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The Performance of Corporate Bond Funds

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Abstract

This thesis investigates the performance of corporate bond funds. In the first part of this thesis, a mathematical equilibrium model is derived to estimate the return of corporate bond funds. The abnormal rate of return of a fund in excess of what would be predicted by this equilibrium model is called the alpha of a fund. This alpha is used as a performance measure for corporate bond funds. In the second part of this thesis, we use a large dataset to fit this model to the data. We show that on average, corporate bond funds manage to obtain a positive alpha. Furthermore, it is shown that funds that performed strongly (weakly) in the past continue to perform strongly (weakly) in the future. Various investment strategies are suggested that exploit this persistence in performance. It is shown a portfolio of past winners can obtain an alpha of 1.77% per year, whereas a strategy of buying past winners and selling past losers can even result in an alpha of 2.77% per year. Investing in the single fund with the highest past alpha can even lead to an alpha of 2.93% per year.

Keywords: corporate bond mutual funds, persistence in performance, equilibrium model, family-wise error rate

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Chapter 1

Introduction

Mutual funds are professionally managed investment programs that pool money from investors and invest this money in stocks, bonds, money-market instruments and other securities. There are two main reasons for investors to invest their money in mutual funds. The first one is risk reduction. Holding a diversified portfolio reduces risk, however, individual investors might not have enough budget for a fully diversified portfolio. The second one is performance. Because mutual funds are professionally managed, one would expect that the performance of these funds is superior to the performance of amateurs. Academic literature agrees that mutual funds do a good job in reducing the risk of investing by diversification (M. C. Jensen (1968)). It is less clear however if actively managed funds also achieve superior performance.

Mutual funds emerged in the 18th century in The Netherlands (Rouwenhorst (2004)) and with over $30 trillion worth of assets under management at the end of 2014 it is arguably the largest financial industry worldwide.[1] Mutual funds can be subdivided into several groups, depending on the assets they invest in. One can for example speak about equity funds, funds investing in stocks, or about fixed income funds, funds investing in corporate or government bonds. This thesis deals with corporate bond funds, a subclass of fixed income mutual funds investing in corporate bonds.

It is not without good reason that we investigate corporate bond funds. One of the dominant trends in the financial industry of today is the growth of the assets under management by these corporate bond funds. The ICI reports that the annual geometric growth rate of assets under management by corporate bond funds from 2000 to 2013 was 14% , which let them to quadruple in size. Comparing this with data reported by Feroli et al. (2014), it can be concluded that corporate bond funds have attracted far greater inflows than equity and money market funds combined. I. Goldstein, Jiang, and Ng (2015) estimate that as much as 23% of all corporate bonds outstanding in 2013 are owned by corporate bond funds. Observing this, one would expect that there is a

[1]Data reported by Statistica. Obtained from: http://www.statista.com/statistics/235553/assets-managed-in-mutual-funds-worldwide/. Retreived on February 28, 2016.

vast literature on the performance of corporate bond funds. It turns out however that surprisingly little research has been done on this topic. Most of the academic research still concentrates on equity funds. This thesis contributes to the academic literature by filling this gap. Several questions related to the perfrmance of corporate bond funds are answered.

First, the average performance of corporate bond funds is investigated. How does the average corporate bond fund perform? Many empirical studies (see for example Duffie, Saita, and Wang (2007), Collin-Dufresne, R. Goldstein, and Martin (2001)) show that the return of corporate bonds is driven by many factors that are difficult to predict. Therefore, one might expect that it is impossible for corporate bond funds to possess a superior corporate bond picking skill and hence that the value added by corporate bond funds is only minor. This thesis shows the opposite, namely that on average, funds generate positive risk adjusted returns in all subsamples.

Next, we turn to the question of persistence. In other words, are some funds able to systematically obtain a higher risk-adjusted return than others? This question is of crucial importance, because persistence in performance indicates that professional managers can add value and a high return is not sheer luck. It is therefore not surprising that much economic literature has been focused on this question. However, most of the papers exclusively focus on equity funds. Evidence from papers that do focus on corporate bond fund is mixed, Gutierrez, Maxwell, and Xu (2008) find persistence in performance for US corporate bond funds, whereas Dietze, Entrop, and Wilkens (2009) do not find any for German corporate bond funds. Our study clearly speaks in favor of active management of corporate bonds; when funds are ranked on their performance over the last 12 months, the top performing funds continue to outperform the worst performing ones in all our subsamples.

The first two research questions are concerned with the performance of bond funds and are very interesting in itself. This new understanding on the existence of skilled fund managers can also be used to form a new investment strategy. We propose an investment strategy where we create a portfolio of corporate bond funds by selecting all the skilled managers, but exclude the non skilled or lucky ones. Differentiating the skilled managers from the non-skilled managers is by no means a trivial task. A manager is considered skilled if a certain performance measure is statistically positive. But by design of statistical tests, there is a $a\%$ chance of a Type I error, where a is chosen by the statistician (usually this is set to 5%). In this case a Type I error means that a manager gets classified as skilled, while it is actually not. When selecting skilled managers out of a dataset of S funds, S tests needs to be performed and one would expect that $a\% * S$ funds get wrongly classified as skilled. As there are many corporate bond funds to choose from, this number can be quite substantial. To minimize the amount of bond fund managers that wrongly get classified as skilled, the algorithm developed by Romano and Wolf (2005) is used. It is shown that the resulting portfolio of funds not only performs excellent, but also reduces risk.

In short, this thesis answers the following three main research questions:

1. How do corporate bond funds perform on average?
2. Is there persistence in the performance of corporate bond funds?
3. Can we construct a profitable investment strategy to exploit this persistence in performance?

The structure of this thesis is as follows. First, the related academic literature is reviewed in Chapter 2. In Chapter 3, the mathematical model we will be working with is derived in detail. Furthermore, it is explained how to estimate this model using a dataset. It is proven that our performance estimator is consistent and asymptotically normal under some assumptions. Afterwards, we describe the dataset. In Chapter 5, the average performance of bond funds is investigated, whereas the persistence in performance of bond funds is explored in Chapter 6. In Chapter 7, a portfolio of skilled fund managers is created. It is shown that this portfolio performs well in our dataset. Finally, in Chapter 8, we conclude and some recommendations for further research are given.

Chapter 2

Literature Review

In this section we will review the literature on corporate bond fund performance. The most common measure assessing a fund manager's performance is called the alpha of a fund. First we review some important results in asset pricing theory that led to this method of evaluating fund performance. Afterwards, the literature on the performance of funds is reviewed. Finally, the literature related to creating a portfolio of funds is reviewed.

As mentioned in the introduction, mutual funds have existed for centuries. However, it wasn't until the 1960s that the first academic studies were conducted on how these mutual funds actually perform. Before this time, academics did not find a suitable method to measure the performance of a mutual fund. One could of course simply look at the return of the fund. However, asset pricing theory tells us risky assets have a higher expected return. A mutual fund can therefore obtain a higher expected return just by holding a portfolio of risky assets. This should be taken into account when evaluating the performance of a fund. Before the 1960s researchers didn't know how to deal with this concept of risk. To take into account this risk, Sharpe (1966) introduced the Sharpe Ratio, which is simply the ratio of expected excess return and volatility, as a risk adjusted performance measure. This simple measure is only based on two statistics. Although the Sharpe Ratio is still used very frequently as performance measure, it is not very suitable for assessing the performance of funds. The reason is that the Sharpe Ratio does not make a difference between active and passive management. One can obtain a high sharp ratio by a passive strategy; for example buying an index and just hold this index. However, every individual can do this and the fund manager does not add any value when doing so. Furthermore, Bayley and López de Prado (2012) show that Sharpe Ratio's often overstate fund performance. The writers show that this is especially a problem in datasets with a smaller return history.

The main breakthrough that made it possible to assess the performance of funds was the CAPM model (Sharpe (1964),Lintner (1965a) and Lintner (1965b)). The model states that investors should only be compensated for risk that cannot be avoided, risk that cannot be diversified away. This type of risk is called systematic risk. An example of systematic risk is the financial crisis of 2008. Anyone saw the value of their investments change because of a worldwide economic crisis, regardless of how many or what types of securities they held. Another type of risk is idiosyncratic risk. This is the risk that affects a single asset or industry, but does not influence every asset in a well diversified portfolio. An example of ideosyncratic risk is one company suffering losses due to a fire. This fire affects the stock price of this single company, but the stock price of other companies is unaffected by this. If an investor has a well diversified portfolio, the drop of the stock price from this one company is offset by a possible rise in value of other securities in the portfolio. Hence, losses due to ideosyncratic risk can be avoided by diversification and the CAPM stipulates that investors should not be reimbursed for bearing such risks. The CAPM model states that securities that have a high systematic risk should have a higher expected return to compensate investors for this risk. In the CAPM, this exposure to systematic risk is measured as the exposure to the market (this makes sense as the whole market is effected by systematic risk). Therefore, in the CAPM framework, the expected return of a security is a function of the return of a portfolio that includes every type of asset available in the financial market, which is called a market portfolio. To be more precise, the expected return of a security $i \in 1, \dots, N$ is modelled as follows:

$$\mathbb{E}R_i = r^f + \beta_{i,MKT}(\mathbb{E}(R_{MKT} - r^f)). \tag{2.1}$$

Here, R_i is the return of stock i, r^f is the return of a risk-free asset, R_{MKT} is the return of the market portfolio and $\beta_{i,MKT}$ the sensitivity of the return of security i to the overall market. It is the extent to which $\mathbb{E}R_i$ is influenced by $\mathbb{E}R_{MKT}$ (and thus by systematic risk). If $\beta_{i,MKT} = 1$, the return of security i moves with the market. If $\beta_{i,MKT} > 1$, systematic risks greatly affect the return of this security; when the return of the market portfolio is expected to drop, the expected return of security i drops even further. If $\beta_{i,MKT} < 1$ it indicates that the return of security i is less volatile than the market. Systematic risks do not influence the return of security i as greatly as the return of the market portfolio.

This simple model gave rise to a whole new stream of literature in asset pricing theory, even resulting in a Nobel Prize in 1990. M. C. Jensen (1968) was the first one to use the CAPM model to create a measure evaluating portfolios. The portfolio of an investor is the set of all its investments. The performance measure of a portfolio is then defined as the constant term in the following time-series regression:

$$R_{p,t} - r^f = \alpha_p + \beta_{p,MKT}((R_{MKT,t} - r^f)) + \epsilon_{p,t}. \tag{2.2}$$

where $R_{p,t}$ equals the return of the portfolio at time t, $\beta_{p,MKT}$ the exposure of the portfolio to the market and $\epsilon_{p,t}$ the error term. Note that Equation (2.1) tells us something about the expected return, whereas Equation (2.2) can only be estimated after the data has been collected. Therefore, the expectation sign drops out. Assuming that the

CAPM indeed accurately describes stock returns, the α_p in the above regression is the return which can be attributed to the portfolio managers, the rest of the return comes from exposure to the market. When the CAPM model is correct, an investor can only increase its expected return by taking more systematic risk (exposure to the market). The easiest method to do this is to buy a market index such as the EAX index in The Netherlands or The Dow Jones index in The United States. Every investor can do this and the return gained from this exposure should not be accounted to the skill of a fund manager. However, to obtain a better α_p, one must obtain return in excess of the return that comes from the exposure to the market, which is the only risk factor in the CAPM model. Therefore, it can be seen as the added value by the portfolio manager.

To use this alpha as a performance measure, the CAPM model must of course be valid. If there are more risk factors than the market factor driving the returns on assets, then Equation (2.1) is wrong and the return of a stock is given by

$$\mathbb{E}R_i = r^f + \beta_{i,MKT}(\mathbb{E}(R_{MKT} - r^f)) + \boldsymbol{\beta}_i'\mathbf{F}. \tag{2.3}$$

Here, $\mathbf{F}$ is a $K \times 1$ vector of the remaining factors influencing the return of a stock and $\boldsymbol{\beta}_i$ a $K \times 1$ vector of factor sensitivities and $\boldsymbol{\beta}_i^{'}$ is its transpose. The entries of $\boldsymbol{\beta}_i$ are defined in a similar matter as $\beta_{i,MKT}$. Hence, the k-th entry of $\boldsymbol{\beta}_i$ is the extent to which factor F_k influences $\mathbb{E}R_i$. Thus, if the returns of assets are generated as in Equation (2.3), but one still uses Equation (2.2) to calculate α, a higher α can be obtained by a passive investment strategy. One can for example invest in the factor F_k and just hold this factor. If $\beta_{i,k} > 0$ this increases the expected return, but one is also exposed to more risk. The return from this passive exposure should again not be seen as the value added by a fund manager as the risk of this portfolio also increased. Besides, individuals can invest in this factor themselves and do not need a portfolio manager for this. Therefore, α is not a good performance measure if the model used to estimate it does not hold.

For this reason, it is not surprising that many empirical studies were performed investigating the validity of the CAPM model. The majority of these studies however firmly reject the CAPM model and suggest that there are more risk factors besides the market factor influencing stock returns, see for example Black, M. Jensen, and Scholes (1972) and Fama and MacBeth (1973). This caused more and more researchers to model the return on assets as a statistical factor model, where the return on assets is a linear function of an arbitrary number of risk factors

$$R_i = a_i + \boldsymbol{\beta}_i'\mathbf{F} + \epsilon_i. \tag{2.4}$$

Here, $\mathbf{F}$ is a $K \times 1$ vector of risk factors and $\boldsymbol{\beta}_i$ is a $K \times 1$ vector of factor sensitivities, $\boldsymbol{\beta}_i'$ is its transpose. The constant term a_i is chosen such that $\mathbb{E}(\epsilon_i) = 0 \ \forall \ i$. In this framework, the return of an asset is modelled as a function of various risk factors. Factor models do not a priori state what these factors should be. Unlike the CAPM, they do not model the expected return of financial products, but it models the return of assets directly. However, to evaluate the performance of funds, an expression for the expected return is essential, because α is defined as the return that is earned above the return one would expect from exposure to these risk factors. Remember that the α_p in Equation (2.2) was defined as the return in excess of the return one would expect from exposure to the market factor, which is the only risk factor in the CAPM framework. Ross (1976) developed an equilibrium model, in which the expected returns of assets whose returns are generated by Equation (2.4) are given by

$$\mathbb{E}R_i = r^f + \sum_{k=1}^{K} \beta_{i,k}(\mathbb{E}F_k - r^f). \tag{2.5}$$

Here, F_k and β_k are the k-th entry of $\mathbf{F}$ and $\boldsymbol{\beta}$ respectively. Because this is the model we use in this study to evaluate bond performance, we prove Equation (2.5) formally in Chapter 3. However, the proof is different than the proof of Ross (1976). Whereas Ross (1976) starts from preferences from investors, we derive this equation from a non arbitrage point of view, this makes the underlying assumptions more easy to verify (investor's preferences are not observed). It is important to stress that this model not only holds for stocks, but also for other types of financial assets and portfolios thereof. For that reason, it is very suitable to evaluate the performance of corporate bond funds. As is the case for the CAPM, the expression for the expected return can be used to create a measure evaluating portfolios. The return of a portfolio can be disentangled in a part that can be attributed to skill (α_p) and a part that can be attributed to exposure to risk factors ($\sum_{k=1}^{K} \beta_{p,k}(F_{k,t} - r^f)$). Therefore, the constant term in the regression

$$R_{p,t} - r^f = \alpha_p + \sum_{k=1}^{K} \beta_{p,k}(F_{k,t} - r^f) + \epsilon_{p,t}. \tag{2.6}$$

can now be used as a performance measure of a portfolio. Because a fund holds a portfolio of securities, it can also be used to assess the performance of a fund. If the α_p in this regression is used to measure the performance of a fund, it is normally referred to as the alpha of a fund. Therefore, calculating the risk-adjusted performance of a fund simply boils down to estimating the constant term in the above regression. Again, it is important that Equation (2.4) is the right model, i.e. that the factors indeed sufficiently explain the return of assets.

Therefore, a large stream of financial literature was devoted to defining what these risk factors exactly should be. This research in building adequate factor models became, and still is, empirical in nature. If the vector $\mathbf{F}$ is a vector of random variables that needs to be estimated, it is called a statistical factor model. In order to estimate this vector of factors, one usually uses maximum likelihood or principal component based factor analyses. For more information on how to estimate these models, one can consult Chapter 8 and 9 of Johnson and Wichern (2006). Many early studies on asset returns use statistical factor models, see for example Roll and Ross (1980) for an application of the return of stocks. In a study by Litterman and Scheinkman (1991) statistical factor analysis is used to find that three factors are enough to explain the return of bonds. The main problem with statistical factor models is however that the estimated factors have no intuitive meaning; they are just vectors of estimates. Consequently, the popularity of these statistical factor models decreased and the majority of recent studies use macroeconomic or fundamental factor models. In these models the factors are macroeconomic variables or company attributes that possibly influence asset prices. The factors either come from economic theory or from empirical observations.

For stocks, the market factor is still viewed as the most important factor, but over the years many factors have been added. Fama and French (1992) and Fama and French (1993) found that small firms consistently perform better than big firms and that firms with a high book to market ratio consistently outperform firms with a low book to market ratio, even when taking into account the stocks exposure to the market (i.e. the difference in these returns could not be explained by the CAPM model). The writers therefore argue that stocks of small firms and firms with a high book to market ratio must carry more systemetic risk with them and exposure to this type of risk increases the expected return of a stock. In line with these findings, the authors put forward a model explaining stock returns with three factors: the market factor as in the CAPM model, the size factor capturing excess return of smaller firms relative to big firms and the value factor capturing the excess returns of high book to market stocks (in models we will abbreviate these factors as SMB and HML). Later on Cahart (1997) discovered that firms with stronger performance in the recent past tend to do better in the future than stocks that have been performing poorly in the recent past, even when taking the beforementioned factors into account. The factor capturing this excess return is called the momentum factor (MOM). These four factors have now become the most popular factors to model stock returns and plugging in these factors in Equation (2.6) gives us the following time series regression:

$$\begin{aligned} R_{p,t} - r^f = \alpha_p &+ \beta_{p,MKT}(R_{MKT,t} - r^f) + \beta_{p,SMB}(SMB_t - r^f) + \beta_{p,HML}(HML_t - r^f) \\ &+ \beta_{p,MOM}(MOM_t - R^f) + \epsilon_{p,t}. \end{aligned} \tag{2.7}$$

The estimate of α_p in this equation is currently one of the most popular performance measures of portfolio's. To evaluate fund performance, the academic literature usually investigates whether funds that obtained a high (low) alpha in the past, also obtain a high (low) alpha in the future. Hence, one investigates if past winners (losers) are also future winners (losers). The reasoning is that one can obtain a high alpha over a certain period by luck, however repeatedly obtaining a high alpha requires skill. Academic evidence on the added value of equity funds is mixed; Cahart (1997), Fama and French (2010) and Busse, Goyal, and Wahal (2010) find little to no evidence of persistence in fund performance. In contrast, Vidal-García (2013) and Huij and Verbeek (2007) show that fund performance can be predicted by past fund performance.

The most popular factor model describing the return of bonds and bond portfolios was introduced by Blake, Elton, and Gruber (1993). Just as in the earlier mentioned study by Litterman and Scheinkman (1991), three factors are used to model the return of bond and bond portfolios. One factor is the market factor, which just as in the CAPM model captures that part of the return of a bond that comes from exposure to systematic risk. Blake, Elton, and Gruber (1993) found that high-yield bonds (bonds with a low credit rating and thus a higher probability of default) and mortgage-backed securities (a special type of bond with an uncertain maturity date[1]), even when taking into account the market factor. The second factor captures the excess return of bonds with a higher probability of default (DEF) and the third factor captures the excess return of mortgage-backed securities (MOR). These can be plugged into Equation (2.6) to obtain the following equation for the return of a bond portfolio:

$$R_{p,t} - r^f = \alpha_p + \beta_{p,MKT}(R_{MKT,t} - r^f) + \beta_{p,DEF}(DEF_t - r^f) + \beta_{p,MOR}(MOR_t - r^f). \quad (2.8)$$

The α_p in this equation is often used to investigate the performance of bond funds. Huij and Derwall (2008) found that corporate bond funds generate a positive alpha on average. Moreover, they find that the performance of bond funds persists; strong performing bond funds are more likely to do well in the future. They create a portfolio of 'past winners' and 'past losers' and find the alpha of the winner portfolio exceeds the one of the loser portfolio by more that 3.5 percent per year.

In our study corporate bond funds are investigated, Fama and French (1993) show that for corporate bond funds and portfolios of corporate bond funds two factors are enough; one factor that captures interest rate risk ($TERM$) and one factor that captures default risk (DEF). As explained before, many factors in Equation (2.9) and (2.8) came from empirical observations. It is for example not clear why small firms perform better than big firms, however it was observed from actual stock returns. The $TERM$ and DEF factor have a clear economic interpretation. It is clear why they determine the return of corporate bond funds. This is explained in detail in Section 3.2. As was the case for equity and bond portfolios, these factors can be plugged into (2.6) to obtain the following time series regression:

$$R_{p,t} - r^f = \alpha_p + \beta_{p,TERM} TERM_t + \beta_{p,DEF} DEF_t + \epsilon_{p,t}. \quad (2.9)$$

The α_p in this equation can now be used to estimate the performance of corporate bond funds. However, this has never been done until now. This study exactly closes this gap by investigating the performance of corporate bond funds.

A summary of the literature on factor models and performance persistence can be found in Table 1. In the academic literature, fund performance is usually evaluated by the constant term in a time series regression, where the return of the funds is regressed

[1]When an investor buys a bond, it is lending money to a company. The maturity date is the date at which the company pays off this debt. This will be explained in detail in Section 3.2

on risk factors. For each fund type these risk factors are different. Performance persistence has been found in the academic literature for bond funds. For equity funds some papers do find persistence in performance, whereas others do not. Performance persistence is not yet researched for corporate bond funds. One of the contributions of this thesis is that we find strong evidence of persistence of corporate bond funds.

Fund Type	Factors	Persistence in Performance
Equity	MKT, SMB, HML, MOM	Mixed
Bond	DEF, TERM	Yes
Corporate Bond	DEF, TERM	?

Table 1: Different types of funds require different factor models. This table gives an overview of the factors used to model returns of various types of funds in the academic literature, as well as of the performance persistence of various fund types. The performance persistence of corporate bond funds is not yet investigated in the current academic literature

Another way in which this thesis contributes to the academic literature is that it considers several investment strategies to exploit this persistence in performance of corporate bond funds. An investor can select a fund with a good past performance, and because performance persists, this fund is likely to perform well in the future too. However, as Chevalier and Ellison (1997) notice, there are incentives for fund managers to take excessive risks. Therefore, investing in a single fund might be risky. To reduce this risk, one can also invest in a portfolio of funds, in the literature this is called a Fund of Funds (FoF). One strategy often considered in the literature is to invest in a portfolio of well performing funds and go short a portfolio of under performing funds. Huij and Verbeek (2007) and Huij and Derwall (2008) show that these strategies can be profitable for stock and bond funds. A more advanced strategy is proposed by Wolf and Wunderli (2009). They create a portfolio of all funds with a statistically significant alpha, i.e. they select all the skilled managers. This is done by estimating the alpha for each fund s (denoted by α_s) and then performing the following hypothesis test for all $s = 1, \ldots, S$:

$$H_0 : \alpha_s \leq 0 \qquad vs. \qquad H_1 : \alpha_s > 0.$$

Let δ denote the significance level of the statistical test. Then, as Wolf and Wunderli (2009) notice, by construction of this test, there is an $\delta\%$ chance of a Type I error for each test; i.e. for every statistical test there is an $\delta\%$ chance a fund manager is incorrectly classified as skilled. Because in total S statistical tests need to be performed, the chance that at least one fund manager is wrongly classified as skilled is given by

$$1 - P(\text{no type one error}) = 1 - (1 - \delta\%)^S.$$

Denote the set of managers that are classified as skilled by $\mathcal{I}$, so $\mathcal{I} = \{s : \alpha_s > 0\}$. If S is as low as 50 and $\delta = 10\%$, the probability that the set $\mathcal{I}$ is too large (i.e. the probability of at least one fund being wrongly classified as skilled) is as high as 99.48%. One possible solution to this is applying the Bonferroni correction (as introduced by Bonferroni (1935)), where the significance level of the individual statistical tests would be adjusted to $\frac{\delta}{S}$. This correction makes sure that the chance that at least one fund is wrongly classified as skilled is smaller or equal to δ%. However, the Bonferoni inequality is too strict. If it is applied in this context, the set $\mathcal{I}$ would be too small. An algorithm developed by Romano and Wolf (2005) is used to exclude all the fund with managers that have been wrongly classified as skilled from $\mathcal{I}$, without excluding the funds with skilled managers. This method is applied on equity funds and it is found that their portfolio achieves a high return and positive alphas. In this study, we apply this investment strategy to corporate bond funds and show how investors can make use of the persistence in performance.

Chapter 3

The Model

This study is concerned with the performance of corporate bond funds. In order to compare the performance of fund managers, a method is needed to quantify the performance of funds. It was mentioned in Chapter 2 that if the expected return of securities is influenced by K risk factors, the performance of fund $i \in 1, \dots, N$ can be evaluated by the α_i in the following regression

$$R_{i,t} - r^f = \alpha_i + \sum_{k=1}^{K} \beta_{k,i}(F_{k,t}) + \epsilon_{i,t}. \tag{3.1}$$

Where r^f denotes the risk free rate and $F_{j,t}$ is the return of the j-th risk factor at time t and $\epsilon_{i,t}$ is the error term in the regression. $\sum_{j=1}^{K} \beta_{j,i}(F_{j,t})$ is the part of the eventual realized return that comes from the exposure to K risk factors, α_i is the return earned in excess of. In this chapter we will mathematically give a mathematical derivation of this model. In Section 3.3 the alpha is introduced as a performance measure for portfolios. Afterwards, in Section 3.2, we list all the factors we use in our model. Finally, in Section 3.5, econometric techniques to estimate Equation (3.1) are discussed.

Before we continue to the mathematical derivation, we will give an economic interpretation of Equation (3.1). It tells us that high returns might not only be the result of a skilled fund manager, but it can also be the result of exposure to common risk factors. The part of the retun in (3.1) that should be attributed to the exposure to risk factors is $\sum_{j=1}^{K} \beta_{j,i}(F_{j,t})$. As exposure to risk factors can be achieved by a passive strategy (just buy an index mimicking the risk factor), this part of the return should therefore not be attributed to the skill of the fund manager. One can therefore interpret the alpha as the part of the return that is not explained by exposure to risk factors, but by the skill of the fund manager.

3.1 Derivation of The Model

Many papers on fund performance just start from Equation (3.1) without giving a mathematical model, they only present the intuition of this equation as given in the introduction of this chapter. In this Chapter, we present a mathematical model in which we mathematically derive Equation (3.1). In the derivations, we partly rely on the approach of Cochrane (2005) and that of Back (2010). In our derivations some standard theorems and notions in mathematical finance are used, such as absence of arbitrage, the stochastic discount factor and the first theorem of asset pricing. Those concepts will first be reviewed in subsection 3.1.1. These notions will then be used in subsection 3.1.2 to formally derive our model of performance of a corporate bond fund.

3.1.1 Basic Concepts in Mathematical Finance

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space. For deriving the model, we assume that there are only two relevant time instances, t and $t+1$. Let $X_{t+1} \in \mathbb{R}$ be a payoff defined on this space. Note that X_t is a random variable, more formally X_{t+1} is a Borel measurable function $X_{t+1} : \Omega \to \mathbb{R}$. All random variables we encounter are real valued and are defined on the same probablity space. This is not explicitely stated in the remainder of this thesis. The expectation of a random variable Y is denoted by $\mathbb{E}(Y)$. The conditional expectation of a variable Y, given the information available at time t is denoted by $\mathbb{E}_t(Y)$. Hence, $\mathbb{E}_t(Y) = \mathbb{E}_t[Y|\mathcal{F}_t]$. These expectations are under the measure $\mathbb{P}$.

Let p_t denote the price of this payoff X_{t+1} at time t. The payoff space, the set of all possible payoffs that investors can obtain is denoted by $\underline{X}$, hence $X_{t+1} \in \underline{X}$. We assume that investors can freely form portfolios to generate new payoffs, i.e.

$$X_1, X_2 \in \underline{X} \Rightarrow aX_1 + bX_2 \in \underline{X} \quad \forall a, b \in \mathbb{R}.$$

We also use the notion of a pricing function, $p : \underline{X} \to \mathbb{R}$, assigning a price to the payoff X_{t+1}. Hence $p_t = p(X_{t+1})$. Hence, if the payoff of a stock at time $t+1$ is given by S_{t+1}, then the price of this stock at time t is given by $p(S_{t+1})$.

The (gross) return of a payoff X_{t+1} is defined as:

$$R_{t+1} = \frac{X_{t+1}}{p_t}.$$

The return of a risk-free asset is denoted by r^f. In the remaining, it is assumed that $r^f > 1$. In the field of mathematical finance, it is often assumed that the market is free of arbitrage. This assumption basically tells us that if a portfolio will certainly not payoff negatively, but might generate a positive payoff, you cannot get this portfolio for free. More formally:

Definition 1 (Absence of Arbitrage). $\forall X_{t+1} \in \underline{X}$: *If* $X_{t+1} \geq 0$ *a.s. and* $\mathbb{P}(X_{t+1} > 0) > 0$ *then* $p(X_{t+1}) > 0$.

Absence of arbitrage is a very standard assumption in the academic literature. If an arbitrage opportunity does exist, it normally only exists for a very short time as many investors will instantly buy such strategy, driving up its price, thereby removing the arbitrage opportunity. Another standard concept in mathematical finance is the stochastic discount factor (also referred to as the pricing kernel).

Definition 2 (Stochastic Discount Factor)**.** *The stochastic discount factor (SDF) is a random variable $M_{t,t+1}$ such that for any $X_{t+1} \in \underline{X}$ the price of this payoff is given by*

$$p_t = p(X_{t+1}) = \mathbb{E}_t(M_{t,t+1}X_{t+1}). \tag{3.2}$$

Hence, the SDF is a random variable generating prices from payoffs. Note that the expectation in (3.2) is defined under the measure $\mathbb{P}$. Harrison and Kreps (1979) proved that the stochastic discount factor always exists in an arbitrage free world. Moreover, he proved that this variable is positive with probability one. This theorem is so important, that it has been called the first fundamendal theorem of asset pricing:

Theorem 1 (First Fundamental Theorem of Asset Pricing)**.** *Let $X_{t+1} \in \underline{X}$. There exists an $M_{t,t+1} > 0$ a.s. such that $p_t = \mathbb{E}_t(M_{t,t+1}X_{t+1})$ if and only if there is absence of arbitrage.*

Proof. It easy to proof that the existence of an $M_{t,t+1} > 0$ implies no arbitrage. Indeed, let $X_{t+1} \geq 0$ a.s. and $X_{t+1} > 0$ for some $w \in \Omega$. Then $p_t = \mathbb{E}_t(M_{t,t+1}X_{t+1}) = \int_{w\in\Omega:X_{t+1}>0} M_{t,t+1}X_{t+1}d\mathbb{P}(w) > 0$. The reverse part of he proof is lengthy and can be found in many books. Therefore, we will not prove it here. One can for example refer to Spreij (2014). Here, a seperating-hyperplane argument is used to prove that the SDF exist in absense of arbitrage. □

Using Theorem 1, it is easy to prove that absence of arbitrage implies that the pricing function p is linear, this is often called the law of one price:

Theorem 2 (Law of One Price)**.** *Assume there is no arbitrage, then $\forall X_{1,t+1}, X_{2,t+1} \in \underline{X}$ and $\forall a, b \in \mathbb{R} : p(aX_{1,t+1} + bX_{2,t+1}) = ap(X_{1,t+1}) + bp(X_{2,t+1})$.*

Proof. Assume absence of arbitrage. Then by Theorem 1 there exists an $M > 0$, such that prices all $X \in \underline{X}$. Using that $\underline{X}$ is linear and the linearity of conditional expectations we have that $p(aX_{1,t+1} + bX_{2,t+1}) = \mathbb{E}_t(M_{t,t+1}(aX_{1,t+1} + bX_{2,t+1})) = a\mathbb{E}_t(M_{t,t+1}(X_{1,t+1})) + b\mathbb{E}_t(M_{t,t+1}(X_{2,t+1})) = ap(X_{1,t+1}) + bp(X_{2,t+1})$. □

Until now, we introduced some basic notions and theorem in mathematical finance. These notions will be used in the next section to derive APT. The remaining of this subsection will be used to provide a link between the risk-neutral probability measure and the stochastic discount factor. Furthermore, we will give an example of a pricing function to make the above definitions and theorems more clear.

As mentioned earlier, every expectation in this section is defined under the measure

$\mathbb{P}$. This is not the risk-neutral measure (denoted by $\mathbb{P}^*$), which is frequently used in mathematical finance (especially in continuous-time finance). The risk-neutral measure is a probability measure such that the price of each payoff is exactly equal to the discounted expectation of the payoff under this measure. Under the assumption of absense of arbitrage, it can be proven that such a measure exists. Hence, another version of Theorem 1 is the following:

Theorem 3 (First Fundamental Theorem of Asset Pricing II)**.** *Let $X_{t+1} \in \underline{X}$. There exists a probability measure $\mathbb{P}*$, such that $p_t = \frac{1}{r^f}\mathbb{E}^*(X_{t+1})$ if and only if there is absence of arbitrage.*

Proof. Assume absence of arbitrage. Then by Theorem 1, the price of a risk-free payoff x_{t+1} is given by

$$p(x_{t+1}) = \mathbb{E}_t(M_{t,t+1}x) = x\mathbb{E}_t(M_{t,t+1}).$$

And thus we have that

$$r^f = \frac{x_{t+1}}{p(x_{t+1})} = \frac{1}{\mathbb{E}_t(M_{t,t+1})}. \tag{3.3}$$

By Theorem 1, the price of a payoff $X_{t+1} \in \underline{X}$ is given by

$$\begin{aligned}
p(X_{t+1}) = \mathbb{E}_t(M_{t,t+1}X_{t+1}) &= \int_{w\in\Omega} M_{t,t+1}(w)X_{t+1}(w)d\mathbb{P}(w|\mathcal{F}_0) \\
&= \int_{w\in\Omega} \frac{M_{t,t+1}(w)}{\mathbb{E}_t(M_{t,t+1})}\mathbb{E}_t(M_{t,t+1})X_{t+1}(w)d\mathbb{P}(w|\mathcal{F}_0) \\
&= \frac{1}{r^f}\int_{w\in\Omega} X_{t+1}(w)d\frac{M_{t,t+1}(w)}{\mathbb{E}_t(M_{t,t+1})}\mathbb{P}(w|\mathcal{F}_0) \\
&= \frac{1}{r^f}\int_{w\in\Omega} X_{t+1}(w)d\mathbb{P}^*(w) \\
&= \frac{1}{r^f}\mathbb{E}^*(X_{t+1}).
\end{aligned}$$

And thus $\mathbb{P}^*(w) = \frac{M_{t,t+1}(w)}{\mathbb{E}_t(M_{t,t+1})}\mathbb{P}(w|\mathcal{F}_0)$. It is straightforward to prove that $\mathbb{P}^*$ is a probability measure. Note that $\mathbb{P}^*(w) > 0$ because $M_{t,t+1} > 0$ almost surely by Theorem 1 and because $\mathbb{P}(w|\mathcal{F}_0) > 0$. Furthermore we have that

$$\int_{w\in\Omega} d\mathbb{P}^*(w) = \int_{w\in\Omega} \frac{M_{t,t+1}(w)}{\mathbb{E}_t(M_{t,t+1})}d\mathbb{P}(w|\mathcal{F}_0) = \frac{1}{\mathbb{E}_t(M_{t,t+1})}\int_{w\in\Omega} M_{t,t+1}(w)d\mathbb{P}(w|\mathcal{F}_0) = 1.$$

Therefore, $\mathbb{P}^*$ is a probability measure. This completes the first part of the proof.

Now assume that there exists a probability measure $\mathbb{P}*$, such that $p_t = \frac{1}{r^f}\mathbb{E}_t^*(X_{t+1})$. Then, by reversing the derivation above, we have that

$$p_t = \frac{1}{r^f}\mathbb{E}_t^*(X_{t+1}) = \mathbb{E}_t(M_{t,t+1}X_{t+1}).$$

Then absence of arbitrage follows by Theorem 1. □

The proof of Theorem 3 shows there is a close link between the stochastic discount factor and the risk neutral probability measure. We have that $\mathbb{P}^*(w|\mathcal{F}_0) = \frac{M_{t,t+1}(w)}{\mathbb{E}_t(M_{t,t+1})}\mathbb{P}(w|\mathcal{F}_0)$.

In the last part of this section, the concepts that we introduced in this subsection are used in the conctext of option pricing. The purpose is not to derive the price of an option, but to use the definitions and theorems introduced in this subsection in a widely known example. A call option gives an investor the right, but not the obligation, to buy a stock for a given price K at time $t+1$. Let the price of the underlying stock at time t be given by S_t. In this case $CO \equiv X_{t+1} = (S_{t+1} - K)\mathbb{1}_{(S_{t+1}-K>0)}$. Black and Scholes (1973) developed a pricing function for this payoff. They derived that the arbitrage free price of a call option is given by

$$p(CO) = N(d_1)S_t - N(d_2)Ke^{-r^f}, \tag{3.4}$$

$$d_1 = \frac{1}{\sigma}(\ln(\frac{S_t}{K}) + (r^f + \frac{\sigma^2}{2})),$$
$$d_2 = d_1 - \sigma.$$

Here, σ is the volatity of the underlying stock (normally this is measured by the standard deviation of this stock) and $N(y) = \int_{-\infty}^{y} \frac{1}{2\pi} e^{\frac{y^2}{2}} dy$ the cumlative distribution function of a standard normal distribution. Note that this function does not depend on the future, the price of the payoff C can be calculated at time t. Therefore, according to Theorem 1 the price of the call option also should be equal to $p(CO) = E_t(M_{t,t+1}C)$. Cochrane (2005) derived that $p(CO) = E_t(M_{t,t+1}C)$ is indeed equal to Equation (3.4).

3.1.2 Arbitrage Pricing Theory

Using the framework introduced in 3.1.1, we can now state and prove the model. As in previous section, there are two relevant time periods, t and $t+1$. Furthermore, there are N basic assets in this economy. The payoffs of these N assets are denoted by $X_{i,t+1}$, $i = 1, \ldots N$. As in Chapter 2, there are k factors influencing the return of these assets, denoted by $R_{i,t+1}$. These factors are bundled in the vector $\mathbf{F}_{t+1}$, hence $\mathbf{F} = [F_{1,t+1}, \ldots, F_{K,t+1}]'$. These basic assets can be used to form portfolios with weights $[w_1, \ldots, w_N]$. The return of a portfolio p is denoted by $R_{p,t+1}$.

Theorem 4 (Arbitrage Pricing Theory)**.** *If the following assumptions hold:*

1. *The returns of a set of assets are generated by the following linear model:*

$$R_{i,t+1} = a_i + \sum_{k=1}^{K} \beta_{i,k} F_{k,t+1} + \epsilon_{i,t+1}. \tag{3.5}$$

where,

$$\mathbb{E}(R_{i,t+1}),\ Var(R_{i,t+1}) < \infty\ \forall i \tag{3.6a}$$

$$\mathbb{E}(\epsilon_{i,t+1}) = 0\ \forall\ i \tag{3.6b}$$

$$\mathbb{E}(\epsilon_{i,t+1} F_{k,t+1}) = 0\ \forall\ i, k \tag{3.6c}$$

$$\epsilon_{i,t+1} \perp \epsilon_{j,t+1}\ \forall\ j \neq i \tag{3.6d}$$

2. *For a portfolio with return $R_{p,t+1}$, one of the following holds (or both):*
   - *$R_{p,t+1}$ is the return on a well diversified portfolio in a large market.*
   - *$Var(R_{p,t+1}) = Var(\boldsymbol{\beta}_p^{'} \mathbf{F}_{t+1})$.*
3. *The market is free of arbitrage.*
4. *No trading costs or taxes.*

Then the unconditional expected return of this portfolio, $\mathbb{E}[R_{p,t+1}]$ is given by:

$$\mathbb{E}(R_{p,t+1}) = r^f + \boldsymbol{\beta_p^{'}\lambda}, \tag{3.7}$$

where $\boldsymbol{\lambda} = -r^f p(\mathbf{F}_{t+1} - \mathbb{E}(\mathbf{F}_{t+1}))$.

Before a mathematical proof of Theorem 4 is given, some economic intuition behind this theorem is provided. APT's core assumption is that the return of securities are determined by three components. A constant term that is specific to each security (a_i). This a_i should not be confused with α, the performance measure that was introduced in Chapter 2. This notation can be somewhat confusing, but it is standard in mathematical finance. The second component comes from exposure to a few systematic factors ($\beta_{i,j} F_{k,t+1},\ k = 1, \ldots, K$). The third term are idiosyncratic shocks ($\epsilon_{i,t+1}$) that affect the return on securities. Every security is influenced by these shocks in a different manner; hence negative shocks to certain securities can be offset by positive shocks to other securities. From this statistical characterization of realized returns, APT derives a mathematical expression for expected returns and prices. APT predicts that idiosyncratic shocks should not be priced as investors can hold portfolios to diversify these risks away. To achieve this diversification, one must hold a well diversified portfolio or the factors must explain all the variation in the asset returns (Assumption 2). Therefore, the price and expected returns of a group of assets should depend on the same common factors. However, not all assets and portfolios perform identically, because different assets and portfolios have different sensitivities to these factors, as captured by the $\beta_{i,k}$'s in Equation (3.5).

Proof. We can subtract expectations from both sides of Equation (3.5) to obtain

$$R_{i,t+1} = \mathbb{E}(R_{i,t+1}) + \sum_{j=1}^{K} \beta_{i,k} \tilde{F}_{k,t+1} + \epsilon_{i,t+1}, \tag{3.8}$$

where $\tilde{F}_{k,t+1} = F_{k,t+1} - \mathbb{E}F_{k,t+1}$. Or in vector notation:

$$R_{i,t+1} = \mathbb{E}(R_{i,t+1}) + \boldsymbol{\beta}_i^{'}\tilde{\mathbf{F}}_{t+1} + \epsilon_{i,t+1}, \tag{3.9}$$

where $\tilde{\mathbf{F}}_{t+1} = [\tilde{F}_{1,t+1}, \tilde{F}_{2,t+1} \ldots, \tilde{F}_{K,t+1}]'$. and $\boldsymbol{\beta}_i = [\beta_{i,1}, \beta_{i,2}, \ldots, \beta_{i,K}]'$. Therefore, for a portfolio $[w_1, \ldots, w_N]$, the following holds:

$$\begin{aligned} R_{p,t+1} &= \sum_{i=1}^{N} w_i R_{i,t+1} \\ &= \sum_{i=1}^{N} w_i \mathbb{E}(R_{i,t+1}) + \sum_{i=1}^{N} w_i \boldsymbol{\beta}_i^{'}\tilde{\mathbf{F}}_{t+1} + \sum_{i=1}^{N} w_i \epsilon_{i,t+1} \\ &= \mathbb{E}(R_{p,t+1}) + \boldsymbol{\beta}_p^{'}\tilde{\mathbf{F}}_{t+1} + \epsilon_{p,t+1} \end{aligned} \tag{3.10}$$

Because of Theorem 2 prices can be taken at both sides of Equation (3.10) to obtain[1]

$$\begin{aligned} p(R_{p,t+1}) &= p(\mathbb{E}(R_{p,t+1}) + \boldsymbol{\beta}_p^{'}\tilde{\mathbf{F}}_{t+1} + \epsilon_{p,t+1}) \\ &= p(\mathbb{E}(R_{p,t+1})) + p(\boldsymbol{\beta}_p^{'}\tilde{\mathbf{F}}_{t+1}) + p(\epsilon_{p,t+1}) \\ &= \mathbb{E}(R_{p,t+1})p(1) + \boldsymbol{\beta}_p^{'}p(\tilde{\mathbf{F}}_{t+1}) + \mathbb{E}_t(M_{t,t+1}\epsilon_{p,t+1}). \end{aligned} \tag{3.11}$$

The structure of the remaining part of the proof is as follows. First, we prove that for any $M_{t,t+1} > 0$, $M_{t,t+1}\epsilon_{p,t+1} \to 0$ under Assumption 2 of Theorem 4, then by continuity of the pricing function, also $p(\epsilon_{p,t+1}) = \mathbb{E}_t(M_{t,t+1}\epsilon_{p,t+1}) \to 0$ as $N \to \infty$. Afterwards, $\mathbb{E}(R_{p,t+1})p(1) + \boldsymbol{\beta}_p^{'}p(\tilde{\mathbf{F}}_{t+1})$ is evaluated.

First, assume that $Var(R_{p,t+1}) = Var(\boldsymbol{\beta}_p^{'}\mathbf{F}_{t+1})$. Because of the assumptions (3.6b) and (3.6c), we also have that

$$Var(R_{p,t+1}) = Var(\boldsymbol{\beta}_{\boldsymbol{p}}^{'}\tilde{\mathbf{F}}_{t+1}) + Var(\epsilon_{p,t+1}). \tag{3.12}$$

It follows that $Var(\epsilon_{p,t+1}) = 0$. Because we assumed $\mathbb{E}(\epsilon_{i,t+1}) = 0$ for all i, $\mathbb{E}(\epsilon_{p,t+1}) = 0$ too. Hence, we have that $\epsilon_{p,t+1} = 0$ almost surely. Because $0 < M_{t,t+1} < \infty$, it must now be the case that $M_{t,t+1}\epsilon_{p,t+1}) = 0$.

Now take the return of a well-diversified portfolio in a market where the number of securities are large (the number of assets having a positive portfolio weight must tend to infinity to be precise). Let again w_i denote the portfolio weight of asset i and let N^* denote the number of weights in this portfolio that are larger than 0. The variance of $\epsilon_{p,t+1}$ is now given by

$$\begin{aligned} Var(\epsilon_{p,t+1}) &= Var(\sum_{i=1}^{N} w_i \epsilon_{i,t+1}) \\ &= \sum_{i=1}^{N} (w_i)^2 Var(\epsilon_{i,t+1}) \to 0 \ as \ N^* \to \infty. \end{aligned} \tag{3.13}$$

[1]By $p(\tilde{\mathbf{f}})$ we mean $[p(f_1), p(f_2), \ldots, p(f_n)]'$. The notation (even though it is clear), is somewhat sloppy because p was defined as $p : \underline{X} \to \mathbb{R}$.

Here we used the independence of $\epsilon_{i,t+1}$ and that $Var(\epsilon_{i,t+1}) < \infty$. Hence, under Assumption 2 of Theorem 4, we have that $\epsilon_{p,t+1} \to 0$. Because $0 < M_{t,t+1} < \infty$, also $M_{t,t+1}\epsilon_{p,t+1} \to 0$ and by the dominated convergence theorem for conditional expectations it follows that

$$p(\epsilon_{p,t+1}) = \mathbb{E}_t(M_{t,t+1}\epsilon_{p,t+1}) \to 0 \tag{3.14}$$

when p is either a diversified portfolio in a large market or the variation in the return of this portfolio is fully determined by the variation in the factors. This should be no surprise. When variations in the return of a payoff are fully determined by the variation in the risk factors, shocks have no influence on this payoff and should therefore not be priced. Furthermore, idiosyncratic shocks also have no influence on the return of a large, diversified portfolio as a positive shock to one asset is offset by a negative shock to another asset.

Therefore, Equation (3.11) can now be reduced to:

$$\begin{aligned} p(R_{p,t+1}) &= \mathbb{E}(R_{p,t+1})p(1) + \boldsymbol{\beta_p'} p(\tilde{\mathbf{F}}_{t+1}) \\ &= \mathbb{E}(R_{p,t+1})\mathbb{E}_t(M) + \boldsymbol{\beta_p'} p(\tilde{\mathbf{F}}_{t+1}). \end{aligned} \tag{3.15}$$

All returns have a price of one, thus the price of $R_{p,t+1}$ is also one:

$$\begin{aligned} p(R_{p,t+1}) &= p\left(\frac{X_{p,t+1}}{p(X_{p,t+1})}\right) \\ &= \mathbb{E}_t\left(M_{t,t+1}\frac{X_{p,t+1}}{p(X_{p,t+1})}\right) \\ &= \frac{1}{p(X_{p,t+1})}\mathbb{E}_t(M_{t,t+1}X_{p,t+1}) \\ &= \frac{p(X_{p,t+1})}{p(X_{p,t+1})} = 1. \end{aligned} \tag{3.16}$$

Furthermore, from Equation (3.3) we get that the risk-free return, r^f can be written as

$$r^f = \frac{1}{\mathbb{E}_t(M_{t,t+1})}. \tag{3.17}$$

Plugging Equation (3.16) and (3.17) in Equation (3.15) we get that

$$1 = \mathbb{E}(R_{p,t+1})\frac{1}{r^f} + \boldsymbol{\beta_p'} p(\tilde{\mathbf{F}}_{t+1}). \tag{3.18}$$

Solving for $\mathbb{E}(R_{p,t+1})$ gives the desired result

$$\begin{aligned} \mathbb{E}R_{p,t+1} &= r^f + \boldsymbol{\beta_p'}[-r^f p(\tilde{\mathbf{F}}_{t+1})] \\ &= r^f + \boldsymbol{\beta_p'}\boldsymbol{\lambda}, \end{aligned} \tag{3.19}$$

where $\boldsymbol{\lambda} = -r^f p(\tilde{\mathbf{F}}_{t+1})$.

□

Equation (3.7) will be used to evaluate the performance of bonds. In this equation, $\boldsymbol{\beta}_{\boldsymbol{p}}'$ is the vector of factor exposures of the bond portfolio and $\boldsymbol{\lambda}$ is the vector of factor premiums. Indeed, a small change in the exposure to factor k, $d\beta_k$, increases the expected return by $d\beta_k\lambda_k$, ceteris paribus.

As explained in the literature review, in most practical applications of APT, the factors are taken to be excess returns. Excess returns are the returns of one payoff minus the return of another payoff. In the Fama-French three factor model for example, the SMB and HML factor were all constructed as excess returns. Also in this thesis, all the factor for corporate bonds are excess returns. The next theorem tells us that when factors are returns, $\boldsymbol{\lambda} = \mathbb{E}\mathbf{F}_{t+1}$. In the proof of the next theorem, we use that the price of excess returns are zero. Indeed, let $R^e_{t+1} = R_{i,t+1} - R_{j,t+1}$ be any excess return. Because returns have a price of 1; the price of this excess return is

$$\begin{aligned} p(R^e_{t+1}) &= p(R_{i,t+1} - R_{j,t+1}) \\ &= p(R_{i,t+1}) - p(R_{j,t+1}) \\ &= 1 - 1 = 0. \end{aligned} \tag{3.20}$$

This result will be used in the proof of the following theorem.

Theorem 5. *If the factors are excess returns, then*

$$\boldsymbol{\lambda} = \mathbb{E}\mathbf{F}_{t+1} \tag{3.21}$$

and hence Equation (3.7) *reduces to*

$$\mathbb{E}R_{p,t+1} = r^f + \boldsymbol{\beta}_p'(\mathbb{E}\mathbf{F}_{t+1}). \tag{3.22}$$

Proof.

$$\begin{aligned} \boldsymbol{\lambda} &= -r^f p(\tilde{\mathbf{F}}_{t+1}) \\ &= -r^f p(\mathbf{F}_{t+1} - \mathbb{E}(\mathbf{F}_{t+1})) \\ &= -r^f p(\mathbf{F}_{t+1}) + r^f p(\mathbb{E}(\mathbf{F}_{t+1})) \\ &= r^f \mathbb{E}(\mathbf{F}_{t+1}) p(\mathbf{1}) \\ &= r^f \mathbb{E}(\mathbf{F}_{t+1}) \mathbb{E}_t(M_{t,t+1}) \\ &= r^f \mathbb{E}(\mathbf{F}_{t+1}) \mathbb{E}_t(M_{t,t+1}) \\ &= r^f \mathbb{E}(\mathbf{F}_{t+1}) \frac{1}{r^f} \\ &= \mathbb{E}\mathbf{F}_{t+1}. \end{aligned} \tag{3.23}$$

In this derivation Theorem 2 was used in the third step. In the fourth step we used that the prices of excess returns are zero. Plugging Equation (3.23) into Equation (3.19) yiels Equation (3.22) and completes the proof. □

Because this thesis (as most academic papers) uses excess returns as factors, we use Equation (3.22) in the remaining part of this work. In this section, there were two relevant time periods, t and $t+1$. Of course, the exact same analysis could also be done for a two period model with periods $t-1$ and t to obtain:[2]

$$\mathbb{E}R_{p,t} = r^f + \boldsymbol{\beta}_p'(\mathbb{E}\mathbf{F}_t). \tag{3.24}$$

This model will be used in this thesis to describe expected bond returns. The remainder of this chapter will be devoted to the content of the vector $\mathbf{F}_t$ (i.e. we explain which factors we use to model the return of a corporate bond portfolio) and it is explained how to fit this model to real life data.

3.2 Factors

In our model, the expected return of a portfolio, $R_{p,t}$, depends linearly on several risk factors. However, this model does not tell us exactly which or how many risk factors to use. The assumptions of Theorem 4 do give us some clues. Assumption 2 of Theorem 3.1.2 tells us that in order for Equation (3.24) to hold exactly the following two conditions must hold:

1. $R_{p,t+1}$ is the return on a well diversified portfolio in a large market.
2. $Var(R_{p,t+1}) = Var(\boldsymbol{\beta}_p'\mathbf{F}_{t+1})$, i.e. the variation in the return of the portfolio should be explained by variation in the factors.

This study is concerned with modeling portfolios of corporate bonds. We will follow the approach of Fama and French (1993), who use two factors to model the return of corporate bond portfolios. A term factor ($TERM$) related to interest rate risk and a default factor (DEF) related to default risk.

Corporate bond funds normally hold a broad portfolio of corporate bonds; which is an enourmous market. Furthermore, in Fama and French (1993) a big econometric study is performed. They conclude that these two risk factors indeed explain most of the variation in bond returns. Therefore, both conditions of Assumption 2 of Theorem 4 are likely to hold when we use this two factor model. As a result, it is likely that Equation (3.24) holds for most portfolios held by bond funds. In this section, an economical and mathematical explanation is given of why these factors influence bond returns. Furthermore, it is explained how to construct a proxy for these risk factors from the data.

To mathematically see why these risk factors influence the returns of corporate bond

[2]In mathematical finance it is unusual to call two periods in a model $t-1$ and t, because it is more intuitive to call a future period $t+1$ and to condition on the information at time t.

funds, we first give a pricing formula for corporate bonds. Investors buying a corporate bond at time t are lending money to a company. In return, the company returns the principal plus a coupon payment at time $t+1$, the maturity date of the bond.[3] Denote the payoff of a bond in time $t+1$ by C_{t+1}. By using Equation (3.2), we get that the price at time t of a corporate bond is given by

$$\begin{aligned} p_t &= p(C_{t+1}) \\ &= \mathbb{E}_t(M_{t,t+1}C_{t+1}) \\ &= \mathbb{E}_t(M_{t,t+1})\mathbb{E}_t(C_{t+1}) + Cov(M_{t,t+1}, C_{t+1}) \\ &= \frac{1}{r^f}\mathbb{E}_t(C_{t+1}) + Cov(M_{t,t+1}, C_{t+1}). \end{aligned} \tag{3.25}$$

In the last step, Equation (3.17) was used. Note that if the payoff is certain (i.e. no default risk), the covariance term in this equation drops out and we are left with a more familiar pricing equation for corporate bonds:

$$p_t = \frac{C_{t+1}}{r^f}.$$

The risk of default decreases $\mathbb{E}_t(C_{t+1})$ as there will be a probability that the company cannot fully fulfill its obligations. Therefore, it is clear from Equation (3.25) that a higher default rate risk will decrease the price of a corporate bond. This is to be expected, as investors require a higher return for a higher risk.

Higher interest rates increase r^f, which in turn leads to a decrease of p_t. The economic reasoning behind the inverse relation of interest rates and corporate bond prices is the following: if interest rates are higher, other economic products become more attractive (for example saving accounts). This drives down the demand for corporate bonds and thus the price of bonds will be lower.

It is clear by now that the risk of changing interest rates and the default probability have an influence on the return of corporate bonds and thereby on the return of corporate bond funds. As explained before, the factors need to capture the extra return investors require for taking those risks. Therefore, the $TERM$ factor should capture the extra return investors require for exposure to interest rate risk and the DEF factor should capture the extra return investors require for exposure to default risk. The construction of such factors is widely documented in the academic literature (see for example Fama and French (1992) and Fama and French (1993)). The most common method to construct a factor (say F_k), is to find two portfolios which are influenced differently by this factor F_k, but are influenced in the same manner by the other factors. The difference in the returns of these two portfolios should be caused by the difference in exposure to this factor.

[3]The analysis in this section can also be done in a multi-period model, where a corporate bond makes multiple coupon payments over time. This analysis can be found in Appendix A

As an estimate for the $TERM$ factor, the difference between the monthly return on a portfolio of long term government bonds and the return on a portfolio of monthly government bonds is used. Government bonds are bonds with the lowest default probability and therefore are hardly influenced by default risk. However, they are differently affected by the $TERM$ factor. The economic reasoning behind this is that the longer the bond's maturity, the more time there is for the interest rate to change and thereby affect the return of a bond. To compensate for this, long term bonds offer a higher return than short term bonds of the same credit quality. As government bonds are relatively riskless, the difference in the return is not a compensation for default risk. The most used proxy for the default risk term is difference between a portfolio of long term risky bonds and a portfolio of long term government bond. It is important that both portfolios contain bonds with the same maturity. Then, the difference in the returns of these two portfolios is not caused by exposure to the $TERM$ factor. A detailed explanation of the construction of these factors can be found in Fama and French (1992). Note that both factors are defined in terms of excess returns; the return of one portfolio over another.

The proxy for the term structure risk is denoted by $TERM$ and the proxy for default risk is denoted by DEF. Plugging these variables in Equation (3.24) gives us the following:

$$\mathbb{E}(R_{p,t}) = r^f + \beta_{p,TERM}\mathbb{E}(TERM_t) + \beta_{p,DEF}\mathbb{E}(DEF_t). \tag{3.26}$$

This has become the most used factor model for the returns of portfolios of corporate bonds in the literature. In this section, we argued that the second assumption of Theorem 4. In the next section, it is explained how to use this model to measure the performance of corporate bond funds.

3.3 Using the model to measure performance

In this chapter, an expression of the expectation of the return on securities was formally derived. In this section it is explained how this expectation can be used to define a performance measure of a portfolio, called the alpha of a portfolio. In this section, it is assumed that there are S corporate bond funds in our economy.

In the previous sections, it was explained that the expected return from a portfolio comes from the exposure to two risk factors, $TERM$ and DEF. Note that the derivation of the model, does not depend on the specific time instances t and $t+1$. The derivation could be done for any two subsequent period in $t \in [0, \dots, T]$. Therefore, for the return of a corporate bond fund $s \in [0, \dots, S]$, the following holds:

$$\begin{bmatrix} \mathbb{E}(R_{s,1}) \\ \vdots \\ \mathbb{E}(R_{s,t}) \\ \vdots \\ \mathbb{E}(R_{s,T}) \end{bmatrix} = \begin{bmatrix} r^f \\ \vdots \\ r^f \\ \vdots \\ r^f \end{bmatrix} + \beta_{s,TERM} \begin{bmatrix} \mathbb{E}(TERM_1) \\ \vdots \\ \mathbb{E}(TERM_t) \\ \vdots \\ \mathbb{E}(TERM_T) \end{bmatrix} + \beta_{s,DEF} \begin{bmatrix} \mathbb{E}(DEF_1) \\ \vdots \\ \mathbb{E}(DEF_t) \\ \vdots \\ \mathbb{E}(DEF_T) \end{bmatrix}. \qquad (3.27)$$

From (3.27) it can be seen that if one wants to estimate the $\boldsymbol{\beta}_s$ of a portfolio s, one can collect data of realized returns of this portfolio and run the following regression:

$$R_{s,t} - r^f = \alpha_s + \beta_{TERM,s} TERM_t + \beta_{DEF,s} DEF_t + \epsilon_{s,t}, \qquad t = 1, \dots, T. \qquad (3.28)$$

Or in vector notation:

$$\begin{bmatrix} R_{s,1} - r^f \\ \vdots \\ R_{s,t} - r^f \\ \vdots \\ R_{s,T} - r^f \end{bmatrix} = \alpha_s \begin{bmatrix} 1 \\ \vdots \\ 1 \\ \vdots \\ 1 \end{bmatrix} + \beta_{s,TERM} \begin{bmatrix} TERM_1 \\ \vdots \\ TERM_t \\ \vdots \\ TERM_T \end{bmatrix} + \beta_{s,DEF} \begin{bmatrix} DEF_1 \\ \vdots \\ DEF_t \\ \vdots \\ DEF_T \end{bmatrix} + \begin{bmatrix} \epsilon_{s,1} \\ \vdots \\ \epsilon_{s,t} \\ \vdots \\ \epsilon_{s,T} \end{bmatrix} \qquad (3.29)$$

It is however not the $\boldsymbol{\beta}$, but the α_s in Equation (3.29) where we are most interested in. This is because this α_s is a performance measure for corporate bond fund s. The idea of alpha is that if a fund manager manages to capture a return in addition to the return gained from exposure to these risk factors, this can be attributed to the skill of the fund manager. Hence, this alpha indicates how the portfolio performed after taking into account the risk involved:

- $\alpha_s > 0$: portfolio s has obtained a return in excess of the reward for the risk it took
- $\alpha_s = 0$: portfolio s has obtained a return that is adaquate for the risk taken
- $\alpha_s < 0$: portfolio s has obtained a return that is too little for the risk it took

This also explains why alpha is the most popular performance measure in the academic literature. One can easily boost ones return by investing bonds with a high default probability (junk bonds) or bonds that are prone to interest rate risk, thereby increasing its $\boldsymbol{\beta}$. But this should off course not be accounted to the skill of manager. A good fund manager manages to obtain a high return without investing in an overly risky portfolio.

3.4 Estimation of The Model

In this chapter, we came up with our final model. Of course, to put this model into practice, it needs to be fitted to the data. This paragraph is concerned with the estimation of Equation (3.29), or more general with the estimation of equations of the form

$$Y_t = \boldsymbol{\beta}_t \mathbf{X}_t + \epsilon_t, \qquad t = 1, \ldots T \tag{3.30}$$

where Y_t and $\mathbf{X}_t$ are defined on $(\Omega, \mathcal{F}, \mathbb{P})$. From Equation (3.29) it follows that in our case $Y_t = R_{s,t}$ and $\mathbf{X}_t = [1, TERM_t, DEF_t]$. However, the proofs in this chapter hold for all equations of the form $Y_t = \boldsymbol{\beta}_t \mathbf{X}_t + \epsilon_t$. The goal is to prove that the OLS estimator is consistent under some assumptions. In usual proofs, one assumes that $\mathbf{X}_t$ is independent and stationary over time. In our case however, the assumption of independence is not reasonable to make. The default probabily in a certain period is very unlikely to be completely independent of the default probability in the past. News that a certain company might default can lead to this company having troubles to find investors in the near future, leading to an even higher probability of default. Instead, we assume our variables to be ergodic. A process is said to be ergodic, if after averaging it is asymptotically independent. In the first part of this section, an important result in ergodic theory is derived. Later we use this result to prove that the OLS estimator is indeed consistent when we assume ergodicity.

In the first part of this section, the approach of Dajani and Dirksin (2008) is followed. From this section onwards it is assumed that $\mathbf{X}_t$ stationary and ergodic. A stationary stochastic process defined on this space is a series of random variables $\mathbf{X} = \{\mathbf{X}_t, t \in \mathbb{N}\}$ taking values in $(X, \mathcal{X})$ for which the joint distribution of $(\mathbf{X}_{t_1}, \ldots, \mathbf{X}_{t_k})$ is the same as that of $(\mathbf{X}_{t_1+t}, \ldots, \mathbf{X}_{t_k+t})$ for any $k \geq 1$ and $t, t_1, \ldots, t_k \in \mathbb{N}$. To derive our limiting result it is convenient to model our stochastic process as a dynamical system $(\tilde{\Omega}, \mathcal{B}, \mathbb{Q}, T)$, where $\tilde{\Omega} = X^{\mathbb{N}}$ and $\mathcal{B} = \mathcal{X}^{\otimes N}$. Furthermore, $T : \tilde{\Omega} \to \tilde{\Omega}$ is the left shift operator, i.e. if $\tilde{\omega} \in \tilde{\Omega}$ is a sequence $(\tilde{\omega}_1, \tilde{\omega}_2, \ldots)$, then $T(\tilde{\omega}_1, \tilde{\omega}_2, \ldots) = (\tilde{\omega}_2, \tilde{\omega}_3, \ldots)$. Note that by the Kolmogorov Existence Theorem, one can indeed construct a measure $\mathbb{Q}$ on the countable product space $X^{\mathbb{N}}$ for which

$$\mathbb{Q}(A) = \mathbb{P}((X_1, X_2, X_3 \ldots) \in A).$$

Furthermore, the following theorem tells us that the stationary stochastic process $X_t, t \in \mathbb{N}$ is equivalent to the measure preserving dynamical system $(\tilde{\Omega}, \mathcal{B}, \mathbb{Q}, T)$.

Theorem 6. *Let $(\tilde{\Omega}, \mathcal{B}, \mathbb{Q}, T)$ be a dynamical system as defined above. The forward shift T is measure preserverving with respect to $\mathbb{Q}$ if and only if X_t is a stationary process on $(\Omega, \mathcal{F}, \mathbb{P})$.*

Proof. Assume X_t is stationary. Now for all $A \in \mathcal{B}$

$$\begin{aligned}
\mathbb{Q}(T^{-1}(A)) &= \mathbb{Q}((T^{-1}(\tilde{X}_1, \tilde{X}_2, \ldots)) \in A) \\
&= \mathbb{Q}((\tilde{X}_2, \tilde{X}_3, \ldots) \in A) \\
&= \mathbb{P}((X_2, X_3, \ldots) \in A) \\
&= \mathbb{P}((X_1, X_2, \ldots) \in A) \\
&= \mathbb{Q}(A).
\end{aligned}$$

And hence T is measure preserving with respect to $\mathbb{Q}$.
Conversely, let $(\tilde{\Omega}, \mathcal{B}, \mathbb{Q}, T)$ be a measure preserving dynamical system. Let $Y : \tilde{\Omega} \to X$. Then the stochastic process defined as $\{Y \circ T^t : t \in \mathbb{N}\}$ is stationary. Indeed, for every k and $t, t_1, \ldots, t_k \in \mathbb{N}$ and cylinder $A \in \mathcal{X}^{\otimes k}$ we have

$$\begin{aligned}\mathbb{Q}((Y \circ T^{t_1+t}, \ldots, Y \circ T^{t_k+t}) \in A) &= \mathbb{Q}(((Y \circ T^{t_1}, \ldots, Y \circ T^{t_k}) \circ T^t) \in A) \\ &= \mathbb{Q}((Y \circ T^{t_1}, \ldots, Y \circ T^{t_k}) \in A).\end{aligned}$$

Hence, this process is stationary. □

We are now ready to give the definition of an ergodic process. A process is ergodic if

Definition 3 (Ergodicity)**.** *A measure preserving dynamical system* $(\tilde{\Omega}, \mathcal{B}, \mathbb{Q}, T)$ *is ergodic if and only if for two events* $A,B \in \mathcal{X}$

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \mathbb{Q}(T^k A \cap B) = \mathbb{Q}(A)\mathbb{Q}(B). \tag{3.31}$$

A stochastic process $\{X_t, t \in \mathbb{N}\}$ *is called ergodic if the corresponding dynamical system* $(\tilde{\Omega}, \mathcal{B}, \mathbb{Q}, T)$ *is ergodic.*

Heuristically, a process is called ergodic when after averaging, it is assymptotically independent. This is of course a much weaker assumption to make than independence. It is not likely that the default probability of tomorrow is independent of the default probability today. It is much more likely that, after averaging, the default probability far in the future is independent of today's default probability. Now, an important theorem is stated that will be used in the second part of this section

Theorem 7. *Let* $\{\mathbf{X}_t, t \in \mathbb{N}\}$ *on* $(\Omega, \mathcal{F}, \mathbb{P})$ *be an stationary and ergodic process with* $\mathbb{E}[\mathbf{X}_t] = \mu < \infty$*. Then*

$$\frac{1}{T} \sum_{t=1}^{T} \mathbf{X}_t \xrightarrow{a.s.} \mu. \tag{3.32}$$

Proof. As shown before, this stochastic process is equivalent to the dynamical system $(\tilde{\Omega}, \mathcal{B}, \mathbb{Q}, T)$, where T is the shift operator. By Theorem 6, stationarity of $\{X_t, t \in \mathbb{N}\}$ implies T is measure preserving with respect to $\mathbb{Q}$. Furthermore, as T is assumed to be ergodic with respect to $\mathbb{Q}$, The Ergodic Theorem (Birkhoff (1931)) now tells us that for the following holds almost surely:

$$\lim_{k\to\infty} \frac{1}{k} \sum_{t=0}^{t=k-1} \tilde{X} \circ T^t = \int_{\tilde{\Omega}} \tilde{X} d\mathbb{Q} = \mu. \tag{3.33}$$

Hence, for the stochastic process $\{X_t, t \in \mathbb{N}\}$, Equation (3.32) holds. □

Now, we are ready to state the assumptions under which the OLS time estimator is consistent and prove this consistency of the OLS estimator. The proof is similar to other proofs of consistency of the OLS estimator, only where in those proofs the law of large numbers is applied, the ergodic theorem is applied in our proof.

Theorem 8. *Let* $Y_t = \mathbf{X}_t\boldsymbol{\beta} + \epsilon_t$, $t = 1 \dots T$, *where* $\mathbf{X}_t$ *is a K-dimensional vector of regressors,* $\boldsymbol{\beta}$ *is a vector of coefficients and* ϵ_t *is an unobservable error term. Then the OLS estimator is consistent under the following assumptions.*

1. *The* K *dymensional process* $\{\mathbf{X}_t, t \in \mathbb{N}\}$ *is stationary and ergodic.*
2. $\mathbb{E}(X_{tk}\epsilon_t) = 0$ *for all* i *and* $k = 1, 2, \dots K$.
3. *The matrix* $\boldsymbol{\Sigma}_{\boldsymbol{XX}} \coloneqq \mathbb{E}[\mathbf{X}_t\mathbf{X}_t']$ *is nonsingular.*

This assumption of ergodicity is much weaker than the usual textbook assumption that $\mathbf{X}_t$ is an independent series of random variables. Note that the second assumption is actually already present in our financial model, see Assumption 3.6c. The proof of the previous theorem is very simple. It just follows standard proofs of consistency of the OLS estimator (see for example Wooldridge (2013)), but in stead of The Law of Large Numbers, The Ergodic Theorem is used.

Proof. Denote $\mathbf{Y} = [Y_1, \dots, Y_T]'$, $\mathbf{X} = [\mathbf{1}, [\mathbf{X}_1, \dots \mathbf{X}_T]']$ and $\boldsymbol{\epsilon} = [\epsilon_1, \dots, \epsilon_T]'$. It is widely known that the OLS estimator is given by $\hat{\boldsymbol{\beta}} = (\mathbf{XX})^{-1}\mathbf{X}'\mathbf{Y}$ and thus we obtain that

$$\begin{aligned}\hat{\boldsymbol{\beta}} - \boldsymbol{\beta} &= (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'(\mathbf{X}\boldsymbol{\beta} + \epsilon) - \boldsymbol{\beta} \\ &= (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\boldsymbol{\epsilon} \\ &= (\frac{1}{T}\sum_{t=1}^{T}\mathbf{X}_t\mathbf{X}_t')^{-1}(\frac{1}{T}\sum_{t=1}^{T}\mathbf{X}_t\epsilon_t).\end{aligned}$$

As we assumed $\mathbf{X}_t$ to be ergodic stationary, $\mathbf{X}_t\mathbf{X}_t$ is ergodic stationary too. Therefore by the Ergodic Theorem $(\frac{1}{T}\sum_{t=1}^{T}\mathbf{X}_t\mathbf{X}_t') \xrightarrow{\text{a.s.}} \boldsymbol{\Sigma}_{XX}$. This matrix is nonsingular by assumption, so it follows that $(\frac{1}{T}\sum_{t=1}^{T}\mathbf{X}_t\mathbf{X}_t')^{-1} \xrightarrow{\text{a.s.}} \boldsymbol{\Sigma}_{\mathbf{XX}}^{-1}$ by the continuous mapping theorem. Similarly $\frac{1}{T}\sum_{t=1}^{T}\mathbf{X}_t\epsilon_t \xrightarrow{\text{a.s.}} \mathbb{E}(\mathbf{X}_t\epsilon_t) = 0$ by assumption. Hence $\hat{\boldsymbol{\beta}} \xrightarrow{\text{a.s.}} \boldsymbol{\beta}$. □

In order to perform T-tests, we need more than consistency. It is also necessary that $\hat{\boldsymbol{\beta}}$ is asymptotically normal. In order to prove asymptotic normality, we use the ergodic stationary martingale difference CLT by Billingsley (1961). He proved that if $\mathbf{g}_t$ is a martingale difference sequence that is stationary and ergodic with $\mathbb{E}(\mathbf{g}_t\mathbf{g}_t^{'}) = \mathbf{S}$, then

$$\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\mathbf{g}_t \xrightarrow{d} N(0, \mathbf{S}). \tag{3.34}$$

Proving asymptotic normality of the estimator is straightforward using this result. First, define $\mathbf{g}_t \equiv \mathbf{X}_t\epsilon_t$.

Theorem 9. *Assume that*

1. *The assumptions of Theorem 8 hold.*
2. $\mathbf{g}_t$ *is a martingale difference sequence and the matrix* $\mathbb{E}(\mathbf{g}_t\mathbf{g}_t)$ *is non sigular.*

Then when $T \to \infty$ *we have that*

$$\sqrt{T}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}) \xrightarrow{d} N(\mathbf{0}, \boldsymbol{\Sigma}_{\mathbf{XX}}^{-1}\mathbf{S}\boldsymbol{\Sigma}_{\mathbf{XX}}^{-1}), \tag{3.35}$$

where $\mathbf{S} = \mathbb{E}(\mathbf{g}_t\mathbf{g}_t^{'})$.

Proof. Let $\bar{\mathbf{g}} \equiv \frac{1}{T}\sum_{t=1}^{T}\mathbf{g}_t$. In the proof of Theorem 8 it was derived that $\hat{\boldsymbol{\beta}} - \boldsymbol{\beta} = (\frac{1}{T}\sum_{t=1}^{T}\mathbf{X}_t\mathbf{X}_t')^{-1}\bar{\mathbf{g}}$. Multiplying both sides with $\sqrt{T}$ gives us

$$\sqrt{T}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}) = (\frac{1}{T}\sum_{t=1}^{T}\mathbf{X}_t\mathbf{X}_t')^{-1}(\sqrt{T}\bar{\mathbf{g}}). \tag{3.36}$$

As $\mathbf{g}_t$ is asummed to be a martingale difference series, we have that $\sqrt{T}\bar{\mathbf{g}} \xrightarrow{d} N(\mathbf{0}, \mathbf{S})$ by the ergodic stationary martingale difference CLT. Therefore, by Slutzky's theorem we have that $\sqrt{T}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}) \xrightarrow{d} N(\mathbf{0}, \boldsymbol{\Sigma}_{\mathbf{XX}}^{-1}\mathbf{S}(\boldsymbol{\Sigma}_{\mathbf{XX}}^{-1})')$. As $\boldsymbol{\Sigma}_{\mathbf{XX}}$ is symmetric, this is equal to Equation (3.35). □

Hence, the estimotor has a normal distribution in the limit. This defends the use of T-tests to test whether a certain coefficient is statistically different from zero.

3.5 Asymptotics

In this chapter, our model was mathematically derived. In the derivation of the mathematical model, one assumption was that $R_{p,t}$ is the return of a well diversified portfolio in a large market. To be precise, it was necessary that $N^* \to \infty$, where N^* are the number of assets in the portfolio. Of course, this also means that the number of assets in our economy needs to tend to infinity; $N \to \infty$. Furthermore, it was shown that the parameters of this model can be consistently estimated using ordinary least squares. To consistently estimate the model, it was necessary that $T \to \infty$. The goal of this section is to shortly discuss these assumptions.

The assumption that $N^* \to \infty$ was made, such that $\mathbb{E}_t(M_{t,t+1}\epsilon_{p,t+1}) \to 0$. The intuition why this holds is in line with our earlier discussion. The term $\mathbb{E}_t(M_{t,t+1}\epsilon_{p,t+1})$ is the price of ideosyncratic risk. In a large portfolio, this type of risk can be diversified away; if an investor holds enough assets, negative shocks to one asset are offset by positive shocks to other assets. Therefore, this type of risk should not be priced. Corporate bond funds normally use very large portfolios of corporate bonds and therefore, this assumption is likely to hold in our case (see also Fama and French (1993)).

The assumption that $T \to \infty$ is not necessary for our model to hold. If T is finite, the expected return on a portfolio is still given by Equation (3.24) as long as the assumptions of Theorem 4 are satisfied. The goal of this thesis is to estimate the alpha of corporate bond funds and this must be estimated from real life data. Here, the data consists of the return of a portfolio of corporate bonds over a period of time. The number of parameters we want to estimate in the model is 3 (α_s, $\beta_{s,TERM}$ and $\beta_{s,DEF}$). Hence the minimum number of observations to theoretically estimate the model is 4. However, using such a small sample might lead to imprecise estimates. The condition that $T \to \infty$ ensures that we have enough data to consistently estimate the model.

There is not an exact answer to how big T exactly should be, but the bigger the dataset, the more precise the estimate. However, in reality, sample sizes are rarely determined by scientific goals; many datasets are still limited in size. Also datasets of fund returns are limited in size. There are many reasons for this; some funds are relatively new and therefore do not have such a long return history, other funds do not report their results and some funds shut down. In the fund performance literature, one normally uses a minimum of 2 years of monthly data (see for example Huij and Derwall (2008)). Also in this thesis, a minimum sample size of 24 months is used to estimate the alpha of individual funds.

Chapter 4

Data

In this study, performance of bonds is measured by the α_s in Equation (3.28). However, to actually run this regression, we must obtain the variables $R_{s,t}$, R^f, $TERM_t$ and DEF_t from actual data. This data is obtained from various databases. In this section, the data we use is described.

This study is the first study on fund performance that fully utilizes the two largest mutual fund databases; the CRSP mutual fund database and the Morningstar database. Both databases report monthly gross total returns of a large number of share classes in the United States and Europe.[1] Besides the monthly returns, also data on fund size and fund expenses are included in these databases. Brown et al. (1992) warns that many datasets suffer from a survivorship-bias. This is a bias that arises when datasets do not include defunct funds and therefore average performance measures are too high (the reason for many funds to go defunct is because they are not successfull). However, as our datasets also include defunct funds, our data is without survivorship-bias. We use return data over the period 1999 to 2014.

All shareclasses that CRSP and morningstar classify as corporate bond mutual funds are selected and combined into one dataset. Double entries arising from this merging of dataset are of course removed. Afterwards, we take the size weighted average over the shareclasses belonging to one fund in order to get the fund return. In total, we are left with 2156 funds over the period 1999-2014. Afterwards, funds with less than 24 observations are deleted. Also, funds with an R^2 lower that 0.6 in the time series regression 3.26 are removed from the dataset. This is because corporate bond returns are highly influenced by these two factors. If the R^2 of a certain fund is low, it is very likely that this fund is investing in other securities than corporate bonds and therefore wrongly classified as corporate bond fund. Excluding funds with a low R^2 is common in

[1] Many funds offer different kind of shares, known as share classes. Each shareclass invests in the same securities. But each class has different kind of distribution arrangement and fees. Investors that invest a big sum of money might for example pay lower management fees. By offering different shareclasses of the same fund an investor can choose the fee and structure that fits their goals best.

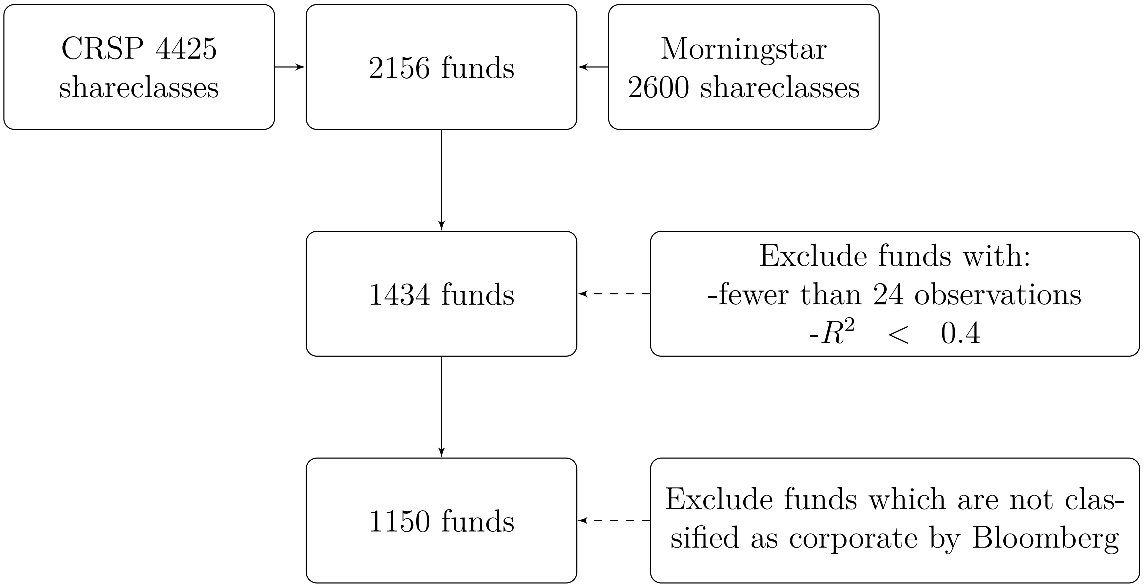

Figure 1: Overview of the data selection process.

the academic literature (see for example Huij and Derwall (2008)). However, we found that, even after excluding funds with a low R^2, still many funds were wrongly classified as corporate bond fund. To battle this issue, we further filtered the dataset by linking our data to the Bloomberg database. Those funds that are not classified as Corporate by Bloomberg are removed from the sample. In the end, we are left with 1150 corporate bond funds. A scemetic overview of the data selection process can be found in Figure 1.

Our dataset consists of funds that invest in corporate bonds traded in the United Stated (US funds) and funds that invest in corporate bonds traded in Europe (EU funds). It is common in the academic liteture to analyze US funds and EU funds seperately because they operate on different markets and therefore their risk factors are also different. We also make a distinction between Investment Grade funds (IG) and High Yield funds (HY). High Yield funds are funds that mainly invest in high risk bonds wheras Investment Grade funds invest in bonds that are considered to be somewhat safer. It is a market convention to treat these market segments as two different asset classes. This can be seen from the availability of market indexes, which normally cover either Investment Grade or High Yield. Also in the academic literature this segmentation is made. Evidence that High Yield funds and Investment Grade funds are indeed different asset classes can be found in Chen et al. (2014). Therefore, as is normally done in academic literature, we perform our analysis for four different subsets; European Investment Grade funds (EUIG), European High Yield funds (EUHY), American Investment Grade funds (USIG) and American High Yield funds (USHY).

As explained in the previous section, the $TERM$ factor is usually calculated as the return on a long term government bond over a monthly government bond. In this thesis, the $TERM$ is calculated as the return over the U.S. Barclays 7-10 year index minus the one month treasury bill return. This return is obtained from the website of Fama-French.[2] The DEF factor is calculated as the difference between a portfolio of long term corporate bonds minus the long term government bond return. As the risk free rate we take the one month treasury bill rate from Ibbotson Associates, again obtained from the Fama French website.

Our dataset contains data of 1150 corporate bond funds, of which 372 are EUIG funds, 158 are EUHY funds, 248 are USIG funds and 372 are USHY funds. Figure 2 plots the sample size over time of our different subsamples. It can be seen that in 1999, our sample size is not that large yet, this is especially the case for EUIG and EUHY funds. But along with the rising popularity of corporate bond funds, our sample size grows over time. At the end of 2013, every subsample contains a decent amount of funds.

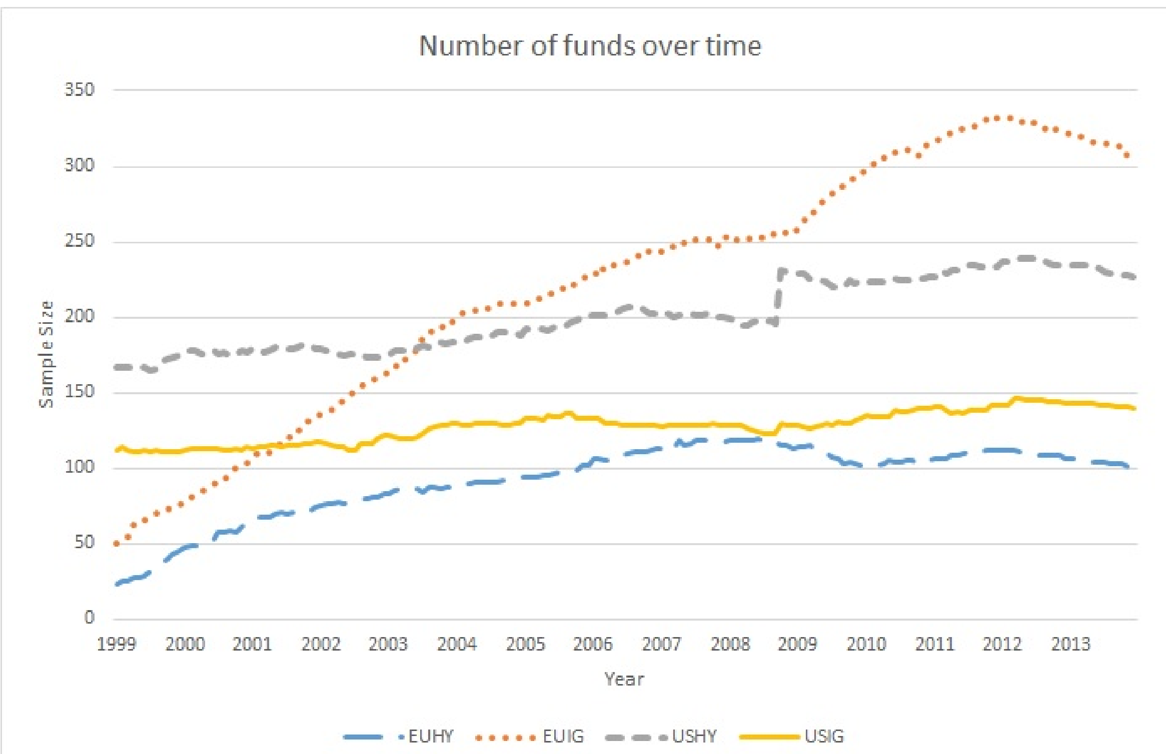

Figure 2: Sample Size over time

[2]http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

Chapter 5

Performance of Corporate Bond Funds

The goal of this chapter is to answer the first research question: How do corporate bond funds perform on average? As explained in the previous sections, performance of funds will be measured by the alpha of a fund. Therefore, we investigate if corporate bond funds can generate a positive alpha on average. If this is the case, the average corporate bond fund obtained a return in excess of the reward for the risk it took and this then suggest corporate bond funds are a valuable investment instruments. Furthermore, it is also investigated how many percent of the corporate bond funds managed to generate a significantly positive alpha.

In order to tell something about the average performance of corporate bond funds, the average alpha in each of our four universes is calculated. Hence, we run Regression (3.26) for each fund s, to obtain $\hat{\alpha}_s$ for every s. Then, if a universe contains S funds, the average alpha is simply given by $\hat{\alpha} = \sum_{s=1}^{S} \hat{\alpha}_s$. Table 2 reports the average annualized alpha, excess return and sharpe ratio for each of our four subsamples.

	Excess Return	Sharpe Ratio	$\hat{\alpha}$
EUIG	2.07	0.67	0.16%
EUHY	3.72	0.39	-0.07%
USIG	3.61	0.70	0.58%
USHY	4.77	0.65	-0.06%

Table 2: Average performance measures for each subsample are shown. Averages are taken over the funds. Sharpe ratio, alpha and return are all annualized.

Table 2 shows that the average excess return for each subsample is positive. The average alpha in both investment grade universes are positive, whereas the average alpha in both high yield universes are negative. Therefore, the average investment grade fund obtained a reward in excess of the risk they took, whereas the average high yield fund obtained a return that is too small for the risk they took.

The average alpha tells us about the average performance of of corporate bond funds. However, it does not tell the whole story. Even if the average alpha is negative for the high yield universes, it is still possible that within these universes there are many funds that obtained a positive alpha. The ratio of funds that obtained a positive alpha is shown in Table 3. This table also shows the ratio of alphas that are significant at the 95% confidence level. In other words, these are the funds for which the p-value of the α was smaller than 0.05.

	$\% \ \alpha > 0$	$\% \ p_\alpha < 0.05$
EUIG	0.72	0.56
EUHY	0.64	0.56
USIG	0.76	0.58
USHY	0.62	0.44

Table 3: The fraction of funds that obtained a positive alpha over the sample period is reported in the first column. The last column reports the fraction of funds that obtained a significantly positive alpha

From this table it can be seen that in every universe, a large percentage of the funds managed to obtain a positive alpha. This result is especially interesting in the light of next chapter. In the next chapter we show that performance of corporate bond funds persists. In other words; funds that did well in the past, continue to do well in the future.

The above analysis was concerned with the performance of funds over the whole sample period. Now, the average performance for each year is investigated seperately. To do this, we first estimate alpha over each year seperately for each fund. Afterwards, the cross-section average is taken to obtain the average alpha for each of the 12 years in our dataset. More precisely, every december, Equation (3.26) is estimated using the data of that year. Therefore, for $t_k = \{tk | k = 1, \ldots, 12\}$, we run the following regression

$$R_{s,t} - r^f = \alpha_{s,k} + \beta_{s,TERM} TERM_t + \beta_{s,DEF} DEF_t + \epsilon_{s,t} \qquad t = t_k - 11, \ldots, t_k. \quad (5.1)$$

for each fund $s = 1, \ldots S$. Where $\alpha_{s,k}$ is the alpha of fund i over year k. Then the average alpha over year k is given by $\hat{\alpha}_k = \sum_{s=1}^{N} \hat{\alpha}_{s,k}$. In Figure 3 the average yearly alpha is plotted for the EUIG and EUHY funds. Figure 4 shows the yearly alpha for the USIG and USHY universes.

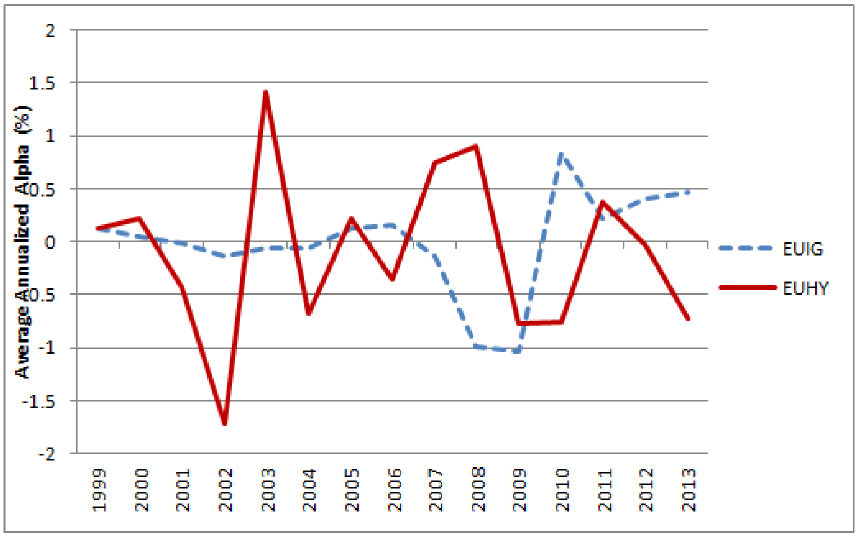

Figure 3: Annualized Alpha per year for the EU universe

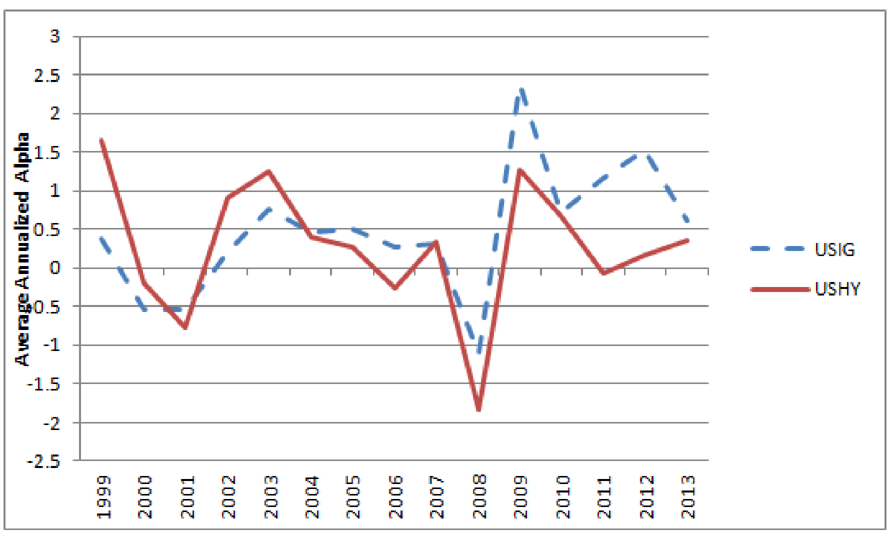

Figure 4: Annualized Alpha per year for the US universe

It can be seen that on average, the investment grade funds performed better than the high yield funds in most of the years, which is in line with our earlier results. Furthermore, it can be seen that US funds performed poorest in 2008, during the financial crisis. The EU funds performed bad in 2009. Investment grade funds performed the best after the 2008 financial crisis, wheras high yield funds performed well in 2003.

This chapter investigated the performance of corporate bond funds. New insights on the performance of corporate bond funds were found. First of all, the average alpha of investment grade funds is positive for both EU and US funds. A negative average alpha was found for the USHY and EUHY universes. However, in all universes, a large fraction of the funds managed to obtain an alpha significantly greater than 0. This means that all our subsamples include funds that on obtained a positive risk adjusted return.

Chapter 6

Persistency of Corporate Bond Funds

In this section the persistency of bond funds is investigated. Simply put, are past winners also future winners? This question is important for several reasons. First of all, this is the most important method in literature to investigate whether fund managers possess superior bond picking skill. However, untill now this is mostly done for stock mutual funds, not for corporate bond funds. From an investor's point of view this question is also interesting; if persistency in bond fund performance is found, an investment strategy based on buying past winners and selling past losers can be a profitable strategy.

The structure of this chapter is as follows. In Section 6.1 the statistical tests we use to test for persistency are described. In Section 6.2 the empirical results of these tests are discussed.

6.1 Persistency Tests

In this study, three statistical methods to test for persistency are used; a Fama MacBeth regression, a Chisquare test and a method that ranks funds according to past performance. All these methods are famous statistical methods that are frequently used in fund performance studies. However, there is no consensus which of these is the best method for testing performance persistence. In all the methods an evaluation period and a post-evaluation period are used. We basically ask the question if the performance in the evaluation period can predict the performance in the post-evaluation period, i.e. if past performance can predict future performance.

The first method we use to test for persistency in performance is a Fama MacBeth regression. This method was first suggested by Fama and MacBeth (1973). First, for each fund s, a measure of recent performance is calculated at every time period t. As a measure of recent performance of fund s at time t, the alpha over the last 24 months is used. This 24-month alpha at time t is denoted by $\alpha_{s,t}^{24}$. Afterwards, it is investigated if $\alpha_{s,t}^{24}$ is correlated with $\alpha_{s,t+1}^{24}$, i.e. if good (bad) past performance in correlated with good (bad) future performance.

To be more precise, at each point of time $t = 24, \ldots, T$, the following regression equation is estimated by OLS:

$$R_{s,u} - r^f = \alpha_{s,t}^{24} + \beta_{s,TERM} TERM_u + \beta_{s,DEF} DEF_u + \epsilon_{s,u}, \qquad u = t-24, \ldots, t \quad (6.1)$$

for each fund $s = 1, \ldots, S$. The OLS estimate of $\alpha_{s,t}^{24}$ is denoted by $\hat{\alpha}_{s,t}^{24}$. Now we are interested in estimating the β_1 in the following regression

$$\hat{\alpha}_{s,t}^{24} = \beta_0 + \beta_1 \hat{\alpha}_{s,t-1}^{24} + \epsilon_{s,t} \quad (6.2)$$

for $s = 1, \ldots, S$ and $t = 24, \ldots, T$. Note that, as is common in panal data estimation, the β_1 is the same for each fund s. A positive β_1 indicates that a high alpha in the previous period generally lead to a high alpha in the current period and that low alphas in the previous period are likely to be followed by a low alpha in the current period. Therefore, if the performance of bond fund managers is indeed persistent, we expect a significantly positive β_1.

Fama and MacBeth (1973) suggest a two step procedure to estimate this β_1. First, Regression (6.2) is estimated at each $t \geq 24$. Hence, $T - 23$ cross-sectional regressions are estimated to obtain β_t for every $t \geq 24$ and then the average of these estimates is taken as an estimator of the β_1 in Regression 6.2. Therefore, the Fama MacBeth estimator, $\hat{\beta}_{FM}$ is given by

$$\hat{\beta}_{FM} = \frac{1}{T} \sum_{t=24}^{T} \hat{\beta}_t = \frac{1}{T} \sum_{t=24}^{T} (\mathbf{X}_t' \mathbf{X}_t)^{-1} (\mathbf{X}_t' \mathbf{Y}_t), \quad (6.3)$$

where $\mathbf{Y_t} = [\hat{\alpha}_{s,t}, \ldots, \hat{\alpha}_{S,t}]'$ and $\mathbf{X_t} = [\hat{\alpha}_{s,t-1}, \ldots, \hat{\alpha}_{S,t-1}]'$. Fama and MacBeth (1973) prove that this estimator is consistent and asymptotically normal under mild regularity conditions. Hence, a simple T-test can be used to statistically test if $\beta_1 > 0$.

The second method used to test for persistency are contingency tables. Based on their alpha relative to the median, funds are assigned to one of the following cells in a two-by-two contingency table: past winner/future winner , past winner/future loser, past loser/future winner, past loser/future loser. A fund is a past (future) winner if its alpha in the preceding (subsequent) year is above the median. A fund is a past (future) loser if its alpha in the preceding (subsequent) year is below the median.
Let p_{WW}, p_{WL}, p_{LW}, p_{LL} denote the probability of each cell, $N_{WW}, N_{WL}, N_{LW}, N_{LL}$ denote the number observations in each cell and N the total number of observations. We are interested in testing the following hypotheses:

$$H_0 : p_{WW} = p_{WL} = p_{LW} = p_{LL} = 0.25 \qquad vs. \qquad H_1 : H_0 \text{ is not true.}$$

It can be proven (see for example section 2.5 of van der Vaart (1995)) that under the H_0

$$C^2 = \frac{(N_{WW} - 0.25N)^2 + (N_{WL} - 0.25N)^2 + (N_{LW} - 0.25N)^2 + (N_{LL} - 0.25N)^2}{0.25N} \tag{6.4}$$

has a χ^2 distribution with 3 degrees of freedom. The null hypothesis is rejected for high values of C^2. Therefore, high values of the test statistic indicates persistence in performance of corporate bond funds.

Our third method was first described by Hendricksson and Merton (1981). It has become the most widely used method to check for performance persistence. We form mutually exclusive portfolios based on past performance and evaluate the return of these portfolios over an evaluation period. Every month, we seperate our 4 universes of funds into quintiles, using the fund's alpha over the last 24 months.
Hence, at every time period $t \geq 24$, $\hat{\alpha}^{24}_{s,t}$ is calculated by estimating Equation (6.1) for each fund s. Afterwards, the funds are ranked based on their $\hat{\alpha}^{24}_{s,t}$ and divided into five equal groups. Therefore, the top quintile consists of the 20 percent top performing funds over the last 24 months. All the portfolios are equally weighted and if a fund disappears the weights are adjusted. After one month, at time $t+1$, the portfolios are sold and new quintile portfolios are constructed based on $\hat{\alpha}^{24}_{s,t+1}$, the 24-month alpha at time $t+1$. This is repeated until time $T-1$. The portfolios formed at time $T-1$ are sold at time T.
In this way, 5 dynamic portfolios are obtained, which are each updated every month. These 5 portfolios can then be evaluated by calculating their average return, sharpe ratio and alpha. If corporate bond funds indeed display persistence in performance, then the alpha, return and sharpe ratio of the quintile portfolios should be decreasing over the quintile portfolios.

6.2 Empirical Results

In this section, the results of the empirical tests described in the previous section are reported. Table 4 reports the Fama MacBeth estimators for all universes. It shows the relation between past and future alphas as in Equation (6.2). The corresponding t-values are reported in brackets. The slope coefficient of the full sample is 0.19 with a t-statistic of 2.99 and is therefore statistically significant at the 1% level. This indicates that there exists a positive association between past and future performance of corporate bond funds.
The analysis is also performed for different subsets of corporate bond funds. We see that the observed results are the same. For every universe, a positive slope coefficent is found. Moreover, all these coefficients are statistically significant at the 1% level. Hence, our Fama MacBeth regressions indicate that there exists a strong positive correlation between past and future performance.

	$\hat{\alpha}$	$\hat{\beta}_{FM}$	R^2
All funds	1.12	0.19***	0.09
	(0.20)	(2.99)	
EUIG	1.02	0.28***	0.07
	(0.40)	(3.23)	
EUHY	0.50	0.17***	0.03
	(0.08)	(3.40)	
USIG	2.84	0.23***	0.20
	(0.96)	(2.63)	
USHY	0.71	0.21***	0.07
	(0.18)	(3.67)	

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table 4: Fama MacBeth regressions

Table 5 presents the contingency tables for the complete sample and the four different subsamples considered in this study. Columns 2 to 5 represent the fraction of the observations in a certain subsample that is assigned to the corresponding cell. For example, 28% of the overall funds are in the LL cell. Furthermore, the C^2 test statistic (calculated as in Equation (6.4)) and its P-value are reported.

The null hypothesis, which as explained in Section 6.1 indicates no relation between past and future performance, is strongly rejected for the full sample and for each subsample. Our results thus provide clear evidence of a relation between past and future performance. From Table 5 it can be seen that a significant portion of funds that are classified as losers (winners) in the past are also classified as losers (winners) in the future.

Also when contingency tables are used, strong evidence is found in favor of persistence in performance of corporate bond funds.

	LL	**LW**	**WL**	**WW**	C^2	**P-value**
All Funds	0.28	0.22	0.22	0.28	8391.00***	0.000
EUIG	0.28	0.22	0.22	0.29	38.48***	0.000
EUHY	0.29	0.22	0.22	0.28	16.93***	0.001
USIG	0.28	0.22	0.21	0.29	43.58***	0.000
USHY	0.28	0.22	0.23	0.28	36.54***	0.000

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table 5: Contingency tables

Our final analysis concerns sorting funds into quintile portfolios based on their alpha over the last 2 years. In this way, 5 portfolios are created which are updated each month as described in Section 6.1. Table 6 presents the post-ranking performance of the five portfolios for each of our subsets. It displayes the yearly return and the annualized Sharpe ratio. Furthermore, the yearly alphas and T-statistics are shown (these are obtained by running regression Equation (3.26) on the returns of the quintile portfolios). For completeness, β_{TERM} and β_{DEF} are also displayed.

One important observation is that we find positive alphas for about half of the portfolios. Investment Grade funds do particularly well; in the USIG subsample, 4 out of 5 portfolios earn positive and statistically significant alphas. The USIG quintile portfolios perform worst, 4 out of 5 quintile portfolios earn negative alphas. However, only one of these alphas is significantly smaller than 0 at the 10% level. This further strengthens our finding that many corporate bond funds are earning a positive risk adjusted return. This is consistent with our findings in Chapter 5, where we found that over 60% of the funds earn a positive alpha in each of our 4 universes.

Another striking observation is that the out of sample performance of the quintile portfolios decreases almost monotonically over the quintiles. No matter if this performance is measured by yearly return, annualized Sharpe Ratio or yearly alpha, the top quintile portfolio performs better than the bottom portfolio and the decrease is surprisingly monotonical for each of the samples. This is very strong evidence in favor of persistence in performance of corporate bond funds. Winners of the past continue to perform well in the future and past losers continue to underperform in the future.

There is no clear pattern in factor loadings across the quintile portfolios. It seems however that for successful funds, the exposure to the TERM factor is lower. An exception is the top portfolio in the USHY universe. For the EUHY universe, better performing funds have less exposure to both the TERM factor and the DEF factor, but the top quintile portfolios in other universes load quite strongly on the DEF factor. In short, the results reported in Table 6 strongly indicate persistence in performance. Although the sensitivities to the factors vary from one quintile to another, the general pattern is that for more successfull funds, the exposure to the TERM factor is lower.

We experimented with different portfolio formation periods and holding periods. Furthermore, we also made portfolios by sorting on return and Sharpe ratio. The results for these different settings remain similar; an almost monotonic decrease in performance is found over the quintile portfolios. To save space, these results are not reported here, but these can be requested if one is interested.
In this section, our second research question was answered. We used three statistical methods to analyze persistency of corporate bond funds and no matter which method was used, strong evidence of persistence in performance was found.

		Excess Return	Sharpe Ratio	$\hat{\alpha}$	T($\hat{\alpha}$)	$\hat{\beta}_{TERM}$	$\hat{\beta}_{DEF}$
EUIG	D1	2.67%	0.77	0.76%	2.87	0.74	1.09
	D2	2.54%	0.70	0.17%	1.08	0.84	1.00
	D3	2.69%	0.74	0.35%	2.08	0.83	1.00
	D4	2.46%	0.63	0.01%	0.04	0.86	1.06
	D5	1.97%	0.37	-0.58%	-0.98	0.81	1.47
EUHY	D1	5.28%	0.55	1.03%	1.12	0.43	0.69
	D2	4.49%	0.44	0.16%	0.14	0.41	0.72
	D3	4.01%	0.40	-0.24%	-0.30	0.37	0.72
	D4	4.41%	0.41	-0.50%	-0.77	0.56	0.78
	D5	3.68%	0.32	-1.73%	-2.37	0.66	0.85
USIG	D1	4.23%	0.83	1.44%	2.74	0.75	1.04
	D2	3.75%	0.77	0.97%	3.55	0.80	0.77
	D3	3.50%	0.76	0.64%	3.00	0.82	0.81
	D4	3.42%	0.76	0.62%	3.01	0.87	0.81
	D5	3.23%	0.62	0.08%	0.26	0.90	0.91
USHY	D1	4.14%	0.82	0.09%	0.19	1.19	0.44
	D2	3.60%	0.83	-0.01%	-0.02	1.13	0.33
	D3	3.77%	0.82	-0.01%	-0.03	1.18	0.34
	D4	3.79%	0.77	-0.11%	-0.17	1.20	0.36
	D5	3.35%	0.61	-1.00%	-1.45	1.30	0.45

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table 6: Running portfolios, alpha's and returns are reported in percentage per year. Sharpe ratio is also annualized.

Chapter 7

Investment Strategies

In Chapter 5 and Chapter 6, the performance of corporate bond funds was analyzed. It was found that many corporate bond funds are able to earn a positive risk adjusted return. Furthermore, it was found that this performance is persistent. Past recent performance of fund s at time t is again measured by $\hat{\alpha}_{s,t}^{24}$, as given by the constant term in Regression (6.1). The previous chapter suggests that it might be a good strategy to invest in bond funds with a high $\hat{\alpha}_{s,t}^{24}$ at time t. In this chapter the performance of four of such strategies are investigated.

The first strategy we investigate is investing in the fund with the highest $\hat{\alpha}_{s,t}^{24}$ at time t. As performance is shown to persist, we expect that this fund continues to do well in the future. Investing in a single fund however brings along some risks, even if the past performance of this fund is good. The return of this investment depends on the choices of a single portfolio manager. This manager is only human and might make mistakes. A fund can also change manager. This new manager might not have such a good track record.
Hence, investment strategies which consists of holding a portfolio of funds have become increasingly popular. Therefore, this chapter also suggests three investment strategies that invest in a portfolio of funds, rather than in a single fund. The first strategy is one that goes long in the top quintile portfolio and goes short in the bottom quintile portfolio. The second and third strategy both invest in funds with a significantly positive alpha over the last 24 months.

7.1 Strategy 1: Highest Alpha Fund

In Chapter 6, strong evidence of persistence of corporate bond funds was found. Funds with a high alpha over the last two year, continued to do well over the next month. If an investor faces the task of selecting one corporate bond fund based on its past track record, a logical choice would therefore be to select the fund with the highest alpha over the last two years. The investment strategy in this section selects the fund with the highest $\hat{\alpha}^{24}_{s,t}$ for each t, $t \geq 24$ (to estimate this alpha, a minimum of 24 past observations are necessary). We invest $I_t > 0$ in this fund and hold this fund for one month. Denote the return of this highest alpha strategy over month t as R^{HA}_t. At time $t+1$, the fund is sold for $I_t R^{HA}_t$ and an amount of $I_{t+1} > 0$ is invested in the the fund with the highest 24-month alpha at time $t+1$. We continue doing so untill time $T-1$. To be precise, this investment strategy consists of the following steps for each t, $t = 24, \ldots, T-1$:

- Calculate $\hat{\alpha}^{24}_{s,t}$ for each fund $s = 1, \ldots, S$.
- Invest $I_t > 0$ in the fund with the highest $\hat{\alpha}^{24}_{s,t}$.
- Sell this fund at time $t+1$ (for a price of $I_t R^{HA}_t$).

Notice that for this strategy, an investor needs to invest an amount I_t at every t. Some investors might prefer a self-financing strategy (a strategy that needs no infusion or withdrawal of money after the initial investment). To do so, one can just put $I_t = \prod_{u=1}^{t} I_{24} R^{HA}_u$. However, as our performance measures are not defined in absolute terms, the exact value of I_t does not influence the result (as long as $I_t > 0$).

	Excess Return	Std. Dev.	Sharpe Ratio	$\hat{\alpha}$
EUIG	3.54%	2.34%	0.44	1.77%
EUHY	4.46%	3.31%	0.39	0.19%
USIG	5.74%	2.15%	0.77	2.93%
USHY	5.51%	2.53%	0.63	2.00%

Table 7: Average performance measures for each subsample are shown. Averages are taken over the funds. Sharpe ratio, alpha and return are all annualized.

This strategy is tested with our own data and Table 7 displays the annualized average excess return, the standard deviation of the return, annualized Sharpe ratio and the annualized alpha for this strategy. In every subsample, a positive alpha is obtained. Furthermore, if we compare the alphas of this strategy with the results in Table 6, it can be seen that, except for the EUIG universe, the α of this highest alpha portfolio surpasses the α of the top quintile portfolio.

The investment strategy of the this section picks only one fund out of the universe of corporate bond funds. However, as explained in the introduction of this chapter, this can be somewhat risky. This can also be seen from the results in Table 7; the standard deviation of the return is large, which leads to a Sharpe ratio that is lower than for example the Sharpe ratio of the top quintile portfolios in Table 6. One can reduce risk by investing in a portfolio of funds. In the remainder of this chapter, we discuss three strategies that invest in a diversified portfolio of funds.

7.2 Strategy 2: Top Minus Bottom Portfolios

In Chapter 6 funds were placed in quintile portfolios based on their 24-month alpha. It was found that the funds in the top quintile continue to outperform funds in the bottom quintile. The investment strategy proposed in this section exploits this finding by buying the funds in the top portfolio and selling the funds in the bottom portfolio.

This strategy buys an equally weighted portfolio of the funds in the top quintile and finances this by going short in an equally weighted portfolio of the funds in the bottom quintile. If the funds in the top quintile keep outperforming the funds in the bottom quintile, one can obtain a profit. Just as in the previous chapter, at each point of time t funds are divided into quintile portfolios based on their 24-month alpha at this point of time, $\hat{\alpha}^{24}_{s,t}$. We denote the number of funds in the top quintile at time t with N^T_t and the number of funds in the bottom quintile at time t with N^B_t. N^T_t and N^B_t are not necessarily equal when the total number of funds cannot be divided by 5. At every point in time $t \geq 24$, we invest $\frac{I_t}{N^T_t}$ in each fund in the top portfolio and $-\frac{I_t}{N^L_t}$ in every fund in the bottom portfolio. The total costs of this portfolio is thus zero. This portfolio is sold at time $t+1$. In summary, this investment strategy consists of the following steps for each t, $t = 24, \ldots, T-1$:

- Calculate $\hat{\alpha}^{24}_{s,t}$ for each fund $s = 1, \ldots, S$.
- Rank funds by $\hat{\alpha}^{24}_{s,t}$ and divide the funds into quintile portfolios based on this ranking.
- Invest $\frac{I_t}{N^T_t}$ in every fund in the top portfolio and $-\frac{I_t}{N^L_t}$ in every fund in the bottom portfolio.
- The portfolio is sold at time $t+1$.

This strategy is again tested on our own data. The results are found in Table 8. As can be seen from Table 8, this strategy leads to a positive alpha in all our four universes. Comparing this to the results to the results from the highest alpha strategy in Table 7, we see that the highest alpha strategy leads to better alphas, except for the EUHY universe. The reason that this strategy performs so well for EUHY funds is because there is a large difference in performance between the top and bottom portfolio, as can be seen from Table 6.

	Excess Return	Std. Dev.	Sharpe Ratio	$\hat{\alpha}$
EUIG	1.00%	0.71%	0.40	1.34%
EUHY	1.60%	0.98%	0.47	2.77%
USIG	1.01%	0.84%	0.35	1.35%
USHY	0.80%	0.65%	0.35	1.10%

Table 8: Average performance measures for each subsample are shown. Averages are taken over the funds. Sharpe ratio, alpha and return are all annualized.

Although this strategy performs very well, there is one serious disadvantage of this strategy. It is not always possible to short sell a fund and if it is possible there might be some serious transaction costs involved in doing so. These costs of course have a negative impact on the return and alpha of this strategy. The next two strategies do not have these shortcomings.

7.3 Strategy 3: Positive Alpha Portfolios

In this section, two investment strategies are proposed that select all the well-performing funds over the last 24 months. As it was shown in Chapter 6 that performance persists, it is expected that this portfolio of funds will continue to do well in the future. A fund manager of fund s is considered well-performing if it obtained a positive alpha over the last 24 months.

At every time period t, we want to select all corporate bond funds with a positive $\alpha_{s,t}^{24}$. Of course, $\alpha_{s,t}^{24}$ is not observed directly, but is estimated by $\hat{\alpha}_{s,t}^{24}$. A positive estimated alpha does not necessarily correspond to a positive actual alpha. Therefore, only the funds with a statistically significant alpha are selected. To be more precise, at every $t = 24, \ldots, T$, the following hypothesis test is performed for every fund s:

$$H_0 : \alpha_{s,t}^{24} \leq 0 \qquad vs. \qquad H_1 : \alpha_{s,t}^{24} > 0. \tag{7.1}$$

Therefore, at every time period t, one calculates the t-statistic for each fund. If the t-statistic is bigger than a critical value chosen by the investor, the fund is included in the portfolio. The critical value, which we denote by d, is chosen to limit the chance of a Type I error. Hence, at every time period t, a fund manager is classified as skilled if

$$t_{s,t} = \frac{\hat{\alpha}_{s,t}^{24}}{\hat{\sigma}_{s,t}} > d. \tag{7.2}$$

The set of funds that are classified as well-performing at time t is denoted by $\mathcal{I}_t$. Hence,

$$\mathcal{I}_t = \{s : t_{s,t} = \frac{\hat{\alpha}^{24}_{s,t}}{\hat{\sigma}_{s,t}} > d\}. \tag{7.3}$$

A famous result in statistics is that, when H_0 is true, the test statistic follows a T distribution with v degrees of freedom, where v is the number of observations in the regression minus the amount of explanatory variables. Therefore, in our case $t_{s,t}$ has a T-distribution with $24 - 3$ de degrees of freedom. For now, d is set to be $t_{21,0.90}$, the 90-th quantile of the T-distribution with 21 degrees of freedom. Hence, for each fund, there is a 10% chance of wrongly rejecting H_0. The number of funds in $\mathcal{I}_t$ is denoted by N^I_t. This leads to an investment strategy with the following steps at time $t = 24, \ldots, T$:

- Calculate $t_{s,t} = \frac{\hat{\alpha}^{24}_{s,t}}{\hat{\sigma}_{s,t}}$ for each fund s.
- Invest $\frac{I_t}{N^I_t}$ in every fund in $\mathcal{I}_t$, i.e. in every fund for which $t_{s,t} > t_{21,0.90}$.
- The portfolio is sold at time $t + 1$.

As before, this investment strategy can easily be turned into a self-financing strategy by setting $I_t = \prod_{u=1}^{t} I_{24} R^{PA}_u$, where R^{PA}_u is the return of this Positive Alpha Portfolio at time u. This strategy is applied on our data. The results can be found in Table 9.

	Excess Return	Std. Dev.	Sharpe Ratio	$\hat{\alpha}$
EUIG	2.27%	1.02%	0.76	0.48%
EUHY	5.03%	2.78%	0.52	0.89%
USIG	3.82%	1.44%	0.77	0.85%
USHY	5.12%	2.22%	0.67	1.05%

Table 9: Average performance measures for each subsample are shown. Averages are taken over the funds. Sharpe ratio, alpha and return are all annualized.

As can be seen from Table 9, this investment strategy results in a positive alpha for each subsample. However, the obtained alphas are lower than those from the Highest Alpha Portfolio and from the Top Minus Bottom Portfolio, i.e. the alphas reported in Table 7 and in Table 8.

In the remainder of this section, a method is introduced to improve upon this Positive Alpha Portfolio. This method is called the Family-Wise Error approach and is introduced by Wolf and Wunderli (2009). As noted in Chapter 2, there is a chance that unskilled managers are classified as skilled by chance. For each hypothesis test performed, this chance of a Type I error is as small as 10%. However, when performing multiple hypothesis tests, the chance of falsely rejecting one H_0 increases. Because at every time period t, $t \geq 24$, testing problem (7.1) is performed for each fund s, there is a big probability that some unskilled managers are wrongly classified as skilled. Our EUIG universe contains for example over 300 funds in 2013. Therefore, the chance that at least one manager is incorrectly classified as skilled in 2013 in this universe is almost 1:

$$1 - P(\text{no type one error}) = 1 - (0.9)^{300} \approx 1.$$

Let F denote the number of tests for which the H_0 is wrongly rejected. The familywise error rate (FWE) is defined as the probability of wrongly rejecting at least one H_0, i.e. the probability of classifying one or more unskilled fund manager as skilled:

$$\text{FWE} \equiv P(F > 0) = P(\text{Reject at least one } H_0 \text{ while } H_0 \text{ is true}). \tag{7.4}$$

The goal of the FWE-approach is not to control the chance of falsely rejecting the H_0 of an individual test, but to control the chance that at least one H_0 gets falsely rejected instead. In other words, the goal of this approach is to make individual decisions about each testing problem (7.1), while controling for the FWE.

Hence, we again calculate $t^*_{s,t}$ for each fund s as in Equation (7.2), but the d is now chosen such that

$$\text{FWE} \leq \delta. \tag{7.5}$$

By controlling this FWE, we control the probability that even one unskilled fund manager gets classified as skilled. Therefore, with a high probability, our portfolio will only contain skilled managers.

There are many ways to accomplish Equation (7.5). One could for example set $\delta = \infty$, such that no fund manager is classified as skilled. Another method is to apply a Bonferroni correction. These methods are too strict; too few fund managers would be classified as skilled. Naturally, we would like to select as many genuinely skilled managers as possible, i.e. we want to determine d in such a way that the power of the test is as high as possible. Romano and Wolf (2005) show that the ideal value of d is given by the $1 - \delta$ quantile of

$$\max_{s=1,\ldots,S} \frac{\hat{\alpha}^{24}_{s,t} - \alpha^{24}_{s,t}}{\hat{\sigma}_{s,t}}. \tag{7.6}$$

The distribution of this random variable is however not known. Therefore, this value of d needs to be estimated. A consistent estimator of d can be obtained by the bootstrap method. This value, denoted by $\hat{d}$ is obtained as the $1 - \delta$ quantile of the following random variable

$$\max_{s=1,\ldots,S} \frac{\hat{\alpha}^{24,*}_{s,t} - \hat{\alpha}^{24}_{s,t}}{\hat{\sigma}^*_{s,t}}. \tag{7.7}$$

Here, $\hat{\alpha}_{s,t}^{24,*}$ and $\hat{\sigma}_{s,t}^{*}$ denote the bootstrap estimators of $\alpha_{s,t}^{24}$ and $\sigma_{s,t}$ respectively. Romano and Wolf (2005) prove that by setting $\hat{d}$ as in Equation (7.7)

$$\limsup_{T\to\infty} \text{FWE} \leq \delta.$$

However, simulation studies by Romano and Wolf (2005) and Romano, Shaikh, and Wolf (2008) show that the FWE is also controlled well for practically relevant sample sizes. How the bootstrap method exactly works in case of regression models is described in Appendix B. In this study, we use $\delta = 10\%$.

Therefore, this investment strategy has the following step at time $t = 24, \ldots, T$:

- Calculate $t_{s,t} = \frac{\hat{\alpha}_{s,t}^{24}}{\hat{\sigma}_{s,t}}$ for each fund s.
- Calculate $\hat{d}$ as in Equation (7.7) by the bootstrap method.
- Invest $\frac{I_t}{N_t^I}$ in every fund in $\mathcal{I}_t$, i.e. in every fund for which $t_{s,t} > \hat{d}$.
- The portfolio is sold at time $t+1$.

Again, by setting $I_t = \prod_{u=1}^{t} I_{24} R_u^{FWE}$, where R_u^{FWE} is the return of this Familywise Error Portfolio at time u, this strategy becomes self-financing. This strategy is applied on our data, the results can be found in Table 10.

	Excess Return	Std. Dev.	Sharpe Ratio	$\hat{\alpha}$
EUIG	2.86%	0.93%	0.90	0.85%
EUHY	5.37%	2.68%	0.57	1.42%
USIG	3.86%	1.46 %	0.76	0.88%
USHY	5.22%	2.21%	0.68	1.17%

Table 10: Average performance measures for each subsample are shown. Averages are taken over the funds. Sharpe ratio, alpha and return are all annualized.

As can be seen from Table 10, the Familywise Error Portfolio leads to positive alphas in all our four universes. Furthermore, when comparing these results with the results of our Positive Alpha Portfolio (see Table 9), it can be seen that the Familwise Error Portfolio generates higher returns, higher Sharpe ratios and higher alphas. These results therefore indicate that the Familywise Error approach indeeds improves upon the Positive Alpha approach. The returns and alphas of the Familywise Error strategy are not as high as those of the Highest Alpha strategy (as reported in Table 7). However, the standard deviation of the returns are much lower for the Familywise Error strategy, leading to higher Sharpe ratios.

Chapter 8

Conclusion

Despite the enormous size of the corporate bond fund market, research on the performance of corporate bond funds is almost non-existent. In this thesis, a rigorous investigation into the performance of corporate bond funds is provided. The performance of funds is measured by the alpha of a fund, which is defined as the excess return of what a two factor equilibrium model would predict. Although this way of measuring performance is nothing new, this thesis provides a rigorous mathematical derivation of this two factor equilibrium model. The alpha is then estmited by utilizing a large dataset of both European and American corporate bond funds over the period 1999-2013.

We find that the average alpha of funds is positive for investment grade funds, but negative for high yield funds. However, even though the average alpha of high yield funds is negative, we show that many high yield funds are still able to obtain a positive alpha. This indicates that corporate bond funds are valuable investment vehicles that earn a high return for the risk they take.

Furthermore, strong evidence of performance persistence of corporate bond funds is found. Three statistical methods are used to investigate persistence in performance and all of these methods consistently show that future performance can be predicted by past performance. Corporate bond funds with a strong performance record continue to perform well in the subsequent period, whereas funds with a poor performance record, repeat their poor performance in the following period. This persistence in performance is found in all of our four subsamples. This persistence in performance indicates that there exist professional fund managers with superior bond selecting skill.

Finally, three investment strategies were introduced that exploit this relation between past and future returns. All these investment strategies earn a high risk adjusted return. Therefore, investors can earn money by investing in corporate bond funds that report strong past performance.

Appendices

Appendix A

A pricing formula for corporate bonds in a multi-period economy

In Section 3.2, the price of a corporate bond fund was derived in a one period economy. It is possible to generalize this result for a multi-period economy. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space and $\mathcal{F}_t$ is the filtration at time t, where $t = 0, \ldots, T$.

In this multi-period setting, the issuer of a corporate bond pays a sum of money to the buyer of a bond in every period t until the bond matures. The payment the company makes to the investor at time t is denoted by C_t. At the maturity date, T, the company also pays back the principal. The payments Therefore C_T consists of a coupon and a principal. We are interested at pricing the bond at $t = 0$. At $t = 1$, the company makes the first payment to the investor.

The risk free rate, r^f is again defined as the return on a risk free payoff after one period. Hence, if currently $t = 0$, then the risk free rate is the return on a risk free payoff at $t = 1$, $X_1 = x$. It is again assumed that $r^f > 1$. Furthermore, $p(X_t)$ denotes the price of payoff X_t at $t = 0$. In this multi-period setting, the definition of arbitrage and the stochastic discount factor must be adjusted to this multi-period setting. The idea of both concepts remains the same.

Definition 4 (Absence of Arbitrage in a Multi-Period Economy)**.** *Let $X = (X_1, \ldots, X_T)$ be a stream of payoffs. If there exists at least one $t^* \in [1, \ldots T]$ for which, $X_{t^*} \geq 0$ a.s. and $\mathbb{P}(X_{t^*} > 0) > 0$ and for all other $t \neq t^*$ it holds that $X_{t^*} = 0$ a.s., then it must be the case that $p(X_t) > 0$.*

Absense of arbitrate tells us that there is no free lunch; one can't make money out of nothing.

The first fundamental theorem of asset pricing tells us that there exists a stochastic discount factor pricing this payoff if and only if there is absense of arbitrage.

Theorem 10 (First Fundamental Theorem of Asset Pricing)**.** *Let X_t be a payoff at time $t \in (1, \dots T)$. Then there exists a random variable $M_{0,t} > 0$ a.s. such that*

$$p(X_t) = \mathbb{E}_0(M_{0,t}X_t) \tag{A.1}$$

if and only if there is absence of arbitrage. This random variable $M_{0,t}$ is called the stochastic discount factor.

The proof of the multi-period version of the First Fundamental Theorem of Asset pricing can be found in Föllmer and Schied (2011). The authors furthermore show that this pricing formula is a gain linear, i.e. if X_t is a payoff at time t and X_{t^*} is a payoff at time t^*, then $p(X_t + X_{t^*}) = p(X_t) + p(X_{t^*})$.

By using Equation (A.1), we get that the price of a payoff of a corporate bond at time t is given by $p(C_t) = \mathbb{E}_0(M_{0,t}C_t)$ and by using the linearity of the pricing function it is obtained that the price of a bond at $t = 0$ is given by the sum of the price of its payoffs:

$$\begin{aligned} p(\sum_{t=1}^{T} C_t) &= \sum_{t=1}^{T} p(C_t) \\ &= \sum_{t=1}^{T} \mathbb{E}_0(M_{0,t}C_t) \\ &= \sum_{t=1}^{T} \mathbb{E}_0[(M_{0,t})]\mathbb{E}_0[(C_t)] + \sum_{t=1}^{T}[Cov_0(M_{0,t}, C_t)], \end{aligned} \tag{A.2}$$

where it was used that for two random variables X and Y, $\mathbb{E}[XY] = \mathbb{E}[X]\mathbb{E}[Y] + Cov(X, Y)$. To rewrite this equation in a way such that the effect of interest rate risk on the bond price becomes more clear, the following theorem is used:

Theorem 11. *Let $1 < k \leq T$. If the market is free of arbitrage*

$$\mathbb{E}_0[M_{0,k}] = \left[\frac{1}{r^f}\right]^k \tag{A.3}$$

Proof. This is proven by induction. For $k = 1$ it is true (see Equation (3.17)). Now assume $\mathbb{E}_0[M_{0,k}] = \left[\frac{1}{r^f}\right]^k$ holds for all k. We show it will also hold for $k + 1$. Consider two strategies:

- At time 0, buy a product that pays 1 at time $k + 1$ with certainty.
- At time k, buy a product that pays 1 at time $k + 1$ with certainty. This product costst $\frac{1}{r^f}$ at time k. Finance this by buying a product at time 0 with a certain payoff of $\frac{1}{r^f}$ at time k.

Both strategies have a certain payoff of 1 at time $k+1$. Therefore, they must have the same price by the law of one price. The price of strategy 1 is $\mathbb{E}_0[M_{0,k+1}]$. The price of strategy 2 is $\mathbb{E}_0[M_{0,k}]\frac{1}{r^f} = \left[\frac{1}{r^f}\right]^k \frac{1}{r^f}$. Hence $\mathbb{E}_0[M_{0,k+1}] = \frac{1}{r^f}^{k+1}$. $\square$

Therefore, we can rewrite the bond price as

$$p_t = \sum_{t=1}^{T} \left[\frac{1}{r^f}\right]^t \mathbb{E}_0(C_t) + \sum_{t=1}^{T} [Cov_0(M_{0,t}, C_t)]. \tag{A.4}$$

Note that if the payoff is certain, i.e. no default risk, the covariance term in this equation drops out and we are left with a more familiar pricing equation for bonds:

$$p_t = \sum_{t=1}^{T} \left(\frac{1}{r^f}\right)^t C_t. \tag{A.5}$$

The influence of the term structure risk and the risk of default remains the same as in Section 3.2. Indeed, if the other variables in (A.5) remain unchanged, an increase in the probability that a company will default lowers $\mathbb{E}_0(C_t)$ and the price of the bond will also drop. In a similar way, an increasing interest rate leads to a decrease in $\left(\frac{1}{r^f}\right)$, thereby decreasing the price of the bond.

Appendix B

Bootstrapping Regression Models

Bootstrapping is an approach to estimate the distribution of a statistic. The bootstrap method is based on building a sampling distribution for a statistic by resampling from the original data. The method was first introduced by Efron (1979). In the bootstrap method, one uses the sample data as a population from which samples are drawn.

Suppose that $\mathbf{S} = \{Y_1, \ldots, Y_N\}$ is a sample from a population $\mathbf{P} = \{y_1, \ldots, y_P\}$, where $P > N$. Furthermore, suppose that we are interested in a statistic $T = t(\mathbf{S})$ as an estimate of $\theta = t(\mathbf{P})$. The traditional approach in statistics is to make assumptions about the population structure, for example by assuming the population follows a certain distribution, and then to use these assumptions to estimate the distribution of T. If the distribution of T cannot be derived, the traditional approach is to try to derive its asymptotic distribution.

The bootstrap method allows us to estimate the sample distribution of T without making assumptions of the underlying population. This is done by drawing a sample of size N^B from the sample $\mathbf{S}$ with replacement. The resulting bootstrap sample is denoted by $\mathbf{S}^*$. This sampling procedure is repeated B times and the bth boothstrap sample is denoted by $\mathbf{S}_b^*$. Then, for each bootstrap sample, the statistic T is computed; $T_b^* = t(\mathbf{S}_b^*)$. Then, the distribution of T_b^* around T is analogous to the sampling distribution of the estimator T around θ. For example, let $G(t) = \mathbb{P}(T \leq t)$ denote the CDF of T. Then the bootstrap estimation of G is given by

$$\hat{G}^*(t) = \frac{1}{B} \sum_{b=1}^{B} I(T_b^* \leq t), \tag{B.1}$$

where I is the indicator function.

In Section 7.3, we want to use the bootstrap method in a regression framework. In this framework, we have the following relation for each Y_i, $i \in \{1, \ldots, N\}$:

$$Y_i = a + \boldsymbol{\beta}' \mathbf{X}_i + \epsilon_i \tag{B.2}$$

From this sample, we want to generate a bootstrap sample $\mathbf{S}^* = \{Y_1^*, \ldots, Y_N^*\}$. There are many procedures to generate such a bootstrap sample. Almost all procedures start with calculating the fitted values, $\hat{\mathbf{Y}} = [\hat{Y}_1, \ldots, \hat{Y}_N]'$, from the model, i.e.

$$\hat{Y}_i = \hat{a}_i + \hat{\boldsymbol{\beta}}'\mathbf{x}_i, \tag{B.3}$$

where $\hat{a}_i$ and $\hat{\boldsymbol{\beta}}$ are the OLS estimates of a_i and $\boldsymbol{\beta}$ respectively. We sample N^B values with replacement from $\hat{\mathbf{Y}}$ to obtain $\hat{\mathbf{Y}}^* = [\hat{Y}_1^*, \ldots, \hat{Y}_{N^B}^*]'$.

Adding a random error term to each $\hat{Y}_i$ produces a bootstrap sample. The errors could be generated from a normal distribution if we assume our errors are IID. However, as discussed in Section 3.5, this is not appropriate in our case. Therefore, errors are sampled from the estimated residuals from the original regression. Hence, from our vector of estimated residuals $\hat{\boldsymbol{\epsilon}} = [\hat{\epsilon}_1, \ldots, \hat{\epsilon}_N]'$, a random sample of N^B estimated residuals is taken. This sample is denoted by $\hat{\boldsymbol{\epsilon}} = [\hat{\epsilon}_1, \ldots, \hat{\epsilon}_{N^B}]'$ Then, the bootstrapped values can be calculated as

$$Y_i^* = \hat{Y}_i^* + \hat{\epsilon}_i \tag{B.4}$$

for $i = 1, \ldots, N^B$.

Appendix C

Matlab Code

```
function out2 = FactorRegression(Data, Settings, returns)
T = length(Data.Dates);
X = ones(T,1);
for f=1:length(Settings.FACTORS)
    X = [X Data.(Settings.FACTORS{f})'];
end

% conduct regression per return series
for n=1:size(returns, 1)
    Y = returns(n,:)';

    if Settings.WINDOW == 0
        select = ~isnan(Y) & ~any(isnan(X),2);
        if sum(select) < Settings.MIN_OBS
            continue;
        end
        out(n,1) = nwest(Y(select), X(select,:), Settings.NR_LAGS);
       else
        for t=(Settings.WINDOW+1):T
            Y2 = Y(t-Settings.WINDOW+1:t);
            X2 = X(t-Settings.WINDOW+1:t, :);
            select = ~isnan(Y2) & ~any(isnan(X2),2);
            if sum(select) < Settings.MIN_OBS
                continue;
            end
            out(n,t) = nwest(Y2(select), X2(select,:), Settings.NR_LAGS);

        end
    end

end
```

```
fields = setdiff(fieldnames(out), {'meth', 'y', 'yhat', 'resid', 'covbeta'});
for f=1:length(fields)
    for t=1:size(out,2)
        for n=1:size(out,1)
            values = [out(n,t).(fields{f})];
            if isempty(values)
                continue;
            end
            out2.(fields{f})(n,t,:) = values;
        end
    end
end

end

function [Results]=AlphaPortfolio(Data,Settings,GrossReturn,Startdate,s,k)
%Startdate is the date from which you want the Evaluation to begin.
%k is the number of portfolios. For quintile portfolios, k=5.
Settings.WINDOW=s;
if s==0 | s>24
    Settings.MIN_OBS=24;
else
    Settings.MIN_OBS=s;
end;

StartDate=datenum(Startdate);

if StartDate<datenum(Data.Dates(1))
    StartDatenum=1;
else
StartDatenum=find(Data.Dates==StartDate);
end
HelpStart=StartDatenum+s-1;
out2=FactorRegression2(Data,Settings,GrossReturn);
Alpha=out2.beta(:,(HelpStart:end),1)*10000;
GrossReturn=GrossReturn(:,(HelpStart:end));
N=size(Alpha);
O=length(Data.Dates);
AlphaPortReturn=NaN(k,O);
for i=1:(N(2)-1)
    Alpha1=Alpha(:,(i));
    Alpha2=Alpha1;
    AlphaIndex=(1:length(Alpha1))';
    AlphaIndex(Alpha1==0)=[];
    Alpha1(Alpha1==0)=[];
    F = ceil(k * tiedrank2(Alpha1) / length(Alpha1));
```

```
    for j=1:k
    AlphaBuckets{j,i}=AlphaIndex(F==k+1-j);
    AlphaExposures{j,i}=Alpha2(AlphaIndex(F==k+1-j));
    AlphaPortReturn(j,HelpStart+i)=nanmean(GrossReturn([AlphaBuckets{j,(i)}],(i+1)));
    end

end
[QuantileResults,HMLresults]=PortfolioEvaluation(Data,Settings,AlphaPortReturn,k);
Results.QuantileResults=QuantileResults;
Results.HMLresults=HMLresults;
end

function [Results]=SignificantAlphaPortfolio(Data,Settings,GrossReturn,Startdate,s)
%Startdate is the date from which you want the Evaluation to begin.
Settings.WINDOW=s;
if s==0 | s>24
    Settings.MIN_OBS=24;
else
    Settings.MIN_OBS=s;
end;

StartDate=datenum(Startdate);

if StartDate<datenum(Data.Dates(1))
    StartDatenum=1;
else
StartDatenum=find(Data.Dates==StartDate);
end
HelpStart=StartDatenum+s-1;
out2=FactorRegression2(Data,Settings,GrossReturn);
tstatmatrix=out2.tstat(:,(HelpStart:end),1);
GrossReturn=GrossReturn(:,(HelpStart:end));
N=size(tstatmatrix);
B=[1:N(1)]';
O=length(Data.Dates);
Critvalue=tinv(0.90,21)
TstatPortReturn=NaN(1,O);
for i=1:(N(2)-1)
    tstatvector=tstatmatrix(:,(i));
    tstatfunds=tstatvector>Critvalue;
    C=B(tstatfunds);
    %TstatPortReturn(1,HelpStart+i)=nanmean(GrossReturn(tstatfunds,(i+1)));
    TstatPortReturn(1,HelpStart+i)=nanmean(GrossReturn(C,(i+1)));
```

```
end
ReturnsPort=nanmean(TstatPortReturn,2);
YearlyReturns=ReturnsPort*12;
StdPort=nanstd(TstatPortReturn,0,2);
Settings.WINDOW=0;
SharpRat=sqrt(12)*ReturnsPort./StdPort;

outport=FactorRegression2(Data, Settings, TstatPortReturn);
alphaout=outport.beta(:,:,1)*100*12;
termout=outport.beta(:,:,2);
defout=outport.beta(:,:,3);
tstatout=outport.tstat(:,:,1);

Results=[YearlyReturns,StdPort,SharpRat,alphaout,tstatout, termout, defout];
end
```

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