Scattered based Marchenko Redatuming by Inversion

Master Thesis

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Abstract

Marchenko redatuming is a recently developed framework, which is used to retrieve up- and down going waveforms from an arbitrary depth in the subsurface. For this, the scheme only needs single-sided surface reflection data and a little information about the medium, for example a conventional velocity model. The obtained Green's function can be used to obtain more detailed images of the subsurface, which is especially important and more difficult for perturbed media, or complex media, e.g. a medium containing a subsalt structure.

This research focuses on two new developments in Marchenko calculations: The first is solving the Marchenko system by means of inversion instead of iteration, which has so far been the main solving method. Secondly a newly proposed variation on the Marchenko system is tested. It focuses specifically on perturbed fields, which will have complex-media reflection data. This will result in a more difficult problem to solve.

This research uses two example models to show that the Marchenko system can be solved by inversion, and to show that the results of this new construction validate the proposed approach. Both these findings will be useful for further use of the Marchenko scheme, especially when the models become more complex, for example, in heterogeneous and/or variable velocity models.

Introduction

Seismic wavefields can be used to construct images of the subsurface, which can help to obtain a better understanding of its structure. A challenge of seismic imaging can be to correctly estimate wavefields, when there is little to no knowledge or observation of the subsurface available. One of the methods that can be used to locate structures in the subsurface, using only reflection data obtained at the surface and a velocity model, is the Marchenko method, which is a recent development in seismic data processing (Broggini and Snieder, 2012). The scheme of Marchenko is made up of the important Marchenko equations, which are based on reciprocity theorems. The focus of the Marchenko scheme is to retrieve up- and down going Green's functions at a desired depth. For this, it uses only single-sided surface reflection data and a smooth background model, which means there is no need for first locating reflectors in order to reconstruct multiple scattering effects. (Wapenaar et al., 2014; Slob et al., 2014, Van der Neut et al., 2015).
The obtained Green's function can eventually be used for various applications such as redatuming, constructing virtual sources and receivers, and target oriented-imaging.
Ravasi et al. (2016) have successfully used this method for the first time to obtain target-oriented images in a North Sea field.

Marchenko focusing is an inverse problem, which, in the research and field examples that have been done so far, has always been solved by means of iteration (e.g. Ravasi et al., 2016; Jia et al., 2018); i.e. the result of the Marchenko inversion can be written as a Neumann's series expansion, which can be iterated to convergence. However, this iterative solution needs strict conditions for it to converge (Staring et al., 2017a), seriously limiting its applicability. The scheme of Marchenko (and its method of solving) that is currently used, is limited. More realistic and/or complex models will have to be simplified and smoothened in order to meet the conditions of the iterative solving method. Secondly, the scheme is limited to only using single arrival inputs. Altogether, the Marchenko method needs further research and updates for it to be able to solve more complex models from more complex media such as in Jia et al. (2018), where a subsalt model in the Gulf of Mexico is analyzed.

It is in this light that this research focuses on two main aspects of Marchenko: the solving technique, and a new variation of the original Marchenko scheme. The first goal is trying to solve the Marchenko scheme by inversion instead of having to use an iterative scheme. Solving by inversion will be computationally more expensive, but when it is used correctly, more complex models can now directly be solved, since they are not subject to the assumptions made by the iterative scheme. This will benefit the quality of the results. An example of a such a complex system are perturbed fields. Vasconcelos and Van der Neut (2016) provided a variation on the Marchenko system, which focuses on perturbed fields. It aims at overcoming some of the limitations that the conventional scheme imposes on initial focusing functions, when looking at a more complex medium. This variation will be very important in targeting scattered fields, such as in highly heterogeneous models containing sharp interfaces. Using inversion as a more convenient way of solving a Marchenko system, this newly proposed framework is tested in this research.

In this research the basic method of Marchenko is discussed in the first section. A clear summary of the original Marchenko scheme can also be found in Cui et al. (2018), but it is useful to describe the method of Marchenko the way it is used in this research. Secondly, the new Marchenko variation as proposed by Vasconcelos and Van der Neut (2016) is explained and tested. Next, the two different solving techniques of iteration and inversion are discussed in detail. Finally, this paper concludes with the discussion on further possibilities and conclusions of this research.

The two outlined goals of this study are tested by using two example models: one basic, layered medium and a more complex medium, containing a subsalt structure. Both media are synthetic models and assume a constant velocity. Figure 1a shows the layered medium. The depth of interest in this case is at 800 m, right below one of the reflection layers. The colourbar indicates density. Next to this layered medium a truncated medium is defined, in which the bottom reflectors below the depth of interest are left out. This is now called the truncated medium (Figure 1b). The red stars are source locations, and the triangles represent receiver locations. It is assumed there are no free-surface multiples.
The calculations in this research are done in MATLAB. Since inverting large matrices is a computationally expensive task, SPOT operators (Ewout van den Berg and Michael P. Friedlander) have been used in the calculations. This is explained in more detail in the section on solving techniques.

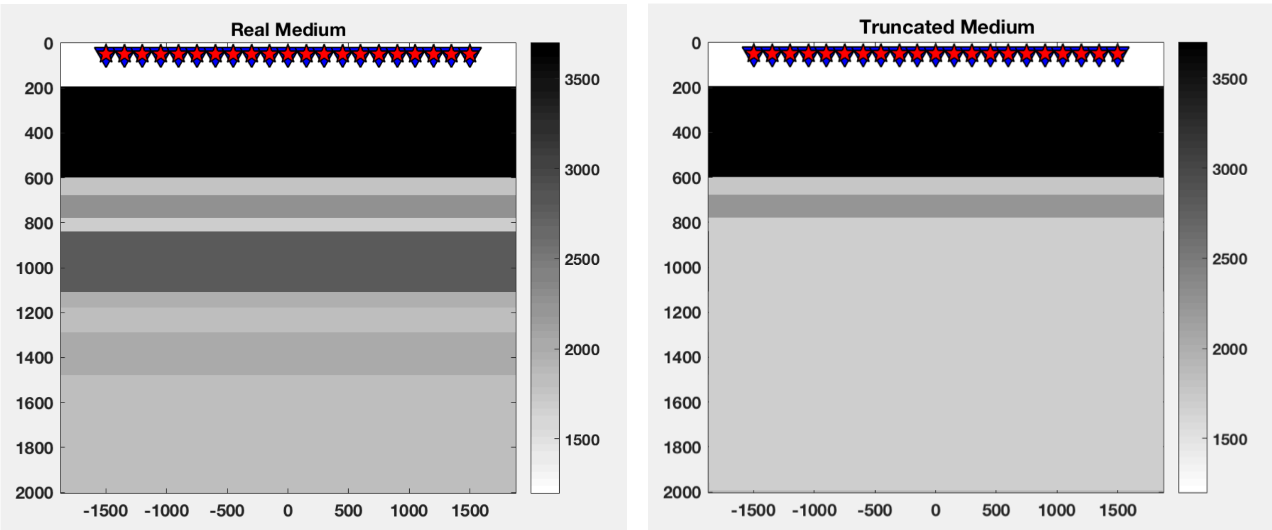


Figure 1: Constant velocity models used in the layered example. The legend indicates density in kg/m^3. a) full medium with reflectors. b) truncated medium.

The Marchenko method

The fundamentals of the Marchenko scheme are explained by means of an overview in a 1D medium. This is displayed in Figures 2 and 4, which are taken from Cui et al. (2018).

They contain depth versus time plots in a 1D medium. Reflection layers are indicated by the black dotted lines. The white dotted line marks the area of interest. Important is the difference between the medium in Figure 2a-b and Figure 2c. Figure 2c shows the full medium with a reflector below the area of interest, whereas that reflector is cut off in Figures 2a and 2b. The cut-off medium is the so-called truncated medium. The eventual goal of Marchenko is to retrieve up- and down going Green's functions at the level of interest.

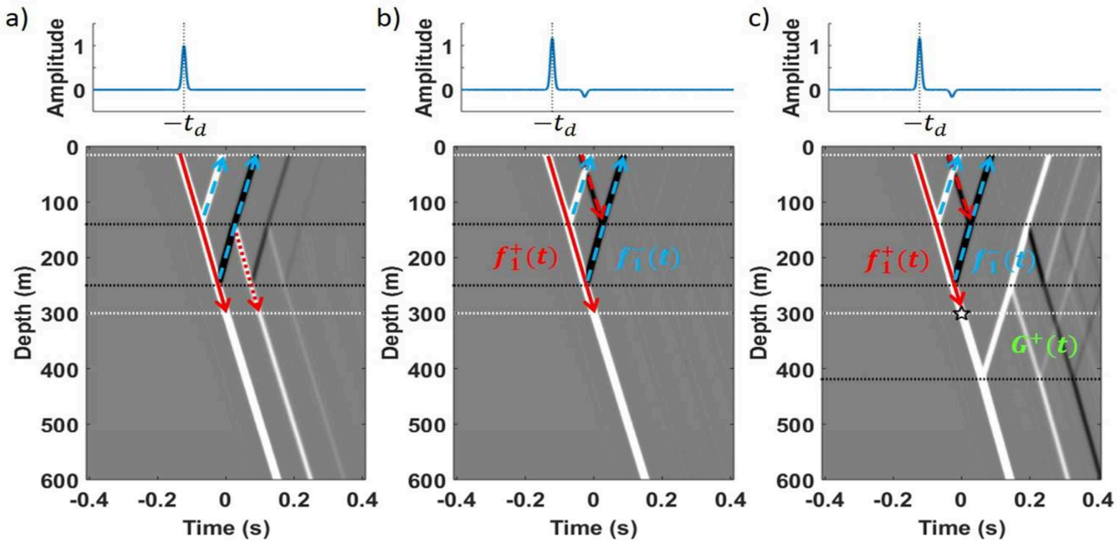


Figure 2 (extracted from Cui et al., 2018): An illustration of concepts behind the Marchenko method in 1D media. a) injecting a wavelet causes multiples. b) down going focusing function as a function of time, resulting in: c) down going Green's function. The star indicates a virtual source.

In this figure a seismic wavelet is 'injected' into the medium (Figure 2a). This is displayed by the down going, solid red line. This wavelet is displayed above the figure, where t_d stands for the direct arrival time. As can be seen, the wavelet is injected at a time of $-t_d$, so that it arrives at the level of interest at $t = 0$. Apart from the waveform that travels directly to the depth of interest, the wavelet will produce multiples due to reflections of the overlying layers. The multiples are indicated by the dashed blue (up going) and dashed red (down going) lines. This coda is undesired. It is with this reason that instead of a wavelet made up of a delta function, a so called focusing wavefield (Slob et al., 2014), or a focusing function (Wapenaar et al., 2014), is injected over time. It is represented by the red lines in Figure 2b. This focusing function cancels out the coda and focuses the wavelet at the desired depth. Again, the corresponding wavelet is displayed above the figure. The $f_1^+(t)$ in the figure stands for the down going focusing function, and it is defined by equation 1. The up going $f_1^-(t)$ is defined in equation 2. They are in fact the focusing wavefields observed at the surface. The $^+$ sign indicates down going wavefields, and the $^-$ sign indicates up going wavefields.

$$T(t) * f_1^+(t) = \delta(t) \qquad (1)$$

$$R_A(t) * f_1^+(t) = f_1^-(t) \qquad (2)$$

From equation 1 it can be seen that the f_1^+ is in fact the inverse of the transmission response of the truncated medium, as is proposed by Vasconcelos et al. (2015). The reflection response of this down going focusing function is the up going f_1^- (equation 2), where R_A is the reflection response of the truncated medium. The reflection response of a medium is based on the single sided surface reflection data of that medium. Figure 3 shows what the reflection response of the full layered medium looks like.

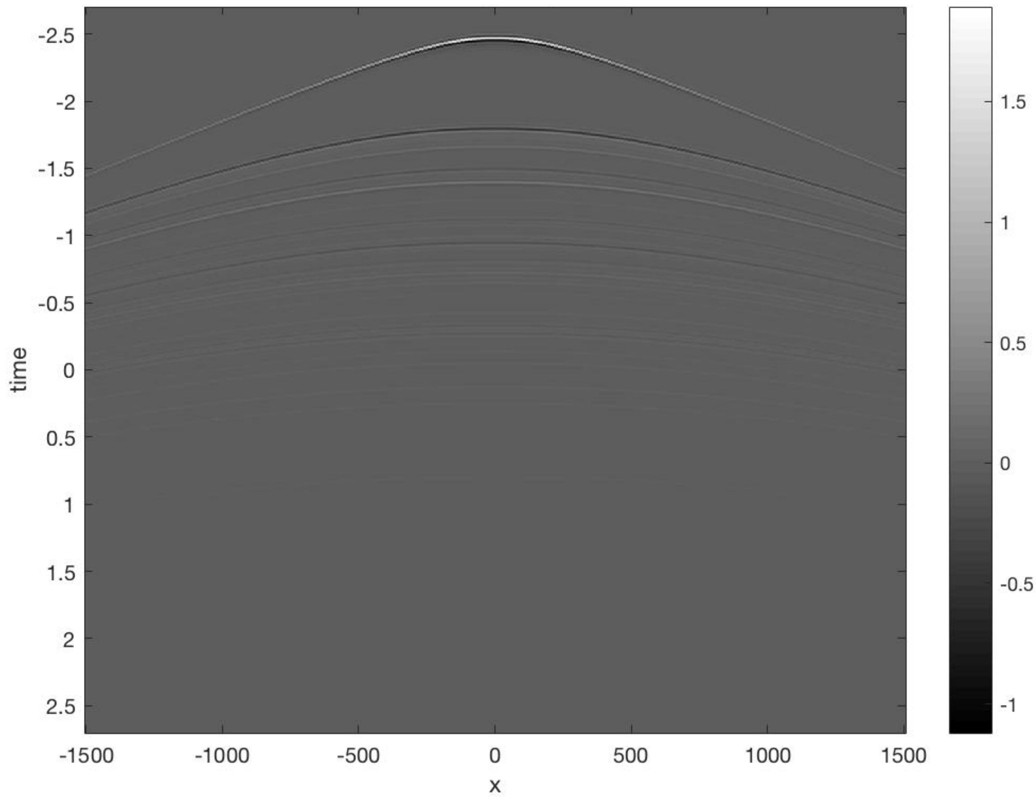
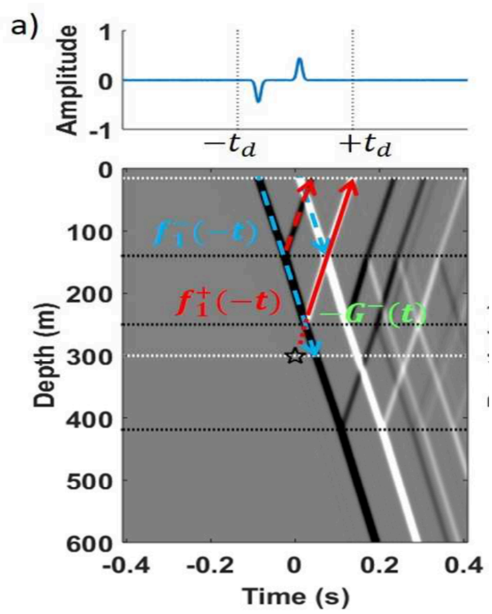


Figure 3: Reflection response of the layered medium

When looking back at Figure 2, it can be seen that the injected f_1^+ 'cancels out' the coda, so that the waveform that remains is now the down going Green's function G^+ in the full medium (Figure 2c). The white star in Figure 2c now acts as a virtual source from with $G^+(t)$ is 'produced'. The way this down going Green's function is retrieved by using reflection data, and both the up- and down going focusing functions is stated in equation 3.

$$G^+(t) = R(t) * f_1^+(t) - f_1^- \qquad (3)$$



In the same way, a time reversed f_1^- can be 'injected' into the full medium. This is represented by the dashed blue lines in Figure 4. The corresponding trace is displayed above the figure. The injected $f_1^-(-t)$ generates the time reversed f_1^+, shown as the solid and dashed red arrows. The following waves propagate as if they were produced by a virtual source (hollow star). This generates the up going Green's function G^-. Equation 4 shows the way the up going Green's function is retrieved from the Reflection data and the time reversed up- and down going focusing functions.

Figure 4 (Cui et al., 2018): a) time reversed up going focusing function results in an up going Green's function G^-.

$$G^-(t) = -R(t) * f_1^-(-t) + f_1^+(-t) \quad (4)$$

Let us recall that retrieving the up- and down going Green's functions is the goal of the Marchenko method. Equations 3 and 4 are therefore key equations, and when they are combined, we get a system which is known as the coupled Marchenko equations (equation 5). Note that the asterisk indicates time convolution.

Since surface reflection data is the only input given in the Marchenko system, only the reflection response of the medium is known. All of the focusing functions and the Green's functions are unknown. It is therefore impossible to solve this system with only two equations and four unknowns. To reduce the number of unknowns, two constrains must be introduced: the windowing operator, and an initial focusing function. The windowing operator is discussed in the next section.

$$\begin{pmatrix} -G^- \\ G^{+*} \end{pmatrix} = \begin{pmatrix} I & -R \\ -R^* & I \end{pmatrix} \begin{pmatrix} F^- \\ F^+ \end{pmatrix} \quad (5)$$

The windowing operator

It is assumed that the Green's functions from the Marchenko equations contain a distinct direct arrival from the level of interest. This direct arrival is calculated by means of a reference velocity model of the medium. All waveforms arriving after the direct arrival are defined as coda. A windowing matrix θ is defined, which will remove the events at and after the first arrival. The window function is symmetric around $t = 0$, meaning that every waveform arriving at or before $-t_d$ is removed as well. The window function θ is displayed in Figure 5.

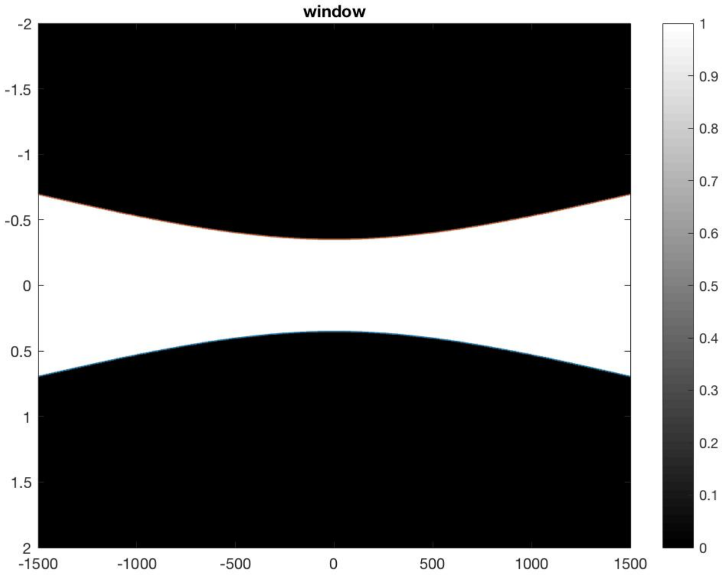


Figure 5: Window function θ for the layered example. Inside the (negative) direct arrival θ = 1. Outside the (negative) direct arrival θ = 0.

The result of windowing the Green's functions is of course that the waveforms that make up the Green's function are all removed, as is made clear from equations 6 and 7. The Green's function can in this way be eliminated from equation 5, already making the Marchenko system a solvable system, since it now has two unknowns.

$$\theta\{G^{+}(t)\} = 0 \quad (6)$$

$$\theta\{G^{-}(t)\} = 0 \quad (7)$$

If a window matrix is applied to the left hand side of the system (i.e. the Green's functions), then it must be applied to the right hand side as well. When looking at how the focusing functions respond to the application of the windowing matrix, it is important to recall that it is assumed that the up going focusing function too, contains a direct arrival ($f_{1,d}^{+}$) and a coda ($f_{1,m}^{+}$). This can clearly be seen from the trace at the top of Figure 2b. The direct arrival of course peaks at $-t_d$, and will therefore be eliminated by the window function. The coda arrives within the window, which ranges from $-t_d$ to t_d, and will therefore be left unchanged by the matrix θ. Altogether this adds up to the results of equation 8. The up going focusing function however, falls completely within the range of window θ, as can be seen from the wavelet in the top of Figure 4. It is therefore that the up going focusing function is unchanged by the window matrix. This is displayed in equation 9. A more extended and detailed explanation on this windowing function and its properties can be found in van der Neut, et al (2015).

$$\theta\{f_1^{+}(t)\} = \theta\left\{f_{1,d}^{+}(t)\right\} + \theta\left\{f_{1,m}^{+}(t)\right\} = f_{1,m}^{+}(t) \quad (8)$$

$$\theta\{f_1^{-}(t)\} = f_1^{-}(t) \quad (9)$$

After now having applied the window matrix to each term in the Marchenko system of equation 5, this scheme now looks as stated in equation 10. This system of two equations is now, in principle, solvable, since it contains only two unknowns: the up going focusing function (F_1^-), and the coda part of the down going focusing function (F_{1m}^+). Again, the direct arrival is calculated by using the assumed velocity structure. This solvable Marchenko system is as follows:

$$\begin{pmatrix} \theta R F_d^+ \\ 0 \end{pmatrix} = \begin{pmatrix} I & -\theta R \\ -\theta R^* & I \end{pmatrix} \begin{pmatrix} F_1^- \\ F_{1m}^+ \end{pmatrix} \quad (10)$$

A new Marchenko system

Though the Marchenko method has improved over the past years (and applied to field data (Ravasi et al., 2016)), there are still more ways in which this method can be researched and used. We have seen now that Marchenko redatuming is capable of providing accurate information on primaries as well as internal multiples. However, with increasing complexity of the subsurface this will become more of a challenge. Jia et al. (2018) show that when applying the Marchenko scheme to a model containing a subsalt structure, it can already produce more continuous images than traditional imaging methods (such as reverse time migration), and that it is effective in suppressing the artifacts caused by internal multiples. Even though these are positive results, with these more complex media it is much more easy to miscalculate Green's functions, for example due to a mismatch between the calculated direct arrivals and the actual reflection data. Also, the inverse transmission matrices that are being used (equation 1), induce artefacts, due to the complexity of the reference model, even though it does produce better results (Vasconcelos et al., 2015). It is for these reasons that a variation on the original Marchenko scheme is set up by Vasconcelos and Van der Neut (2016). This variation focuses on media with high heterogeneities and/or large contrast structures such as subsalt. The system follows the original Marchenko rules but it leaves out some of the inconveniences that made the system hard to solve for complex media. Altogether, the new Marchenko system is designed for application to scattered/perturbed fields. It is explained in this section.

To evaluate the perturbed medium correctly, the Marchenko scheme must be solved twice. Once for the full medium, as displayed in equation 11, and once for a known reference medium as displayed in equation 12. The $_0$ subscript indicates that the function or matrix is evaluated in the reference medium. This reference medium is often represented by a velocity model which should resemble the true velocity model (which in itself is of course unknown, only the true reflection response is known). In the cases of both the layered example and the subsalt example, this reference model is smoothened, so that smaller reflectors below the area of interest will not be as prominent as in the full medium. Figure 6 shows the reflection responses of both the real medium and the reference medium of the subsalt example.

$$\begin{pmatrix} -G^- \\ G^{+*} \end{pmatrix} = \begin{pmatrix} I & -R \\ -R^* & I \end{pmatrix} \begin{pmatrix} F^- \\ F^+ \end{pmatrix} \quad (11)$$

$$\begin{pmatrix} -G_0^- \\ G_0^{+*} \end{pmatrix} = \begin{pmatrix} I & -R_0 \\ -R_0^* & I \end{pmatrix} \begin{pmatrix} F_0^- \\ F_0^+ \end{pmatrix} \quad (12)$$

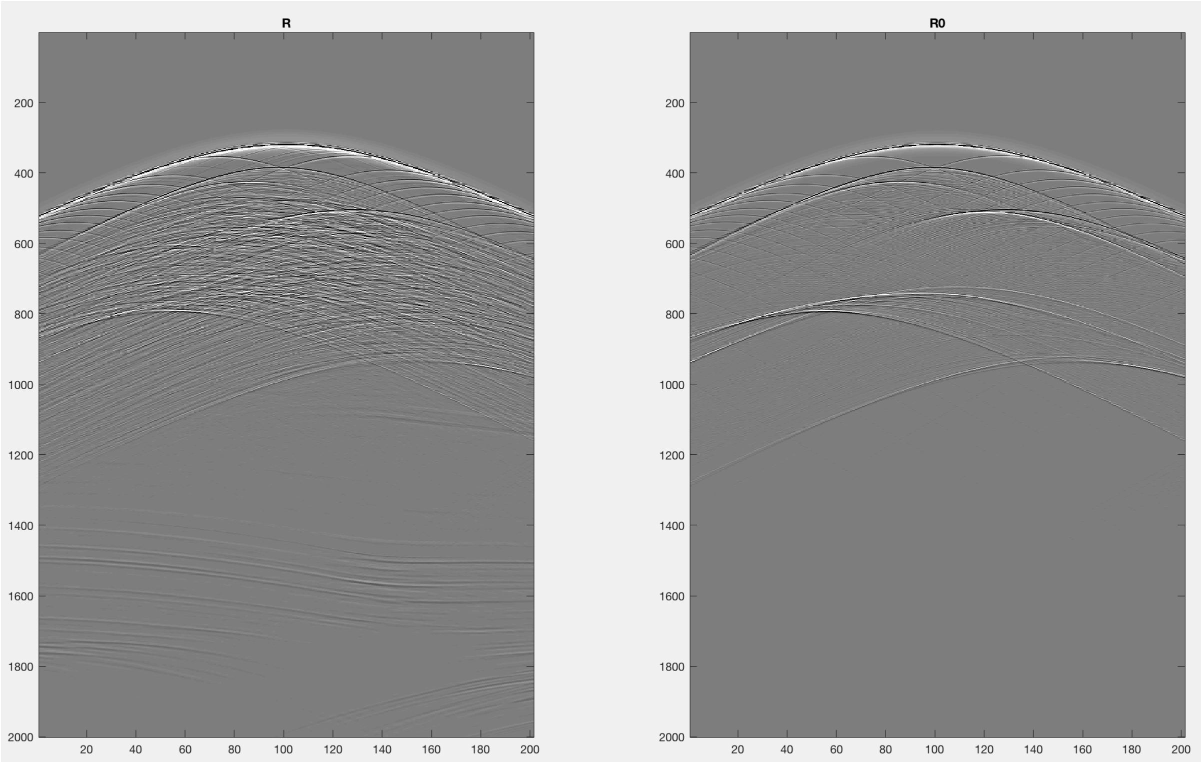


Figure 6: Reflection responses of: a) The full medium in the subsalt example. b) the reference medium in the subsalt example. The real medium shows a more detailed structure than the reference medium. The concave shape at the lower half of the image is not visible in the reflection response of the reference medium.

To obtain the goal of calculating the focusing functions and the Green's functions of the coda, it is possible to subtract the reference medium from the real medium. This would result in the system displayed in equation 13, where the δR denotes the reflection response of the reference medium subtracted from the reflection response in the real medium. Again, the system contains two equations with four unknowns, since in the real medium only the reflection response is known. After applying the window matrix, we get the system displayed in equation 14.

$$\begin{pmatrix} -\delta G^{-} \\ \delta G^{+*} \end{pmatrix} = \begin{pmatrix} 0 & -\delta R \\ -\delta R^{*} & 0 \end{pmatrix} + \begin{pmatrix} I & -R \\ -R^{*} & I \end{pmatrix} \begin{pmatrix} \delta F^{-} \\ \delta F^{+} \end{pmatrix} \quad (13)$$

$$\begin{pmatrix} 0 & -\theta\delta R \\ -\theta\delta R^{*} & 0 \end{pmatrix} \begin{pmatrix} F_0^{-} \\ F_0^{+} \end{pmatrix} = \begin{pmatrix} I & -\theta R \\ -\theta R^{*} & I \end{pmatrix} \begin{pmatrix} \delta F^{-} \\ \delta F^{+} \end{pmatrix} \quad (14)$$

The window function θ will be identical for both the real model and the reference model, since the reference model is built in such a way that it is assumed that the direct arrival in the two models will be the same. Solving equation 14 for the focusing functions is the main problem this research focuses on. When this is successfully done, the up- and down going Green's functions can then be reconstructed with the calculated focusing functions. The importance of proving that this new system (equation 14) can be solved is that it allows for the use of arbitrarily complex reference models, such as those used in e.g. subsalt imaging practice. This is not possible under the conventional Marchenko framework.

For finding the up going focusing function in the reference medium, F_0^+, again the inverse of the transmission matrix in the reference medium can be used, just like in equation 1. From there, its reflection response in the reference medium is F_0^-, just like in equation 2. In this research however, both the up- and down going *F0's* are calculated by solving the reference model separately. This is done by inversion, which will be discussed in the next section. The additional down going F_0^- was not present in the original Marchenko scheme that was solved (equation 10). It accounts for reflections due to F_0^+, and together with the δR it accounts for scattering effects due to sharp contrasts or discontinuities.
In the following examples it is shown that the results of this new system are indeed identical to the result of calculating the two systems separately and subtracting them. Now that it is proved that it can be solved, especially with inversion, this new system will be a next step towards accurately solving perturbed fields, or fields with large contrasts and sharp discontinuities. These results show both the new system solved in the layered example (Figure 7) and in the subsalt example (Figure 8).

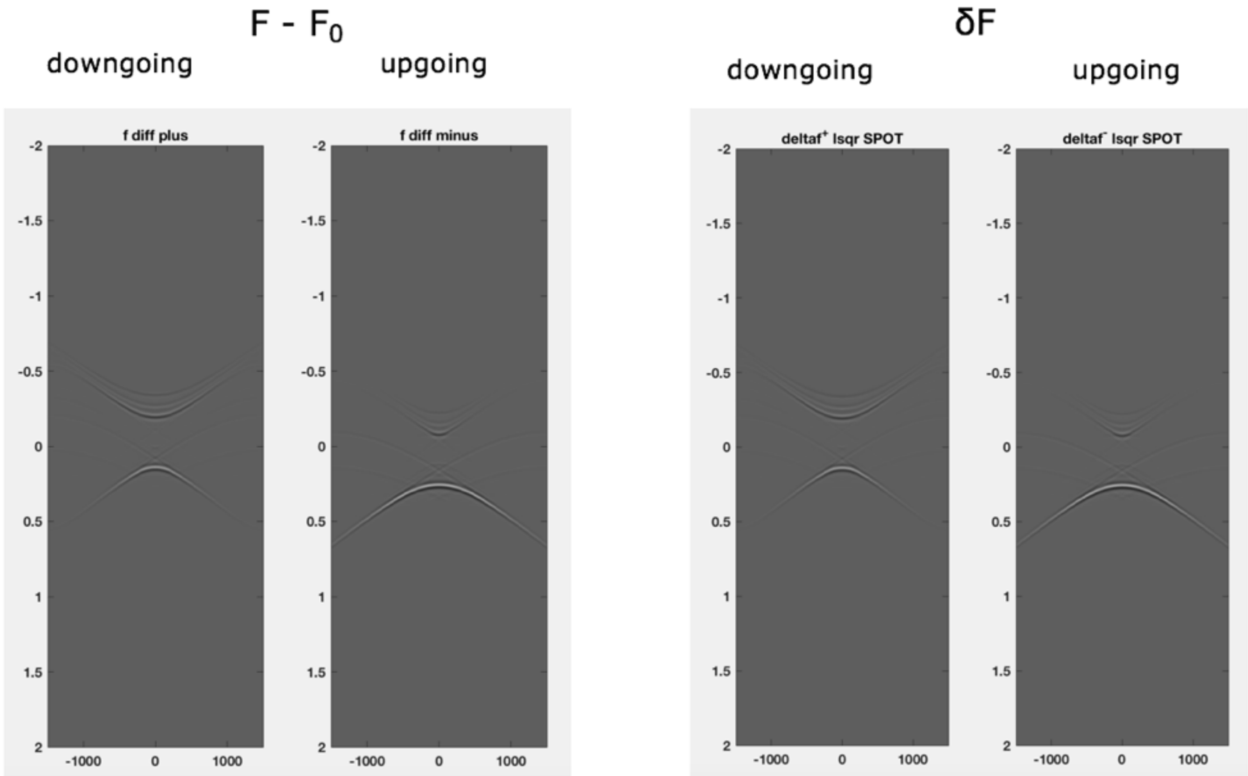


Figure 7: focusing functions in the layered example: a) difference between the focusing functions of the real medium and the reference medium. b) focusing functions calculated with δR. The two plots are identical. When subtracting them from each other the result is 0.

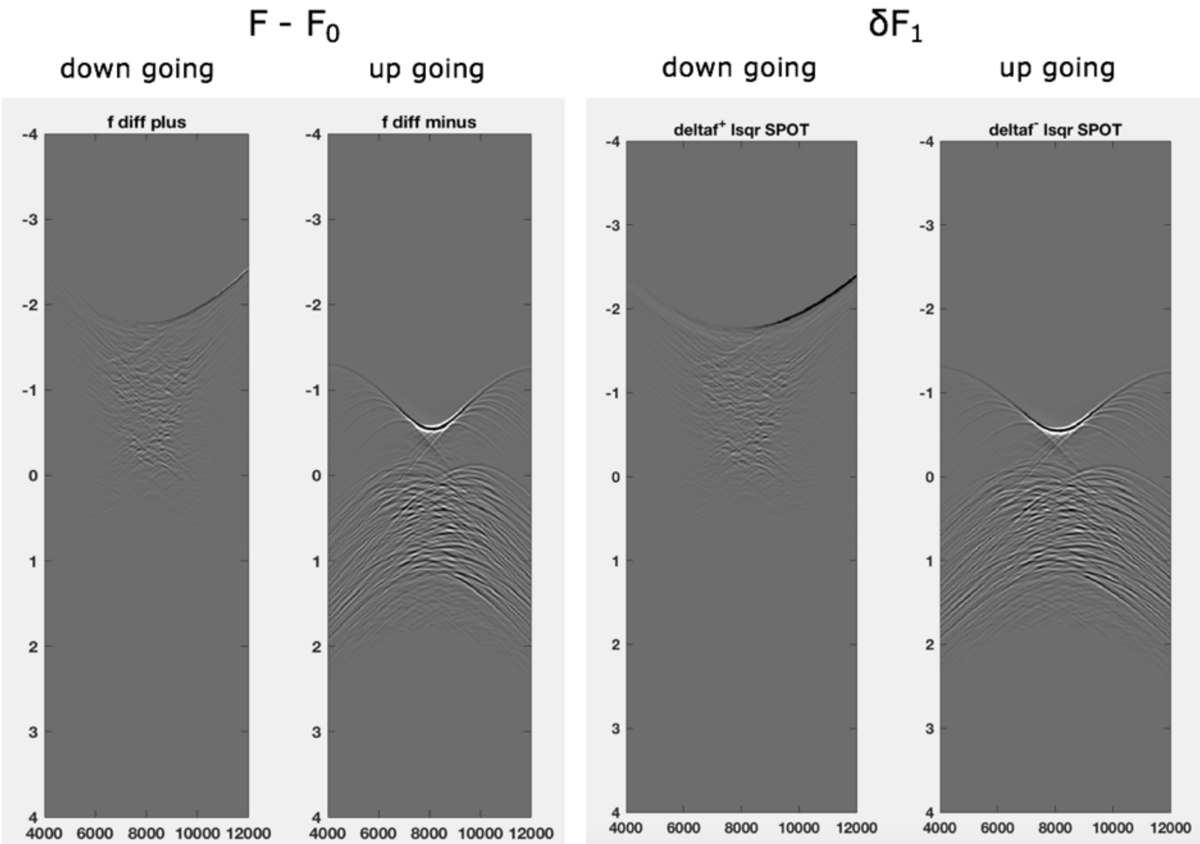


Figure 8: focusing functions in the sub-salt example: a) difference between the focusing functions of the real medium and the reference medium. b) focusing functions calculated with δR. Again, the two ways of calculating produce the exact same focusing functions.

Iteration versus Inversion

As mentioned in the previous sections, the Marchenko system can either be solved by an iterative scheme, or by inversion. The layered example has been used to test and compare the two different techniques. Once proven that the results of the two calculations of this simple model were the exact same, inversion has been the method that is used for the new Marchenko scheme and all the subsalt calculations, since inversion has the advantage of solving more complex problems in more complex media. This section discusses what that advantage is, and why iteration will fail to perform certain calculations.

The solution of the system in equation 14 can be written as a summation (equation 15), where *k* indicates the number of summations and thus iterations. For this series to converge, one necessary condition is that the norm of the Marchenko operators containing the two reflection responses of the real medium has to be smaller than one. More complex media will have more complicated reflection responses, with norms potentially larger than one. Therefore, this iterative solver will not be accurate for just any model.

$$\begin{pmatrix}\delta F_1^-\\ \delta F_{1m}^+\end{pmatrix} = \sum_{k=0}^{\infty}\begin{pmatrix}0 & -\theta R\\ -\theta R^* & 0\end{pmatrix}^k \begin{pmatrix}0 & -\theta\delta R\\ -\theta\delta R^* & 0\end{pmatrix}\begin{pmatrix}F_0^-\\ F_0^+\end{pmatrix} \qquad (15)$$

In the models used in this research, both iterative substitution and inversion lead to the same results. However, future work may involve stronger heterogeneities in the subsurface or more complex background models. Staring et al. (2017a) makes a comparison between solving the Marchenko system by inversion and by iteration for three different kind of models. All three models are evaluated with and without consideration of reflections of the free surface. It can be seen in Figure 9 that when the free surface reflection is included, the more complex the models become, the harder it is to solve them by means of iteration. The results of Model 3 will eventually become unstable. It can be seen that for those calculations, solving by iteration does not yield correct results. A solver using inversion is needed for such calculations.

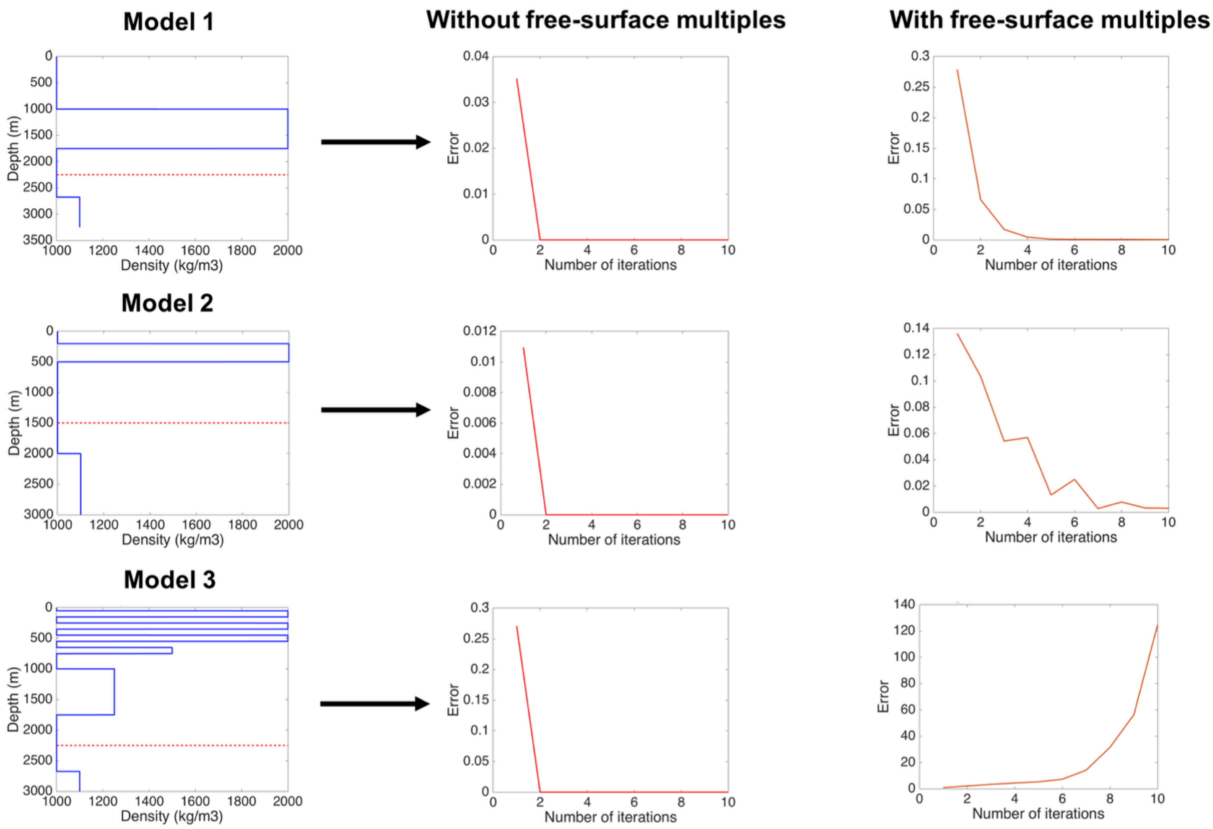


Figure 9 (extracted from Staring et al, 2017a): Three models that differ in complexity solved by iteration. The red dashed line shows the level of interest. The more complex the models are, the higher the error. Also, adding free surface reflections increases the complexity. Model 3 with the free surface multiples can not be solved by iteration: the result becomes unstable.

In order to overcome the limits that go with using the iterative system, it is necessary to find a system that is capable of dealing with the convergence issue. Equation 14 can be seen as a simple linear equation such as $A\,x = b$. This is displayed in equation 16. In this case the δF^- and δF^+ are represented by *x*, the variable that we are interested in. This can then of course be calculated by taking the inverse of this equation: $x = A^{-1}\,b$. Finding inverse matrices for

these larger and complex models is computationally more expensive than using an iterative solver. However, it is necessary in order to overcome the limits explained in the previous paragraph. The solution of this direct inversion can be found in several different ways. The complexity of the models and matrices used is an important factor when you choose a method of solving. In this research we propose the use of the LSQR method, which is well suited for over-determined equations. In this method an error is defined as $r = A\,x - b$. The solution *x* is chosen so that $\|r\|$ is minimized.
The results of the focusing functions calculated by inversion were identical to the iterative solutions in the layered model. This proves that the inversion calculation works correctly. It can be used for more complex calculations, for example in perturbed fields and models with sharp contrasts. It is for this reason that after testing inversion on the layered model, all the calculations in the subsalt example and the calculations of the new Marchenko scheme described in the previous section, are done by means of inversion, even though it is computationally more expensive than iteration.

$$\underbrace{\begin{pmatrix} 0 & -\theta\delta R \\ -\theta\delta R^* & 0 \end{pmatrix}\begin{pmatrix} F_0^- \\ F_0^+ \end{pmatrix}}_{b} = \underbrace{\begin{pmatrix} I & -\theta R \\ -\theta R^* & I \end{pmatrix}}_{A}\underbrace{\begin{pmatrix} \delta F^- \\ \delta F^+ \end{pmatrix}}_{x} \qquad (16)$$

The calculations in this research are done in MATLAB. Inverting matrices and finding LSQR solutions with large reflection responses is computationally expensive. It is therefore that the matrices found in equations 14 and 16 are stored as SPOT operators (Ewout van den Berg and Michael P. Friedlander). SPOT is a linear operator toolbox provided for MATLAB calculations. In our calculations, the data operator R consists of a multidimensional convolution (MDC) operator: even for these relatively small examples, the time-domain version of this MDC operator would yield too large a matrix to be explicitly stored and used in the calculations. Using the SPOT library, we can handle the time-domain Marchenko operators in the time domain efficiently, while maintaining the convenience of matrix-like syntax in our scripts.

Conclusion

This research has shown two important aspects in the ongoing goal of improving calculations with the Marchenko system. It has shown that there is a new Marchenko scheme which is able to focus on complex and perturbed fields. It does so by adding a reference medium to the Marchenko calculations. As a result of this, not only an up going focusing function is used as known term in the calculation, but also a down going focusing function, which can account for the scattering effects of the medium of interest. Next to that the difference between the reflection response of the full medium and the known reference medium is used in the calculations. The results of this research show that the proposed variation on the Marchenko system produces the same results as subtracting separate calculations for the truncated medium from results of the full medium calculations. This is convenient for any future calculations with more complex setups or velocity structures. The example models in this research have all used a constant velocity. For complex models, such as sub-salt, and/or models that use a variable velocity, the new Marchenko scheme will provide a potentially better calculation of the up- and down going waveforms.
Secondly, this research explains why solving the Marchenko scheme by iteration is limited. For convergence, the norm of the matrix that is summed must eventually approach zero.

Again, with complex reflection responses this term will be more difficult to be met. This research suggests solving the system by inversion. The results show that for the layered, constant velocity example the iterative results are identical to the results obtained by using the LSQR method. This is used an argument for only using inversion for the remaining tests of the research.
Overall this research will hopefully contribute to further research and improvement of the Marchenko system, and the use of it. Some of the related research are briefly discussed in the final section below.

Related Research
The Marchenko scheme has been subject of multiple research the past few years. It has been tested with (complex) field data, and further improvements on the system can still be made. One of the examples of further improving the system is by using an adaptive double focusing-method for source-receiver redatuming, as proposed by Staring et al. (2017b). Here, source redatuming is done by taking the calculated up going Green's function and the down going focusing function to obtain a redatumed reflection response of the real medium. This is done instead of the usual calculation of the Green's functions as presented in this research.
A second way to apply Marchenko redatuming is by combining the coupled Marchenko equations with a Rayleigh integral representation. This is called Rayleigh-Marchenko redatuming, and it is explained by Ravasi (2017). His newly proposed scheme can handle internal as well as free-surface multiples.

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