Improving the mathematical reading skills of students who read braille by scaffolding reading strategies

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Abstract

Students who are braille-dependent use an electronic braille display to access mathematical expressions. The linear and extensive notation makes it challenging for these students to read and comprehend mathematics. Previous research revealed that substantial more time is needed to solve mathematical problems, partly due to inefficient finger movements. In this research the influence of an intervention on braille reading strategies for mathematics is investigated. The developed reading strategies, scanning, tracing and fixating, are expected to help to respectively explore text, find specific symbols in expressions, and focus on these symbols. Three students, experienced braille readers from grade 1, 2 and 5, participated in the study. The intervention consisted of five lessons of 30 minutes and used a scaffolding approach. The reading strategies of the students were evaluated with help of finger tracking during pre, post and retention tests. After the intervention, the students needed only 45% of the initial reading and solving time and the reading movements became more efficient. The effects were still present during the retention test, six weeks after the post test. Conclusion of this research is that teaching explicit reading strategies for mathematics can be really effective. With a compact intervention, it is possible to decrease the needed time drastically. This gives these students more time to practice and develop their mathematical skills.

Keywords: braille reading, mathematical expression, reading strategies, intervention study, finger tracking.

Introduction

In today's world, mathematics is mandatory in almost all secondary educational programs. Both mathematical knowledge and mathematical problem solving skills play a role in lots of different jobs, this is the reason for entry requirements on mathematics in higher educational programs. Although most students are able to fulfill these requirements, mathematics is often seen as a challenging course for various reasons. For some individuals, the personal capacities or motivation are not the limiting factor, but their lack of visual capacities. Nowadays, these students read braille on an electronical refreshable braille display, which makes it possible to access all different kinds of information, including mathematical expressions. Despite of these possibilities and improvements, only a small percentage of these students is able to learn mathematics at a level comparable to their sighted peers (Fajardo Flores & Archambault, 2012).

One of the reasons for the difference in performance between students that are braille dependent and their peers, is that working with mathematical expressions is difficult for these students. This becomes visible in the research of Van Leendert, Doorman, Pel, Drijvers, and Van Der Steen (2017), where students who are braille dependent need much more time than students who are sighted need for solving mathematical exercises. Several reasons can be found for these differences. One of them is the linear representation of mathematical expressions on the braille display. This makes the expressions easily accessible, but also brings difficulties along. An example to demonstrate the challenge of the linear notation is given below. This equation uses more than one level and several symbols to represent the mathematical expression in a structured way:

$$\frac{(x+2)^2+1}{\sqrt{3}} = \frac{3(x+2)+1}{\sqrt{3}}$$

To show this expression on a braille display, a linear representation has to be used. This requires the use of additional brackets and descriptive notations. The students read all elements of expressions in one movement from left to right, it is not possible to overlook parts, which is helpful for braille readers. On the other hand, it is challenging to obtain and maintain an overview of expressions, in contrast with reading regular language. Reasons for difficulty with overview are for example corresponding brackets which are far apart from each other. This becomes clear with the linear notation of the example:

$$((x+2) \wedge 2+1)/(\mathrm{sqrt}(3)) = (3(x+2)+1)/(\mathrm{sqrt}(3))$$

The above linear notation is not the only obstacle for students who read braille, tactile reading is also a more complicated process than reading by sight (Steinman, LeJeune, & Kimbrough, 2006). The learning process of reading braile is similar to that of learning sighted reading. Sighted children learn the basic concepts of language and start recognizing letter patterns at a young age. In a certain stage children get training in recognizing letters and making words. If these skills are settled, they learn to acquire and receive information from texts (Chall, 1983). This process is similar for braille learners (Steinman et al., 2006), although automatic decoding may take longer to learn and process. The students who read braille have less spontanous exposure to language representations during the prereading phase, this is particularly the case for numbers and symbols. Also the learning process is primarily focused on processing letters, and secondary on symbols and numbers. In general, braille readers start reading braille on paper to become familiar with textual layouts and structures, these are more difficult to recognise on the braille display where only one line is shown. At a certain stage pupils start to use the braille display. In the Netherlands, all students at the age of 12 work with a refreshable braille display linked to a computer. Through this device, students have access to digital information (D'Andrea, 2012).

On both paper and braille display, braille is used as writing method, but on the display various types of braille alphabet are available. For reading regular texts the differences between the alphabets are small and do not cause specific errors (Argyropoulos, Kouroupetroglou, Martos, Nikolaraizi, & Chamonikolaou, 2014). The differences are mostly related to numbers and symbols and therefore the chosen alphabet influences the transfer of mathematics. Traditionally, every letter and punctuation mark is represented by a specific combination of raised dots in a 2×3-matrix. Numbers are represented by a signalling number sign (in Figure 1 represented as "#") followed by the corresponding letter(s) (a=1, b=2, etc) in the alphabet. It is possible to use this alphabet on the braille display, but this device also facilitates other alphabets because of the extended 2×4-matrix (Kacorri & Kouroupetroglou, 2013). Multiple alphabets were developed that use the eight dots and all of them use the same combinations for letters. Numbers are a variation on the numbers in literature braille, whereas capitals, italics and other distictions make use of the extra two lower dots. Figure 1 shows the differences between three alphabets. In European braille, numbers are represented by the corresponding letter plus the sixth dot. In American braille, numbers are characterised by the corresponding letter lowered by one dot. Symbols get easily mixed up because of similarities, especially in mathematical contexts where it is often not possible to guess or correct reading errors with help of the context.

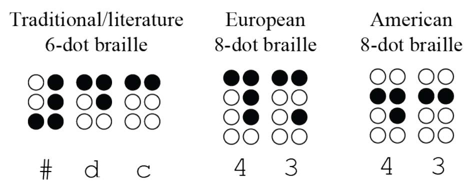


Figure 1: Writing the number 43 in various braille alphabets

During tactile reading the fingers move along the braille line to gather information, the fingers can move together or independently to gather information (Breidegard et al., 2008). Bertelson, Mousty, and D'Alimonte (1985) studied the patterns of handmovements when regular text is being read, and distinguished two main types of hand cooperation: the conjoint and disjoint pattern. In conjoint reading the two reading fingers proceed side by side along the same passage, about one or two cells apart. In disjoint reading the two fingers simultaneously or successively explore different parts of the text (Breidegard, Jönsson, Fellenius, & Strömqvist, 2006). Most participants in their study showed a mixed pattern, which combines the two exploration patterns. Movements from left to right match the linear character of written text and language. Although mathematics is linearly displayed in braille, these movements do not match the structure of mathematical expressions, which are two dimensional in nature. In addition, braille dependent students read one element at a time, which gives them a small perceptual view. This makes it difficult to get an overview of expressions.

Several investigations focused on possibilities for improving the access to mathematics for braille readers. For example, in the research of Fajardo Flores and Archambault (2012), an interface is designed for algebra that facilitates better comprehension, editing and collaboration. The research of Van Leendert et al. (2017) compared the finger movements of students who read braille with eye movements of sighted students, while they are reading and comprehending mathematical expressions. This research made clear that students who read braille need almost five times as much time for solving exercises than sighted students. The extra time was due to problems like recognizing mathematical symbols and correcting initial reading errors. The finger movements are often not efficient, for example, the participants reread a lot and these regressions always begin at the start of the expression. Suggested is that developing more effective reading strategies can help to prevent rereading and decrease the needed time. This provides a clear starting point for the current research. In a short intervention, we want to develop hypothetical more efficient reading strategies and want to teach students who are braille dependent to integrate them in their mathematical reading behaviour. The lessons will contain an introduction of possible reading strategies and exercises to use them. This brings us to the following research question:

How and to what extent can a short intervention improve the mathematical reading strategies of braille-dependent students?

This question will be answered through the following subquestions:

1. To what extent does an intervention oriented on mathematical reading strategies in braille improve the students' performance in reading and solving mathematical expressions?
2. How does the intervention support the development of independent use of mathematical reading strategies?
3. To what extent does the intervention change the students' awareness about mathematical reading strategies?

Theory on reading strategies and instruction

For designing an appropriate intervention, we first explore the topics of reading comprehension, mathematics reading and braille reading in more detail. We are interested in the comprehension process with the specific circumstances where mathematics is read by touch. With this knowledge, we can construct an overview of hypothetically efficient reading strategies. Combined with theory about effective instruction methods, this will form the basis for the intervention.

Conprehension of regular and mathematical text

Text comprehension is often the ultimate goal of reading instruction. During this process the meaning of written text is extracted and constructed ("Chapter Six - Reading Comprehension for Braille Readers: An Empirical Framework for Research", 2014). This is also the case for the comprehension of mathematical expressions: students construct the meaning of expressions, so that they can manipulate and simplify it. In more advanced exercises with more than one solving step, this process is repeated several times. Mathematical structures and symbols play an important role in the construction of meaning. For example, in the expression $6 + 3(4^2 - 7)$, the brackets structure the expression and indicate the part that gets higher precedence. Futhermore, it is not necessary to mention the multiplication symbol between the " 3 " and " (" and for the square we use a " 2 " in superscript.

Giving meaning to mathematical expressions is often perceived as a difficult task. Despite the difference in meaning between regular text and mathematical expressions, similarities are found between both comprehension processes (Jansen, Marriott, & Yelland, 2007). To demonstrate, parts of texts or expressions that contain content, need more comprehension time than parts that give structure. This can be nouns or numbers, respectively prepositions or brackets and operators. This similarity in comprehension processes shows that generally readers can extract the structure quickly, and have to fill this framework with the content. A big difference between regular and mathematical text is that expressions represent an abstract meaning, which is portrayed as two dimensional expressions. This spatial alignment assists in the recognition of structures of mathematical expressions (Jansen et al., 2007), for example in fractions and square roots. This is helpful for the comprehension process, as it supports the recognition of the global structure and local elements (Drijvers, Goddijn, & Kindt, 2011).

The usefulness of the two dimensional notation is detectable in the eye movements of sighted mathematics readers. They only need a few seconds to get an overview of the expression (Van Leendert et al., 2017). The eye movements follow the structure of expressions immediately (Jansen et al., 2007). This is the case for all levels of exercises, from arithmetic to complex abstract equations (Duval, 2006). Nevertheless, the symbols and alignment that display the structure so clearly for sighted mathematics readers, is not accessible to braille readers. Compared to their sighted peers, braille students face a challenge when comprehending expressions. This is visible in the hand movement observations of Van Leendert et al. (2017). The braille readers needed a lot of movements to explore and solve mathematical exercises, these movements were needed for exploring the structure and content of the expression. Only after this time consuming exploration, students can start solving the exercises.

Handmovements for reading braille

Where the comprehension process for mathematics shows several influential differences for regular and braille readers, the process is more similar for regular texts. Texts tell a story in a sequential way, which matches the linearly written words and letters. Braille readers learn to read with fluent movements from the start of the line till the end, which matches the structure of texts. In the learning process of reading, usually a conjoint pattern is taught. This pattern could lead to deep and precise reading (Breidegard et al., 2006). When developing personal reading habits, most readers show a mixed pattern, with characteristics of both conjoint and disjoint reading. The disjoint pattern could be useful for quickly scanning text (Breidegard et al., 2006). Another pattern, "assisted-one-handed reading", was observed by Bertelson et al. (1985) where the left hand marks the beginning of each line and the right hand explores the text.

The research of Van Leendert et al. (2017) studied the patterns of hand movements for mathematics. The three participants of this research showed a more conjoint pattern in the beginning of the process, and a more disjoint pattern in the end. Probably this approach is due to the conjoint reading strategy they initially learn for reading and exploring regular text, followed by movements that contribute searching and rereading. Sometimes one finger lingers at strategic positions, like brackets. Regressions are often ineffective, since they often start at the first braille cell. The occurring errors mostly concern the recognition of symbols. Such errors form the first level of Newman's (1983) error analysis. The other levels are comprehension and process errors, where problems arise with respectively understanding symbols and structures, and performing mathematical procedures, such as removing brackets. Errors at the reading level often cause difficulties in the comprehension and processing levels (Newman, 1983).

The occurring reading errors and the extensive movements needed for exploring and solving expressions give clues about the difficulties that the students face. The student reads and decodes all seperate numbers and symbols of the expression, but it is hard to make sense of it immediately. Also remembering and structuring all elements is very demanding. A lot of rereading is needed to get all the elements in place. Only after that, the solving process can start. Hopefully, no errors are made, because in that case the process has to start over.

Mathematical solving process

Braille students have difficulties with getting an overview of expressions, while sighted students get this overview in a few seconds. Our hypothesis is that braille students do not focus on structure of the exercise first, but try to construct the complete expression right away. With easy arithmetics, like $3 + 7 - 2$, this approach is sufficient. But when expressions get more difficult, like $4^2 - 2 \times \frac{5+1}{2}$, it is difficult to recall and structure all elements at the same time. Again, this is especially difficult for braille readers, who only have access to the linear notation: $4 \wedge 2 - 2 * (5 + 1)/2$.

We can learn from the approach that sighted readers use when reading and solving mathematics. Although it takes only a few seconds, their first step is to explore the structure of the expression. When the structure is clear, it is possible to determine the solving steps and explore content information. Awareness of the structure of the expression is often a prerequisite to appropriately select an operation or a solution strategy (Drijvers et al., 2011). With a better sense of the structure the solving process is quicker and more accurate, opportunities for mistakes that occur in long calculations are avoided (Hoch & Dreyfus, 2004). This is in line with the problem solving process described by Polya (1957). This process starts with getting an overview of the problem. During the second and third phase the students make a solving plan and carry it out. In the last phase they evaluate the exercise and their solution. For algebra the third phase takes most time and gets the most attention. In the example of the previous paragraph, the fraction is a right part to start. After forming a plan, the elements are decoded and simplified, in this case a good first step is executing $5 + 1$. In this way the exercise is solved.

Hypothetical reading strategies

For braille readers, the global structure is more difficult to grasp. Our hypothesis is that a first explorational movement with a focus on mathematical symbols will improve the understanding of the global structure. A strategy with this focus, from now on called scanning, is the first reading strategy of the intervention. The goals is that braille students extract the structure of expressions with less movements and in less time. Mathematical symbols, like brackets and the equals sign, are important for identifying the global structure, numbers hardly play a structuring role. Brackets are often important (Hoch & Dreyfus, 2004), but they can function on global as well as more local levels, especially in the linear notation (Jansen et al., 2007). The most straightforward movement for this strategy seems to be conjoint reading, where both fingers scan the expression together.

With such a scanning strategy, the first phase of the solving process is executed. In the second phase the solving steps get planned out. Based on the retrieved structure, students can plan their solving steps. This does not require reading. Probably this is good to mention, because braille students are used to keeping their hands in motion. In the third phase, the solving steps are executed. Therefore it could be necessary to read at a specific place - in the example from the last page, this would be the $5 + 1$ between the brackets. Regressions are needed to arrive at the desired location. In previous research (Van Leendert et al., 2017) regressions are often ineffective, since they mostly start at the first braille cell. To optimize this process, the second strategy, tracing, is introduced. For efficient tracing, the element and physical location should be remembered so that the fingers can return to that position. Possible corresponding movements are repositioning with both fingers or with one finger, in that case the other finger can linger or read.

After repositioning, it is possible to aim for two positions, one for each finger. In the example it could be useful to mark both the 5 and the 1. Although motion is needed to perceive information (Breidegard et al., 2006), it could be useful to linger the fingers in a position to have time to think and calculate. Lingering is unusual in text reading, therefore this is new for the students. It is also possible to fixate one finger on a specific location. To simplify the fraction in the example, it could be useful to fixate on the multiplication sign as a reference. The other finger can delve into the fraction, if the value is found the next steps can get addressed. This is the third and last reading strategy that is carried out in this research. In the last evaluative phase the students can scan the expression once again to check. The conjecture is that the three reading strategies, scanning, tracing and fixating, could help students to structure their mathematical way of working (see Table 1).

Table 1: Content of the intervention

Reading strategy	Definition	Reading strategy movements
Scanning	The student scans to explore the expression until the structure of the expression is clear.	Conjoint
Tracing	The student traces to select a part of the expression and moves back to that position.	Reposition One finger assisted Disjoint
Fixating	The student fixates one or two fingers to focus on parts of the expression for closer reading and deeper understanding.	One finger assisted Lingering

Instruction method

The core of the intervention is now defined. The next thing to determine is an appropriate way to teach the students to integrate the strategies in their own mathematical working process. Stanfa and Johnson (2015) indicate that guided reading and individual feedback are elements of an instructional strategy that effectuates strong improvements on braille reading and reading comprehension. To implement these elements, a teacher can summarize, clarify or help in case of mistakes or inconveniences (Liang & Dole, 2006). Individual feedback can be provided during one-to-one sessions about processing of text or about hand movements. In the research of Breidegard et al. (2006) similar conjectures are found. In this research hand movements are observed with new technology, but participants reported that they started changing their hand movements in response to the investigation. Probably the awareness of possibilities encourages them to experiment and revise their reading. This finding suggests to investigate the influences of developing reading techniques, individual feedback on reading styles and creating awareness about personal preferences. One-to-one lessons can improve reading and prove to have long-term effects (Wasik & Slavin, 1993).

Individual assistance in reading and problem solving that is oriented on future independence often involves a scaffolding process. In this process the student is guided to solve a task which would be beyond his unassisted efforts, ultimately the student is able to perform the task without assistance (Wood, Bruner, & Ross, 1976). The instruction method of scaffolding supports students to construct mathematical ideas and solutions, in a way that the student develops individual thinking as well as the generation of mathematically valid understandings (Anghileri, 2006). The process starts with the students' representations, attempting to gradually move it to the representation desired by the teacher (Goldman, 1989). The teacher has several roles, including simplifying the task and maintaining direction toward the goals (Wood et al., 1976). In the scaffolding process, important elements are responsiveness, fading and transfer of responsibility. Responsiveness is demonstrated when the teacher's support adapts to the current level of the student's performance, the support is either at the same or a slightly higher level. Diagnosis of the current level is important for adjusted support and feedback (Van de Pol, Volman, & Beishuizen, 2010). This support has a temporary nature and in the course of time it fades away. With a successful fading process the student becomes independent of external support and is responsible for his or her own performance (Smit, Van Eerde, & Bakker, 2013).

Methods

In order to investigate the research questions, a series of five lessons is designed that focuses on the three mathematical braille reading strategies with scaffolding as instruction method. The guiding on reading strategies fades during the lesson series, which is evaluated by studying the practice during the lessons. To determine the change in reading pattern, a test is given before, right after and a month after the intervention. With small interviews at the start of every lesson and a reflectional interview after the last lesson, the awareness of students is investigated.

Participants

Three braille dependent students participate in our study. The participants are students at a secondary special school for visually impaired students from grade 7, 8 and 11 respectively. The students all developed blindness at a very young age and have no comorbidities. At an age of 6 they started learning braille and are fluent braille readers right now. They are good students and their mathematical skills are at or above average. All students use the index fingers of both hands while reading. In regular mathematics lessons, these students use braille in combination with speech when working with mathematical expressions. All three use the traditional literature braille (a variation on 6-dot braille) on their braille display.

Design of the intervention

The previous section describes a way of working with mathematics on the braile display that is expected to be more efficient than their current way of working. During the intervention of five lessons of 30 to 45 minutes, the teacher scaffolds the students' practice in the direction of this hypothetical way. Attention to current habits and awareness of reading strategies are elements to activate the student to improve their working process. The three reading strategies scanning, tracing and fixating, form the core of the first two lessons (Table 2). These strategies are introduced and explained, whereafter focused exercises are practised (Table 3). These exercises are designed to practise the reading strategy movements in an isolated way. Scanning is for example practised when the student reads an exercise and focuses only on retrieving the structure. Guiding questions are asked during this task, like: 'how can you describe the structure?' or 'how many symbols does this expression contain?'. If the students can carry out this task, they expand their range of reading strategies. Important to note is that efficient tracing requires a scan to determine which elements should be traced. This is also the case for fixating, which is only possible when the right place is tracked down. During the first two lessons, guidance and feedback on the execution of reading strategies are explicitly given.

Table 2: Content of the intervention

Lesson 1	Scanning & recognising
Lesson 2	Tracing & fixating, natures of exercises
Lesson 3	Expressions & strategies
Lesson 4	Equations & strategies, typing answers
Lesson 5	Mixed exercises

The reading strategies that these students practised during the first two lessons are implemented in the problem solving process during the third and fourth lesson. In these lessons expressions respectively equations give the mathematical context. The teacher assists the students in their problem solving process, with questions like 'what is the structure?', 'can you make a solving plan?' or 'what is your first solving step?' When needed, explicit feedback is given on physical movements. The assistance fades during the fourth and fifth lessons, the goal is to let the students work without strategy assistance during the last lesson. The scaffolding process is succesfully executed if the student can solve exercises with the reading strategies independently at the end of the series. The aim of the intervention is that the mathematical reading skills of braille students improve if they can adopt the hypothetical more efficient reading strategies. The execution of the scaffolding intervention is evaluated by research question 2. The improvement of the students' reading skills and awareness of mathematical working are examined by questions 1 and 3.

Table 3: Examples of exercises on reading strategies

Scanning	Find the symbols $+, -, *$.	$3432 + 628 * 3$
	Where is the variable?	$3 + (2p - 5) - 2p$
Tracing	What is the first step?	$10 - 5 * 4 + 2$
	Fill in the gap and calculate.	$2;\;\; 3 * .. \wedge 2 + .. - 1$
Fixating	Solve the equation, use the = as focus point.	$5 + 1 = 2(x - 1)$
	Calculate, fixate on corresponding decimals.	$1,293 - 2,193 + 4,182$

Instruments & data analysis for research question 1

The answer to the first research question (evaluation of the intervention) can be found by studying the learning gain of the intervention. Therefore, the students worked on a pre, post and retention test. The pre test took place in a session before the first lesson, the post test was given in a session after the last lesson. The retention test took place 6,5 weeks after the post test. The exercises were similar on each test, but for each student the exercises were adapted to their own level. The test existed of a total of 22 mathematical expressions. In this study, we present the results from four of the items, see Table 4. These items were selected because the three participants have similar exercises and these items did not require interim writing, so reading movements represent the solving steps. In the first two items the order of operations is important for the solving steps. In the other two items selecting and combining symbols has influence on the solving and reading process. This offers interesting similarities and differences to evaluate.

Table 4: test items

Item number	Item		
Item 1	$4 + 2 * (10 - 3)$	or	$4 + 2(10 - 3)$
Item 2	$2 + 1 - (2 \wedge 2 + 1)$	or	$2 + 1 - (5 - 3) \wedge 2$
Item 3	$45,7 + 13,4$	or	$45,7 - 13,4$
Item 4	$3/4 * 2/5$	or	$5/7 * 2/9$

The exercises were typed in a Word document and converted to braille by the screen reader NVDA. During the tests, the exercises were displayed on the Active Braille braille display (Handytech) provided by the researchers. This device saves the displayed text, which is helpful for analysing. All students use literature braille, so "Dutch grade 1" was selected as braille alphabet. A videocamera was placed above the braille display, so that the movements of the hands and the conversation were captured, see Figure 2.

Figure 2: Capturing hand movements with the camera

The pre, post and retention test generate videos of hand movements and recordings of the conversation. First of all, the students' answers are verified. After that, the hand movements are analysed with video analysis software (Kinovea). For every 200 milliseconds the position of both index fingers in Excel is notated. The position is not documented during repositioning movements. That is the case when a finger has a higher speed, during these moments the image of the finger(s) is fuzzy. This research focuses on improving reading movements, therefore also the slower repositioning movements from right to left are taken into account. With this information the movements can be analysed. The time and scan path are simple to measure. By using Excel it is possible to determine the way of reading for every 200 milliseconds, according to Table 5. Part of this study focuses on the exploration phase. For the evaluation of this phase, we define it as the first progressive movement(s) from the first braille cell to the end of the line. The exploration phase usually consists of one quick or slower scan. Sometimes the exploration contains another scan from start to end, often the first movement is quick, the following one is slower and occasionally contains small regressions for deeper reading. The intervention focuses on the development of a exploration phase consisting of one clear explorative movement. This gives information to make a solving plan and to search and combine elements of the expression, till the answer is found. The hypothesis is that the intervention improves the reading patterns that initially show many regressions from the beginning of the line. After the intervention the pattern is more structured and regressions also start at other location, for example brackets. We expect a decrease in in time and scan path.

Table 5: Classification of ways of reading derived from the finger position and movements

Left \ Right	Absent	Fixated	Moving
Absent	finished (fin) or reposition (rep)	reposition (rep)	one finger reading (1fr)
Fixated	retake (ret)	lingering (lin)	one finger assisted (1fa)
Moving	one finger reading (1fr)	one finger assisted (1fa)	conjoint (con) or disjoint (dis)

Instruments & data analysis for research question 2

The second research question (the scaffolding practice of the intervention) can be dealt with by comparing the theory-based design characteristics and hypotheses with the observed learning process of the students. To verify the scaffolding process of the three reading strategies, the lessons are transcribed and analysed. The protocol of the lessons is split up in fragments where one topic is addressed, such as the various reading strategies or mathematical subjects. A new fragment starts when a new exercise starts or when the topic or support changes. Every fragment gets one code. For each lesson the codes are examined, so that the implementation of scaffolding becomes visible if the number of strategies addressed decreases during the last lessons. To ensure the reliability, 20% of the lessons is double coded: this is compared and verified. In Table 6 examples from each category are given. Some illustrative situations are presented to demonstrate the way of working and teaching during the intervention. The lessons are designed in a way that explicit attention is given to reading strategies in the first two lessons, while the other three lessons, the support for reading strategies fades. The hypothesis is that the analysis will confirm the fading practice of the intervention which demonstrates the development of independent use of the reading strategies.

Table 6: Results of the analysis of the classroom practice: examples

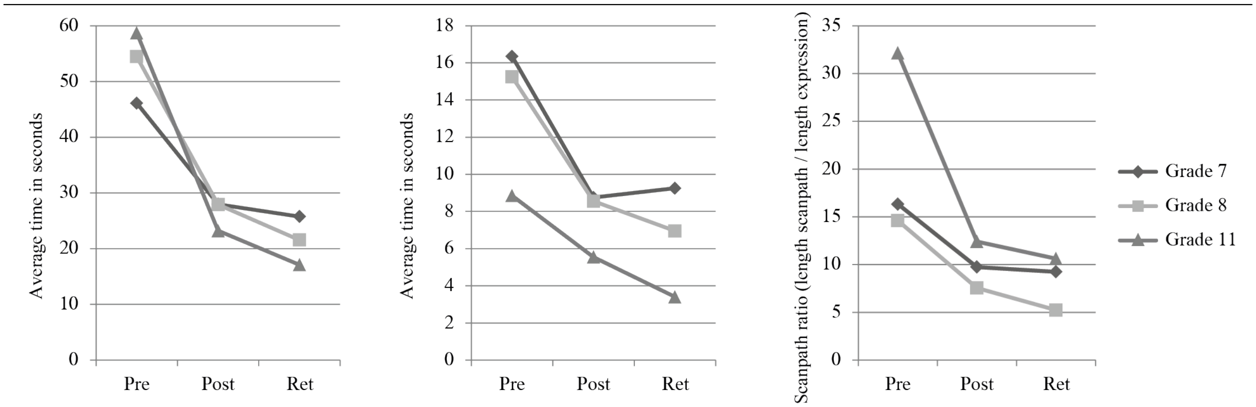
Categories	Examples
Scanning	Are you searching for the dots first?
	Can you tell me more about the structure of this exercise?
Tracing	Now you know the structure of this exercise. What will be your first step?
	In a situation with decimal numbers, what is the best place to start reading?
Fixating	How can you make steps to the solution. Where can you fixate?
	Is it helpful for you to fixate on a location?
Mathematics	You should multiply the 6 in front of the brackets with the sum inside the brackets
	And how can you solve this?
Others	Why are you moving the cursor every time?
	Do you prefer to have spaces around the slash in this fraction?
	You can copy this to the next line with a shortcut...

Instruments & data analysis for research question 3

To find an answer on the last research question (the students' awareness), the small interviews at the start of the lessons, as well as the evaluative interview are analysed. Discussed subjects during the short interviews are the content of the last lesson, the opinion of the student about the utility of the content, the level and the use of the strategies in regular lessons. The evaluative interview takes about fifteen minutes and is semistructured. Discussed subjects are the utility of learning reading strategies, the students' opinion about the content and instruction and possibilities for implementation in mathematics lessons. The transcribed interviews are analysed and compared over time and between students. Quotes that explicitely refer to the students' learning process are selected to map the changes in awareness. The hypothesis is that after some lessons the students start to talk about how reading strategies can be helpful for themselves, they can describe elements of the lessons that are most usable for themselves.

Results

For answering the first research question about the influence of the intervention, the results of the pre, post and retention test are summed up. In the selected items of the test, the students did not make major mistakes and there are no extreme deviations. The average time of the four items for each test moment is shown in Figure 3a. All three students needed less time after the intervention, on average they only needed 45% of the original time. The retention test showed that the obtained progress is not temporary. The time needed for the exploration phase is presented in Figure 3b. The principles for extracting this phase can be found in the method section, the selection is demonstrated by the gray backgrounds in Figure 5 and 6. After the intervention students only needed 56% of the initial time. These improvements still existed longer period after the intervention as is shown through the retention test. Another way of comparing the mathematical reading skills is to compute the length of the touch paths. The ratio between the length of the touch path and the length of the expression is a measure for the amount of rereading. In Figure 3c the ratios during the pre, post and retention test are shown. The average touch path ratio of the three students during the pre test was 21.0, after the intervention this was 9.1.



a. Time for exploration and solving phase (total time)

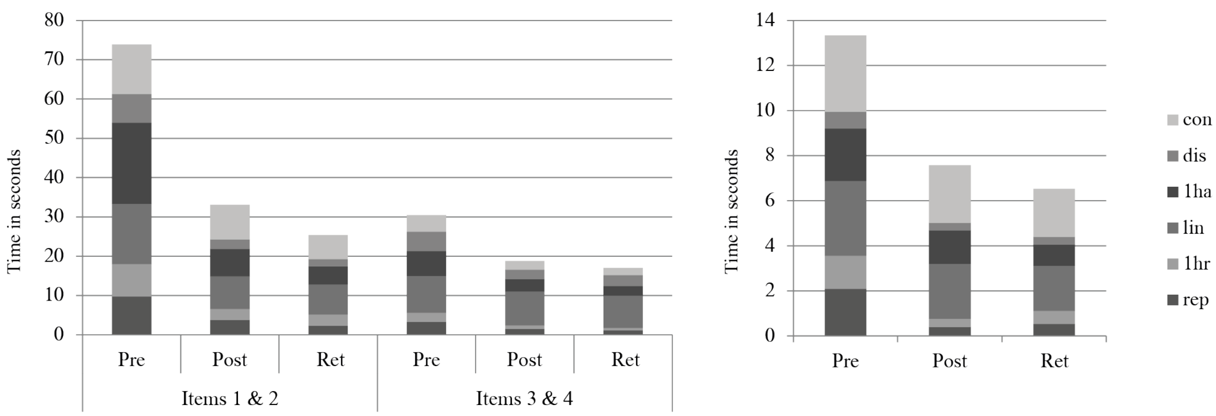
b. Time for exploration phase

c. Touch path ratio

Figure 3: Evaluation of the pre, post and retention test: development in time and scan path

The intervention was focused on reading strategies with corresponding hand movements. These ways of reading during the three tests are presented in Figure 4. In Figure 4a a comparison is made between the average time per way of reading for items 1 and 2, and those for items 3 and 4. In the first category, the order of operations affects the hand movements. In the other category selecting and combining elements of the expressions determines the hand movements. For items 1 and 2, all ways of reading are represented during the pre test. After the intervention the time for all ways of reading is drastically reduced, lingering and conjoint reading play a proportional big role. In items 3 and 4 another change is noticeable. Where lingering already is a big contributor during the pre test, it dominates the hand patterns during the post and retention tests. The time spend lingering remains the same during all test moments, while time for other ways of reading reduced. Figure 4b presents the ways of reading during the exploration phase. Conjoint reading and lingering represent the biggest part of reading during the exploration. This is already the case during the pre test, and after the intervention the share of these ways of reading increases.

In Figures 5 and 6 the finger movements of two exercises during the three test moments are displayed. The vertical axis represents time (sec), the horizontal axis represents the location in the expression. The dark



a. During the exploration and solving phase, split up over items 1 & 2 and items 3 & 4.

b. During the exploration phase, all items together.

Figure 4: Ways of reading

grey line represents the movement of the left finger, the light grey line represents the right. These two graphs are selected as illustration because these patterns illustrate the development in a clear way. Additionally, the students are from different grades and the items are different. In Figure 5 the three test moments start with a similar conjoint scan. During the pre test, another slower scan with interruptions follows the quick scan. In the post and retention test, one quick scan is enough to start the solving process. The hand movements reveal the student's solving steps according to the order of operations. In the graph of the pre test this pattern can be recognised after 27 seconds. This path is predominantly executed by the left finger, the right finger lingers around the last symbol of the expression. In Figure 6 the changes in the exploration phase are visible. The pre test shows a conjoint exploration phase with several interruptions for rereading, while the scan is a progressive movement during the post and retention test. Lingering is the main way of reading in the solving phase, preceded by exploratory and orientating movements. During the pre test, this phase takes substantially more time.

Verification of the scaffolding practice during the lessons answers the second research question about the scaffolding practice. The contribution of each topic to each seperate lessons follows from the analysis of the transcribed and coded lessons, which is presented in Figure 7a. Working and talking about mathematics is part of every lesson. Support in the reading strategies is given during the first two lessons, while the third lesson onwards this support decreases. This matches the design of the lessons, in which the reading strategies were planned during the first and second lesson. The category "others" contains the subcategories "technology", "braille" and "typing". The braille subcategory was the biggest component during the first lessons. In the fourth lesson the contribution of this category is remarkably high. An explanation for this is the later added content for this lesson about typing answers and solving steps. This subject is also addressed several times during the last lesson. In Figure 7b the category of the reading topics is split up in the subcategories "scanning", "tracing" and "fixating". During the first lesson the instruction and support is focused on scanning, this decreases during the following lessons. The support for tracing and fixating is given during the second lesson. This matches the lesson designs.

The practice of supporting students in implementing the reading strategies is illustrated by transcribed pieces of the lessons. The first two lessons are designed to explain the hand movements and to give specific

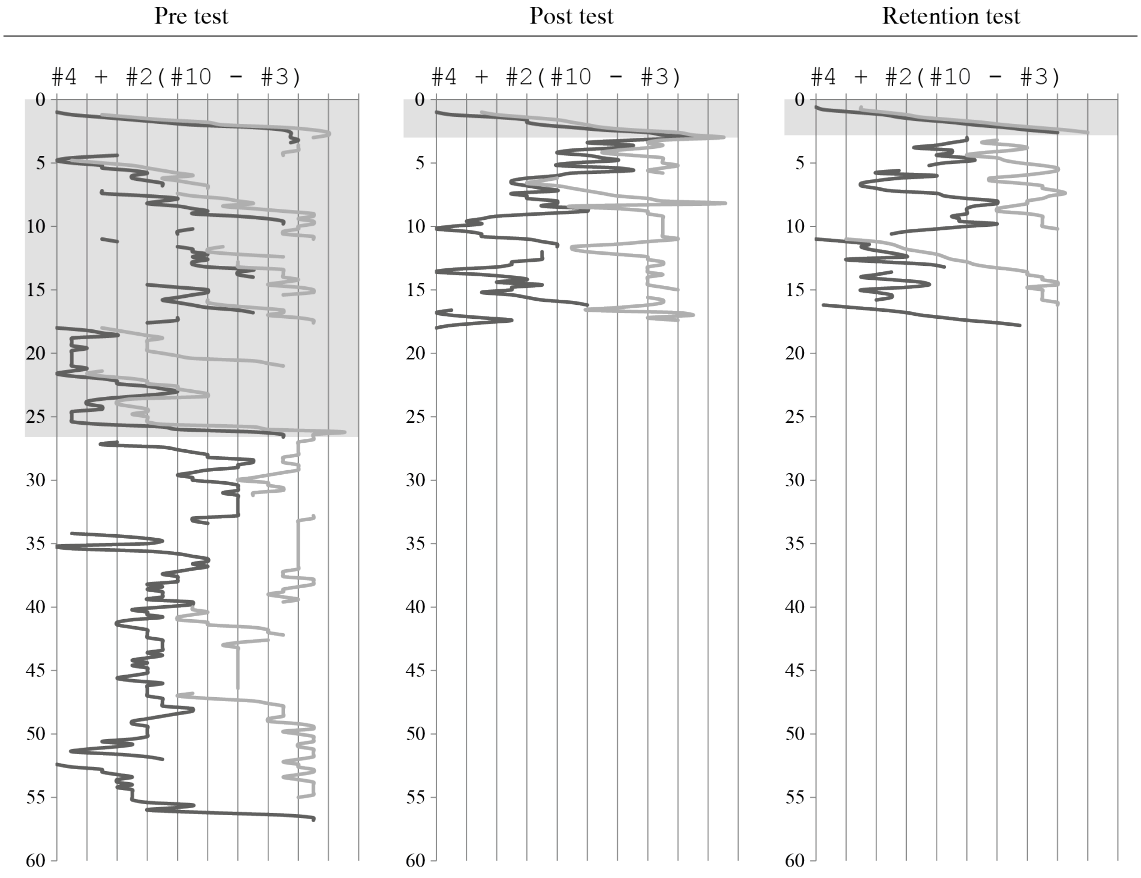


Figure 5: Illustration of development in reading. Item 1, grade 11.

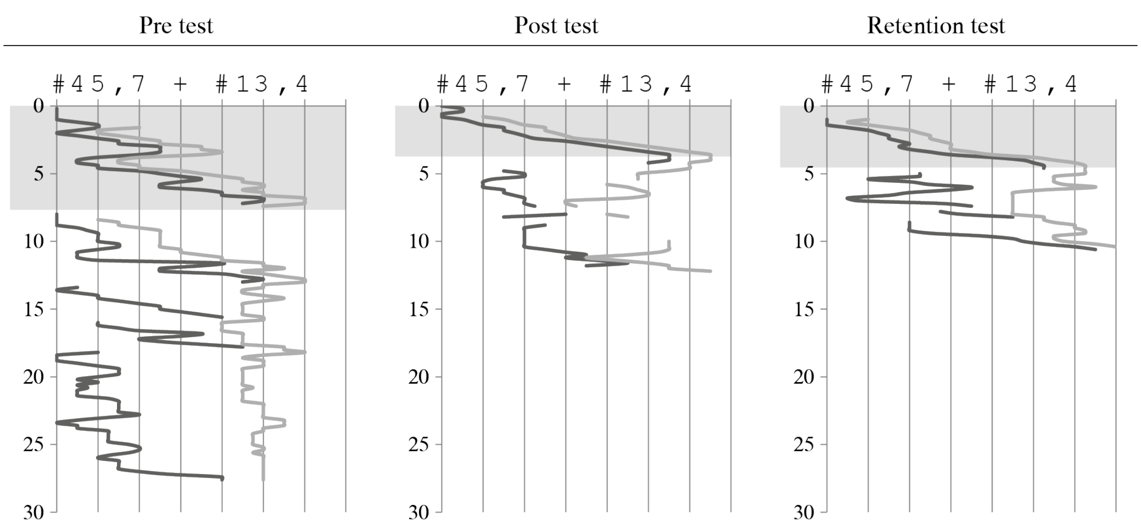
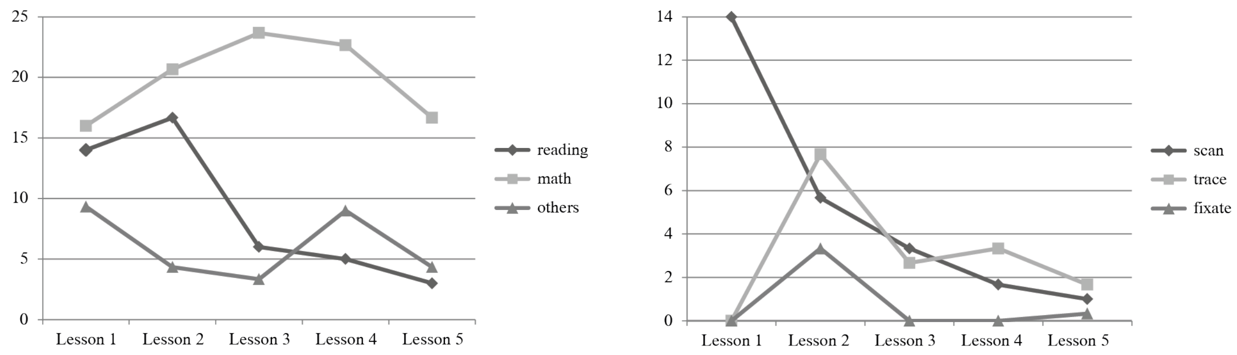


Figure 6: Illustration of development in reading. Item 3, grade 8.



a. Topics addressed

b. Support of individual reading strategies

Figure 7: Remarks

feedback. In the first lesson the strategy "scanning" is explained. The first exercise, focused on the scanning movement, is carried out in the following fragment of the transcribed recording.

TEACHER : We are going to practise this scanning technique. Try to search for the plus, the minus and the multiplication symbol.

STUDENT : Here is the plus. And here is the minus.

TEACHER : Yes. Do you still read everything?

STUDENT : No, I do not read the numbers.

TEACHER : O alright, so you can move along the display while focusing on the symbols. Good.

Another example demonstrates the way of guiding students through exercises. This examples in from the second lesson on exercise $4(5 * 2 - 7) + 8 - 4, 5$. The conversation is not specifically about finger movements, but about the structure of the expression and physical locations. This kind of guiding is often used during the lessons. In the last lessons less or no guiding was needed.

STUDENT : Here and here are the brackets, and there is a minus, a plus, another minus and that's it.

TEACHER : Can you now tell me more about the structure of this exercise?

STUDENT : A bit. First there is a part between brackets, and after that you have to add something else.

TEACHER : Yes, and there was also something in front of the brackets, right? That is important. So where do you start?

STUDENT : In the brackets.

TEACHER : Yes, definitely.

To answer the last research question about the students' awareness, the executed interviews provide insight in the students' awareness about reading strategies. Each lesson starts with some questions about the previous lesson. During the start of the series, the students are in general quite neutral about the content of the lesson. They recognized the ideas behind the reading strategies, but do not think that the lessons influence or improve their own way of working in mathematics. Like the student from grade 11 said: "I already used a scanning technique, because in an expression I have to know where the x is anyway". After two lessons the grade 9 student mentioned that the strategies were useful during the mathematics lessons:

“It is difficult to keep applying it, because it is easier to start reading everything than to focus on specific symbols”. During the last lessons they started mentioning the value of it. This also became clear during the evaluative interview. The three students mentioned different components of the series when we asked for the most useful parts. One student said the reading strategies, especially scanning, are a helpful tool for doing mathematics. She already applied it in the regular mathematics lessons. Another student mentioned that learning to type solving steps was very useful, both for organised thinking as for practising that skill. The more experienced student from grade 11 appreciated the knowledge about characteristics of expressions the most.

Conclusions and discussion

The first research question concerned the evaluation of the intervention. The initial average reading and solving time per exercise is 53 seconds. After the intervention the students only needed 45% of the time they needed during the pre test. For the exploration phase this is 56%. In general the exploration phase changed from one or more scans from start to end with interruptions and parts of rereading, to one regressive scan. For all test moments, a solving path is visible in the graphs of the hand movements, which starts with the part between brackets. In the post and retention test this starts quickly after the first explorative scans, while in the pre test additional exploration took place. The decrease in touch path ratio indicates the same development. The research of Van Leendert et al. (2017) remarked that reading patterns of braille readers are only to a small extent determined by the structure of expressions, in contrast to the gaze path of sighted students. The earlier start of the solving path and more differentiated reading strategies indicate progress in this area. The retention test that took place six weeks after the post test, has similar or slightly improved results. The improvements in time and reading strategies seems to be long-lasting rather than short-term. This is in line with the statement of Wasik and Slavin (1993) about long-term effects of one-to-one lessons on reading skills of students.

The second research question is about the support on the reading strategies during the lessons. Scaffolding was one of the principles on which the series was built. An important goal of scaffolding instruction is that the student is able to carry out specific tasks independently in the end of the process (Smit et al., 2013). This is achieved when it is possible to decrease the initial support and feedback. The lesson design reflects this process, the first lessons offer specific exercises to practise the movements in an isolated way without mathematical context. In later lessons the strategies were implemented in exercises of the student's personal level with fading support for reading strategies, as demonstrated by the lesson analysis. In the last lesson the students were responsible for the use of reading strategies, and partly for the mathematical process. The subject of typing answers and solving steps was later added because this creates an overview of mathematical steps, and especially the younger students missed those skills. For the student in grade 11 later lessons offered more challenging items. In this way, the level and content of the lessons were adapted to the students' level and interest. In the results it becomes clear that the support indeed fades after the first two sessions, at the end the students are able the perform the task independently. According to the definitions of Van de Pol et al. (2010) scaffolding is successfully implemented in the series.

The last research question focused on the awareness of the students about the lesson series, in particular the reading strategies. It took some time before the students could name elements they appreciated. Those elements were not exclusively the reading strategies, but also added components of lessons that focus on the general solving process. Especially the student from grade 11 mentioned that other components were more useful, because this student had more mathematical experience and already developed personal habits. Although the students mention various elements, they all improved their comprehending and solving process. Developing a more structured and effective way of solving exercises, is a good combination with developing more efficient reading movements. This agrees with the theories of scaffolding on mathematics, where students develop mathematical thinking by starting with known representations moving to representations desired by the teacher (Anghileri, 2006; Goldman, 1989).

It is important to bear in mind that this study has some limitations. First of all, the number of students is small and all students are in a different grade. Moreover, all three participants are good students who do not have specific difficulties with mathematics. All participants use the literature braille alphabet and the influences of braille alphabets on mathematical reading are unknown. Therefore, it is possible that an intervention like this has slightly different effects with other alphabets. During the lessons the initial instructions and exercises are specifically focused on hand movements, later instructions are more focused on the solving process ("move your left finger back to the bracket" versus "where do you start solving?"). This makes it difficult to ascribe the positive results to individual elements of the intervention. The implications of exclusively learning other hand movements remain ambiguous. A last constraint of this research is the absence of the function of speech in the lessons while doing mathematics. The participants normally use braille in combination with speech when doing mathematics. Due to a lack of prior knowledge about this function, it was not possible to find an established way to implement it in the lesson design.

Notwithstanding these limitations, the results of this research seem very promising. With a relatively small investment of five lessons, it is possible to regulate the hand movements and to make the mathematical solving process more efficient. Especially the time reduction and more structured solving process will help students to get more mathematical experience and to overcome difficulties in more advanced exercises. Considering the given attention to the mathematical process in this intervention, the results are also interesting for mathematics learners in general. Scaffolding with particular attention for problem solving can improve the mathematical process and understanding (Anghileri, 2006).

This study has provided some insight into the effects of specific support for reading strategies regarding braille. Although other ways of teaching these skills are not examined, giving attention and support to the reading strategies is recommended because of the promising results. The intervention can have the same format as in this research. An alternative is (partially) implementing the intervention in regular mathematics lessons. After practising the hand movements, students can use them in the regular exercises with decreasing support of the teacher. The knowledge of the teacher about braille technology, hand movements and mathematical structures is important in this intervention. Teaching physical navigation through expressions is rare, for teachers as well as for braille students. An informed teacher can scaffold these skills and gives students the opportunity to ask various question. This research does not give clear indications about the moment of exposing students to mathematical reading strategies. Younger students will take more advantage of the lessons, because they can implement it in their mathematical habits for the coming years. On the other hand, in lower grades mathematical expressions are still quite transparant and the structures do not cause specific problems. Presumably, it is good to expose students to reading strategies in lower grades, and recall it when mathematical expressions become more complicated.

This research shows that a short individual intervention on mathematical reading strategies and the solving process has positive influences on the efficiency of hand movements and mathematical solving process of braille students. The time reduction and more structured way of working can help students to deal with the course of mathematics in secondary education. More advanced reading skills make the lack of vision a less limiting factor. The ultimate goal is, of course, that the students' personal capacities are the only limiting factor, just like their sighted peers. This research is a promising step in the right direction, but further research is required to approach the ultimate goal. The limitations of current research give directions for further research. Especially in the field of algebra, the influence of the chosen braille alphabet on mathematical understanding and comprehension is an interesting topic. Another unexplored subject is the use of speech in mathematics, both in word problems as in algebra. This needs to be explored in future research.

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