# University of Utrecht 

Debye Institute
Nanophotonics Group

# Interference in a Bose-Einstein Condensate 

Author:
Bsc. Ambrosius Vermeulen

Supervisor:
Prof. dr. Peter van der Straten


#### Abstract

A BEC can be viewed as a giant matter wave, this means that it can show interference. By creating a double-well potential we can split a BEC into two parts. After turning off the potential the BEC will start to expand because of its chemical potential. As the BEC's expand and the two matter waves superpose they show an interference pattern.

In this thesis it is described how to succesfully create such a double-well potential. The potential created is stable, however not symmetric. The reason why it is not symmetric is investigated and it can be concluded that for a symmetric potential an optical dipole potential with a crosssection of around $10 \mu \mathrm{~m}$ is preferable.

A description is given of the interference pattern that arises from different expansion time and different height of the potential barrier. Because the potential is not symmetric a dependence arises that causes the interference pattern not to be constant in space as observed in previous experiments. This might be an advantage for determining the phase difference between two BEC's.


## Contents

1 Introduction ..... 1
2 Theory ..... 2
2.1 The potential ..... 2
2.1.1 Optical dipole potential ..... 2
2.1.2 Double-well potential ..... 3
2.2 The wavefunction inside the double-well potential ..... 3
2.3 Imaging ..... 4
2.3.1 Absorption imaging ..... 5
2.3.2 PCI imaging ..... 5
2.4 Interference ..... 6
2.4.1 Interference in the atomic density ..... 6
2.4.2 Description of an expanding condensate ..... 7
2.4.3 Combining two expanding wavefunctions ..... 7
2.5 Calculating $\omega_{z}$ ..... 8
2.6 A difference in $\omega_{z}$ ..... 9
3 Experiment ..... 11
3.1 The experiment ..... 11
3.2 The setup ..... 11
3.3 Alignment ..... 12
3.4 Controlling the experiment ..... 13
3.5 Include SLM ..... 14
4 Results ..... 16
4.1 In-situ measurements ..... 16
4.2 Interference at different expansion time ..... 17
4.3 Interference at different green laser power ..... 19
5 Analysis ..... 20
5.1 In-situ measurements ..... 20
5.1.1 Modelling the dipole potential ..... 22
5.2 In expansion ..... 23
5.2.1 Getting the wavelength ..... 23
5.2.2 Extracting $\omega_{z}$ from the wavelength ..... 24
5.2.3 Interference at different expansion time ..... 24
5.2.4 Interference at different green laser power ..... 24
6 Discussion ..... 26
6.1 Difference in $\omega_{z}$ ..... 26
$6.2 \omega_{z}$ is lower than expected ..... 27
6.3 Increase in $\mu$ and laserwidth ..... 28
7 Conclusion ..... 29
8 Outlook ..... 30
8.1 Smaller cross-section of the focused part of the beam ..... 30
8.2 Determine phase difference ..... 30
8.3 Incorporate a SLM in the setup ..... 31
8.4 Dragging the condensate through the beam profile ..... 31
9 Acknowledgements ..... 32
Appendix A Interference in the MT ..... 33
Appendix B SLM python code ..... 34

## 1 Introduction

A Bose-Einstein Condensate (BEC) can be viewed as a giant matter wave. An interesting property of waves is that they can interfere. As waves superpose they form a wave of greater, lower or the same amplitude. For this the waves need to be coherent with each other, meaning that they originate from the same source or that they nearly have the same frequency.

This behaviour has been shown for many types of waves, such as, light, radio, acoustic and surface water waves. Using a setup in which we can create a BEC it is possible to show this behaviour for matter waves. For this an experiment needs to be conducted where two separate condensates come into contact to show matter wave interference.

By creating a double-well potential containing one condensate in each of the wells they can be let to interfere. When the potential is turned off the BEC will start to expand. The condensate will expand with a rate depending on its chemical potential. In the case of a double-well potential we have two condensates which both will expand, demonstrating an interesting property because at some point the condensates will start to superpose. From observation of the evolution of the wavefunction we can learn about the interference pattern.

The interference pattern will start to exist as the two maxima in the combined condensate wavefunction move towards each other because of their chemical potential. It is possible to give a description of the interference pattern based on the shape of the potential. From the description of the interference pattern we can then extract the wavelength of the interference, which can be compared with measurements.

From interference measurements we are interested to observe the different properties of the interference pattern. We are especially interested in the parameters on which the wavelength depends. Measurements that vary the expansion time and the distance between the two minima in the potential have been conducted to search for their dependence. Another parameter of interest is the phase of the interference pattern.

The phase of a BEC is the argument of a complex number in the macroscopic wavefunction, because of this it is unfortunatly not an observable. It is an interesting property of a BEC though, because it contributes to the wave like nature of the system. The relative phase between two BECs is something that can be measured and will be visible in the interference pattern. From the phase difference we might be able to learn about how a BEC acquires its phase.

## 2 Theory

### 2.1 The potential

In this research the goal is to split a BEC in order to see how the two parts interfere. In order to do that it is first needed to create a potential that can split the BEC in two separate parts, which can be turned off to let the BECs interfere. This potential consists of the usual cigar-shaped magnetic trap (MT) [1] and an optical dipole force centered on the magnetic trap, forming a double-well potential.

### 2.1.1 Optical dipole potential

An optical dipole potential is used to trap the Sodium atoms in the two different wells. The mechanism depends on the interaction between light and the induced electric dipole moment in the atom. The light field needs to be far detuned from atomic resonance to keep scattering of photons by the atoms to a minimum.

The force the atoms feel arises from the dispersive interaction of the induced dipole moment in the atom with the intensity gradient of the light field. The potential is thus linear with the intensity of the light field [2],

$$
\begin{equation*}
U_{d i p}=\frac{3 \pi c^{2}}{2 \omega_{0}^{3}} \frac{\Gamma}{\Delta} I, \tag{2.1}
\end{equation*}
$$

while the scattering rate is given by

$$
\begin{equation*}
\Gamma_{s c}=\frac{3 \pi c^{2}}{2 \hbar \omega_{0}^{3}}\left(\frac{\Gamma}{\Delta}\right)^{2} I . \tag{2.2}
\end{equation*}
$$

Here $\omega_{0}$ is the Sodium resonance frequency, $\Gamma$ is the natural linewidth (FWHM), $\Delta$ is the detuning from atomic resonance and $I$ is the light intensity.

In order to choose the right wavelength for the light field it is important to consider the detuning. Except for the light intensity the rest of the parameters, $\omega_{0}=2 \pi \cdot 5.088 \times 10^{14} \mathrm{~Hz}$ and $\Gamma=2 \pi \cdot 9.795 \times 10^{6} \mathrm{~Hz}$, are already determined by the original setup.

In the setup a laser is chosen with $\lambda=532 \mathrm{~nm}$ giving it a detuning with respect to the atomic resonance of natrium of $\left(\lambda_{0}-\lambda\right)=57 \mathrm{~nm}$, which translates into a detuning of $\Delta=5.26 \times 10^{15} \mathrm{~Hz}$. For a laser with a $I_{\max }=1.6 \mathrm{~W}$ the scattering rate is below $\approx 1 \%$ and there is sufficient interaction strength.

### 2.1.2 Double-well potential

The optical dipole potential is aligned at the center of an axially symmetric magnetic trap along the $z$ direction. This will give an effective potential for the atoms,

$$
\begin{equation*}
V(x, y, z)=\frac{1}{2} m \omega_{\perp}^{2}\left(x^{2}+y^{2}\right)+\frac{1}{2} m \omega_{z}^{2} z^{2}+A e^{-\frac{1}{2}\left(\frac{z}{\sigma}\right)^{2}} \tag{2.3}
\end{equation*}
$$

which is a combination of the magnetic trap and the optical dipole potential. Here $\omega_{\perp}$ is the trapping frequency in the $x$ and $y$ direction and $\omega_{z}$ is the trapping frequency in the $z$ direction.

A cross-section of the potential along the $z$ direction is shown in figure 2.1. The atoms will want to seek the minimum in the potential and the BEC will split in two parts both centered at one of the minima, if the height of the potential is larger as compared to the chemical potential.


Figure 2.1: Cross-section of the potential in the $z$ direction showing the magnetic trap in addition with the optical dipole trap.

### 2.2 The wavefunction inside the double-well potential

The wavefunction of a BEC obeys the non-linear Schrödinger equation,

$$
\begin{equation*}
\left(\frac{\hat{p}^{2}}{2 m}+V+U_{0}|\Psi|^{2}\right) \Psi=\mu \Psi \tag{2.4}
\end{equation*}
$$

where $\hat{p}$ is the momentum operator, $V$ the potential, $U_{0}$ is a constant that contains the interactions between atoms in the BEC and $\mu$ is the chemical potential of the BEC.

For a BEC the Thomas-Fermi approximation can be applied,

$$
\begin{equation*}
\left(V+U_{0}|\Psi|^{2}\right) \Psi=\mu \Psi, \tag{2.5}
\end{equation*}
$$

this approximation states that the momentum of the condensate is sufficiently small compared to the interactions such that it can be neglected. From the Thomas-Fermi approximation we can derive the density of the condensate,

$$
\begin{equation*}
n(\vec{r})=|\Psi(\vec{r})|^{2}=\max \left(\frac{\mu-V(\vec{r})}{U_{0}}, 0\right) \tag{2.6}
\end{equation*}
$$

where $V(\vec{r})$ is given by equation 2.3. It is relevant to look at the relation between $\mu$ and $V(\vec{r})$.

In the $x$ and $y$ direction the potential has a minimum in the center of the magnetic trap. The trap will just fill up with atoms until it reaches the chemical potential.

However, when considering the cross-section of the potential in the $z$ direction as shown in figure 2.1, it demonstrates that the optical dipole potential creates a barrier in the center of the magnetic trap with a maximum height in the potential we call $V_{\text {dipole }}$. At the points where $\mu<V_{\text {dipole }}$ the density of the condensate will be zero and atoms will be absent, meaning that the condensate is split. This behavior is shown in figure 2.2.


Figure 2.2: Density of the condensate for different values of $\mu$. (a) $\mu>V_{\text {dipole }}$. (b) $\mu<V_{\text {dipole }}$.

### 2.3 Imaging

In the experiment two different imaging techniques are applied. The techniques used are absorption imaging and phase-contrast imaging (PCI). Both work by aiming
light, called probe light, on the atoms which is imaged on a CCD chip. Absorption imaging allows measurements of the condensate in expansion and PCI allows in-situ measurements. From in-situ measurements it is possible to extract the shape of the potential and a value for the chemical potential, $\mu$.

When light interacts with the atoms it gathers information about the atomic density as it propagates through the condensate. This information is integrated over the propagation direction. Hence, it is necessary to consider the column density of the atoms,

$$
\begin{equation*}
\rho(x, z)=\int n(\vec{r}) d y g=\frac{4}{3 U_{0}}(\mu-V(x, z))^{3 / 2}, \tag{2.7}
\end{equation*}
$$

which is equation 2.6 integrated over the probe axis, the $y$ direction.

### 2.3.1 Absorption imaging

In absorption imaging a beam of resonant light is aimed at the atoms. Resonant light is absorbed by the atoms and the absorbtion casts a shadow on the image. In this process the atoms are excited and the measurement therefore destroys the condensate. The light intensity will behave according to Lambert-Beer's law,

$$
\begin{equation*}
I=I_{0} e^{-\int \sigma \rho d x}=I_{0} e^{-O D}, \tag{2.8}
\end{equation*}
$$

where $\sigma=C_{g, e} \frac{3 \lambda^{2}}{2 \pi}$ is the cross-section for absorption, $\rho$ is the atomic density and OD the optical density. $C_{g, e}$ is the relative transition strength from the ground state $g$ to the excited state $e$.

Three measurements are conducted in order to construct the transmission and to correct for the background. In the first measurement, $I_{\text {atoms }}$, light is shone on the atoms. In the second measurement, $I_{\text {probe }}$, light is shone through the same path as the first measurement, but now without atoms present. The last measurement is done without any probe light, this measurement is called $I_{\text {background }}$. To get the transmission we divide $I_{\text {atoms }}$ by $I_{\text {probe }}$, both corrected for $I_{\text {background }}$,

$$
\begin{equation*}
T=\frac{I_{\text {atoms }}-I_{\text {background }}}{I_{\text {probe }}-I_{\text {background }}} \tag{2.9}
\end{equation*}
$$

From the transmittance, the column density $\rho(x, z)$ can be determined,

$$
\begin{equation*}
\rho(x, z)=\frac{1}{\sigma} \ln \frac{1}{T} \tag{2.10}
\end{equation*}
$$

### 2.3.2 PCI imaging

PCI imaging is based on the phase shift light picks up as it travels through the condensate as a consequence of the refractive index of the condensate. This means
that it is not necessary to use resonant light, presenting several advantages. Because the light in not on resonance it is non destructive, meaning several measurements can be done on the same condensate. It also means that the zeeman shift caused by the magnetic field is still far from resonance allowing measurements in-situ.

After light has travelled though the condensate and has picked up the phase shift, it passes through a phase spot. At the phase spot the light that passed through the condensate is let to interfere with the light that travelled around the condensate.

In the experiment a constant phase shift $\theta=\pi / 3$ is applied at the phase spot. The intensity of the light then behaves according to

$$
\begin{equation*}
I=I_{0}(2-\cos \phi+\sqrt{3} \sin \phi) \tag{2.11}
\end{equation*}
$$

where $\phi$ is the induced phase by the condensate. From the induced phase the column density $\rho(x, z)$ can be determined,

$$
\begin{equation*}
\phi(x, y)=k \frac{\alpha}{2 \epsilon_{0}} \rho(x, z) \tag{2.12}
\end{equation*}
$$

where $k$ is the wavenumber and $\alpha$ is the polarizability of the atoms. Here the equation is not inverted because we are only able to determine the phase from all the pixels.

### 2.4 Interference

When the potential is turned off we want to observe the evolution of the condensate wavefunction in time. Under normal conditions the condensate will expand with a velocity depending on its chemical potential. In our case we have two condensates which both will expand. We are interested in the point where the condensates will start to overlap. It is expected that the condensates will interfere at this point.

### 2.4.1 Interference in the atomic density

If there is coherence, in the sense that there is a constant phase difference between the two BECs, the state may be described by a single condensate wave function [3],

$$
\begin{equation*}
\Psi(\vec{r}, t)=\sqrt{N_{1}} \psi_{1}(\vec{r}, t)+\sqrt{N_{2}} \psi_{2}(\vec{r}, t) \tag{2.13}
\end{equation*}
$$

where N is the expectation value for the number of particles. As explained in section 2.2 the quantity we measure is the particle density. This can be calculated from equation 2.13 and is given by

$$
\begin{equation*}
n(\vec{r}, t)=|\Psi(\vec{r}, t)|^{2}=N_{1}\left|\psi_{1}(\vec{r}, t)\right|^{2}+N_{2}\left|\psi_{2}(\vec{r}, t)\right|^{2}+2 \sqrt{N_{1} N_{2}} \operatorname{Re}\left[\psi_{1}(\vec{r}, t) \psi_{2}^{*}(\vec{r}, t)\right] \tag{2.14}
\end{equation*}
$$

The quantity of interest is the product of the two wavefunctions, this is where the phase difference can be observed. So we want to analyze the evolution in time of,

$$
\begin{equation*}
n_{\text {interference }}(\vec{r}, t)=\operatorname{Re}\left[\psi_{1}(\vec{r}, t) \psi_{2}^{*}(\vec{r}, t)\right] \tag{2.15}
\end{equation*}
$$

in expansion. This will give a result that we can compare to measurements.

### 2.4.2 Description of an expanding condensate

The wavefunction of an expanding condensate in a magnetic trap is given by

$$
\begin{equation*}
\psi(\vec{r}, t)=e^{-i \beta(t)} e^{i m \sum_{j} r_{j}^{2} \dot{\lambda}_{j}(t) / 2 \hbar \lambda_{j}(t)} \times \frac{\tilde{\Psi}\left[\left\{r_{k} / \lambda_{k}(t)\right\}_{k=1,2,3}, t\right]}{\sqrt{\lambda_{1} \lambda_{2} \lambda_{3}}} \tag{2.16}
\end{equation*}
$$

where there is scaling in $\vec{r}$ and $e^{-i \beta(t)}$ is a global phase factor [4]. The dynamics of the expansion of the BEC are contained within the $\lambda$ parameters. For an axially symmetric harmonic trap these are given by,

$$
\begin{equation*}
\lambda_{\perp}(\tau)=\sqrt{1+\tau^{2}} \tag{2.17}
\end{equation*}
$$

and

$$
\begin{equation*}
\lambda_{z}(\tau)=1+\epsilon^{2}\left(\tau \arctan \tau-\ln \sqrt{1+\tau^{2}}\right) \tag{2.18}
\end{equation*}
$$

where $\tau=\omega_{\perp} t$ and $\epsilon=\omega_{z} / \omega_{\perp}$.
Since the BEC is split along the $z$ direction the interference between the two condensates will only be in this direction. In the $x$ and $y$ direction there will be regular expansion. For describing the interference the only direction of interest is thus the $z$ direction, which is given by

$$
\begin{equation*}
\psi(z, t) \propto e^{-i \phi} e^{i m z^{2} \dot{\lambda}_{z}(t) / 2 \hbar \lambda_{z}(t)} \tag{2.19}
\end{equation*}
$$

where we have assumed that the phase $\beta(t)$ is constant in time.

### 2.4.3 Combining two expanding wavefunctions

Using equation 2.19 it is possible to describe two BECs, one centered at $-d / 2$ and one centered at $d / 2$ with respect to the geometric center of the trap in the $z$ direction by,

$$
\begin{equation*}
\psi_{1}(z, t) \propto e^{-i \phi_{1}} e^{i m\left(z-\frac{d}{2}\right)^{2} \lambda_{z, 1}(t) / 2 \hbar \lambda_{z, 1}(t)} \tag{2.20}
\end{equation*}
$$

and

$$
\begin{equation*}
\psi_{2}(z, t) \propto e^{-i \phi_{2}} e^{i m\left(z+\frac{d}{2}\right)^{2} \lambda_{z, 2}(t) / 2 \hbar \lambda_{z, 2}(t)} \tag{2.21}
\end{equation*}
$$

From substituting equation 2.20 and equation 2.21 into equation 2.15, we obtain $n_{\text {interference }}$ in expansion,

$$
\begin{equation*}
n_{\text {interference }}(\vec{r}, t)=\cos \left[\phi_{1}+\frac{m}{2 \hbar}\left(\left(z-\frac{d}{2}\right)^{2} \frac{\dot{\lambda_{z, 1}}(t)}{\lambda_{z, 1}(t)}-\phi_{2}-\frac{m}{2 \hbar}\left(\left(z+\frac{d}{2}\right)^{2} \frac{\dot{\lambda_{z, 2}}(t)}{\lambda_{z, 2}(t)}\right],\right.\right. \tag{2.22}
\end{equation*}
$$

which is an equation of the form,

$$
\begin{equation*}
\cos [k z+\Delta \phi] \tag{2.23}
\end{equation*}
$$

from this relation we can thus extract the wavenumber of the interference pattern. The only terms of interest for the wavenumber are those proportional to $z$,

$$
\begin{equation*}
k=\left|\frac{m}{2 \hbar}\left((z-d) \frac{\dot{\lambda}\left(\epsilon_{1}, t\right)}{\lambda_{z}\left(\epsilon_{1}, t\right)}-(z+d) \frac{\dot{\lambda}\left(\epsilon_{2}, t\right)}{\lambda_{z}\left(\epsilon_{2}, t\right)}\right)\right| \tag{2.24}
\end{equation*}
$$

and accordingly we can derive the wavelength of the interference,

$$
\begin{equation*}
\lambda=\left|\frac{2 \hbar}{m}\left(\frac{1}{(z-d) \dot{\lambda_{z}\left(\epsilon_{1}, t\right)} \dot{\lambda}_{z}\left(\epsilon_{1}, t\right)}-(z+d) \frac{\dot{z}\left(\epsilon_{2}, t\right)}{\lambda_{z}\left(\epsilon_{2}, t\right)}\right)\right| . \tag{2.25}
\end{equation*}
$$

This result can be compared to measurements. As can be observed it depends on the distance between the two BECs, the two values for $\epsilon$ and the time of expansion. The distance between the two BECs, $d$ can be measured in-situ and the time of expansion, $t$, is known with a precision of the order of $1 \mu \mathrm{~s}$. For $\epsilon=\omega_{z} / \omega_{\perp}$ the transverse trap frequency, $\omega_{\perp}$, is also precisely known from previous measurements. The values for the trap frequency in the $z$ direction, $\omega_{z}$, are the only free parameters, which are to be determined by fitting equation 2.25 to the data.

### 2.5 Calculating $\omega_{z}$

The expected value for $\omega_{z}$ can be been calculated from equation 2.3. For the calculation the approximation has been made that the potential is still a harmonic trap of the form,

$$
\begin{equation*}
V_{z}=\frac{1}{2} m \omega_{z}^{2} z^{2} . \tag{2.26}
\end{equation*}
$$

If this is differentiated twice with respect to $z$ we obtain,

$$
\begin{equation*}
\frac{d^{2}}{d z^{2}} V_{z}=m \omega_{z}^{2} \tag{2.27}
\end{equation*}
$$

and we thus have the value for omega

$$
\begin{equation*}
\omega_{z}=\sqrt{\frac{m}{\frac{d^{2}}{d z^{2}} V_{z}}} . \tag{2.28}
\end{equation*}
$$

We subsitute the cross-section of equation 2.3 along the $z$ direction into this equation to arrive at figure 2.3 and we expect a value of around $2 \pi \cdot 20 \mathrm{~Hz}$ for $\omega_{z}$.

Trapfrequency (Hz)


Figure 2.3: Trap frequency calculated for a laser width of $56.7 \mu \mathrm{~m}$.

### 2.6 A difference in $\omega_{z}$

The trap frequencies, $\omega_{z}$, are expected to be the same for the two condensates due to the alignment of the dipole trap with respect to the magnetic trap. However, experimentally it will be very hard to have an exactly symmetric potential, so it will most likely be of the form $\omega_{z 1} \approx \omega_{z 2}$.

In the case of a symmetric trap, $\omega_{z 1}=\omega_{z 2}$, indicates that $\epsilon_{1}=\epsilon_{2}$ and hence that $\lambda_{z}\left(\epsilon_{1}, t\right)=\lambda_{z}\left(\epsilon_{2}, t\right)$. This would simplify equation 2.25 , leading to

$$
\begin{equation*}
\lambda=\frac{4 \hbar}{m d} \frac{\lambda_{z}(\epsilon, t)}{\dot{\lambda_{z}}(\epsilon, t)}, \tag{2.29}
\end{equation*}
$$

which gives a constant interference pattern in the $z$ direction. This is what has been observed by Andrews et al. [5].

Close examination of equation 2.25 shows that a difference in $\omega_{z}$ causes a $1 / z$ dependence in the wavelength. This difference in behavior of the interference pattern is shown in figure 2.4.

It can also be observed that the center of the interference pattern moves in time when there is a difference in $\omega_{z}$. It moves to the left, if $\omega_{z}$ on the left is smaller and to the right, if $\omega_{z}$ on the right is smaller.


Figure 2.4: Interference pattern for different values of $\omega_{z}$ showing (a) $\omega_{z 1}=\omega_{z 2}$ and (b) $\omega_{z 1} \approx \omega_{z 2}$, here $\omega_{z 1}=2 \pi \cdot 3 \mathrm{~Hz}$ and $\omega_{z 1}=2 \pi \cdot 5 \mathrm{~Hz}$.

## 3 Experiment

### 3.1 The experiment

In the experiment a Bose-Einstein condensate is created containing about $10^{7}$ atoms. The chemical potential of our condensate is around $3 \times 10^{-30} \mathrm{~J}$. The condensate is created in an axial symmetric magnetic trap. The magnetic trap has a radial trap frequency of 16 Hz and an axial trap frequency of $95 \mathrm{~Hz}[1]$.

After the condensate has been created in the magnetic trap a beam in the form of an optical sheet is switched on, generating a repulsive optical dipole potential. The sheet is switched on for about 500 ms , long enough for the condensate to get into equilibrium with the magnetic and optical forces. It is aligned at the center of the magnetic trap creating a double-well potential which can be directly observed in in-situ images in the magnetic trap using PCI.

### 3.2 The setup

In this experiment the laser used has a wavelength of 532 nm , yielding a detuning of $57 \mathrm{~nm}\left(\Delta=+5.26 \times 10^{15} \mathrm{~Hz}\right)$ from atomic resonance. In this regime the scattering rate is of the order $10^{-4} \mathrm{~s}^{-1}$ whilst maintaining a optical dipole potential in the order $3 \times 10^{-30} \mathrm{~J}$ for output power in the order 1 W . The laser has an adjustable output power with a maximum of 1.6 W .

| $\lambda$ | 532 nm | Power | 1.6 W |
| :--- | :---: | :--- | :---: |
| Bandwidth | $<40 \mathrm{GHz}$ | Mode | TEM $_{00}$ |

Table 3.1: Technical data of the used Laser Quantum ventus laser. [6]
The green laser beam is focused into a thin sheet with a cross section at the BEC of about $55 \mu \mathrm{~m}$ by $200 \mu \mathrm{~m}\left(1 / e^{2}\right.$ radii) using a cylindrical lens. The beam profile at the focus of the cylindrical lens is shown in figure 3.1. The beam profile has been detected by diverting the beam and placing a camera at the BEC position.


Figure 3.1: The beam profile of the 532 nm laser at the focus.

Figure 3.2 shows the setup. The green laser beam passes through an AOM and is fiber ported to the setup. It passes through a cylindrical lens with $\mathrm{f}=50 \mathrm{~cm}$. The focal length is the distance between the cylindrical lens and the condensate in order to generate a focused optical sheet in the center of the magnetic trap. After the cylindrical lens a mirror is placed, which can be used for aligning the beam at the center of the magnetic trap. The beam then passes through the window into the experimental chamber.


Figure 3.2: Experimental setup for creating the optical dipole potential and aligning on the BEC.

### 3.3 Alignment

The setup is aligned on the center of the magnetic trap by observing the interaction of light with atoms. The size of atomic cloud decreases in the cooling process as the atoms get pushed further down in the magnetic trap as the temperature decreases. A BEC has the size of $\approx 250 \mu \mathrm{~m}$ whilst the atomic cloud before reaching BEC measures $\approx 6 \mathrm{~mm}$. Because it is easier to align the beam on a large atomic cloud this procedure is chosen.


Figure 3.3: Alignment of the beam on the atomic cloud.
The atomic cloud of dimensions described above has a chemical potential of the order $4 \times 10^{-27} \mathrm{~J}$. An optical dipole force with a much larger potential is needed to show
effect for a system with such a chemical potential. Our green laser does not have this power, it only reaches the order of $10^{-30} \mathrm{~J}$, so it can not be used for this purpose.

In the alignment a workaround is used. Light with a detuning of $\Delta=27 \mathrm{MHz}$ with respect to the atomic resonance is flashed on the condensate. In figure 3.3 a measurement is shown in which this near resonance light is used to align the beam at the center of the magnetic trap.

After centering the beam on the atomic cloud the sheet is centered on the magnetic trap more precisely by using the green laser on the condensate. At this point the centering can be fine tuned. In figure 3.4 a condensate split in two is shown.


Figure 3.4: The interaction between the BEC and the green laser at 1.6 W .

### 3.4 Controlling the experiment

The beam separating the two condensates and the magnetic trap needs to be switched off instantly in order to be able to incorperate the theoretical description in the analysis. A ramp in the potential is problematic as it makes the atoms still feel a slight potential as the two condensates start moving towards each other in expansion. As can been seen from figure 3.5 a mechanical shutter has a response time in the order of ms. The magnetic trap is switched off in time of the order of $\mu \mathrm{s}$, so the reaction time from just the mechanical shutter is too low.

In the experiment we complement the mechanical shutter with an Acousto-Optic Modulator (AOM), which has the response time in the order of ns. An AOM uses the acousto-optic effect to diffract and shift the frequency of light using sound waves. The incoming light is diffracted into several orders with an increasing frequency. The increment of frequency per order is negligible for our purposes. We only use the different orders for switching faster. The first order of the AOM is coupled into the fiber port that goes to the setup and this allows us to switch the beam off with ns precision.

In order to properly synchronize the experiment a program based on Wordgenerator 3.14, developed at MIT, is used [7]. The program allows to time the inhibit of the


Figure 3.5: Image from an oscilloscope showing a mechanical shutter response time of 12.6 ms . The orange shows the trigger and the blue/green shows intensity of a beam measured after the shutter.

AOM and trigger for the mechanical shutter with switching off the magnetic trap.

### 3.5 Include SLM

A next step would be incorperating a Spatial Light Modulator in the setup. A spatial light modulator (SLM) can be used to control the light field of the green laser. The type of SLM to be used in this research is a digital mirror device (DMD) and it comprises of 786,432 small mirrors arranged on a rectangular grid. These mirrors can be controlled seperately by a computer to be in an on- or off-state ( $\mp 12^{\circ}$ relative to the DMD plane). Around the active mirrors, a band of 10 mirrors is situated which are always in the off-state $\left(+12^{\circ}\right)$ and which cannot be controlled. When light hits the DMD, the amplitude, phase and direction of the reflected light field can be shaped. The technical data of the used SLM is presented in table 3.2.

| Pixels | $1024 \times 768$ | Mirror Reflectivity | $88 \%$ |
| :--- | :---: | :--- | :---: |
| Pitch | $10.8 \cdot 10^{-6} \mu \mathrm{~m}$ | Mirror Fill Factor | $92 \%$ |
| Tilt Angle | $\pm 12^{\circ}$ | Window Transmitance | $97 \%$ |
| Range of $\lambda$ | $420 \mathrm{~nm}-700 \mathrm{~nm}$ | Data Clock Rate | 200 MHz |

Table 3.2: Technical data of the used Texas Instruments DLP5500 DMD [8].
For the incoming light the many mirrors act as a diffraction grating. The diffracted
light will have maxima at angles $\theta$ given by

$$
\begin{equation*}
m \lambda=d \sin (\theta) \tag{3.1}
\end{equation*}
$$

with $m$ the order of the diffraction, $\lambda$ the wavelength of the incident light, $d$ the distance between the mirrors. This can be used to optimize the efficiency of the SLM [9].


Figure 3.6: Setup of figure 3.2 including a SLM.
The experiment will have almost the same setup as in figure 3.2, but includes the SLM to mask the center of the laserfield. When using this mask a weak link can be created in the thin sheet. In this experiment the condensate could be split and come into contact again through the weak link. This should give interference as the two condensates come into contact.


Figure 3.7: The cross-sections of the shaped lightfield in both directions. The top figure shows the $y$ direction. Here the weak link is shown at different sizes, ranging from a row of 25 mirrors turned off on the SLM to a row of 2 mirrors. The bottom figure shows the $x$ direction which is not shaped.

## 4 Results

The expansion time and the power of the green laser are the main free parameters in the experimental setup. As a consequence measurements for different expansion time, as well as different green laser power have been done to see if this has an effect on $\omega_{z}$.

In order to analyse the BEC interference pattern several additional measurements are done. We need to fix the parameters for the distance between the two BECs, $d$, to make sure that the only free parameter in equation 2.25 is the value for trap frequency in the z-direction, $\omega_{z}$. In order to obtain $d$ in-situ measurements are done. This has been done for different green laser power, determining $d$ indirectly. After the in-situ measurements, measurements in expansion are done from which $\omega_{z}$ is determined.

This sums up to the following measurements:

- In-situ measurements at different green laser power, to relate the green laser power to $d$;
- Interference of the BECs at different expansion time at a single green laser power;
- Interference of the BECs at a constant expansion time but with different green laser power.


### 4.1 In-situ measurements

We can determine $d$ from the shape of the potential. The geometrical mean of the system is also of interest, since this is needed to determine the origin of the z -axis for equation 2.25 . Other parameters are also extracted from the in-situ measurements to assess the stability and performance of the experiment, which can be used for discussion of the accuracy of the data.

The measurement is done for green laser powers of $700-1600 \mathrm{~mW}$ with increments of 50 mW . Each measurement has been repeated 10 times. From the results it can be seen that with increasing green laser power the distance between the two BECs increases. The behavior is as expected, a larger green laser power translates to a larger optical dipole potential, meaning that the minima in the potential are further apart.


Figure 4.1: In-situ measurements at different green laser power (a) 1600 mW (b) 1550 mW (c) 1500 mW
(d) 1450 mW (e) 1400 mW (f) 1350 mW (g) 1300 mW (h) 1250 mW (i) 1200 mW (j) 1150 mW (k)

1100 mW (I) 1050 mW (m) 1000 mW (n) 950 mW (o) 900 mW (p) 850 mW (q) 800 mW (r) 750 mW (s) 700 mW

### 4.2 Interference at different expansion time

With the values for $d$ known the only free parameters are the values for $\omega_{z}$, which can be determined from the interference pattern of the BEC in expansion. The measurement is done for an expansion time of 5-70 ms with increments of 5 ms and a green laser power of 1000 mW . Each measurement has been repeated 15 times.

From the results it can be seen that the BECs start moving towards each other at 5 ms and slowly start to show signs of interference in the region where they overlap. At 30 ms of expansion time the behavior becomes more distinct.

We can clearly see that the interference pattern is not constant in the $z$ direction, which indicates a $1 / z$ dependence as discussed in section 2.6. A $1 / z$ dependence would tell us that the $\omega_{z}$ for the two BECs are different. The images are cropped and are not shown in a coordinate system with the same origin. From the full figures it can be observed that the condensate is moving towards the left on the CCD. This would hint that the $\omega_{z}$ of the BEC on the right is bigger than on the left.


Figure 4.2: Seperation at a green laser power of 1000 mw . (a) 5 ms (b) 10 ms (c) 15 ms (d) 20 ms (e) 25 ms (f) 30 ms (g) 35 ms (h) 40 ms (i) 45 ms (j) 50 ms (k) 55 ms (I) 60 ms (m) 65 ms ( n$) 70 \mathrm{~ms}$

### 4.3 Interference at different green laser power

We also observed the interference pattern for different green laser power. This will change the value for $d$ in equation 2.25. It might also give rise to a change of values for $\omega_{z}$ because a steeper potential will increase $\omega_{z}$.

The measurement is done for a green laser power of $700-1300 \mathrm{~mW}$ with increments of 25 mW and an expansion time of 40 ms . Each measurement has been repeated 10 times.

At first glance the interference pattern does not seem to change with different green laser power. This would mean that the green laser power does not effect $\omega_{z}$.


Figure 4.3: Seperation at 40 ms of expansion time. (a) 1300 mW (b) 1000 mW (c) 974 mW (d) 950 mW (e) 924 mW (f) 900 mW (g) 874 mW (h) 850 mW (i) 826 mW (j) 800 mW (k) 774 mW (I) 750 mW (m) 726 mW (n) 700 mW

## 5 Analysis

### 5.1 In-situ measurements

From the results of the in-situ measurements the parameters of interest can be obtained from,

$$
\begin{equation*}
\rho(x, z)=a \cdot \max \left[\mu-\left(V_{\text {magnetic }}(x, z)+V_{\text {dipole }}(z)\right), 0\right]^{3 / 2} \tag{5.1}
\end{equation*}
$$

the potentials are given by,

$$
\begin{align*}
& V_{\text {magnetic }}(x, z)=\frac{1}{2} m \omega_{\text {mag }, \perp}^{2}\left(x-x_{\text {center }}\right)^{2}+\frac{1}{2} m \omega_{\text {mag }, z}^{2}\left(z-z_{\text {center }}\right)^{2} \\
& V_{\text {dipole }}(z)=\text { gauss }  \tag{5.2}\\
& \text { height }
\end{align*} \cdot \operatorname{Exp}\left[-\frac{1}{2}\left(\frac{z-z_{\text {center }}-\text { gauss }_{\text {shift }}}{\text { gauss }}\right)_{\text {width }}^{2}\right], ~ \$
$$

where $\omega_{m a g, \perp}$ is $2 \pi \cdot 95.5 \mathrm{~Hz}, \omega_{m a g, z}$ is $2 \pi \cdot 16 \mathrm{~Hz}, a$ is a scaling factor proportional to $U_{0}, x_{\text {center }}$ determines the center of the magnetic trap in the x direction, $z_{\text {center }}$ determines the center of the magnetic trap in the z direction, gauss $_{\text {height }}$ determines the height of the optical dipole potential, gauss shift determines the position of the optical dipole potential with respect to the center of the magnetic trap in the z direction and gauss width determines the width of the optical dipole potential.

The parameters $x_{\text {center }}$ and $z_{\text {center }}$ should stay constant throughout all the measurements as the center of the magnetic trap does not change. These parameters are a check on the stability of the experiment. Gauss shift is an additional check on the stability of the experiment as we do not expect the green laser to move during the experiment.


Figure 5.1: (a) Fluctuation of $x_{\text {center }}$ and $z_{\text {center }}$ around their mean, representing the position of the MT. (b) Fluctuation of gauss shift, representing the position of the dipole potential with respect to the center of the MT.

The monitoring of $x_{c e n t e r}$ shows a clear movement during the measurement. The position of the magnetic trap is expected to be constant. From $z_{\text {center }}$ we can also determine to origin of the z-axis defining the geometrical mean of the BEC. We will need this later for fitting the interference pattern of the BEC in expansion.


Figure 5.2: (a) Width of the dipole potential versus laser power. (b) $\mu$ versus laser power.

In figure 5.2 we see the width of the dipole potential, being the width of the green laser beam, and the chemical potential, $\mu$, of the condensate. The width of the green laser beam is not expected to change with increased power. It seems to be an artefact of the fit and hints that something else might be going on. Although the increasing $\mu$ can be expected, both plots show large correlation.

If we put the found parameters into $V=V_{\text {magetic }}+V_{\text {dipole }}$ and determine the minima, the distance between these points will be our $d$ parameter. In figure 5.3 the $d$ parameter used in the analysis for expansion is shown.


Figure 5.3: Distance versus laser power.

### 5.1.1 Modelling the dipole potential

Another check for consistency of the data is to look if Gaussheight is proportional to the power of the green laser. Gauss height can be determined using equation 2.1 where $I_{0}$ corresponds to the amount of power we put into the green laser. The rest of the parameters are known, $\omega_{0}=2 \pi \cdot 508.8487162 \cdot 10^{12} \mathrm{~Hz}, \Gamma=2 \pi \cdot 9.7946 \cdot 10^{6}$ Hz and $\Delta=5.263 \cdot 10^{15} \mathrm{~Hz}$.

We first determine $I_{0}$,

$$
\begin{equation*}
I_{0}=\frac{P}{\iint e^{-x^{2} / 2 \sigma_{x}^{2}} e^{-y^{2} / 2 \sigma_{y}^{2} d x d y}}, \tag{5.3}
\end{equation*}
$$

if we use the values for the beam profile of the green laser, $\sigma_{x}=56.70 \pm 10 \mu \mathrm{~m}$ and $\sigma_{y}=795 \pm 15 \mu \mathrm{~m}$ and we obtain

$$
\begin{equation*}
I_{0}=\frac{P}{2.83 \pm 0.05 \cdot 10^{-7}}, \tag{5.4}
\end{equation*}
$$

which we can use to compare to the values of Gaussheight.


Figure 5.4: Dipole potential strenght versus laser power compared to the expected value. The green area is within $1 \sigma$ and the yellow area is within $2 \sigma$.

From figure 5.4 we can see that the Gaussheight is in good agreement with the expected value.

### 5.2 In expansion

### 5.2.1 Getting the wavelength

From the data it is not directly possible to extract the wavelength. Assuming the interference pattern is sinusoidal, we can find the wavelength by determining the difference between the minima and maxima.

The method used for determining the minima and maxima is by cutting the signal up into the different regions with a isolated minimum or maximum. The maxima or minima are found iteratively by looking at three data points on each side of a datapoint to account for noise. For a maximum this means that it is bound between datapoints that are lower on the left and right. For the minimum this means that it is bound between datapoints that are higher on the left and right.

In these regions a parabola is used to model the minimum or maximum,

$$
\begin{equation*}
f(x)=a+b \cdot(x-c)^{2} \tag{5.5}
\end{equation*}
$$

where $a$ is used to scale the height, $b$ is the amplitude and $c$ is the position. The number of datapoints used for fitting is half the datapoints within the region, with a minimum of 4 datapoints, in order to have sufficient datapoints for a fit. When c is known for each minimum and maximum the difference can be calculated and we arrive at figure 5.5.


Figure 5.5: (a) The measured interference pattern after 40 ms of expansion. The top plot shows the raw signal with a Thomas-Fermi fit. In the plot in the middle the Thomas-Fermi fit is subtracted from the raw signal and the minima and maxima are determined iteratively. In the bottom plot the minima and maxima have been divided into isolated regions and a parabola has been fit in each region. (b) The difference between the position, $c_{j}-c_{i}$, of the minima and maxima is shown versus the position of the wavelength at that point, $\left(c_{i}+c_{j}\right) / 2$

### 5.2.2 Extracting $\omega_{z}$ from the wavelength

Once we obtained the distribution for the wavelength, equation 2.25 is used to fit the interference patterns. In order to extract $\omega_{z}$ the following function has been used,

$$
\begin{equation*}
\lambda=2 \pi \cdot \frac{2 \hbar}{m}\left(\frac{1}{\left(z-z_{\text {center }}-d\right) \cdot \beta\left(\omega_{z 1}\right)-\left(z-z_{\text {center }}+d\right) \cdot \beta\left(\omega_{z 2}\right)}\right) \tag{5.6}
\end{equation*}
$$

where $\beta$ contains $\omega_{z}$,

$$
\begin{equation*}
\beta\left(\omega_{z}\right)=\frac{1+\left(\frac{\omega_{z}}{\omega_{\perp}}\right)^{2}\left(\omega_{\perp} t \cdot \arctan \left(\omega_{\perp} t\right)-\ln \sqrt{1+\left(\omega_{\perp} t\right)^{2}}\right)}{\left(\frac{\omega_{z}}{\omega_{\perp}}\right)^{2} \omega_{\perp} \arctan \left(\omega_{\perp} t\right)} \tag{5.7}
\end{equation*}
$$

where $z_{\text {center }}$ and $d$ are known from equation $5.2, t$ is known in the experiment and $\omega_{\perp}$ is known from previous measurements. This leaves $\omega_{z}$ the only parameter to be determined.

The parameter $\omega_{z}$ has been determined for different measurements in expansion. Both expansion time and the green laser power have been varied.

### 5.2.3 Interference at different expansion time

For different expansion time, $t$, equation 5.6 has been fitted to the data. The expansion time, $t$, is known within ms precision, the distance, $d$, is known from the in-situ data and the axial trap frequency, $\omega_{\perp}$, is taken from previous experiments. The only free parameter to be determined is $\omega_{z}$ which is shown in figure 5.6

The trap frequency is smaller on the left side which tells us the condensate moves to the left. In the data this can be clearly seen, as the condensate has an overal movement of approximately $5.5 \mu \mathrm{~m} / \mathrm{ms}$.

We have estimated the trap frequency to be around 20 Hz for a $56.7 \mu \mathrm{~m}$ laser width. The measured trap frequency is smaller than expected. This may be due to the fact that the condensate is not fully split.

### 5.2.4 Interference at different green laser power

Equation 5.6 has also been fitted to the data for different green laser power. In the equation this corresponds to a different value for $d$. Again $t, d$ and $\omega_{\perp}$ are known. The free parameter $\omega_{z}$ is shown in figure 5.7.

The measured trap frequency at different green laser power is consistent with the trap frequency at different expansion time. This results demonstrate that the interference


Figure 5.6: Trap frequency in Hz of the two different wells versus expansion time.
pattern does not change with different green laser power. Or within the region of dipole strengths in the measurements the change in steepness of the potential may not be visible. The width of the light sheet is probably a much stronger experimental parameter for increasing the value of $\omega_{z}$ as this will make the potential much steeper.


Figure 5.7: Trap frequency in Hz of the two different wells versus power of the green laser.

## 6 Discussion

### 6.1 Difference in $\omega_{z}$

It has been observed that there is a $1 / z$ dependence in the interference pattern we measure. This dependence arrises from the relative difference in $\omega_{z}$. In simular experiments [5] this behaviour has not been observed. One difference between our experiment and other experiments is the cross-section ( $1 / e^{2}$ radius) of the focused part of the beam profile. In our experiment this is around $55 \mu \mathrm{~m}$ while in other experiments this is around $12 \mu \mathrm{~m}$. This causes a difference in the dipole potential. The difference in the dipole potential can be observed in terms of a different trap frequency as shown in figure 6.1.


Figure 6.1: Trap frequency from different cross-sections of the focused part of the beam profile. Figure (a) shows a cross-section of $56.7 \mu \mathrm{~m}$ and figure (b) of $12 \mu \mathrm{~m}$.

We can observe whether this difference in trap frequency is the cause of our large dependence on $1 / z$ by rewriting equation 2.25 ,

$$
\begin{equation*}
\lambda=\left|\frac{2 \hbar}{m}\left(\frac{1}{z\left(\frac{\dot{\lambda_{z}}\left(\epsilon_{1}, t\right)}{\lambda_{z}\left(\epsilon_{1}, t\right)}-\frac{\dot{\lambda}_{z}\left(\epsilon_{2}, t\right)}{\lambda_{z}\left(\epsilon_{2}, t\right)}\right)+d\left(-\frac{\dot{z}_{z}\left(\epsilon_{1}, t\right)}{\lambda_{z}\left(\epsilon_{1}, t\right)}-\frac{\dot{\lambda}_{z}\left(\epsilon_{2}, t\right)}{\lambda_{z}\left(\epsilon_{2}, t\right)}\right)}\right)\right| \tag{6.1}
\end{equation*}
$$

and consequently by scaling the prefactor of $z$ to the prefactor of $d$,

$$
\begin{equation*}
\alpha(\omega)=\frac{\frac{\dot{\lambda}(\omega)}{\lambda(\omega)}-\frac{\dot{\lambda}(\omega+\delta)}{\lambda(\omega+\delta)}}{-\frac{\dot{\lambda}(\omega)}{\lambda(\omega)}-\frac{\dot{\lambda}(\omega+\delta)}{\lambda(\omega+\delta)}} \tag{6.2}
\end{equation*}
$$

For constant $t$ and a constant $\delta$ of 2 Hz , this factor is shown in figure 6.2. From the figure we can see that in our case the dependence on $1 / z$ is much larger. At the point where we get to a trap frequency of 60 Hz we can see that the dependence on $1 / z$ has become neglegible. This might explain why other experiments did not observe the same behaviour.


Figure 6.2: For a constant $\delta$ of 2 Hz the fraction $\alpha$ is shown. It can be seen that for a larger trap frequency the contribution of $1 / z$ seems to become neglegible.


Figure 6.3: The factor of the contribution of $1 / z$ to the wavefunction for two different cross-sections of the focused part of the beam profile, both for have a $\delta$ of 2 Hz . Figure (a) shows a beam with a cross-section of $56.7 \mu \mathrm{~m}$ and figure (b) of $12 \mu \mathrm{~m}$.

## $6.2 \omega_{z}$ is lower than expected

We expect a trap frequency of around 20 Hz from figure $5.5(\mathrm{a})$ on both sides of the trap. However, we observe a trap frequency of around 5 Hz (see figure 5.6 and 5.7). The reason for this may be the value for $V_{\text {dipole }}$ used when calculating the trap frequency of figure 5.5(a).

In the calculation we have used that $V_{\text {dipole }}$ is $2.5 \times 10^{-30} \mathrm{~J}$. The measurements of the interference pattern in expansion have been done at a laser intensity of 1 W , which according to figure 5.4 would mean a $V_{\text {dipole }}$ of $1.8 \times 10^{-30} \mathrm{~J}$. The figure with a $V_{\text {dipole }}$ of $1.8 \times 10^{-30} \mathrm{~J}$ resembles the figure of $2.5 \times 10^{-30} \mathrm{~J}$.

However, in figure 6.4 a $V_{\text {dipole }}$ of $1.2 \times 10^{-30} \mathrm{~J}$ has been used. Here, we observe a condensate that is not fully separated into two parts and the trap frequency in the center is much lower. The trap frequency shown in figure 6.4 seems to coincide with the measured trap frequency.


Figure 6.4: Trap frequency from a crosssection of $56.7 \mu \mathrm{~m}$ with a height of $1.2 \times 10^{-30} \mathrm{~J}$.

From the analysis figures $5.2(\mathrm{~b})$ and figure 5.4 we observe that $\mu>V_{\text {dipole }}$ for all of the measurements making the condensate appear not to be fully split. Also from the fact that a large fraction of the atoms seems to stay in the center in expansion hints that it is not fully split (see figure 4.2 and 4.3). The case of figure 6.4 thus seems to give a sensible explanation for the measured trap frequency.

### 6.3 Increase in $\mu$ and laserwidth

The increase in $\mu$ in figure $5.2(\mathrm{~b})$ might also be an artefact of the fit. We see an increase in the laser width with increasing laser power which seems to be strongly correlated to the increase in $\mu$. Both figures show almost the exact same behaviour. On top of that an increase in the laser width is something that we do not expect with increasing laser power. The laser width is expected to stay constant. If the increase in $\mu$ can not be correctly judged from figure 5.2(b) than our comparison to $V_{\text {dipole }}$ is thus also incorrect.

However, from a phenomenological point of view $\mu$ is expected to increase when the trap frequency increases, which seems to be the case when we increase laser power and thus $V_{\text {dipole }}$. Taking this into account, I would judge that the increase in $\mu$ is correct.

## 7 Conclusion

It can be concluded that a double-well potential has been created succesfully and that this potential is stable. There is no great variation in the center of the magnetic trap and in the position of the dipole potential with respect to the center of the magnetic trap.

However, in a perfect setup the potential is expected to be symmetric. From the measurements in expansion it can be concluded that this is not the case. In the measurements $\omega_{z}$ is determined from the interference pattern for both sides of the potential. For a condensate with similar $\omega_{z}$ on both sides a constant wavelength is expected in the interference pattern. It is clear that there is a $1 / z$ dependence in the measurements, indicating that the two BECs do not have the same $\omega_{z}$. The right side of the double-well potential has a higher $\omega_{z}$ than the left side.

It can be seen that when the green laser power is increased the distance between the two BECs increases as expected. We expect this from the potential, because when we increase the height of the potential the minima move further apart.

Finally, it can be concluded that interference has been observed. The observation has been made at different expansion times and at different power of the green laser. In all cases the same $\omega_{z}$ for the different wells has been observed. A difference in power of the green laser does not give a change in $\omega_{z}$. The parameters stay constant in all the measurements. However, possibly the difference in dipole strengths that we are probing is not sufficient for showing a change in $\omega_{z}$. Despite minor aberrations, the experiment demonstrates unmistakingly that interference has taken place.

## 8 Outlook

### 8.1 Smaller cross-section of the focused part of the beam

In order to determine whether the $1 / z$ behaviour is caused by the $56.7 \mu \mathrm{~m}$ crosssection of the focused part of the beam it would be interesting to try the same experiment with a $12 \mu \mathrm{~m}$ cross-section as in experiments, where a constant interference pattern was observed. If we observe that the $1 / z$ behaviour disappears we can conclude that we understand this correctly and that we have correctly probed a different part of the equation for interference in a BEC.

A better focused laser should result in a constant interference pattern. This is something which might be desirable as a lot of the analysis becomes more straightforward. The $1 / z$ dependence can be neglected making the equation for interference in a BEC much simpler. It would also mean that we are able to determine the parameters of the equation of the interference a lot better. For example it would no longer be necessary to determine the center of the interference pattern as it is constant throughout.

A smaller cross-section can be implemented in setup by using a larger beam that goes through the cylindrical lens in the setup. For this a larger cylindrical lens is needed as well.

### 8.2 Determine phase difference

A constant interference pattern might make it easier to determine the phase difference between the two BECs. As we can now look at the shift of the interference pattern by using all the points. However an adventage of the $1 / z$ behaviour is that the center is well defined. A well defined center gives other advantages if we want to determine the phase. A simplified expression for the interference pattern can be given by,

$$
\begin{equation*}
I=\operatorname{Cos}[k \cdot z+\phi] . \tag{8.1}
\end{equation*}
$$

If we were to look at the center of the interference pattern where $z=0$ we can determine the phase by,

$$
\begin{equation*}
\phi=\operatorname{Cos}^{-1}[I] \tag{8.2}
\end{equation*}
$$

Determining the phase difference between the condensates is of great interest. From this we can see how the phase difference is distributed, whether this is a random process or that there is some underlying mechanism. It is not clear how a BEC acquires phase. From the phase difference we might get a handle on this process.

### 8.3 Incorporate a SLM in the setup

In section 3.5 the use of a SLM in the setup is explained. In the setup the beam profile shown in figure 3.1 contains a weak link in the center. In appendix B a python code that can be used to control the SLM is shown. Basically, all the ingredients to do the experiment have been looked into apart from the theory side.

It is expected, that similar behaviour will be observed as in research done by Brantut et al. [10]. In this experiment fermions are let to diffuse through a narrow channel. The proposed experiment would be the bosonic analog of this experiment.

### 8.4 Dragging the condensate through the beam profile

Another approach can be to drag the condensate through the light field. The goal in this case is to look at tunneling through the laserfield. An experimental challenge is the width of the sheet in the focussed direction, this width needs to be sufficiently narrow $(<10 \mu \mathrm{~m})$. By precise tuning of the magnetic field the condensate can be dragged through the light field at different speeds, which should have an impact on the rate of tunneling.

## 9 Acknowledgements

Most of the experimental work was actually already finished half a year before handing in this thesis. The interpretation though has proven to be a tough nut to crack. A characteristic behavior was present in the results that previous experiments did not observe. Whilst already having left the group I kept contact with my supervisor Peter van der Straten to interpret these results, which after a lot of different approaches has in the end proven to be successful. After a few months whilst already living in Amsterdam we found out what the reason was for the deviation from previous experiments. For the final clues that were needed to make this thesis feel like a complete work I would like to thank Peter a lot for his guidance in this process.

In the BEC group I would like to thank Jasper, Qiao and Koen for helping out with doing the experiment. Jasper and Qiao did a lot of hard work for achieving BECs of a sufficient size after the transfer to a new configuration of the laser setup. I would like to thank Koen for all the work that we have done regarding the SLM.

There was also a lot of fun in the BEC group. Qiao would fall a sleep some of the time in the lab near the oven for the setup, which is not strange if you think of it as a cozy fireplace in the midst of all the wires and computer screens. This also happened one evening in another setting after we had dinner with the BEC group in the city. After we were joking because this had happened again we switched places with another group of people sitting in the restaurant. And guess who woke up...

I have received a lot of help on the SLM from Robbert and Gordian. We had many discussions on how to improve the efficiency and how to control it using software. In the end not one of us has used the SLM for their thesis, which was supposed to be the binding factor for our SLM group. Where we started with weekly meeting full of enthusiasm these turned into monthly meetings with less and less people present. Nevertheless it was fun finding out how it works and to work on this project with the entire group.

The remainder of the Nanophotonics group I would like to thank for making my master thesis such an enjoyable experience under supervision of the Papa Bear a.k.a. Dries van Oosten which are Ole, Arjon, Sandy, Sebastiaan, Marcel, James, Jasper, Anne, Karindra and Kostas.

Another major thanks goes out to the technical staff. Without them it would not have been possible to create the experimental setup. Thanks to Cees de Kok for helping with all the optics, Paul Jurrius for securing the Ventus laser to the optical table as well as the SLM and Frits Ditewig\&Dante Killian for getting the AOM to work.

Appendix A Interference in the MT


Figure A.1: Interference in MT

## Appendix B SLM python code

```
import pyglet
import numpy as np
from pyglet.window import key
import matplotlib.pyplot as plt
import mat time
import flycapture2 as fc2
import fly
#functions
def initialize_cam():
    print "Initializing camera"
    #get the relevant properties
    auto_exp = c.get_property (fc2.AUTO_EXPOSURE)
    gamma = c.get_property (fc2.GAMMA)
    pan = c.get_property (fc2.PAN)
    tilt = c.get_property (fc2.TILT)
    shut = c.get_property (fc2.SHUTTER)
    gain = c.get_property (fc2.GAIN)
    frame_rate = c.get_property (fc2.FRAME_RATE)
    temp = c.get_property (fc2.TEMPERATURE)
    #turn off all the auto stuff
    auto_exp['on_off']= True
    auto_exp[',auto_manual_mode']= False
    auto_exp['abs_value']=0
    c.set_property(**auto_exp)
    gamma[',on_off']= False
    gamma[',auto_manual_mode']= False
    c.set_property (**gamma)
    pan[',on_off']= False
    pan['auto_manual_mode'] = False
    c.set_property (**pan)
    tilt['on_off']= False
    tilt['auto_manual_mode']= False
    c.set_property(** tilt)
    gain['on_off'']= True
    gain[',auto_manual_mode']= False
    gain[',abs_value'] = 0 #(-5)#(-5.630) #-5,630-24
    c.set_property(**gain)
    print "gain =", gain['abs_value']
    frame_rate[',on_off']= False
    frame_rate[',auto_manual_mode']= False
    frame_rate['abs_value'] = 15
    c.set_property(**frame_rate)
    shut['on_off']= True
    shut['auto_manual_mode']}=\textrm{False
    shut['abs_value']=18 #30#88#71 #use this variable (shutter time) to prevent flickering
    c.set_property(**shut)
    print "shutter time = ", shut['abs_value']
def diagonal(start,end):
    #define the array size
    len-x=1024
    len-y=768
    rest = float('nan')
    # lengths of the lists after rotation (which will be used for mapping), and append
    len1 = np.arange(1, len-y + 1)
    len2 = np.linspace(len_y, len_y, len_x - len_y)
    len3 = np.arange(len-y - 1, 0, -1)
    lengths = np.concatenate((len1, len2, len3))
    # SLM as a rectangle, filled with ones (all pixels on), initial input
    SLM_rect = np.ones([len_y, len_x])
    # Number of arrays needed for the transformation (becomes length in y-direction)
    n_lists = lengths.shape[0]
    # Length of longest arrays, made an integer (becomes length in x-direction)
    max_length = int(np.amax(lengths))
    # New SLM, rotated over 45 degrees, all values set to nan
    SLM_rotated = rest * np.zeros((n_lists, max_length))
```


## Appendix B SLM python code

```
    for i in range(0, n_lists):
        for j in range(0, int(lengths[i]) )
                        if i < len_y
                            SLM_rotated [i,j] = SLM_rect[i-j,j]
                            SLM_rotated [i,j] = SLM_rect[len_y -1 - j, j + (i - len_y + 1)]
SLM_rotated[start:end] = 0
    SLM_new = np.ones([len_y, len_x])
    for i in range(0, n_lists):
        for j in range(0, int(lengths[i]) )
            if i < len_y:
                        lse. SLM_new [i-j,j] = SLM_rotated [i, j]
                            SLM_new[len_y - - j j, j + (i - len_y + 1)] = SLM_rotated [i,j]
    return SLM_new
def background_func():
    window.dispatch_events()
    window.switch_to()
    window.clear()
    ones = np.ones((1024,768))
    ones255 = (ones*255).astype('uint8')
    image_data = ones255.data._-str__()
    image = pyglet.image.ImageData(1024, 768, 'L', image_data)
    #take snapshot
    while True:
        image.blit(0,0
        window. flip()
        camdatapoint=np.array(c.retrieve_buffer(fc2.Image()))
        cv2.imshow('test, camdatapoint)
        k=cv2.waitKey(1)
        if k==32
            break
    background=np.sum(camdatapoint [:,:,0],1). astype(np.int32)
    plt.plot(background)
    plt.show()
    return background
def mappingpoint(begin, end, background):
    ###################
    ##first diagonal##
    ##################
    #print info
    print "Position %d" % begin
    #pyglet stuff
    window.dispatch_events()
    window.switch_to()
    window.clear()
    #create pattern
    ones = diagonal(begin, end)
    ones255 = (ones*255). astype('uint8')
    mage_data = ones255.data._-str
    image = pyglet.image.ImageData(1024, 768, 'L', image_data)
    #take snapshot
    while True:
        image.blit (0,0)
        window.flip()
        camdatapoint=np.array(c.retrieve_buffer(fc2.Image()))
        cv2.imshow('test', camdatapoint)
        k=cv2.waitKey(1)
        if k==32
    datapoint=np.sum(camdatapoint[:,:,0],1). astype(np.int32)
    #process data
    minimum=0
    subtract=np.subtract(datapoint, background). astype(np.int32)
    for }x\mathrm{ in range(0,960)
        if (subtract[x]==np.amin(subtract)):
        minimum}=\textrm{x
    print "Minimum %d" % minimum
    #plot results
```

```
plt.plot(background)
plt.plot(datapoint)
plt.plot(subtract)
plt.show()
return minimum
#connect to the camera
c=fc2. Context()
c.connect(*c.get_camera_from_index (0))
initialize_cam()
c.start-capture()
im0=fc2.Image()
img0 = np.array(c.retrieve_buffer(im0))
#create a SLM
#get the right screen
platform = pyglet.window.get.platform()
display = platform.get_display(':0.0')
screens = display.get_screens()
window = pyglet.window.Window(screen=screens[1], fullscreen=True)
####################
##start mapping###
#####################
mapping_array=np.zeros ((10,2))
background=background_func()
mapping_array [0,0]=700
mapping_array[0,1]=mappingpoint(700,705, background)
mapping_array [1,0]=750
mapping_array [1,1]== mappingpoint (750,755, background )
mapping_array [2,0]=800
mapping_array[2,1]== mappingpoint(800, 805, background)
mapping_array [3,0]=850
mapping_array [3,1]== mappingpoint(850,855,background)
mapping_array [4,0]=900
mapping_array [4,1]== mappingpoint(900,905, background )
mapping_array [5,0]=950
mapping_array [5,1]== mappingpoint (950,955,background )
mapping_array [6,0]=1000
mapping_array[6,1]== mappingpoint (1000,1005,background}
mapping_array [7,0]=1050
mapping_array[7,1]== mappingpoint (1050,1055, background )
mapping_array [8,0]=1100
mapping_array[8,1]== mappingpoint(1100,1105, background}
mapping_array [9,0]=1150
mapping_array [9,1]== mappingpoint(1150,1155,background)
#save array
np.save("mapping",mapping_array)
#exit the event loop
@window. event
def on_key_press(symbol,modifiers)
    if symbol == key.Q:
                                    pyglet.app.exit()
```


## References

[1] R. Meppelink, Hydrodynamic Excitations in a Bose-Einstein Condensate, PhD Thesis, Utrecht University, October 2009.
[2] Pieter Bons, Probing the properties of quantum matter, Utrecht University, May 2015.
[3] C.J. Pethick and H.Smith, "Bose-Einstein Condensation in Dilute Gases," Cambridge University Press, 2002, p. 375.
[4] Y. Castin and R. Dum, "Bose-Einstein Condensates in Time Dependant Traps," Physical Review Letters, vol. 77, no. 27, 1996.
[5] M. R. Andrews et al., "Observation of Interference Between Two Bose Condensates," Science, vol. 275, 1997.
[6] Specification sheet Laser Quantum Ventus 532
[7] A. Keshet and W. Ketterle, "A distriubuted, graphical user interface based, computer control system for atomic physics experiments," Review of Scientific Instruments, vol. 84, no. 1, pp.-, 2013.
[8] Specification sheet Texas Instruments DLP5500 DMD
[9] Milton Roy, 'Diffraction grating handbook," 1989.
[10] J.P. Brantut et al., "Conduction of Ultracold Fermions Through a Mesoscopic Channel," Science, vol. 337, 2012.

