Effectiveness of Dynamic Visualisation in Video-based Animated Algebra Instruction

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"Knowledge is only a rumour until it lives in the muscle."

— The Asaro Tribe —

Abstract. Findings indicate that learning biologically secondary abilities, such as algebra, can be supported by using dynamic visualisation, and by referring to biologically primary abilities, such as bodily motions. By taking on a metaphorical approach, we conducted an investigation towards the effectiveness of algebra instruction by virtue of video-based animation. In two distinct experimental settings, we provided in total 161 secondary education students a short video-based animated instruction of elementary algebraic operations, following the so-called object collection metaphor. Individuals were assigned to one of three conditions, containing a video with static visualisation (control), dynamic visualisation (dynamic) or dynamic visualisation using bodily motion (embodied). To determine a change in the level of comprehension prior to and after the intervention, a pre- and post-test was administered. Over two different settings, no significant difference ($F(2, 128) = 0.459$, $p = 0.633$ and $F(2, 88) = 0.238$, $p = 0.789$) between conditions was found. In the first setting, a small effect ($d = 0.15$, $d = 0.20$) was found for the dynamic and embodied condition, and in the second setting no effect ($d = 0.01$) was found for the dynamic condition and an adverse effect ($d = -0.14$) for the embodied condition. Results suggest that an extensive approach is needed to establish a connection between a biological secondary ability and bodily motions, by means of a metaphorical approach. Possible limitations in relation to this outcome, and recommendations for future research are discussed.

Keywords: algebra instruction, dynamic visualisation, video-based animation, embodied cognition, object collection metaphor.

1 Introduction

When talking about mathematics instruction, most people instinctively construct an image of a teacher working examples on a blackboard, who is standing in front of a classroom. This view towards teaching has been around for many decades and can nowadays still be seen throughout education. In consequence, learning is determined by the way in which classroom-based schooling mandates how, when and at which pace is being worked. At the same time, numerous instances of video-based instruction can be found online, which offer individuals the opportunity to learn about a topic without having to experience the rigours of traditional schooling. Within this growing body of educational material, we perceive a significant share that relies on animation to deliver its education message. Due to an increase in computing power, it has become relatively easy to produce instructional animations, that have the potential to precisely model interactions between elements, in space and over time. Especially for the subject of mathematics, we see great education benefit in using animation, when looking at, for example, the tantalizing instances of video-based mathematics instruction that have been made by Grant Sanderson (known as *3blue1brown*[1] on Youtube). Even so, in a lot of the existing material, a predominant tendency towards creating aesthetically pleasing visuals can be seen, without any clear signs of the adherence to empirical-based principles (Chandler, 2009). There are, as far as we know, no established guidelines related to the design of video-based animation for instructional means (within the subject of mathematics). We do however know that there is an extensive and increasing body of literature that appoints the effectiveness of using animation for this purpose (see e.g., Berney & Bétrancourt, 2016; Castro-Alonso, Ayres, & Paas, 2016; Höffler & Leutner, 2007; Tversky, Morrison, & Betrancourt, 2002).

According to the work of Höffler and Leutner (2007) there are reasons to believe that animation is especially effective for the explanation of procedural knowledge and tasks. In conventional secondary mathematics curricula, we recognise elements that can unambiguously be associated with this finding (e.g., certain aspects of geometry), but also parts that are not as clearly connected at all. In case of the latter, the topic of algebra comes forward to us in particular, due to its seemingly abstract and symbolic character. Findings suggest that for less concrete subject matter in particular, instruction by means of animation can have a negative effect on learning outcomes, due to it impairing the development of understanding, by only showing content for a short period of time (Ayres & Paas, 2007). Related to this issue, it has been pointed out that using motions similar to human movements can be used to circumvent unwanted effects (Van Gog, Paas, Marcus, Ayres, & Sweller, 2009). In spite of this, doing algebra is typically not something that is associated with the idea of engaging in concrete motions, let alone bodily actions. Nonetheless, based on a extensive theory by Lakoff and Núñez (2000), we have reason to believe that it is appropriate to associate this aspect with the topic. By means of their embodied view on the subject of mathematics, we are provided several suggestions that can be used to ground the explanation of algebraic procedures in concrete (and bodily) actions. In this investigation, we employ the so-called object collection metaphor, which is a perspective that is especially suitable for the explanation of elementary algebra. It consists of the idea that formulas are a representation of a particular metaphorical arrangement of collections of objects, that can manipulated by certain actions, which in turn is reflected in the composition of the formula. By using this conceptualisation, we want establish principles for the effective use of animation for algebra instruction. Henceforth, we want to establish empirical principles that make it possible to produce video-based animation with proven effectiveness. For this we formulated the following leading hypothesis:

a) Animated video-based instruction of elementary algebra, based on the object collection metaphor, is more effective when using dynamic visualisation than static visualisation.

Additionally, it has been suggested by Van Gog et al. (2009) that learning can be facilitates by using visualizations of actual human movement. The involvement of bodily motion in dynamic visualisation, is said to better support individuals in reflecting on actions from an animation, compared to static or mere dynamic visualisation. Based on this idea, we foresee a beneficial effect on learning, when human gestures are introduced in the visualisation of the object collection metaphor. This will be realised by extending the mere dynamic visualisation by showing the bodily movements that are involved in performing the action related to the underlying object metaphor. Related to this, we formulated the following additional hypothesis:

b) Animated video-based instruction of elementary algebra, based on the object collection metaphor, is more effective when using dynamic visualisation supported by bodily motions, than solely dynamic or static visualisation.

[1] For an overview of Sandersons' work, see: http://www.3blue1brown.com/

In accordance with the two aforementioned hypotheses, in this research we aim at finding validation for the effectiveness of dynamic visualisation, when offering instruction of elementary algebra by means of animated video, based on the object collection metaphor. Additionally, we aim at obtaining validation for the effectiveness of extending said visualisation with human-based motions to embody the used metaphor. Based on these two objectives, we formulated the following two research questions:

a) What effect does using dynamic visualisation in animated video-based instruction of elementary algebra, based on the object collection metaphor, have on the learning outcome, compared to using static visualisation?

b) What effect does using dynamic visualisation in animated video-based instruction of elementary algebra, based on the object collection metaphor that is embodied through human movement, have on the learning outcome, compared to using static or (non-embodied) dynamic visualisation?

2 Theoretical Background

Research into the effectiveness of animation seems to produce varying results. A meta-analysis by Höffler and Leutner (2007) brought for example forward that there is more benefit in using animation for teaching procedural tasks and acquiring skill, than conceptual or declarative knowledge. A finding by Ayres, Marcus, Chan, and Qian (2009) adds to this that video-based animation can be a very effective method for acquiring tasks related to hand gestures. However, it appears that animation is not in all cases the most effective method. In their meta-analysis Berney and Bétrancourt (2016) for example conclude that animation had a beneficial effect on learning in only 30.7% of the studies, while in 59.3% no significant difference was found. This small percentage of successful applications, is according to Castro-Alonso et al. (2016) likely due to the difficulty that studies have with effectively controlling different variables and factors. It suggests that there are many aspects that can determine the effectiveness of animation as a teaching instrument. This is confirmed in the work of Berney and Bétrancourt (2016), which for example mentions that learning can be influenced by pacing control, signalling, abstraction of the visual representation and modality of commentary (e.g., speech or text).

In literature, we found five principles that are specifically relevant in relation to the didactics of video-based animation. Firstly, according to the *congruency principle* it is important to design an external representation that corresponds to that of the desired internal representation, both structurally and content-wise (Tversky et al., 2002). By exploiting the spatial and temporal character of animation, a representation should reflect the relevant changes in a phenomenon. Secondly, conforming to the *apprehension principle*, an external representation needs to be composed in a way that can be perceived and comprehended in a readily and accurate fashion (Tversky et al., 2002). An representation should allow the observer the opportunity to see and understand change in a phenomenon in a timely manner. Thirdly, in line with the *multiple representation principle*, an explanation can better be given with words and pictures, than words alone (Mayer & Moreno, 2002). For explanations by means of multimedia (e.g., video-based animation) the visual capabilities of animation should be used whenever it can. Fourthly, according to the *contiguity principle*, a multimedia explanation is best given by presenting words and pictures simultaneous and not separate (Mayer & Moreno, 2002). A video-based animation needs to be composed carefully, to prevent doing harm to learning, due to misalignment of words and pictures. Fifthly, following the *worked examples principle*, an explanation should consist of procedures that work step by step, when a new cognitive skill is being acquired (Renkl, 2005). Animation is especially effective for introducing a learner in a new domain, by providing examples with discernible mental steps.

Related to the visualisation of animated instruction, we found five relevant principles in literature. Firstly, in line with the *split-attention principle*, compositions should be avoided that require learners to pay attention to and integrate multiple sources at once (Ayres & Sweller, 2005). Having separate sources of information in a video, can have a diminishing effect on the perception of the whole. Secondly, according to the *coherence principle*, video-based animation should contain as few as possible, extra and unnecessary, words and sounds, to improve comprehension (Mayer & Moreno, 2002). By having audio-visual elements not needed for the explanation of a phenomena, acquiring understanding can be compromised. Thirdly, following the *modality principle*, words can best be presented through auditory narration and not as visualised text (Mayer & Moreno, 2002). Due to the difference in modality between words and pictures, more time is needed to comprehend on-screen text, which makes learning is less effective. Fourthly, conforming the *redundancy principle*, animation and narration is most effective when presented without having additional visualised text (Mayer & Moreno, 2002). Because of the cognitive difference between hearing and sight, having on-screen text can cause interference while learning. Fifthly, according to the *transiency principle*, instruction through animation requires the employment of certain compensation methods, due to the depictment of elements that continually disappear (Castro-Alonso, Ayres, & Paas, 2014). Taken that attention is only limited, it is important to use certain methods (e.g., pace-control, modality, cueing) to reduce cognitive strain.

Using animation for learning, can be beneficial when individuals need to compensate for insufficient aptitude or skill when motions need to be imagined (Salomon, 2012). The realistic presentation of a process or procedure, permits individuals to obtain a deeper understanding of a phenomenon. By displaying a dynamic visualization of actions or motions, an external representation is provided, that can be used to construct an internal (or mental) representation. On a technical level, animation can be seen as a series of frames that depict a procedural state that is an alteration of a foregoing one, that follows a sequential and temporal trajectory defined by the user or designer (Bétrancourt & Tversky, 2000). It is important to note, that it should not be considered as a sequence of frames depicting the movement of real objects (e.g., video) as defined by for example Mayer and Moreno (2002). As opposed to observing real phenomena, animation-based visualisation intent to provide a representation of specific dynamics of something, typically based on the idea that it is hard to observe, difficult to realize in a learning situation or not visual on itself (Betrancourt, 2005).

When motion is being learned from an animation, a fundamental difference exists between human movement tasks and more abstract ones performed in, for example, mathematics (Castro-Alonso et al., 2016). In case of the former, humans have the ability to learn specific movements and corresponding actions relatively easy, by having an evolutionary disposition for them. The latter, non-human and a-typical motion, has, in contrast, not been emphasized by evolution and is therefore much harder to learn. These two opposing faculties, have been characterized by Geary (1995) as respectively biologically primary and secondary abilities. It relates to the distinction made by the dual-process theory from psychology, which states that two purpose-specific systems co-exist in our brains. One is fast and more intuitive, and evolved early, and one is slow and more analytic, and developed later (Barrouillet, 2011).

Research suggests that using animation for the instruction of procedural-motor tasks is more effective, than using static pictures (Höffler & Leutner, 2007). Learning biologically primary abilities rely on the presence of an evolutionary context, which is aided by for example the possibility to depict movements in an animation in a timely and fluent manner (Kilner, Paulignan, & Blakemore, 2003). In case of biologically secondary tasks, the opposite seems to be the case. When learning motions in a non-evolved context from animation, humans are less effective in acquiring them, than they are with biologically primary tasks (Paas & Sweller, 2012; Sweller, 2011).

Due to having working memory with limited processing and retention capacity, humans are typically not able to cognitively handle more than a certain amount of elements being introduced at a time (e.g., Cowan, 2001; Miller, 1956) or over a prolonged period of time (e.g., Peterson & Peterson, 1959). When animation is used for learning biologically secondary abilities, issues can occur when it is not possible to perceive things within an allowed period of time, due to pacing or a lack of control thereof. This so-called transient character of depictions, prevents learners from fully processing and integrating the information in an given educational message. Henceforth, this character of animation has the potential to negatively affect learning outcome (see Ayres & Paas, 2007).

Looking at the transient nature of animation, it could be suggested that using static pictures is more effective when learning a biologically secondary ability (Castro-Alonso et al., 2016). A meta-analysis done by Berney and Bétrancourt (2016) does however contradict this. It concludes that static material was in only 10% of investigations in STEM-topics more effective, while in 59% no significant difference was found. Taken that it was found that animation was not predominantly more effective in 69% of the cases, it could again be argued that static is more effective for acquiring biologically secondary abilities. This is however not an entirely viable conclusion, given that a lot of investigations are affected by inconsistencies coming from a lack of control over confounding variables (Castro-Alonso et al., 2016).

To counteract the negative effect of transiency in teaching biologically secondary abilities through animations, Van Gog et al. (2009) suggests the use of motion related to human movement. By involving the mirror-neuron system (Rizzolatti & Craighero, 2004), learning is steered towards an evolutionary meaningful pathway. It is based on the *common coding principle*, that states that similar neural structures are involved in perceiving the outcome of an action, as actually planning the action in the future (Prinz, 1997). Subsequently, the same neural areas are involved in observation of an action, as in executing the action. It is likely a method to prime the brain for the execution of the same or a similar action (see e.g., Iacoboni, Woods, Brass, Bekkering, Mazziotta, & Rizzolatti, 1999), hence to imitate the execution of the action.

When considering biologically secondary abilities, it is according to Barsalou (1999) important to recognize that a grounding in perception and action exists, which does not involve any abstract phenomena such as symbol manipulation. This idea is represented by the theoretical framework *embodied cognition*, which claims that the cognitive processes of organisms are developed from goal-directed interaction with their environment (Barsalou, 1999, 2008; Glenberg, 1997; Rueschemeyer, Lindemann, Van Elk, & Bekkering, 2009). Widespread findings indicate that cognitive states can be caused by a bodily state, and can also cause a bodily state, meaning that cognition is grounded in perception and action (e.g., Barsalou, Simmons, Barbey, & Wilson, 2003; Lakoff & Johnson, 1980a). According to Barsalou (1999) embodied cognition is not limited to concrete operations, such as walking, but also applicable to abstract higher order levels of cognition, that for example relate to the processing of language and mathematics.

In their article de Koning and Tabbers (2011) suggest that during learning, by means of *embodied simulation*, previously acquired sensorimotor experiences are made available for the construction of knowledge. It is seen as the re-enactment of perceptual and motor states that have been acquired through real-world experiences (de Koning & Tabbers, 2011). By integrating patterns of neural activity, originating from different modalities, a multimodal representation is constructed in the brain. When an experience is retrieved from memory, it is reactivated and used to simulate the way in which perception and action are mentally represented. Subsequently, this mechanism is facilitated to make abstract conceptions, based on perceptual experience and interaction with the environment (see e.g., Barsalou, 1999). It is seen as a mapping of elements of concrete concepts on more abstract concepts, making the latter inherently situated in everyday experiences by virtue of the former (Barsalou, Wiemer-Hastings, Pecher, & Zwaan, 2005). In this view, cognitive activity is supported by simulation mechanisms that use the same neural basis as conventional perception and action do, in a real-world context (de Koning & Tabbers, 2011).

In the field of cognitive linguistics a theory about human cognition can be found, which is closely tied to the above mentioned idea of embodied simulation. By looking at the structure and use of language, Lakoff and Johnson (1980b) hypothesized the existence of cerebral functionality very similar to that in the article by de Koning and Tabbers (2011). From this, a potent theory called Conceptual Metaphor Theory (CMT) has been conceived, that found a permanent place in its respective field. At its core is the idea that in everyday language, metaphors are used widespread, consistent and often unaware. A metaphor is however not considered as a pure linguistic thing, but as a representation of the way in which the brain organises perceptual and conceptual knowledge. This implies, based on the aforementioned metaphorical nature of language, that the human comprehension of concepts is also metaphorical. The suggestion is raised that there are metaphorical mappings between certain source and target concepts, that are used to come to understanding. Hence, it facilitates a mapping between perception and linguistic reasoning. According to Johnson (1987), source concepts are usually concrete and present in the real world, and are therefore embodied in some way, shape or form. In contrary to this, target concepts are often abstract, and it is not possible to directly experience or perceive them. This can for example be seen in the metaphor of containment, which relates the idea of a perceptually closed-of area (e.g., a circle) to the linguistic concept of a 'container'. Taken that many concepts are metaphoric, conceptual understanding effectively originates in the human body and its interaction with the physical environment.

By means of an elaborate book, Lakoff and Núñez (2000) describe the embodied character of mathematics utilizing the ideas from CMT. They state that the field, traditionally seen as rather abstract and symbolic, can be considered as embodied by having an inherently metaphorical nature. To underpin this, they discuss *grounding metaphors*, which are concrete conceptualization of fundamental mathematical ideas by means of several concrete metaphors, amongst which the 'object collection', 'measuring stick' and 'moving along a path' metaphor. Each of them derived from language related to real-world experiences performing certain human actions. The theory relies on specific cognitive faculties, or schemas, recognised as *image schemas* by Johnson (1987), of which is hypothesized that a multitude of collective similar bodily experiences has lead to their formation. They are characterized as being pre-linguistic multi-modal phenomena, that represent certain elements of real-world perceptual experiences. Their composition offers a generalisable structure that can be used for learning, by projecting in a unidirectional fashion a mapping onto new concepts. By providing a specific inference structure, reasoning is supported in, amongst other things, building metaphorical structures. A common occurring instance of this principle, is the *containment image schema*, which expresses the idea of a bordered of area that can contain something. Looking at the way in which this phenomena is used in language, a connection can be perceived with the way in which embodied simulation (de Koning & Tabbers, 2011) facilitates knowledge construction, using previous acquired sensorimotor experience. An general overview of the image schema and its relation to the conceptual metaphor is given in figure 1.

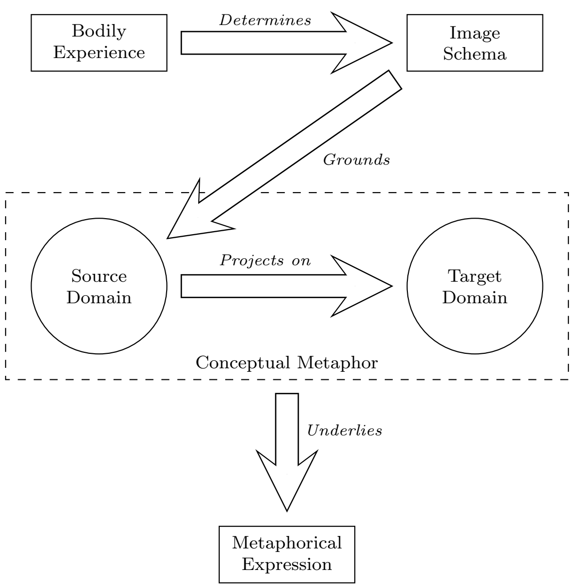


Figure 1. Conceptual Metaphor (Johnson, 1987)

The theory of Lakoff and Núñez (2000) mentions a metaphor than can be used to compose an embodied theory towards the topic of algebra. This so-called *object collection metaphor*, gives rise to the idea that algebraic expressions are representations of a collection of arbitrary objects, which can be variables or collections within this context. The image schema underlying this metaphor, that of 'containment', is represented by the idea that there are certain objects that are put inside a closed of area. By the character of this schema, this view on algebra is very much grounded, and therefore only suited for natural amounts of objects. It is therefore perfectly possible to represent $2a + 3b + a$, but not so much to represent $\frac{1}{2}a$. In figure 2 an extensive example is given of the dynamics of this metaphor in relation to the distributive law in algebra.

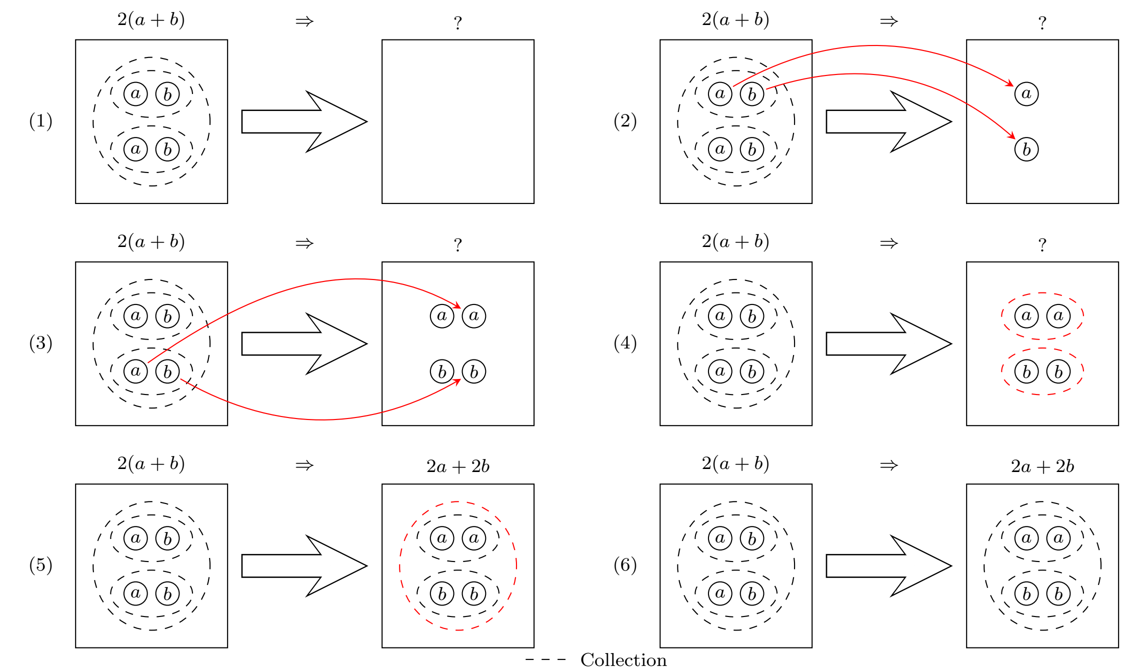


Figure 2. Example of object collection metaphor (Lakoff & Núñez, 2000) in algebra

3 Methodology

3.1 Intervention

In our investigation, we employed an experiment containing three conditions and involving a specific intervention per experimental condition. At its basis, our research consisted of an learning experience in which students were asked to master a set of operations required for the simplification of limited set of algebraic formulas. Shortly before the intervention, a pre-test was administered to rate the initial understanding of participants related to the content matter. Consequently, by plenary watching a short instructional video, individuals received an explanation of a set of procedures that can be used for algebraic simplification. Directly after watching the video, a post-test was administered to rate the understanding of participants again. By differing the video per experimental condition, results from varying interventions were obtained to compare the effectiveness of the particular conditional differences.

The videos in the experiment contained an explanation for four different algebraic operations. First, a multiplication of a variable was represented as a repeated addition to clarify the meaning of using a variable in such a manner, e.g., $a + a + a + a = 5a$. Second, a formula of two variables was used to explain the commutative property of variables, e.g., $5a + 3b = 3b + 5a$. Third, a formula of three variables was shown to exemplify the addition of two variables, e.g., $5a + 3a + 3b = 8a + 3b$. Fourth, a multiplication of a nested formula was used to clarify the multiplication of a formula, e.g., $3(5a + 3b) = 5a + 3b + 5a + 3b + 5a + 3b = 15a + 9b$. For simplicity, examples were used that can be reduced in a minimal number of steps. In case of the last two, operations were present that were explained in the first two instances.

Based on the object collection metaphor by Lakoff and Núñez (2000), we designed an initial intervention that resulted in the first experimental condition. In it, an alteration of the first example is used to show that a scalar variable is composed of a number of instances of that variable, e.g. $5a = a, a, a, a, a$. This example intends to make the principle of object collection in relation to variables explicit. It relies on the idea that a variable is a referent to a specific container with an arbitrary number of things in it. The existence of an unique object is referred to by a specific variable name, and different objects are reflected by the existence of different variable names. Ultimately, the metaphor brings the idea forward, that algebra entails operations of variables as objects and collections of such[2]. This is reflected by the structure of algebraic formulas in a number of ways. Firstly, the presence of brackets suggests that there is an arbitrary collection of objects or collections of objects in between. Secondly, a multiplier signifies a repetition of an object or a collection thereof. Thirdly, the use of addition implies the coexistence of a number of objects. For the sake of clarity an example of these characteristics is given in figure 3.

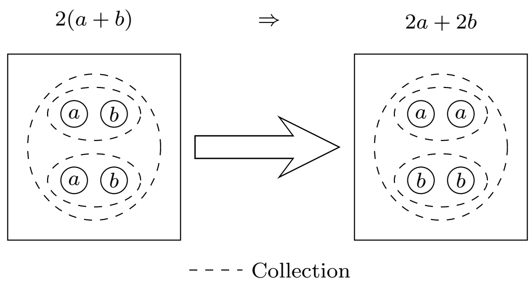


Figure 3. Example of object collection metaphor in relation to distributivity.

By extending the initial intervention, we constructed the intervention for the first experimental condition. In it we employ the finding by Höffler and Leutner (2007), that states that animation is especially effective for learning, when procedural knowledge is involved. We altered the animations by introducing dynamics, such as moving (parts of) formulas, to emphasize the use of the object collection metaphor by Lakoff and Núñez (2000) as principle for instruction.

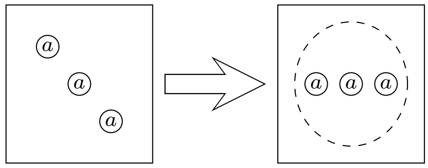
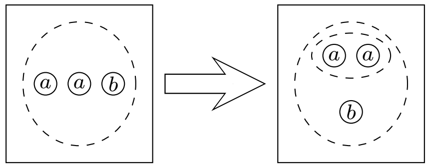
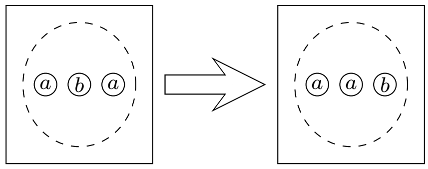
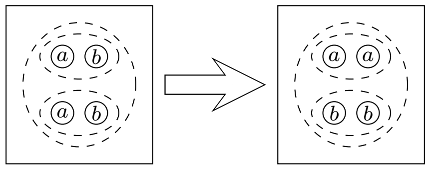
In the third intervention, we added bodily movements, to implement the finding by Van Gog et al. (2009) that involving human movement reduces the effect of transiency when learning movements. The addition is based on the idea that movement is explicitly visualised to refer to the motor-operations

[2] We choose for this specific view on algebra based on its didactic simplicity, and recognise that it has its limitations and can not be used when for example working with decimals.

associated with the execution of specific actions. To realize this, we added the silhouette of the upper body of a person to the first two examples of the video. In the remaining two examples, we omitted this torso to prevent having to many elements, which could work distracting to the viewer. For clarity, in table 1 each example is discussed from a visualisation, action and metaphorical perspective.

Table 1

Visualisation and action per exemplary algebraic operation.

Operation	*Visualisation*	*Action*	*Object Collection Example*[3]
Scalar multiplication	Move similar variables together to get scalar times variable term.	Move similar objects together in a collection.	$a, a, a \Rightarrow 3a$ 
Addition of similar terms	Move terms with similar variables together to get scalar times similar variable term.	Put similar objects together in a collection.	$a + a + b \Rightarrow 2a + b$ 
Commutativity	Move terms around to obtain new arrangement.	Move collection around to obtain new arrangement.	$a + b + a \Rightarrow a + a + b$ 
Distributivity	Move expressions together to obtain sum of variables.	Put objects together to obtain collection of similar objects.	$2(a + b) \Rightarrow 2a + 2b$ 

3.2 Participants

Within our investigation, we conducted several experiments within moderately different settings. This makes that a distinction exists between groups of participants, based on for example different experimental aims and method of measurement. We involved participants in the following three settings: a pilot, a leading experiment and an additional experiment.

Based on the character of our investigation, we distinguished a suitable participant primarily on their level of understanding of the topic on hand. Essentially, some minor experience with using basic mathematical operations (e.g. multiplication) and symbols (e.g., the dot operator) was required. This of course only applied up to a certain point, because no familiarity with the content of the intervention (e.g., distributive law) was desirable. By choosing a specific group students, we pursued an experiment with individuals that had the right amount of prior knowledge, but not to much.

To attract the right participants, we asked a number of mathematics teachers at specific schools, whether they would like to cooperate. They were selected based on the level and prior knowledge of their students. By reserving a time-slot during regular hours, we were able to conduct our experiment with limited impact. We controlled the variety in participants, by working within each setting with individuals of the same schools and at the same school level, whenever possible.

3.2.1 Participant Assignment

To reduce variability within each setting and between conditions, we assigned participants using a block-based approach. By allocating persons based on perceived mathematics level, we pursued an outcome in which each block ideally consisted of a low, medium and high achieving student. Consequently, all blocks were assigned in such a manner that each condition had approximately the same composition. For practical and privacy reasons, we asked each teacher to autonomously assign each of their pupils to one of these three levels. To aid this process, we provided a method in which blocks are created automatically, by numbering students in an appropriate manner. In this approach, first the factor 'perceived level' was integrated by putting students in one of the aforementioned three levels. Consequently, the first group was given the numbers 1, 4, 7, ..., the second group the numbers 2, 5, 8, ..., and the third group the numbers 3, 6, 9, which resulted in a two level block. By working in this way, individuals could end up in any of the three conditions by virtue of the teachers ordering, which means that a degree of randomness was introduced in the procedure. For clarity, an example of this approach with 9 participants is given in figure 4.

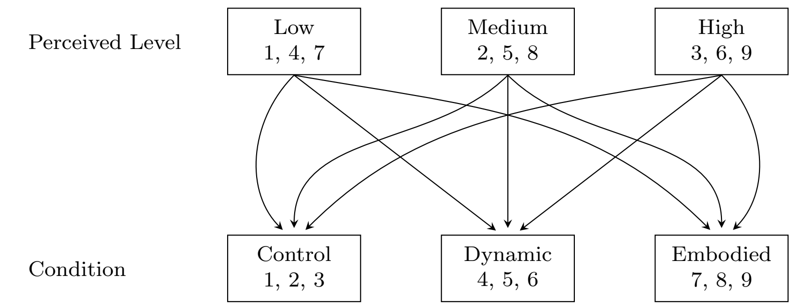


Figure 4. Example of used blocking approach with nine participants

By dividing each class, numbered from 1 to n, in three uniform groups and assigning each group to a condition, a degree of variability is removed from the experiment and possible effects of confounding variables are reduced. An additional advantage is the ability to conceal the pattern behind the assignment of individuals to specific conditions. This could potentially prevent affecting the outcomes, due to students noticing an specific ranking that could be associated with the existence of expectations related to their/others skill and/or level.

3.3 Material

As part of our intervention, a video clip was shown to the participants in which a number of algebraic concepts are explained in an (pseudo) animated manner. By means of four examples, specific operations intended for the simplification of algebraic formulas were discussed. In each condition, an altered version of the clip was used that visualised certain operations in a specific way (see 3.1 for an in-depth explanation). We tried to control the visual and auditory stimuli as much as possible, by using the same algebraic expressions, typesetting, colouring, timing and overall design for the former and the same audio track for the latter. Subsequently, we checked the applicability of the principles for we found in literature, as discussed in 1 By taking this approach, we tried to circumvent the introduction of possible biases, as suggested by Castro-Alonso et al. (2016), that could act as confounding factors in our intervention. Additionally, we avoided the use of visuals with a decorational function (Carney & Levin, 2002) as much as possible. This resulted in three videos that were very similar and mostly differed in the way in which each algebraic operations are visualised.

The visualisations in our video clips are produced using screen recordings of Powerpoint 2016, and the narration using Windows recording software and a basic USB-microphone. After recording the audio, we edited and improved it with Audicity 2.1.3 by applying noise reduction, compression and normalisation. We subsequently edited the screen recordings using Lightworks version 14 and added and synchronised the audio recordings. The resulting videos are about 92 seconds long, have a resolution of 720 x 1280 (30 fps) and an audio sample rate of 48 kHz.

The video clips that we used for each condition, are constructed using the principles discussed in 1 and using the work by Höffler and Leutner (2007), Lakoff and Núñez (2000) and Van Gog et al. (2009). For a more in-depth explanation, see 3.1. Firstly, we designed a video clip for the *control* condition that does not contain any dynamic animations. Operations are explained in the voice over, but transitions are not visualised using a dynamic animation. This effectively means that the visual part of the explanation only deals with the outcomes and not the procedural character of the underlying operation. Secondly, we adapted the initial video to produce a clip for the *dynamic* condition. This modification consisted of making transitions visible using dynamic animation. In this case, depictions of moving and/or merging of terms/formulas were used to make specific operations more apparent. Thirdly, we made an extension of the second clip to fit on the *embodied* condition. By adding an upper body/or pair of hands manipulating the terms or formulas, the second video was altered. Using this approach, we made it apparent that the involved transitions can be associated with actual human actions. In table 2 an overview is given that shows a transitional frame per example and condition.

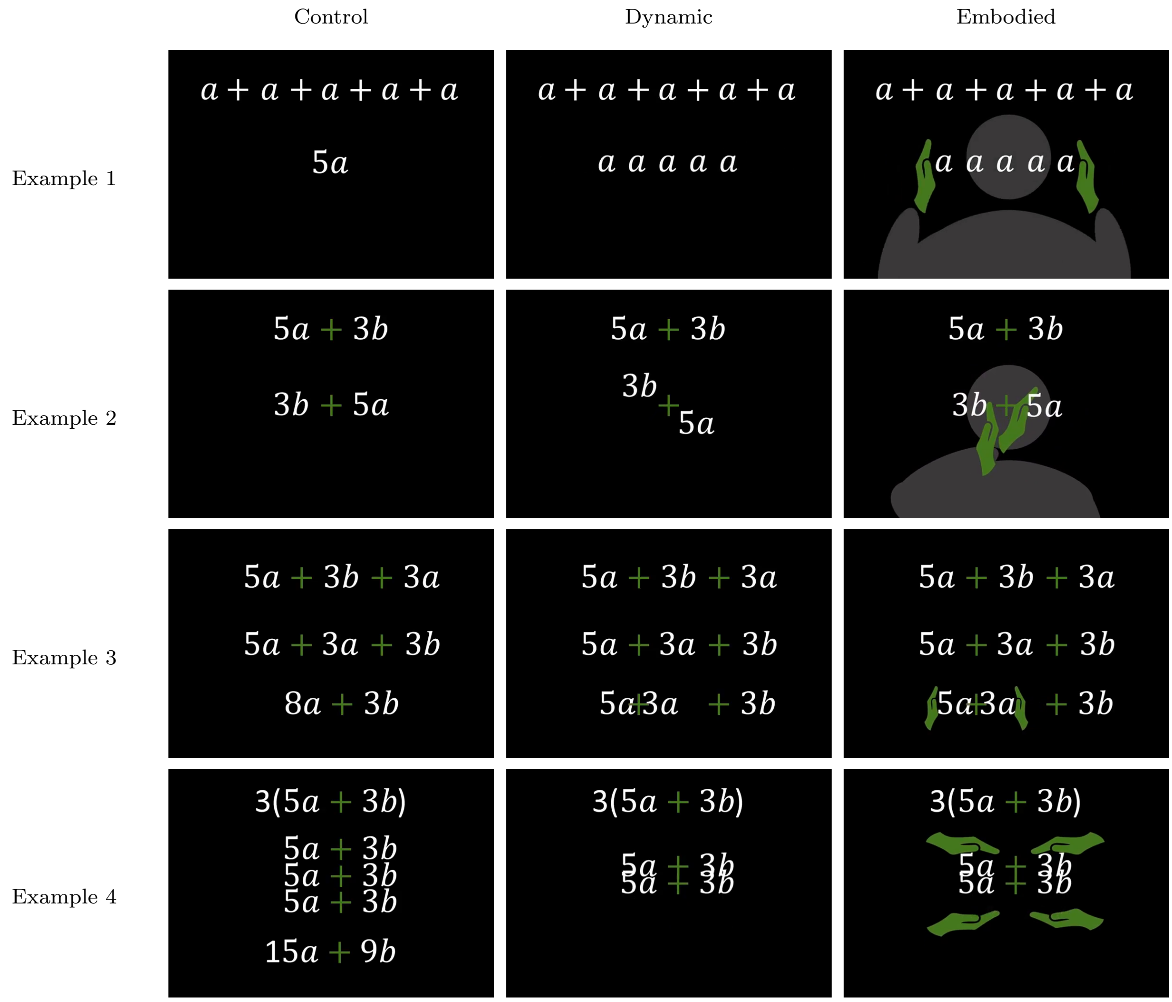


Table 2: Visualisation of example per condition

3.4 Measurement

To determine the effect of each video clip on the participants, we administered a pre- and post-test among them. By applying the former, a baseline measurement of the knowledge and skill of involved individuals was obtained, which was to be used with results of the post-test to attain a per condition effect. It also made it possible to correcting outcomes for any prior knowledge, when for example a person obtained a high score in the pre-test.

The test items used for measurement, consisted of a number of algebraic formulas participants had to simplify. In case of the pre-test only prior knowledge about the topic, if any present, could be used. For the post-test, additional knowledge provided during the intervention, by means of the video clip, could be used.

In our investigation, we initially did measurements using a short test containing items that relate to specific algebraic characteristics. They were constructed with a varying complexity, using a study that focused on the types of errors made during simplification of algebraic expressions by Lim (2010). This resulted in the following guidelines towards the design of the test and each item:

- An easy item based on the assumption that students can lack comprehension of the use of brackets, hence the use of the distributive law.
- A moderate item based on the assumption that students can have trouble multiplying variables and ordering them, hence the use of the commutative law.
- A hard item composed of a combination of the other two items, hence requiring both the use of the distributive and commutative law.

By following the aforementioned structure, we composed a pre- and post-test consisting of 3 items. We choose to move the hardest item to the second position, to make it less likely that for example time-constraints would entice participants to not answer the question. The actual tests than came out of this structure can be found in appendix A.

Based on the outcomes of our leading experiment, we conducted an additional experiment with an alternative approach towards measuring the effect of our intervention. By using an extensive test containing items that could be answered through reproduction and items that require a little understanding, we targeted at obtaining a more distinctive measurement. This resulted in a pre- and post-test consisting of 16 items, which can be found in appendix B.

4 Results

For our investigation, we attracted in total 223 participants, attending various secondary educational institutions. We attempted to keep the type of students relatively similar within each setting by working with individuals of the same schools and at the same school level when possible. Based on the outcomes of our initial (and leading) experiment, which involved 132 individuals, we did an additional investigation involving 91 persons. This was done under the same experimental conditions, but amongst students at a lower school level than those in the initial experiment, and by administering a more elaborate pre- and post-test. The rationale for choosing this distinct group of students and a more extensive test, is, on the one side, based on the finding by Spanjers, Wouters, Van Gog, and Van Merrienboer (2011) that animations like the kind we are using, can be more effective for students with a lower level of prior knowledge, and on the other side on the idea that an exhaustive test allows for a scoring with a higher granularity. Because of the apparent dissimilarity of the groups and the difference in measurement, the results are discussed per experiment.

4.1 Leading Experiment

In our leading experiment, in total 132 students participated, originating from five classes from a single pre-university school for secondary education. Each was assigned to one of three conditions by their respective teacher, using the block-based approach as discussed in the methodology section. The resulting distribution of participants can be seen in table 3.

Table 3

Mean post-test scores in leading experiment

Condition	n	%	M	SD
Control	43	32.6	5.6047	1.9413
Dynamic	45	34.1	5.8667	1.8291
Embodied	44	33.3	5.9773	1.7319
Totals	132	100.0	5.8182	1.8277

Note: Maximum score: 7

Using the outcomes of the pre- and post-tests, we conducted a one-way ANCOVA to compare the effectiveness of three conditions whilst controlling for prior knowledge. For this we carried out Levene's test and normality checks and the assumptions met. There was no significant difference in mean score $F(2, 128) = 0.459, p = 0.633$ between the conditions. Based on the Partial Eta Squared value, we computed Cohen's d value (Cohen, 2013) which pointed to a small effect ($d = 0.15$) for the dynamic, and also a small effect ($d = 0.20$) for the embodied condition.

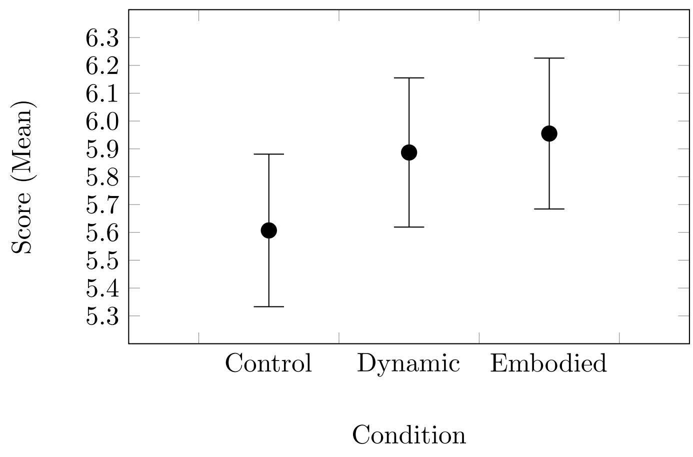


Figure 5. Mean post-test scores in leading experiment, corrected for prior knowledge. Error bars represent standard errors.

4.2 Additional Experiment

In our additional experiment, in total 91 students participated, originating from four secondary vocational education classes at two schools for secondary education. Each was assigned to one of three conditions by their respective teacher, using the block-based approach as discussed in the methodology section. The resulting distribution of participants can be seen in table 4.

Table 4

Mean post-test scores in additional experiment

Condition	n	%	M	SD
Control	31	34.1	8.2097	6.2474
Dynamic	29	31.8	8.2414	5.9801
Embodied	31	34.1	7.2258	7.3074
Totals	91	100.0	7.8846	6.4932

Note: Maximum score: 29

Using the outcomes of the post-tests, we conducted a one-way ANOVA to compare the effectiveness of the three conditions. We did not use ANCOVA, as no controlling for prior knowledge was required, due to non-existent scores on the pre-test. We carried out Levene's test and normality checks and the assumptions met. There was no significant difference in mean score $F(2, 88) = 0.238, p = 0.789$ between the conditions. Using the results, we computed Cohen's d value (Cohen, 2013) which pointed to no effect ($d = 0.01$) for the dynamic and an adverse effect ($d = -0.14$) for the embodied condition.

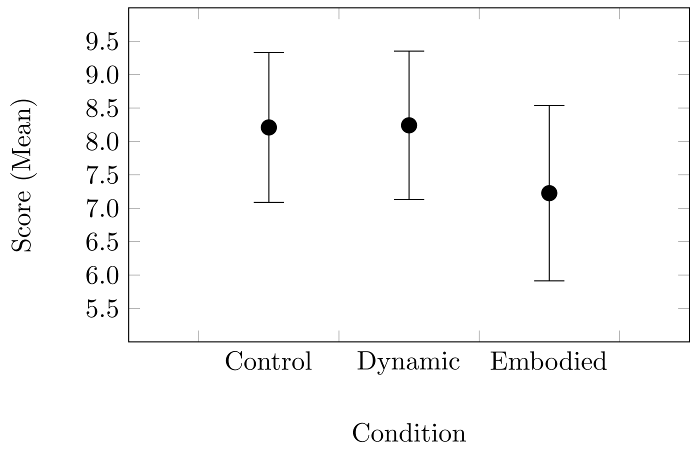


Figure 6. Mean test scores in additional experiment. Not corrected for prior knowledge. Error bars represent standard errors.

5 Discussion

In this investigation, we sought to validate the effectiveness of dynamic visualisation, when offering instruction of elementary algebra by means of video-based animation, based on the object collection metaphor. For this, we formulated the following two hypothesis:

a) Animated video-based instruction of elementary algebra, based on the object collection metaphor, is more effective when using dynamic visualisation than static visualisation.

b) Animated video-based instruction of elementary algebra, based on the object collection metaphor, is more effective when using dynamic visualisation supported by bodily motions, than solely dynamic or static visualisation.

Based on these, we came to the following two research questions:

a) What effect does using dynamic visualisation in animated video-based instruction of elementary algebra, based on the object collection metaphor, have on the learning outcome, compared to using static visualisation?

b) What effect does using dynamic visualisation in animated video-based instruction of elementary algebra, based on the object collection metaphor that is embodied through human movement, have on the learning outcome, compared to using static or (non-embodied) dynamic visualisation?

For our research questions we composed two experimental conditions, which were deployed in two relatively similar settings.

In the end, we did not find a significant difference between conditions in any of the settings, which means that we did not obtain confirmation for the aforementioned hypotheses. We furthermore found a small effect for both experimental conditions in the first setting, and no effect and an adverse effect for respectively the dynamic and embodied condition in the second setting. Considering the lack of significance, we believe there is no conclusive confirmation for these outcomes.

Looking at the character of our work, being a comparison between a static and two dynamic variants of video-based animated instruction, and based on several findings in literature, we perceive the outcome of our work as one that was to be anticipated for. Berney and Bétrancourt (2016) do for example mention in their meta-analysis that in 59.3% of the investigations they included, no significant difference was found. Additionally, according to Castro-Alonso et al. (2016) it can be difficult to find significance due to the potential presence of various biases (e.g., appeal, variety), an issue which is difficult to control for. Considering our experiment, we do not perceive any obvious causes (e.g., biases or confounding factors) that could explain the outcome of our work, but we do recognise a number of limitations in relation to the result.

For our leading experiment, we administered a pre- and post-test amongst 132 pre-university students. After analysis of the outcomes, we observed the occurrence of a ceiling effect in the post scores. We believe that shortcomings in measurement might have led to this result, as there were clear indications that our test had a to small number of items. In our view, this could explain why we did not find a significant difference between conditions in this setting. There was however a slight difference between the mean of the control and the two dynamic conditions, indicating that on average a higher learning outcome was established when dynamic visualisation was used, which can also be seen in the effect sizes found for both conditions.

Based on our initial findings, we conducted an additional experiment, with a longer pre- and post-test, amongst 91 students attending secondary vocational education. By employing a more extensive measurement in a group of individuals who we conceived as less knowledgable of and biased by prior-knowledge than those participating in the leading experiment, we hoped to get more insightful results. We chose to work with a group of students in which we anticipated very little prior knowledge, following a finding by Spanjers et al. (2011) which suggests that dynamic animations can have a negative effect on learners with a higher level of prior knowledge. However, looking at the overall outcome of the pre-test, and based on observations during the experiments, we infer that related to the content matter there was possibly not enough prior knowledge present amongst participants, which makes it feasible that the goal of the procedure was not apparent enough to the participants. This might have consequently have affected the outcomes, as students were not able to comprehend the content matter offered in the intervention. We think that this could explain why no significance was found and that the distribution of means found in this setting was greatly different to that of the leading experiment.

Looking at the aforementioned limitations, we see an important finding that comes forward in our work, that relates to the recognition of goals-directed behaviour. It is mentioned by Van Gog et al. (2009), that it is much harder to discern actions from cognitive skills, such as algebraic reduction, because of it being less observable than motor skills. We hypothesized that using an idea embedded in a theoretical framework about embodied cognition, would enable us to facilitate setting up a straightforward connection between motion in the real-world and that needed for algebraic operation, by facilitating the mirror neuron system. There is however no clear indications that we were able to support learners in establishing a connection between the former, being a biologically primary ability (Geary, 1995), and the latter, being a biologically secondary skill (Geary, 1995). Remarkably, this finding is in line with the idea that states that understanding actions entails much more than activating similar motor structures in the brain (Gazzola, Rizzolatti, Wicker, & Keysers, 2007). It additionally complies to the Indexical Hypothesis, which mentions that comprehension (of language) requires that a (directly) perceived object is mapped onto action experiences (Glenberg & Kaschak, 2002; Glenberg & Robertson, 1999). We hypothesize that due to an apparent unfamiliarity of participants with working with real-life instances of the object collection metaphor, it is less likely that a connection is made. Only by interaction with a physical object or through prior interaction with an object (e.g., simulation), activation of related neural structures finds place. Consequently, patterns of how something can and should be used are triggered in the brain (Gibson, 1979), something which does not seem to have happened in our work. We believe that making a connection entails more than merely visualising a metaphor. In our view, it could have been beneficial if students were prepared during teaching, by gradually moving from motor execution to simulation, and consequently to abstraction. Taking such an approach, could in our view have supported learners in making the connection between human motion and abstract concepts such as those in algebra.

6 Suggestions for Future research

Towards future research, we perceive a number of possible avenues. It might for instance be interesting to conduct an investigation similar to this one, without narration (or any other textual information). Leaving out this aspect can potentially affect learning outcomes in a positive way, according to a finding by Berney and Bétrancourt (2016). We argue that it might avoid split attention effects or reduce cognitive load (Berney & Bétrancourt, 2016) and that it could emphasize the procedural character of the content matter.

When it comes to controlling prior-knowledge, we see value in taking on an approach in which procedural knowledge is separated from conceptual and declarative knowledge as much as possible. In accordance with the finding by Höffler and Leutner (2007), which suggests that animation is less effective for the latter, than for the former, we conceive possible benefits in distinctly separating the types. Literature shows that prior-knowledge can have a strong influence on the effectiveness of animated instruction (see e.g., Kalyuga, 2008), and that it is therefore important to control for it as much as possible. When doing so, Berney and Bétrancourt (2016) suggest to take a common approach for determining and distinguishing knowledge, by using a revised version of Bloom's taxonomy (Krathwohl, 2002) that makes a distinction between factual and procedural knowledge. Based on our findings and this recommendation, we think it could be helpful for future research within a context similar to ours, to teach and test for the former (e.g., what constitutes a variable) before actually conducting an experiment involving the latter (e.g., how to merge two arbitrary variables).

Looking at the per participant results, we suggest future research to thoroughly control for individual differences. We do for example think that having high spatial ability, might influence the ease of comprehension of the object-based approach we used. It is a factor, which in general is recognized as playing an important role when learning from dynamic visualisation. Indications suggest for example that using animation can affect, both positively and negatively, the learning outcomes of learners with high spatial ability, and have a compensating effect for those with lower spatial ability (Höffler, 2010). Findings like these emphasize the importance of stringently controlling for specific individual differences.

Considering the conceivable limitations of the object collection metaphor used in this investigation, we propose doing similar research using more elaborate instances. We believe the instance we used, to be a very accessible one, by it being very close to everyday perceptual-motor skill, but also very limited, due to it being distant from abstract notions such as decimals or fractions. In our view, it might for example be interesting to find out whether the measuring stick metaphor (Lakoff & Núñez, 2000) offers a more effective way to express the same ideas.

In relation to the object collection metaphor, it might be beneficial to conduct a similar investigation that starts with establishing a connection between motion in the real-world and the intended concept used in the visualised material. Creating such a real-world context, is according to Duijzer, Van den Heuvel-Panhuizen, Veldhuis, Doorman, and Leseman (2019) an important mediating factor in relation to embodied cognition. In our work, we tried to make a connection between the everyday perceptual-motor skill of moving collections around, with a specific procedural algebraic operation. Based on the outcome and the apparent novelty of the approach we believe that a significant number of participants were not able to establish a functional connection. Looking at this result, we perceive potential benefit in facilitating participants in developing a more metaphorical way of reasoning.

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A Pre- and post-tests - Leading Experiment

Opgaven 1

Instructie:

- Gebruik een pen.
- Vul je deelnemersnummer in.
- Maak de opgaven.
- Leg dit papier met de tekst naar beneden op tafel als je klaar bent.

Deelnemersnummer: ....

1. Vereenvoudig. Schrijf de formule zo kort mogelijk.

$$2a + 8b + 2a + 2b$$

2. Werk de haakjes weg en vereenvoudig.

$$6a + 5(10b + 5a) + 6b$$

3. Werk de haakjes weg en vereenvoudig.

$$6(4a + 6b)$$

Opgaven 2

Instructie:

- Gebruik een pen.
- Vul je deelnemersnummer in.
- Maak de opgaven.
- Leg dit papier met de tekst naar beneden op tafel als je klaar bent.

Deelnemersnummer: ....

4. Vereenvoudig. Schrijf de formule zo kort mogelijk, zoals in de video.

$$5a + 4b + 8a + 3b$$

5. Werk de haakjes weg en vereenvoudig.

$$8a + 6(2b + 9a) + 9b$$

6. Werk de haakjes weg en vereenvoudig.

$$3(7a + 2b)$$

B Pre- and post-tests - Additional Experiment

Pretest Algebra Animatie

Instructie

- Gebruik een pen.
- Vul je deelnemersnummer in rechts in het kader.
- Maak **zo veel mogelijk** van de onderstaande opgaven.
- Tussenstappen opschrijven mag, maar hoeft niet.

Deelnemersnummer:

Vereenvoudig de formules: werk de haakjes weg (als die er zijn) en schrijf zo kort mogelijk op.

- $2a + 3b + 6a$ ....

  ....
- $2(2a + 3b)$ ....

  ....
- $3 \cdot 4b + 2a$ ....

  ....
- $2b + 2(4a + 3b)$ ....

  ....
- $2 \cdot 3a + 6b + 2a$ ....

  ....
- $5(a + 3b + 2a)$ ....

  ....
- $3 \cdot 3 \cdot 2a + 2b + a$ ....

  ....
- $5a + 3b + a + 7b$ ....

  ....
- $4(2b + 5a)$ ....

  ....
- $2a + 2 \cdot 6a$ ....

  ....

Zie omme zijde

- $a + 3(a + b) + 4b$ ....

  ....

- $a + 2a + 3a + 4a$ ....

  ....

- $2(a + 3b) + 3(2a + b)$ ....

  ....

- $5 \cdot 3b + 7 \cdot 2a + 4b$ ....

  ....

- $5(3a + 2 \cdot 2b)$ ....

  ....

- $3b + 9b$ ....

  ....

Posttest Algebra Animatie

Instructie

- Gebruik een pen.
- Vul je deelnemersnummer in rechts in het kader.
- Maak **zo veel mogelijk** van de onderstaande opgaven.
- Tussenstappen opschrijven mag, maar hoeft niet.

Deelnemersnummer:

Vereenvoudig de formules: werk de haakjes weg (als die er zijn) en schrijf zo kort mogelijk op.

- $3a + 6b + 2a$ ....

  ....
- $3(3a + 2b)$ ....

  ....
- $5 \cdot 2a + 3b$ ....

  ....
- $3b + 2(2a + 4b)$ ....

  ....
- $3 \cdot 4a + 3b + 3a$ ....

  ....
- $4(2a + b + 3a)$ ....

  ....
- $2 \cdot 5 \cdot 2b + 3a + b$ ....

  ....
- $2b + 3a + 4b + 2a$ ....

  ....
- $3(4b + 2a)$ ....

  ....
- $2 \cdot 7a + 4a$ ....

  ....

Zie omme zijde

- $b + 4(a+b) + 2a$ ....

  ....

- $5b + 4b + 3b + 2b$ ....

  ....

- $4(2a+b) + 2(3a+2b)$ ....

  ....

- $4 \cdot 4a + 6 \cdot 3b + 2a$ ....

  ....

- $3(2a + 2 \cdot 2b)$ ....

  ....

- $5b + 6b$ ....

  ....