Gaze Data as a Metacognitive Tool to Support the Development of Structure Sense in Algebra

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Abstract

This paper focuses on the development of structure sense in high school mathematics, which is fundamental for students' algebraic skills. The purpose of this study was to investigate whether an intervention that connects gaze data, metacognition, and attention for structures would support high school students in their development of structure sense. A quasi-experimental approach was taken to answer this question. Both the experiment group ($n = 37$) and the control group ($n = 45$) took a pre- and posttest. Between these tests, the experiment group received three interventions. The results indicated that the intervention had helped the students in their development of structure sense. The test scores in the experiment group improved significantly whereas the test scores in the control group did not. Furthermore, the students in the experiment group chose a wider variety of strategies to solve the problems after the intervention and they showed more recognition of structures. The question remains whether an actual shift in thinking has taken place for the students in the experiment group or that after the intervention they only knew better what to do with equations whose structures were also in the intervention. The results, however, seem to be promising for the development of structure sense in high school mathematics.

Keywords: equation solving, eye tracking technology, mathematics education, metacognition, structure sense

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It is not uncommon for high school students to struggle with solving mathematical equations. Algebra in general seems to be a topic that students experience trouble with (Knuth, Alibali, McNeil, Weinberg, & Stephens, 2005). The difficulties include a lack of understanding of the equality sign (Kieran, 1981), working with variables (Knuth et al., 2005), and working with brackets (Hoch & Dreyfus, 2004). Mastery of algebra is not only necessary to succeed in mathematics, but it is also a determining factor for succeeding in future education (Knuth, Stephens, McNeil, & Alibali, 2006). Many students have to follow at least one algebra course in their first year of university and this happens to be very hard for most of them (Steen, 1992). Surely, for most jobs, students will need more than just plain algebra to be successful, but several basic mathematical skills are required in many of them. Therefore, it is valuable to keep striving for improvement in high school students' algebraic skills.

It has been argued before that students' difficulty with algebra can be traced back to a lack of structure sense (Novotná & Hoch, 2008). One can think of a mathematical structure as the way in which a mathematical entity (such as an equation) is made up of its parts and how these parts are connected (Hoch & Dreyfus, 2004). Misunderstanding or misinterpreting this structure hinders solving the equation efficiently (Linchevski & Livneh, 1999). The power of structure sense can for instance be seen when solving the following equation:

$$\frac{x^2+x-1}{x-2} = \frac{x}{x-2} \tag{1}$$

When students fail to recognize the structure and think in learnt procedures, it is expected that they use cross-multiplication and remove the brackets, which is the standard approach for solving equations with fractions. However, when one recognizes the structure of this equation, which is

$$\frac{A}{B} = \frac{C}{B} \tag{2}$$

it follows easily that $A = C$ (provided that $B \neq 0$). Cross-multiplication (which gives $A \cdot B = C \cdot B$) and removing the brackets might also lead to the correct answer, but takes a lot more time and calculations. Students are more prone to make errors when they have to perform many calculations which is why structure sense helps to solve equations not only more efficiently, but also more accurately. The lack of structure sense is mainly due to procedural thinking, as opposed to relational thinking, which is what structure sense requires (Novotná & Hoch, 2008). A shift in thinking is needed from immediately applying a procedure to first determining structure characteristics that might help to solve the problem.

In order to establish this shift in thinking the students must first become aware of their own solving processes. A focus on metacognition in order to improve mathematical problem solving skills has received increasing attention in recent years, since it proves to be valuable for making the learning more effective and sustainable (Cohors-Fresenborg, Kramer, Pundsack, Sjuts, & Sommer, 2010). One recent technology that researchers use to gain insight into students' solving processes is that of eye tracking (Susac, Bubic, Kaponja, Planinic, & Palmovic, 2014). Eye tracking technology detects and tracks the movements of the eyes while persons are involved in specific tasks or contexts (Duchowski, 2007). Eye tracking has already been used in studies on mathematical problem solving and in this way, these studies have provided some insight into the strategies students use (e.g. Chesney, McNeil, Brockmole, & Kelley, 2013; Susac et al. 2014). However, gaze data have only been used in mathematics education to give the *researchers* more insight into the strategies students use, not the students themselves. One research that uses gaze data as a metacognitive tool for the students is performed in the context of Science Technology Engineering Mathematics (STEM) literacy (Sommer, Hinojosa, & Polman, 2016). This study revealed that confronting students with their own gaze data helped them to get more insight into their own perceptual and interpretive processes. It seems valuable to investigate whether such an approach has similar positive results in the context of developing structure sense in algebra.

The lack of existing studies that use and investigate gaze data in a similar way makes the current study more explorative than experimental. The main aim is to investigate whether an intervention that makes connections between gaze data, metacognition, and attention for structures can support students in their development of structure sense.

Theoretical background

Structure sense

The term structure sense was used first by Linchevski and Livneh (1999), when they described the difficulties that students have with algebraic structures because of a lack of understanding of arithmetical structural notions. A useful definition of structure sense is given by Hoch and Dreyfus (2005). They define structure sense as a combination of the following abilities:

> "A student is said to display structure sense (SS) if s/he can:
>
> - Deal with a compound literal term as a single entity. (SS1)
> - Recognise equivalence to familiar structures. (SS2)
> - Choose appropriate manipulations to make best use of the structure. (SS3)" (p. 146)

Examples are given of all three abilities. Students display SS1 if they deal with $3x - 14$ as a single entity in the following equation:

$$5\,(3x - 14)^{2} = 20 \tag{3}$$

Students display SS2 if they recognize the structure $A \cdot B = 0$ in the following equation:

$$(x + 17)(x - 12) = 0 \tag{4}$$

And finally, students display SS3 if they first simplify both terms separately when simplifying the following product:

$$(4x^2 - 25x + 13x^2)(7x - 4 - 7x) \tag{5}$$

Hoch and Dreyfus (2005) argue that these abilities are partially hierarchical, for in order to display SS2 it is necessary to have mastered SS1 and in order to display SS3 both SS1 and SS2 are required. Structure sense in this study will be tested according to the abilities specified in this definition.

Metacognition

In this study, it is investigated whether attention for metacognition supports students in their development of structure sense. This is expected since an insight into their own cognitive processes is necessary for students to make the shift in thinking that is needed to apply structure sense. Metacognition refers to “an individual’s awareness of his own thinking processes and his ability to control these processes” (Özsoy & Ataman, 2009). Note that these are two different aspects of metacognition: awareness of thinking processes on the one hand and being able to control these on the other. Considering metacognition in relation to mathematics, awareness of one’s own thinking processes refers to the mathematical strategies and techniques one knows and metacognitive control refers to the ability to use this metacognitive knowledge strategically (Özsoy & Ataman, 2009). A metacognitive student thus knows what strategies they can choose from and knows when and how to use these strategies. There are several ways in which this can be seen in their work: they successfully apply chosen strategies, they use organizing operators, and they try to break complex problems into simple parts (Özsoy & Ataman, 2009).

An example of how metacognition can be beneficial in mathematical problem solving, is given by Schoenfeld (1987). He gave his students the following calculus problem:

$$\int \frac{x}{x^2-9}\,dx \tag{6}$$

Most students found the correct answer, but very different strategies were used. Many students found the answer by using partial fractions or trigonometric substitutions. Although these strategies are not wrong, they cost a lot of time. The most efficient way to solve this problem is by rewriting it into:

$$\frac{1}{2}\int \frac{2x}{x^2-9}\,dx \tag{7}$$

Now only a simple substitution is left to find the answer. The amount of extra work that many students undertook, could have been avoided if the students had known other strategies (metacognitive awareness) and how to use them to solve this problem in an easier way (metacognitive control). As Schoenfeld (1987) states: "Never use any difficult techniques before checking to see whether simple techniques will do the job" (p. 191). This idea is highly related to structure sense, since using the structure in mathematical equation solving often leads to a relatively simple solving process. Relating the two aspects of metacognition to structure sense, we could see metacognitive awareness as awareness and recognition of general structures and metacognitive control as the ability to make efficient use of these structures during the solving process.

The effects of promoting metacognition on educational outcomes, although not specifically related to structure sense, have been studied in several mathematics education studies. Özsoy and Ataman (2009) found students who received metacognitive strategy training performed better in mathematical problem solving than students who did not. Cardelle-Elawar (1992) found similar results for students with low mathematical ability.

These studies suggest that metacognition can be beneficial for mathematical performance. In the current study, structure sense serves as the link between metacognition and performance. The promotion of metacognition is done in a new way, namely by making use of results from eye tracking technology: gaze data.

Gaze data as a metacognitive tool

Eye tracking technology is not new, since techniques to measure eye movements have been developed since the 1950s (Duchowski, 2007). More recently, research has proposed the link between visual behavior and cognition, for example in the study by Thomas and Lleras (2007). Their participants had to perform a problem-solving task on radiation, in combination with a tracking task which was meant to guide the participants' eyes. They found that the participants whose eyes moved in a pattern related to the solution were the ones who were most successful in finding it. Their research suggests that patterns of visual behavior can epitomize cognitive processes during a problem-solving task. Similar results have been found in mathematics education as well. Chesney, McNeil, Brockmole, and Kelley (2013) monitored their participants' visual behavior during a mathematical equation solving task. They found that the participants who chose one strategy showed different visual behavior than the ones who chose another, which also led to differences in test results. Susac et al. (2014) performed an eye tracking study on simple equation solving and found a correlation between the number of eye fixations of the students and the efficiency in their equation solving. These studies emphasize that visual behavior can only be seen as an *indicator* of thought processes, but still, it follows that gaze data can provide some insight into these processes.

Unfortunately, in most studies, gaze data have only been used as a window to cognitive processes *for the researchers*, not the students. The study by Sommer et al. (2016) proved that it can be very useful for the students to be confronted with gaze data in order to get a better understanding of their visual behavior, and thereby their cognitive processes. In mathematics education studies, students have not been confronted yet with gaze data in order to develop their metacognition, which is what the current study wants to undertake. In this study students will be reflecting on other students' gaze data, because of the time it takes to process and analyze eye tracking results and because these results are already available. It is unknown whether looking at other students' gaze data has similar effects to being confronted with your own, but neither have been researched extensively. Because of the argued link between visual behavior and strategy choosing, it is expected that being confronted with any gaze data will challenge students to think about their own strategic choices when solving mathematical equations. The intervention will be set up in a way to stimulate this.

Research questions and hypotheses

Based on the theory discussed above, the current study investigates possibilities of using other students' gaze data as a metacognitive tool to support the development of high school students' structure sense in mathematical equation solving. In order to investigate this, the following research questions need to be answered.

1. To what extent does an intervention that uses gaze data as a metacognitive tool, aimed at improving structure sense, improve students' results on an algebra test?
2. Does the intervention make students choose strategies that require more structure sense?
3. Does the intervention help students develop their metacognitive awareness and control for strategy choosing in mathematical equation solving?

It is expected that students' test results improve because of the intervention. The intervention is aimed at developing structure sense, which, according to the theory, should lead to improved results (Hoch & Dreyfus, 2004). The second and third question help to make sure that the improvement in results is actually because of developed metacognition and improved structure sense and not just because of more practice. After all, we only expect improvement in results because of the development of structure sense. We expect structure sense to be developed because of attention for metacognition: the intervention challenges students to think about (and change) their own strategic choices, which is what structure sense requires (Novotná & Hoch, 2008). It is expected that students make different strategic choices after the intervention and that they become more aware and in control of these choices as well.

Methods

Design

In order to answer the proposed research questions, a quasi-experimental approach was taken, using a pretest, a posttest, and semi-structured interviews. An experiment group received an intervention between the pre- and posttest, using gaze data as a metacognitive tool, whereas a control group did not. The results from the pre- and posttest serve to answer the first question, where the control group makes sure that improvements in the posttest are not just because of experience from the pretest. Students' written answers to the test items are also used to answer the second question, with additional information on the strategic choices coming from the semi-structured interviews with four students from the experiment group, directly following the tests. The third research question is answered by analyzing both the interviews and the materials from the intervention.

Participants

Four classes from two different high schools participated in the study, resulting in a total of 82 participants. The students were all in tenth grade (all fifteen to seventeen years old) in *vwo*, which is the pre-university track. From each school, one class was assigned to the experiment group and one was assigned to the control group, resulting in an experiment group of 37 students and a control group of 45 students. The assignment of each class to one of the groups, as well as the selection of the schools, was done because of convenience. The researcher teaches at one of the schools and specifically, one of the participating classes, which became part of the experiment group. The other school was chosen because of acquaintance with one of the teachers, whose class became part of the experiment group. Another class of each school became part of the control group. The level and grade of the students were carefully chosen. These upper-secondary pre-university students have had enough algebra to show at least some structure sense and they also have some years in high school left to further develop it and to feel the need of mastering these skills.

Instruments

Intervention. The intervention was based on the connections between gaze data, metacognition, and attention for structures. The gaze data were collected from a co-researcher who had recently performed an eye tracking study with ten high school students, focusing on structure sense. The figure below shows how these gaze data were presented to the experiment group from the current study.

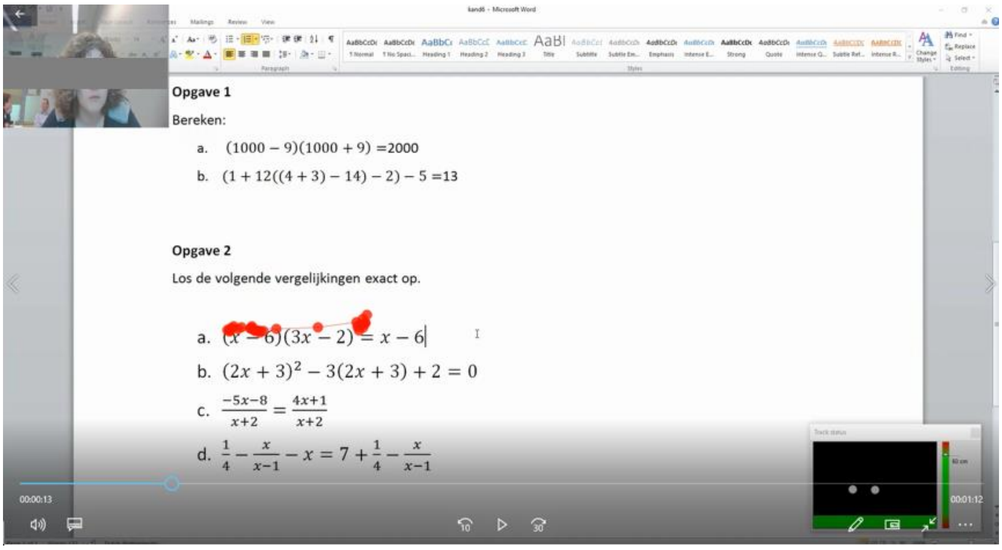

Figure 1. Still from one student’s gaze data, as shown during the intervention.

The student in this video started to remove brackets on the left side of the equation without having seen what was on the right side of the equation. Students who search for structure show traversing (looking back and forth across the equal sign) in order to search for the relations between the different parts of the structure (Chesney et al., 2013). This student was definitely not searching for structure and her visual behavior was coherent with her strategic choice of removing the brackets. During the intervention of the current study, the students were challenged to think about the presented gaze data in this way. The gaze data were the foundation from which the teacher made connections with strategic choices and recognizing mathematical structure, in order to stimulate both metacognitive awareness and control in relation to structure sense. Students also had to answer questions on paper, which included:

- Consider the following equation. Can you write down the mathematical structure of this equation by using capital letters?
- Here you see the visual behavior of student X while solving this equation. Write down whether you think this student recognizes the structure of the equation. How can you tell?
- Now think about your own visual behavior when solving equations. How do you think one should look in order to recognize a mathematical structure? Do you do that yourself?

It was expected that, by looking at the visual behavior and strategic choices of other students, the students in the current study would become more aware of the different strategies that are available and that there is a difference in efficiency between these strategies. The teacher stimulated this in the discussion. Apart from making the students more metacognitively aware, metacognitive control was gradually stimulated by explaining how recognizing a structure often helps to solve an equation easily and is therefore an efficient strategy to choose. Students' notes from the intervention, including the written answers to the questions asked by the teacher, were collected at the end of the intervention. Based on the theory on metacognition and how students show this, the notes were coded into different levels of metacognition concerning structure sense: low level (superficial notes, showing no awareness of available strategies), mid-level (notes hinting at structure or recognizing similar parts of an equation, thereby showing metacognitive awareness but no control), high level (using organizing operators to denote structure, showing metacognitive awareness and control), and undecided. This coding system was chosen, because it was expected that during the intervention, students would gradually learn to show metacognitive awareness and at the end also metacognitive control. Unit of analysis was a “topic” in the intervention, which was either a video or a collection of reflective questions, following which was a written task. Each intervention contained several topics, resulting in a total of seven different topics. A second coder was asked to review part of the data from the intervention and Cohen’s κ was run to determine the level of agreement between the researcher and the second coder. This resulted in $\kappa = 0.773$ ($p < 0.001$), which, according to Landis and Koch (1977), means that there is substantial agreement.

Pre- and posttest. The pre- and posttest have been created in order to test the mathematical equation solving abilities of the students and they can be found in appendices A and B. All items on the pretest share the same mathematical structure with an item on the posttest, and thereby they test the same algebraic skills (Gierl & Haladyna, 2012). An answer sheet had been made beforehand with point allocation per test item. Apart from regular point allocation, a structure point could be earned for each item, since each item tests one specific aspect of structure sense. Additionally, an analysis was made per test item of the strategies that the students chose to solve it. This analysis was performed in order to answer the second research question on the possible change in strategic choices on the part of the students. An independent researcher reviewed an amount of the tests in order to determine interrater reliability, for both the point allocation and the strategic analysis. The point allocation resulted in $\kappa = 0.707$ ($p < 0.001$) and the strategic analysis in $\kappa = 0.753$ ($p < 0.001$). Both indicate substantial agreement between the researchers (Landis & Koch, 1977).

The first three test items test the three abilities that define structure sense according to Hoch and Dreyfus (2005) and are based on the example exercises they provide for each ability. The numbers used in the equations on the tests are not too difficult, to not let arithmetical complexity distract from the algebraic structure. Consider the following example from the pretest:

$$3(3x-3)^{2}=27 \tag{8}$$

If students recognize that they can deal with $(3x-3)$ as a single entity (SS1) and therefore divide both sides by three and take square roots, they earn a structure point for this equation. By solving the equation, irrespective of the strategy chosen, they can earn up to three regular points as well.

The fourth test item contains two variables: *u* and *v*, and asks the students to solve for *v*. It was based on an example from the article "Symbol Sense Behavior in Digital Activities" (Bokhove & Drijvers, 2010). In the article it is explained that students often have difficulty dealing with an algebraic entity without caring for the content, such as in the following example, where the students are asked to solve for *v*:

$$v\cdot\sqrt{u}=1+2v\cdot\sqrt{1+u} \tag{9}$$

They do not actually have to do anything with the expressions within the square roots, but many students will try to do this anyway. Recognizing algebraic entities and using them is related to SS1, in the used definition of structure sense. This test item served to give the students another format to work with than the first three test items. However, it turned out that the students had no idea what to do with this equation. Not a single point was earned for this test item, which is why it was taken out of the analysis.

These test items have been reviewed by co-researchers and by four teachers who are currently teaching in the tenth grade. The teachers were also asked to check the difficulty level of the items. They all thought that these students should be able to solve equations one to three, although some questioned the students' algebraic skills to actually find the correct answers for these equations. This was mainly because they thought students would not recognize the structures of these equations and would therefore end up in extensive calculations.

The fifth and final test item served a different purpose than the others. For this item the students could not earn regular points nor structure points. The students were confronted with an equation and were asked to write down what struck them when looking at the equation (a) and what strategy they would use to solve it (b). This test item was meant to give us more insight into the students' (awareness of their) strategic choices and therefore it was included in the strategic analysis.

Semi-structured interviews. The semi-structured interviews have been conducted with two students together from each class that was in the experiment group, to make it possible for them to react on each other. The teachers of these classes chose one stronger and one weaker student whom they thought could express themselves verbally well. The interviews were semi-structured, which means there were predetermined questions from the researcher and the interviewees were given the opportunity to go into detail when answering these questions, as well as to go beyond them (Whiting, 2008). The interviews were recorded and transcribed verbatim. The questions asked were aimed at getting a better idea of what the students thought of the tests and why they chose certain strategies. Examples of questions that that were asked during the interview:

- Did you find the test easy or difficult? Why?
- Did you have enough time to make the test?
- What strategy did you use to solve this equation? Why did you use this strategy?

Procedure

The research followed this order: pretest, semi-structured interviews, intervention, post-test, semi-structured interviews, all within a maximum of eleven days. The intervention was split into three sessions of fifteen minutes. The pre- and posttest took fifteen minutes as well. Both the tests and the interventions were scheduled within the regular mathematics lessons and were carried out by the researcher. For the experiment group the first fifteen minutes of five consecutive lessons were used for the research: pretest, thee interventions, and posttest. After these fifteen minutes the students continued their regular mathematics lessons. The semi-structured interviews with two students were conducted directly after both tests, because at that time the students would remember best what they did and why they did it. The control group took the same pre- and posttest in the same timeframe, but without receiving any intervention in between. They just followed their regular mathematics lessons.

Results

The results from the pre- and posttest are summarized in Table 1. In total, the students could earn eight regular points and three structure points per test.

Table 1

Results from the pre- and posttest

		Pretest	Posttest	Pretest SP [a]	Posttest SP
Experiment group (n = 37)	M	1.43	2.89	0.08	0.73
	SD	1.214	1.776	0.277	0.693
Control group (n = 45)	M	2.93	3.40	0.18	0.11
	SD	2.093	2.280	0.442	0.318

[a] SP denotes structure points.

It follows from the table that the mean scores in the experiment group doubled from the pretest to the posttest (1.43 to 2.89), whereas the mean scores in the control group increased only slightly. A Wilcoxon signed-rank test revealed that the increase in the experiment group is statistically significant ($Z = -4.252$, $p < 0.001$), whereas the increase in the control group is not ($Z = -1.797$, $p = 0.072$). Furthermore, the mean scores of structure points in the experiment group increased significantly ($Z = -4.523$, $p < 0.001$), while the mean scores of structure points in the control group decreased. However, both the regular scores and the scores of structure points remain low, despite the intervention being focused on the development of structure sense.

Next to the test scores, a strategic analysis has been made of every test item except item four, which can be found in appendices C and D. A comprehended version for the experiment group is provided in Table 2. The answers fall into different categories, representing a strategy using the structure (A), using learnt procedures without using the structure (B), otherwise (C), and not answered (D). The numbers indicate how many students chose a certain strategy.

Table 2

Strategic choices by the students in the experiment group

		Pretest					Posttest				
		1	2	3	5a	5b	1	2	3	5a	5b
Category	A	0	3	0	12	0	1	22	4	21	23
	B	34	31	4	17	29	34	14	6	13	7
	C	2	3	19	5	3	1	1	14	0	3
	D	1	0	14	3	5	1	0	13	3	4

It follows from the table that the strategic choices of the students changed from the pretest to the posttest. The strategic choices in the control group hardly changed. Students from the experiment group clearly made more strategic choices using mathematical structure in the posttest than in the pretest. This is most evident for the second and fifth test items, which is not that surprising, since the structure of equation two appeared in the intervention at one of the schools and the structure of equation five appeared in the intervention at both schools. The structures of equations one and three did not and we see that the strategic choices for these test items did not change as much as the strategic choices for items two and five. Still, there were some students who did change their strategy for these equations. Table 3 shows the answers that one student provided to the third test item on both the pre- and posttest. Both answers are incomplete, but they do illustrate how the strategic choices for this item changed for this student after the intervention.

Table 3

Example of a different strategic choice in the pre- and posttest

Pretest	Posttest
$\frac{-x+10}{x+3}=\frac{8x}{2x+6}$ $\frac{-x+10\cdot 8x}{x+3\cdot 2x+6}=\frac{-2x^2+80}{2x^2+18}$	$\frac{6x-7}{x-2}=\frac{-2x}{2x-4}$ $\frac{12x-14}{2x-4}=\frac{-2x}{2x-4}$ $12x-14=-2x$ $14x=14$ $x=1$

Directly following the tests, two students from each class in the experiment group were interviewed by the researcher. One student already recognized structure at an early stage. She said that when starting the second item on the pretest, she wanted to remove the brackets, but then she changed her mind. “Because what it says here is actually that $2x + 8$ equals zero or $x – 3$ equals zero.” Because of this already high level of metacognition, the two interviews at this school did not reveal much improvement. However, at the other school, there was a notable difference between answers given after the pretest and answers given after the posttest. For example, one student said that, overall, she “recognized more structure” when making the posttest. In the pretest she removed brackets when she saw them, but now she noticed similar parts in the equations and knew what to do with them. For example, in equation five, she recognized “$x + 6$ is the same as $x + 6$ so that makes it A and B is then $2x – 7$”. The other student agreed. He already noticed similar parts in the pretest, but did not know what to do with them and still went on to remove brackets. Now, for equation 5, he said he recognized the form “A times B equals A” and used that in his solution. The first student did admit that this was mainly because of the examples given in the intervention.

The student notes from the three sessions of the intervention were analyzed per topic. There were seven different topics in total, one from the first session and three from both the second and third session of the intervention. The notes were coded into four different categories. In total, there were 230 notes by the students, of which 54 fell into the category “undecided”. The results from the other notes are presented in Figure 2, where the amount of answers within one category are presented as a percentage of the whole.

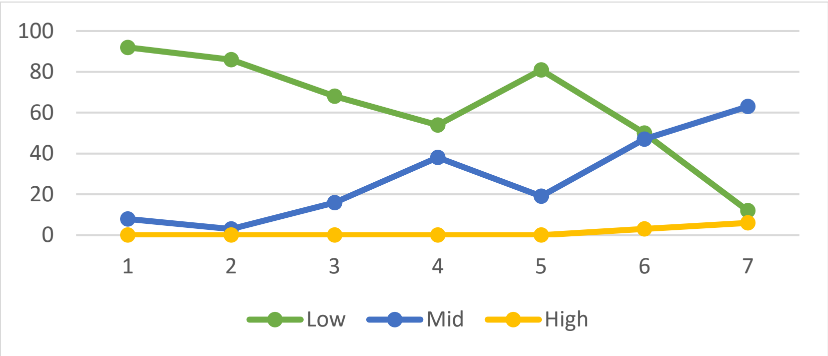

Figure 2. Level of metacognition from student notes during the intervention.

It follows from the figure that, at the start of the intervention, most of the written answers by the students fell into the lowest category. However, as the intervention proceeded, students began to give more answers that fell into mid-level metacognition and at the end there were even some answers that fell into a high level of metacognition. This indicates that during the intervention students became more reflective of their strategies, that they began to see structure as a possible strategy to quickly solve equations and that some of them also learnt how to use this structure in the solving process.

In Table 4, an example is given of three different answers to part of the sixth topic, which was the following task: “write down aspects of the visual behavior a student should have in order to recognize the structure of an equation.” The answers fall into three different categories, varying from not hinting at structure (low level) to describing structure and how to denote it (high level).

Table 4

Example student notes from the intervention (translated by author)

Low level	Mid-level	High level
"Look at the whole exercise. The beginning of the exercise is what you usually look at first."	"See whether there are similar elements in the formula."	"Whether there are parts of the exercise that are the same, because then they get the same letter."

Conclusions and discussion

The aim of this study was to investigate whether an intervention that utilizes other students' gaze data as a metacognitive tool supports the development of high school students' structure sense. The first research question concerned an effect on the students' test results. We have seen that the test results in the experiment group improved significantly, while the results in the control group did not, which suggests that the intervention had an effect on these scores. This was expected from the beginning, since attention for structure sense should lead to improved results (Hoch & Dreyfus, 2004).

However, to make sure that the improved test results were actually the result of improved structure sense and not caused by more practice, the second question concerned the strategies used by the students and how these were related to structure sense. From the test results we can conclude that also the amount of structure points that the students scored increased significantly in the experiment group, which indicates that structure sense was indeed applied more often after the intervention. The strategic analysis further illustrated how the strategic choices by the students changed from the pretest to the posttest towards using more strategies that require structure sense. However, we also saw that this was mainly true for the equations whose structures were also dealt with in the intervention. This indicates that students still have trouble recognizing structures when they have not seen them before. It might even imply that no shift in thinking has taken place and that students have come to see structure as another procedure that they can only apply to familiar structures.

The third question concerned students’ metacognition: their awareness of the strategies they can choose from and their ability to choose the most efficient one. After all, the intervention was aimed at changing the way students think, necessary to develop structure sense (Novotná & Hoch, 2008). The student notes from the intervention indicated that, as the intervention proceeded, the students became more aware of the structure of mathematical equations and that they could use this in their solving strategies. Some also showed that they could denote this structure by using organizing operators (in this case: capital letters to denote different parts of the structure), but not as many students as we expected. At the end, there were only some student notes that reflected high metacognition, the others remained in the lower levels. The interview at one of the schools was very indicative of improved awareness of strategic choices. The students gave different answers during the second interview that were more concerned with structure and recognizing similar parts. However, one student also expressed awareness that this was mainly because of similar examples in the intervention. The interview at the other school showed that one of the students was already aware of her strategic choices and this did not further improve during the intervention.

In conclusion, we see that the intervention turned out to be effective for the students’ test scores and their strategic choices. This was expected because of the connections that the intervention makes between gaze data, metacognition, and structure sense. An important factor for the effectiveness of the intervention was the role of the researcher. The researcher provided that the discussion of the videos showing gaze data was aimed at developing metacognitive awareness and later on also metacognitive control, thereby monitoring the carefully chosen build-up of the intervention. However, students’ development of structure sense was confined to the equations whose structures were also in the intervention. There were only few students who could transfer their gained knowledge to other structures. Therefore, for the largest part of the students, the question remains whether the change in thinking that is needed to develop structure sense has actually taken place, or that the students still think procedurally, where they learnt to use structure as another procedure only for the equations that were discussed in the intervention.

Furthermore, the results from this study are based on interventions in the two participating schools, which are quite similar in size and level, and on the limited time frame of the research. The actual study took place within two weeks, which means that the long-term effects are not measured within this study. It would be interesting to see what the effects of this kind of intervention are if it is used over a longer period of time. We also saw that the test scores in the current study, even though they improved after the intervention, are quite low. This indicates that the tests were very difficult for these students. However, the students have seen similar equations before and the teachers that were involved agreed that tenth-graders should be able to make these tests. The explanation could be that it has been a while since the students saw these types of equations. One student pointed this out during the interview as well. She did not know what to do because it had been too long since she had to do something like this. Therefore the results might have turned out better at another time during the year, when this was also the topic students were discussing during the regular lessons. Finally, the students were reflecting on other students' gaze data instead of their own. This was mainly because of the time frame of the current research and the time it takes to develop and analyze eye tracking results. However, doing the same research but with eye tracking during both tests, could reveal much more about the visual behavior of the students and thereby their strategic choices. It would be interesting to see whether the gaze data from the posttest were notably different to the gaze data from the pretest. The study by Sommer et al. (2016) indicates that students can learn a lot about their own cognitive processes when reflecting on their own gaze data. The fact that the students were reflecting on other students' gaze data was compensated by the questions asked during the intervention, where the students had to link the presented gaze data to their own visual behavior. The bigger question that arises is whether the gaze data were at all a useful tool in this intervention. The gaze data were meant as a window for reflection on strategic choices, which was supposed to result in a shift in thinking so that students could apply structure sense to many different equations. However, we saw that the students had trouble transferring their knowledge of structures to structures they had not seen before, so it could be that the videos were not enough, or simply not suitable, for the achievement of this goal.

The current study has provided a new angle to helping high school students develop structure sense. The intervention was based on the connections between gaze data, metacognition, and recognizing mathematical structure, which is quite a new approach to structure sense development. The results indicate that it is an approach to keep trying to improve, as it seems that students did develop some structure sense during the intervention. Practically, the results indicate that a focus on metacognition might support this development and improve students' results. This is in line with other research where metacognition is a point of focus and performance improves (Cardelle-Elawar 1992; Özsoy and Ataman, 2009).

Further research could develop a similar intervention but spread over a longer period of time, revealing more about its possibilities as a long-term teaching method. Additionally, it would be very informing to use eye tracking during both tests, which would also give some more insight into the students' cognitive processes. The current study seems to be a promising start and further research could further investigate whether more extensive and enriched attention for metacognitive skills really helps to develop structure sense, and in general, students' algebraic skills.

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Appendix A

Pretest

- Write down your name on the answer sheet.
- The test must be made individually and without a calculator.
- Make clear what you are doing and write down all the steps that you take.
- After 15 minutes the test and the answer sheet will be collected.

Exercise 1

Solve the following equation algebraically.

$3(3x-3)^{2}=27$

Exercise 2

Solve the following equation algebraically.

$(2x+8)(x-3)=0$

Exercise 3

Solve the following equation algebraically.

$\frac{-x+10}{x+3}=\frac{8x}{2x+6}$

Exercise 4

Solve the following equation algebraically for v.

$2-v\cdot\sqrt{u+1}=v\cdot\sqrt{u+2}$

Exercise 5

Have a look at the following equation: $(3x+7)(x-4)=x-4$.

a. What catches your eye first when you see this equation?

b. What strategy do you use in order to solve this equation?

Appendix B

Posttest

- Write down your name on the answer sheet.
- The test must be made individually and without a calculator.
- Make clear what you are doing and write down all the steps that you take.
- After 15 minutes the test and the answer sheet will be collected.

Exercise 1

Solve the following equation algebraically.

$2(2x-6)^2 = 8$

Exercise 2

Solve the following equation algebraically.

$(x+4)(3x-6) = 0$

Exercise 3

Solve the following equation algebraically.

$$\frac{6x-7}{x-2} = \frac{-2x}{2x-4}$$

Exercise 4

Solve the following equation algebraically for v.

$v \cdot \sqrt{u} = v \cdot \sqrt{u+2} + 3$

Exercise 5

Have a look at the following equation: $(x+6)(2x-7) = x+6$.

a. What catches your eye first when you see this equation?

b. What strategy do you use in order to solve this equation?

Appendix C

Strategic Analysis for the Experiment Group ($n = 37$)

Exercise	Strategy	Number of students	
		Pretest	Posttest
1	Structure – divide by three and take square roots (A)	0	1
	Procedural – remove brackets (B)	34	34
	Otherwise (C)	2	1
	Not tried (D)	1	1
2	Structure – one of the factors must be zero (A)	3	22
	Procedural – remove brackets (B)	31	14
	Otherwise (C)	3	1
	Not tried (D)	0	0
3	Structure – equal the denominators, equal the numerators (A)	0	4
	Procedural – cross-multiplication (B)	4	6
	Manipulation of the fractions (C)	10	5
	Equal the denominators to combine the fractions (D)	5	5
	Otherwise (E)	4	4
	Not tried (F)	14	13
5a	Structure – $x - 4$ appears two times (A)	12	21
	Procedural – brackets (B)	17	13
	The right side is not zero (C)	2	0
	Otherwise (D)	3	0
	Not answered (E)	3	3
5b	Structure – $A \times B = B$ (A)	0	23
	Procedural – remove brackets (B)	29	7
	Otherwise (C)	3	3
	Not answered (D)	5	4

Appendix D

Strategic Analysis for the Control Group ($n = 45$)

Exercise	Strategy	Number of students	
		Pretest	Posttest
1	Structure – divide by three/two and take square roots (A)	2	1
	Procedural – remove brackets (B)	43	44
	Otherwise (C)	0	0
	Not tried (D)	0	0
2	Structure – one of the factors must be zero (A)	5	4
	Procedural – remove brackets (B)	39	40
	Otherwise (C)	1	1
	Not tried (D)	0	0
3	Structure – divide right side by two, equal the numerators (A)	1	0
	Procedural – cross-multiplication (B)	10	11
	Manipulation of the fractions (C)	19	16
	Equal the denominators to combine the fractions (D)	5	3
	Otherwise (E)	3	7
	Not tried (F)	7	8
5a	Structure – $x - 4$ / $x + 6$ appears two times (A)	9	8
	Procedural – brackets (B)	22	29
	The right side is not zero (C)	2	0
	Otherwise (D)	7	2
	Not answered (E)	5	6
5b	Structure – A x $B = B$ (A)	0	0
	Procedural – remove brackets (B)	37	37
	Otherwise (C)	3	3
	Not answered (D)	5	5