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Modeling Self-Protective Responses in Randomized Response

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Abstract

Randomized response (RR) is an interview technique developed to ask sensitive questions. Confidentiality is assured, since the answers to the questions depend partly on the respondent's true status and partly on the outcome of a randomizing device. Despite this protection not all respondents answer by the rules of the RR method and consistently give self-protective answers instead. The basic model that takes self-protective answering into account, assumes that this behaviour is independent of the true status of the respondent. Since this model shows a lack of fit, models assuming dependence between self-protective answering and the true status are studied using empirical data from a social welfare study. The results show that the model fit does not improve by using these models. Furthermore, the results show that the models give identification problems. It is concluded that the basic model is a robust model and is therefore preferred for now.

Keywords: randomized response, self-protective responses, SP-No parameter, log-linear model.

1 Introduction

A problem in research is that some respondents are reluctant to answer questions about sensitive behaviour directly. This results in underestimation of the prevalence of the behaviour under study. Randomized response (RR) is an interview technique developed to ask sensitive questions in a less direct way. Within this method the answer of the respondent is partly determined by a randomizing device and partly by the respondent's true status. The use of a randomizing device results in protection of the privacy of the respondent, because the outcome of the randomizing device is kept hidden from the interviewer. Therefore, the interviewer will not know whether a given answer is a true answer or not. Hence, this method leads to better estimates of the prevalence of the behaviour under study.

There are several designs for the RR method. Within this report the Forced Choice Randomized Response (FCRR) and the Kuk design are used, for other designs see Fox and Tracy (1986). In the FCRR design, the respondent rolls two dice for every sensitive question and keeps the outcome hidden from the interviewer. The outcome of the dice indicates whether the respondent has to answer truthfully or has to give a forced answer. If the outcome of the dice is 2, 3 or 4 the respondent is forced to answer *yes* and if the outcome of the dice is 11 or 12 the respondent is forced to answer *no*, regardless of the true status of the respondent. If the outcome of the dice is 5 through 10 the respondent is instructed to answer truthfully. The misclassification probabilities of the responses by using the FCRR method then are $P(\text{observing yes}|\text{true yes})= 5/6$, $P(\text{observing yes}|\text{true no})= 1/6$, $P(\text{observing no}|\text{true yes})= 1/12$ and $P(\text{observing no}|\text{true no})= 11/12$.

The Kuk design uses two stacks of cards, both containing red and black cards. In the right stack the proportion red cards is 0.8 and in the left stack 0.2. The respondent is asked to draw a card from each stack and to keep the color hidden

from the interviewer. Then the sensitive question is asked. If the (true) answer is *yes*, the respondent names the color of the card drawn from the right stack. If the answer is *no*, the respondent names the color of the card drawn from the left stack. Within this method the color red is associated with a *yes* response. Hence the misclassification probabilities of the responses of this method are $P(\text{observing yes}|\text{true yes})= 4/5$, $P(\text{observing yes}|\text{true no})= 1/5$, $P(\text{observing no}|\text{true yes})= 1/5$ and $P(\text{observing no}|\text{true no})= 4/5$.

It is possible to model the observed response to the true status of the respondent, by taking into account the misclassification probabilities of the used method. This will result in estimates of the prevalence of the behaviour under study. The general RR model is given by

$$\boldsymbol{\pi}^* = \mathbf{P}\boldsymbol{\pi} \quad (1)$$

where $\boldsymbol{\pi}^*$ is a vector containing the observed response profile probabilities, $\boldsymbol{\pi}$ is a vector containing the true-response profile probabilities and \mathbf{P} is the transition matrix containing the misclassification probabilities. For the univariate design, the transition matrix is given by

$$\mathbf{P} = \begin{pmatrix} p_{yes|yes} & p_{yes|no} \\ p_{no|yes} & p_{no|no} \end{pmatrix}. \quad (2)$$

By taking the Kronecker product over of the univariate transition matrices, the transition matrix for the multivariate design is found. Model 1 is estimated by maximizing the kernel of the log-likelihood

$$\ln \ell(\boldsymbol{\pi}|n^*, P) = \sum_{i=1}^D n_i^* \ln \left(\sum_{j=1}^D p_{i|j} \pi_j \right), \quad (3)$$

where D is the number of response profiles and n^* are the observed response profile frequencies. The general RR model is a saturated model in the sense that the number of independent parameters equals the number of independent observed response frequencies.

It has been shown that the randomized response method results in more valid answers than direct questioning (van der Heijden et al. (2000); Elffers et al. (2003); Lensvelt-Mulders et al. (2005)). In the study of van der Heijden et al. (2000) respondents who were known to have committed fraud, were asked if they had committed fraud. By using direct questioning only 25% admitted fraud, while by using the RR method 43% to 49% of the participants admitted fraud. Elffers et al. (2003) studied regulatory noncompliance under two Dutch laws; the law on individual rent subsidy and the law on agricultural chemicals. They found that the noncompliance estimates under the randomized response method were twice as high as those obtained under the locked box method, which is a technique for direct self-report.

Boeije and Lensvelt-Mulders (2002) showed that participants found it difficult to give a false *yes* response and some of them admitted that they had given a *no* response instead. This study, together with the study of van der Heijden et al. (2000), indicates that it is unlikely that all respondents follow the instructions of the RR method. Furthermore, the general RR model often shows a lack of fit, which is unexpected since the general RR model is a saturated model. Böckenholt and van der Heijden (2007), Böckenholt et al. (2009), and Cruyff et al. (2007) have shown that there is strong evidence for self-protective (SP) answering in randomized response data by using item response models, latent class models and log-linear models.

They propose a model in which self-protective answering is recognized by a response profile containing only *no* answers, referred to as the basic SP-No model. This model takes SP answering into account by adding a parameter to the general

RR model (see Section 2.1), which results in better fitting models and more realistic estimates of the prevalence of the behaviour under study. Although these models fit much better, the model fit is not perfect. A possible explanation is the underlying assumption of the basic SP-No model, namely it is assumed that SP and the presence or absence of the behaviour under study, are independent. So, one group of respondents always answers by the rules of the RR method and the other group always gives an SP answer, regardless of the outcome of the randomizing device or the true status of the respondent. This assumption implies that the probability of giving an SP answer is independent of the true-response profile of the respondent and the sensitivity of the questions. Though, the independence assumption may be too simplistic.

There are other possible assumptions. The probability of SP may actually depend on the sensitivity of the questions (Böckenholt et al. (2009)). Respondents will be more reluctant to answer *yes* on highly sensitive questions than on less sensitive questions. So the probability of SP will be higher for more sensitive questions than for less sensitive questions.

Furthermore, it is possible that SP answering is related to the true-response profile of a respondent. Some respondents may not trust the RR method and feel threatened by answering *yes*. Respondents with a sensitive characteristic have a higher probability on a *yes* response than respondents without a sensitive characteristic. Therefore, these respondents may show more SP answering than respondents without a sensitive characteristic.

This report focusses on this last possible assumption. It will be tested whether modeling additional SP-No parameters results in a better model fit than the basic SP-No model. Adding SP-No parameters will be done in two ways. First, one parameter will be added to the basic SP-No model to distinguish between respondents whose true-response profiles consist of only *no* responses and respondents whose true-

response profiles consist of at least one *yes* response. So the respondents are split up in two groups, one group of respondents without sensitive characteristics and one group of respondents with at least one sensitive characteristic. Second, SP-No parameters will be added in such a way that the probability of SP is linearly related to the number of sensitive characteristics of a respondent.

Besides studying the model fit, the prevalence estimates under the basic SP-No model and the models with additional parameters will be presented.

Adding SP-No parameters to the general RR model results in over-parameterized models, since the number of independent parameters is larger than the number of independent observed response profiles. Therefore, the SP-No parameters need to be fixed. Hence, the models are estimated by using the log-linear randomized response model. It is possible to win degrees of freedom within this model by restricting interaction terms to zero (see Section 2).

The models are fitted on several datasets to see whether the results are stable over these datasets. The datasets come from a nationwide survey conducted by the Department of Social Affairs in the Netherlands in 2000, 2002, 2004 and 2006. Furthermore, the power of the analyses will be analyzed by performing a simulation study.

The remainder of this paper is structured as follows. In Section 2, the basic SP-No model and the models with additional parameters will be described, as well as the log-linear randomized response model. Section 3 will be devoted to the datasets. The results will be shown in Section 4 and paper will be concluded with a discussion in Section 5.

2 Models

2.1 The Basic SP-No Model

For a multivariate RR design with k dichotomous sensitive questions, let random variable Y denote the true status of a sensitive characteristic, for $y \in \{0 \equiv \text{absent}, 1 \equiv \text{present}\}$, let Y^* denote the response to the sensitive question, for $y^* \in \{0 \equiv \text{no}, 1 \equiv \text{yes}\}$ and $p_{y^*|y}$ the conditional misclassification probabilities of observing response y^* given true status y . Then for the variables Y_1, \dots, Y_k , let the 2^k response profiles be denoted by D , with $D = 1 \equiv 0, \dots, 0$ and $D = 2^k \equiv 1, \dots, 1$ and let θ denote the probability of observing a self-protective response profile ($Y_j^* = 0$ for all $j \in \{1, \dots, k\}$). The basic SP-No model is then given by:

$$\begin{aligned} \boldsymbol{\pi}^* &= (1 - \theta)\mathbf{P}\boldsymbol{\pi} + \mathbf{I}\theta \\ &= \mathbf{Q}\boldsymbol{\pi} \end{aligned} \tag{4}$$

where $\boldsymbol{\pi}^*$ is a vector containing the observed response profile probabilities, $\boldsymbol{\pi} = (\pi_1, \dots, \pi_D)^t$ is a vector containing the true-response profile probabilities, \mathbf{I} is an indicator that is one for the self-protective response profile and zero otherwise, \mathbf{P} is the transition matrix found by taking the Kronecker product over the univariate transition matrices, and \mathbf{Q} is the transition matrix containing the elements:

$$q_{i|j} = \begin{cases} (1 - \theta)p_{i|j} & \text{for } i \neq 1, j \in \{1, \dots, D\} \\ (1 - \theta)p_{i|j} + \theta & \text{for } i = 1, j \in \{1, \dots, D\} \end{cases} \tag{5}$$

For the univariate Kuk design, the transition matrix is given by:

$$\mathbf{P} = \begin{pmatrix} p_{0|0} & p_{0|1} \\ p_{1|0} & p_{1|1} \end{pmatrix} = \begin{pmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{pmatrix} \tag{6}$$

In the empirical datasets obtained by using the FCRR method used in this report, the dice are biased due to a mistake made by the programmer. Therefore, for the univariate FCRR design, the transition matrix is given by:

$$\mathbf{P} = \begin{pmatrix} p_{0|0} & p_{0|1} \\ p_{1|0} & p_{1|1} \end{pmatrix} = \begin{pmatrix} 0.8132 & 0.0671 \\ 0.1868 & 0.9329 \end{pmatrix} \quad (7)$$

2.2 Extended SP-No Models

Let us take a closer look at the transition matrix \mathbf{Q} . Writing out this matrix gives:

$$\mathbf{Q} = \begin{pmatrix} \theta_1 + (1 - \theta_1)p_{1|1} & \theta_2 + (1 - \theta_2)p_{1|2} & \dots & \theta_D + (1 - \theta_D)p_{1|D} \\ (1 - \theta_1)p_{2|1} & (1 - \theta_2)p_{2|2} & \dots & (1 - \theta_D)p_{2|D} \\ \vdots & \vdots & \ddots & \vdots \\ (1 - \theta_1)p_{D|1} & (1 - \theta_2)p_{D|2} & \dots & (1 - \theta_D)p_{D|D} \end{pmatrix} \quad (8)$$

As mentioned before, within the basic SP-No model it is assumed that the probability of SP and the presence or absence of the behaviour under study are independent, so $\theta_1 = \theta_2 = \dots = \theta_D$. Now, by modifying the \mathbf{Q} -matrix, i.e. include additional SP-no parameters, it is possible to find out if SP answering is related to the true-response profile of respondents.

First one SP-No parameter will be added to the basic SP-No model, that makes a distinction between the true-response profile containing only *no* answers and the true-response profiles containing at least one *yes* answer. So within the \mathbf{Q} -matrix $\theta_1 \neq \theta_2 = \dots = \theta_D$. This model will be referred to as the two-parameter SP-No model.

Second, k SP-No parameters will be added to the basic SP-No model, where the probability of SP linearly increases or decreases with the number of *yes* responses in

the true-response profile. Within the \mathbf{Q} -matrix this can be modeled with the log-odds, that is $\text{logit}(\theta_j) = \theta_1 + \beta v_j$, where subscript j indicates the column in the \mathbf{Q} -matrix, θ_1 is the SP parameter for the true-response profile containing only zeros, β is the amount of increase in θ , and v is a vector indicating the number of *yes* responses in every true-response profile ($v = \{0, 1, \dots, k\}$). This model is referred to as the multiple-parameter SP-No model.

2.3 The Log-Linear Randomized Response Model

The log-linear randomized response model estimates the association between the sensitive questions and the prevalence of the behaviour under study corrected for SP answering. In a design with k variables, the k -factor interaction term is set to zero to get an identified model for the basic SP-No model. When modeling additional SP parameters, an equal number of interaction terms need to be set to zero as well in order to get these models identified. For three variables A, B, C , the saturated SP-No model is denoted by $\theta, [AB, AC, BC]$ and is given by:

$$\pi_j = \exp(\lambda_0 + \lambda_a^A + \lambda_b^B + \lambda_c^C + \lambda_{ab}^{AB} + \lambda_{ac}^{AC} + \lambda_{bc}^{BC}) \quad (9)$$

Note that the λ terms are restricted to sum to zero over any subscript. When the vector π_j contains the response profile probabilities, the basic and extended SP-No models are estimated by maximization of the kernel of the log-likelihood:

$$\ln \ell(\lambda, \theta | n^*, P) = \sum_{i=1}^D n_i^* \ln \left(\sum_{j=1}^D q_{i|j} \pi_j \right) \quad (10)$$

This will be done by using the `nlm`-function in R. To force the θ parameters to be in the interval $(0, 1)$ they are estimated by the logistic functions $1/(1 + \exp(-\alpha))$ for the basic SP-No model, for the two-parameter model θ_1 is estimated by $1/(1 + \exp(-\alpha))$

and $\theta_{2,\dots,16}$ is estimated by $1/(1 + \exp(-\alpha - \beta))$ and for the multiple-parameter model the θ 's are estimated by $1/(1 + \exp(-\alpha - \beta * v))$.

It can be seen that the basic model is nested within the extended models, since the only difference is the parameter β . If this parameter is estimated to be zero, the extended models reduce to the basic model. In the remainder of this paper, this model will be referred to as the log-linear model.

3 The Social Welfare Study

The empirical datasets used in this report come from a nationwide survey conducted by the Department of Social Affairs in the Netherlands in 2000, 2002, 2004 and 2006. In the Netherlands, employees are insured under various social welfare acts against loss of income due to redundancy, disability, or sickness. Within this report we focus on respondents who receive a Disability Benefit Income. People receiving this form of benefit are chronically ill or disabled and are unable to work fulltime. The respondents receive up to 70% of their last earned wages and have to comply to the rules and regulations of this benefit. For example, they are obliged to report additional income and improvements in their health status. Noncompliance with these rules and regulations is considered fraud and can have serious consequences.

Within this report, the following four questions about health related regulations are used:

1. Has a doctor or specialist ever told you that the symptoms that your disability classification is based on have decreased without you informing the Department of Social Services of this change?
2. At a Social Services checkup, have you ever acted as if you were sicker or less able to work than you actually are?

3. Have you yourself ever noticed an improvement in the symptoms causing your disability, for example, in your present job, in volunteer work, or the chores you do at home, without informing the Department of Social Services of this change?
4. For periods of any length at all, do you ever feel stronger, healthier, and able to work more hours without informing the Department of Social Services of this change?

In 2000, these questions were answered according to the Kuk design and in the remaining years the questions were answered according to the FCRR design. The four questions above are the variables $Y_1^*-Y_4^*$ with observed-response profiles 0000, 0001, \dots , 1111. The frequencies for all possible response profiles and the total number of respondents for every year are shown in Table 1.

Table 1: Response profile frequencies and total number of respondents for the health related questions of the social welfare study conducted in the years 2000, 2002, 2004 and 2006

| Response profiles | 2000 | 2002 | 2004 | 2006 |
|-------------------|------|------|------|------|
| 0000 | 533 | 769 | 373 | 597 |
| 0001 | 146 | 189 | 80 | 82 |
| 0010 | 86 | 142 | 82 | 78 |
| 0011 | 60 | 78 | 33 | 21 |
| 0100 | 91 | 132 | 58 | 76 |
| 0101 | 40 | 56 | 20 | 22 |
| 0110 | 29 | 40 | 22 | 16 |
| 0111 | 30 | 31 | 13 | 3 |
| 1000 | 93 | 137 | 62 | 80 |
| 1001 | 40 | 31 | 15 | 30 |
| 1010 | 31 | 32 | 20 | 25 |
| 1011 | 20 | 30 | 6 | 7 |
| 1100 | 34 | 34 | 18 | 15 |
| 1101 | 10 | 26 | 13 | 6 |
| 1110 | 22 | 15 | 5 | 5 |
| 1111 | 43 | 18 | 10 | 7 |
| Total N | 1308 | 1760 | 830 | 1070 |

4 Results

The data are analyzed by using the log-linear randomized response model, where this model is restricted to main effects only and a model with first order interaction terms added. First, the results of the main effects only model will be discussed. For this model the estimates for α and β are extreme for the two-parameter SP-No model (see appendix Table 5) with a high negative value for α and a high positive value for β . For the multiple-parameter model the parameter for α is estimated equal to α in the basic SP-No model, while the parameter for β is estimated to be zero. So the multiple-parameter model is reduced to the basic model when only main effects are modeled.

Table 2 shows the model fit of the basic and extended SP-No models. The main effects only model is identified, since the model fit for the two-parameter SP-No model is somewhat better than for the basic SP-No model. So an extra SP parameter in the model does add something. Though, the fit for the multiple-parameter model is equal to the fit of the basic model. Furthermore, it can be seen that the main effects only model does not fit the data well.

The estimated SP parameters are shown in Table 4. For the two-parameter model using the main effects only log-linear model, the SP parameter for respondents who do not possess any of the characteristics under study is estimated to be zero, while the SP parameter for respondents possessing at least one characteristic is estimated extremely high (about 36 to 64%).

For the multiple-parameter SP-No model, it can be seen that the SP parameters are all estimated equal to the SP parameter of the basic model, when the main effects only log-linear model is used. So in this situation, the multiple-parameter model is reduced to the basic model.

However, since the main effects only model does not fit the data, the estimated

SP parameters actually cannot be interpreted. Furthermore, the prevalence estimates under this model cannot be interpreted as well.

In the log-linear model with interaction terms added, the parameter estimates for α and β are only extreme for the two-parameter model fitted on the data of 2002 (see appendix Table 6 and 7). For the other datasets, the parameter estimate for α of the extended models is close to the estimate of the basic model, though the parameter estimates for β differ per model. The standard error of the estimate of β is extremely high for the data of 2000 and 2004 when using the two-parameter model and for the multiple-parameter model this is also the case for the data of 2000 and 2006.

The model fit is good for the log-linear model with interaction terms added for all SP-No models, but does not distinguish the SP-No models from each other (Table 2). However, the estimates for the SP parameters (Table 4) and the prevalence estimates (Table 4) are different for the three SP-No models. These results indicate that the log-linear model with interaction terms is not identified. This identification problem can be explained by that in the log-linear models with interaction terms, the log-linear parameters together with the two SP parameters in the extended models, explain as much variance as the log-linear parameters together with one SP parameter in the basic model. It is not possible to decide on a model by looking at the model fit, because they are all almost the same. Therefore, the basic model, the most parsimonious model, is chosen. The prevalence estimates for this model are given in Table 4 and show that the lowest prevalence is 1.4% for question 2 in the social welfare study of 2006 and the highest prevalence is 24.7% for question 4 in the social welfare study of 2000.

Table 2: Model fit for the log-linear model with only main effects and the log-linear model with interaction terms added, of the basic, two-parameter and multiple-parameter SP-No models on the data of the social welfare study

| Year | Model | Main effects only | | | Interaction terms added | | |
|------|-------|-------------------|----------|----|-------------------------|----------|----|
| | | -logl. | χ^2 | df | -logl. | χ^2 | df |
| 2000 | Basic | 2859.40 | 107.31 | 10 | 2821.45 | 6.30 | 4 |
| | Two | 2857.09 | 99.34 | 9 | 2821.45 | 6.30 | 3 |
| | Multi | 2859.40 | 107.32 | 9 | 2821.45 | 6.30 | 3 |
| 2002 | Basic | 3614.82 | 42.68 | 10 | 3598.70 | 9.97 | 4 |
| | Two | 3611.75 | 36.13 | 9 | 3598.58 | 9.65 | 3 |
| | Multi | 3614.82 | 42.68 | 9 | 3598.70 | 9.97 | 3 |
| 2004 | Basic | 1673.46 | 26.56 | 10 | 1663.28 | 1.76 | 4 |
| | Two | 1672.50 | 23.55 | 9 | 1663.27 | 1.75 | 3 |
| | Multi | 1673.46 | 26.56 | 9 | 1663.28 | 1.76 | 3 |
| 2006 | Basic | 1825.58 | 20.39 | 10 | 1819.10 | 2.00 | 4 |
| | Two | 1824.48 | 16.43 | 9 | 1819.10 | 2.00 | 3 |
| | Multi | 1825.58 | 20.39 | 9 | 1819.10 | 2.00 | 3 |

Table 3: Values of the SP parameters under the basic, two-parameter and multiple-parameter SP-No models for the four questions of the social welfare study conducted in 2000, 2002, 2004 and 2006 for the log-linear model with main effects only and with interaction terms added

| Year | Basic SP-No | Two-parameter | | | Multiple-parameter ^a | | | | |
|------------------------------------|-------------|---------------|-----------------------|------------|---------------------------------|----------------|---------------|------------|--|
| | θ | θ_1 | $\theta_{2,\dots,16}$ | θ_I | θ_{II} | θ_{III} | θ_{IV} | θ_V | |
| Model with main effects only | | | | | | | | | |
| 2000 | .2718 | 0 | .3651 | .2718 | .2718 | .2718 | .2718 | .2718 | |
| 2002 | .2351 | 0 | .4257 | .2351 | .2351 | .2351 | .2351 | .2351 | |
| 2004 | .2329 | 0 | .4507 | .2329 | .2329 | .2329 | .2329 | .2329 | |
| 2006 | .3075 | 0 | .6447 | .3075 | .3075 | .3075 | .3075 | .3075 | |
| Model with interaction terms added | | | | | | | | | |
| 2000 | .1449 | .1790 | .0473 | .1861 | .0438 | .0091 | .0018 | .0004 | |
| 2002 | .1290 | 0 | .4252 | .1295 | .1279 | .1264 | .1249 | .1234 | |
| 2004 | .1349 | .1596 | .0008 | .1375 | .1280 | .1190 | .1106 | .1027 | |
| 2006 | .1996 | .1969 | .2320 | .2104 | .0734 | .0230 | .0069 | .0021 | |

^atrue response profile containing 0 to 4 *yes* responses

Table 4: Prevalence estimates corrected for self-protective response behaviour under the basic, two-parameter and multiple-parameter SP-No models for the four questions of the social welfare study conducted in 2000, 2002, 2004 and 2006 given the log-linear model with interaction terms added

| | Model | Question 1 | Question 2 | Question 3 | Question 4 |
|------|--------------------|------------|------------|------------|------------|
| 2000 | Basic SP-No | .1062 | .1094 | .1450 | .2469 |
| | Two-parameter | .0954 | .0982 | .1301 | .2216 |
| | Multiple-parameter | .0916 | .0937 | .1244 | .2162 |
| 2002 | Basic SP-No | .0367 | .0568 | .0862 | .1511 |
| | Two-parameter | .0556 | .0872 | .1315 | .2289 |
| | Multiple-parameter | .0365 | .0566 | .0860 | .1507 |
| 2004 | Basic SP-No | .0317 | .0442 | .1055 | .1041 |
| | Two-parameter | .0275 | .0418 | .0920 | .0902 |
| | Multiple-parameter | .0308 | .0471 | .1047 | .1023 |
| 2006 | Basic SP-No | .0502 | .0143 | .0255 | .0551 |
| | Two-parameter | .0523 | .0149 | .0265 | .0574 |
| | Multiple-parameter | .0410 | .0116 | .0207 | .0458 |

5 Discussion

Randomized response is an interview technique used to ask sensitive questions. By using a randomizing device, the respondent's privacy is protected and more realistic estimates of the prevalence of the behaviour under study are obtained. Despite this protection, not all respondents will answer the questions by the rules of the RR method. These respondents give a self-protective answer to every question, which results in underestimation of the prevalence of the behaviour under study. The prevalence estimates are obtained by modeling the observed response to the true status of the respondent, taking into account the misclassification of the responses. To correct for self-protective answers, an SP-parameter can be added to the general RR model.

Within the basic SP-No model, one SP-parameter is added, independent of the true response profiles of the respondents. Although this model fits much better than the general RR model and gives more realistic estimates, this model still does not fit the data very well all the time. The aim of the current study was to investigate whether the model fit can be improved by modeling more than one SP-parameter. The two-parameter SP-No model assumes that there is a difference in SP between respondents who do not possess any of the characteristics under study and respondents who possess at least one characteristic. Within the multiple-parameter SP-No model it is assumed that there is a linear relationship between the number of characteristics a respondent possesses and SP. So, the extended SP-No models assume that there is a relationship between the true-response profile of the respondent and SP, while the basic SP-No model assumes that SP is independent of the true-response profile.

The results showed that deciding on a model is more complicated than expected. The log-linear model containing only main effects is identified and shows a better model fit for the two-parameter model than for the basic SP-No model. This indicates that it is useful to add an extra SP parameter. However, these models show a

lack of fit, so the estimated SP parameters and the prevalence estimates cannot be interpreted. When first order interaction terms are added to the log-linear model, the model fit improves a lot. Though, within this log-linear model all three SP-No models fit equally well, but result in different SP parameters and prevalence estimates for the three models. This indicates that these models are not identified. So, in the extended models the log-linear parameters together with the extra SP parameter estimate as much variance as the parameters of the basic SP-No model. Since the basic model is the most parsimonious model, this model is preferred.

Within this report, it is tested whether SP is related to the true-response profile of respondents. However, it may also be possible that SP is related to the observed response profiles or a combination of the true and observed response profiles. For example, it can be hypothesized that the probability of SP for the fourth question will be higher when the respondent had to give a *yes* response on the first three questions than when the answers to these questions were *no*, because a respondent may feel more threatened by answering *yes* to a lot of questions than when he or she only has to answer *yes* to a few questions. This effect may be larger for persons who possess some or all of the characteristics under study than for those who do not possess the characteristics, since their probability on a *yes* response is higher. It would be interesting to model these assumptions in future research.

The aim of this study was to improve the model fit of the RR method by modeling assumptions different from the assumption of the basic SP-No model. The studied assumptions can be summarized to the assumption that SP is related to the true-response profile of respondents. The results showed that the extended models in which this assumption is modeled, did not fit the data better than the basic model. Furthermore, the results showed that there are some identification problems. Though the basic SP-No model is robust and is therefore preferred over the extended models.

Besides, the basic SP-No model is a parsimonious model and quite easy to explain to the applied researcher, which make this model appealing as well.

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A Tables for parameter estimates of the log-linear models

Table 5: Parameter estimates (and standard errors) of the log-linear model containing main effects only

| Year | Parameter | Basic Model | Two-parameter Model | Multiple-parameter Model |
|------|-------------|---------------|---------------------|--------------------------|
| 2000 | λ_A | .760 (.087) | .708 (.093) | .760 (.087) |
| | λ_B | .725 (.084) | .605 (.089) | .725 (.084) |
| | λ_C | .609 (.075) | .531 (.078) | .609 (.075) |
| | λ_D | .315 (.061) | .231 (.065) | .315 (.061) |
| | α | -.985 (.084) | -30.230 (.089) | -.985 (.087) |
| | β | | 29.677 (.088) | 0 (.074) |
| 2002 | λ_A | 1.284 (.121) | 1.203 (.141) | 1.284 (.121) |
| | λ_B | 1.098 (.091) | .972 (.100) | 1.098 (.091) |
| | λ_C | .933 (.073) | .784 (.079) | .933 (.073) |
| | λ_D | .673 (.055) | .505 (.060) | .673 (.055) |
| | α | -1.180 (.089) | -129.943 (.095) | -1.180 (.089) |
| | β | | 129.643 (.095) | 0 (.124) |
| 2004 | λ_A | 1.347 (.194) | 1.241 (.228) | 1.347 (.194) |
| | λ_B | 1.192 (.153) | 1.031 (.165) | 1.192 (.153) |
| | λ_C | .861 (.098) | .688 (.108) | .861 (.098) |
| | λ_D | .869 (.099) | .691 (.108) | .869 (.099) |
| | α | -1.192 (.135) | -23.541 (.146) | -1.192 (.135) |
| | β | | 23.343 (.146) | 0 (.208) |
| 2006 | λ_A | 1.141 (.128) | .799 (.137) | 1.141 (.128) |
| | λ_B | 1.497 (.222) | 1.292 (.288) | 1.497 (.222) |
| | λ_C | 1.299 (.162) | 1.012 (.185) | 1.299 (.162) |
| | λ_D | 1.109 (.122) | .772 (.133) | 1.109 (.122) |
| | α | -.812 (.112) | -12.262 (.150) | -.812 (.112) |
| | β | | 12.858 (.150) | 0 (.227) |

Table 6: Parameter estimates (and standard errors) of the log-linear model containing main effects and interaction terms

| Year | Parameter | Basic Model | Two-parameter Model | Multiple-parameter Model |
|------|----------------|---------------|---------------------|--------------------------|
| 2000 | λ_A | .644 (.076) | .681 (.074) | .685 (.074) |
| | λ_B | .392 (.069) | .391 (.068) | .402 (.067) |
| | λ_C | .039 (.063) | .040 (.061) | .051 (.060) |
| | λ_D | .123 (.059) | .160 (.055) | .163 (.055) |
| | λ_{AB} | 2.693 (.302) | 2.287 (.291) | 3.445 (.289) |
| | λ_{AC} | 1.832 (.143) | 1.367 (.136) | .267 (.134) |
| | λ_{AD} | -3.273 (.083) | -2.365 (.076) | -2.420 (.074) |
| | λ_{BC} | 2.649 (.190) | 2.859 (.184) | 2.961 (.182) |
| | λ_{BD} | .033 (.089) | -.583 (.083) | .479 (.082) |
| | λ_{CD} | 4.009 (.115) | 3.754 (.107) | 2.750 (.107) |
| | α | -1.775 (.158) | -1.523 (.178) | -1.476 (.183) |
| | β | | -1.479 (1.130) | -1.607 (2.229) |
| 2002 | λ_A | .852 (.123) | .820 (.127) | .853 (.123) |
| | λ_B | .918 (.100) | .766 (.104) | .919 (.100) |
| | λ_C | .892 (.075) | .739 (.079) | .893 (.075) |
| | λ_D | -.400 (.056) | -.343 (.062) | -.399 (.056) |
| | λ_{AB} | .827 (.159) | .757 (.172) | .827 (.158) |
| | λ_{AC} | .533 (.090) | .458 (.097) | .533 (.090) |
| | λ_{AD} | .180 (.069) | .122 (.078) | .180 (.069) |
| | λ_{BC} | -.455 (.074) | -.360 (.084) | -.455 (.074) |
| | λ_{BD} | .693 (.074) | .556 (.084) | .692 (.074) |
| | λ_{CD} | .814 (.094) | .672 (.110) | .814 (.093) |
| | α | -1.910 (.163) | -19.307 (.144) | -1.906 (.163) |
| | β | | 19.006 (.144) | -.014 (.262) |

Table 7: Parameter estimates (and standard errors) of the log-linear model containing main effects and interaction terms (continued)

| Year | Parameter | Basic Model | Two-parameter Model | Multiple-parameter Model |
|------|----------------|---------------|---------------------|--------------------------|
| 2004 | λ_A | 3.769 (.152) | 4.495 (.151) | 3.787 (.152) |
| | λ_B | -2.326 (.128) | -3.017 (.126) | -2.334 (.127) |
| | λ_C | .802 (.109) | .844 (.104) | .807 (.109) |
| | λ_D | .029 (.102) | .037 (.098) | .034 (.101) |
| | λ_{AB} | 4.024 (.315) | 4.773 (.312) | 4.037 (.314) |
| | λ_{AC} | -.726 (.108) | -.791 (.103) | -.726 (.108) |
| | λ_{AD} | .358 (.141) | .401 (.134) | .358 (.140) |
| | λ_{BC} | .775 (.124) | .845 (.117) | -.776 (.124) |
| | λ_{BD} | .614 (.171) | .605 (.161) | .614 (.170) |
| | λ_{CD} | .433 (.134) | .473 (.123) | .433 (.134) |
| | α | -1.858 (.232) | -1.661 (.265) | -1.836 (.235) |
| | β | | -5.406 (93.787) | -.083 (.454) |
| 2006 | λ_A | -1.143 (.126) | -2.773 (.127) | -1.406 (.123) |
| | λ_B | 3.097 (.226) | 2.220 (.226) | 2.428 (.225) |
| | λ_C | 2.059 (.171) | 3.638 (.172) | 2.381 (.169) |
| | λ_D | -1.657 (.133) | -.791 (.134) | -.929 (.130) |
| | λ_{AB} | -2.476 (.154) | -2.464 (.154) | -2.476 (.149) |
| | λ_{AC} | 5.110 (.210) | 6.678 (.211) | 5.427 (.207) |
| | λ_{AD} | 1.612 (.220) | 1.602 (.222) | 1.640 (.211) |
| | λ_{BC} | 5.375 (.248) | 14.058 (.249) | 10.450 (.244) |
| | λ_{BD} | 5.449 (.181) | 13.267 (.183) | 9.829 (.177) |
| | λ_{CD} | -3.431 (.141) | -12.127 (.142) | -8.487 (.136) |
| | α | -1.389 (.172) | -1.406 (.170) | -1.323 (.182) |
| | β | | .209 (.778) | -1.213 (4.037) |