postcode Loterij is independent of your choice of buying a ticket or not. This is the case in the regret theory examples as well.

In Table 5.2, Problem 4 is shown as a normal form game. Each state of the world has the outcome of both lotteries, because fully resolved outcomes are assumed. The lotteries are independent, so the probabilities of the two outcomes are multiplied. Here, winning a lottery means that the highest pay-off is the case, when you chose that lottery. For example, winning lottery $X_9$ leads to the proposition $W_9$ and has a probability of 0.20, while winning lottery $X_{10}$ leads to the proposition $W_{10}$ and has a probability of 0.25, therefore winning both lotteries $W_9 \wedge W_{10}$ has a probability of $0.20 \times 0.25 = 0.05$. Here, winning does not mean that the agent will receive the jackpot of both lotteries, but rather that he would have won if he chose that particular lottery.

| Nature's choice | Agent's choice: $X_9$ | Agent's choice: $X_{10}$ |
|---|---|---|
| $W_9 \wedge W_{10}$ 0.05 | 4000 | 3000 |
| $W_9 \wedge \neg W_{10}$ 0.15 | 4000 | 0 |
| $\neg W_9 \wedge W_{10}$ 0.20 | 0 | 3000 |
| $\neg W_9 \wedge \neg W_{10}$ 0.60 | 0 | 0 |

Table 5.2: Problem 4 in normal form.

Since we can normalise an extensive form game, we can model them as a single time step in STIT logic, as has been done by Kooi and Tamminga (2006, 2008). Bentzen (2010) proves that Horty's (2001) utilitarian STIT can be reduced to a single time step. Then, the choices of the agent and *nature* are independent and it is possible to reason about counterfactuals. There are other ways to formalise these kinds of examples in STIT logic, in which the extensive form game is not reduced to a single moment in time (Horty, 2001, pp.91-95). For the current examples, the reduction to a single moment is simpler and more intuitive than Horty's (2001) solution.

## 5.2 Maximum Expected Utility

Related to the current extension of probabilistic XSTIT with regret theory, is the extension by Bartha (2014), where maximum expected utility is added to Horty's

(2001) utilitarian STIT logic. The main difference between these two is that the goal of Horty (2001) was to reason about uncertainty, rather than risk. He therefore did not use probabilities, while Bartha (2014) did add probabilities to STIT logic to be able to calculate the *expected* utilities of choices. Since probabilistic XSTIT is used in this thesis, it is not possible to simply copy the definitions Bartha (2014) gave for the preference ordering and ought operator based on maximum expected utility. Instead, two new definitions are proposed. Here a moment $m$ is an element of the intersection of a choice $K \in Choices(m, \alpha)$ and a state $S \in States(m, \alpha)$, while Bartha (2014) adds the outcome $O$ to this intersection. This is done, because he does not assume "uniform utilities" for the conjunction of a choice and a state. In other words, if an agent decides to gamble and nature decides to let him win, the prize won can be $4000, but might just as well be higher or lower instead, depending on future events. Since the pay-off is associated to a moment here, instead of a history (Bartha, 2014), there can be only one moment $m$ in the intersection $K \cap S$ and the outcome $O$ does not have to be included. The expectation function $B(m_0, \alpha, S_j)$ gives the probability of a state $S_j$ in the next time step after moment $m_0$ for an agent $\alpha$. This probability will be multiplied with the value of the moment $m_{ij}$ that is the result of choice $K_i$ and state $S_j$. The sum over all states $S_j$ is then used to calculate the expected utility of a choice $K_i$. This will be compared to the expected utility of a choice $K_k$ with moment $m_{kj}$ to find the preference based on expected utility, where $K_i \succcurlyeq_e K_k$ if and only if

$$\sum_{j=1}^{n} B(m_0, \alpha, S_j) \times Value(m_{ij}) \geq \sum_{j=1}^{n} B(m_0, \alpha, S_j) \times Value(m_{kj})$$

Which can be simplified as follows

$$\sum_{j=1}^{n} B(m_0, \alpha, S_j) \times Value(m_{ij}) - \sum_{j=1}^{n} B(m_0, \alpha, S_j) \times Value(m_{kj}) \geq 0$$

$$\sum_{j=1}^{n} B(m_0, \alpha, S_j) \times Value(m_{ij}) - B(m_0, \alpha, S_j) \times Value(m_{kj}) \geq 0$$

$$\sum_{j=1}^{n} B(m_0, \alpha, S_j) \times (Value(m_{ij}) - Value(m_{kj})) \geq 0$$

This formula can then be used to define the preference ordering on the choices available to the agent based on the expected utility.

**Definition 5.3.** Expected utility preference ordering ($\succcurlyeq_e$ and $\succ_e$) on $Choices(m_0, \alpha)$. If $K_i$ and $K_k$ are members of $Choices(m_0, \alpha)$ and $S_j \in States(m_0, \alpha)$, then

1. $K_i \succcurlyeq_e K_k$ if and only if for all moments $m_{ij} \in K_i \cap S_j$ and $m_{kj} \in K_k \cap S_j$, it is the case that $\sum_{j=1}^{n} B(m_0, \alpha, S_j) \times (Value(m_{ij}) - Value(m_{kj})) \geq 0$; and

2. $K_i \succ_e K_k$ if $K_i \succcurlyeq_e K_k$ but not $K_i \preccurlyeq_e K_k$.

Bartha's (2014) dominating expectation ought operator uses Chellas's STIT operator, but since we ordered choices for next states, $[\alpha\ \texttt{xstit}^{\geq c}]\varphi$ will be used here. Another difference is that Horty (2001) and Bartha (2014) see the ought operator as something that tells the agent what he must do in general, over the entire histories. In the current work, only local choices and outcomes are considered, the goal is to see what the agent should choose *now*. Therefore, the ought operator will not be defined over all histories, but only for the current moments on the basis of the expected utility preference ordering.

**Definition 5.4.** Maximum expected utility ought operator $\bigcirc_e$.
$\langle m, h\rangle \models \bigcirc_e[\alpha\ \texttt{xstit}^{\geq c}]\varphi$ if and only if there is a choice $K' \in Choices(m, \alpha)$ such that

1. $\forall h' \in K'$ we have $\langle m, h'\rangle \models [\alpha\ \texttt{xstit}^{\geq c}]\varphi$, and

2. $\forall h'' \in K''$ belonging to $Choices(m, \alpha)$ it is the case that $K' \succcurlyeq_e K''$.

This ought operator $\bigcirc_e$ will be used to tell which choice is the most sensible for the agent in its current context. This means that it should not be interpreted as a strict law or obligation to act in a certain way. It only applies to actions and is therefore an 'ought-to-do' operator, and not an 'ought-to-be' operator. If a particular choice $K$ means that the agent gambles, the proposition *Gamble* will be true in all moments $m$ belonging to that choice $K$. The chance of success $c$ is therefore equal to 1. It is possible that $K \sim K'$ when the expected utilities of the choices are equal. In such a case, we want to say that both choices are equally sensible. This means that there is always at least one action that the agent ought to do. This is represented by the weak preference $\succcurlyeq_e$ in the definition. If the strict preference $\succ_e$ would have been used, as is done by Horty (2001), it would be possible that there is no ought-action if $K \sim K'$. In a situation where the decision making agent only has one choice available, this choice is trivially the most sensible one. For this ought operator, it is not necessary that proposition $\varphi$ holds in any of the other moments that do not belong to the preferred choice $K$. This ought operator can be used to tell which proposition belongs to the preferred action.

## 5.3 Regret Theory

Now that it is clear how a problem from regret theory should look like in a probabilistic XSTIT-frame, and maximum expected utility is defined in this logic as well, it is time to formally define the extension of regret theory in probabilistic XSTIT. The above definitions will be adapted such that the preference ordering is not based on expected utility, but on regret theory instead and that the ought operator can be used as a regret-ought.

**Definition 5.5.** Regret theory preference ordering ($\succcurlyeq_r$ and $\succ_r$) on $Choices(m_0, \alpha)$. If $K_i$ and $K_k$ are members of $Choices(m_0, \alpha)$ and $S_j \in States(m_0, \alpha)$, then

1. $K_i \succcurlyeq_r K_k$ if and only if all moments $m_{ij} \in K_i \cap S_j$ and all moments $m_{kj} \in K_k \cap S_j$, it is the case that $\sum_{j=1}^{n} B(m_0, \alpha, S_j) \times Q(Value(m_{ij}) - Value(m_{kj})) \geq 0$; and

2. $K_i \succ_r K_k$ if $K_i \succcurlyeq_r K_k$ but not $K_i \preccurlyeq_r K_k$.

The only difference between the two preference orderings of Definition 5.3 and Definition 5.6 is that the function $Q$ is added here. The shape of this function was discussed in Section 3.4. With this preference ordering based on regret theory, another ought operator can be defined.

**Definition 5.6.** Regret theory ought operator $\bigcirc_r$.
$\langle m, h \rangle \models \bigcirc_r[\alpha\ \texttt{xstit}^{\geq c}]\varphi$ if and only if there is a choice $K' \in Choices(m, \alpha)$ such that

1. $\forall h' \in K'$ we have $\langle m, h' \rangle \models [\alpha\ \texttt{xstit}^{\geq c}]\varphi$, and

2. $\forall h'' \in K''$ belonging to $Choices(m, \alpha)$ it is the case that $K' \succcurlyeq_r K''$.

Again, this ought operator considers the local choices and outcomes to prescribe what would be a sensible choice for the agent now, rather than an overall optimal choice considering the entire histories. As mentioned in Chapter 3, regret theory allows intransitivity of pairwise choices. Therefore, this ought operator $\bigcirc_r$ allows intransitivity as well.

## 5.4 Examples

With these new definitions, the extension of maximum expected utility and regret theory in probabilistic XSTIT will be evaluated with the help of two examples. In Figure 5.1, Problem 3 (Loomes & Sugden, 1982) is formalised. Two colours are used in the squares that represent moments. Red is used for the propositions where $G$ stands for gamble and its negation for the sure prize. Blue is used for the pay-offs. The agent $\alpha$ is presented with a choice between a lottery $K_5$ and a sure prize $K_6$, which correspond to the columns. These are numbered 5 and 6 to be in line with Table 3.1. The rows represent the states of the world, the choices *nature* has in moment $m_0$. The lottery gives a pay-off of \$4000 with a probability of 0.8 in state $S_1$, while the sure prize has a pay-off of \$3000.

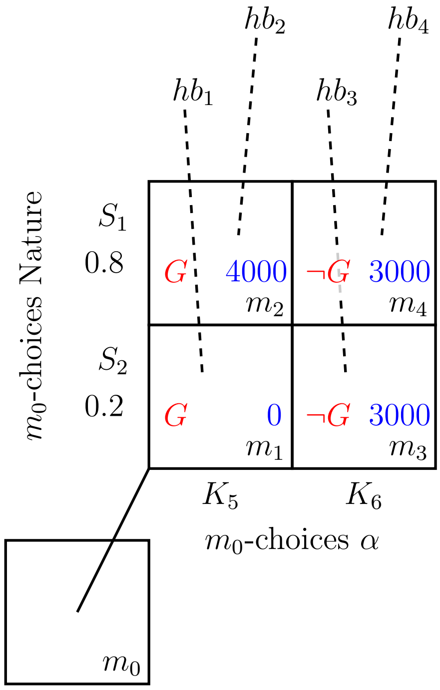


Figure 5.1: Problem 3 in the proposed XSTIT extension.

Here, we have that $\langle m_0, h_2 \rangle \models [\alpha \ \texttt{xstit}^{\geq 1}]G$, since agent $\alpha$ ensures that proposition $G$ is true in the next time step by choosing $K_5$. Similarly, it is the case that $\langle m_0, h_4 \rangle \models [\alpha \ \texttt{xstit}^{\geq 1}]\neg G$ for choice $K_6$. The expected utility of choice $K_5$ is $0.8 \times 4000 + 0.2 \times 0 = 3200$, and for choice $K_6$ it is a sure 3000. With definition 5.3, maximising expected utility would tell that $K_5 \succ_e K_6$ since $3200 > 3000$. Therefore, we have that $\langle m_0, h_2 \rangle \models \bigcirc_e[\alpha \ \texttt{xstit}^{\geq 1}]G$ and $\langle m_0, h_4 \rangle \not\models \bigcirc_e[\alpha \ \texttt{xstit}^{\geq 1}]\neg G$, since $K_5 \succ_e K_6$ and $h_2 \in K_5$ and $h_4 \in K_6$. Based on expected utility, it would be the most sensible choice for agent $\alpha$ to gamble. Similar reasoning can be done on the basis of regret theory. With definition 5.5, we have that $K_5 \prec_r K_6$. This leads to $\langle m_0, h_2 \rangle \not\models \bigcirc_r[\alpha \ \texttt{xstit}^{\geq 1}]G$ and $\langle m_0, h_4 \rangle \models \bigcirc_r[\alpha \ \texttt{xstit}^{\geq 1}]\neg G$. Thus, based on regret theory, it would be the most sensible choice to go for the sure prize. Note that both ought operators ($\bigcirc_e$ and $\bigcirc_r$) are moment determinate, which means that the truth value is equal for each history through a specific moment. In Figure 5.1, this means that $\langle m_0, h \rangle \models \bigcirc_e[\alpha \ \texttt{xstit}^{\geq 1}]G$ for all histories $h$ going through moment $m_0$. Similarly, we have that $\langle m_0, h \rangle \models \bigcirc_r[\alpha \ \texttt{xstit}^{\geq 1}]\neg G$ for all histories $h$ going through moment $m_0$.

Another example in our extension of probabilistic XSTIT can be seen in Figure

5.2, which shows Problem 4. Here, we have four choices for *nature*, each row ($S_1$, $S_2$, $S_3$, $S_4$) represents a state of the world. Since fully resolved outcomes are assumed and the two lotteries are independent, the probabilities are multiplied for each combination of winning or losing one of the two lotteries. This means that state $S_1$ consists of winning both gambles with a probability of $0.20 \times 0.25 = 0.05$. State $S_2$ means winning lottery $X_9$ but losing lottery $X_{10}$ and has a probability of $0.20 \times (1 - 0.25) = 0.15$. State $S_3$ represents losing lottery $X_9$ but winning lottery $X_{10}$ with a $(1 - 0.20) \times 0.25 = 0.20$ probability. In the last state, $S_4$, both lotteries are lost, this state has a $(1 - 0.20) \times (1 - 0.25) = 0.60$ probability.

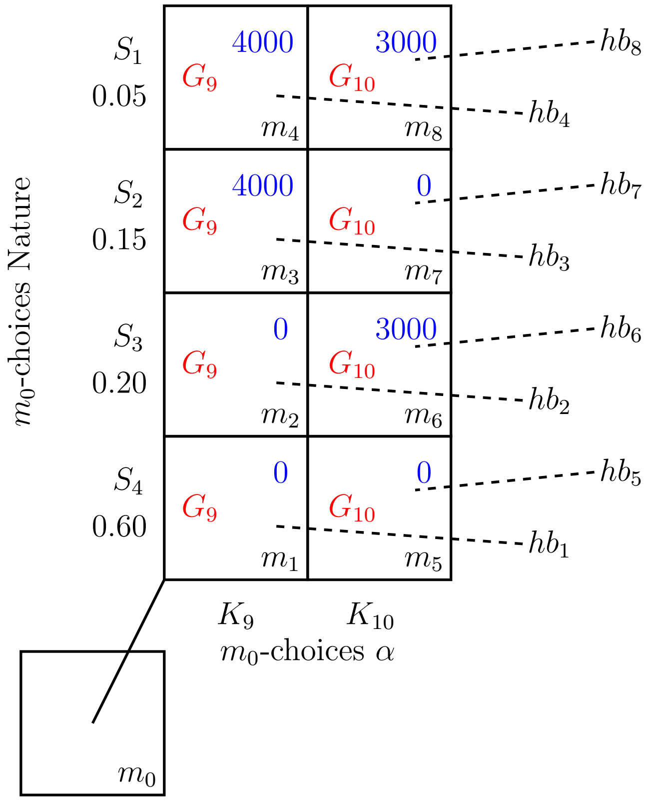


Figure 5.2: Problem 4 in the proposed XSTIT extension.

Agent $\alpha$ has two choices: $K_9$ and $K_{10}$. The propositions shown in red can be interpreted as follows: $G_9$ means that lottery $X_9 = (4000, 0.20)$ is chosen and $G_{10}$ means that lottery $X_{10} = (3000, 0.25)$ is chosen by the agent. These propositions are only written down in the moments when their valuation is true, their negations are implicit. Because we use the propositions $G_9$ and $G_{10}$ to describe the two choices available to agent $\alpha$, we have that $G_9 \leftrightarrow \neg G_{10}$, but that is not necessarily the case in all STIT-models. Again, the pay-off of each moment is written down in blue.

It is now possible to evaluate some formulas in the proposed extension. We have that $\langle m_0, h_4\rangle \models [\alpha\ \texttt{xstit}^{\geq 1}]G_9$ since agent $\alpha$ ensures that proposition $G_9$ is true by choosing $K_9$. Similarly, by choosing $K_{10}$, agent $\alpha$ ensures that proposition $G_{10}$ is true in the next moment, thus we have $\langle m_0, h_8\rangle \models [\alpha\ \texttt{xstit}^{\geq 1}]G_{10}$. The expected utility of choice $K_9$ is $0.20 \times 4000 + 0.80 \times 0 = 800$ and of choice $K_{10}$ it is $0.25 \times 3000 + 0.75 \times 0 = 750$. Therefore, $K_9$ is preferred over $K_{10}$ when the agent wants to maximise the expected utility. This means that $\langle m_0, h\rangle \models \bigcirc_e[\alpha\ \texttt{xstit}^{\geq 1}]G_9$ holds for all histories $h$ going through moment $m_0$. In other words, at moment $m_0$, the most sensible choice is $K_9$ for agent $\alpha$. At the same time, we have that $\langle m_0, h\rangle \not\models \bigcirc_e[\alpha\ \texttt{xstit}^{\geq 1}]G_{10}$, because $K_{10}$ is not the most sensible choice based on expected utility. As we have seen in Table 3.1, regret theory would result in the same preference $K_9 \succ_r K_{10}$ as we had with expected utility. This leads to $\langle m_0, h\rangle \models \bigcirc_r[\alpha\ \texttt{xstit}^{\geq 1}]G_9$ for all histories $h$ going through moment $m_0$.

In the formalisation of these two examples, propositions are used to describe choices. We have chosen to only use propositions to describe the choices of the decision making agent. It is possible to describe the choices for the other agent, in this case *nature*, with propositions as well. In that case, the truth value of additional formulas can be evaluated. In Problem 4 (Figure 5.2) for example, we could use the proposition $W_9$ for nature's choice of winning lottery $X_9$. This proposition would be true in the moments of state $S_1$ and state $S_2$, while its negation, $\neg W_9$ would be true in the moments of states $S_3$ and $S_4$. It is then possible to say that $\langle m, h_4\rangle \models \bigcirc_r[\alpha\ \texttt{xstit}^{\geq 0.20}]W_9$, since choice $K_9$ is the most sensible choice based on regret theory and agent $\alpha$ can see to it that he wins lottery $X_9$ with a probability of 0.20. This is not very informative. In this thesis, we focus on one decision making agent, instead of taking on a more game theoretic approach. Therefore, the probabilities describing *nature*'s choices are not necessary here.

# 6 Regret Theory and Moral Luck

Now that the extension of probabilistic XSTIT with maximum expected utility and regret theory has been proposed, we can formalise the two moral luck examples. The models in our framework can then help to compare regret theory with moral luck. As we have seen in Chapter 2, the feeling of regret plays a role in the examples of moral luck. In example 2.1, the lorry driver that hit the child regrets the fact that he has been negligent about checking his brakes. Suppose that there was a third lorry driver, $\gamma$, that did check his brakes regularly and everything was working properly (Nagel, 1979). Lorry driver $\gamma$ was driving correctly and was paying attention to other traffic on the road. Suddenly a person jumps in front of the lorry, the driver does not have enough time to stop before colliding with this person. Since $\gamma$ did everything correctly and was driving safely, it is not his fault that the person got hit by his lorry. In such a case, the lorry driver may still be punished on the basis of the law for the accident, but since it was not his fault, he will probably not regret anything. He cannot blame himself for not checking his brakes or any other mistake he made (Nagel, 1979). We now have a case in which there is no regret, because the morally right decision has been made. The situation of lorry driver $\gamma$ can be compared to the ones of $\alpha$ and $\beta$.

Both the lorry driver example 2.1 and the Gauguin example 2.2 show that the feeling of regret is involved with moral luck. The most straightforward similarity of moral luck and regret theory is, of course, this involvement of regret. In the moral luck examples, regret is felt afterwards and correlates to an agent being morally unlucky. On the other hand, the feeling of regret is anticipated in regret theory, it is used in the decision making process beforehand. Of course, regret is only felt after the outcomes are revealed and another choice would have been more successful. Regret theory, in contrast to moral luck, is about general decision making instead of moral decision making. The examples in Chapter 3 involve lotteries with monetary outcomes, but no morality. The relation between moral luck and regret theory can be further explored by a formalisation in probabilistic XSTIT logic. By formalising moral luck examples into the proposed framework, it is possible to see how regret theory may be involved in moral decision making. In addition, the violations of expected utility theory as discussed in Section 3.3

will be used to gain a better understanding of the decision making process in the moral luck examples.

## 6.1 Problems

To formalise the examples of moral luck, some issues need to be solved first. Four problems have been identified:

1. Probabilities are not given
2. Pay-off unknown and involves more than just money
3. Non-resolution of rejected lotteries
4. Simultaneous actions and independent resolution

Solutions will be proposed for each of these problems, all requiring certain assumptions.

The first problem is that the moral luck examples do not mention any probabilities. Broersen (2014) uses *subjective* probabilities to express the belief the agent has about the choices of other agents in an XSTIT formalisation of moral luck. The probabilities in regret theory may be interpreted as objective probabilities, or subjective ones (Loomes & Sugden, 1982). The subjective probabilities seem suitable for the current examples as well. It is then possible to choose probabilities such that they capture the idea of the moral luck examples.

The second problem consists of choosing the right pay-off for an outcome. Outcomes in regret theory usually involve money, but the outcomes of the moral luck examples are not expressed in numerical values. To be able to calculate the regret preference, pay-offs of the outcomes are necessary. Some assumptions about these pay-offs have to be made.

The third problem arises when the outcome of a rejected lottery (or choice in general) remains unknown. The simple examples of regret theory have fully resolved outcomes, which means that if the sure prize is chosen instead of the lottery (in Problem 3 of Table 3.1 for example), the outcome of the lottery is revealed as well. In real life, we often do not have fully resolved outcomes. Suppose Gauguin (example 2.2) decides to stay at home with his family instead of going to Tahiti, then he will never find out whether he would have been more successful in Tahiti or not. This makes the problem somewhat more complex to formalise, since there is less information available to the agent. It can be argued that the value of the actual pay-off can be compared to the *unresolved* foregone lottery to find the regret theory preference (Bell, 1983), but this approach is beyond the scope of this thesis. The lorry drivers example 2.1 will be formalised as a simple comparison with fully resolved outcomes.

Finally, the fourth problem involves simultaneous actions and independent lotteries. In Chapter 3, it was assumed that the choices of the agent and of *nature* were simultaneous and independent to simplify the situations. Unfortunately, that is not a suitable assumption for both of the examples of moral luck. To stay with the Gauguin example, only *after* he has decided to go to Tahiti, *nature* will make her choice to give him a successful career or let him fail. The choices are therefore not simultaneous. The choice *nature* can make, is only available if Gauguin actually goes to Tahiti. This means that the choices are not independent either. This is easily modelled in STIT logic, but calculating regret might be more problematic. This seems to be correlated to the unresolved lotteries discussed above. Again, Bell (1983) would compare the actual outcome with the unresolved foregone lottery. Luckily, the lorry drivers example 2.1 can be formalised with simultaneous and independent actions of the agents and *nature*.

## 6.2 Formalisation

Based on the problems above, some assumptions have to be made in order to formalise the examples in our probabilistic XSTIT extension. In the lorry drivers example 2.1 we will assume that there are two choices for agent $\delta$, checking the brakes of his lorry regularly represented by proposition $B$, and neglecting to check them represented by proposition $\neg B$. This means that the agents $\alpha$, $\beta$, and $\gamma$ in the example are combined into a single agent $\delta$ with two choice options. A special agent called *nature* will be used to represent the combined choices of other agents involved, such as the child (Nagel, 1979; Broersen, 2014). This is done to simplify the situation to a decision making problem for one agent, instead of a game theoretic problem for the lorry driver and the child. For *nature*, there are two choices, one where there is no child on the road, one where there is. Assumed probabilities here are 0.1 for the state of the world where the child appears, and 0.9 for the situation without a child on the road. Next, pay-offs of the possible outcomes have to be considered. We will assume a pay-off of $-1$ for not checking the brakes regularly to model the involvement of making a morally bad decision. For hitting a child with malfunctioning brakes, a pay-off of $-100$ is assumed. In the case where the brakes are working properly and hitting a child, a pay-off of $-40$ will be used, since it was not the agent's fault. Checking the brakes does cost some time and money, therefore $-10$ will be the pay-off in that situation.

As mentioned earlier, the lorry drivers example is a simple case, the choices of agent $\delta$ and *nature* are independent and simultaneous, and outcomes are fully resolved. These assumptions are used in probabilistic XSTIT-model of Figure 6.1.

The agents mentioned in the text are combined into one agent $\delta$ in the model. Here, agent $\alpha$ corresponds to moment $m_3$ and agent $\beta$ to moment $m_4$ in which they both did not check their brakes. Agent $\gamma$ corresponds to moment $m_1$ in which the agent did check his brakes regularly, but still could not avoid hitting the child that suddenly appeared on the road.

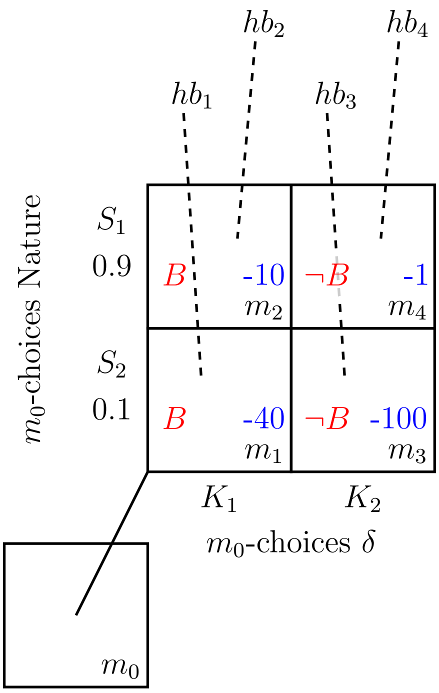


Figure 6.1: XSTIT-model of the lorry driver example.

The Gauguin example 2.2 is somewhat more complex to formalise. The agent has the choice to go to Tahiti represented by proposition $T$, or he can stay in France, which is represented by proposition $\neg T$. In the next time step, *nature* will only have a choice if Gauguin decided to go to Tahiti, then *nature* can choose to let him be successful or not. Gauguin believes that moving to Tahiti will be beneficial for his artwork, he therefore thinks that his chance of success is 0.7 when he goes to Tahiti. The pay-off is \$50 for selling his art in France, \$150 when he is successful in Tahiti, and \$0 when he fails in Tahiti. A moral pay-off of $-50$ will be used to model the fact that he leaves his family behind when he moves away. In the problems above, it was illustrated that the outcomes are not fully resolved. If Gauguin decides to stay in France, he will not be able to find out what would have happened if he did go instead. This formalisation can be seen in Figure 6.2.

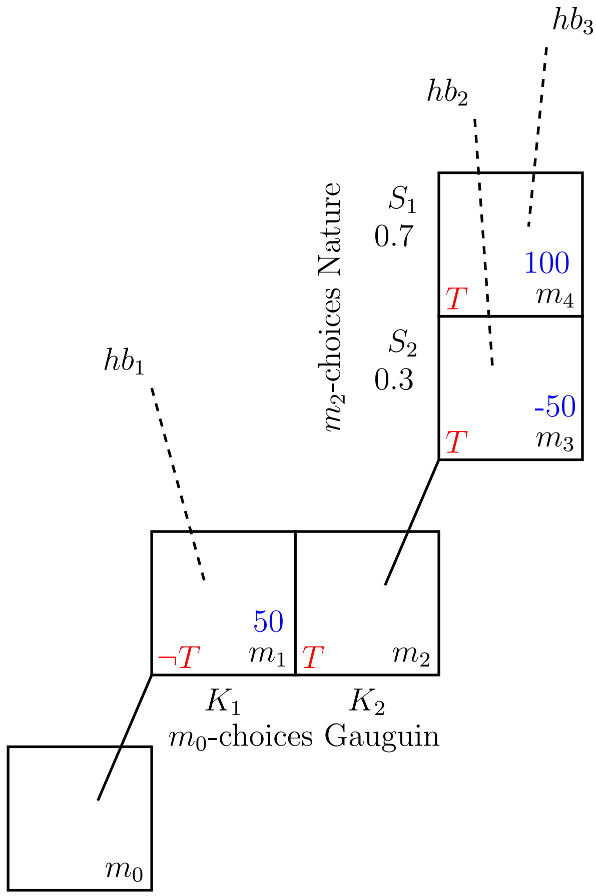


Figure 6.2: XSTIT-model of the Gauguin example.

Unfortunately, it is not possible to use this basic version of regret theory for the Gauguin example. This is due to the dependence of *nature*'s decision on the decision by the agent and the non-resolution of the rejected choice. If Gauguin stays in France, he will never find out what would have happened if he did move Tahiti. The outcome of the "lottery" is only resolved when the agent does not choose the sure prize. Bell (1983) addresses these types of more complex situations. His 'non-resolution of foregone lotteries' could be used to improve the current extension, but is beyond the scope of this thesis.

## 6.3 Evaluation

The formalisation of the two moral luck examples will be evaluated in this section. First, the truth of certain formulas will be discussed by using the proposed extension of probabilistic XSTIT with maximum expected utility and regret theory.

This shows some interesting patterns that will then be discussed to reveal part of the relationship between moral luck and regret theory. Next, we will revisit the feeling of regret and rejoice and how they occur in the examples. Finally, some limitations of the formalisations will be mentioned before moving on to the discussion in Section 7.1.

### 6.3.1 Truth of Formulas

In the above probabilistic XSTIT-models of the moral luck examples, truth of certain formulas can be evaluated. As in the problems from regret theory that were formalised in Section 5.4, we want to know which choice is the most sensible for the decision making agent. In the lorry drivers example 2.1, we have that $\langle m_0, h1\rangle \models [\delta \texttt{ xstit}^{\geq 1}]B$ and $\langle m_0, h3\rangle \models [\delta \texttt{ xstit}^{\geq 1}]\neg B$, which correspond to the two choices the agent faces. The expected utility of checking the brakes is $0.9 \times -10 + 0.1 \times -40 = -13$, while it is $0.9 \times -1 + 0.1 \times -100 = -10.9$ for not checking the brakes. Therefore, it is the case that $K_1 \prec_e K_2$ and not checking the brakes is preferred when maximising expected utility. The formula $\langle m_0, h\rangle \models \bigcirc_e[\delta \texttt{ xstit}^{\geq 1}]\neg B$ is then true for all histories $h$ going through moment $m_0$, since not checking the brakes is the most sensible choice, based on expected utility.

The function $Q(x)$ is described as having a certain shape, but Loomes and Sugden (1982) do not give a specific formula that we can use to calculate the preference. We have that $K_1 \succcurlyeq_r K_2$ if and only if

$$0.9 \times Q(-10 - -1) + 0.1 \times Q(-40 - -100) \geq 0$$
$$0.9 \times Q(-9) + 0.1 \times Q(60) \geq 0$$

but we do not know whether this is indeed the case, since the exact behaviour of the function is unknown. On the basis of the shape that Loomes and Sugden (1982) describe, it can be argued that $Q(-9)$ is negative and closer to zero than the positive result of $Q(60)$. Multiplying with the probabilities causes difficulties, because the result depends on the derivative of $Q(x)$. An agent that does not feel any regret or rejoice has an increasing linear function for $Q(x)$. When an agent does feel regret and rejoice, the function will become convex for all positive values of $x$. As the function becomes more convex, the result of $Q(60)$ will be so much further away from zero than $Q(-9)$, that even multiplying by these probabilities could not make the entire equation negative.

To find the regret theory preference, a specific function for $Q$ will be assumed to make it possible to actually plug in the numbers and calculate the outcome. For this example, it is not possible to use the definitions of the violations of expected utility, since they use just three different pay-offs ($x_1$, $x_2$ and zero) (Loomes & Sugden, 1982). The function $Q(x)$ is argued to be strictly increasing and convex for all positive values of $x$, and $Q(0) = 0$. It is symmetrical such that $Q(x) = -Q(x)$. For now, the function $Q(x) = x^3$ is assumed, which is in line with the description[1]. That way, we have that $K_1 \succcurlyeq_r K_2$ if and only if

$$\begin{gathered}
0.9 \times Q(-10 - -1) + 0.1 \times Q(-40 - -100) \geq 0 \\
0.9 \times (-10+1)^3 + 0.1 \times (-40+100)^3 \geq 0 \\
0.9 \times (-9)^3 + 0.1 \times 60^3 \geq 0 \\
0.9 \times -729 + 0.1 \times 216000 \geq 0 \\
-656.1 + 21600 \geq 0 \\
20943.0 \geq 0
\end{gathered}$$

which is indeed the case and regret theory tells us that checking the brakes is preferred over neglecting to check them. Now, the formula $\langle m_0, h\rangle \models \bigcirc_r[\delta\ \mathtt{xstit}^{\geq 1}]B$ is true for all histories $h$ that go through moment $m_0$. This represents that checking the brakes is the most sensible choice when one follows regret theory. In this example, maximising expected utility and following regret theory lead to different preferences.

Nagel (1979) points out that the agent only has moral bad luck in the case where it was negligent and this contributed to the death of the child (moment $m_3$). If the agent did check the brakes of his lorry regularly and everything was working properly, but suddenly a child appeared (moment $m_1$), then the agent would feel terrible about the accident, but it was not his fault. This situation is not a case of *moral* bad luck (Nagel, 1979). Moral good luck occurs when the agent made a morally bad decision, but things turn out fine (moment $m_4$). This seems to correspond to the assumed pay-offs. The highest pay-off possible is $-1$ in moment $m_4$ and the agent is morally lucky. The lowest pay-off is $-100$ in moment $m_3$ where the agent has moral bad luck. The middle two pay-offs are $-40$ and $-10$ in moments $m_1$ and $m_2$ respectively, in these moments, the agent made the morally right decision and there is no moral luck. Here, it seems to be the case that maximising expected utility would lead to the immoral decision, while following regret theory would result in the agent choosing the morally right decision.

Although the Gauguin example is too complex for the current extension, it is possible to see that the pay-offs correspond to the agent being morally lucky or not. As in the lorry drivers example, it is in the Gauguin example again the case that the lowest pay-off corresponds to the agent having moral bad luck. This is in moment $m_3$ where the pay-off is $-50$. Gauguin left his family behind but failed to

[1] Loomes and Sugden (1982, p. 816) assume the function $R(x) = 1 - 0.8^x$ which makes $Q(x) = x + (1 - 0.8^x) - (1 - 0.8^{-x})$ and has the proper shape as well. Our assumption of $Q(x) = x^3$ is simpler to calculate and is less steep than theirs. A steeper slope results in more extreme feelings of regret and rejoice.

become successful in Tahiti. The highest pay-off is awarded in moment $m_4$ when his paintings of Tahitian women are easily sold and Gauguin has moral good luck. The choice to go to Tahiti can be seen as the morally wrong choice, since he leaves his family behind. On the other hand, when Gauguin stays in France, he makes the morally right choice. This corresponds to moment $m_1$ with the median pay-off of 50. When staying in France, the agent does not experience any moral luck.

Suppose the Gauguin example would be simplified to a single time step, then the agent would be able to reason about counterfactuals even if he does not go to Tahiti. It is then possible to find the preference ordering over the agent's choices for both expected utility and regret theory. Maximising expected utility would lead to the preference of going to Tahiti, since $0.7 \times (50 - 100) + 0.3 \times (50 - -50) = -5$, so $K_1 \prec_e K_2$. The preference based on regret theory will be approached in two different ways. First, a specific function for $Q$ will be assumed to make it possible to actually plug in the numbers and calculate the outcome. Second, the reverse common ratio effect (Loomes & Sugden, 1982) will be used.

We will again assume $Q(x) = x^3$ to find the regret preference ordering such that $K_1 \succcurlyeq_r K_2$ if and only if

$$\begin{gathered}
0.7 \times Q(50 - 100) + 0.3 \times Q(50 - -50) \geq 0 \\
0.7 \times (50 - 100)^3 + 0.3 \times (50 - -50)^3 \geq 0 \\
0.7 \times (-50)^3 + 0.3 \times 100^3 \geq 0 \\
0.7 \times -125000 + 0.3 \times 1000000 \geq 0 \\
-87500 + 300000 \geq 0 \\
212500 \geq 0
\end{gathered}$$

which is indeed the case and we have that staying in France is preferred over going to Tahiti.

Next, the definition for the common ratio effect (Loomes & Sugden, 1982) will be used to argue the preference ordering. The simplified Gauguin example is very similar to Problem 3 in Table 3.1. In the common ratio effect, there are two pay-offs larger than zero, and one that is equal to zero. To have the same situation for Gauguin, we will add 50 to each of the pay-offs, so the lowest pay-off is zero. This way, maximum expected utility and regret theory with the above assumed function for $Q$ stay exactly the same, and in addition, we can now use the definition for the common ratio effect. The Gauguin example can be written as two independent lotteries $K_1 = (100, 1.0)$ and $K_2 = (150, 0.7)$. The two probabilities can be rewritten with $\lambda = 0.7$ and $p = 1.0$. This is now in line with Definition 3.1, where $X_i = (x_1, \lambda p)$ and $X_k = (x_2, p)$ and $0 < x_2 < x_1$. Loomes and Sugden (1982) argue that there must exist some probability $\bar{p}$. Since we have $p = 1$ it must be the case that $\bar{p} < p$, therefore $X_i \preccurlyeq_r X_k$. Translating $X_i$ and $X_k$ back to

$X_2$ and $X_1$ respectively, gives the preference $K_1 \succcurlyeq_r K_2$ as before. These two ways show that regret theory tells us that staying in France is preferred over going to Tahiti in the simplified version.

### 6.3.2 Generalisation of the results

In both the lorry drivers example and the simplified Gauguin example, maximising expected utility tells the agent to pick the morally wrong decision, while regret theory prefers the morally right decision. Of course, this depends on the assumed pay-offs and probabilities. For example, when changing the probability of the lorry driver colliding with a child from 0.1 to 0.2, both strategies prefer checking the brakes over neglecting to check them. These are choices one has to make when formalising such examples. In the philosophical texts there were no specific probabilities or pay-offs mentioned (Nagel, 1979; Williams, 1981). In order to formalise these moral luck examples and analyse them in our logic, it was necessary to make these assumptions.

Without introducing even more examples and additional figures, we do want to mention briefly that the pattern mentioned above seems to occur in two other examples as well. Nagel (1979) says that a drunk driver will experience moral bad luck when he swerves onto the pavement and hits a pedestrian. On the other hand, he may have moral good luck when there are no pedestrians on the pavement. Another example involves an attempted murder where the agent is considered morally lucky when a bird flies into the path of the bullet and the victim is not killed (Nagel, 1979). Although it is difficult to find realistic pay-offs for the attempted murder, a simplified version shows the same patterns of correspondence between moral luck and the pay-off. In these two quick analyses, it is again the case that maximising expected utility leads to preferring the morally wrong action, while regret theory would suggest to perform the morally right action.

### 6.3.3 The Feeling of Regret

Williams (1981) says that Gauguin would only wish to have acted otherwise if he was unsuccessful. He would regret his choice of going to Tahiti. The same holds for the driver that did not check his brakes and runs over the child with his lorry. He will regret his negligence and wish that he had acted differently. When an agent has moral bad luck, regret can be felt. In the examples for regret theory, we have seen that regret can be felt in other situations as well. When you do not buy a ticket for the Dutch Postcode Loterij and your postcode turns out to have won, you missed out on a lot of money and regret your earlier decision. On the other hand, if you did buy a ticket but you did not win anything, it is possible that you regret wasting your money on this ticket. Participating in a lottery does not (really) involve any morality, therefore, the feeling of regret does not imply moral luck. On the other hand, it seems that moral bad luck does somehow imply the feeling of regret (Williams, 1981). Depending on the chosen probabilities and pay-offs, regret theory seems to prefer the least risky choice and therefore avoid moral bad luck in cases where an agent could be morally blamed.

Nagel (1979) and Williams (1981) only discuss examples of resultant moral luck in which a risky and morally bad decision has been made. Although they do say that an agent can be morally praised or blamed, they do not give any examples in which we would morally praise the agent. An interesting question here is if moral good luck implies rejoice. Remember that rejoice is the opposite of regret, it is the feeling of pleasure, of happiness when something went better than if one had chosen otherwise. In situations where an agent could be morally praised, regret theory seems to prefer the most risky choice, hoping that the agent will have moral good luck. Since we do not have any convincing examples of situations where an agent can be morally praised, the relation between moral luck and the feeling of rejoice remains unknown.

### 6.3.4 Limitations

The absence of examples in which an agent is morally praised limits the possible connections we can find between moral luck and regret theory. That is not the only type of situation that is left unknown. As discussed before, we have only considered simple situations in regret theory and it would be interesting to see what more there is to find when using Bell's (1983) definitions for more complex cases of regret theory.

Another open question relates to the violations of maximum expected utility that were explained in Section 3.3. This hints at the possibility that resultant moral luck could be separated into three different cases corresponding to these violations. It is quite difficult to come up with moral choice situations that correspond with the examples from regret theory. With such examples, it would be possible to explore whether these violations still exist in moral choice situations.

# 7 Conclusion

## 7.1 Discussion

The aim of this thesis was to research the relation between moral luck and regret theory. To accomplish this, we integrated maximum expected utility and regret theory into probabilistic XSTIT logic. The resulting framework was then used to formally analyse examples of moral luck and find the relation with regret theory.

The first research question proposed in Chapter 1 is: How can maximum expected utility and regret theory be incorporated into probabilistic XSTIT logic? To achieve this, we defined the pay-off and value of a moment. This is in contrast to Horty (2001), who defines a value for a history. This choice was made to make clear that the pay-off is a result of a specific choice at a particular time. Events in the past or future may change the amount of money the agent actually has. Propositions were used to describe the choices available to the decision making agent. We did not use propositions to describe the choices of *nature*, since this would lead to uninformative formulas in our examples. The propositions are necessary to describe choices, since the base logic does not have a way to express the chosen action directly. Next, we discussed how normal form games can be translated to STIT logic.

These preparations were necessary before defining the two preference orderings and the two ought operators, one of each for maximum expected utility and for regret theory. These preference orderings and ought operators were inspired by Bartha (2014), but differ in some aspects. The preference ordering over choices is defined with the assumption that a combination of a particular choice of the agent and state of the world could only lead to a single pay-off, since pay-offs are defined on moments in the current work, instead of on entire histories. The ought-to-do operators are defined as prescribing the most sensible choice for the agent, based on either one of the preference orderings. In contrast to Horty (2001), we used a weak preference here, since we want that multiple actions can be reasonable if one is not preferred over the other.

In Section 5.4, some examples were given in the proposed extension of probabilistic XSTIT logic with maximum expected utility and regret theory. These examples are not about *moral* decision making yet, but were used to explain the observations captured by regret theory (Loomes & Sugden, 1982). The two examples have shown how the proposed extension works and which formulas can be derived in such models.

The second research question was: How do regret theory and moral luck relate to each other? To find this relation, we formalised two moral luck examples in our proposed extension of probabilistic XSTIT with maximum expected utility and regret theory. There were some complications in formalising these examples. The lorry drivers example was a simple case, since the choice for the decision making agent and for nature were independent and the outcome of the rejected choice is known after the event, which makes reasoning about counterfactuals possible. Therefore, we only had to assume probabilities for the states of the world and pay-offs for the possible outcomes. Unfortunately, the Gauguin example was somewhat more complex, because this is a case with non-resolution of rejected choices and the actions available to Gauguin and *nature* are dependent and sequential. Here, *nature* only has the choice to make Gauguin successful or not after he chooses to move to Tahiti. If Gauguin stays in France, he will never find out if he would have been successful if he had chosen otherwise. It was therefore impossible to analyse the Gauguin example in our proposed logic.

We attempted to find the relationship between moral luck and regret theory. Some connections could be found in the way pay-offs correspond to the existence of moral luck. Another connection could be found in comparing the preference orderings of expected utility and regret theory. These observations are based on just a few examples. To formalise these examples, a number of assumptions had to be made. The observed patterns depend on these assumptions for the probabilities and pay-offs. The existence of these patterns supports the reason to think that moral luck and regret theory are related in a certain way. Unfortunately, we do not believe that this is enough information to conclude that there is a strong connection between moral luck and regret theory.

## 7.2 Summary

This thesis formally analyses the relation between moral luck and regret theory. For this an extension of probabilistic XSTIT with maximum expected utility and regret theory is proposed.

Moral luck is the philosophical problem where an agent can be morally judged based on things that were beyond its control. This goes against the ethical principle that one should not be held morally responsible for factors beyond your control. In cases of moral bad luck, regret plays an important role. When an agent makes a morally wrong decision and things turn out to be unlucky, the agent will feel that this outcome could have been avoided and will regret its decision.

In computer science and economics, game theory is a widely discussed topic. The goal in game theory, is to find the optimal strategy for one agent, when its opponents plays optimally as well. Decision theory can be seen as a subset of game theory where only one agent makes a decision. This usually involves gambles and other games that are based on chance. In decision theory, maximising expected utility is often assumed to be the most rational strategy. This strategy considers the pay-off and probabilities of certain outcomes based on the choices available to the agent. Empirical experiments have shown that humans do not always follow this rational strategy. Regret theory has been proposed to explain such observations in human behaviour. This is done by not only considering the outcome, but comparing it to the counterfactual outcome as well. The feeling of regret can play a role in the decision making process, when the actual outcome is less desirable than what could have been if one had chosen differently.

In logics, STIT is often used to reason about agent's actions and decisions. The acronym STIT stands for "sees to it that" in which an agent (or group of agents) sees to it that some proposition will be true. This thesis uses probabilistic XSTIT to formalise the examples of moral luck. In contrast to other STIT logics, probabilistic XSTIT includes probabilities and the effects of actions are not instantaneous, but can be made true in the next time step.

Both moral luck and regret theory involve the feeling of regret. To analyse the relation between these two, an extension of probabilistic XSTIT has been proposed. Maximum expected utility and regret theory were added to this logic. The resulting framework was used to formalise two moral luck examples in order to find the relation with regret theory. Unfortunately, no strong connection between the occurrence of moral luck and the regret-rejoice function was found.

## 7.3 Future Research

There are several directions for further research. The proposed extension considers decisions made by a single agent, this decision theoretic view could be generalised to a game theoretic view. The definitions for pay-off and value can easily be adapted to include the agent. That way, different agents could each get their own pay-off and value for a particular moment. This can then be used to investigate Nash equilibria and mixed strategies in different situations. Bartha (2014) briefly covers this topic and his propositions could be used as a basis for adapting the current framework to be better suited for game theory.

This thesis was confined to analysing simple decision theory situations in which all lotteries are resolved independently and simultaneously. As mentioned in Chapters 3 and 6, more complex situations are discussed by Bell (1983). Especially the Gauguin example 2.2 would be interesting to analyse with the help of a regret theory for such complex cases.

In the discussions about moral luck, responsibility is a recurring subject. Both Nagel (1979) and Williams (1981) relate the responsibility of an agent for a particular outcome to the existence of moral luck. Zimmerman (2001) argues that we should only consider responsibility for the action, instead of for the outcome. Degrees of responsibility are added to probabilistic XSTIT by Doriot and Broersen (2016), which is based on the ideas of Vallentyne (2008) that responsibility is not an all or nothing property. Using the responsibility operator in the logical framework proposed by Doriot and Broersen (2016) could lead to interesting analyses of moral luck and responsibility. This could be combined with the attempt operator in the logical framework (Broersen, 2014), since it could be used to better formalise the determination of an agent. An agent such as Gauguin may attempt to become a successful painter by moving to Tahiti, but his attempt is not necessarily successful.

In the current work, the choices for an agent were ordered by preference. Horty (2001), in contrast, defines a preference ordering on propositions. We have considered to do this as well, but found some complications. Such as two moments that have the same propositional valuations, but are not equally preferred in regret theory. It was beyond the scope of this thesis to further investigate the possibilities of such a preference ordering on propositions instead of choices and is therefore left for further research. Since we defined the preference ordering on choices, it would have been helpful if there was a way to express the choices of the agent in the probabilistic XSTIT-models. As a solution, we used propositions to describe these choices, in order to use the XSTIT-operator to express the most sensible choice for an agent. In other words, it would be more convenient to directly say 'agent $\alpha$ ought to perform action $K$' instead of 'agent $\alpha$ ought to see to it that proposition $\varphi$' where $\varphi$ describes the same action $K$. But an attempt operator could overcome this difficulty as well. It would then be possible to define an ought operator that means 'agent $\alpha$ ought to attempt to $Win$'.

Another direction is to research the possibilities of using epistemic operators, since an agent is not always aware of all the possible choices or outcomes. Deontic epistemic STIT logic can be used to formalise different levels of legal culpability (Broersen, 2011) and could possibly be a basis for expressing the knowledge of agents in examples from regret theory or moral luck as well.

Finally, the logical properties of the proposed extension are left for further research. An axiomatisation as well as proofs for soundness and completeness of the proposed extension are uninvestigated. Unfortunately, we will have to leave this interesting topic for further research.

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