Universiteit Utrecht

Bachelor Thesis

Bell's inequality and the nature of quantum theory

A brief review of the principle of local causality and associated concepts with respect to quantum theory

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Preface

I am convinced that it is possible up to a certain level to think and speak about modern physics in a conceptual way without losing physical precision. In the future, I will try to do so as much as I can, in the hopes of luring more people towards the field of physics in general and quantum physics in particular. I hope that one day the topics that are still subject to debate within the field of modern physics, will be topics of heated discussions in public space. Over a coffee in the café; that would be nice.

I wrote this bachelor thesis on one of the topics in quantum theory that already appealed to me when I was not too familiar with the matter; *nonlocality* and *Bell's theorem*. Of course, this prior appeal was lead on by false analogues and hidden agendas of scientists promoting this interpretation or another. It was during the course on foundations of quantum mechanics at Utrecht University that I got a glimpse of all nuance and precision that is needed when dealing with the fundamentals of modern physics. In a way, all romance leaked away, but after reading more, it came back even stronger. It is in the politics behind the proofs, in the details of thought experiments, in the attempts for correct analogues, in the choice of words. It is in challenging ourselves to try and understand that what we have cooked up physics-wise in the past decennia.

> "[...I]t seems to me that in the course of time one may find that because of technical pragmatic progress the 'Problem of Interpretation of Quantum Mechanics' has been encircled. And the solution, invisible from the front, may be seen from the back. For the present, the problem is there, and some of us will not be able to resist paying attention to it." (Bell, 1989, [1])

Acknowledgements

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Introduction

Does a scientific theory describe reality? Or does it describe the appearance of reality and nothing but the appearance? Are we humans able to recognize what is going on a fundamental level of the real world? These questions arise with every statement that is made about the nature of reality. I will not give a detailed answer to this question, the more so because it can depend on personal conviction. There is, however, a very interesting debate about the nature of reality and our ability to pin it down, that evolves around the theory of quantum mechanics, henceforth quantum theory, that shows it pays to not only think about what we can know about nature, but also to think of how nature is. Whether in the end we will be able to know if we are right or wrong.

There are various topics in quantum theory that lead to debate. Amongst them, we find the concept of local causality, which seems not to fit in quantum theory. Local causality comes with a huge load of other concepts; elements of reality, (non)contextuality, complementarity and completeness for example. In this report, I will try to give an overview of how these concepts are related throughout historical and modern literature and what conclusions could be made with respect to the nature of quantum theory and in the end perhaps a conclusion about the nature of Nature. While reading the report, it is important to keep in mind that the sources for my statements are carefully picked from a much wider range of articles I have read, which are less carefully picked from an even wider range of literature on the subject. So, although 'completeness' is discussed with respect to theories, my presentation is by far not based on a complete description of all there is to quantum theory. However, my approach is in my opinion 'internally' consistent, which is the highest possible aim, considering that writing down things and thoughts is always to make a selection of what to say and what to leave out.

To sketch what I mean with 'the nature of Nature', I start out with an example of how Isaac Newton wrote about relative and absolute space and quantities and what his assumptions were on the behavior of absolute space (chapter 1). I will proceed in chapter 2 with a description of the concepts mentioned above, that can be found puzzling, nonintuitive and food for foundational debate and how they follow from quantum theory. In this chapter the concept of hidden variables will be briefly mentioned and Bell's original inequality is explained. Finally, in chapter 3, I discuss how a violation of Bell's inequality is interpreted with respect to quantum theory according to different authors. In this final chapter I conclude with how, in my opinion, Bell's work (and much of the commentary on it by others) makes us wiser about quantum theory and is therefore useful and indeed necessary for the development of scientific theories.

Chapter 1

A historical example of a demand on the nature of Nature

1.1 Isaac Newton and his demand on absolute reality

Looking back in the history of science, there are some theories that seem to be of revolutionary kind. One of them was brought to me since kindergarten; Isaac Newton and his discovery of gravity, which came to him when he saw an apple fall from the tree. Later on, I learned it had a bit more to it than this fairy tale like representation of the efforts and accomplishments of the man, but nevertheless the theory was revolutionary[1]. Newton observed certain structural behavior of the world surrounding him and captured the structure of this behavior, calling them universal laws. An example are his three laws of motion that describe how particles exert forces on each other, induced by their motions and resulting from those motions. In the first book of the *Principia* [2], in the scholium of Definitions, Newton declares:

> "Although time, space, place, and motion are very familiar to everyone, it must be noted that these quantities are popularly conceived solely with reference to the objects of sense perception. And this is the source of certain preconceptions; to eliminate them it is useful to distinguish these quantities into *absolute and relative*, *true and apparent*, mathematical and common." (emphasis added)(Newton, [2, p. 408])

Further on, he continuously distinguishes between the relative and the absolute. Interpreting this quote, one could say that the mathematical structure of his laws of motion describe the underlying structure that explains the apparent motions of objects. Those motions are directly related to the absolute motions

[1] There is so much to the scientific ongoings in the 18th century; descriptions are found in every book about the history of science. The mentioning here serves as an illustration for a more general point.

in absolute space which is "the same in species and magnitude, but [it does] not always remain the same numerically" [2, p. 409]. As an example, he gives the 'air' surrounding the earth, which moves along with the body of the earth and therefore seems to have no relative motion, but it has an absolute motion with respect to the absolute space surrounding both the earth and the air.

Bluntly speaking, the laws of motion deal for a great part with forces that are exerted by bodies physically touching each other. This all is explained by Newton in the contexts of both relative and absolute quantities. But with his other famous law, the universal law of gravitation, there is no touching of bodies and hence problems arise in explaining the fundamentals - the structure underlying the appearance of gravity that is embedded in absolute space. Newton in a letter to Richard Bentley:

> "That gravity should be innate inherent and essential to matter so that one body may act upon another at a distance through a vacuum without the mediation of any thing else by and through which their action or force may be conveyed from one to another is to me so great an absurdity that I believe no man who has in philosophical matters any competent faculty of thinking can ever fall into it. Gravity must be caused by an agent acting constantly according to certain laws, but whether this agent be material or immaterial is a question I have left to the consideration of my readers." (Newton, [3])

Over a century after this letter, Newton's call for an underlying structure for gravitational effects was answered by Albert Einstein in his theory of general relativity, which was brought to the public in 1915. From his special theory of relativity, ten years earlier, Einstein brought a general description of space-time in which gravity indeed is explained, namely as an effect of the curvature of space-time[2].

1.2 A jump to modern physics

So had Einstein then succeeded to give a true description of reality, when he came as far to construct a theory in which those things that seemed fundamental, fundamentally *absolute* to us (space, time), are not absolute at all, but still are predictable if one has the right descriptive tools? It is very hard to imagine there is a structure underneath something so fundamental as space, but hey, if not imaginable, is that the same as that thing not being there at all?

One could say that Newton was convinced that his theory was not complete, because there was some agent missing to completely describe the fundamentals of gravity. Einstein's theory of general relativity confirmed this conviction, and provided us with a deeper explanation. However, another appearance of action at a distance had risen in the early 20th century, and analogues to Newton's classification as "absurd", Einstein called this situation "spooky" [4].

[2]Although up to date, the theory of general relativity is not extended to the microscopical level.

Chapter 2

Implications of quantum theory

Around the turn of the 19th century, new phenomena were observed due to technological and industrial advancements, especially at atomic scale. Classical theories could not account for these phenomena. Quantum theory was constructed as a tool to predict the probabilities of certain measurement outcomes that fitted the new experimental data, and therefore as a statistical theory. People were already familiar with statistical theories; statistical mechanics was used since the 1870s to deal with large numbers of molecules in gases. Quantum theory was, however, fundamentally different from statistical mechanics, as it was a fundamentally statistical theory, as often advocated by some of its founders (for example Niels Bohr and Werner Heisenberg). What a theory states about the fundamentals of reality it describes, is to interpret the formalism of the theory. This is not as straightforward for quantum theory, as it has been for classical theories (although also not without disagreement). For firstly, the theory is entirely about the results of measurements (and therefore the representatives of physical quantities are called 'observables'). What is exactly comprised by the concept of measurement differs per author and is one of those other topics of debate.

Furthermore, quantum theory makes use of abstract mathematical structures. Where classical theories handle physical, conceivable quantities[1] like velocity, force, momentum, all in a space with the same number of spatial dimensions as the one we observe, quantum theory is about hermitian operators acting on – if you like – infinitely many dimensional Hilbert spaces. According to quantum theory, a system is in a *state* that is characterized by the *wave function* ψ. The wave function is a description of the system (object) in such a way that its absolute square $|\psi|^2$ gives the probability density of finding a certain quantity value upon measurement. For example, to find a particle in a one dimensional space on a position between $x = a$ and $x = b$ (so the probability that quantity X has a value $x \in [a, b]$ is found by integrating $|\psi(x)|^2$ – the wave function now being a function of position x – over interval $[a, b]$.

[1]Although many of these concepts were, when they were introduced, topic of debate, perhaps similar to present day debates concerning topics from quantum theory.

2.1 On the completeness of the wave function; Einstein, Podolsky and Rosen

In quantum theory ψ tells us everything there is to say about a system, i.e. it is a complete description of the system at hand. But the wave function provides us with probabilities, not with certainties. Furthermore, there is the concept of *complementarity*; two observables are complementary when they are not measurable at the same time, like momentum and position of a particle, or two non-(anti)parallel spin directions of a spin particle. Because the wave function gives a complete description of the system, is must do so also of those complementary observables. However, to account for different observables, the wave function needs to be expanded in the bases of the *eigenstates* of the observables. For many observables, this expansion can be done in a simultaneous base of eigenstates. The *operators* of complementary observables do not commute – and there the issue starts. Because of their noncommutativity, it is not possible to find a simultaneous basis of eigenstates. However, for the system to have a defined value for an observable, it needs to be in an eigenstate of the observable. I just showed how that is not possible to be in an eigenstate for both of these complementary observables at the same time. So if one expands the wave function in a basis that fits the eigenstates of one of the observables, this observable has a defined value – but the other observable is in a superposition and hence has no defined value. And because ψ is complete, the two observables – according to quantum theory – cannot have simultaneous defined values.

There is a lot more to quantum theory, but the above are two concepts (complementarity and the impossibility for some pairs of observables to have simultaneously defined values) Einstein found irreconcilable with a physics theory that attempts to describe the structure of reality. In several thought experiments, he tried to show that in reality it is possible to construct a measurement that could give us both values – hence showing that quantum theory does not tell us all there is to know about reality, otherwise quantum theory could tell us both values at the same time. Now, because quantum theory is especially about the microscopic level, measurement disturbance is a fundamental problem. Einstein tried to get around this disturbance, by trying to design an experiment in which a particle is not disturbed – an indirect measurement. In 1935, together with Boris Podolsky and Nathan Rosen, Einstein published a paper in which he presented a way to make an indirect measurement on a particle, thus without disturbing the particle through interaction with a measurement apparatus.

The argument

Instead of describing a system existing of a single particle, in the Einstein-Podolsky-Rosen (EPR) paper[2] under study (see [5])there is a system consisting of two particles that have interacted with each other for a certain time. After this interaction they are spatially separated in such a way that there cannot be any influence on one particle by meddling with the other particle, as a consequence of special relativity. The particles are furthermore assumed to be *entangled*; they have interacted physically in such a way that when they become separated

[2]For a fine account of the formation of the argument by EPR and the aftermath, I advise the interested reader to check out the online Stanford Encyclopedia of Philosophy of Stanford University, California: http://plato.stanford.edu.

after the interaction, each particle is described by the same quantum state. This state is a superposition and thus complementary observables like momentum and position, or different orientations of spin have no defined values. Still it is so, because of the entanglement, that the outcome of a measurement of observable O_a on particle 1 gives us with certainty what the outcome of a subsequent measurement of observable O_a on particle 2 will be.

The aim of the thought experiment is to show that the wave function does not provide a complete description of reality. It is possible to measure observable O_a on particle 1, whereafter the outcome of a subsequent measurement of observable O_b on particle 2 is certain. EPR state that this certainty is a satisfactory criterion for the value of observable O_a for particle 2 to be 'an element of physical reality', i.e. that it is real. It is also possible to measure complementary observable O_b (so an observable of which the operator does not commute with the operator of observable O_a) and with the same reasoning, the value of observable O_b of particle 1 is then real. Now there are two more or less tacit assumptions:

- After the interaction period, there is no influence from one particle to another. Although they are part of a conjoint system, both particles exist individually, separately after the interaction.
- The locality principle holds. The measurement (or absence of measurement) on one particle does not directly affect the reality of the other particle, because they are spatially separated.

It is possible to measure *either* observable O_a or O_b of particle 1, or make no measurement at all. So it is possible to predict *either* the value of observable O_a or the value of observable O_b of particle 2 with certainty. But because the particles both have a separate reality after the interaction and the principle of locality is assumed, it must make no difference for the values of the observables of particle 2 what measurement is done on particle 1. So complementary observable O_a and O_b must have simultaneous definite values for particle 2. The wave function does not describe these values simultaneously and therefore does not provide a complete description of reality.

2.2 Contextuality

To explain another concept that plays a role in discussing local causality, let us consider the following situation[3]: an experimenter has boxes each with nine chambers in it – three by three – and each with a lid to cover up all the rooms. Each box has a sign 'this side up', so the experimenter can compare the boxes on orientation. See also figure 1. The experimenter is allowed to open either one row or one column for every box – which one is completely up to him. He does so for a number of boxes and finds that each of the chambers he opens are filled with one ball, sometimes blue, sometimes red. Even within one chamber, for example chamber number 8, the color of the ball is sometimes blue, sometimes red. See figure 2 for some examples. So he finds that it is certainly not so that there is always the same distribution of red and blue balls in the chambers, but there *is* a pattern:

[3]For this analogue, I was for the greatest part inspired by a version on http://scienceblogs.com/pontiff/2008/01/contextuality_of_quantum_theor.php.

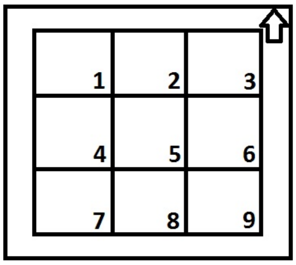


Figure 1: A box with nine chambers, organized in three rows and three columns. For further convenience, I labeled the chambers with numbers 1 through 9.

- Every row that is opened, contains an even number of blue balls. So either zero or two.
- Every column that is opened, contains an even number of blue balls, except for the third column. The third column always contains an odd number of blue balls, either one or three.

Intuitively, these balls have colors, no matter what row or column the experimenter opens. Even if the experimenter decided to not open anything at all, the balls still have either a red or a blue color. But if the experimenter were to lift the lid of the whole box, it is impossible for him to find a distribution that satisfies both summarized measurement results listed above. For to have an even number of blue balls on all three rows, there has to be an even number of blue balls in the whole box. But to fulfill the second observation, there has to be an odd number of blue balls in the whole box. There cannot be both an even number and an odd number of blue balls in the whole box at the same time. What does this mean for the observations the experimenter makes when he only opens single rows or columns? Perhaps that by coincidence he picks the rows and columns in such a way, that he never opens the rows or columns that show that there is always an even/odd number of blue balls in the boxes. But even if the experimenter does this experiment an infinite number of times, he finds the patterns listed above. So this coincidence becomes more and more unlikely. To create such a pattern every time, the creator must have known beforehand what row or column will be opened by the experimenter. In the situation under study, a situation of macroscopic proportions, this clairvoyance of some creator is not a part of scientific theories. It does not qualify as a (classical) scientific explanation. The act of opening a row or a column however is a physical act, so does this act play some role? A scientific explanation could be that the choice of opening (*measurement*) somehow affects the color of the balls.

For balls in boxes, I assume the listed statistics will not be found, although I know not of any serious experiments that support this common sense assumption. The reasoning is that someone has to actually pack the boxes and – as explained – this person cannot do that the desired way without knowing the future choice of the experimenter. Furthermore, it is very nonintuitive that the color of a ball is affected by the measurement, for we take a ball to have a defined color no matter if we look at ball in chamber 8 together with the balls

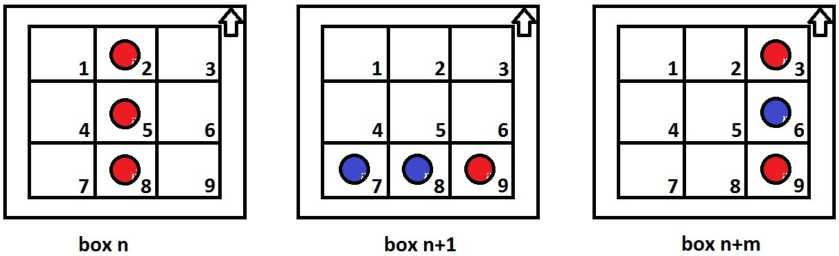


Figure 2: Some examples of what the experimenter finds, when opening randomly rows and columns on different boxes.

in chambers 2 and 5, or together with the balls in chamber 7 and 9. Indeed, we take a ball to have a defined color even if we don't look at it at all.

Nonetheless, the statistics listed above can easily be predicted by quantum *theory*. David Mermin (Mermin, 1993, [6]), inspired by the Bell-Kochen-Specker theorem[4], described various ways to show the dependence of measurement outcomes on the choice of measurement, by constructing geometrical forms of measurement order, one of them being similar to the box with nine chambers[5]. For a quantum theoretical version of this box, one can use two independent spin-$\frac{1}{2}$ particles 1 and 2. In every chamber, there is a spin observable, represented by their according operator. The operators in a row are mutually commutative, as are the operators in the columns, to account for 'simultaneous measurement'. Simultaneous in the sense that the outcomes per measurement in a row or column do not depend in which order you make the measurements. The observables in for example chamber 2 and chamber 4 do not need to commute. The box then can look like:

$$
\begin{array}{ccc|c}
\sigma_x^1 & \sigma_x^2 & \sigma_x^1\sigma_x^2 & +1 \\
\sigma_y^2 & \sigma_y^1 & \sigma_y^1\sigma_y^2 & +1 \\
\sigma_x^1\sigma_y^2 & \sigma_x^2\sigma_y^1 & \sigma_z^1\sigma_z^2 & +1 \\
\hline
+1 & +1 & \text{-1} &
\end{array}
$$

Figure 3: Quantum theoretical version of the box with nine chambers. Figure adapted from Mermin (1993, [6, p. 811]).

The $+1$s stand for the product of the three preceding operators, as stands the -1. Since the triplets of operators are mutually commutative, they have a common basis of eigenvectors. Evaluating the product of these operators in

[4]Again, a detailed treaty of the (Bell-)Kochen-Specker theorem can be found on the website of the online Stanford Encyclopedia of Philosophy of Stanford University, California: http://plato.stanford.edu.

[5]The version on http://scienceblogs.com/pontiff/2008/01/contextuality_of_quantum_theor.php. is actually based on Mermin's article.

this common basis results in a product of diagonal matrices. And therefore the eigenvalues of their product are the products of the eigenvalues. In other words: the product of the three sets of eigenvalues within a row or column, must satisfy the same product as is satisfied by the corresponding operators, hence observables. For all nine observables in the box to have a defined value simultaneously, the product of all nine must be $+1$ for the row-products, and -1 for the column-products. This is impossible and analogous to the situation of the blue and red balls. So through quantum theory, we have filled the box with observables in such a way that the conflicting statistics per row and column are predicted. But beware: the fact that quantum theory can provide us with these statistics, depends on the fact that the observables that are not in the same row or column do not all commute. It is thus not possible to 'open the box as a whole', because it is not possible to make simultaneous measurements on all the observables. The order of measurement does make a difference for noncommuting observables.

So in quantum theory, it is possible to construct a situation in which the value of a single observable depends on the choice of measurement. Still, when looking at just the outcomes for the observable in for example chamber 8, the distribution of the values is predicted to be completely random. It is only when looking at the context of measurement, a correlation between the other observables in the same row/column is found. It is then said that the measurement of a row or column is contextual. Because it is possible to construct this situation from quantum theory, it can be said that quantum theory is contextual in nature. However, the values of a single observable are randomly distributed; the distribution of the values of a single observable does not depend on whether it is measured with its row-companions or column-companions. Therefore, the measurement of the single observable seems to be noncontextual. I will elaborate on this in the next section.

2.3 Interlude: hidden variables

When one thinks quantum theory to be incomplete in the way Einstein thought it to be, but also correct in the sense that the statistical predictions are consistent with experimental data, one would expect that a description of an underlying, deeper structure under these data and this correct but incomplete theory should be possible. So one could say (and many people, like Einstein, have said) that there should be some additional variable that accounts for (at least for a part) the outcomes that appear to us not defined before measurement, as seems to be the case in quantum theory, shown by Bell-KS and Mermin. Alternative theories with respect to quantum theory that incorporate this concept of the yet unseen, additional variable, are called hidden variable theories.

To construct an alternative theory for quantum theory out of the desire to obtain a more complete description of nature, a constraint to this alternative is that it at least should make as good predictions of measurement outcomes as quantum theory provides. So, a hidden variable theory should account also for the noncontextuality of the measurement results on single observables as mentioned by Mermin. But in some cases, as the case of the box with nine chambers, it is impossible to assign consistent values to all observables at the same time. So, only a contextual theory would then be possible, although all single measurements yield noncontextual values (when looking only at the values of one observable, one cannot deduce when or how many times the observable was measured in combination with one set or the other). In Mermin's words:

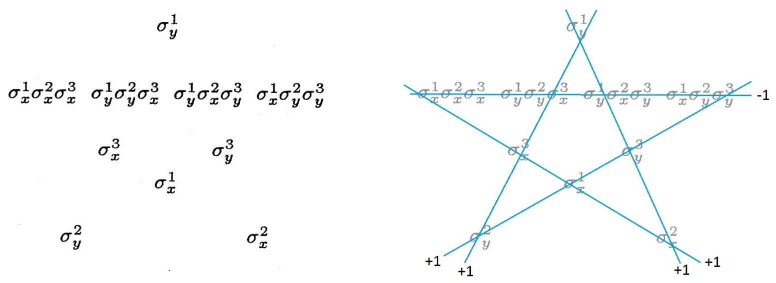
> "[...] a *contextual* hidden-variables account of this fact would be as mysteriously silent as the quantum theory on the question why nature should conspire to arrange for the marginal [isolated – ed.] distributions to be the same for the two different experimental arrangements." (Mermin, [6])

To think what it means for a (hidden variable) theory to be contextual, is in the case of looking for a theory that describes a deeper structure of reality than quantum mechanics does, to think of this more fundamental level of reality to be contextual. What it means is that reality at some deeper level might be contextual and, as Mermin put it so eloquently, still presents itself to us in almost all cases as noncontextual, is perhaps more graspable when we put a restriction on (non)contextuality, namely locality.

2.4 Locality

In Mermin's paper (1993, [6]), the author makes a conceptual link between (non)contextuality and locality. Here, I will follow his steps. To do so, we assume a similar experimental construction as in figure 3. This time, there are three independent spin-$\frac{1}{2}$ particles 1, 2, and 3, and we consider a set of 10 observables, arranged in the lines of a pentagram as is shown in figure 4. Due to (anti)commutation rules (see for details [6]), the observables on each line are all four mutually commuting. The products of the operators associated with the observables are for all lines +1, except for the horizontal line of the pentagram, where the product is -1. This leads to the same problem as the previous example: it is not possible to assign consistent values to all 10 observables simultaneously. This set-up, too, therefore reveals the contextual nature of quantum theory.

The convenience of this pentagram set-up is that each group, except the one on the horizontal line, consists of an observable associated with all three particles (the observable that is also a member of the horizontal group) and *three observables associated with a single particle, for all the three particles respectively.* This was not the case in the previous example. From here on, it is very easy to see how (non)contextuality is linked to locality. Every observable in the pentagram is a member of two different groups. Although the observables within one group are all mutually commuting, this is not the case for all the observables that are *not* in the same group. This is how contextuality plays a part: to measure the observables of a certain group means that the experimenter has to choose one setting out of a certain range of settings (for example by choosing the angles of a Stern-Gerlach magnet). Take for example the two groups that contain observable σ_y^1 (top of the pentagram). The measurement of particle 1 is for both groups along the y-axis. But for the measurement of particle 3, the experimenter has to decide whether to measure along the x-axis or along the y-axis. And for particle 2, he has to decide whether to measure along the y-axis or along the x-axis (consistently with his former choice). This is how a measurement context is chosen and this is what seems to be influencing the outcome of



(a) Mermin's pentagram.

(b) The products of the operators, similar to figure 3.

Figure 4: "[...] The observables are arranged in five groups of four, lying along the five legs of a five-pointed star. Each observable is associated with two such groups. The observables within each of the five groups are mutually commuting, and the product of the [four] observables in each of the [five] groups is $+1$ except for the group of four along the horizontal line of the star, where the product is -1." Figure and caption adapted from Mermin (1993, [6, p. 811]).

the series of measurement; contextuality. Niels Bohr already pointed something similar out in *On the notions of Causality and Complementarity* (1950, [7]);

> "The very fact that quantum phenomena cannot be analysed on classical lines [...] implies the impossibility of separating a behaviour of atomic objects from the interaction of these objects with the measuring instruments which serve to specify the conditions under which the phenomena appear."

So no surprise in contextuality for Bohr. Still, to think of what it implies: because the observables are mutually commuting within a group, it is possible to make the choice of measuring subsequently either the one group or the other *after* measurement of the observable that is in both – still the contextuality holds. Besides, also the remark of Mermin concerning the conspiring nature of reality is not undermined by this reply. So would Bohr then mean to say that the *choice* of the experimenter lays within the context of the measurement? What does that imply about the freedom of this choice? More on this in section 3.2.

On with the connection of noncontextuality with locality. Suppose we have a measurement apparatus that consists of three spatially separated devices; one part for each of the three particles. Looking again at the two groups that contain observable σ_y^1, the device that measures particle 1 can be left untouched, no matter which group of observables is measured. The settings of the other two devices depend on the choice of measurement (contextuality). But this time, these two devices are located at such a distance that no interaction can reasonably take place with particle 1 when the experimenter makes a choice of measurement – this we call *locality*. Otherwise, there would be action at a distance, which is a violation of special relativity. As was already shown before: it is not possible to assign consistent values to all the observables in the pentagram simultaneously. So the choice of measurement has an effect on the measurement outcomes, but this time, the choice of measurement for particle 2 is made far away from particle 1 and 3. I.e. the context of particle 2 is chosen far away from the context of particle 1 and 3. So, now contextuality is a matter of a violation of locality.

Is it action at a distance? How this 'action' is defined, is not within the scope of this writing. But I showed how the condition of noncontextuality can be translated to locality and that locality in the sense of this section is violated by the predictions of quantum theory.

2.5 Three sorts of independence: Bell's inequality

All of the above (measurement outcomes are predicted to be both contextual and noncontextual, violation of locality) is just a confirmation of the issues in quantum theory pointed out by EPR. There is a correlation between measurement outcomes that is *stronger* than one of pure coincidence, but a mechanism (noncontextual values) behind this correlation is not described by the theory. In the EPR-experiment, the experimenters (let us name them Alice and Bob) at the spatially located measurement sites both can choose to measure either spin along the x-axis or spin along the y-axis. At their particular location, both will find a 50/50 percent chance for spin-up/spin-down (noncontextuality). Both Alice and Bob tally their measurements: they make a note for every measurement for the measurement direction and for the measurement outcome. When putting their results together after a series of measurements, it is that the correlations spring to the eye - according to quantum mechanics. When comparing the measurement results for the particle pairs, they will find for all situations where they simultaneously measured along the same axis, there is a 100 percent anti-correlation (a result of the entanglement). For all situations where they simultaneously measured along different axes, there is no or less correlation. According to quantum mechanics – as illustrated by Mermin's example – it is not possible to assign a priori values to the spins of the particles, so, assuming that the wave function gives a complete description of the particle system, they have no a priori values. Then *how* does the particle at Bob's site *know* to behave in a way that Bob will find these anti-correlations when Bob chooses – unknowingly – to measure along the same axis as Alice, and that he will find no (or weaker) correlation when he decides to measure along a different axis?

Is it possible to construct a theory that makes the same predictions as quantum mechanics, but can explain contextuality and perhaps this way can preserve locality through some additional variable? So far, it seems that it will not be a very obvious theory.

John S. Bell (1964, [8]) came with a very elegant proof of how the aforementioned desired theory leads to a violation by the predictions of quantum theory *and* several results of actual experiments. This proof is known as Bell's theorem or Bell's inequalities. Abner Shimony (1990, [9]) gave a version of Bell's inequality that I find useful to present here, so I will follow Shimony's steps.

As Bell points out in *Bertlmann's Socks and the Nature of Reality* (1981, [10]), the proof doesn't require a quantum theoretical context. The proof is a test for the nature of certain correlations and can, for example, be used to check whether the answers on a survey were truly given independently by the responder. However, for brevity, I will not hesitate to use words that are associated with quantum theory.

Let us start with a probability distribution that could describe the situation of EPR. The distribution is of the outcomes A and B at the two spatially separated measurement sites 'Alice' and 'Bob'. Furthermore, we have to deal with the choice of measurement at each site; let a and b denote the parameter settings for the two measurement devices. Finally, let λ describe the complete state of the pair of particles (note that λ could denote any number of other variables that might be relevant [8, 10]). The *joint* probability distribution then looks like

$$p(A, B|a, b, \lambda). \tag{2.1}$$

Equation (2.1) gives the probability of joint outcomes A, B given circumstances a, b and λ. In his publication of 1964 (see [8]), Bell does not linger too long in explaining how to factorize the joint distribution (for more on this, see section 3.1), but let us look at how it is appropriate to go from (2.1) to a factorized distribution. Firstly, the following product rule is one following from the general principles of probability theory:

$$p(A, B|a, b, \lambda) = p_1(A|a, b, \lambda)p_2(B|a, b, \lambda, A) = p_2(A|a, b, \lambda)p_1(A|a, b, \lambda, B) \tag{2.2}$$

For complete factorizability we need certain assumptions:

- **Outcome Independence** The distribution of outcome A does not depend on what outcome B is found at the other measurement site and vice versa.

$$\begin{aligned} p_1(A|a, b, \lambda, B) &= p_1(A|a, b, \lambda) && (2.3) \\ \text{and } p_2(B|a, b, \lambda, A) &= p_2(B|a, b, \lambda). && (2.4) \end{aligned}$$

- **Parameter Independence** The distribution of outcome A does not depend on the parameter setting b of the measurement apparatus at the other measurement site, nor does the distribution of outcome B depend on the parameter setting a.

$$\begin{aligned} p_1(A|a, b, \lambda) &= p_1(A|a, \lambda) && (2.5) \\ \text{and } p_2(B|a, b, \lambda) &= p_2(B|b, \lambda). && (2.6) \end{aligned}$$

Inserting (2.3)-(2.6) in equation (2.2) then gives us:

$$p(A, B|a, b, \lambda) = p_1(A|a, \lambda)p_2(B|b, \lambda), \tag{2.7}$$

the desired factorization of the joint probability distribution for the two particles. It is the analysis of equation (2.7) that leads to Bell's inequality[8][9], which I here will jump to, just to have an idea what is the case numerically when some theory is said to be violated by the predictions of quantum theory and experimental results. Bell's inequality is an inequality between the expectation values for the possible combinations of the parameter settings. In the version of Clauser-Holt-Horne-Shimony (CHHS), it takes the form of

$$|E(a', b') \pm E(a', b'')| - |E(a'', b') \mp E(a'', b'')| \leq 2, \tag{2.8}$$

where a', a'' are two different possible settings for a and likewise b', b'' for b. $E(a', b'')$ is then the expectation value in the case that Alice has parameter setting a' (for example measuring spin along the x-axis) and Bob has parameter setting b'' (for example measuring spin at angle θ from the x-axis).

There is another independence Mermin does not mention, which Bell does, albeit it not very pronounced in his 1964 article. I will call it

- **Free Variables** Parameter settings a and b are independent of the state description λ, and therefore to be considered free variables.

This is captured in the denoting of the probability distribution of λ as

$$(p(\lambda|a, b) =) \;\; p(\lambda). \tag{2.9}$$

In *La nouvelle cuisine* (1990, [11]), Bell states it with more emphasis (in the notation of the present writing):

> The variables a and b can be considered to be *free*, or *random*. [...]
> $p(\lambda|a, b, c) = p(\lambda|c)$,

where c stands "for the values of any number of other variables describing the experimental set-up, as admitted by ordinary quantum mechanics", which, together with λ gives "a complete specification" of the observables that form the past cause of A and B (for example some setting of the particle emitter). The condition of free variables is included in Bell's inequality.

The predictions of quantum theory on specially prepared systems (such as the spin-$\frac{1}{2}$ singlet) violate Bell's inequality for certain parameter settings. So the assumption made above cannot be valid in quantum theory.

Chapter 3

The meaning of a violation

What concepts are at stake, knowing that quantum theory violates Bell's inequality, as do several experimental results? And does that make us any wiser, and if so, about what? Here, it is appropriate to let the mind linger on what the aim could be of a scientific theory. EPR state immediately at the beginning of their article that "[a]ny serious consideration of a physical theory must take into account the distinction between objective reality, which is independent of any theory, and the physical concepts with which the theory operates." (See [5].) Bell is very explicit as well, and it is his formulation that is very important when looking at the meaning of a violation of Bell's inequality:

> "I would insist [...] on the distinction between analyzing various physical theories, on the one hand, and philosophising about the unique real world on the other hand."
> Bell (1977) [12]

To Bell, a theory is at best a candidate for the description of nature. Instead of the term 'observables' as is used in quantum theory, Bell speaks of *beables* as the elements "which might correspond to elements of reality" (see 1984, [13]). Philosophy professor Bas van Fraassen introduced a view called *constructive empiricism* through his book *The Scientific Image* [1980, [14]], which means as much as that "science aims to give us theories which are empirically adequate; these theories may speak about underlying structures and principles and laws, but in the end, it is solely possible to test whether its predictions are true within an acceptable (explainable by the theory at hand) margin of error. This empiricist view gives room for EPR's questioning of the completeness of quantum theory. But speaking about the nature of reality seems to be outside the range of constructive empiricism. Without going in any deeper on the views of Van Fraassen, I will return on this subject in the last section of this report (section 3.3).

3.1 Local causality

As is visible in Shimony's approach (section 2.5), two steps are taken to factorize the probability distribution of the joint measurement values into the measurement value per measurement device at two spatially separated sites. Travis

Norsen (2009, [15]) comments on how Bell's formation of this factorization is often not recognized:

> "There is, in particular, a tendency for a relatively superficial focus on the relatively formal aspects of Bell's arguments, to lead commentators astray. For example, how many commentators have too-quickly breezed through the prosaic first section of Bell's 1964 paper (pp. 14-21) - where his reliance on the EPR argument "from locality to deterministic hidden variables" is made clear - and simply jumped ahead to Sect. 2's equation [[8]- ed.], hence erroneously inferring (and subsequently reporting to other physicists and ultimately teaching to students) that the derivation "begins with deterministic hidden variables"?" (Norsen, [15])

In his 1964 paper, Bell begins with a summary of the reasoning in EPR about how quantum theory cannot be complete. In section 2, he takes as an hypothesis for his proof a quote from Einstein (see [16, 8]):

> "But on one supposition we should, in my opinion, absolutely hold fast: the real factual situation of the system S_2 is independent of what is done with the system S_1, which is spatially separated from the former."

So I have to agree with Norsen here on how this can be interpreted as the starting point leading to the factorization, rather than the factorization itself being the beginning of the proof. Bell's most recent (and thus last) writing, *La nouvelle cuisine* (1990, [11]) is fully dedicated to define local causality, supported by a visualization of the principle and, indeed, in the article is enclosed how local causality leads to factorization of the conjunct probability distribution (and expectation values) of the observables of two particles in entanglement. And it is here, Bell states that "[v]ery often such factorizability is taken as the starting point of the analysis. Here we have preferred to see it not as the *formulation* of "local causality", but as a consequence thereof."

Now how does Bell's local causality relate to Einstein's and Mermin's locality? All three 'respecting' the correctness of special relativity, it can taken to be the same principle. It is just that Bell expresses that what is not allowed to go faster than light in terms of causality (see [11]):

> "The direct causes (and effects) of events are near by, and even the indirect causes (and effects) are no further away than permitted by the velocity of light."

Having a more precise idea what is meant by local causality, then how are the first two steps of 'independency' in section 2.5 to be interpreted, then? In 1985 Jon Jarrett (1985, [17]) gave an interpretation of each of these steps (see also [15]). According to him, Parameter Independence (equation (2.5) and (2.6)) represents a 'simple locality' condition, and Outcome Independence (equation (2.7) and (2.8)) stands for 'completeness'. The conjunction of the two results in a 'strong locality' condition. So the local causality of Bell is then subdivided in a locality condition and a completeness condition. When asking Jarrett what exactly is violated by quantum mechanics (and, moreover, by various experimental results) when the strong locality is violated, he reasons as follows:

> "The interpretation that strong locality inherits from locality and completeness may be exploited to help to understand the significance of the Bell arguments. Those arguments, together with the relevant experimental results, are generally taken to provide excellent evidence that strong locality cannot be satisfied by any empirically adequate theory. Since locality is contravened only on pain of a serious conflict with relativity theory (which is extraordinarily well-confirmed independently), it is appropriate to assign the blame to the completeness condition." (Jarrett (1985) [17])

In his analysis, violation of the completeness condition does not leave us with a conflict between quantum theory and special relativity - which is at the basis of EPR - and therefore it is preferable to choose the completeness condition to be the one that is violated. Norsen (2009) in his turn is very critical about this perception. He argues that to Bell the whole of the conjunction made up local causality, not just one of the parts. The different interpretations of the role of the outcome independence rest on a different understanding of the role of λ.

3.2 The role of λ

What does this variable λ exactly comprise? It can be seen as a representative of the general hidden variable that should hold the information about reality that quantum theory has missed. It could be the variable that 'completes' the wave equation in the way EPR inferred.

In section 2.5, the third independency is the one of the freedom of certain variables, namely the variables (beables) that describe the parameter settings of the measurement apparatus. What is meant by this freedom, is maybe best illustrated by analyzing what it would mean if these particular values were not free, but fixed. Although these fixed values may be unknown to the experimenters themselves, allowing for the *illusion* of free choice, they would have indeed none in setting the parameters for the measurement. And even if the experimenters would try their very best to make their choices random by using some randomizer, still, this 'randomness' would be an illusion, for the indications given by the randomizer would be fixed beforehand in λ. Is it out of place in science to think that a hidden variable could be representing an underlying structure of the appearing reality that is so complete that the behavior of at least two experimenters and some randomizer is determined by it? This so-called (super)determinism is seriously considered by some scientists, like Gerard 't Hooft (see 't Hooft, 2007, [18]). Thinking of this truly complete description of reality automatically leads to a discussion about the concept of free will, and then it is as nonintuitive to me as a reality that has (or consists of) fundamentally random processes. So it is a conceptual solution for preserving local causality as defined by Bell, but on the other hand, the concept of causality has no meaning anymore; causality itself is then an illusion. I prefer not to elaborate any further on free will and superdeterminism in this writing, for I have not enough knowledge of its mathematical structures and detailed implications. I will leave it for a good thought in sleepless nights and continue here with assuming the free variable condition.

To get our feet back on the ground, let us turn to figure 5. To really appreciate the details of the figure, it is best to read Bell's *La nouvelle cuisine* (1990,

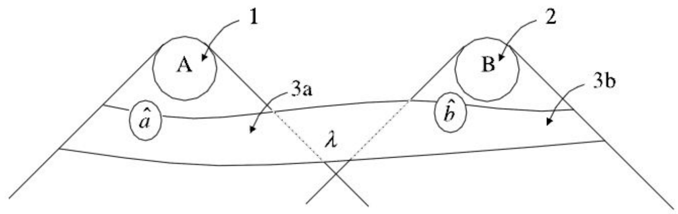


Figure 5: "Space-time diagram illustrating the various beables of relevance for a discussion of factorization. Separated observers Alice and Bob make spin-component measurements (using apparatus characterized by variables $\hat{a}$ and $\hat{b}$ respectively) of a pair of entangled spin-$\frac{1}{2}$ particles. The state of the particles (and/or any other appropriately associated beables) is denoted λ, and the outcomes of the two measurements are represented by the beables A (in region 1) and B (in region 2). Note that λ and $\hat{a}$ jointly constitute a complete specification of beables in space-time region $3a$, which shields off region 1 from the overlap of the past light cones of 1 and 2. Likewise, λ and $\hat{b}$ jointly constitute a complete specification of beables in space-time region $3b$, which shields off region 2 from the overlap of the past light cones of 2 and 1. Thus the joint specification of λ and $\hat{a}$ will - in a locally causal theory - make $\hat{b}$ and B redundant for a theory's predictions about A (and likewise, specification of λ and $\hat{b}$ will render $\hat{a}$ and A redundant for predictions about B)." Figure and caption taken from Norsen (2009 [[15], p. 282])

[11]), where he introduces the picture of a backward (and forward) lightcone, and Norsen (2009) (see [15]). Here, I will only discuss the details that are of direct use in the present analysis. As illustrated by figure 5 and commented by Norsen, λ together with the setting of the parameter setting of the apparatuses, give a *complete* specification of beables in the space time regions that are in the backward lightcones of the two events of measuring A and B. This way, the parameter settings can have the role of free variables and completeness of λ is somehow not a completeness that includes these parameter settings. Then how complete must λ be? Michael Seevinck and Jos Uffink introduce a new way to look at the completeness of λ (Seevinck and Uffink, 2010, [19]). In short, they state that in a candidate theory, the description of λ should be sufficient

(α) "relative to a specific class R_λ of beables (i.e., the beables have a particular space-time specification, and they need not include all such beables the theory in fact allows for!).";

(β_1) "for a specific purpose, namely"

(β_2) "to render some other variables redundant for the task of determining some particular quantity."

So when constructing a candidate theory (to compete with quantum theory, or to make quantum theory 'complete'), it make sense to construct it so that λ satisfies (α), (β_1) and (β_2). The number of 'other variables' that have to be rendered redundant are, in the case of incorporating the principle of local causality into the theory, outcome B and parameter setting b when determining particular quantity A and vice versa (see also figure 5). Jarrett, however, takes λ to be possibly incomplete in such a way that relevant beables could be left out, thus that it is at least not possible anymore for the description of λ to fulfill (β_1) and (β_2). With such an incomplete description given by λ, the Bell's inequality can still be constructed from this theory. But now the λ in the independence relations, is not the λ that is sufficient for the purpose of defining the local causality principle. A violation of the inequality now is merely a violation of some sort of no-correlation condition. So to define which of the two independencies that make up local causality stand for which principle and how ('simple locality', 'completeness') and the reason why we prefer the one to be violated and not the other, then makes no sense.

Seevinck and Uffink furthermore analyze the different theoretical roles of settings and outcomes, a difference that was already pointed out by Bell himself, and combine this analysis with the concept of sufficiency, which results in a new formulation of the steps needed for factorization. In their analysis, they call the two conditions making up the conjunction: sufficiency conditions. Without going into any further detail, it can be stated that this new formulation leads to Bell's inequality as well, but now 'mathematically 'sharp and clean''. So again, when constructing a candidate theory (to compete with quantum theory, or to make quantum theory 'complete'), now we want this theory to respect the local causality principle. Then λ has to satisfy (α), (β_1) and (β_2). With this definition of λ and the two conditions of sufficiency, when Bell's inequality is violated, for physical interpretation of this violation it does not matter which of the two conditions is violated. For (see [19])

> "[...] despite this theoretical difference [between the two sufficiency conditions - ed.], the physical status of the two conditions is exactly the same. Both are a consequence of local causality [...]."

This makes Jarrett's conclusion about a violation of local causality not being the result of a conflict with special relativity stand on loose ground.

Summing up: It is easily concluded from Bell's 1964 paper that the argument is inspired on the EPR thought experiment. The EPR experiment in turn was constructed to assess whether the wave function in quantum theory gives a complete description of a system, and through the merits of quantum theory, EPR concluded that it should be possible to complete the theory with additional variables, perhaps in a way that would restore local causality. This possibility of a theory that, through an extra (with respect to quantum theory) variable λ, would conserve the principle of local causality as formulated by Albert Einstein and later by Bell himself, is exactly what Bell assessed. To assume a theory with a λ that possibly leaves out already a description of the wanted (relevant) beables, in a way Jarrett did, is then a silly assumption. And therefore, as Norsen (2009) and Seevinck and Uffink (2010) explain in detail, no matter which of the two steps leading to the factorization of the probability distribution is violated, the violation of the 'strong locality' condition remains a violation of local causality and the violating theory is fundamentally in conflict with special relativity.

3.3 The usefulness and necessity of Bell's inequality

As pointed out by different authors and throughout this report, quantum theory is a *theory*. In the introduction of this chapter, I already briefly went by some views on the aim of scientific theories. Now let us mount Bell's inequality in his own view that a theory at best is a candidate for describing Nature. The theorem of Bell explicitly is a test for theories. The recipe is: construct a theory, define its beables that might correspond to elements of reality, and find that a theory that assumes local causality leads to Bell's inequality. So a theory assuming local causality is in conflict with the predictions of quantum mechanics, and, perhaps 'moreover', is in conflict with various experimental data. So Nature, received through these experimental data, cannot be described by a theory that assumes local causality. I cannot help but conclude that the nature of Nature is not locally causal. As Norsen (2009) puts it:

> "That is (and leaving aside the various experimental loopholes), no locally causal theory in Bell's sense can agree with experiment, can be empirically viable, can be true. Which means the true theory (whatever it might be) necessarily violates Bell's locality condition. Nature is not locally causal."

So leaving open the answer if it is possible to ever really understand the ways of Nature, it certainly is so that thinking about the nature of our environment leads to sharper definitions of current theories, to new parts of theories and probably to new theories as a whole. So far, quantum theory has provided us with a great basin of not yet precisely formulated features, that for some people scream for interpretation. In the wild forest of definitions, alternative (metaphysical) theories, vague arguments and for-all-practical-purposes solutions, Bell's inequality stands out as a useful tool. By assessing the foundation of this theorem, Seevinck and Uffink (2010) and Norsen (2009) lead us away from the 'conservative' interpretation by Jarrett and others, that Bell's inequality does not point out fundamental issues between quantum theory and special relativity. This way, instead of the theorem being a definite closing of the door for a certain class of hidden variable theories, it could be at the beginning of (renewed) research of what it means for local causality to be violated and of the usefulness and possibilities of non-local theories.

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