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Game Theory of Loss Aversion

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1 Introduction

In game theory, expected utility theory is one of the most known theories for decision making under risk. However, Kahneman and Tversky demonstrated in their article [2] that several classes of choice problems violate the axioms of expected utility theory. The three basic tenets of utility theory; expectation, asset integration and risk aversion are violated by several lotteries as Kahneman and Tversky showed. A good example for this is the Allais paradox. This paradox was introduced by the French exonomist Maurcice Allais in 1953 [4]. Allais' example was similair to the following pair of choice problems:

Lottery 1: Choose between

A:	2,500 with probability	.33,	**B** : 2,400 with certainty
	2,400 with probability	.66,	
	0 with probability	.01,	

Lottery 2: Choose between

A:	2,500 with probability	.33,	**B** :	2,400 with probability	.34,
	0 with probability	.67,		0 with probability	.66,

Kahneman and Tversky did an experiment with these lotteries. They presented these lotteries to students of the University of Stockholm and the University of Michigan. [2] The data [1] show that 82 per cent of the students, who had to choose between the lotteries, chose option B in lottery 1 and 83 per cent of the students chose option A in lottery 2. So in lottery 1, students choose for certainty and in lottery 2, students choose for the lottery with the highest amount they can possible win. If we calculate the expected values of the lotteries, we can see that in lottery 1, option A has a higher expected value. This would mean that it is more obvious for the student to choose option A in lottery 1 since she would earn more than by option B. However, the certainty factor in option B is more appealing, and the 0 in option A is very terrifying for the student, she does not want to get 0. This pattern of choices, choosing 1A and 2B is inconsistent with expected utility theory. According to expected utility theory, the student should choose either 1A and 2A or 1B and 2B. Mathematical proof for the inconsistency of the lotteries is given in the Appendix.

There are a lot of possible explanations for these choices. [1] One of these explanations is the fact that these lotteries are hypothetical. If these lotteries were done with real money and you would really get that amount of money you won, than the choices would possible be different. Experimenters, who study game theory, are aware of the fact that this is virtually not possible to test. Another explanation for why these lotteries are chosen is the fact that small probabilities are overweighted. The difference between 0 and 0.01 will be seen as a bigger difference than between 0.33 and 0.34. In words this is quite obvious to see because with a probability of 0 there is no chance and with a probability of

[1] Data collected by Kahneman and Tversky

0.01 there is actually a chance, but this is a quite small one. This in comparison with 0.33 and 0.34, these probabilities are almost the same but both as likely to happen. There is a possible third explanation, but this explanation will come further in my thesis in section 6.

To come up for these violations of utility theory, a new theory was created: prospect theory [2]. Prospect theory is a behavioral economic theory that describes the decision making process of people who have to choose between probabilistic problems that involve risk. In these probabilistic alternatives, the probabilities of outcomes are known. The theory evaluates the effect that people make there decisions based on the potential value of gains and losses rather than the final outcome. This so called reference dependent utility is the main theory for prospect theory. The outcomes are evaluated with respect to a reference point. With this new investigation about reference-dependent utility functions, a new kind of concept is born, Loss aversion. Loss aversion refers to people's tendency to minimize losses much more than to maximize gain, all relative to a reference point. [1] Loss aversion implies that one who loses 100 dollar will lose more satisfaction than another person who will gain satisfaction from a 100 dollar windfall. [7]

We want to investigate loss aversion more and more by theory. We will do this in my thesis by looking at games and lotteries. First we have to look at game theory. We have to adjust games that we will be using. We have to do this so we can analyze loss aversion with the right conditions in the games. We will be proving some propositions about loss-aversion equilibria, like myopic and non-myopic equilibria. Eventually, we will give some examples that show the principle of loss aversion in games. In my thesis I will come up for an answer about how loss aversion affect the outcomes in the games.

2 Reference points

Expected utility theory was not consequent and a new theory was found by Kahneman and Tversky, Prospect theory [2]. Prospect theory is about the decision making in which decisions are based on the potential value of gains and losses rather than the final outcome. This can be described as reference dependent utility. The outcomes of the games are evaluated with respect to the reference point. This means that reference points are very important in prospect theory and also very important in research into loss aversion.

For our analyses about loss aversion. Reference dependency plays an important role. We want to incorporate reference points in our model. In this way we can look at the affect of the reference point on the utility of the player. Reference points are widely used in prospect theory. In particular, a person, who wants to play a game, has to look at the game first, so she can set a reference point. An outcome above the reference point will be seen as a gain and an outcome under the reference point will be seen as a loss. [5]

If we talk about games, we do have strategies to play for the players and each strategy has its own outcome. This outcome will be seen as payoffs. However, loss-aversion characteristics can not be embodied in the payoffs of the game. This is because of the fact that the utility of the outcome, the payoffs, depend on the reference point. This reference point usually depends on all the possible outcomes of the lottery. For instance, lottery 1 in section 1. If we look at all the possible outcomes of the lottery, 0, 2400 and 2500, and we take the probabilities into consideration, we would say that the reference point of the lotteries is 2400. There is a very high chance, almost 1, of achieving at least 2400. Another lottery with different outcomes would have a different reference point for the player. Thus an outcome will usually have different utility values for different lotteries of which it is a component.

We have to incorporate reference points in our model as well. We assume that the reference points for all the players are given. We are given an extended game, which includes the standard game, a loss aversion coefficient and the reference point for all the players. This game will be transformed into a standard game with the final utilities for the players. These final utilities are the utilities according to the reference points. However, a problem could arise. The reference points are said to be given, but these reference points could be manipulated by the experimenters. An example could be that your reference point is manipulated by experiences and anticipations. Kahneman [6] has done much relevant work about reference points where he talks about this manipulation about reference points. For our investigation, we say that the reference point of a player is fixed at the beginning of the game. We endogineze the reference points into the solution concept. [1] In this solution, the utility of the reference points will be equal to the utility of the outcomes. This is the basic value of an outcome, the utility of the outcome when the reference point is equal to the outcome.

3 Games

Like we said before, we want to investigation loss aversion in games and lotteries. To do this, we need to have a closer look at games. Which factors do have an effect on the outcome of the game and what do we have to take into consideration furthermore. That are things we have to explore to have a good vision about loss aversion in games.

If we look at a game in normal form, the outcomes for the players ignore the possibility of reference dependent utility functions. In the matrix of the game, each player gets a single number to represent the outcome of each possible strategy that can be played. These numbers are the von Neumann-Morgenstern utilities of the players for the outcome given by the strategy profile. Expected utility theory and prospect theory suggest that for any lottery a person faces, the person is assumed to prefer the lottery with the highest expected utility. [2] In these calculations, risk aversion characteristics, like the concavity of the utility function, are taken into consideration, but the loss-aversion characteristics are not. This means that the loss-aversion characteristics can't be incorporated in the payoff numbers of the game. This can easily been proven, because the utility of an outcome in a lottery is depending on the reference point. Since the reference point has to be taken seriously, it would depend on all the possible outcomes of the lottery. Thus an outcome of a game will have different utility values for different lotteries since the outcome depends on the reference point.

We will enlarge the analysis of games to incorporate the loss-aversion characteristics and reference dependance into our games. We will give a formula which relates the outcomes and reference points of the players to there utilities. In my thesis, I will use a loss aversion coefficient λ which will indicate the level of loss aversion. This loss aversion coefficient of all players will be common knowledge in my results, this is because the utilities of all possible outcomes of all players are also common knowledge. With this new coefficient, the final utility outcome dealing with the reference point can be calculated, so the reference point and the loss aversion coefficient are taken into consideration. Now we have created a loss aversion coefficient, we can extend our normal game with this coefficient for all players and there reference points. We can transform this new called extended game into a new game with the final utilities. In my thesis, I will analyse this new game in the standard fashion.

4 Loss Aversion

In decision theory, loss aversion refers to the fact that people prefer avoiding losses much more than they are motivated to get the equivalent gain. [7] This kind of reasoning is almost the same as risk aversion. What distinguishes loss aversion from risk aversion is the reference dependence, so what has happened and will be experienced in the future.

In game theory, the Nash equilibrium is a well-known term. A pair of strategies (p^*, q^*) in a bimatrix game (A, B) is a *Nash equilibrium* if p^* is a best reply of player 1 to q^* and q^* is a best reply of player 2 to p^*. A Nash equilibrium (p^*, q^*) is called *pure* if both p^* and q^* are pure strategies. [3] p. 39. The current set of strategies and the corresponding payoffs constitutes a Nash equilibrium. Since we talk about normal games, which we will transform the standard game to an extended game in which the loss aversion coefficient and reference points are included, we want to find an equilibrium as well. This equilibrium will be defined as a loss-aversion equilibrium.

Definition 1 *(Loss aversion equilibrium)[1]: A strategy profile in which for each player the expected outcome, using loss aversion evaluation, is equal to her reference point, and no unilateral deviation of a player from this strategy can increase her utility.*

Every person has a natural and healty aversion to losing money. Money is besides health one of the most important goods of the human. This loss aversion is in our mind and it helps us to avoid for falling for financial scams. Loss aversion also helps the people for over-spending and everybody likes to save money for days when you need it the most. However, there is another type of loss aversion which we have to define. We define two types of loss-aversion equilibria, which are myopic loss-aversion equilibrium and non-myopic loss-aversion equilibrium.[1] The term myopic loss-aversion refers to the face that people lose sight of the bigger picture, they are way too much focused on what lies immediately in front of us. They focus only at the short-term. In mathematics we refer myopia to the updating of reference points as situations change. Non-myopic equilibrium is the opposite of the myopic equilibrium. Since with these two types of equilibria we are dealing with time, it is easy to see that the two types are the same in simultaneous games because there is no time involved. In my thesis I will show that this is the case in simultaneous games and that they can differ in extensive form games. This is because we talk about reference points and the points can have a multiple interpretations in a loss aversion equilibrium. We use reference point to evaluate payoffs, while we take the values and loss aversion coefficients into consideration. And we use the reference point to the expectation of the evaluated payoffs for every player sinsce these are equivalent to the reference point.

5 Game theory of loss aversion

In my thesis, we want to analyse the effect of loss aversion to people in games and lotteries. The games studied in my thesis and in general in game theory are well-defined objects. The games will mainly be in extensive form. The extensive form can be used to describe games with a time move. The game is presented as a tree with nodes in it and a node represents a point of choice by a player or by nature. We define a game G by the set of players and het von Neumann-Morgenstein utilities. The set of players ,φ, is a finite set. We denote the *pure-strategy space* as S_i for every player i, and payoff functions u_i give player $i's$ von Neumann-Morgenstein utility $u_i(s)$ for every pure strategy profile $s \in S$, where $S = \prod_{i \in \varphi} S_i$. We make the assumption that the utilities $u_i(s)$ are the utility of the outcome when the reference point is equal to the outcome. The utilities are then modified according to their relation to the reference points and the loss aversion characteristics of the players. The outcomes of the games will be pure outcomes and not like lotteries. This all is without loss of generality. The pure outcomes in lotteries will be presented as an additional level in the tree with a move by nature, the lottery move.

Like we said before, we deal mainly with games in extensive form. But we can also represent it as an extended game. We denote this extended game as $(G, (\lambda_i)_{i \in \varphi}$. This game has an extra element, the loss-aversion coefficients of the players. λ_i represents the level of loss aversion of player i by a positive real number, so $\lambda_i \in \mathbb{R}_+$. High values of λ_i represent a high level of loss aversion and $\lambda_i = 0$ means no loss aversion so it characterizes an expected utility maximizer because the utility function of player i is not reference dependent. We define the reference point as $r_i \in \mathbb{R}$. Given this reference point, we take a basic utility value $x_i \in \mathbb{R}$, then the utility of the player with taking loss aversion into consideration is given by:

$$v_i(x_i, r_i) = \begin{cases} x_i & \text{if } x_i \geq r_i \\ x_i - \lambda_i(r_i - x_i) & \text{if } x_i < r_i \end{cases} \tag{1}$$

The valuation of a utility value of x_i if $x_i \geq r_i$ is quiet obvious. The valuation is higher then your reference point so you are very happy with this outcome. However, if $x_i < r_i$ then the outcome is below your reference point, so you are less happy. You expected to earn at least r_i but you earned less than that. The valuation of this outcome will therefore be less than the utility value x_i. The valuation will then depend on your level of loss aversion and your reference point. Is x_i close to the reference point and is your level of loss aversion not that high, then the valuation will still be close to x_i only a little bit less. By using Formula (1) and the von Neumann-Morgenstern utilities, we can now see that the utility function depends on the aspect of loss aversion. When experiencing a loss, the utility will be lower than your reference point, which means that the utility function for losses will be depending on λ_1. This loss aversion aspect is the reason for the fact that the utility function is steeper for losses than for gains, relative to the reference point. This S-shaped value function is proven in prospect theory.[2] The value function is definded on deviations from the reference point. However, we don't get the same value function as in prospect theory. This is because of risk aversion and risk seeking in prospect theory. Risk aversion and risk seeking are both included as possible theories of our specification, but they are not included as a function of the reference point, which is needed in the case of loss-aversion. The loss aversion coefficients in our model are unique for different players to reflect their level of loss aversion.

Reference points are in our model given. This means that we can transform an extended game into a standard game using Formula (1). We evaluate the utility of each outcome by Formula (1). We will do this transformation by using a vector of reference points. So given an extended game $(G, (\lambda_i)_{i\in\varphi})$ and a vector of reference points $r \in \mathbb{R}^{\varpi}$. We now define the transformed game as G' by $L(G, \lambda, r) = (G')$. The new standard game differs from the extended game G only in the utility of the outcomes for the players. These new utilities of each outcome are calculated using Formula (1) and the appropiate reference points and loss aversion coefficients. To illustrate this we take S, the set of pure strategy profiles, then for $s \in S$, $u_i^{'}(s) = v_i(u_i(s), r_i)$ gives the final utilty from the outcome s for player i.

We have now introduced a new game which is the extended game with the loss aversion coefficient in it. We will give an example, battle of the sexes, in which the concept given above is illustrated. First we want to take another look at the Allais paradox that was given in section 1.

5.1 Example1 : Allais paradox with loss aversion.

We have seen before that the modal choices made in the two lotteries are not consistent with expected utility theory. This is proven in the appendix. We now want to see what happens when we calculate the expected outcome of the two lotteries with loss aversion characteristics. We will use formula (1) to do this. In equilibrium, the expected value will be the same as the reference point. Therefore we will take 2,400 as our reference point in lottery 1 and 2. We now want to calculate the expected outcome of both A and B.
Lottery 1:
The expected value of lottery 1A is:

$$A = 0.33 \cdot 2500 + 0.66 \cdot 2400 + 0.01 \cdot (-2400\lambda) = 2409 - 24\lambda$$

The expected value of lottery 1B is: 2400
We can now calculate for which values of λ you choose A and for which you choose B.
$A - B = 9 - 24\lambda$, so if $\lambda < \frac{3}{8}$ then the player chooses A over B, if $\lambda = \frac{3}{8}$ the player is indifferent between the two lotteries and if $\lambda > \frac{3}{8}$ the player chooses B over A. It is interesting to see that when you are very loss averse, high λ, you

will choose option B and for low values of λ you will choose option A.
We will now see what happens at lottery 2. If expected utility theory is correctly used, we will get the same result for the same values of λ.
The expected value of lottery 2A is:

$$A = 0.33 \cdot 2500 + 0.67 \cdot -2400\lambda = 825 - 1608\lambda$$

The expected value of lottery 2B is:

$$B = 0.34 \cdot 2400 + 0.66 \cdot -2400\lambda = 816 - 1584\lambda$$

$A - B = 9 - 24\lambda$ which is the same as we got by lottery 1. This means that if we take loss aversion into consideration, the lotteries are consistent for expected utility theory for the same values of λ. We have calculated the loss aversion utilities for the lotteries. This means that if a person is loss averse, he has a value for his λ, he can calculate which lottery, A or B, he has to choose. However, in contradiction to the example in section 1, if the person chooses A in lottery 1, he also has to choose, according to his λ, A in lottery 2.

5.2 Example2 : The Battle of the Sexes

Figure 1: Game of battle of the sexes

The concept given above will be illustrated by another example with a bimatrix game. We use the famous game: Battle of the sexes. This is a two-player coordination game. It is about a man and a woman who discuss about what to do. The man wants to go boxing and woman wants to go shopping. The payoffs are illustrated in the matrix above where man is the row player and woman the column player. The battle of the sexes has two pure-strategy Nash equilibria, which are given in the matrix: (Boxing, Boxing) and (Shopping, Shopping). Besides the pure-strategy equilibra, there is also a mixed strategy equilibrium with each player playing the strategy with his/her most preferred outcome with probability $\frac{2}{3}$.

Besides being a Nash equilibrium, the pure strategy equilibria are also a loss aversion equilibrium. This will be proven in Proposition 2 later in my thesis.

There is also a mixed strategy loss aversion equilibrium. This equilibrium can be calculated by solving the following equations. In these equations, every player is indifferent between both of his/her strategies (the pure strategies), given his/her reference point. We take the reference point as the utility of the expected outcome. This reference point will be a bit higher then 0. We denote p and q as probabilities, where p represents the probability that the man plays Boxing and q the probability that the woman plays Boxing. We define the player man as player 1 and woman as player 2. The equations for the mixed strategies using formula (1) are:

$$2q + (-\lambda_1 r_1)(1-q) = -\lambda_1 r_1 q + (1-q) = r_1 \tag{2}$$

$$p + (-\lambda_2 r_2)(1-p) = (-\lambda_2 r_2)p + 2(1-p) = r_2 \tag{3}$$[2]

Since p and q are probabilities, they are in [0,1]. Given this interval, we can calculate a unique solution to these equations. These solutions[3] are:

$$p = 1 - \frac{-3 - 2\lambda_2 + \sqrt{9 + 8\lambda_2(2+\lambda_2)}}{2\lambda_2} \tag{4}$$

$$q = \frac{-3 - 2\lambda_1 + \sqrt{9 + 8\lambda_1(2+\lambda_1)}}{2\lambda_1} \tag{5}$$

$$r_i = \frac{-3 + \sqrt{9 + 8\lambda_i(2+\lambda_i)}}{2\lambda_i(2+\lambda_i)} \qquad i = 1,2 \tag{6}$$

Out of these solutions, we can easily calculate the pure mixed strategies as given above. We fill in $\lambda_i = 0$ and we get $p = \frac{2}{3}$ and $q = \frac{1}{3}$. We now want to analyse these solutions to see what happens when we change the level of loss aversion, in other words we want to see what will happen to the solutions for p and q if we increase λ_i. We can plot p, q and r and see what happens.

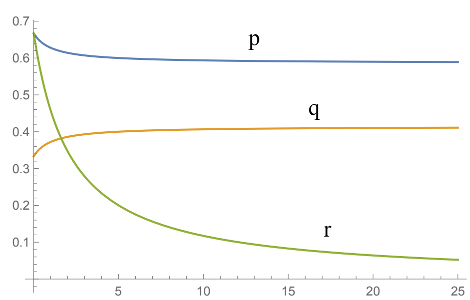


Figure 2: Plot of p,q and r is λ increases

[2]Shalev used the wrong equation for player 2 in his article, this is the correct one
[3]Calculations to come for these solutions are given in the appendix

The solution for p is decreasing as a function of λ_2 and q is increasing as a function of λ_1. Every r_i is decreasing as a function of λ_i. This means that a player who becomes more loss averse, higher λ_i, has a higher probability of receiving his/her preferred outcome in the mixed-strategy equilibrium. However, since r_i is decreasing as a function of λ_i, the player will receive a lower utility when he becomes more loss averse. So being loss averse, helps you to receive your most preferred outcome, but this outcome will be lower since you receive a lower utility because r_i decreases. An intuition for this result is that as a player becomes more loss averse, the uncoordinated outcome that is part of the lottery including his/her most preferred outcome loses more than the other uncoordinated outcome, and the rebalancing necessary to keep the player indifferent between the two lotteries increases the probability of the most preferred outcome. Every player has a unique loss aversion coefficient which is for every player different. A change of the coefficient of a player will have no affect to the other player's payoff in the mixed strategy loss aversion equilibrium, since we have λ_i with $i \in \varphi$.

However, there are a lot of other examples to think of which have unique loss aversion equilibria which show that increasing loss aversion of a player either increase or decrease the payoffs for the player herself and for the other players. If we turn the battle of the sexes into a three player game as you can read in [1], then we get a different result. As player 1's loss aversion increased, her payoff increased, that of player 3 decreased and that of player 2 remained the same.

5.3 Strategies

We have talked about the pure strategy space S, which contains every pure strategy of the game. Besides pure strategy, mixed strategies are also possible. We have seen them in Example 2. A mixed strategy is a strategy in which a player makes a random choice among two or more possible actions, based on a set of chosen probabilities. For instance, a mixed strategy could be that a player chooses strategy 1 with probability $\frac{1}{3}$ and strategy 2 with probability $\frac{2}{3}$ We have to take these mixed strategies in our model as well. We have to extend our utilty function to include the mixed strategies. We will do this by first defining the mixed strategies. We define Σ_i as the set of player $i's$ mixed strategies. All possible mixed strategies of all players will be denoted as: $\Sigma = \prod_{i \in \varphi} \Sigma_i$. In our games we define mixed strategies as a mixed strategy profile $\sigma \in \Sigma$ which gives probability $p_\sigma(s)$ to each pure action profile $s \in S$. We now can give the new utility given by the mixed strategies and given a reference point r_i. The utility of player i is given by

$$w_i(\sigma, r_i) = \sum_{s \in S} p_\sigma(s) v_i(u_i(s), r_i) \tag{7}$$

When we evaluate this equation, we can see that first the payoffs for player i are defined on the outcomes as a function of the players's reference point: $v_i(u_i(s), r_i)$ and then multiplied by $p_\sigma(s)$ because of the mixed strategy. This order is needed, since the payoffs are not linear in the reference points. And this is important because we use the reference point to evaluate every pure outcome and not the expected value of the outcome. In equation (7), we need r_i which is the reference point of player i and we see that this reference point is fixed. Since we have that the reference point is fixed, we can not deal with extensive games that have more than one information set for a player. These information sets will lead to changes in the reference points for the playes as the get new information about the actions of the other players or moves by nature.

Definition 2 *[1] We say a reference point is* consistent *with a lottery, if the utility evaluation of the lottery with respect to the reference point is equal to the reference point. Formally, for a lottery* x *giving outcomes $x^1, ..., x^n$ with respective probabilities $p^1, ..., p^n$, a reference point r_i is a* consistent reference point *for the lottery* x *for a player* i *with loss-aversion coefffecient λ_i if*

$$r_i = \sum_{j=1}^{n} p^j v_i(u_i(x^j), r_i) \tag{8}$$

The set of consistent reference points for an extended game (G, λ) and a mixed strategy profile $\sigma \in \Sigma$ is given by $R_i(\sigma) = \{r_i \in \mathbb{R} | w_i(\sigma, r_i) = r_i\}$. All reference points that are consisent for player i with the lottery are in this set and they are implied by the mixed strategy profile σ.

We define for an extended game (G, λ): $\overline{r}$ and $\underline{r}$. These are given by $\overline{r} = max_{i \in \varpi}\{max_{s \in S}\{u_i(s)\}\}$ and $\underline{r} = min_{i \in \varpi}\{min_{s \in S}\{v_i(u_i(s), \overline{r})\}\}$. This means that for any lottery over outcomes, if the reference points are all in the interval $[\underline{r}, \overline{r}]$, the evaluated utilities of all players will also be in this interval. This is easily seen since we take as a lower bound for the interval the minimum of all utilities and $\overline{r}$ and as an upper bound the maximum.

We will now give a lemma that shows that for any player and any mixed strategy profile, $R_i(\sigma)$ contains a single value:

Lemma 1 *[1] If $(G, (\lambda_i)_{i \in \varphi})$ is an extended game, then for all $i \in \varphi$, and for all $\sigma \in \Sigma$, the correspondence $R_i(\sigma)$ is single-valued, and the value is in the interval $[\underline{r}, \overline{r}]$.*

Proof: Take $i \in \varpi$ and $\sigma \in \Sigma$. $w_i(\sigma, r_i)$, viewed as a function of $r_i \in \mathbb{R}$, is non-increasing and continious, since $w_i(\sigma, r_i) = r_i$. The following three relations hold, defining $u_i = \sum_{s \in S} p_\sigma(s) u_i(s)$:

$$u_i(\sigma) \in [\underline{r}, \overline{r}] \tag{9}$$

$$w_i(\sigma, \underline{r}) = u_i(\sigma) \geq \underline{r} \tag{10}$$

$$w_i(\sigma, \overline{r}) \leq u_i(\sigma) \leq \overline{r} \tag{11}$$

Relation (9) is true since $\underline{r}$ is the global minimum of all utilities $u_i(\sigma)$ and $\overline{r}$ the maximum, so $u_i(\sigma)$ must be in the interval of $[\underline{r}, \overline{r}]$ Inequalities (10) and (11) follow directly out of (9). Therefore, since $w_i(\sigma, \overline{r})$ is non-increasing and continious in r_i, there exists a unique $r_i^* \in \mathbb{R}$ satisfying $w_i(\sigma, r_i^*) = r_i^*$. Moreover, $r_i^* \in [\underline{r}, \overline{r}]$

We can now use Lemma 1 for more defenitions and proofs. Since $R_i(\sigma)$ is single-valued, we can define $r_i(\sigma)$ as a function with this value we just have proven exists. We can calculate this function for a strategy profile σ. This calculation is not just done at the root of the game tree, but can be done at any information set in the game. This will give the consistent reference point for a player at that information set, taking in mind for this player the belief that the strategy profile σ is being played.

5.4 Loss aversion equilibria

We have talked before about loss aversion equilibria, but there are two kinds of equilibria, myopic and non-myopic. We will now define these equilibria correctly so we can use them in our investigation.

Definition 3 *[1] A strategy profile $\sigma \in \Sigma$ is a* myopic loss aversion equilibrium *of (G, λ) if there exists $r \in \mathbb{R}^{\varphi}$ such that σ is a Nash equilibrium of the transformed game $L(G, \lambda, r)$, and the payoff to the players from using σ in $L(G, \lambda, r)$ is r.*

This defenition is a bit more technical than defenition 1. We now have the vector r of reference points for all players and we say that the strategy profile σ is a Nash equilibrium, so no deviation from this strategy will benefit the outcome, if the payoff of the transformed game is exactly the reference point. In our defenition of myopic loss aversion equilibrium, we see two aspects of myopia. First one is that since the reference points of all players are fixed, the evaluation of the strategy profile is done at the root of the tree. The players do not take into account changes of the reference point as the game proceeds. This is an assumption that is sometimes different in real life, but we take the reference points as fixed. The second aspect of myopia is that when we evaluate a deviation, than still the player will not change her reference point. This will be eventhough the distribution of outcomes may change is she changes her reference point. Again, we see that the reference point is fixed and we do not take deviations into consideration. Kahneman [6] discussed how multiple reference points and deviations between them might be used, but we do not take this into consideration.

Definition 4 *[1] A strategy profile σ is a* non-myopic loss aversion equilibrium *of (G, λ) if for all $i \in \varphi$, all $\sigma_i' \in \Sigma_i$, and for all information sets μ of player i that are reached with possitive probability under σ, the evaluation at μ satifies*

$$r_i(\sigma) \geq r_i(\sigma_{-i}, \sigma_i')) \tag{12}$$

where (σ_{-i}, σ_i') signifies that all players $j \in \varphi \setminus \{i\}$ play σ_j and player i plays σ_i'.

This is the defenition of a non-myopic loss-aversion equilibrium. These equilibria mean that if a player takes a deviation into consideration, an appropiate change in her reference point has to be made to be consistent with this deviation. Since the defenition says for all information sets μ of player i that are reached, we evaluate also at every information set that is reached. This means that all availaible information, given by the information sets, is used to evaluate the lottery and the strategy that has to be played. Because we have talked about changing the reference point, non-myopic loss-aversion equilibria are very usable in games where a small change in reference point occur.

We have now introduced two defenitions of loss-aversion equilibria. These defenitions both concern about reference points. Here again we can see that reference points are very important in the analyse of loss aversion.

5.5 Nash equilibrium

We have been talking in this thesis about loss aversion equilibria. The most famous equilibrium is ofcourse the Nash equilibrium. A pair of strategies (p^*, q^*) in a bimatrix game (A,B) is a *Nash equilibrium* if p^* is a best reply of player 1 to q^* and q^* is a best reply of player 2 to p^*.[3] Nash proved that if we allow mixed strategies, then every game with a finite number of players in which each player can choose from finitely many pure strategies has at least one Nash equilibrium. We could now look at the loss aversion equilibrium as well as the Nash equilibria in a game. There is a similarity between these two equilibria. We take any extended game (G, λ) with $\lambda_i = 0 \forall i \in \varphi$. Then the set of loss aversion equilibria, myopic and non-myopic, of (G, λ) is the same as the set of Nash equilibria of the standard game G. This is quite straightforward since out of the defenitions, we can conclude that if for a player $\lambda_i = 0$ then any lottery gives the expected utility of this lottery.

6 Propositions

We have introduced in previous sections several loss aversion equilibria. In this section we will take a closer look at these equilibria and see if they coincide in particular situations or if a equilibrium is always findable in the game, like we can say about several games and Nash equilibria. The first proposition we prove shows that the myopic and non-myopic loss-aversion equilibrium are exactly the same for games where each player has only one information set, which is always reached. For these games, since each player has only one information set, time is not important because every player knows what he has to play. The information for the player at the beginning of the game is the same as the information at the end of the game so deviations in strategy are not possible, since the information at every node in the tree is the same.

6.1 Proposition 1

Proposition 1 *[1] For any extended game where each player has exactly one information set, which is reached on every path of play (a simultaneous game), the set of myopic loss-aversion equilibria coincides with the set of non-myopic loss-aversion equilibria.*

Proof: We take an extended game (G, λ) in which every player has one information set that is reached on every path of play, so it is a simultaneous game. From the defenition of myopic loss-aversion equilibrium and Lemma 1, the set of myopic loss-aversion equilibrium is the set of $\sigma \in \Sigma$ that satisfy

$$w_i(\sigma, r_i(\sigma)) \geq w_i((\sigma_{-i}, \sigma_i'), r_i(\sigma)) \qquad \forall i \in \varphi, \forall \sigma_i' \in \Sigma_i \tag{13}$$

From the defenition of non-myopica loss-aversion equilibrium and Lemma 1, the set of non-myopic loss-aversion equilibrium is the set of $\sigma \in \Sigma$ that satisfy

$$w_i(\sigma, r_i(\sigma)) \geq w_i((\sigma_{-i}, \sigma_i'), r_i((\sigma_{-i}, \sigma_i'))) \qquad \forall i \in \varphi, \forall \sigma_i' \in \Sigma_i \tag{14}$$

The difference between equation (13) and (14) is only the last bit of the equation of r. This difference is that by myopic loss aversion, the reference point is taken by evaluation the vector σ and by non-myopic loss aversion the reference point is taken by evaluating the vector of strategy profile σ but there is a small difference in the vector of myopic loss aversion namely at place i of the vector. This is that for non-myopic the reference point with the vector σ is $r_i((\sigma_{-i}, \sigma_i'))$ where (σ_{-i}, σ_i') signifies that all players $j \in \varphi \setminus \{i\}$ play σ_j and player i plays σ_i'. We used that fact that each player has exactly one information set which is always reached. This means that the information at this point, every point, is the same as the information set at the root of the tree. From the defenition of consistent reference points and of the function $r_i(\sigma)$, we have

$$w_i(\sigma, r_i(\sigma)) = r_i(\sigma) \qquad \forall i \in \varphi, \forall \sigma \in \Sigma \tag{15}$$

and with substitution we get

$$w_i((\sigma_{-i},\sigma'_i),r_i((\sigma_{-i},\sigma'_i))) = r_i((\sigma_{-i},\sigma'_i))$$
$$\forall i\in\varphi,\forall\sigma'_i\in\Sigma_i,\forall\sigma_{-i}\in\Sigma_{-i} \tag{16}$$

We have to prove that equation (14) is the same as equation (13). To do this To do this we have to prove that (13) $\Leftrightarrow$ (14).
First $\Leftarrow$:
If we substitute equations (15) and (16) into inequality (14) we get that $r_i(\sigma)\geq r_i((\sigma_{-i},\sigma'_i))$. Out of the defenition of w_i we get that w_i is continuous and monotonically non-increasing in its second parameter. Given this we get that:

$$w_i((\sigma_{-i},\sigma'_i),r_i((\sigma_{-i},\sigma'_i)))\geq w_i((\sigma_{-i},\sigma'_i),r_i(\sigma)) \qquad \forall i\in\varphi,\forall\sigma'_i\in\Sigma_i \tag{17}$$

Taking inequality (17) together with (14) we get (13), so (14) $\Rightarrow$ (13)
Now $\Rightarrow$: If we substitute equation (15) into inequality (13) we get that $r(\sigma)\geq w_i((\sigma_{-i},\sigma'_i),r_i(\sigma))$, for all $i\in\varphi$ and for all $\sigma'_i\in\Sigma_i$. We use again that w_i is continuous and monotonically non-increasing with respect to its second paramater. We can conclude out of this that $r_i(\sigma)\geq r_i((\sigma_{-i},\sigma'_i))$., which implies (14), using equations (15) and (16). So we have proven that (13) $\Rightarrow$ (14).
Since we have that (13) $\Rightarrow$ (14) and (14) $\Rightarrow$ (13), we have proven that (13) $\Leftrightarrow$ (14).

Like we said before, the information available to the player when she has to choose what she wants to play is no different to the information available to her further in the game. This means that when we talk about game where each player has one information set no time is involved. And when no time is involved, we showed that the myopic and non-myopic loss-aversion equilibria are the same. So the difference between myopic and non-myopic loss-aversion equilibria comes from differences in the moment of updating the reference points. So the difference comes not from that fact that you take another look at the reference point when you consider to deviate.

■ (Proposition 1)

6.2 Proposition 2

Proposition 2 *[1] For any game G with perfect information, any pure-strategy Nash equilibrium of G is both a myopic and a non-myopic loss-aversion equilibrium of (G,λ) for any λ.*

Proof: We take a pure strategy equilibrium σ giving payoffs $x=(x_i)_{i\in\varphi}$, then σ is also a pure strategy equilibrium in the standard game $L(G,\lambda,x)$. We can say this because any deviation to another strategy from σ by a player i in G is not profitable, because we are in a Nash equilibrium. In terms of paysoffs, any deviation gives no more than x_i. This is the same for the standard game $L(G,\lambda,x)$ because in this game the strategy also gives no more than x_i because the payoffs in $L(G,\lambda,x)$ are not higher than the payoffs in G. Given this all, we say that σ gives also x in $L(G, \lambda, x)$ and can not be higher because of a deviation in strategy, therefore we can say that x is a loss-aversion equilibrium. This is a myopic loss-aversion equilibrium, but to prove the proposition, we have to show the same for a non-myopic loss-aversion equilibrium.

The proposition is about any game G with perfect information. And in a game of perfect information, each information set is reached with probability one or probability zero. We have a profile of pure strategies σ which are than reached with probability one or zero, so playing the strategy yes or no. Eventually exactly one terminal node of the tree is reached with probability one, with certainty. The payoffs of this terminal node are exactly the consistent reference points of the players of the profile σ. Now we suppose that a player has a deviation with a consistent reference point that possibly gives her more, then this deviation would give her a higher expected payoff than we she would have played the pure strategy σ, without loss aversion evaluation. However, this is in contradiction with the assumption that σ is a Nash equilibrium of G. So we have proved that for any game with perfect information, a pure-strategy Nash equilibrium is both a myopic and a non-myopic loss-aversion equilibrium of (G, λ) for any λ.

■ (Proposition 2)

6.3 Proposition 3

Proposition 3 *[1] For any extended game (G, λ), there exists a myopic loss-aversion equilibrium.*

To prove proposition 3, we first have to define a new theorem that helps us in our proof. It is the Kakutani's fixed point theorem.

Definition 4 *[9] (Kakutani's fixed point theorem) Let A be a non-empty, compact[4] and convex subset of $R^n and \phi : A \longrightarrow P(A)$such that*

- $\phi(x)$ is nonempty and convex $\forall x \in A$
- the graph of ϕ, so the set $\{(x, y)|y \in \phi(x)\}$, is closed

Then A has a fixed point: $x^* \in \phi(x^*)$

Proof of Proposition 3: For our proof we use the Kakutani's fixed point theorem that we defined above. To use the theorem, we have to check if all assumptions needed for the theorem are correct in our proof. First we assume an exteded game (G, λ) like in the proposition. We define the correspondence [5] f from $\Sigma \times [\underline{r}, \overline{r}]^{\varphi}$ to itself as follows. $(\sigma', r') \in f(\sigma, r)$ if σ'_i, the set of strategies, is a best response to σ_{-i} in the game $L(G, \lambda, r)$ for all $i \in \varphi$, and r'_i is the payoff to i

[4] A set A $\subseteq \mathbb{R}^n$ is *compact* if it is closed and bounded. A set A $\subseteq \mathbb{R}^n$ is bounded if there is an $M > 0$ such that $\|x\| < M \forall x \in A$

[5] A correspondence is a function that returns for each input many outputs

from (σ_i', σ_{-i}) in the game $L(G, \lambda, r)$ for all $i \in \varphi$. So $f : \Sigma \times [\underline{r}, \overline{r}]^{\varphi} \longrightarrow 2^{\Sigma \times [\underline{r}, \overline{r}]^{\varphi}}$

To apply the fixed point theorem of Kakutani as in defenition 9, we need to show that the domain is non-empty, compact and convex and that the correspondence is nonempty, convex valued, and has a closed graph. First, we can see that the interval of $[\underline{r}, \overline{r}]^{\varphi}$ is a closed interval. By defenition this means that the interval is nonempty. The interval is compact because it is a closed and bounded subset of $\mathbb{R}^n$. The interval is convex because it is an interval in $\mathbb{R}^n$. Now we have to show that the strategy space is nonempty, compact and convex. By defenition: $\Sigma = \prod_{i \in \varphi} \Sigma_i$ where each $\Sigma_i = \{x | \Sigma_j x_j = 1\}$ is a simplex of dimension $|S_i| - 1$, thus each Σ_i is closed and bounded, and thus compact. And so is their product compact. So Σ is compact. Σ is also convex, proof of this can be found in the appendix. Both $[\underline{r}, \overline{r}]^{\varphi}$ and the strategy space are nonempty, compact and convex, and therefore so is their product: $[\underline{r}, \overline{r}]^{\varphi} \times \Sigma$.

Now we have to show that the correspondence is nonempty, convex and has a closed graph. For every strategy (σ, r) and each player i there is a best response strategy to the strategy that is being played. Their is always at least one pure strategy that is a best response. This best response is the strategy σ_i' to σ_{-i} in $L(G, \lambda, r)$. We take r_i' as the payoff from playing (σ_i', σ_{-i}) in $L(G, \lambda, r)$. This means that we have an element (σ', r') in $f(\sigma, r)$ so $f(\sigma, r) \neq \emptyset$. So the correspondence is nonempty. We have said before that there is always a best response to a strategy that is being played. It is possible that there is more than one best response for a player i. In this case, the payoffs of these best responses will be the same, and so does any convex combination of the best responses. Therefore we can say that the correspondence is convex valued. For the correspondence to be a closed graph, we need that the correspondence must be continuous. In the transformed game, we see that the payoffs as a function of r are continuous. Furthermore, we have that $\Sigma \times [\underline{r}, \overline{r}]^{\varphi}$ is closed and we have that the best-response function has a closed graph. Given this we can conclude that the correspondence has a closed graph.

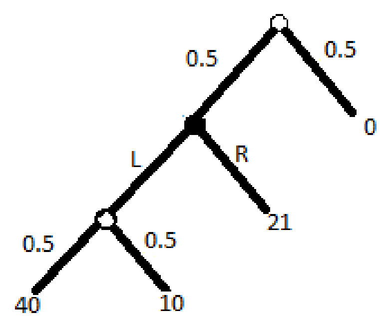
We have now shown that we have fulfilled all assumptions needed for the fixed point theorem of Kakutani, so we can use it. Applying the theorem, there exist σ^* and r^* such that $(\sigma^*, r^*) \in f(\sigma^*, r^*)$. From the defenition of f, we have that σ^* is a myopic loss-aversion equilibrium of (G, λ), giving payoffs of r^*.

■ (Proposition 3)

7 Myopic vs. Non-myopic loss-aversion equilibria

We have talked about two different types of loss aversion equilibria, myopic and non-myopic. The difference between them is that by myopic loss aversion, when you evaluate a deviation, the player does not change her reference point, even though the outcome will be different if she deviates. By non-myopic loss aversion, when a deviation takes place, she will change her reference point that is consistent with the deviation. Also in making choices in a tree, there is a difference between the two. In myopic loss aversion all evaluation is done at the root of the game, this in different as by non-myopic loss aversion where at every decision node the player comes, an evaluation is done. We have seen that the two equilibria are the same in games with only one information set for each player. We now want to show with an example the exact difference between the two equilibria. We will calculate the myopic loss aversion equilibrium and the non-myopic loss aversion equilibrium.

Figure 3: Tree for representing the example



We use the tree given in figure 1. There is one player in this game and he has a loss aversion coeffecient of 1, $\lambda = 1$. The tree consists of two moves by nature, indicated by the hollow circles, both with chance 0.5. The solid circle is a decision node for the player where he has to choose R or L. We will first calculate the myopic loss-aversion equilibrium. Suppose the player chooses R, we then could make a lottery out of the tree which is that he gets 0 with probability 0.5 and 21 with probability 0.5. In lottery form: $(0, 0.5, 21, 0.5)$. The reference point that is consistent with this lottery is 7. Now suppose the player chooses L. Then she faces the lottery $(0, 0.5, 40, 0.25, 10, 0.25)$. The reference point consistent with this lottery is $8\frac{1}{3}$. It is best to choose the lottery with the highest consistent reference point so the player would choose to play L. Thus choosing L is the unique myopic loss aversion equilibrium. We will now calculate the non-myopic loss aversion equilibrium. By calculating this equilibrium, we do not start at the root of the game but at the decision node where the player has to choose R or L. Choosing R gives the player 21 for sure. This is consistent with a reference point of 21. Choosing L gives the player the following lottery: $(40, 0.5, 10, 0.5)$. This lottery is consistent with a reference point of 20. Now again, we choose the lottery with the highest consistent reference point. This means that choosing R is the best strategy, so this is the non-myopic loss aversion equilibrium.

It is now quite obvious that the both equilibria are not the same. This difference can be explained. If we look at the way we calculated the consistent reference point for the non-myopic loss aversion equilibrium, we can see that we do not take into account that it is possible that the outcome will be 0. The player immediately looks at the decision node where she can not get 0. Taken this into consideration, her reference point is calculated. However, in the myopic loss aversion equilibrium, the outcome of 0 is taken into consideration in the calculation of her reference point when she has to choose R or L, although when she has to make this choice, the outcome of 0 can not be reached anymore. This is because we slowly adjust our reference points.

We have now seen that it is possible to have both a myopic and non-myopic loss aversion equilibrium in a game that are not same. Proposition 3 however proved that for any extended game (G, λ), there exists a myopic loss aversion equilibrium. Therefore, it is possible that a game has a myopic loss aversion equilibrium but not a non-myopic loss aversion equilibrium.

8 Conclusion

In this thesis, we have taken a closer look at Loss Aversion. This phenomena is very important as you have seen in game theory. Game theory is about finding out what strategy is the best for you to play and what will be the outcome of this play. You want to maximize your utility in any situation. To do this, you have to take a lot of factors in mind that play a role in the decision making about which strategy to play. One of this factors is loss aversion. We want to know what happens if we give losing a higher weight in your decision making. The most important factor in analysing loss aversion, as we have seen, is reference dependency. In every game, your reference point is deciding what your action is going to be in the next decision node. You have to evaluate this reference point and choose the highest utility that depends on this reference point. Before you can do this, you have to measure the level of loss aversion of every player individually. This is quite hard to measure since we talk about games and lotteries which we play virtually, but it gives a quite precise indication in which loss aversion affects the play of the game.

We have analysed the games we play in game theory and how we must transform these games to incorporate the loss aversion characteristics and the reference points. We transformed the standard game G into an extended game $L(G, \lambda, r)$ which we analysed in our examples. We used formula (1) to change the utilities of the original game into the final utilities for the extended game. These utilities now depend on the reference point and the loss aversion coefficient.

Furthermore, we have analysed these games and we want to find an equilibrium in these games, the loss aversion equilibrium. We have seen that there is always a loss aversion equilibrium in any extended game, a myopic loss aversion equilibrium. Besides that, it is also possible that a non-myopic loss aversion equilibrium can be found. Just like Nash equilibria, we do have an optimal strategy in a game with loss aversion. This helps us to see what happens when we change λ. We have seen in several examples that when you become more loss aversion, λ will be higher, than the player has a higher probability of receiving his preferred outcome in the equilibrium. However, the utility will be lower. This is an interesting result because becoming more loss averse means that you will indeed get your preferred outcome, but your utility will be lower. In words you can say that you play it safe, you know what you will receive out of the lottery, but you will not be as happy if you get the same outcome but you did not know if your preferred strategy was actually played. In the article of Shalev [1], there are other examples presented in which increasing λ for player 1 got the result that the utility of player 1 also increased, that of player 3 decreased and the utility of player 2 remained the same.

The main conclusion of my thesis is that if you look at games and lotteries and you analyse the behaviour of the people, you also have to look at loss aversion. This is a very important factor in decision making. As we have seen in the Allais paradox with loss aversion, it also can give guidance for the player what strategy to play in games and lotteries. A lot of more projects could be done about loss aversion. Now all the work on loss aversion has looked for averages and not dealt with variations between individuals. This kind of work is done for risk aversion and could also be done for loss aversion. You could look at differences in age, gender and culture, do these factors have an impact on loss aversion. This could be interesting for further research.

9 Appendix

In this appendix, some extra information or calculations are given that were too long to put in the original text.

A Allais paradox inconsistency

In section 1 we showed the Allais paradox. We had two lotteries in which you had to choose for option A or B. The modal choices were lotteries 1B and 2A. These choices are inconsistent with expected utility theory. We will proof this by using simple mathematics. Using the values of the paradox and a utility function $U(W)$ where W is the wealth, we can demonstrate how the paradox works.
Lottery1 :
$1.00U(2400) > 0.33U(2500) + 0.01U(0) + 0.66U(2400)$
Lottery2 :
$0.33U(2500) + 0.67U(0) > 0.34U(2400) + 0.66U(0)$
These are representing the choices made in the paradox. We now will use the equation of lottery 2 to show that this leads to a contradiction.
$0.33U(2500) + 0.01U(0) > 0.34U(2400)$
$0.33U(2500) + 0.01U(0) > 1.00U(2400) - 0.66U(2400)$
$0.33U(2500) + 0.01U(0) + 0.66U(2400) > 1.00U(2400)$
which contradicts with the equation of lottery 1. We have proven that these choices, 1B and 2A, are not consistent with expected utility theory.

B Battle of the sexes

In my thesis, we used the example of the battle of the sexes. In this example, some equations for p and q were given directly. Here in this appendix we will take another look at these equations and we will show how these equations were made up.
We do have these four equations for p and q:

$$2q + (-\lambda_1 r_1)(1-q) = -\lambda_1 r_1 q + (1-q) = r_1 \tag{18}$$

$$p + (-\lambda_2 r_2)(1-p) = (-\lambda_2 r_2)p + 2(1-p) = r_2 \tag{19}$$

With the restriction that p and q are in $[0, 1]$, there is a unique solution to these equations, which is given by:

$$p = 1 - \frac{-3 - 2\lambda_2 + \sqrt{9 + 8\lambda_2(2+\lambda_2)}}{2\lambda_2} \tag{20}$$

$$q = \frac{-3 - 2\lambda_1 + \sqrt{9 + 8\lambda_1(2+\lambda_1)}}{2\lambda_1} \tag{21}$$

$$r_i = \frac{-3 + \sqrt{9 + 8\lambda_i(2+\lambda_i)}}{2\lambda_i(2+\lambda_i)} \qquad i = 1, 2 \tag{22}$$

The mathematics to get to equations (20), (21) and (22) will be given by the following:
We can rewrite equation (18) which gives an equation of q with variable r_1

$$2q - \lambda_1 r_1 + \lambda_1 r_1 + \lambda_1 r_1 q = r_1$$
$$(2 + \lambda_1 r_1)q = (1 + \lambda_1)r_1$$
$$q = \frac{(1 + \lambda_1)r_1}{2 + \lambda_1 r_1}$$

We substitute q in equation (18), which will give us the following equation:

$$-\lambda_1 r_1 \frac{(1 + \lambda_1)r_1}{2 + \lambda_1 r_1} + 1 - \frac{(1 + \lambda_1)r_1}{2 + \lambda_1 r_1} = r_1 \tag{23}$$

Using simple mathematical skills, we can rewrite equation (23) to the following quadratic equation:

$$(\lambda_1^2 + 2\lambda_1)r_1^2 + 3r_1 - 2 = 0 \tag{24}$$

We can solve equation (24) for r_1 with the ABC-formula and this gives us the right equation for r_1 like in equation (22)

We now want to use r_i to get to a solution for the equation for q. Out of equation (18), we get that $q = \frac{1+\lambda_1 r_1}{3+2\lambda_1 r_1}$. We will rewrite q as followed:

$$q = \frac{1}{2}\left(\frac{2 + 2\lambda_1 r_1}{3 + 2\lambda_1 r_1}\right) = \frac{1}{2}\left(\frac{2 + 2\lambda_1 r_1 + 1 - 1}{3 + 2\lambda_1 r_1}\right) = \frac{1}{2}\left(\frac{3 + 2\lambda_1 r_1 - 1}{3 + 2\lambda_1 r_1}\right)$$
$$q = \frac{1}{2}\left(1 - \frac{1}{3 + 2\lambda_1 r_1}\right)$$

We have rewritten the equation for q so it will be easier to substitute r_1 into q. To do this substitution we calculate q bits by bits as given below:

$$3+2\lambda_1 r_1 = \frac{3(2 + \lambda_1)}{(2 + \lambda_1)} + \frac{-3 + \sqrt{(9 + 8\lambda_1(2 + \lambda_1)}}{(2 + \lambda_1)} = \frac{3\lambda_1 + 3 + \sqrt{(9 + 8\lambda_1(2 + \lambda_1)}}{2 + \lambda_1}$$

This is part of q, but now we can calculate the fraction easier.

$$\frac{1}{3 + 2\lambda_1 r_1} = \frac{2 + \lambda_1}{3\lambda_1 + 3 + \sqrt{(9 + 8\lambda_1(2 + \lambda_1)}}$$

Now we have part of q, but we have a square root in the denominator which we want to get rid of. We do this by multiplying the fraction with 1. This gives us:

$$\frac{1}{3 + 2\lambda_1 r_1} = \frac{2 + \lambda_1}{3\lambda_1 + 3 + \sqrt{(9 + 8\lambda_1(2 + \lambda_1)}} \times \frac{(3\lambda_1 + 3) - \sqrt{(9 + 8\lambda_1(2 + \lambda_1)}}{(3\lambda_1 + 3) - \sqrt{(9 + 8\lambda_1(2 + \lambda_1)}}$$

$$= \frac{(2+\lambda_1)(3\lambda_1+3) - (2+\lambda_1)\sqrt{9+8\lambda_1(2+\lambda_1)}}{(3\lambda_1+3)^2 - (9+8\lambda_1(2+\lambda_1))}$$

$$= \frac{3(2+\lambda_1)(\lambda_1+1) - (2+\lambda_1)\sqrt{9+8\lambda_1(2+\lambda_1)}}{\lambda_1^2+2\lambda_1} = \frac{3(\lambda_1+1) - \sqrt{9+8\lambda_1(2+\lambda_1)}}{\lambda_1}$$

We now have a nice expression for part of the eqaution of q. We still need some further calculations to have succesfully substituted r_1 into q.

$$1 - \frac{1}{3+2\lambda_1 r_1} = \frac{\lambda_1}{\lambda_1} - \frac{3(\lambda_1+1) - \sqrt{9+8\lambda_1(2+\lambda_1)}}{\lambda_1} = \frac{-3-2\lambda_1+\sqrt{9+8\lambda_1(2+\lambda_1)}}{\lambda_1}$$

The final substitution into q will give us the right solution for q:

$$\frac{1}{2}\left(1 - \frac{1}{3+2\lambda_1 r_1}\right) = \frac{-3-2\lambda_1+\sqrt{9+8\lambda_1(2+\lambda_1)}}{2\lambda_1} = q$$

We now have the solution for q. The solution for p can be found in the same way, but then you have to use equation (19) to get the right solution. The solution for p will be:

$$p = 1 - \frac{-3-2\lambda_1+\sqrt{9+8\lambda_1(2+\lambda_1)}}{2\lambda_1}$$

C Proof of proposition 3

Proof for Σ being convex.

We first want to show that Σ_i is convex and than immediately it follows by defenition that Σ is convex. Let $M_1, \ldots, M_k$ be mixed strategies out of Σ_i. $\mu_1, \ldots, \mu_k$, with $\mu_j \in [0,1]$ for all $j \in 1, \ldots, k$ and $\mu_1 + \ldots + \mu_k = 1$. We want to show that $M' = \mu_1 M_1 + \ldots \mu_k M_k$ is also a mixed strategy. Let R be the set of responses and for $r \in R$ let r_i be the chance that strategy M_i response r chooses. Than we obtain for all i that $\sum_{r\in R} r_i = 1$. Let $r \in R$ be a response, than is the chance for this response in M' equal to $\mu_1 r_1 + \ldots + \mu_k r_k$. We want to show that $\sum_{r\in R}(\mu_1 r_1 + \ldots + \mu_k r_k) = 1$.

$$\begin{aligned}
&\textstyle\sum_{r\in R}(\mu_1 r_1 + \ldots + \mu_k r_k) = \\
&\textstyle\sum_{r\in R}\sum_{i=1}^k \lambda_i r_i = \\
&\textstyle\sum_{i=1}^k \sum_{r\in R} \lambda_i r_i = \\
&\textstyle\sum_{i=1}^k \lambda_i \sum_{r\in R} r_i = \\
&\textstyle\sum_{i=1}^k \lambda_i = \\
&1.
\end{aligned}$$

So M' is also a mixed strategy, so Σ_i is convex and by defenition of Σ_i and Σ we can say that Σ is convex.

■

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