A problem-posing approach to the teaching of the Special Theory of Relativity

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ABSTRACT
The Special Theory of Relativity is based on two principles: the relativity principle that says that the laws of physics must be the same in all inertial reference frames and the light postulate that says that the speed of light is constant in all inertial reference frames. From these two postulates we obtained 5 important ideas that we consider are crucial for secondary school students that want to learn the topic, in particular the two relativistic effects: time dilation and length contraction. These ideas are: Movement is not absolute, it is defined by comparing with something else; There are more than one possible reference frames; All of them are equally valid; The speed of light is defined relative to the observer; and All observers measure the same value for the speed of light. Research shows that secondary school learners tend to regard the relativistic effects as a matter of perception and they tend to consider that only one of the reference frames is right. Addressing the first 3 ideas (the ones related to the relativity postulate) might solve this problem. In this work we show how we tried to teach those ideas through a problem posing approach. Furthermore we show that after following the proposed lesson plan students no longer considered one reference frame as the right one and that, to some degree, they do not exhibit concepts of absolute motion.

KEYWORDS
Special Relativity; Problem-posing approach

1. Introduction

The Special Theory of Relativity is one of the core ideas of modern physics. It changed our understanding of space and time and caused a revolution in the paradigms of 20th century physics. According to Sanchez and Ron (1985) STR has become a requirement for any physical theory more than a simple characteristic of them. Being the STR such an important topic it is only natural that it should be part of any compulsory physics curriculum. Nevertheless the teaching and learning of the STR has been exclusive to physics in higher education. Only recently, efforts have been made towards its introduction in secondary education.

This relentlessness to teach the STR in secondary schools is understandable when one considers that even after instruction undergraduate physics students tend to use arguments that correspond more to spontaneous reasoning than to the instructed theory (Villani & Pacca, 1987), and that they seem to construct a personal framework

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in which the instructed theory coexists with spontaneous ideas (Scherr, Shaffer, & Vokos, 2001).

However there exist exemplary works for the teaching of STR in secondary education. It is worth mentioning the works of (Dimitriadi & Halkia, 2012), the contextualized approach of (Arriassecq & Greca, 2010), and the work of (Kamphorst, 2018a). In these three examples students become familiar with the core concepts of the theory through different methodologies.

Dimitriadi and Halkia (2012) make an extensive use of stories and thought experiments. Their aim was manly exploratory, and they tried to investigate what students can do and what kind of difficulties that they face when dealing with the basic ideas of the STR. They found that students can successfully deal with most of the ideas of the STR. However they also found that students tend to believe that there is an absolute reference frame, that objects have fixed properties (such as length) and that the way events happen is independent of what the observers perceive. All of this leads students to regard the relativity effects (time dilation and length contraction) as a matter of perception. Some of these results are shared by Kamphorst (2018a) whom through a problem-posing approach observed that students make similar assertions about the reality of the relativistic effects. Just as Dimitriadi's students, Kamphorst's students tended to talk in terms of real and apparent times and distances.

Arriassecq and Greca (2010), on the other hand, introduce the STR in context, paying special attention to the historical and cultural value of it. Their approach is focused on teaching the STR from a modern historical and epistemological point of view. Arriassecq and Greca also obtained positive results regarding students familiarity with the concepts of the STR. On top of that, they reported that their students did not regard the relativistic effects as apparent. According to them "It seemed the students were clear about the reality of length contraction" (Arriasecq & Greca, 2010, p.843). They achieved this by devoting some time to Aristotelian cosmology, the definition of movement, the definition of speed and to Galilean relativity. However, this translates to a much longer lesson series, to the point that it is not incorporable in a normal physics curriculum for secondary education.

From these examples, it is clear that it is indeed possible to teach the STR in secondary education, but there are still problems to be addressed. In this work we will try to address one of them, namely students conception about the reality of relativistic effects. To do this, we will take what we believe made successful the approach of Arriassecq and Greca (2010) (the emphasis on Galilean relativity and the definitions of movement and speed) and we will try to incorporate it into an existing approach for the teaching of the SRT that falls within an acceptable time frame for secondary education. We will use Kamphorst's lesson plan (2018) as our starting point.

In the following the elements that are needed for a good understanding of the STR from the point of view of physics will be briefly presented. Afterwards the characteristics of the lesson plan will be drawn from the philosophy of the problem-posing approach. With these two elements (the physics content and the educational perspective) a lesson plan will be presented. Finally the implementation of the proposed lesson plan will be discussed and the results obtained from it will be evaluated in terms of whether or not students still consider relativistic effects as apparent after having followed the lesson plan.

The two postulates of the STR for secondary education

The theory of special relativity is built upon two main ideas or postulates: the light postulate and the relativity postulate. The relativity postulate establishes that the laws of physics have the same form in all inertial reference frames; and the light postulate establishes that the speed of light has a constant value (c) in all inertial reference frames. Following both postulates carries great consequences for our conceptions of space and time. In order to preserve the constant speed of light and all the laws of electrodynamics and optics in all inertial reference frames, the length of time intervals and distances must change accordingly in each one of them. These redefinitions of time intervals and lengths are commonly known as time dilation and length contraction and are two of the most revolutionary ideas of the last century.

For secondary education the two postulates can be rephrased to a more simple working definition that allows us to reach the same shocking conclusions. The relativity postulate, for example, can be rephrased as "the laws of physics must be always the same in all reference frames". This phrasing leaves out some important information such as mentioning that the reference frames must be inertial (meaning non accelerating), and that it is actually the functional form of the laws of physics the one that must remain equal. While this may be important for an undergraduate course of special relativity we consider that these details are not relevant for secondary school learners who are at this point only grasping the essence of the theory.

What is relevant for secondary education is not the postulate itself, but its implications. The relativity postulate implies in the first place that there exist more than one reference frame and that all of them are equally valid to describe phenomena. Here lays the key to students ideas about the apparentness of relativity effects. Students tend to consider that there exists one reference frame that is the correct one, or in which distances and times are right (Villani & Pacca, 1987). From students perspective the other reference frames are just wrong and the observers in them think or see that things last longer or have a shorter length, but those measurements are wrong and only the observer that doesnt move at all is right. Who might be the observer that "doesn't move at all" depends on how the situation is depicted and on the preferred reference frame of the student.

However the relativity postulate indicates that there is no right reference frame. And it goes beyond that, it also tells us that there is not such a thing as absolute motion. In other words there is no way to know whether an object is not moving at all. What is moving in one reference frame could not be moving in another one. Motion has to be defined as compared with something else. An object moves relative to something or it moves in one of the reference frames.

From the above discussion three important ideas can be recognized as fundamental for the theory from the relativity postulate:

(1) Movement is not absolute, it is defined by comparing with something else.
(2) There are more than one possible reference frames.
(3) All of them are equally valid.

These will be the ideas that we want students to obtain from the relativity postulate.

The second postulate of special relativity can also be rephrased to something more accessible for secondary education, namely: "The speed of light is always the same relative to all observers". Here the concept of reference frame has been replaced by an observer. Panse, Ramadas, and Kumar (1994), have shown that students tend to associate reference frames to specific objects. This can be used to ease the concept for students by placing an object (the observer) that is consistent with the theory and the concepts to be developed (the measurement of times and distances).

Once again important information regarding the nature of the reference frames associated to the observers is left out, and the value of the speed of light is also not mentioned but we maintain that that information is unnecessary at this level. Moreover research suggest that students can reason spontaneously with a finite and constant speed of light (Kamphorst, 2018b) but that reasoning with a speed of light relative to the observers is not as straight forward. Students tend to consider the speed of light as constant in only one reference frame depending on how the situation is represented. The focus of introducing the light postulate should be then in reasoning with a speed of light relative to the observer in different situations.

In short, from the light postulate there are two ideas that students should be able to follow in order to reason within the STR:

(4) The speed of light is defined relative to the observer
(5) All observers measure the same value for the speed of light.

Reasoning with these five ideas, two from the light postulate and three from the relativity postulate, will be the main goal of our lesson plan. In what follows some didactical guidelines will be drawn from the problem-posing approach for teaching.

The Problem-Posing Approach

As it has been mentioned before, students spontaneous ideas tend to survive instruction (Klaassen, 1995; Scherr et all, 2001), then it seems worthwhile to take them into account when designing a lesson plan. The problem-posing approach, as many other constructivist approaches, acknowledges that students arrive to the classroom with ideas and experiences on their own, so the learning process in not one of passive acquisition, but one of active construction of knowledge. However most constructivist approaches consider students previous knowledge as incorrect or far away from the accepted physical theory. In consequence these previous ideas are to be changed into the correct scientific ones.

The problem-posing approach, on the other hand, considers that students ideas about certain topics in physics are not necessarily wrong and it is assumed that these ideas (either spontaneous or result of previous instruction) are closer to the accepted physical theories that what is commonly recognized (Klaassen, 1995; Vollebregt, 1998). The goal of education then is not to replace wrong ideas, but to use (correct) existing ideas to lead students to the accepted physical ones. In order to do so students should recognize that their current frameworks have some limitations and that there are reasons to extend it. In Klaassen's (1995) words "the emphasis of a problem-posing approach is on bringing pupils in such a position that they themselves come to see the point of extending their existing conceptual resources, experiential base and belief system (with accompanying changes of meaning) in a certain direction" (p. 105).

An important component of the problem-posing approach is that students see the point of extending their current knowledge. If they do so it will be easier that they accept new knowledge because they see why is it important to do it and where this new knowledge comes from. In this approach problems are the tool trough which designers provide students with motives to extend their conceptual resources. Vollebregt (1998) gives some guidelines as to what conditions problems and subproblems should meet:

- The solution of each subproblem gives rise to the next subproblem in the series;
- Subsequently solving the subproblems in a series eventually solves the main problem;
- When the main problems are solved, finally, the educational goals intended by the designer are reached (Vollebregt, 1998, p.33).

In this work we aim to use these guidelines and the core ideas of the problem-posing approach (students ideas are mostly right, they should realize that their own ideas are not enough sometimes, they should want to expand their current resources and they should see the point of what they are doing) to help students realize that reasoning with a single preferential reference frame is not enough to deal with the STR.

We believe that by reasoning with the three important ideas about reference frames students will not regard the relativity effects as apparent anymore. This hypothesis leads us to the following research question: **Does addressing the important ideas of the relativity postulate in a lesson plan about the STR for secondary education prevent students from considering the relativistic effects as apparent?** In order to answer this question we will first show how the lesson plan we used looks like.

A problem posing-approach to teaching the STR

As it was mentioned in the previous section, there are five main ideas that we want students to acquire.

(1) Movement is not absolute, it is defined by comparing with something else.
(2) There are more than one possible reference frames.
(3) All of them are equally valid.
(4) The speed of light is defined relative to the observer
(5) All observers measure the same value for the speed of light.

We will try to teach these ideas through a problem-posing approach in the context of a STR lesson.

Kamphorst (2018a) has already designed a lesson plan that successfully introduces the last three ideas with a strong focus on the ones related to the light postulate [1]. In this work we will use Kamphorst's lesson plan as our base and we will make modifications and additions to introduce the ideas related to the relativity postulate. We believe that these ideas will help students to stop regarding the relativistic effects as apparent.

Our lesson plan will consist of 5 phases:

[1] In Kamphorst's lesson the 3rd important idea is used, but the main focus of the design is to carefully introduce ideas 4 and 5

- In the first phase the definition of movement as well as the existence of multiple equivalent reference frames will be introduced (goals 1, 2 and 3). This will be done throughout a classical situation using only Galilean relativity. This phase is an addition to Kamphorst lesson plan.
- In the second phase the important ideas introduced earlier will be applied to a situation that involves light, setting the first steps towards the light postulate. This phase is a modification to Kamphorst lesson.
- During the third phase the 4th idea will be introduced and students will formulate their own version of the light postulate. During this phase our work aligns back with the one of Kamphorst and the following steps mimic the ones designed by Kamphorst.
- In the fourth phase the remaining idea will be introduced: the constant nature of the speed of light in all reference frames (5th). All of this is used to obtain time dilation. From students'
- Finally time dilation (and all conceptual tools developed before) are used to obtain length contraction.

In the following it will be described how these phases take place and how the problem-posing approach is used so students can develop them.

Phase 1. Relative to what?

The goal of this phase is to introduce the important ideas of the relativity postulate through a problem-posing approach.

As the problem-posing approach dictates we will part from the idea that students' previous knowledge and spontaneous reasoning are mostly right. In this lesson series we will work with students who haven't had any previous instruction on the STR. Because of that we can assume that students have no previous knowledge on the topic and we will work only with their spontaneous reasoning.

From student's spontaneous reasoning we will obtain their preferred (and implicit) reference frame. As it has been mentioned before students' tend to define motion as absolute in this single reference frame, things "really move" (or don't) in this reference frame and in all others it only looks like it is moving. By contrasting their own preferred reference frame with the one of their peers students will recognize that there are more possible reference frames (conceptual goal 2) and that they are all valid descriptions of the situation (conceptual goal 3). Simultaneously, during the discussion with their peers students will have to make explicit their, until now, implicit reference points. Students will not longer be able to say wether things are really moving, since in the preferred reference frame of their peers the definitions of movement differ. This will force them to make a more appropriate definition of movement: things move relative to something (conceptual goal 1).

Throughout the lesson series we will make use of events diagrams as the one depicted in Figure 1. Diagram 0 is the first diagram that students encounter and it will serve to bring students' spontaneous reasoning into play.

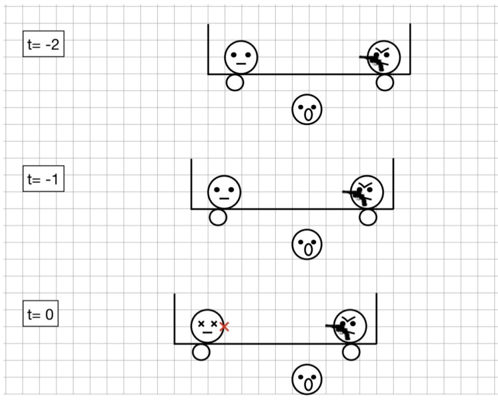


Figure 1. Fragment of Diagram 0. First diagram used in the lesson series.

This diagram represents a situation in which only Galilean relativity plays a role. In this, as in all event diagrams used throughout the lessons, a situation is shown at different moments of time. In this way the events can be read as if one reads a comic book but in a vertical way. Movement in these diagrams is depicted as a change of position in the grid that spans the whole diagram. By comparing the current frame with the previous one it is easily recognizable which elements in the diagram are moving.

In the starting situation presented on phase 1 there is an event happening inside a moving train. One of the passengers of the train was shot. Students will be given the task of finding out when the murderer shot, given a known speed for the bullet. Students will work individually in this task. Students' spontaneous reasoning will lead them through one of the two possible solutions to the task. Either they have as preferred reference frame the one of the cart, or they have the one of the outside observer (the ground or the grid). These two different reference frames allow two different interpretations of the speed of the bullet which in turn generates two different outcomes.

The solution they give and the procedure they follow will show which is their preferred reference frame or more concretely what did they use as their reference point. After having found an answer students will compare their results with their peers. If students had different results they will explain them to their classmates. If not the teaching will introduce the other answer and together they will compare both.

We expect that students begin their explanations and comparisons in terms of their own implicit reference frame. Whenever a students speaks about a certain speed they will do it without specifying the reference point/frame that they used. However through discussion with their peers it will become necessary to specify their reference frame, since a speed of 2 relative to the cart becomes a speed of 3 relative to the outside observer and their peers might have a different preferred reference frame.

This will result in acknowledging that it is important to explicitly indicate relative to what are speeds given. Otherwise they will have different results to the same situation.

With these experiences students should arrive to the point in which they start wondering relative to what speeds are defined. They should do so whenever they encounter a a situation in which the reference frame is not explicitly given. With this students will start to construct a personal idea of the concept of reference frame. Furthermore we believe that the acknowledgement of the validity of descriptions made from different reference will bring them a step closer to recognize that the measurements made in different reference frames (of times and distances) are all equally valid.

To test whether these ideas were successfully transmitted, at the end of the phase we will try to answer the following questions.

- *Do students accept that there are solutions other than the original one they presented?*
  This question can be answered when the second RF is introduced (either by their peers or by the teacher) by observing how students react to the second solution. Do they accept it and make sense of it? or do they reject it? If students accept the second solution we can say that they had made the first step towards conceptual goals 2 and 3, accepting that there are more possible reference frames.

- *Do students go from "the speed is" to "the speed relative to xxx is"?*
  This question is related to conceptual goal 1 and will help us to evaluate whether students come closer to a better definition of movement. We can find an answer to this question in students utterances when discussing and comparing their solutions with the ones of their peers.

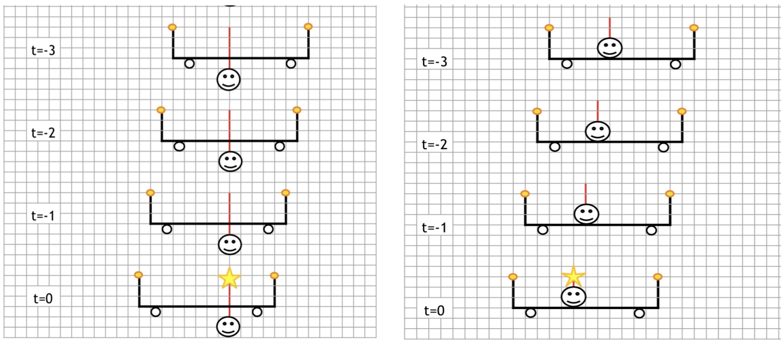
Phase 2. How is it with light?

The aim of the second phase is that students transfer the concepts acquired in the first phase to a situation involving light. In that way this phase serves as a bridge to connect the important concepts of the relativity postulate (that students worked with in phase 1) with the important concepts of the light postulate that will be introduced in phase 3. From the previous phase students should have started wondering relative to what speeds are given. In this phase students should also start wondering how is it in the particular case of light.

After having dealt with Diagram 0, students will be introduced to situations that involves light. This will be done by the hand of diagrams A and B (See Figure 2). In these diagrams there are three elements that have different relative movements. One researcher with its measuring instrument (the smiley with the red vertical line), a cart with lamps in the corners (the box-like figure, with the yellow circles representing the lamps), and the grid (that as we saw before spans the whole diagrams and helps us to define positions and displacements).

Students will be indicated that the lamps turned on at unknown times and that the light emitted by both lamps arrived at the same time to the measuring instrument of the researcher. Given a fixed speed of light, students will be asked to find when did the lamps turn on.

From the previous activity students have learnt that it is important to determine relative to what speeds are defined. In this activity students will be given a certain speed of light, but they will not be given a reference point for that speed. As a result of the previous phase students should wonder what is the reference point for the given speed of light. Students will then propose possible reference points and they will



(a) Diagram A. The researcher and the lamps move relative to each other.

(b) Diagram B. The researcher and the lamps do not move relative to each other

Figure 2. Diagrams used during phase 2

explore the results obtained from the suggested possibilities.

The two possible reference frames are the one of the ground and the one of the cart, and as in the previous exercise this will generate two answers for each diagram. At the end of the activity students will have two possible options for each one of the diagrams (see figure 3).

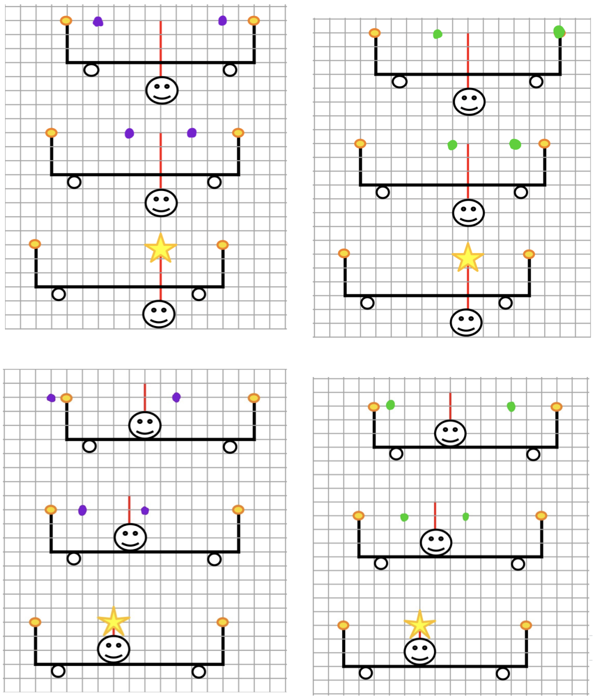


Figure 3. The solutions to Diagram A are shown in the two upper diagrams. The solutions to Diagram B are shown in the two lower diagrams. To the left the solution for a constant speed relative to the grid (purple dots) and to the right the solutions for a constant speed relative to the lamps (green dots).

During the whole lesson students never, or rarely use the words "reference frame", however all the activities are built in such a way that the properties and characteristics of reference frames are something students can intuitively work with. Up to this point, students should be able to identify different reference frames (conceptual goal 2) and to define speeds in all of them (conceptual goal 1).

At the end of the phase we will try to answer the following questions to assess whether students have reached the position described in the previous paragraph.

- *Do students question what is the reference point of the given speed of light?* This question checks whether students can transfer conceptual goal (1) from the previous phase to the particular case of light. We will be able to give answer to this question at the moment students are given the speed of light. It is then when they should ask for the reference point used.
- *Do students find the two possible reference points for the speed of light in Diagram A and Diagram B?* This question checks whether students can transfer conceptual goal (2) from the previous phase to the particular case of light. Once again students should be able to recognize and accept as valid both reference frames in each Diagram (A and B).

At the end of this phase students are looking for the correct reference point for the speed of light. They have already suggested possible reference points (the grid and the cart) and explored how the solutions look like in different situations. However they still don't know which one is the correct one. Finding out the answer to that questions will be the procedural goal of the following phase.

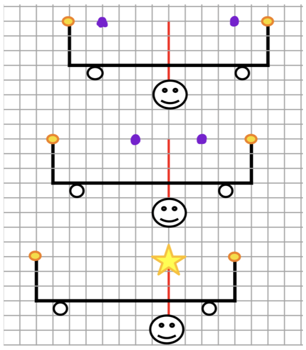
Phase 3. Relative to the observer

The aim of this phase is that, based on experimental results, students can arrive to the first idea about the light postulate. Phase 1 gave students the tools to find a problem in the set up of the main question of the second phase ("when did the lamps turn on given certain speed of light"). Students were given a speed but were not indicated the reference frame in which that speed was defined. Throughout the second phase students looked for possible ways of solving that problem. That was done by proposing possible RF for the speed of light and observing the different solutions obtained. In this third phase students will make a evidence based decision for the appropriate reference frame for the speed of light, namely the one of the observer.

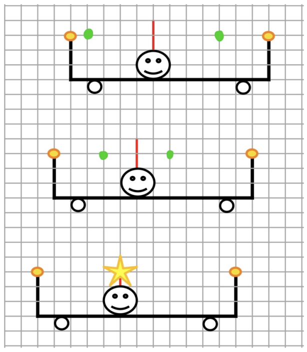
At the beginning of phase 3 students will be given information regarding two physics experiments and their results. Each one of the experiments have a set up that parallels the relative movements depicted in diagrams A and B.

Students will link each experiment to the diagram that has the same relative movement set up. From the results and conclusions of each experiment students will be able to choose which of the solutions proposed before (shown in figure 3) is in agreement with the experiments. One of the experiments points towards the speed of light being defined relative to the ground and the other experiment towards the speed of light being defined relative to the cart (see figure 4).

These results may seem contradicting at first sight. Students will be prompted



(a) The right solution for Diagram A seems to indicate that the speed of light should be relative to the grid



(b) The right solution for Diagram B seems to indicate that the speed of light should be relative to the cart

Figure 4. Right solutions according to the experiments. The "correct" reference frame for the speed of light is the one of the observer.

to find a solution that can accommodate both answers. By looking at both answers students should be able to find what is the common factor in both of them (see figure 4). Light moves relative to the researcher. This is the light postulate.

With this procedure students have covered the first important idea about the light postulate. The speed of light is defined relative to the observer. The second one (all observers measure the same value for the speed of light) will be directly addressed in the following phase.

Guided by the philosophy of the problem-posing approach the lesson series is such that students have arrived to the solution of a problem in terms of things they understand. They were not simply told what the light postulate is, instead students "constructed" it. The light postulate will become then something students can hold onto since it is based on their intuitive ideas and sounded experimental results by following a series of steps that make sense for them.

To evaluate whether the build up of the lesson plan was successful and students have a grasp of conceptual goal 4 we will try to answer the following questions at the end of this phase:

- *Based on the information provided of the two experiments, are students able to find the right solution for each diagram?*
  Interpreting the experimental results in terms of the diagrams requires making explicit the reference frame in which the experimental results were obtained, moreover it requires and transferring the information to the graphical working form of the diagrams. This will evaluate the easiness (or difficulty) students have when working with reference frames.

- *Do students formulate their own version of the light postulate?*
  This questions checks for conceptual goal 4 and the answer will be obtain at the end of the phase when students arrive (or not) to the correct reference frame for the speed of light: the observer.

At the end of this phase students have finally found the correct reference frame for the speed of light which gives one single answer to the first task: "When did the lamps turn on?". When applying the light postulate to other situations in which light is involved, students should be able to obtain a single answer to the given task. However difficulties arise when we consider more than one observer, and that will be the central problem of the next phase.

Phase 4. Time dilation

This phase concludes the introduction of important ideas and sets the starting point of application and evaluation of our hypothesis. Because of that the fourth phase has 2 aims. The first one is to introduce the last important idea of the light postulate, and with it obtain the time dilation effect. The second aim, that is also shared with the fifth phase, is to look for evidence to give an answer to our research question: *Does addressing the important ideas of the relativity postulate in a lesson plan about the STR for secondary education prevent students from considering the relativistic effects as apparent?*. During the activities in which the time dilation effect (and later length contraction) is obtained and worked with, we will pay special attention to students' ways of referring to time (and later distance). Do they talk about a real time, and other wrong or apparent times?

To work towards the first aim, students will work with diagram D in which a second researcher is introduced. Both researchers receive light from the two lamps at the same time and place (t=0) (see figure 5). By using their newly discovered rule for light propagation students will have to determine when did the lamps turned on. However, since the two researchers are moving relative to each other, they will observe that the lamps turned on at different times (see figure 5).

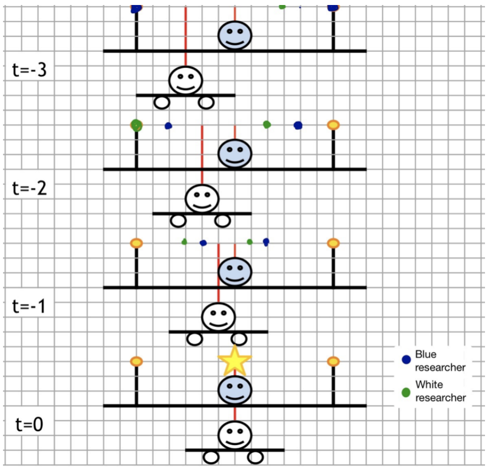


Figure 5. In this diagram the white researcher will determine that the lamps turned on at t=-2 and t=-6. The blue researcher, on the other hand, will determine that both lamps turned on at t=-3

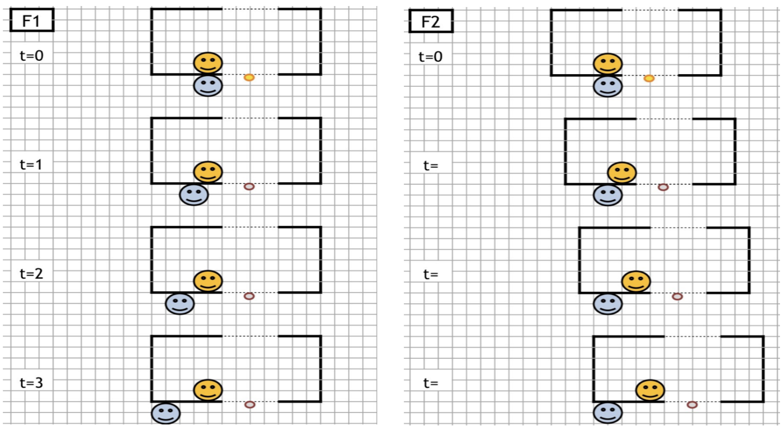
It is at this point where we can assess whether students hold to the light postulate or not. Accepting the light postulate, together with all the important ideas introduced before, lead to very surprising and "unnatural" consequences. If both researchers are to be right, there must be something else going on. After all, it is the same light and the same lamps that they are studying, but they still measure that the lamps turned on at different moments. What has to change is their definition of time. Is this realization about the difference in times what opens towards the understanding of the time dilation effect and finishes with the introduction of the five important ideas.

To assess whether both the relativity postulate and the light postulate were accepted by students we will try to answer the following question:

- *Do students hold to the light postulate when the second researcher is introduced?* Holding to the light postulate in diagram D also implies that students are applying the equivalence of reference frames. Students have to accept that there is no right or wrong researcher here. They both are equally right (both reference frames are equally valid). We will consider that students hold to the light postulate if they consider both researchers to be right. Not holding to it will be evidenced when students reject the rule (saying that it is wrong) or when they say that one of the researchers must be correct and the other had made a mistake.

Once again we can see how the guidelines of the problem-posing approach lead the design of the class. The solution of the last set of problems (the observer being the correct reference point for the definition of the speed of light) gave rise to the new problem (time acquires a different definition for each observer).

In order to explore the problem with time, and make sense of it, students will now work with two diagrams that show a light clock (see figure 6). These diagrams represent the same situation but from different points of view (different reference frames), the ones of two observers that move relative to each other, one of them not moving relative to the light clock. Here students encounter once again a situation that can be studied from two reference frames, only this time the reference frames are made explicit by giving one diagram to each RF [2] .



(a) Light clock in the reference frame of the yellow researcher (b) Light clock in the reference frame of the blue researcher

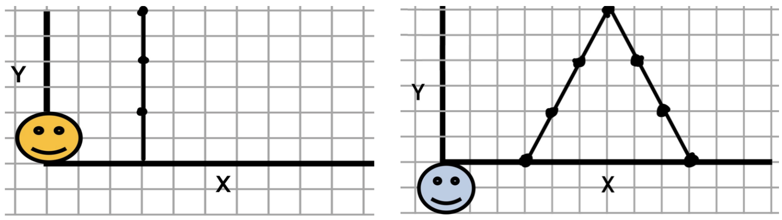
Figure 6. Diagrams used to obtain time dilation

A light clock is a mechanism used to measure time by studying the bouncing of

[2]Students also worked with Diagrams G1 and G2 that are inverted versions of F1 and F2. In the G diagrams the blue researcher is the one holding the mirrors and the yellow researcher is inside the box without mirrors. G1 is drawn from the point of view of the blue researcher and G2 from the point of view of the yellow one

light between two mirrors. In this activity students are asked to determine the time that it took light to bounce between the mirrors from both reference frames. Solving this in the reference frame of the researcher that is stationary relative to the mirrors, is straight forward and students should not have any difficulty with it. Light bounces between up and down following a vertical path.

However in the reference frame of the observer that moves relative to the mirrors light covers a diagonal path, which means that it covered a longer distance (see figure 7). Is at this point that students have to show their conviction to the light postulate. In order to preserve the constant speed of light something else must change, namely the time. The researcher that does not move relative to the light clock will measure a shorter time than the one moving relative to it. The former (and always shorter time) is usually called proper time. This is the first big conceptual step that students must give. Abandoning classical ideas about the universality of time has proven difficult not only for students but also for physicist during the past century.



(a) Right Solution Diagram A

(b) Solutions relative to the cart. Diagram A above and Diagram B below

Figure 7. Path followed by light in its bouncing up between the mirrors as seen from different reference frames

Students have been equipped with the tools to make this step. They know where the light postulate comes from and they know that experimental evidence supports it. The obtained consequences also come from a logical and well supported procedure.

Once students have qualitatively understood that time passes differently for different observers, the quantitative relation between the time duration of an event from different reference frames will be derived by the teacher.

In this phase there are several things that can be tested. By observing how students talk about the researchers and time we will try to give an answer to our research question, for that we will make use of the following sub-questions:

- *Do students still talk in terms of absolute movement? (Is there a researcher that is not moving at all?)*
- *Do students say that only one of the researchers is right?*
- *Do students mention that one of the times is the correct one?*

Phase 5. Length Contraction

The fifth and last phase of this lesson series has both an instructional goal and a research one. The instructional one is that students apply the time dilation effect to obtain the length contraction one. The research aim is to provide opportunities to check for our research question. Once again, in the activities in which students are working with length contraction, we will pay special attention to students' expressions in order to observer whether they consider the length contraction effect as real or not.

The first step in this phase is that students recognize that there is also something going on with time. In order achieve that, students will be asked how to measure the speed of light. To measure the speed of light it is only necessary to measure the distance it covered in a determined period of time. As usual this will be done from the point of view of two different researchers. Students should agree that the value for the speed of light must be the same for both researchers, however, as they saw in the previous activity the time that each of them will measure will be different which translates in an accordingly different distance.

With this idea in mind students will explore a new situation in which two researchers moving relative to each other measure the distance between two stars. The stars are at a fixed distance from each other and one of the researchers move relative to the stars while the other doesnt. This time students will not be given a new diagram, instead they should draw the situation themselves, from the point of view of each one of the researchers.

Obtaining the length contraction effects in this scenario strongly depends on identifying the time dilation effect. Because of that students should be able to work with all the previously used concepts, with an special emphasis on proper time.

Identifying who measures the proper time is an easy way to observe students hold to the important ideas from relativity postulate. The proper time is the one measured by whoever is not moving relative to what is being measured. As it was mentioned before students tend to think in absolute terms so their preferred reference frames will come into play and they will be inclined to assign one of the observers as the one that doesnt move at all. However if what has being done has been effective students should be able to recognize that there is no observer that doesnt move at all. At this point students should also be able to easily recognize who and what is moving (or not) relative to other objects and events.

Finally, guided by the teacher, the equation for the ratio between distances measured from different reference frames will be derived.

Since this phase shares the research aim of the previous one, we will ask ourselves the same questions but now in the context of length contraction. Once again by observing how students talk about the researchers and lengths we will try to give an answer to our research question.

Subquestions:

- *Do students still talk in terms of absolute movement? (Is there a researcher that is not moving at all?)*
- *Do students say that only one of the researchers is right?*
- *Do students mention that one of the lengths is the correct one?*

With this phase our lesson plan for the teaching of STR for secondary education concludes.

Participants

The lesson plan was implemented in two schools. A Dutch school that offers bilingual pre-university education (tweetalig gymnasium) and an international school that follows the International Baccalaureate program. Students were volunteers from grades VWO 5 (in the Dutch system) and DP1 and DP2 from the international one. There was a total of 17 students between the two schools.

Special Relativity is an elective topic in both physics curriculums (the Dutch and the International one). In both of them teachers can choose out of 4 topics which 2 they teach. From those two students can later decided which one they will follow. In the Dutch school Special Relativity is one of the chosen optional topics, but in the international school it isn't.

At the moment the lessons took plan the Dutch students had not decided yet which optional topic they would follow. So none of the participants had had any previous instruction on the STR.

Setting and data collection

The 12 students from the Dutch school were divided into 3 groups of 4 students, and the 5 volunteers of the international school into 3 groups, 2 of 2 students and one student who chose to participate on his own. This particular division responded to fluctuations in students schedules because all of them were participating on voluntary bases.

The relativity lessons took place in the respective students' schools. 4 groups (the 3 Dutch plus the individual international student) participated in 6 to 7 lessons of 50 minutes, while the remaining 2 groups (from the international school) participated in 3 lessons of 1.5 hours.

Since we wanted to observe how students interact with the teaching sequence and how their reasoning is influenced by it the lessons were taught through a teaching experiment. The Teaching experiment is a variation of the clinical interview (Engelhardt, Corpuz, Ozimek, & Rebello, 2004). It consist of a series of teaching episodes in which the researcher plays the role of both a teacher and an interviewer. In the teaching experiment a series of experiments and phenomena that require explanations are discussed with the students (teacher role), the teacher/researcher must then provide answer to conceptual difficulties students may have (Komorek & Duit, 2004). The second role of the researcher is the one of an interviewer or observer. The teacher/researcher asks students questions that try to elicit as much of their reasoning and thought processes as possible in order to model students responses into a coherent picture of students progress over an extended period of time (Steffe & Thompson, 2000; Engelhardt et al., 2004). Through the teaching experiment we aim to learn how the teaching and learning activities in the lesson plan helped students to add to their conceptual framework.

All episodes of the teaching experiment were video recorded. The camera was placed overhead in such a way that students interactions with the teaching materials as well as their procedures to solve the task were visible. Students annotations and solutions to the task were recorded and stored.

The recorded material, as well as students solved diagrams and annotations were used to answers all the previously presented questions. In order to give an coherent answer we used an interpretative analysis. An interpretative analysis tends to use large sections of transcripts followed by the researchers inferences concerning the thought processes of the subjects, the researchers interpretation will then form a model of the students learning processes (Clement, 2000). We were guided by the idea that students beliefs form a coherent and complete framework that produces conclusions that are logical within those frameworks. Under that assumption students frameworks have to be understood and interpreted in such a way that "we can see for ourselves that what he [the student] did was the reasonable thing to do for him" (Lijnse, 2010).

Results

Introducing the important ideas of the light and relativity postulates

Phase 1. Relative to what?
Introducing the important ideas from the relativity postulate

As it was mentioned before in this phase students worked with diagram 0 (see figure 1). In the following, extracts of the interactions of two groups with the diagram will be shown.

- Group 1

In this group students used different strategies to find when the bullet left the gun. Student 1 used the reference frame of the grid and Students 2 and 3 the reference frame of the cart. When they finished the task Student 1 said aloud his answer and the following exchange happened:

STUDENT 1: t is -6
STUDENT 2: [*Towards the teacher*] You said 2 boxes... [*referring to the given speed of light: 2 squares/boxes per time unit*]
STUDENT 3: Right, so t is -3
STUDENT 1: But the box is also moving
STUDENT 3: Oh!
STUDENT 2: Yeah...
STUDENT 1: If we go two steps to the right here [*pointing at two square to the right of the gun at t=0*] and the box was... well the bullet was here at t -1 [*moving his hand directly upwards from the previous point*] then 2 steps to the right [*moving his hand two squares to the right of the point he was holding before*] it was here. I drew the line.
STUDENT 2: What is it? Is it relative... because the bullet is all... The way I drew it is that the bullet is already moving with one box per second because it is in the cart, and then it has like an additional speed of 2 boxes from the gun itself, so then it would actually be 3 boxes. I do understand what STUDENT 1 is saying, but I dont know whether my explanation is right or his.

- Group 2

In this group all students used the same strategy to solve the task, they both used the reference frame of the cart and therefore they obtained the same answer.

STUDENT 1: t is -3

STUDENT 2: Yes. I agree.

TEACHER: Can you explain me what you did?

STUDENT 2: You said it was 2 squares per time unit so I moved here back to t is minus 1 and 2 squares to the right and then here also

STUDENT 1: Yes I did the same. I measure, it was 2 squares per time unit so I moved back every time and then ended up in t -3 that the bullet was in the gun.

TEACHER: What if I say... [*showing the other option*] would this also be a speed of 2?

STUDENT 1: Yes, because the car is moving with one square per second to the left so thats... thats the opposite direction for the same direction of the bullet so the bullet is less fast in comparison to the car. So it takes longer for it to hit the person.

TEACHER: [*To* STUDENT 2] Do you agree?

STUDENT 2: I think I do agree but then this [*pointing to her original solution*] isn't right.

TEACHER: Why?

STUDENT 2: Because then it moves here [*one square away from the shot person*], then it has to be there [*she construct the other option in her own paper*] then it's there [*at t=-3*]. The car is moving... then it's not right. Then this is 3 squares per time unit [*her own previous diagram*]. So this isn't right.

TEACHER: But this was 2 as well, right?

STUDENT 2: Yeah but this is... from the person, from this [*pointing to the "victim" inside the cart*]. So I did this and the person is actually here, and then it's there. Then it has to be here, and then there, [STUDENT 2 *takes a pen of another color and constructs the whole answer from the other RF, now she has two answers in two colors: blue-cart and red-grid*]. Because what he said is right, that the car is also moving and then it takes longer to reach. So if that [*pointing to the solution suggested by the teacher*] is two squares per time unit this is three [*pointing to her original solution*]. The blue one [*her original solution - relative to the cart*] aren't right.

TEACHER: But the blue one is also 2, right?

STUDENT 2: No, actually no, because here its moving three squares pertime unit. But two squares to the person, but the person is also moving and I didn't take it into account.

From the dialogue of Group 1 we can observe that Student 1 is using his hands and the diagram itself to explain his answer. He is not vocalizing his reference points but merely pointing at them. He has the grid as his preferred reference frame.

Afterwards we can see that Student 2 is able to recognize and make sense of answers obtained by using different reference points. Originally he had used the reference frame of the cart to obtain his own answer, but after Student 1 explained his solution (and with it showed that he is using the reference frame of the grid), Student 2 explained his answer from the reference frame of the grid as well.

From the dialogue that took place in Group 2 we can see that Student 2 (in a similar fashion as Student 1 from Group 1) used her hands and the diagram to show her preferred reference frame. Since Student 2 stablished the shared reference frame for the group (the one of the cart) Student 1 had no need to look for a way to explain his and simply said "I moved back every time".

When the teacher introduced the answer that used the reference frame of the grid, Student 1 moved to the reference frame of the grid *"the car is moving with one square per second to the left"* and then he gave the first step towards translating the speed of the bullet to the reference frame of the cart *... so the bullet is less fast in comparison to the car*. In this sentence students are already making explicit their original reference point.

Moreover Student 2 goes even further. When asked about her original answer she says *"Yeah but this is... from the person, from this (pointing to the "victim" inside the cart)*. In this Student 2 not only pointed to the diagram, but in order to make her point clear she had to make explicit the reference point she used (*from the person*). Later, we see that she is also capable of translating speeds from one reference frame to the other. "*So if that (the solution obtained defining the speed in the reference frame of the grid) is 2 squares per time unit this* (her original solution defining the speed in the reference frame of the cart) *is three.*

Now to respond to the questions we set for this phase:

- *Do students accept that there are solutions other than the original one they presented?*
  Yes. As expected all students solved Diagram 0 by using their spontaneous reasoning and with it they evidenced their implicit preferred reference frame. When students compared their answers with the ones of their classmates, all of them seemed to be able to make sense of the other answer (the one that corresponded to the reference frame that they did not use originally). Although they recognized both options as possible solutions they did not seem to realize that they were both valid. Instead they were looking for the right solution: *"... but I don't know whether my explanation is right or his"* (Student 1, Group 1); *"I think I do agree, but then this isn't right"* (Student 2, Group 2).

- *Do students go from "the speed is" to "the speed relative to xxx is"?*
  No yet. Students did start to make the reference point that they use more explicit *"in comparison with the car"*, *"from the person"*, but they have not make the step towards "the speed relative to". Only one group found both solutions and automatically assigned a reference point to each one of them.

STUDENT: Therefore there are two answers to the question. Because the first answer would be related to this smiley [*pointing to the smiley in left inside the car*], the second answer would be related to this smiley [*pointing to the smiley outside the cart*], because if we take the car into account moving then the distance is going to be different from the distance inside of the car itself.

Phase 2. How is it with light?
Bridging towards the light postulate

In this phase, students were to look for possible reference points for the speed of light. This was to be done by the hand of Diagrams A and B (see figure 2). However in the previous phase students did not fully defined speeds as relative to something. They merely began to make explicit the reference points they were using. In consequence when presented with Diagram A students did not recognize the need of defining the speed of light as relative to something. They simply worked with Diagram A from the bottom. They started by working from their preferred reference frame, and then by discussing with their peers they opened to other options.

By working with Diagram A and B students could finally start to describe movement as relative to something. However this was not an easy step. The following extract from one of the groups illustrates this.

The discussion emerged after students had found when the lamps turned on in Diagram A. Student 1 worked in the reference frame of the grid, and Student 2 worked in the reference frame of the cart.

STUDENT 1: We did different things again. So... What I did is the same I did in this diagram [*showing Diagram 0*]. I thought, light is now here, so at one time step before t is 0 then the time must have moved, oh yeah, so the light must have been 2 boxes to the right so at one time step earlier the time must have been again 2 steps to the right and one time step earlier again 2 steps, so I ended up here. I did the same to the left side so, 2 steps to the right, 2 steps to the right. To the left. 2 steps to the left.

STUDENT 2: Again, I said, just like the last assignment I thought the light would be moving two boxes per time unit, from the light source, so each time, I drew the dot where the light would be like, two boxes to the left or to the right, depends on which one I drew. But like... I didn't actually look at the speed of the cart. I just looked at what the distance from the light source will be and then per time step by time unit I could figure out how far the light... So that it would result in three to the left each time but one to the right for the other light source.

Later on in this same group.

TEACHER: Then, what are the differences between the 2 options?

STUDENT 1: [*after looking at both options for several seconds*] I think mine is relative... STUDENT 1's is relative to the cart, the speed of the light is relative to the cart, and mine is relative to... You know... relative to the... to its destination actually. I don't know what to say what is relative to in my drawing

STUDENT 2: I guess that I did it was just for the speed of light and then added or subtracted the speed of the cart and then just drew, so from the right side it will be three boxes and from the left side it was one box for each time unit. [...] I think I am doing it, like according to my basic knowledge what I knew that... Yeah, what is logical, because we are used to add these velocities. For example when you are standing on a train and then you throw a ball the direction the train is already moving, then I would just add the velocity of the train and the velocity which I threw the ball, and then the velocity of the ball relative to the point like the ground. [...]

TEACHER: So relative to what is it 2?

STUDENT 2: To the scientist in this case.

STUDENT 1: No, is to the light in your case. It's 2 boxes relative to the light because that is what you were showing here, every time subtracting 2. Mine is actually from the ground. 2 boxes relative to the ground. I think.

STUDENT 2: Yeah, indeed.

We can see that in the first extract Students 1 and 2 express having followed the same strategy that when solving Diagram 0. Student 1 points towards the objects and do not vocalize his reference points. Student 2 does. He says *I thought the light would be moving two boxes per time unit, from the light source*. Through later discussion Student 2 repeatedly emphasizes the fact that he is counting from the light source. However when Student 1 starts to introduce the "relative to" description Student 2 seems not to recognize that he is using the lamp as his reference point. The strategy he follows (described in the first extract) does not match the conceptualization he does of the problem (described in the second extract). What is more, Student 2 could actually describe speeds as relative to something (*and then the velocity of the ball relative to the point like the ground*) but he could not recognize that in his own solution he hasd defined the speed relative to the car/lamps.

We can observe that even if Student 2 could easily name the reference point used by Student 1, he could not do so with his own. By analyzing his descriptions one can see that he is counting from the researcher and his measuring instrument *mine is relative to... [...] to its destination actually*. But after listening to Student 1 (probably to this part *and then the velocity of the ball relative to the point like the ground*), he arrived to a reference point that made sense (after all in Diagram A the researcher is not moving relative to the grid/ground).

Now, responding to the questions we set to this phase:

- *Do students question what is the reference point of the given speed of light?*
  No. Students were not able to make the conceptual step of recognizing that every motion has to be defined relative to something in the previous phase. It is only natural that they do not make it for light either and in consequence they cannot ask themselves relative to what the speed of light must be defined.

  However it is still useful to question ourselves whether students came closer to defining speeds properly. For that reason we will make the following question:
  - *Do students move forward in their understanding of speeds having to be defined as relative to something?*
    Yes, but this process was not straight forward and it took a considerable amount of time. Although in all their discussions students have started to use reference points to explain their ideas most of them found it very difficult to move from explaining what they were doing to actually defining and naming their answers in terms of the used reference frame.

- *Do students find the two possible reference points for the speed of light in Diagram A and Diagram B?*

Yes. Although not in the way intended. At this point we were expecting that students were already aware of the need of defining speeds as compared to something and in consequence they would question themselves about possible reference points for light. However this did not happen and students proceed to solve the diagram in the same way they solved Diagram 0.

Phase 3. Relative to the observer
The light postulate

In this phase students used information about two experiments to make an informed choice for the correct reference frame of light. The experimental results were worded in such a way that the used reference frame was implicit. To be able to use the results students had to identify the implicit reference frame (the one of the observer) and link it to the solutions to the diagrams they constructed.

When reading the information one student said *Ok but, here (in the information provided) they say that the speed of light is always the same, but in relation to what?*. This is one example of students' utterances that suggest that it is in this phase when students start to be aware of the need of defining speeds properly. In the following paragraphs more extracts will be shown to support this claim.

The following dialogue took place when students were asked to use the results from one specific experiment (the Michelson-Morley experiment) to choose one of the two solutions they had found for Diagram B (the two solutions that corresponded to different reference frames - see figure 3).

TEACHER:
STUDENT 1: This one [*pointing to the relative to the grid option*]. It just moves 2 to the right. 2 to the left each time on the grid and not relative to the researcher.
TEACHER: What is the researcher measuring in here? For the researcher what is the speed of the light in this side?
STUDENT 2: One. One grid square per time unit.
TEACHER: And in the left side?
STUDENT 2: Three
TEACHER: Is that in agreement with what MM found?
STUDENT 1: No
STUDENT 2: No. [*pause*] Well it depends on what is it relative... in the speed I mean... I believe we can assume that the speed is relative to the researcher, then this one [*the one relative to the lamps/observer*] is correct. But if they are talking about the speed relative to the universe, to the star... wait wait, wait... What are they talking about? Oh wait. Never mind. Never mind.
TEACHER: So what speed did they measure?
STUDENT 2: Eh... Relative to the researcher speed
TEACHER: So which one corresponds to what Michelson and Morley concluded?
STUDENT 2: This one [*the one relative to the lamps*].

Students then found the right solution for Diagram A with the help of another experiment, and afterwards they were asked to find a general rule.

TEACHER: Can we make a general rule that responds to both results.

STUDENT 2: [*After thinking for a while*] Yeah I think we can because the speed relative to the researcher is always 2, so the rule should have something to do with that. Here [*pointing to Diagram A*] it is 2 squares to the left, to the left 2 squares. Here [*pointing to Diagram B*] you it is 2. So... the rule... [...] Yeah the only speed you can... the light you can have the... measure from the researcher's perspective is *c*. So you would see the speed *c* here [*Diagram A*] and the speed of light here [*Diagram B*]. But I don't know how to phrase it into such a nice rule.

TEACHER: Something in your words

STUDENT 2: The speed of light you measure is always c. Yeah... Eh... the speed relative to the observer is always *c*.

From the first extract we can see that, just as the student from the in text dialogue before, Student 2 has started wondering the reference point of given speeds *"It depends on what it is relative"*. Later he recognizes that the speed is *"relative to the researcher speed"*. Using this Student 2 was able to figure out what solution was correct for Diagram B.

Having named it *"relative to the researcher speed"*, makes it easier for him to find the common factor between the two correct solution for the diagrams (*the speed relative to the researcher is always 2, so the rule should have something to do with that*). Most of the groups had a similar process as the one described here. And they could easily formulate their own version of the light postulate.

There was one group that developed a group-shared preferred reference frame. Meaning that they implicitly agreed in using one reference frame and they no longer had to make explicit the used reference points, since they all did the same. For a more detail description of this see Appendix A.

Now, to respond to the questions we set for this phase:

- *Based on the information provided of the two experiments, are students able to find the right solution for each diagram?*
  Yes. Students correctly identified the reference frame used in the solutions provided and then they choose one of the two options they had for each diagram. During this phase students came closer to fulfill one of our initial goals: that students start to define speeds as relative to something.
- *Do students formulate their own version of the light postulate?*
  Yes. From the previous activity students promptly identified that the correct speed in both diagrams was relative to the researcher. So they could easily make their own version of the light postulate. Other phrasings for the light postulate produced by students were: *"The speed of light is always the same relative to the one... to the person who's or the thing that is receiving"* and *"If observer moves, is again relative to the observer, so it depends on the movement of the observer, or if it's stationary doesn't depend on the movement outside the box, because then it doesn't count"*.

Phase 4. Time dilation
The last idea from the light postulate

In this part of the phase students used their recently formulated rule (the light postulate) in a situation involving two researchers. This was supposed to help them realize that, if we use the light postulate, there is a problem with how each observer defines time. However this did not happen.

Most of the students seemed indeed confused or surprised for finding a different answer for each observer, however in thinking how to solve this conflict students followed different approaches. Only one of the groups chose to review the rule, other groups tried to think about travel time or distances. When they realized that both researchers received light at the same time and place they also agreed that it wasnt a problem of distances. However they did not offer more explanations and the teacher had to introduce the possible solution: that times goes differently for each researcher. In the following the interactions of the group that suggested to review the rule will be shown. To observe to more detail some examples of how the other groups interacted with the solution see Appendix B.

The following dialogue took place when students found that the two researchers moving relative to each other in Diagram D (see figure 5) assigned different times to the moment the lamps turned on.

STUDENT 1: According to this rule there are several options, because we have t is -2 and t is -3 for the different observers. yeah.

STUDENT 2: Yeah. It depends on who is the observer.

TEACHER: If we have the 2 observers what is going on? Is one too slow or didn't measure right? or...?

STUDENT 3: One is moving too slow.. to the left I mean

STUDENT 4: No, to the right. It starts here. [*pointing at t=0*]

STUDENT 1: Hmm... Yeah... it's not logical. It's eh yeah. I don't know what to say about it. It's not logical that the light source turn on at two different time periods and they still see the light at the same time... together. It's weird.

TEACHER: Do you think the rule is wrong?

STUDENT 1: Yeah. I do.

TEACHER: Does this provoke any questions in you?

STUDENT 1: Yeah of course. What's wrong with the rule? and what's the correct answer?

STUDENT 4: Yeah. I don't get it because if you just don't look at the researcher on the car beneath Eh... you get like 3 different speeds for the light. Either is one box or it's three boxes per time unit or it's 2 boxes per time unit if you just look at the... relative to the universe.

STUDENT 1: I think this is not but... the conclusion we concluded from the experiments doesn't apply here so the conclusion must be wrong because... oh... well... it can be correct but then it's just very illogical

TEACHER: Then we agree that the rule is wrong? [*Students respond with surprise and hesitation*]

STUDENT 1: Well... you can't be sure about it. I'm not sure if it's right, but also I'm not sure if it's wrong. I mean this is not proved, is just thinking that is wrong.

TEACHER: Our rule was supported by experiments and we follow like... logical steps. We think it's right, isn't it?

STUDENT 1: Yeah

TEACHER: To obtain our conclusion. So we could say our rule is right, and this is weird. But that it is weird doesn't mean it is wrong, is it?
STUDENT 1: No, it doesn't mean it is wrong.
TEACHER: It just tell us that the blue guy and the white guy see different times for when the lights turned on. So it can tell us that if our rule is right something weird is happening with time.
STUDENT 1 AND STUDENT 2: Yeah!
STUDENT 4: There is something we don't understand yet or that is wrong. And it has to do with the time or either it has to do with our rule.
TEACHER: The consequence of our rule for now is?
STUDENT 1: That they both see that lights turn on at different moments.

We can observe that students found illogical having two solutions for the diagram when the light and lamps were the same and the observers where in the same place at the same time. Their first reaction was to reject the rule. Student 2 says *he conclusion we concluded from the experiments doesn't apply here so the conclusion must be wrong because...* but the he goes on and say *oh... well... it can be correct but then it's just very illogical.*

When the teacher suggested that the rule was simply wrong students responded with hesitation *Well you can't be sure about it* and instead decided that there was something goin on and that they had no enough information to make a decision at that point.

To respond to the question we set for this part of the phase:

- *Do students hold to the light postulate when the second researcher is introduced?* We don't have enough data to answer this question. Only one group discussed the light postulate as the reason of the difference in time. Although all students found shocking that there was an answer for each researcher, only one group refer back to the rule, and their first reaction was to reject it. Most of the groups did not express any relation with the light postulate.

Looking for evidence to answer the research question

Phase 4 and 5. Time dilation and length contraction

From now on we will not discuss how students arrived to time dilation and length contraction effects since those points are not the focus of this research. This has already been addressed by Kamphorst (2018) and as the lesson plan we implemented follows such work we will consider such point already addressed.

In the following we will only discuss small fragments that can help us to answer the questions we set for this phase and that will help us to give an answer to our research question.

- Group 1

This dialogue took place when students started to work with Diagrams G1 and G2 (see Phase 4).

STUDENT 1: I see the yellow person/observer in the box and there's I think two mirrors below it and the blue observer is moving relative to the box, and the mirrors are also moving relative to the box with the same speed as the blue observer so there is no relative speed between the mirrors and the blue observer.
STUDENT 2: I think the box is moving because the blue observer and the mirrors are at the same place [*in the grid*]
STUDENT 1: Yeah that's true. [...] I was mixing this two up [*pointing to different times*]
STUDENT 2: The box with the yellow observer in it is moving one to the right each time unit
TEACHER: In both diagrams?
STUDENT 2: No, in diagram G1 and in G2 the blue observer is seemingly moving to the left each time
TEACHER: Does it matter which one moves? Is the one that really moves?
STUDENT 2: No. They are both like in the same situation and it is just how you draw them. From a different point of view that you get two different results
STUDENT 1: Yeah.

- Group 2

This dialogue took place when students were reflecting on the recently obtained equation for time dilation.

TEACHER: What have we learnt about the duration of a process according to different observers with speed relative to each other?
STUDENT 1: I think the faster the other one is moving in relation to the object more the time differs, t prime differs from t
TEACHER: Which researcher is right? What is the correct time duration?
STUDENT 1: You mean from the two experiments?
TEACHER: Yeah
STUDENT 2: When the speed is zero, that's the correct one, when the researcher isn't moving
TEACHER: So the other one is wrong?
STUDENT 2: No, because I think if you measure a little bit bigger time you also have to take into consideration the speed and with this thing [*the equation*] you go back to normal time.

From the first dialogue we can see that Student 1 is already defining movements relative to something *"the blue observer is moving relative to the box, and the mirrors are also moving relative to the box with the same speed as the blue observer so there is no relative speed between the mirrors and the blue observer"*. Then later student 2 uses the grid to express that the blue observer is moving, although he does not make explicit that he is comparing with the grid to define movement.

Then Student 2 mentions that in G2 the blue observer is *seemingly* moving to the left. This could be interpreted as the perspective of the blue observer (shown in

Diagram G1) being the correct one. However when prompted by the teacher *Does it matter which one moves? Is the one that really moves?* Student 2 quickly acknowledges that that both situations are equally valid *They are both like in the same situation and it is just how you draw them.*

From the second dialogue we can observe that Student 1 also defines speed as relative to something *the faster the other one is moving in relation to the object.* When prompted by the teacher students still speak of a right time *When the speed is zero, that's the correct one, when the researcher isn't moving*, however they do not say that the other time is wrong, instead they mention using the equation to go back to *"normal time"*

Now, to answer to the questions set for phases 4 and 5:

- *Do students still talk in terms of absolute movement? (Is there a researcher that is not moving at all?)*
  No. Students generally make reference to things moving relative to something else. There are many times in which they leave out their reference points and they seem to talk in terms of an absolute movement. However when inquired by the teacher they quickly acknowledge that you can choose "who is not moving". The following extract took place when students were looking for the proper time to use it to obtain length contraction, and it clearly illustrates how students recognize that in each reference frame there is a proper time.

TEACHER: So what would be the proper time?
STUDENT 1: I still think is A because is more logical for a rocket to move than for a planet to move towards the rocket
STUDENT 2: I think in this situation [*A's perspective*] it's what he says and A is the proper time and B not, and in this situation [*B's perspective*] I think it's B the proper time, cause here he is in the same place

- *Do students say that only one of the researchers is right?* We do not have enough data to respond this question. Students on their own do not express preference for any researcher. When they are directly inquired about it they usually say that it is not that one of them is wrong.
- *Do students mention that one of the times (or lengths) is the correct one?* Yes. Although they do not say that only one researcher has the right time. Students say that each researcher has their own correct or normal time *in this situation (A's perspective) [...] A is the proper time and B not, and in this situation (B's perspective) I think its B the proper time.*

Conclusions

In this work we tried to answer the following research question:

- **Does addressing the important ideas of the relativity postulate in a lesson plan about the STR for secondary education prevent students from considering the relativistic effects as apparent?**

To give an answer to that question we had to ask ourselves first whether the important ideas of the relativity postulate were properly implemented in the lesson plan.

The three ideas that we were focus on were:

(1) Movement is not absolute, it is defined by comparing with something else.
(2) There are more than one possible reference frames.
(3) All of them are equally valid.

In the previous section we shown that students were able to recognize that there are more than one possible reference frame. Although it seemed that they did not consider the solutions as equally valid. They were always looking for a right solution. However they intuitively used the equivalence of the reference frames to a level that allowed them to obtain the consequences of the theory.

The most difficult idea to transmit was the need of a reference frame to define movement. At the end of the lesson plan students indeed began describing movement as relative to something although they would repeatedly go back to using an implicit reference frame or point.

We believe that something that forced students to define movement in a more appropriate way (making explicit their used reference point) was the discussion with their peers. Since not everyone had the same preferred reference frame it was necessary to start defining movement in a way that other people could also interpret.

Only one of the three intended conceptual goals was timely achieved, the existence of more reference frames. The definition of movement was also achieved but only to a certain degree. Students could make use of it in certain occasions and in many other teacher intervention was needed. The equivalence of reference frames was also achieved only to a certain extent. The equivalence was simply used intuitively, but at a level that allowed students to obtain the consequences of the theory. Students still seem to have a preferred reference frame but it seems that they have shifted that preferred reference frame to the one of the observer.

It seems that students do not consider one of the researchers to be right, but that they are always right in their own preferred reference frame. In which they can measure proper times and distances.

Since we have estimated that the important ideas were transmitted to a certain degree now we can ask ourselves whether our research question was solved.

Does addressing the important ideas of the relativity postulate in a lesson plan about the SRT for secondary education prevent students from considering the relativistic effects as apparent? We believe it does.

During the implementation of the lesson plan students could deal with the time dilation and length contraction effect. Students still make use of a preferred reference frame, but, as it was mentioned before they shifted to have as preferred reference frame the rest frame. From the in group discussion it seems that students are aware of the existence of different reference frames and seem not to have one as the "correct one". However they still choose to work on one, the rest frame.

This is of course only one step towards a better teaching practice for the topic of the STR. The group size in which this lesson plan was small (from 1 to 4 students) and it deeply depends on student participation and teacher contributions.

All students in this study were volunteers and hence we can assume that they were highly motivated students. Because of that it is possible that they participated more actively and more freely since the results of the lessons had no impact on their grades.

Research should still be done in how the lesson scales to bigger groups and how it can be improved to fit the time constraints of a common high school program. This lesson plan took 6 to 7 hours. Much time is spend on making the transition from ”moving from” to ”moving relative to”. Because of that more support should be offer to students so they can give this step more smoothly.

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Appendix A.
A group-shared reference frame

TEACHER: And we had 2 options for A and B as well, right? From Michelson-Morley we know that it's similar to situation B. What was the conclusion of MM?

STUDENT 1: That the speed of light is always the same

TEACHER: According to that statement which of the options in diagram B is the right one?

STUDENT 2: This one because here the speed of the light is the same and here the speed of light differs. I think it is this one because the speed of the light is the same here

STUDENT 1: And the same also here independently of the direction in which the researcher and the light source move in the universe so it doesn't matter. The other moving objects don't matter.

STUDENT 3: Yes. I agree

STUDENT 2: Yes, because here the speed of light is two this way and two that way but here they are different because here the speed of light is three times as big that way and this way

Later in this same group

TEACHER: For De Sitter we agreed that the experiment was similar to diagram A, right? And from the De Sitter experiment can we do the same thing we did with MM, cross out one of the options and choose which one is the right one?

STUDENT 1: But he also says that the speed of the light is also always the same. Hm...

STUDENT 3: Mm... I think that eh... That would be this one with the dots because its moving two squares at the sides on both sides...

STUDENT 1: I agree with him

STUDENT 3: In comparison with the person

STUDENT 2: And the speed would be the same?

STUDENT 3: Yes

STUDENT 1: I agree

STUDENT 2: I agree

TEACHER: Those are the right ones, can we draw a general conclusion from what MM thought us? Which would be the general rule that tell us how light behaves and moves?

STUDENT 1: That the speed of light is always the same and it doesn't matter what objects are moving

STUDENT 2: Yeah

TEACHER: But the speed of light was always the same from the beginning, right? It was 2. So can we add a bit more?

STUDENT 3: The distance it travels doesn't increase if the objects around it are moving so it always stays the same, not like here the distance becomes bigger because the car is moving in a different direction [*All students agree*]

TEACHER: Then it's always the same relative to...

STUDENT 2: Distance. Right? So it doesn't matter what the distance if the speed moves is always the same.

STUDENT 3: So relative to the grid right?
TEACHER: Is always relative to the grid? Also in this one? [*pointing to the one relative to the cart*]
STUDENT 1: No but in here is relative to the person
STUDENT 2: Yeah
TEACHER: And in here?
STUDENT 3: It's also relative to the person
TEACHER: So our rule is?
STUDENT 3: Always relative to the person.
STUDENT 1: The speed of light is always the same relative to the one... to the person who's or the thing that is receiving

In the first extract we can observe that students could also choose one option by using the experimental results. However, this group developed a preferred reference frame that was shared within the group. In consequence they lost the need of making the reference frame used explicit.

In the second extract we can observe that Student 1 seems to see a problem with the speed being always the same. However Student 3 quickly solves it by (implicitly) choosing the reference frame of the observer and explaining how that way the speed is the same (*"2 squares at the sides on both sides"*. Then Student 3 goes on to say *"in comparison with the person"*, making explicit the reference point he used. The group agrees and the discussion about this diagram ends.

From the above we can see that depending on the discussion by the students has a downfall. Students can develop a group-chosen preferred reference frame, and from then they will not make explicit anymore their reference points. We can see that even when they make their first version of the "rule": *"That the speed is always the same and it doesn't matter what objects are moving"* they make use of this shared preferred reference frame.

Appendix B.
Other strategies used to explain time dilation

In the following it will be shown how students in different groups dealt with having found two solutions for Diagram D.

- Group 1

STUDENT 1: This situation [*pointing to his diagram*] is not possible, right? It's not possible for the lights to turn on a single time and for two reference frames to move a different speeds and then observe it the same way, it's not possible.
TEACHER: What is not possible?
STUDENT 1: If two lights turn on, right let's say at the same time, and the blue puppet observed it at the star and the white puppet wouldn't see it the same way, because it's moving from this direction and if two lights turn on here, which makes it correct for this puppet [*the blue*], it would see the left light over here, so a bit earlier, this is not possible. For this to happen the light should have turn on at two different times.
STUDENT 2: If you think about it, if you observe the light at different times in the universe it can be because there are different factors...
STUDENT 1: Yeah, but we are still in the same reference frame on Earth. These two people, they are like... so for example, if you do like, if you overcomplicate it, if you have the red shit from a planet and a person is stationary they will see the red shift to a certain amount but if the other person is moving past them it will see a different kind of red shift because it's cancelling, that's sort of happening here, you can't have two of the scenarios because this is correct for this one and these are correct for this one, so this cannot happen because it will mix the two realities
TEACHER: Ok, yeah. There is a conflict, of course.
STUDENT 1: Yeah
TEACHER: ...but we won't talk about different realities and stuff... there is an easier solution, special relativity offers an easier solution
STUDENT 1: Alright, hopefully

- Group 2

STUDENT 1: I think that... I think it's from where you look... it's relative to... yeah... person 1 or 2... I don't think one of the two is really wrong, but I think it is also strange that if the light would go on at the time that he... that they receive it on the same time but that would be strange if the lights go on in a different time that they arrive at the same time but they have the same speed so that would be strange
STUDENT 2: I don't think it is strange that if for example the light has a certain speed and it goes on earlier... that it has to be coming from farther away to receive it at the same moment. You understand what I'm saying? ok. So the... the guy that is moving is receiving light from t -6 and t -2 in my diagram and he is moving in such a way that the light from t-6 has to travel farther then the light for t-2 right?

STUDENT 3: Yes

STUDENT 2: [*continues*] and therefore it has to go on earlier if he wants to receive it at t -0 at the same moment. So I don't think it is strange that the light goes on at different times.

STUDENT 1: No, that is not what I mean. I meant that the persons are on the same place, but... and they receive the light at the same moment. So if the lights would go on at t -6 and the other at t -2 They would arrive at (for me) the red person [*Student 1 had previously colored the white smiley in red*]. But also for the blue one. But for the blue on it isn't right that is t -3.

STUDENT 4: Yeah! You mean why the person does not receive while he could also receive the light that went on at t -3

STUDENT 2: Yeah...

STUDENT 4: Yeah... I think that is the problem here

STUDENT 2: Yeah...

STUDENT 3: Yes. I agree.

STUDENT 4: That the blue person could also review the light that went on at t -2 and t - 6

STUDENT 2: Well t - 2 not yet but t -6 would be possible because if t -2 it wouldnt receive it

STUDENT 4: Yeah but it arrives here in time for him to see. And he is exactly in the same spot. So why couldn't he receive it?

STUDENT 2: The t - 4 or which one?

STUDENT 4: No, the blue person. He doesn't receive the light from t - 2 at the same time as the other person is receiving it and he is in the exact same spot

TEACHER: It is the same light, isn't it? It's the same light from the same spots. The problem in here is the time. They see that the lights go on at different times. And that's weird. [...] and that's one consequence of our rule, our rule was that light travels with the same speed for all observers. So we have to find out what is going on. Let's explore other situations with 2 observers.

We can observe that in both groups students respond with surprise, but they respond to it in different ways. In Group 1 Student 1 directly rejects the results *this situation is not possible, right?*. Student 2 is more open to the solution and she points that the solution could be somewhere there *it can be because there are different factors*.

In Group 2 students take a while to realize that there is a problem. Student 2 says it is a problem of travel time. Once it has been stablished that it is not, the students do it offer a solution, nor think back to the rule.

From this two examples we can see that students do not think about the rule nor time to solve the inconsistency they found.