History of the Mercator projection

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Preface

The Mercator projection is one of the most well-known projections, and nearly everyone has probably seen a map in this projection, e.g. in an atlas or as wall map (maybe without noticing or recognizing it as such though). Although often criticized nowadays, mainly because of the distortion in the areas towards the pole (leading to reactions like: "I didn't realize Greenland is that large!"), it has been of immense value in the past, especially for navigation at sea. Also many of the sea charts used nowadays still make use of the Mercator projection.

The concept behind the Mercator projection seems pretty straightforward, and the projection itself rather simple, especially when compared to several of the more recently developed projections. However, as we will see in this thesis, after the first map in this projection was developed by Mercator, more than a century passed until the mathematics underlying the projection was fully understood and proved.

The thesis is written in such a way that people with some basic knowledge in calculus should be able to follow.

Chapter 1 starts with an introduction about the historical context in which the Mercator projection was developed. In chapter 2, formulas for the Mercator projection are derived from a modern perspective. Chapter 3 then follows with an overview how Mercator created his map in the 16^{th} century. Chapter 4 is about the first more mathematical description of Mercator's map provided by Wright. Chapter 5, finally, briefly touches upon the influence that Mercator's map had on the development of mathematics, and discusses an inconsistency in literature with regard to the role of Henry Bond.

Most of this thesis is based on secondary literature by modern scholars. Chapter 4, however, also contains a small analysis which I did to analyze the accuracy of the method that Wright used. Furthermore, figures and tables for which no source is provided were created by the author of this thesis. As far as we are aware, the remarks on the mathematician Henry Bond in chapter 5 have not appeared before in the literature.

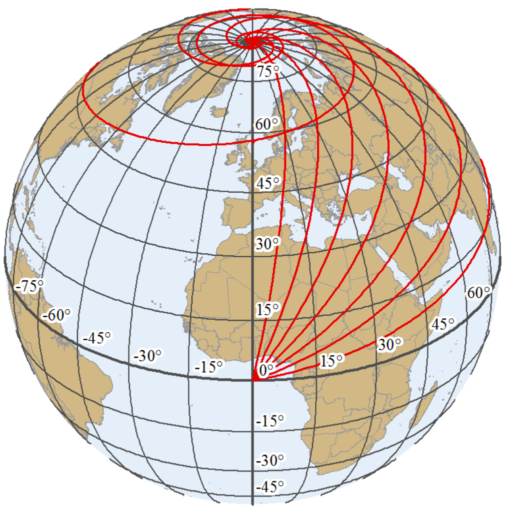
Contents

1 Introduction

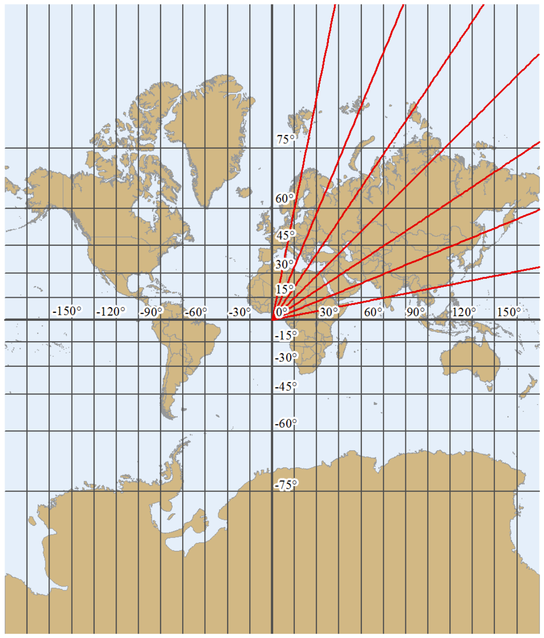
Geography and navigation were very important in the time of Mercator (i.e. 16^{th} century). Many European countries were exploring the world, creating colonies, and trading. Knowing the ship's position was important. The latitude could be relatively easy determined by measuring the altitude of either the pole star or the sun (at noon, and using tables with the declination of the sun for every day of the year). Longitude was much more difficult, and required accurate time measurement, a place with known longitude, so that one could measure e.g. local noon, and one could then compare the difference in time with the reference location. Each hour difference equals 15 degrees longitude. The (mechanical) clocks available around that time were not accurate enough though, making it a challenge for seamen to know the exact location of their ship (Katz 1998, pp. 393-394). These difficulties made the availability of good maps even more important for navigation at sea, since maps could potentially tell how different places were located relative to each other, and therefore how far and in which direction one had to go to reach a certain destination.

The first maps simply used a rectangular grid of parallels and meridians, using the same scale for both of them (Katz 1998, p. 396). The name we use nowadays for this projection is 'Plate Carrée projection' (Snyder & Voxland 1989). See figure 1(c) for an impression how meridians and parallels are located relative to each other within this projection. In the literature of the $16^{\text{th}}/17^{\text{th}}$ century several different names were used to refer to maps in the Plate Carrée projection, e.g. the plaine or ordinary Sea-chart (Norwood 1645) or the general or common Sea-Chart (Wright 1657). However, although commonly used around these times, maps with this projection had some disadvantages in the sense that the projection tends to distort shapes and directions (Snyder & Voxland 1989). These distortions made that people sometimes had so little confidence in the maps that they preferred the safer alternative of first sailing to the desired latitude, and from there continue in Eastern or Western direction until the destination was reached, also if this route would be (much) longer and more time consuming (Katz 1998, p. 394), (Wright 1657, pp. B-B2).

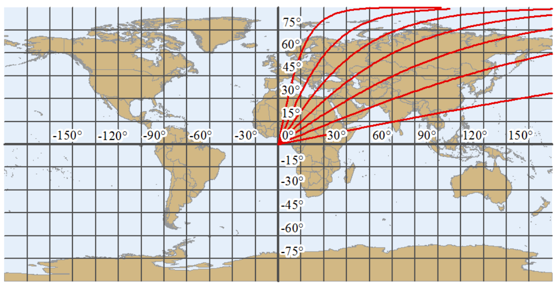
Pedro Nunes investigated the lines if one would keep following a constant compass bearing, called 'loxodromes' or 'rhumb lines', and discovered that these lines spiral around the globe towards the poles (Katz 1998, p. 396). Note that loxodromes are in general not the shortest route between two places. The shortest route between two places is always lying on the great circle that is going through both places. Only the loxodromes that make an angle of 0° or 90° with the equator are great circles themselves. Loxodromes were important for navigation, because it was convenient if one could just find the bearing between starting point and destination, and follow the same direction with the compass throughout the whole journey (Snyder 1997, p. 45). As such, following a loxodrome was a good compromise between the shortest route and a safe route. However, since the Plate Carrée projection distorts angles, loxodromes were not represented by straight lines on the common Sea-Chart. It was Mercator who then created the first map where loxodromes were represented by straight lines (Katz 1998, p. 397). Figure 1 shows 7 loxodromes starting from the point with spherical coordinates $(0,0)$ in positive longitudinal and latitudinal direction on a sphere 1(a), on the Mercator projection 1(b) and the Plate Carrée projection 1(c).



(a) Globe



(b) Mercator projection

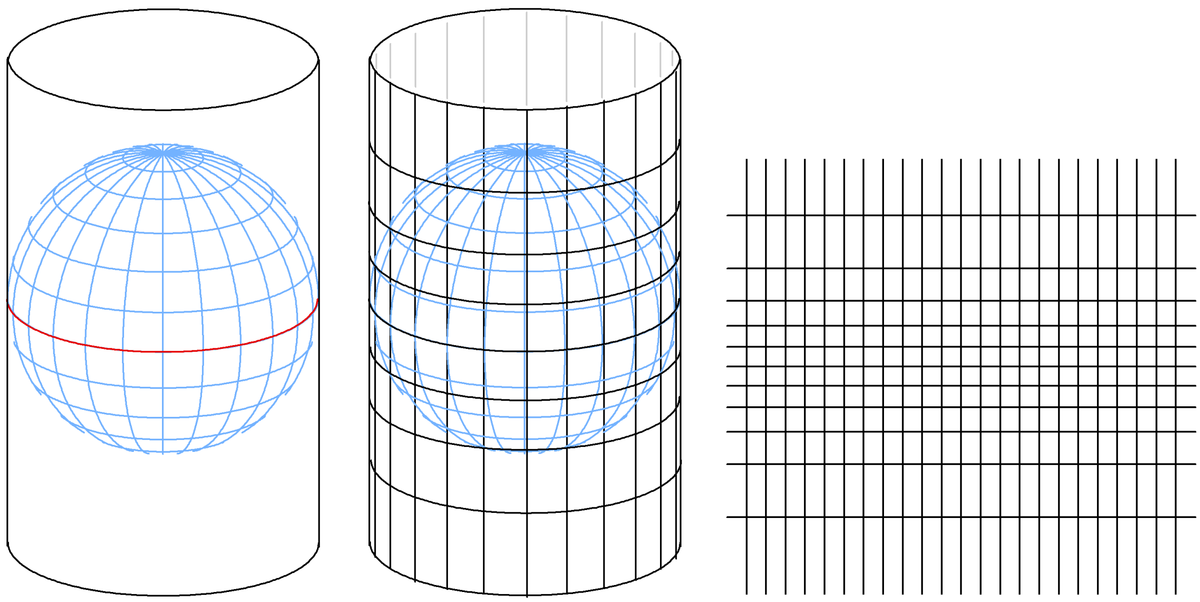


(c) Plate carrée projection

Figure 1: Loxodromes (red lines) visualized on the globe, on the Mercator projection and on the Plate Carrée projection

2 The Mercator projection from a modern perspective

As we have seen in the previous chapter, the Mercator projection was developed as improvement of the common sea chart. Both are so-called cylindrical projections, i.e. in both cases the Earth is projected onto a cylinder, which is then 'cut open' and 'unrolled' to a flat sheet. Thereby the assumption is made that the earth has the shape of a perfect sphere. The sphere is inscribed into a hollow cylinder that is open at both ends, in such a way that the equator of the sphere coincides with the cylinder (see figure 2(a)). In other words, the intersection of the sphere and the cylinder is exactly the equator.



(a) Inscribed sphere (b) Coordinates on cylinder (c) Cylinder flattened

Figure 2: Cylindrical projection

To derive the Mercator projection, imagine that the sphere grows, and as soon as a point of the surface of the sphere coincides with the cylinder, it sticks to the surface, which is then the projection of that point. How the growing is done exactly will be made clear later on (but the basic concept is to make sure that for each point the scaling factor in latitude is the same as the scaling factor in longitude. Wright already used a similar description in 1599, where he speaks about the swelling of a bladder, in every point as much in longitude as in latitude (Wright 1599, p.C3)). The equator touches the cylinder already from the beginning, and hence, has its true scale on the projection surface of the cylinder. The point where the cylinder touches the spherical coordinates $(0,0)$ of the surface of the sphere will be the origin of the coordinate system on the cylinder, and the intersection with the sphere will serve as x-axis. The x-coordinates correspond to the longitudes on the equator of the sphere, so they go from $-180°$ till $180°$, whereby the points with $-180°$ and $180°$ coincide (at least until the cylinder has been cut open). While the sphere is growing, points on a meridian, say with longitude λ, will make contact with the cylinder on a point straight above the point on the x-axis with x-coordinate λ, i.e. meridians on the sphere will be represented by straight lines on the cylinder, perpendicular to the equator. Hence, the x-coordinate of the projection of a point depends only on the longitude that it had on the sphere and not on its latitude.

Although we didn't specify in detail yet how the growing is done exactly, it will be done in such a way that for each point the distance to the projected equator on the cylinder will only depend on the latitude and not on its longitude. In other words, two points with the same latitude will be projected at the same distance from the equator. So each parallel on the sphere will be projected as a circle on the cylinder which is parallel to the equator. It follows that (after the cylinder has been cut open along the 180° (= $-180°$) meridian and 'flattened') parallels on the sphere will be projected as straight lines parallel to the equator. As a result the Mercator projection plots meridians and parallels as straight lines that intersect at right angles.

Now we get to the point where we will make clear how the growing/scaling will be applied exactly. The goal is to make sure that loxodromes will be represented by straight lines on the map. Since loxodromes are those lines on a globe which cross each meridian under the same angle, and we saw that the way the projection was defined so far makes that meridians are parallel lines on the map, it follows that loxodromes will be straight lines on the map if and only if in each point on the map angles on the map correspond to angles on the sphere, i.e. the projection needs to be conformal, i.e. (locally) there are no shape distortions. (Note that there are also conformal projections where meridians are not parallel and loxodromes will therefore not be straight lines).

Since parallels and meridians are perpendicular, they form a basis, i.e. every line can be described as linear combination of longitude and latitude. If longitude and latitude have the same scale factor in each point (which may differ from point to point though), every line will be scaled similar 'in horizontal and vertical' direction, and therefore preserve its angle.

Consider the radius of a parallel at latitude ϕ. On the sphere the radius of this parallel is $\cos(\phi)$ times the length of the equator (see the red lines in figure 3). On the cylinder, however, the circumference is always the circumference of the cylinder, i.e. the length of the equator. So on the map the distance between two meridians at latitude ϕ is enlarged by a factor $1/\cos(\phi) = \sec(\phi)$. To make sure that the map is conformal, the same scale needs to be applied in each point in 'vertical' direction as well.

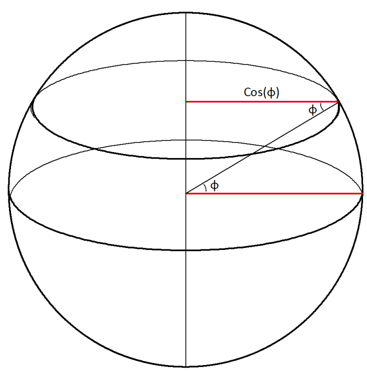


Figure 3: Scaling factor

So we now know the scaling factor at each individual latitude, but we are actually interested in the overall scaling factor between the equator and a certain latitude ϕ, because then we can easily calculate the distance between the equator and latitude ϕ by multiplying the distance that a similar arc would have at the equator with the overall scaling factor. This overall scaling factor can be calculated by taking the integral over the scaling factors at each latitude:

$$\int_0^\alpha \sec\phi d\phi \tag{1}$$

Assume for example that one degree on the equator has a distance g on the map, and that there are a degrees in between the equator and latitude α. Then the distance from the equator to latitude α on the Mercator map would be $a \cdot g \cdot \int_0^\alpha \sec\phi d\phi$.

The integral can be calculated as follows (note that in the derivation ϕ will be in radians):

$$\int \sec(\phi) d\phi = \int \frac{d\phi}{\cos(\phi)}$$

Multiplying numerator and denominator by $\cos(\phi)$ gives:

$$\int \sec(\phi) d\phi = \int \frac{\cos(\phi) d\phi}{\cos^2(\phi)}$$

This can be rewritten using the equality $\cos^2(\phi) + \sin^2(\phi) = 1$:

$$\int \sec(\phi) d\phi = \int \frac{\cos(\phi) d\phi}{1 - \sin(\phi)^2}$$

Substitution of u for $\sin(\phi)$ can now be used. Since $\frac{du}{d\phi} = \sin'(\phi) = \cos(\phi)$, i.e. $du = \cos(\phi) d\phi$ we get:

$$\int \sec(\phi) d\phi = \int \frac{du}{1 - u^2}$$

Then, since $1 - u^2 = (1+u)(1-u)$:

$$\int \sec(\phi) d\phi = \int \frac{du}{(1+u)(1-u)}$$

Using $1 = \frac{1}{2}(1-u) + \frac{1}{2}(1+u)$ gives

$$\begin{aligned}
\int \sec(\phi) d\phi &= \int \left(\frac{\frac{1}{2}(1-u) + \frac{1}{2}(1+u)}{(1+u)(1-u)} \right) du \\
&= \int \left(\frac{\frac{1}{2}}{(1+u)} + \frac{\frac{1}{2}}{(1-u)} \right) du \\
&= \frac{1}{2} \int \frac{1}{(1+u)} du + \int \frac{1}{(1-u)} du
\end{aligned}$$

Now using that $\int \frac{1}{(1+u)} du = \ln|1+u| + C$ and $\int \frac{1}{(1-u)} du = -\ln|1-u| + C$ gives

$$\int \sec(\phi) d\phi = \frac{1}{2} (\ln|1+u| - \ln|1-u|) + C$$

Substituting $u = \sin(\phi)$ back gives:

$$\int \sec(\phi)d\phi = \frac{1}{2}\left(\ln|1+\sin(\phi)| - \ln|1-\sin(\phi)|\right) + C$$

Since $\ln a - \ln b = \ln \frac{a}{b}$:

$$\int \sec(\phi)d\phi = \frac{1}{2}\ln\left|\frac{1+\sin(\phi)}{1-\sin(\phi)}\right| + C$$

Combining the equalities $\sin(\phi) = -\cos(\phi+\frac{\pi}{2}) = -\cos(2(\frac{\phi}{2}+\frac{\pi}{4}))$ and $\cos(2\phi) = 1-2\sin^2(\phi)$ gives $\sin(\phi) = -1+2\sin^2(\frac{\phi}{2}+\frac{\pi}{4})$, which can then be used to continue as follows:

$$\begin{aligned}
\int \sec(\phi)d\phi &= \frac{1}{2}\ln\left|\frac{2\sin^2(\frac{\phi}{2}+\frac{\pi}{4})}{2-2\sin^2(\frac{\phi}{2}+\frac{\pi}{4})}\right| + C \\
&= \frac{1}{2}\ln\left|\frac{\sin^2(\frac{\phi}{2}+\frac{\pi}{4})}{\cos^2(\frac{\phi}{2}+\frac{\pi}{4})}\right| + C \\
&= \frac{1}{2}\ln\left|\tan^2(\frac{\phi}{2}+\frac{\pi}{4})\right| + C \\
&= \ln\left|\tan(\frac{\phi}{2}+\frac{\pi}{4})\right| + C
\end{aligned}$$

Latitudes ϕ are within the interval $[-\frac{1}{2}\pi, \frac{1}{2}\pi]$. From $\lim_{\phi\to\frac{1}{2}\pi}\tan(\frac{\pi}{4}+\frac{\phi}{2}) = \infty$ and $\lim_{\phi\to-\frac{1}{2}\pi}\ln\left|\tan(\frac{\pi}{4}+\frac{\phi}{2})\right| = -\infty$, it follows that the poles cannot be shown on the map when using the Mercator projection. For latitudes of ϕ within the interval $(-\frac{1}{2}\pi, \frac{1}{2}\pi)$ it holds that $0 < \frac{\phi}{2}+\frac{\pi}{4} < \frac{1}{2}\pi$, and from that it follows that the tangent is always positive, and the absolute value brackets can therefore be removed.

So finally we get for latitude α:

$$\int_0^\alpha \sec(\phi)d\phi = \ln\tan(\frac{\alpha}{2}+\frac{\pi}{4}) - \ln\tan(\frac{\pi}{4}) = \ln\tan(\frac{\alpha}{2}+\frac{\pi}{4})$$

This brings us to the definition of the Mercator projection as is common nowadays, where R depends on the scaling factor of the map, i.e. R is the radius of the so-called generating globe:

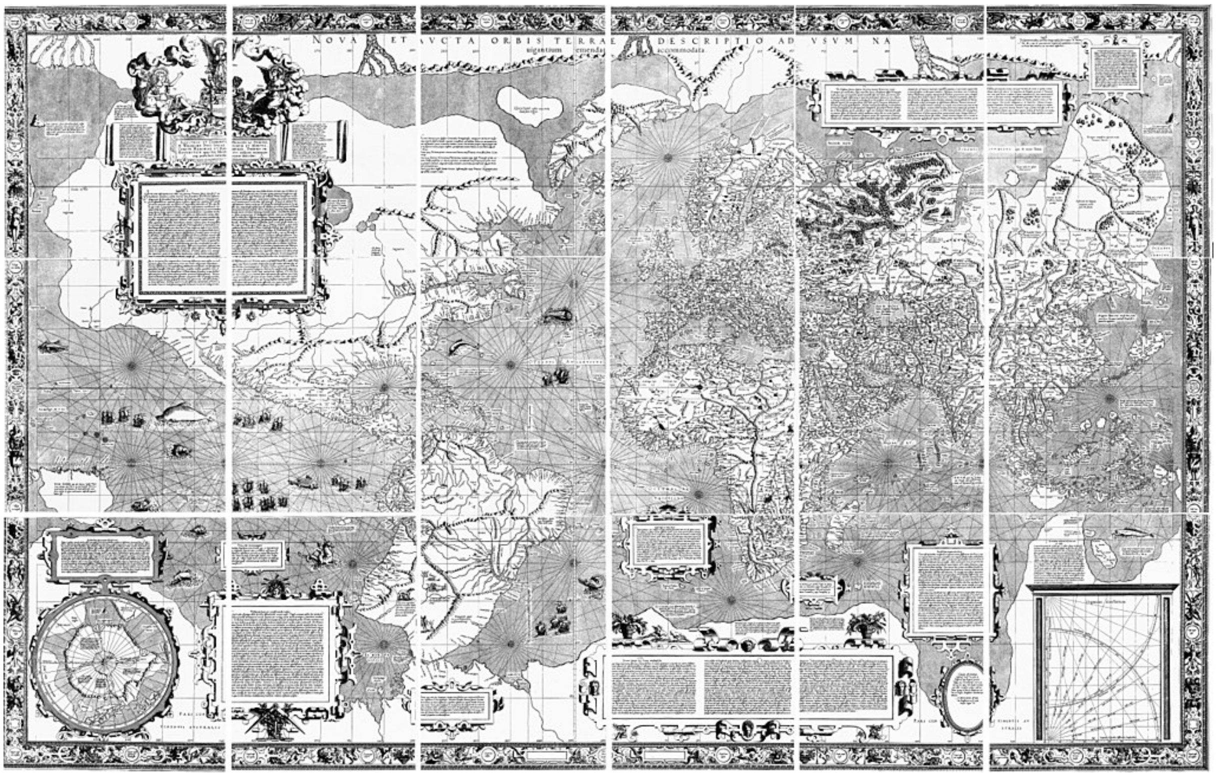
$$x = R\lambda \tag{2}$$

$$y = R\ln\tan(\frac{\pi}{4}+\frac{\phi}{2}) \tag{3}$$

Note that there is sometimes the misconception that the Mercator projection is the projection one would get by putting a light in the center of the earth that projects the earth on the cylinder. However, this would result in $y = \tan(\phi)$, and is therefore not the same as the Mercator projection.

An interesting detail worth mentioning is that logarithms were only discovered around 1615-1620 and therefore unknown in the time of Mercator (see also chapter 5).

3 How did Mercator define his projection?



Source: `https://commons.wikimedia.org/wiki/File:Mercator_1569_world_map_composite.jpg`

Figure 4: Mercator's world map (1569)

Gerhardus Mercator (1512 - 1594), born as Gerhard Kremer, studied philosophy and theology at the University of Louvain. It was only after his studies that he started to get more acquainted with mathematics and astronomy. Besides his knowledge in these fields, as well as in geography, he had also artistic talents (calligraphy and engraving), which he used for making globes, maps and scientific instruments (Kish 1975).

After Pedro Nunes had shown in 1537 that loxodromes are different from great circles, and spiral towards the poles (Katz 1998, p. 396), Mercator drew loxodromes on a globe he produced in 1541 (Krücken & Milz 1994). However, globes were not very practical for navigation at a ship (not only because the curved surface makes measurements harder, but also because maps could, in contrary to globes, be at sufficiently large scales (Katz 1998, p. 396)). In 1569 Mercator then published his famous world map named "Nova et Aucta Orbis Terrae Descriptio ad Usum Navigantium Emendate Accommodata" ("New and more complete representation of the terrestrial globe properly adapted for use in navigation") using a new projection, which is since then known as the Mercator projection (Krücken & Milz 1994).

In the decades after 1569 many copies of Mercator's map were sold (the map had 14 years of copyright). Nowadays, of only 3 copies it is known that they still exist (known as the 'Paris', 'Rotterdam' and 'Basel' versions). Furthermore there are reprints of a Breslau version, from which it is not clear if the original still exists (Krücken & Milz 1994).

Little is known about how Mercator produced his map. Besides the text on the map itself, there are no other sources by Mercator himself about (the production of) the map. However, from the legends on the map one gets the impression that Mercator knew exactly what he was doing. He mentions that one of the goals is to project the sphere in such a way on the map that places are located correctly in latitude, longitude, as well as direction and distance in comparison with each other. To do that, he writes that he has gradually increased the degrees of latitude towards the poles in proportion to the increase in length of the parallels relative to the equator, and he furthermore mentions that the poles cannot be visualized on the map, since their latitudes would reach infinity (Krücken & Milz 1994).

Several studies have been carried out in which different hypotheses of how Mercator could have made his map have been analyzed (see e.g. Nordenskiöld (1889) for such a study, or Kyewski (1962) for an overview of studies). Many of those studies include an error assessment where the accuracy of the 'mesh' on one of the still existing copies of Mercator's map is compared with the exact values which the hypothesized method would theoretically result in.

One such a hypothesis is that Mercator calculated a table with distances between parallels in a way Wright did, and which is discussed in more detail in the next chapter. However a more likely scenario probably is that Mercator made use of (available) tables of rhumbs, as suggested by (a.o.) Gaspar & Leitão (2014). Tables of rhumbs usually contained a standard set of rhumb lines corresponding to the main compass directions, including the rhumbs which are crossing the meridians at angles of $11.25°$ (EbN), $22.5°$ (ENE), $33.75°$ (NEbE), $45°$ (NE), $56.25°$ (NEbN), $67.5°$ (NNE) and $78.75°$ (NbE) (see also figure 1(a)). For each rhumb line, the table contains a series of coordinates (which most likely were calculated using a method based on spherical triangles), whereby coordinates were provided at equal intervals of latitude or equal intervals of longitude. (See figure 5 for an example of such a table, where the coordinates are given with an interval of $1°$ longitude).

With the help of such a table, one could proceed as follows. First draw the equator and the (equally spaced) meridians. Select a rhumb line, and draw a straight line starting in the origin and making an angle with the meridians that corresponds to the specific rhumb line. For each coordinate pair in the rhumb table, say (λ, ϕ), the longitude λ can be easily linked to a specific point at the equator. From there, move straight up until reaching the rhumb line. The point where the line is crossing the rhumb line has latitude ϕ, and a horizontal line can be drawn representing that latitude. After this procedure has been applied to all coordinates in the rhumb table, parallels are drawn on the map with a resolution similar to that of the rhumb table. (Tables of rhumbs using equal intervals of longitudes can be used in a similar way, though this would require some interpolation).

If this method has been applied correctly, all other rhumb lines, when plotted on the map based on the coordinates in their rhumb tables, should also appear as straight lines. This could be used to double check if the stretching of the latitudes had been applied correctly.

Noteworthy is that not only the correct scaling of meridians and parallels was important for practical usage of Mercator's map. As important was that the content of the map, e.g. countries, cities and ports, were placed correctly on the map, and Mercator states this clearly as the second important goal for creating his map. For this part Mercator was dependent on other sources, and he has used several other maps and sources to construct the content of the map itself (Krücken & Milz 1994, p. 20). The importance of this second goal also follows from a discussion by (Norwood 1645, p.13), who mentioned that some people had as little trust in maps with the Mercator projection as they had in the common Sea-Chart. Not because the parallels, meridians or rhums would be inaccurate, but because if content was transferred from the common Sea-Chart, errors in there were copied as well.

The fourth rumbe from the Equinoctiall.

The rumbs of { *Northeast, Northwest:* / *Southeast, Southwest.* }

Lon.	Latitu.		Lon.	Latitu.		Lon.	Latitu.		Lon.	Latitu.		Lon.	Latitu.		Lon.	Latitu.	
Deg.	De.	Mi.	Deg.	De.	Mi.	Deg.	De.	Mi.	Deg.	De.	Mi.	Deg.	De.	Mi.	Deg.	De.	Mi.
1	0	59	31	29	35	61	51	56	91	56	55	121	76	11	151	81	47
2	1	59	32	30	27	62	52	33	92	97	17	122	76	25	152	81	56
3	2	59	33	31	18	63	53	8	93	67	40	123	76	39	153	82	4
4	3	59	34	32	9	64	53	45	94	68	3	124	76	53	154	82	12
5	4	59	35	33	0	65	54	20	95	68	25	125	77	7	155	82	20
6	5	59	36	33	50	66	54	55	96	68	47	126	77	20	156	82	28
7	6	58	37	34	41	67	55	29	97	69	8	127	77	33	157	82	36
8	7	58	38	35	29	68	55	3	98	69	30	128	77	46	158	82	43
9	8	5	39	36	17	69	56	36	99	59	50	129	77	58	159	82	51
10	9	5	40	37	5	70	57	9	100	70	11	130	78	11	160	82	58
11	10	5	41	37	53	71	57	41	101	70	31	131	78	23	161	83	6
12	11	54	42	38	40	72	58	13	102	70	51	132	78	35	162	83	13
13	12	52	43	39	27	73	58	44	103	71	10	133	78	46	163	83	20
14	13	51	44	40	13	74	59	15	104	71	30	134	78	58	164	83	27
15	14	49	45	40	58	75	59	46	105	71	48	135	79	9	165	83	33
16	15	47	46	41	43	76	50	16	106	72	7	136	79	21	166	83	40
17	16	45	47	42	28	77	50	45	107	72	25	137	79	32	167	83	47
18	17	42	48	43	17	78	61	14	108	72	43	138	79	42	168	83	53
19	18	39	49	43	38	79	61	43	109	73	1	139	79	53	169	83	59
20	19	36	50	44	21	80	62	11	110	73	18	140	80	3	170	84	6
21	20	32	51	45	21	81	52	39	111	73	35	141	80	14	171	84	12
22	21	28	52	46	3	82	63	6	112	73	52	142	80	24	172	84	18
23	22	24	53	45	44	83	63	33	113	74	9	143	80	34	173	84	24
24	23	19	54	47	25	84	64	0	114	74	25	144	80	43	174	84	29
25	24	14	55	48	5	85	64	26	115	74	41	145	80	53	175	84	35
26	25	9	56	48	45	86	64	51	116	74	57	146	81	2	176	84	41
27	25	3	57	49	24	87	65	17	117	75	12	147	81	12	177	84	46
28	26	56	58	50	3	88	65	41	118	75	27	148	81	21	178	84	52
29	27	50	59	50	41	89	66	6	119	75	42	149	81	30	179	84	57
30	28	42	60	51	19	90	56	30	120	75	57	150	81	38	180	85	2

Figure 5: The first page of a table with coordinates of the 4th rhumb (Wright 1599, p. n.d.)

4 Tables of meridional parts

4.1 Wright

Edward Wright (1561 - 1615) was the first who gave an explicit (mathematical) description of the Mercator projection, and explained how maps with this projection could be computed . Wright was a mathematician and cartographer, and in his early years he had joined some expeditions by ship. The experiences he gained during these expeditions probably inspired him to write his major work "Certaine Errors in Navigation" (1^{st} version published in 1599, 2^{nd} version in 1610 and a 3^{rd} version (by Joseph Moxon) in 1657) (Wallis 1975).

In his book Wright criticizes the common Sea-Chart, which was commonly used for navigation in those days, stating that it has some fundamental errors, consisting of errors in direction as well as in distance. Wright mentions that directions might be wrong by several points on a compass, and distances had the tendency to be (significantly) overestimated (Wright 1657, p.B).

Consider for example the distance between two places both located at 60° latitude, and 1° of longitude apart from each other. Since $\cos(60) = 0.5$, the distance between those two places is only half of the distance between two places that are 1° apart from each other at the equator. However, by using the common sea-chart, the distance between meridians was considered to be equal to the distance that it has at the equator, and therefore the distance at 60° North would be overestimated by a factor two, and even more further North.

Wright uses a similar example in a way that nicely shows how people were struggling with the common sea chart in those times, using figure 6. In short, he reasons as follows (Wright 1599, p.B2). Assume that one is sailing from D to B, around the parallel located 60° North, where B is located North-East of D, and their difference is one degree in latitude. Then on the common sea chart the distance DC is one degree as well. However, in reality it is only half a degree. So if DC is only half a degree, B should actually be located at A. But CA has a distance on the map of 2 degrees. According to the direction from D to B it follows that D should be located at E (with a distance EC of two degrees), but the chart only shows half the distance.

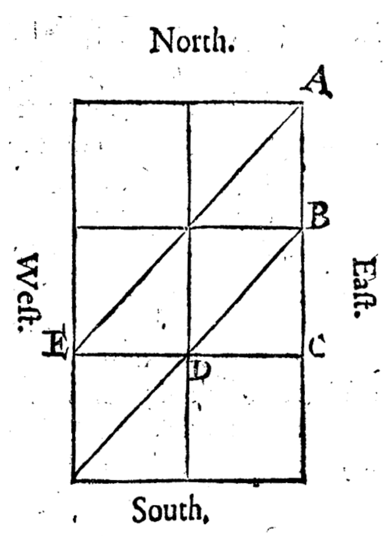


Figure 6: Example of the error made at 60° latitude (Wright 1599, p.B2)

The Mercator map avoided such errors by increasing the distance between the parallels towards the poles, and was therefore a huge improvement. It was not clear though, how such a map could be constructed, or more precisely, how much the distance between each of the parallels should be exactly. Wright explains that at each point of latitude, the meridians need to be (locally) stretched by a factor $\sec(\phi)$ (see the graph in figure 7(a) for the relation between the latitude and the 'stretching factor' $\sec(\phi)$). However, it was not straightforward how to translate this knowledge into exact distances between parallels. Nowadays we know, as we have seen in the previous chapter, that in order to get the overall scaling factor between the equator and a certain latitude, one needs to take the integral of the secant (e.g. the gray area under the graph in figure 7(a) represents the total scaling factor for the distance between the equator and the 60° parallel). However, the integral calculus, and in particular the integral of the secant, was not known yet by that time (Katz 1998, p.416-417, 468). Wright explains how a table with (approximations of) these distances can be calculated.

The table that Wright produced has a resolution of 1 minute of latitude, i.e. for each minute α in between the equator and the pole, Wright calculated the distance from the equator. Since there are 90 degrees in between the equator and the pole, each consisting of 60 minutes, this resulted in a table with a total of 5400 distances. To approximate the distance between the equator and latitude α, Wright divided the distance in n parts $\Delta\phi$, i.e. $n \cdot \Delta\phi = \alpha$. For $\Delta\phi$ he used a value of one minute.

Wright did not calculate the secant values himself, but used a table which provided secant values at a resolution of 1 minute. The secant values are slightly different from the secant values we use nowadays, in the sense that they were 10,000,000 times as large, i.e. $\mathrm{Sec}(x) = L \cdot \sec(x)$, where the notation $\mathrm{Sec}(x)$ is used for the secant values as used by Wright, the notation $\sec(x)$ for the secant values we use nowadays, and furthermore $L = 10,000,000$.

The method that Wright used for calculating the distances can be formulated in modern notation as follows:

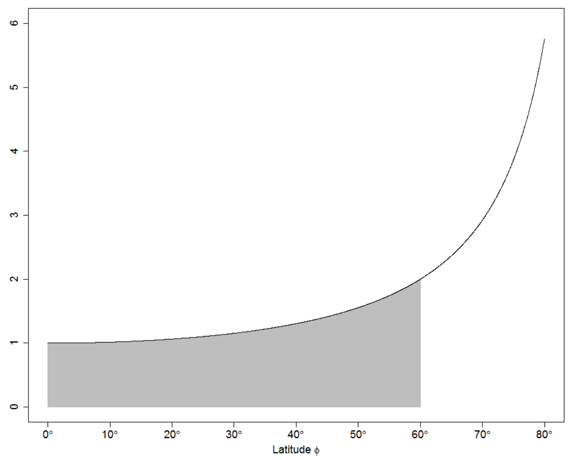
$$D(\alpha) = \sum_{k=1}^{n} \mathrm{Sec}(k \cdot \Delta\phi) = \sum_{k=1}^{n} L \cdot \sec(k \cdot \Delta\phi) \tag{4}$$

This can be interpreted as the distance from the equator to latitude α if one minute at the equator has a length of 10,000,000. Note that for each interval $(\phi_{n-1}, \phi_n]$ Wright assumed the secant to be constant and equal to $\mathrm{Sec}(\phi_n)$. See figure 7(b) for a visualization of the method.

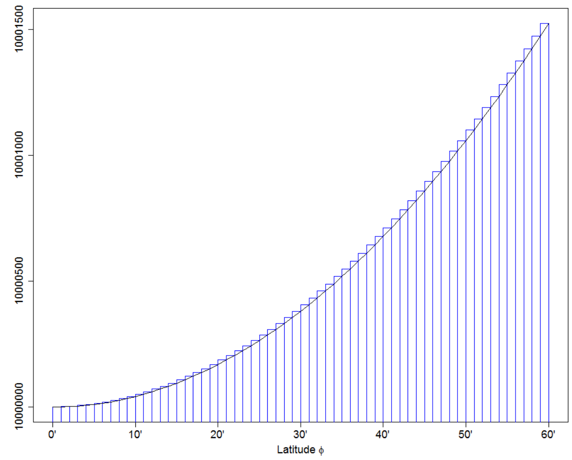
Distances from the equator to latitude ϕ, like these calculated by Wright, were called 'meridional parts', and a table listing these distances a 'table of meridional parts'.

In the passage of "Certaine errors in Navigation" shown in figure 8, Wright explains how he proceeded. (See also table 1 for a small summary). He writes that the secant of one minute equals 10,000,000, which is indeed $L \cdot \sec(1 \cdot \frac{1}{2} \cdot \pi/5400)$ (note that 1 minute corresponds to $\frac{1}{2} \cdot \pi/5400$ radians), and used as the approximation for the distance from the equator to the first minute. He continues by adding the secant of 2 minutes, which equals 10,000,002, giving a total distance from the equator to the second minute of $10,000,000 + 10,000,002 = 20,000,002$. The secant of 3 minutes equals 10,000,004, giving a total distance from the equator to the third minute of $20,000,002 + 10,000,004 = 30,000,006$, and so on.

As follows from the last part of the passage in figure 8, Wright was aware that his method resulted in an overestimation. The overestimation can also be clearly seen from figure 7(b). Since $\sec(\phi)$ is a monotoni-

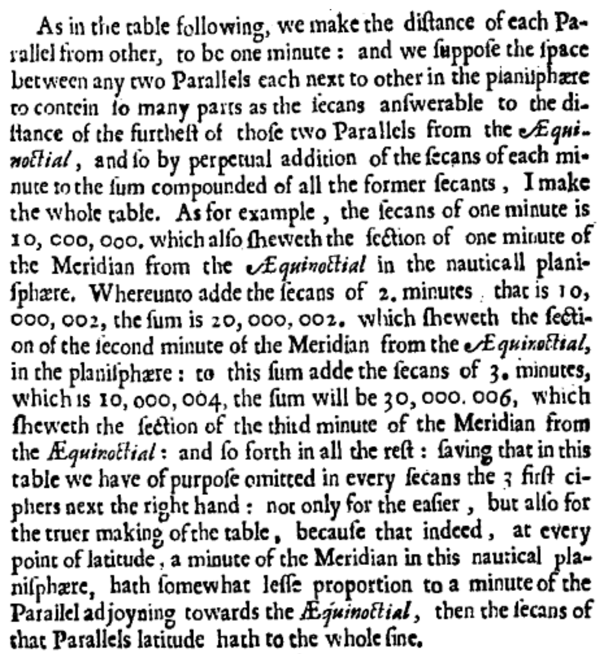


(a) $1/\cos(\phi)$



(b) Approximation by Wright

Figure 7: Secant



As in the table following, we make the diſtance of each Parallel from other, to be one minute : and we ſuppoſe the ſpace between any two Parallels each next to other in the planiſphære to contein ſo many parts as the ſecans anſwerable to the diſtance of the furtheſt of thoſe two Parallels from the *Æquinoctial*, and ſo by perpetual addition of the ſecans of each minute to the ſum compounded of all the former ſecants , I make the whole table. As for example , the ſecans of one minute is 10, 000, 000. which alſo ſheweth the ſection of one minute of the Meridian from the *Æquinoctial* in the nauticall planiſphære. Whereunto adde the ſecans of 2. minutes that is 10, 000, 002, the ſum is 20, 000, 002. which ſheweth the ſection of the ſecond minute of the Meridian from the *Æquinoctial*, in the planiſphære : to this ſum adde the ſecans of 3. minutes, which is 10, 000, 004, the ſum will be 30, 000. 006, which ſheweth the ſection of the third minute of the Meridian from the *Æquinoctial* : and ſo forth in all the reſt : ſaving that in this table we have of purpoſe omitted in every ſecans the 3 firſt ciphers next the right hand : not only for the eaſier , but alſo for the truer making of the table , becauſe that indeed , at every point of latitude , a minute of the Meridian in this nautical planiſphære, hath ſomewhat leſſe proportion to a minute of the Parallel adjoyning towards the *Æquinoctial* , then the ſecans of that Parallels latitude hath to the whole ſine.

Figure 8: Certain Errors In Navigation (Wright 1657, p.12)

Minute	Distance	Secant
1	$10,000,000$	$10,000,000$
2	$20,000,002$	$10,000,002$
3	$30,000,006$	$10,000,004$

Table 1: Summary of example provided by Wright

cally increasing function on the interval $[0, \frac{1}{2}\pi]$, and Wright used the secant value at the upper end of each minute interval as estimation for the whole interval, the area under the graph, and therefore the approximation of the distance to the equator, would be overestimated. Wright decided to omit the last three digits in his calculations, which, as well as for making the calculations slightly easier, would partly compensate for the overestimation. As a result he basically produced his table using a value of $10,000$ for the scaling factor L, thereby always rounding values down (see equation 5).

$$D(\alpha) = \sum_{k=1}^{n} \lfloor \mathrm{Sec}(k \cdot \Delta\phi)/1,000 \rfloor = \sum_{k=1}^{n} \lfloor L \cdot \sec(k \cdot \Delta\phi) \rfloor \tag{5}$$

Figure 9 shows the first page of the table as published in Wright (1657). The first line of the header contains the number of degrees. The first column further subdivides the degrees into minutes. For each degree there are two columns. The first column contains the meridional parts, whereas the second column contains the difference between two consecutive meridional parts, which is basically the secant multiplied by 10,000 (Wright 1657). Note that the table starts with the distance of latitude 0° to the equator, i.e. $00,000$. The following three records for minutes 1, 2 and 3 correspond to those in table 1, except that the last three digits have been omitted.

In Wright (1599), Wright already intended to publish the same table which in the end was published in Wright (1657). However, although he had carried out the exact same calculations, he decided to publish only the meridional parts for every 10^{th} minute, and to omit two more digits of every value.

4.2 Table of Wright recomputed

To get an impression of how accurate Wright calculated his table, I have recomputed the table which he published in the 1657 version of his book. Thereby I applied the same method as he did, using equation 5, with the only difference that I used the version based on the modern secant. By doing so, not only the inaccuracies of Wright's calculations, but also the inaccuracies in the meridional parts resulting from possible inaccuracies in the secant values could be analyzed. Since Wright actually included the differences between meridional parts in his table, i.e. the values for $\mathrm{Sec}(\phi)$, these could be easily compared with the values based on $L \cdot \sec(\phi)$. Figure 10 shows the first part of the reproduced values, corresponding to the first page of the table of meridional parts shown in figure 9. Marked in gray are the differences between the table in Wright (1657) and the recomputed table.

In total I could identify 213 errors in the table (see Appendix). On a total of 5400 minutes in between equator and pole, this corresponds to 4 percent of all minutes, whereby the error can be either in the value of the secant or the meridional part itself. (Note that values which could not be read due to the quality of the copy of the book were assumed to be correct). The errors which were identified can be roughly subdivided into the following categories (between brackets the number of occurrences within each category are given):

- Typographical and/or 'local' calculation errors, i.e. values that do not effect any other values in the table (121).
- Inaccuracies of the secant values, affecting the meridional parts (35).
- Calculation errors in the meridional parts which affect all further values 'in the direction of the poles' (57).

14 *A Table of Latitudes &c.*

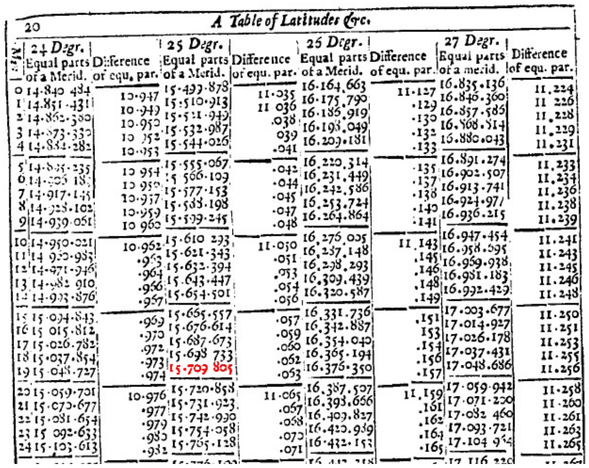
Min.	0 Degr. Equal parts of a Merid.	Difference of equ. par.	1 Degr. Equal parts of a Merid.	Difference of equ. par.	2 Degr. Equal parts of a Merid.	Difference of equ. par.	3 Degr. Equal parts of a Merid.	Difference of equ. par.
0	00.000	10.000	600.012	10.001	1.200.196	10.006	1.800.749	10.013
1	10.000	10.000	610.013	10.001	1.210.202	10.006	1.810.762	10.014
2	20.000	10.000	620.014	10.001	1.220.208	10.006	1.820.776	10.014
3	30.000	10.000	630.015	10.001	1.230.214	10.006	1.830.790	10.014
4	40.000	10.000	640.016	10.001	1.240.220	10.006	1.840.804	10.014
5	50.000	10.000	650.017	10.001	1.250.226	10.006	1.850.818	10.014
6	60.000	10.000	660.018	10.001	1.260.232	10.006	1.860.832	10.014
7	70.000	10.000	670.019	10.001	1.270.238	10.006	1.870.846	10.014
8	80.000	10.000	680.020	10.002	1.280.244	10.007	1.888.860	10.015
9	90.000	10.000	690.022	10.002	1.290.251	10.007	1.890.875	10.015
10	100.000	10.000	700.024	10.002	1.300.258	10.007	1.900.890	10.015
11	110.000	10.000	710.026	10.002	1.310.265	10.007	1.910.905	10.015
12	120.000	10.000	720.028	10.002	1.320.272	10.007	1.920.920	10.015
13	130.000	10.000	730.030	10.002	1.330.279	10.007	1.931.935	10.015
14	140.000	10.000	740.032	10.002	1.340.286	10.007	1.940.950	10.016
15	150.000	10.000	750.034	10.002	1.350.293	10.007	1.950.966	10.016
16	160.000	10.000	760.036	10.002	1.360.300	10.007	1.960.982	10.916
17	170.000	10.000	770.038	10.002	1.370.307	10.008	1.970.998	10.016
18	180.000	10.000	780.040	10.002	1.380.315	10.008	1.981.014	10.016
19	190.000	10.000	790.042	10.002	1.390.323	10.008	1.991.030	10.016
20	200.000	10.000	800.044	10.002	1.400.331	10.008	2.001.046	10.017
21	210.000	10.000	810.046	10.002	1.410.339	10.008	2.011.063	10.017
22	220.000	10.000	820.048	10.002	1.420.347	10.008	2.021.080	10.017
23	230.000	10.000	830.050	10.002	1.430.355	10.008	2.031.097	10.017
24	240.000	10.000	840.052	10.003	1.440.363	10.008	2.041.114	10.017
25	250.000	10.000	850.055	10.003	1.450.371	10.009	2.051.131	10.017
26	260.000	10.000	860.058	10.003	1.460.380	10.009	2.061.148	10.018
27	270.000	10.000	870.061	10.003	1.470.389	10.009	2.071.166	10.018
28	280.000	10.000	880.064	10.003	1.480.398	10.009	2.081.184	10.018
29	290.000	10.000	890.067	10.003	1.490.407	10.009	2.091.201	10.018
30	300.000	10.000	900.070	10.003	1.500.416	10.009	2.101.220	10.018
31	310.000	10.000	910.073	10.003	1.510.425	10.009	2.111.238	10.019
32	320.000	10.000	920.076	10.003	1.520.434	10.009	2.121.257	10.019
33	330.000	10.000	930.079	10.003	1.530.443	10.010	2.131.276	10.019
34	340.000	10.000	940.082	10.003	1.540.453	10.010	2.141.295	10.019
35	350.000	10.000	950.085	10.003	1.550.463	10.010	2.151.314	10.019
36	360.000	10.000	960.088	10.003	1.560.473	10.010	2.161.333	10.019
37	370.000	10.000	970.091	10.004	1.570.483	10.010	2.171.352	10.020
38	380.000	10.000	980.095	10.004	1.580.493	10.010	2.181.372	10.020
39	390.000	10.000	990.099	10.004	1.590.503	10.010	2.191.392	10.020
40	400.000	10.000	1.000.103	10.004	1.600.513	10.010	2.201.412	10.020
41	410.000	10.000	1.010.107	10.004	1.610.523	10.011	2.211.432	10.021
42	420.000	10.000	1.020.111	10.004	1.620.534	10.011	2.221.452	10.021
43	430.000	10.000	1.030.115	10.004	1.630.545	10.011	2.231.473	10.021
44	440.000	10.000	1.040.119	10.004	1.640.556	10.011	2.241.494	10.001
45	450.000	10.000	1.050.123	10.004	1.650.567	10.011	2.251.515	12.021
46	460.000	10.000	1.060.127	10.004	1.660.578	10.011	2.261.536	12.021
47	470.000	10.000	1.070.131	10.004	1.670.589	10.011	2.271.557	11.002
48	480.000	10.001	1.080.135	10.005	1.680.600	10.012	2.281.579	10.022
49	490.000	10.001	1.090.140	10.005	1.690.612	10.012	2.291.601	10.022
50	500.000	10.001	1.100.145	10.005	1.709.624	10.012	2.301.[illegible]	10.022
51	510.000	10.001	1.110.150	10.005	1.710.636	10.012	2.311.[illegible]45	10.022
52	520.000	10.001	1.120.155	10.005	1.720.648	10.012	2.321.667	10.023
53	530.000	10.001	1.130.160	10.005	1.730.660	10.012	2.331.690	10.023
54	540.000	10.001	1.140.165	10.005	1.740.672	10.012	2.341.712	10.023
55	550.000	10.001	1.150.170	10.005	1.750.684	10.013	2.351.736	10.023
56	560.000	10.001	1.160.175	10.005	1.760.697	10.013	2.361.759	10.023
57	570.000	10.001	16170.180	10.005	1.770.710	10.013	2.371.782	10.024
58	580.000	10.001	1.180.185	10.006	1.780.723	10.013	2.381.806	10.024
59	590.000	10.001	1.190.190	10.006	1.790.736	10.013	2.391.830	10.024

Figure 9: First page of the table of meridional parts in (Wright 1657, p.14)

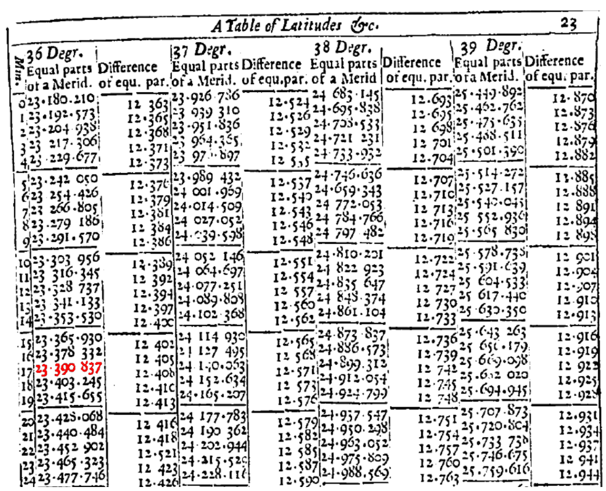
	0 Degr.		1 Degr.		2 Degr.		3 Degr.	
Min.	Equal parts of a Merid.	Difference of equ. par.	Equal parts of a Merid.	Difference of equ. par.	Equal parts of a Merid.	Difference of equ. par.	Equal parts of a Merid.	Difference of equ. par.
0	0	10,000	600,012	10,001	1,200,196	10,006	1,800,749	10,013
1	10,000	10,000	610,013	10,001	1,210,202	10,006	1,810,762	10,013
2	20,000	10,000	620,014	10,001	1,220,208	10,006	1,820,776	10,014
3	30,000	10,000	630,015	10,001	1,230,214	10,006	1,830,790	10,014
4	40,000	10,000	640,016	10,001	1,240,220	10,006	1,840,804	10,014
5	50,000	10,000	650,017	10,001	1,250,226	10,006	1,850,818	10,014
6	60,000	10,000	660,018	10,001	1,260,232	10,006	1,860,832	10,014
7	70,000	10,000	670,019	10,001	1,270,238	10,006	1,870,846	10,014
8	80,000	10,000	680,020	10,001	1,280,244	10,006	1,880,860	10,014
9	90,000	10,000	690,022	10,002	1,290,251	10,007	1,890,875	10,015
10	100,000	10,000	700,024	10,002	1,300,258	10,007	1,900,890	10,015
11	110,000	10,000	710,026	10,002	1,310,265	10,007	1,910,905	10,015
12	120,000	10,000	720,028	10,002	1,320,272	10,007	1,920,920	10,015
13	130,000	10,000	730,030	10,002	1,330,279	10,007	1,930,935	10,015
14	140,000	10,000	740,032	10,002	1,340,286	10,007	1,940,950	10,015
15	150,000	10,000	750,034	10,002	1,350,293	10,007	1,950,966	10,016
16	160,000	10,000	760,036	10,002	1,360,300	10,007	1,960,982	10,016
17	170,000	10,000	770,038	10,002	1,370,307	10,007	1,970,998	10,016
18	180,000	10,000	780,040	10,002	1,380,315	10,008	1,981,014	10,016
19	190,000	10,000	790,042	10,002	1,390,323	10,008	1,991,030	10,016
20	200,000	10,000	800,044	10,002	1,400,331	10,008	2,001,046	10,016
21	210,000	10,000	810,046	10,002	1,410,339	10,008	2,011,063	10,017
22	220,000	10,000	820,048	10,002	1,420,347	10,008	2,021,080	10,017
23	230,000	10,000	830,050	10,002	1,430,355	10,008	2,031,097	10,017
24	240,000	10,000	840,052	10,002	1,440,363	10,008	2,041,114	10,017
25	250,000	10,000	850,055	10,003	1,450,371	10,008	2,051,131	10,017
26	260,000	10,000	860,058	10,003	1,460,380	10,009	2,061,148	10,017
27	270,000	10,000	870,061	10,003	1,470,389	10,009	2,071,166	10,018
28	280,000	10,000	880,064	10,003	1,480,398	10,009	2,081,184	10,018
29	290,000	10,000	890,067	10,003	1,490,407	10,009	2,091,202	10,018
30	300,000	10,000	900,070	10,003	1,500,416	10,009	2,101,220	10,018
31	310,000	10,000	910,073	10,003	1,510,425	10,009	2,111,238	10,018
32	320,000	10,000	920,076	10,003	1,520,434	10,009	2,121,257	10,019
33	330,000	10,000	930,079	10,003	1,530,443	10,009	2,131,276	10,019
34	340,000	10,000	940,082	10,003	1,540,453	10,010	2,141,295	10,019
35	350,000	10,000	950,085	10,003	1,550,463	10,010	2,151,314	10,019
36	360,000	10,000	960,088	10,003	1,560,473	10,010	2,161,333	10,019
37	370,000	10,000	970,091	10,003	1,570,483	10,010	2,171,352	10,019
38	380,000	10,000	980,095	10,004	1,580,493	10,010	2,181,372	10,020
39	390,000	10,000	990,099	10,004	1,590,503	10,010	2,191,392	10,020
40	400,000	10,000	1,000,103	10,004	1,600,513	10,010	2,201,412	10,020
41	410,000	10,000	1,010,107	10,004	1,610,523	10,010	2,211,432	10,020
42	420,000	10,000	1,020,111	10,004	1,620,534	10,011	2,221,452	10,020
43	430,000	10,000	1,030,115	10,004	1,630,545	10,011	2,231,473	10,021
44	440,000	10,000	1,040,119	10,004	1,640,556	10,011	2,241,494	10,021
45	450,000	10,000	1,050,123	10,004	1,650,567	10,011	2,251,515	10,021
46	460,000	10,000	1,060,127	10,004	1,660,578	10,011	2,261,536	10,021
47	470,000	10,000	1,070,131	10,004	1,670,589	10,011	2,271,557	10,021
48	480,000	10,000	1,080,135	10,004	1,680,600	10,011	2,281,579	10,022
49	490,001	10,001	1,090,140	10,005	1,690,612	10,012	2,291,601	10,022
50	500,002	10,001	1,100,145	10,005	1,700,624	10,012	2,301,623	10,022
51	510,003	10,001	1,110,150	10,005	1,710,636	10,012	2,311,645	10,022
52	520,004	10,001	1,120,155	10,005	1,720,648	10,012	2,321,667	10,022
53	530,005	10,001	1,130,160	10,005	1,730,660	10,012	2,331,690	10,023
54	540,006	10,001	1,140,165	10,005	1,740,672	10,012	2,341,713	10,023
55	550,007	10,001	1,150,170	10,005	1,750,684	10,012	2,351,736	10,023
56	560,008	10,001	1,160,175	10,005	1,760,697	10,013	2,361,759	10,023
57	570,009	10,001	1,170,180	10,005	1,770,710	10,013	2,371,782	10,023
58	580,010	10,001	1,180,185	10,005	1,780,723	10,013	2,381,806	10,024
59	590,011	10,001	1,190,190	10,005	1,790,736	10,013	2,391,830	10,024

Figure 10: Recomputed values corresponding to the first page of the table of meridional parts in Wright (1657)

The errors contained in the first category might not seem very exciting at first sight. It is not always clear, however, if an error is simply a typo or a calculation error. Since the errors are 'local' in the sense that they do not affect any other values in the table, it seems rather likely that they are simply typos. Errors in the secants might have occurred when copying them from their source, while in some or another way the correct value was used for the actual calculation of the meridional part. There are a couple of errors in the meridional part, however, which are less likely to be typos. An example is given in figure 11(a). The meridional part of $25°18'$ is $15,698,733$. Adding the secant of $25°19'$ which has a value of $11,062$ (note that the outlining of the table is somewhat unfortunate, by which it might falsely appear that $11,062$ belongs to $25°18'$ instead of $25°19'$) resulted in $15,709,805$, where $15,709,795$ would be the correct value. In this case there are two digits involved, making it more likely to be a computation error then two typos in the same number. However, in that case one might expect that the error, once introduced, will affect all further values. Surprisingly though, the next meridional part has a value of $15,720,858$, which is the correct sum of $15,709,795$ and $11,063$ (the secant of $25°20'$). One can wonder what has happened exactly. It is not likely that Wright made another calculation error, which exactly compensated for the previous one. Errors like these could potentially give us some insight in the way Wright calculated his table.



(a) Typographic or calculation error?



(b) Error affecting all further values

Figure 11: Examples of errors in table of meridional parts (Wright 1657, p.14-30)

The second category, consisting of inaccuracies in the values of the secants, can tell us something about the quality of the secants data used by Wright. For all except four of the cases where there is a difference between the secants that were used by Wright and the values based on the modern secants, the (absolute) difference is only 1. In modern units this is equal to a difference of only 0.0001. From the four cases where differences are larger, three of them occur within 3 degrees from the pole, a region where secant values 'are changing rapidly'. In case the errors in the secants were already present in the data which Wright used, they can possibly be of help to identify from which source Wright used the secants. Unfortunately, a quick investigation has not resulted in a match yet.

The last category with calculation errors in the meridional parts which affect all further values is potentially the most severe one, since errors might accumulate and affect the usefulness of the table, especially for usage in the polar regions. However, most of the errors are so small that they have hardly any effect at all, and errors partly cancel each other out. The (by far) largest error made is an error of $100,000$ and occurs around the 87^{th} degree. Compared to the value of the meridional part, which is $124,078,381$, the relative error is less than 0.001. Figure 11(b) shows an example where the value in red, $23,390,837$, is one hundred too large. Since this value is used to calculate the meridional parts further down in the table, all those meridional parts inherit the error.

Overall most of the errors can be ignored with regard to the accuracy of the table. Some of the typos result in significantly different values, but many of them somehow 'break the pattern' (e.g. a meridional part becoming smaller instead of larger), and an attentive user of the table might notice the error and be warned to not blindly trust it.

Interestingly it seems that more errors occur on the first line of a page than one would expect based on a random distribution of the errors (i.e. 10 times, where the expected value is 3.6).

4.3 Comparing Wright's table of meridional parts with the modern values

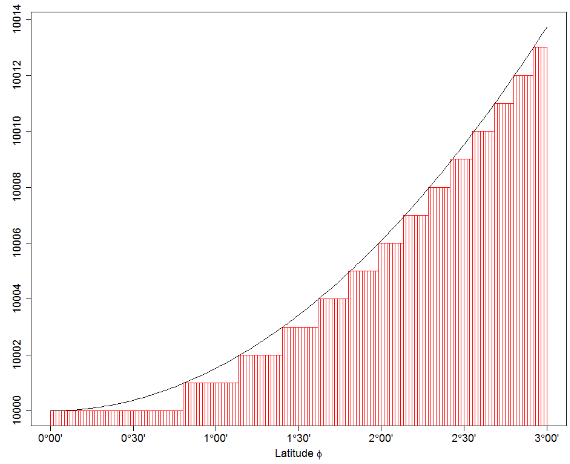
In the previous chapter I have analyzed the accuracy of the calculations carried out by Wright. However, for reproducing his table I used a similar method as Wright had used, i.e. approximating the integral of the secant by a finite sum. A more interesting question is how accurate his method actually was compared to the modern solution. Therefore, in this chapter Wright's method will be compared with the meridional parts which I calculated using equation 6, with $L = 10,000$, $c = \frac{10800}{\pi}$ a conversion factor from radians to minutes (and ϕ in radians).

$$y = L \cdot c \cdot \ln\tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right) \tag{6}$$

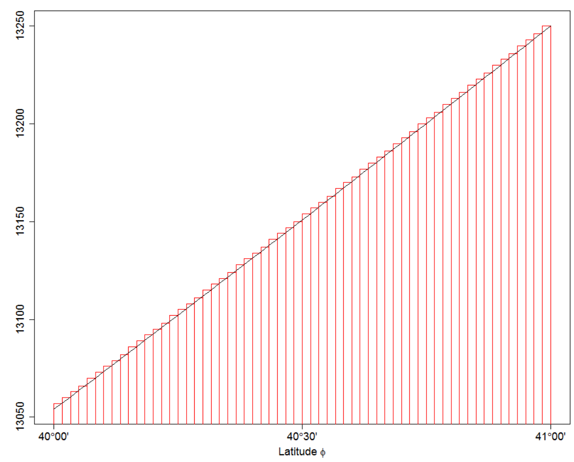
Figure 12(c) shows the difference between the approximation of equation 5 and equation 6. Since Wright consistently overestimated the integral by assuming for each interval $(\phi_{n-1}, \phi_n]$ the secant to be constant and equal to $\mathrm{Sec}(\phi_n)$ it is remarkable that the differences are negative for lower latitudes. However, the negative values are the result of cutting of the last three digits instead of rounding the values. This effect was intended by Wright to compensate (at least partly) for the overestimation. Figure 12(a) clearly shows that for lower latitudes, where secants only slightly differ between minutes, the way Wright rounds results even in an underestimation. For medium to higher altitudes, where secants change more rapidly from minute to minute, Wright's way of rounding hardly has any effect on the values anymore (see as example figure 12(b)), and the overestimation inherent to his method becomes larger and larger. Around 75 degrees the overestimation has reached a total of about $12,000$. Compared to the length of $10,000$ that 1 degree has at the equator, the overestimation might seem rather significant. However, when looking at the relative difference by dividing the difference of equation 5 and 6 by equation 6 it becomes clear that the errors are rather minor (see figure 12(d)).

4.4 Comparing tables of meridional parts of different people

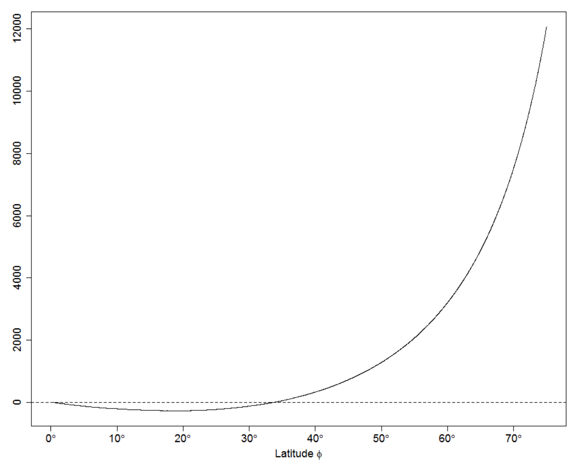
After Wright many more people have generated tables with meridional parts, partly making use of different methods. Table 2 shows a selection of them, where only the meridional part for every fifth degree is given, as well as two parallels very close to the pole. Furthermore, for each column with meridional parts, the differences with the values calculated using equation 6 is given in the column directly right of it. The last two columns, finally, provide values based on the common sea-chart as a comparison. Note that, since most



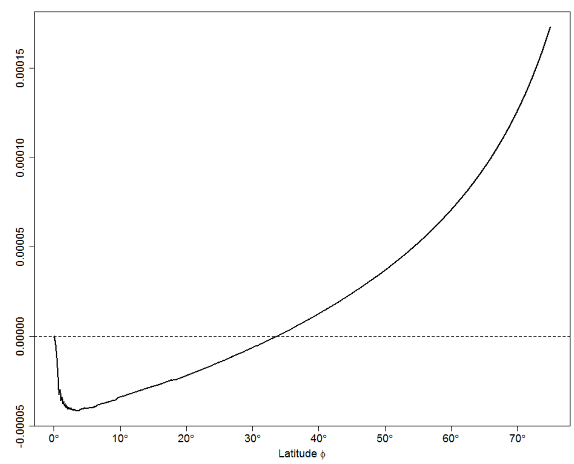
(a) Effect of rounding down (1)



(b) Effect of rounding down (2)



(c) Difference



(d) Relative difference

Figure 12: Wright's method compared to real solution

Distance from equator	$\ln\tan(\frac{\phi}{2}+\frac{\pi}{4})$	Wright (1599)[1]		Snellius (1624)[2]		Norwood (1645)[3]		Wright (1657)[4]		Sir Jonas Moore (1681)[5]		Mendoza (1809)[6]		Common Sea-Chart	
0°	0	-	-	-	-	0.00	0.00	0.0000	0.0000	0.0	0.0	0.00	0.00	0.0	0.0
5°	300.38150	300.4	0.0	300.3796	-0.0019	300.36	-0.02	300.3694	-0.0121	300.4	0.0	300.38	0.00	300.0	-0.4
10°	603.06958	603.0	-0.1	603.0618	-0.0078	603.06	-0.01	603.0475	-0.0221	603.1	0.0	603.07	0.00	600.0	-3.1
15°	910.46058	910.4	-0.1	910.4428	-0.0178	910.44	-0.02	910.4325	-0.0281	910.5	0.0	910.46	0.00	900.0	-10.5
20°	1225.13905	1225.1	0.0	1225.1068	-0.0323	1225.14	0.00	1225.1292	-0.0099	1225.1	0.0	1225.14	0.00	1200.0	-25.1
25°	1549.99521	1550.0	0.0	1549.9434	-0.0518	1549.98	-0.02	1549.9878	-0.0074	1550.0	0.0	1549.99	-0.01	1500.0	-50.0
30°	1888.37542	1888.4	0.0	1888.2980	-0.0774	1888.32	-0.06	1888.3768	0.0014	1888.4	0.0	1888.38	0.00	1800.0	-88.4
35°	2244.28684	2244.3	0.0	2244.1764	-0.1104	2244.24	-0.05	2244.3047	0.0179	2244.3	0.0	2244.29	0.00	2100.0	-144.3
40°	2622.69019	2622.8	0.1	2622.5374	-0.1528	2622.60	-0.09	2622.7559	0.0657	2622.7	0.0	2622.69	0.00	2400.0	-222.7
45°	3029.93920	3030.0	0.1	3029.7320	-0.2072	3029.82	-0.12	3030.1271	0.1879	3030.0	0.1	3029.94	0.00	2700.0	-329.9
50°	3474.47287	3474.6	0.1	3474.1919	-0.2810	3474.30	-0.17	3474.6045	0.1316	3474.5	0.0	3474.47	0.00	3000.0	-474.5
55°	3967.96611	3968.2	0.2	3967.5943	-0.3718	3967.74	-0.23	3968.1879	0.2218	3968.0	0.0	3967.97	0.00	3300.0	-668.0
60°	4527.36776	4527.7	0.3	4526.8678	-0.5000	4527.06	-0.31	4527.7106	0.3428	4527.4	0.0	4527.37	0.00	3600.0	-927.4
65°	5178.80819	5179.3	0.5	5178.1250	-0.6832	5178.42	-0.39	5179.3079	0.4997	5178.8	0.0	5178.81	0.00	3900.0	-1278.8
70°	5965.91787	5966.7	0.8	5964.9560	-0.9619	5965.20	-0.72	5966.6811	0.7632	5966.0	0.1	5965.92	0.00	4200.0	-1765.9
75°	6970.33899	6971.5	1.2	-	-	6969.60	-0.74	6971.5485	1.2095	6970.3	0.0	6970.34	0.00	4500.0	-2470.3
80°	8375.19700	8377.3	2.1	-	-	8373.60	-1.60	8377.3416	2.1446			8375.20	0.00	4800.0	-3575.2
85°	10764.62104	10769.6	5.0	-	-	-	-	10769.6200	4.9990	10764.7	0.1	10764.62	0.00	5100.0	-5664.6
89°50′	22459.25656	22622.3	163.0	-	-	-	-	22623.2506	163.9940	22458.0	-1.3	22459.26	0.00	5390.0	-17069.3
89°59′	30374.96343	-	-	-	-	-	-	32348.5279	1973.5645	30364.3	-10.7	30374.96	0.00	5399.0	-24976.0

Table 2: Meridional parts from different sources

of the authors used different units, values have been transformed to a value relative to one minute at the equator where needed.

Without going into too much detail about the exact methods that were used, one can clearly observe some differences in the meridional parts. Where we have seen that Wright's method resulted (at least for the higher altitudes) in an overestimation, both the methods of Snellius and Norwood tend to result in underestimated values. Furthermore, both Snellius and Norwood did not specify values for the highest altitudes, possibly being aware of increasing inaccuracies for these regions (or simply to reduce the computational efforts, since values for regions so far to the north were of little practical use). The table of meridional parts published by Sir Jonas Moore is very accurate, and has only some minor inaccuracies for the highest altitudes. Although it is not stated explicitly how the values in the table were calculated, equation 3 was known to Sir Jonas Moore (Moore 1681, p. 208). It is therefore likely that he used this method, although it is not clear in that case why the last values in the table are less accurate. According to (Bathe 1915), Moore used the secant of the mean of each interval instead of the upperbound as Wright did, and furthermore he used a resolution of 0.1 to calculate the meridional parts. Also in that case the last values in the table should be more accurate. Moore's table has been in use till the 19th century. The extremely accurate values in Mendoza's table are a result of using the method based on the logarithmic tangents to calculate the meridional parts (Wallis 1685).

Overall one can clearly observe that meridional parts, as calculated by the different people, are in general rather accurate up to about 85°, after which, depending on the method that is used, errors sometimes rapidly increase. For navigation purposes the accuracy of the tables was in general more than sufficient. Especially when comparing the errors for the different versions of meridional parts with the errors in the common sea-chart which are of a complete different order, and already much more inaccurate at lower altitudes.

[1] Values from (Wright 1599, pp. n.d.)

[2] Values from (Snellius 1624, pp. 206-233). Table contains values for each minute up to 70°.

[3] Values from (Norwood 1645, pp. A-A2). Table contains values for every 10th minute up to 80°.

[4] Values from (Wright 1657, pp. 14-36)

[5] Values from (Moore 1681, pp. 67-93). Table contains values for each minute up to 89°59′.

[6] Values from (Rios 1809, pp. 585-592). Table contains values for each minute up to 90°.

5 The mystery of Henry Bond

Mercator created his map in 1569, and in 1599 Wright published a table with meridional parts. As we have seen, the meridional parts are equivalent to the logarithmic tangent of the sum of $\frac{\pi}{4}$ and half the latitude. However, neither logarithms nor integral calculus were known in the 16$^{\text{th}}$ century. Logarithms were only discovered by Napier around 1615-1620, and also the development of calculus had to wait till the 17$^{\text{th}}$ century. An interesting question is when, how and by whom the link was made between the meridional parts and the logarithmic tangents. There is a lot of literature available, which can be consulted by the interested reader, discussing the roles of people like Thomas Harriot, James Gregory, Isaac Barrow, John Wallis, Edmond Halley and others in proving the equivalence, e.g. Rickey & Tuchinsky (1980) and Pepper (1968). But there is one interesting detail that we came across during the work on this thesis, which might be worth mentioning.

According to a.o. Rickey & Tuchinsky (1980), Pepper (1968), Hofmann (1950) and Roy (1990), the early 17$^{\text{th}}$ century English mathematician Henry Bond compared (tables with) meridional parts and logarithmic tangents and noticed the similarity between them. Bond would then have published this as a conjecture in Norwood (1645). However, checking Norwood (1645), the correspondence between the meridional parts and the logarithmic tangent is indeed mentioned [see p.14], however, without any reference to Bond. In (Norwood 1644, p. A2) (which is included in Norwood (1645)), the name of Bond is mentioned, but in this part no link is made with the relation between meridional parts and logarithmic tangents. According to Cajori (1915) Bond included his conjecture also in Gunter (1653). However, although in this work the correspondence is mentioned as well [p.99], it is again without any reference to Bond.

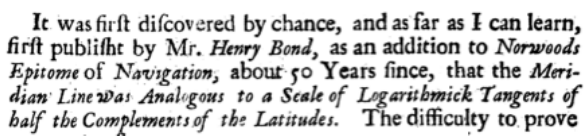

It was firſt diſcovered by chance, and as far as I can learn, firſt publiſht by Mr. *Henry Bond*, as an addition to *Norwoods Epitome* of *Navigation*, about 50 Years ſince, that the *Meridian Line was Analogous to a Scale of Logarithmick Tangents of half the Complements of the Latitudes.* The difficulty to prove

Figure 13: Excerpt from Halley (1696)

This raises the question what the role of Bond actually has been. It seems that the source of the story about the conjecture of Bond is Halley (1696) (see figure 13). A few authors mention that they have searched for a copy of Norwood (1645), unfortunately without any success. Some refer to Halley (1696) as a confirmation of the role of bond with regard to the linkage between meridional parts and the logarithmic tangent (Monmonier 2010), where there are others who do not mention Bond at all, but only that the similarity seems to have been noticed around 1645 (Carslaw 1924).

Collections like the Early English Books Online (accessed June 10, 2018) make it easier nowadays to check for original sources. More research is needed to investigate if Bond played a role in discovering the link between the meridional parts and the logarithmic tangents, and in case Bond was not involved, who else first noticed the similarity. However, one thing is for sure, many people were involved in unraveling the mathematics behind the Mercator projection, which has all in all taken more than a century to be fully understood and proved.

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Appendix

Degrees	Minutes	Secant (calculated)	Meridional part (calculated)	Secant (Wright)	Meridional part (Wright)	Local error secant	Local error meridional part	Accumulating error secant	Accumulating error meridional part	Accumulated error
0	49	10001	490001	10001	490000		-1			0
0	50	10001	500002	10001	500000		-2			0
0	51	10001	510003	10001	510000		-3			0
0	52	10001	520004	10001	520000		-4			0
0	53	10001	530005	10001	530000		-5			0
0	54	10001	540006	10001	540000		-6			0
0	55	10001	550007	10001	550000		-7			0
0	56	10001	560008	10001	560000		-8			0
0	57	10001	570009	10001	570000		-9			0
0	58	10001	580010	10001	580000		-10			0
0	59	10001	590011	10001	590000		-11			0
1	57	10005	1170180	10005	16170180		15000000			0
2	50	10012	1700624	10012	1709624		9000			0
3	8	10014	1880860	10014	1888860		8000			0
3	29	10018	2091202	10018	2091201		-1			0
3	41	10020	2221452	10021	2221452	1				0
3	44	10021	2251515	10001	2251515	-20				0
3	45	10021	2261536	12021	2261536	2000				0
3	46	10021	2271557	12021	2271557	2000				0
3	47	10022	2281579	11002	2281579	980				0
4	25	10030	2662551	10039	2662551	9				0
4	52	10036	2923400	10036	2923300		-100			0
5	16	10042	3174375	10052	3174375	10				0
6	36	10066	3968634	10066	3968934		300			0
7	36	10089	4583349	10889	4583349	800				0
7	38	10089	4593438	10089	4543438		-50000			0
7	38	10089	4603527	10099	4603527	10				0
7	46	10092	4674163	10092	4674136				-27	-27
8	44	10117	5260219	10117	5260193		1			-27
9	0	10124	5422150	10124	5422133				10	-17
10	4	10156	6071112	10156	6071098		3			-17
10	16	10163	6203186	30163	6203169	20000				-17
10	20	10164	6233677	10165	6233661			1		-16
10	34	10172	6376035	10172	6379019		3000			-16
11	0	10187	6640705	10187	6640679				-10	-26
11	8	10191	6722219	10191	6722219		26			-26
11	58	10222	7232549	10222	7232323		-200			-26
13	35	10287	8227191	10287	8227169		4			-26
16	46	10444	10206470	10443	10206443			-1		-27
18	1	10515	10992412	10515	10992383				-2	-29
19	4	10580	11656913	10580	21656884		10000000			-29
19	31	10609	11942977	10609	11912948		-30000			-29
19	53	10633	12176655	10633	12176636		10			-29
20	0	10641	12251121	10641	12251292				200	171
21	47	10768	13396486	10768	12396657		-1000000			171
22	13	10801	13676909	10801	13677070		-10			171
22	14	10803	13687712	10803	13687873		-10			171
22	15	10804	13698516	10804	13698677		-10			171
22	45	10844	14034084	100844	14034255	90000				171
23	7	10873	14262124	10873	14262275				-20	151
23	26	10898	14468960	10898	14499111		30000			151
24	15	10967	15004692	10967	15094843		90000			151
24	18	10972	15037603	10972	15037854		100			151
25	19	11062	15709644	11062	15709805		10			151
25	28	11076	15809271	11076	15809432		10			151

Degrees	Minutes	Secant (calculated)	Meridional part (calculated)	Secant (Wright)	Meridional part (Wright)	Local error secant	Local error meridional part	Accumulating error secant	Accumulating error meridional part	Accumulated error
27	24	11263	17104833	11263	17104964				-20	131
29	5	11444	18262857	11448	18262988	4				131
29	18	11466	18400332	11466	18400403		-60			131
30	36	11617	19300613	11617	19300734				-10	121
30	39	11625	19347101	11425	19347222	-200				121
31	14	11695	19743568	11695	19743699		10			121
31	15	11697	19755265	11697	19755396		10			121
31	34	11736	19977896	11736	19678017		-300000			121
32	37	11872	20721577	11872	20721702				4	125
33	57	12055	21678642	12055	21687767		9000			125
33	59	12059	21702758	12059	21720883		18000			125
34	37	12151	22162783	12150	22162907			-1		124
35	5	12220	22503997	12220	22504221				100	224
36	13	12394	23340907	12394	23341133				2	226
36	17	12405	23390511	12405	23390837				100	326
36	22	12421	23464997	12521	23465323	100				326
36	29	12437	23539578	12437	22539904		-1000000			326
36	52	12499	23826370	12499	23226696		-600000			326
37	48	12658	24543390	12648	24543716	-10				326
38	6	12707	24759017	12707	24659343		-100000			326
39	10	12898	25578402	12898	25578738		10			326
39	11	12901	25591303	12901	25591639		10			326
39	44	13003	26018756	13003	26019072		-10			326
39	45	13006	26031762	13006	26032078		-10			326
40	11	13089	26371034	13089	2637136		-23734224			326
40	31	13154	26633489	13154	26633842				27	353
40	38	13177	26725656	13177	26729009		3000			353
40	42	13190	26778395	13190	26778648				-100	253
41	59	13452	27804137	13452	27804380				-10	243
42	0	13456	27817593	13456	27817839		3			243
42	38	13592	28331554	13592	28331807				10	253
42	40	13599	28358749	13599	28358902				-100	153
42	52	13647	28535872	13646	28536025	-1				153
43	33	13797	29084806	13797	29084969		10			153
43	42	13835	29222986	13839	29223139	4				153
43	43	13839	29236825	13835	29236978	-4				153
44	50	14101	30158887	14100	30159039			-1		152
45	0	14142	30300119	14142	30301271		1000			152
45	58	14386	31127493	14386	31127635				-10	142
46	0	14395	31156279	-9999	31156423				2	144
46	36	14554	31677421	14554	31677545				-20	124
46	39	14567	31721109	14567	31721133				-100	24
47	56	14925	32856576	14925	32856595				-5	19
48	19	15037	33201188	15037	33201227				20	39
49	23	15361	34173963	15361	34173102		-900			39
49	35	15423	34358697	15423	34358733				-3	36
49	44	15471	34497746	15471	34487782		-10000			36
50	39	15771	35356982	15771	35357028				10	46
51	8	15935	35816799	15936	35816846			1		47
52	44	16520	37390755	16420	37390802	-100				47
52	49	16546	37456900	16546	57456947		20000000			47
53	14	16706	37872622	16706	3772669		-34100000			47
53	19	16739	37956251	-9999	37959298		3000			47
53	49	16938	38461489	16948	38461546			10		57

Degrees	Minutes	Secant (calculated)	Meridional part (calculated)	Secant (Wright)	Meridional part (Wright)	Local error secant	Local error meridional part	Accumulating error secant	Accumulating error meridional part	Accumulated error
53	50	16951	38495385	16501	38495442	-450				57
53	59	17006	38631241	17006	38631268		-30			57
54	21	17157	39007101	17157	38007158		-1000000			57
54	22	17164	39024265	17164	39024324				2	59
54	36	17262	39265296	17262	39265356				1	60
54	42	17305	39369019	17305	39369179				100	160
54	44	17319	39403650	-9999	39403800				-10	150
54	58	17419	39646867	17420	39647018			1		151
55	11	17514	39873981	17514	39874122				-10	141
55	13	17529	39909031	17529	34909172		-5000000			141
55	24	17610	40102334	17610	40102465		-10			141
56	0	17882	40741302	-8888	40741423				-20	121
56	13	17983	40974479	17983	40974601				1	122
57	0	18360	41828668	18360	41828810				20	142
57	21	18535	42216150	18535	42216392				100	242
57	25	18569	42290374	-9999	42290608				-8	234
58	24	19084	43401222	19084	43401458				2	236
58	27	19111	43458528	19111	43458794		30			236
58	28	19120	43477648	19120	43477880				-4	232
58	57	19387	44036119	19387	44936351		900000			232
59	0	19416	44094338	19416	44094550				-20	212
59	36	19761	44799646	19761	44799857				-1	211
59	55	19949	45176979	19949	45177199		9			211
60	0	20000	45276875	20000	45277106				20	231
60	30	20307	45881603	20307	45881827				-7	224
60	53	20551	46351580	20551	46351814		10			224
60	55	20572	46392713	20572	46382937		-10000			224
61	5	20680	46599030	20680	46599250				-4	220
61	28	20934	47077713	20934	47077923		-10			220
61	43	21104	47393083	21104	47393383		80			220
61	48	21161	47498774	21161	47498984		-10			220
62	50	21901	48833787	21901	48834017				10	230
62	51	21914	48855701	21914	48855821				-110	120
62	56	21976	48965457	21976	48965557		-20			120
63	37	22503	49877449	22503	49877568				-1	119
64	2	22838	50444362	22838	50444461				-20	99
64	11	22976	50673500	22676	50673599	-300				99
64	16	23031	50765541	23031	50765660				20	119
64	36	23313	51229112	23313	51229233				2	121
64	39	23370	51322507	23270	51322628	-100				121
66	23	24961	53810383	24961	53810514		10			121
67	41	26334	55810552	26334	55810670		-3			121
68	32	27325	57179019	27325	37179140		-20000000			121
68	57	27840	57868800	27841	57868922			1		122
70	9	29449	59930883	29449	59931015				10	132
70	12	29521	60019374	29521	60019596		90			132
70	19	29689	60226693	29689	60220825		-6000			132
70	55	30586	61311915	30586	61311947				-100	32
71	26	31406	62273075	31406	62273097		-10			32
71	41	31819	62747456	31819	62747478		-10			32
72	36	33440	64542119	33440	64542171		20			32
72	43	33659	64777071	-9999	64777102				-1	31
74	38	37736	68874984	37736	68874915		-100			31
75	7	38932	69987091	38932	69987142				20	51

Degrees	Minutes	Secant (calculated)	Meridional part (calculated)	Secant (Wright)	Meridional part (Wright)	Local error secant	Local error meridional part	Accumulating error secant	Accumulating error meridional part	Accumulated error
75	15	39276	70300095	39277	70300147			1		52
75	39	40347	71255998	40347	71256051				1	53
75	42	40485	71377315	40485	71377358		-10			53
76	28	42733	73291495	42732	73291547			-1		52
76	39	43308	73764993	43309	73765046			1		53
77	37	46630	76372441	46630	76372394		-100			53
79	49	56561	83145068	-9999	83149121		4000			53
80	38	61443	86035256	61443	86035209		-100			53
81	35	68319	89729896	68319	89729749		-200			53
81	48	70112	90630491	70111	90630543			-1		52
81	59	71704	91411207	71704	91411159		-100			52
82	1	72001	91555060	72001	91555113				1	53
82	10	73371	92209883	73371	62209936		-30000000			53
82	30	76612	93710871	76612	93710914		-10			53
82	47	79604	95039877	79603	95039929			-1		52
83	7	83438	96671611	83439	96671664			1		53
83	34	89248	99004010	89248	94004063		-5000000			53
83	51	93343	100557552	93342	100557604			-1		52
83	55	94362	100933463	94361	100933514			-1		51
84	29	104020	104305415	104019	104305465			-1		50
84	57	113604	107352978	113604	107353128				100	150
85	4	116283	108158850	116283	108159002				2	152
85	12	119505	109103498	119506	109103651			1		153
85	23	124240	110446131	124240	110445628		-656			153
85	23	124689	110570820	124989	110570973	300				153
85	42	133371	112895950	133371	112895803		-300			153
85	51	138182	114120092	138183	114120246			1		154
85	52	138739	114258831	138739	114258685		-300			154
85	58	142173	115103200	142172	115103353			-1		153
85	59	142762	115245962	142761	115246114			-1		152
86	8	148291	116558149	148291	116558001		-300			152
86	11	150231	117006890	150230	117007041			-1		151
86	49	180079	123263431	180078	123263581			-1		150
86	50	181026	123444457	181025	123444608		2	-1		149
86	54	184915	124178232	184915	124078381				-100000	-99851
87	4	195411	126084153	195411	125984102		-200			-99851
87	33	233931	132295420	233936	132195574			5		-99846
87	37	240471	133247381	240470	133147534			-1		-99847
87	42	249179	134475611	249178	134375763			-1		-99848
88	7	304280	141375628	304281	141275781			1		-99847
88	12	318362	142938762	318361	142838914			-1		-99848
88	16	330603	144242523	330602	144142674			-1		-99849
88	33	395185	150411493	395185	150311944		300			-99849
88	39	429757	153312762	429752	153212913	-5				-99849
89	1	582697	163857870	582696	163758020			-1		-99850
89	51	3819722	230152078	3819723	230052229			1		-99849
89	53	4911070	239360335	4911090	239260506			20		-99829
89	55	6875495	251965410	6875496	251865582			1		-99828
89	56	8594368	260559778	8594338	260459920			-30		-99858

Table 3: Overview of differences between the table with meridional parts of Wright and those calculated using equation 5. Here the local errors have been further subdivided into local errors in the secant values and local errors in the meridional parts. The accumulated error is the cumulative sum of both the accumulating errors in the secants and those in the meridional parts. Note that the difference between the calculated meridional part and the meridional part of Wright equals the sum of the accumulated error and the local error in the meridional part.