

Bachelor Thesis

The strategy of a central authority with respect to income tax evasion

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Abstract

In this thesis, we consider a gametheoretical model of income taxation. We take a closer look at the strategy of the central authority with respect to the audit probability of tax returns, given the fact that taxpayers may underdeclare their income. Different cut-off rules are compared to investigate which one optimizes the net tax revenue of the central authority. We conclude by giving the optimal strategy, depending on specific conditions, and apply this result to the Ukrainian and Dutch taxation systems.

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1 Introduction

Income tax evasion is a subject on which much research has been done. Allingham and Sandmo (1972) provided one of the first models, analyzing the decision of an individual taxpayer whether to avoid taxes by underreporting [1]. Their model, however, does not take into account the strategy of the central authority, whose net tax revenue is formed by the paid income taxes. The central authority is able to determine a strategy, depending on several factors. First, the taxation system that they handle can be proportional, which means that every taxpayer pays the same proportion on taxes, independent of their income. On the other hand, the proportion that should be paid on taxes could increase or decrease by an increasing income, characterizing a respectively progressive or regressive system. The fourth option covers a system in which every taxpayer pays the same constant amount to the central authority, independent of their income. Around the same time that Allingham and Sandmo published their model, Srinivasan (1973) wrote a model that does take into account the possibility for the central authority to choose a tax system [8]. He showed that, given a fixed income distribution, a proportional tax system that yields the same net tax revenue as a progressive tax system if every taxpayer declares his true income, will yield a higher net tax revenue in the presence of tax evasion.

Besides the nature of a tax system, central authorities often choose to organize audits to check if taxpayers underdeclared their income. If a taxpayer gets caught for tax evasion, there is a corresponding fine. Of course, there is no time and money to check every taxpayer. Therefore, in determining their strategy, the central authority picks an audit probability. This probability could be constant, as in the model of Srinivasan (1973) and Singh (1973) [8][7]. Also, Singh (1973) shows that if the central authority wants to optimize the net tax revenue of a progressive tax system, the audit probability should be greater than $1/3$, independent of the level of income.

On the other hand, arguments can be formulated that advocate an audit probability depending on the declared income. Reinganum and Wilde (1985) were the first one to introduce a 1-step cut-off rule, where taxpayers with a declared income below a specific threshold get audited with probability p_0 , while taxpayers with a declared income above this threshold encounter an audit probability p_1 . They claim that the optimal policy for a central authority reduces to the form where $p_0 = 1$ and $p_1 = 0$ and that this strategy dominates random audits [5]. Vasin and Panova (2000) showed also that the optimal strategy is a 1-step cut-off rule, but then with $p_0 = \hat{p}$, where \hat{p} is the audit probability such that taxpayers declare

their true income [10]. Both articles assume decreasing audit probabilities as the income increases, which seems to coincide with the findings of Yitzhaki (1974) and Dzhumashev and Gahramanov (2011). They found that there is a positive relationship between tax rate and tax evasion [11][3]. Assuming that this relationship also holds for due tax and tax evasion, taxpayers with a higher income are less inclined to evade tax, which leads to the optimal audit policy to be decreasing. On the other hand, Levaggi and Menoncin (2013) found evidence that the mentioned relationship is under specific conditions negative instead of positive [4].

In this thesis, we will follow the line of reasoning of Vasin and Panova (2000). They provide a tax model in which both the due tax and the penalty have a proportional structure, and in which the audit probability depends on the declared income [10]. The purpose of this thesis is to investigate the optimal audit probability, such that the net tax revenue of the central authority is optimized. First, we will explain the general model and the corresponding strategies of the taxpayer and the central authority. Here, we will also introduce the mathematical framework of a cut-off rule with respect to the audit probability. After that, we will focus on the proportionate tax and fine structure. We will compute the audit probability such that there is no tax evasion and proof a theorem which gives us the net tax revenue optimizing strategy of a central authority.

2 The model

In this thesis, we consider a game theoretical tax evasion model with two players, namely the individual taxpayer and the central authority that collects tax. We assume that the group of taxpayers has a distribution of income $I \geq 0$ which is given by an arbitrary density $v(I)$. The declared income I_d represents the behaviour of the taxpayer and depends on the true income I and the auditing probability. The central authority determines this auditing probabilities $P(\cdot)$, where an audit always reveals the true income. If a taxpayer gets caught cheating, the due penalty is determined by the function $F(I, I_d)$, which includes the unpaid tax.

2.1 Strategies of players

First, consider the strategy of an individual taxpayer. The taxpayer can choose between declaring his true income I , or declaring a lower income I_d . Depending on if he gets caught by the central authority, this leads to the following two possible revenues:

- a) If the taxpayer gets caught cheating, his revenue equals $I - T(I_d) - F(I, I_d)$.
- b) If a taxpayer doesn't get caught, his revenue is equal to $I - T(I_d)$. Note that this could either mean that he cheated, but didn't get audited, or that he declared his true income.

The rule of divide and conquer [6] tells us the following:

$$E(B) = E(B|D)P(D) + E(B|D^C)P(D^C). \quad (1)$$

Because the chance of getting caught equals $P(\cdot)$, we get:

$$\begin{aligned} E(A) &= E(A|\text{caught})P(\text{caught}) + E(A|\text{not caught})P(\text{not caught}) \\ &= [I - T(I_d) - F(I, I_d)]P(\cdot) + [I - T(I_d)](1 - P(\cdot)) \\ &= I - T(I_d) - P(\cdot)F(I, I_d). \end{aligned} \quad (2)$$

The optimal strategy of a taxpayer is choosing a $I_d \in [0, I]$ such that his expected revenue and thus (2) is maximized. I_d is therefore determined by the solution to the problem:

$$I_d(I, P(\cdot)) \rightarrow \max\{I - T(I_d) - P(\cdot)F(I, I_d)\}. \quad (3)$$

The central authority, on the other hand, chooses its strategy by picking an audit probability $P(\cdot)$ under a specific tax and fine structure. If c equals the cost of an audit, the net tax

revenue R of the central authority, depending on the choice of $P(\cdot)$, is given by:

$$R(P(I_d)) = \int_I \left\{ T(I_d(I, P(I_d))) + P(I_d) [F(I, I_d(I, P(I_d))) - c] \right\} dv(I). \quad (4)$$

The central authority wants to find the $P(\cdot)$ such that (4) is maximized. In this thesis, we focus on this problem.

2.2 Cut-off rule

The central authority could handle a specific audit policy, based on the idea of a cut-off rule. As we saw in the introduction, a cut-off rule gives a prescription which audit probability to handle in which case, depending on the income. Given a 1-step cut-off rule, the audit probability equals p_0 if the income is below a specific threshold I' , and is equal to p_1 if the income is larger or equal than I' . In other words, a general 1-step cut-off rule looks as follows:

$$P(I') = \begin{cases} p_0 & \text{if } I_d < I', \\ p_1 & \text{if } I_d \geq I'. \end{cases} \quad (5)$$

This idea can be generalized to a k -step cut-off rule:

$$P(I_1, \dots, I_k) = \begin{cases} p_0 & \text{if } I_d < I_1, \\ p_l & \text{if } I_l \leq I_d < I_{l+1}, \\ p_k & \text{if } I_d \geq I_k, \end{cases} \quad l = 1, \dots, k - 1. \quad (6)$$

In this thesis, we focus on decreasing audit probabilities. In other words, we assume that $p_0 \geq \dots \geq p_k$.

3 Proportionate tax and fine structure

We will consider the following proportionate tax and fine structure:

$$T(I_d) = tI_d, \quad F(I, I_d) = (f + t)(I - I_d), \quad (7)$$

with $t, f \in \mathbb{R}_+$. This leads to the following expected revenue of a taxpayer after tax and fines:

$$E(A) = I - tI_d - P(\cdot)(f + t)(I - I_d). \quad (8)$$

Given (7), the net tax revenue for the central authority equals:

$$R(P(\cdot)) = \int_I \left\{ tI_d + P(\cdot)[(f + t)(I - I_d) - c] \right\} dv(I). \quad (9)$$

3.1 No tax evasion

The central authority could strive after different goals. One possibility is to make sure that there is no tax evasion. The following proposition determines the minimum audit probability, such that every taxpayer declares his true income.

Proposition 1. *If $P(\cdot) \geq \frac{t}{f + t}$ for any $I_d < I$, then $I_d(I, P(\cdot)) = I$.*

Proof. Let's take a look at the expected revenue $E(A)$ of a taxpayer after taxes and fines, which is given by (8). If a taxpayer declares his true income, then $I_d = I$ and (8) equals $I(1 - t)$. If, however, a taxpayer underdeclares his income by declaring I_d , his expected income after taxes and fines equals $I - tI_d - P(\cdot)(f + t)(I - I_d)$. Making sure that a taxpayer declares his true income is therefore achieved by satisfying the following inequality:

$$I(1 - t) \geq I - tI_d - P(\cdot)(f + t)(I - I_d). \quad (10)$$

Rewriting this condition gives us $P(\cdot)(f + t)(I - I_d) \geq t(I - I_d)$ and so:

$$P(\cdot) \geq \frac{t}{f + t}. \quad (11)$$

Therefore, if $P(\cdot) \geq \frac{t}{f + t}$ for any $I_d < I$, then $I_d(I, p(\cdot)) = I$. ■

Let us denote the given threshold probability by $\hat{p} := \frac{t}{f + t}$. Then, for any $P(\cdot) \geq \hat{p}$,

taxpayers declare their true income. Because every audit brings costs c , the central authority wants to choose an audit probability as small as possible. Therefore, if the goal is to prevent tax evasion, the optimal strategy is $P(\hat{p})$, where every taxpayer gets audited with chance \hat{p} .

3.2 Optimizing the net tax revenue

Of course, the absence of tax evasion seems like a good target for the central authority. However, it is not said that this strategy will maximize their net tax revenue. As we saw in section 2.2, another strategy is using a specific cut-off rule. Consider the following class of 1-step cut-off rules:

$$\tilde{P}(I') = \begin{cases} \hat{p} & \text{if } I_d < I', \\ 0 & \text{if } I_d \geq I', \end{cases} \quad (12)$$

with \hat{p} as defined in section 3.1. The following theorem shows us that the optimal strategy of the central authority always belongs to this class of 1-step cut-off rules. In other words, the theorem shows that any k -step strategy of the form (6) is dominated by a 1-step strategy (12).

Theorem. *If*

$$\int_{I \geq I'} \{t(I - I') - \hat{p}c\} dv(I) \geq 0 \quad (13)$$

for any I' , then the strategy $P(\hat{p})$ is optimal for the central authority. If (13) doesn't hold, then the optimal strategy is a 1-step cut-off rule of the form (12).

Proof. Consider the k -step cut-off rule, given by (6), with $I_{k+1} = \infty$. We assume that $p_i \leq \hat{p}$ for $i = 0, \dots, k$ and thus $\hat{p} \geq p_0 \geq \dots \geq p_k$. For $l = 1, \dots, k - 1$, we define \bar{I}_l as the true income value, such that the expected income of a taxpayer after taxes and fines is the same whether he declares I_l or I_{l+1} . Therefore, looking at (8),

$$tI_l + p_l(f + t)(\bar{I}_l - I_l) = tI_{l+1} + p_{l+1}(f + t)(\bar{I}_l - I_{l+1}). \quad (14)$$

Rewriting this to get an expression for \bar{I}_l , we get:

$$\bar{I}_l [p_l(f + t) - p_{l+1}(f + t)] = tI_{l+1} - tI_l + p_l(f + t)I_l - p_{l+1}(f + t)I_{l+1},$$

and so

$$\begin{aligned}\bar{I}_l &= \frac{tI_{l+1} - tI_l + p_l(f+t)I_l - p_{l+1}(f+t)I_{l+1}}{(p_l - p_{l+1})(f+t)} \\ &= \hat{p} \frac{I_{l+1} - I_l}{p_l - p_{l+1}} + \frac{p_l I_l - p_{l+1} I_{l+1}}{p_l - p_{l+1}}.\end{aligned}\quad (15)$$

Note that $\bar{I}_l > I_l$, because

$$\begin{aligned}\bar{I}_l &= \frac{tI_{l+1} - tI_l + p_l(f+t)I_l - p_{l+1}(f+t)I_{l+1}}{(p_l - p_{l+1})(f+t)} \\ &= \frac{I_l[(f+t)p_l - t] - I_{l+1}[(f+t)p_{l+1} - t]}{(p_l - p_{l+1})(f+t)} \\ &> \frac{I_l[(f+t)p_l - t] + I_{l+1}t}{(p_l - p_{l+1})(f+t)} \\ &> \frac{I_l[(f+t)(p_l - p_{l+1}) - t] + I_l t}{(p_l - p_{l+1})(f+t)} \\ &= \frac{I_l(p_l - p_{l+1})(f+t)}{(p_l - p_{l+1})(f+t)} \\ &= I_l.\end{aligned}\quad (16)$$

Let $\bar{I}_1 < \bar{I}_2 < \dots < \bar{I}_{k-1}$ and $\bar{I}_k = \infty$. Given \bar{I}_l , we would like to variate the audit probabilities and see which values of p_i optimize the net tax revenue for the central authority. Therefore, we determine a variation $\mathbf{d} = (d_0, \dots, d_k)$ such that \bar{I}_l stays the same for any k -step strategy

$$P_x(I_1, \dots, I_k) = \begin{cases} p_0(x) & \text{if } I_d < I_1, \\ p_l(x) & \text{if } I_l \leq I_d < I_{l+1}, \end{cases} \quad l = 1, \dots, k, \quad (17)$$

with $p_i(x) = p_i + xd_i$. Assume that $p_0(x) \geq \dots \geq p_k(x) \geq 0$.

Filling in this strategy in the expression for \bar{I}_l gives us the following equation:

$$\bar{I}_l = \hat{p} \frac{I_{l+1} - I_l}{p_l + xd_l - (p_{l+1} + xd_{l+1})} + \frac{(p_l + xd_l)I_l - (p_{l+1} + xd_{l+1})I_{l+1}}{p_l + xd_l - (p_{l+1} + xd_{l+1})}. \quad (18)$$

Rewriting this in terms of p_l gives us:

$$p_l = \hat{p} \frac{I_{l+1} - I_l}{\bar{I}_l - I_l} + p_{l+1} \frac{\bar{I}_l - I_{l+1}}{\bar{I}_l - I_l} + x \frac{d_{l+1}(\bar{I}_l - I_{l+1}) - d_l(\bar{I}_l - I_l)}{\bar{I}_l - I_l}. \quad (19)$$

Because \bar{I}_l has to remain the same for any x , so does (19). This only holds if the term

multiplied by x equals zero. Therefore,

$$p_l = \hat{p} \frac{I_{l+1} - I_l}{\bar{I}_l - I_l} + p_{l+1} \frac{\bar{I}_l - I_{l+1}}{\bar{I}_l - I_l} \quad (20)$$

and

$$d_l = d_{l+1} \frac{I_{l+1} - \bar{I}_l}{I_l - \bar{I}_l}, \quad (21)$$

with $l = 1, \dots, k - 1$. For these values of l , we get the following expression for $p_l(x)$:

$$\begin{aligned} p_l(x) &= p_l + x d_l \\ &= \hat{p} \frac{I_{l+1} - I_l}{\bar{I}_l - I_l} + p_{l+1} \frac{\bar{I}_l - I_{l+1}}{\bar{I}_l - I_l} + x d_{l+1} \frac{I_{l+1} - \bar{I}_l}{I_l - \bar{I}_l} \\ &= \hat{p} \frac{I_{l+1} - I_l}{\bar{I}_l - I_l} + p_{l+1}(x) \frac{\bar{I}_l - I_{l+1}}{\bar{I}_l - I_l}. \end{aligned} \quad (22)$$

To take a look at the net tax revenue of the central authority under strategy $P_x(I_1, \dots, I_k)$, we first have to determine the behaviour of a taxpayer under this strategy:

$$I_d(P_x(I_1, \dots, I_k)) = \begin{cases} I_d^{(0)} & \text{if } I < I_1, \\ I_d^{(1)} & \text{if } I_1 \leq I < \bar{I}_1, \\ I_d^{(l+1)} & \text{if } \bar{I}_l \leq I < \bar{I}_{l+1}, \end{cases} \quad l = 1, \dots, k - 1. \quad (23)$$

For the central authority, this yields the following net tax revenue:

$$\begin{aligned} R(P_x(I_1, \dots, I_k)) &= \int_0^{I_1} \{tI_d^{(0)} + (p_0 + x d_0)[(f + t)(I - I_d^{(0)}) - c]\} dv(I) \\ &+ \int_{I_1}^{\bar{I}_1} \{tI_d^{(1)} + (p_1 + x d_1)[(f + t)(I - I_d^{(1)}) - c]\} dv(I) \\ &+ \int_{\bar{I}_1}^{\bar{I}_2} \{tI_d^{(2)} + (p_2 + x d_2)[(f + t)(I - I_d^{(2)}) - c]\} dv(I) + \dots \\ &+ \int_{\bar{I}_{k-1}}^{\infty} \{tI_d^{(k)} + (p_k + x d_k)[(f + t)(I - I_d^{(k)}) - c]\} dv(I). \end{aligned} \quad (24)$$

We will compute the derivative of this revenue with respect to x .

$$\begin{aligned} \frac{dR(P_x(I_1, \dots, I_k))}{dx} &= \int_0^{I_1} d_0 [(f + t)(I - I_d^{(0)}) - c] dv(I) \\ &+ \int_{I_1}^{\bar{I}_1} d_1 [(f + t)(I - I_d^{(1)}) - c] dv(I) \end{aligned}$$

$$\begin{aligned}
& + \int_{\bar{I}_1}^{\bar{I}_2} d_2 [(f+t)(I - I_d^{(2)}) - c] \, dv(I) + \dots \\
& + \int_{\bar{I}_{k-1}}^{\infty} d_k [(f+t)(I - I_d^{(k)}) - c] \, dv(I). \tag{25}
\end{aligned}$$

This derivative is independent of x , and therefore $R(P_x(I_1, \dots, I_k))$ is maximized at one of the boundaries of x , which is x_{\max} or x_{\min} . Let's determine the strategies of the central authority at those boundaries.

By definition, $p_i(x) = p_i + xd_i$ for $i = 0, \dots, k$. Because all the d_i are multiplied by the same factor x , we may divide them by d_k and absorb this in the factor x . This allows us to take $d_k = 1$ without loss of generality, giving us $p_k(x) = p_k + x$. We will first determine the value of $p_k(x)$ in the boundaries of x , which will lead us to the values of $p_i(x)$ in this boundaries for $i = 0, \dots, k - 1$.

Because of the assumption that $p_0(x) \geq \dots \geq p_{k-1}(x) \geq p_k(x) \geq 0$, the value of x_{\max} makes the inequality $p_{k-1}(x) \geq p_k(x)$ an equality. Rewriting (22) gives us:

$$\begin{aligned}
\hat{p} &= \frac{\bar{I}_l - I_l}{I_{l+1} - I_l} p_l(x) + \frac{I_{l+1} - \bar{I}_l}{I_{l+1} - I_l} p_{l+1}(x) \\
&= \lambda p_l(x) + (1 - \lambda) p_{l+1}(x), \tag{26}
\end{aligned}$$

with $\lambda = \frac{\bar{I}_l - I_l}{I_{l+1} - I_l}$. Picking $l = k - 1$ and $x = x_{\max}$, (26) gives us $p_k(x_{\max}) = p_{k-1}(x_{\max}) = \hat{p}$. By backwards induction with base case k , we will show that $p_l(x_{\max}) = \hat{p}$ for $l = 0, \dots, k$. Suppose that $p_{l+1}(x_{\max}) = \hat{p}$. Then,

$$\begin{aligned}
p_l(x_{\max}) &= \hat{p} \frac{I_{l+1} - I_l}{\bar{I}_l - I_l} + \hat{p} \frac{\bar{I}_l - I_{l+1}}{\bar{I}_l - I_l} \\
&= \hat{p}, \tag{27}
\end{aligned}$$

for $l = 0, \dots, k$. Therefore, if $R(P_x(I_1, \dots, I_k))$ is maximized at boundary x_{\max} , it is maximized by strategy $P(\hat{p})$.

Because $p_k(x) \geq 0$, the minimum value of x corresponds to $p_k(x_{\min}) = 0$. The following lemma gives us the value for any $p_l(x)$, corresponding to x_{\min} .

Lemma 1. In a k -step strategy, given that $\alpha_l = \frac{I_{l+1} - I_l}{\bar{I}_l - I_l}$, we have:

$$p_l(x_{\min}) = \hat{p}\beta_l, \quad (28)$$

with

$$\beta_l = \sum_{i=l}^{k-1} \alpha_i - \sum_{\substack{i,j=l \\ i \neq j}}^{k-1} \alpha_i \alpha_j + \sum_{\substack{i,j,m=l \\ i \neq j \neq m}}^{k-1} \alpha_i \alpha_j \alpha_m - \dots + (-1)^{k+1-l} \alpha_l \dots \alpha_{k-1}. \quad (29)$$

Notice that $\beta_k = 0$. Without loss of generality, we may take $\beta_0 = 1$ and absorb this value in the β_l . Therefore, if $R(P_x(I_1, \dots, I_k))$ is maximized at boundary x_{\min} , it is maximized by strategy

$$P_\beta(I_1, \dots, I_k) = \begin{cases} \hat{p} & \text{if } I_d < I_1, \\ \beta_l \hat{p} & \text{if } I_l \leq I_d < I_{l+1}, \quad l = 1, \dots, k-1. \\ 0 & \text{if } I_d \geq I_k, \end{cases} \quad (30)$$

Let's take a look at the behaviour of a taxpayer under strategy $P_\beta(I_1, \dots, I_k)$. We separate 4 cases.

- a) Suppose $I \in [0, I_1)$. Because the audit probability is equal to \hat{p} for a declared income in this interval, a taxpayer declares his true income by proposition 1.
- b) Suppose $I \in [I_1, \bar{I}_1)$, so the income is between I_1 and the true income for which it doesn't matter if a taxpayer declares I_1 or I_2 . In this case, the taxpayer will pick a declared income $I_d = I_1$. Because of (30), this corresponds to an audit probability of $\beta_1 \hat{p}$.
- c) Suppose $I \in (\bar{I}_l, \bar{I}_{l+1})$ for $l = 0, \dots, k-1$. Because $I > \bar{I}_l$, it is better for a taxpayer to declare I_{l+1} than I_l . Because $I < \bar{I}_{l+1}$, it is also better for him to declare I_{l+1} than I_{l+1} . Therefore, the declared income equals $I_d = I_{l+1}$. By (30), this corresponds to an audit probability $\beta_{l+1} \hat{p}$. Note that $\beta_k = 0$.
- d) If $I = \bar{I}_l$, it doesn't matter if the taxpayer declares I_l or I_{l+1} . We assume that he declares the value closest to the actual income:

$$I_d = \begin{cases} I_l & \text{if } |I - I_l| \leq |I - I_{l+1}|, \\ I_{l+1} & \text{if } |I - I_l| > |I - I_{l+1}|. \end{cases} \quad (31)$$

Next, we can compute the net tax revenue under strategy $P_\beta(I_1, \dots, I_k) := P_\beta$.

$$\begin{aligned}
R(P_\beta) &= \int_0^{I_1} \{tI - \hat{p}c\} dv(I) + \int_{I_1}^{\bar{I}_1} \left\{ tI_1 + \beta_1 \hat{p}[(f+t)(I - I_1) - c] \right\} dv(I) \\
&\quad + \int_{\bar{I}_1}^{\bar{I}_2} \left\{ tI_2 + \beta_2 \hat{p}[(f+t)(I - I_2) - c] \right\} dv(I) + \dots \\
&\quad + \int_{\bar{I}_{k-2}}^{\bar{I}_{k-1}} \left\{ tI_{k-1} + \beta_{k-1} \hat{p}[(f+t)(I - I_{k-1}) - c] \right\} dv(I) + \int_{\bar{I}_{k-1}}^{\infty} tI_k dv(I) \\
&= \int_0^{I_1} \{tI - \hat{p}c\} dv(I) + \int_{I_1}^{\bar{I}_1} \left\{ (1 - \beta_1)tI_1 + \beta_1(tI - \hat{p}c) \right\} dv(I) \\
&\quad + \int_{\bar{I}_1}^{\bar{I}_2} \left\{ (1 - \beta_2)tI_2 + \beta_2(tI - \hat{p}c) \right\} dv(I) + \dots \\
&\quad + \int_{\bar{I}_{k-2}}^{\bar{I}_{k-1}} \left\{ (1 - \beta_{k-1})tI_{k-1} + \beta_{k-1}(tI - \hat{p}c) \right\} dv(I) + \int_{\bar{I}_{k-1}}^{\infty} tI_k dv(I). \tag{32}
\end{aligned}$$

To continue the rewriting, we will make use of the following lemma.

Lemma 2. For $l = 1, \dots, k$,

$$\begin{aligned}
(1 - \beta_l)tI_l + \beta_l(tI - \hat{p}c) &= (1 - \beta_1)tI_1 + (\beta_1 - \beta_2)t\bar{I}_1 + \dots \\
&\quad + (\beta_{l-1} - \beta_l)t\bar{I}_{l-1} + \beta_l(tI - \hat{p}c). \tag{33}
\end{aligned}$$

We will rewrite (32) using lemma 2. Because $\beta_k = 0$, we see:

$$\begin{aligned}
R(P_\beta) &= \int_0^{I_1} \{tI - \hat{p}c\} dv(I) + \int_{I_1}^{\bar{I}_1} \left\{ (1 - \beta_1)tI_1 + \beta_1(tI - \hat{p}c) \right\} dv(I) \\
&\quad + \int_{\bar{I}_1}^{\bar{I}_2} \left\{ (1 - \beta_1)tI_1 + (\beta_1 - \beta_2)t\bar{I}_1 + (\beta_2 - \beta_3)t\bar{I}_2 + \beta_2(tI - \hat{p}c) \right\} dv(I) + \dots \\
&\quad + \int_{\bar{I}_{k-2}}^{\bar{I}_{k-1}} \left\{ (1 - \beta_1)tI_1 + (\beta_1 - \beta_2)t\bar{I}_1 + \dots + (\beta_{k-2} - \beta_{k-1})t\bar{I}_{k-2} \right. \\
&\quad \quad \left. + \beta_{k-1}(tI - \hat{p}c) \right\} dv(I) + \int_{\bar{I}_{k-1}}^{\infty} tI_k dv(I) \\
&= (1 - \beta_1 + \beta_1 - \beta_2 + \beta_2 - \dots + \beta_{k-1}) \int_0^{I_1} \{tI - \hat{p}c\} dv(I)
\end{aligned}$$

$$\begin{aligned}
& + (1 - \beta_1) \int_{I_1}^{\bar{I}_{k-1}} tI_1 \, dv(I) + (\beta_1 - \beta_2) \int_{\bar{I}_1}^{\bar{I}_{k-1}} t\bar{I}_1 \, dv(I) + \dots \\
& + (\beta_{k-2} - \beta_{k-1}) \int_{\bar{I}_{k-2}}^{\bar{I}_{k-1}} t\bar{I}_{k-2} \, dv(I) \\
& + \int_{I_1}^{\bar{I}_1} (\beta_1 - \beta_2 + \beta_2 - \dots - \beta_{k-1} + \beta_{k-1})(tI - \hat{p}c) \, dv(I) \\
& + \int_{\bar{I}_1}^{\bar{I}_2} (\beta_2 - \beta_3 + \beta_3 - \dots - \beta_{k-1} + \beta_{k-1})(tI - \hat{p}c) \, dv(I) + \dots \\
& + \int_{\bar{I}_{k-2}}^{\bar{I}_{k-1}} \beta_{k-1}(tI - \hat{p}c) \, dv(I) + \int_{\bar{I}_{k-1}}^{\infty} tI_k \, dv(I) \\
= & (1 - \beta_1) \left[\int_0^{I_1} \{tI - \hat{p}c\} \, dv(I) + \int_{I_1}^{\bar{I}_{k-1}} tI_1 \, dv(I) \right] \\
& + (\beta_1 - \beta_2) \left[\int_0^{\bar{I}_1} \{tI - \hat{p}c\} \, dv(I) + \int_{\bar{I}_1}^{\bar{I}_{k-1}} t\bar{I}_1 \, dv(I) \right] + \dots \\
& + (\beta_{k-2} - \beta_{k-1}) \left[\int_0^{\bar{I}_{k-2}} \{tI - \hat{p}c\} \, dv(I) + \int_{\bar{I}_{k-2}}^{\bar{I}_{k-1}} t\bar{I}_{k-2} \, dv(I) \right] \\
& + \beta_{k-1} \int_0^{\bar{I}_{k-1}} \{tI - \hat{p}c\} \, dv(I) + \int_{\bar{I}_{k-1}}^{\infty} tI_k \, dv(I). \tag{34}
\end{aligned}$$

By lemma 2 for $l = k$, with $\beta_k = 0$, we know:

$$tI_k = (1 - \beta_1)tI_1 + (\beta_1 - \beta_2)t\bar{I}_1 + \dots + (\beta_{k-2} - \beta_{k-1})t\bar{I}_{k-2} + \beta_{k-1}t\bar{I}_{k-1}. \tag{35}$$

Therefore, we continue rewriting (34) as follows:

$$\begin{aligned}
R(P_\beta) & = (1 - \beta_1) \left[\int_0^{I_1} \{tI - \hat{p}c\} \, dv(I) + \int_{I_1}^{\infty} tI_1 \, dv(I) \right] \\
& + (\beta_1 - \beta_2) \left[\int_0^{\bar{I}_1} \{tI - \hat{p}c\} \, dv(I) + \int_{\bar{I}_1}^{\infty} t\bar{I}_1 \, dv(I) \right] + \dots \\
& + (\beta_{k-2} - \beta_{k-1}) \left[\int_0^{\bar{I}_{k-2}} \{tI - \hat{p}c\} \, dv(I) + \int_{\bar{I}_{k-2}}^{\infty} t\bar{I}_{k-2} \, dv(I) \right] \\
& + \beta_{k-1} \left[\int_0^{\bar{I}_{k-1}} \{tI - \hat{p}c\} + \int_{\bar{I}_{k-1}}^{\infty} t\bar{I}_{k-1} \, dv(I) \right] \\
& = (1 - \beta_1)R(\tilde{P}(I_1)) + \sum_{l=1}^{k-1} (\beta_l - \beta_{l+1})R(\tilde{P}(\bar{I}_l)). \tag{36}
\end{aligned}$$

We notice that $R(P_\beta)$ is a convex combination of 1-step strategies of the form (12). If we

define:

$$R(\tilde{P}(I^*)) = \max\{R(\tilde{P}(I_1)), R(\tilde{P}(\bar{I}_1)), \dots, R(\tilde{P}(\bar{I}_{k-1}))\}, \quad (37)$$

then:

$$\begin{aligned} R(P_\beta) &\leq (1 - \beta_1)R(\tilde{P}(I^*)) + \sum_{l=1}^{k-1} (\beta_l - \beta_{l+1})R(\tilde{P}(I^*)) \\ &= R(\tilde{P}(I^*)). \end{aligned} \quad (38)$$

We see that strategy P_β is dominated by the 1-step strategy $\tilde{P}(I^*)$. Therefore, the net tax revenue is either optimized by $P(\hat{p})$ (for boundary x_{\max}) or $\tilde{P}(I^*)$ (for boundary x_{\min}). Let's take a look at the difference of the revenues of these strategies.

$$\begin{aligned} R(P(\hat{p})) - R(\tilde{P}(I^*)) &= \int_0^\infty \{tI - \hat{p}c\} dv(I) \\ &\quad - \left[\int_0^{I^*} \{tI - \hat{p}c\} dv(I) + \int_{I^*}^\infty tI^* dv(I) \right] \\ &= \int_{I^*}^\infty \{t(I - I^*) - \hat{p}c\} dv(I). \end{aligned} \quad (39)$$

If (13) holds, then (39) ≥ 0 and strategy $P(\hat{p})$ yields the highest net tax revenue of all possible k -step strategies. If, however, (13) doesn't hold, then (39) < 0 and strategy $\tilde{P}(I^*)$ is the optimal strategy of all possible k -step strategies. This completes the proof of the theorem. ■

Now that we have proven the theorem, we will prove the lemmas used to prove it.

Proof of lemma 1. We will prove the lemma by backwards induction. From the lemma, we notice directly that $p_k(x_{\min}) = 0$, which equals the value that we found ourselves. Also, $p_{k-1}(x_{\min}) = \hat{p}\alpha_{k-1}$ which equals the value that we find computing $p_{k-1}(x_{\min})$ using equation (22). Next, assume that (28) holds for $p_{l+1}(x_{\min})$. Then, because of (22),

$$p_l(x_{\min}) = \hat{p}\alpha_l + p_{l+1}(x_{\min})(1 - \alpha_l)$$

$$\begin{aligned}
&= \hat{p}\alpha_l + (1 - \alpha_l)\hat{p} \left(\sum_{i=l+1}^{k-1} \alpha_i - \sum_{\substack{i,j=l+1 \\ i \neq j}}^{k-1} \alpha_i\alpha_j + \sum_{\substack{i,j,m=l+1 \\ i \neq j \neq m}}^{k-1} \alpha_i\alpha_j\alpha_m - \dots \right. \\
&\quad \left. + (-1)^{k-l}\alpha_{l+1}\dots\alpha_{k-1} \right) \\
&= \hat{p} \left(\sum_{i=l}^{k-1} \alpha_i - \sum_{\substack{i,j=l+1 \\ i \neq j}}^{k-1} \alpha_i\alpha_j + \sum_{\substack{i,j,m=l+1 \\ i \neq j \neq m}}^{k-1} \alpha_i\alpha_j\alpha_m - \dots + (-1)^{k-l}\alpha_{l+1}\dots\alpha_{k-1} \right) \\
&\quad - \hat{p} \left(\sum_{i=l+1}^{k-1} \alpha_l\alpha_i - \sum_{\substack{i,j=l+1 \\ i \neq j}}^{k-1} \alpha_l\alpha_i\alpha_j + \sum_{\substack{i,j,m=l+1 \\ i \neq j \neq m}}^{k-1} \alpha_l\alpha_i\alpha_j\alpha_m - \dots \right. \\
&\quad \left. + (-1)^{k+1-l}\alpha_l\alpha_{l+1}\dots\alpha_{k-1} \right) \\
&= \hat{p} \left(\sum_{i=l}^{k-1} \alpha_i - \sum_{\substack{i,j=l \\ i \neq j}}^{k-1} \alpha_i\alpha_j + \sum_{\substack{i,j,m=l \\ i \neq j \neq m}}^{k-1} \alpha_i\alpha_j\alpha_m - \dots + (-1)^{k+1-l}\alpha_l\dots\alpha_{k-1} \right), \tag{40}
\end{aligned}$$

which equals (28). This finishes the proof of lemma 1. ■

Proof of lemma 2. We will prove lemma 2 by induction. We see directly that the claim holds for the base case $l = 1$. Suppose that (33) holds for $l = i$. Then,

$$(1 - \beta_i)tI_i = (1 - \beta_1)tI_1 + (\beta_1 - \beta_2)t\bar{I}_1 + \dots + (\beta_{i-1} - \beta_i)t\bar{I}_{i-1}. \tag{41}$$

Note that, by lemma 1,

$$\beta_i = \alpha_i + (1 - \alpha_i)\beta_{i+1}, \tag{42}$$

with $\alpha_i = \frac{I_{i+1} - I_i}{\bar{I}_i - I_i}$. We will use this to show that (33) holds for $l = i + 1$.

$$\begin{aligned}
(1 - \beta_{i+1})tI_{i+1} &= (1 - \beta_{i+1})t(I_{i+1} - I_i) + (1 - \beta_{i+1})tI_i \\
&= \alpha_i(1 - \beta_{i+1})t(\bar{I}_i - I_i) + tI_i - \beta_{i+1}tI_i
\end{aligned}$$

$$\begin{aligned}
&= (\alpha_i + \beta_{i+1} - \alpha_i\beta_{i+1})t(\bar{I}_i - I_i) - \beta_{i+1}t(\bar{I}_i - I_i) + tI_i - \beta_{i+1}tI_i \\
&= \beta_i t(\bar{I}_i - I_i) - \beta_{i+1}t\bar{I}_i + tI_i \\
&= (1 - \beta_i)tI_i + (\beta_i - \beta_{i+1})t\bar{I}_i \\
&= (1 - \beta_1)tI_1 + (\beta_1 - \beta_2)t\bar{I}_1 + \dots \\
&\quad + (\beta_{i-1} - \beta_i)t\bar{I}_{i-1} + (\beta_i - \beta_{i+1})t\bar{I}_i,
\end{aligned} \tag{43}$$

which equals (33) for $l = i + 1$. This completes the proof of lemma 2.

■

4 Application of the theorem

We will look at the consequences of the theorem that we proved in the previous section, by applying it to two taxation systems: the Ukrainian and the Dutch system.

4.1 Ukrainian taxation system

In this section, we will show the influence of the theorem on the optimal taxation system in Ukraine. The currency in Ukraine is the Ukrainian hryvnia and the average income is 41.000 UAH per year. A proportionate taxation system is used with a taxation rate of $t = 0.2$ [9]. Because the fine rate f is unknown, we assume $f = 0.5$. Because $\hat{p} = \frac{t}{f+t}$, we have $\hat{p} = \frac{0.2}{0.7} \approx 0.29$. Consider the following 2-step cut-off rule:

$$P(I_1, I_2) = \begin{cases} p_0 & \text{if } I_d < I_1, \\ p_1 & \text{if } I_1 \leq I_d < I_2, \\ p_2 & \text{if } I_d \geq I_2, \end{cases}$$

with $I_1 = 30.000$, $I_2 = 50.000$, $p_1 = 0.22$ and $p_2 = 0.15$. Because of (15),

$$\bar{I}_1 = \hat{p} \frac{I_2 - I_1}{p_1 - p_2} + \frac{p_1 I_1 - p_2 I_2}{p_1 - p_2} = 70.000. \quad (44)$$

Therefore, under $P(I_1, I_2)$, the strategy of the taxpayer is as follows:

$$I_d = \begin{cases} I & \text{if } I < 30.000, \\ 30.000 & \text{if } 30.000 \leq I < 70.000, \\ 50.000 & \text{if } I \geq 70.000. \end{cases}$$

However, because of the theorem, $P(I_1, I_2)$ is either dominated by $P(\hat{p})$ or $\tilde{P}(I^*)$, with I^* such that $R(\tilde{P}(I^*)) = \max\{R(\tilde{P}(I_1)), R(\tilde{P}(\bar{I}_1))\}$. If (13) holds, $P(\hat{p})$ yields the highest net tax revenue:

$$R(P(\hat{p})) = \int_0^\infty \{0.2I - 0.29c\} dv(I). \quad (45)$$

Strategy $\tilde{P}(I^*)$ corresponds to the following net tax revenue:

$$R(\tilde{P}(I^*)) = \int_0^{I^*} \{0.2I - 0.29c\} dv(I) + \int_{I^*}^{\infty} 0.2I^* dv(I). \quad (46)$$

Given the income density in Ukraine and the auditing costs c , these net tax revenues can be computed.

4.2 Dutch taxation system

Because the Ukrainian taxation system has a proportionate nature, the theorem applies directly to it. The Dutch taxation system, on the other hand, is progressive. The possible incomes are divided into 3 intervals [2]:

- (0) If $I \in [0, 19922.5)$, the due tax is 36.55% of the income.
- (1) The second interval actually consists of two intervals whose tax rates are equal, for reasons which we will not explain here. Therefore, we combine them into one interval. If $I \in [19922.5, 66421.5)$, the due tax is 40,4% of the income that falls in this interval plus 0.3655×19922.5 of the income in the first interval.
- (2) If $I \in [66421.4, \infty)$, the due tax is 52% of the income that falls in this interval plus 0.404×46499 of the income that falls in the second and third interval plus 0.3655×19922.5 of the income in the first interval.

The penalty that an individual taxpayer has to pay, only depends on the underdeclared income and on the intention of the taxpayer. For the conscious tax evasion that we consider in this thesis, the fine is equal to $0.5(I - I_d)$ [2]. In addition, of course, the taxpayer has to pay the evaded tax.

We could interpret the Dutch taxation system as a variant on our model, with $I_1 = 19922.5$ and $I_2 = 66421.5$. Because the taxation rates differ for the three intervals, this gives us three different values of \hat{p} . First, we will determine the \hat{p}_i such that individuals with a income in interval $i = 0, 1, 2$ declare their true income. In interval (0), we have the same situation as in proposition 1, with $t = t_0$. For that reason, $\hat{p}_0 = \frac{t_0}{f + t_0} = \frac{0.3655}{0.5 + 0.3655} \approx 0.42$. Having an income in interval (1), gives a taxpayer two options: declaring an income in interval (0) or (1). This gives us two different values for \hat{p}_1 .

- The second option, declaring an income $I \in [I_1, I_2)$, gives the most easy solution. The expected revenue of a taxpayer is in this case equal to

$I - t_0 I_1 - t_1(I_d - I_1) - P(\cdot)(f + t_1)(I - I_d)$. The central authority makes sure taxpayers declare their true income if the following inequality holds:

$$I - t_0 I_1 - t_1(I_d - I_1) - P(\cdot)(f + t_1)(I - I_d) \leq I - t_0 I_1 - t_1(I - I_1). \quad (47)$$

Rewriting this gives us:

$$P(\cdot) \geq \frac{t_1}{f + t_1}. \quad (48)$$

- Declaring, however, an income $I \in [0, I_1)$ gives the following inequality:

$$\begin{aligned} I - t_0 I_d - P(\cdot)[(f + t_0)(I_1 - I_d) + (f + t_1)(I - I_1)] \\ \leq I - t_0 I - P(\cdot)[(f + t_0)(I_1 - I) + (f + t_1)(I - I_1)], \end{aligned} \quad (49)$$

corresponding to

$$P(\cdot) \geq \frac{t_0}{f + t_0} \frac{I - I_d}{I_1 - I_d}. \quad (50)$$

To find out under which conditions the theorem of section 3.1 applies to the Dutch taxation system, we would like \hat{p}_i to be of the same form as \hat{p} in that section. Therefore, we assume that a taxpayer always declares a income in the same interval as his true income. In this case, $\hat{p}_1 = \frac{t_1}{f + t_1} = \frac{0.404}{0.5 + 0.404} \approx 0.45$. For a true income in interval (2) and assuming that an underdeclared income lies in the same interval, a taxpayer declares his true income if the following inequality is satisfied:

$$\begin{aligned} I - t_0 I_1 - t_1(I_2 - I_1) - t_2(I_d - I_2) - P(\cdot)(f + t_2)(I - I_d) \\ \leq I - t_0 I_1 - t_1(I_2 - I_1) - t_2(I - I_2). \end{aligned} \quad (51)$$

Rewriting this gives us again the condition $P(\cdot) \geq \frac{t_2}{f + t_2}$. Therefore, $\hat{p}_2 = \frac{t_2}{f + t_2} = \frac{0.52}{0.5 + 0.52} \approx 0.51$.

Next, we define \bar{I}_1 as the income for which it doesn't matter if a taxpayer declares I_1 or I_2 . If a taxpayer declares I_1 with a true income \bar{I}_1 , he pays an amount $t_0 I_1$ on tax. The chance of getting audited equals p_1 , in which case he has to pay the evaded tax $t_1(\bar{I}_1 - I_1)$ and a fine f over the underdeclared amount: $f(\bar{I}_1 - I_1)$. The expected revenue is in this case equal to $t_0 I_1 + p_1(f + t_1)(\bar{I}_1 - I_1)$. If, however, the taxpayer declares I_2 , the due tax equals $t_0 I_1 + t_1(I_2 - I_1)$. The expected revenue is equal to $t_0 I_0 + t_1(I_2 - I_1) + p_2(f + t_1)(\bar{I}_1 - I_2)$.

Therefore, for \bar{I}_1 , the following equation holds:

$$t_0 I_1 + p_1(f + t_1)(\bar{I}_1 - I_1) = t_0 I_1 + t_1(I_2 - I_1) + p_2(f + t_1)(\bar{I}_1 - I_2). \quad (52)$$

Rewriting this gives us an expression for \bar{I}_1 :

$$\begin{aligned} \bar{I}_1 &= \hat{p}_1 \frac{I_2 - I_1}{p_1 - p_2} + \frac{p_1 I_1 - p_2 I_2}{p_1 - p_2} \\ &= \frac{20780.40 + 19922.5p_1 - 66421.5p_2}{p_1 - p_2}. \end{aligned} \quad (53)$$

Note that the only difference between equation (53) and (15) is that \hat{p} is now replaced by \hat{p}_1 . However, we note that $\hat{p}_0 \leq \hat{p}_1 \leq \hat{p}_2$, which is in contradiction with our assumption of decreasing audit probabilities. It seems like the theorem needs a modification if we want to apply it to the Dutch taxation system.

5 Conclusion and discussion

In this thesis, we considered a gametheoretical model of a taxation system with a proportional tax and fine structure. We showed that, if (13) holds, the optimal strategy for the central authority equals $P(\hat{p})$ with $\hat{p} = \frac{t}{f+t}$. If, on the other hand, (13) doesn't hold, then the optimal strategy is a 1-step cut-off rule of the form (12).

First, we will take a look at the consequences of this result. For a central authority, it seems profitable to invent a k -step cut-off rule for the audit probabilities. However, under the given conditions, there is by the theorem no need to determine such a k -step cut-off rule, because the revenue is either optimized by $P(\hat{p})$ or $\tilde{P}(I^*)$. We provided formulas to compute the value of \hat{p} and given the income density and auditing costs, the value of I^* can be determined. In the application of the Ukrainian taxation system, this is pointed out clearly because the theorem applies directly on this system.

There are, however, a few aspects of this thesis that could be discussed. First of all, we assumed that the audit probabilities are decreasing as the declared income increases. On the one hand, reasoning from the idea that people with high incomes underdeclare their income as much as possible, this seems logical. In this case, the lower the interval of declared incomes is, the more tax evasion the central authority will find. On the other hand, we could argue that most people evading tax will give up a declared income in the same interval or a lower interval close to theirs. The reasoning behind this idea is that the central authority has certain expectations about the income of a taxpayer, depending for example on their job, residence and spending patterns. Because there is more money to win for individuals with high incomes than low incomes if they evade tax, a central authority could choose to use increasing audit probabilities. As we saw in the introduction, there is no consensus on this topic.

Next, we assumed that the auditing probabilities are known to the taxpayer, while this is not very usual. Taxpayers mostly don't know the chances of getting audited, and therefore can't adjust their declared income to this as easily as we made it appear.

Also, we only considered a proportionate tax and fine system, while a lot of authorities handle a progressive system. An example is the Dutch tax system, which divides the incomes into three groups with increasing tax rate. As we saw, the theorem doesn't apply directly to this system. An idea for future research could be to investigate if the results that we found

hold for such a progressive system.

Finally, we didn't take into account the possibility for taxpayers to bribe an auditor. Unfortunately, this is a real issue in most countries. Vasin and Panova (2000) take this into account in their model and it could be an idea to incorporate this in our model too.

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