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One-loop effects of fermions on gravitons in power-law inflation

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Contents

Abstract

We compute to one-loop order the quantum corrections to the graviton propagator due to massive fermions in FLRW spacetimes with constant ϵ parameter i.e. power-law inflation. We determine the fermion propagator in such spacetimes. The graviton is treated as a small perturbation around a classical FLRW background. The quartic vertex contribution to the graviton self-energy is computed using the 1PI effective action formalism. The quartic contribution to the self-energy is partially renormalized using a dimensional regularization procedure and appropriate counterterms. We anticipate to use this result to quantum correct the linearized Einstein field equations by using the Schwinger-Keldysh formalism. This could lead to secular corrections to dynamical gravitons during inflation and the force of gravity (at least to one-loop order).

Introduction

In 2017 the Nobel prize in physics was awarded to Kip Thorne, Rainer Weiss, and Barry Barish ”for decisive contributions to the LIGO detector and the observation of gravitational waves.” This marked the beginning of a new era for observations in cosmology. Coupled with the increasing precision of land-based experiments and a bright future for extraodinary space-based endeavours such as LISA, BBO etc. can only be a good thing as cosmology transitions from a ”pen-and-paper” science to a science backed by honest-to-goodness empirical observations.

Cosmologists around the globe are sitting on the edge of their seats in expectation for the plethora of data which will either rule out or confirm our models: from primordial inflation, to dark matter, to dark energy. Of particular interest to us is primordial inflation. A period of exponential expansion shortly after the Big Bang singularity introduced by Guth, Starobinski, and Linde [1, 2, 3] to resolve some issues with the Big Bang. Many models of inflation are floating around in the literature, but all with one thread in common; that quantum fluctuations during inflation are enhanced by the expansion of the universe. Which makes it crucial that we study quantum effects during this period of time as they formed the source for local density fluctuations and eventually large-scale structure and the temperature fluctuations in the cosmic microwave backround (CMB) spectrum. Moreover, particles are produced in abundance during inflation by gravitons and scalars present then [4, 5, 6]. These particles interact and generate precisely the quantum fluctuations we know are there. Thus, we arrive at the main motivation for this thesis: to study the effects of fermions on graviton propagation during inflation to 1-loop order. This field has been extensively studied by the likes of Woodard, Park, Prokopec, Miao and others [7, 8, 9, 10, 11, 12] who have studied such things as the interactions between gravitons and minimally coupled massless scalar fields, scalar field QED during inflation, massive non-minimally coupled scalar fields interacting with gravitons and many more.

Of particular interest is the work of Miao and Woodard who computed the 1-loop contribution to the massless/massive fermion self-energy $\left[{}_i\Sigma_j\right]$ due to interactions with gravitons [7, 12]. They used their renormalized results to quantum correct the linearized Dirac equations using the so-called Schwinger-Keldysh formalism. Their results show fermion mode functions are enhanced by a factor of $\sim \ln a$ which grow with time. Similarly Woodard, Park, Prokopec [10] computed the renormalized graviton self-energy $[{}^{\mu\nu}\Sigma^{\rho\sigma}]$ to 1-loop order in a massless minimally coupled scalar field in de Sitter. Quantum corrections come about by solving the linearized Einstein field equations by using the Schwinger-Keldysh formalism

$$\sqrt{-g}\mathcal{L}^{\mu\nu\rho\sigma}h_{\rho\sigma}(x) - \int \mathrm{d}^4x'[{}^{\mu\nu}\Sigma^{\rho\sigma}](x;x')h_{\rho\sigma}(x') = 0$$

where $\mathcal{L}^{\mu\nu\rho\sigma}$ is known as the Lichnerowicz operator (which in general spacetimes is rather complicated) and $h_{\mu\nu}(x)$ the full graviton field which is treated in linear perturbation theory as a small fluctuation around some background metric (usually de Sitter or FLRW). They concluded no significant effects from this interaction [9].

Park and others also applied these studies to look at gravitational potentials sourced by a point mass M during de Sitter inflation [10] and found the gravitational potentials also acquire corrections,

$$\Phi_{\rm dS}(x) = \frac{-GM}{ar}\left\{1 + \frac{\hbar}{20\pi c^3}\frac{G}{(ar)^2} + \frac{\hbar G H^2}{\pi c^5}\left[-\frac{1}{30}\ln a\right] - \frac{3}{10}\ln\left(\frac{Har}{c}\right) + \mathcal{O}\left(G^2, \frac{1}{a^3}\right)\right\},$$
$$\Psi_{\rm dS}(x) = \frac{-GM}{ar}\left\{1 - \frac{\hbar}{60\pi c^3}\frac{G}{(ar)^2} + \frac{\hbar G H^2}{\pi c^5}\left[-\frac{1}{30}\ln a - \frac{3}{10}\ln\left(\frac{Har}{c}\right) + \frac{2}{3}\frac{Har}{c}\right] + \mathcal{O}\left(G^2, \frac{1}{a^3}\right)\right\}.$$

If one compares these with the flat space results computed in [8, 13],

$$\Phi_{\rm Mink}(x) = -\frac{GM}{r}\left\{1 + \frac{\hbar}{20\pi c^3}\frac{G}{r^2} + \mathcal{O}(G^2)\right\},$$
$$\Psi_{\rm Mink}(x) = -\frac{GM}{r}\left\{1 - \frac{\hbar}{60\pi c^3}\frac{G}{r^2} + \mathcal{O}(G^2)\right\},$$

one notes again the appearance of secular corrections $\sim \ln a$ which grow with time. Initially these corrections are suppressed by a very small factor $\frac{\hbar G H^2}{\pi c^5} \sim 10^{-10}$, however $\ln a$ grows linearly with the number of e-foldings during inflation, as a result these secular correction eventually become significant after about $\ln a \sim \frac{1}{GH^2}$ e-foldings. An important interpretation of this result: because these secular corrections contribute equally one can reinterpret this result as a time dependent renormalization of the mass M or equivalently the Newton's constant G,

$$M \to M\left[1 - \frac{\hbar}{c^5}\frac{GH^2}{30\pi}\ln a\right],$$
$$G \to G\left[1 - \frac{3\hbar}{c^5}\frac{GH^2}{10\pi}\ln\left(\frac{Har}{c}\right)\right],$$

i.e. these secular corrections lead to a screening of the point mass M or equivalently Newton's constant G.

This work will attempt to apply this same formalism to interactions between massive fermions and gravitons which oddly enough has yet to be studied in this way eventhough most particles we know of are fermionic in nature! However, this work intends to be more general as we choose to study the more general FLRW spacetime with constant ϵ parameter i.e. power-law inflation $(0 < \epsilon < 1,\ \dot{\epsilon} = 0)$.

This thesis is structured as follows. In the first chapter we sketch the current standard cosmological paradigm; FLRW spacetime, classical inflation, and cosmological perturbations are discussed. In the second chapter we provide the necessary background to do fermionic field theory in general curved backgrounds and we discuss the case of FLRW spacetime and compute the propagator there. In chapter three we introduce perturbative quantum gravity and compute the main focus of this thesis: one loop corrections to the graviton self-energy. In chapter four we detail how to renormalize the self-energy, particularly the choice of counter-terms and difficulties we encountered throughout. Finally in chapter five we discuss our results and detail areas in which future work is needed.

Note some conventions, we work in units $\hbar = c = 1$ and the signature of the metric $(-,+,+,+)$.

Chapter 1

The Cosmological Paradigm

This chapter serves as a basic introduction to standard cosmology. We introduce FLRW cosmology and derive the Einstein field equations. We also inject matter into a toy model by assuming it behaves like a perfect fluid on cosmological scales. With this we derive the useful Friedmann equations. Big bang cosmology is introduced and its pitfalls discussed. Finally inflation is introduced to resolve the issues with big bang cosmology and the topic of cosmological perturbations is discussed. Note, we follow quite closely [14, 15].

1.1 Friedmann-Lemaître-Robertson-Walker spacetime

If we look out into the cosmos the universe seems to be the same in all directions (at least as far as we can see) everywhere. Since we are not in any way special we must assume that this is the case for all "inertial" observers at every point in spacetime thus we arrive at the first assumption in cosmology; the universe is isotropic. Furthermore, the universe must be homogeneous (the second assumption in cosmology) since the geometric properties of the universe must be the same everywhere. Together these two assumptions (known as the cosmological principle) lay the groundwork for the large scale properties of our universe. These assumptions may seem trivial but it severely restricts the possible configurations our universe can inhabit. Modern observational data further restricts these configurations by requiring that the universe is expanding and be mostly flat [16].

These basic assumptions force us to choose a (luckily convenient) spacetime coordinate system, first described by Friedmann, Walker, and Robertson, in which the line element takes the form:

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\mu = -\mathrm{d}t^2 + a^2(t)\delta_{ij}\mathrm{d}x^i \mathrm{d}x^j = a^2(\eta)\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu, \tag{1.1}$$

where $g_{\mu\nu}$ is the global spacetime metric, $\eta_{\mu\nu}$ is the flat space Minkowski metric $\mathrm{diag}(-1, 1, \dots, 1)$, a is the scale factor, t is the cosmological time, and η is known as the conformal time which is related to cosmological time by $\mathrm{d}t = a\mathrm{d}\eta$. An important quantity to define is the Hubble rate,

$$H \equiv \frac{\dot{a}}{a} = \frac{a'}{a^2}, \tag{1.2}$$

with overdots signifying derivatives w.r.t physical time (t) and dashes conformal time (η). The Hubble rate is positive for expanding spacetime and negative for a collapsing spacetime.

The dynamics of this universe are governed by the Einstein field equations. The purely gravitational sector of which is given by the Einstein-Hilbert action,

$$S_{EH} = \frac{1}{\kappa^2}\int \mathrm{d}^D x\sqrt{-g}R, \tag{1.3}$$

with $\kappa^2 = 16\pi G_N$, G_N is Newton's gravitational constant, R the Ricci scalar. Varying the action (1.3) with respect to the inverse metric tensor $g^{\mu\nu}$ one arrives at the vacuum Einstein field equations

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu} = 0, \tag{1.4}$$

where $G_{\mu\nu}$ is the familiar Einstein tensor, and the geometric quantities are given by,

$$R^{\rho}{}_{\sigma\mu\nu} = \partial_{[\mu}\Gamma^{\rho}{}_{\nu]\sigma} + \Gamma^{\rho}{}_{\lambda[\nu}\Gamma^{\lambda}{}_{\nu]\sigma}, \tag{1.5}$$

$$R_{\mu\nu} = R^{\lambda}{}_{\mu\lambda\nu} = 2\partial_{[\lambda}\Gamma^{\lambda}{}_{\nu]\mu} + 2\Gamma^{\lambda}{}_{\rho[\lambda}\Gamma^{\rho}{}_{\nu]\mu}, \tag{1.6}$$

$$R = g^{\mu\nu}R_{\mu\nu}, \tag{1.7}$$

$$\Gamma^{\rho}{}_{\mu\nu} = \frac{1}{2}g^{\rho\sigma}\Big(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu}\Big). \tag{1.8}$$

Where $R^{\rho}{}_{\mu\nu\sigma}$ is the Riemann tensor, $R_{\mu\nu}$ the Ricci curvature tensor, R the Ricci scalar, and $\Gamma^{\rho}{}_{\mu\nu}$ the metric compatible Levi-Civita connection and parentheses (square brackets) denote (anti)-symmetrization of indices,

$$A_{(\mu\nu)} \equiv \frac{1}{2}\Big(A_{\mu\nu} + A_{\nu\mu}\Big), \quad A_{[\mu\nu]} \equiv \frac{1}{2}\Big(A_{\mu\nu} - A_{\nu\mu}\Big). \tag{1.9}$$

If one wishes to also include matter fields then the total action is given by,

$$S_T = S_{EH} + S_M, \tag{1.10}$$

then the Einstein field equations become:

$$G_{\mu\nu} = \frac{\kappa^2}{2}T_{\mu\nu}, \tag{1.11}$$

with $T_{\mu\nu}$ is the stress-energy tensor associated with the matter fields and is defined by,

$$T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta S_M}{\delta g^{\mu\nu}}. \tag{1.12}$$

A typical assumption in cosmology is that the matter sector (at least on cosmological scales) behaves like a perfect fluid whose stress-energy tensor can be cast into the form,

$$T_{\mu\nu} = (\rho_M + P_M)u_\mu u_\nu + P_M g_{\mu\nu}, \tag{1.13}$$

with ρ_M being the matter energy density, P_M the pressure, and u_μ the plasma four velocity, which in the reference frame of the fluid becomes $u_\mu = \big(-1, 0, 0, 0\big)$, such that $u_\mu u^\mu = -1$. The evolution of the universe i.e. the dynamics of the scale factor are given by the Einstein field equations (1.11).

Solving for the (00)-component of (1.11) in $D = 4$ we obtain the first Friedmann equation,

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa^2}{2}\frac{\rho_M}{3} - \frac{k}{a^2}, \tag{1.14}$$

with k is the spacial curvature paramter ($k = 0$ for a flat universe). Taking the trace of (1.11) we get the second Friedmann equation,

$$\dot{H} + H^2 \equiv \frac{\ddot{a}}{a} = -\frac{1}{6}\frac{\kappa^2}{2}(\rho_M + 3P_M) \tag{1.15}$$

Combining (1.14), (1.15) with the covariant energy conservation condition $\nabla_\mu T^{\mu\nu} = 0$ one finds the continuity equation

$$\dot{\rho}_M = -3H(1 + w)\rho_M, \tag{1.16}$$

with $w = P_M/\rho_M$ is the equation of state parameter. Thus, we find the time evolution of the scale factor $a(t)$

$$a(t) \propto \begin{cases} t^{2/3(1+w)} & w \neq -1, \\ e^{Ht} & w = -1. \end{cases} \tag{1.17}$$

Thus for a flat universe ($k = 0$) we find the following forms of the scale factor for different eras:

- Matter domination: $a(t) \propto \sqrt{t}$,
- Radiation domination: $a(t) \propto t^{2/3}$,
- Cosmological constant domination: $a(t) \propto e^{Ht}$.

1.2 Big Bang Cosmology

We know the universe is expanding, therefore, if we work with this logic backwards in time we conclude that the universe had a beginning of infinite density, energy, and pressure, all confined to a singular point.

This is known as the Big Bang singularity. However, this simple logical argument does not agree well with observations unless we assume very fine-tuned initial conditions which lead to what we see today. This is of course something which makes many physicists incredibly uncomfortable as it implies our universe and our existence to be a mear unlikely accident. These are philosophical considerations we as physicists do not wish to deal with so we seek solutions away from fine-tuning. This section will present simple arguments detailing these fine-tuning problems and the standard way to resolve them: inflation.

1.2.1 Fine-tuned universe: Issues

Horizon problem

It is convenient to define the so called *comoving particle horizon* as the maximum comoving distance light can propagate in a given time-interval

$$\tau = \int_0^t \frac{\mathrm{d}t'}{a(t')} = \int_0^t \mathrm{d}\ln a \left(\frac{1}{aH}\right). \tag{1.18}$$

Recall, we model the universe as dominated by a perfect fluid. Thus,

$$(aH)^{-1} = H_0^{-1} a^{\frac{1}{2}(1+3w)}. \tag{1.19}$$

As a result, it is clear since the comoving Hubble rate increases monotonically with time that the comoving horizon τ (which describes the ratio of the universe in causal contact) also grows in the same way. If we reverse this argument the conclusion is that the universe was less causally connected than it is now. However, if we look at the CMB for example we see that it is extremely homogeneous. The question is: how can causally disconnected regions at the moment of last-scattering reproduce the homogeinity we see in the CMB?

We either accept that these completely different regions of the universe all had the same inital conditions and evolved in the exact same way i.e. extreme fine-tuning, or there is another mechanism at play here.

Flatness problem

Consider for a moment the first Friedmann equation (1.14) (where we have set $\kappa^2/2 \equiv 1$ for simplicity)

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho_M}{3} - \frac{k}{a^2}. \tag{1.20}$$

We can rewrite this as,

$$1 - \Omega = -\frac{k}{(aH)^2}, \tag{1.21}$$

with $\Omega \equiv \rho_M/3H^2$. The quantity Ω clearly grows with time if the comoving Hubble rate increases and thus must diverge today. However, current observational data tells us Ω is of order one [17]. Again, for this to be the case we require extreme fine-tuning of Ω in the early universe. Remember, this makes physicists very nervous.

1.3 Inflation

The cause of the horizon and flatness problem is precisely the fact that the comoving Hubble radius increases monotonically with time. Logically a "simple" solution is: What if there is a period early in the universe where the comoving Hubble radius *decreases*? This is what we call the period of inflation.

With this assumption in mind, let us re-examine the horizon and flatness problems.

- **Horizon problem:** A decreasing comoving Hubble radius implies that large scales entering the horizon today were inside the horizon before inflation and thus in causal contact and homogeneity of the CMB is explained.

- **Flatness problem:** Clearly a decreasing comoving Hubble radius itself drives the universe toward flatness,

$$1 - \Omega = \frac{-k}{(aH)^2}. \tag{1.22}$$

  i.e. $\Omega \to 1$ if $(aH)^{-1}$ decreases, and the flatness problem is solved.

This is nice, a rather simple assumption resolves both of the issues we detailed previously. A logical follow-up question is: Under what conditions can this be achieved? Let us first look at the crucial assumption

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{aH}\right) = \frac{-\ddot{a}}{(aH)^2}. \tag{1.23}$$

Thus from this relation we require *accelerated expansion* ($\ddot{a} > 0$). We can relate accelerated expansion to the first derivative of the Hubble rate H by,

$$\frac{\ddot{a}}{a} = H^2(1-\epsilon), \quad \text{with } \epsilon \equiv -\frac{\dot{H}}{H^2}. \tag{1.24}$$

So we can characterize accelerated expansion by the so-called principal parameter for inflation which must satisfy,

$$\epsilon < 1. \tag{1.25}$$

Furthermore, the condition $\ddot{a} > 0$ also requires that the strong energy condition be violated

$$\rho_M + 3p_M < 0 \to \rho_M(3w+1) < 0 \tag{1.26}$$

or equivalently the equation of state parameter $w < -1/3$.

1.3.1 A model for inflation: Slow-rolling scalar field

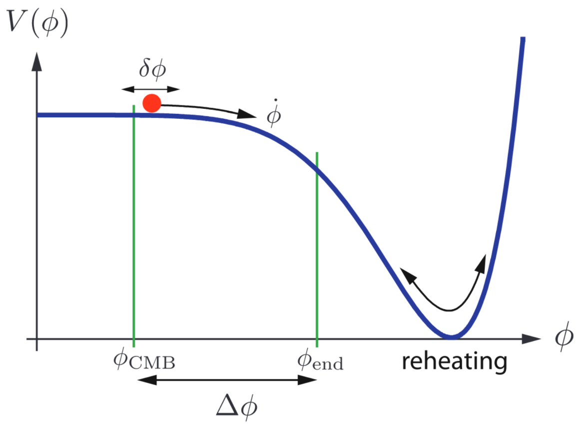

Figure 1.1: An example of inflaton potential which satisfy the conditions set for inflation. Inflation occurs while the field ϕ slowly rolls into the minimum at which point inflation ends and a period of reheating begins. Notice that quantum fluctuations $\delta\phi$ can modify when inflation starts creating the seeds for density perturbation $\delta\rho$.

In cosmology the simplest models of inflation assume some scalar "inflaton" field[1] ϕ coupled to gravity and whose dynamics is characterized by the action,

$$S = S_{EH} + \int \mathrm{d}^4x\sqrt{-g}\Big\{-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\Big\}. \tag{1.27}$$

The potential $V(\phi)$ describes the self-interactions of the scalar field. The features of this potential which will be constructed such as to generate inflation. If we compute the stress-energy tensor $T_{\mu\nu}$ and the equations of motion for this scalar field, assume the FLRW metric, and use that the energy-momentum tensor takes the perfect fluid form we find,

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{1.28}$$

$$P_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{1.29}$$

Resulting in the equation of state parameter

$$w_\phi \equiv \frac{P_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot{\phi}^2 + V(\phi)}{\frac{1}{2}\dot{\phi}^2 - V(\phi)}, \tag{1.30}$$

which clearly implies that accelerated expansion $w_\phi < -1/3$ can be achieved if the potential energy V dominates over the kinetic energy of the inflaton field. This is known as slow-roll inflation i.e. the inflaton field slowly rolls down the potential V Figure 1.1 carrying out inflation as it does so, and inflation

[1] The precise nature of the scalar field is not particularly important here.

ends when the particle falls into the minimum of the potential (when the conditions are violated). The slow-roll parameter ϵ is related to the inflaton field

$$\epsilon = \frac{1}{2M_P^2}\frac{\dot{\phi}^2}{H^2} \tag{1.31}$$

A final consideration is that inflation should last a "sufficient" amount of time which can be characterized by introducing the second slow-roll parameter η. One can find this condition from looking at the equation of motion for the scalar field, which in FLRW is given by,

$$\ddot{\phi} + 3H\dot{\phi} - \partial_\phi V = 0 \Rightarrow \ddot{\phi} \ll 3H\dot{\phi},\ -\partial_\phi V. \tag{1.32}$$

Which defines the second slow-roll parameter

$$\eta = -\frac{\ddot{\phi}}{\dot{\phi}H} = \epsilon - \frac{1}{2\epsilon}\frac{\mathrm{d}\epsilon}{\mathrm{d}N}. \tag{1.33}$$

i.e. the second slow-roll parameter should satisfy $\eta < 1$. Expressed in terms of first slow-roll parameter (ϵ) its function is clear: the fractional change of ϵ per e-fold N is to be kept small, this quantity is often called the second *geometric* slow-roll parameter $\epsilon_2 \equiv \partial_N \ln(\epsilon)$.

The first and second slow-roll parameter can also be related to the potential by

$$\epsilon_V = \frac{M_\mathrm{P}^2}{2}\left(\frac{\partial_\phi V}{V}\right)^2, \tag{1.34}$$

$$\eta_V = M_\mathrm{P}^2\frac{\partial_\phi^2 V}{V}. \tag{1.35}$$

Using this we can restrict the possible potentials that can produce inflation.

Power law inflation

At this point we find it pertinent to mention that in this thesis we "strictly speaking" are not working in slow-roll inflation but *power-law* inflation which assumes the slow-roll parameter ϵ satisfies,

$$0 < \epsilon < 1, \quad \dot{\epsilon} = 0. \tag{1.36}$$

If we assume inflation is sourced by the aforementioned inflaton field ϕ then the potential in power-law inflation becomes exponential, and the scale-factor $a \sim t^{1/\epsilon}$ (hence power-law). Such theories are no longer favorable in light of bounds on the tensor-to-scalar ratio r set by CMB data [17] at least insofar as the scalar field models are concerend. Nonetheless studying such models simplifies things and an argument can be made that one can promote ϵ to be a function of time after the fact, however to compute quantum corrections complications arise with the renormalization scheme, for more details on this we refer the reader to [18] where such an argument is presented.

Remark:

There are still many questions surrounding the precise nature of the inflaton field: what is the inflaton? what is the shape of the potential? is a scalar field the only way to produce inflation? how does inflation affect cosmological perturbations? Which naturally leads us into the next section.

1.4 Cosmological Perturbations

Quantum fluctuations generated during inflation become amplified. One can see this by consider quantum fluctuations of e.g. the inflaton field $\delta\phi$ around a classical background $\bar{\phi}(t)$. These fluctuations lead to inflation coming to and end at slightly different times in different parts of the universe i.e. resulting in large-scale relative density perturbations $\delta\rho$. These are evident when we look at the CMB sky and the distribution of structure. We can extract the quantum effects of our models on cosmological perturbations and test these against observable data by the approach of linear perturbation theory. This section will concern the topic of these cosmological perturbations, how we can make predictions given a model by computing the scalar and tensor power spectra, and finally the most recent bounds on such quantities from modern cosmological observations.

1.4.1 Linear perturbation theory

In linear perturbation theory we choose to expand space-time dependent quantities $X(t, \mathbf{x})$ into a homogeneous background part $\bar{X}(t)$ that depends only on time and a *small* perturbation from the background $\delta X(t, \mathbf{x}) \lll \bar{X}(t)$. For example consider a scalar field ϕ (usually the inflaton) coupled to gravity. We expand the fields as,

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}, \quad \phi = \bar{\phi} + \delta\phi \tag{1.37}$$

with $\delta g_{\mu\nu} \ll 1,\ \delta\phi \ll \bar{\phi}$. These fluctuations are naturally generated by quantum effects.

Remark: The issue of gauge choice

It is important to note that such an expansion is not unique i.e. it is a *gauge choice*. This is of course a problem for us since we would like to determine truths about the universe and truth cannot be relative. Thus we would like to define quantities which are invariant of our own gauge choice such that we are assured our results are not muddied by our own perspectives.

1.4.2 Scalar-Vector-Tensor decomposition

The FLRW metric is endowed with some nice symmetries which allows us to decompose it and perturbations of it into 4 independent scalar quantities, 2 divergence-free vector quantities, and 1 transverese-traceless tensor quantity. One can check the validity of such a decomposition by a simple counting argument 4 scalars + 2 divergence-free vectors which each carry 2 degrees of freedom and 1 tensor which carries 2 polarizations $D.o.F. = (4 + 2 \times 2 + 2) = 10$ which is precisely the number of degrees of freedom the metric carries. An important consequence of this decomposition as that each individual quantity evolves independently from the others i.e. scalars decouple from vectors and tensors, and vectors decouple from the tensor, and as a result so do their perturbations. This coupled with linear perturbation theory greatly simplifies computations of cosmological perturbations.

1.4.3 Metric and Matter perturbations

One can decompose the metric peturbations using SVT as follows:

$$\mathrm{d}s^2 = -(1+2\Phi)\mathrm{d}t^2 + 2a(\partial_i B - S_i)\mathrm{d}x^i\mathrm{d}t + a^2\Big[(1-2\Psi)\delta_{ij} + 2\partial_i\partial_j E + 2\partial_{(i}F_{j)} + h_{ij}^{TT}\Big]\mathrm{d}x^i\mathrm{d}x^j \quad (1.38)$$

The quantities are:

- 4 scalars quantities Φ, Ψ, B, E,
- 2 divergence-less vector quantities S_i, F_i satisfying $\partial_i F^i = \partial_i S^i = 0$,
- 1 transverse-traceless tensor quantity h_{ij}^{TT} satisfying $\partial^i h_{ij} = \partial^j h_{ij} = h^i{}_i = 0$.

Note, it can be shown the vector perturbations S_i, F_i are subdominant during inflation i.e. they go like $1/a$ if the stress-energy tensor does not contain anisotropic stress (which is the case in scalar models of inflation), thus these will not be relevant for us. We are left with,

$$\mathrm{d}s^2 = -(1+2\Phi)\mathrm{d}t^2 + 2a\partial_i B\mathrm{d}x^i\mathrm{d}t + a^2\Big[(1-2\Psi)\delta_{ij} + \partial_i\partial_j E + h_{ij}^{TT}\Big]\mathrm{d}x^i\mathrm{d}x^j. \quad (1.39)$$

As discussed previously these quantities are dependent on our gauge choice (except the tensor perturbation which is invariant). Therefore we would like to remove this dependence. Consider a general coordinate transformation

$$t \to t' = t + \alpha, \quad (1.40)$$
$$x^i \to x'^i = x^i + \partial^i\beta, \quad (1.41)$$

then the scalar quantities transform as,

$$\Phi \to \Phi' = \Phi - \alpha, \quad (1.42)$$
$$B \to B' = B + a^{-1}\alpha - a\dot{\beta}, \quad (1.43)$$
$$E \to E' = E - \beta, \quad (1.44)$$
$$\Psi \to \Psi' = \Psi + H\alpha \quad (1.45)$$

Perturbations in the matter content of the universe is given by the perturbations to the stress-energy tensor which in the perfect fluid form amount to perturbation in the pressure δP, density $\delta\rho$, and momentum density δq (defined as the scalar part of the 3-momentum density $T^0_i = \partial_i \delta q$). Under gauge transformation these transform as,

$$\delta\rho \to \delta\rho' = \delta\rho - \dot{\bar{\rho}}\alpha, \tag{1.46}$$

$$\delta P \to \delta P' = \delta P - \dot{\bar{P}}\alpha, \tag{1.47}$$

$$\delta q \to \delta q = \delta q + (\bar{\rho} + \bar{P})\alpha. \tag{1.48}$$

Constructing gauge invariant quantities

By combining matter and metric perturbations one can construct quantities which are invariant under gauge transformations (which is what we want). We will discuss here one such quantity used in the literature:

- The so-called *comoving curvature perturbation*

$$\mathcal{R} \equiv \Psi - \frac{H}{\bar{\rho} + \bar{P}}\delta q, \tag{1.49}$$

  Which, for a scalar field during inflation is given by,

$$\mathcal{R} = \Psi + \frac{H}{\dot{\bar{\phi}}}\delta\phi, \tag{1.50}$$

  or alternatively using (1.31),

$$\mathcal{R} = \Psi + \frac{1}{\sqrt{2\epsilon}M_P}\delta\phi. \tag{1.51}$$

  Note that under gauge transformations $\mathcal{R}$ is indeed invariant. Th comoving curvature is a measure of the spatial curvature of a comoving hypersurface Σ. Furthermore, choosing this gauge reduces the dynamics of the problem to only two physical fields, the graviton h^{TT}_{ij} and the gravitational potential Ψ.

One important property of this quantity is that it is conserved on super-Hubble scales, $k < aH$, in slow-roll inflation. In other words, the comoving curvature perturbation is frozen after crossing the Hubble radius and one can be assured it stays the same at late times when it reenters.

1.4.4 Scalar and Tensor Power Spectra

We now turn our attention to computing the spectrum of the scalar and tensor fluctuations during inflation. As mentioned before, scalar fluctuations are important because they describe for example the temperature fluctuations in the CMB sky. Tensor fluctuations on the other hand are far more fascinating as these generate primordial gravitational waves which are encoded with (in principal) information about the energy scale of inflation. These have as of yet not been discovered but we are now in a new era of observational science with breakthroughs such as LIGO and in future LISA and others.

The Scalar Power Spectrum

Recall the slow-roll model of inflation which is given by the action

$$S = S_{EH} + \int \mathrm{d}^4x\sqrt{-g}\Big[-\frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - V(\phi)\Big]. \tag{1.52}$$

If we choose to work in the comoving gauge i.e. we leave the inflaton unperturbed $\delta\phi = 0$ and push all the degrees of freedom onto the metric $g_{ij} = a^2(1-2\Psi)\delta_{ij}$. Notice, since $\delta\phi = 0$ we can make the replacement $\Psi \to \mathcal{R}$ then the action expanded to second order in $\mathcal{R}$ [19] to

$$S^{(2)} = \frac{1}{2}\int \mathrm{d}^4x a^3 2\epsilon M_P^2\Big[\dot{\mathcal{R}}^2 - a^{-2}(\partial_i\mathcal{R})^2\Big]. \tag{1.53}$$

Note that this action is not in canonical form and is therefore not ideal for quantization. However, if we define a suitable Mukhanov variable

$$v \equiv z\mathcal{R}, \quad \text{with } z^2 = 2a^2\epsilon M_P^2, \tag{1.54}$$

and turn to conformal time τ, then the action becomes

$$S^{(2)} = \int \mathrm{d}\tau\mathrm{d}^3x\Big[(v')^2 + (\partial_i v)^2 + \frac{z''}{z}v^2\Big]. \tag{1.55}$$

Varying with respect to the Mukhanov field v gives the equation of motion

$$\Big(\partial_\tau^2 - \nabla^2 - \frac{z''}{z}\Big)v = 0 \tag{1.56}$$

Canonical quantization now proceeds normally. Namely, one promotes the field $v \to \hat{v}$ to a quantum operator then expands in terms of mode functions

$$\bar{v} = \int \frac{\mathrm{d}^3k}{(2\pi)^3}\Big[v(k,\tau)\hat{a}_{\mathbf{k}}e^{i\mathbf{k}\cdot\mathbf{x}} + v^*(k,\tau)\hat{a}^\dagger_{\mathbf{k}}e^{-i\mathbf{k}\cdot\mathbf{x}}\Big] \tag{1.57}$$

where the creation and annihilation operators obey,

$$\hat{a}_{\mathbf{k}}\left|\Omega\right\rangle = 0, \quad \Big[\hat{a}_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{k}'}\Big] = (2\pi)^3\delta^3(\mathbf{k}-\mathbf{k}'), \quad \big[\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{k}'}\big] = \Big[\hat{a}^\dagger_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{k}'}\Big] = 0. \tag{1.58}$$

Using this one can solve[2] for the comoving curvature perturbation $\mathcal{R}$.

[2]For a detailed treatment of this we refer the reader to [14, 15].

The primordial scalar power spectrum $\mathcal{P}_{\mathcal{R}}(k)$ is related to the two-point function by,

$$\langle\Omega|\,\hat{\mathcal{R}}(t,\mathbf{x})\hat{\mathcal{R}}(t,\mathbf{x}')\,|\Omega\rangle = \int \frac{\mathrm{d}k}{k}\mathcal{P}_{\mathcal{R}}(k,\tau)\frac{\sin(kr)}{kr}, \tag{1.59}$$

with $r = |\mathbf{x}-\mathbf{x}'|$. One finds the scalar power spectrum on super-Hubble scales, $(1-\epsilon)k|\tau| \ll 1$,

$$\mathcal{P}_{\mathcal{R}}(k,\tau) = \mathcal{P}^*_{\mathcal{R}}\left(\frac{k}{(1-\epsilon)aH}\right)^{n_s-1}, \tag{1.60}$$

with

$$\mathcal{P}^*_{\mathcal{R}} = \frac{1+2\epsilon(5-3\ln 2-3\gamma_E)-2\eta(2-\ln 2-\gamma_E)}{8\pi^2}\frac{H^2}{M_P^2}\frac{1}{\epsilon} \approx \frac{H^2}{8\pi^2M_P^2}\frac{1}{\epsilon}+\mathcal{O}(\epsilon^0), \tag{1.61}$$

$$n_s-1 = (-6\epsilon+2\eta)+\frac{2}{3}(-13\epsilon^2+4\epsilon\eta+\eta^2) \approx (-6\epsilon+2\eta)+\mathcal{O}(\epsilon^2,\eta^2,\dots). \tag{1.62}$$

where n_s is the so-called scalar spectral index, ϵ,η are the first and second slow-roll parameters respectively, and $\gamma_E \approx 0.577\dots$ the so-called Euler-Mascheroni constant.

Remarks:

- Notice the dependence on the slow-roll parameters (ϵ,η) which shows clearly the dependence on the particular potential during inflation.
- It is also evident how the energy scale of inflation comes into play here through the dependence on the Hubble rate H.
- Finally the scalar power spectrum is nearly scale invariant. Namely, we know the spectral index from observations to be $n_s \approx 0.96$ i.e. $n_s = 1$ corresponds to a scale invariant power spectrum.

The Tensor Power Spectrum

Recall tensor fluctuations are given by expanding the Einstein-Hilbert action to second order in the fluctuation h_{ij}^{TT},

$$S_{EH}^{(2)} = \frac{M_P^2}{8}\int \mathrm{d}\tau\mathrm{d}^3x a^2\Big[((h_{ij}^{TT})')^2-(\nabla h_{ij}^{TT})^2\Big]. \tag{1.63}$$

In the same vein as for the scalar we promote the graviton $h_{ij}^{TT} \to \hat{h}_{ij}^{TT}$ to a quantum operator, and decompose into Fourier modes as,

$$\hat{h}_{ij}^{TT} = \frac{2}{M_P}\sum_{\alpha=+,\times}\int\frac{\mathrm{d}^3k}{(2\pi)^3}\Big[\epsilon_{ij}^{\alpha}(\mathbf{k})h(k,\tau)\hat{a}_{\mathbf{k},\alpha}e^{i\mathbf{k}\cdot\mathbf{x}}+\epsilon_{ij}^{\alpha}(\mathbf{k})^*h^*(k,\tau)\hat{a}^\dagger_{\mathbf{k},\alpha}e^{-i\mathbf{k}\cdot\mathbf{x}}\Big] \tag{1.64}$$

where $\hat{a}_{\mathbf{k},\alpha}, \hat{a}^{\dagger}_{\mathbf{k},\alpha}$ are the annihilation and creation operators satisfying the standard commutation relations, and $\epsilon^{\alpha}_{ij}(k)$ are the two graviton polarisation tensors which satisfy,

$$\sum_{ij} \epsilon^{\alpha}_{ij}(\mathbf{k})\epsilon^{\alpha'}_{ij}(\mathbf{k})^* = \delta_{\alpha,\alpha'}, \quad \sum_{\alpha} \epsilon^{\alpha}_{ij}(\mathbf{k})\epsilon^{\alpha'}_{kl}(\mathbf{k})^* = \frac{1}{2}\Big[P_{ik}P_{jl} + P_{il}P_{jk} - P_{ij}P_{kl}\Big], \tag{1.65}$$

with $P_{ij} \equiv \delta_{ij} - k_i k_j / k^2$ is the momentum space transvere projection operator.Furthermore, the graviton mode functions $h(k,\tau)$ satisfy the differential equation

$$(\partial_0^2 + \mathbf{k}^2 - \frac{a''}{a})(ah(k,\tau)) = 0. \tag{1.66}$$

We will solve this for the case of power-law inflation i.e.

$$a(\tau) = ((\epsilon - 1)H_0\tau)^{\frac{1}{\epsilon-1}}, \quad H = H_0 a^{\epsilon} \tag{1.67}$$

with $\epsilon \ll 1$ and $\dot{\epsilon} \ll 1$. Thus, one can write,

$$\frac{a''}{a} = \frac{2-\epsilon}{(1-\epsilon)^2}\frac{1}{\tau^2} + \mathcal{O}(\dot{\epsilon}). \tag{1.68}$$

We recognize the differential equation is of the familiar Bessel type whose solutions (after properly normalizing) can be written in terms of the Hankel functions of the first and second kind

$$h(k,\tau) = \frac{1}{a}\sqrt{-\frac{\pi\tau}{4}}H^{(1)}_{\nu}(-k\tau), \quad h^*(k,\tau) = \frac{1}{a}\sqrt{-\frac{\pi\tau}{4}}H^{(2)}_{\nu}(-k\tau), \quad \nu = \frac{3-\epsilon}{2(1-\epsilon)}. \tag{1.69}$$

Again the power spectrum for tensor fluctuations is related to the two-point function,

$$\langle\Omega|\, h^{TT}_{ij}(t,\mathbf{x})h^{TT}_{ij}(t,\mathbf{x}')\,|\Omega\rangle = \frac{4}{M_P^2}\int \frac{\mathrm{d}^3k}{(2\pi)^3}|h(k,\tau)|^2 \sum_{\alpha} \epsilon^{\alpha}_{ij}\epsilon^{\alpha}_{kl} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}\frac{\sin kr}{kr} \tag{1.70}$$

$$\equiv \int \frac{\mathrm{d}k}{k}\mathcal{P}_h(k,\tau)\frac{\sin kr}{kr}\frac{1}{4}\Big[P_{ik}P_{jl} + P_{il}P_{jk} - P_{ij}P_{kl}\Big] \tag{1.71}$$

Thus, after much work we find the tensor power spectrum

$$\mathcal{P}_h(k,\tau) = \frac{4k^3}{\pi^2}\frac{|h(k,\tau)|^2}{M_P^2}. \tag{1.72}$$

On super-horizon scale $(-k\tau \ll 1)$ the spectrum becomes

$$\mathcal{P}_h(k,\tau) = \frac{H_0^2}{\pi^3 M_P^2} 2^{\frac{3-\epsilon}{1-\epsilon}} \Gamma^2(\frac{3-\epsilon}{2(1-\epsilon)})(1-\epsilon)^{\frac{2}{1-\epsilon}} \left(\frac{k}{H_0}\right)^{\frac{-2\epsilon}{1-\epsilon}}, \tag{1.73}$$

which to leading order in slow-roll,

$$\mathcal{P}_h(k,\tau) = \mathcal{P}_h^* \left(\frac{k}{k_*}\right)^{n_t}, \quad \mathcal{P}_h^* = \frac{2H_0^2}{\pi^2 M_P^2}\Big\{1 + 2\epsilon\big[1 - \gamma_E - \ln(2)\big]\Big\}, \quad n_t = -2\epsilon. \tag{1.74}$$

Note, as in the scalar case if we were to detect any deviation from perfect scale-invariance ($n_t = 0$). This would be indirect evidence for inflation.

Tensor-scalar ratio

A common measurable quantity mentioned in the literature is the ratio of the scalar to tensor power spectra r defined as

$$r \equiv \frac{\mathcal{P}_h^*}{\mathcal{P}_\mathcal{R}^*} \tag{1.75}$$

or to leading order in slow-roll,

$$r = 16\epsilon = -8n_t \tag{1.76}$$

Remarks:

We now have at our disposal two of the (arguably) most important measurable quantities for the study of the early universe. I will now take the time to discuss some experimental bounds on these observables.

1.4.5 Experimental bounds and support for inflation

Courtesy of the Planck 2018 results [16, 17] we have:

- $n_s = 0.9649 \pm 0.0042$ at 68% CL a sufficient deviation from perfect scale invariance $n_s = 1$. This is indirect evidence for inflation,
- $r < 0.064$, which indicates that the tensor power spectrum is far weaker than the scalar. This explains why we have yet to detect them, however the sensitivity of our experiments is increasing rapidly. It is only a matter of time!
- Parameter space plot r vs. n_s shown in Figure 1.2. Note power-law inflation is nowhere near an acceptable theory, and that the most favorable theories at the moment are of the attractor type (the yellow area), and hilltop (the green area).

Further remarks:

Recall that the aim of this thesis is to compute quantum corrections to the **full** graviton $h_{\mu\nu}$ due to the presence of fermions during inflation which naturally will provide corrections to both the scalar and

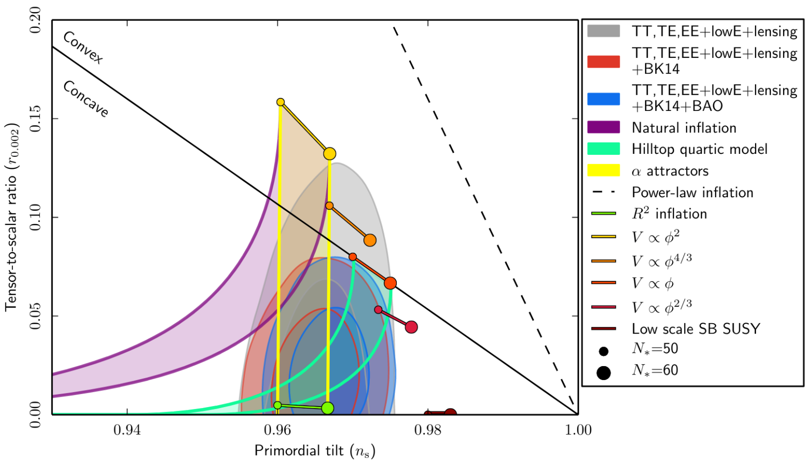

Figure 1.2: Planck 2018 [17] results plotting the bounds on the spectral index n_s and tensor-to-scalar ratio r at $k_* = 0.002$ Mpc^{-1} as compared to estimates from a variety of inflationary models.

tensor perturbations!

This will be done by first computing the renormalized graviton self-energy, then solving perturbatively the Einstein field equation without a source. If we expand the graviton into Fourier modes as was done for the tensor power spectrum we can arrive at one-loop corrections to the tensor and scalar power spectrum.

Chapter 2

Quantum Field Theory in Curved Spacetime

From the previous chapter we have clearly seen that in even the simplest cosmological settings we encounter spacetimes with curvature. In fact, solutions to the flatness and horizon problems require this. Thus, if we wish to do quantum field theory during inflation we must work in this setting. This of course brings with it many challenges. For example, common techniques developed for quantum field theory on flat spacetimes simply do not work in a cosmological framework. The notion of "in-out" scattering is problematic for example. A particularly relevant complication is that fermionic theory needs to be modified, namely Dirac theory assumes a flat background. This chapter will serve as a basis for extending quantum field of fermions to general D-dimensional curved spacetimes. In the last section we detail a quantum field theory of fermions in FRW cosmologies and present a derivation of the propagator for this case. Note that we keep most quantities in D-dimensions anticipating a dimensional regularization procedure.

2.1 Quantum field theory of fermions in a general curved spacetime

In this section we follow quite closely [20, 21].

Fermions in Minkowski spacetime are described by spinor fields ψ which transform in a representation of the Lorentz group. It is not immediately obvious how to describe these spinor fields in a full theory which couples to gravity when the standard approach to introducing curvature is to change ordinary derivatives to covariant ones. Namely, we do not know how the covariant derivative acts on spinors. This section serves to set up the language which allows us to "covariant-ize" the Dirac fields we know in Minkowski.

2.1.1 Towards a formal description: the vierbein

In attempting to generalize to curved spacetimes we lose the inherent connection our theory has to the Lorentz group. In order to recover this connection we recall that there exists for any metric tensor $g_{\mu\nu}(x)$ a local inertial frame (Riemann normal coordinates y^a_X) at each point X in spacetime in which the metric is locally flat. In this spirit we can describe a (generally complicated) metric tensor $g_{\mu\nu}$ in terms of the Minkowski metric $\eta_{\mu\nu}$ by,

$$g_{\mu\nu}(x) = e^a_{\ \mu}(x) e^b_{\ \nu}(x) \eta_{ab}, \qquad e^a_{\ \mu}(x) = \frac{\partial y^a_X}{\partial x^\mu} \tag{2.1}$$

where the Greek indices are Lorentz indices $(0, 1, 2, 3, \dots)$ and the Roman indices are coordinate indices. The objects $e^a_{\ \mu}$ are known as the vierbein or frame fields. Note that,

$$e^\mu_{\ a}(x) e_{\mu b}(x) = \eta_{ab}, \tag{2.2}$$

and that one can raise or lower Lorentz indices by contracting with the Minkowski metric similarly one raises or lowers coordinate indices by contracting with the global metric $g_{\mu\nu}, g^{\mu\nu}$. An important question to note at this time is; how does one parallel transport a quantity with a Lorentz index? We can extend the notion of covariant differentiation to such quantities by introducing the Lorentz or *spin connection* $(\omega_{\mu cd})$ defined by demanding the covariant derivative be compatible with the vierbein,

$$\nabla_\mu e^\nu_{\ a} = \partial_\mu e^\nu_{\ a} - \Gamma^\nu_{\mu\lambda} e^\lambda_{\ a} - \omega^b_{\ a\mu} e^\nu_{\ b} = 0 \Rightarrow \omega^b_{\ a\mu} = e^b_{\ \rho}(\partial_\mu e^\rho_{\ a} + \Gamma^\rho_{\ \mu\sigma} e^\sigma_{\ a}), \tag{2.3}$$

or written with all indices down,

$$\omega_{\mu cd} = e^\nu_{\ c}(\partial_\mu e_{\nu d} - \Gamma^\rho_{\ \mu\nu} e_{\rho d}). \tag{2.4}$$

The spin connection is also anti-symmetric under exchange of its Lorentz indices $(\omega_{\mu(cd)} = 0)$, this is required for Minkowskian metric compatibility to hold.

2.1.2 The Dirac Action in Curved Spacetime

We can now proceed with our treatment of fermions in general curved spacetimes. Our first step is to "covariant-ize" the familiar Dirac theory in flat space. The tangent space Dirac action is given by:

$$S_D = \int \mathrm{d}^D x \ \bar{\psi}(i\gamma^a \partial_a - m)\psi, \tag{2.5}$$

where the gamma matrices are defined by the standard commutations relations.

$$\left\{\gamma^a, \gamma^b\right\} = -2\eta^{ab}, \tag{2.6}$$

where η^{ab} is the usual Minkowski metric. We can naively write a covariant version of this by simply replacing ordinary derivatives with a covariant derivative $\partial_\mu \to \nabla_\mu$ and the invariant measure $\mathrm{d}^D x \to \mathrm{d}^D x\sqrt{-g}$,

$$S_D = \int \mathrm{d}^D x\sqrt{-g}\ \bar{\psi}(i\gamma^\mu \nabla_\mu - m)\psi, \tag{2.7}$$

where the anticommutation relations for the Dirac gamma matrices naturally generalizes to

$$\{\gamma^\mu, \gamma^\nu\} = -2g^{\mu\nu}, \tag{2.8}$$

where $g^{\mu\nu}$ is the global coordinate metric. Notice that the coordinate gamma matrices acquire space-time dependence through the vierbein,

$$\gamma^\mu = e^\mu_{\ a}(x)\gamma^a, \tag{2.9}$$

where γ^a are the familiar flat space gamma matrices. This is a good place to start, except we have no idea what this means! Namely, we don't know how a covariant derivative should act on a spinor. This will be the focus of the next section.

The spinor connection

In a similar manner to how one would determine the Dirac covariant derivative in QED we will determine the action of a "geometric" covariant derivative on a spinor by demanding that the quantity $\nabla_\mu \psi$ transform as a spinor i.e. is itself a spinor. Recall that a spinor under general coordinate transformations transforms as,

$$\psi \to \psi' = L\psi, \tag{2.10}$$

where $L = e^{i\epsilon_{ab}(x)J^{ab}}$ is the *spinor representation* of the Lorentz group, ϵ_{ab} are functions of space-time, and J^{ab} are the generators of the Lorentz algebra. Similarly the adjoint spinor $\bar{\psi}$ transforms as,

$$\bar{\psi} \to \bar{\psi}' = \bar{\psi}L^{-1}. \tag{2.11}$$

The ordinary derivative of a spinor does not transform as a spinor:

$$\partial_\mu \psi \to \partial_\mu \psi' = L\partial_\mu \psi + \partial_\mu L\psi. \tag{2.12}$$

We can introduce a *spinor connection* Γ_μ which transforms as,

$$\Gamma_\mu \to \Gamma'_\mu = (\partial_\mu L)L^{-1} + L\Gamma_\mu L^{-1}, \tag{2.13}$$

such that we can define a covariant derivative which when applied to a spinor ψ transforms like a spinor under general coordinate transformations,

$$\nabla_\mu \psi \to (\nabla_\mu \psi)' = L \nabla_\mu \psi. \tag{2.14}$$

The explicit form of the spinor connection can be determined from the metricity condition ($\nabla_\mu g_{\rho\sigma} = 0$) which defines the Christoffel connection, the tetrad postulate ($\nabla_\mu e^a_\nu = 0$) which defines the *spin* connection, and demanding the covariant derivative of the dirac matrices to vanish, since there always exists a coordinate transformation which takes γ^μ to a spacetime independent form. With this in mind one can show that the spinor connection is given by,

$$\Gamma_\mu = \frac{i}{2} \omega_{\mu cd} J^{cd}, \tag{2.15}$$

with,

$$\omega_{\mu cd} = e^\nu{}_c (\partial_\mu e_{\nu d} - \Gamma^\rho_{\mu\nu} e_{\rho d}), \tag{2.16}$$

$$J^{cd} = \frac{i}{4} \big[\gamma^c, \gamma^d\big]. \tag{2.17}$$

2.2 Fermions in FRW spacetimes

Using the language from the previous section we can turn our attention to a specific case for fermions in FRW cosmologies with constant deceleration. Recall that the action for fermions in general curved background is given by,

$$S_D = \int \mathrm{d}^D x \sqrt{-g}\ \bar{\psi}(i\gamma^\mu \nabla_\mu - m)\psi. \tag{2.18}$$

Note that the metric in FRW spacetime can be expressed in conformal time as,

$$g_{\mu\nu} = a^2(\eta)\eta_{\mu\nu}. \tag{2.19}$$

As a consequence it is clear that the vierbein field become functions of conformal time only and are given by,

$$e^a{}_\mu(\eta) = a(\eta)\delta^a{}_\mu, \quad e^\mu{}_a = a^{-1}(\eta)\delta^\mu_a. \tag{2.20}$$

Furthermore the covariant derivative reduces to a partial derivative

$$i\gamma^\mu \nabla_\mu \psi \to a^{-\frac{D+1}{2}} i\gamma^b \partial_b \Big(a^{\frac{D-1}{2}} \psi \Big). \tag{2.21}$$

Note that this implies the kinetic term is Weyl invariant (in arbitrary dimension) i.e. one can perform a classical conformal rescaling of the fermion field $\psi \to \psi_{cf} = a^{\frac{(D-1)}{2}}\psi$, as a result any dependence on the expansion disappears from the kinetic term.

To compute the massive fermion propagator we also need a few additional constraints. Namely, we assume the first slow-roll parameter ϵ to be constant and that the mass of the fermion is proportional to the Hubble parameter H,

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \mathit{constant}, \tag{2.22}$$

$$\frac{m}{H} = \mathit{constant}. \tag{2.23}$$

This can be accomplished by requiring the mass of the fermion to be generated by a suitable coupling to a Yukawa scalar field ϕ which goes like the Hubble parameter H up to a constant. The action for this scalar field (which is itself coupled to gravity) is given by:

$$S_\phi = \int \mathrm{d}^D x \sqrt{-g} \Big\{ -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi, R) - Y \bar{\psi} \psi \phi \Big\}, \tag{2.24}$$

with Y the Yukawa coupling constant and the potential for the field,

$$V(\phi, R) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{2}\xi^2 R\phi^2 + \frac{\lambda}{4!}\phi^4. \tag{2.25}$$

We can now identify the fermion mass,

$$m_\psi(\eta) = Y\phi(\eta). \tag{2.26}$$

The equations of motion for this field can be derived by variation of the action,

$$\Box\phi - \partial_\phi V(\phi, R) = 0. \tag{2.27}$$

The solution to this equation, if one assumes the mass of the scalar to be small as compared to the Hubble rate $(\mu^2) \ll \xi R$, is given by:

$$\phi = \pm H \sqrt{\frac{6}{\lambda}\big(\epsilon(3 - 2\epsilon) - 6\xi(2 - \epsilon)\big)} \tag{2.28}$$

Clearly the condition $\phi \sim H$ is satisfied. Note, this is true for the case of power-law inflation $\dot{\epsilon} = 0$ in slow-roll inflation this condition generalizes $\phi = \alpha(\epsilon, \dot{\epsilon})H$.

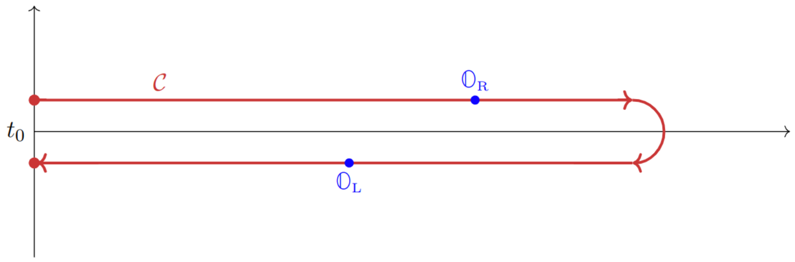

Figure 2.1: The Schwinger-Keldysh contour, notice the (+) and (−) operator insertion (in this figure R for right and L for left), operators inserted on the (+)-contour are said to be time ordered and operators inserted on the (−)-contour are anti-time ordered. Taken from [22].

Remark:

In this work we will be using the Schwinger-Keldysh [23, 24] formalism for defining appropriate Green's functions. Keldysh introduced this formalism to generalize time-ordering for non-equilibrium systems in which the traditional "in-out" prescription does not hold [1]. In this formalism one splits the Schwinger complex integration contour $\mathcal{C}$ (shown in Figure 2.1) over complex time into two parts one in the upper-half plane $\mathcal{C}_+$ which is related to time ordering of operators and one in the lower-half plane $\mathcal{C}_-$ which is related to anti-time ordering. One can thus define the Schwinger-Keldysh Green's function as,

$$G_{SK}(x;x') = \langle\Omega|\, \mathcal{T}_{\mathcal{C}}(\hat{O}_{\mathcal{C}}(x)\hat{O}^{\dagger}_{\mathcal{C}}(x'))\, |\Omega\rangle \tag{2.29}$$

with the subscript $\mathcal{C}$ denoting the place of operator insertion (either + or −). It is clear that the Schwinger-Keldysh Green's function can be denoted by a (2×2) matrix,

$$G_{SK}(x;x') = \begin{pmatrix} G_{++} & G_{+-} \\ G_{-+} & G_{--}. \end{pmatrix} \tag{2.30}$$

where G_{++}, G_{--} are the familiar Feynman and anti-Feynman propagators (also knows as the Dyson propagator) and two new cross-contour correlators are known as the positive and negative frequency Wightman functions [2]. For the purposes of our calculations it suffices to only consider the Feynman propagator since one can reconstruct the other three from it. Furthermore, for notational convenience we drop the (++) subscripts henceforth.

We also find it convenient to define the de Sitter length functions (with appropriate $i\varepsilon$) prescriptions:

[1] The Schwinger-Keldysh formalism is also known as the "in-in" formalism.

[2] For more information on this topic we refer the reader to an the introductory section of [22]

$$y_{++}(x;x') = \frac{1}{\eta\eta'}\big(-(|\eta-\eta'|-i\varepsilon)^2 + |\mathbf{x}-\mathbf{x}'|\big), \tag{2.31}$$
$$y_{--}(x;x') = \frac{1}{\eta\eta'}\big(-(\eta-\eta'+i\varepsilon)^2 + |\mathbf{x}-\mathbf{x}'|\big), \tag{2.32}$$
$$y_{-+}(x;x') = \frac{1}{\eta\eta'}\big(-(\eta-\eta'-i\varepsilon)^2 + |\mathbf{x}-\mathbf{x}'|\big), \tag{2.33}$$
$$y_{+-}(x;x') = \frac{1}{\eta\eta'}\big(-(|\eta-\eta'|+i\varepsilon)^2 + |\mathbf{x}-\mathbf{x}'|\big). \tag{2.34}$$

where here the $(\pm)$ subscripts are related to the Keldysh operator insertions i.e. $(++)$ yields the Feynman propagator, $(--)$ yields the Dyson propagator, and $(-+),(+-)$ the positive and negative frequency Wightman functions, respectively. As stated before we only need the Feynman propagator and thus we will adopt the shorthand $y_{++}(x;x') \equiv y(x;x')$. With this in mind we can move on to computing the fermion propagator in FRW spacetimes.

2.2.1 Feynman fermion propagator in FRW spacetime

The next ingredient we need for the main calculation performed in this thesis is the Feynman fermion propagator in FRW spacetime, since this will come into play in all one-loop diagrams. In this section we follow very closely the paper by Koksma and Prokopec [18], however we choose the less general Bunch-Davies vacuum instead of the more general α-vacuum chosen by the authors.

We are interested in the time ordered Feynman fermion propagator defined by,

$$iS_F^{ij}(x;x') = \langle\Omega|\,\mathrm{T}(\psi_i(x)\bar{\psi}_j(x'))\,|\Omega\rangle \tag{2.35}$$
$$= \Theta(\eta-\eta')\,\langle\Omega|\,\psi_i(x)\bar{\psi}_j(x')\,|\Omega\rangle - \Theta(\eta'-\eta)\,\langle\Omega|\,\bar{\psi}_j(x)\psi_i(x')\,|\Omega\rangle\,. \tag{2.36}$$

The Feynman propagator at tree level satisfies the equation of motion,

$$\sqrt{-g}[i\gamma^\mu\nabla_\mu^x - m]iS_F^{ij}(x;x') = i\delta^D(x-x')I^{ij}, \tag{2.37}$$

where ij are spinor indices.

The massless propagator

For massless (conformal) fermions in FLRW spacetime the equation of motion reduces significantly and the propagator can be dealt with nicely. Namely, since the kinetic term is Weyl invariant the problem reduces to solving the propagator in conformal Minkowski, using (2.21) the propagator satisfies:

$$a^{-\frac{D+1}{2}}(\eta)i\gamma^b\partial_b^x\left(a^{\frac{D-1}{2}}(\eta)iS_F^{ij}(x;x')\right) = \frac{i}{a^D}\delta^D(x-x'). \tag{2.38}$$

The solution of (2.38) is:

$$iS^{ij}(x;x') = (aa')^{-\frac{D-1}{2}} \frac{\Gamma(D/2-1)}{4\pi D/2} i\gamma^b \partial_b \frac{1}{\Delta x_{++}^{D-2}(x;x')}, \tag{2.39}$$

where we note that this is simply the Weyl rescaled Minkowski propagator, with

$$\Delta x_{++}(x;x') \equiv \eta^{\mu\nu}(x-x')_\mu (x-x')_\nu, \tag{2.40}$$

which is related to the de Sitter invariant lenght y by,

$$y(x;x') = (1-\epsilon)^2 aa' H^2 \Delta x_{++}^2(x;x'). \tag{2.41}$$

The massive propagator

The fermion propagator for massive fermions is much more complicated since fermions are no longer conformal due to the mass term. To circumvent this we solve for the fermionic mode functions in a helicity and chirality decomposition. Firstly to make things a bit neater we define conformally rescaled fermion fields [3]

$$\chi \equiv a^{\frac{D-1}{2}} \psi, \quad \bar{\chi} \equiv a^{\frac{D-1}{2}} \bar{\psi},$$

then the Dirac equation can be written as:

$$i\gamma^b \partial_b \chi - am\chi = 0. \tag{2.42}$$

The following procedure goes exactly like that of any ordinary quantum field theory. Namely, we promote the fields to operators which satisfy anti-commutation relations:

$$\{\hat{\chi}_i(\mathbf{x},t),\ \hat{\chi}_j^*(\mathbf{x}',t)\} = \delta^{D-1}(\mathbf{x}-\mathbf{x}')\delta_{ij}, \tag{2.43}$$

$$\{\hat{\chi}_i(\mathbf{x},t),\ \hat{\chi}_j(\mathbf{x}',t)\} = 0, \tag{2.44}$$

$$\{\hat{\chi}_i^*(\mathbf{x},t),\ \hat{\chi}_j^*(\mathbf{x}',t)\} = 0. \tag{2.45}$$

Similarly to the standard treatment we Fourier transform the fields $\hat{\chi}, \hat{\bar{\chi}}$ and expand in terms of creation and annihilation operators. We write,

[3] As a result all quantities are with respect to these rescaled fields

$$\hat{\chi} = \int \frac{\mathrm{d}^{D-1}\mathbf{k}}{(2\pi)^{D-1}} \sum_h \Big[\hat{a}_{\mathbf{k},h}\gamma^{(h)}(\mathbf{k},\eta)e^{i\mathbf{k}\cdot\mathbf{x}} + \hat{b}^\dagger_{\mathbf{k},h}\nu^{(h)}(\mathbf{k},\eta)e^{-i\mathbf{k}\cdot\mathbf{x}}\Big], \tag{2.46}$$

$$\hat{\bar{\chi}} = \int \frac{\mathrm{d}^{D-1}\mathbf{k}}{(2\pi)^{D-1}} \sum_h \Big[\hat{b}_{\mathbf{k},h}\bar{\nu}^{(h)}(\mathbf{k},\eta)e^{i\mathbf{k}\cdot\mathbf{x}} + \hat{a}^\dagger_{\mathbf{k},h}\gamma^{(h)}(\mathbf{k},\eta)e^{-i\mathbf{k}\cdot\mathbf{x}}\Big]. \tag{2.47}$$

Note that the (anti)-fermion creation and annihilation operators are of helicity $h = \pm 1$ and that we are splitting the D-spinor into two $2^{(D-4)/2}$-spinors (left and right-handed) of helicity h. Furthermore, $\gamma^{(h)}$ and $\nu^{(h)}$ are the particle, anti-particle mode functions respectively,

$$\gamma(\mathbf{k},\eta) = \sum_h \gamma^{(h)}(\mathbf{k},\eta) = \sum_h \begin{pmatrix}\gamma_{L,h}(\mathbf{k},\eta)\\ \gamma_{R,h}(\mathbf{k},\eta)\end{pmatrix} \otimes \xi_h \tag{2.48}$$

$$\nu(\mathbf{k},\eta) = \sum_h \nu^{(h)}(\mathbf{k},\eta), = \sum_h \begin{pmatrix}\nu_{L,h}(\mathbf{k},\eta)\\ \nu_{R,h}(\mathbf{k},\eta)\end{pmatrix} \otimes \xi_h. \tag{2.49}$$

This is useful because in contrast to the fermionic mode decomposition in Minkowski where we choose to do the mode expansion in spin-space (since spin is there a conserved quantity), however in time-dependent spatially homogeneous spacetimes (like FLRW) helicity is conserved and is therefore a more appropriate quantum number. Furthermore normalisation of the commutation relations between the creation and annihilation operators proceeds normally.

We turn our attention now to solving the Dirac equation (2.42) for the mode functions. To start with we look at only the particle mode function $\gamma^{(h)}(\mathbf{k},\eta)$. Combining the mode expansion (2.46) and the helicity decomposition (2.48) into the Dirac equation reduces it to a system of coupled linear first-order differential equations,

$$i\gamma'_{L,h} + hk\gamma_{L,h} - am\gamma_{R,h} = 0 \tag{2.50}$$

$$i\gamma'_{R,h} - hk\gamma_{R,h} - am\gamma_{L,h} = 0 \tag{2.51}$$

To solve these differential equations we employ a linear combination $u_\pm$ to transform (2.50) to:

$$iu'_{+h} + hku_{-h} - amu_{+h} = 0, \tag{2.52}$$

$$iu'_{-h} + hku_{+h} - amu_{-h} = 0 \tag{2.53}$$

where,

$$u_{\pm h}(k,\eta) \equiv \frac{\gamma_{L,h}(k,\eta) \pm \gamma_{R,h}(k,\eta)}{\sqrt{2}}. \tag{2.54}$$

Using $a = \frac{1}{H(1-\epsilon)}\frac{1}{\eta}$ and defining $\zeta = \frac{m}{H(1-\epsilon)}$ and after some algebra one can decouple this system of first-order differential equations to one second-order differential equation,

$$u''_{\pm h} + \left(k^2 + \frac{\zeta^2 \pm i\zeta}{\eta^2}\right) u_{\pm h} = 0 \tag{2.55}$$

the solution to which is given by the Hankel function of the first kind:

$$u_{\pm h} = \alpha^h_{\pm k}\sqrt{k\eta}H^{(1)}_{\nu_\pm}(-k\eta), \tag{2.56}$$

where $\nu_\pm \equiv \frac{1}{2} \mp \zeta$ is the order of the Hankel function.

Vacuum considerations: In the deep UV (asymptotic past) fermions become effectively massless thus we require the mode functions to take the standard conformal vacuum solutions:

$$\lim_{\eta\to\infty} u_{+h} = \frac{1}{\sqrt{2}}e^{-ik\eta}, \tag{2.57}$$

$$\lim_{\eta\to\infty} u_{-h} = \frac{-h}{\sqrt{2}}e^{-k\eta}. \tag{2.58}$$

Equipped with this one can fix the coefficients $\alpha^h_{\pm k}$ and construct the solutions:

$$u_{+h}(-k\eta) = c\mathcal{H}(\eta,\zeta) + d\tilde{\mathcal{H}}(\eta,\zeta), \tag{2.59}$$

$$u_{-h}(-k\eta) = -hc\tilde{\mathcal{H}}^*(\eta,\zeta) + hd\mathcal{H}^*(\eta,\zeta). \tag{2.60}$$

where we have taken account of the fact that Hankel functions of the second kind are also admitted as solutions to (2.55) and matched the same vacuum considerations as before. The functions $\mathcal{H}, \tilde{\mathcal{H}}$ are given by,

$$\mathcal{H}(\eta,\zeta) = e^{i\frac{\pi}{2}(\nu_+ + 1/2)}\sqrt{-\frac{k\pi\eta}{4}}H^{(1)}_{\nu_+}(-k\eta) \tag{2.61}$$

$$\tilde{\mathcal{H}}(\eta,\zeta) = e^{-i\frac{\pi}{2}(\nu_+ + 1/2)}\sqrt{-\frac{k\pi\eta}{4}}H^{(2)}_{\nu_+}(-k\eta). \tag{2.62}$$

Similarly one finds the anti-particle mode functions:

$$v_{+h}(-k\eta) = f\mathcal{H}(\eta,\zeta) + f\tilde{\mathcal{H}}(\eta,\zeta), \tag{2.63}$$

$$v_{-h}(-k\eta) = -hf\tilde{\mathcal{H}}^*(\eta,\zeta) + hg\mathcal{H}^*(\eta,\zeta), \tag{2.64}$$

Where the constant coefficients are related by $|c| = |g|, |d| = |f|$.

Recalling the transformation in (2.54) one recovers the form of the fermion mode functions,

$$\gamma_{L,h}(k,\eta) = \frac{1}{\sqrt{2}}[c\{\mathcal{H} - h\tilde{\mathcal{H}}^*\} + d\{\tilde{\mathcal{H}} + h\mathcal{H}^*\}], \tag{2.65}$$

$$\gamma_{R,h}(k,\eta) = \frac{1}{\sqrt{2}}[c\{\mathcal{H} + h\tilde{\mathcal{H}}^*\} + d\{\mathcal{H} - h\mathcal{H}^*\}]. \tag{2.66}$$

for the particle and for the anti-particle,

$$\nu_{L,h}(k,\eta) = \frac{1}{\sqrt{2}}[f\{\mathcal{H} - h\tilde{\mathcal{H}}^*\} + g\{\tilde{\mathcal{H}} + h\mathcal{H}^*\}], \tag{2.67}$$

$$\nu_{R,h}(k,\eta) = \frac{1}{\sqrt{2}}[f\{\mathcal{H} + h\tilde{\mathcal{H}}^*\} + g\{\mathcal{H} - h\mathcal{H}^*\}]. \tag{2.68}$$

Our final task is to compute the Feynman propagator by combining the fermion mode function expansion with equation (2.35) and carry out the integration over momentum [4].

$$iS_F^{ij}(x;x') = (aa')^{-\frac{D-1}{2}} \Bigg\{ \Theta(\eta - \eta') \int \frac{\mathrm{d}^{D-1}\mathbf{k}}{(2\pi)^{D-1}} \sum_h \gamma_i^{(h)}(\mathbf{k},\eta)\bar{\gamma}_j^{(h)}(\mathbf{k},\eta')e^{i\mathbf{k}\cdot(\mathbf{x}-\tilde{\mathbf{x}})} \tag{2.69}$$

$$-\Theta(\eta' - \eta) \int \frac{\mathrm{d}^{D-1}\mathbf{k}}{(2\pi)^{D-1}} \sum_h \bar{\nu}_j^{(h)}(\mathbf{k},\eta')\nu_i^{(h)}(\mathbf{k},\eta)e^{-i\mathbf{k}\cdot(\mathbf{x}-\tilde{\mathbf{x}})} \Bigg\} \tag{2.70}$$

We only quote the result of the integration:

$$iS_F^{ij}(x;x') = a(i\gamma^\mu\nabla_\mu + m)\frac{(a\eta a'\eta')^{-\frac{D-2}{2}}}{\sqrt{aa'}}\left[iS_+(x;x')\frac{1+\gamma^0}{2} + iS_-(x;x')\frac{1-\gamma^0}{2}\right], \tag{2.71}$$

with,

$$\begin{aligned} iS_\pm(x;x') = \frac{\Gamma(\frac{D}{2} \pm i\zeta)\Gamma(\frac{D-2}{2} \mp i\zeta)}{(4\pi)^{D/2}\Gamma(\frac{D}{2})} \Bigg\{ & |c|^2 {}_2F_1\left(\frac{D}{2} \pm i\zeta, \frac{D-2}{2} \mp i\zeta; \frac{D}{2}; 1 - \frac{y_{++}(x;x')}{4}\right) \\ & \mp icd^* e^{i\pi\frac{D-1}{2}} {}_2F_1\left(\frac{D}{2} \pm i\zeta, \frac{D-2}{2} \mp i\zeta; \frac{D}{2}; 1 - \frac{y_{+-}(x;x')}{4}\right) \\ & \mp ic^* d e^{-i\pi\frac{D-1}{2}} {}_2F_1\left(\frac{D}{2} \pm i\zeta, \frac{D-2}{2} \mp i\zeta; \frac{D}{2}; 1 - \frac{y_{-+}(x;x')}{4}\right) \\ & - |d|^2 {}_2F_1\left(\frac{D}{2} \pm i\zeta, \frac{D-2}{2} \mp i\zeta; \frac{D}{2}; 1 - \frac{y_{--}(x;x')}{4}\right) \Bigg\}, \end{aligned} \tag{2.72}$$

[4] For more detail on this procedure we direct the reader to the appendix in the paper by Koksma and Prokopec [18].

and ${}_2F_1(a,b;c;z)$ is the *Gauss hypergeometric function.*

Remark: In this thesis we will consider the less general Bunch-Davies vacuum instead of the authors' choice of α-vacua which corresponds to setting the coefficients $c = 1, d = 0$. Thus, for this work the scalar functions ($iS_\pm$) in equation (2.72) are given by,

$$iS_\pm = \frac{\Gamma(\frac{D}{2} \pm i\zeta)\Gamma(\frac{D-2}{2} \mp i\zeta)}{(4\pi)^{D/2}\Gamma(\frac{D}{2})} \, {}_2F_1\left(\frac{D}{2} \pm i\zeta, \frac{D-2}{1} \mp i\zeta; \frac{D}{2}, 1 - \frac{y}{4}\right) \tag{2.73}$$

One can check this propagator with known results, namely if we take the massless limit $m \to 0$ we recover the propagator computed in (2.39). We can also check the de Sitter limit $\epsilon \to 0$ for which we recover the well known result of Candelas and Raine [25].

Chapter 3

Quantum corrections at one-loop order

In this chapter we will compute corrections to the graviton propagator due to the presence of minimally coupled fermions. We employ a perturbative expansion of the metric around a constant epsilon FRW background characterized by the metric (1.1) in Section 1.1. We determine the graviton self-energy using the 1PI effective action formalism. We take the matter action to be the standard Dirac action describing fermions in a general D-dimensional curved spacetime given by,

$$S_D = \int \mathrm{d}^D x \sqrt{-g}\bar{\psi}\Big[ie^{\mu}_{\ b}\gamma^b\nabla_\mu - m\Big]\psi.$$

Where the covariant derivative is defined as in Section 2.1. Our first order of business is to find the cubic and quatic interaction vertices from perturbing the action up to second order in the graviton field $h_{\mu\nu}$.

3.1 Perturbative quantum gravity: cubic and quartic interactions

A first step which will simplify our calculations extensively is to rescale the metric by the square of the scale factor $a(\eta)$.

$$g_{\mu\nu} \to a^2(\eta)\bar{g}_{\mu\nu}, \quad g^{\mu\nu} \to a^{-2}(\eta)\bar{g}^{\mu\nu}. \tag{3.1}$$

This implies a rescaling of the metric dependent quantities:

$$e^{\mu}_{\ a} \to a^{-1}\bar{e}^{\mu}_{\ a}, \quad e^{a}_{\ \mu} \to a\bar{e}^{a}_{\ \mu} \tag{3.2}$$

$$\sqrt{-g} \to a^D\sqrt{-\bar{g}} \tag{3.3}$$

The kinetic part of the action is invariant under such a transformation since, $\psi \to a^{-\frac{D-1}{2}}\chi$ (an intended consequence). Notice however that the mass term is not but this is nonetheless a useful thing to do.

Thus, the action transforms as,

$$S_D \to \bar{S}_D = \int \mathrm{d}^D x \sqrt{-\bar{g}}\bar{\chi}\Big[i\bar{e}^{\mu}_{\ b}\gamma^b\bar{\nabla}_\mu - am\Big]\chi. \tag{3.4}$$

We can now treat the graviton as a small perturbation around the flat background as,

$$\bar{g}_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu} \tag{3.5}$$

where, $\kappa^2 = 16\pi G_N$, $\eta_{\mu\nu}$ is the Minkowski metric with the mostly plus signature which is responsible for raising and lowering indices: $h \equiv h^{\mu}_{\ \mu} \equiv \eta^{\mu\nu}h_{\mu\nu}$. We furthermore identify κ^2 as the loop counting parameter of perturbative quantum gravity. We expand all the relevant operators and fields in terms of the graviton field h up to second order.

$$\bar{e}^{\mu}_{\ a} = \delta^{\mu}_{\ a} - \frac{1}{2}\kappa h^{\mu}_{\ a} + \frac{3}{8}\kappa^2 h^{\mu\lambda}h_{\lambda a} + \mathcal{O}(\kappa^3), \tag{3.6}$$

$$\bar{g}^{\mu\nu} = \eta^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu}_{\ \lambda}h^{\lambda\nu} + \mathcal{O}(\kappa^3), \tag{3.7}$$

$$\sqrt{-\bar{g}} = 1 + \frac{1}{2}\kappa^2 + \frac{1}{8}\kappa^2 h^2 - \frac{1}{4}h^{\mu\nu}h_{\mu\nu} + \mathcal{O}(\kappa^3). \tag{3.8}$$

Using this we can write the expanded conformally rescaled Dirac action as,

$$\bar{S}_D = S^{(0)} + S^{(3)} + S^{(4)},$$

with,

$$S^{(0)} = \int \mathrm{d}^D x\ \bar{\chi}\big(i\partial\!\!\!/ - am\big)\chi, \tag{3.9}$$

is the tree-level action. The first order part, which contains the cubic interaction i.e. one graviton and two fermions, is given by

$$\begin{aligned} S^{(3)} &= \int \mathrm{d}^D x \frac{\kappa}{2}\Big[h\bar{\chi}(i\partial\!\!\!/ - am)\chi - h^{\mu\nu}\bar{\chi}i\gamma_\mu\partial_\nu\chi - \partial_d h_{\mu c}\bar{\chi}\gamma^\mu J^{cd}\chi\Big] \\ &= \int \mathrm{d}^D x \frac{\kappa}{2}\mathcal{T}^{\mu\nu}h_{\mu\nu}, \end{aligned} \tag{3.10}$$

here we identify $\mathcal{T}^{\mu\nu}$ as the stress-energy tensor

#	$\mathcal{V}^{\mu\nu\rho\sigma}_{Iab}$
$I = 1$	$\frac{1}{4}\eta^{\mu\nu}\eta^{\rho\sigma}(i\gamma^{\lambda}\partial_{\lambda} - am)$
$I = 2$	$-\frac{1}{2}\eta^{\mu(\rho}\eta^{\sigma)\nu}(i\gamma^{\lambda}\partial_{\lambda} - am)$
$I = 3$	$-\frac{1}{4}\left(\eta^{\mu\nu}\gamma^{(\rho}\partial^{\sigma)} + \eta^{\rho\sigma}\gamma^{(\mu}\partial^{\nu)}\right)$
$I = 4$	$\frac{3}{8}\left(\eta^{\rho)(\mu}\gamma^{\nu)}\partial^{(\sigma} + \eta^{\mu)(\rho}\gamma^{\sigma)}\partial^{(\nu}\right)$
$I = 5$	$-\frac{1}{4}\left(\eta^{\mu\nu}\gamma^{(\rho}J^{\sigma)\lambda}\partial^{h'}_{\lambda} + \eta^{\rho\sigma}\gamma^{(\mu}J^{\nu)\lambda}\partial^{h}_{\lambda}\right)$
$I = 6$	$\frac{1}{8}\left(\eta^{\rho)(\mu}\gamma^{\alpha}J^{\nu)(\sigma}\partial^{h}_{\alpha} + \eta^{\mu)(\rho}\gamma^{\alpha}J^{\sigma)(\nu}\partial^{h'}_{\alpha}\right)$
$I = 7$	$\frac{1}{4}\left(\delta^{(\mu}_{\beta}\eta^{\nu)(\rho}\delta^{\sigma)}_{\alpha} + \delta^{(\rho}_{\beta}\eta^{\sigma)(\mu}\delta^{\nu)}_{\alpha}\right)\gamma^{\alpha}J^{\beta\lambda}(\partial^{h}_{\lambda} + \partial^{h'}_{\lambda})$
$I = 8$	$\frac{1}{4}\left[\gamma^{(\mu}J^{\nu)(\sigma}\partial^{\rho)}_{h'} + \gamma^{(\rho}J^{\sigma)(\mu}\partial^{\nu)}_{h}\right]$

Table 3.1: Table of vertex operators $\mathcal{V}^{\mu\nu\rho\sigma}_{I}$, note the sub/superscript h is to indicate that the derivative acts on an external graviton leg (i.e. the delta function).

$$\mathcal{T}^{\mu\nu} \equiv \frac{2}{\kappa}\frac{\delta S}{\delta h_{\mu\nu}(x)}\bigg|_{h\to 0} = \eta^{\mu\nu}\bar{\chi}(i\partial\!\!\!/ - am)\chi - \bar{\chi}i\gamma^{(\mu}\partial^{\nu)}\chi + \partial_{\lambda}(\bar{\chi}\gamma^{(\mu}J^{\nu)\lambda}\chi). \tag{3.11}$$

The second order part, which contains the quartic interaction i.e. two gravitons and two fermions, is given by

$$S^{(4)} = \int \mathrm{d}^D x\ \kappa^2\Bigg\{\left[\frac{1}{8}h^2 - \frac{1}{4}h^{\rho\sigma}h_{\rho\sigma}\right]\bar{\chi}(i\partial\!\!\!/ - am)\chi + \left[-\frac{1}{4}hh^{\mu\nu} + \frac{3}{8}h^{\mu\rho}h_{\rho}^{\ \nu}\right]\bar{\chi}i\gamma_{\nu}\partial_{\mu}\chi \tag{3.12}$$

$$+\left[-\frac{1}{4}h\partial_d h_{\mu c} + \frac{1}{8}h^{\lambda}_{\ c}\partial_{\mu}h_{\lambda d} + \frac{1}{4}\partial_d(h^{\lambda}_{\ \mu}h_{\lambda c}) + \frac{1}{4}h^{\lambda}_{\ d}\partial_{\lambda}h_{\mu c}\right]\bar{\chi}\gamma^{\mu}J^{cd}\chi\Bigg\} \tag{3.13}$$

$$= \frac{1}{2}\int \mathrm{d}^D x \int \mathrm{d}^D x'\ \kappa^2 h_{\mu\nu}(x)V^{\mu\nu\rho\sigma}(x - x')h_{\rho\sigma}(x'), \tag{3.14}$$

where,

$$V^{\mu\nu\rho\sigma}(x - x') \equiv \frac{1}{\kappa^2}\frac{\delta^2 S}{\delta h_{\mu\nu}(x)\delta h_{\rho\sigma}(x')} = \sum_{I=1}^{8}\mathcal{V}^{\mu\nu\rho\sigma}_{I}\delta^D(x - x') \tag{3.15}$$

The various vertex factors are given in Table 3.1. We notice as well that our interaction vertexes agree with those found in the literature [7, 12] up to symmetrizations and a mass term.

3.2 Effective action formalism: one-loop self-energy

The 1PI effective action for this theory is given by the path integral,

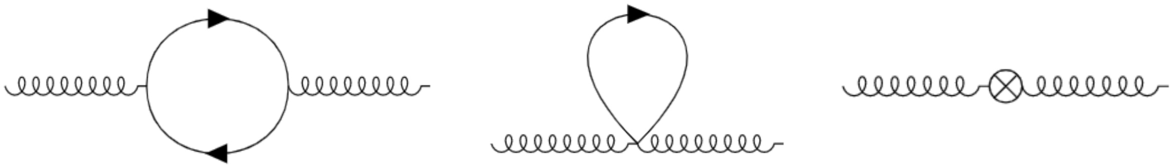

Figure 3.1: Diagrammatic representation of the graviton self energy Σ. It contains a one-loop diagram with two cubic vertexes (one graviton and two fermions) $\propto \langle \mathcal{T}^{\mu\nu}(x)\mathcal{T}^{\rho\sigma}(x')\rangle$, a diagram with one quartic vertex (two gravitons and two fermions), and the counter-term diagram.

$$
\begin{aligned}
e^{i\Gamma[\bar{g}]} &= \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \; e^{iS[\eta+h,\bar{\psi},\psi]} = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \; e^{i(S_0+S^{(3)}+S^{(2)})} && (3.16)\\
&= \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \; e^{iS_0}\left[1 + iS^{(1)} + iS^{(4)} + \frac{1}{2}(iS^{(1)})^2 + \dots\right] && (3.17)\\
&= 1 + \langle iS^{(3)}\rangle + \langle iS^{(4)}\rangle + \frac{1}{2}\langle (iS^{(3)})^2\rangle + \dots && (3.18)
\end{aligned}
$$

where we have made use of the expanded action computed in (3.9)-(3.12). Technically speaking the graviton field is also a quantum field and as a result should appear in the path integral, however in our case we are expanding around the classical background. Furthermore, note that there is no fermion condensate $\langle\psi\rangle$ Expanding the left-hand side,

$$
i\Gamma[\bar{g}] = i\Gamma[\eta] + i\Gamma_h[\eta] + i\Gamma_{hh}[\eta] + \mathcal{O}(h^3) \tag{3.19}
$$

where the object of interest is $i\Gamma_{hh}$ which contain the one-loop corrections/diagram as shown in Figure 3.1 to the effective action. This term is given by:

$$
i\Gamma_{hh} = -\frac{1}{2}\int d^Dx \int d^Dx' h_{\mu\nu}(x) i \left[{}^{\mu\nu}\Sigma^{\rho\sigma}\right](x;x') h_{\rho\sigma}(x'), \tag{3.20}
$$

with the bi-tensor quantity $-i\left[{}^{\mu\nu}\Sigma^{\rho\sigma}\right]$ being the graviton self-energy which is the sum of all unique i.e. the diagram topologies cannot be constructed from other diagrams in the sum [1] one-loop corrections to the graviton propagator and appropriate counter-term diagrams,

$$
-i\left[{}^{\mu\nu}\Sigma^{\rho\sigma}\right](x;x') = -i\left[{}^{\mu\nu}\Sigma^{\rho\sigma}_{(3)}\right](x;x') - i\left[{}^{\mu\nu}\Sigma^{\rho\sigma}_{(4)}\right](x;x') - i\left[{}^{\mu\nu}\Sigma^{\rho\sigma}_{ct}\right](x;x'). \tag{3.21}
$$

The expectation values on the right-hand side are given by

[1] These are known as the one-particle irriducible (1PI) diagrams.

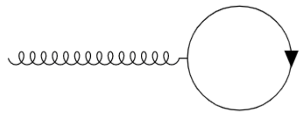

Figure 3.2: The tadpole diagram $\propto \langle \mathcal{T}^{\mu\nu}(x) \rangle$ which quantum corrects the classical background.

$$\langle iS^{(3)} \rangle = i \int \mathrm{d}^D x \frac{\kappa}{2} h_{\mu\nu} \langle \mathcal{T}^{\mu\nu}(x) \rangle, \tag{3.22}$$

$$\langle iS^{(4)} \rangle = i \int \mathrm{d}^D x \int \mathrm{d}^D x' \frac{\kappa^2}{2} h_{\mu\nu}(x) \langle V^{\mu\nu\rho\sigma}(x - x') \rangle h_{\rho\sigma}(x'), \tag{3.23}$$

$$\frac{1}{2} \langle (iS^{(3)})^2 \rangle = -\frac{1}{2} \int \mathrm{d}^D x \int \mathrm{d}^D x' \frac{\kappa^2}{4} h_{\mu\nu}(x) \langle T\{\mathcal{T}^{\mu\nu}(x) \mathcal{T}^{\rho\sigma}(x')\} \rangle h_{\rho\sigma}(x') \tag{3.24}$$

The quantity $\langle \mathcal{T}^{\mu\nu}(x) \rangle$ is the so called tadpole diagram Figure 3.2 and will serve to quantum correct the classical background around which we are expanding, we will not consider this contribution in our work. The contribution containing two cubic vertexes is $\langle (iS^{(3)})^2 \rangle$ and corresponds to the left diagram in Figure 3.1, computing this diagram will be left to a future work. The quantity of focus for this thesis will be the self-energy term including quartic interaction vertexes (the middle diagram in Figure 3.1),

$$-\,i \left[{}^{\mu\nu}\Sigma^{\rho\sigma}_{(4)} \right] (x; x') = -i\kappa^2 \langle V^{\mu\nu\rho\sigma}(x - x') \rangle. \tag{3.25}$$

In the next sections we will concern ourselves with the details for computing the quartic self-energy. After which, in Chapter 4 we will attempt to renormalize them.

3.2.1 The graviton self energy: quartic contribution

The 4-vertex contribution to the graviton self-energy in the effective action formalism is given by the vacuum expectation value of the operator $\langle V^{\mu\nu\rho\sigma} \rangle$. Which can be computed diagramatically by taking the vertex operators and acting them on the fermion propagator (2.71) then taking the coincidence limit $x' \to x$, taking the trace over the spinorial part, and summing over the eight different vertex types. This is given algebraically by,

$$-\,i \Big[{}^{\mu\nu}\Sigma^{\rho\sigma}_{(4)} \Big] (x; x') = -i\kappa^2 \sum_{I=1}^{8} \mathrm{Tr} \Big[\mathcal{V}_I^{\mu\nu\rho\sigma} iS_F(x; x) \Big] \delta^D(x - x'). \tag{3.26}$$

A subtlety that is not directly discernable from this notation is that we must keep in mind that some of the derivative operators $\mathcal{V}$ act externally on a graviton leg instead of on the loop (i.e. on the delta function, this will be important). As a start to this rather long calculation we will first take a look at properties of the scalar functions ($iS_\pm$) which make up the fermion propagator (2.71) in the coincidence limit. Since the vertex operators have at most one derivative hitting the fermion propagator and the fermion propagator intrinsically has a derivative hitting the scalar functions, we need $iS_\pm$, $\partial_\mu iS_\pm$, and $\partial_\mu \partial_\nu iS_\pm$ in the coincidence limit. Recall that these functions are given by Gauss hypergeometric functions of the de Sitter invariant length $y = (1-\epsilon)^2 aa'H^2\Delta x^2$ which vanishes at coincidence,

$$iS_\pm(x;x') \propto {}_2F_1\left(a,b;c;1-\frac{y}{4}\right), \tag{3.27}$$

where $a = \frac{D}{2} \pm i\zeta,\ b = \frac{D-2}{2} \mp i\zeta,\ c = \frac{D}{2}$.

One can show that in the coincidence limit hypergeometric functions of this type reduce to,

$${}_2F_1\big(a,b;c;1\big) = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}. \tag{3.28}$$

Using this, the general properties of derivatives of the hypergeometric function,

$$\partial_\mu\ {}_2F_1\left(a,b;c;1-\frac{y}{4}\right) = -\frac{ab}{4c}(\partial_\mu y){}_2F_1\left(a+1,b+1;c+1;1-\frac{y}{4}\right), \tag{3.29}$$

and derivatives of the de Sitter length,

$$\partial_\mu y = \partial_\mu\big(aa'(H(1-\epsilon)^2\Delta x^2\big) = (1-\epsilon)(aH)y + 2aa'H^2(1-\epsilon)^2\Delta x_\mu, \tag{3.30}$$

we can determine the three quantities we need. Namely,

$$iS_\pm(x;x) = \frac{1}{(4\pi)^{D/2}}\frac{\Gamma\left(\frac{D}{2}\pm i\zeta\right)\Gamma\left(\frac{D}{2}\mp i\zeta-1\right)}{\Gamma(\mp i\zeta)\Gamma(1\pm i\zeta)}\Gamma\left(1-\frac{D}{2}\right), \tag{3.31}$$

$$\partial_\mu iS_\pm(x;x) = 0, \tag{3.32}$$

$$\partial_\mu\partial_\nu iS_\pm(x;x) = \frac{1}{(4\pi)^{D/2}}\frac{1}{D}\frac{\Gamma\left(\frac{D}{2}\pm i\zeta+1\right)\Gamma\left(\frac{D}{2}\mp i\zeta\right)}{\Gamma\left(\mp i\zeta\right)\Gamma\left(1\pm i\zeta\right)}\Gamma\left(1-\frac{D}{2}\right)\bar{H}^2 g_{\mu\nu}. \tag{3.33}$$

Note that single derivatives hitting the hypergeometric functions do not contribute in the coincidence limit. As a second consideration we will rewrite the fermion propagator (2.71) in such a way that it is explicitly proportional to the scalar function and derivatives thereof. Recall the fermion propagator is given by,

$$iS_F(x;x') = a(i\gamma^\mu\nabla_\mu + m)\frac{\bar{H}^{D-2}}{\sqrt{aa'}}\big[iS_+P_+ + iS_-P_-\big], \tag{3.34}$$

with $\bar{H} = H(1-\epsilon)$. We quote the intermediate result for the covariant derivative on a spinor quantity in FRW spacetimes:

$$i\gamma^\mu\nabla_\mu \to i\partial\!\!\!/ - i\frac{D-1}{2}(\partial_\mu u)\gamma^\mu. \tag{3.35}$$

Where we used $u = \ln a$, and $\gamma_\mu J^{\mu\nu} = -\frac{i}{2}(D-1)\gamma^\nu$. Note as well that a $\partial\!\!\!/$ operator hits the prefactor $\frac{\bar{H}^{D-2}}{\sqrt{aa'}}$ which delivers an extra term,

$$i\partial\!\!\!/\left(\frac{\bar{H}^{D-2}}{\sqrt{aa'}}\right) = -i\frac{\bar{H}^{D-2}}{\sqrt{aa'}}\left[\left(\frac{1}{2} + (D-2)\epsilon\right)\right](\partial_\mu u)\gamma^\mu \tag{3.36}$$

Finally writing out the fermion propagator we have,

$$iS_F(x;x') = \frac{\bar{H}^{D-2}}{\sqrt{aa'}}\Big\{-i\alpha(\partial_\mu u)\gamma^\mu + am + ia\partial\!\!\!/\Big\}[iS_+P_+ + iS_-P_-] \tag{3.37}$$

with $\alpha \equiv D/2 + (D-2)\epsilon$.

Using this and the properties of the scalar functions at coincidence. We write two important quantities necessary for the graviton self-energy,

$$iS_F(x;x) = \frac{\bar{H}^{D-2}}{(4\pi)^{D/2}}\Big[-i\alpha(\partial_\mu u)\gamma^\mu + m\Big](Q_+P_+ + Q_-P_-)\Gamma\left(1-\frac{D}{2}\right), \tag{3.38}$$

$$\partial_\alpha iS_F(x;x) = \frac{\bar{H}^{D-2}}{(4\pi)^{D/2}}\Bigg\{[i\alpha(1+\beta)(\partial_\alpha u)(\partial_\mu u)\gamma^\mu + \beta m(\partial_\alpha u)](Q_+P_+ + Q_-P_-) + \bar{H}^2(i\gamma_\alpha)(R_+P_+ + R_-P_-)\Bigg\}\Gamma\left(1-\frac{D}{2}\right), \tag{3.39}$$

with,

$$\alpha = \frac{D}{2} + (D-2)\epsilon, \tag{3.40}$$

$$\beta = \frac{3}{2} + (D-3)\epsilon, \tag{3.41}$$

$$R_\pm = \frac{1}{D}\frac{\Gamma(\frac{D}{2} \pm i\zeta + 1)\Gamma(\frac{D}{2} \mp i\zeta)}{\Gamma(\mp i\zeta)\Gamma(1 \pm i\zeta)}, \tag{3.42}$$

$$Q_\pm = \frac{\Gamma(\frac{D}{2} \pm i\zeta)\Gamma(\frac{D}{2} \mp i\zeta - 1)}{\Gamma(\mp i\zeta)\Gamma(1 \pm i\zeta)}. \tag{3.43}$$

We can now continue with our calculation of the graviton self-energy given by (3.26). As an example we will illustrate the calculation for the $I = 1, 2$ vertex operator which includes both $iS_F(x;x)$ and $\partial_\mu iS_F(x;x)$ [2].

[2] Explicit evaluation for other vertexes is included in Appendix A.

$$
\begin{aligned}
-i[^{\mu\nu}\Sigma^{\rho\sigma}](x;x')_{4,1+2} &= -i\kappa^2 \,\mathrm{Tr}\Big[\Big(\mathcal{V}_1^{\mu\nu\rho\sigma} + \mathcal{V}_2^{\mu\nu\rho\sigma}\Big) iS_F(x;x)\Big]\delta^D(x-x') \\
&= -i\kappa^2\left(\frac{1}{4}\eta^{\mu\nu}\eta^{\rho\sigma} - \frac{1}{2}\eta^{\mu(\rho}\eta^{\sigma)\nu}\right)\mathrm{Tr}\Big[\big(i\gamma^\lambda\partial_\lambda - am\big) iS_F(x;x)\Big]\delta^D(x-x') \quad (3.44)
\end{aligned}
$$

Note that we also need to trace over the (implicit) spinor indices. Using the identities:

$$
\gamma_\mu\gamma^\mu = -DI_N, \quad (3.45)
$$

$$
\mathrm{Tr}\big[P_\pm\big] = \frac{N}{2}, \quad (3.46)
$$

$$
\mathrm{Tr}\big[\gamma^\mu P_\pm\big] = \pm\frac{N}{2}\delta^\mu_0, \quad (3.47)
$$

$$
\mathrm{Tr}\big[\gamma^\mu\gamma^\nu P_\pm\big] = -\frac{N}{2}\eta^{\mu\nu}. \quad (3.48)
$$

Where $N \equiv 2^{D/2}$ is the number of degrees of freedom for a spinor in D dimensions [26]. We find,

$$
-i\Big[^{\mu\nu}\Sigma^{\rho\sigma}_{4,1+2}\Big](x;x') = -i\kappa^2\frac{\bar{H}^D}{(4\pi)^{D/2}}\frac{N}{2}\left\{b_1 + b_2 a + b_3\frac{a^2}{m}\right\}\left[\frac{1}{4}\eta^{\mu\nu}\eta^{\rho\sigma} - \frac{1}{2}\eta^{\mu(\rho}\eta^{\sigma)\nu}\right], \quad (3.49)
$$

with,

$$
b_1 = -\left(\frac{\alpha(1+\beta)}{(1-\epsilon)^2} + \tau\right)\Delta, \quad (3.50)
$$

$$
b_2 = \frac{(D-2)\beta}{2(1-\epsilon)}, \quad (3.51)
$$

$$
b_3 = -\frac{(D-2)\alpha}{2(1-\epsilon)}, \quad (3.52)
$$

$$
\tau = \frac{1}{4}(D-2)^2 + \zeta^2, \quad (3.53)
$$

$$
\Delta \equiv Q_+ + Q_-. \quad (3.54)
$$

With that we quote the final result which we have opted to split into four contributions the subscripts $I = 1, 2, 3, 4, 5, 7, 8$ correspond to the vertex operators 3.1.

$$
\begin{aligned}
-i[^{\mu\nu}\Sigma^{\rho\sigma}](x;x')_{4,1+2} &= -i\kappa^2\sqrt{-g}\frac{\bar{H}^D}{(4\pi)^{D/2}}\frac{N}{2}a^{4-D}\left\{b_1 + b_2 a + b_3\frac{a^2}{m}\right\} \\
&\quad\times\left[\frac{1}{4}g^{\mu\nu}g^{\rho\sigma} - \frac{1}{2}g^{\mu(\rho}g^{\sigma)\nu}\right]\Gamma\left(1-\frac{D}{2}\right)\delta^D(x-x'), \quad (3.55)
\end{aligned}
$$

$$
\begin{aligned}
- i\Big[{}^{\mu\nu}\Sigma^{\rho\sigma}_{4,3}\Big](x;x') = {} & -i\kappa^2\sqrt{-g}\frac{N}{2}\frac{\bar{H}^{D-2}}{(4\pi)^{D/2}}a^{2-D}\Gamma\Big(1-\frac{D}{2}\Big) \\
& \times\Bigg\{\Big(b_4+b_5\frac{1}{a}\Big)\Big[g^{\mu\nu}(\partial^{(\rho}u)(\partial^{\sigma)}u)+(\partial^{(\rho}u)(\partial^{\sigma)}u)\Big]+b_6\bar{H}^2a^2\big[g^{\mu\nu}g^{\rho\sigma}\big]\Bigg\}\delta^D(x-x') \qquad (3.56)
\end{aligned}
$$

$$
\begin{aligned}
- i\Big[{}^{\mu\nu}\Sigma^{\rho\sigma}_{4,4}\Big](x;x') = {} & -i\kappa^2\sqrt{-g}\frac{3N}{2}\frac{\bar{H}^{D-2}}{(4\pi)^{D/2}}a^{2-D}\Gamma\Big(1-\frac{D}{2}\Big)\Bigg\{-\Big(b_4+b_5\frac{1}{a}\Big)\Big[g^{\mu)(\rho}(\partial^{\sigma)}u)(\partial^{(\nu}u)\Big] \\
& -b_6\bar{H}^2a^2\Big[g^{\mu(\rho}g^{\sigma)\nu}\Big]\Bigg\}\delta^D(x-x'). \qquad (3.57)
\end{aligned}
$$

$$
\begin{aligned}
- i\Big[{}^{\mu\nu}\Sigma^{\rho\sigma}_{4,5+7+8}\Big](x;x') = {} & -i\kappa^2\sqrt{-g}\frac{N}{2}\frac{\bar{H}^{D-2}}{(4\pi)^{D/2}}a^{2-D}\Gamma\Big(1-\frac{D}{2}\Big) \\
& \times\Bigg\{\Big(b_7+b_8\frac{1}{a}\Big)\Big[a^2(g^{\mu\nu}g^{\rho\sigma}-2g^{\mu(\rho}g^{\sigma)\nu})(\partial_\lambda u)(\partial^\lambda+\partial^{\lambda'})-2g^{\mu\nu}(\partial^\rho u)\partial^{\sigma)'} \\
& -2g^{\rho\sigma}(\partial^{(\mu}u)\partial^{\nu)}+g^{\mu)(\rho}(\partial^{\sigma)}u)(\partial^{(\nu'}+2\partial^{(\nu})+g^{\rho)(\mu}(2\partial^{\sigma)'}+\partial^{\sigma})\Big]\Bigg\}\delta^D(x-x') \qquad (3.58)
\end{aligned}
$$

The various D-dependent coefficients b_4, b_5, b_6, b_7, b_8 are detailed in Appendix A. Note that these D-dependent coefficients are all finite in $D = 4$, however the self-energy is still divergent due to the gamma function $\Gamma\Big(1-\frac{D}{2}\Big)$. To renormalize this will be the focus of the next chapter where we introduce appropriate counter-term Lagrangians to subtract these divergences and hopefully deliver in the end a finite contribution in $D = 4$.

Chapter 4

Renormalization

In this chapter we regularize the four-point self-energy contribution in the spirit of dimensional regularization we take the limit $D \to 4$ and employ a counter-term subtraction scheme. We will first introduce the possible counter-terms, then show that these are suitable for renormalization of the self-energy. Finally we give a partially renormalized result for the quartic interaction which contributes to the full graviton self-energy and discuss the difficulties we were faced with in renormalizing the theory.

4.1 The counter-terms

We begin by noting that the self-energy is of canonical dimension four, thus we also need counter-terms of a dimension four. We now remind the reader that we assumed the mass of the fermion to be generated by a suitable Yukawa scalar ($\phi \propto H$). Therefore there are six general dimension four Lagrangians which we can write down which can serve as counter-terms for this theory. Three of these are fully geometric ($R^2, (R^2_{\mu\nu}, R_{\mu\nu\rho\sigma})^2$) and three are matter-like. For the purposes of this thesis we will most likely need only the matter counterterms, since in the limit $D \to 4$ the geometric terms all vanish. The matter counterterms are:

$$S_{ct}^{4,a} = -\frac{\alpha_{4,a}}{2} \int \mathrm{d}^D x \sqrt{-g} R \phi^2, \tag{4.1}$$

$$S_{ct}^{4,b} = -\frac{\alpha_{4,b}}{2} \int \mathrm{d}^D x \sqrt{-g} g^{\mu\nu} (\nabla_\mu \phi)(\nabla_\nu \phi), \tag{4.2}$$

$$S_{ct}^{4,c} = -\frac{\alpha_{4,c}}{2} \int \mathrm{d}^D x \sqrt{-g} \phi^4. \tag{4.3}$$

To determine how these counterterms come into play to renormalize the self-energy we must take the second variation of the action with respect to the metric $g_{\mu\nu}$.

$$\frac{\delta^2 S_{ct}^{4,a}}{\delta g_{\rho\sigma}(x')\delta g_{\mu\nu}(x)} = -\frac{\alpha_{4,a}}{4}\sqrt{-g}\Big\{\phi^2(x)\left[g_{\mu\nu}R_{\rho\sigma} + g_{\rho\sigma}R_{\mu\nu} + \left(g_{\mu(\rho}g_{\sigma)\nu} - \frac{1}{2}g_{\mu\nu}g_{\rho\sigma}\right)R\right]\delta^D(x-x')$$
$$+ \big[3g_{\mu\nu}\nabla_{(\rho}\delta^D(x-x')\nabla_{\sigma)} + g_{\rho\sigma}\nabla_{(\mu}\delta^D(x-x')\nabla_{\nu)}$$
$$- 3g_{\rho)(\mu}\nabla_{\nu)}\delta^D(x-x')\nabla_{(\sigma} - g_{\mu\nu}g_{\rho\sigma}\nabla^\lambda\delta^D(x-x')\nabla_\lambda\big]\phi^2(x)\Big\} \tag{4.4}$$

$$\frac{\delta^2 S_{ct}^{4,b}}{\delta g_{\rho\sigma}(x')\delta g_{\mu\nu}(x)} = \frac{\alpha_{4,b}}{2}\sqrt{-g}\Big\{g^{\mu\nu}(\nabla^\rho\phi)(\nabla^\sigma\phi) + g^{\rho\sigma}(\nabla^\mu\phi)(\nabla^\nu\phi)$$
$$- \left[\frac{1}{2}g^{\mu\nu}g^{\rho\sigma} - g^{\mu(\rho}g^{\sigma)\nu}\right](\nabla^\lambda\phi)(\nabla_\lambda\phi)\Big\}\delta^D(x-x') \tag{4.5}$$

$$\frac{\delta^2 S_{ct}^{4,c}}{\delta g_{\rho\sigma}(x')\delta g_{\mu\nu}(x)} = -\frac{\alpha_{4,c}}{2}\sqrt{-g}\left[\frac{1}{4}g^{\mu\nu}g^{\rho\sigma} - \frac{1}{2}g^{\mu(\rho}g^{\sigma)\nu}\right]\phi^4\delta^D(x-x') \tag{4.6}$$

Notice the $(4,a)$-counterterm was obtained after many integrations by parts and dropping boundary terms, this was done to generate terms like those we find in the graviton self-energy [1]. In order to simplify matching the counter-terms to terms that appear in the self-energy we will make use of the following identitifications. Namely, recall that the Yukawa scalar is proportional to the Hubble rate i.e.

$$\phi = qH. \tag{4.7}$$

Furthermore, the identities:

$$\nabla_\mu\phi = -\epsilon(\partial_\mu u)\phi, \tag{4.8}$$
$$\partial_\mu u\partial^\mu u = -H^2 \tag{4.9}$$

prove useful.

4.1.1 The $(1,2)$ contribution

Let us consider for a moment the contributions to the graviton self-energy coming from the vertexes $(1,2)$. These can be fundamentally summarised as,

$$-i[^{\mu\nu}\Sigma^{\rho\sigma}](x;x')_{4,1+2} = -i\kappa^2\sqrt{-g}\frac{\bar{H}^D}{(4\pi)^{D/2}}\frac{N}{2}a^{4-D}\left\{b_1 + b_2 a + b_3\frac{a^2}{m}\right\}$$
$$\times\left[\frac{1}{4}g^{\mu\nu}g^{\rho\sigma} - \frac{1}{2}g^{\mu(\rho}g^{\sigma)\nu}\right]\Gamma\left(1-\frac{D}{2}\right)\delta^D(x-x'), \tag{4.10}$$

with the constants (b_1, b_2, b_3) given as in 3.50 and we have chosen to reintroduce the determinant of the metric by $a^{-D}\sqrt{-g}$, and have identified $\eta^{\mu\nu} = a^2 g^{\mu\nu}$. We can make the divergence manifest by

[1] The validity of this counter-term is unclear.

expanding the Gamma functions in powers of $(D-4)$ and introducing some renormalization scale μ,

$$\Gamma\left(1-\frac{D}{2}\right) = \frac{2}{D-4} + \gamma_E - 1 + \mathcal{O}(D-4), \tag{4.11}$$

$$\bar{H}^D = \bar{H}^4\mu^{D-4}\left(\frac{\bar{H}}{\mu}\right)^{D-4} \tag{4.12}$$

$$= \bar{H}^4\mu^{D-4}\left[1 + \frac{D-4}{2}\ln\left(\frac{\bar{H}}{\mu}\right) + \mathcal{O}((D-4)^2)\right], \tag{4.13}$$

$$a^{4-D} = 1 - \frac{D-4}{2}\ln a + \mathcal{O}(D-4), \tag{4.14}$$

then from looking at the tensor structure, part of this self-energy term can be renormalized by the $(4,c)$-type counter-term, after using the identifications (4.7)-(4.8), if we take the coupling constant to be

$$\alpha_{4,c} \equiv -\frac{2}{\pi^{D/2}}\frac{1}{(1-\epsilon)^4 q^4}\frac{\mu^{D-4}}{D-4}b_1. \tag{4.15}$$

Thus, the partially renormalized result for this (after we take the unregulated limit $D \to 4$) is given by,

$$\left[{}^{\mu\nu}\Sigma^{\rho\sigma}_{4,ren}\right]_{(1,2,b_1)}(x;x') = \frac{H^4}{8\pi^2}\left(\ln\left(\frac{\bar{H}}{\mu}\right) - \ln a + \gamma_E - 1\right)b_1^f\left[\frac{1}{4}g^{\mu\nu}g^{\rho\sigma} - \frac{1}{2}g^{\mu(\rho}g^{\sigma)\nu}\right]\delta^4(x-x'), \tag{4.16}$$

with

$$b_1^f = \frac{m^2}{H^2}\left(\frac{m^2}{H^2} - (19 + \epsilon(30 + 7\epsilon))\right). \tag{4.17}$$

Unfortunately the other two terms b_2, b_3 are coming with factors of the scale factor which means they cannot be renormalized simultaneously with the b_1-term using the $\alpha_{4,c}$-type counter-term. These terms are curious for another reason. Namely, unless there has been an error along the way (a very likely possibility due to the sensitivity in these calculations) we may need non-covariant counterterms to renormalize them. Of course, another possibility is that this scale-factor dependence is somehow cancelled by the terms contained in the TT-correlator.

4.1.2 The $(3,4)$ contributions

These contributions suffer from similar issues. In principal one can mold the $(3,4)$ contributions into a part which can be renormalized by the $(4,b)$-type counter-term plus some other part. However, the coefficients again acquire a dependence on the scale factor which cannot be renormalized by the covariant counterterms (4.4) i.e. we need counterterms which are explicitly proportional to the scale-factor.

4.1.3 The $(5, 7, 8)$ contributions

These contributions suffer from the most egregious issues. They again contain terms which gain a dependence on the scale factor as well as containing tensor structures $\propto \partial\delta^D(x-x')$. Renormalizing these may be possible with the $4, a$-type counter-term which contains $\propto \partial\partial\delta^D(x-x')$ which can in principle be integrated-by-parts into terms like the ones in the self-energy, however it would require dropping some surface terms. It is unclear if this is a justifiable thing to do i.e. whether this is compatible with taking the second variation of the counterterm action.

Remarks:

Clearly renormalization is not possible at this point. Many individual parts of the calculation could be suspect. Note for example the $(5, 7, 8)$-contributions contain ∂' operators, it is unclear how to generate these terms from the $4, a$-counterterm. The scale-factor dependence of some coefficients is also highly troublesome, it could be an artifact of the conformal rescaling performed in Chapter 3, or these terms could somehow be cancelled by structures in the TT-correlator. Nonetheless, an important intermediate result is the partially renormalized term illustrated in (4.16). Future work will attempt to address these issues.

Chapter 5

Discussion and Outlook

The main focus of this thesis has been to compute the quartic contribution to the graviton self-energy due to massive fermion fields in the FLRW background. We were successfull in computing this contribution. However, renormalizing this term has proved more cumbersome than expected because of a surprise dependence on the scale factor in many coefficients of the self-energy. This could mean one of three things; we need non-covariant counter-terms, a mistake was made along the way, or the uncomputed qubic vertex contributes in such a way as to cancel this.

Our goal is to use the quantum corrected linearized Einstein field equations,

$$\sqrt{-g}\mathcal{L}^{\mu\nu\rho\sigma} - \int \mathrm{d}^4x' \left[{}^{\mu\nu}\Sigma^{\rho\sigma}_{ren}\right](x;x')h_{\rho\sigma}(x') = T^{\mu\nu}_{lin}(x)$$

for which we need the renormalized graviton self-energy. Before we can get to that point we must properly understand the emergence of the scale-factor dependence in the quartic contribution. We must also compute the cubic contribution which amounts to computing the TT-correlator. We can expect structures very similar to those described in [27, 28] i.e. terms $\propto y^{1-D/2}, y^{-D/2}$. The procedure for dealing with these is to reduce the degree of divergence by extracting a d'Alembertian operator and localize these onto delta function terms as is detailed in a myriad of works [7, 8, 9, 10, 11, 12]

Once we have the renormalized graviton self-energy we can apply it to quantum correct the Einstein field equations using the Schwinger-Keldysh formalism by replacing the graviton self-energy with its retareded version. We do this perturbatively to order $\hbar$ since we only have the 1-loop corrections i.e. we expand the graviton as,

$$h_{\mu\nu} = h^{(0)}_{\mu\nu} + \hbar\kappa^2 h^{(1)}_{\mu\nu} + \mathcal{O}(\hbar^2)$$

with

$$h^{(0)}_{\mu\nu} = \epsilon_{\mu\nu}(\mathbf{k})v_0(\eta,\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}$$

solves the classical Einstein equations, $e_{\mu\nu}$ is the graviton polarization tensor which satisfies

$$e_{\mu 0}(\mathbf{k}) = 0, \quad k^i \epsilon_{ij}(\mathbf{k}) = 0, \quad \epsilon^i{}_i(\mathbf{k}) = 0.$$

and $v_0(\eta, \mathbf{k})$ are the classical mode function solutions.

We then solve the linearized Einstein equations without a source $T_{\mu\nu} = 0$ perturbatively

$$\hbar\kappa^2 \sqrt{-g} \mathcal{L}^{\mu\nu\rho\sigma} h^{(1)}_{\mu\nu} - \int \mathrm{d}^4x' \left[{}^{\mu\nu}\Sigma^{\rho\sigma}_{(1)}\right] h^{(0)}_{\rho\sigma}(x') = 0$$

If we include a source, for example a point particle of mass M, by introducing the stress-energy tensor $T^{\mu\nu}_{lin}(x, M)$ on the RHS we can compute quantum corrections to the gravitational potentials Φ, Ψ as was done in [10].

If we accept the partial renormalization we have done, we expect to see a similar secular effect to that found by Miao and Woodard [7] for the massless fermion self-energy in de Sitter. Namely, the self-energy picks up a dependence on $\sim \ln a$ which can grow with time.

Finally we note the computations in this work concern the full graviton. Thus, one can in principle consider corrections to the graviton mode functions. Using these we can construct both the scalar and tensor power spectra with one-loop corrections due to fermions, the latter being a currently measurable quantity, and with the tensor power spectrum on the horizon. This work could prove to be significant in this way.

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Appendices

Appendix A

Computing the quartic self-energy terms

This appendix serves to show the vertex contractions and spinor traces needed for the quartic contribution to the graviton self-energy for index factors $I > 2$.

A.1 The $I = 3$ and $I = 4$ contributions

The procedure here proceeds fairly similarly to the one for the $(1, 2)$ vertex operator contributions. The only extra thing we need to keep in mind here is the presence of symmetrizations in the indices

$$-i\Big[{}^{\mu\nu}\Sigma^{\rho\sigma}_{4,3}\Big](x;x') = -i\frac{\kappa^2}{4}\,\mathrm{Tr}\Big[\Big(\eta^{\mu\nu}\gamma^{(\rho}\partial^{\sigma)} + \eta^{\rho\sigma}\gamma^{(\mu}\partial^{\nu)}\Big)iS_F(x;x)\Big]\delta^D(x-x') \tag{A.1}$$

Using the previously derived expressions for the fermion propagator in the coincidence limit and performing the trace. For which we need the following identities:

$$\mathrm{Tr}\big[\gamma^\rho\gamma^\sigma P_\pm\big] = -\frac{N}{2}\eta^{\rho\sigma}, \tag{A.2}$$

$$\mathrm{Tr}\big[\gamma^\rho P_\pm\big] = \pm\frac{N}{2}\delta^\rho_{\ 0}, \tag{A.3}$$

$$\mathrm{Tr}\big[\gamma^\rho\gamma_\lambda P_\pm\big] = -\frac{N}{2}\delta^\beta_\alpha. \tag{A.4}$$

$$\tag{A.5}$$

With that we find

$$
\begin{aligned}
-i\Big[{}^{\mu\nu}\Sigma^{\rho\sigma}_{4,3}\Big](x;x') = -\frac{i}{\kappa^2}\frac{\bar{H}^{D-2}}{(4\pi)^{D/2}}\frac{N}{2}\Big\{&\frac{\alpha(1+\beta)}{4}\Delta\Big[\eta^{\mu\nu}(\partial^{(\rho}u)(\partial^{\sigma)}u)+\eta^{\rho\sigma}(\partial^{(\mu)}u)(\partial^{\nu)}u)\Big] \\
&-\frac{(D-2)\beta m}{8\zeta}\Delta\Big[\eta^{\mu\nu}(\partial^{(\rho}u)\delta^{\sigma)}_{0}+\eta^{\rho\sigma}(\partial^{(\mu}u)\delta^{\nu)}_{0}\Big] \\
&-\frac{\bar{H}^2\tau}{2D}\Delta\big[\eta^{\mu\nu}\eta^{\rho\sigma}\big]\Big\}\Gamma\Big(1-\frac{D}{2}\Big)\delta^D(x-x'). \qquad (A.6)
\end{aligned}
$$

As with the $(1,2)$ contributions we opt to rewrite this in the most covariant way possible by making the identifications $\eta^{\mu\nu}\to a^2g^{\mu\nu}$, $\delta^{\mu}_{0}\to-(aH)^{-1}(\partial_{\mu u})$, and reintroducing the determinant of the metric $\sqrt{-g}a^{-D}=1$. We do this to make identifying the counterterms easier.

$$
\begin{aligned}
-i\Big[{}^{\mu\nu}\Sigma^{\rho\sigma}_{4,3}\Big](x;x') = -i\kappa^2\sqrt{-g}\frac{N}{2}\frac{\bar{H}^{D-2}}{(4\pi)^{D/2}}a^{2-D}\Gamma\Big(1-\frac{D}{2}\Big) \\
\times\Bigg\{\Big(b_4+b_5\frac{1}{a}\Big)\Big[g^{\mu\nu}(\partial^{(\rho}u)(\partial^{\sigma)}u)+(\partial^{(\rho}u)(\partial^{\sigma)}u)\Big]+b_6\bar{H}^2a^2\big[g^{\mu\nu}g^{\rho\sigma}\big]\Bigg\}\delta^D(x-x') \qquad (A.7)
\end{aligned}
$$

with

$$
\begin{aligned}
b_4 &= \frac{\alpha(1+\beta)}{4}\Delta, & (A.8)\\
b_5 &= -\frac{(D-2)(1-\epsilon)\beta}{8}\Delta, & (A.9)\\
b_6 &= -\frac{\tau}{2D}\Delta, & (A.10)
\end{aligned}
$$

and the constants α,β,Δ,τ defined as before.

Similarly, one can compute the $I=4$ contribution where the traces are essentially the same, only we need to take care with the symmetrizations of the $I=4$ vertex. With that we quote the result,

$$
\begin{aligned}
-i\Big[{}^{\mu\nu}\Sigma^{\rho\sigma}_{4,4}\Big](x;x') = -i\kappa^2\sqrt{-g}\frac{3N}{2}\frac{\bar{H}^{D-2}}{(4\pi)^{D/2}}a^{2-D}\Gamma\Big(1-\frac{D}{2}\Big)\Big\{-\Big(b_4+b_5\frac{1}{a}\Big)\Big[g^{\mu)(\rho}(\partial^{\sigma)}u)(\partial^{(\nu}u)\Big] \\
-b_6\bar{H}^2a^2\Big[g^{\mu(\rho}g^{\sigma)\nu}\Big]\Big\}\delta^D(x-x'). \qquad (A.11)
\end{aligned}
$$

A.2 The $I=5,6,7,8$ contributions

Here we will only concern ourselves with the $I=5$ part as $I=7,8$ follow the same spinor structure but have different symmetrizations. Note after taking the spinor trace that $[I=6]=0$ [1].

[1]This is seemingly consistent with the corresponding vertex contractions in [7] where the $I=6$ also does not contribute

The trace identities,

$$\mathrm{Tr}\big[\gamma^\rho J^{\sigma\lambda}\gamma^\alpha P_\pm\big] = -\frac{iN}{4}\big(\eta^{\alpha\sigma}\eta^{\rho\lambda} - \eta^{\alpha\lambda}\eta^{\rho\sigma}\big), \tag{A.12}$$

$$\mathrm{Tr}\big[\gamma^\rho J^{\sigma\lambda} P_\pm\big] = \pm\Big(\delta^\rho_{\ 0}\eta^{\sigma\lambda} - \eta^{\rho\sigma}\delta^\lambda_{\ 0}\Big), \tag{A.13}$$

prove useful. With that we quote the result

$$\begin{aligned} -i\Big[{}^{\mu\nu}\Sigma^{\rho\sigma}_{4,5+7+8}\Big](x;x') &= -i\kappa^2\sqrt{-g}\,\frac{N}{2}\frac{\bar{H}^{D-2}}{(4\pi)^{D/2}}a^{2-D}\Gamma\Big(1-\frac{D}{2}\Big) \\ &\times\Big\{\Big(b_7 + b_8\frac{1}{a}\Big)\Big[a^2(g^{\mu\nu}g^{\rho\sigma} - 2g^{\mu(\rho}g^{\sigma)\nu})(\partial_\lambda u)(\partial^\lambda + \partial^{\lambda'}) - 2g^{\mu\nu}(\partial^\rho u)\partial^{\sigma)'} \\ &- 2g^{\rho\sigma}(\partial^{(\mu}u)\partial^{\nu)} + g^{\mu)(\rho}(\partial^{\sigma)}u)(\partial^{(\nu'} + 2\partial^{(\nu)}) + g^{\rho)(\mu}(2\partial^{\sigma)'} + \partial^{\sigma})\Big]\Big\}\delta^D(x-x') \end{aligned} \tag{A.14}$$

with

$$b_7 = \frac{\alpha}{8}\Delta, \tag{A.15}$$

$$b_8 = (D-2)(1-\epsilon)\Delta. \tag{A.16}$$

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