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Bachelor thesis

Simulating a glacier: application to Engabreen

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Abstract

In this thesis, a 1D flow model has been applied to Engabreen to look at the sensitivity of Engabreen to climate forcing. First the effect of various parameters, e.g. the slope and the width are examined. These effects can be explained by a balance between accumulation and ablation. In equilibrium, the amount of accumulation and ablation exactly balance each other. Next, the input data for Engabreen are determined. When this is done the model is applied to Engabreen. In the present day configuration, the length of Engabreen is around 12 km. At the end of Engabreen, the slope is relatively large and the width relatively small compared to the rest of Engabreen. A large slope causes a glacier to be less sensitive, while a relatively small width causes a glacier to be more sensitive. There is, therefore, a balance between these two processes. The glacier is found to be more sensitive for lengths around 12 km, so the effect of the smaller width is dominating. Hysteresis is found when changing the climate forcing. This is due to an overdeepening in the bed in the present accumulation area.

Contents

Chapter 1

Introduction

Ice is a very special material. It exhibits fluid-like behaviour and this is the reason that ice ”flows”. It is a non-newtonian fluid which means that the flow depends on the geometry. The ”flowing” ability of ice is the reason that glaciers are able to advance or retreat in reaction to changes in the mass budget and thus to changes in the climate. Research on the flow of glaciers tells us a lot about the climate in the past and the sensitivity of a specific glacier to climate change. This can be used to make predictions for future climate scenarios.

Glaciers can survive in regions where the snow that has fallen in the winter does not completely melt away in the summer. The interaction between a glacier and the climate happens at the surface of the glacier, assuming there is no calving of ice in an ocean or a lake. In this case the surface mass balance is the key link between variables concerning the flow of the glacier and variables related to the climate. The surface mass balance is the amount of mass that has accumulated or melted on a specific surface in a certain period of time. It is the net change in mass for that surface in a certain period. It is positive when the amount of mass is increased and negative when there is a decrease in mass. The surface mass balance includes all processes that can add or remove mass, but accumulation of snow and surface melt will in general be the most important processes.

Glacier flow is determined by gravity. In general, a glacier can be split up in two regions: the accumulation region and the ablation region. In the accumulation region, snow is being accumulated and the mass increases, while in the ablation region, mass decreases due to surface melt. Glaciers always strive towards a geometry in which the glacier is in equilibrium. However, this equilibrium is never reached. This is because glaciers reach equilibrium only on long time scales, but in our continuously changing climate, there is no time to reach this. In equilibrium, the amount of accumulation in the accumulation region would be exactly balanced by the amount of melt in the ablation region. To divide the glacier in these two regions, an equilibrium line is used. This line is the seperation between the accumulation region and the ablation region. This is illustrated in figure 1.1.

The glacier studied here is Engabreen. Engabreen is a glacier in central Norway (see figure 2) and is an oulet glacier of the Svartisen ice cap. It is 12 km long and covers an altitudinal range of 10 to 1575 m (NVE [2016]). It covers an area of 39.6 km^2. Most of its area is located on altitudes between 1200 and 1400 m. Its tongue goes down to a lake called Engabrevatnet at 7 m. This is the lake visible on the front page. For studying Engabreen, a 1D flow model will be used here. The model will be applied to Engabreen and it will be calibrated in order to agree with present day measurements. In the end, the sensitivity of Engabreen to a change in climate forcing will be investigated.

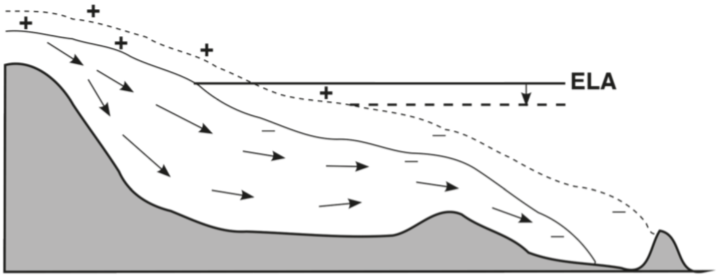


Figure 1.1 – Figure from Kuipers Munneke et al. [2015] illustrating a typical configuration for the surface mass balance of a glacier with the corresponding equilibrium line. Above the equilibrium line there is a positive mass balance indicated with a + and below the equilibrium line the negative mass balance is indicated with a −. The dashed lines correspond to a new configuration, when there has been a period of a more positive mass balance. As a result, the ablation region extends to reach equilibrium again and the glacier becomes longer.

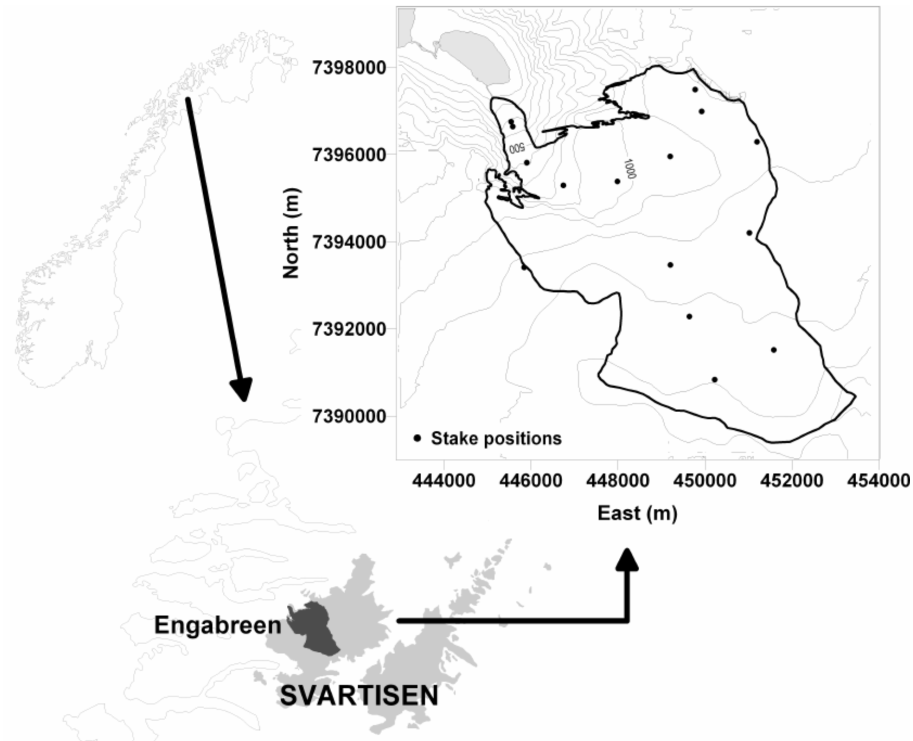


Figure 1.2 – Map showing the location of Engabreen (Jackson et al. [2005]). As one can see, it is situated in the northern part of central Norway. It is an outlet glacier of the Svartisen ice cap. Engabreen covers an area of 39.6 km^2(NVE [2016]). Most of Engabreen is located on altitudes between 1200 and 1400 m, with the tongue going all the way to a lake called Engabrevatnet at 7 m.

Chapter 2

Theory and model description

Glacier flow can be described using conservation laws, specifically conservation of mass and conservation of momentum. The theory leading to the equations governing glacier flow is stated below.

2.1 Theory

In general, conservation laws can be formulated as follows (van de Berg [2015]):

$$\frac{\partial}{\partial t}\int_V \Psi dV = -\int_S \Psi\vec{u}\cdot\vec{n}dS + \int_V \upsilon(\psi)dV \tag{2.1.1}$$

Here Ψ is a characteristic of a fluid, for example the density or the temperature. The first term is the change of a Ψ in time in a volume V. The second term is the flux through the surface and the last term respresents possible sources or losses. With the use of Gauss law, this can be written as folllows:

$$\int_V \Upsilon(\psi)dV = \int_V \left(\frac{\partial\Psi}{\partial t} + \nabla\cdot(\Psi\vec{u})\right)dV \tag{2.1.2}$$

$$= \int_V \left(\frac{\partial\Psi}{\partial t} + \vec{u}\cdot\nabla\Psi + \Psi\nabla\cdot\vec{u}\right)dV \tag{2.1.3}$$

$$= \int_V \left(\frac{D\Psi}{Dt} + \Psi\nabla\cdot\vec{u}\right)dV \tag{2.1.4}$$

Where $\frac{D}{Dt}$ is the material derivative. The above equation is the general conservation law for a certain variable in a fluid of volume V. By choosing $\Psi = \rho$ and assuming no mass is created or destroyed ($\Upsilon = 0$), the following is equation is obtained:

$$\int_V \left(\frac{D\Psi}{Dt} + \Psi\nabla\cdot\vec{u}\right)dV = 0 \tag{2.1.5}$$

The above equation has to hold for every volume, so the integral can be removed. This implies:

$$\frac{D\Psi}{Dt} + \Psi\nabla\cdot\vec{u} = 0 \tag{2.1.6}$$

Which is known as the continuity equation. The momentum equation can be found in a similar way, by setting $\Psi = \rho\vec{u}$. In the momentum equation, there will be external forces, so $\Upsilon \neq 0$. The term including sources and losses can be split in external volume forces, F_{vol}, and external surface forces, F_S:

$$\int_V \Upsilon\left(\rho\vec{u}\right) dV = \int_S \vec{F}_S dS + \int_V \vec{F}_{vol} dV \tag{2.1.7}$$

$\vec{F}_S$ can be written as $\sigma \cdot \vec{n}$ where the stress tensor σ is a matrix and $\vec{n}$ is the vector normal to the surface. A dot product between a vector and a scalar is again a vector, confirming the fact that the above equation is a vector equation. In total, the equation for conservation of momentum becomes:

$$\int_V \left(\frac{D\left(\rho\vec{u}\right)}{Dt} + \rho\vec{u}\nabla\cdot\vec{u}\right) dV = \int_V \left(\nabla\cdot\sigma + \vec{F}_{vol}\right) dV \tag{2.1.8}$$

Where Gauss' law is used to get the first term on the right hand side. This equation has to hold for any volume as well, so the integrads should again be equal to each other:

$$\nabla\cdot\sigma + \vec{F}_{vol} = \frac{D\left(\rho\vec{u}\right)}{Dt} + \rho\vec{u}\nabla\cdot\vec{u} \tag{2.1.9}$$

$$= \rho\frac{D\vec{u}}{Dt} + \vec{u}\left(\frac{D\rho}{Dt} + \rho\nabla\cdot\vec{u}\right) \tag{2.1.10}$$

Here the product rule is used in the last step to write the material derivative in two different terms. The term in the brackets is exactly the equation we had for mass conservation and is consequently equal to 0. What is left is:

$$\rho\frac{D\vec{u}}{Dt} = \nabla\cdot\sigma + \vec{F}_{vol} \tag{2.1.11}$$

This is the equation for the conservation of momentum. To simplify things even more, σ can be divided into surface forces that deform a body and surface forces that only cause compression or expansion. The latter is related to the pressure p and corresponds to the diagonal parts of the matrix: σ_{xx}, σ_{yy} and σ_{zz}. The diagonal parts of σ are defined to be positive when a force is away from the body, while p is positive when a force is towards a body and causing compression. This means that there should be a sign difference between σ and p. Using this, the stress tensor σ can be divided in a part containing the shear stresses τ (causing deformation) and a pressure part p (related to compression):

$$\sigma = -pI + \tau \tag{2.1.12}$$

Here I is the identity matrix and τ is the shear stress tensor. To ensure that the shear stress tensor τ does not lead to compression or extension, the condition for τ is that it has to be traceless, i.e. $\tau_{11} + \tau_{22} + \tau_{33} = 0$. After dividing σ in two parts, the momentum equation becomes:

$$\rho\frac{D\vec{u}}{Dt} = \nabla\cdot\left(-pI + \tau\right) + \vec{F}_{vol} \tag{2.1.13}$$

In Einstein index notation, the equation for the stress tensor σ is:

$$\sigma_{ij} = -p\delta_{ij} + \tau_{ij} \tag{2.1.14}$$

The part concerning the pressure on the right hand side of the momentum equation can be formulated in an easier way:

$$\nabla\cdot\left(-pI\right) = -\frac{\partial p\delta_{ij}}{\partial x_j} = -\frac{\partial p}{x_i} = -\nabla p \tag{2.1.15}$$

We end up with:

$$\rho \frac{D\vec{u}}{Dt} = -\nabla p + \nabla \cdot \tau + \vec{F}_{vol} \tag{2.1.16}$$

In this equation, $\frac{D\vec{u}}{Dt}$ can be neglected, because accelerations are very small when considering glacier flow. Furthermore, $\vec{F}_{vol} = -\rho g$ where the gravity g acts only in the z-direction. Eventually, the momentum equations become:

$$\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = \frac{\partial p}{\partial x} \tag{2.1.17}$$

$$\frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} = \frac{\partial p}{\partial y} \tag{2.1.18}$$

$$\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} = \frac{\partial p}{\partial z} + \rho g \tag{2.1.19}$$

2.2 Model description

Next, we will consider the flowline model. In this model there are two variables concerning height, so it's probably best to clearly state the difference. The first parameter $b(x, y)$ is the bed topography, also called the orography. The second parameter is $h(x, y)$ which is the topography of the surface. The foundation of the flowline model is the conservation of mass, which is stated in eq 2.1.6. In this model, the density of the ice is assumed to be constant. This implies that $\frac{D\rho}{Dt} = 0$. What is left is the continuity equation for an incompressible medium (Oerlemans [2015]):

$$\nabla \cdot \vec{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \tag{2.2.1}$$

This equation can be integrated from the bed $b(x, y)$ to the surface $h(x, y)$ to obtain an expression for the difference in the vertical velocity w:

$$w(h) - w(b) = -\int_b^h \frac{\partial u}{\partial x} dz - \int_b^h \frac{\partial v}{\partial y} dz \tag{2.2.2}$$

The integration rule of Leibniz can be used to write the integral over $\frac{\partial u}{\partial x}$ as:

$$-\int_b^h \frac{\partial u}{\partial x} dz = u(h)\frac{\partial h}{\partial x} - u(b)\frac{\partial b}{\partial x} - \frac{\partial}{\partial x}\int_b^h u dz \tag{2.2.3}$$

Using this formula for both terms on the right hand side of the equation for w, the equation for w can be written as follows:

$$w(h) - w(b) = u(h)\frac{\partial h}{\partial x} - u(b)\frac{\partial b}{\partial x} - \frac{\partial}{\partial x}\int_b^h u dz + v(h)\frac{\partial h}{\partial y} - v(b)\frac{\partial b}{\partial y} - \frac{\partial}{\partial y}\int_b^h v dz \tag{2.2.4}$$

The velocities $w(h)$ and $w(b)$ are non zero when there is a change in surface or due to accumulation or melt, so $w(h)$ and $w(b)$ can be stated as:

$$w(h) = \frac{Dh}{Dt} - \dot{b} = \frac{\partial h}{\partial t} + u(h)\frac{\partial h}{\partial x} + v(h)\frac{\partial h}{\partial y} - \dot{b} \tag{2.2.5}$$

$$w(b) = \frac{Db}{Dt} - \dot{b}_b = \frac{\partial b}{\partial t} + u(b)\frac{\partial b}{\partial x} + v(b)\frac{\partial b}{\partial y} - \dot{b}_b \tag{2.2.6}$$

Where $\dot{b}_b$ is the mass balance rate at the base and $\dot{b}$ the mass balance rate at the surface of the glacier h. The expressions for $w(h)$ and $w(b)$ can be used to simplify equation 2.2.4:

$$\frac{\partial h}{\partial t} - \frac{\partial b}{\partial t} - \dot{b} - \dot{b}_b = -\frac{\partial}{\partial x} \int_b^h udz - -\frac{\partial}{\partial y} \int_b^h vdz \tag{2.2.7}$$

The thickness of the ice, H, is equal to the difference between the surface topography and the base topography $H = h - b$. Using this, an expression for the ice thickness can be found:

$$\frac{\partial (h-b)}{\partial t} = -\frac{\partial}{\partial x} U (h-b) - \frac{\partial}{\partial y} V (h-b) + \dot{b} + \dot{b}_b \tag{2.2.8}$$

$$\frac{\partial H}{\partial t} = -\frac{\partial (UH)}{\partial x} - \frac{\partial (VH)}{\partial y} + \dot{b} + \dot{b}_b \tag{2.2.9}$$

$$\tag{2.2.10}$$

The above expression is an expression for the evolution of the ice thickness in time. If the mass balances $\dot{b}$, $\dot{b}_b$ and the velocity field are known, the change of the ice thickness in time can be calculated. For the flowline model, the following equation is used:

$$\frac{\partial H}{\partial t} = -\frac{\partial (UH)}{\partial x} - \frac{\partial (VH)}{\partial y} + \dot{b} \tag{2.2.11}$$

This is the same as the equation we just got for $\frac{\partial h}{\partial t}$, the only difference is the absence of $\dot{b}_b$. This term has been neglected, because the processes governing the mass balance are assumed to only act on the surface of the glacier. In the flowline model used here,the x-direction is chosen to coincide with the flowline, and is therefore not a straight line. The y-direction is perpendicular to the x-direction and therefore also perpendicular to the flowline. The expression above is found by vertically integrating the continuity equation. Equation 2.2.11 can also be integrated in the y-direction. This gives a similar result:

$$\frac{\partial S}{\partial t} = -\frac{\partial (US)}{\partial x} + \dot{b} \tag{2.2.12}$$

Where S is the cross section at a certain point along the flowline, see figure 2.1. In general, the cross section of a glacier will not have the same width everywhere along the flowline. The width of the base can in principle vary, as well as its width at the surface. The cross sections will typically have a trapezoidal shape. This is illustrated in figures 2.1 and 2.2. The area of a cross section of trapezoidal shape can be calculated in the following way:

$$S = H \left(w_0 + \frac{1}{2} \lambda H \right) \tag{2.2.13}$$

Here w_0 is the width of the base, H is the thickness of the ice and λ is the tangent of the angle that the valley makes with the walls, measured with respect to the vertical (see figure 2.2). Thus, λ can be calculated in the following way:

$$\lambda = \frac{1}{2} \left(tan\,(\alpha_L) + tan\,(\alpha_R) \right) \tag{2.2.14}$$

Where α_L and α_R are the angles of the valley walls to the left and right of the glacier, measured with respect to the vertical. λ is the average of the tangents to the left and right of the glacier. The term containing λ is added to enable the width at the surface to

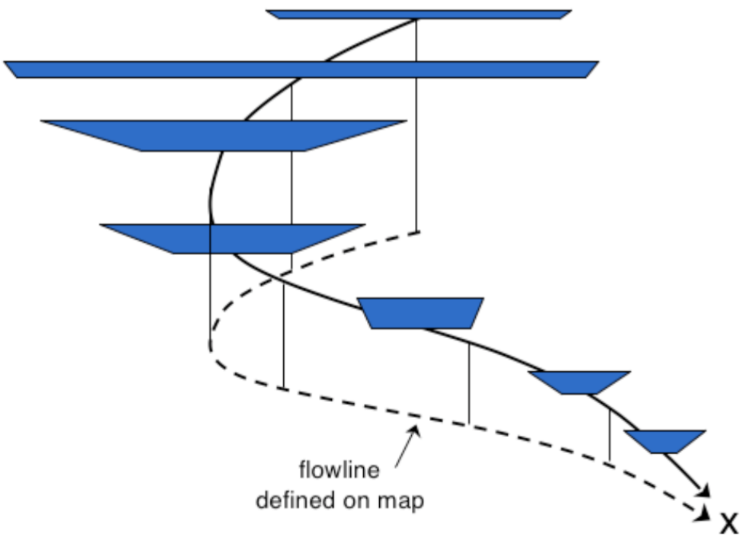


Figure 2.1 – An example of trapezoidal shaped cross sections along a flowline that is orientated in such a way to coincide with the x-axis. The cross sections could in principle be different at every point, with varying width of the bed, w_0, and varying λ. The figure is from Oerlemans [2015].

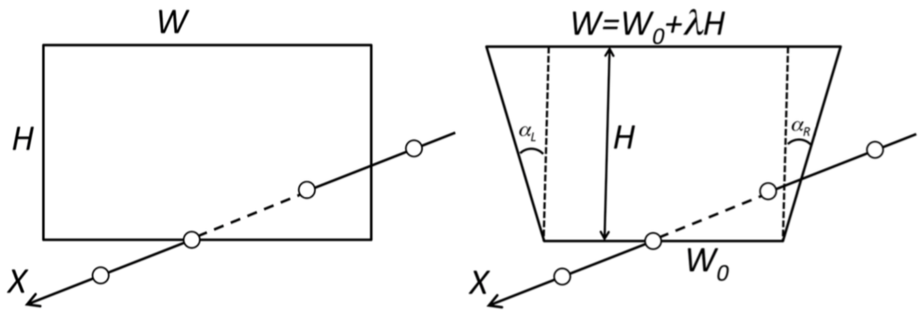


Figure 2.2 – The difference between a cross section with vertical walls and a cross section where there is an angle with respect to the vertical. w_0 is the width of the bed, H the ice thickness and λ is the sum of the tangent of angles the valley walls to the left and to the right make with the vertical. The figure is from Kuipers Munneke et al. [2015].

be different from the width at the base. In one of the first experiments vertical columns will be used where $w_S = w_0$. In this case, $\lambda = 0$ and the cross section S is just Hw_0. By substituting the equation for S in equation 2.2.12, the following is obtained:

$$\frac{\partial S}{\partial t} = -\frac{\partial (US)}{\partial x} + \dot{b}(w_0 + \lambda H) \tag{2.2.15}$$

$$\frac{\partial \left(H\left(w_0 + \frac{1}{2}\lambda H\right)\right)}{\partial t} = -\frac{\partial \left(UH\left(w_0 + \frac{1}{2}\lambda H\right)\right)}{\partial x} + \dot{b}(w_0 + \lambda H) \tag{2.2.16}$$

The first term on the left hand side can be simplified:

$$\frac{\partial \left(H\left(w_0 + \frac{1}{2}\lambda H\right)\right)}{\partial t} = \frac{\partial (Hw_0)}{\partial t} + \frac{\partial \left(\frac{1}{2}H^2\lambda\right)}{\partial t} \tag{2.2.17}$$

$$= w_0\frac{\partial H}{\partial t} + H\frac{\partial H}{\partial t}\lambda \tag{2.2.18}$$

$$= \frac{\partial H}{\partial t}(w_0 + \lambda H) \tag{2.2.19}$$

Using this we obtain:

$$\frac{\partial H}{\partial t}(w_0 + \lambda H) = -\frac{\partial \left(UH\left(w_0 + \frac{1}{2}\lambda H\right)\right)}{\partial x} + \dot{b}(w_0 + \lambda H) \tag{2.2.20}$$

$$\frac{\partial H}{\partial t} = -\frac{1}{(w_0 + \lambda H)}\frac{\partial \left(UH\left(w_0 + \frac{1}{2}\lambda H\right)\right)}{\partial x} + \dot{b} \tag{2.2.21}$$

The only variable that is still unknown is the horizontal velocity U. In our 1d flow model we will make use of the shallow ice approximation. In this approximation the stress field is dominated by the vertical shear stresses, τ_{xz} and τ_{yz}. This approximation is valid when the horizontal extent of the glacier is at least ten times as large as the vertical extent of the glacier. The horizontal velocity U can be seperated into a deformation velocity U_d and a sliding velocity U_S. If a Weertman-type sliding law is used, in which U_d and U_S are proportional to τ^3 (Budd, Keage, and Blundy [1979]), we get the following:

$$U = U_d + U_S = f_d H\tau^3 + f_s\frac{\tau^3}{H} \tag{2.2.22}$$

Where f_d is the flow parameter for deformation and f_s is the flow parameter for sliding. Based on eq. 2.1.17, an expression for τ can be found. If the x-axis makes an angle η with the vertical, the force ρg can be split into $-\rho g \sin(\eta)$ and $\rho g \cos(\eta)$. Using this and the fact that the stress field is dominated by vertical shear stresses, we get:

$$\frac{\partial \tau_{xz}}{\partial z} = -\rho g \sin(\eta) \tag{2.2.23}$$

This can be integrated with respect to z:

$$\tau_{xz}(H) - \tau_{xz}(z) = -(H - z)\rho g \sin(\eta) \tag{2.2.24}$$

The shear stress due to air flow is negligible, so $\tau_{xz}(H)$ can be neglected. The following is left:

$$\tau_{xz}(z) = (H - z)\rho g \sin(\eta) \tag{2.2.25}$$

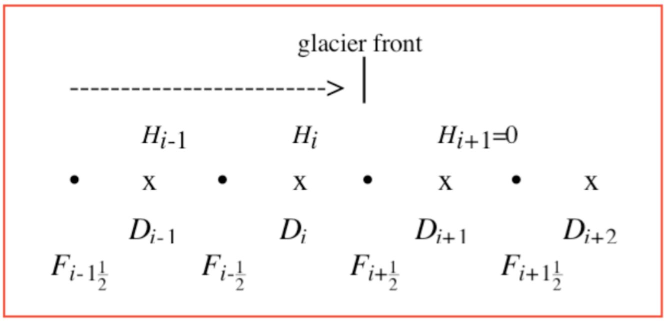


Figure 2.3 – Figure from Oerlemans [2015] showing where the grid points are evaluated. The height and diffusion are evaluated at integer points, whereas F is calculated at half integer grid points. The truncation error makes sure that the glacier front can advance.

At the bed (z=0), this expression becomes:

$$\tau_{xz}(z) = H\rho g \sin(\eta) \tag{2.2.26}$$

Note that $\sin(\eta)$ can be approximated as $\left|\frac{\partial h}{\partial x}\right|$ if the angle η is small. Eventually, an expression for τ is found:

$$\tau = \rho g H \left|\frac{\partial h}{\partial x}\right| \tag{2.2.27}$$

Equation 2.2.21 can be expressed even further by substituting the values for U and τ in it. After some algebra, the following expression is obtained:

$$\frac{\partial H}{\partial t} = -\frac{1}{(w_0 + \lambda H)}\frac{\partial}{\partial x}\left[D\frac{\partial h}{\partial x}\right] + \dot{b} \tag{2.2.28}$$

Where D is a diffusion parameter and is given by:

$$D = \left(w_0 + \frac{1}{2}\lambda H\right)(\rho g)^3 H^3 \left(\frac{\partial h}{\partial x}\right)^2 \left(f_d H^2 + f_s\right) \tag{2.2.29}$$

The last two equations are the equations to be solved in our 1D flow model.

2.2.1 Numerical modelling

Solving the equations analytically is not possible in our model. The diffusion D and the ice thickness H are therefore discretized to be able to solve the equations numerically. A staggered grid is used in order to avoid instabilities. The discretized version of D has the following form:

$$D = \left(w_0 + \frac{1}{2}\lambda H\right)(\rho g)^3 H^3 \left(\frac{h_{i+1} - h_{i-1}}{2\Delta x}\right)^2 \left(f_d H^2 + f_s\right) \tag{2.2.30}$$

To make things easier, the term $D\frac{\partial h}{\partial x}$ in eq. 2.2.28 will be called F:

$$F = D\frac{\partial h}{\partial x} \tag{2.2.31}$$

In calculating F, two adjacent points of D are used, D_i and D_{i+1}. This implies that F is calculated exactly between two normal gridpoints (see figure 2.3). We will denote this by $F_{i+\frac{1}{2}}$:

$$F_{i+\frac{1}{2}} = \frac{D_{i+1} - D_i}{2} \frac{h_{i+1} - h_i}{\Delta x} \tag{2.2.32}$$

In order to get an equation for the ice thickness H, the equation for $\partial H/\partial t$ needs to be integrated over time. Numerically, this can be done in the following way:

$$H_{i,t+\Delta t} = H_{i,t} + \Delta t \left(\frac{1}{w_{0,i} + \lambda_{w,i} H_{i,t}} \frac{F_{i+\frac{1}{2},t} - F_{i-\frac{1}{2},t}}{\Delta x} + \dot{b} \right) \tag{2.2.33}$$

At the glacier front, $\dot{b}$ will be negative. The thickness of the glacier will therefore only increase if the mass flux exceeds the ablation. If this happens, the ice will built up and as a result of this, the glacier front will advance.
The equation for $\partial H/\partial t$ is not valid at the glacier front, because here the diffusion $D = 0$. This problem can be overcome in numerical modelling by doing the following. Due to the truncation error caused by termination of ice, D will have a positive value at the gridpoint just after the glacier front, allowing the glacier front to advance. One last thing to keep in mind is the boundary condition that is set at the top of the glacier:

$$\left[\frac{\partial h}{\partial x} \right]_{x=0} = 0 \tag{2.2.34}$$

This implies that there is no flux at the head of the glacier, i.e. the head is either a wall or a division of ice into seperate ice flows. Numerically, this is easy to realise, we just have to set h_1 equal to h_2.

2.3 Data used

The input data used in this model is data for three different parameters. For an interval of Δx=100 m the bed topography, width of the glacier and the value of λ are determined individually. The first thing that has to be done in order to get a dataset is to define a flowline. A flowline needs to be defined in such a way to always be perpendicular to the contour lines of equal height. It starts at the heighest point and goes all the way down, beyond the glacier tongue and in this case all the way to the sea. In order to define a flowline, an outline of the specific glacier is needed. In this case, the outline illustrated in figure 2.4 is used (Jackson et al. [2005]).
In this figure, the colored line is the determined flowline. Once a flowline has been defined, the different input variables can be measured. All the input variables will be measured on a map. Note that, because this is a 1D flow model, every variable is measured along the flowline. First, the surface topography and the distance along the flowline will be measured, with x=0 starting at the top. Because it is easier to use the same interval in suface height, we will measure the distance between two contour lines (corresponding to an interval of Δh = 50m). Later, this will be interpolated, so the value of Δx is held constant.
The next variable to be measured is the width. This measurement is pretty straightforward. Using the outline of Engabreen and a curvimeter, the length of the contour lines is measured. Using the same interpolation algorithm, this can be converted to widths at fixed intervals of Δx = 100 meters. Thirdly, the value of the λ parameter needs to be measured. This is done by measuring the angles based on measured differences in height at a fixed distance x from the glacier. Once Δx and Δh are measured, the angle can be calculated in the following way:

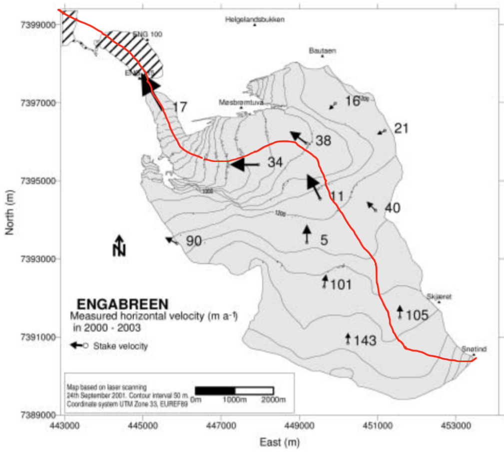


Figure 2.4 – The outline of Engabreen. The colored line is the flowline. This line has to be perpendicular to the contour lines of equal height. The outline is from Jackson et al. [2005]

$$\lambda = \tan(\alpha) = \frac{1}{2}\big(\tan(\alpha_L) + \tan(\alpha_R)\big) = \frac{1}{2}\left(\frac{\Delta x_L}{\Delta h_L} + \frac{\Delta x_R}{\Delta h_R}\right) \tag{2.3.1}$$

Since the distance along the flowline and the surface topography are now measured, the slope can be calculated. To calculate the bed topography, it is easiest to get a first guess by calculating the ice thickness using a rewritten form of equation 2.2.27 and subtract this from the surface topography:

$$H = h - b = \frac{\tau_0}{\rho g}\frac{1}{\big(\partial h/\partial x\big)} \tag{2.3.2}$$

Where b is the bedrock topography or orography as explained earlier. In regions where the slope is small, this formula does not work very well, because $1/\big(\partial h/\partial x\big)$ will go to infinity. Next, the value for τ_0 needs to be known. The basal stress reaches its maximum in the middle and gradually decreases to small values at the ends of the glacier. Using this information, an approximate formula for τ has been used:

$$\tau = \frac{\tau_m}{(1+\delta)}\left\{1 - \left[\frac{|x/L - 0.5|}{0.5}\right]^3 + \delta\right\} \tag{2.3.3}$$

Here τ_m is the maximum basal stress and L is the length of the glacier. A value of 200 kpa for τ_m and 0.15 for δ is used here. This value of τ can be substituted for the τ_0 in the equation for the ice thickness. Once this new value of τ is calculated, the value of the ice thickness H can be calculated. We now have the surface topography and the ice thickness as a function of the distance x along the flowline, so now we can simply substract the ice thickness from the surface topography to find the orography (see equation 2.3.2).
Because the ice thickness is related to the slope $\partial h/\partial x$ and the slope is measured in finite intervals, the ice thickness can exhibit sudden changes. To overcome this problem, Gaussian smoothing has been used. This means that the value of a point and nearby points are weighted by a certain factor calculated by integrating the Gaussian exponential formula given in equation 2.3.4. The equation below is the Gaussian exponential and this is the formula that needs to be integrated to obtain values that can be used to weigh the points. Once this is done, the ice thickness will be smoothened resulting in a smoother orography (see figure 2.5).

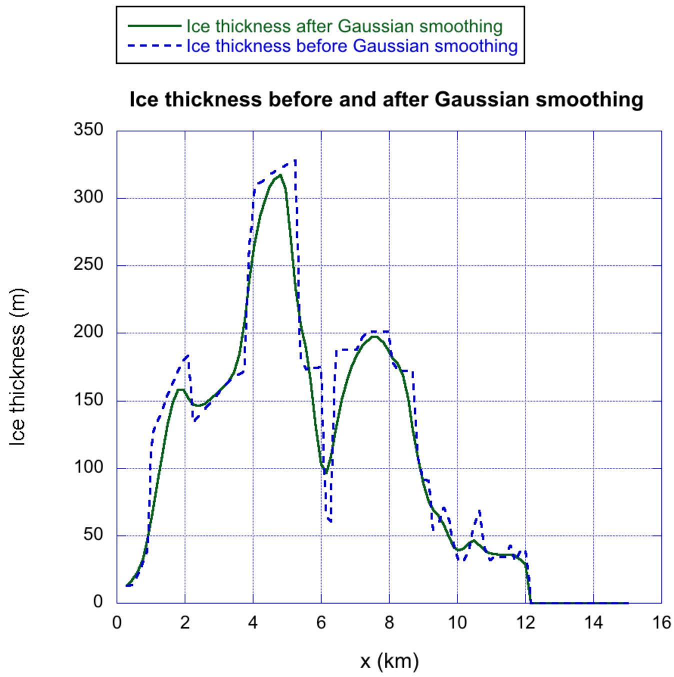


Figure 2.5 – The ice thickness as a function of x before (dotted line) and after Gaussian smoothing (solid line). In Gaussian smoothing, the value of a point and nearby points are weighted by a certain factor calculated by integrating the Gaussian formula. The points will be smoothed, and the result is the solid line.

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-x^2}{2\sigma^2}} \tag{2.3.4}$$

Now all the input data are known and the model can be used to make simulations for various scenario's.

Chapter 3

Results

3.1 Simple model tests

First we will do some tests with a basic model, that is not yet applied to Engabreen. The basic configuration of our first test is illustrated in figure 3.1. Here the height of the glacier is visible as a function of the distance x. The orography has a constant slope, starting at a height of 2000 m descending to 0 m over a horizontal distance of 20 km. The dashed green line is the equilibrium line, set at 1600 meter. Above this line, in the accumulation zone, there is a positive mass balance, while the mass balance is negative in the ablation zone below the equilibrium line. The width is assumed to be constant and the walls are assumed to be vertical, so the width at the surface is the same as the width of the bed. The formula for calculating the change in ice thickness reduces to:

$$\frac{\partial H}{\partial t} = -\frac{\partial}{\partial x}\left[D\frac{\partial h}{\partial x}\right] + \dot{b} \tag{3.1.1}$$

Where D is now given by:

$$D = (\rho g)^3 H^3 \left(\frac{\partial h}{\partial x}\right)^2 \left(f_d H^2 + f_s\right) \tag{3.1.2}$$

3.1.1 Changes in orography

We want to know how the glacier reacts to a change in value of certain variables. The first variable we will be looking at is the slope. First we will look at the effect of the slope on the glacier length. The observation is that a smaller slope corresponds to a longer glacier length (see figure 3.2). As is visible, the relation between the slope and the glacier length is non linear.
In the next experiment, the equilibrium line is changed from 1600 meter to 1700 meter to 1800 meter and then in the same way back to 1600. Figure 3.3 is obtained when plotting the change in glacier length as a function of time for two different slopes of the bed. Here the change in glacier length is visible for various values of the slope. After a certain amount of time steps (enough for the glacier to reach its equilibrium length), the equilibrium line is increased with 100 meters. As a consequence of this, the glacier length decreases (more ablation, less accumulation). We see that the glacier length is the same before and after the changes in equilibrium line. Concerning the slopes, the observation is that the variation in the equilibrium line has more impact on a glacier on a smaller slope than on one lying with a much steeper slope, because there is a larger change in accumulation / ablation area when changing the height of the equilibrium line, so the glacier length has to compensate more. Furthermore, the glacier with a larger slope reaches its equilibrium length quicker than a glacier with a smaller slope.
Next, a small bump is added in the orography. The configuration of the glacier is illustrated in figure 3.4. This is done at a horizontal distance, x, of 10 km. The consequence is

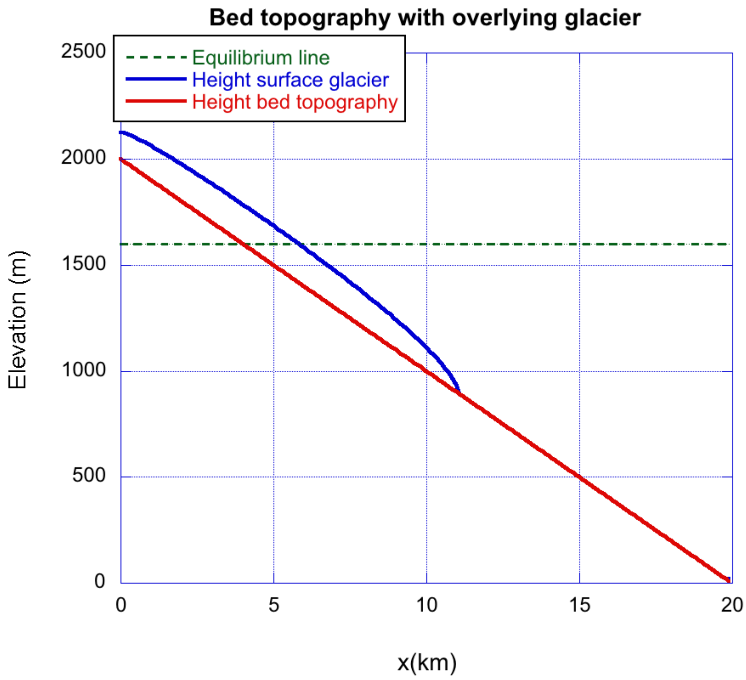


Figure 3.1 – The configuration used for the basic tests. The red line cornsponds to the height of the bed topography, while the blue line ccorresponds to the height of the surface of the glacier. The dashed line in green is the equilibrium line.

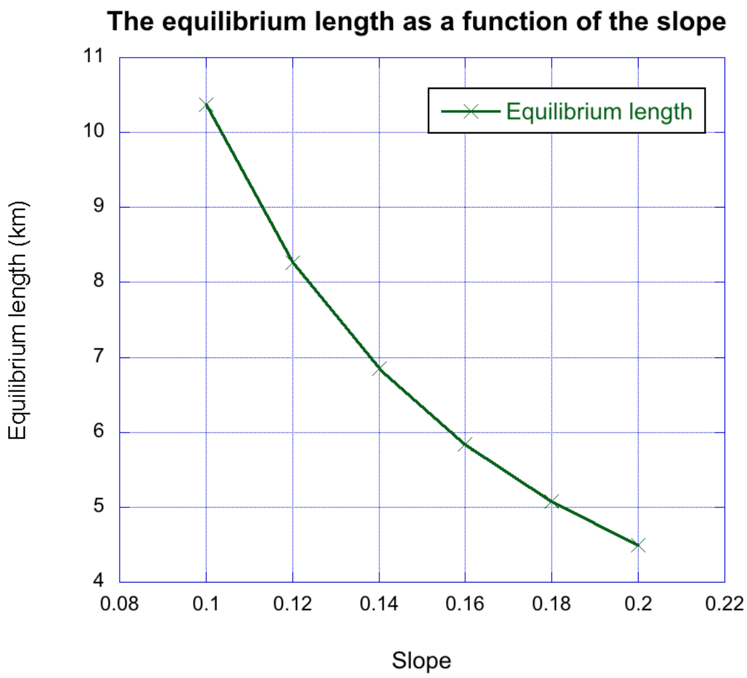


Figure 3.2 – The equilibrium length as a function of the slope. The length decreases non linearly as the slope increases.

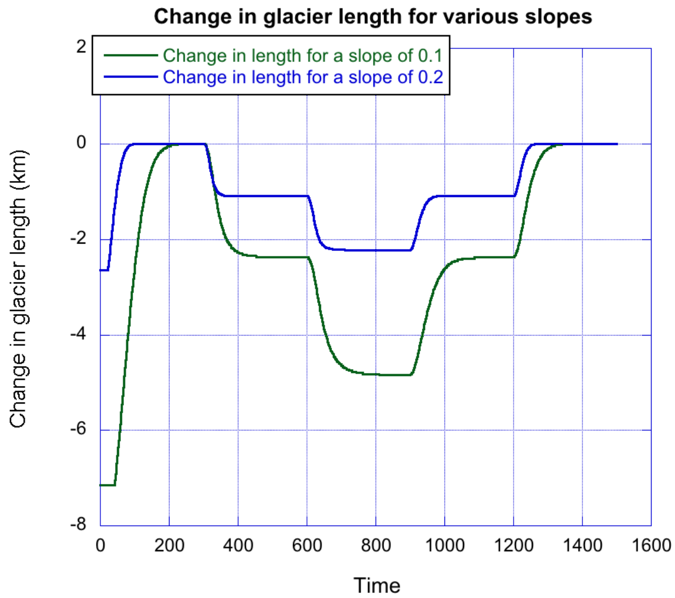


Figure 3.3 – Change in glacier length with respect to the length at an equilibrium line of 1600 m. In of 100 m, the equilibrium line is upped to 1800 and later lowered back to 1600 m. A glacier with a higher slope responds faster to a change in equilibrium length than a glacier with a smaller slope. The differences in length are larger for the glacier with the smaller slope.

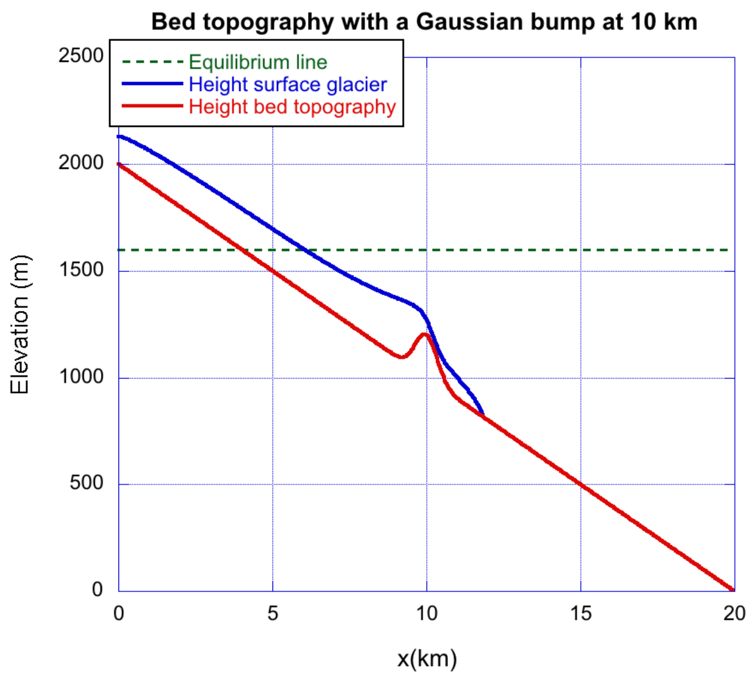


Figure 3.4 – The configuration when a bump in the orography is present. The red line correnspsonds to the height of the bed topography, while the blue line ccorresponds to the height of the surface of the glacier. The dashed line in green is the equilibrium line. The glacier becomes thicker just in front of the bump and flattens just after the bump.

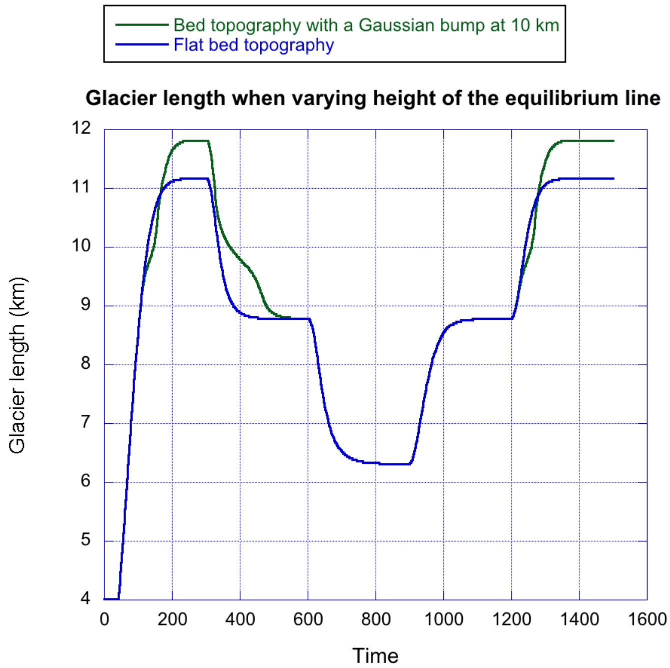


Figure 3.5 – Figure showing the glacier length, starting with an equilibrium line at 1600 m. In steps of 100 m, the equilibrium line is upped to 1800 and later lowered back to 1600 m. The green line respresents the length of the glacier where a Gaussian bump is present in the bed topography. The bump causes the length of the glacier to grow or drop more slowly.

that the length of the glacier increases. The bump causes an increase in glacier thickness just before the 10 km mark. The glacier piles up. After the bump the glacier thickness decreases quickly (only the highest part can make it over the hill) and later increases again. The latter is due to the fact that the hill has been passed and the bed is flat again. In a similar way to the slope, the equilibrium line can be changed to look at the effect of the bump on the glacier adapting to a change. Figure 3.5 is obtained. Here the equilbrium length is plotted. The green line illustrates the glacier length where a bump is added in the orography at 10 km. For an equilibrium line of 1600 meter, the glacier length is higher for the orography with the added bump. When changing the equilibrium length to 1800 meter, the glacier length suddenly become equally long. This is because the glacier is shorter than 10 km in this configuration, implying that the glacier has not reached the added bump in the orography. As a result, the two configurations are equal and thus are the lengths of the glaciers. The bump causes the length to increase or drop more slowly, as is illustrated in the figure.
Figure 3.6 illustrates the increase in ice thickness just before the bump. Here the thickness of the glacier is visible as a function of the horizontal distance x. The blue line shows the thickness for a flat orography. The green line shows the thickness when a bump is present in the orography. One can see that the thickness increases when approaching the bump from the left. The glacier basically piles up here. After passing the bump, the glacier thickness sharply decreases to a minimum after which it increases, because the orography is flat again. Another observation is that the glacier has a larger length when a bump is present in the orography. This is because there needs to be a balance between accumulation and ablation and the bump is the reason that the ablation region needs to extend a bit more, resulting in an increase in glacier length.

3.1.2 Changes in width

Next, the effect of a change in the width will be tested. Take for example a glacier with a small snout and a larger upper part (see figure 3.7). Here the width is varied at a

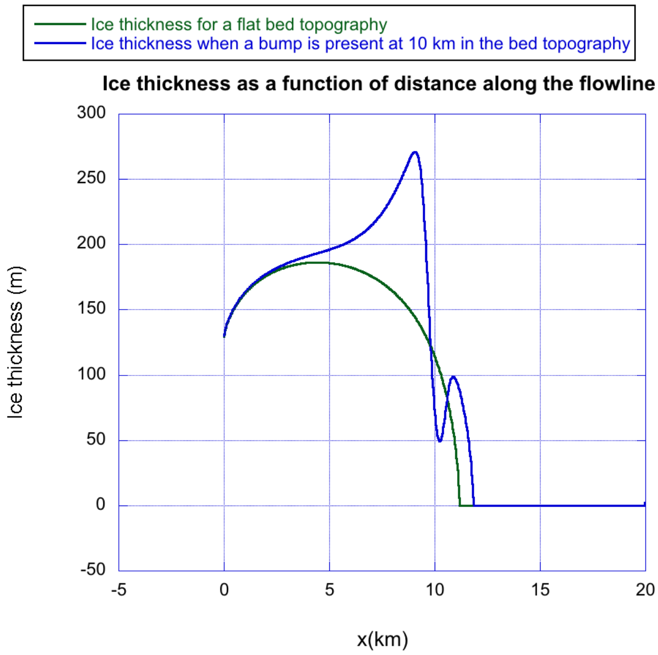


Figure 3.6 – The ice thickness as a function of distance along the flowline. The green line corresponds to a situation where a Gaussian bump is present in the bed topography. The glacier becomes thicker just in front of the bump and flattens just after the bump.

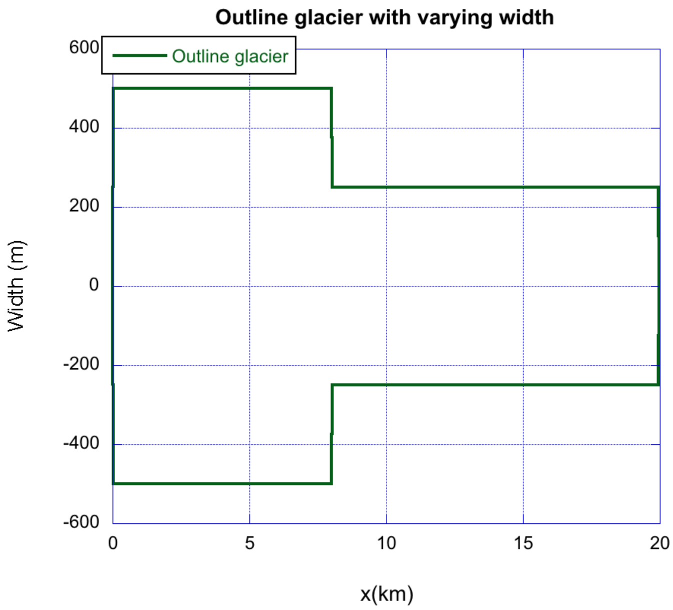
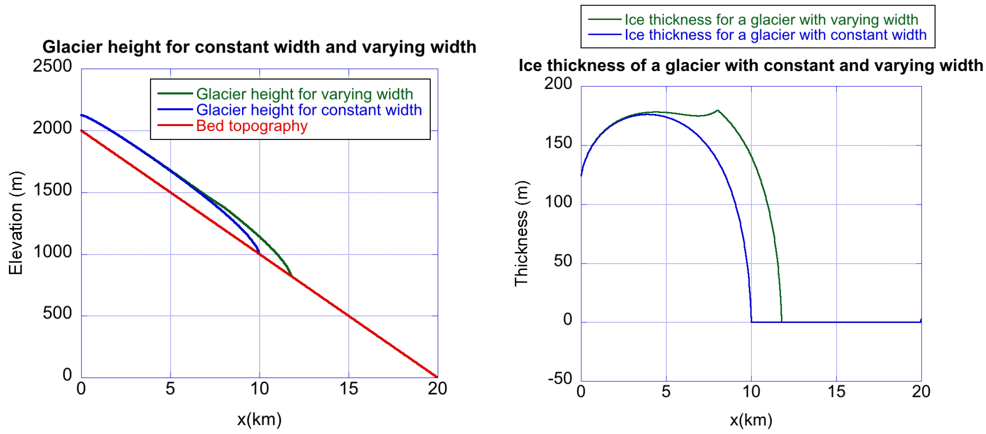


Figure 3.7 – Outline of a glacier with a width of 1 km for values of x smaller than 8 km and a width of 500 m for values of x greater than 8 km. As is visible, there is a sudden jump in the width, this is not possible in real situations.

horizontal distance of 8 km. By comparing the configuration of a glacier with a constant width to a glacier with a varying width, the effect of the width is clearly visible (see figure 3.8).



(a) **(b)**

Figure 3.8 – The configuration (figure 3.8a) and the ice thickness (figure 3.8b) for a glacier with constant width compared with a glacier with varying width. The glacier with varying width is longer, because of the smaller area in the ablation zone. The discontinuity in the ice thickness is due to the sudden jump in width.

The effect of a changing width is that the surface is mostly smaller in the ablation zone. This implies that less ice can melt away, so the ablation zone needs to extend, resulting in a larger glacier length. The glacier thickness increases before a horizontal distance of 8 km. The discontinuity at 8 km is due to the sudden jump in width. In reality, the width will vary gradually and such a discontinuity will not occur. The increase in ice thickness close to 8 km is most likely due to the fact that the ice piles up due to the discontinuity at 8 km where the width is suddenly decreased.

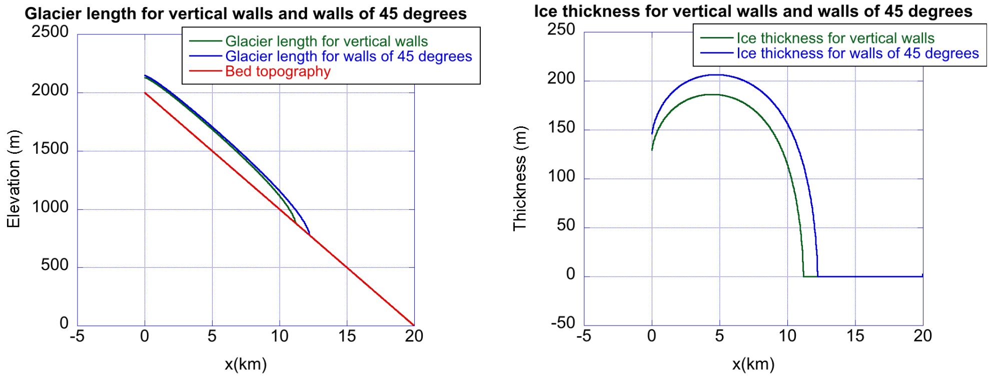
3.1.3 Angle variation

Lastly we will look at the effect of angle variations of the valley walls to the left and right of the glacier. Due to variations in the angles, the surface width can be higher than the width of the bed. This is described in figure 2.2. Plots for the ice thickness and the configuration are illustrated in figure 3.9. Here we see the glacier for the case of an angle of zero degrees. This corresponds to vertical walls. The blue line shows the configuration for an angle of 45 degrees. This means that we now have walls under an angle of 45 degrees. Because there is more surface for snow to accumulate on, the accumulation will increase. If the accumulation is increased, then the glacier extends over a longer distance in the x direction, because the ablation region needs to extend in order to balance the increase in accumulation. When the thickness decreases, the surface decreases as well.

3.2 Application to Engabreen

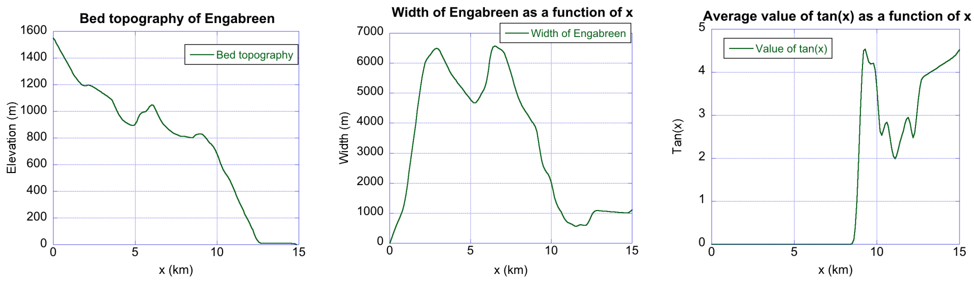
The model can now be applied to Engabreen. In doing this, reasonable estimates of the equilibrium line have to be made in order to generate reasonable results. The literature value of the equilibrium line for Engabreen is 1085 meter (NVE [2016]), so the first estimate will be around 1100 meter. The flowline is going all the way to the sea and this is useful because higher values of the glacier length are allowed. This is also useful when possible future scenarios are considered. First of all, let's take a look at the value of the different variables in the input data illustrated in figure 3.10.

Figure 3.10a contains the bed topography. There are quite a few things to note here. First of all, there are multiple overdeepenings. Secondly, it is clearly visible that the



(a) (b)

Figure 3.9 – The configuration (figure 3.9a) and the ice thickness (figure 3.9b) for a glacier with vertical walls compared with a glacier with walls at an angle of 45 degrees. The glacier with walls at an angle of 45 degrees is longer and thicker. This is because there is a greater area at the surface for snow to accumulate on.



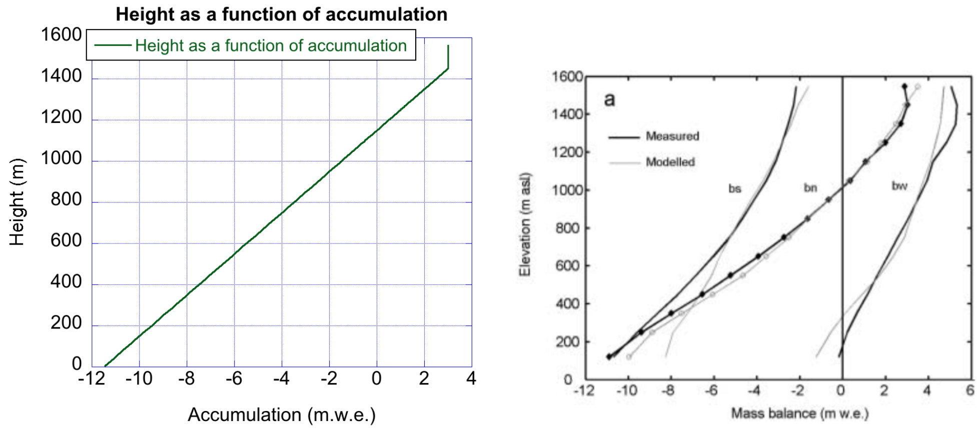
(a) (b) (c)

Figure 3.10 – The orography (3.10a), the width (3.10b) and the tangent of the angle (3.10c) as measured for Engabreen. Note that there are two overdeepenings in the orography. The width is large in the upper part of the glacier and decreases towards the snout. It gets a little larger after the snout has been passed. For the figure with the tangent the walls are assumed to be vertical in the upper part, corresponding to an angle of 0 degrees.

slope is higher in the snout of the glacier and that it flattens out when it reaches the lake. In figure 3.10b, the width has been plotted versus the distance along the flowline. In the first few kilometers, the width is clearly larger than in the narrow snout. When following the flowline through the lake the width gets a little bit larger again until it reaches the sea. Figure 3.10c contains the angle, everywhere in the upper part the walls are for simplicity chosen to be vertical. This is because the boundaries are other glacier basins, instead of mountains. From the snout onwards non zero angles are determined. Note that the values visible here are the values of λ, so this is actually the average tangent of the angle the valley walls make with the vertical. The profile that was initially chosen for the accumulation is linear, folllowing the relation:

$$\dot{b} = \beta\,(h - ela) \tag{3.2.1}$$

Here $\dot{b}$ is the accumulation and ela is the equilibrium line altitude. β can be seen as the gradient in this relation. The relation is in such a way to ensure that there is accumulation above the equilibrium line ($\dot{b} > 0$) and ablation below it ($\dot{b} < 0$). By looking at data collected in the past (Schuler, Hock, Jackson, Elvehøy, Braun, Brown, and Hagen [2005]), an accumulation profile is visible in which the accumulation no longer grows after reaching a maximum value (see figure 3.11). By looking at these plots, the maximum value for the accumulation has been set to 3 m.w.e., where m.w.e. is an abbreviation for meters water equivalent. From the same plots the value of the gradient β can also be estimated. The estimation is that β is 0.01 m.w.e. $a^{-1}m^{-1}$.



(a) (b)

Figure 3.11 – Figure (3.11a) shows the height versus the accumulation. Figure (3.11b) shows the observed accumulation profile (Schuler et al. [2005]), where b_n is the year average. For the most part, this is just a linear relation. By looking at the observed accumulation profile, the decision has been made to set the maximum value of the accumulation to 3 m.w.e., explaining the behaviour for values of the elevation higher than 1450 meter.

Now that the accumulation profile is defined, tests can be run using all the input data and the 1d flow model. By comparing with the profile for the input data, the observation is that an equilibrium line of approximately 1160 meter gives the same configuration. The literature value is around 1085 meter, so there is some deviation between the measurements and the reality, although this is within the error margin, so we conclude that there are no major errors in the model. The reason for the differences will be elaborated on in the discussions section. For comparison, both the model output of the glacier and the current situation as measured are plotted in figure 3.12. The green line corresponds to the configuration of the glacier as calculated by our 1d flow model and the blue line is the measured data. To which extent these configurations agree is a measure of how accurate the topography of the bed is determined. The ice thickness as calculated by the model is larger than the ice thickness calculated by the measured data for all values of x. This is due to the fact that the bed topography is a simplified 1d version of the real bed topography.
To test climate sensitivity, a plot of the equilibrium length as a function of the height of the equilibrium line can be made. This is often a very insightful plot containing a lot of information about the sensitivity of the glacier to climate variantions and possible hysteresis. Hysteresis is a time dependent process in which the initial state has influence on the outcome of the current state. In the case of glaciers, this means that a glacier that is gradually being shortened shows different behaviour than when the glacier is gradually made longer. This means that there are multiple length possible for the same value of the equilibrium line, depending on whether the glacier has been longer or shorter in the past. Hysteresis are e.g. caused by overdeepenings in the bed topography of the glacier. Physically this can be illustrated in the following way: a glacier that is shorter will not grow longer, because of the height of the equilibrium line. If it has to overcome a hill, the height of the bed topography increases, resulting in an increase in surface area available for accumulation. When this happens, the glacier suddenly gets longer again. While a glacier that was originally longer on the other hand, has a greater accumulation area in the first case and loses this once the height of the equilibrium line is heigher than the top of the hill. When this happens, the length of the glacier will be the same, independent from the initial state, because the hill is no longer an issue. For Engabreen, a plot of the equilibrium length versus the height of the equilibrium line is shown in figure 3.13.

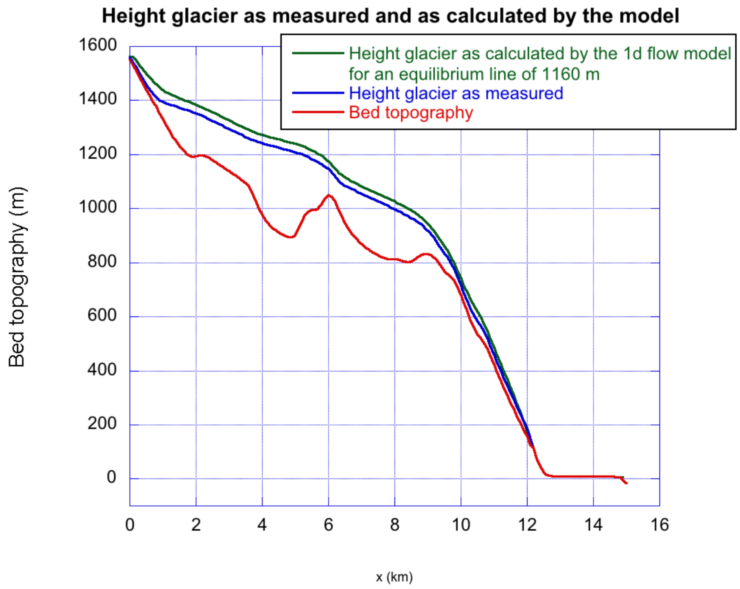


Figure 3.12 – Figure showing the configuration of the glacier as measured and as calculated by the model. The model does have a higher thickness at all times, although the difference is not that large.

The second process causing hysteresis is the flattening of the slope. This is when the equilibrium line is higher than the elevation of the bed in the flattened area. If there has not been ice before, there will not be ice. This makes sense, since there is only ablation and no accumulation, while if there has been ice before (in the initial state) there will still be ice, because the surface has a higher elevation than the equilibrium line and an accumulation area is still present.

Concerning the hysteresis, an important process here will be the overdeepening. Both hysteresis correspond to region with an overdeepening, with the large hysteresis corresponding to the large overdeepening and the small hysteresis corresponding to the small overdeepening.

Engabreen reacts more sensitive when the glacier length falls below 12 km. This is illustrated by the larger slope in figure 3.13 for a glacier length of 12 km. This is because the end of the glacier is then situated in the small snout. For glacier lengths higher than 12 km, the width is larger. This means that when the wider accumulation region becomes shorter, the same surface has to be shortened in the ablation zone to get equilibrium. This area extends however over a much longer distance along the flowline, simply because the width is smaller here. A process working against this is the slope. The slope increases in this part, and an increase in slope implies a less sensitive glacier. From our plot we can conclude that the smaller width is clearly the dominant process here, since the glacier does get more sensitive in this part. The smaller width is thus the reason for the increase in sensitivity around the 12 km mark.

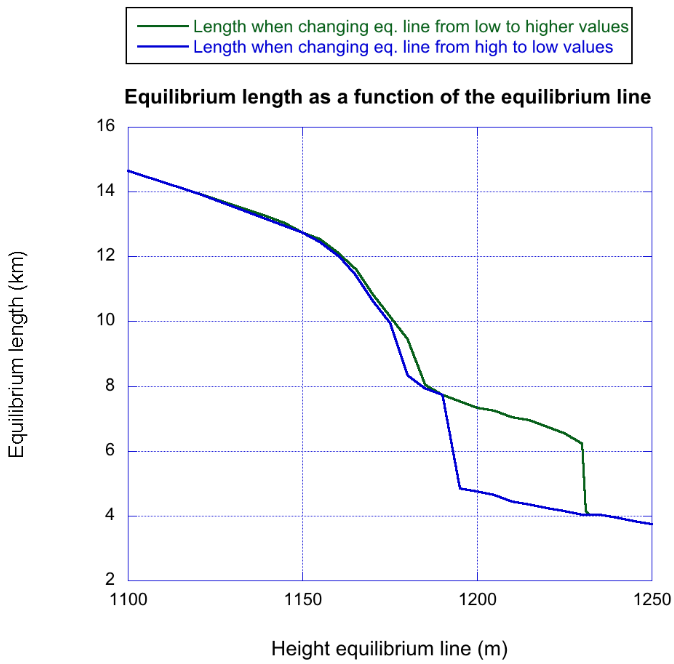


Figure 3.13 – Figure showing the equilibrium length versus the equilibrium line. Between an equilibrium line of 1190 meter and 1230 meter, a large hysteresis is visible. This corresponds with two processes. The first process is as explained earlier the overdeepening. This is explained later in this section. One other thing to notice is the sensitivity once the length of the glacier becomes shorter than 12 km. This is the current configuration and this is where the snout endsand the broader lake starts. The glacier reacts more sensitive here due to the small width of the snout. It can be shorter over a longer distance along the flowline due to the fact that the glacier is much wider in the accumulation zone than in the snout. There is also a small hysteresis for an equilibrium line between 1150 meter and 1190 meter.

Chapter 4

Discussion and conclusions

4.1 Discussion

By comparing the value of the equilibrium line of 1160 meter needed to generate the current situation with the literature value of 1085 meter(NVE [2016]), it is clearly visible that the value of the equilibrium line used in the model deviates quite a bit from the literature value. There are multiple reasons for this deviation.
The first and probably most important reason is that the assumption in this model is that glacier reaches an equilibrium length. In our model, the model is run as long as needed to get to the equilibrium length. In reality, a glacier is not given enough time to reach an equilibrium length, because the climate changes constantly. The current glacier length is consequently not the length that it would have in equilibrium. This implies that the glacier length in equilibrium is shorter than it is today, since the glacier is retreating. Thus the results found here deviate even further from the real situation.
Secondly, the geometry of the glacier is very simply represented. The assumption made here is that there are no variations in the bed perpendicular to the flowline. In reality there are definitely variations and the geometry is therefore more complicated than the geometry used here. Furthermore, as described in section 2.3 Gaussian smoothing has been applied. So the bed topography is also a simplification of the real bed topography. This is, as said, partly due to the fact that a 1D model is used. A 2D model would probably work better with complicated geometries.
Thirdly, there are quite a few assumptions that have a minor effect. First of all, the valley walls are assumed to be vertical in the upper and more shallow part of Engabreen. Furthermore, a typical value for the maximum of τ is assumed. Regarding the accumulation, a linear profile has been assumed, with a maximum value for the accumulation. In literature, a linear approximation is pretty accurate up until the accumulation reaches a maximum value after which a hyperbolic profile is assumed. Due to time, this has not been implemented here, but this can still provide some improvement.
Lastly, all values for the width, the height, the distance along the flowline and the angles to the left and to the right of the glacier are all measured by hand using a curvimeter or a ruler. This in itself brings in an error. It would definitely help if these values could be measured more precisely. This can in theory be done by using the shape of the glacier outline and measure distances digitally using GIS (geographic information software). In this project, there has not been enough time to get familiar with this program and the decision has been made that it would have been more instructive to leave this part out. This would, without a doubt, improve the import data.

4.2 Conclusions

In this thesis a 1D flow model has been used to desribe the sensitivity of Engabreen to changes in climate forcing. First the effect of a change in various parameters, e.g. in the slope and the width, has been examined. Next, the data of Engabreen has been determined and the model is applied to Engabreen. Afterwards the knowledge acquired in the basic tests is used to explain the results obtained for Engabreen.
Glacier are never in equilibrium and this limits the ability to draw conclusions. In reality, the current length is not an equilibrium length, so it is difficult to compare the results in the model (values in the case of an equilibrium) with the real configuration. The value of the equilibrium line needed to agree with the present day situation does definitely vary, but this is within the error margin. Because this is within the error margin, conclusions can still be drawn regarding the sensitivity of Engabreen.
Some conclusions can be made about the plot of the equilibrium length versus the equilibrium line. It is first of all difficult to apply the model to reality, but there are definitely some things that can be said regarding the sensitivity of engabreen. Engabreen will in general be relatively sensitive to changes in climate forcing, due to the small snout dominating over the increase in slope. It will be relatively sensitive until it reaches an equilibrium length of approximately 8 km. Here Engabreen will be less sensitive due to the large hysteresis. This will slow down the retreat of Engabreen and in this part Engabreen will be less sensitive. One thing to note is that Engabreen has to be about 4 km shorter than it is currently to reacht the hysteresis, so this will most likely not occur in the near future.
For future experiments, different models, for example a two dimensional model, can be used to acquire more information about Engabreen. As explained in the discussions section, the input data can be improved, although this probably doesn't matter that much when drawing qualitative conclusions. Furthermore, there are a lot more characteristics to study on Engabreen, for example the response time and the sliding and deformation velocity, these could be studied as well.

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