Utrecht University

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Master Thesis

Magnetic Catalysis and Holography

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ABSTRACT

In this thesis we will study the effects of a magnetic field on the quark condensate. To do this we'll look at Quantum Chromo Dynamics, the breaking of chiral symmetry and the formation of the chiral condensate and how the magnetic field affects this. We'll discuss two different models, first the Improved Holographic QCD model with an added flavour sector, and the AdS-soliton model. While we could show a constructive effect of the magnetic field on the quark condensate in the soliton model, this proved to be out of our reach in the IHQCD theory due to the inability to get the proper tachyon divergence.

CONTENTS

1

INTRODUCTION

Quantum Chromodynamics has been an important theory ever sinze its first formulation. The phenomena of asymptotic freedom and confinement make this theory very interesting, but due to the strong coupling it requires different methods of analysis. Lattice QCD has been tried with varying succes, but Maldacena's conjecture of the AdS/CFT correspondense [1] seems to be the most promising alternative method thus far.

Like many theories, QCD comes with several different phases and phase transitions. There is the first order transition between a confining and a deconfining phase, responsible for the formation of a Quark Gluon Plasma in extreme conditions. Another second order phase transition is the breaking of the chiral symmetry, leading to the existence of a quark-antiquark condensate. This condensate shows non-trivial behaviour when an external magnetic field is applied. Studies on this subject [2] indicate that such a magnetic field should have a constructive effect on the condensate, but more recent LQCD studies also indicate destructive effects. The main purpose of this study is to determine how the Quark Condensate will react to the presence of a uniform magnetic field by using the gauge-gravity duality. The first part of this study gives the theoretical framework required, while the second part deals with the calculations and numerical methods. Chapter 1 will contain a review of Quantum Chromo Dynamics, the theory that describes quarks, gluons and their interactions. In chapter 2 we will take a look at the topic of magnetic catalysis, both the predictions from QCD and some results from Lattice QCD. Chapter 3 will give a review and motivation of the gauge-gravity duality, the technique we will use to analyze the system. In the second part of this thesis we'll discuss the results gained from two different models, chapter 4 and 5 respectively.

Part I

THEORY

2

QUANTUM CHROMO DYNAMICS

Quantum Chromo Dynamics (QCD) is the fundemental theory of the strong interactions. According to this theory, all matter is made up out of two kinds of elementary particles: gluons and quarks. Similar to how molecules are made up of atoms, atoms in turn are made up of protons, neutrons and electrons. It's no strange step to then postulate the existence of even smaller elementary particles: Quarks. The quarks would have to have non-integer electric charges but would be able to explain the systematics seen in the masses and spins of baryons.

People have put a lot of effort in searching and isolating these quarks, unfortunately no isolated quarks have ever been spotted. In contrast, the research into deep in-elastic scattering experiments was very succesful, it was shown that protons have fragmentally charged constituents that would behave like point-like particles at short distances.

This seeming contradiction between strongly interacting quarks inside hadrons and weakly interacting quarks seen in high-energy scattering was resolved with the discovery that non-abelian gauge theories can be asymptotically free. This suggests that the strong force responsible for the interactions between quarks is propagated by a (non abelian) gauge group. The force carriers of this force are called gluons which, like quarks, have not been directly obserbed in nature, yet.

2.1 QUARKS AND GLUONS

So far, six different types of quarks are *known* to exist, three with electric charge $\frac{2}{3}$ and 3 with electric charge $-\frac{1}{3}$. Quarks belong to the group of fermionic matter, and the six variants were given the names: up (u), charm (c), top (t) and down (d), strange (s), bottom (b). The different kinds of quarks are called quark flavours. As mentioned, they have not been detected as isolated particles and as such, their masses are not exactly known. They can however be estimated from hadron spectroscopy once the composition of the hadron (in terms of the quarks) is given. The Quark Flavours and their estimated masses, as well as their electric charge, are given in tabel 2.1.

Flavour	mass GeV/c^2	Q
u	0.01	$\frac{2}{3}$
d	0.01	$-\frac{1}{3}$
c	1.5	$\frac{2}{3}$
s	0.1	$-\frac{1}{3}$
t	172	$\frac{2}{3}$
b	5	$-\frac{1}{3}$

Of course, every quark has a corresponding anti-quark: $\bar{u}$, $\bar{c}$, $\bar{t}$ and $\bar{d}$, $\bar{s}$, $\bar{b}$ which have opposite electric charge but equal mass.

The interactions of the quarks are to be described by a non-abelian gauge group, this automatically means each quark (with a particular flavour) has to be extended, otherwise they cannot transform according to a representation of the gauge group. The smallest representation is the three dimensional reprentation of $SU(3)$, which means that every quark of a given flavour must appear three fold (although other representations might require more). We'll be dealing with a triplet of quark fields denoted as a three component vector: $q^i = (q_1(x), q_2(x), q_3(x))$. These three components are called *colours* and are usually denoted as red, green and blue, obviously this does not refer to any actual color. Any composite material made from quarks has the property that it's colorless. In Baryons three quarks with each a different colour combine into a colourless particle in the same way that white light can be split into red, blue and green. Mesons, particles made up from a quark and an anti-quark, are colourless because anti quarks have *anti*-colour.

The group $SU(3)$ is eight dimensional and so there must be eight gauge fields, denoted by V^a_μ. Under $SU(3)$ the quark fields then transform in the fundamental triplet representation:

$$q(x) \to q'(x) = \exp(\xi^a(x) t_a) q(x), \tag{1}$$

with $\xi^a(x)$ denoting the eight space-time dependent transformation parameters of $SU(3)$, the three by three matrices t_a denote the generators of $SU(3)$. The t_a matrices are often expressed in terms of eight hermitian traceless matrices λ_a, which are just generalizations of the Pauli Spin matrices.

The Invariant Lagrangian now takes the standard form [3]:

$$\mathcal{L} = -\frac{1}{4}(G^a_{\mu\nu})^2 - \bar{q} D_\mu \gamma^\mu q - m\bar{q}q, \tag{2}$$

with

$$G^a_{\mu\nu} = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu - g f^a_{bc} V^b_\mu V^c_\nu \tag{3}$$

$$D_\mu q^i = \partial_\mu q^i - g V^a_\mu t^i_{aj} q^j \tag{4}$$

where the structure constants are given by: $f^c_{ab} t_c = [t_a, t_b]$.

For all quark flavours, the QCD Lagrangian looks like 2, except that the actual value of the quark mass is different. The full Lagrangian thus depends on the QCD coupling constant g and on the mass parameters m. While the mass of the quarks differs between different quark flavours, it should not differ between quark colours, otherwise the $SU(3)$ gauge symmetry is broken, and the whole construction becomes useless. Of course, the parameter m in the above lagrangian cannot be identified with the masses from table 2.1, these should follow from solving the full QCD field equations at the non-peturbative level.

The flavour of quarks will not be changed via the strong interaction. However, the weak interactions can change the flavour of quarks. Gluons do not carry flavour, but they do have color (since they transform under the $SU(3)$ gauge group). Therefore, it's possible to probe the quark make-up of hadrons with the electroweak force. Approximately 50% of the momentum of the proton is carried by the quarks, the other 50% must be carried by constituents that do not interact electroweakly, according to the above theory, these must be the gluons [3].

The masses of the u and d quarks are not only nearly equal, they're also very small. This implies that the lagrangian has an approximate flavour symmetry (other than a simple u $\leftrightarrow$ d symmetry): for vanishing quark masses the lagrangian is invariant under chiral transformations, which are given by the group $U(2) \times U(2)$. These are unitary transformations of the u and d fields that involve the gamma matrix γ^5. This chiral symmetry is realized in a sponteneously broken way and this leads to the quark condensate.

2.1.1 *Asymptotic Freedom*

A unique property of non-abelian gauge theories is that they can be asymptotically free, this implies that the strength of the interaction vanishes with (vanishingly) small distances. Accepting QCD as a good representation of nature then offers a natural explanation why one only observes weakly bound, point-like constituents in the hadrons when probing them with high-energy leptons.

The important point is that an asymptotically free theory one can test these ideas in a quantative fashion, because one may use pertubation theory for highly energetic reactions, using the small gauge field coupling as expansion parameter. This method of analysis is called Pertubative QCD and it can be made precise enough to justify seeing QCD as the fundamental theory of hadrons.

2.1.2 *Colour confinement*

Nonperturbative QCD is more difficult to understand. One of the major problems is to explain the fact that only color singlets are seen in nature. This is actually a long distance phenomenon: when a quark of gluon tries to break out from a hadron, the force that binds it increases indefinitely with the distance, like an elastic band, preventing its escape.

This model is sketched in figure 1. To fully understand it one can compare it with Quantum Electrodynamics (QED). In this theory, the electric charges are not confined (after all, it's described by an Abelian Gauge Theory) and the theory is not asymptotically free. Additionally the force carriers of QED, the photon gauge fields, do not self interact. In contrast, the self interacting gluon gauge fields makes the theory non linear at low energies. As a result, the interactions between quarks are not simply the exchange of a gluon. Effectively, these nonlinear terms surpress any spread of the gluons in the transverse directions, so that the stringlike field configurations dominate. These stringlike gluons, with a quark and anti-quark at the end then describe why colour is confined: when the two quarks are forced apart, the strings are stretched, corresponding to an increase in the strong force. This force will increase untill the string breaks by making quark-antiquark pair. In such a way, a colourless hadron breaks up in to two colourless hadrons. The Lund String Model is based on this idea and accurately describes many aspects of hadron formation and fragmentation in scattering processes. Note that the String in this model is not the same strings used in String Theory. The strings in the latter are fundemental particles of the theory while in the former they are just a collection of field-lines that appear to behave as a string.

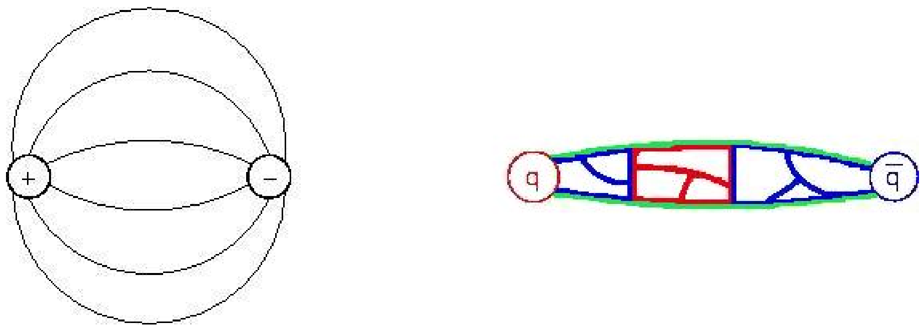

Figure 1: On the left is a sketch of QED, the gauge fields (photons) between two charged particles do not interact and are free to spread out in any direction. On the right is a sketch of QCD, here the gauge fields (gluons) do interact, and as a result, they're kept tightly bound in between the two quarks.

2.2 METHODS OF INVESTIGATING QCD

There are several ways to investigate the results and intricacies of QCD. Like with most things, each method comes with its advantages and disadvantages. Here we'll mention a few techniques, some important results and shortcomings.

2.2.1 *Perturbative QCD*

As the name implies, perturbative QCD is the study of perturbative effects in QCD. Like with other field theories the pertubative expansion comes as a series in the coupling constant (α_s). However, as mentioned in subsection 2.1.1, this coupling constant is only a small parameter when one is dealing with high-energy (or short distance) interactions. For low energy or long distance interactions the coupling constant is large and hence cannot be used as an expansion parameter. In most cases, making testable predictions with QCD is difficult due to an infinite number of topologically inequivalent interactions. However, there is an area with short distance interactions where one can make decent approximations. One of the most important results from pertubative QCD is the measurement of the R ratio [4]:

$$R = \frac{\#(e^+ + e^- \to \text{Hadrons})}{\#(e^+ + e^- \to \mu^+ + \mu^-)}, \tag{5}$$

which is the ratio between the production rate of Hadrons and the production rate of muons, in an electron-positron annihilation. One of the shortcomings of perturbative QCD is that Hadrons are non-perturbative in nature. Since it's impossible to observe quarks and gluons due to color confinement, this makes it impossible to solve most processes.

2.2.2 *Lattice QCD*

Lattice QCD is a method that is used to describe non-perturbative effects in QCD. The word lattice in the name comes from the fact that it formulates QCD on a discrete space-time grid. One can show that in the theoretical limit of an infinitely large lattice and infinitesimally small lattice spacing the continuum QCD is recovered. As such one can expect that lattice QCD gives reasonable approximations to reality.

Lattice QCD has been able to make verifiable predictions about several experiments. Amongst them is the mass of the proton which was determined theoretically with an error less than 2 percent [5].

Unfortunately, lattice QCD is computationally intensive. The amount of lattice points in a 4D grid becomes very large very soon and a lot of memory is needed to have it work. Such a problem could of course be fixed with the usage of more powerful computers, but other problems play a role too. The addition of a chemical potential leads to complex eigenvalues which in turn make it impossible to give a statistical interpretation. This is called the *sign problem*.

There are more ways to get results in QCD, one of them will be used as a motivation for a string duality in chapter (4).

2.3 QUARK CONDENSATE

Consider QCD with two massless quarks, one quark is up, the other is down. Like the Lagrangian given in section 2.1, we have:

$$\mathcal{L} = \bar{u} D_\mu \gamma^\mu u + \bar{d} D_\mu \gamma^\mu d + \mathcal{L}_{gluons}. \tag{6}$$

where $\mathcal{L}$ is lagrangian for the gluons which hasn't been specified, terms without a quark field were ignored. This Lagrangian can be expanded in terms of left and right handed spinors, it will read:

$$\mathcal{L} = \bar{u}_L D_\mu \gamma^\mu u_L + \bar{u}_R D_\mu \gamma^\mu u_R + \bar{d}_L D_\mu \gamma^\mu u_L + \bar{d}_R D_\mu \gamma^\mu u_R + \mathcal{L}_{gluons}. \tag{7}$$

Defining $q = (u, d)$ this Lagrangian can be rewritten again as:

$$\mathcal{L} = \bar{q}_L D_\mu \gamma^\mu q_L + \bar{q}_R D_\mu \gamma^\mu q R + \mathcal{L}_{gluons}. \quad (8)$$

This Lagrangian is invariant under any rotation of q_L by a 2×2 unitary matrix L, and under any rotation of q_R by a 2×2 unitary matrix R. This symmetry is called Flavour symmetry and is denoted as $U(2)_L \times U(2)_R$.

There are two other $U(1)$ symmetries. One acts as: $q_n \rightarrow e^{i\theta} q_n$, and corresponds to Baryon number conservation. The second acts as: $q_n \rightarrow e^{(-1)^n i\theta} q_n$, but this does not correspond to a conserved quantity because it's broken due to a quantum anomaly.

The chiral group $SU(2)_L \times SU(2)_R$ breaks sponteneously by a quark condensate, $\langle \bar{q}^a_R q^b_L \rangle = v\delta^{ab}$, formed though nonperturbative action of the QCD gluons, into the diagional vector subgroup $SU(2)_V$ generally known as isospin. The goldstone bosons corresponding to the three broken generators are the three pions. As a consequene, the effective theory of QCD bound states (like the baryons), must now include mass terms for them. As a result, the chiral symmetry breaking induces the bulk of the hadron masses. In reality, the masses of the quarks are not equal, nor zero, so $SU(2) \times SU(2)_R$ is only an approximate symmetry. The pions are hence not massles and are called pseudo-goldstone bosons.

3

MAGNETIC CATALYSIS

We'll now take a look at Magnetic Catalysis, the process where a magnetic field enhances the quark condensate [2], see graph (3) for predicted behaviour from Lattice QCD.

Consider first the action of a relativistic fermion in a four dimensional space-time:

$$\mathcal{L} = \frac{1}{2} \int d^4x [\bar{\psi}(i\gamma^\mu D_\mu - m)\psi]. \tag{9}$$

One can inroduce an external magnetic field by setting the gauge field to:

$$A_\mu^{ext} = \{0, 0, Bx, 0\}, \tag{10}$$

where B is the magnetic field strength. Solving the dirac equations, it can be shown that the energy spectrum is given by:

$$E_n(k_3) = \pm\sqrt{m^2 + 2|eB|n + k_3}, \quad n = 0, 1, 2, ... \tag{11}$$

with the Landau index n. The Landau levels are degenerate and parameterized by the momentum parallel to the magnetic field and a parameter n which will encapture the dynamics of the plane orthogonal to the magnetic field when this is large. Effectively, this comes down to the system reducing to a (1+1)D system. This is a phenomen called Dimensional Reduction. The degeneracy factors are $\frac{|eB|}{2\pi}$ for the lowest level and $\frac{eB}{\pi}$ for the remaining levels.

We now take a look a chiral symmetry breaking. The chiral condensate is as usual defined by:

$$\langle\bar{\psi}\psi\rangle \equiv \lim_{x\to y} -\mathrm{Tr}[S(x, y)], \tag{12}$$

Where $S(x, y)$ is the propagator that takes the usual form [2]:

$$S(x, y) = (i\gamma^\mu D_\mu^x + m)\langle x|\frac{-1}{\gamma^{(\mu}D_\mu)^2 + m^2}|y\rangle \tag{13}$$

$$= (i\gamma^\mu D_\mu^x + m) \int ds \langle x|e^{-is[(\gamma^\mu D_\mu)^2 + m^2}|y\rangle. \tag{14}$$

These Matrix elements can be calculated using Schwinger's proper time approach, the result is given by:

$$S(x,y) = e^{ie\int_y^x A_\nu^{ext} dx^\nu} \tilde{S}(x-y) \tag{15}$$

with

$$\tilde{S}(x-y) = -i\int_0^\infty \frac{ds}{16(\pi s)^2}\Big(e^{-ism^2} e^{\frac{i}{4s}[(x^0)^2 - x_A^2(eBs)\cot eBs - (x_3)^2]} \tag{16}$$

$$(m + \frac{1}{2s}(\gamma^0 x^0 - y^A x_A(eBs)\cot(eBs) - \gamma^3 x^3) - \frac{eB}{2}\epsilon_{AB}\gamma^A x^B) \tag{17}$$

$$((eBs)\cot eBs - \gamma^2\gamma^2(eBs))\Big), \quad A = 1,2 \quad \epsilon_{12} = 1. \tag{18}$$

In order to use this, one has to perform a wick tranformation and look at the expression in Fourier space ($k^0 \to ik_4, s \to is$) to get:

$$\langle\bar{\psi}\psi\rangle = \frac{-i}{(2\pi)^2}\mathrm{Tr}\int d^4k \tilde{S}_E(k) \tag{19}$$

$$= \frac{4m}{(2\pi)^2}\int d^4k \int_{\frac{1}{\Lambda}}^\infty ds e^{-s(m^2 + k_4^2 + k_3^2 + k_A^2(\frac{1}{eBs}\tanh eBs)} \tag{20}$$

$$= \frac{eBm}{(2\pi)^2}\int_{\frac{1}{\Lambda}}^\infty \frac{ds}{s} e^{sm^2}\coth eBs \tag{21}$$

This result can be computed as a series expansion, giving:

$$\langle\bar{\psi}\psi\rangle \simeq -\frac{m_0}{(2\pi)^2}[\Lambda^2 + |eB|\log\left(\frac{|eB|}{\pi m_0^2}\right) - m_0^2\log\left(\frac{\Lambda^2}{2|eB|}\right) + ...] \tag{22}$$

If we take this condensate to the limit of zero mass, then it will vanish. In the mean field approximation, the gap equation for a dynamically generated mass looks like:

$$m = G\mathrm{Tr}S(x,x) \tag{23}$$

Using all the information so far, one arrives at an explicit form of the mass gap formula:

$$m \simeq \frac{G}{(2\pi)^2}[\Lambda^2 + |eB|\log\left(\frac{|eB|}{\pi m^2}\right)] \tag{24}$$

This mass gap equation can be solved and gives:

$$m \simeq \sqrt{\frac{|eB|}{\pi}}\exp\left(\frac{\Lambda^2}{2|eB|}\right)\exp\left(\frac{-2\pi^2}{G|eB|}\right) \tag{25}$$

It's clear that perturbatively the magnetic field reinforces the dynamically generated mass term. Hence *Magnetic Catalysis* is a proper name.

3.1 LATTICE QCD

At low energies, QCD describes strongly coupled systems, this makes it impossible to do pertubation theory and hence people have spent time to develop new techniques. One of these techniques is Lattice Quantum Chromo Dynamics (LQCD) which was constructed by K. Wilson who proved the confinement of quarks when QCD was put on a Euclidean lattice in his paper [6]. Once it was shown that this procedure could also produce quantitative results [7] it became possible to obtain quantitative results in the strongly coupled domain, enabling studies into confinement-deconfinement, the quark gluon plasma, etc.

In LQCD one formulates the theory of QCD on a lattice of discrete space-time points rather than on a space-time continuum. The lattice automatically provides a cut-off for both the UV and the IR regions of the theory

3.1.1 *Construction*

LQCD is started by creating a four dimensional lattice with size $N_\sigma^3 \times N_\tau$ and lattice spacing a. The volume of the system is then defined by $V = (N_\sigma a)^3$ with the temperature of the system determined by $T = \frac{1}{N_\tau a}$. While in principle any lattice is viable, for simplicity sake a cubic lattice is usually considered. The Lorentz invariance is then given by a discrete subgroup of rotations over angles of $\frac{\pi}{2}$. It's expected that the limit $a \to 0$ will return the standard QCD on the continuum.

Any dynamic theory will contain fields and derivatives of those fields and since we're looking at a discretized version of the system, these need to be converted. Coordinates in the lattice can be specified by $x^\mu = an^\mu$ with $n^\mu = (n^1, n^2, n^3, n^4)$ with integer components. Dimensionless fields are now defined as:

$$\psi_n = a\psi(na), \quad \text{so that} \quad \partial_\mu \psi(x) = a^{-1}(\psi_{n+\mu} - \psi_n) \tag{26}$$

where μ is the unit vector that points in the positive μ direction, connecting two neighbouring lattice sites.

When looking at the lattice, it appears natural to place any quarks on lattice points and for the gluons to be the link between neighbouring points. This directly represents the fact that the gluons are the mediating particles of this theory. This requires a link variable [8]:

$$U_{x,\mu} = P\exp\left(ig\int_x^{x+\mu a} dx^\mu A_\mu(x)\right) \tag{27}$$

which will describe the path ordered parallel transport of the gauge field from site x to $x+\mu$. A product of link variables that will form a closed loop is called a Wilson loop which is an integral part of QCD and can be used to show that a theory is confinement.

Since the work is done on a lattice, every observable quantity calculated will also depend on the lattice constant. Since the actual QCD theory is on a continuum it's required that a smooth limit to regain the continuous case (up to discretization errors of order $\mathcal{O}(a^2)$ exists. As mentioned before, this is done by taking the limit of $a \to 0$, but it needs to be done with some care. The temperature $T = \frac{1}{N_\tau a}$ will diverge in this limit unless $N_\tau \to \infty$. This is impossible to do in numerical calculations, but one can calculate results for different lattice spacings a and extrapolate those results to keep the temperature constant.

3.1.2 *Results*

As is typical in thermodynamics, observables are given in terms of the partition function $\mathcal{Z}$, which can be written as [9]:

$$\mathcal{Z} = \int \mathcal{D}U e^{-\beta S_g} \prod_f \det(a\gamma^\mu D_\mu + am_f), \tag{28}$$

where $f = u, ds$ is the label for the different quark flavours, a is the lattice spacing, $\beta = \frac{6}{g^2}$ is the inverse coupling, the quark masses are given by am_f and the flux of a external eB is given by Φ. We can then identify observables with the partial derivatives of the partition function.

$$f = -\frac{T}{V}\log\mathcal{Z} \tag{29}$$

gives the freen energy density while the energy density and pressure are given by:

$$\epsilon = \frac{T^2}{V}\frac{\partial\mathcal{Z}}{\partial T} \tag{30}$$

$$p = T\frac{\partial\mathcal{Z}}{\partial V}. \tag{31}$$

One of the most characteristic properties of QCD is of course the confinement-deconfinement phase transition. The phase transition is directly related to the chiral symmetry breaking and the formation of pseudo goldstone bosons. To study this quantitatively one needs to define the appropriate orderparameter that governs this phase transition. This turns out to be the Polyakov loop for the deconfinement phase transition and the chiral condensate for the chiral condensate symmetry breaking. The quark condensate is given by:

$$\langle \bar{\psi}_f \psi_f \rangle = \frac{\partial \mathcal{Z}}{\partial (am_f)}, \tag{32}$$

and one can calculate the susceptibilities with:

$$\chi_L = N_\sigma^3 \left(\langle L^2 \rangle - \langle L \rangle^2 \right), \quad \chi_m = \frac{\partial}{\partial} \langle \bar{\psi}_f \psi_f \rangle. \tag{33}$$

3.2 LQCD IN AN EXTERNAL MAGNETIC FIELD

Once the fundation of LQCD has been built, it becomes important to take into account interactions and external influences. One of the simplest additions to a theory would be to consider a system under the influence of an external magnetic field. In nature, such systems would occur naturally in heavy-ion collisions in the Quark Gluon Plasma.

Below we give results of an important analysis of Magnetic field related behaviour done by Bali et al. First is the critical temperature T_c which plays an important role in thermodynamics and the deconfinement transition. As one can see in graph (2) the critical temperature deceases with an increased magnetic field.

Another important result is of course the behaviour of the quark condensate under the influence of the magnetic field, see figure (3). Originally this influence was expected to be constructive, see section (3), which was called magnetic Catalysis. However, when looking at sufficiently high temperatures, the magnetic catalysis effect tends to break down. At first the magnetic field still has a constructive influence, albeit a weakening one. However increasing the magnetic field, or the temperature, will reveal that eventually the effects of the magnetic field become destructive, the condensate becomes weaker than it would have been without magnetic field. This new phenomenon is called Magnetic Decatalysis.

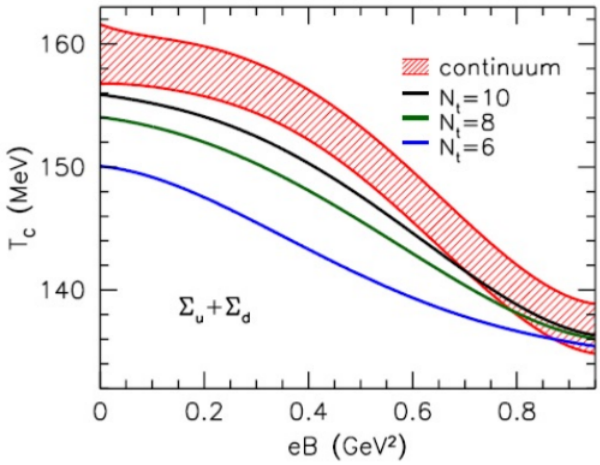

Figure 2: The critical temperature as a function of the magnetic field in 1+1+1 flavoured QCD [10]

While Lattice QCD is a decent way to procure results for all kinds of QCD effects, it unfortunately does not reveal why or through what mechanism these effects occur. Another method is needed to analyse QCD and magnetic decatalysis.

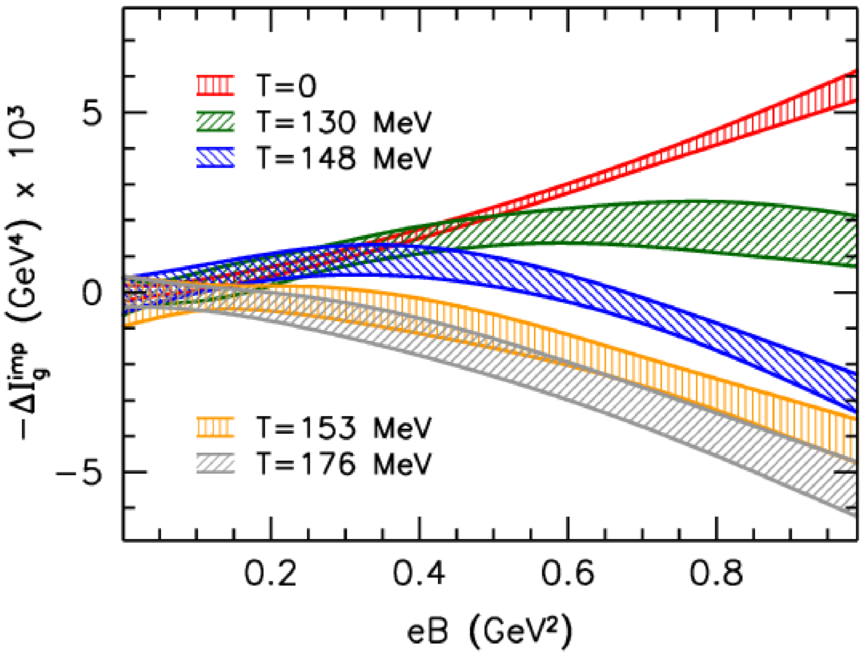

Figure 3: Magnetic Catalysis, for sufficiently large magnetic fields, at nonzero temperatures, the magnetic field will have a descructive effect on the condensate. [10]

4

HOLOGRAPHY

So far, we've seen one indication (fig (1)) that QCD should be dual to some gravity theory. In this chapter we will look into some more motivation as well as consider a specific constructed case of the duality.

4.1 MOTIVATION

There are certain properties of QCD that indicate the existence of some string dual theory. One convincing argument comes from considering the 't Hooft large N_c limit see [11] and [12]. As mentioned before, QCD is a gauge theory with a $SU(3)$ symmmetry and because of dimensional transmutation it has no expansion parameter. 't Hooft idea was to consider a generalized theory of QCD, rather than having 3 colors and the $SU(3)$ symmetry group, take N_c colors with therefore a $SU(N_c)$ gauge symmetry. Consider the limit of $N_c \to \infty$ and perform an expansion in $\frac{1}{N_c}$ which would then be a proper expansion parameter.

We can take a look back at equation (2) and simply change the values over which some indices run. The gluon fields, given by $A^i_{\mu j}$ and the quark fields q^i_a now have indices running over $i, j = 1, ..., N_c$ and $a = 1, ..., N_f$, where N_f is then the number of different quark flavours.

Since the gauge fields A have a $SU(N_c)$ symmetry they have $N_c^2 - 1$ degrees of freedom, however, since N_c is taken to be very large, the -1 is usually ignored. There are now N_c^2 gluons but only $N_f N_c$ quarks. As a consequence, the dynamics of the system are dominated by the gluons. In the limit of $N_c \to \infty$ one can thus look at the effects due to quarks as a small pertubation on the dominant gluon dynamics.

To start, consider the one loop glyon self-energy Feynman diagram of fig (4). The diagram has two vertices, and one free colour index, hence it scales as $g_{YM} N_c$ where g_{YM} is the Yang

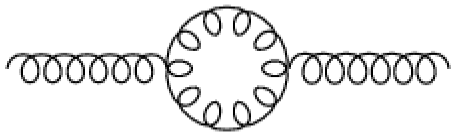

Figure 4: A one loop gluon self-energy diagram.

Mills Coupling. If this diagram is to have a finite limit when $N_c \to \infty$, we must demand that $g_{YM} \to 0$ in such a way that $\lambda = g_{YM}^2 N_c$ remains fixed. This comes down to demanding that the confinement scale, denoted by Λ_{QCD}, remains fixed in the large N_c limit. To see this, consider the one loop β-function:

$$\mu \frac{d}{d\mu} g_{YM}^2 \propto -N_c g_{YM}^4, \tag{34}$$

this becomes independent of N_c when it is written in terms of λ:

$$\mu \frac{d}{d\mu} \lambda \propto -\lambda^2 \tag{35}$$

In order to easily determine how Feynman diagrams scale with N_c we introduce the so called double-line notation. Basically, this means that the feynman propagator that's usually associated with the gluon is now written as two propagators, one associated to the quark and the other to the anti-quark, see fig (5). In fig (6) and (7) we show a few more diagrams in the double-line notation. Notice that the factor of N_c associated to the free internal line, carrying the index k in these diagrams.

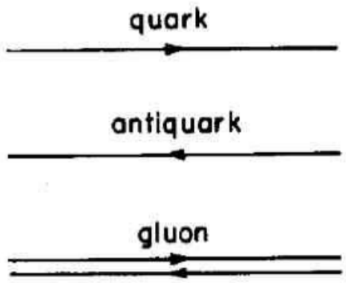

Figure 5: The double-line notation for (anti-)quarks and the gluon [13].

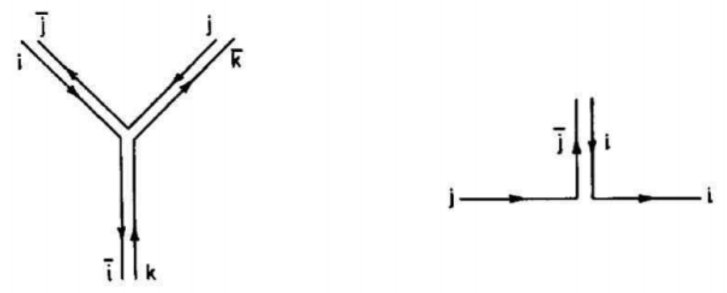

Figure 6: Vertices in the double-line Notation [13].

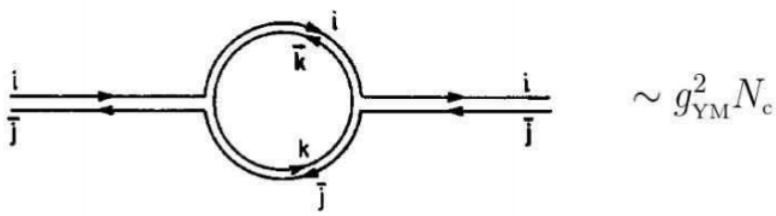

Figure 7: The gluon self energy loop again, this time in double-line notation [13].

It turns out that Feynman diagraons organise themselves in a double-series expansion in powers of $\frac{1}{N_c}$ and λ. While it should of course be proven for every diagram, we only consider some of the simple vacuum diagrams here. See fig (8).

It's clear that all diagrams scale with the same factor of N_c but with different factors of λ. In fact, the scaling with regards to that is λ^{l-1} where l is the number of loops. Of course, there are diagrams that scale with different factors of N_c^2, consider figure (9). This diagram scales as λ^2 and is thus surpressed with the diagrams of figure (8) by a power of $\frac{1}{N_c^2}$.

The major difference between the diagrams in fig (8) and fig (9) are that the latter are not planar diagrams: in order to draw them we need to actually cross lines. This gives a clear indication that the diagrams are classified by their topology, and that non-planar diagrams are supressed in the large N_c limit.

The fact that the diagrams are influenced by the topology is a direct connection to string theory. It can be show more precies by associating a Riemann surface to each Feynman di-

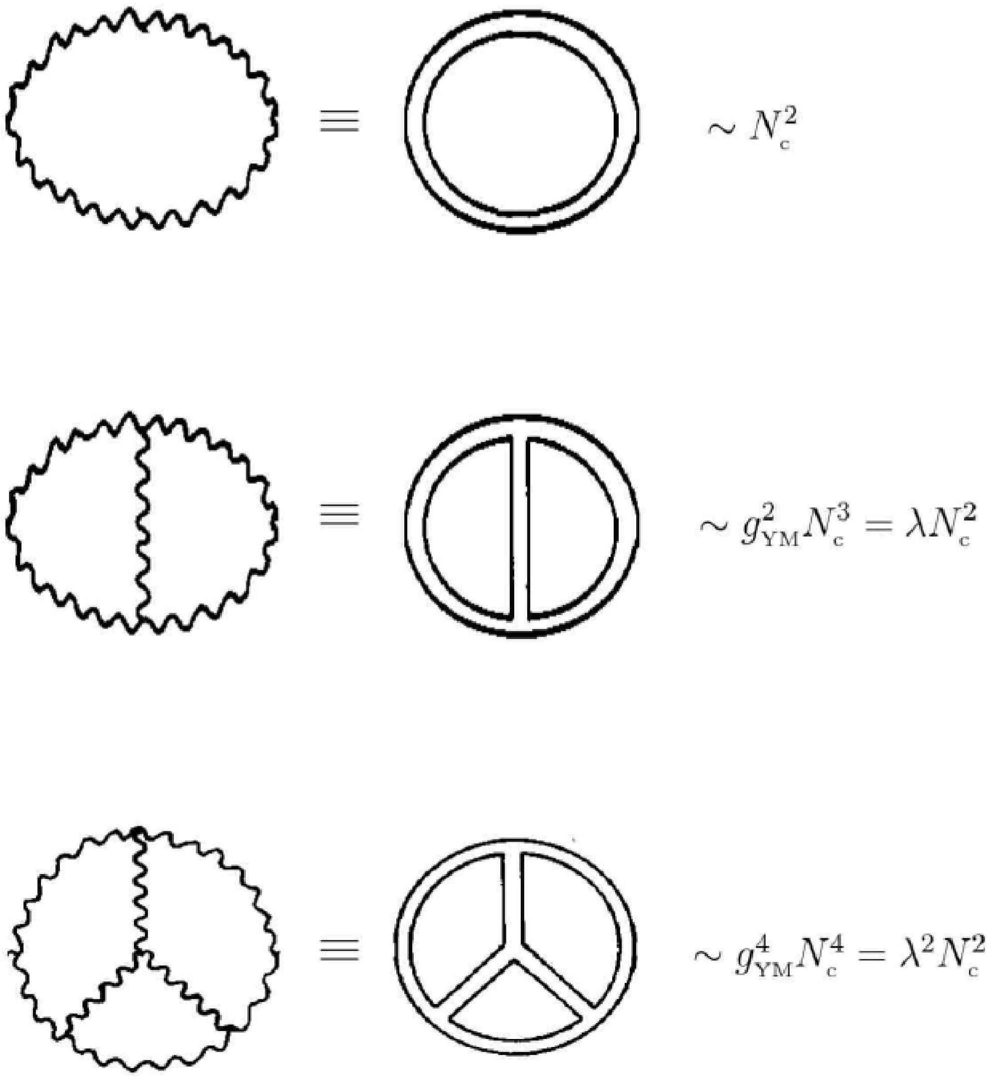

Figure 8: The Planar loops and how they scale with regards to N_c [13].

agram. This means that, in the double line notation, each line in a Feynman diagram is a closed loop that is thought of as the boundary of a two dimensional surface. The Riemann surface is obtained by glueing these surfaces together along their boundaries. To compact the diagram, we add a point at infinity that's associated to the external line in the diagram. See fig (8) for an illustration of a planar diagram and fig (9) for a non-planar diagram.

The planar diagram we obtain a sphere while the non-planar diagram gives a torus. One can show that the power of N_c associated with a given Feynman diagram is given by N_c^{χ}, where

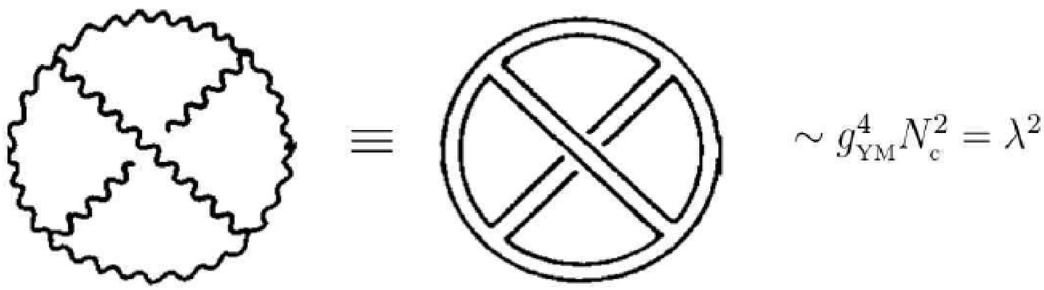

Figure 9: A Non Planar diagram [13].

χ is, as usual, the Euler number of the corresponding Riemann surface. For a compact, orientable surface of genus g, without boundaries, $\chi = 2 - 2g$. So for the sphere $\chi = 2$ and for the torus $\chi = 0$. Which is exactly what was derived before.

The conclusion is then that the expansion of any gauge theory amplitude in Feynman diagrams takes the form of:

$$\mathcal{A} = \sum_{g=0}^{\infty} N_c^{\chi} \sum_{n=0}^{\infty} c_{g,n} \lambda^n. \tag{36}$$

Where $c_{g,n}$ are constants. The first sum is the loop expansion in Rieman surfaces for a closed string theory with coupling constant $g_s \sim \frac{1}{N_c}$. As a result, the expansion parameter is therefore $frac1N_c^2$. It's shown in [13] that the second sum is associated with the α' expansion in string theory.

In the above analysis we never really used the specifics of QCD, or the fact that the gauge and matter fields were gluons and quarks respectively (we only used the double-line notation, but that was more of an ease of notation than any demand on the system). We can therefore say that the analysis should hold for any gauge theory with Yang-Mills fields, and probably for all theories with matter that belongs to the adjoint representation (since this is described by fields that carry two colour indices).

We still need to analyse what happens when we introduce quarks (or to be more general, matter in the fundamental representation). To get an idea about what happens we take a look at the two diagrams in fig (10). The difference between the two diagrams rests soley on the fact that a gluon internal loop has been replaced by a quark loop. As a consequence there is one less colour line, and hence the diagram has one factor of N_c

less. However, since the diagram works for all quark flavours we also need to sum over those, getting a factor N_f. Hence, diagrams that have internal quark loops are surpressed by a factor of $\frac{N_f}{N_c}$ with respect to diagrams that have a gluon loop instead of that quark loop. When we look again at the Riemann surface associated to a Feynman diagram, the quark loop corresponds to the introduction of a boundary. The power of N_c associated to the Feynman diagram is still N_c^{χ} but now $\chi = 2 - 2g - b$ where b is the number of boundaries. So the introduction of quarks to the Feynman diagrams reveals the need to sum over all values of b in equation (36), which makes this an expansion for a theory that deals with both closed and open strings. The open strings are associated to the boundaries and they have coupling constant of $g_{open} \sim \frac{N_f}{N_c}$.

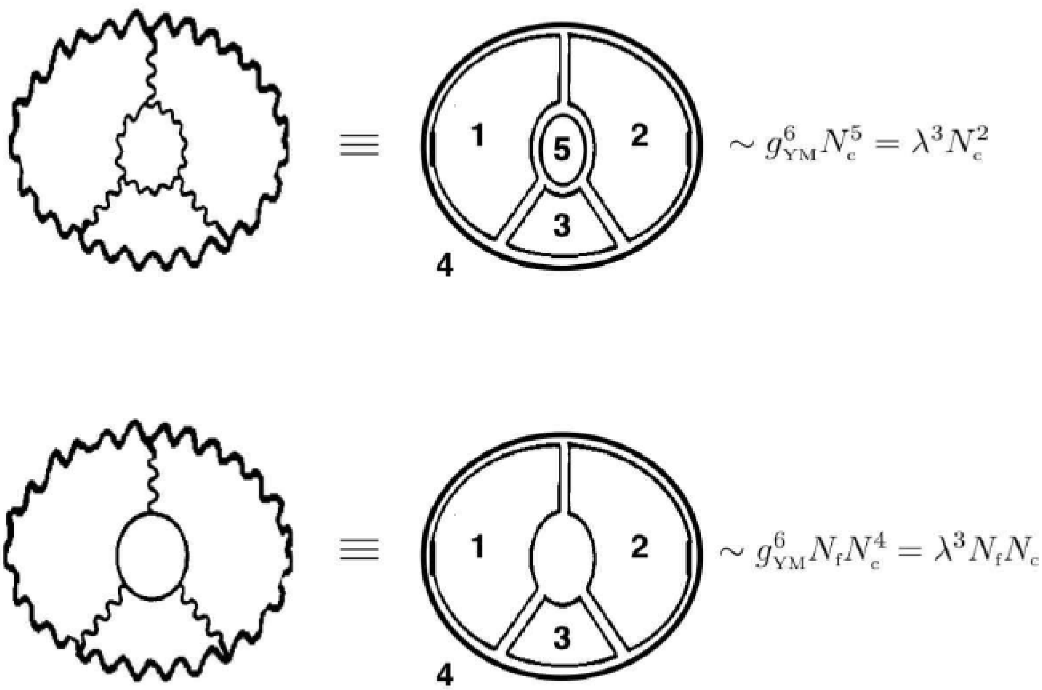

Figure 10: Two Planar loops, one without an internal quark loop (top) and one with such a loop (bottom) [13].

The above analysis clearly shows that the large N_c expansion of a gauge theory can be identified with the genus expansion of a string theory. Through this identification the planar limit of the gauge theory corresponds to the classical limit of string theory. Of course, this does not show us how to construct an explicit duality between a string theory and a gauge theory, this will explored in the next section.

4.2 THE ADS/CFT CORRESPONDENCE

In this section we will study one of the easier examples of a gauge/gravity duality: the equivalence between type IIB string theory on $AdS_5 \times S^5$ and $\mathcal{N} = 4$ Yang-Mills theory on four dimensional Minkowsky space. Later on we will see how non-AdS/non-conformal examples can be constructed.

4.3 DECOUPLING LIMIT

To start with some motivation, consider the ground state of type IIB string theory in the presence of N_c D3 branes (see fig (11)). Of course, the D-branes carry mass and charge, so they curve the space-time around them, schematically shown in fig (12).

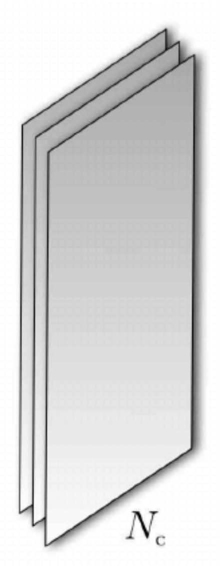

Figure 11: A stack of N_c branes [13].

Far away from the branes/mass the spacetime will be flat, ten-dimensional Minkowski space. The space towards to the branes will curve more and will take the shape of $AdS_5 \times S^5$ close to the branes. This is not the practical way to construct this spacetime, but one could conceptually obtain it by resumming an infinite number of tadpole diagrams with boundaries of the form in fig (12). For a closed string propagating in the presence of the D3 branes.

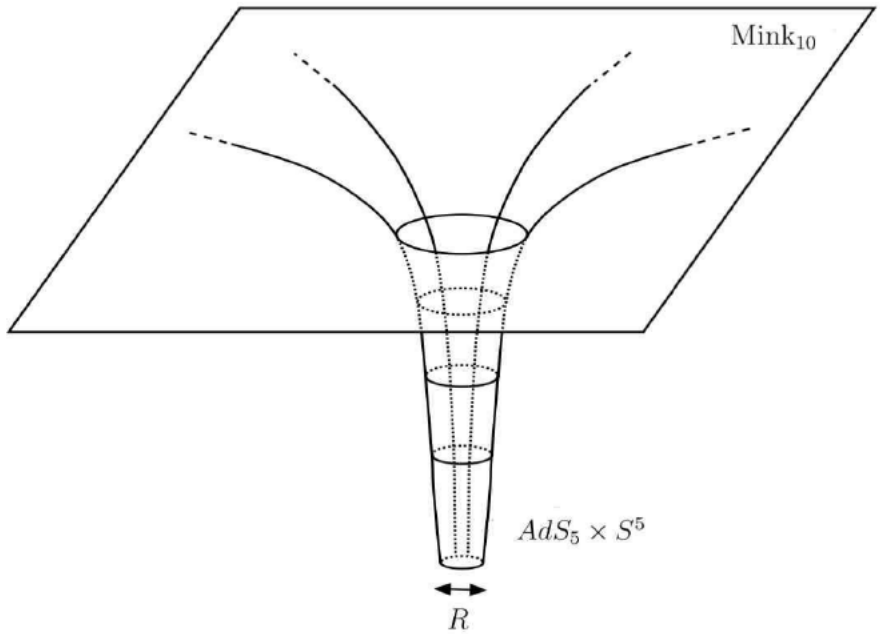

Figure 12: A schematic view of the spacetime around D3 branes [13].

We would like to compare the gravitational Radius R of the D3 branes with the string length. Generally it's expected that some power of R will be proportional to Newton's constant, the number of D3 branes and their tension. Newton's constant is given by:

$$16\pi G = (2\pi)^7 g_s^2 l_s^8 \tag{37}$$

with l_s the string length. Newton's constant is proportional to g_s^2, which has dimensions of length in 10 dimensions. The D3 branes are solitonic objects whose tension scales as the inverse power of the coupling $T_{D3} \sim \frac{1}{g_s l_s^4}$. It follows that the gravitational radius in string units must scale as $g_s N_c$. To be precise:

$$\frac{R^4}{l_s^4} = 4\pi g_s N_c \tag{38}$$

As a consequence, if $g_s N_c \ll 1$ then we can describe the stack of D branes as essentially zero-thickness objects in an otherwise flat spacetime. In this limit the D3-branes are well described as a defect in spacetime, they will be the goundary condition of open strings. That is to say, the Neumann Boundary conditions of an open string allow the endpoint of that string to sweep out a certain surface in the spacetime. Note that this only works because the tension of the D-branes scales as $\frac{1}{g_s}$ instead of the more usual $\frac{1}{g_s^2}$.

In reverse, if $g_s N_c \gg 1$ then one cannot ignore the effect of the D-branes on the surrounding spacetime. However, it turns out that the description in terms of an effective geometry for closed strings becomes simple, in this limit the size of the $AdS_5 \times S^5$ space-time near the brane will become large in terms of string units.

We now consider the groundstate of the two descriptions and take a look at the low energy (or decoupling) limit. In the first description, the lowest excitations consist of open and closed strings, check fig (13) for a schematic depiction.

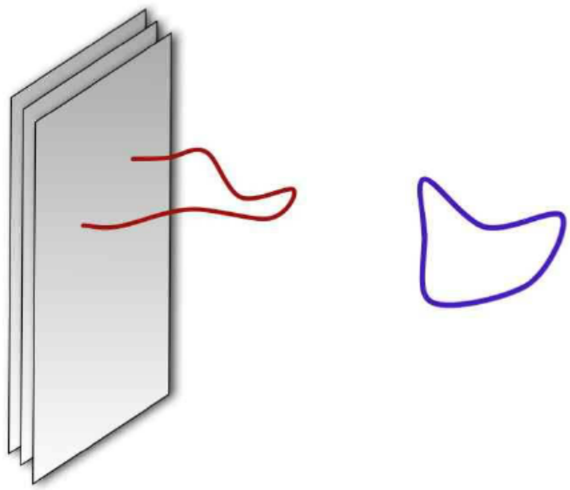

Figure 13: A stack of N_c branes with an open and a closed string excitation [13].

At low energies we can focus on the light degrees of freedom. Quantisation of the open strings leads to a spectrum consisting of massless $\mathcal{N} = 4$ $SU(N_c)$ SYM multiplet plus a tower of massive string excitations. Since the open string endpoints are constrained to to lie on the D3-branes all these modes propagate in $3+1$ flat dimensions, the world volume of the branes. Similarly, quantisation of the closed strings leads to a massless graviton supermultiplet plus a tower of massive string modes, all of which propagate in flat ten dimensional spacetime. The string of interactions of closed string modes with eachother is controlled by Newton's constant G, so the dimensionless coupling constant at an energy E is GE^8. This vanishes at lower energies and so in this limit closed string become non-interacting, which is essentially the statement that gravity is infrared free. Interactions between closed and open strings are also controlled by the same parameter, since gravity couples universally to all forms of matter. Therefore at low energies closed decouple from open strings. In contrast, interactions between open strings are controled by the $\mathcal{N} = 4$ SYM coupling constant in 4 dimensions which is given by $g_{YM}^2 \sim g_{op}^2 \sim g_s$ Note that this relation is consisten with the fact that g_{YM} is dimensionless in four dimensions.

In the second description the low energy limit consists of focusing on excitations that have an arbitrarily low energy with respect to an observer in the asymptotically flat Minkowski region. As before, there are now two distinct sets of degrees of freedom, those propagating in the Minkowsku region and those in the space around the throat, see fig (14). In the minkowski region the only modes are those of the massless ten dimensional graviton supermultiplet. Moreover, at lower energies these modes decouple from eachother since their interactions are also governed by GE^8 like before. They also decouple from modes in the throat region since at lowe energies the wavelength of those modes becomes much larger than the size of the throat, they simply cannot enter it.

Inside the throat, however, the whole tower of massive string excitations survive. This is because in order to reach the asymptoticall flat region, a mode must climb up a gravitational potential. As a consequence, a closed string of arbitrarily high proper energy in the throat may have an arbitrarily low region as seen by an observer at asymptotic infinity, provided the string is located sufficiently deep inside the throat. If we focus on lower and lower energies these modes become supported deeper and deeper in the throat so they decouple from those of the asymptotic region. Hence the conclusion is that at low energies the second description of the system reduces to interacting closed strings in $AdS_5 \times S^5$ plus free gravity in flat ten-dimensional spacetime.

At the start there were two apparently different descriptions and yet they seem to have the same underlying physics, hence the conjecture that these two theories are dual to eachother.

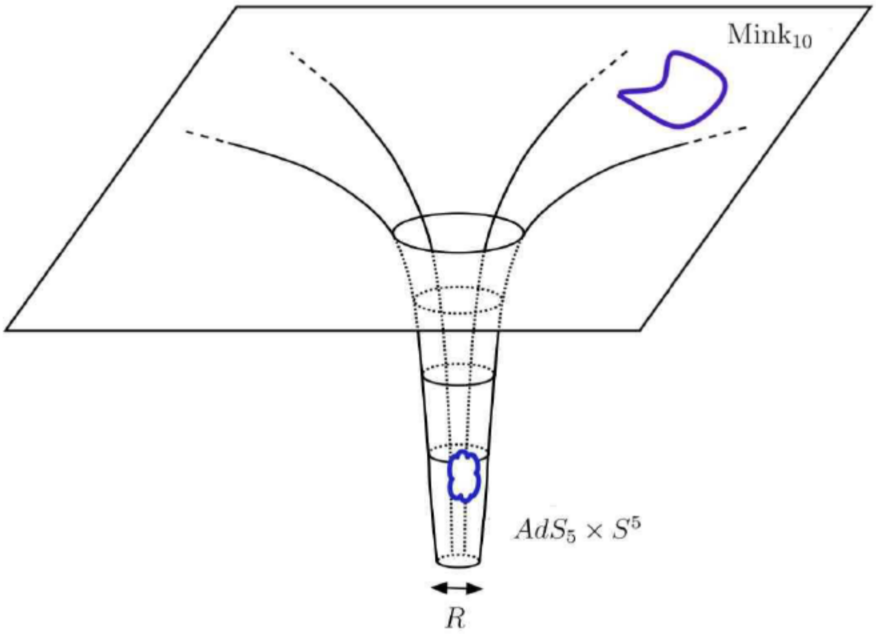

Figure 14: The curved spacetime near D3 branes with excitations [13].

4.4 THE ADS/CFT CORRESPONDENCE

So far, we've seen three different ideas that lend credit to QCD having a gravity dual theory. First of all was the string like behaviour of the quark-antiquark pair, then we derived an open-and-closed string expansion by looking at Feynman diagrams in the double-line notation together with the large N_c limit. And finally we took a look at the decoupling limit of a stack of D branes.

While one might argue that there were a lot of approximations made, and even a bit of handwaving was involved, we feel we've given enough motivation to make the statement:

$$\mathcal{N} = 4 \text{ SYM with gauge group } SU(N_c) \iff \text{type IIB superstring theory on } AdS_5 \times S^5 \quad (39)$$

Of course, it was shown that this could be correct only in the case of the large N_c limit and at fixed 't Hooft coupling $\lambda = g_s N_c$. Maldacena generalized this idea to the conjecture that the duality goes beyond the approximations made. Typically, the ADS/CFT correspondence is divided into forms of different strengths:

The strongest form of the AdS/CFT correspondence states that the duality between the supersymmetric $SU(N_c)$ gauge theory and type IIB supergravity holds for any value of N and any value of the coupling g_s. This basically means that $\mathcal{N} = 4$ SYM theory is exactly equal to the full type IIB superstring theory on $AdS_5 \times S^5$. Because there is no known way to (non-perturbatively) quantize string theory yet, let alone in a curved space-time, there is no way to test the strong form.

The weak form of AdS/CFT is the one stated above. The correspondence is only valid when $N_c \to \infty$ and λ becomes very large. It then relates the $\mathcal{N} = 4$ SYM theory at strong coupling and $N_c \to \infty$ with classical supergravity.

4.4.1 *The mapping between the two theories*

We've motivated and stated the Maldacena conjecture so now it's time to develop the mapping between the two theories. To be more precise, we need to work out a dictionary between objects of the two theories, in particular between representations of the common symmetry groups.

CFT correlation functions

Correlation functions are important in any field theory, and hence it's no surprise they can be used to test the AdS/CFT correspondence. Let's consider a general n point function of composite, regularized gauge invariant operators $\mathcal{O}_k$:

$$\langle \mathcal{O}_1(x_1)...\mathcal{O}_n(x_n)\rangle. \tag{40}$$

The most general way to compute such correlators is of course by using the generating functional ($\mathcal{Z}[\mathcal{J}]$ ($W[J]$ for connected diagrams), defined by:

$$\mathcal{Z}[\mathcal{J}] \equiv \langle \exp\left(-\int d^D x \mathcal{L}_J\right)\rangle = e^{-W[j]}, \tag{41}$$

where $\mathcal{L}_J$ is the lagrangian (density of a given field theory with added source terms coupled to a basis of $\mathcal{O}_i$ of the gauge invariant operators:

$$\mathcal{L}_J = \mathcal{L} + \sum_i J_i \mathcal{O}_i. \tag{42}$$

The n point function is given by:

$$\langle \mathcal{O}_1(x_1)...\mathcal{O}_n(x_n)\rangle = \frac{\delta^n \log(\mathcal{Z}[J])}{\delta J_1(x_1)...\delta J_n(x_n)}|_{J_i=0}. \tag{43}$$

In order to calculate these correlation functions in $AdS_5 \times S^5$ we wick rotate to get a Euclidean signature rather than a Minkowski one. In the Poincare coordinates we then write:

$$H = \{(z_0, \vec{z}), z_0 > 0, \vec{z} \in \mathbb{R}^4\}, \quad \partial H = \mathbb{R}^4. \tag{44}$$

The metric, given by:

$$ds^2 = \frac{1}{z_0^2}\left(dz_0^2 + d\vec{z}^2\right), \tag{45}$$

will diverge at the boundary $z_0 = 0$, but this is only a coordinate singularity, not a curvature one. Weyl scaling can be used to remove this divergence but it's sometimes more useful to consider a cutoff at fixed $z_0 = \epsilon$. The UV cutoff $\lambda = \frac{1}{\epsilon}$ is mapped to the IR cutoff ϵ in AdS. The $\mathcal{N} = 4$ SYM can be understood to live on the boundary of AdS_5.

A typical gauge invariant operator in $SU(N)$ SYM with $\mathcal{N} = 4$ is given by:

$$\mathcal{O}_\Delta(x) = \text{sTR}\{X^{i_1}...X^{i_\Delta}\} = N^{\frac{1-\Delta}{2}} C_{i_1...i_\Delta} \text{Tr}\{X^{i_1}...X^{i_\Delta}\}. \tag{46}$$

Here Δ is the conformal dimension of the operator, X^i are the elementary scalar fields of the $\mathcal{N} = 4$ SYM that transform in the representation of $SO(6) \cong SU(4)$ and $C_{i_1...i_\Delta}$ fall into the totally symmetric rank Δ tensor representation of $SO(6)$. The trace is of course taken over the color indices and the normalization is chosen such that all planar graphs scale with N_c^2

Dual fields of supergravity

On the AdS side of the duality one expands all fields in spherical harmonics $Y_\Delta(\vec{y})$ of S^5:

$$\phi(z, \vec{y}) = \sum_{\Delta=0}^{\infty} \phi_\Delta(z) Y_\Delta(\vec{y}). \tag{47}$$

Consider the ten dimensional Klein Gordon equation, which implies a massive wave equation in the five dimensions of the AdS_5 sector:

$$\left(\partial_\mu \partial^\mu + m_\Delta^2\right) \phi_\Delta, \quad m_\Delta^2 = \Delta(\Delta - 4) \tag{48}$$

Like any second order differential equation, this one has two solutions, they can be characterized by their asymptotics as $z_0 \to 0$:

$$\phi_\Delta(z_0, \vec{z}) \sim \begin{cases} z_0^\Delta : \text{normalizable} \\ z_0^{4-\Delta} : \text{non-normalizable} \end{cases} \tag{49}$$

The non-normalizable fields define associated boundary fields like:

$$\bar{\phi}_{\Delta}(\vec{z}) = \lim_{z_0 \to 0} \phi_{\Delta}(z_0, \vec{z}) z_0^{\Delta-4}. \tag{50}$$

One can now identify the normalizable AdS modes ϕ_{Δ} as vacuum expectation values of the field theory operators $\mathcal{O}_{\Delta}$ and the non-normalizable modes $\bar{\phi}_{\Delta}$ as sources for thse operators:

$$\phi_{\Delta}(z_0, \vec{z}) \sim \langle \mathcal{O}_{\Delta} \rangle z_0^{\Delta} + \bar{\phi} z_0^{4-\Delta}. \tag{51}$$

We can now map the correlation functions in SYM theory and the supergravity theory. It's defined as: the generating functional $W[\bar{\phi}]$ for all correlators of single trace operators $\mathcal{O}_{\Delta}$ in SYM is given in terms of the source fields $\bar{\phi}$. the boundary values of these supergravity fields become the sources for the field theory. So on the field theory side we have:

$$e^{-W[\bar{\phi}_{\Delta}]} = \langle \exp\left(-\int_{\partial H} d^4 z \bar{\phi}_{\Delta} \mathcal{O}_{\Delta} \right) \rangle \tag{52}$$

The other side of the duality, the AdS spacetime, is governed by an action in terms of the bulk fields $\mathcal{S}[\phi_{\Delta}]$ in the framework of type IIB supergravity on $AdS_5 \times S^5$. The AdS/CFT conjecture for correlation functions says that precisely this classical gravity action enters the generating functional for the subclass $\{\mathcal{O}_{\Delta}\}$ of operators in the $\mathcal{N} = 4$ QFT. The AdS boundary conditions have to be adjusted to meet the field theory values of the source fields:

$$W[\bar{\phi}] = \mathcal{S}[\phi]|_{lim_{z_0 \to 0} \phi_{\Delta}(z_0, \vec{z}) z_0^{\Delta-4} = \bar{\phi}(\vec{z}}. \tag{53}$$

The action $\mathcal{S}$ is the generating functional for tree diagrams on the AdS space, which is the classical expansion of correlators. These tree level graphs in AdS are called Witten diagrams. To complete this section, we give the Feynman rules that correspond with what we derived: Each external source $\bar{\phi}(\vec{z})$ is located at the boundary. Propagators depart from the exernal sources either to another boundary point or to an interior interaction point (the latter are then called bulk-to-boundary propagators). The interior interactions are governed by the interaction vertices derived from supergravity. Two interior interaction points may be connected by so called bulk-to-bulk propagators.

Below is a table with the holographic dictionary.

Bulk/Gravity	Boundary/Field Theory
Metric Tensor $g_{\mu\nu}$	Energy Momentum Tensor $T_{\mu\nu}$
Scalar Field ϕ	Scalar Operator $\mathcal{O}$
Dirac Field ψ	Fermionic Operator $\mathcal{O}_f$
Gauge Field A_μ	Global Symmetry Current J_μ
Mass of the Field	Conformal Dimension of the Operator
Hawking Temperature	Temperature
Local Isometry	Global Spacetime Symmetry

4.5 GENERALIZATIONS OF ADS/CFT

So far we've given a description of the duality between AdS and CFT, however, the correspondence is not very realistic in the sense that we had to take the limit of $N_c \to \infty$. A conformal field theory has, of course, the conformal symmetry and in the case we described also supersymmetry. Lastly we had fields that transformed under the adjoint representation of the gauge group.

However QCD, as described in chapter (2), is not a $\mathcal{N} = 4$ SYM theory because: The symmetry of QCD is $SU(3)$ so the number of colors is not only finite, it's also quite small. QCD has no supersymmetry. One of the most important features of QCD, confinement, is incompatible with conformal symmetry. And last but not least, the quarks transform in the fundamental representation of the gauge group.

More general models will have different backgrounds in order to facilitate confinement/deconfinement and matter will have to be added in the fundamental representation. As was explained in [14]

4.6 INTRODUCING FLAVOURS

Adding matter in the fundamental representation, called the flavour sector of QCD, to the boundary theory can be done by adding an extra part to the action. This part, called the Sen's action, a generalization of the DBI action, represents N_f overlapping flavour branes and antibranes [26]. This part of the action includes the dynamics of the lowest open string mode, the tachyon. It reads:

$$S = -\int d^{p+1}x \mathbf{STr}[e^{-\phi} V(TT^\dagger, Y_L^I - Y_R^I, x)(\sqrt{-\mathrm{Det}(A_L)} + \sqrt{-\mathrm{Det}(A_R)})] \tag{54}$$

Where **STr** is the symmetric trace as defined in [21] and the fields are:

$$A_{(i)MN} = g_{MN} + B_{MN} + F^{(i)}_{MN} + \partial_M Y^I_{(i)} \partial_N Y^I_{(i)} + \frac{1}{\pi}(D_M T)^*(D_N T) + \frac{1}{\pi}(D_N T)^*(D_M T) \quad (55)$$

$$(56)$$

$$F^{(i)}_{MN} = dA^i - iA^i \wedge A^{(i)}, \quad D_M T = (\partial_M + iA^L_M - iA^R_M)T$$

Here $V(TT^\dagger, Y^I_L - Y^I_R, x)$ is the tachyon potential. The transverse scalars $Y^I_{L/R}$ and B_{MN} will be assumed to be zero in this thesis, they have no analogue in QCD so there is no need to include them. There will also be another term to the action which is coming from the WZ coupling of the flavour branes with the RR potentials, it's described by:

$$S_{WZ} = T_p \int_{\Sigma_{p+1}} C \wedge \mathbf{Str} e^{i2\pi\alpha' \mathcal{F}} \quad (57)$$

where Σ_{p+1} is the worldvolume of the branes, C is the formal sum of the RR potentials and $\mathcal{F}$ is the curvature of the superconnection $\mathcal{A}$ which is expressed as:

$$i\mathcal{A} = \begin{pmatrix} iA_L & T^\dagger \\ T & iA_R \end{pmatrix}, \quad i\mathcal{F} = \begin{pmatrix} iF_L - TT^\dagger & DT^\dagger \\ DT & iF_R - TT^\dagger \end{pmatrix} \quad (58)$$

The super connection is then defined as:

$$\mathcal{F} = dA - iA \wedge A \quad (59)$$

and satisfies the bianchi identity:

$$d\mathcal{F} - i\mathcal{A} \wedge \mathcal{F} + i\mathcal{F} \wedge \mathcal{A} = 0 \quad (60)$$

4.6.1 *Tachyon*

Now, the most important field from this added action is the Tachyon. The tachyon is the lowest string mode and it transforms in the bifundamental of $U(N_f)_L \times U(N_f)_R$ and therefore it's the natural candidate to be dual to the chiral condensate, which describes chiral symmetry breaking after all.

It was shown in [15] that in a confining background, with no blackholes, the tachyon must divere in the deep IR of the bulk. The diverging tachyon can be thought of as a recombination of the brane-antibrane. If the tachyon was finite until the very end of the space, one would have an open brane, antibrane. It was argued in [15] that this would lead to bulk flavour anomalies that do not match QCD.

We can then use the AdS/CFT dictionary to see that the renormalizable component of the tachyon in the UV region (close to the boundary) must be dual to the quark mass, while the non-renormalizable component is the quark bilinear of dimension three: the quark condensate.

We now know that the information of the quark condensate lies inside the solution of the tachyon. We have the action for the tachyon and hence the differential equation for it. We also have the action for the bulk theory, which will allow us to calculate the background. In principle, we now have all the tools we need to find an expression for the condensate.

The idea is then to derive all the equations of motion from the action, find their boundary conditions via their UV and IR expansions and physical arguments, solve everything at once and gain the expression for the quark condensate and the mass. We can then redo this calculation for different values of the external magnetic field and that way we can derive the way the condensate depends on the magnetic field

After that, we can change our ansatz metric from a thermal gas to a black hole solution, compactify the space-time to gain a Hawking Temperature and then proceed to redo all the calculations for different temperatures.

It's important to note though, that the different fields in the full action do not come on equal footing. The bulk fields all come with a factor of N_c^2, while the fields from the DBI action, the tachyon and the magnetic field for instance, come with a factor $N_c N_f = N_c^2 x_f$.

The whole idea behind AdS/CFT came originally from the large N_c expansion, while the strong maldacena conjecture proposes the duality works for all N_c, it's a rather unprovable statement. The weaker conjecture seems far more likely to hold. This leads to the idea that we might be able to solve the tachyon in a fixed background, and still gain some nontrivial condensate behaviour.

In the next section we'll discuss two different models. The first model, so called IHQCD, will be a model in the Veneziano limit. This means that the factor of $x_f = \frac{N_f}{N_c}$ will remain finite: the amount of flavours is of the same order as the amount of colors. This means that in the equations of motion, the contributions from the tachyon and the magnetic field are not insignificantly small. In other words, they affect the space-time in a signficant fashion. This means we're dealing with a model where there is backreaction.

The second model, called the AdS Soliton will be in the quenched limit: $x_f \rightarrow 0$. This means that the effect of the tachyon/magnetic field on the space-time is negligibly small. We can thus calculate the background fields on their own and afterwards introduce the tachyon that will not affect them.

Part II

CALCULATIONS

5

IMPROVED HOLOGRAPHIC QCD

The AdS/CFT correspondence was formulated for a Super symmetric, conformal Yang-Mills theory. However, QCD does not have super symmetry, neither can it be conformal because that prohibits confinement. In order to still make use of the correspondence, the actual statement made must be deformed so it applies to more physical systems. Since QCD has very important IR physics, these need to be considered when using holography.

AdS/QCD is a phenomonological approach that has been developped and used with mixed success, mainly for analysis of the meson sector. The approach is based on a UV and IR cut-off and a constant dilaton. Confinement can be realised with requiring specific boundary conditions at the IR cut-off [27]

More recent bottom up models were developped by Kiritsis et all [24] and [25]. These models offer an effective action that is constructed from ideas in string theory, but matched to the behaviour of QCD. This model is called Improved Holographic QCD and it describes the glue sector of QCD.

The IHQCD model has two different background solutions, one translates to a confined phase (a thermal gas) and the other to a deconfined phase (black hole solution) which allows to study both zero and non-zero temperatures. The system will show a first order Hawking Page phase transition at a particular temperature, this will correspond to a confinement/deconfinement phase transition.

5.1 SET-UP

The glue sector of the 5D bulk theory is given by:

$$S = M^3 N_c^2 \int d^5x \sqrt{-g} \left(\mathcal{R} - \frac{4}{3} \left(\frac{\partial \lambda}{\lambda} \right)^2 + V_g(\lambda) \right). \tag{61}$$

Here $\mathcal{R}$ is the ricci scalar, $\lambda = e^{\phi}$ is the exponential of the dilaton field which is dual to the the $\text{Tr}(F^2)$ operator and its boundary value is equivalent to the holographic 't Hooft coupling. $V_g(\lambda)$ is the gluon potential and it is responsible for non-trivial dilaton dynamics which, in turn, give a running coupling constant. We take the metric to be of the form:

$$ds^2 = e^{2A(r)} \left(\frac{dr^2}{f(r)} - f(r)dt^2 + dx_1^2 + dx_2^2 + e^{2W(r)} dx_3^3 \right). \quad (62)$$

Here x, y, z, t are the ordinary 4 dimensional space-time coordinates and r represents the AdS coordinate, hence we have that the UV boundary lies at $r = 0$ (which corresponds to $A \to \infty$).

For the flavour sector, we have the generalized Sen's action as described in chapter (4). We are only interested in the vacuum solutions, so we can simplify the original expression to gain:

$$S_f = -xM^3 N_c^2 \int d^5 x V_f(\lambda, \tau) \sqrt{-\text{Det}(g_{\mu\nu} + w(\lambda, \tau) F_{\mu\nu} + \kappa(\lambda, \tau) \partial_\mu \tau \partial_\nu \tau)} \quad (63)$$

where $x = \frac{N_f}{N_c}$. When x_f is considered to be small one works in the so called quenched limit, in this limit the tachyon and the magnetic field do not backreact on the metric. However, we are interested in this backreaction, that's why we introduced the factor of $e^{2W(r)}$ in the metric. Hence we take x_f to be finite, this is called the Veneziano limit. The potential is chosen to be [16]:

$$V_f(\lambda, \tau) = V_{f_0}(\lambda) e^{-a(\lambda)\tau^2}, \quad (64)$$

and $w(\lambda, \tau)$ and $\kappa(\lambda, \tau)$ are coupling functions that are in general allowed to depend on both the Tachyon and the Dilaton as long as they transform covariantly under the flavour symmetry.

5.2 EQUATIONS OF MOTION

The complete action is then given by:

$$S = M^3 N_c^2 \int d^5 x \sqrt{-g} \left(R - \frac{4}{3} \frac{\partial \lambda}{\lambda} + V_g(\lambda) \right) - xV_f(\lambda, \tau) \sqrt{-\text{Det}(g_{\mu\nu} + w(\lambda, \tau) F_{\mu\nu} + \kappa(\lambda, \tau) \partial_\mu \tau \partial_\nu \tau)}. \quad (65)$$

We can now determine the equations of motion. We start with the Einstein Equations of motion and derive that:

$$R^{\mu}_{\nu} - \frac{4}{3}\frac{\partial^{\mu}\lambda\partial_{\nu}\lambda}{\lambda^2} - \frac{1}{2}\delta^{\mu}_{\nu}\left(R - \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda,\tau) - xV_f(\lambda,\tau)\sqrt{D}\right) - xg^{\mu\alpha}\frac{V_f(\lambda,\tau)}{2\sqrt{D}}\frac{dD}{dg^{\alpha\nu}} = 0. \quad (66)$$

Here we used that $D = \text{Det}\left(\delta^{\rho}_{\sigma} + w(\lambda,\tau)g^{\rho\beta}F_{\beta\sigma} + \kappa(\lambda,\tau)g^{\rho\beta}\partial_{\beta}\tau\partial_{\sigma}\tau\right)$. From this condensed expression we extract the equations of motion in terms of the metric functions given by (62).

$$-\frac{e^{2A(r)}V_g(\lambda(r),\tau(r))}{2f(r)} + 3A''(r) + \frac{f'(r)\left(3A'(r) - W'(r)\right)}{2f(r)} + 3A'(r)^2 + \frac{2\lambda'(r)^2}{3\lambda(r)^2} x\frac{e^{2A(r)}G(r)\left(2Q(r)^2 - 1\right)\text{Vf}(\lambda(r),\tau(r))}{2f(r)Q(r)} = 0 \quad (67)$$

$$3A'(r)W'(r) - \frac{f'(r)W'(r)}{f(r)} + W''(r) + W'(r)^2 + x\frac{e^{2A(r)}G(r)\left(1 - Q(r)^2\right)\text{Vf}(\lambda(r),\tau(r))}{2f(r)Q(r)} = 0 \quad (68)$$

$$f'(r)\left(3A'(r) + W'(r)\right) + f''(r) - x\frac{e^{2A(r)}G(r)\left(1 - Q(r)^2\right)V_f(\lambda(r),\tau(r))}{Q(r)} = 0 \quad (69)$$

And the constraint equation given by:

$$\frac{e^{2A(r)}V_g(\lambda(r),\tau(r))}{2f(r)} - \frac{f'(r)\left(3A'(r) + W'(r)\right)}{2f(r)} - 3A'(r)W'(r) - 6A'(r)^2 + \frac{2\lambda'(r)^2}{3\lambda(r)^2} - x\frac{e^{2A(r)}Q(r)V_f(\lambda(r),\tau(r))}{2f(r)G(r)} = 0 \quad (70)$$

We then consider the dilaton equation:

$$\frac{\lambda''(r)}{\lambda(r)} - \frac{\lambda'(r)^2}{\lambda(r)^2} + \left(3A'(r) + W'(r) + \frac{f'(r)}{f(r)}\right)\frac{\lambda'(r)}{\lambda(r)} + \frac{3}{8}\frac{\lambda(r)e^{2A(r)}}{f(r)}\partial_{\lambda}V_g(\lambda) - x\frac{3B^2e^{-2A(r)}G(r)\lambda(r)V_f(\lambda,\tau)w(\lambda,\tau))}{8f(r)Q(r)}\partial_{\lambda}w(\lambda,\tau) - x\frac{3e^{2A(r)G(r)\lambda(r)Q(r)}}{8f(r)}\partial_{\lambda}V_f(\lambda,\tau) - x\frac{3\lambda(r)Q(r)V_f(\lambda,\tau)\tau'(r)^2}{16G(r)}\partial_{\lambda}\kappa(\lambda,\tau) = 0, \quad (71)$$

the tachyon equation:

$$
\begin{aligned}
&\tau''(r) - \frac{e^{2A(r)}G(r)^2}{f(r)\kappa(\lambda,\tau)}\partial_\tau \log(V_f(\lambda,\tau)) + e^{-2A(r)}f(r)\kappa(\lambda,\tau)\Bigg(W'(r) + \frac{1}{2}\frac{f'(r)}{f(r)} \\
&+2A'(r)\frac{1+Q(r)^2}{Q(r)^2} + \frac{1}{2}\lambda'(r)\partial_\lambda \log(\kappa(\lambda,\tau)V_f(\lambda,\tau)^2)) - \lambda'(r)\frac{1-Q(r)^2)}{Q(r)^2}\partial_\lambda \log(w(\lambda,\tau))\Bigg) \\
&\Bigg(W'(r) + \frac{f'(r)}{f(r)} + \lambda'(r)\partial_\lambda \log(V_f(\lambda,\tau)\kappa(\lambda,\tau)) - \lambda'(r)\frac{1-Q(r)^2}{Q(r)^2}\partial_\lambda \log(w(\lambda,\tau)) \\
&A'(r)\frac{2+Q(r)^2}{Q(r)^2} + \Bigg)\tau'(r) + \frac{\tau'(r)^2}{2}\partial_\tau \log(\kappa(\lambda,\tau)) + \frac{e^{2A(r)}G(r)^2(1-Q(r)^2)}{f(r)\kappa(\lambda,\tau)Q(r)^2}\partial_\tau \log(w(\kappa,\tau))
\end{aligned}
$$

and the equation for the magnetic field:

$$\partial_\mu \left(V_f(\lambda,\tau) w(\lambda,\tau)\sqrt{-G}G^{xy} \right) = 0 \tag{73}$$

.

Note that the Magnetic Field equation is trivially satisfied by a magnetic field of the shape of:

$$V_\mu = \{0, -\frac{y}{2}B, \frac{x}{2}B, 0, 0\} \tag{74}$$

As discussed earlier, the main interest we have here is to look for the influence of the magnetic field B on the strength of the quark condensate. We want to incorperate the backreaction of the magnetic field on the metric and that's why we've added the factor of $e^{2W(r)}$ to the metric 62. The choice of the gauge field V_μ is such that it breaks the symmetry between the x, y and z axes. It will create a constant magnetic field pointing in the z direction and this is reflected in the metric with that factor of $e^{2W(r)}$.

5.3 THE IR ASYMPTOTICS

We will now start with finding the quark condensate. In order to do this we first look at the IR asymptotics of the system. We know from [15] the general behaviour of the tachyon. In general it will show some $\sqrt{r_0 - r}$ like behaviour: the derivative will diverge but the function itself will not. However, there is one specific solution we are interested in, the solution where the tachyon will diverge at the end of the spacetime: $r \to \infty$.

First we need to find how the Tachyon will diverge in the infrared.

$$\tau'(r)\left(\frac{\left(B^2e^{-4A(r)}+3\right)A'(r)}{B^2e^{-4A(r)}+1}+3W'(r)\right)$$
$$+e^{2W(r)-2A(r)}\tau'(r)^3\left(\frac{2\left(B^2e^{-4A(r)}+2\right)A'(r)}{B^2e^{-4A(r)}+1}+2W'(r)\right)$$
$$+2a\tau(r)e^{2A(r)-2W(r)}\left(e^{2W(r)-2A(r)}\tau'(r)^2+1\right)+\tau''(r)=0 \quad (75)$$

We'll now research two different cases

5.3.1 $\tau(r)$ diverges but $\tau'(r)$ stays finite

In this case, we know that $\tau \to \infty$ and hence that $\tau \gg \tau'$. This means that in equation (75) we only need to take care of the terms that scale with the highest order of τ:

$$\tau\left(e^{2A(r)-2W(r)}+\tau'(r)^2\right)=0. \quad (76)$$

Note that both A and W are expected to be real fields, hence for this equation to be consistent, we'd require a complex τ', which would mean a complex τ field. We're only interested in a real field, so we can conclude that the derivative of the tachyon should diverge as well.

5.3.2 Both $\tau(r)$ and $\tau'(r)$ diverge

In this case, we need to look at the terms with the highest order in $\tau'(r)$ together with the highest order of any mixed terms:

$$\left(e^{2W(r)-2A(r)}\tau'(r)\left(\frac{2\left(B^2e^{-4A(r)}+2\right)A'(r)}{B^2e^{-4A(r)}+1}+2W'(r)\right)+2a\tau(r)\right)\tau'(r)^2 \quad (77)$$

It can be shown that, as long as A and W behave as:

$$A_{IR}=-\left(\frac{r}{R}\right)^{\alpha} \quad \alpha>1 \quad (78)$$

and

$$W_{IR}=-\left(\frac{r}{R}\right)^{\alpha}+\frac{1-\alpha}{2}\log\frac{r}{R} \quad \alpha>1. \quad (79)$$

The Tachyon will then show the IR behaviour of:

$$\tau_{IR} = e^{\frac{aRr}{2l^2\alpha}} \tag{80}$$

The W_{IR} behaviour turns out to be consistent with equation (68).

5.4 THE UV ASYMPTOTICS

In order to actually get an expression for the quark condensate we also require the UV asymptotics of the tachyon. We take another look at equation (75), to then consider an expansion around $r = 0$. Up to order r^2 this becomes:

$$ar^2\tau(r)\left(\frac{1}{R^4} - \frac{1}{6}W^{(4)}(0)\right) + \frac{2a\tau(r)}{r^2} + \left(2a\tau(r)\left(\tau'(r)^2 - \frac{1}{R^2}\right) + \tau''(r)\right) + r\left(-\frac{3\tau'(r)}{R^2} - 4\tau'(r)^3\right) - \frac{3\tau'(r)}{r} = 0 \tag{81}$$

The appropriate solution to this is of the form:

$$\tau_{UV} = mr + \sigma r^3 \tag{82}$$

Note that this last solution is in complete accordance with (49).

5.4.1 *UV of W*

We'll also need the UV behaviour of the W function. Taking a look at equation (68) and expanding around $r = 0$ will give the expansion:

$$-\frac{B^2r^2x}{2l^4} + W''(r) - \frac{3W'(r)}{r} = 0 \tag{83}$$

which is solved by:

$$W_{UV} = -\frac{B^2r^4x}{32l^4} + \frac{B^2r^4x\log(r)}{8l^4} + \frac{c_1r^4}{4} + c_2 \tag{84}$$

The two integration constants can be taken to be zero: one can be absorbed in the other term proportional to r^4 and the other can be ignored due to a redefinition of W and the z coordinate.

5.5 NUMERICAL SOLUTION

We now have the needed ingredients to numerically solve the tachyon equation. To reiterate: We know how the tachyon behaves in the infrared domain (diverges as an exponential function). We also know its behaviour in the ultraviolet and are able

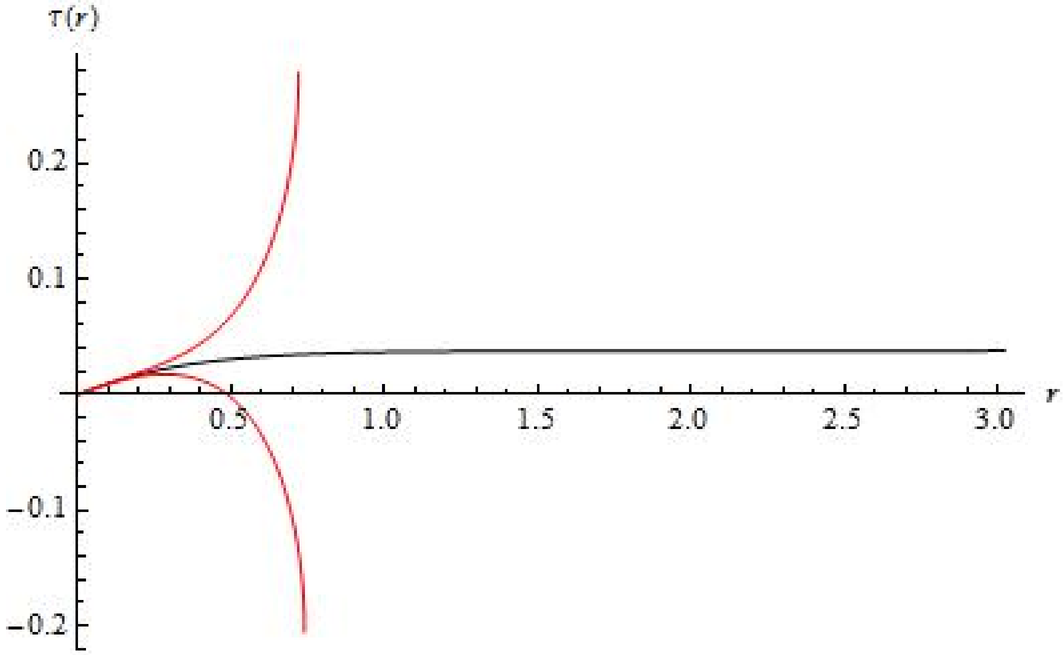

Figure 15: The numerical solutions for the tachyon. The two red curves are of the general $C + \sqrt{r_* - r}$ behaviour which has a diverging derivative, at which point the numerical calculation ends. Continuously changing the parameter σ, one will find solutions that shift the upper red curve towards the lower one. This makes it appear reasonable there might be a solution that does not actually show the square root behaviour and actually runs all the way to infinity, the black curve is an attempt at finding the right σ for that, unfortunately the precision required for σ quickly outruns the numerical precision of the computer.

to tweak the boundary conditions because of this ultraviolet behaviour. The tweaking of the constants m and σ will then allow us to connect the UV behaviour with the appropriate IR divergence, hence allowing us to get the right behaviour together with the quark mass and condensate.

Introducing a nonzero magnetic field will also include a non trivial solution to the W differential equation whose boundary conditions were determined with the help of its UV expansion.

The solutions found for $\tau(r)$ are given in graph (15). Unfortunately we weren't able to get the proper IR behaviour by tweaking the UV boundary conditions, even without a magnetic field this proved to be impossible.

6

ADS SOLITON

Since we didn't manage to connect the UV behaviour with the IR behaviour in the IHQCD model, we couldn't determine the quark condensate. Therefore we consider a simpler model called the AdS soliton model: a minimal, non critical approach to holographic large N_c QCD type theories. The 6D string theory dual has, as low energy excitation, the duals of the lowest dimension gauge invariant operators. This translates to: the six-dimensional metric $g_\mu \nu$ (which is dual to the YM stress tensor); a scalar field ϕ called the dilaton (and which is dual to the YM operator $\mathrm{Tr}F^2$) and an axion, dual to $\mathrm{Tr}FF$. The scalar ϕ encodes the running of the YM coupling and is naturally identified with the 6D string Dilaton.

It should be mentioned that this bulk theory is considered to be a non critical string theory, it's not just a gravity theory. However, we still constrict ourselves to the two derivative effective action.

Hence, the string frame action describing the low-lying excitations is [28]:

$$S = \int d^6x \sqrt{-\mathrm{Det}(g_{\mu\nu})} \left[e^{2\phi} \left(\mathcal{R} + 4(\partial\phi)^2 + \frac{c}{\alpha'} \right) - \frac{1}{2}\frac{1}{6!} F_{(6)}^2 \right]. \tag{85}$$

Where $\mathcal{R}$ is the Ricci Scalar, ϕ is the dilaton field and $F_{(6)}$ is the only considered RR-form, c is a constant.

The Ansatz metric looks like:

$$\begin{aligned} ds_6^2 &= -g_{tt}dt^2 + g_{rr}dr^2 + g_{xx}dx^2 + g_{yy}dy^2 + g_{zz}dz^2 + g_{\eta\eta}d\eta^2 \\ &= e^{2X(r)} \left(-dt^2 + dx^2 + dy^2 + e^{2W}dz^2 + \frac{1}{f}dr^2 + f d\eta^2 \right), \end{aligned} \tag{86}$$

where $t, \vec{x}$ are the usual 4 space-time coordinates, r is the AdS coordinate and η is a coordinate on the boundary of the AdS cone. It's important to note that compared to 62, the factor $e^{2W(r)}$ in front of the dz^2 is missing. Effectively, this means that we give up trying to take the backreaction of the magnetic field into account. This greatly reduces the complexity of the problem, but unfortunately also its touch with reality.

It's very important to note that this Ansatz metric reveals a large difference between this model and the previous. In the AdS-Soliton model there is no $e^{2W(r)}$ factor present. This means that any backreaction of the magnetic field on the metric is ignored. The background is considered to be fixed and the effects of the tachyon nor the magnetic field will influence the space-time.

6.1 EQUATIONS OF MOTION

We start with the equation of motion for the RR form F. A form like this is given by:

$$F_{(6)} = \partial_{[a_1} A_{a_2 a_3 a_4 a_5 a_6]} \tag{87}$$

for some field $A_{a_2 a_3 a_4 a_5 a_6}$, note that this expression is completely antisymmetric in all indices, by construction. The only relevant term in the lagrangian is then given by:

$$\sqrt{-\mathrm{Det}(g_{\mu\nu})} F_{(6)}^2 = \sqrt{-\mathrm{Det}(g_{\mu\nu})} \partial_{[a_1} A_{a_2 a_3 a_4 a_5 a_6]} \partial_{[b_1} A_{b_2 b_3 b_4 b_5 b_6]} g^{a_1 b_1} ... g^{a_6 b_6}. \tag{88}$$

Simply using the Euler Lagrange Equation for A then gives the equation:

$$\frac{\delta S}{\delta A_{a_2 a_3 a_4 a_5 a_6}} = -\partial_{a_1} \left(\sqrt{-\mathrm{Det}(g_{\mu\nu})} g^{a_1 b_1} ... g^{a_6 b_6} \partial_{[b_1} A_{b_2 b_3 b_4 b_5 b_6]} \right). \tag{89}$$

The solution to this differential equation is obvious once you know of it, simply pick

$$F_{(6)} = \partial_{[a_1} A_{a_2 a_3 a_4 a_5 a_6]} = C \sqrt{-\mathrm{Det}(g_{\mu\nu})} \epsilon_{a_1 a_2 a_3 a_4 a_5 a_6}, \tag{90}$$

and plug it into the differential equation to gain

$$\begin{aligned} &\partial_{a_1} \left(\sqrt{-\mathrm{Det}(g_{\mu\nu})} g^{a_1 b_1} ... g^{a_6 b_6} C \sqrt{-\mathrm{Det}(g_{\mu\nu})} \epsilon_{b_1 b_2 b_3 b_4 b_5 b_6} \right) \\ &\quad = -\partial_{a_1} \left(C \cdot \mathrm{Det}(g\mu\nu) * (\mathrm{Det}(g\mu\nu)^{-1} \right) \\ &\quad = \partial_{a_1} C = 0 \end{aligned} \tag{91}$$

which holds by default for any value of C. Hence the solution to F is given by

$$F_{(6)} = \frac{Q}{\sqrt{\alpha'}} \sqrt{-\mathrm{Det}(g_{\mu\nu})} \epsilon_{a_1 a_2 a_3 a_4 a_5 a_6} \tag{92}$$

with Q a constant related to the number of colors, but it's not important here. Note that with this choice of F, the term in the lagrangian becomes $F_{(6)}^2 = \frac{Q^2}{\alpha'} 6!$.

6.1.1 *Einstein Equations*

Variating the action 85 with respect to the metric gives the Einstein Field equations. Note that for this part, $W(r) = 0$, this function will become nonzero later, when the magnetic field is introduced. The equations of motion are then given by the expression

$$e^{2\phi}\left(g^{\mu\alpha'}\mathcal{R}_{\alpha'\nu} - \frac{1}{2}\mathcal{R}\delta^\mu_\nu + 4g^{\mu\alpha'}\partial_{\alpha'}\phi\partial_\nu\phi - 2g^{\alpha'\beta}\partial_{\alpha'}\phi\partial_\beta\phi\delta^\mu_\nu - \frac{1}{2}\delta^\mu_\nu\frac{c}{\alpha'}\right) + \frac{1}{4}\frac{Q^2}{\alpha'}\delta^\mu_\nu \tag{93}$$

Taking several linear combinations, this can be more explicitly written as

$$2\alpha'\left(8X'(r)f'(r) + 4f(r)\left(2X''(r) + 3A'(r)^2 - \phi'(r)^2\right) + f''(r)\right) + e^{2x(r)}\left(Q^2e^{2\phi(r)} - 2c\right) = 0 \tag{94}$$

$$f''(r) + 4f'(r)X'(r) - 8f(r)\left(-X''(r) + X'(r)^2 + \phi'(r)^2\right) = 0 \tag{95}$$

$$-X''(r) + X'(r)^2 + \phi'(r)^2 = 0 \tag{96}$$

$$f''(r) + 4f'(r)X'(r) = 0 \tag{97}$$

The last equation can be solved by choosing $f(r) = 1 - \frac{r^5}{r_0^5}$ and $X(r) = \frac{1}{2}\log\left(\frac{R^2}{r^2}\right)$, plugging this into the third equation immediately enforces that $\phi(r) = \phi_0$. The Dilaton equation will then determine wheher this is consistent.

6.1.2 *Dilaton*

The Dilaton equation is derived by the simple use of the Euler Lagrange equations for the ϕ fields. The equation of motion is given by

$$\frac{2c}{\alpha'} - 2e^{-2X(r)}f''(r) - 20e^{-2X(r)}f'(r)X'(r) - 20f(r)e^{-2X(r)}X''(r) \\ - 40f(r)e^{-2X(r)}X'(r)^2 + 48X'(r)\phi'(r) + 8\phi''(r) - 8\phi'(r)^2 = 0 \tag{98}$$

Entering the equations that were found for f and X gives

$$\frac{2c}{\alpha'} + 8\phi''(r) - \frac{8\phi'(r)\left(r\phi'(r)+6\right)}{r} - \frac{60}{R^2} = 0, \tag{99}$$

which obviously allows for the constant solution as long as the AdS radius $R^2 = \frac{30}{c}\alpha'$

Note that the very first Einstein equation can be used to determine what constant the dilaton should be. Plugging in the choices for f, X and ϕ simplifies the equation to

$$\frac{10\alpha'\left(3Q^2e^{2\phi_0} - 2c\right)}{cr^2} \tag{100}$$

or $\phi_0 = \frac{1}{2}\log\left(\frac{2c}{3Q^2}\right)$

Since the Dilaton is constant, its dynamics are trivial and we do not have a running coupling constant as we know QCD to have, it should be clear that this model is not an accurate description of reality.

6.2 TACHYON

We can now introduce the Tachyon. As described in chapter 4 one adds the Sen's action to the bulk action:

$$S = -\int d^4x dr V(|T|)\left(\sqrt{-\text{Det}A_L} + \sqrt{-\text{Det}A_R}\right) \tag{101}$$

with A_i as given in equation (56). The complex tachyon will be denoted by $T = \tau e^{i\theta}$. The indices M, N run over the 5 world volume dimensions while μ, ν will be used for the Minkowski directions. There are two new constants, g_v and λ, both are related to the normalization but can be set to 1 for now. For the tachyon potential we take

$$V = \mathcal{K}e^{-\frac{\mu^2}{2}\tau^2} \tag{102}$$

where $\mathcal{K}$ is a constant which is, in principle, related ot the tension of the D4 branes. The gaussian choice for the tachyon potential is a simple choice that's been discussed before in [16], [17] and [17]. First we look for the vacuum of the theory, $\theta = 0$ because of Lorentz Invariance, making the tachyon a real valued field. With these simplifications we can reduce the actions to:

$$S = -2\mathcal{K} \int d^4 dr e^{0\frac{\mu^2}{2}\tau^2} \sqrt{g_{tt}}\sqrt{g_{xx}}^3 \sqrt{g_{zz} + 2\pi\alpha'\lambda(\partial_r\tau)^2} \quad (103)$$

with the corresponding equation of motion:

$$\begin{aligned} &\frac{1}{2}\tau'(r)\frac{\partial}{\partial r}\log\left(\frac{g_{tt}(r)g_{xx}(r)g_{yy}(r)g_{zz}(r)}{g_{rr}(r)}\right) + \\ &\quad \frac{\pi\alpha'\lambda}{g_{rr}(r)}\frac{\partial}{\partial r}\left(\log[g_{tt}(r)g_{xx}(r)g_{yy}(r)g_{zz}(r)]\right)\tau'(r)^3 + \\ &\qquad \mu^2\tau(r)\left(\frac{g_{rr}(r)}{2\pi\alpha'\lambda} + \tau'(r)^2\right) + \tau''(r) = 0 \quad (104) \end{aligned}$$

Of course, with metric (86) being quite simple, one can even further simplify this expression. We simply do this by changing the g_{ii} terms with their field expressions. In this case we get:

$$\begin{aligned} &\frac{\pi\alpha'\lambda f'(r)\tau'(r) + \mu^2\tau(r)e^{2X(r)}}{2\pi\alpha'\lambda f(r)} + 8\pi\alpha'\lambda f(r)e^{-2X(r)}\tau'(r)^3 X'(r) + \\ &\qquad \mu^2\tau(r)\tau'(r)^2 + \tau''(r) + 3\tau'(r)X'(r) = 0 \quad (105) \end{aligned}$$

In this equation we can then fill in the explicit formulae for $X(r)$ and $f(r)$ to get:

$$\begin{aligned} &-\frac{8\pi\alpha'\lambda r\left(1 - \frac{r^5}{\mathrm{ro}^5}\right)\tau'(r)^3}{R^2} + \frac{\frac{\mu^2 R^2\tau(r)}{r^2} - \frac{5\pi\alpha'\lambda r^4\tau'(r)}{\mathrm{ro}^5}}{2\pi\alpha'\lambda\left(1 - \frac{r^5}{\mathrm{ro}^5}\right)} + \mu^2\tau(r)\tau'(r)^2 \\ &\qquad - \frac{3\tau'(r)}{r} + \tau''(r) = 0 \quad (106) \end{aligned}$$

6.2.1 *UV asymptotics*

This equation is to be studied in the confined background of metric (86). To do this we explicitly give the metric functions. The first thing we do is to find the UV asymptotics of the tachyon equation. To do this, we expand the tachyon differential equation for the near boundary limit $r \to 0$ and get

$$\frac{\mu^2 R^2\tau(r)}{2\pi\alpha'\lambda r^2} + \left(\mu^2\tau(r)\tau'(r)^2 + \tau''(r)\right) - \frac{8r\left(\pi\alpha'\lambda\tau'(r)^3\right)}{R^2} - \frac{3\tau'(r)}{r}. \quad (107)$$

Since this UV differential equation is still too complicated to solve, an estimated guess has to be made as to what terms can be ignored and which terms are dominant. Alternatively, an ansatz solution can be tried to see if it removes the lowest orders of r. Done properly, the solution

$$\tau = c_1 r + \frac{1}{6} c_1^3 \mu^2 r^3 \log(r) + c_3 r^3 \tag{108}$$

can be shown to work. However, for this to work, the relationsip

$$\frac{R^2\mu^2}{2\pi\alpha'\lambda} = 3 \tag{109}$$

was imposed.

This means that the scalar bifundamental operator dual to the scalar field (with mass $m_\tau^2 = -\frac{\mu^2}{2\pi\alpha'\lambda}$) has UV dimension of 3, matching the dimension of $\bar{q}q$ in QCD. This agrees with the fundamental AdS-CFT rule $\Delta(\Delta - 4) = m_\tau^2 R^2$ as mentioned in chapter (4). Note that this relationship should be taken as a restraint on μ, α', λ and not on R, since one should not think of the flavour branes backreacting on the closed string background.

It's now possible to start working on a numerical solution for the tachyon. Filling in the known equations for f, X, ϕ gives the following differential equation:

$$\tau''(r) - \frac{4\mu^2(1 - \frac{r^5}{r_0^4})}{3}\tau'(r)^3 + \left(\frac{-3}{r} + \frac{5r^4}{r^5 - \mathrm{ro}^5}\right)\tau'(r) + \left(\frac{3}{r^2(1 - \frac{r^5}{\mathrm{ro}^5})}\right)\tau(3) = 0 \tag{110}$$

First note that this equation only depends on two constants, r_0 and μ. This dependence is artificial, because it's possible to rescale the field τ and the radial coordinate r to get rid of these constants. In this section however, it's assumed that $r_0 = 1$ and $\mu^2 = \pi$. As was explained in [15], a confining background means that the tachyon must diverge somewhere. One can think of this as the recombining of the brane and anti-brane: a finite tachyon would have an open brane (and anti-brane). In the referenced paper it was mentioned that would lead to bulk flavour anomalies that do not match those of QCD. The fac tthat confinement requires brane recombination (which corresponds to chiral symmetry breaking) is a Coleman-Witten like theorem, described in [19].

This tachyon equation allows for the tachyon to diverge precisely at the end of the space when $z = z_0$.

In the IR, there are generically two independent solutions that behave as a constant and as $\sqrt{z - z_0}$ and are regular at the tip of the cigar. However, there is one solution that depends on a single parameter and diverges at the tip. This is the solution we are looking for. The solution is plotted below:

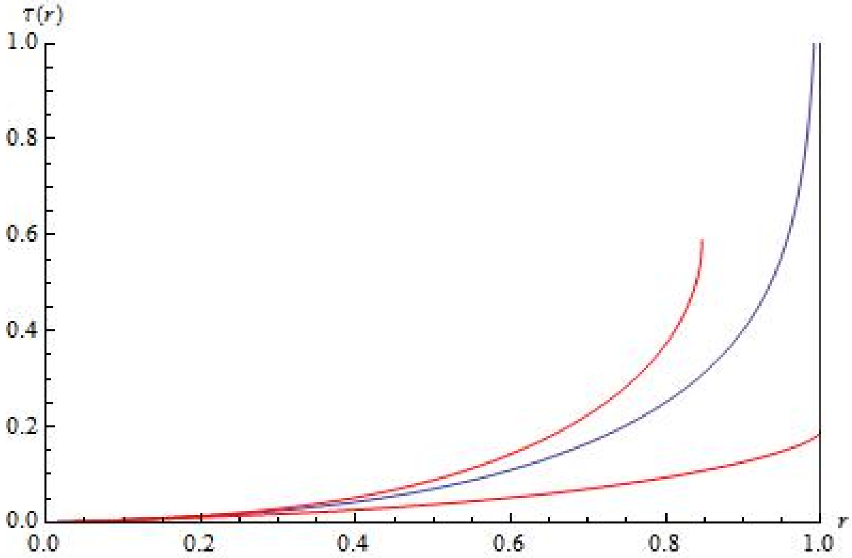

Figure 16: We see the Tachyon solution plotted for different initial conditions. The upper red curve is of the typical $\sqrt{r_0 - r}$ behaviour and has a diverging derivative at some point. The lowest red curve simply does not diverge anywhere on this space time. The blue curve has inital conditions that exactly fit the behaviour we want: divergence of the tachyon at the end of the space-time.

As one can see, for general values of C_3 that are not approximately 0.3579 the solution to the differential equation either stops early, showing the before mentioned $\sqrt{r_0 - r}$ behaviour, which means the derivative diverges at a certain point. Or the solution does not diverge at all at $r = r_0$

6.2.2 *Renormalization, the quark mass and the quark condensate*

We now have the correct behaviour for the tachyon, and the initial conditions, specifically the values of c_1 and c_3. From general AdS/CFT correspondence, we know of the relationship between c_1, c_3, $\bar{q}q$ and m. We'll now make this relationship precise.

The quark condensate is defined as

$$\langle \bar{q}q \rangle = -\frac{\delta S_{ren}}{\delta m_q} \tag{111}$$

where m_q is the quark mass, S_{ren} is the holographically renormalized action.

First, we regularize the action by placing a UV cut-off at $r = \epsilon$: $S_{reg} = \int_{\epsilon}^{z_0} \mathcal{L}$, here

$$\mathcal{L} = -2\mathcal{K}e^{0\frac{\mu^2}{2}\tau^2}\sqrt{g_{tt}}\sqrt{g_{xx}}^3\sqrt{g_{zz} + 2\pi\alpha'\lambda(\partial_r\tau)^2}. \tag{112}$$

We are only really interested with the variation of the regularized action with respect to m_q, we can simplify this by first variating with respect to τ

$$\delta S_{reg} = \int_{\epsilon}^{z_0} \left(\delta\tau \frac{\partial \mathcal{L}}{\partial \tau} + \delta\tau' \frac{\partial \mathcal{L}}{\partial \tau'} \right) dr = \int_{\epsilon}^{z_0} \frac{d}{dr} \left(\delta\tau \frac{\partial \mathcal{L}}{\partial \tau'} \right) \tag{113}$$

and hence

$$\frac{\delta S_{reg}}{\delta \tau} = -\frac{\partial \mathcal{L}}{\partial \tau'}\bigg|_{z=\epsilon}. \tag{114}$$

We can now use the chain rule to find $\frac{\delta S_{reg}}{\delta c_1} = \frac{\delta S_{reg}}{\delta \tau}\frac{\delta \tau}{\delta c_1}$. It's important to note that c_3 is some non trivial function of c_1. Plugging in the UV expansion of τ gives us:

$$\frac{\delta S_{reg}}{\delta c_1} = \mathcal{K}R^5\mu^2 \left(\frac{2c_1}{3\epsilon^2} + \frac{2}{3}c_1^3\mu^2\log(\epsilon) + 2c_3 - \frac{1}{3}c_1^3\mu^2 + \frac{2}{3}c_1\partial_{c_1}c_3 \right) \tag{115}$$

where terms that vanish as $\epsilon \to 0$ were ignored. Now we have to write the appropriate covariant counterms, to be added to S_{reg}, to define the subtracted action $S_{sub} = S_{reg} + S_{ct}$:

$$S_{ct} = -\mathcal{K}R \int d^4x\sqrt{-\gamma} \left(-\frac{1}{2} + \frac{\mu^2}{3}\tau^2 + \frac{\mu^4}{18}\tau^4\log(\epsilon) + \frac{\mu^2}{12}\alpha'\tau^4 \right), \tag{116}$$

where γ corresponds to the induced metric at $r = \epsilon$, ie. $\sqrt{-\gamma} = \frac{R^4}{\epsilon^4}$. The constant α'' captures the scheme dependence of the condensate and reflects an analogous scheme dependence in field theory. The renormalization action is then $S_{ren} = \lim_{\epsilon\to 0} S_{sub}$. So we then find:

$$\frac{\delta S_{ren}}{\delta c_1} = -(2\pi\alpha'\mathcal{L}R^3\lambda)\left(-4c_3 + c_1^3\mu^2(1+\alpha)\right) \tag{117}$$

It's usefull to note that the term $c_1 \partial_{c_1} c_3$ drops out against the same term (with negative sign) in $\frac{\delta S_{ren}}{\delta c_1}$. We now want to evaluate the quark condensate. The quark mass is known to be proportional to c_1 and so we just define:

$$m_q = \beta c_1 \tag{118}$$

for some constant β. The fact that there is an arbitrary factor β has been stressed in [20]. The final expression we get is:

$$\langle \bar{q}q \rangle = \frac{1}{\beta}(2\pi\alpha' \mathcal{K} R^3 \lambda) \left(-4c_3 + \frac{m_q^3}{\beta^3} \mu^2 (1+\alpha) \right) \tag{119}$$

The condensate depends on the value of α, which is a renormalization scheme dependend constant. In order to get rid of this dependence, we only look at values between the condensate $\langle \bar{q}q \rangle$ at a certain magnetic field, and the value of the condensate without magnetic field. Since the term proportional to α is only dependent on m, which was kept constant, and not on σ, which changed due to the magnetic field, the result:

$$\Delta = \langle \bar{q}q \rangle_B - \langle \bar{q}q \rangle_0 \tag{120}$$

is renormalization scheme independent.

6.3 INTRODUCING THE MAGNETIC FIELD

Like in the previous chapter, we are interested in the behaviour of the quark condensate as a function of the magnetic field. As done previously, this means that we have to write A as:

$$A_{(i)MN} = g_{MN} + \frac{2\pi\alpha'}{g_v} F_{MN} + \partial_M \tau \partial_N \tau \tag{121}$$

In principle, the presence of a magnetic field will cause a backreaction on the underlying space-time. The space-time would then again change the equations for the tachyon, etc. This model, however, is taken in the quenched approximation: we assume that x_f is very small and so any backreactions will be ignored.

When we turn on the external magnetic field, the term F_{MN} becomes nonzero. That has the consequence that the Flavour part of the action is now given by:

$$S = -2\mathcal{K} \int d^4 dr e^{0\frac{\mu^2}{2}\tau^2} \sqrt{g_{tt}} \sqrt{g_{zz}} \sqrt{g_{xx} g_{yy} + \frac{2\pi\alpha'}{g_v^2} B^2} \sqrt{g_{zz} + 2\pi\alpha' \lambda (\partial_r \tau)^2} \tag{122}$$

We notice that the main difference with the original action, can be described by the change of:

$$g_{xx}(r)g_{yy}(r) \to g_{xx}(r)g_{yy}(r) + \frac{2\pi\alpha'}{gv2}B^2. \tag{123}$$

Then of course, the equation of motion will have the same transformation so it's easily written as:

$$\frac{1}{2}\tau'(r)\frac{\partial}{\partial r}\log\left(\frac{g_{tt}(r)\left(g_{xx}(r)g_{yy}(r) + \frac{4\pi^2\alpha^2}{g_v^2}B^2\right)g_{zz}(r)}{g_{rr}(r)}\right) +$$
$$\frac{\pi\alpha\lambda}{g_{rr}(r)}\frac{\partial}{\partial r}\left(\log[g_{tt}(r)\left(g_{xx}(r)g_{yy}(r) + \frac{4\pi^2\alpha^2B^2}{g_v^2}\right)g_{zz}(r)]\right)\tau'(r)^3 +$$
$$\mu^2\tau(r)\left(\frac{g_{rr}(r)}{2\pi\alpha\lambda} + \tau'(r)^2\right) + \tau''(r) = 0 \tag{124}$$

It's now important to see whether the magnetic field influences the UV behaviour of the system. To check this we take a look at the terms that have gained a B dependence. A quick calculation shows that:

Which means that there is no present B term in equation of motion for small r, hence the UV behaviour of the tachyon is not explicitly dependent on B. However, the B field does have a presence in the IR region, shifting some behaviour. This will couple back to the UV behaviour in the form of different values for m and σ.

We now have the differential equation, it's just a matter of filling in the constants and we can shoot from the UV. The results are given in the next section.

6.4 NON-ZERO TEMPERATURE

As was seen in section (3), magnetic decatalysis occurs only for sufficiently high temperatures. In order to look at the system at some non-zero temperature we need to change the metric. We know that the metric (86) solved the equations of motion for the background. As such, the metric given by:

$$ds_6^2 = e^{2X(r)}\left(f(r)d\tau^2 + \frac{dr^2}{f(r)} + dx^2 + dy2 + dz^2 + d\eta 2\right) \tag{125}$$

also solves these equations. This is a black hole solution with corresponding Hawking Temperature of:

$$t_E \sim t_E + \delta t_E, \quad \delta t_E = \frac{4\pi}{5}r_0 = \frac{1}{T}. \tag{126}$$

It should be noted that while the explicit solutions for the metric didn't change, their meaning did. The constant r_0 is now directly related to the temperature of the system, while it originally was an indication of the size of the space-time. Hence we'll call it r_T from now on.

6.4.1 *Tachyon equation*

The new metric (125) is responsible for a change in the tachyon equation:

$$\frac{r\left(25\pi^2B^2r^4+225\right)\tau''(r)}{25\pi^2B^2r^5+225r}-\frac{2\pi r\left(1-r^5\right)\left(25\pi^2B^2r^4+450\right)\tau'(r)^3}{3\left(25\pi^2B^2r^4+225\right)}+$$
$$\left(-\frac{675}{25\pi^2B^2r^5+225r}-\frac{25\pi^2B^2r^4}{25\pi^2B^2r^5+225r}-\frac{5r^4}{2\left(1-r^5\right)}\right)\tau'(r)+$$
$$\tau(r)\left(\frac{\pi r\left(25\pi^2B^2r^4+225\right)\tau'(r)^2}{25\pi^2B^2r^5+225r}+\frac{3}{r^2\left(1-r^5\right)}\right)=0 \quad (127)$$

We note, however, that the only change is added factors of $f(r)=1-\frac{r^5}{r_T^5}\approx 1+\mathcal{O}(r^5)$, which will clearly not interfere with the UV solution of the tachyon. However, the IR physics of the tachyon do change, as was argued in [21] the tachyon is now not allowed to diverge anywhere on the space-time, it must instead be constant. The proper tachyon solution then looks like:

The two red graphs represent undesireable tachyon behaviour, the solutions tend to diverge before the coordinate r_T is even reached. The black solution shows the tachyon solution with a proper constant value. There were multiple values for σ that granted a proper solution, and so an average value was taken.

Now we know how to solve the tachyon equation for non-zero temperature we can solve for different values of B at different temperatures, the results are plotted below.

The black line is the solution for zero temperature, calculated in the previous section. The coloured graphs are then condensate behaviour at different temperatures. While we do see that the effects of the magnetic field seems to decrease in strength, it never becomes a desctruvtive effect. So unfortunately, we were able to show magnetic catalysis in this system, but not magnetic decatalysis.

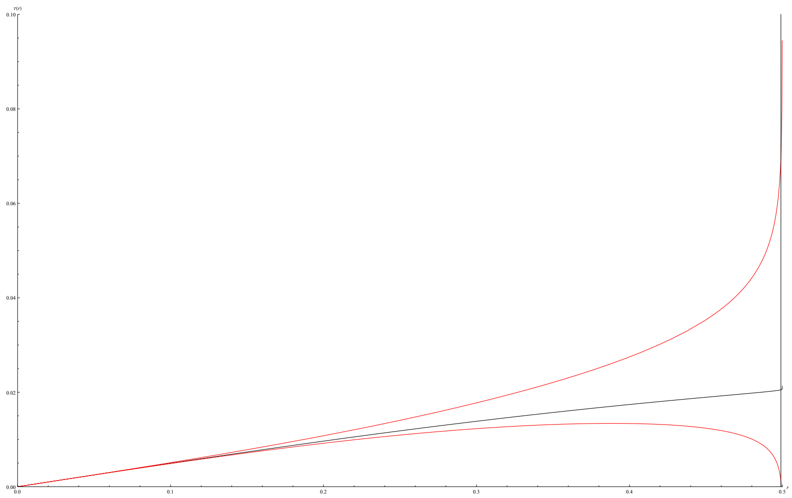

Figure 17: The solution of $\tau(r)$. The two red functions diverge towards the end and unlike in the zero temperature theory, we do not desire this behaviour this time. Rather, we want the behaviour of the black curve: a constant value.

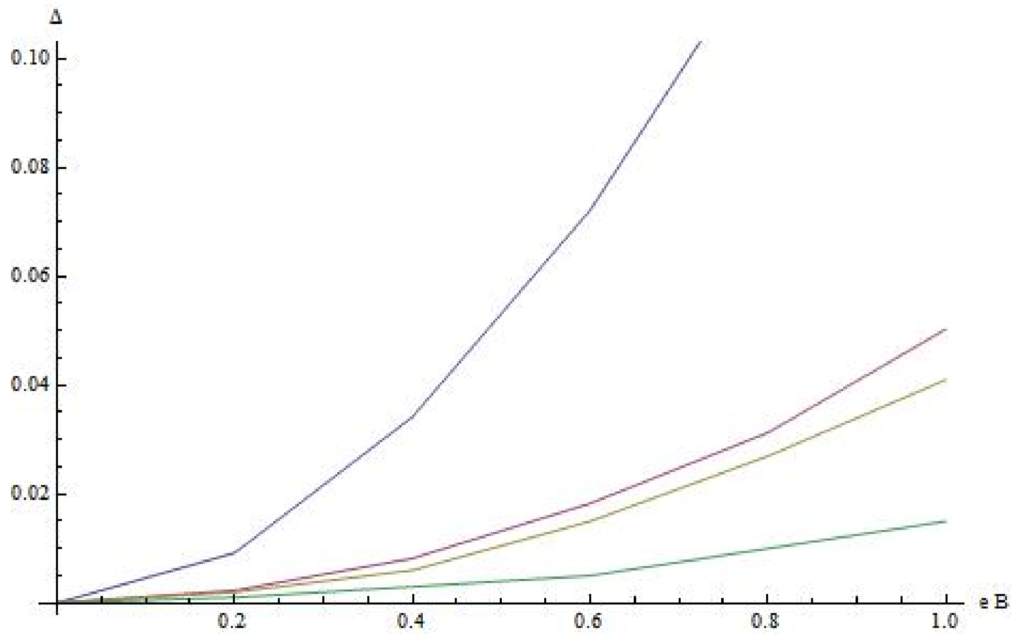

Figure 18: The behaviour of the condensate $\Delta = \langle \bar{q}q \rangle_B - \langle \bar{q}q \rangle_0$ for different temperatures. The black curve gives the $T = 0$ behaviour, it's essentially the same behaviour as seen in figure 3. The coloured curves (from top to bottom: red, yellow and green) are curves for increased temperatures. The red curve is at the critical temperature T_c, the yellow curve is at $T = 1.1T_c$ and the green curve at $T = 2T_c$. While the increased temperature does dampen the constructive effects of the magnetic field, it does not cause this effect to become destructive. So unfortunately, even for higher temperatures, we were unable to show the phenomenon of magnetic decatalysis

7

DISCUSSION

In this thesis we wanted to study the effects of an external magnetic field on the quark condensate created by spontaneous symmetry breaking. Lattice QCD predicts both an enhancing effect, magnetic catalysis, and for sufficiently high enough temperatures a destructive effect, magnetic decatalysis. We set out trying to explain this phenomenon with looking at the Improved Holographic QCD model. This model works in the Veneziano limit, which means that the parameter x_f remains finite. Physically, this means we take into account the backreaction of the magnetic field and the tachyon on the action. Unfortunately, we were unable to even get the right tachyon behaviour, so we switched to a new model.

The second model, the AdS Soliton, is a far simpler model. It does not take into account the backreaction. As a consequence, the background can actually be solved analytically, which greatly reduced the calculation of the tachyon solution. Another benefit of this model is that the horizon, $f(r_h) = 0$, actually lies at a finite point. This makes it possible to find the right tachyon behaviour with a mere look. Unfortunately these simplifications meant that we did not see the phenomenon of magnetic decatalysis. While we did note a dampening effect of the temperature on the effects of the external magnetic field on the condensate, it wasn't the behaviour of magnetic decatalysis. The main reason for this is probably the lack of backreaction. Unfortunately including this in the AdS model would take too much time.

8

OUTLOOK

There are several things that can be improved upon in both models. The main problem we faced with the IHQCD model was the numerical calculation and, perhaps, the ambition to take on too much too fast. Anyone that will take another look at this model will encounter the same problem we did: the tachyon behaviour is extremely complicated to get right. While both asymptotic regions are easily calculated analytically it becomes a problem to then connect the two. Shooting from the UV, which is the technique we used, turns out to be inviable. Perhaps shooting from the IR, or from somewhere in the middle, will actually give the right behaviour. It's also very possible that changing the shape of couplings and potentials will somehow simplify these calculations. There is a grand range of possibilities and it may be very naive to think that setting everything to unity will give the easiest to solve model.

The AdS Soliton model is rather simplistic in its nature, but one could imagine that certain generalizations might produce interesting results. Going away from the quenched approximation will be interesting regardless of the results: if magnetic decatalysis remains beyond the grasp of this model, then perhaps it can be compared with models that do show decatalysis and one might be able to find what exactly is responsible for this. If magnetic decatalysis is actually in range of this model, then this means we have a very simple, easy to understand, albeit nonphysical, model that might be able to explain the mechanism behind decatalysis, this might in turn result in more physical-relevant models with this behaviour.

9

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