Universiteit Utrecht

Opleiding Natuur- en Sterrenkunde

Measurements on electrical properties of water while acoustically levitated

Bachelor Thesis

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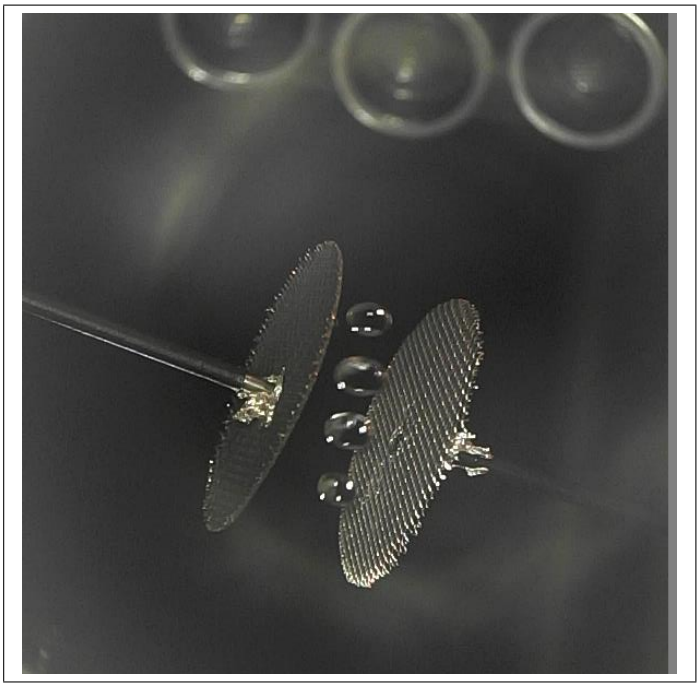

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Abstract

With the use of an acoustic levitator, I remotely measured the capacitance of water, with the idea of obtaining the dielectric constant. A water droplet was trapped inside the central node of the acoustic field and its capacitance measured, the field was then turned on and off to allow the measurement of the electrodes without the water and to calculate the difference. The capacitance was measured with the help of a self build LC-circuit that relies on the resonance between a capacitor and inductor. To verify measurements a Fluke PM6306 RCL-meter was used with the same measurement routine. Resistance was also measured using the same RCL-meter at different frequencies and salt concentrations to see the effect on the resistivity. The droplet was placed at the same node and two small electrodes were brought into the droplet on either side.

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1 Introduction

The use of an acoustic levitator allows us to measure properties of materials without touching them. This technique has gotten a big boost with the advent of 3D-printers, allowing people to more easily and cheaply build them [1]. Because liquids take the shape of the container they are in, measurements on a liquid without deforming the surface are hard to do. A levitated droplet will take the shape of a sphere or oblate spheroid since this is the shape with the lowest surface area to volume ratio. In liquids there is a disconnection between bulk and surface charge, changing one of them alters the electrical properties of the liquid. Because when a drop is levitated it hardly has any outside interference. Thus the electrical effects can be studied without having to account for discrepancies at a boundary of another material. Things to measure are the complex impedance which can lead us to expressions for the carrier density, charge mobility, Debye length or dielectric constant. Two types of measurements are done, one measurement of the capacitance to obtain the dielectric constant and another to measure the resistance and obtain conductivity.

Measuring the capacitance is done by floating a droplet in the acoustic levitator. First measuring the capacitance with a droplet between two electrodes and later measuring the capacitance with out the liquid in between. Subtracting the two values will give us the capacitance of the droplet. Consequently measuring the capacitance allows for a measurement of the dielectric constant of that liquid. Two ways of measuring the capacitance are explored in this thesis, using a Schering bridge or with a LC-circuit, the self-build devices are also checked with a more conventional RCL-meter. The Schering bridge works on the principle of balancing two arms of a parallel circuit and the LC-circuit on the resonance frequency that exists when a capacitor and inductor are in the same circuit. I measured the resistance by placing two small electrode points on either side of the droplet while it is levitated. The resistance is measured at different frequencies and salt concentrations to see the effect of these changes. Especially changing the salt concentration is interesting since this changes the bulk properties of the liquid. The resistance is measured with the RCL-meter.

Following are, first a section of theory on how capacitance can be measured, then a section about the instruments and set-up that was used. Finally the results will be packed in with the discussion.

2 Theory

Unless referenced otherwise all information in the theory section comes from the book: The art of electronics by Paul Horowitz and Winfield Hill[2]

2.1 Complex way of writing impedance

A capacitor(C) consists of two conductors closely separated by an insulator. Putting a voltage across the two conductors charges the capacitor with a charge Q on one side and -Q on the other. This leads to the familiar equation: $Q = CV$ with C a constant that depends on the shape of the capacitor and the dielectric constant of the material in between. There is a simple equation for the capacity of two parallel plates of surface area A with distance d between them,

$$C = \frac{\epsilon A}{d}. \tag{1}$$

With ϵ the absolute permittivity which can be written as, $\epsilon = \epsilon_0 \epsilon_r$ with ϵ_0 the permittivity of a vacuum ($8.85 \times 10^{-12} Fm^{-1}$[3]) and ϵ_r the relative permittivity.

Because current is a change of charge over time, if we take the derivative of $Q = CV$ we get: $I(t) = C\frac{dV}{dt}$. If the current were to be oscillating as $I(t) = I_0 \sin(\omega t)$, then by equating these two we get: $\frac{dV}{dt} = \frac{I_0 \sin(\omega t)}{C}$. Integrating this gives us equation 2 which can be used to find something that resembles a resistance in equation 3, this is called the reactance. What can also be seen from the equation for voltage and current is that the current is 90° ahead of the voltage. A similar relation holds true for an inductor only with voltage ahead of the current and a different reactance.

$$V(t) = -\frac{I_0 \cos(\omega t)}{C\omega} \tag{2}$$

$$X_c = \frac{|V|}{|I|} = \frac{\frac{I_0}{\omega C}}{I_0} = \frac{1}{\omega C} \tag{3}$$

As can be seen from the above formulations we have to solve differential equations to explain the behaviour of capacitors. The same holds for inductors, thus circuits with these elements in them have increasingly harder differential equations to solve. To combat this problem it is easier to write them in a complex form and use something called the Impedance(**Z**), this is the generalized resistance. Because capacitors and inductors have a 90° phase shift between voltage and current they are reactive and have a reactance(X). Resistors have voltage and current in phase and are resistive, they have a resistance(R). The impedance is: impedance = resistance + reactance or Z = R + jX, with j the imaginary number.

The voltage and current can also be written as complex numbers, $V(t) = V_0 \cos(\omega t + \phi) = V_0 \Re(e^{j(\omega t + \phi)})$. This, combined with the impedance allows us to use a more general form of Ohm's law on every circuit containing linear components like resistors, capacitors and

inductors. The impedance for the various components are,

$$\mathbf{Z}_R = R \tag{4}$$

$$\mathbf{Z}_L = j\omega L \tag{5}$$

$$\mathbf{Z}_C = \frac{1}{j\omega C}. \tag{6}$$

Knowing this we can more easily get the equations that describe the circuits that were used.

2.2 Ways of measuring capacitance

Two ways of measuring capacitance were tested in the experiments: a resonant frequency- and a bridge balancing-circuit. The resonance frequency circuit works on the principle that when a inductor and capacitor are in parallel there is a frequency were the impedance goes to infinity.

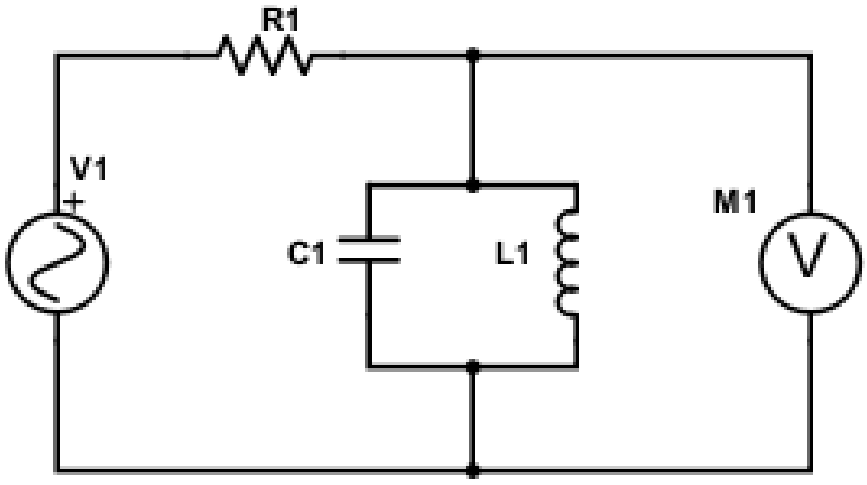


Figure 1: LC resonance circuit

The equation describing the circuit in figure 1 can be obtained using the generalized Ohm's law. The impedance from the LC part is just the two added in parallel, which is done the same way as with "regular" circuits.

$$\mathbf{Z}_{LC} = \frac{1}{\frac{1}{\mathbf{Z}_L} + \frac{1}{\mathbf{Z}_C}} = \frac{1}{j\omega C + \frac{1}{j\omega L}} = \frac{j}{\frac{1}{\omega L} - \omega C} \tag{7}$$

Equation 7 goes to infinity when $\omega C = \frac{1}{\omega L}$ which happens at a frequency of,

$$\omega = \frac{1}{\sqrt{LC}}. \tag{8}$$

At this frequency the measured voltage will spike and using a known inductor the capacitance can be calculated from the obtained frequency.

The second method involves a bridge type circuit, the idea of such a circuit is to balance both parallel arms. This is done by measuring the voltage or current between the two arms and adjusting components in the circuit to bring the voltage or current to zero. To measure a capacitance a Schering bridge is often used[4], and is also what was used in this thesis.

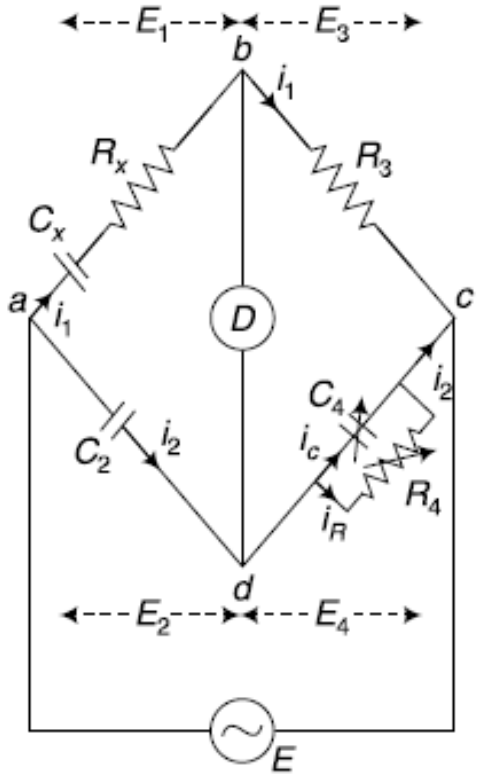


Figure 2: Schering bridge

Figure 2 is a schematic of a possible schering bridge, the main differences between versions are how many of the components are variable, or if there are measures taken to counter parasitic capacitance, more on that later in the discussion. Because no capacitor is perfect it works to segment the problem by describing the measured capacitance as a perfect capacitor with the loss described as a series resistance. To find the equations that govern what happens when there is balance between the arms, so no voltage measured, it is beneficial to write each arm as a complex number. The top arm of the bridge can be written as a single complex number by means of dividing E_1 by E_3. If then there is a balance between the two arms the ratio of the other arm is equal to this complex number. We can thus say, at balance: $\frac{E_1}{E_3} = \frac{E_2}{E_4}$ which gives us the easier balance equation of,

$$E_1 E_4 = E_2 E_3. \tag{9}$$

Knowing that: $E_1 = R_x + \frac{1}{j\omega C_x}$, $E_2 = \frac{1}{j\omega C_2}$, $E_3 = R_3$, $E_4 = \frac{1}{1/R_4 + j\omega C_4} = \frac{R_4(1/j\omega C_4)}{1/(j\omega C_4)+R_4}$. Together with equation 9 we can solve for R_x and C_x. Doing this gives us the following equations which we can use to find what we want.

$$R_x = C_4 \frac{R_3}{C_2}, \quad C_x = R_4 \frac{C_2}{R_3} \tag{10}$$

2.3 Parasitic capacitance

As mentioned above, when dealing with circuits no component is perfect and each has always a certain loss or side-effect associated with them. One of these side-effects is parasitic capacitance, prevalent when there are two wires or other conducting material close to each other with alternating current flowing through them. Parasitic capacitance is proportional to the frequency of the power source, just as with normal capacitance. The higher the frequency the lower the impedance of such a capacitor which can lead to unwanted losses. The distance between the wires is very important as well, since this determines the constant C.

3 Experiment

3.1 Acoustic trap

The acoustic levitator is made using two domes that are cut-outs of a sphere with a radius of 110.5 mm. On the domes 72 transducers are placed following the curvature. The transducers have a strong resonance at 40 kHz. Both domes are sent a 40 kHz block signal by a signal generator, the signal is amplified by an amplifier with a 16V power supply. The two domes are placed on top of one another with such a distance in between that there is a central node roughly in the middle. The trap is shown in figure 3b, with a perspex casing surrounding it to limit the air flow that could potentially disturb measurements.

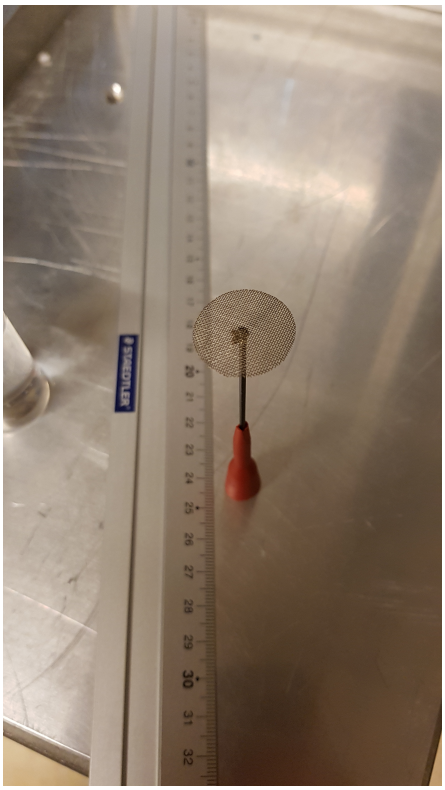
3.2 Electrodes

To measure the capacitance electrodes were created that wouldn't interfere with the acoustic field. This was done with a metallic mesh of the right grid size as to not disturb the field. Two electrodes were bought from panoma that had very fine metallic tips, they are 10 cm long but the is tip only $\sim$1 mm in diameter. The metallic mesh, with a radius of 1.1 cm, was then glued to end of the tip using a conducting silver glue. Only a small amount was needed and after testing we could see that with the glue the electrodes still did not, or hardly, interfere with the acoustic field. Figure 3a is a picture of one of the electrodes.

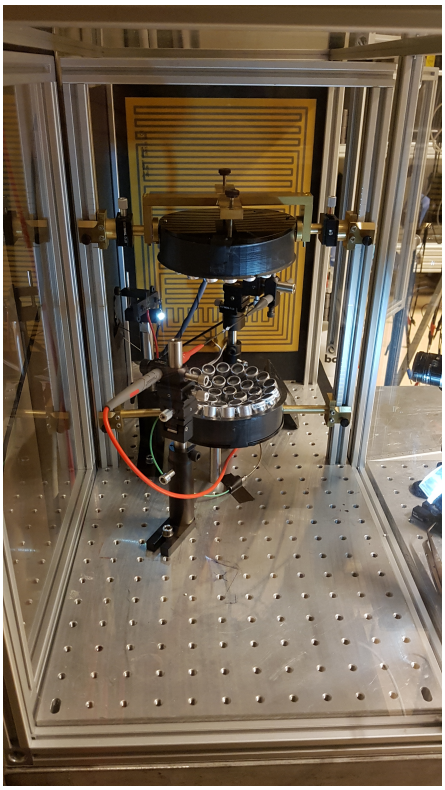
To measure the resistance the same type of electrodes were used but this time without a mesh glued to the tip. Since the electrodes are made from stainless steel we expect that there will hardly be a concentration change due to corrosion of the metal. This chance is further reduced by the modification made to the tip. To reduce the wetting of the point and the subsequent creeping of the droplet onto the side of the tip. The entire tip, save the very point, was coated with paraffin wax a very good insulator[5] and insoluble in water[6]. This stopped the creeping and made the experiment easier to execute and also more reproducible.

The wires coming from the electrodes(for convenience called probes) into either the RCL-meter or the LC-circuit were twisted to reduce the amount of noise from the electric field of the acoustic trap. Because the probes go through the same tiny hole on the base of the trap as the wires that go to the speaker(for convenience just called wires). These wires have a voltage of around 14 V which is quite high compared to either the 5 V from the LC-circuit or the 2.5 V from the RCL-meter. Because of the close proximity the probes might wrap around the wires, forming a loop around the wires and therefore having an electric field induced in them. Twisting the probes ensures that if such a thing might occur the next wrap is in the opposite direction in the field, thus cancelling out. The undesired side effect of twisting to wires is that it also increases the influence of parasitic capacitance. When untwisted the electrodes had a capacity of around 1 pF(measured with the RCL-meter), when twisted this increased to around 15 pF.

The electrodes are mounted on movable stands to allow them to be moved in and out of the centre of the acoustic field where the droplets will be. This helps with putting water droplets into field, but makes obtaining a consistent distance between measurements very hard. The stands are placed in a way that they are as perpendicular to the camera as can be done by eye. Since the mesh is not perfectly perpendicular to the electrode tips and the tips



(a)
One of the electrodes used to measure the capacitance. The ones used to measure the resistance are the same model but with a sharp point at the end.



(b)
The set-up with the acoustic trap, behind it is a heater mat not relevant for our experiment, the stands on which the electrodes are attached are also visible the silver coloured knobs allow the top part to move. The electrodes are positioned in a way that the led and the camera are on one line in between the electrodes.

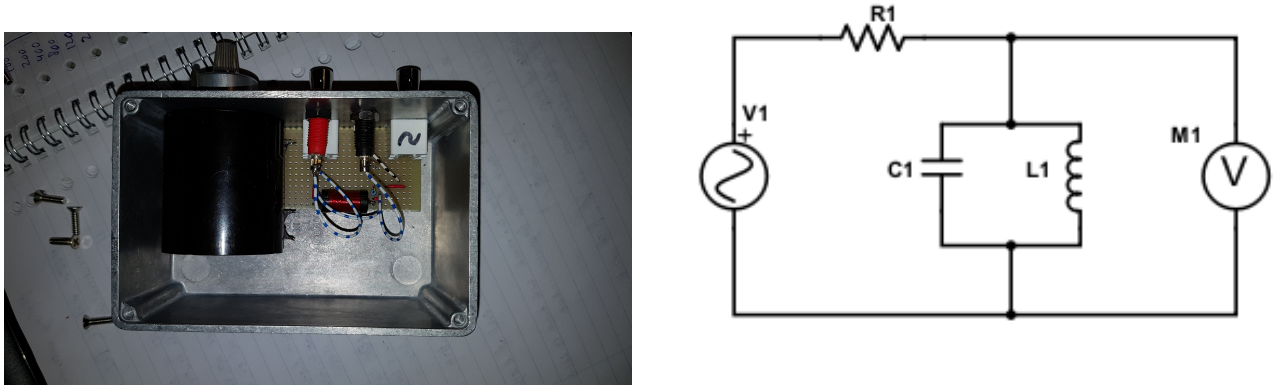
Figure 3

of the other kind of electrode are very thin it was assumed that a perfect 90° angle wasn't necessary. The 90° angle would be important because it allows the camera to determine the distance between the electrodes more accurately. The distance between the electrodes was measured by first calibrating a picture of a ruler and then converting the distance in pixels into meters. The ImageJ program was used for this task, using the plot profile function to get an accurate distance between marks on the ruler.

3.3 Apparatus

For the measurement of the capacitance a self-made device was used, a professional device was used to verify measurements and be able to get more accurate results. Two circuits were made, a Schering bridge and a resonant LC-circuit, how these work is explained in the

theory. These circuits were soldered onto a PCB(printed circuit board) and then placed in a metal box to shield them from outside interference. In figure 4a the inside of the box can be seen, both circuits can be seen in figure 4 and 5 together with a schematic and values of each component. The component values for the Schering bridge were measured by a multimeter which had an error of 0.20% + 2d, meaning each value deviates by 0.2% plus the last digit shown. The components in the LC-circuit were measured with the RCL-meter whose error will be shown later. The values for the used components were chosen with the guess that the capacity of our electrodes/water droplet would be around 1-2 pF and have a very high resistance. The guess came from the simple equation for capacitance in the beginning of the theory, using a cube of water with sides of 2 mm and filling that value into equations 1 and 10. Both the Schering bridge and the LC-circuit were operated by a Digilent analog discovery 2. This device has four BNC ports, two are signal generators and the other two are oscilloscopes.



(a) LC-circuit inside box, the large variable resistor is not hooked up

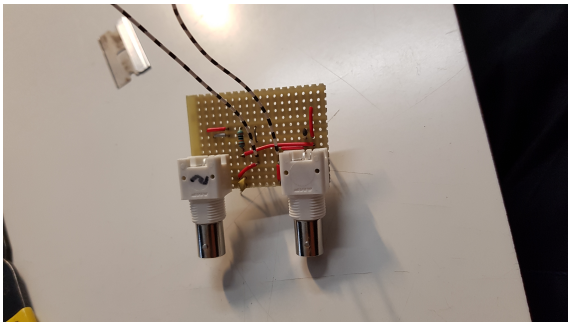
(b) schematic of an LC-circuit, as shown before

Figure 4: The values for the components are as follows: R1= 987.8 ± 1.1 kΩ at 1 kHz, L1 = 14.793 ± 0.01 mH at 1 kHz, C1 is of course to be determined

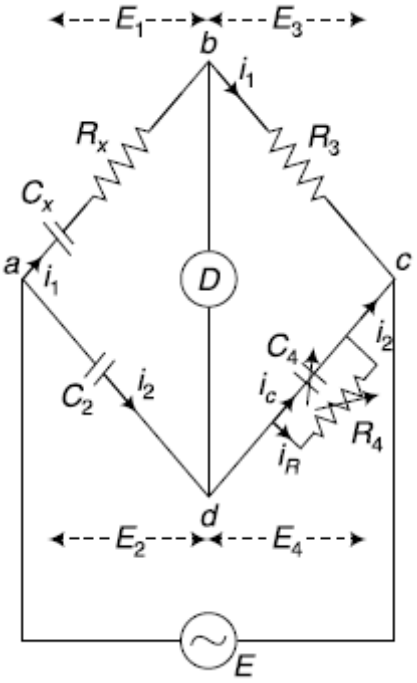
The RCL meter is a Fluke PM6306 which can measure reactance and resistance at frequencies of up to 1 MHz, it is shown in figure 6. The measurements are based on the current and voltage technique. It measures the voltage and current of the components at different phases and can calculate the reactance and resistance from those values. A detailed description on how these values are obtained can be found in the manual of the meter or online[7]. In the measurement region used during the capacitance measurements(500 kHz) the accuracy is ±1% ±1 digit. In general the accuracy below 50 kHz is ±0.1% ±1 digit and above 50 kHz it is $\pm 0.1\% \times (f/50\text{ kHz})$ ±1 digit.

3.4 Measuring capacitance

Two different measurements were done regarding capacitance, first a capacitance with distance was performed to validate the LC-circuit, secondly a measurement of a water droplet inside the capacitor was performed.



(a) The Schering bridge, the large variable resistor shown in figure 4a is connected to this circuit via the two wires coming out of it



(b) schematic of an Schering bridge, as shown before

Figure 5: The values for the components are as follows: R3 = 295.2 ± 0.8 kΩ, C2 = 15 pF (could not be measured with multimeter), C4 = 68 pF (""), R4 is variable up to 300 kΩ

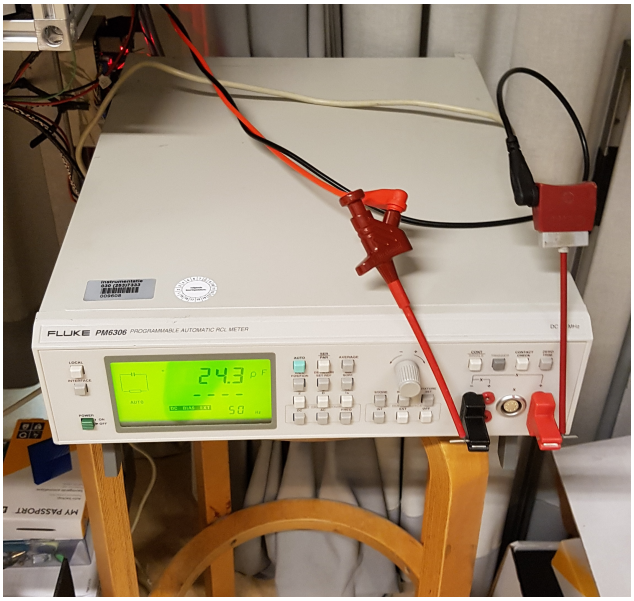


Figure 6: The RCL-meter with the electrodes hooked up, the twist in the wires is visible on top of the device.

3.4.1 Capacitance with distance

Because the distance between the two electrodes can be altered with a turning knob this was used as a frame of reference to move the two plates together at a roughly consistent pace. Since this measurements is just to validate the LC-circuit and see if our mesh works as a plate the acoustic field was turned off. The electric signals to the speakers would just interfere with the measurements. Now a description of how a measurement is done, the initial part is shared among both the LC-circuit measurements and those with the RCL-meter.

First the electrodes are both placed at their maximum distance from each other. From that point both electrodes are alternately moved towards each other till they touch. After every distance change the box surrounding the acoustic trap is closed and a small video is made to determine the distance between the electrodes afterwards. When measuring with the LC-circuit the network analyser option is used. This sweeps across a certain frequency domain and plots the measured voltage. The settings used are; Start: 90 kHz; Stop: 120 kHz; Samples: 5k; Wavegen: [settle: 5 ms; 16 periods; ampl: 5V]. The measurement produces a set of data points that can be fitted to determine the location of the peak, an example of the data the program produces is shown in figure 7. The fitting is done using python with the curve_fit function of the scipy package.

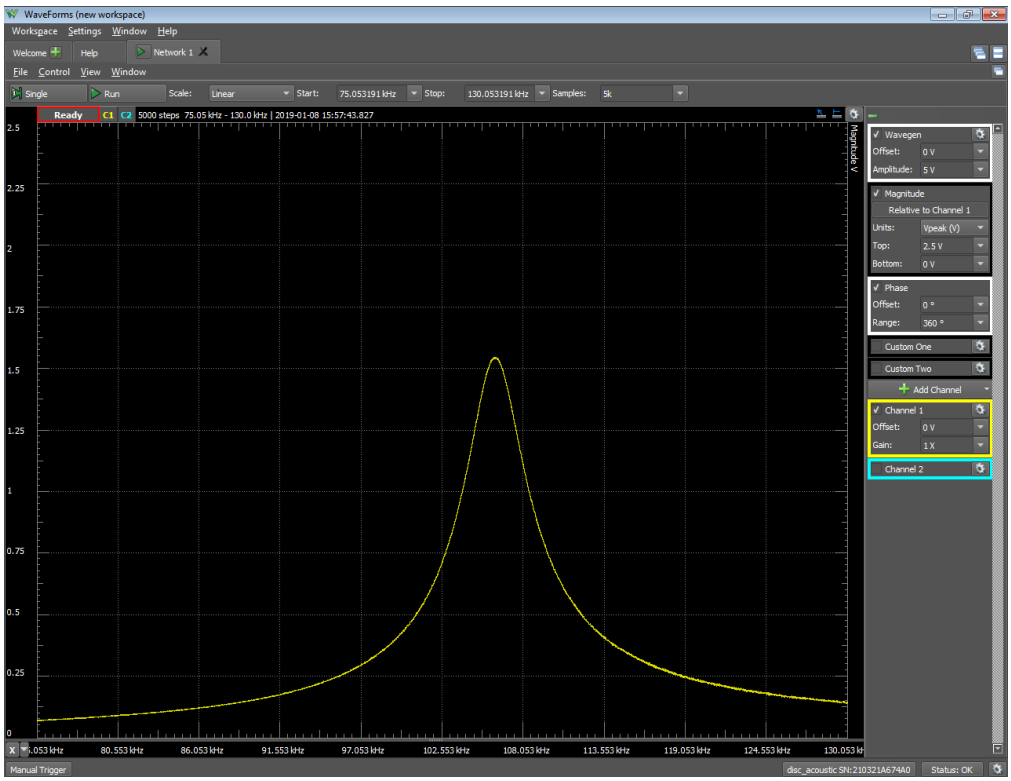


Figure 7: The interface of the waveforms program and how the dataset looks like. Python is used to determine the position of the peak.

As stated before, the first part of a measurement with the RCL-meter is the same, the big difference is in data collection. Since, with the acoustic trap turned off, the RCL-meter produces a fairly constant value this can just be read of the display on the apparatus. This

greatly reduces measurement time. Since the RCL-meter can measure at different frequencies a certain frequency has to be chosen to measure at. Some considerations have to be taken into account when choosing such a frequency. Too low and the output might be too low to be reliably detected, since a capacitor can act like a high-pass filter. Using a high frequency will also mitigate any potential interference with the 40 kHz signal from the speakers. For those reasons 500 kHz was selected as the measuring frequency of choice. Because the uncertainty of the device is known, data processing is straight forward.

Even though the Schering bridge was build it proved insufficient in measuring any data. A brief rundown of why the Schering bridge did not work can be found in the results and discussion section.

3.4.2 Capacitance of a water droplet

To measure the capacitance of a water droplet first a droplet is placed inside the acoustic trap, the electrodes are then brought into close proximity. After the doors are closed and a picture has been taken a measurement is done with either the LC-circuit or the RCL-meter. Because moving the wires from the electrodes slightly alters the measured capacitance it is best to leave them be till after the complete measurement. Once the measurement is done the acoustic field is turned off and the droplet will drop down. Put the field back on to do a second measurement, this time without the water droplet. Since no other things were touched the only variable that has changed is whether there is a water droplet between the capacitor plates. The measurements are then processed in a similar way as described above.

3.5 Measuring resistance

Measuring resistance is done with the electrodes without a plate glued to the end. This measurement is done with the RCL-meter, this time using different frequencies to get the frequency response of a water droplet. The measurement is repeated with different NaCl concentrations in demi water to also get the salinity dependence. The frequencies and salt concentrations at which were measured are shown in table 1.

f (Hz)	50	100	200	400	800	1.2k	2.4k	5k	10k	25k	50k	70k	310k	570k	970k
c(g/L)	200	50	20	10	1	0.1									

Table 1: The different frequencies(f) and concentrations(c) measured at

To make the salt solutions a stock solution of 200 g/L is made using a volumetric flask the other concentrations are made on the spot using Eppendorf tube and an eppendorf pipet. The water droplets are placed in the acoustic trap using a Nichipet ex plus ii of 10-100μL, only 12μL is used. Five data points are taken at each frequency with the RCL-meter and the average is later calculated with python. Below in figure 8 it is shown how the electrodes look like with a drop in between.

Figure 8: Pictures taken during the measurement of 200 g/L.

4 Results and discussion

Due to errors made during the building of devices and being unsuccessful in measuring what was initially the plan the results and discussion will be written in tandem. The initial question was to measure the impedance and relate this to the surface and bulk charges but due to time concerns only test measurements were possible. Because the measurements of the resistance came after the measurements of capacitance these are less accounted for. This is noticeable in both the method of measuring and what was measured, picking the data points more on interest than relevance. These can be seen as test measurements.

The section about the Schering bridge is there because a lot of time was put into making it, yet it did not work due to errors made in planning.

4.1 Capacitance

4.1.1 Distance dependence

To validate the electrodes and the build LC-circuit the change in capacitance with distance was measured. This was done in a procedure as stated in the experiment section. With the LC-circuit 27 measurements were taken, with the RCL-meter 21. During the measurements with the RCL-meter an error was made while turning the knobs, this caused the measured distances to be different with the LC-circuit. A plot of the data can be found in figure 9a, shown in the left picture is a fit made using a Gaussian function to find the position of the peak. Since all we need to measure is the x-position of the peak, it is possible to only take a data sample around the top. Both the method of taking all the data and only from around the top are tried. To find which one is better the data will later be fit to the function for a plate capacitor, equation 1. Another way of fitting would be to fit the data to the function that describes the circuit. This can be easily found since the LC-circuit is basically

a voltage divider with the second resistance exchanged for the LC part. Doing this gives us the equation,

$$V_u = \Re(\frac{\mathbf{Z}_{LC}}{R + \mathbf{Z}_{LC}})V_{in}. \tag{11}$$

With V_{in} the input voltage, V_u the measured voltage and $\mathbf{Z}_{LC}$ being the same as equation 7. In theory this equation describes the behaviour of our circuit, yet when we try to fit this equation to our data we get the fit and corresponding Capacitance shown in figure 10. Even though the fit looks better than those made with a Gaussian function, which is to be expected, the found C values don't seem to make sense. This can be explained by a difference in determining the C-values. Rather than measuring the resonance frequency and using that to determine the capacitance with a known inductance, it fits the parameters in the function to the data. If no good fit can be found the values of those parameters will also be of poor quality. In the Appendix A, figure 14, the plot can be seen with the standard deviation of the values obtained from the direct fit. In both plots the measured capacitance is scaled to itself, subtracting the value found when the plates are furthest apart from each subsequent data-point. This is done to highlight the change in capacitance just from the two plates moving away. Because each component in the LC-circuit has some parasitic capacitance and twisting the wires creates capacitance as well, a value will always be found regardless of how far the plates are apart. The assumption that at max distance the capacitance from just the electrodes is too small to measure is backed up by the fact that we only see an upwards trend after a movement of about 10 mm. We can then say that the capacitance displayed in these plots is the capacitance of just the electrodes.

With the data from the Gaussian fit, figure 9, it is possible to fit the data points to the simple equation for a capacitor found in the theory, equation 1. Fitting this data gives the function shown in figure 11.

Calculating the χ^2_{red} for each of the two fits gives us 45 for the fit using just the data points around the top and 3.2 for the one using the whole data set. This hints that the fit using the whole data set is more precise in the calculation of the resonance frequency. Saying this implies that the equation for a plate capacitor holds well enough for our case, and this is not a bad assumption looking at the shape of the electrodes. More hints that using the whole data set is better come from figure 9 and the fact that the C values using this type of fit lie more closely to the measurements done with the RCL-meter. Using the whole data set thus turns out to be the preferred method of fitting even though when looking at figure 9a no real difference can be seen.

4.1.2 Measurements with water

Following the distance measurements, the effect of water in between the capacitor plates was studied. First a measurement with one droplet was done as a test, the effect of the single water droplet was unmeasurable. From Ernst *et al*[8] a relation between the change in capacitance and the distance between the capacitor plates can be found, this relation also relies heavily on the radius of the droplet. The relation is,

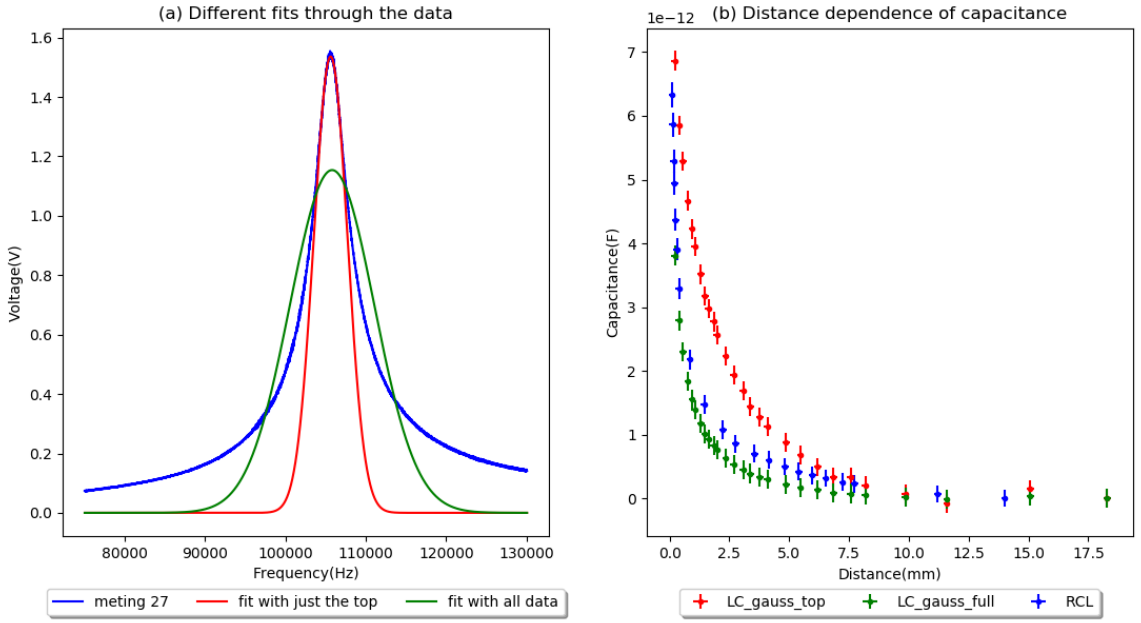


Figure 9: a) Obtained data with the LC-circuit of measurement 27, when the plates were nearest. b) The capacitance at each distance obtained from the fit, the blue dots are data obtained with the RCL-meter

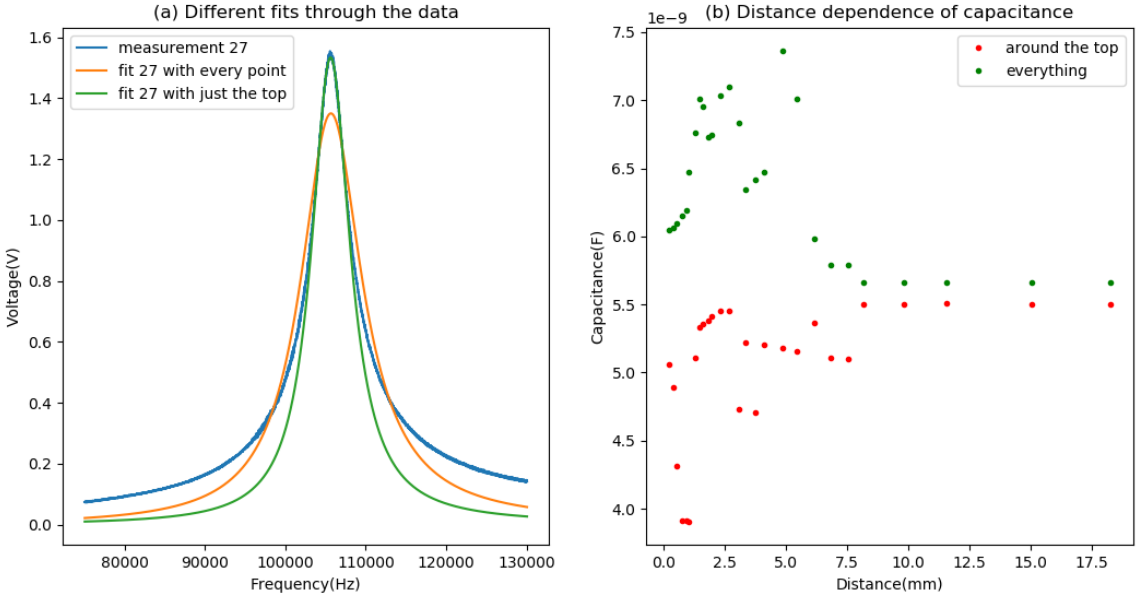


Figure 10: a) Obtained data with the LC-circuit of measurement 27, when the plates were nearest. b) The capacitance at each distance

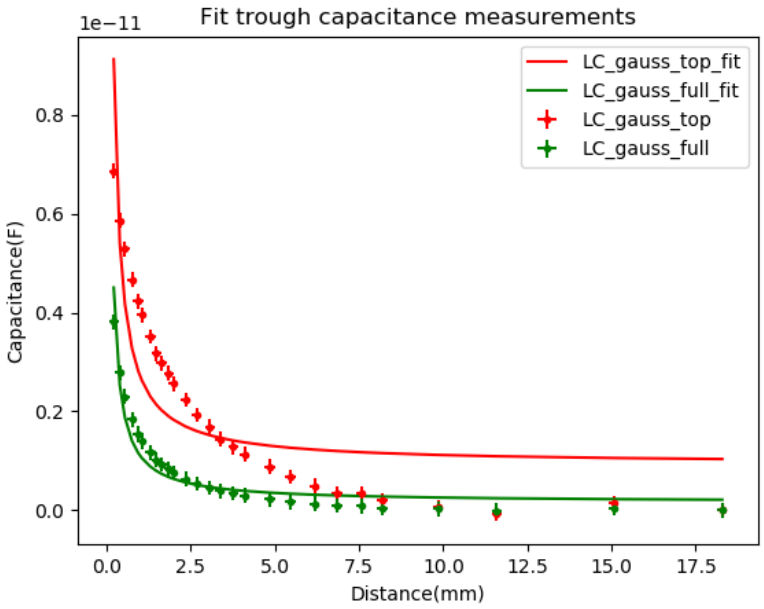


Figure 11: Fit of the function $C = \frac{b}{x} + a$ through the data

$$C = \epsilon_0 \epsilon_r \frac{A}{s} + \Delta C, \tag{12}$$

$$\Delta C = 4\pi\epsilon_0 \frac{r^3}{s^2} \frac{\epsilon_r - 1}{\epsilon_r + 2}, \tag{13}$$

with r the radius of the droplet, s the distance between the plates and ϵ_r the relative permittivity of water(the ϵ_r of air is taken as 1). Because the change in capacitance relies heavily on r and s a small distance and large droplet is preferred. However there is a practical limit to how close the plates can be placed before the droplets will touch and stick to them. This is due to wiggling of the droplet in place when it is first put in a node, and afterwards each time a door is opened to move an electrode, this effect also increases with the size of the droplet. Even though the relative permittivity of water is around 80, so 80 times higher than that of air this alone was not enough the make a difference.

To go around this problem four, rather large, droplets were placed between the plates in the hopes of increasing the effect of another permittivity . This did produce a repeatable change in capacitance when the droplets were removed between the plates, but only in the last digit on the RCL-meter, going from 9.52 pF to 9.50 pF, all the data points are in table 2.

Even though a value was found the error in the value is a lot larger than the value itself. Looking at the errors each value could be both positive or negative and so no true conclusion can be taken about the change in capacitance of water between the plates, other than that it is very small. Using some estimation we can try to see if this should have been expected. Looking at figure 12 taken during the second measurement with the RCL-meter we can get an idea of how big the water droplets were.

#	ΔC measured with LC-circuit	ΔC measured with RCL-meter
1	$6 \pm 25 \times 10^{-14}$	$1 \pm 10 \times 10^{-14}$
2	$-1 \pm 25 \times 10^{-14}$	$2 \pm 10 \times 10^{-14}$
3	$2 \pm 25 \times 10^{-14}$	$2 \pm 10 \times 10^{-14}$

Table 2: The measured data

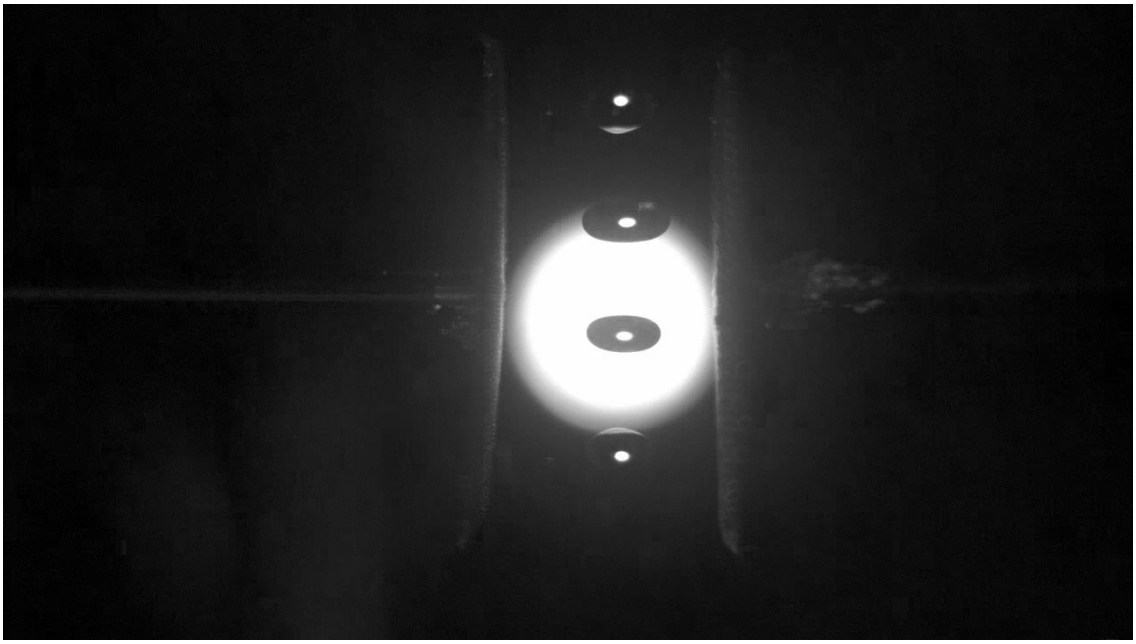


Figure 12: A picture taken from the measurement of the capacitance of a water droplet. This particular picture is from the second measurement taken with the RCL-meter

We know the scale in this picture from a previous calibration. Using the the third droplet as the benchmark since it is easy to measure and fairly representable for the average. Assuming the shape of an oblate spheroid, the long axis(a) is: 3.1 mm; the short axis(c) is: 1.6 mm. This gives a volume of 8.1 mm^3 (V=$\frac{4}{3}\pi ca^2$) for each droplet. If we measure the distance between the the plates we can use equation 13 to find the expected ΔC. Finding a distance of 8.6 mm and using a ϵ_r of 80 which is an estimate since the droplet was not DI water nor was it exactly 25°C. Since the function calls for spherical droplets we have to use the found volume to create a radius for a spherical droplet with the same volume. A sphere with a volume of 8.1 mm^3 has a radius of 1.2 mm. The total equation (12) can easily be scaled to incorporate 4 droplets by multiplying the ΔC by four. Filling all values into equation 13 gives us a change in capacitance of 1.1×10^{-14} F. This is the same order of magnitude that we measured, suggesting that our measured values are in agreement with what would be expected. To reduce the errors on the measurements either more measurements would have to be conducted or a different device should be used. The problem with more measurements is that the total volume of water would have to be very consistent each time, right now each droplet is placed by hand with using a small needle. An eppendorf pipet might be precise in its dispensed volume but the bigger tip creates problems with placing the droplets in such a configuration. A different device might be a more thought out version of the Schering bridge,

or another device build for measuring very small capacitance.

4.2 Resistance

Before the measurements could begin a salt solution had to be created, this was done as described above. Weighing 10 grams of NaCl using a Mettler P1210N and dissolving this in an erlenmeyer with 40 mL of demi water. After dissolving, the solution was transferred to a 50 mL volumetric flask were it was brought op to 50 mL. During/after homogenization some solution was lost due to a bad cork, following this experiment we assume that the concentration was not affected. If it was affected this would only be a systematic error in every following solution and not a problem for a difference in concentration. The solution of 200 g/L was made on the 13th of December and placed in a plastic container sealed with laboratory wax. The actual measurements were done on the seventh of January, again the assumption is made that this does not alter the concentration in any significant way. The concentration should be high enough that any solvating gasses do not introduce a significant change.

The other concentrations were made as described in the experiment section, using a nichipet ex ii to make the other solutions. Since the range is between 10 and 100 μL to make the 1 and 0.1 g/L one respectively two extra steps were needed. Going first to 10 g/L and from there to 1 g/L doing either an experiment with 1 g/L or continuing further down to 0.1 g/L. The voltage of the acoustic trap was at 12 V and a volume of 12 μL was used for each droplet. The distance between the tip of the electrodes is approximately 1.6 mm. With these parameters the data presented in figure 13 could be collected.

For the plot of resistance as a function of salinity the measurement at 100 Hz is omitted because one of the data points is made at 120 Hz. The fact that when resistance is plotted against the frequency there is asymptotic behaviour at low frequencies, not clearly visible on this figure, can be attributed to the reactance. The two electrodes act like a capacitor and the reactance behaves as in equation 3.

There are some general remarks to be made about the way the measurement was performed. Because the RCL-meter could not be hooked to a computer every data point had to be written down on paper from the small display on the device. If the measured value was constant with time this would not have been a big issue, merely a nuisance. But because CO_2 dissolves into the droplet and the droplet evaporates the concentration changes over time. The increase of concentration over time corresponds to a decrease in resistance over time, the extend of this effect was studied. Using DI water and a frequency of 50 kHz to measure every 3 minutes or so. In table 3 the measurement data can be found, after 10 minutes the measured resistance is only 40% of what it was in the beginning.

	∼1 minute	∼4 minutes	∼7 minutes	∼10 minutes
R(MΩ)	6.5 ± 0.3	4.4 ± 0.1	3.39 ± 0.06	2.65 ± 0.04

Table 3: Measurements over time of DI water

This is significant since due to way the measurements had to be obtained, each measurement took around 9 minutes which meant it experienced almost the full 40% reduction in value. This makes the measured difference between frequencies less substantial.

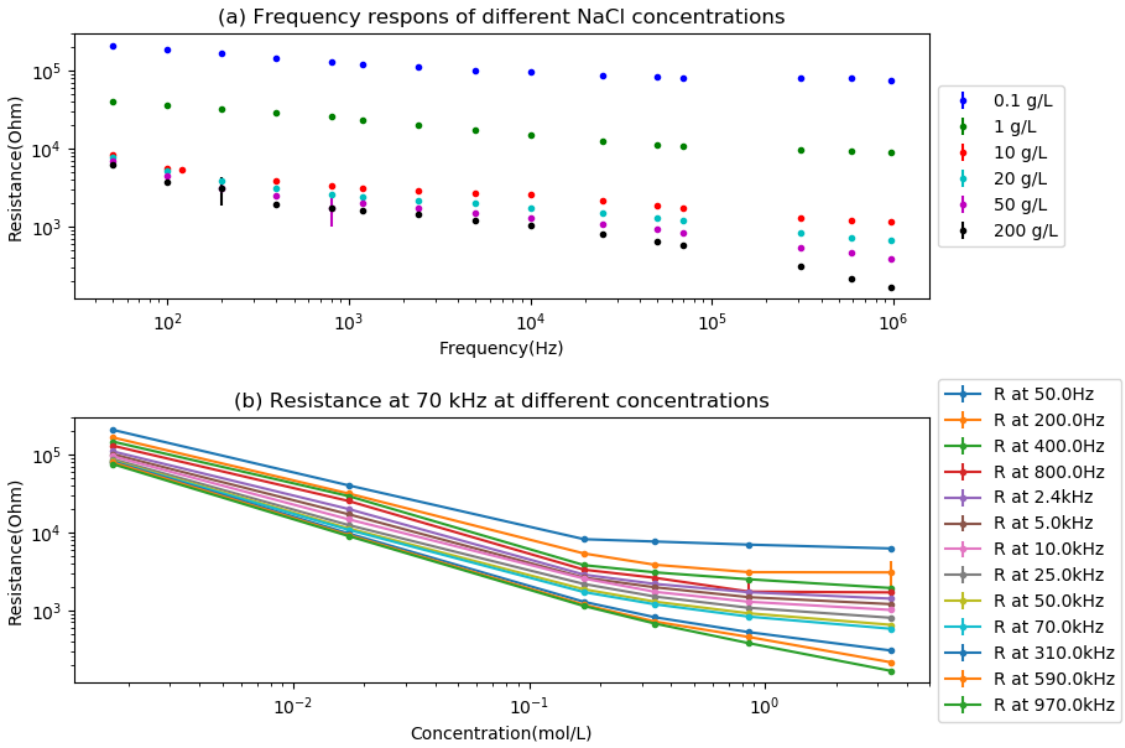


Figure 13: a) Resistance as a function of frequency for various concentrations. b) Resistance as a function of salinity at various frequencies

The equation to go from resistance to conductivity,

$$\sigma = \frac{l}{AR}, \tag{14}$$

with l the distance between the electrodes, A the surface area and R the resistance relies on the surface area A of the electrode. Unfortunately the electrodes that were used had a pointy tip and thus no well defined surface area. This can be circumvented by using a liquid with a known conductivity or resistivity or flattening the tip, but due to time constraints this was not pursued. Without a well defined area to work with this means no reliable information can be extracted from the measured data except that it is intuitively correct. Lower resistance for higher concentration and higher resistance for lower frequencies.

4.3 Schering bridge

Initially the plan was to measure the capacitance using a Schering bridge, since this came up often as a good way of measuring during an initial search. Instead of only being able to measure the change in capacitance a Schering bridge would allow the direct measurement of an unknown capacitance. Yet after construction it turned out that it was not possible to balance the device when the desired capacitance was hooked up. While trying to see why the Schering bridge did not work numerous problems were discovered. Some problems stem

from bad design and planning of the circuit while others are more profound and require a more substantial change.

A problem that was immediately noticeable was the fact that due to the low value capacitors used in the circuit the circuit would act like a highpass-filter. Having a too low voltage output at lower frequency to be easily measured. This limited the range of frequencies that could be used to higher frequencies only. Because parasitic capacitance is larger at higher frequencies the use of lower frequencies is preferred. Another problem was the wrong use of a BNC/coax connection, the measurement between the two arms was with a connection to a single coax cable. Thus connecting one of the arms to the ground of the oscilloscope. This in of itself rendered the device unusable since it would be impossible to know if the arms would be balanced. And finally a design flaw was made in the use of components, using only one variable component. This makes it almost impossible, unless the unknown component has just the right value, to balance the two arms. In the equations 10 we can see this, if an equation has no variable component this equation can not be balanced. And without a balance no measurement can be done.

The more substantial problem is the parasitic capacitance. Not putting enough variable components in the circuit or connecting the ground in the circuit are problems that can easily be fixed, but a different solution is needed for the parasitic capacitance. For a Schering bridge to work the value of each component must be known to measure the unknown value, if these differ due to parasitic capacitance it is not possible to reliably determine the unknown component. There are ways around this problem, like a Wagner earth, with this addition to the circuit one can first balance the circuit without the unknown component and compensate for the parasitic capacitance. After introducing the unknown component another balance gives the value of the desired component. To explain this circuit in more detail is beyond the scope of this thesis, but for further research or experiments a Schering bridge with a Wagner earth is worth considering.[4]

5 Conclusion

The complex impedance and following that the permittivity of water was investigated using an acoustic levitator. Two possible ways of measuring were build, the Schering bridge and the LC resonance circuit, a third pre-build device was also used, a Fluke PM6306 RCL-meter. After testing the Schering bridge proved insufficient in measuring the capacitance and was thus dropped, the failure was mainly due to poor build design. For measuring capacitance the LC circuit proved adequate in measuring the distance variations of a plate capacitor, being able to measure differences in the order of 0.5 pF. Even though the prediction of the capacitance of a water droplet was guessed at 1 pF the LC circuit and the pre-build RCL-meter proved unable to reliably measure the capacitance of water droplets placed between the plates. A measurement of the relation between resistance, frequency and salinity was also done. Due to a badly defined surface area of the electrodes no further information could be extracted from the measurement, a flatter electrode tip or the use of a calibration solution could make this measurement work better.

6 References

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A Appendix

This figure shows the uncertainty in the obtained data from a direct fit, the error bars are taken directly from the scipy fit function.

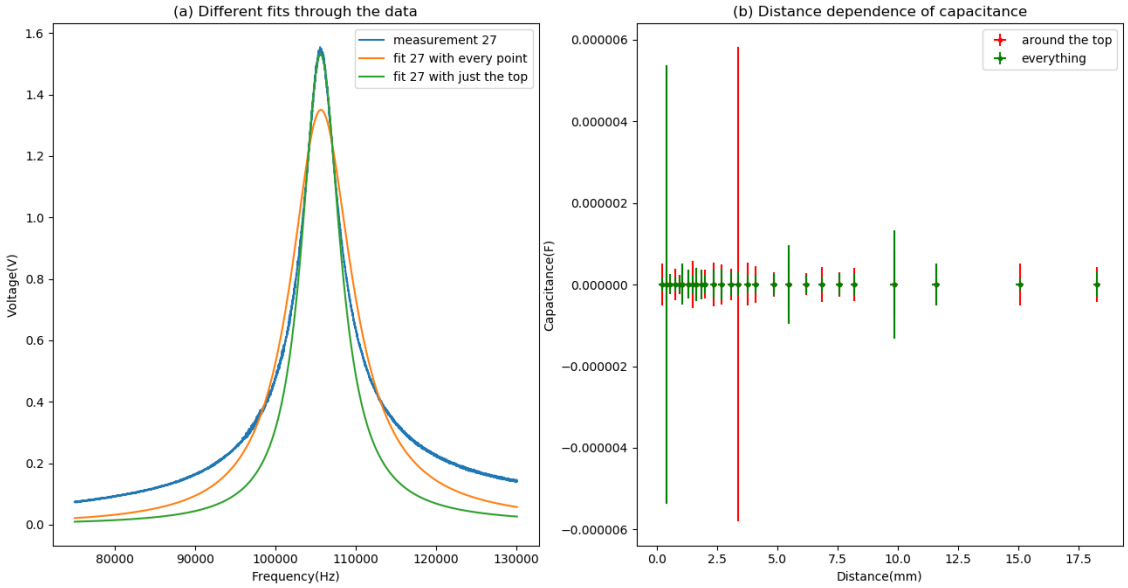

Figure 14: a) Obtained data with the LC-circuit of measurement 27, when the plates were nearest. b) The capacitance at each distance with error bars.