Version Control Systems: Diffing with Structure
Thesis

Giovanni Garufi
5685109

Supervisors:
Wouter Swierstra
Victor Cacciari Miraldo

Department of Computing Science
University of Utrecht

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Abstract

Today's version control systems rely on the Unix *diff* utilities to detect which lines in a file have been changed and to merge different changes to the same file. Not all such changes, however, are best represented in terms of modifications to lines of code. This may lead to unnecessary conflicts that must be resolved manually by developers. This paper explores the usage of an alternative algorithm for merging the *syntax trees* of the programming language Clojure. As a result, a significant number of conflicts drawn from existing Clojure repositories may be merged automatically, providing evidence that tree-based algorithms offer better precision than the traditional line-based approach in determining which changes give rise to conflicts.

1 Introduction

Version control systems (VCS) have been steadily becoming an ubiquitous and central tool in programming. Many big projects, with hundreds of collaborators, fundamentally rely on these kind of tools to enable different users to interact with one another and to keep a structured log of the units of change. At the heart of most modern VCS is the Unix *diff* utility. This tool computes a line-by-line difference between two files of text, determining the smallest set of insertions or deletions of lines that transform one file into the other. This sequence of transformations produced by *diff* is called an edit script. When two collaborators have modified the same source file in independent ways and the VCS wants to reconcile the two independent changes into a single one it will attempt to produce a single edit script that encompasses both changes, this is commonly referred to as a merge. This operation is clearly not always possible: if two collaborators have modified the same line the two resulting edit scripts will "overlap", it is not clear which one of the two modifications should be picked. The algorithm used to calculate these merges in most VCS is *diff*3, which works by calculating the two edit-scripts with *diff*, and attempting to merge them.

In general not all merges can be performed automatically; some conflicts will always require a manual intervention: when two people change the same thing in two different ways there is no general way of deciding which should be the resulting transformation; however the diff tool makes a big assumption in considering lines to be the basic units in which change is observable.

The shortcomings of the approach in *diff*3 can be seen in this simple example. Suppose we have the following function in any lisp-like language:

```
(defn head [l]
  (first l))
```

Suppose we make two independent modifications to this function obtaining two distinct versions. The first adds a default parameter to be returned in case the list is empty

```
(defn head [l, d]
  (if (nil? l)
    (d)
    (first l)))
```

The second one changes the name of the function from `head` to `fst`.

```
(defn fst [l]
  (first l))
```

When attempting to reconcile these two changes with the *diff*3 algorithm we will run into problems; despite the two patches are modifying disjoint pieces of the actual code, the result will be a conflict. The reason is that both changes occur on the same line; this is a nuisance as manual intervention is required to solve the conflicts. Furthermore it also introduces some level of non-determinism, as the presence of conflicts may depend on things like indentation instead of being a fundamental property of the transformation.

The core idea that will be explored in this thesis is to design an alternative to *diff* which offers a finer grained control over the units of change. By attempting to extend the *diff* algorithm to operate on an AST that represents the parsed program, we are able to focus on smaller units of change; this allows us to produce more accurate patches.

The drawback of this approach is that it lifts the problem from one on strings to one on trees – computing a patch between two elements suddenly becomes computationally more expensive. Approaches similar to this one have already been explored in previous literature [5, 4, 24]. In these papers, the problem of computing the difference between two trees was always reduced to the problem of computing the difference of a flattened representation of the tree. While the flattened representation makes the problem of computing a difference easier, it makes reconstructing a valid tree from the representation more complex, and patches are more likely to generate ill-structured data. The novelty in the work of Dagand, Miraldo and Swierstra [6] lies in the idea of enforcing a structure-preserving, type-directed approach. On one hand structural information is directly encoded in the patches, so that applying a patch will always produce well-formed tree. On the other hand, the type information encoded in the grammar of the AST is exploited in the creation of a patch, making the process potentially more efficient.

The paper introduces a theoretical and practical framework to define and compute patches between structured data. This framework is generic and makes extensive use of dependent types in order to guarantee structure preserving transformations.

The main contributions of this thesis are the following

- Porting the algorithm presented by Miraldo et al. from Agda to Haskell. This requires some non-trivial setup as we need to replicate the more advanced type-level features that are present in Agda.

- The authors define a non-deterministic specification of the algorithm to compute all possible patches between two ASTs, this implies the solution space is still too large when dealing with data collected from bigger projects. We present different heuristics which can be combined to prune the search space and make the problem tractable.

- Mining data from real-world Clojure repositories. This consists of picking suitable repositories to be investigated and devising techniques to extract real-world conflicts from their history branches.

- Finally, we want to measure the performance of our type-directed structured-diff against the standard *diff*. Competing against *diff* in terms of speed is definitely unfeasible as *diff* has linear complexity, in contrast to the exponential behavior of our implementation. However, we want to investigate if the quality of patches produced by our algorithm is better than the ones produced by *diff*. For this reason we will measure performance in terms of the conflicts that may arise when applying two patches with the same origin one after the other; intuitively more accurate patches should lead to a smaller chance of conflicts in this kind of scenario.

1.1 Overview

The chapters are structured as following: We start by showing a full Haskell implementation of the algorithm presented by Miraldo et al. [6], which is presented in Agda in the original paper. The original algorithm is implemented generically and described through dependent types in the paper; one of the main contributions of this thesis is to provide a Haskell implementation, instantiated for the Clojure language, that is suitable to run experiments on real-world data. Haskell is scheduled to land full support for dependent types in the next couple of years [9], in the meanwhile, they can only be partially supported and require some additional effort to encode. Both Generics and Dependent Types require some language extensions and machinery which are non-trivial in Haskell.

The following section starts with an introduction to dependent types in Haskell as they will be crucial in encoding the structure we want to express in our data types and their transformations. Essentially we want to characterize patches by the transformation they operate on the source code (e.g. the patch that adds an extra argument to a function) as such, we need dependent types to reflect onto the type system the action of a patch on a

certain value. This will assure that any code produced by the algorithm is structurally valid by construction.

Following dependent types we will need to introduce sums of products: these give us a general way to view types and will allow us to define an algorithm that is independent of the representation of the AST for the language we are treating. In this regard there is a fine balance between having a core algorithm which is generic and can be applied to any language, and the use of domain-specific strategies to guide the generation of patches by using knowledge specific to the language in question. Despite the generality of the algorithm, we have instantiated it for the Clojure programming language [8]; this choice is motivated by the general simplicity of parsing languages that derive from LISP and by the need to select a language that is popular enough to have large active projects, available on Github, that will provide us a good sample of data to test.

After presenting the algorithm for a type-directed diff, alongside its Haskell implementation, we will analyze the performance of the non deterministic specification and conclude that it is still to slow for real-world data. To solve this problem we will introduce different heuristics to guide the process of patch generation and analyze their performance and shortcomings. Finally we will introduce the notion of *disjointedness*, a predicate that attempts to capture the intuition that two patches that are not "overlapping" should be mergeable. We will use this predicate to run experiments on conflicts gathered from public repositories on Github and compare the amount of merge conflicts obtained by our approach compared to *diff*3.

2 Background

With time, Haskell's type system has kept evolving from its Hindley-Miller origins and through the use of different language extensions it has gained the ability to express more complex types. In particular, many efforts have gone to add some support for dependently typed programming in the latest years. A dependent type is a type whose definition depends on a value. For example, a list of `Int` is a standard type, but a vector of `Int` of a fixed length `n` is a dependent type, as the static type depends on the dynamic information about how many elements are actually in the list.

One major stepping stone in this direction is the `DataKinds` [3] extension which duplicates an ordinary data type, such as

```
data Nat = Z | S Nat
```

at the kind level; this means that from this declaration we automatically get two new types, namely `Z` of kind `Nat` and `S` of kind `Nat -> Nat`.

We can use the `Nat` kind to index generalized algebraic data types (GADTs) in a way that allows us to create an analogue of a dependent type. In the case of `Nat`, we can use it to define a GADT for vectors of a given length.

```
data Vec :: * -> Nat -> * where
  Vn :: Vec x Z
  Vc :: x -> Vec x n -> Vec x (S n)
```

Such a vector is either the empty vector, of type `Vec x Z`, or a vector which is built by adding an element of type `x` in front of a vector of `xs` of length `n`, yielding a vector of `xs` of length `S n`.

This allows us to define safer alternatives to some of the functions which operate on lists. The infamous `head` function will crash our program when passed an empty list; equipped with `Vec`, we can rule this out by construction.

This is how we can define a `head` function on vectors.

```
head :: Vec x (S n) -> x
head (Vc h t) = h
```

Informally, we are saying that the `head` function takes as argument a vector with length strictly greater than 0. In this way, if we try to pass an empty vector to `head` we will get a compile time error instead of the usual run time one.

Another extension which plays a crucial role in dependent types is TypeFamilies: informally it allows us to write functions which operate on types, we will use this to define concatenation between `Vec`s.

The following type family can be seen as a function that takes two types of kind `Nat` and returns another type of kind `Nat` representing the result of adding those two types.

```
type family (m :: Nat) :+ (n :: Nat) :: Nat where
 Z     :+ n = n
 (S m) :+ n = S (m :+ n)
```

Equipped with this type family we can now define concatenation between vectors.

```
vappend :: Vec x n -> Vec x m -> Vec x (n :+ m)
vappend Vn          ys = ys
vappend (x `Vc` xs) ys = x `Vc` (vappend xs ys)
```

One interesting thing to note is that up to this point, we never use the `Nat` part of a vector at run time. That information is only used at compile time to check that everything "lines up" the way it should be, but it could actually be erased at run time.

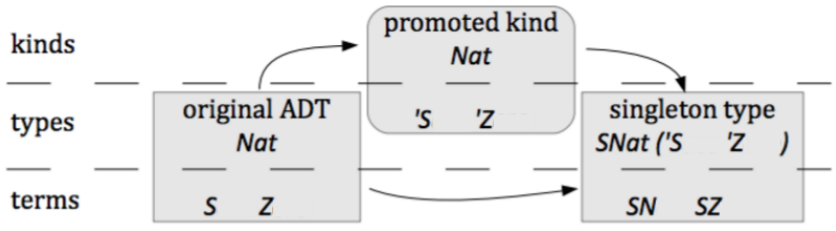
Suppose we want to write a `split` function; this function takes an `n` of kind `Nat`, a vector of length `n :+ m` and splits it into a pair of vectors, with respectively `n` and `m` elements.

The first problem is that we can not pass something of kind `Nat` to our `split` function, in fact the types `Z` and `S` have no inhabitants, so we can not construct any term of those types. Furthermore, we want to express that this `n` of kind `Nat` that we pass as a first argument, is the same `n` as in the vector length. The idea is to wrap this type into a singleton data type, giving us a dynamic container of the static information.

```
data SNat :: Nat -> * where
  SZ ::           SNat Z
  SN :: SNat n -> SNat (S n)
```

The name singleton comes from the fact that each type level value of kind `Nat` (namely `Z` or `S` applied to another type of kind `Nat`) has a single representative in the `SNat` type.

To sum things up: DataKinds extension promotes members of the type `Nat` to inhabitants of the kind `Nat`. Singletons allow us to take one step back in this ladder, associating to every thing of kind `Nat` a term from the singleton type `SNat`. The following picture, borrowed from Eisenberg et al. [2], gives a good representation of this process.



We can think of the DataKinds extension as a way of embedding dynamic information into the static fragment of the language. Singletons, on the other

hand, are a way to reflect this static information back to the dynamic level, and make run-time decisions based on the types we obtain.

Singletons solve the two problems outlined above: they have kind $*$ and contain a `Nat` that we can later refer to in the function type. We can now define `split` as follows:

```
split :: SNat n -> Vec x (n :+ m) -> (Vec x n, Vec x m)
split SZ     xs          = (Vn, xs)
split (Sn n) (x `Vc` xs) = (x `Vc` ys, zs)
  where
    (ys, zs) = split n xs
```

With these three tricks up our sleeve (data kind promotion, type level functions and singletons) we can emulate some of the features that are present in dependently typed languages such as Agda. These features allow us to emulate explicit dependent quantification. We can actually go even further in Haskell: Lindley et McBride [1] show us how to emulate implicit types via type classes and ultimately how all kinds of quantification, modulo some boilerplate, are possible in Haskell. Much like the encoding of a dynamic property such as the length of a vector in its type, we will use the same techniques to represent values of the data type we construct patches on. This extra information encoded in the type will be crucial in guaranteeing the transformations to be structurally and type preserving.

2.1 Sum of Products

The basic idea of the Sum of Products (SOP) approach is to define a "normal form" for all generic representations of data types and to define generic functions by induction on this form. It is, in some sense, similar to the Disjunctive Normal Form for propositional logic in the sense that every proposition can be expressed in DNF and we can write theorems (functions) which assume the expression is represented in DNF. The bulk idea is to view each type as a choice between a constructor and all the arguments that are passed to that constructor. It is useful to think about the two different levels: on the first level we make a choice about which constructor to pick; this corresponds to a sum over all the constructors of the type. On the second level, we are choosing a certain number of arguments to feed to that constructor and this can be viewed as a list, or product, of those arguments. Clearly the products will depend on the choice of constructor, each of which could take a different number of arguments of possibly different types. This

motivates the need to represent the product as some sort of heterogeneous list. A constructor can also take no arguments, in which case we can simply use an empty list to represent that, but can also take an argument of the same type it is trying to construct (like the `S` constructor from the previous section). In this case, the recursive argument can itself be encoded as an SOP and the same encoding can be used all the way down to the leaves.

For example, given the simple data type

```
data T = A Int Int | B String | C String Bool
```

We can view a value of type `T` as a choice between one of the three constructors, together with the list of arguments the constructor requires.

- an `A` constructor and a `[Int, Int]`
- a `B` constructor and a `[String]`
- a `C` constructor and a `[String, Bool]`

Since every regular data type can be encoded as a SOP, we can write functions that act on this representation. For each data type we can first convert it to the SOP representation and then pass it to the desired function. Since the SOP representation is isomorphic to the original type, we can eventually reconstruct the desired term after we are done working with the representation. This encoding is presented by Loh et al. [7] and represents one of the several different ways to approach generic programming in Haskell. In the remainder of this section we will try to formalize the intuition presented above by building the SOP representation of a data type.

2.2 Building our Universe

An AST consists of a family of data types, with a main one which represents the outer structure of the language, and a number of other possibly mutually recursive data types appearing as arguments to the constructors of the main data type. We can define a very simple language which consists of only one data type, `IntTree` which is isomorphic to binary trees of integers. This choice is just for ease of presentation, the following construction can be applied to a family of mutually recursive data types.

```
data IntTree = Node IntTree IntTree | Leaf Int
```

For each type in our family of data types, or any type appearing as an argument to the constructors in our family of data types, we want to introduce a kind representing that type.

Following the example above, we will define the following data type. Keep in mind we only define the type so that the DataKinds extension will automatically generate the corresponding kinds we are interested in.

```
data U = KInt | KIntTree
```

We will also need to associate each type with a singleton, which will allow us to relate terms of our language to their type level representation.

```
data Usingl :: U -> * where
  UInt :: Int -> Usingl KInt
  UIntTree :: IntTree -> Usingl KIntTree
```

Now, we can group all the constructors appearing in our original data types under the `Constr` type in a similar way as we did for the atoms.

```
data Constr = CNode | CLeaf
```

We have now deconstructed our original data type into two levels: `Constr` corresponding to the level of sums (the constructor) and `U` corresponding to the level of products (the atoms).

Since we had to define two additional data types that have no relation between each other we need a way to tie them together on the type level. To achieve this, we define the `ConstrFor` data type, which can be viewed as a proof that a certain constructor builds element of a certain family. In general an AST consists of a family of possibly recursive data types, hence we will need the additional information about what family the constructor `Constr` belongs to.

```
data ConstrFor :: U -> Constr -> * where
  NodeProof :: ConstrFor KIntTree CNode
  LeafProof :: ConstrFor KIntTree CLeaf
```

Finally we must encode one last bit of information: the "shape" of each constructor. To do so, we can use a closed type family which can be viewed as a function on types. This function takes a `Constr` and returns a list of atoms representing the arguments the constructor accepts.

```
type family TypeOf (c :: Constr) :: [U] where
  TypeOf CNode = '[KIntTree, KIntTree]
  TypeOf CLeaf = '[KInt]
```

Since `TypeOf` returns something of kind `[U]` we will define another GADT named `All`, that maps a type constructor `k -> *` over an argument of kind `[k]` giving us something of kind $*$ to quantify over a list of singletons.

The definition for `All` is straightforward:

```
data All (k -> *) :: [k] -> * where
  An :: All p '[]
  Ac :: p x -> All p xs -> All p (x : xs)
```

With this setup we can finally construct the `View` data type; this loosely corresponds to a generic view as sum of products of a data type, and simply deconstructs each term of a type into a constructor and a list of arguments applied to that constructor

```
data View u where
 Tag :: ConstrFor u c -> All Usingl (TypeOf c) -> View u
```

An element of type `View u` represents an element of type `u` deconstructed into its SOP view; the first argument records the choice of the constructor for that data type and the second is the heterogeneous list of arguments that are required to build that constructor, note that we are dependently relating the shape of the product to the actual choice of constructor (the c).

Finally, we want to define a pair of functions that allow us to move from an `Usingl` to a `View` and from that, back to the singleton representation.

The `view` function, defined only for the recursive elements of the language can be implemented as follows.

```
view :: IsRecEl r => Usingl r -> View r
view (UIntTree t) = viewTree t

viewTree :: IntTree -> View KIntTree
viewTree (Node t1 t2)
  = Tag NodeProof (UIntTree t1 `Ac` UIntTree t2 `Ac` An)
viewTree (Leaf i)
  = Tag LeafProof (UInt i `Ac` An)
```

We parametrize this function by the `IsRecEl` constraint which is an empty type-class used to distinguish recursive atoms from non-recursive one. We need this distinction as there is no way to make a `ConstrFor KInt`, hence the type family would not reduce and we would be stuck.

The same holds for the inverse function `inj`: it will take as arguments the constructor, the list of arguments for that constructor and produce the corresponding `Usingl`.

```
inj :: IsRecEl r => ConstrFor r c -> All Usingl (TypeOf c)
  -> Usingl r
inj NodeProof (t1 `Ac` t2 `Ac` An)
  = UIntTree (Node (UIntTree t1) (UIntTreet2))
inj LeafProof (i `Ac` An) = UIntTree (Leaf (UInt i))
```

In the next section we will show the implementation of the type-directed diff. Our functions will operate on the concrete type `Usingl u` which we have shown how to instantiate. It is worth noting that to achieve full-generality we could represent `Usingl` as a type-class instead of a concrete type. This would allow us to keep the implementation of the algorithm generally extensible as to new data types other that the ones we mention it would suffice to give their `Usingl` instance. Techniques for lifting the concrete singleton data type to a type-class are well-known and they were first showed by Eisenberg et al. [2]. Since investigating the performance across different languages was never in the scope of this thesis, the code and the presentation are tied to the concrete `Usingl u` data type.

3 Type-directed diff

The approach presented by Miraldo, Swierstra and Dagand [6] takes advantage of the structure encoded in types to define a generic type-directed diff algorithm between typed trees. The inspiration comes from the diff utility present in Unix; it is at the heart of the current methodologies employed by VCS to attempt to compute a patch between two different versions of the same file. The limitations of the diff algorithm, as it currently stands, is that it does not employ any structural information about the data on which it is trying to calculate a patch.

The underlying idea to the approach presented in the article is to employ the generic SOP view presented to obtain a view of any well defined program as a structured tree of data. We then try to compute a transformation, from one tree to the other, that respects the structural information we acquired.

In the process of transforming one tree into another we need to keep track of three different things:

- Transformations between constructors – These are transformation between internal nodes of the trees.
- Transformations on the product level – When transforming an internal node to another, we might end up in the situation where the nodes have a different number of children. We somehow have to align the children of the source node with the destination.
- Transformation between atoms – The atoms can be either other internal nodes, in which case we recurse down the tree, or leaves in which case we record the pair of source and target leaf.

We will use the simple binary tree language presented in previous sections as a working example to show the construction of a patch. The transformation we will walk through is the following: given the following AST:

```
t1 = Node (Leaf 1) (Node (Leaf 1) (Leaf 1))
```

we want to characterize the patch that transforms it to

```
t2 = Node (Leaf 1) (Node (Leaf 2))
```

The first structure we will employ is the *spine*, this can be thought of as a common skeleton between the two trees which captures the parts that do not change under the transformation.

3.1 Spine

We will define Spines only between two elements of the same family, and defer to later the discussion on how to perform transformation between elements of different families. Calculating a spine for two elements of the same family loosely corresponds to calculating the longest common prefix between two strings. Recall that the two elements x and y are viewed as SOP, in this sense calculating the spine between x and y corresponds to capturing the common co-product structure between them. Let x and y be two such elements, we will have three cases to consider.

- $x = y$
- x and y have the same constructor on the sum level but differ in the arguments

- x and y have different constructors

This gives rise to the following three different constructors for the Spine GADT, each corresponding to one of the cases described above.

```
data Spine (at :: U -> *)(al :: [U] -> [U] -> *) u where
  Scp  :: Spine at al u
  Scns :: ConstrFor u s -> All at (TypeOf s) -> Spine at al u
  Schg :: ConstrFor u s -> ConstrFor u r
       -> al (TypeOf s) (TypeOf r)
       -> Spine at al u
```

Let's not worry about the `al` and `at` parameters for the time being, these will later be used to close the recursive knot and generate the full patch by interleaving the construction showed here and in the following sections.

If the two elements are the same, the spine is a copy. If the top level constructors match, the spine consists of this information and a function to transform the constructor fields pairwise. Lastly, if two constructors don't match, the spine must record this and also contain a description of a transformation the list of source fields into the list of destination fields.

The `Scp` constructor corresponds to the first case, in which we need to record no additional information other than the fact that the two elements are equal. Before looking at the other two constructors, let us focus our attention for a moment to the three arguments that a spine takes: the third parameter of the spine (the `U`) represents the underlying type for which we are trying to compute a patch, the sum-type to which x and y both belong. The other two, `al` and `at` are respectively a description of a transformation between products – that describes what to do with the different constructor fields – and a transformation between atoms, which describes what to do with the paired fields in case we have the same constructor. These parameters are needed for the remaining constructors: the `Scns` constructor corresponds to the case where the constructor is left untouched but some of the arguments have changed. For this reason its second argument consists of the predicate `at` applied to the list of arguments which describes how to transform them.

Finally, `Schg` represents a change of constructor on the sum level: the first two arguments record the source and destination constructors, the third argument is the `al` function applied to the constructor fields of the source and destination constructor respectively.

For now we can simply observe that if we were to calculate a spine between the two programs introduced above we would proceed by constructing

their view as presented in 2.1 obtaining the following

```
v1 = Tag NodeProof (UIntTree (Leaf 1) `Ac` UIntTree (Node
↪  (Leaf 1) (Leaf 1)))
v2 = Tag NodeProof (UIntTree (Leaf 1) `Ac` UIntTree (Leaf 2))
```

These views are not completely equal, but the choice of constructor is, this makes sense as we are transforming a `Node` into another one. The spine produced by these two views will then start with an `Scns` recording the fact that the outer constructor has stayed the same but there are some changes in its arguments.

```
spine = Scns NodeProof _
```

We ignored the second argument up to this point (representing it as an underscore); let's turn our attention to that now: since we know that the constructor is unchanged in the transformation, we also know that both for the source and destination tree, the number and types of its arguments will be the same. For this reason we can simply pair up the corresponding arguments and calculate the diff between every pair. The second argument to `Scns` can be read as: the function `at` applied to a list of pairs of elements of the same type. The type of each pair is specified by `TypeOf s`, which in our working example is equal to `[KIntTree, KIntTree]`. Essentially we have a list of `UIntTree` pairs and are left with the problem of calculating patches between the elements contained in each pair. We can easily see that the first pair of our example will give rise to an `Scp`, the two sub-trees are in fact the same and we can simply copy the information along the transformation. The second pair is more interesting though: this is the case where we have a change on the constructor level and the spine we produce will be the following

```
Schg NodeProof LeafProof _
```

However the remaining argument to fill is not as simple as the `Scns` case since `Node` and `Leaf` expect completely different arguments. In the case where the constructor had remained the same, we could pair up the arguments and proceed from there; however, when the external constructor has changed, there is no obvious way of pairing up the arguments. Indeed they might be completely in different numbers and types, which motivates the following definition of alignments.

3.2 Alignment

The spine takes care of matching the constructors of two trees, alignments handle the products packed within the constructors. Recall that this alignment has to work between two heterogeneous lists corresponding to the fields associated with two distinct constructors. The approach presented below is inspired by the existing algorithms based on the edit distance between two strings. The problem of finding an alignment of two lists of constructor fields can be viewed as the problem of finding an edit script between them. An edit script is simply a sequence of operations which describes how to change the source list into the destination. In the case of *diff* the source and destination lists are the lists of lines in the source and destination files; in our context the source and destination lists are the products of fields of the source and destination constructors respectively. To compute an edit script we simply traverse the lists, from left to right, considering one element from each list. At each step we are presented with three choices:

- We can match the two elements (`Amod`) of the list and continue recursively aligning the rest
- We can insert the destination element before the current element in the source list (`Ains`) and recursively compute an alignment between whatever we have in the source list and the tail of the destination.
- We can delete the element from the source list (`Adel`) and recursively compute the alignment between the rest of the source and the destination.

The following GADT models the sequence of operations that represent an alignment.

```
data Al (at :: U -> *) :: [U] -> [U] -> * where
  A0   :: Al at '[] '[]
  Ains :: Usingl u -> Al at xs ys -> Al at xs (u : ys)
  Adel :: Usingl u -> Al at xs ys -> Al at (u : xs) ys
  Amod :: at u -> Al at xs ys -> Al at (u : xs) (u : ys)
```

For alignments, the `at` parameter plays the same role as in the spine. `A0` represents the empty alignment, `Ains` and `Adel` take as first argument a singleton representing the element being inserted or deleted. These two, together with an alignment for the rest of the list, give us the alignment with an insertion (resp. deletion) as explained in the section above. In the

`Amod` case the first argument is the predicate on the underlying atom that describes how to transform it and the second one, as for the case of insertions and deletions, represents an alignment between the rest of the lists.

One key difference to keep in mind, between the edit scripts produced by diff and the alignments is the atomicity of the elements being aligned (lines and sub-trees respectively). In the case of strings we can assume deletions and insertions to be somewhat equivalent in cost thus we can safely try to minimize one of the two. However, in our case, the elements we are inserting or deleting are sub-trees of arbitrary size, therefore it is not obvious if we should try and prune insertions or deletions.

This poses the problem that when enumerating alignments we have no guiding heuristic to cut the number of solutions, for the time being we will simply ignore the problem and resort to enumerate all possible alignments, to not skew the algorithm into preferring insertions over deletions or viceversa.

Let us walk through calculating the alignment for our running example: we have to produce an alignment between `[KSExpr, KSExpr]` and `[Kint]` (recall that these are the shapes of the `Node` and `Leaf` constructors). Because of the way we define the `Amod` constructor, more precisely because of the `at` function that describes how to transform an atom into the other, we restrict ourselves to only attempt an `Amod` between two singletons of the same underlying type.

This means that in a case like this one, we will only be able to transform one list into the other by repeated applications of `Ains` or `Adel`. It is worth noticing that this restriction is not mandatory and we could in principle allow these transformations as well, the choice here is purely pragmatical and the underlying reason is, this will be a recurring theme, to reduce the sheer amount of combinations that must be checked. What we are left with are the three possible alignments that can be formed by inserting the `Uint` and deleting the two `UIntTree`s, we will generate all of them.

3.3 Atoms

Having figured out all the alignments between two lists of constructor fields, we still have to decide what to do in the case where we match two elements. We need to make a distinction between the possibly recursive fields and the constant ones. In the case of constant fields like `Int`s or `String`s, a transformation between two values of this type consists of a pair recording the source value and the destination value. In the case of a recursive data type we are essentially left with the problem we started from: transforming a value of a data type into another. To do so, we simply start all over again,

recursively computing a spine and an alignment between constructor fields.

To represent pairs of constant atoms we introduce a helper `Diagonal` data type which lifts f over a pair of xs.

```
newtype Diagonal (f :: k -> *) (x :: k)
  = Diagonal { unDiag :: (f x , f x) }
```

To distinguish between recursive and non recursive elements of the language we define a typeclass with no additional methods, and add instances of this typeclass only for the recursive atoms.

Once again, borrowing the language definition from the previous section, we will have the following class and instances defined

```
class IsRecEl (u :: U) where
instance IsRecEl KIntTree where
```

With this we can define the following data type to represent diffs between atoms of our language.

```
data At (recP :: U -> *) :: U -> * where
  Ai :: (IsRecEl u) => recP u -> At recP u
  As :: Diagonal Usingl u -> At recP u
```

Here the `At` data type is parametrised by a predicate that describes how to transform the recursive atoms. The first constructor, `Ai`, which represents the recursive case is parametrised by this predicate, the constraint is added to ensure by construction that when we build an `Ai` we can only do so for the elements of the language that actually are recursive. The other case is covered by the `As` constructor, recall that in this case `Diagonal` simply lifts `Usingl` to a pair of elements of type u, so the first parameter can be read as: a pair of `Usingl u` where the first element is the source and the other is the destination.

In our example we have already seen the case for `Ai`, the atoms paired by the first spine were all `Kexpr`s which we recursively calculated spines on. The `As` will be produced when we match two `Leaf`s that contain different integers, in that case we produce a pair of `Usingl Kint` that record the transformation from one `Int` to the other.

3.4 Recursive alignments

Starting by computing the spine is not necessarily the optimal choice, this can be seen from the following simple example between lists:

```
[ 1, 2, 3, 4 ] -> [ 2, 3, 4 ]
```

Intuitively the optimal patch will proceed to delete the first element and then copy over any remaining one. Our definition, however, does not allow for such deletions. Deletions (resp. insertions) are only handled by alignments at the product level. To handle such cases we can extend our spines and alignments with the data type `Almu` that allows insertions or deletions to happen on the sum level.

A match of constructors will be represented as a spine while insertions and deletions will record the constructor being inserted (resp. deleted) and a `Ctx` which records which fields are associated to that constructor. `Ctx`s are inspeired by Huet zippers [10]: they can be thought as a representation of a type with a hole somewhere; the hole represents the place where we plug in the rest of the tree to continue the computation.

```
data Ctx (r :: U -> *) :: [U] -> * where
  Here :: (IsRecEl u) => r u -> All Usingl l  -> Ctx r (u : l)
  There :: Usingl u -> Ctx r l -> Ctx r (u : l)
```

The `Here` constructor represents the hole, or the recursive position in which we want to carry on the computation. With this definition of contexts, we can finally define `Almu u v`, the data type that represents structured patches between `u` and `v`

```
data Almu :: U -> U -> * where
  Alspn :: Spine (At AlmuH) (Al (At AlmuH)) u -> Almu u u
  Alins :: ConstrFor v s -> Ctx (AtmuPos u) (TypeOf s)
        -> Almu u v
  Aldel :: ConstrFor u s -> Ctx (AtmuNeg v) (TypeOf s)
        -> Almu u v
```

The `AlmuH`, `AtmuPos` and `AtmuNeg` are wrappers around `Almu` s to make source and destination types line up correctly. This gives rise to another occasion for non-determinism, as `Almu` has the same shortcomings of `Al` meaning that we have no obvious choice of what operation should be maximized over the others. As for alignments, we decide to proceed non-deterministically and compute every possibly choice.

3.5 Putting everything together

Now that we have defined all the types we need to represent our patches we are ready to define a function that given two `Usingl u` and `Usingl v`

produces an `Almu u v` – a patch that describes how to transform an `u` into a `v`. We can write this function from the "bottom up", starting from the atoms and working our way up through spines and recursive alignments. Notice how all these functions return a list of results, as non-determinism comes into play at every step. The signatures have been slightly simplified from the actual implementation where, for example, the result is parametrised by a monad, making it more general.

For some functions, we will only present the signatures of the functions in this section, as it allows us to simplify some details of the implementation. The bodies of each function will be replaced by a short description which, given the types introduced up to this point and the supplied signature, will hopefully make it easy to fill in the gaps. The full implementation, which fills in the details for the missing functions, can be found at [**?**]. The diff function for atoms should have the following signature

```
diffAt :: (forall r . IsRecEl r => Usingl r -> Usingl r
          -> [rec r]) -> Usingl a -> Usingl a -> [At rec a]
```

This function is parametrised by a function that describes the treatment for recursive atoms. By inspecting the first singleton we learn whether the atom is recursive or not; if that is the case, the function that deals with the recursive elements can be used to build the corresponding `At`. In the other case, when the element is non recursive, we can simply pair up the two constant atoms with a `Diagonal` and build the non-recursive `At`.

We need to define a function that given a pair of singletons produces all the spines between those two singletons. Remember that spines are parametrised by the alignment that handles the product structure, for this reason we will end up with as many spines as the number of valid alignments. Recall that in a `Spine al at u`, `al` and `at` are respectively two functions that respectively describe how to handle the paired atoms in case the external constructor has not changed (`Scns`), and how to handle the products in the case where the external constructor has changed (`Schg`). These parameters are of kind `at :: U -> *` and `al :: [U] -> [U] -> *`. In the most simple case we can imagine that the treatment of atoms and products consists of simply pairing them up. We can define "trivial" alignments for atoms and products which simply consist of this pairing up. For atoms we can reuse the `Diagonal` type we introduced earlier, for products we need to define a custom `Pair` type.

```
type TrivialA = Diagonal Usingl
data TrivialP :: [U] -> [U] -> * where
```

```
Pair :: All Usingl l -> All Usingl r -> TrivialP l r
```

We compute the diff in two steps: first we produce the spine with the trivial alignments as parameters, then we will map over this spine with our `diffAt`, and another function to generate all the alignments to obtain all possible choices. We can start by defining the function that computes the trivial spine.

```
spine :: IsRecEl r => Usingl r -> Usingl r
      -> Spine TrivialA TrivialP r
spine x y | x == y = Scp
spine x y | otherwise = case (view x, view y) of
  ((Tag c1 l1), (Tag c2 l2)) -> case testEquality c1 c2 of
    Just Refl -> Scns c1 (zipP l1 l2)
    Nothing -> Schg c1 c2 (Pair l1 l2)
```

Here `zipP` is a function that simply zips the two lists together by constructing a `Diagonal` for each pair. In the `Scns` branch, we use the `Pair` constructor to record the two lists of arguments. Recall an `Al at s d` is also parametrised by a function that describes how to handle the atoms. We can start by defining the trivial alignment that uses `TrivialA` to simply pair up elements with `Diagonal`, as for the spine, we will then map over this trivial alignment with a function that takes the pair of recorded elements and produces all patches between them, thus closing our recursive loop.

```
align :: All Usingl p1 -> All Usingl p2
      -> [ Al TrivialA p1 p2 ]
align An           An           = pure A0
align An           (a `Ac` p)   = Ains a <$> align An p
align (a `Ac` p)   An           = Adel a <$> align p An
align (a1 `Ac` p1) (a2 `Ac` p2) = case testEquality a1 a2 of
 Just Refl -> Amod (Diagonal (a1, a2)) <$> align p1 p2
          <|> Adel a1 <$> align p1 (a2 `Ac` p2)
          <|> Ains a2 <$> align (a1 `Ac` p1) p2
 Nothing   -> Adel a1 <$> align p1 (a2 `Ac` p2)
          <|> Ains a2 <$> align (a1 `Ac` p1) p2
```

The implementation is straight forward as we non-deterministically explore all the possible choices, the only exception being that we choose to pair-up two elements only if the `testEquality` test succeeds, which indicates the elements belong to the same member of the family of data types.

Finally we can define `diffS`. The definitions of `mapAlM` and `mapSpineM` are omitted, these combinators are used to map the recursive functions over their trivial counterparts.

```
diffS :: IsRecEl a => (forall r . IsRecEl r => Usingl r
                    -> Usingl r -> [rec r])
       -> Usingl a -> Usingl a
       -> [Spine (At rec) (Al (At rec)) a]
diffS diffR s1 s2 =
       mapSpineM (uncurry diffAt . unDiagonal)
            (uncurryPair $ alignP diffR)
            (spine s1 s2)
  where
    alignP :: (forall r . IsRecEl r => Usingl r -> Usingl r
           -> [rec r]) -> All Usingl s -> All Usingl d
           -> [Al (At rec) s d]
    alignP diffR p1 p2 = do
      al <- align p1 p2
      (mapAlM (uncurry diffAt . unDiagonal) al)
```

We finally have to define a function that computes the diff in terms of `Almu`s. This will call `diffS` in case of two matching constructors from which we can compute the spine wrapping that with the corresponding `Alspn` constructor. In the other cases it will attempt the insertion (resp. deletion) at the constructor level by recording the constructor being inserted (deleted) and producing a `Ctx` which describes where the original tree is attached in respect to the added (deleted) constructor.

This function will have the following signature

```
diffAlmu :: (IsRecEl u, IsRecEl v)
         => Usingl u -> Usingl v -> [Almu u v]
```

As is the case for the alignment between products, here we will simply proceed by enumerating all possible recursive alignments, attempting at each level the alignment of spines, insertions and deletions. One shortcoming of this approach lies in the great combinatorial explosion of possibilities that arises in computing the alignments for constructors and products. We will see in the next sections what we will employ different techniques to keep this in check.

3.6 Applying Patches

Now that we have constructed these type-safe patches we can define how to apply them to an expression to produce a transformed expression.

Application will be defined between a patch of type `Almu u v` and a singleton `Usingl u`. Again, we will proceed defining our functions from the atoms all the way up to recursive alignments. As before, our functions will be parametrised by one or more functions to deal with the recursive elements.

```
applyAt :: (IsRecEl a => rec a -> Usingl a -> Maybe (Usingl a))
        -> At rec a -> Usingl a -> Maybe (Usingl a)
applyAt appRec (Ai r) x = appRec r x
applyAt appRec (As c) x = if old == new then pure x
                          else if old == x then pure new
                          else Nothing
    where (old, new) = unDiag c
```

If we are dealing with a recursive element we can apply the supplied function and proceed the recursive application with that. If the element is non-recursive, we check if it is a copy, in which we can return whatever argument we got. If it is a change instead, we will check if the argument matches the source and return the target.

Alignments will be applied to heterogeneous list of `Usingl u`. The function is parametrised by the previous function we defined over atoms.

```
applyAl :: (forall a . at a -> Usingl a -> Maybe (Usingl a))
        -> Al at p1 p2 -> All Usingl p1
        -> Maybe (All Usingl p2)
applyAl appAt A0 An
  = pure An
applyAl appAt (Amod p a) (Ac x xs)
  = Ac <$> appAt p x <*> applyAl appAt a xs
applyAl appAt (Ains k a) xs
  = Ac <$> pure k <*> applyAl appAt a xs
applyAl appAt (Adel k a) (Ac x xs) = do
  Refl <- testEquality x k
  applyAl appAt a xs
```

Applying an alignment is straightforward, the types guarantee that we can only apply lists of `Usingl`s and alignments which are compatible. This

is reflected by the fact that the supplied alignment has type `Al at p1 p2` which matches the type in `All Usingl p1` and the result produced by applyAl has type `All Usingl p2`. We step through the alignment applying each `Al` to the head of the list until we are done.

```
applyS :: IsRecEl r =>
          (forall a . at a -> Usingl a -> Maybe (Usingl a))
       -> (forall p1 p2 . al p1 p2 -> All Usingl p1
          -> Maybe (All Usingl p2))
       -> Spine at al r
       -> Usingl r
       -> Maybe (Usingl r)
applyS appAt appAl Scp x = pure x
applyS appAt appAl (Schg i j p) x = case view x of
  Tag c d -> do
    Refl <- testEquality c i
    inj j <$> appAl p d
applyS appAt appAl (Scns i p) x = case view x of
  Tag c d -> do
    Refl <- testEquality c i
    inj i <$> sAll appAt p d
```

Application for spine is parametrised by a function to apply atoms and one to apply alignments. We inspect the spine and proceed accordingly. If it is an `Scp` we return the argument, if it is an `Schg` we need to inspect the argument and convert it to the SOP view. If the constructor on the sum level matches the source constructor of the `Schg` then we can construct the element with the target constructor and the result of the application on the alignment. Finally, if the constructor is an `Scns`, we start by peeling the argument and turning it into its SOP view. If the constructors match, then we can construct the element with the old constructor plus the result of applying `appAt` to every pair of atoms found by pairing the product in the `Scns` and the view of `x`.

Finally, for the case of recursive elements, we can give the following implementation.

```
applyAlmu :: (IsRecEl u, IsRecEl v) => Almu u v -> Usingl u
          -> Maybe (Usingl v)
applyAlmu (Alspn s) x
  = applyS (applyAt applyAlmu)
```

```
          (applyAl (applyAt applyAlmu))
          s x
applyAlmu (Alins constr ctx) x = inj constr <$> ctxIns ctx x
applyAlmu (Aldel constr ctx) x = case view x of
  (Tag c1 p1) -> do
    Refl <- testEquality constr c1
    ctxDel ctx p1
```

The case of `Alspn` amounts to simply calling the function we have defined to apply spines with the correct arguments. In case of an `Alins`, we still have to construct an element, it will be obtained by an injection of the inserted constructor and the context. `ctxIns` walks through the context collecting all the singletons and calling the application recursively when it finds the hole. In case of a deletion, we don't have to build anything. We check if the element matches the constructor we intend to delete and simply carry on if it does. `ctxDel` walks through the context, throwing away every singleton it finds and only calling the recursive application once it finds the corresponding hole.

3.7 Disjointedness

The idea that we want to capture with disjointedness is that two disjoint patches should always commute; this means that we can apply them in any order to the source and always get the same result. We want to define this notion only for pair of patches that share the same source, as patches from completely different sources are incomparable to each other. This property is crucial in reconciling patches coming from two different branches. When trying to compute a three way merge, if we can determine that the two patches are disjoint, then we know we can pick an arbitrary order to apply them, and will not end up with a conflict. In the case of *diff3*, two patches are not disjoint when they modify the same line in two different ways. If that is the case, then the two patches must not commute, as whatever patch is applied last, will overwrite the conflicting line with it's own version.

We saw an example of a pair of disjoint patches in the introduction; suppose we have the following Clojure function

```
(defn head [l]
  (first l))
```

If two users modify this function in two independent ways

```
(defn fst [l]
  (first l))
```

```
(defn head [l, d]
  (if (nil? l)
    (d)
    (first l)))
```

then the resulting pair of patches should be disjoint. We have seen that these patches give a conflict under *diff3* because they both happen to modify the same line, albeit being independent.

We can start by attempting to define what disjointedness is on the recursive level. Disjointedness should model the fact that two patches are acting on different parts of the source. This suggests we want to impose the condition that any patch different from the trivial one, is disjoint from itself.

Following this line of though we can start by defining

```
disjointAlmu _ _ (Alins _ _) (Alins _ _)
  = False
disjointAlmu _ _ (Aldel _ _) (Aldel _ _)
  = False
```

When matching an `Alins` with anything else, we extract the focus from the context, and recursively call the disjointedness predicate on the focus and whatever the other argument is. In other words, insertions are always allowed.

```
disjointAlmu (Alins constr ctx) almu
  = disjointFromCtxPos ctx almu
disjointAlmu almu (Alins constr ctx)
  = disjointFromCtxPos ctx almu
```

When we match an `Aldel` with an `Alspn`, we will inspect the spine contained in the `Alspn`. If it is an `Scp` then the two are trivially disjoint.

```
disjointAlmu _ _ (Aldel c ctx) (Alspn Scp)
  = True
disjointAlmu _ _ (Alspn Scp) (Aldel c ctx)
  = True
```

If it is an `Scns` then they are disjoint if the recursive changes within the `Scns` do not change the deleted context and the focus of the context is disjoint from the corresponding changes in the `Scns` product.

```
disjointAlmu  (Aldel c ctx) (Alspn (Scns c' ats))
  = case testEquality c c' of
      Just Refl -> disjointFromCtxNeg ctx ats
      Nothing   -> False
disjointAlmu (Alspn (Scns c' ats)) (Aldel c ctx)
  = case testEquality c c' of
      Just Refl -> disjointFromCtxNeg ctx ats
      Nothing   -> False
```

If the spine is an `Schg` then they are not disjoint: as one patch is deleting a constructor, where the other one is trying to change it to something else.

The only case left is when we have two `Alspn`. In this case we have to look into the pair of spines to decide whether they are disjoint. As before, we can walk through all the cases. `Scp` is disjoint from any other node.

```
disjointS Scp s'
  = True
disjointS s'  Scp
  = True
```

A pair of `Scns` is disjoint if the constructor they fix is the same, and if their fields are pairwise disjoint.

```
disjointS (Scns c p) (Scns c' p')
  = case testEquality c c' of
          Just Refl -> disjAts p p'
          Nothing   -> False
```

A pair of `Schg` is never disjoint, as the two patches are trying to modify the same constructor in two different ways. Finally, when we have an `Scns` and a `Schg`: they are disjoint if the constructor fixed by `Scns` is the source constructor for the `Schg` and if the fields of the `Scns` are disjoint from the alignment in `Schg`.

```
disjointS disjointAt (Scns c p) (Schg i j p')
 = case testEquality c i of
         Just Refl -> disjAtAl p p'
         Nothing   -> False

disjointS disjointAt (Schg i j p') (Scns c p)
 = case testEquality c i of
```

```
        Just Refl -> disjAtAl p p'
        Nothing   -> False
```

Since we learned that the constructor of the `Scns` and the source constructor of `Schg` are the same, we know that `p` and `p'` are acting on the same product of arguments. The predicate `disjAtAl` follows the same pattern outlined up to this point, we step through the elements of the alignment and the list of paired atoms, considering them in pairs. Insertions are always fine as long as the rest of the patches are disjoint. If we find an `Adel` we have to check that the patch on the field being deleted is the identity patch. Lastly, when the alignment contains an `Amod` we can check if the argument of the `Amod` is disjoint from the field.

To do so we need to introduce a function that tells us when two atoms are disjoint, the recursive case can be dealt with a function which will be taken as the first argument to close the recursive loop. Finally, two non-recursive atoms are disjoint if either one of them is the identity (represented by a pair containing the same element).

```
disjointAt :: (IsRecEl a => rec1 a -> rec2 a -> Bool)
           -> At rec1 a -> At rec2 a -> Bool
disjointAt disjointR (Ai r) (Ai r') = disjointR r r'
disjointAt disjointR (As p) (As p')
  = old == new || old' == new'
  where
    (old, new) = unDiag p
    (old', new') = unDiag p'
```

3.8 Clojure

Having developed a general framework to compute patches between typed trees, we know want explore its performance in the context of a real programming language. To test this, we developed a parser for Clojure; the implementation of this parser can be somewhat different to one designed to interpret and run Clojure code, this is because in this context we are more concerned with capturing the syntactical structure rather than the semantical one. One further consideration is that the parser should try to capture as much syntactical information as possible, in order to produce code that strives to respect any syntactical convention embraced by the authors.

The AST will be composed by a family of data types, with the `Expr` type representing the "entry point" for each parse.

```
data Expr = Special FormTy Expr
          | Dispatch Expr
          | Collection CollType SepExprList
          | Term Term
          | Comment String
          | Seq Expr Expr
          | Empty

data SepExprList = Nil
  | Cons Expr Sep SepExprList

data Term = TaggedString Tag String

data Sep = Space | Comma | NewLine | SEmpty
data FormTy = Quote | SQuote | UnQuote | DeRef
data CollType = Vec | Set | Parens
data Tag = String | Metadata | Var
```

This is enough to parse all Clojure code obtained from the test data that we collected; it can actually parse even more than legal Clojure as it does not consider the semantical correctness of the parsed code, only its syntactical coherence. The definition of `SepExprList` can represent lists of expressions separated by either newlines, commas or spaces, which are all legal legal and interchangeable separators in Clojure, we also have an `SEmpty` separator for the `Nil` case.

We can apply the same procedure outlined before to generate the required singletons and type families. The only difference from before is that in this case is that our language is represented by a family of mutually recursive data types. If you recall from section 2 we introduced `ConstrFor` to relate our singleton constructors to the type they were constructing. In case of a single data type representing the whole AST there was no real necessity for this type, and we could have resorted so simply passing `Constr` around. In the case of the Clojure AST we need this additional information, and the `ConsrFor` type becomes (slightly) more interesting.

```
data ConstrFor :: U -> Constr -> * where
  NilProof :: ConstrFor KSepExprList Nil
  ConsProof :: ConstrFor KSepExprList Cons

  SpecialProof :: ConstrFor KExpr Special
```

```
DispatchProof :: ConstrFor KExpr Dispatch
CollectionProof :: ConstrFor KExpr Collection
TermProof :: ConstrFor KExpr Term
CommentProof :: ConstrFor KExpr Comment
SeqProof :: ConstrFor KExpr Seq
EmptyProof :: ConstrFor KExpr Empty

[...]
```

As the constructor name suggests, we can think of these as proofs that that a certain `Constr` maps to a specific `U`. These are straightforward to generate manually, but can also be generically derived.

Finally we will have to add `IsRecEl` instances for the recursive elements of the family, we can define these instances for `Expr`, `SepExprList` and `Term` only, and treat all the other types as atomic.

4 Heuristics

When moving from our toy language to a more complex one like Clojure, we soon run into the limitations of our non-deterministic approach. Non-determinism explodes when we have to compute an alignment, either on the recursive or on the atomic level.

It is easy to see that in some cases, prioritizing deletions can be more profitable and in other it may be better to do the opposite; this uncertainty stems from the fact that at the time we are calculating the alignment we have no information about the size of the sub-trees we are considering.

Since we don't know a priori which alignment is more efficient, in the original specification we simply decide to enumerate all possible ones. This number can grow very quickly and, to make things worse, we are dealing with alignments of arbitrarily large subtrees, which prevents us from optimising towards insertions or deletions.

In this section we will define some heuristics we can use to guide this process of enumeration in order to trim down the amount of computations that need to be carried on. These heuristics fall under two categories.

- One is to explore the possibility of using the standard unix `diff3` algorithm as an oracle to prune the alignment trees that are being generated. The idea is that instead of enumerating all possible alignments between two trees, we can check and see how *diff*3 treats the sequence of lines in which that tree resides in the source. This can

allow us to prune the search space based on the information we can derive from *diff*3 and may be able to speed up the computation to handle larger inputs. This idea can be taken a step further. We can generalise the approach to add an Oracle which, based on some internal state, generates the next branches that should be explored.

We can define different Oracles and explore different strategies to reduce the combinatorial explosion. One of the upsides of this approach is that it will allow us to test different kinds of optimisations in a clean and flexible way; we could even imagine an oracle that interacts with the user, occasionally asking her for guidance into which branches to pursue. Furthermore, we could define a notion of composition between oracles that will give us the chance to combine different optimisations into one.

- Another approach that we could take in the attempt to speed up the algorithm is to define an heuristic to score patches which will allow us to greedily prune the search space. With this approach, the question that arises is: what are the properties of patches for which we can compare and score them? The answer is not clear yet. Informally we want to prefer patches that make minimal modifications and encourage copying as much as possible. This is because if a patch consists of a copy on a certain sub-tree, we can be sure that we can safely merge this with any patch that modifies that same sub-tree. In other words, want to define an heuristic that picks the patch that maximises the chances of it being disjoint from any other patch from the same source.

4.1 Basic Oracles

The goal for Oracles is to be able to have a uniform interface to implement different kinds of optimisations, heuristic and possibly even human interaction in the process of generating all the possible patches.

The key idea, is to extend the algorithm to perform a monadic action at each non-deterministic "junction". The result of this action will be a list that encodes which branches should be explored and which should be cut from the enumeration of patches.

We can start by observing that in both places where we have a non-deterministic choice, we always have to pick between three possible paths: namely to insert or delete something or to match source and destination in a pair. We can model this with a very simple data type

```
data Path = I | M | D
```

Where the three constructors respectively stand for: Insert, Modify and Delete. We want to give our oracles the possibility to inspect the history of issued paths on each branch, this can be modeled with a reader monad.

```
type HistoryM = ReaderT [Path]
```

We can now define our Oracle class

```
class Oracle o m where
  callP :: o -> All Usingl p1 -> All Usingl p2
           -> HistoryM m [Path]
  callF :: (IsRecEl u, IsRecEl v)
        => o -> Usingl u -> Usingl v -> HistoryM m [Path]
```

The Oracle class has two functions, one for the choice on the constructor level (`callF`) (we call this the *Fixpoint* too, hence the 'F' in the suffix) and one for the choice on the product level (`callP`). At each choice, the oracle has access to the history of paths issued on the branch, and has also access to it's internal state (the *o* type). It is important to keep in mind the difference in role between the two lists of paths. The one that is threaded through by the history monad keeps track of the path taken on the current branch. The output of the oracles, on the other hand, is still a list of paths, but this represents possible paths we can take at each non-deterministic choice. As such, it is actually treated as a set, with each different element appearing at most once, and the empty list representing the fact that the oracle can not make a choice at that step.

Notice that the signatures differ in the arguments they take, `callP` is meant to handle alignments on the product level, as such it takes the two heterogeneous lists that are being aligned. The function `callF` on the other hand, is meant to handle the recursive alignments on the constructor level and takes two singletons as input.

4.1.1 NoOracle

To warm up we can start by defining the "unit" oracle.

```
data NoOracle = NoOracle
instance (Monad m) => Oracle NoOracle m where
  callP _ An         An         = return []
  callP _ An         (_ `Ac` _) = return [I]
  callP _ (_ `Ac` _) An         = return [D]
```

```
  callP _ _          _          = return [I , M , D]

  callF _ _ _ = return [I , M , D]
```

This oracle will not contain any global information and will ignore the history of issued paths. It will simply output all possible choices in any non-trivial case.

4.1.2 NoDupBranches

The first optimisation we want to encode is to limit the duplicate branches being explored. It is based on the observation that the order of sequential insertions and deletions does not matter: we can avoid performing an insertion if the last step was a deletion and vice-versa, It is easy to see that an insertion followed by a deletion is equivalent to a deletion followed by an insertion, furthermore – if we can match two elements – then the match will never be worst than an insertion followed by a deletion.

We can define the following function that looks at the history of issued paths to avoid performing an insertion if the last step was a deletion and vice-versa. The last step is attached in front of the list as it is more efficient that traversing the list every time to add it at the end.

```
nextPaths :: [Path] -> [Path]
nextPaths (I:_)     = [I, M]
nextPaths (D:_)     = [D, M]
nextPaths (M:_)     = [I, M, D]
```

Given this function we can implement the `NoDupBranches` oracle

```
data NoDupBranches = NoDupBranches

instance (Monad m) => Oracle NoDupBranches m where
  callP _ An         An         = return []
  callP _ An         (_ `Ac` _) = return [I]
  callP _ (_ `Ac` _) An         = return [D]
  callP _ (s `Ac` _) (d `Ac` _) = ask >>= return . nextPaths

  callF _ s d = ask >>= return . nextPaths
```

4.2 Oracle composition

With the oracles we gain the possibility to tweak the run-time behaviour of the algorithm without having to change any parts of the actual implementation. An advantage of this is that we can define a notion of composition between oracles; this way we can layer different processes, each of which is independent of the other in terms of implementation.

The composition we define wants to model a stack of oracles, the oracles are consulted in the order in which they appear on the stack. When an oracle is called, only if the answer is an empty list we will go down the stack and ask the oracle underneath.

This means that we can build other optimisations on top of `NoDupBranches`, these optimisations can also be partial or based on heuristics, as long as we have a "safe" oracle at the bottom of the stack we can always fallback to the ones below in the cases when it is not clear which choice should be done.

```
  data ComposeOracle a b = ComposeOracle a b

-- Give it a nice constructor
(<°>) :: a -> b -> ComposeOracle a b
a <°> b = ComposeOracle a b

instance (Monad m, Oracle a m, Oracle b m)
  => Oracle (ComposeOracle a b) m where
  callF (ComposeOracle a b) s d = do
    o1 <- callF a s d
    case o1 of
      [] -> callF b s d
      o1 -> return o1

  callP (ComposeOracle a b) s d = do
    o1 <- callP a s d
    case o1 of
      [] -> callP b s d
      o1 -> return o1
```

4.3 DiffOracle

A considerable speedup can be obtained by using *diff* to prune the search space. We can define a data type, which mirrors the definition of a path and

identifies which lines are copied, which are deleted and which are inserted according to *diff*.

We will call this data type `DiffAction` and it will have the following definition.

```
data DiffAction =
  OMod LineRange LineRange
| OIns LineRange
| ODel LineRange
```

Where `LineRange` is a pair of `Int` which represent the first and last line of the corresponding region. We can think of `DiffAction` as some sort of Edit Script, with the difference that in addition to copies, insertions and deletions we introduce modifications. We will produce an `OMod` every time we have two contiguous regions of insertions or deletions between the source and destination file. The regions tagged by an `OMod` are the ones where we attempt to produce a more accurate patch than *diff*.

Note that regions that are not covered by any `DiffAction` are implicitly considered copies. For example, given this pair of source and destination

```
1  (defn function
2    [a b]
3    return a)
```

```
1  ;; A comment
2  (defn function
3    [a b]
4    doSomethingElse
5    return b)
```

The pre-processing will produce `[OIns (1,1), OMod (3,3) (4,5)]`. The first line can be definitely ruled as an insertion, as it is adjacent to a copy on both the source and destination. *Diff* records line 3 of the source as a deletion and lines 4 and 5 of the destination as insertions. This gives us two contiguous regions of insertions and deletions between source and destination and are accordingly recorded as a modification.

Since traversing the whole list of `DiffAction` every time we want to call the oracle is not very efficient we can encode the same information in a pair of maps from `Int` to `Path` (one for the source and one for the destination files). Given this pair of maps, the oracle will extract the `LineRange` contained in each `Usingl u` and lookup the line that corresponds to the beginning of the `LineRange` in the corresponding map. We want to only look at the beginning of the `LineRange` as we want to locally exploit the global solution provided by *diff*. An alternative approach might be to look at the whole `LineRange` and observe how *diff* behaves on the corresponding set of lines.

This however, introduces too much noise at the top-level, not giving us enough chances to prune the tree early on which is where we gain the most from the heuristic. We can define the following helper predicate

```
isMod :: LineRange -> M.IntMap Path -> Bool
isMod lr m = case M.lookup (takeStart lr) m of
  Just M -> True
  _      -> False
```

with the corresponding `isIns` and `isDel`, we can finally define the `giveAdvice` function.

```
giveAdvice :: DelInsMap -> Usingl u -> Usingl v -> [Path]
giveAdvice (srcMap, dstMap) src dst =
  if (isMod srcRange srcMap && isMod dstRange dstMap)
    then [] -- Fall back to underlying oracle
  else if (isDel srcRange srcMap || isMod srcRange srcMap)
    then [ D ]
  else if (isIns dstRange dstMap || isMod dstRange dstMap)
    then [ I ]
  else [ M ]
  where
    srcRange = fromJust $ extractRange src
    dstRange = fromJust $ extractRange dst
```

In case in which the ranges do not match with anything in our maps, we simply want to emit an `M`, as these expressions lie in lines that *diff* identified as copies (we will later see how this may give rise to some problems, as *diff* can copy in a way that our algorithm does not support). When changes in the source and destination overlap, we are essentially in a range where we know that *diff* can not reconcile the changes between them; it will simply delete everything in the source and insert everything in the destination. This is the case where we can attempt to generate a more efficient patch, so we return an empty list to fall back to any underlying oracle that will compute a more precise solution in that range.

Alternatively, if the source is marked as a deletion, or the destination is marked as an insertions, we will emit the corresponding instruction. In the else cases we also check if either the source or the destination is marked as modification (but the other one isn't as the first condition was already showed to be false at this point). We have to handle the case that when transforming a source into the destination, we end up on a branch that has

inserted (resp. deleted) everything it had to, and it simply needs to delete (rep. insert) the remaining range.

Finally, the full oracle can be defined as follows:

```
instance (Monad m) => Oracle DiffOracle m where
  callF o s d = return (askOracle o s d)

  callP _ An         An         = return []
  callP _ An         (_ `Ac` _) = return [ I ]
  callP _ (_ `Ac` _) An         = return [ D ]
  callP o (s `Ac` _) (d `Ac` _) = return (askOracle o s d)

askOracle :: DiffOracle -> Usingl u -> Usingl v -> [Path]
askOracle (DiffOracle da) src dst
  = case (extractRange src, extractRange dst) of
      (Nothing, Nothing)         -> [ M ]
      (Just sRange, Nothing)     -> [ D ]
      (Nothing, Just dRange)     -> [ I ]
      (Just sRange, Just dRange) -> giveAdvice da src dst
```

With this definition, we exploit the fact that the `LineRange` is only defined for the recursive elements of the family, which means that in the case we can not extract it, we can short-circuit the computation since we never pair up non-recursive elements with recursive ones.

4.3.1 Edge cases

One of the issues with this optimisation is that not every modification between source and destination file is correctly identified by this procedure. Since the changes detected by *diff* on the lines do not always map directly to changes on the AST there are some corner cases we have to account for.

Suppose that we didn't include empty expressions in our AST and modeled the top-level parse as either a single expression or a sequence of multiple ones. What happens when try to compute the patch between the following files?

```
1 (keep
2   old keep)
```

```
1 (keep
2   new keep)
3 (new new)
```

The pre-processing will produce `[OMod (2,2) (2,3)]`. However, to perform the optimal patch between these two programs we clearly want to

insert a `Seq` on the first line and proceed calculating the diff from there, but our `DiffAction` does not contain any insertion there. By following the *diff* instead, we realize we had to change the external node when it is too late, we are now looking at line 2 and have already decided to copy over the first line. At its core, the problem lies in the fact that the external change is invisible to *diff*, since it only deals with lines, and the addition of lines at the end does not influence in any way the lines at the beginning of the file. In our case, we have the whole expression tree, and adding a line at the end of the file does not necessarily only trigger changes on the leaves of the tree, but changes may bubble up to the root, as in the example shown.

The problem presented in the previous snippet arises from the fact that – when transforming a top level expression from a single `Expr` to a `Seq` – we only realize that this change has to be made when we get to the second `Expr`. Whenever we change a single expression to a sequence of two by adding one at the end, we will inevitably mark the first expression as a copy, and the second as an insertion. This means it will be "too late" when we get to the inserted line: we already decided to copy a node which leaves us no space to insert the new expression.

A possible solution is to design the AST in a way that prevents this problem from ever coming up. If we modeled the top-level of a program so that the top level is always a `Seq`, and we add an `Empty` constructor to `Expr` an example like the previous one, would not be problematic anymore. Indeed, when we get to the second line, we don't have to change the previous constructor anymore, as the source will conveniently contain an `Empty` expression slot in which we can insert our new expression. This solution has the problem of being very ad-hoc. We have to modify the parser and add support for the new constructors throughout the implementation. Empty expressions turned out to be useful in any case, as we need them to model the patch that inserts into an empty file, but the solution is clearly not satisfactory as it requires us to model our AST in a very ad-hoc way.

A better solution would be to design oracles that can deal with the problematic cases. Sticking to the example presented earlier, we could define a simple oracle which ignores everything, except the case when it gets as input a single expression from the source and a sequence of multiple ones in the destination. In that case we know that – no matter what the other oracles say – we must insert a `Seq` node on the top-level; in that situation we can directly return `I` and skip the other oracles.

This approach is better than the previous one, as it is less invasive. It is still troublesome however that the correctness of the oracle depends on other oracles being present. Also, if we want to add support for other languages

in the future we will have to carefully re-implement the ad-hoc oracles for the new languages. To solve these last lingering issues, we can adopt a different strategy. Ultimately the problem lies in the fact that there are two distinct actors at play: on one hand we have the AST, on the other the lines of code, which can be thought of as a pretty-printed representation of the AST. We are using the line based solution over the pretty-printed AST to derive a solution on the original AST. Our problems arise from the fact that changes to the pretty-printed AST don't always map one-to-one to changes in the AST. If we could somehow run the line based diff on the AST directly there would be no discrepancies. We can obtain a printable representation of the AST that does not collapse the representation of changes like the pretty-printed version does. Since every element of the AST is annotated with the line range which tells us on what line it appears, we can simply traverse the tree and print every constructor on the line the corresponding syntactical element originally appeared. We can now run *diff* to pre-process the ASTs printed in this way. Each change that was detected by running on the original files will also be detected by looking at the ASTs, as they are representations of the content of those files. Below we show the results of printing the ASTs from the two previous examples with this strategy.

```
1  (Collection Parens (Cons (Term (TaggedString "keep" ...  NewLine
2  (Cons (Term (TaggedString Var "old")) ... )
```

```
1  (Seq (Collection Parens (Cons (Term (TaggedString "keep" ... NewLine
2  (Cons (Term (TaggedString Var "new")) ... )
3  (Collection Parens (Cons (Term (TaggedString "new" ... )
```

The problematic case we showed earlier will now be correctly recognized. Indeed, by printing the ASTs of the two files showed earlier, we will immediately detect that on line 1 the constructor has changed from `Collection` to `Seq`. Running the pre-processing on this representation of the AST nets us the desired `[OMod (1,2) (1,3)]`

Another problem, perhaps even more troubling, is that *diff* can mark some parts as copies even though these copies are illegal for our algorithm. Imagine a case where we have this pair of source and destination files

```
1  ((keep1
2    del
3  keep2))
```

```
1  ((keep1
2    ins)(
3  keep2))
```

In this case, according to `diff3` we are supposed to copy lines 1 and 3 and modify line 2. The problem is that – in the source file – we have a `Cons` node that contains all of the copied lines in one of its children. In the destination

however, some of these lines are copied to the left child and others to the right. In the example above this can be seen with the `keep` appearing on line 1 and 3 in the source. Despite belonging to the same expression in the source, they will end up in two different ones in the destination. What happens is that *diff*3 can copy nodes across adjacent sub-trees, but our algorithm does not: we have to pick one of the two sub-trees and attempt the copies only into that.

For example, when we are transforming a `Cons a Nil` into a `Cons c d` the only option we have is to insert `d` in the place of `Nil`. However, according to *diff*, lines in the sub-tree `a` may get copied both to `c` and `d`. In practice this means that when we are at the step in which we have to calculate the patch between `a` and `c`, we are left with inconsistent information, in particular we marked some nodes as copies into `d` when we actually want to delete them.

The problem may occur when matching any two expressions that have multiple children, these expressions will be referred to as sub-trees in this section. In the most simple case we can consider only binary sub-trees, but the same approach can be generalized for arbitrary sub-trees.

To solve this problem we must find a way of detecting when a situation like the one described above arises. Suppose we are computing a patch between two nodes as they appear in the picture above, with a, b, c and d being arbitrary children of these nodes. We will write $i \in a$ to denote the fact that line i is contained in the sub-tree a. Let $CopyLines(P, P')$ be the set of pairs of lines (s, t) that are marked as copies by *diff* when calculating a patch between P and P'. If we can find a pair of lines $s \in a$ and $t \in c$, where $(s, t) \in CopyLines(P, P')$ together with another pair $s' \in a$ and $t' \in d$, where $(s', t') \in CopyLines(P, P')$ then we know we are in a conflicting situation. This condition deals with the case where we had to insert a sub-tree which contains at least one expression marked as copy, to deal with the deletion as well, we must also check that the converse condition does not hold going from c to a and b.

Once we detected the conflicting copy across sub-trees, we can simply remove pairs from the set $CopyLines(P, P')$ until the previous condition is satisfied. In other words, after we detect that a node contains copies over sub-trees, we pick one of the sub-trees and remove all the copies contained in it. The only step that is still missing is about which criteria can be used to pair up the sub-trees we must check. One way to formulate the problem is the following: we run into this issue whenever we have a pair of candidate nodes that are marked as copy on their outer level, but would then later attempt to copy across sub-trees. The crucial observation is that the candidate nodes to check will always start on a line that *diff* will have

marked as copies (this is exactly the issue, *diff* can issue a copy there and move across sub-trees later, our algorithm can not).

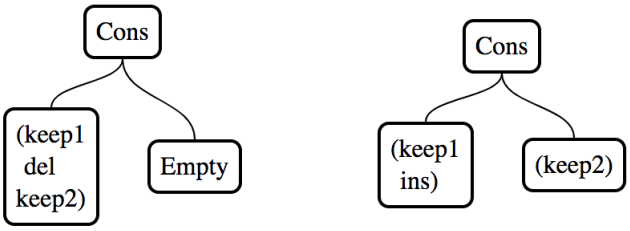
A possible solution, could be to try and solve the problem with some pre-processing. This criteria identified to match nodes suggests an enumeration strategy that we could use to break any eventual conflicts in the `[DiffAction]`. The idea is to identify which lines *diff* marked as copies and collect all the expressions that start on lines (from the source and destination respectively) so that we can carry out the check on them. The final observation we need is that, when collecting expressions starting on a pair of lines (s, t) – where s is a copied line from the source file that ends up on line t in the destination – then we will collect the same number of sub-trees from s and t. This happens because, since the line is unchanged between source and destination, it must contain the same number of expressions starting from it, regardless of what happens in subsequent lines. For example, we can see that, in the example presented above, the source file has two sub-trees starting on the first line: the whole expression and the inner one contained in parenthesis. The destination has two sub-expressions as well, with the first one being the same, but the other one stopping on line 2.

Moreover, we observe that by following the DiffOracle we will never pair these expressions with any other expression coming from a different line; meaning that we can zip up the two lists of collected sub-trees and simply check each pair. Given, a function `CollectST(P,s)` that takes as input a program `P` and collects all sub-trees starting on line `s` in `P`. For each pair $(s, t) \in CopyLines(P, P')$ we can perform the following steps:

```
let src = CollectST(P,s)
let dst = CollectST(P',t)
map check (zip src dst)
```

Where `check` is the predicate we defined earlier to check if two expressions are conflicting. At this point, if check fails, we have a pair `(s,t)` of lines which *diff* identified as copies but our algorithm can not. Once identified the critical pairs of lines we can remove pairs from $CopySet(P, P')$ until the predicate is true.

For example, the set of copied lines in the example showed above is simply $\{(1, 1), (3, 3)\}$, meaning that line 1 in the source will get copied to line 1 in the destination, the same holds for line 3. By zipping the expressions obtained by `CollectST(source,1)` and `CollectST(dest,1)` we will eventually consider the following pair.



We can easily see that this pair of sub-trees will fail the check as the left child in the source contains lines 1 and 3, and these lines end up to the left and right respectively in the destination.

4.4 Cost

Up to this point we have seen how the algorithm generates a list of patches, we have explored some techniques to reduce the size of this output list but we still have never considered the problem of which patch choosing from the output. Given the notion of application defined in 3.6, we can consider patches as partial functions. That is: a patch `Almu u v` is a partial function where the domain is the set of values of type `Usingl u` and the codomain is `Maybe (Usingl v)`, where the function is obtained through patch application. A partial function is more accurate than another, if it succeeds in producing a result to more input data – if one domain is a subset of the other. We can formally define this as

$$p1 \leq p2 \iff \forall x.\ p1\, x \leq p2\, x$$

Where the point-wise comparisons follows the canonical partial order on `Maybe`. This definition – while capturing our intuition faithfully – is too extensional to be practical. The approach we will take is to assign a natural number to every patch, representing its cost. In this way we can compute this value by just traversing the patch, ideally this ordering should respect the extensional definition. Intuitively the best patch between x and y is the one that fixes as little elements as possible in the domain and range of its application.

We will count the elements that are fixed by performing insertions, deletions or replacement of non-recursive atoms. With this in mind we can start defining the cost function for trivial patches.

```
costK :: TrivialA u -> Int
costK c = if old == new then 0 else 2
  where (old, new) = unDiag c
```

If the patch is a copy then no element is fixed, if it contains a replacement instead we fix one item in the domain and one in the range giving a cost of 2.

In the general case of atoms, we just need to distinguish between recursive and non-recursive elements and call the appropriate function.

```
costAt :: (IsRecEl a => rec a -> Int)
       -> At rec a -> Int
costAt costR (As pair) = costK pair
costAt costR (Ai spmu) = costR spmu
```

With this function we can now define the cost for alignments.

```
costAl :: (forall a . at a -> Int)
       -> Al at p1 p2 -> Int
costAl costAt A0 = 0
costAl costAt (Adel a al) = costUsingl a + costAl costAt al
costAl costAt (Ains a al) = costUsingl a + costAl costAt al
costAl costAt (Amod at al) = costAt at + costAl costAt al
```

Here we need a cost function over `Usingl` s, intuitively we want to assign a higher cost to a patch that deletes (resp. inserts) bigger elements. We can assign a cost to each `Usingl` and to each type in the family that constitute our target language by counting the number of choices that are fixed by each element. In the case of BinaryTrees – which have been our running example – this translates to

```
costUsingl :: Usingl u -> Int
costUsingl (UInt u) = 2
costUsingl (UIntTree t) = 1 + costIntTree t

costIntTree :: IntTree -> Int
costIntTree (Node t1 t2) = costIntTree t1 + costIntTree t2
costIntTree (Leaf i)     = 1
```

We can now define cost for spines.

```
costS :: (forall a . at a -> Int)
      -> (forall p1 p2 . al p1 p2 -> Int)
      -> Spine at al u -> Int
costS costAt costAl Scp = 0
costS costAt costAl (Scns c p) = sumAll costAt p
costS costAt costAl (Schg i j p) = costAl p
```

The definition for the function `sumAll` is omitted: it simply computes `costAt` over each pair of elements and sums all these together.

Finally, we are only left with the recursive alignments to handle. The case of `Alspn` has been handled already, if we match on an `Alins` or an `Aldel`, then we want to add 1, which represents fixing the choice of the external constructor being inserted or deleted, and the cost of the context.

```
costAlmu :: Almu v u -> Int
costAlmu (Alspn sp)
  = costS (costAt costAlmuH) (costAl (costAt costAlmuH)) sp
costAlmu (Alins c ctx) = 1 + costCtxPos ctx
costAlmu (Aldel c ctx) = 1 + costCtxNeg ctx
```

The cost of the context, as in the case for alignments, sums the cost of all the `Usingls` in the context with the result of recursively calling `costAlmu` when we reach the hole.

4.5 Bounded Search

The last optimisation we are going to present is very simple but will allow us to gain just enough benefit to be able to handle the majority of our test data. Bounded search is a widespread technique employed when we have an exponential number of solutions to a problem and a *quality* function which assigns a value to any (possibly partial) solution. We can start the search process by establishing an upper bound to the quality of solutions we want to consider.

As we move through the solution space we want to keep track of the current quality of the solution, and prune every branch that exceeds the imposed bound. This process is not guaranteed to generate a solution, the upper bound to the solution quality might be too low. For this reason, bounded search implementations usually have some strategy to restart the search in such cases, increasing the bound by a suitable amount. On the other hand, if the bounded search terminates, we know that the optimal

solution (where optimal means minimal according to the quality function) must be in the produced results.

A perfect candidate for the quality function is the notion of cost defined above. However we want to adapt it so that it can work on partial solutions, furthermore we want to be sure to apply the pruning as soon as possible and reduce the number of steps performed in a branch that will be pruned. We can extend our History monad to store an `Int` which will represent the cost of each branch. Now we just have to change the algorithm to update this local cost at every step and check if the bound hasn't been exceeded at each step. We can omit the details of how this cost is assigned, as it mirrors exactly the definition of the cost function over patches presented in 4.4. We will thread the current cost across the computation at each alignment step (recursive and non-recursive); we will add the cost of performing that step to the total and, if we exceed the supplied upper bound, prune the branch immediately.

4.6 Visualization

Patch objects produced by the algorithm are isomorphic to trees. For this reason, inspecting them by hand is often slow and error-prone. To make the inspection of these objects easier we wrote a simple interactive visualizer. The visualization relies on treantJS [11], a library for the creation and manipulation of tree structures. To take advantage of the library we only have to define `ToJSON` instances for our patch type and – depending on how we want to represent insertions and deletions in the tree – possibly for each type in the family that constitutes our language.

Deletion and insertion nodes (both recursive and in alignments) are color-coded to be respectively red and green – and this coloration is inherited by child nodes and edges. Patches on non-recursive elements are represented as a single element if the patch was a copy, or by the pair of elements in the other case. Finally, nodes are collapsible, and the visualization defaults to collapse every node that only contains copies among its children. This allows us to keep the size of the visualized tree restrained, and allows us to read off the important information contained in a patch with ease.

This tool has proven to be very useful in the development process, allowing quick analysis and comparison of the produced patches. Deriving `ToJSON` instances can also be automated, making the visualization easily extensible to the treatment of other target languages. Figure 1 shows a representation of the transformation between the following pair of expressions

```
Node (Leaf 1) (Node (Leaf 1) (Leaf 1))

Node (Leaf  1) (Node (Leaf 2))
```

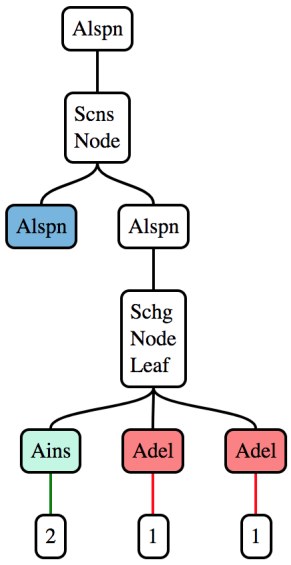


Figure 1: A patch between Binary Trees

The produced patch correctly shows a copy on the left sub-tree and an `Schg` on the right one: from a `Node` containing two leaves, to a single `Leaf` containing a 2. The alignment produced must change the pair of 1s supplied to the source `Node` to the single 2 that is the argument to `Leaf`. It does so by inserting the 2 and deleting the 1s.

5 Experimentation

5.1 Domain specific conflict resolution

Before presenting the results we want to introduce a relaxation of the definition of disjointedness. Many of the conflicts appearing in the collected test data share a common pattern. These are conflicts that arise from modifications to configuration files, which in Clojure are often expressed in the language itself via the `defproject` macro.

For example:

```
(defproject project-name "1.2.3"
  :dependencies [ [dependency-1 "0.1.1"]
    [dependency-2 "1.1.0"] ] )
```

It is very common for these files to give rise to conflicts due to modifications in the required version of the different dependencies by commits which are either merged into, or rebased on top of the development branch.

Suppose this code gets changed in two different ways

```
(defproject project-name "1.2.4"
  :dependencies [ [dependency-1 "0.1.1"]
    [dependency-2 "1.1.0"] ] )

(defproject project-name "1.3.0"
  :dependencies [ [dependency-1 "1.0.1"]
    [dependency-2 "1.3.0"] ] )
```

We can define two patches to be structurally-disjoint if they only differ on non-recursive atoms. Given a pair of structurally disjoint patches we can employ some domain specific merging strategies which may automate or drastically reduce the user effort required to perform the merge. In a case like the one described above, most of the conflicts are resolved by picking the highest version number every time there is a conflict consisting of a choice between two strings encoding version numbers. Nowadays, most projects adhere to SEMVER [12] which defines a standard total-ordering for strings representing versions which partially encode the semantics of the change.

This suggests that conflicts arising from structurally-disjoint patches can often be automatically resolved via some user-defined partial ordering between the atoms of the language. For the case of SEMVER specifically, we could imagine encoding different specific resolution strategies; e.g. we could decide to automatically resolve conflicts between PATCH or MINOR version changes and still notify the user when there is a conflict on a MAJOR version change.

This idea can be pushed even further. By exploiting domain specific knowledge, and the extra information we now have encoded inside a patch, we can come up with custom merging strategies. We can detect certain patters of changes and treat them accordingly in a merge. For instance, suppose that the expression `(map (fn [x] (+ x 10)) l)` was changed into:

```
(map (fn [y] (+ y 10))
     l)
```

```
(map (fn [idx] (+ idx 10))
     l)
```

Knowing that the conflict happened on an identifier, it is conceivable to think of programmable strategies to solve those cases. For instance, the tool could arbitrarily opt for choosing the longer identifier name. Whenever there is a conflict on an identifier named `x` becoming `y` and `idx`, as long as the tool performs this choice consistently, we can solve that conflict.

5.2 Results

In order to test the framework in a real world context we need to find some suitable data. To acquire this we explored all the Clojure repositories on Github and extracted the ones with the best combination of stars and collaborators. A high number of collaborators will possibly imply a higher chance for conflicts in the source tree, the high number of stars is a good indicator of the quality of the Clojure code and hopefully provides a selection of repositories from different domains.

We have collected data from twenty popular Clojure repositories. For each repository, we counted the number of *merge points*. A merge point indicates that files have been changed in two different ways, requiring a merge to reconcile the changes. Merge commits are identified by the simple fact of being commits that have more than one parent, we will restrict our attention to the case of two parents. This is the most common and sensible case for our scenario, as most teams will develop features on different branches and eventually merge each branch into master. This process will generate a merge commit with exactly two parents: one of which is the master branch and the other is the feature branch.

1. For each of these commits we reproduced the process of performing the merge within branches and extracted any Clojure files that were marked as conflicts from this merge.

2. For each of these files we want to extract three different versions of it. The original version `O.clj` represents the snapshot of the file at the moment the branches initially diverged. It is the last version the two branches agreed on. We also want to extract the versions `A.clj` and `B.clj`, which capture the current state of the file on each of the two branches.

From this process we obtained 652 folders – each containing the three different snapshots of the conflicting file. These files have been shrunk with

a pre-processing that relies on *diff3* to remove all top level expressions that are are not involved in a conflict in any way. Each test consists in generating the pair of patches (OA, OB) and checking if they are disjoint, all the tests are run with a one minute time-out. The patch OA is the best patch generated between `O.clj` and `A.clj` according to the cost function, and OB is generated in the same way but with `B.clj` as destination.

Out of the 616 conflicts, 164 time-out while constructing the OA or the OB patch. By excluding these ones we are left with 452 valid conflicts that are reported in the following table, showing results of running the tests for disjointedness and compatibility, both in the full and the structurally-respecting variations.

Table 1 shows the results of our experiment. The first columns describe the number of contributors (Contributors), lines of Clojure code (LOC), the total number of commits (Commits), the number of conflicts that *diff*3 has encountered (Conflicts). Note that for some repositories, only counting the Clojure code present leads to skewed statistics. For example, the **circleci**/frontend repository contains only a fraction of Clojure code, compared to the other languages used.

We classified each result in one of three ways: (A) structural merging gives no conflicts, as the patches are disjoint. As a result, *diff*3 signaled a *false conflict*; (B) the patches are structurally disjoint, as such they can be automatically resolved by using domain specific knowledge to automate the conflict resolution process, we call these *resolved* conflicts; or (C) even merging syntax trees would require human intervention to resolve the conflict. This last category are what we classify as *true conflicts*.

6 Conclusion

From these numbers, we can see that the despite the large number of commits, conflicts are still fairly rare. Of the tens of thousands of commits we considered, only slightly more than 600 resulted in a conflict. Despite these numbers, it is clear that structure-aware diff and merge algorithms manipulating syntax trees gave rise to significantly fewer conflicts than line-based diff algorithms. This provides evidence that employing and developing structure-aware algorithms is a worthwhile pursuit.

There are a few caveats associated with these numbers. Firstly, the structure-aware diff and merge algorithms are significantly slower than their *diff* and *diff*3 counterparts. The finer granularity of change that the structure-aware algorithms may observe results in a significantly larger search space

Name	Contributors	LOC	Commits	Conflicts	A	B	C
marick/Midje	35	14,693	2,416	18	8	2	8
ztellman/aleph	62	4,557	1,064	17	6	5	6
boot-clj/boot	66	9,370	1,271	8	2	2	4
nathanmarz/cascalog	43	8,028	1,366	46	17	14	15
dakrone/clj-http	109	5,193	1,111	5	1	0	4
metosin/compojure-api	36	6,604	1,818	12	1	4	7
wit-ai/duckling-old	65	28,790	586	12	3	2	7
cemerick/friend	33	803	227	1	1	0	0
circleci/frontend	92	894	18,857	27	5	2	20
incanter/incanter	82	16,478	1,282	40	9	22	9
jonase/kibit	47	1,099	401	4	2	0	2
bhauman/lein-figwheel	86	6,515	1,464	6	4	0	2
technomacy/leiningen	315	10,669	4,484	28	12	4	12
clojure-liberator/liberator	42	2,965	347	8	6	1	1
onyx-platform/onyx	46	23,778	6,641	90	46	11	33
overtone/overtone	55	27,935	2,996	50	21	6	23
pedestal/pedestal	59	1,1206	1,403	24	13	5	6
quil/quil	34	1,341	960	10	1	8	3
riemann/riemann	114	16,586	1,654	6	3	0	3
ring-clojure/ring	99	4,909	958	40	4	31	5
Total				452	165	117	170

Table 1: The results collected

of patches between trees. In some cases, the algorithms failed to find or merge patches within the one minute time-out we provided. There is a clear need for further work to optimize these algorithms before they are truly competitive with existing technology.

We have restricted our study to a single programming language, Clojure. It is still unclear if similar studies targeting other languages would produce similar results. We believe that structure-aware diff algorithms have the biggest potential in functional languages, such as Clojure, Haskell, or OCaml [**?**], where code consists of expressions that may be split into lines in very different ways. In imperative languages, on the other hand, the most common unit of code is a statement; programmers typically have a single statement per line of code. To repeat this study for other languages would require some work, such as writing a parser together with diff and

merge algorithms on the syntax trees – yet doing so would provide further insight into *how* code in different languages is organized.

Finally, we observe that – given this specific set of conflicts – we perform approximately 35% better than *diff3* in producing patches that can be safely and automatically merged. We can also see that these numbers drastically increase if we are willing to relax the notion of disjointedness we are investigating. This points to the fact that another strong advantage of this approach – alongside improving the number of patches which can be automatically merged – is in the quality of the conflicts that can be produced. Given more accurate information about what is changing between two files enables us to employ stronger conflict resolution rules, which would have been cumbersome, or almost impossible, to express given the "opaque" representation of conflicts from *diff*

6.1 Related Work

In this thesis we focus on converting the theoretical approach presented in [6] into a practical implementation that can be tested against real-world data. Previous attempts to tackle this problems with similar approaches had, nonetheless, some key differences. The Untyped approach has been extensively studied: with authors focusing both on the linear [16, 17] and the tree [15, 18, 19, 20, 21, 22] variation.

In recent years other authors explored the typed approach [24, 23] in a generic setting. However they restricted their attention to the linear variation, by considering a flattening of the tree consisting in a pre-order traversal. The downside is that this flattening makes it harder to guarantee that the transformation encoded in a patch is structurally preserving and thus correct.

Several pieces of related work exist in the literature: from VCS systems built on strong theoretical foundations like Darcs and Pijul, respectively based on work by Roundy [13] and Mimram [25]. In our experimentation we have used Git to extract the information required for a merge (e.g. picking the common ancestor between two files). It is interesting to note how O'Connor [26] and other authors point out how the strategy adopted by Git in identifying and picking merge points, is inherently inconsistent and can lead to some surprising outcomes compared to other Version Control Systems.

Remarkably there is a formalization of the theory of patches through the lens of Homotopy Type Theory. This has been explored by Licata et al. [14] with the key idea of modeling patches as paths in a suitable topological

space.

Finally, Swierstra et al. [5] showed Hoare calculus augmented with separation logic can be used to reason about patches, in particular in terms of characterizing the relationship between patches and defining safe ways in which they can be combined.

6.2 Future Work

The goal of this work was to explore the performance of the typed, tree-structured diff between data types. The result we managed to obtain are surely encouraging, but are still too sparse and specific when confronted with one of the venerable Unix tools as *diff*.

Strategies for automatic or semi-automatic conflict resolution where just hinted to in this thesis, exploring them in full generality is probably one of the most important and substantial next steps that can be taken. The definition of application should be modified to convey more information, namely the reason why a certain application failed. As mentioned in the section 5.2, we could imagine that some classes of conflicts could be automatically resolved with a custom set of directives specified by end users. Identifying the key use-cases, designing and integrating a framework which allows these custom user-supplied rules is definitely a non-trivial, but possibly very fruitful task.

Finally one of the remaining pieces to fill the puzzle is to explore the level of generality that can be achieved with this approach; while the algorithm is presented generically, the implementation is concretely tied to a specific language: both for convenience of presentation and efficiency. Regardless, for each language, we still need a custom parser to obtain the abstract representation we operate on, which hinders our claim to generality. Of course we can always use a generic strategy to parse a language for which we don't have a more specific parser; in particular one of these generic parsers could be the parser that consumes all characters until a newline. We can imagine that, presented in this context, the original *diff* can be collocated at the left end of a scale; by moving to the right we add structural information, which enables us to characterize transformations, and produce more accurate patches. The cost we pay for this is some loss of generality and the added computational complexity. Thus, the final question is ultimately about investigating this balance, the trade-off between adding information and complexity and the (possibly several) "sweet spots" that can be identified in this spectrum.

References

[1] Lindley, Sam, and Conor McBride. "Hasochism: the pleasure and pain of dependently typed Haskell programming." ACM SIGPLAN Notices 48.12 (2014): 81-92.

[2] Eisenberg, Richard A., and Stephanie Weirich. "Dependently typed programming with singletons." ACM SIGPLAN Notices 47.12 (2013): 117-130.

[3] Yorgey, Brent A., et al. "Giving Haskell a promotion." Proceedings of the 8th ACM SIGPLAN workshop on Types in language design and implementation. ACM, 2012.

[4] Miraldo, Victor Cacciari, and Wouter Swierstra. "Structure-aware version control: A generic approach using Agda." (2017).

[5] Swierstra, Wouter, and Andres Loh. "The semantics of version control." Proceedings of the 2014 ACM International Symposium on New Ideas, New Paradigms, and Reflections on Programming and Software. ACM, 2014.

[6] Miraldo, Victor Cacciari, Pierre-Évariste Dagand, and Wouter Swierstra. "Type-directed diffing of structured data." Proceedings of the 2nd ACM SIGPLAN International Workshop on Type-Driven Development. ACM, 2017.

[7] de Vries, Edsko, and Andres Löh. "True sums of products." Proceedings of the 10th ACM SIGPLAN workshop on Generic programming. ACM, 2014.

[8] Hickey, Rich. "The clojure programming language." Proceedings of the 2008 symposium on Dynamic languages. ACM, 2008.

[9] Eisenberg, Richard A. "Dependent types in Haskell: Theory and practice." arXiv preprint arXiv:1610.07978 (2016). APA

[10] Huet, Gérard. "The zipper." Journal of functional programming 7.5 (1997): 549-554.

[11] "Treant.js." http://fperucic.github.io/treant-js/

[12] "Semantic Versioning 2.0.0." https://semver.org/

[13] Roundy, David. "Darcs: distributed version management in haskell." Proceedings of the 2005 ACM SIGPLAN workshop on Haskell. ACM, 2005.

[14] Angiuli, Carlo, et al. "Homotopical patch theory." ACM SIGPLAN Notices. Vol. 49. No. 9. ACM, 2014.

[15] Akutsu, Tatsuya, Daiji Fukagawa, and Atsuhiro Takasu. "Approximating tree edit distance through string edit distance." Algorithmica 57.2 (2010): 325-348.

[16] Hunt, James Wayne, and M. D. MacIlroy. An algorithm for differential file comparison. Murray Hill: Bell Laboratories, 1976.

[17] Bergroth, Lasse, Harri Hakonen, and Timo Raita. "A survey of longest common subsequence algorithms." String Processing and Information Retrieval, 2000. SPIRE 2000. Proceedings. Seventh International Symposium on. IEEE, 2000.

[18] Klein, Philip N. "Computing the edit-distance between unrooted ordered trees." ESA. Vol. 98. 1998.

[19] Demaine, Erik D., et al. "An optimal decomposition algorithm for tree edit distance." ACM Transactions on Algorithms (TALG) 6.1 (2009): 2.

[20] Bille, Philip. "A survey on tree edit distance and related problems." Theoretical computer science 337.1 (2005): 217-239.

[21] Autexier, Serge. "Similarity-Based Diff, Three-Way Diff and Merge." International Journal of Software and Informatics 9.2 (2015).

[22] Chawathe, Sudarshan S., and Hector Garcia-Molina. "Meaningful change detection in structured data." ACM SIGMOD Record. Vol. 26. No. 2. ACM, 1997.

[23] Lempsink, Eelco, Sean Leather, and Andres Löh. "Type-safe diff for families of datatypes." Proceedings of the 2009 ACM SIGPLAN workshop on Generic programming. ACM, 2009.

[24] Vassena, Marco. "Generic Diff3 for algebraic datatypes." Proceedings of the 1st International Workshop on Type-Driven Development. ACM, 2016.

[25] Mimram, Samuel, and Cinzia Di Giusto. "A categorical theory of patches." Electronic notes in theoretical computer science 298 (2013): 283-307.

[26] "Git is Inconsistent." http://r6.ca/blog/20110416T204742Z.html