Truth and lies in election campaigns.

A game theoretic approach.

Bachelorthesis

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Abstract

Political candidates do not infrequently lie during the campaign. Afterwards, when a candidate has won the election, the difference between the campaign announcements and the implemented policy of the candidate is made clear. There are candidates for whom the difference between the campaign announcement and the implemented policy will be quite large, while the difference may be zero for more truthful candidates.

In this thesis we will transform the political election into a game theoretic model. In the model different candidates can differ in the amount they are willing to lie. We find that the presence of liars in an election affects the behavior of the more truthful candidates. Also, the presence of truthful candidates changes the behavior of candidates who are more willing to lie. We conclude that liars have a slight advantage over truthful candidates regarding the chance of being elected into office.

Contents

1 Introduction

Oftentimes, promises made during electoral campaigns are not quite the same as the policy implemented if the candidate is elected. Voters have to choose between candidates in an election, while they are not sure what the policy will be when a candidate is elected. This resembles a signaling game between political candidates and voters. The political candidates send a signal, the policy platform a candidate announces in the campaign, and based on this announcement (the signal) a voter chooses a candidate. In order to attract as many voters as possible, candidates can lie about their true policy intentions to try and make their announcement as attractive as possible.

Different candidates differ in the amount that they are able to and willing to lie. There are more truthful candidates who almost speak the truth about their policy intentions and on the flip side there are some candidates who will say anything to get elected. Candidates can also differ in their policy intentions. We will take a closer look at the effect of having both liars and truthful candidates with different policy intentions present in the election on the behavior of different candidates during the campaign. We will do this using a game theoretical perspective.

This thesis is based on the article "Lies, damned lies, and political campaigns" by Callander and Wilkie (2006). We will examine which strategy a candidate will choose in an election equilibrium. One of the central questions will be: Do better liars always win an election?

First, we will take a look at the theoretical background that is needed to answer the question; this will be the subject of part two. Then we will introduce some notation and under some assumptions transform the political election in a game theoretic model in part three. The election equilibria are determined in part four. In part five we will take a closer look at the characteristics of the election equilibria. Finally, we will end with the conclusion and answer the preceding question. Throughout this text we will be using the same manner of notation as Callander and Wilkie (2006).

2 Theory

The 'political game' between voters and candidates is a signaling game. This signaling game is a game of incomplete information, because the voters are uninformed about the characteristics of the candidates. Voters do not know whether or not a particular candidate is willing to lie about future implemented policy, nor do they know the severity of the lies made by the candidates. They therefore have no way of knowing what a candidate will actually do when elected into office. We want to determine the strategy of the candidates in this game employed in equilibrium. Only using the perfect Bayesian equilibrium requirements gives rise to many equilibria in most signaling games. To get a better idea of how the candidates will behave, we will introduce a refinement of the perfect Bayesian equilibrium. In a perfect Bayesian equilibrium, players have beliefs about what kind of type the other player is, and a player chooses an optimal strategy given these beliefs. A belief is a probability distribution over the different types a candidate can be, and these beliefs are updated with Bayes' rule at every possible moment in the game. A refinement of the perfect Bayesian equilibrium is the intuitive criterion, which we will discuss in the next section.

2.1 Intuitive Criterion

As previously stated, the intuitive criterion is a refinement of the perfect Bayesian equilibrium. We will largely adopt the notation of [4]. Let T denote the set of possible types for player one and let $t_1 \in T$ be player one's type with probability distribution $\mu(t_1) \in [0, 1]$. Nature decides the type of player one, and t_1 is only known to player one. After being assigned a type by Nature, player one sends a message, a signal m. Player two receives this message while not knowing player one's type. Player two does know the probability distribution $\mu(t_1)$ of player one, because player two knows about the message player one has send him, he adjusts his beliefs about player one being type t_1 from $\mu(t_1)$ to $\mu(t_1|m)$. Given these beliefs and message m, he chooses a best reply, a, with $A^*(T, m)$ being the set of best replies.

In any given equilibrium let $u_1(m^*, a^*, t_1)$ be the equilibrium payoff for player one, and let m^* be the equilibrium message for player one. We use a similar notation for player two; a^* being the equilibrium best reply for player two and $u_2(m^*, a^*, t_1)$ being the equilibrium payoff for player two. A pair of strategies (m^*, a^*) with beliefs $\mu(t_1|m)$ is a perfect Bayesian equilibrium if the beliefs of player two are consistent under consideration of strategy m^*, and given these beliefs strategy a^* is the best reply of player two to m^*.

A perfect Bayesian equilibrium violates the intuitive criterion under certain conditions. We will specify these conditions next. First, one has to check whether or not there is a non-empty set which only contains types of player one that can benefit from sending an off-the-equilibrium message m. Then, given these types, one should check if the minimum payoff received by a type out of this set of types of player one by sending the off-the-equilibrium message is greater than the payoff received in equilibrium. If this is the case, then the perfect Bayesian equilibrium will not 'survive' the intuitive criterion. Formally the intuitive criterion is:

Intuitive criterion ([4], p.3)**.**

First step: *For any off-the-equilibrium message* m *we define a set* $T^{**}(m) \subset T$*, such that for every type* t *in* $T^{**}(m)$*,*

$$T^{**}(m) = \left\{ t \in T | u_i^*(t) \leq \max_{a \in A^*(T,m)} u_i(m, a, t) \right\}.$$

Second step: *Given the set* $T^{**}(m)$*, if the equilibrium-payoff is dominated by any type in* $t \in T^{**}(m)$*, and so*

$$\min_{a \in A^*(T^{**}(m),m)} u_i(m, a, t) > u_i^*(t),$$

then the equilibrium does not survive the intuitive criterion.

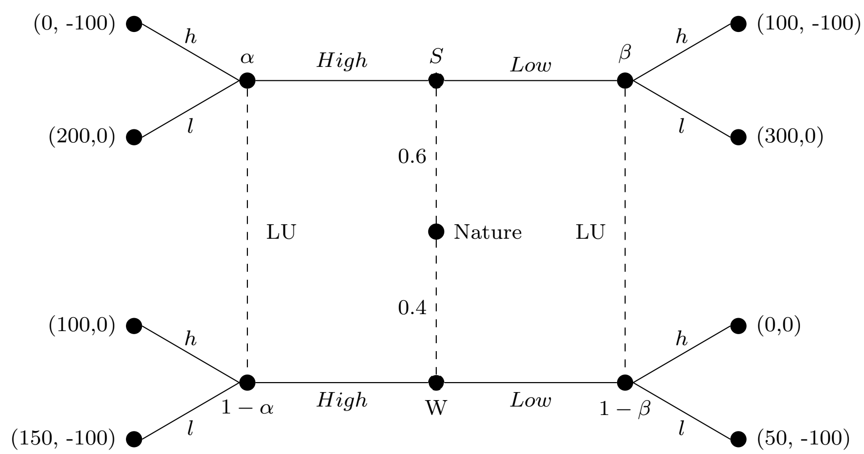


Figure 1: ([4], p.5) S=strong monetary authority, W=weak monetary authority, LU=labor union, high=high inflation, low=low inflation, l=low wage growth and h=high wage growth.

Let us have a look at an example also known as the 'beer-quiche' game. In our particular example, it is a game between a monetary authority and a labor union (this example is from [4]). The monetary authority is either strong or weak; meaning it will either have a strong commitment to keeping inflation levels low or it won't. The type of the monetary authority is decided by 'Nature' and is only observed by the monetary authority. The probability that the monetary authority is of the strong type is 0.6 and the probability that the monetary authority is of the weak type is 0.4. Knowing its type, the monetary authority announces either low or high inflation. After observing this 'signal', the labor union will demand either high or low wage growth. Though the labor union does not know the type of the monetary authority, it does know the probabilities of the weak and strong type and has observed the signal sent by the monetary authority. This is a signaling game of incomplete information. The extensive form is shown in figure 1. This game has two perfect Bayesian equilibria:

1. A pooling equilibrium. Both types of monetary authority will announce high inflation in this equilibrium. The labor union reacts to the announcement of high inflation by announcing low wages, taking into account consistency of beliefs (meaning that α will be equal to 0.6). Because the expected utility of high wage growth is smaller than the expected utility of low wage growth,$-60 < -40$.

2. A separating equilibrium. The strong monetary authority announces low inflation and the weak monetary authority announces high inflation. Therefore, by consistency of beliefs, α will be equal to zero and β will be equal to one. The labor union will respond by demanding low wage growth if it observes a message of low inflation. When observing an announcement of high inflation, the labor union will demand high wage growth.

In the first perfect Bayesian equilibrium, the strong and weak monetary authority both announce high inflation. This pooling equilibrium does not seem rational, because the strong monetary authority can obviously gain more payoff by announcing low inflation. Let us have a look at whether or not the pooling equilibrium will survive the intuitive criterion.

Step 1: Which monetary authority type(s) can gain more payoff from an off-the-equilibrium message of low inflation? We use the abbreviation MA to denote monetary authority and we use LU to denote labor union.

For the strong type of monetary authority, the following is true:

$$\max_{a\in(l,h)} u_{MA}(low, a, strong) = \max\{300, 100\}$$

$$= 300 > u^*_{MA}(High, a^* = low, strong) = 200.$$

For the weak monetary authority, the following holds:

$$\max_{a\in(l,h)} u_{MA}(low, a, weak) = \max\{0, 50\}$$

$$= 50 < u^*_{MA}(High, a^* = low, strong) = 150.$$

This means that only the strong type of monetary authority can gain more payoff from a deviation from the equilibrium message; $T^{**}(Low) = Strong$. The labor union will be certain that a message of low inflation can only be made by the strong monetary authority, because this is the only type that can profit from a deviation from the off-the-equilibrium message. This means that the labor union will have the beliefs that α is equal to zero and β is equal to one.

Step 2: Given $T^{**}(Low) = Strong$ and the beliefs that α is equal to zero and β is equal to one, the labor union will demand low wage growth when observing a low inflation message.

$$\min_{a \in A^*(strong,low)} u_{MA}(low, low, strong) = 300 > 200 = u^*_{MA}(high, low, strong).$$

The strong type monetary authority can improve its payoff by deviating from announcing high inflation, because $300 > 200$. This means that the pooling perfect Bayesian equilibrium does not survive the intuitive criterion. The separating perfect Bayesian equilibrium will survive the intuitive criterion, because there is no off-the-equilibrium message. In this equilibrium both possible message, low or high inflation, is used by a type of the monetary authority [4].

However, the intuitive criterion does not always restrict the set of equilibria when the player who sends the signal can be one of two or more types [4]. A further refinement (which is used in the article of Callander and Wilkie (2006)) to restrict the number of equilibria is the universal divinity criterion. This criterion can reduce the number of equilibria when the intuitive criterion does not.

2.1.1 Refinement of the intuitive criterion: universal divinity

The difference between the intuitive criterion and universal divinity criterion is in the first step. The universal divinity criterion only looks at the types of T who can gain the most from an off-the-equilibrium message. If various types can gain a higher pay-off from an off-the-equilibrium message m, all these types will be in $T^{**}(m)$ in the intuitive criterion. However, only the types that gain the most from the off-the-equilibrium message m will be in $T^{**}(m)$ if you follow the universal divinity criterion. The second step of the universal-divinity criterion is equal to the second step of the intuitive criterion and this step therefore needs no explaining.

2.2 Median voter theorem

According to the median voter theorem, the candidate that wins in an election, where the majority of the votes determine the winner, is the candidate most preferred by the median voter. The assumptions needed for the Median voter theorem to hold are ([2] and [5]) :

- Preferences of the voters are single-peaked. The voter prefers one policy above all others, and the further a given policy is removed from his particular preference, the more he dislikes it.

- There are only two candidates running for office.

- The policy space has one dimension. This means that only one subject is considered in the election and candidates make only campaign announcements about this subject.

The above applies in a situation of complete information. However, our signaling game is one of incomplete information. Later on it will be made clear that the median voter still plays an important role, although the most preferred candidate of the median voter may not win.

3 The Model

To be able to transform the election game into a game theoretic model, we assume that there are only two political candidates to choose from; candidate A and candidate B. They both make their campaign announcements, p_A respectively p_B, at the same time. Only one subject is considered in the election, therefore our policy space is one-dimensional and can be viewed as an interval on the real line. We let P denote our policy space with $P = [-D, D] \subseteq \mathbb{R}$. Where 0 is the midpoint of the policy space and candidates make their campaign announcements and have their implemented policies in the interval $P = [-D, D]$. Candidate A makes an announcement in the interval $p_A \in [0, D]$ and candidate B does so in the interval $p_B \in [-D, 0]$. The fact that a candidate can only make an policy announcement on one side of the interval of the policy space P can be interpreted as this politician being more on the right (respectively left) side of the political spectrum equivalently, because a candidate belongs to a specific party, he or she would never make a policy announcement outside $[0, D]$ (respectively $[-D, 0]$). Both candidates know beforehand which policy they will implement if they are elected. This is the policy intention of the candidate. The policy intention of candidate A is α and the policy intentions of candidate B is β. We assume that these policy intentions (α and β) are independent and $\alpha \in [0, D]$ and also that $\beta \in [-D, 0]$. The variable α has cdf $F(x)$ and probability density $f(x)$, ($f(x) > 0$ for $x \in [0, D]$). For β it is symmetric to α for $x \in [-D, 0]$ the cdf is $F(-x)$ and the density function is $f(-x)$, with $f(-x) > 0$. The cumulative distribution F is common knowledge for both the voters and candidates.

Besides a policy intention, a candidate incurs costs for differences between his campaign announcement and the policy intention (lying in the campaign). Therefore we introduce the parameter k as our cost parameter. The parameter k shows how much a candidate 'suffers' (this can be from both external and internal factors such as his own moral) from lying in the campaign. If $k = 0$, a candidate can say whatever he wants without having to face consequences for lying, the candidate is a zero-cost type. We will call a candidate with a zero-cost type a cheap talker. We assume that with probability q a candidate is a zero-cost type ($k = 0$) and with probability $1 - q$ a candidate is a high-cost type with ($k = K$). In addition, the probability q is known to everyone.

We can now see that our candidates have two-dimensional types consisting of the policy intention and the cost parameter. The type of candidate A is (α, k_A). The type (or characteristics) of a candidate is only known by the candidate himself and not known by the other candidate or the voters.

Now we will look at how to measure the utility of the candidates. If a candidate loses the election, he will have a utility of zero. If candidate A wins, the utility he derives from winning is (for candidate B use β and p_B):

$$\psi(\alpha, k, p_A) = y - k \cdot (\alpha - p_A)^2, \text{with } y > 0.$$

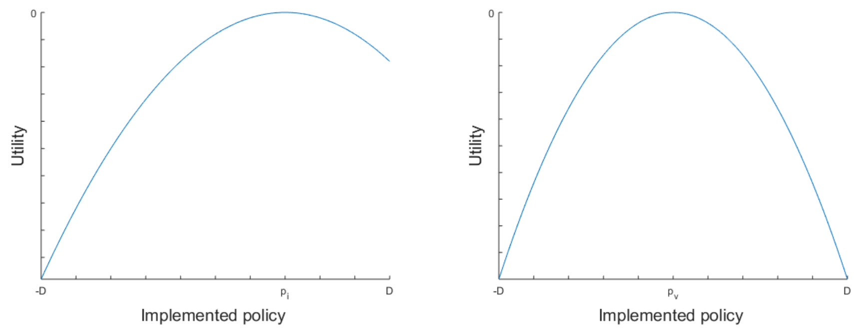
If a candidate wins he derives a positive utility y from winning and negative utility $k \cdot (\alpha - p_A)^2$ from lying. The net payoff in this case is equal to $y - k \cdot (\alpha - p_A)^2$. If a candidate loses, the difference between the announcement and the policy intention will not be known. Therefore, there is no cost for lying and no positive utility from winning the election; it follows that for a losing candidate utility is indeed always zero. We assume that making any kind of announcement and winning as a result of that announcement is always better than losing, so $\psi \geq 0$.

Furthermore, it can be seen from the utility formula for candidates that utility is not affected by differences between campaign announcements and policy intentions, and only depends on $y > 0$ and winning for a zero-cost candidate. For every announcement the utility for a zero-cost candidate is, given he wins the election:

$$\psi(\alpha, k, p_A) = \psi(\alpha, 0, p_A) = y. \tag{1}$$

Further, we assume that we have an finite odd number of voters; we will call this set of voters N, with $N = \{1, 2, \cdots, n\}$. Every voter is obligated to vote; it is not possible to abstain from voting. How 'happy' a voter is with the final implemented policy of the winning candidate is measured in utility. The utility of voter i if candidate A wins is $u_i = -(\alpha - p_i)^2$, where p_i is the best policy according to voter i and $\alpha \in [0, D]$ is the implemented policy (if candidate B wins the election replace α in the utility formula by $\beta \in [-D, 0]$). The consequence of the quadratic formula for the utility is, that it does not matter whether a deviation is to the right or to the left of p_i, deviations of the same size have the same effect on the utility of a voter. Furthermore, it follows from the utility formula that every implemented policy other than p_i implies a negative utility. The utility of the voter has a maximum value of 0 and it only depends on the implemented policy; it does not depend on the promises in the campaign.

We let $v \in N$ be the median voter and we assume that $p_v = 0$. The ideal policy, according to the median voter is precisely in the middle of all possible policies. See figure 2.



(a) Voter i

(b) Median voter

Figure 2: Utility of voter i (a) and the median voter (b) for all possible different implemented policies.

From figure 2 it can clearly be seen that the voters' preferences are single-peaked. The winner of the election will be the candidate with the most votes.

We have two candidates and n voters; the voters do not know what the policy intention and the cost of lying is for either candidate. We assume that everyone behaves rationally and every voter has access to the same information. In extensive form this is a game where Nature makes the first step and decides if candidate A is of high- or zero-cost type and what the implemented policy will be (same for B). Then the candidates (knowing their own type) can choose a policy platform to announce; the options here are infinite. Observing this policy announcement the voter can try to deduce what he thinks the type of the candidate is and then decide whether to vote for candidate A or B.

3.1 Voter strategies

A voter wants to maximize his utility, so he will vote for the candidate who will maximize his expected utility. Expected utility is used because a voter does not know the implemented policy beforehand. Let $\mu_A(\alpha|p_A)$ be the probability of the implemented policy being equal to α, given that A makes campaign announcement p_A. We assume that all voters have the same beliefs about the distribution of μ_A. The expected utility for voter i if A wins the election is, observing announcement p_A and given beliefs μ_A :

$$E_A[u_i(\alpha)] = \int_{\alpha\in[0,D]} -(\alpha - p_i)^2 \mu(\alpha|p_A)\mathrm{d}\alpha$$

$$= \int_{\alpha\in[0,D]} -(\alpha^2 - 2\alpha p_i + p_i^2)\mu_A \mathrm{d}\alpha$$

$$= -p_i^2 - E(\alpha^2|p_A) + 2p_i E(\alpha|p_A)$$

$$= -p_i^2 - \mathrm{Var}(\alpha|p_A) - [E(\alpha|p_A)]^2 + 2p_i E(\alpha|p_A)$$
$$= -p_i^2 - \mathrm{var}(\alpha) - \bar{\alpha}^2 + 2p_i\bar{\alpha} = -(\bar{\alpha} - p_i)^2 - \mathrm{var}(\alpha).$$

In the preceding formula, $E(\alpha|p_A) = \bar{\alpha}$ is equal to the mean of μ_A and $\mathrm{Var}(\alpha|p_A) = \mathrm{var}(\alpha)$ is equal to the variance of μ_A. Similarly, we can show that the expected utility for voter i if candidate B wins the election equals $E_B[u_i(\beta)] = -(\bar{\beta} - p_i)^2 - \mathrm{var}(\beta)$, with conditional probability $\mu_B = \mu_B(\beta|p_B)$, mean $\bar{\beta}$ and variance $\mathrm{var}(\beta)$.

Based on this expected utility a voter can decide on a strategy. Voter i will vote for the candidate he prefers, given that he has a strict preference. We further assume that the voter will use a mixed strategy and vote for candidate A with probability $\frac{1}{2}$ and for B with probability $\frac{1}{2}$ if a voter is indifferent between the two candidates or does not strictly prefer one over the other. This means that the strategy of voter i is the function ([1], p.311):

$$r_i : [0, D] \times [-D, 0] \to \left\{0, \frac{1}{2}, 1\right\}.$$

Let $r_i(p_A, p_B)$ be the chance that voter i votes for candidate A and, because we made the assumption that abstaining from voting is not allowed, it follows that $1 - r_i(p_A, p_B)$ is the probability that voter i votes for candidate B.

The median voter plays an important role, because he decides the election. If the median voter prefers candidate A, then a voter with $p_i > 0$ also will prefer A. If the median voter v prefers candidate A then $-\bar{\beta}^2 - \mathrm{var}(\beta) < -\bar{\alpha}^2 - \mathrm{var}(\alpha)$. For $p_i > 0$ (remember $\beta \leq 0$ and $\alpha \geq 0$), we get the following inequalities:

$$-(\bar{\beta} - p_i)^2 - \mathrm{var}(\beta) < -\bar{\beta}^2 - \mathrm{var}(\beta) < -\bar{\alpha}^2 - \mathrm{var}(\alpha) < -(\bar{\alpha} - p_i)^2 - \mathrm{var}(\alpha).$$

This means that voter i with $p_i > 0$ will prefer candidate A if the median voter prefers candidate A, and so A will win in this case. This is because there are n voters and at least $\frac{1}{2}(n-1) + 1$ will vote for A, meaning that candidte A will win. If the median voter prefers candidate B, then so will all voters with $p_i < 0$, meaning that candidate B will win. If the median voter is indifferent between A and B, then $-\bar{\beta}^2 - \mathrm{var}(\beta) = -\bar{\alpha}^2 - \mathrm{var}(\alpha)$. Every voter with $p_i > 0$ will vote for A and every voter with $p_i < 0$ will vote for B. In this case, the median voter decides the election and $r_v(p_A, p_B)$ is the probability of candidate A winning the election.

3.2 Political candidates strategy

It was already assumed that during the campaign candidate A will make a campaign announcement $p_A \in [0, D]$ and candidate B will make a campaign announcement $p_B \in [-D, 0]$. We will denote a mixed strategy of candidate A with $\sigma_A(\alpha, k)$. Given policy intention α and cost variable k, σ_A gives a probability distribution over possible campaign announcements A can make; we call this a strategy. We let $\sigma(p|\alpha, k)$ be the probability that A announces campaign platform p and we let the set $s_A(\alpha, k) \subseteq [0, D]$ be the possible campaign announcements, so $p \in s_A(\alpha, k)$. The preceding can be summed up in the following function:

$$\sigma_A : [0, D] \times \{0, K\} \rightarrow \sigma(p|\alpha, k) \text{ with } p \in s_A(\alpha, k).$$

σ_A denotes the function that maps candidate A's characteristics to a probability distribution over possible campaign announcements for A. In the same vein, $\sigma_B(p|\beta, k)$ equals the chance that candidate B announces policy platform p given policy intention β and cost variable k with $p \in s_B(\beta, k)$. We assume that the strategies of candidate A and B are symmetric with respect to each other and the origin; this means that $\sigma_A(p|\alpha, k) = \sigma_B(-p|-\alpha, k)$ for $\alpha \in [0, D]$.

Like the voter, the political candidate also wants to maximize his utility. A candidate can only have a utility greater than 0 if he wins. Let $\lambda(p_A, p_B)$ be the chance that A wins the election, with p_A and p_B being the policy announcements made by the candidates. It follows that $\lambda(\sigma_A, \sigma_B)$ is the probability A wins the election, when A plays mixed strategy σ_A and B plays mixed strategy σ_B. The expected utility candidate A gains from making announcement p will be the chance of wining the election times the utility gained from winning the election given announcement p:

$$\lambda(p, \sigma_B)\psi(\alpha, k, p).$$

Similarly, the expected utility of candidate B is $(1 - \lambda(\sigma_A, p))\psi(\beta, k, p)$ with $p \in [-D, 0]$. In any equilibrium, a candidate will only use a strategy that maximizes his own expected utility. We will give the condition for candidate A in equilibrium, the equilibrium condition for candidate B is constructed in a similar manner. Given a strategy played by candidate A in equilibrium for all $p \in s_A(\alpha, k)$, it follows that:

$$\lambda(p, \sigma_B)\psi(\alpha, k, p) \geq \lambda(p', \sigma_B)\psi(\alpha, k, p'), \forall p' \in P.$$

For a zero-cost type candidate the utility gained does not depend on the type of announcement made and instead only depends on winning or losing the election, see equation 1. The expected utility of a zero-cost candidate will be maximized by announcing a policy platform that maximizes the chances of winning the election. The equilibrium condition will become, for all $p \in s_A(\alpha, 0)$:

$$\lambda(p, \sigma_B) \geq \lambda(p', \sigma_B), \forall p' \in P.$$

This means that in an election equilibrium a zero-cost type candidate will only make an announcement if it maximizes his chance of winning. This gives rise to proposition 1; see below. Because the equilibrium condition only depends on the probability of winning the election, a zero-cost type candidate will play the same strategy in an equilibrium, independent of the policy intention in $[0, D]$. The same conditions apply to a zero-cost type candidate B.

Proposition 1 ([3], p.269)**.** *In an equilibrium, strategies of a zero-cost candidate A will be* $s_A(\alpha, 0) \subseteq \arg\max_{p_A \in P}[\lambda(p_A, \sigma_B)]$. *The strategies of zero-cost candidate B will be* $s_B(\beta, 0) \subseteq \arg\max_{p_B \in P}[\lambda(\sigma_A, p_B)]$.

This means that $s_A(\alpha, 0)$ is the same for every α. Therefore, we will denote the possible policy platforms that can be announced in equilibrium by a zero-cost candidate A as s_A^0 and for a zero-cost candidate B as s_B^0.

4 Electoral equilibria

We want to know what the 'actions' of political candidates and voters will be in an electoral equilibrium. If everyone gives a best reply to the others' best reply of actions. What is special about the article by Callander and Wilkie (2006) is the possibility that a candidate is either high-cost or zero-cost. We will first look at what happens in an equilibrium when all the political candidates have the zero-cost type (so $k = 0$ and $q = 1$). Which strategy will the candidate choose in this case? Following this we will have a look at what happens in an election with only high-cost type candidates (so $k = K$ and $q = 0$). Finally, we will have a look at one of the more interesting cases, namely what happens if there are both zero-cost and high-cost candidates (so $q \in (0, 1)$). The election equilibra we are going to look at now are all universally-divine equilibria. The proofs that these equilibria are indeed universally-divine will not be treated here, for proofs see ([1] and [3]). Furthermore, because we assumed that the strategies are symmetric with respect to the origin and with respect to the candidates, we only have to define the electoral equilibria for one of the candidates and then we will also know the equilibria for the other candidate.

In our case, the universal divinity criterion means that a voter only has positive beliefs that a candidate will be of type (α, k) given observed announcement p_A, so $\mu(\alpha|p_A) > 0$ if type (α, k) is among the types that are most likely to defect to the off-the-equilibrium message p_A.

4.1 Zero-cost candidates

If q is equal to one then all our candidates are zero-cost types. The distribution of q is known to the candidates but also to the voters. The voters therefore know that the promises made by the candidates during the campaign are meaningless, because of the fact that candidates do not incur any costs for lying. In the preceding, we have seen that the chances of winning for a candidate are the same as the chances that the median voter votes for this candidate. The median voter wants an implemented policy as close to zero as possible. Since campaign promises are meaningless, the median voter cannot form a belief about the true policy intention given the observed policy announcement. So every strategy $\sigma_A(\alpha, 0)$ for candidate A and every strategy $\sigma_B(\beta, 0)$ for candidate B are equally good. This means that every strategy of a candidate is a 'good' strategy and can be played in equilibrium, see figure 3. It can be seen that candidate A can make any campaign announcement p_A in the interval $[0, D]$ independent of the policy intention α in equilibrium.

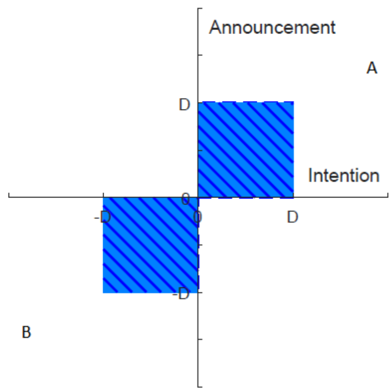


Figure 3: Campaign announcements in a universally-divine equilibrium in an election consisting of only zero-cost candidates. For candidate A and B.

4.2 High-cost candidates

When there are only high-cost type candidates in the election, q is equal to zero. Let K^* be the cost such that the most extreme candidates on the political spectrum are indifferent between either making a campaign announcement equal to zero, or losing the election. This is equivalent to the formula $\psi(D, K^*, 0) = 0$, with $K^* = \frac{y}{D^2}$. The next proposition shows that the true intention of more extreme candidates will be known to the voters if the cost of lying for a candidate is sufficiently high ($K > K^*$).

Proposition 2 ([3], p.271)**.** *If $q = 0$ and $K > K^*$, then the universally-divine equilibrium will take the following form:*

(i) $\forall \alpha \in [0, \alpha(K)], s_A(\alpha, K) = 0,$

(ii) $\forall \alpha \in (\alpha(K), D], s_A(\alpha, K)$ *is strictly increasing, and therefore separating, with* $\alpha(K) \in (0, D)$.

Proposition 2 shows that candidates with policy intention around the median (zero) pool at announcement zero and that the true policy intention for candidates with a policy intention more extreme than $\alpha(K)$ is revealed. These are the equilibrium strategies for candidate A; because of symmetries one can deduce the strategies for candidate B from the strategies of candidate A (see also figure 4).

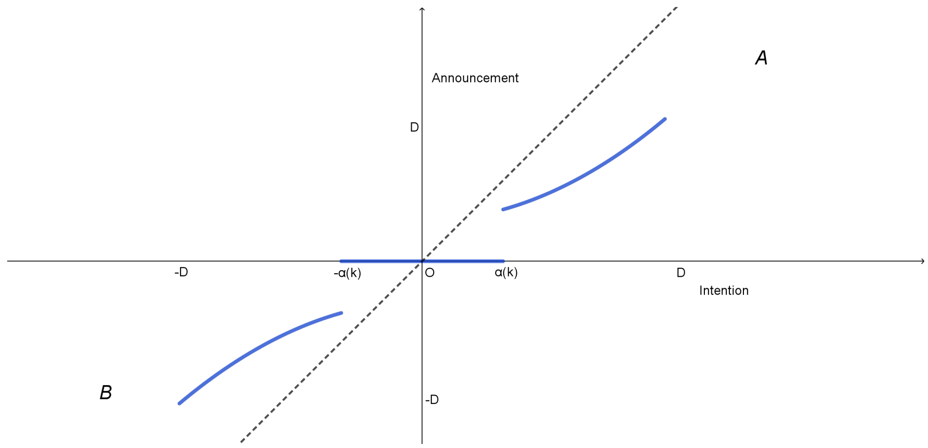


Figure 4: ([3], p.273) Policy announcements in a universally-divine equilibrium in an election consisting of only high-cost candidates ($K > K^*$), for candidate A and B.

The median voter will vote for the candidate that announces a policy platform zero. If a candidate announces policy platform zero the median voter knows that the true policy is in the interval $[-\alpha(K), \alpha(K)]$. For a campaign announcement other than zero the median voter knows the true policy intention of the candidates making these announcements. Of these candidates, the one with a policy intention slightly higher than $\alpha(K)$ is the closest to the preference of the median voter $p_v = 0$. The utility of the median voter of an implemented policy in the interval $[-\alpha(K), \alpha(K)]$ is higher than the utility gained from a policy implemented outside this interval. Therefore, the median voter will vote for the candidate with campaign promise zero, and a candidate with campaign promise zero will win the election. This is despite the fact that the campaign promise of zero gives uncertainty about the real policy intention, while a campaign promise other than zero gives assurance about the prospective implemented policy.

If both candidates announce zero in the campaign then the median voter is indifferent between both candidates and $r_v = \frac{1}{2}$, and therefore $\lambda(p_A, p_B) = \lambda(0, 0) = \frac{1}{2}$.

A candidate with a more extreme policy intention than $\alpha(K)$ or $-\alpha(K)$ will only win if the other candidate has an even more extreme policy intention. Of these candidates, the one with the least extreme policy will win for certain, because of the separating equilibrium (the median voter knows for both types the exact policy intention from there announced campaign platforms).

The separating type $\alpha(K)$ is the policy intention for which a candidate with cost of lying K is indifferent between announcing a median platform zero and announcing $p \in s(\alpha(K), K)$. For candidate A the separating type satisfies:

$$\lambda(p, \sigma_B)\psi(\alpha(K), K, p) = \lambda(0, \sigma_B)\psi(\alpha(K), K, 0);$$

for candidate B (with $p \in s(-\alpha(K), K)$) this equation is written as

$$\lambda(\sigma_A, p)\psi(-\alpha(K), K, p) = \lambda(\sigma_A, 0)\psi(-\alpha(K), K, 0).$$

Because we assumed symmetry of strategies, the following is true: $|\alpha(K)| = |-\alpha(K)|$.

4.2.1 'Low' high-cost

What if $K < K^*$ (and $q = 0$)? Even candidates with the most extreme policy intentions D and $-D$ will gain a positive utility from getting elected when announcing the median policy zero, $\psi(D, K, 0) > \psi(D, K^*, 0) = 0$. The cost K is sufficiently low such that even the candidates with extreme policy intentions will announce policy platform zero rather than losing the election. In this case, all candidates will make the same policy platform announcement of zero during the campaign, independent of their policy intentions. The preceding gives rise to proposition 3.

Proposition 3 ([3], p.271)**.** *If $q = 0$ and $K < K^*$, then the universally-divine equilibrium will take the following form:*

$$s_A(\alpha, K) = s_B(\beta, K) = 0, \forall \beta, \alpha.$$

Therefore, this universal divine equilibrium is a pooling equilibrium. Voters do not gain any information from the policy announcements made during campaign (see also figure 5). A candidate with an extreme policy intention could increase his utility by announcing a policy platform $p_A > 0$, respectively $p_B < 0$, but then the median voter would think that this announcement must come from the most extreme policy intention type of candidate ([3], p.271). This candidate type stands to gain the most from deviating to an announcement p other than zero and is the 'most likely' to defect. The beliefs of the median voter imply that he would assume that the announcement will have been made by a candidate with the most extreme policy intention, therefore the median voter will not vote for a candidate that made an announcement other than zero. This means that also candidates with the most extreme policy intentions, D or$-D$, will announce a policy platform of zero during the campaign. The probability that candidate A wins the election is exactly the same as the probability that B wins the election, because everyone announces a policy platform of zero. The median voter is indifferent between the announcements made by the candidates, so $r_v(0, 0) = \lambda(0, 0) = \frac{1}{2}$.

Therefore, in an election with only high-cost candidates with $K < K^*$, every candidate has the same chances of winning the election independent of the policy intention of that particular candidate.

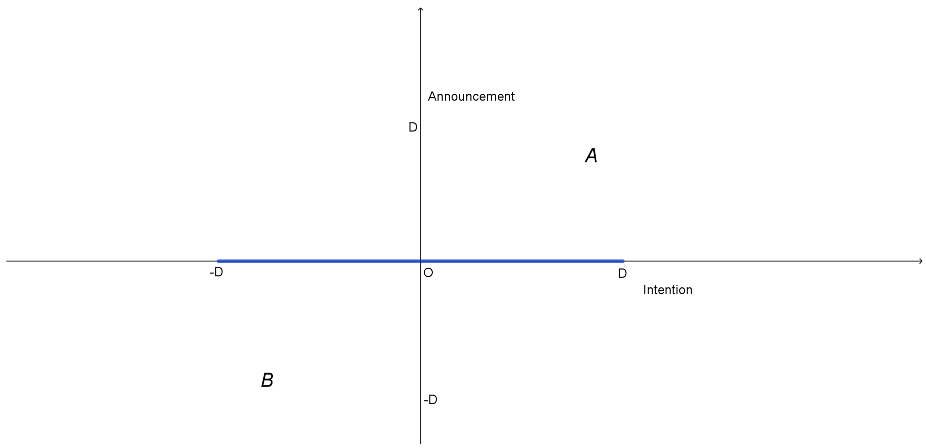


Figure 5: ([3], p.272) Policy announcements made by candidate A and B in a universally-divine equilibrium consisting of only high-cost candidates, with $K < K^*$.

4.3 High- and zero-cost

What happens when $q \in (0, 1)$ and there are high-cost as well as zero-cost candidates present in the election? Let us imagine that we start with an election with only high-cost candidates with cost parameter ($K > K^*$) and $q = 0$. Iif the median voter observes an announcement of zero in the campaign, then the median voter knows the policy intention of these candidates is in the interval $[-\alpha(K), \alpha(K)]$. Let us now enter a zero-cost candidate into the election, so $q > 0$. From proposition 1 it follows that the zero-cost type candidate who entered the election will make an announcement that maximizes his chances of winning and this is a campaign announcement of policy platform zero. A vote by the median voter for a candidate with announcement zero no longer guarantees that the implemented policy is in the interval $[-\alpha(K), \alpha(K)]$. The probability that a policy announcement of zero will be made by candidate A is the probability that a candidate is high-cost and the policy intention is in the interval $[0, \alpha(K)]$, plus the probability that the candidate is zero-cost. Therefore, the probability of a policy announcement zero by candidate A is equal to $(1 - q)F(\alpha(K)) + q$. The probability that candidate A with campaign announcement zero is high-cost and thus will have a policy intention in the interval $[0, \alpha(K)]$ is:

$$\frac{(1 - q)F(\alpha(K))}{(1 - q)F(\alpha(K)) + q}.$$

The probability that, given the campaign announcement of zero, candidate A is zero-cost and thus will have a policy intention in $[0, D]$ is:

$$\frac{q}{(1 - q)F(\alpha(K)) + q}.$$

These probabilities are the same for candidate B with policy intentions in $[-D, 0]$. A vote for a candidate with announcement zero now incurs a risk that the policy intention may not be in the interval $[0, \alpha(K)]$, but somewhere in the interval $[0, D]$. For the median voter, a policy intention somewhere in the interval $[0, D]$ can give much a smaller utility than the certain utility of the first separating type $\alpha(K)$. If q is below a given threshold value, the probability of a policy intention in the interval $[0, D]$, given announcement zero, is small. In this case, the median voter will still vote for a candidate with campaign announcement of zero. If q becomes larger there will be a turning point where the median voter (election decider) prefers a certain policy intention of $\alpha(K)$ to a chance that there will be a policy intention closer to zero. The risk that a vote for a campaign announcement of zero is a vote for a zero-cost type candidate, who possibly has an extreme policy intention, becomes too large.

The certain outcome the median voter prefers over a gamble for announcement zero depends on q. However, when the median voter prefers the separating type over announcement zero, then zero-cost types will no longer announce platform zero. Otherwise, they do not maximize their chances of winning and behave according to proposition 1.

What will the behavior be of high- and zero-cost candidates in a universally-divine equilibrium? Proposition 4 describes the behavior for candidate A given that the candidate is high-cost. The behavior for candidate B is defined symmetrically (see also figure 6).

Proposition 4 ([3], p.275). *Let $q \in (0, 1)$, then in every universally-divine equilibrium there exists an α' and an α'' such that for high-cost candidates the universally-divine equilibrium will take the following form:*

(i) $\forall \alpha \in [0, \alpha'), s_A(\alpha, K) = \alpha,$

(ii) $\forall \alpha \in [\alpha', \alpha''], s_A(\alpha, K) = \alpha',$

(iii) $\forall \alpha \in (\alpha'', D]$, *with* $\frac{\mathrm{d}s_A(\alpha,K)}{\mathrm{d}\alpha} > 0$, *so* $s_A(\alpha, K)$ *is strictly increasing on* $(\alpha'', D]$.

This proposition shows that high-cost candidates with policy intention around the zero value will truthfully announce their policy intention; the campaign announcement is the real policy intention. In an interval $[\alpha', \alpha'']$ which lies away from zero there is a pool of high-cost candidates. Candidates with the most extreme policy intentions will be separated in equilibrium. See figure 6.

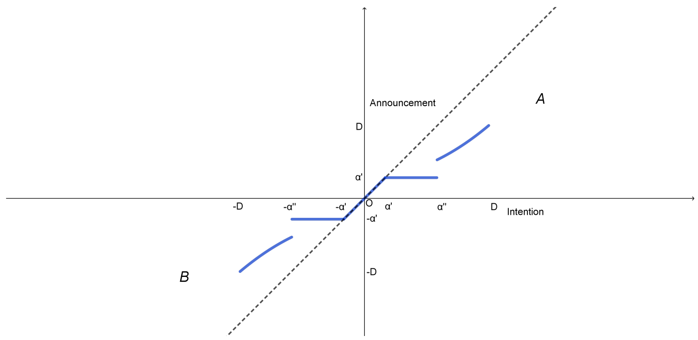


Figure 6: ([3], p.275) Policy announcements made by high-cost type candidates in universally-divine equilibrium of election with $q > 0$.

Now that we know what high-cost candidates will do in equilibrium, we will take a look at the behavior of zero-cost candidates.

Lemma 5 ([3], p.276)**.** *Let* $q \in (0, 1)$ *and* $K > K^*$*, then in every universally-divine equilibrium* $[0, \alpha') \subseteq s_A(\alpha, 0) \subseteq [0, \alpha']$.

Lemma 5 states that zero-cost candidates will announce a campaign platform which imitates the behavior of the high-cost candidates with centrist policy intention in addition to sometimes imitating a high-cost candidate with intentions $[\alpha', \alpha'']$, the behavior of candidate B is defined symmetrically, see also figure 7. The zero-cost types are distributed over the interval $[0, \alpha')$ or over the interval $[0, \alpha']$, following a particular distribution ([3], p. 276). The strategies given by lemma 5 will only exist in equilibrium if the chance of winning for each candidate is the same for every announcement in the interval $[-\alpha', \alpha']$. Otherwise, the strategies do not satisfy proposition 1. Therefore, the median voter must be indifferent between every announcement in the interval $[-\alpha', \alpha']$.

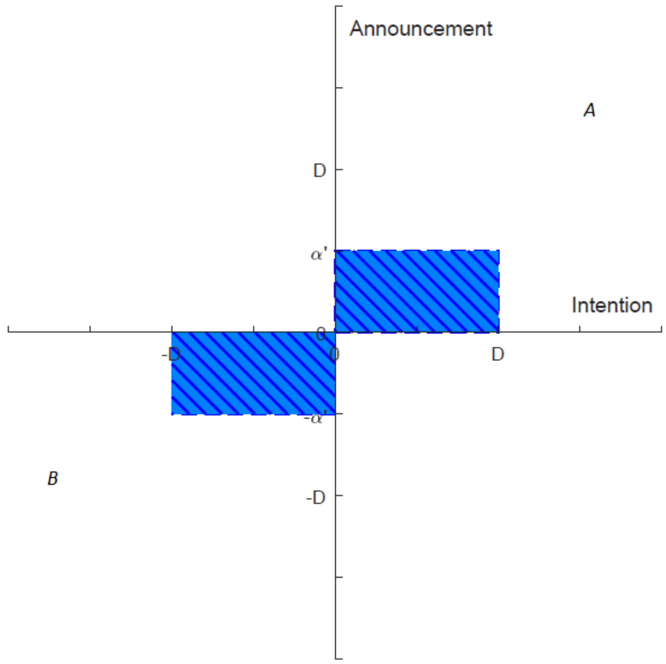


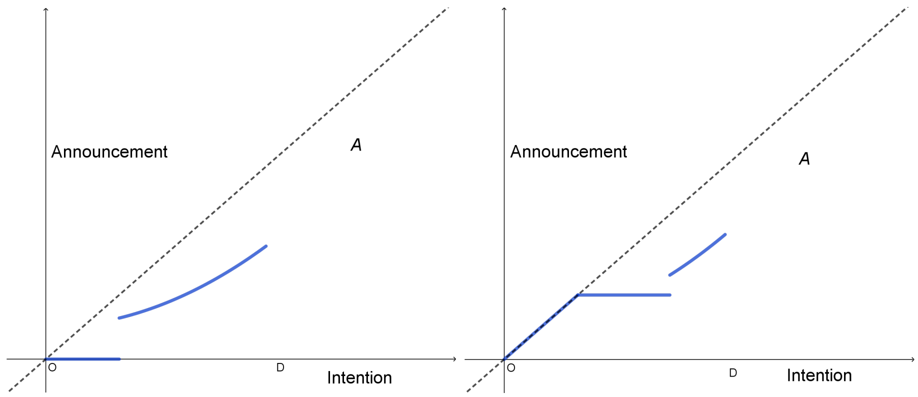
Figure 7: Policy announcements made by zero-cost type candidates A and B in a universally-divine equilibrium in an election with $q > 0$.

It can be shown that for every election with $q \in (0, 1)$ that there exists a universally-divine equilibrium [3]. The behavior of high-cost candidates, with $K > K^*$, will be as described in figure 6 and the behavior of zero-cost candidates will be as described in figure 7, given that the high-cost candidates have cost parameter $K > K^*$.

5 Discussion

5.1 Election with High-cost types vs. election with high- and zero-cost types.

The most striking result of the paper by Callander and Wilkie (2006) is that the honesty of high-cost types increases if you start with an election consisting of only high-cost types (with $K > K^*$) and then add some zero-cost types. Therefore, adding zero-cost types to an election makes the high-cost candidates (if $K > K^*$) more true to their words. See figure 8; there is no pooling around zero in (b), so the centrist candidates will announce their policy intention truthfully.



(a) (b)

Figure 8: Election with only high-cost type candidates $q = 0$ and $K > K^*$ (a) vs. election with both high- and zero-cost type candidates ($q > 0$) (b).

High-cost candidates with the most extreme policy intention will behave the same.

5.2 What if $q \to 1$?

If q approaches one, the election becomes an election consistent of almost only zero-cost type candidates. Despite the fact that this almost resembles the situation which arises when q equals one, where every candidate is of the zero-cost type, there is a big difference between them ([3], p.278). In the case of $q \to 1$, the equilibrium will take the form of figure 6 for the high-cost type candidate and the true policy intention of centrist candidates will be known to the voters. Voters will also know the true policy intention of the high-cost candidates with the more extreme policy intentions in this case. The zero-cost candidates will make their announcements around the center and the pooling announcements α' and $-\alpha'$. If $q \to 1$ the probability that a campaign announcement both has any value and is meaningful is low, because voters almost know for certain that the campaign announcement will have been made by a zero-cost candidate. However, the small chance that a campaign announcement both matters and has meaning determines the behavior of high- and zero-cost type candidates in equilibrium; this makes a big difference. While in the case of $q = 1$ the zero-cost type candidates can make any announcement in equilibrium, the equilibrium behavior in the case $q \to 1$ is restricted. The presence of high-cost type candidate alters the behavior of the zero-cost type candidates.

5.3 Median voter

The median voter theorem states that the candidate most preferred by the median voter will be the winner of the election, if our game is a game of perfect information. However, this is not the case in this thesis. We have seen that under our assumptions the median voter still plays an important role. The median voter ultimately decides the election. Therefore, the political candidates try to make their announcements more appealing to the median voter, in order to increase their chances of winning.

5.4 Winner

Do liars win an election more often than those candidates that tell the truth (do zero-cost type candidates win more often than high-cost type candidates)? The equilibrium results show that it depends on the policy intention of the high-cost type candidate. The median voter will have a preference to vote for a candidate with a campaign announcement in the interval $[-\alpha', \alpha']$. If a high-cost candidate has a policy intention in the interval $[0, \alpha'')$ or in the interval $(-\alpha'', 0]$, the candidate will make an announcement in the interval $[0, \alpha']$, respectively $[-\alpha', 0]$. In this case, the high-cost type candidate has the same chance of winning the election as a zero-cost type candidate. This is because of the fact that a zero-cost type candidate will always make a policy announcement in the interval $[0, \alpha']$, respectively $[-\alpha', 0]$. If both candidates A and B make an announcement anywhere in the interval$[-\alpha', \alpha']$ then the median voter is indifferent between either candidate and, $r_V(p_A, p_B) = \lambda(p_A, p_B) = \frac{1}{2}$. Both candidates therefore have the same chances of winning. If there is one zero-cost candidate and one high-cost candidate, the high-cost candidate has the same probability of winning the election as the zero-cost candidate. This means that high-cost type candidate with policy intentions ($\alpha < \alpha''$ and $\beta > -\alpha''$) have the same chance of winning as zero-cost type candidates.

High-cost candidates with more extreme policy intentions, ($\alpha > \alpha''$ and $\beta < -\alpha''$), will only win the election if the other candidate is also a high-cost candidate and has a more extreme policy intention. If the other candidate is a zero-cost type candidate, the zero-cost type candidate will make an announcement in the interval $[0, \alpha']$, respectively $[-\alpha', 0]$, and the median voter will vote for the zero-cost candidate. If the median voter observes two policy announcements which are both outside the region $[-\alpha', \alpha']$, then the median voter will know that the announcements will have been made by high-cost candidates and, more importantly, the median voter will then know the true policy intentions of both candidates. The median voter, who prefers a centrist implemented policy of zero, will vote for the candidate with the least extreme policy intention.

Therefore, the chance that a high-cost type candidate will win the election depends on the chance that the other candidate is zero-cost type candidate,q, and depends on α' and α''. Both α' and α'' depend on q; meaning that both α' and α'' will increase if q increases ([3], p.278). For larger q, the chance that a candidate is a zero-cost type candidate, and therefore that his promises are worthless, is high; this is also known to the voter. If q increases, a vote for a candidate with an announcement in the interval $[-\alpha', \alpha']$ has an increasing probability of being a vote for a zero-cost candidate. The election equilibrium of figure 6 will only hold if α' and α'' increase. Otherwise, there will be a turning point where the median voter will vote for the first separating type. Therefore, if the value of q changes, the values of α' and α'' will also change.

Finally we can see that zero-cost type candidates has some advantage over high-cost candidates in the election, but this advantage is limited.

6 Conclusion

Surprisingly enough, adding zero-cost types to an election increases the honesty of the entire election. This follows from the fact that adding zero-cost type candidates to an election changes the behavior of high-cost candidates in the election. In addition to this, the behavior of zero-cost candidates changes if there is a chance (however small) that another candidate is of the high-cost type. Thus campaign announcements may matter.

One may expect that better (or more willing) liars (our zero-cost candidates) win every election. However, this is not the case. If a high-cost candidate has not overly extreme policy intention, ($\alpha < \alpha''$ or $\beta < -\alpha''$), the candidate has the same chances of winning as a zero-cost candidate. A zero-cost candidate may have more chances of winning an election than an arbitrary high-cost candidate, but will certainly not win every election.

Nomenclature

α	Policy intention of candidate A
β	Policy intention of candidate B
λ	Probability candidate A wins the election
μ_j	Beliefs of voters about policy intention of candidate $j \in \{A, B\}$
ψ_j	Utility for candidate $j \in \{A, B\}$
σ_j	Mixed strategy for candidate $j \in \{A, B\}$
F	Cumulative distribution function of policy intention
f	Probability density of policy intention
K	A candidate is high-cost
k	Cost parameter
P	Policy space
p_A	Campaign announcement of candidate A
p_B	Campaign announcement of candidate B
q	Probability a candidate is zero-cost
r_i	Probability that voter i votes for candidate A
r_v	Probability that the median voter votes for candidate A
$s0_j$	Possible campaign announcements in equilibrium for candidate $j \in \{A, B\}$
s_j^0	Possible campaign announcements in equilibrium for candidate $j \in \{A, B\}$
s_j	Support of σ_j for candidate $j \in \{A, B\}$
u_i	Utility of voter i

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