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The effectiveness of the SaLVO pedagogy on students’ performance on proportionality

Master’s Thesis (30 ECTS)

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Abstract[1]

The aim of this study is measuring the effectiveness of the SaLVO pedagogy, coherent educational material. The main research question was: *To what extend is the SaLVO pedagogy effective on cognitive level for dealing with proportionality problems?* This was tested with a Dutch 8th grade SaLVO maths group and a maths control group that used the Numbers & Space material. Both groups took the same newly developed test. No differences in test scores were found, overall and on question-level. The SaLVO pedagogy was not more effective on cognitive learning gains than the Numbers & Space material for proportionality problems.

Keywords: SaLVO pedagogy ~ Coherent education ~ Proportionality

[1] A maximum of 100 words is permitted by *Teaching and Teacher Education*.

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1. INTRODUCTION

The coherence of science and mathematics education has been an important topic for many years (Geraedts, Boersma, Huis, & Eijkelhof, 2001). This can be established in three different ways: on organizational level, on content level and on pedagogical level[2] (Lake, 1994; Mooldijk, 2013). Some examples of organizational coherence are found in American schools (Kali, Linn, & Roseman, 2008). Some examples of content coherence are found in the Netherlands. There are not many examples of pedagogical coherent education. When the curriculum is not pedagogically coherent, students will not apply the acquired knowledge in another course (Mooldijk, 2013).

Due to the need for pedagogical coherent education, the Freudenthal Institute (Utrecht University) started the SaLVO project in 2004. SaLVO stands for Coherent Learning Secondary Education[3]. In the SaLVO project this educational material is developed for the topic proportionality, a topic students were found to have difficulties with (Mooldijk & Sonneveld, 2010). For example, students sometimes have the tendency to reason from a direct proportional (linear) scheme in situations where this is not appropriate (de Bock, van Dooren, Janssens, & Verschaffel, 2003; van Dooren, de Bock, Hessels, Janssens, & Verschaffel, 2005). Furthermore, they have difficulties with applying the acquired problem solving methods. In science courses the underlying mathematical conceptual knowledge about proportion is taken for granted (van der Valk, Wijers, & Frederik, 2000). Since the methods in science textbooks often differ from those in the mathematics textbooks, students do not know how to solve proportionality problems.

Expected is that the SaLVO material offers a solution for these conceptual and transfer problems since it is pedagogical coherent. There is some evidence for this claim; three years after the SaLVO material was implemented at various schools teachers noticed improvements in students' work on proportionality problems (Mooldijk & Sonneveld, 2010). A few years later, some teachers saw improvements on final exam scores for questions about proportion, which they claim to be the effect of the SaLVO material.

The research question in this study is: *To what extend is the SaLVO pedagogy effective on cognitive level for dealing with proportionality problems?*

This study aims to measure the effectiveness of the SaLVO pedagogy, where effectiveness is defined as cognitive learning outcomes. Can students better deal with proportionality problems after the same amount of instruction? The relevance of this study is twofold. When the SaLVO pedagogy turns out to be an effective way of learning proportionality, it can be implemented at more schools in the Netherlands. Generalizing, more pedagogical coherent educational material like SaLVO can be developed and implemented. This links to the maths test *(rekentoets)* and the mathematical thinking activities *(WDA)* that are interwoven in the high school examination. On the other hand, this study contributes to the knowledge base about pedagogical coherent education.

The remainder of this thesis is split up into seven chapters. Chapter two presents the theoretical foundation of this study, with sections on coherent education; the observed problems with proportionality and pedagogies for teaching proportionality. Chapter three focusses on the SaLVO

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[2] Pedagogical is in this context best translated as *didactisch* in Dutch.
[3] *Samenhangend Leren Voortgezet Onderwijs*

pedagogy. Chapter four analyses the SaLVO material and eventually presents the hypotheses for this study. Chapter five describes the qualitative and quantitative approaches that are used in this study. The results of this study are shown in chapter six. This study ends with its conclusion and discussion in chapter seven and eight.

Some Dutch terms in this thesis were difficult to translate. To make it more understandable for Dutch readers, sometimes the original Dutch terms are added *(like this)*.

2. THEORY

This chapter consists of five paragraphs. Section 2.1 gives an overview of the curriculum reforms in the Dutch secondary education, in relation to coherency. Section 2.2 follows with an overview of the different types of coherent education. Section 2.3 takes a short side path where arguments and effects of coherent education are mentioned. Section 2.4 continues with an illustration of the pedagogical coherent material that will be researched in this study (the SaLVO material). Section 2.5 gives the theoretical background of proportionality.

2.1. BACKGROUND

In 1986 the Dutch Scientific Council for Government Policy[4] advised to reform the first three years of secondary education, now known as the "basic education" *(basisvorming)*. They advised to offer all students the same 14 separate courses and not to integrate multiple subjects into one, since the advantages of integrated subjects were unclear (Geraedts et al., 2001; Geraedts, Boersma, & Eijkelhof, 2006). The "basic education" was introduced in 1993. For higher streams, another curriculum reform was introduced in 1998-1999, called *"Studiehuis"*. In 10th grade *(4 havo/vwo)* students choose between four clusters *(profielen)* instead of choosing a number of freely chosen subjects. There are two science and two social clusters and each cluster includes a combination of (clustered) subjects. For example: chemistry + physics and geography + history. *Studiehuis* was implemented to prepare students better for different fields of study in higher/academic education, as well as to help them acquire a more independent way of learning, and coherency. The need for co-operation between the science and mathematics teachers in the Netherlands further increased due to the reform of 1998-1999 (van der Valk et al., 2000).

Since the 1978 conference titled "Integrated science education worldwide" by the International Council of Associations for Science Education (ICASE) was held in Nijmegen, the Netherlands, coherent science education became more and more topic of discussion in the Netherlands (Geraedts et al., 2001). After the introduction of the reform in 1998-1999, the first five years of "basic education" were evaluated in 1999. Two major points were that 1) students thought of their curriculum as incoherent and shredded (Geraedts et al., 2001) and 2) students did not see the relevance of mathematics for their science courses (Mooldijk & Sonneveld, 2010).

Point 1 aligns with the early 20th century research results by Thorndike & Woodworth (1901) that interactions between knowledge areas malfunction because students store their knowledge in different places in the memory – a process described by Van Parreren (1982) as "system separation" (as cited in Vos, den Braber, Roorda, & Goedhart, 2010, p. 38). As for point 2 (the relevance), "Mathematics ... provides the tools by which quantitative relationships in the natural sciences can be modelled, calculated, represented, and predicted, and the natural sciences provide relevant contexts in which mathematical and statistical knowledge can be applied", according to Dierdorp, Bakker, van Maanen, & Eijkelhof (2014). Thus students should experience coherence between mathematics and the sciences, to make these courses more meaningful to them (Dierdorp et al., 2014).

A series of 50 school visits in 2000-2001 showed that students in the higher grades also see the lack of alignment between science and mathematics teachers as a problem (Zegers, Boersma, Wijers, Pilot, & Eijkelhof, 2002). The reforms caused less coherence than expected. Yet, there

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[4] WRR, *Wetenschappelijke Raad voor het Regeringsbeleid.*

remains resistance against integrating multiple courses into one course as this might lead to a decreasing educational quality since teachers should teach in areas they were not trained for (Geraedts et al., 2001). However, there was still enthusiasm for coherent education.

This need for co-operation in the sciences and the strive for coherent education was the reason the SONaTE-project was started in 2001 by the Freudenthal Institute[5], the Foundation for Curriculum Development *(SLO)* and the Axis foundation. SONaTE stands for "coherent education in science and technology"[6]. In the project, first existing coherent educational projects in the Netherlands ("good practice") were mapped in lower secondary education (Geraedts et al., 2001), then in the science clusters in upper secondary education (Zegers et al., 2002), and finally an overview was given of how five foreign countries formed their education to be more coherent (van Engelen, Boersma, & Eijkelhof, 2003).

2.2. Forms of coherent education

Coherent education is in this thesis defined as a form of integration, in which there are still separate courses, but where teachers made agreements about the alignment of materials, skills or ways of instruction (Geraedts et al., 2001; Lake, 1994). In the words of Lederman and Niess "the subject specific concepts remain recognizable" (as cited in Dierdorp et al., 2014, p. 3). One can aim for coherence in different aspects of education: on organizational, content and pedagogical level. These forms of coherence can be obtained in the regular lessons or in special projects in which multiple courses are involved (Geraedts et al., 2001).

Examples of coherence on organizational level are: a shared science lab where physics, chemistry or biology lessons are given or where students can work independently on assignments; agreements on how to use measuring equipment and calculators; and a school timetable where the science subjects are clustered in for example each Tuesday afternoon.

On the content level, sometimes a distinction is made between the subject matter *(vakinhoud)* and skills *(vaardigheden)* taught (Huijs & Bruning, 2008). Examples of coherence on subject matter are agreements on which topics are treated when, how and by whom (mathematics, physics, etc.). The wish for this type of coherence was one of the reasons the examination programs changed (KNAW, 2003). In upper secondary school mathematics students first learn how to differentiate a power function before they need this knowledge in economics class. This is not always the case, as vectors are usually introduced in 11th grade mathematics but already needed for physics in 10th grade (Mooldijk & Sonneveld, 2010). A lack of time may be causing this lack of coherence.

Other examples on content level are interdisciplinary courses and skills, for example research and arithmetic skills. Some schools offer the elective courses "Science", "NaSk"[7], or "Research & Design"[8] (SLO, 2014). Some schools offer arithmetic lessons (next to – or integrated in – the mathematics lessons). In the Netherlands students approaching their graduation (9th, 10th or 11th grade) work on a big assignment about something that fits in their chosen cluster *(profielwerkstuk)*. Since 2007, 10th grade students can choose the elective course Nature, Life and

[5] At that time known as the Centre for Pedagogy of Mathematics and Natural sciences *(CD-β)*.
[6] *Samenhangend Onderwijs in Natuur en Techniek*
[7] Contraction of the Dutch abbreviations for Science *(Natuurkunde)* and Physics *(Scheikunde)*
[8] *Onderzoek & Ontwerpen*

Technology *(NLT)*, which is an integrated science and mathematics course in the upper secondary education. It is a two year course for the senior general level *(havo)* and a three year course for the pre-university education level *(vwo)*.

Examples of coherence on pedagogical level, such as teachers from different courses using the same terminology, the same teaching materials or teaching the same problem solving strategy, are scarce in the Netherlands (Geraedts et al., 2001; Huijs & Bruning, 2008). In fact, the opposite is observed. For example, with no mention or introduction of the concepts in mathematics textbooks, in economic textbooks references are made to *'difference quotient'* and *'differential quotient'* whereas in physics textbooks the term *'differentials'* is used (Den Braber, 2007, as cited in Vos et al., 2010, p. 39). For the lower secondary part of the SONaTe project nine school visits were made. Two schools had a school-wide pedagogical/didactical vision: in one school students were clustered in groups in the classroom and in one school students worked independently a lot (Geraedts et al., 2001). Although these are pedagogical examples, this falls within 'teaching style' more than in 'pedagogical coherent education'. The SONaTe project concluded that in Canada, the USA and Israel, some concepts (for example change, energy, systems, interaction, model, scale) are approached in an interdisciplinary way (van Engelen et al., 2003).

2.3. WHY COHERENT EDUCATION?

Proponents of coherent education use amongst others the following arguments, according to Eijkelhof (1999, as cited in Geraedts et al., 2001, p. 25-26).

- Through coherent education the subject matter is broadened or widened. (content level)
- The students develop a coherent knowledge base, instead of separate knowledge elements. (content level / pedagogical level)
- Coherent education is more efficient, since no overlapping content or skills are thought (content level)
- Students do not experience confusion about the meaning of concepts or the learning/using of skills (pedagogical level)

For the sciences and mathematics, there are two main reasons for coherency: mathematics offers the toolbox for the quantitative relationships in the sciences and the sciences offer contexts in which the mathematical knowledge can be applied (Boersma, Bulte, Krüger, Pieters, & Seller, 2011).

Teachers and headmasters of the schools visited in the SONaTe project give the following motives for coherent education (Geraedts et al., 2001, p. 111-112).

- Creating a less fragmented curriculum, whereby a subject can be approached from different angles and thus better connects with the career perspectives (or higher education) and the everyday environment.
- Providing students more insight in the coherence on content level between the learning areas *(leergebieden*)
- Making it easier for students to apply the learnt skills
- Increasing student motivation

Although there are a few comparison studies available thus far, the literature reports some positive effects of coherent education. For example, a meta-analysis of 50 studies found positive effects on student achievements on respectively science and mathematics grades (effects sizes

.37 and .27) (Hurley, 2001). Moreover, most students appreciate it when teachers collaborate in interdisciplinary courses (Boersma et al., 2011). Teachers having "the same expectations across subject areas" is a key factor for the performance of students (Lake, 1994, p. 9). Furthermore, students will get motivated when they see connections between different subjects (Lake, 1994). However, a previous master's thesis about the SaLVO material did not find a significant relation between using the SaLVO material and seeing the relevance of mathematics for science (Wolthoff, 2013).

2.4. Pedagogical coherent education: The SaLVO material

After two years of analysing the existing coherent science educational materials, the SONaTE-project continued with design research since there was not much material to be found. In 2004 this resulted in the SaLVO material, developed by the Freudenthal Institute in collaboration with teachers from four Dutch secondary schools. SaLVO stands for "coherent learning in the secondary education"[9].

As mentioned before, the topic proportionality was chosen. This is a core topic in mathematics education, but also a common topic in the science subjects as many quantities (density, speed, concentration, etc.) have proportional properties (van der Valk et al., 2000). Research showed that proportional reasoning is required for students to understand concepts in science such as scale, and that it can be an indicator of students' success in learning chemistry and biology (several authors as cited in Taylor & Jones, 2009, p. 1233). Furthermore, proportional reasoning is a difficult skill needed in daily life, but not mastered by many adults (Tourniaire & Pulos, 1985).

The SaLVO material consists of 17 booklets *(modules)* which together form a continuous teaching/learning trajectory *(doorlopende leerlijn)* for students in 8th – 11th grade on senior general level *(havo)* and pre-university education level *(vwo)*.

Considering the resistance for integrating courses each booklet can be used in one course; mostly in the mathematics and physics classes, but also some in chemistry, economics or Nature, Life and Technology class. The booklets partly replace and partly complement paragraphs in the textbooks used.

2.5. Proportionality

This section gives a theoretical background for the concept 'proportionality'. In section 2.5.1 the concepts 'ratio', 'proportionality' and 'proportional reasoning' are defined. Section 2.5.2 discusses three types of proportionality problems and three types of assessment problems. In section 2.5.3 six strategies for solving proportionality problems are discussed. Section 2.5.4 continues with (the background of) problems that students encounter when solving proportionality problems and with some remarks on what was found in literature on teaching approaches.

2.5.1. Defining Ratio, Proportionality and Proportional reasoning

Ratio

The ratio $a : b$ is the relationship between two numbers a and b. The numbers a and b are called terms, a being the antecedent and b being the consequent. It may be numbers of the same nature (e.g. 2 amounts of money) or numbers of a different kind (4 books : 20 dollars). Ratios of

[9] *Samenhangend Leren in het Voortgezet Onderwijs*

quantities of the same kind are called **internal ratios**, ratios of quantities of different kinds are called **external ratios** (Tourniaire & Pulos, 1985).

Ratios are often converted into fractions $\frac{a}{b}$. Turning the ratio into a single number (for example $2 : 5$ as 0.4) is loaded with difficulty, since the ratio then loses its meaning '*for every … we have …*' (Watson, Jones, & Pratt, 2013). There exists also ratios with three or more terms, the ratio of flour to sugar to eggs can be expressed as $3 : 1 : 2$. In this case the fraction or decimal notation cannot be used.

Proportionality

When two ratios are equal they are referred to as a *proportion expression,* $a : b = c : d$ or in fraction notation $\frac{a}{b} = \frac{c}{d}$. This proportion $\frac{a}{b} = \frac{c}{d}$ has two types of multiplicative relationships in it (Steinthorsdottir, 2006). The multiplicative relationship **within** the given ratio ($b : a$) and **between** the ratios ($c : a$).

An **integer ratio** is a ratio in which both the within and the between ratio are an integer. For example, $\frac{2}{4} = \frac{12}{x}$ is an integer ratio problem since the within multiple is integer ($4 : 2 = 2$) and the between multiple is integer ($12 : 2 = 6$). When at least one of the multiplicative relationships is not an integer, it concerns a **non-integer ratio**. For example, the problem $\frac{8}{5} = \frac{48}{x}$ has an integer multiple between the ratios ($48 : 8 = 6$) but the within ratio does not $\left(5 : 8 = \frac{5}{8}\right)$.

The proportionality $a : b = c : d$ is called direct proportionality *(recht evenredig verband)*. Direct proportionality can also be represented as a function of the form $y = kx$, with $k \neq 0$. If $k > 0$, this means that if one term of the ratio increases with a factor, the other one also increases with this factor. This is mostly the case in the contextual proportionality problems used in school and in research assessment. If $k < 0$ this means that if the one term increases with a factor, the other term decreases with this factor.

More advanced forms of proportionality can be represented by functions but are not represented in the fraction or ratio notation. Inverse proportionality is represented by the formula $y = k/x = kx^{-1}$, with $k > 0$. When one variable decreases, the other variable increases, given that their product stays constant ($yx = k$). Other more abstract examples of proportionality are square proportionality ($y = kx^2$) and exponential proportionality ($y = k \times g^x$). All types of proportionality can be represented by a graph or in a table.

Proportional reasoning

Proportional reasoning is "the human ability to make use of an effective form of the proportional scheme" (Ben-Chaim, Keret, & Ilany, 2012, p. 49). This means being able to construct and solve proportion problems with algebra (Lamon, 1993).

2.5.2. *Types of Problems and Assessment problems*

Ratio and Proportionality problems

A ratio problem exists of comparing two ratios, with one of the terms being the unknown x. Ratio problems can be divided into three general categories (Ben-Chaim et al., 2012).

1. **Comparing two parts of a single whole**.
   The terms of the ratio are two disjoint subsets forming one set together. Most of the times the terms have the same unit. An example problem: a class consists of boys and girls. The ratio girls to boys is 3 ∶ 2. In another class with the same ratio girls to boys are 18 boys. How many girls are in this class?
2. **Comparing quantities of the same nature**
   The terms of the ratio propose two magnitudes that are related, but are not 1 when added together. The terms can have the same unit. An example problem: paper sheets[10] have the same ratio length to width. You want to enlarge a piece of paper that is 297 mm long and 210 mm wide to one size bigger that is 420 mm long. What will be the width?
3. **Comparing quantities of different natures**
   The two terms in the ratio are not conceptually related, for example $4\ books : 20\ dollars$. They may have an interesting connection, then a new dimensional unit is often created, for example the ratio $42\ km : 1.5\ hour$ (distance over time) represents speed (28 km/h).

Lamon found that the first type of problem (she called this "Part-Part-Whole problems") did not elicit any proportional reasoning from 6th grade students because the problems could be solved using less sophisticated methods (1993). Type 3 problems with the rate not being a new entity (she called this "Associated sets") presented with concrete pictures elicit the most sophisticated thinking (Lamon, 1993).

Ratio and Proportionality Assessment problems

The literature reports three types of a problem for assessing / evaluating the proportional reasoning (Ben-Chaim et al., 2012; English & Halford, 1995; Tourniaire & Pulos, 1985).

1. **Missing value problems.** Two ratios are given with three of the terms known. The task is to find the fourth missing term. For example, what is x when $\frac{2}{4} = \frac{12}{x}$?
2. **Numerical comparison problems.** Two ratios are given with the question if they are equal, or if one is smaller/larger. For example, is $\frac{2}{7}$ smaller/equal/bigger than $\frac{9}{28}$?
3. **Estimation problems.** A comparison problem in which no numerical values are used. Another answer possibility is added: not enough information to decide. For example, if I use less oranges and more water today, will my lemonade be stronger, weaker, the same as yesterday, or is there not enough information?

In the missing value and numerical comparison problems, a memorized skill/trick can be used (English & Halford, 1995). In assessment problems types 2 and 3 the students should understand the meaning of proportion in order to solve it (English & Halford, 1995, p. 248). Lamon distinguished another type of assessment problems that she called "Strechters and Shrinkers": situations that involved scaling up/scaling down (with a fixed ratio) a quantity that is typically measured as a distance (length, width, height). She found that these problems were the most difficult for 6th grade students since they failed to recognize the multiplicative nature of the problems (1993).

Another distinction can be made when looking at the semantics: symbolic problems and word problems (Steinthorsdottir, 2006; Tourniaire & Pulos, 1985). In **symbolic problems** the ratios

[10] The A-series that is used in Europe (amongst others) have ratio $\sqrt{2} : 1$.

are presented in mathematical symbols $\left(\frac{3}{7} = \frac{x}{28}\right)$ without context. Word problems can be given with or without illustrations. Tourniaire and Pulos distinguish two types of **word problems**: rate problems and mixture problems (1985).

In **rate problems** two ratios of dissimilar objects are compared (comparable with problem type 3). In **mixture problems** two ratios of mixtures have to be compared, which is a type 1 problem in which the 'whole' is a new object (water + orange makes juice; red paint + white paint makes pink). In this case, the student should understand what happens when two elements are mixed in order to solve the problem (Tourniaire & Pulos, 1985). In most mixture problems the two terms of the ratio are given in the same unit, which may be more confusing for students than for example the problem of $4\ books : 20\ dollar$ (Tourniaire & Pulos, 1985).

2.5.3. STRATEGIES FOR SOLVING PROPORTION PROBLEMS

Research mentions six strategies used by students for solving proportion problems (Avcu & Avcu, 2010; Cramer, Post, & Currier, 1993; Tourniaire & Pulos, 1985; Watson et al., 2013). Proportional reasoning develops from qualitative thinking to additive strategies to multiplicative strategies, each following strategy being a more sophisticated and abstract way of thinking about proportionality (Steinthorsdottir, 2006). Words as smaller, bigger, more and less characterize the qualitative thinking phase. It was found that the semantics and/or the context influenced the choice of strategy (Tourniaire & Pulos, 1985).

Additive and multiplicative strategies have a quantitative nature. Below, each strategy will be further explained by the use of the sample problem *"If 3 apples cost 60 cents, find the costs of 12 apples"* (to the idea of Cramer et al., 1993, p. 167).

Additive strategy

In primary school students initially use a **building-up strategy (1)** (Tourniaire & Pulos, 1985). In this reasoning, the students use their knowledge of addition to solve the proportion problem. The students iterate the pattern found within the ratio to the unknown quantity. For example, for the earlier proposed problem the student would reason as follows. Three apples cost 60 $cents$. So three apples more makes 6 apples that will cost $60 + 60 = 120\ cents$. Three apples more is 9 apples for $120 + 60 = 180\ cents$. Three apples more is 12 apples for $180 + 60 = 240\ cents$.

The building up strategy works well for problems with integer ratios. You may expect problems when three apples cost 57.25 $cents$ (the addition becomes more complicated) or when the students are asked to find the costs of 10 apples (which only works well if the ratio $cents : apples$ is an integer or perhaps a simple fraction like ½ or ¼). The building-up strategy is used in higher grades for simple problems or for familiar contexts (Tourniaire & Pulos, 1985).

Multiplicative strategies

Identifying a unit rate (2) *(terugrekenen naar '1')*. The key in this method is finding the multiplicative relationship **within** the given ratio (b/a). This ratio is the amount that stands for one unit of the quantity. This can be multiplied with the other known quantity to attain to the answer (Avcu & Avcu, 2010). For the sample problem, the cost for one apple is $60 : 3 = 20\ cents$. So the costs for twelve apples is $20 \times 12 = 240\ cents$. In a research including 12 to 14-year olds, this method was more used by the younger students (Cramer and Post, 1993, as cited in Watson et al., 2013, p. 54).

The unit rate method can easily be used when the within ratio is an integer. When using a calculator, this method also can be used with non-inter within ratios. Errors can occur when students round the calculated within ratio to 1 or 2 decimal numbers and then continue their calculation with this rounded ratio. For example when the 3 apples cost 62 cents the correct answer should be 248 cents for 12 apples. A student might reason that $62 : 3 = 20.67\ cents = 21\ cents\ per\ apple$. The context influences the rounding of the numbers in the problem.

Identifying the scale factor (3) (also known as factor of change). The students look for the scale factor, i.e. "how many times larger" the quantity should be (Watson et al., 2013). They compute this factor by comparing the known parts of both ratios (that is to say or both the numerators or both the denominators). Then they multiply the factor with the value of the given quantity. For the sample problem, 12 apples are 4 times 3 apples. So the scale factor is 4. Thus the answer is $4 \times 60\ cents = 240\ cents$.

This approach is easy to use when the scale could be expressed as an integer (English & Halford, 1995; Watson et al., 2013).

Matching equivalent fractions (4). This strategy is a standalone method that does not rely on any context (Cramer et al., 1993). The pairs of ratios are treated as fractions. The multiplication rule is used $\left(\frac{a}{b} \times \frac{c}{c} = \frac{ac}{bc}\right)$ for matching the known part of one ratio with the corresponding known part of the other ratio by multiplying it with a fraction of the form $\frac{c}{c} = 1$. The product ratio will have a term equal to the desired answer (Avcu & Avcu, 2010). In the sample problem, the student first writes the problem as $\frac{3}{60} = \frac{12}{?}$. The multiplication rule tells the student that the numerator is multiplied by 4, so the denominator should also be multiplied by 4. In algebra: $\frac{3}{60} \times \frac{4}{4} = \frac{12}{240}$, so the answer is $240\ cents$.

This method also works if the students writes the fraction as $\frac{60}{3} = \frac{?}{12}$, meaning 60 cents per 3 apples. This method can be used disregarding any units. So the problem "we ride 3 km per 2 hours, how many km in 5 hours?" may also be written as $\frac{2}{3} = \frac{5}{?}$, turning the unit of the fraction not into speed but into hours/km.

In the 1993 study by Cramer and Post, this method was used more by the older students in the 12 to 14-year old group (as cited in Watson et al., 2013, p. 54).

Matching equivalence class (5). This strategy also starts with writing the given rate as a fraction. Instead of multiplying at once, the student uses a number of equivalent fractions until the desired answer arises (Avcu & Avcu, 2010). For this problem, the fraction will be $\frac{3}{60}$ and with one intermediate step the student will arrive at the answer. Since $\frac{3}{60} = \frac{6}{120} = \frac{12}{240}$.

Cross-multiplying (6). This is a method that does not arise from the meaning of proportionality, but from combining several actions in the thinking process (Watson et al., 2013). If the proportion problem is $\frac{a}{b} = \frac{c}{x}$ and the student has to solve for x, he would reason $x = \frac{cb}{a}$ (Avcu & Avcu, 2010). In our example, the student might write down $\frac{3}{60} = \frac{12}{x}$ so $x = 60\ cents \times 12\ apples : 3\ apples = 240\ cents$, or, more likely, without the units: $60 \times 12 : 3 = 240$.

This strategy is tented to be misapplied: cross-multiplying often is a "trick" that students do not understand (Broekman, van der Valk, & Wijers, 2000). In the before mentioned research, this method was only used by the 14-year olds (as cited in Watson et al., 2013, p. 54).

Summary of the strategies for the sample problem

Table 1 gives a summary of each strategy by giving a solution for the sample problem: *"If 3 apples cost 60 cents, find the costs of 12 apples"* (to the idea of Cramer et al., 1993, p. 167).

Table 1 – Summary of different problem solving strategies

Strategy	Problem solved in the following way
1. Building up	60 cents for 3 apples, 3 more is 120 cents for 6 apples, 3 more is 180 cents for 9 apples, 3 more is 240 cents for 12 apples.
2. Unit rate	The cost for 1 apple is $60 : 3 = 20$ cents The cost for 12 apples is $20 \times 12 = 240$ cents
3. Scale factors	I want four times as many apples, thus the cost will be four times as much. $60 \times 4 = 240$ cents
4. Equivalent fractions	$\frac{3}{60} \times \frac{n}{n} = \frac{12}{?}$ thus $\frac{3}{60} \times \frac{4}{4} = \frac{12}{240}$ thus 240 cents
5. Matching equivalence class	$\frac{3}{60} = \frac{6}{120} = \frac{12}{240}$ thus 240 cents
6. Cross-multiplying	$60 \times 12 : 3 = 240$ cents

2.5.4. *Problems with proportional reasoning and teaching approaches*

Proportional reasoning ability is correlated with the developmental stage of the student (Taylor & Jones, 2009). Understanding informal methods for solving proportionality problems will "strengthen the intuitive foundation of the proportional scheme" and will "encourage students to solve problems using informal strategies before formal instruction is given" (Ben-Chaim et al., 2012, p. 52). In order to understand proportionality, the student has to understand the relationship $\frac{a}{b} = \frac{c}{d}$ and to understand what happens to the other terms if one term changes (Watson et al., 2013). Incorrect use of strategies may be due to flawed procedural skills or due to lack of understanding proportional reasoning (Ben-Chaim et al., 2012).

The experienced problems have different causes. Formal methods tend to be misapplied when dealing with non-integer ratios, co-prime denominators and unfamiliar contexts (Watson et al., 2013). Students can use a comparing strategy or difference strategy as fallback strategy (Steinthorsdottir, 2006). In the comparing strategy, students only compare the denominators of the proportion (Tourniaire & Pulos, 1985). For the sample problem in the previous section, a student might say that the apples will cost 6 cents more so $60 + 9 = 69$ cents since $12 - 3 = 9$. In the difference strategy the student does not work with the fraction but with the difference of two parts of one ratio (Ben-Chaim et al., 2012). The student will argue that for 3 $apples : 60\ cents$ it gives $60 - 3 = 57$, so the 12 apples will cost $12 + 57 = 69$ cents.

Another mistake can be ignoring part of the information in the problem (Steinthorsdottir, 2006). This of course leads to difficulties when solving a proportionality problem. The last problem arises when students learn the more advanced forms of proportionality. Students can stick with following the direct proportional scheme, when they are dealing with inverse proportion (Ben-Chaim et al., 2012) or square proportionality (van Dooren et al., 2005).

Literature gives no clear answer on what is the best way to teach proportionality. However, some pedagogical considerations are found in different studies, covering different aspects of proportional reasoning and proportionality problems.

Confrey found that **time** is a key factor when one wants to understand proportionality (1995, as cited in Watson et al., 2013, p. 61). The students need a repertoire of ideas, models, ways of talking and past experience. A **range of question types** is necessary for fully understanding proportional reasoning. Furthermore, **language** plays a role. Kaput & Maxwell-West found doing case studies that using the phrases 'for each' and 'for every' were critical in supporting understanding (1994, as cited in Watson et al., 2013, p. 56).

Confrey started teaching with **scaling recipes** for different numbers of people, since it was found that this approach led to a strong understanding of proportional reasoning and the concept of distributivity. It was also found that an approach requiring the use of **several different calculation methods**, while maintaining the focus on ratio, improves this understanding (Watson et al., 2013). A way to do this is to pose a series of questions about a giant or a toy and their relationship to human dimensions.

There has been some research about the **contexts** of the proportionality problems. Watson et. al found that children who had previously worked with measurement in contexts learnt decimals and fractions more easily than expected (2013, p. 61). Contexts where the quantities cannot be counted by hand encourage the imaginary qualities of proportional reasoning (Watson et al., 2013, p. 65).

Furthermore, students are more likely to use unitary methods ('the unit rate method') with contextual problems where they can choose the **unit** themselves, i.e. the unit does not have to be 1 (Watson et al., 2013, p. 65). Hino found that when using a unit that gives insight into the situation, rather than just as a label, students can use multiplicative strategies in novel situations (instead of falling back to additive methods) (2002, as cited in Watson et al., 2013, p. 56).

Lastly, one remark about **calculators** needs to be made. Calculators that give simplified answers hide the associated structure of the proportion (Watson et al., 2013). For example a slope is $\frac{3}{2} = 1.5$. The fraction 3/2 illustrates the meaning 'for every 2 steps to the right we go 3 steps upwards', while 1.5 does not give any meaning especially in the beginning of learning proportionality.

Teaching approaches for proportionality did not gained enough attention as research subject. The SaLVO pedagogy matches some of the considerations mentioned above. There is a lot of time for the topic, since the booklets form a continuous teaching/learning trajectory. The booklets are used in several courses, leading to (meaningful) contexts and a range of question types. Whether the SaLVO pedagogy matches the other pedagogical considerations mentioned above, will become clear in the course of this research. In the next chapter the SaLVO pedagogy will be described in detail. For now we can assume that the SaLVO material meets sufficient conditions to be included in this research.

3. THE SALVO PEDAGOGY

This remaining part of this research consists of three phases. The first phase of this study focuses on defining the SaLVO pedagogy and comparing it with the mathematics textbook used on school. A list of learning goals and the survey are developed in phase 2. The intervention took place in the third and final phase. For a schematic overview, see Figure 1. This chapter covers the SaLVO pedagogy.

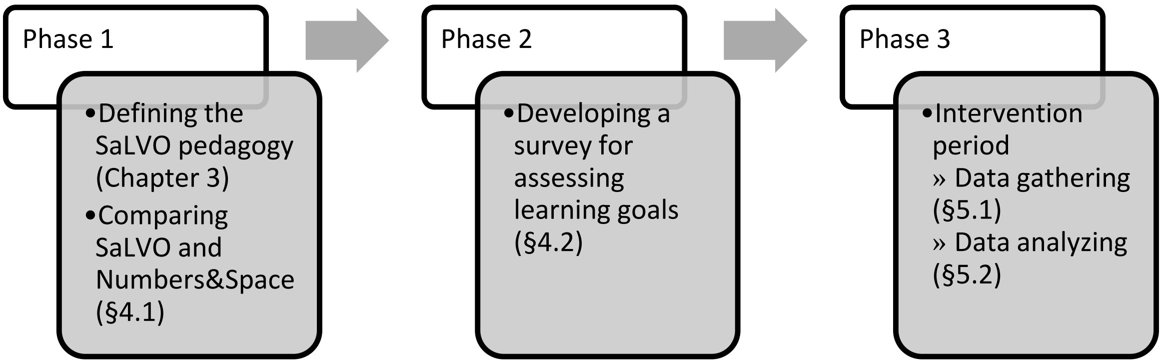


Figure 1 – Overview phases of this study

As explained before, the main goal of the SaLVO material was providing coherent educational material for the topic of proportionality. Five pedagogical assumptions underlie the SaLVO pedagogy. These were found studying the SaLVO material. To ensure completeness and accuracy the assumptions have been checked by three of the main developers[11] of the SaLVO material.

The assumptions *learning by experiencing*, *the ratio table* and *group work* may have a positive impact on the effectiveness (cognitive learning gains) of the SaLVO material.

1. Learning by experiencing

When the students start learning a new concept in one of the SaLVO booklets, this does not start with reading a text explaining the concept. The SaLVO pedagogy is that learning by experiencing suits the process of learning and shaping a new concept *(conceptontwikkeling)*. Students should not only apply knowledge, but also understand why the formula works the way it works. Every paragraph in every booklet is structured in the same way. It starts with one "paragraph question" that triggers the students' thinking about the topic and gives them the feeling that they need more knowledge on the subject to answer the question. The students will be able to answer the "paragraph question" at the end of that paragraph.

After this "paragraph question" an "entry level" question *(instapvraag)* follows. This is a question that the students are able to answer with their pre-knowledge. Mostly it is a context question that has numbers that are suitable for easy calculations. By answering this question, the students have already worked with the principle (e.g. formula, concept) that they will learn in the paragraph. It is the teacher's job to start a discussion afterwards in which he/she reflects on the question and the different problem solving strategies. The teacher guides the students from their (mostly) informal strategies to a general applicable formal principle/formula.

---

[11] Kees Hooyman, Ad Mooldijk and Wim Sonneveld.

Another way of learning by experiencing is doing experiments. For physics and chemistry classes this is common, bur for mathematics it is generally not. For example in booklet 3, the students perform several experiments to relate the diameter of steel balls and the dimensions of a package of rice to the volume. After the experiments, the students are asked to formulate a general applicable formula: *"If all dimensions become a factor k larger, then the volume will become ……..".*

2. Ratio table

The SaLVO pedagogy offers the ratio table as a tool for solving proportion problems. If a student wishes to solve the example problem from paragraph 2.5.3 (*"If 3 apples cost 60 cents, find the costs of 12 apples"*) he would draw the two-row table below. Each row gets a label (quantity and unit) indicating the meaning of the numbers in that row (Broekman et al., 2000). The columns can now be filled with pairs of numbers that have the same ratio.

Number of apples	3	12
Costs in cents	60	

The students then essentially seeks the scale factor (12 : 3 = 4) and multiplies 60 with this factor. The reasoning behind this table is that students make less calculation errors and they always write down the units. The ratio table can also be used for the building up, unit rate and cross-multiplying strategy. An extensive description can be found in Broekman et al. (2000).

The ratio-table is not limited to two rows. Van der Valk (2001) explains that multiple rows can be used when dealing with one quantity in several units, which can be the case in physics or chemistry. His example: calculating the combustion heat in J/g with information about the density in kg/l and the combustion heat in J/m^3. In this case, it is extremely important that the student writes down the unit in the label for each row. The ratio table serves the coherent nature of the SaLVO material.

The ratio table is not only used for questions on basic proportionality, fractions and percentages (through standardizing to 100), this is also done in some Dutch schoolbooks. The ratio table can also be used for questions on percentage change; for examining which kind of proportional relationship is presented in the table; and the mathematical transformation of coordinates.

An example on percentage change: an increase of 150% is related to a scale factor / enlargement factor of 1.5. The students also use this method in the economics booklets and chemistry booklets. See Figure 2 for using the ratio table on calculations on VAT.

	prijs zonder BTW	BTW	prijs inclusief BTW
prijs in euro	€ 18,28		
in procenten	100 %	19%	

Figure 2 – The ratio table for calculations on VAT (Hooyman & Sonneveld, 2010, p. 16)

3. Group work

The "entry level" question is a question that the students will answer in groups. First they think about it themselves; then they discuss their answers and, more important, their calculation methods. Students will learn that different methods lead to the same answer. Additionally, students can explain concepts to each other.

Group work is not limited to the "entry level" question; also in other questions students are encouraged to discuss their work with other students. For example, students are asked for alternative calculation methods, or asked to compare their method with their classmates and choose the most convenient method. Most of the experiments will be done in groups as well.

Following the SaLVO pedagogy; learning by experiencing leads to a better and more meaningful understanding and thus to better cognitive learning. The ratio table helps students to collate the information and causes less calculation errors. Due to the group process of comparing calculation methods, students will learn the underlying principles of the calculation and thus this influences the effectiveness of the SaLVO material.

The assumptions *continuous teaching/learning trajectory* and *coherency* cover key elements of the pedagogy, but do not influence the cognitive effectiveness measured in this study. These last assumptions will not be referred to in the remainder of this study.

4. Continuous teaching/learning trajectory

The SaLVO material is a continuous teaching/learning trajectory *(doorlopende leerlijn)* existing of 17 booklets. These booklets can be used next to the mathematics and physics textbooks, since SaLVO does not cover all topics for the final examination. The SaLVO booklets cover the topic of proportionality (with its formulas and graphs) with sideways to contexts in which proportionality is used. For example, there is a booklet on planets with a link to the scientific notation and density: the direct proportional relation between mass and volume.

The booklets can be offered throughout the whole senior education. There are three booklets for 8th grade, three for 8th or 9th grade, five for 9th grade, four for 10th grade and two for 11th grade[12]. The skills and insights are build up step by step. The material starts with ratio and direct proportionality ($y = ax$) and goes via inverse proportionality ($y = ax^{-1}$), square proportionality ($y = ax^2$), inverse square proportionality ($y = ax^{-2}$), exponential proportionality *($y = b \times g^x$)* and percentages, to square root proportionality *($y = a\sqrt{x}$),* to end with periodic functions *($y = a\sin\big(b(x - c)\big) + d$).*

5. Coherency

Coherency is the most important characteristic of the material. The material is both coherent on content level as on pedagogical level. On content level, there are sets of consecutive books that directly connect to one another. These sets of books support the knowledge transfer from

[12] Two of the 10th grade booklets are only for senior general level *(havo)*, and the two 11th booklets are only for pre-university education level *(vwo)*.

mathematics to physics. These sets are booklet 1 & 2; booklet 4 & 8 and booklet 11 & 12. That these sets are indeed coherent is shown by the following descriptions.

> *Booklet 1 and 2.* Booklet 1 is dealt with in the mathematics lessons. An introduction is given on the enlargement factor and the understanding that the ratio remains constant when enlarging a picture or recipe. In the end the students learn how to recognize a direct proportional relation by using the ratio table, and how to construct an appropriate formula $y=ax$. Booklet 2 is dealt with in the physics lessons. In an experimental setting, the students apply their knowledge of the proportional relation on the data they gather on mass and volume of several materials.
>
> *Booklet 11 and 12*. Here the first booklet is dealt with in the mathematics lessons also. It introduces different proportionalities to the students (inverse proportionality, square proportionality, square root proportionality and inverse square proportionality). It trains the students in converting formulas with the calculation rules for power functions. In the physics lessons booklet 12 continues with these proportionalities in visual form. Students should recognize the proportionality by its graphs and should construct a formula if they only get some numbers. The focus in this booklet is on practical situations and experiments.

On pedagogical level the coherency manifests itself in teaching the same problem solving strategy (the ratio table), using the same terminology or making connections explicit if slightly different terminology is used. For example, in booklet 3 the relation between the mathematical "multiplication factor k " *(vermenigvuldigingsfactor)* and the "enlargement factor N " *(vergrotingsfactor)* from physics is made explicit.

In booklet 6 students see that calculating with percentages (VAT or discount) can be done the same way in their mathematics classes as in their economics lessons. This is also done in booklet 7, where students use the ratio table for their exercises in the chemistry lessons.

4. ANALYSIS

4.1. COMPARING THE SALVO PEDAGOGY

To be able to answer the research question, the performance of a group of students that use the SaLVO material will be compared to the performance of a group of students that do not use it. The content of both materials should be equal or that is to say: the students should reach the same learning goals. The SaLVO material needs to be compared to the educational material used on the test school.

Some boundaries need to be set on which SaLVO booklets to include in this thesis. Due to time limits not all booklets can be included. For the remainder of this research (comparing the content and the pedagogy) a limitation will be made to one or two booklets.

A theoretical requirement is that all of the above mentioned underlying pedagogies are reflected in the chosen material. This is possible for all except the assumption *coherency*, since studying the entire SaLVO trajectory over multiple years is ruled out. Practical limitations are that this research took place in the first months of the academic year and that the research is carried out in the mathematics lessons.

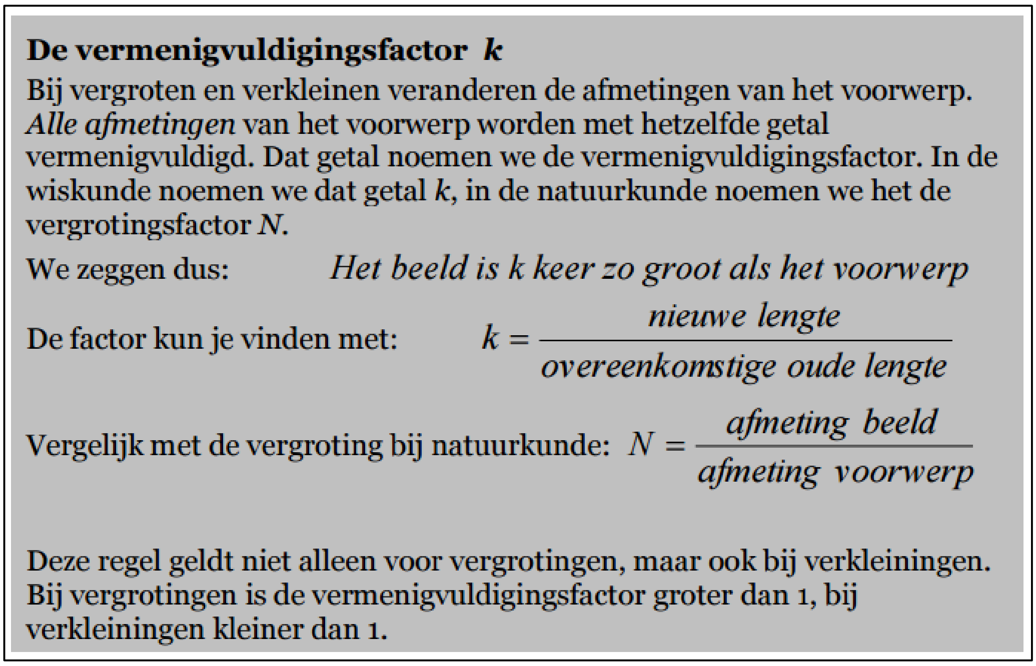
SaLVO booklet 1 was first selected for this research. All assumptions (except coherency) are reflected in this booklet. The booklet suits the first months of the academic year, the time period in which this research takes place. Furthermore, being the first booklet in the series, the students do not lack any pre-knowledge from previous SaLVO booklets. Part of the first SaLVO booklet matches with paragraph 8.4 of the mathematics textbook used at school[13]. For the non-SaLVO group it will be desirable to also include the (on content connecting) paragraphs 8.5 and 8.6. Since these two paragraphs are covered in the mathematics part of SaLVO booklet 3, these are also included in this research. An additional advantage, is that this booklet has coherent characteristics on its own.

The text box in Figure 3 explains how to calculate the enlargement factor k. The comparison to the enlargement factor N used in physics textbooks is made explicit. The Dutch text reads:

The factor can be found using: $k = \frac{new\ length}{corresponding\ old\ length}$

Compare this to the enlargement in physics: $N = \frac{size\ image}{size\ object}$

[13] The (Dutch) 2008 edition of Numbers & Space.



De vermenigvuldigingsfactor *k*
Bij vergroten en verkleinen veranderen de afmetingen van het voorwerp. *Alle afmetingen* van het voorwerp worden met hetzelfde getal vermenigvuldigd. Dat getal noemen we de vermenigvuldigingsfactor. In de wiskunde noemen we dat getal *k*, in de natuurkunde noemen we het de vergrotingsfactor *N*.

We zeggen dus: *Het beeld is k keer zo groot als het voorwerp*

De factor kun je vinden met: $k = \frac{\text{nieuwe lengte}}{\text{overeenkomstige oude lengte}}$

Vergelijk met de vergroting bij natuurkunde: $N = \frac{\text{afmeting beeld}}{\text{afmeting voorwerp}}$

Deze regel geldt niet alleen voor vergrotingen, maar ook bij verkleiningen. Bij vergrotingen is de vermenigvuldigingsfactor groter dan 1, bij verkleiningen kleiner dan 1.

Figure 3 – The enlargement factor *k* (Hooyman & Raterink, 2008, p. 17)

4.1.1. Comparing SaLVO and Numbers & Space on content level

To ensure that both groups of students have gained the same set of skills when assessed, the content of the SaLVO material is compared to the mathematics textbook used in the control group. The textbook used on the school where the intervention took place is the Dutch 2008 edition of “Numbers & Space” (Reichard et al., 2008a; Reichard et al., 2008b).

First, every concept in the first and third SaLVO booklet is made explicitly in first column of Table 2. The second column indicates where the concept is handled in the material: in theory text boxes or in exercises. The number between brackets refers to exercises in the paragraph. The third column indicates if and where to find this concept in the Numbers & Space material. Numbers & Space is shortened to N&S. In the end, the paragraphs 4.1, 4.2, 8.3, 8.4 and 8.5 of Numbers & Space were checked whether some concepts were missing in the first column.

The outcome of this comparison is that the content of the SaLVO material (§1.1, 1.2, 1.3, 1.4 and §3B, D) is to large extent similar to the content of the Numbers & Space material (§4.1, 4.2 and §8.3, 8.4, 8.5). There are five mismatches, indicated with the blue text. The asterisks refer to the solutions presented here:

* The term “ratio” is not mentioned explicitly in the N&S material. This will be introduced to the N&S group after exercise 34 in §8.3.

** Paragraph 1.5 and 1.6 of the SaLVO booklet as well as the concept “cube root” will be excluded from this research.

**** In the N&S book there are two exercises on calculating hours into minutes (how many minutes is 0.3 hours?). In the SaLVO material there is no exercise on this calculation. The exercise will be skipped in the N&S material and this skill will not be tested.

This comparison leads us to a list of learning goals, which will be used for designing the test and which will be assessed in the test. This list can be found in Appendix A.

Table 2 – Comparison of the SaLVO material with the Numbers & Space material on content level

Content (concepts)	SaLVO paragraph	N&S paragraph
Enlargement factor k	§1.1 and §1.2	§8.3
Linking a given scale (1 : 200) to enlarging with a certain factor and vice versa	§1.1 (5)	§8.4 (43)
Ratio stays unchanged when enlarging	§1.2	*
Writing a percentage as a decimal number	§1.3 (13)	§4.1 (2)
Calculate x percent of y	§1.3 (15)	§4.1 (3, 4, 5)
Calculate how many percent x is out of y	§1.3 (16)	§4.2 (20, 22)
Linking a percentage change of x percent to enlarging with a certain factor and vice versa *(150% means multiplying with 1.5)*	§1.3 (14) §1.4 (22)	§4.1 (9, 15)
Calculate the 'new value' y with a given percentage change from x (both percentage increase as decrease)	§1.3 (18, 19, 20)	§4.1 (10, 11, 12, 16)
Calculate the percentage change from x to y (both increase/decrease)	§1.4	§4.2 (26 - 34)
Recognizing a direct proportional relation by using the ratio table	§1.5	Not in 8th grade books. In 7th grade in §10.4. **
Constructing a formula for the direct proportional relation: $y = \ldots x$	§1.6	Not in 8th grade books. **
Draw an enlargement of an image with a given enlargement factor k	§3B (17)	§8.3 (29)
Calculating enlargement factor k from two images using a ruler	§3B	§8.3 (30 – 33, 35)
Relating enlargement factor to change in surface *(k and k^2)* and calculate the new surface from a calculated or given k *($k \rightarrow k^2$)*	§3B	§8.4 (38 – 42)
Doing 'backwards' calculations: what is the enlargement factor k (and the new length/width) if the surface is enlarged with factor p *($k^2 \rightarrow k$)*	§3B (25, 26)	§8.4 (46, 47, 48, 49, 50)
Relating enlargement factor to change in volume *(k and k^3)* and calculate the new volume from a calculated or given k *($k \rightarrow k^3$)*	§3D	§8.5 (52 – 58)
Doing 'backwards' calculations: what is the enlargement factor k (and the new length/width) if the volume is enlarged with factor p. Using the cube root. *($k^3 \rightarrow k$)*	Not in SaLVO material **	§8.5 (59, 60, 61, 62, 63)
Calculations on measurement of volume $m^3 \leftrightarrow dm^3$ (L) $\leftrightarrow cm^3$ (mL)	§3D (43, 44)	§8.5 (54)
Calculations on measurement of time *0.3 hours = 0.3 x 60 = 18 minutes*	****	§4.1 (6, 7)

4.1.2. COMPARING SALVO WITH NUMBERS & SPACE ON PEDAGOGICAL LEVEL

The Numbers & Space material differs also on pedagogical level from the SaLVO material. It differs on the order in which the concepts are dealt with, how the concepts are explained, how the concepts are dealt with in the classroom (*werkvormen*), and which calculation methods are taught. This list is further discussed below.

The order of concepts

The order in which the different subjects are offered are different. In the Numbers & Space material chapter 4 is on percentages and on diagrams. Normally the students do this chapter in the beginning of January. The enlargement factor is covered in chapter 8, which also covers the formulas for calculating the volume of cones and cylinders. This is normally done in May/June. The difference with the SaLVO material is that the enlargement factor is introduced in an early stage. In booklet 1 percentages are introduced after the enlargement factor. The SaLVO material offers students one way of calculating both enlargements as percentage changes.

Calculation methods

In fact, the SaLVO material offers the ratio table as the only one calculation tool students need. In this table they should write down the numbers and think about what to calculate. They only start calculating after this is clear. For their calculations they can use a building up, unit rate or cross-multiplying strategy[14].

Numbers & Space offers different calculation methods for percentages and for the enlargement factor. Both are based on the scale factor strategy. For percentages the students convert it first to a decimal number (the scale factor) and then calculate. The scale factor or enlargement factor $\frac{NEW}{OLD}$ is also 'hidden' in the rule $\frac{NEW-OLD}{OLD} \times 100\%$, which they learn to use for percentage change. Since $\frac{NEW-OLD}{OLD} = \frac{NEW}{OLD} - \frac{OLD}{OLD}$ they calculate the scale factor minus one. Multiplying this by 100% it gives the students their answer right away. With the ratio table the student will get the answer '113%' or '88%' and has to calculate that this comes down to a increase of 13% and a decrease of 12% respectively. The Numbers & Space book does not make it explicit to the students that this calculation is based on the scale factor. It justifies the formula by explaining that the value has increased with $NEW - OLD$ and that this number divided by OLD gives us how many percent the increase is of the total.

When enlarging, Numbers & Space teaches to first calculate the enlargement factor (the scale factor) and then the new height/volume/surface.

Outline of the book and included learning-activities

The outline of the SaLVO booklets has been described in chapter 3. Here the outline of the Numbers & Space books is described and after this the differences are marked.

In the Numbers & Space book every paragraph begins with an Orientation exercise (labeled 'O'). This exercise aims to activate pre-knowledge and to introduce the new subject to the students. In this exercise the students discover parts of the new theory by themselves. After this the book presents the theory. See the next subparagraph for more details on the theory boxes. Then there follow some exercises in which the student can practice with the new learnt knowledge, and one

[14] See paragraph 2.5.3.

or more closing exercises *(afsluitend)* labeled 'A' which are generally of a higher level than the normal ones. The exercises are done by the students individually. Sometimes there is an exercise labeled 'S' which means this is a game *(spel)* students can do in groups. This combination of orientation exercises, theory, exercises and closing exercises can be repeated multiple times in one paragraph. Then more paragraphs follow with the same structure.

The paragraphs are followed by one paragraph called "mixed questions". The exercises in this paragraph cover multiple parts of the theory, so that the student need to combine the knowledge learnt. Then follows a paragraph called "Summary" which sums up all the theory from the chapter. The chapter ends with two more paragraphs with extra questions to practice.

The SaLVO booklets have a similar start as the Numbers & Space books with the "entry level" question *(instapvraag)*. The difference is that the students have to do this exercise individually, compare their answers with their classmates and then discuss with the teacher. The *group work* assumption is not found in the Numbers & Space pedagogy.

In some SaLVO paragraphs there follows a box with theory, in some paragraphs the theory is explained in text and in some paragraphs the "entry level" question continues so that the student discovers the theory by himself.

There is only one "entry level" question per paragraph; but there can be more than one theory box in one paragraph. In general the SaLVO paragraphs are shorter than the Numbers & Space paragraphs.

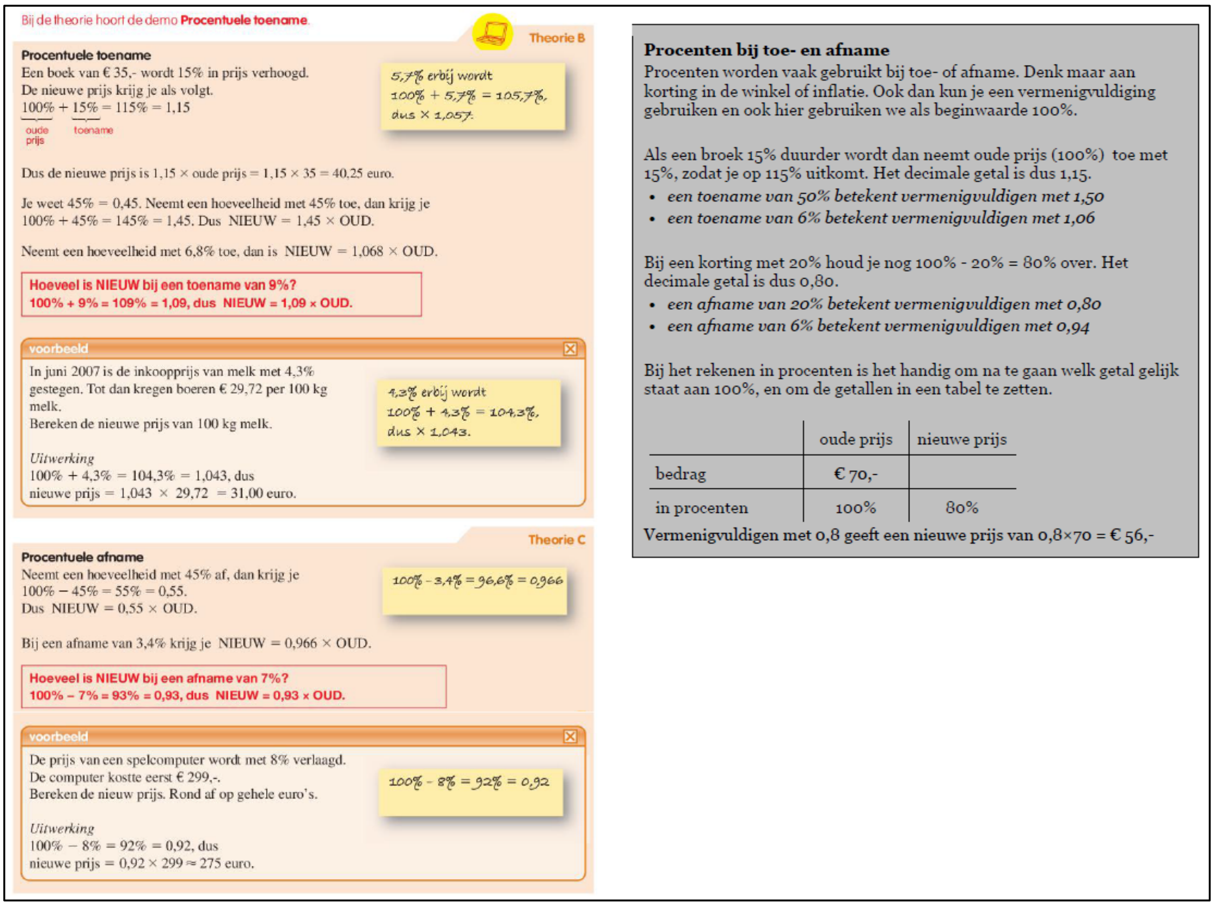
The SaLVO exercises in the end of each paragraph are not labeled as "closing" exercises, but they are more difficult than the ones in the beginning. What differs with the Numbers & Space exercises is that SaLVO also has experiment-exercises. See the assumption *learning by experiencing* in chapter 3. After all paragraphs with theory there are no paragraphs with extra exercises to practice and no "Summary" of all theory is given.

Explaining the theory

Both materials have boxes that explain the theory. In the Numbers & Space material there are more of these boxes. The SaLVO material has less theory boxes since the students only need to do calculations in the ratio table. Once they have learned how to do calculations in this table they can also do calculations on percentages and on k^2 and k^3.

The Numbers & Space theory boxes are more extensive than the SaLVO boxes. The Numbers & Space boxes are supplemented with an example on how to do the calculations. Only one theory box in the SaLVO material contains a calculation example. See the comparison in Figure 4. Numbers & Space uses two different theory boxes for percentage increase and percentage decrease. In SaLVO this is discussed in one theory box. Both the Numbers & Space as the SaLVO box explain that for example $115\% = 1.15$. The Numbers & Space adds the calculation 1.15 times the price of the book and states this in a general formula: $NEW = \ldots\ldots \times OLD$.

This is the case for all theory boxes. The Numbers & Space boxes display the theory with numerical examples and then give an example on how to solve a problem that the student will encounter in the exercises. Also emphasis is placed on the general rule or formula by displaying it in bold red text in a red framing.



Bij de theorie hoort de demo **Procentuele toename**

Theorie B

Procentuele toename

Een boek van € 35,- wordt 15% in prijs verhoogd.
De nieuwe prijs krijg je als volgt.
100% + 15% = 115% = 1,15
oude prijs toename

5,7% erbij wordt 100% + 5,7% = 105,7%, dus × 1,057.

Dus de nieuwe prijs is 1,15 × oude prijs = 1,15 × 35 = 40,25 euro.

Je weet 45% = 0,45. Neemt een hoeveelheid met 45% toe, dan krijg je 100% + 45% = 145% = 1,45. Dus NIEUW = 1,45 × OUD.

Neemt een hoeveelheid met 6,8% toe, dan is NIEUW = 1,068 × OUD.

Hoeveel is NIEUW bij een toename van 9%?
100% + 9% = 109% = 1,09, dus NIEUW = 1,09 × OUD.

voorbeeld

In juni 2007 is de inkoopprijs van melk met 4,3% gestegen. Tot dan kregen boeren € 29,72 per 100 kg melk.
Bereken de nieuwe prijs van 100 kg melk.

4,3% erbij wordt 100% + 4,3% = 104,3%, dus × 1,043.

Uitwerking
100% + 4,3% = 104,3% = 1,043, dus
nieuwe prijs = 1,043 × 29,72 = 31,00 euro.

Theorie C

Procentuele afname

Neemt een hoeveelheid met 45% af, dan krijg je
100% − 45% = 55% = 0,55.
Dus NIEUW = 0,55 × OUD.

100% − 3,4% = 96,6% = 0,966

Bij een afname van 3,4% krijg je NIEUW = 0,966 × OUD.

Hoeveel is NIEUW bij een afname van 7%?
100% − 7% = 93% = 0,93, dus NIEUW = 0,93 × OUD.

voorbeeld

De prijs van een spelcomputer wordt met 8% verlaagd.
De computer kostte eerst € 299,-.
Bereken de nieuw prijs. Rond af op gehele euro's.

100% − 8% = 92% = 0,92

Uitwerking
100% − 8% = 92% = 0,92, dus
nieuwe prijs = 0,92 × 299 ≈ 275 euro.

Procenten bij toe- en afname

Procenten worden vaak gebruikt bij toe- of afname. Denk maar aan korting in de winkel of inflatie. Ook dan kun je een vermenigvuldiging gebruiken en ook hier gebruiken we als beginwaarde 100%.

Als een broek 15% duurder wordt dan neemt oude prijs (100%) toe met 15%, zodat je op 115% uitkomt. Het decimale getal is dus 1,15.

- *een toename van 50% betekent vermenigvuldigen met 1,50*
- *een toename van 6% betekent vermenigvuldigen met 1,06*

Bij een korting met 20% houd je nog 100% - 20% = 80% over. Het decimale getal is dus 0,80.

- *een afname van 20% betekent vermenigvuldigen met 0,80*
- *een afname van 6% betekent vermenigvuldigen met 0,94*

Bij het rekenen in procenten is het handig om na te gaan welk getal gelijk staat aan 100%, en om de getallen in een tabel te zetten.

	oude prijs	nieuwe prijs
bedrag	€ 70,-	
in procenten	100%	80%

Vermenigvuldigen met 0,8 geeft een nieuwe prijs van 0,8×70 = € 56,-

Figure 4 – Theory boxes on percentage increase and decrease in Numbers & Space (left) and SaLVO (right)

4.2. ASSESSING THE LEARNING GOALS: DEVELOPING A SURVEY

These learning goals will be assessed with a survey test, which is based on already existing tests for the selected SaLVO booklet and the corresponding parts of the regular textbook. Tried was to make a well-balanced test, which assesses as many learning goals as possible. An overview of this analysis can be found in Appendix D. The test is also based on literature about testing (see next subparagraph on RTTI) and on the literature about ratio and proportionally.

In paragraph 2.5.2 three types of ratio and proportionality problems were discussed: comparing two parts of a single whole, comparing quantities of the same nature and comparing quantities of different natures. Since there was found that the first type did not elicit any proportional reasoning, no questions of this type were included in the survey. The other questions are of type 2 and 3, see Appendix E. As type of assessment, almost all questions are of type 1: missing value problems. The students need to calculate one missing value (the enlargement factor, the percentage change, etc.) and can do so with a memorized skill. Two questions are of the estimation type; here students need to compare information to make a statement. For this type students need to understand the meaning of proportion. All survey questions are word problems (in contrast to symbolic problems).

The systematics used for developing the test is the Dutch system RTTI[15] which is comparable to Bloom's taxonomy. This is a way of distinguishing different levels of thinking skills assessed in

[15] Dutch abbreviation for *Reproductie, Toepassen1, Toepassen2, Inzicht*.

questions. The R stands for *reproduction*: this are questions which can be answered by remembering. The first T stands for *applying in familiar contexts*: this are questions where the student should apply his knowledge in comparable situations as the ones he practiced with in the lessons. The second T stands for *applying in new contexts*: this are similar questions to T1 questions, only this are unfamiliar contexts for the students. The I stands for *insight*: this are questions where the student himself should construct a problem solving method that fits the given context.

Drost & Verra (2013) have suggested guidelines for composing a test: at least 5% of the points should be earned with R-questions; at least 5% with I-questions. Furthermore, R-questions and T1-questions together should cover at least 30%; and T2-questions and I-questions should also cover at least 30%. Our survey has 20% R-questions, 48% T1-questions, 20% T2-questions and 12% I-questions and thus meets the criteria above. See Appendix F for a detailed analysis of all survey questions.

The test is repeatedly checked by the two teachers who participated in this research, on appropriateness to assessing the learning goals in the SaLVO booklet and the regular textbook and on appropriateness for the chosen target audience (textual level). This final survey can be found in Appendix B.

A score form was developed which shows how many points the students earns for their answers. Regardless of the solving strategy used, an equal amount of points could be gained by the SaLVO and Numbers & Space students. All points added up results in a final test score per student. For convenience there were made two versions of the score form since there is a difference in strategy, this helps classifying the answers the students gave in SaLVO pedagogy / Numbers & Space pedagogy / other pedagogy. This classification and score system will be used to credit possible differences in final test scores to the chosen pedagogy for answering the questions. The two versions of the score can be found in Appendix C.

4.3. HYPOTHESES

The SaLVO pedagogy matches some of the pedagogical *considerations* (see paragraph 2.5.4) that were found to improve rational thinking. The SaLVO material matches these considerations:

- The SaLVO material starts with scaling recipes which should lead to a strong understanding of proportional reasoning.
- The students gain experience with several different calculation methods in the “entry level” questions which also improves the understanding.
- The material contains a range of question types, in meaningful and realistic contexts.
- Due to the structure the ratio table offers the students only use their calculators in the end of the solution process; they do not randomly multiply and divide numbers.

Furthermore, these *assumptions* (see chapter 3) underlying the SaLVO pedagogy may also improve rational thinking:

- Learning by experiencing: discussing problem solving strategies, measuring and doing experiments.
- Using the ratio table as main problem solving strategy
- Learning by doing group work, for example in the “entry level” questions but also in experiments.

For the Numbers & Space group the two topics (percentages & enlarging) remain separate. For the SaLVO group they form a coherent entity. We expect that the students in the SaLVO group can draw from one coherent knowledge base which makes it easier to apply the learnt skills, and we expect that there is less confusion about the meaning of concepts in this group. Given the knowledge we have now about proportionality and about the SaLVO pedagogy, we expect that the SaLVO group performs better on the survey test than the Numbers & Space group.

The main research question *"To what extend is the SaLVO pedagogy effective on cognitive level for dealing with proportionality problems?"* is broken down in two sub-questions.

The first sub-question is: *Did the SaLVO group performed better than the Numbers & Space group on answering the questions?*

Following the reasoning above, we expect the SaLVO students to perform better on the test in general, whereby this difference is expressed in the questions that ask more insight from students. This leads to hypotheses 1, 2 and 3.

H1: The SaLVO group performs better on the test than the Numbers & Space group.

H2: The SaLVO group and the Numbers & Space group perform equally on the *Reproduction* and *Applying 1* questions.

H3: The SaLVO group performs better than the Numbers & Space group on the *Applying 2* and *Insight* questions.

Three more hypotheses are composed to better interpret possible differences in test scores: to account a difference in test score to using the SaLVO pedagogy. This is done to be able to answer the second sub-question: *Did the SaLVO group performed better due to answering the questions using the SaLVO pedagogy?* This gives us hypotheses 4, 5 and 6.

H4: A SaLVO approach leads to a higher score than a Numbers & Space approach.

H5: A SaLVO approach leads to an equally high score as a Numbers & Space approach on *Reproduction* and *Applying 1* questions.

H6: A SaLVO approach leads to a higher score than a Numbers & Space approach on *Applying 2* and *Insight* questions.

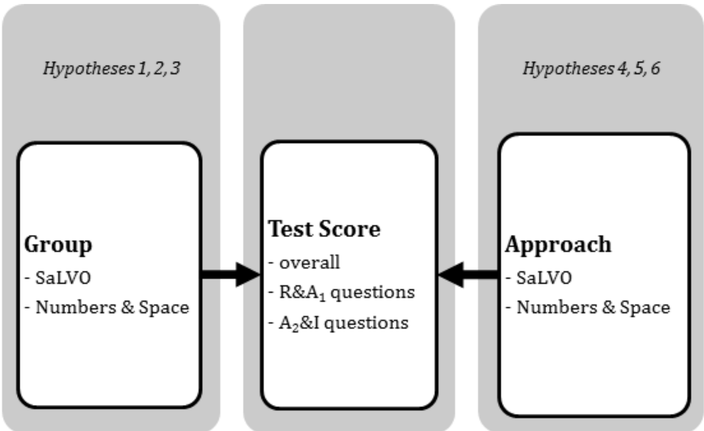


Figure 5 – Schematic overview of the variables used in the hypotheses

5. METHOD

This chapter describes phase 3 of this research: data gathering in the intervention period and the way this data is analyzed.

5.1. DATA GATHERING

Selecting participants

Data was gathered on the *Sint Bonifatiuscollege* in Utrecht, The Netherlands. Besides practical reasons, this school was selected because of its involvement in developing the SaLVO material. It is a Roman Catholic comprehensive school with approximately 1500 students in senior general secondary education *(havo)* and pre-university education *(vwo)*. The school is located in an urban area: 95% of all students live in the city Utrecht (VO-raad, 2015).

The intervention took place in two 8th grade pre-university level classes; one using the SaLVO material and one using the Numbers & Space material. From now on these classes are referred to as the SaLVO group and the N&S group.

First the two teachers were chosen to participate in this study. There is explicitly chosen for one teacher per group, since the risk exists with one teacher for both groups that the pedagogies get mixed. Now each teacher could focus on her ascribed pedagogy. Both teachers are female, and have comparable experience (20+ years) in teaching the subject matter. The teacher teaching the SaLVO group has affinity with this pedagogy: she partly developed the material and has used it for several years. The N&S teacher has worked with the Numbers & Space textbooks for many years and therefore knows this pedagogy well.

Both teachers had already one 8th grade class assigned to them by the school timetable. These classes were chosen for this study. In total 56 students aged 12 – 14 years participated in this study. The SalVO group had 30 students (17 boys and 13 girls), the N&S group had 26 students (13 boys and 13 girls). The N&S group was a little bit older (M = 13.63, SD = .35) than the SaLVO group (M = 13.50, SD = .48). There was no inducement in any form to participate.

The intervention period

In the intervention period from 16th September – 14th October the students followed 9 lessons of 50 minutes and 2 lessons of 40 minutes[16]. Both groups had their lessons in the late morning/early afternoon on three consecutive days. The SaLVO group had class on Wednesday 3rd hour, Thursday 6th hour and Friday 5th hour. The N&S group had class on Monday 7th, Tuesday 6th and Wednesday 5th hour.

The students knew they were participating in an experiment. They were told that the other group had another way of teaching and that the researcher was in the classroom to make observations and to eventually look at their tests. The SaLVO group received the (modified[17]) SaLVO booklets in which they worked the entire intervention period. They did not know which paragraphs from their Numbers & Space textbook matched these booklets and thus could not use the textbook for their homework problems. With other words: the SaLVO group had to draw on SaLVO pedagogies to solve the homework problems.

[16] Due to a shortened timetable *(verkort lesrooster)*.

[17] See section 4.1.1 for a rationale on which parts to include/exclude.

Both teachers are instructed to stick to their pedagogy as much as possible. If students came up with other problem solving solutions they would acknowledge these but when explaining in front of the classroom they only worked with their specific pedagogies.

The researcher observed the lessons to ensure that the teachers stick to their pedagogies. Also the researcher timed how long each teacher took the time to explain the concepts to the students; this instruction time included discussing homework. This is done to ensure that possible results cannot be ascribed to large differences in teaching style, since Confrey (1995) found that the length of instruction on proportionality was related to the success rate (as cited in Watson et al., 2013). These field notes can be found in Appendix G. These field notes show that both teachers used their designated pedagogies. See for some examples the notes students made during class.

The intervention period ended with both groups doing the same test, the survey developed for this study. See Appendix B for the survey and Appendix C for the two versions of the score form. This resulted in two types of data for each student: a score per question (varying from zero to the maximum amount of points for that question) and a classification for their answer (SaLVO / Numbers & Space / non-discriminating or other pedagogy). To illustrate, the dataset looked like Table 3.

Table 3 – Illustrating the dataset

Student	Score01	Type01	Score02	Type02	... etc.
01	1	SaLVO	0	Numbers & Space	
02	0	Non-discr.	1	Numbers & Space	

5.2. ANALYZING THE DATA

Inter-rater reliability

To ensure inter-rater reliability three coders independently rated four randomly chosen surveys with each 13 items, two from the SaLVO group and two from the N&S group. The raters coded each question with one of the following five codes: *SaLVO*, *SaLVO?*, *non-discriminating*, *N&S?* and *N&S*. Since this is nominal data and there were more than two raters involved a generalized version of Cohen's Kappa was used to analyze the inter-rater agreement (Geertzen, 2012). Three raters coding $4 \times 13 = 52$ items resulted in a high agreement, *Fleiss' Kappa = .883*, which is above the threshold.

Comparability of the two groups

To ensure that the SaLVO group and the N&S group can be compared three factors were taken into account. The first factor is the heterogeneity of the two groups. If the groups are not heterogeneous, a difference found in test scores could possibly be due to the previous year (teacher) and the previous knowledge of the student. *LastYearGroup* is a nominal variable with seven possible outcomes. This variable is explored using a histogram which can be found in Appendix H.

The second factor is the groups starting level. As an estimator of this level, last year's average grade of each student is used. *LastYearGrade* is an interval variable. *LastYearGrade* satisfies the normality condition for both groups. The groups' means are compared using an independent *t*-test. On average, last year's grade in the SaLVO group (M = 7.71, SE = 0.73) did not differ significantly from the N&S group (M = 7.37, SE = 0.83), $t(52) = -1.61$, $p > 0.10$.

As a third factor the test score on the first three questions is compared: *Score123*. There is no difference in answering method for these three questions, so the groups should score equally on these questions. The test score on the first three questions is an interval variable. However, *Score123* does not meet the assumptions for a parametric test. The test score on the first three items is significantly non-normal for both the SaLVO group, $D(29) = .19$, $p < .05$, and the N&S group, $D(25) = .31$, $p < .001$. Furthermore, the variances in the two groups were significantly different, $F(1, 52) = 5.70$, $p < .021$, this made the Mann-Whitney test unsuitable. For the analysis the Kolmogorov-Smirnov test is used since it has no assumption on the distribution of the data. This test compares the cumulative distribution functions for the two populations. The Kolmogorov-Smirnov test shows no significant difference between the two distributions of the test score on the first three items (*K-S* = .738, *ns*).

Comparing the two groups

To answer hypotheses 1, 2 and 3 the overall test scores are compared. The test score is computed by adding up all scores for each questions. This results in a number between 0 and 30. *TestScore* is an interval variable. The test scores for the *Reproduction* & *Applying1* questions (*TestScoreRA1)* are computed by adding up all scores for these types of questions. The same is done for the *Applying2 & Insight* questions, which results in *TestScoreA2I*.

TestScore meets all assumptions for a parametric test. To test hypothesis 1 an independent *t*-test is used. *TestScoreRA1* and *TestScoreA2I* are not normally distributed. However, the variances in the two groups are not significantly different for *TestScoreRA1* ($F(1, 52) = .271$, *ns*) and for *TestScoreA2I* ($F(1, 52) = 1.942$, *ns*). Thus for testing hypotheses 2 and 3 the Mann-Whitney 2 independent samples test is used.

Comparing the two approaches

For hypotheses 4, 5 and 6 the analysis is more complicated. The variable *Approach* is not independent since the students choose which method to use. Since they can switch at every question, the number of students using a specific method differs per question. It is impossible to predict how large the N-values are for each question. Even if the numbers stay constant, it is very likely not the same group of students. Hypotheses 4, 5 and 6 will thus be analyzed on question-level with the most suitable option: a Chi-Square test.

This test requires the conversion of the dependent variables *ScoreXX*[18] into binary variables. Chosen was to recode zero points into 0 and one or more points into 1. This way all variables were recoded the same way. The Chi-Square test has the underlying assumption that in 2×2 tables all expected frequencies should be greater than 5.

Other tests were excluded due to two statistical reasons. The dependent variables were all not normally distributed because of the little variation in possible scores (0, 1, 2 or 3 points). Since there is so little variation (a lot of students with the same score), a rank sum test like the Mann-Whitney U would make no sense either.

[18] XX being the question number, e.g. *Score01* for the points scored on question 1.

6. Results

6.1. The effect of 'Group' on performance

Hypothesis 1 is not supported. There is no statistical mean difference in test score between the SaLVO group and the Numbers & Space group, see Table 4. The SaLVO group did not perform better than the Numbers & Space group.

Table 4
Results of t-test and Descriptive Statistics for overall test score

	Group						95% CI for Mean Difference		
	SaLVO			Numbers & Space					
	M	SD	n	M	SD	n		t	df
Test score	13.03	3.448	25	13.60	2.958	29	-1.204, 2.335	.641	52

Hypothesis 2 is supported. There is no statistical difference in test scores on the Reproduction and Applying 1 questions between the SaLVO group and the Numbers & Space group, see Table 5. The SaLVO group and the Numbers & Space group performed equally on these questions.

Hypothesis 3 is not supported. There is no statistical difference in test scores on the Applying 2 and Insight questions between the SaLVO group and the Numbers & Space group. The SaLVO group did not perform better on these questions, in fact there is a small sized effect in favor of the Numbers & Space group.

Table 5
Results of Mann-Whitney U test and Descriptive Statistics for Test Score on RA1 and A2I questions

	Group								
	SaLVO			Numbers & Space					
	Mdn	Mean Rank	n	Mdn	Mean Rank	n	U	Z	r
Test score Reproduction&Applying1	11.0	26.50	25	12.0	28.66	29	333.5	-.507	-.07
Test score Applying 2 & Insight	2.0	26.02	25	2.0	29.22	29	319.5	-.764	-.10

6.2. The effect of 'Approach' on performance

Questions 6, 7, 9, 10, 11 and 13 will be taken into account for the analysis on question-level. On these questions more than 40% of the students used the approach from the pedagogy ascribed to them. On some questions the SaLVO students did not use the SaLVO approach. Questions 4, 5, 8 and 12 are excluded from this analysis for this reason. Question 1, 2 and 3 are excluded since no difference in answering approach was possible. Detailed information on the distribution of the chosen approaches per group can be found in Appendix I.

To give the reader an idea of how the approaches effected the performance on the test, Table 6 shows the mean, standard deviation and N[19] for each question. We can already see that these averages do not differ that much. To see these numbers visually and in comparison with the 'non-discriminating' approach, see Appendix J.

Table 6
Descriptive statistics for several questions

	Approach						RTTI type
	SaLVO			Numbers & Space			
	M	SD	n	M	SD	n	
Question 6	1.85	.464	26	1.74	.619	23	*T1*
Question 7	1.63	.565	27	1.50	.590	24	*T1*
Question 9	.92	.277	13	.86	.356	28	*R*
Question 10	.85	.376	13	.93	.254	30	*R*
Question 11	.47	.514	17	.43	.504	28	*R*
Question 13	.93	.475	14	1.13	.743	15	*I*

It turned out that for all questions the expected frequencies are too low to perform the Chi-Square test, except for question 11. There was no significant association between the type of approach and whether or not the students would gain points on question 11, $\chi^2(1, N=45) = .076$, *ns*. For this question, the odds of gaining points after using the SaLVO approach were 1.19 times higher than after using the Numbers & Space approach.

Hypotheses 4, 5, and 6 are not supported for lack of statistical evidence.

Two remarks on the numbers in Appendix J. When looking at the approaches used by students, we see that the SaLVO group only used the SaLVO approach on questions about percentages. For the three questions (4, 5, 8) on enlarging surfaces and volumes only in 6 out of 87 answers (7%) from the SaLVO group the SaLVO approach was found. For the questions on percentages (6, 7, 9, 10, 11, 12, 13) the SaLVO approach was found in 109 out of 203 answers (54%) from the SaLVO group. We do not see a clear distinction when looking at the RTTI-level of the questions. The SaLVO approach is used for all levels of questions.

[19] The N-values do not add up to $25 + 29 = 54$ since the 'non-discriminating' approach is left out of consideration.

7. CONCLUSION

There was some evidence for the claim that the SaLVO material offers a solution for the conceptual and transfer problems students encounter on the topic of proportionality: teachers claimed they saw improvements in students' work. This study presents a theoretical background which also gives rise to the claim that students would benefit from the SaLVO material.

However this claim was never empirically tested. This study is testing this claim by an intervention period in which there were two groups. One group worked with the SaLVO material and one group worked with the Numbers & Spaces books. After several lessons on the topic proportionality both groups of students did the same test. This test was analysed on possible differences in scores.

The aim of this study is measuring the effectiveness of the SaLVO pedagogy. The main research question was: *To what extend is the SaLVO pedagogy effective on cognitive level for dealing with proportionality problems?* This question was split into two sub-questions.

1) *Did the SaLVO group performed better than the Numbers & Space group on answering the questions?*
2) *Did the SaLVO group performed better due to answering the questions using the SaLVO pedagogy?*

The results of this study show that there is no significant difference in the performance of both groups. There is no difference when looking at the overall score and there is no difference when looking at reproductive questions only and at insightful questions only. For the analysis of the approaches used by students (in contrast to in which group they were placed) the dataset was too small.

The SaLVO pedagogy is equally effective on cognitive level as the Numbers & Space textbook used on schools for students dealing with proportionality problems.

The results do however show that students do not use the SaLVO approach for questions on enlarging volumes and surfaces: they only use it on problems with percentages. Can we even draw conclusions if the SaLVO pedagogy is not used consistently?

8. Discussion

In this chapter three topics will be covered: 1) the SaLVO approach not being applied, 2) the research setting (survey test) and 3) suggestions for further research.

8.1. Not using the SaLVO approach

The results showed that the majority of the SaLVO students did not use the SaLVO approach on their tests (especially on the questions on the enlargement factor). This may partially be attributed to the design of the survey test (see paragraph 8.2), but mostly to the teaching.

Appendix G shows that the SaLVO teacher gave instructions to the students following the SaLVO booklets conscientiously. The teacher always used a ratio table to explain and to answer the questions on percentages, as this is done in the booklet as well. The students got two notes with exercises to practice where they should fill in ratio tables. See Figure 6 below for an example.

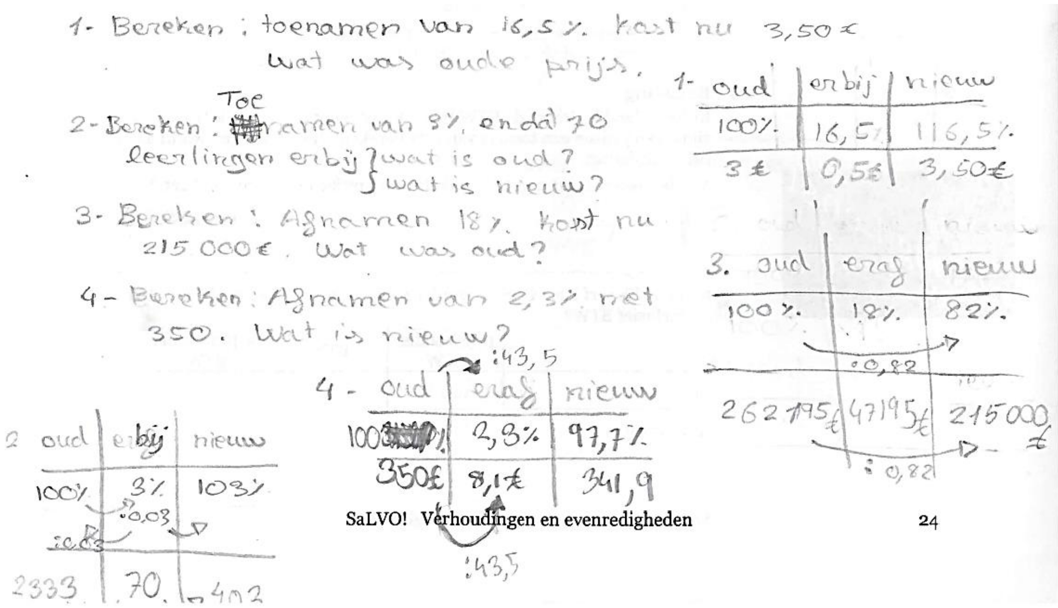


Figure 6 – Student work, calculating with ratio tables

Booklet 3 is not that strict anymore on using the ratio table. The ratio table is only twice shown as a way to present the data and to do the calculations. For the other exercises, the students can choose their own methods. Figure 7 shows that the students did not use the SaLVO approach in their exercises; and logically also not on their tests. Doing only the calculations and not writing down the ratio table was scored as 'Numbers & Space pedagogy' in the score form. The researcher had not seen the student work when making the score form and when scoring the tests, since the booklets were not available to the researcher at that time.

The teacher did followed the *group work* and *learning by experiencing* assumptions of the SaLVO pedagogy when teaching about enlargement factors. When explaining k^2 she brought DUPLO building blocks to school so the students would 'see' the calculation rule. One of the other lessons the students did an experiment about k^3 for enlarging surfaces. There was too little time left in class to discuss how to write down your answers for enlargement problems.

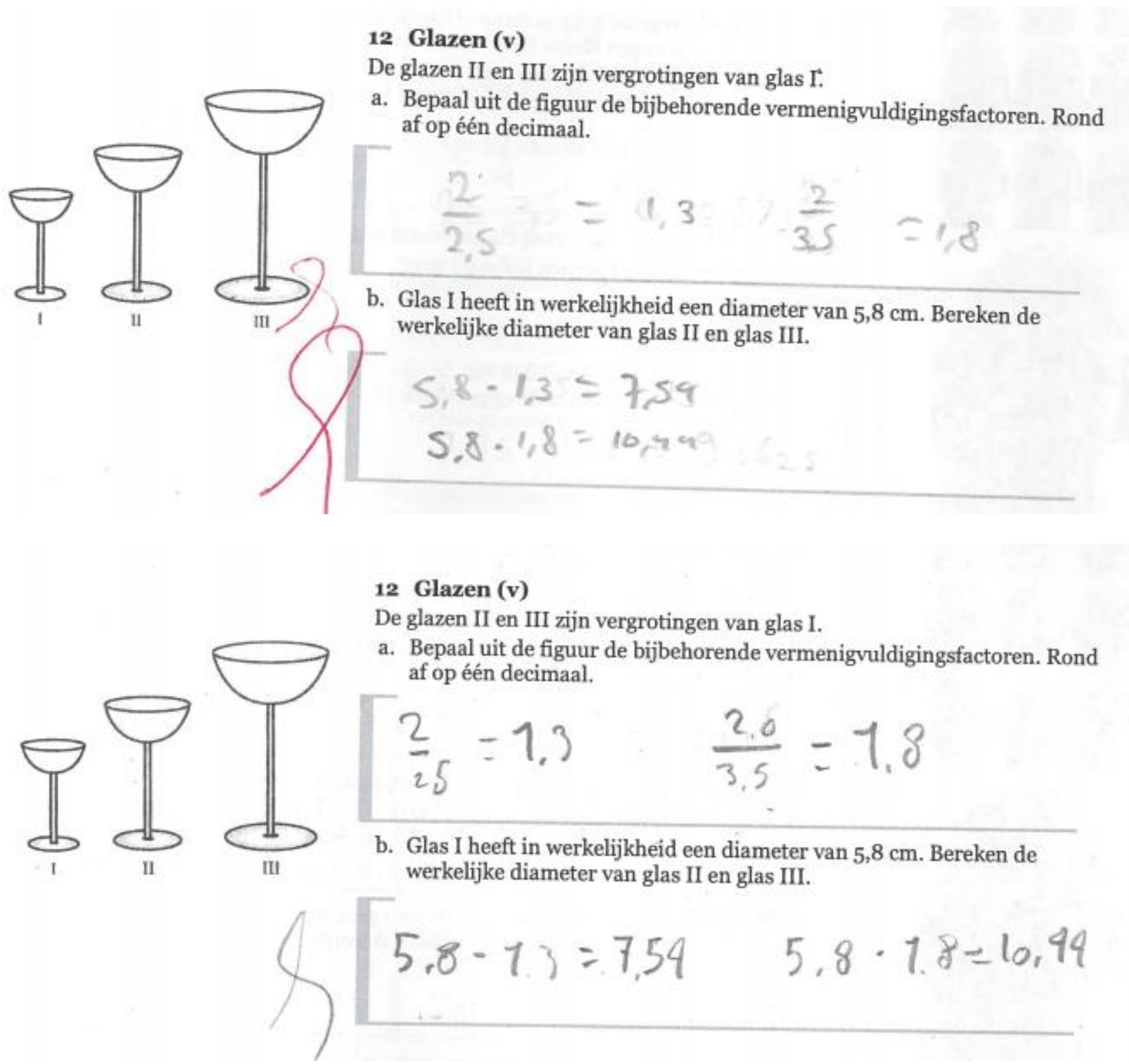


Figure 7 – Work of two students on exercise 12 of the 3rd SaLVO booklet

That the students did not use the SaLVO approach the way it was labeled and scored in the score form caused that no conclusions could have been draw in this study. The main suggestion arising from this research is thus for the booklet and the teacher to be more strict on using the ratio table, especially for the enlargement problems. The booklet would have to be rewritten and future SaLVO teachers would have to be informed better about consistent use of the ratio table. Another option would be to give a (formative) test to the SaLVO group in between their lesson series to check whether they properly apply the SaLVO approach.

8.2. THE RESEARCH SETTING: LIMITATIONS

Besides the non-using of the SaLVO approach, there are also some limitations on the setting of this research. For starters, the survey test has not been tested on (other) students before the final examination in this study. This should have been done one year before the final examination to the previous group of 8th graders. This has not been done due to time-limitations. On the one hand this testing would have made the survey test better since minor ambiguities may have come to light; on the other hand no one of the students had any questions on the questioning in the survey.

Moreover the survey test has been checked several times by the researcher and the two senior teachers participating in this research.

The survey test was not sufficient in discriminating between the two approaches. The test took place in the start of the second year of pre-university education, relatively early in the students school career. This is one of the reasons that many students used strategies they learnt in primary school for answering the questions on the test. Another reason is that the subject and the test gave the students the opportunity to do so, since doing calculations on percentages is a topic that is also covered in the elementary school curriculum.

The learning goals were well distributed among the different questions in the test. Also there were different levels of thinking represented by the questions. The fact that the Numbers & Space students scored a lot higher on question 12 (see Appendix B) may have been that they recognized the context of this question more than the SaLVO students did. In both pedagogies there was a question on calculating a percentage of a percentage, but in the Numbers & Space booklet the context was a company that made profit as in the SaLVO booklet the context was on a photocopying machine.

8.3. SUGGESTIONS FOR FURTHER RESEARCH

An alteration that could have been made in this research setting was the **timeframe**. Now there only was a short period of time between the lessons and the test. The last lessons the SaLVO students had was on a Friday with the test following the next Wednesday. For the Numbers & Space group this was a Tuesday and that same Wednesday. The hypothesis being that SaLVO students perform better on the insight questions, you would expect them to perform better than the Numbers & Space students after a certain amount of time passed. An underlying hypothesis is then that the SaLVO pedagogy sinks in *(beklijft)* better.

Another alteration could be to test only after several SaLVO booklets and not after two; since the benefits of using the same method may only become visible after a longer time period in which students worked with the method in several courses. This way the **coherency** assumption is also taken into account.

If this research was repeated more students (**bigger dataset**) should participate in the study to make sure that analyzing on question-level is possible.

Since the claims teachers made about improvements in students' work were stories about **students in 10th grade** on the senior general secondary education level *(4 havo)* and not on 8th grade pre-university grade level *(2 vwo)*; it would be better to repeat this research on this group of students. In this research setting it was not possible due to practical reasons to do research on older students, since the 10th grade students did not used the SaLVO booklets in 8th and 9th grade.

This study has yielded on theoretical level: an overview of possible strategies for proportional problems (paragraph 2.5.3) and a description of the SaLVO pedagogy by means of five underlying assumptions (chapter 3). On practical level: this study was the first to empirically test the claim that the SaLVO pedagogy was more effective for proportionality problems. This study has not found statistical differences in effectiveness. However it can be used for further research as a theoretical and practical starting point implementing the suggestions described above.

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APPENDICES

A. LIST OF LEARNING GOALS

After working through the SaLVO booklets or through the Numbers & Space book the students are able to:

1) Doing calculations on ratios (e.g. a recipe with several ingredients)
2) Calculate the enlargement factor k by
- **a)** Dividing two given or measured lengths/widths/heights/numbers
- **b)** Converting a given scale and vice versa
- **c)** Converting a given growth percentage
- **d)** Taking the root of a given or calculated k^2

3) Calculate the squared enlargement factor k^2 by
- **a)** Squaring a given or calculated enlargement factor k
- **b)** Dividing two given or calculated surfaces

4) Calculate the cubed enlargement factor k^3 by
- **a)** Cubing a given or calculated enlargement factor k
- **b)** Dividing two given or calculated volumes

5) Calculate the asked length/width/height by using k (and knowing that k has to be used)
6) Calculate the asked surface by using k^2 (and knowing that k^2 has to be used)
7) Calculate the asked volume by using k^3 (and knowing that k^3 has to be used)
8) Draw an enlargement of a given image using
- **a)** A given enlargement factor k
- **b)** A given percentage

9) Write a percentage as
- **a)** a decimal number
- **b)** an enlargement factor

10) Calculations with percentages
- **a)** Calculate x% of y (e.g. what is 24% of 336?)
- **b)** Calculate how many percent x is out of y (e.g. how many % is 24 of 500?)
- **c)** Calculate the 'new value' y with a given percentage change from x (both percentage increase as decrease)
- **d)** Calculate the percentage change from x to y (both increase/decrease)

B. SURVEY (TEST)

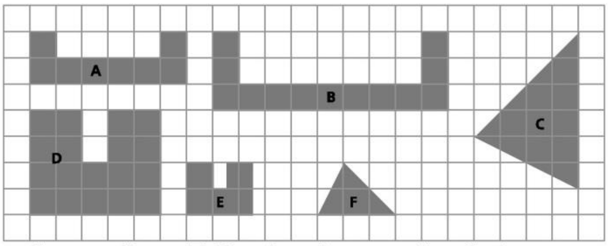
Repetitie 2 vwo H4.1, 4.2 en H8.3, 8.4 en 8.5

- Geef altijd een berekening.
- Rond indien nodig je antwoord af op 2 decimalen.

Je kunt voor deze toets 25 punten halen. Veel succes!

Vergroten (4 punten)

Hiernaast zie je zes figuren.



1p. **1.** Figuur C is een vergroting van figuur F. Bereken de vergrotingsfactor.

1p. **2.** Figuur E is een verkleining van figuur D. Bereken de vergrotingsfactor.

2p. **3.** Is figuur B een vergroting van figuur A? Zo nee, leg uit. Zo ja, bereken de vergrotingsfactor.

Russische babushka poppen (5 punten)

Hiernaast zie je drie gelijkvormige babushka poppen.

Babushka **2** is 12 cm hoog en babushka **3** is 30 cm hoog.

3p. **4.** De inhoud van babushka **2** is 0,8 liter. Bereken de inhoud van babushka **3**.

2p. **5.** De vergrotingsfactor van babushka **1** naar babushka **2** is 1,4. De oppervlakte van de onderkant van babushka **1** is 14,5 cm². Bereken de oppervlakte van de onderkant van babushka **2**.

Vliegtickets (4 punten)

Een vliegreis maken in de vakantieperiode is duur. Een reis naar Canada kost in november €500. Dezelfde reis kost in de kerstvakantie €705.

2p. **6.** Hoeveel procent bedraagt de prijsverhoging?

Twee weken na de kerstvakantie kost de reis van €705 nog maar €606,30.

2p. **7.** Hoeveel procent bedraagt de korting?

DE TOETS GAAT VERDER OP DE ACHTERZIJDE

Tuinontwerp op schaal (4 punten)

Hiernaast zie je een ontwerp van een tuin getekend.

De schaal is helaas onleesbaar geworden door de modderhanden van de tuinman.

In de tuin zie je een cirkelvormige vijver. De oppervlakte van de vijver op de plattegrond is 7 cm². De oppervlakte van de echte vijver is 15,75 m².

4p. **8.** Bereken de schaal van het ontwerp, schrijf je antwoord in de vorm 1: ... .

CD verzamelaar (3 punten)

Johan, een liefhebber van muziek, luistert alleen nog maar muziek via Spotify. Hij besluit daarom zijn collectie van 480 CD's te verkopen via een winkel. Alle CD's samen worden voor 2500 euro verkocht.

Johan heeft de CD's een paar jaar geleden op een beurs voor verzamelaars gekocht voor 20% meer.

1p. **9.** Hoeveel had Johan destijds betaald voor zijn collectie CD's?

De winkeleigenaar houdt zelf 15% van het verkoopbedrag van 2500 euro.

1p. **10.** Bereken hoeveel de winkeleigenaar krijgt.

De nieuwe eigenaar van de CD's is niet zo blij met zijn aankoop. Er blijken 54 CD's beschadigd te zijn.

1p. **11.** Hoeveel procent van de collectie CD's is beschadigd?

Omzet (5 punten)

In 2011 ging het goed met de PLUS aan de A. van Ostadelaan. Twee jaar achter elkaar nam de omzet met 10% per jaar toe. Door de komst van de Jumbo nam de omzet het jaar erop in één keer met 20% af.

3p. **12.** Bereken of leg uit met hoeveel procent de omzet van de PLUS is gestegen/gedaald/gelijk gebleven na 3 jaar.

In 2014 bedroeg de omzet van deze PLUS 3,4 miljoen euro. Dit was 26% van de totale omzet van alle PLUS filialen in de hele stad Utrecht.

2p. **13.** Bereken de omzet van 2014 van alle PLUS filialen uit Utrecht samen.

EINDE TOETS

C. SCORE FORM

The first score form covers the SaLVO approach, the second one covers the Numbers & Space approach. Both score forms give an equal amount of points for the same steps in the solution process.

SaLVO approach

Vergroten (4 punten)

Schrijf als decimaal getal

1p. **1.** Figuur C is een vergroting van figuur F. Bereken de vergrotingsfactor.

$k = \frac{6}{3} = 2$

1p. **2.** Figuur E is een verkleining van figuur D. Bereken de vergrotingsfactor.

$$k = \frac{linker\ zijkant\ E}{linker\ zijkant\ D} = \frac{2}{4} = \frac{1}{2}$$

2p. **3.** Is figuur B een vergroting van figuur A? Zo nee, leg uit. Zo ja, bereken de vergrotingsfactor.

Nee, want de zijkant (of onderkant) is vergroot met factor $k = \frac{3}{2} = 1{,}5$, maar de dikte van de uitsteeksels (of de ruimte er tussen) niet.

Russische babushka poppen (5 punten)

Hiernaast zie je drie gelijkvormige babushka poppen.

Babushka **2** is 12 cm hoog en babushka **3** is 30 cm hoog.

3p. **4.** De inhoud van babushka **2** is 0,8 liter. Bereken de inhoud van babushka **3**.

	Babushka 2	**Babushka 3**
Hoogte (cm)	12	30
Factor k	$k = 1$	$k = 2{,}5$
Inhoud (l)	0,8	12,5

-1p. bij vergeten eenheden in de tabel

2p. **5.** De vergrotingsfactor van babushka **1** naar babushka **2** is 1,4. De oppervlakte van de onderkant van babushka **1** is 14,5 cm².

Bereken de oppervlakte van de onderkant van babushka **2**.

	Babushka 1	**Babushka 2**
Factor k	$k = 1$	$k = 1{,}4$
Oppervlakte (cm²)	$14{,}5$	$28{,}42$

-1p. bij vergeten eenheid 'cm²'

Vliegtickets (4 punten)

Een vliegreis maken in de vakantieperiode is duur. Een reis naar Canada kost in november €500. Dezelfde reis kost in de kerstvakantie €705.

2p. **6.** Hoeveel procent bedraagt de prijsverhoging?

	Oude prijs	Nieuwe prijs
in procenten	100%	141%
Bedrag in euro	500	705

De prijsverhoging is 41%

Twee weken na de kerstvakantie kost de reis van €705 nog maar €606,30.

2p. **7.** Hoeveel procent bedraagt de korting?

	Oude prijs	Nieuwe prijs
in procenten	100%	86%
Bedrag in euro	705	$606{,}30$

De korting is $100 - 86 = 14\%$

Tuinontwerp op schaal (4 punten

Hiernaast zie je een ontwerp van een tuin getekend.
De schaal is helaas onleesbaar geworden door de modderhanden van de tuinman.

In de tuin zie je een cirkelvormige vijver. De oppervlakte van de vijver op de plattegrond is 7 cm². De oppervlakte van de echte vijver is 15,75 m².

4p. **8.** Bereken de schaal van het ontwerp, en schrijf je antwoord in de vorm: de schaal is 1: ... .

	Ontwerp	**Werkelijkheid**
Oppervlakte (cm^2)	7	157500
Factor k^2		$k^2 = 22500$
Factor k		$k = 150$

De schaal is $1 : 150$ (1p.)

CD verzamelaar (3 punten)

Johan, een liefhebber van muziek, luistert alleen nog maar muziek via Spotify. Hij besluit daarom zijn collectie van 480 CD's te verkopen via een winkel. Alle CD's samen worden voor 2500 euro verkocht.

Johan heeft de 480 CD's een paar jaar geleden op een beurs voor verzamelaars gekocht voor 20% meer.

1p. **9.** Hoeveel heeft Johan toen betaald voor zijn collectie CD's?

	Oude prijs	Nieuwe prijs
in procenten	120%	100%
Bedrag in euro	3000	250

Hij betaalde toen $1{,}20 \cdot 2500 = 3000$ euro.

De winkeleigenaar houdt zelf 15% van het verkoopbedrag van 2500 euro.

1p. **10.** Bereken hoeveel de winkeleigenaar krijgt.

	Hele bedrag	Winkeleigenaar
in procenten	100%	15%
Bedrag in euro	2500	375

De winkeleigenaar houdt $0{,}15 \cdot 2500 = 375$ euro

De nieuwe eigenaar van de CD's is niet zo blij met zijn aankoop. Het blijkt dat 54 van de CD's beschadigd is.

1p. **11.** Hoeveel procent van de collectie CD's is beschadigd?

	Alle CD's	Beschadigd
in procenten	100%	$11{,}25\%$
Bedrag in euro	480	54

Omzet (5 punten)
In 2011 ging het goed met de PLUS aan de A. van Ostadelaan. Twee jaar achter elkaar nam de omzet met 10% per jaar toe. Door de komst van de Jumbo nam de omzet het jaar erop in één keer met 20% af.

3p. **12.** Bereken of leg uit met hoeveel procent de omzet van de PLUS is gestegen/gedaald/gelijk gebleven na 3 jaar.

	Oude prijs	Nieuwe prijs	Nieuwe prijs	Nieuwe prijs
in procenten	100%	110%	121%	96,8%
Bedrag in euro				

De omzet is dus gedaald met $100 - 96{,}8 = 3{,}2\%$.

Alleen conclusie 'gedaald' zonder percentage = 1p.

OF

Uitleg + conclusie zonder percentage = 2p.

De 20% gaat van een groter bedrag af, dus dit is meer dan twee keer 10% over een kleiner bedrag. De omzet is dus gedaald na drie jaar.

OF

Berekening + conclusie = 3p.

De omzet is nog $1{,}1 \cdot 1{,}1 \cdot 0{,}8 = 0{,}968 = 96{,}8\%$ van de oude omzet
De omzet is dus gedaald met $100 - 96{,}8 = 3{,}2\%$

In 2014 bedroeg de omzet van deze PLUS 3,4 miljoen euro. Dit was 26% van de totale omzet van alle PLUS filialen in de hele stad Utrecht.

13. Bereken de omzet van 2014 van alle PLUS filialen uit Utrecht samen.

2p.

	PLUS	Alle filialen
in procenten	26%	100%
Bedrag in euro	3,4 miljoen	13,07 miljoen

Vergroten (4 punten)

Schrijf als decimaal getal

1p. **1.** Figuur C is een vergroting van figuur F. Bereken de vergrotingsfactor.

$k = \frac{6}{3} = 2$

1p. **2.** Figuur E is een verkleining van figuur D. Bereken de vergrotingsfactor.

$k = \frac{linker\ zijkant\ E}{linker\ zijkant\ D} = \frac{2}{4} = \frac{1}{2}$

2p. **3.** Is figuur B een vergroting van figuur A? Zo nee, leg uit. Zo ja, bereken de vergrotingsfactor.

Nee, want de zijkant (of onderkant) is vergroot met factor $k = \frac{3}{2} =$ 1,5 , maar de dikte van de uitsteeksels (of de ruimte er tussen) niet.

Russische babushka poppen (5 punten)

Hiernaast zie je drie gelijkvormige babushka poppen.

Babushka **2** is 12 cm hoog en babushka **3** is 30 cm hoog.

3p. **4.** De inhoud van babushka **2** is 0,8 liter.

Bereken de inhoud van babushka **3.**

De vergrotingsfactor is $k = \frac{30}{12} = 2{,}5.$ (1p)

We rekenen met inhoud dus nodig is k^3 (1p voor gebruik k^3)

De inhoud van babushka 3 is $0{,}8 \cdot 2{,}5^3 = 12{,}5$ liter (1p)

-1p. bij vergeten eenheid 'liter'

2p. **5.** De vergrotingsfactor van babushka **1** naar babushka **2** is 1,4. De oppervlakte van de onderkant van babushka **1** is 14,5 cm².

Bereken de <u>oppervlakte</u> van de onderkant van babushka **2**.

We rekenen met oppervlakte dus nodig is k^2 (1p voor gebruik k^2)

De oppervlakte is $14{,}5 \cdot 1{,}4^2 = 28{,}42$ cm² (1p)

-1p. bij vergeten eenheid 'cm²'

Vliegtickets (4 punten)

Een vliegreis maken in de vakantieperiode is duur. Een reis naar Canada kost in november €500. Dezelfde reis kost in de kerstvakantie €705.

2p. **6.** Hoeveel procent bedraagt de prijsverhoging?

De prijsverhoging is $\frac{705-500}{500} \cdot 100 = 0{,}41 \cdot 100 = 41\%$

Twee weken na de kerstvakantie kost de reis van €705 nog maar €606,30.

2p. **7.** Hoeveel procent bedraagt de korting?

De prijsverandering is $\frac{606{,}30-705}{705} \cdot 100 = -0{,}14 \cdot 100 = -14\%$

Dus de korting is 14%

Tuinontwerp op schaal (4 punten)

Hiernaast zie je een ontwerp van een tuin getekend.

De schaal is helaas onleesbaar geworden door de modderhanden van de tuinman.

In de tuin zie je een cirkelvormige vijver. De oppervlakte van de vijver op de plattegrond is 7 cm². De oppervlakte van de echte vijver is 15,75 m².

4p. **8.** Bereken de schaal van het ontwerp, en schrijf je antwoord in de vorm: de schaal is 1: ... .

De oppervlakte van de vijver is $15{,}75\ m^2 = 157500\ cm^2$ (1p.)

We weten $k^2 = \frac{157500}{7} = 22500$ (1p.)

De vergrotingsfactor is dus $k = \sqrt{22500} = 150$ (1p.)

De schaal is 1 : 150 (1p.)

CD verzamelaar (3 punten)

Johan, een liefhebber van muziek, luistert alleen nog maar muziek via Spotify. Hij besluit daarom zijn collectie van 480 CD's te verkopen via een winkel. Alle CD's samen worden voor 2500 euro verkocht.

Johan heeft de 480 CD's een paar jaar geleden op een beurs voor verzamelaars gekocht voor 20% meer.

1p. **9.** Hoeveel heeft Johan toen betaald voor zijn collectie CD's?

Hij betaalde toen $1{,}20 \cdot 2500 = 3000$ euro.

De winkeleigenaar houdt zelf 15% van het verkoopbedrag van 2500 euro.

1p. **10.** Bereken hoeveel de winkeleigenaar krijgt.

De winkeleigenaar houdt $0{,}15 \cdot 2500 = 375$ euro

De nieuwe eigenaar van de CD's is niet zo blij met zijn aankoop. Het blijkt dat 54 van de CD's beschadigd is.

1p. **11.** Hoeveel procent van de collectie CD's is beschadigd?

Er is $\frac{54}{480} \cdot 100 = 11{,}25\%$ beschadigd.

Omzet (5 punten)

In 2011 ging het goed met de PLUS aan de A. van Ostadelaan. Twee jaar achter elkaar nam de omzet met 10% per jaar toe. Door de komst van de Jumbo nam de omzet het jaar erop in één keer met 20% af.

3p. **12.** Bereken of leg uit met hoeveel procent de omzet van de PLUS is gestegen/gedaald/gelijk gebleven na 3 jaar.

Alleen conclusie 'gedaald' zonder percentage = 1p.

OF

Uitleg + conclusie zonder percentage = 2p.

De 20% gaat van een groter bedrag af, dus dit is meer dan twee keer 10% over een kleiner bedrag. De omzet is dus gedaald na drie jaar.

OF

Berekening + conclusie = 3p.

De omzet is nog $1{,}1 \cdot 1{,}1 \cdot 0{,}8 = 0{,}968 = 96{,}8\%$ van de oude omzet

De omzet is dus gedaald met $100 - 96{,}8 = 3{,}2\%$

In 2014 bedroeg de omzet van deze PLUS 3,4 miljoen euro. Dit was 26% van de totale omzet van alle PLUS filialen in de hele stad Utrecht.

2p. **13.** Bereken de omzet van 2014 van alle PLUS filialen uit Utrecht samen.

$3{,}4$ miljoen euro $= 26\%$

Dus $1\% = 0{,}1307692 \ldots$ miljoen euro (1p)

Dus $100\% = 13{,}07$ miljoen euro (1p)

D. SURVEY ANALYSIS – LEARNING GOALS

Table 7 provides an overview of how the learning goals (see Appendix A) are reflected in the survey test. The number between brackets refers to the number of points that can be earned for each learning goal.

Table 7 – Analysis of presence of learning goals in survey test questions

Learning goals		**Question** [points]
1	Doing calculations on ratios	
2	Calculate the enlargement factor k by	
a	Dividing two given or measured lengths/widths/heights/numbers	**1, 2, 3** [4] **4** [1]
b	Converting a given scale and vice versa	**8** [1]
c	Converting a given growth percentage	
d	Taking the root of a given or calculated k^2	**8** [1]
3	Calculate the squared enlargement factor k^2 by	
a	Squaring a given or calculated enlargement factor k	**5** [1]
b	Dividing two given or calculated surfaces	**8** [1]
4	Calculate the cubed enlargement factor k^3 by	
a	Cubing a given or calculated enlargement factor k	**4** [1]
b	Dividing two given or calculated volumes	
5	Calculate the asked length/width/height by using k	
6	Calculate the asked surface by using k^2	**5** [1]
7	Calculate the asked volume by using k^3	**4** [1]
8	Draw an enlargement of a given image using	
a	A given enlargement factor k	
b	A given percentage	
9	Write a percentage as	
a	a decimal number	
b	an enlargement factor	
10	Calculations with percentages	
a	Calculate x% of y	**10** [1] **13** [2]
b	Calculate how many percent x is out of y	**11** [1]
c	Calculate the 'new value' y with a given percentage change from x	**9** [1]
d	Calculate the percentage change from x to y	**6, 7** [4] **12** [3]

Furthermore, 1 point can be gained by calculating cm^2 to m^2.

E. SURVEY ANALYSIS – TYPES OF PROBLEMS

The first column of Table 8 refers to the numbers on the survey (see Appendix B). The numbers in the second and third column refer to the lists of problem types and the assessment problem types respectively. These lists are copied in a shortened way below. For a further description see paragraph 2.5.2 (p.10).

Type of Problem:

1. Comparing two parts of a single whole
2. Comparing quantities of the same nature
3. Comparing quantities of different natures

Type of Assessment:

1. Missing value problems
2. Numerical comparison problems
3. Estimation problems

Table 8 – Analysis of type of (assessment) problems

Question	Type of Problem	Type of Assessment
1	2	1
2	2	1
3	2	3
4	2	1
5	2	1
6	3	1
7	3	1
8	3	1
9	3	1
10	3	1
11	2	1
12	2	3
13	3	1

F. Survey analysis – RTTI

Table 9 – RTTI characterization for each survey questionTable 9 shows for every question the RTTI label. Table 10 shows how many points can be gained in what level of the RTTI systematic. The question numbers refer to the numbers on the survey (see Appendix B). In total 25 points can be gained.

Table 9 – RTTI characterization for each survey question

Question	RTTI Type
1	R
2	R
3	T2
4	T1
5	T1
6	T1
7	T1
8	T1 & T2
9	R
10	R
11	R
12	I
13	T2

Table 10 – Analysis of RTTI points for each survey question

Question	points for **Reproduction**	points for **Applying 1** *(familiar context)*	points for **Applying 2** *(new context)*	points for **Insight**
1	1			
2	1			
3			2	
4		3		
5		2		
6		2		
7		2		
8		3	1	
9	1			
10	1			
11	1			
12				3
13			2	
Total (%)	5 (20%)	12 (48%)	5 (20%)	3 (12%)
	68%		32%	

G. FIELD NOTES

SaLVO group

In total: 114 minutes of explaining.

1. **Wednesday September 16th 6 minutes**
   The teacher explained that the class is part of a study. After this follows an introduction on the concept “proportional”. Together they start with the SaLVO booklet on page 5.
   Students’ homework: exercise 1 – 5.

2. **Thursday September 17th 11½ minutes**
   Discussing the concept “multiplying factor” that the students encountered in their homework. The students do the “Entry level” on page 9 and after this they discuss it with the class.
   Students’ homework: exercise 6 – 11.

3. **Friday September 18th 5 minutes**
   Explaining that the multiplying factor is smaller than one when you are reducing instead of enlarging.
   Students’ homework: exercise 13 – 20.

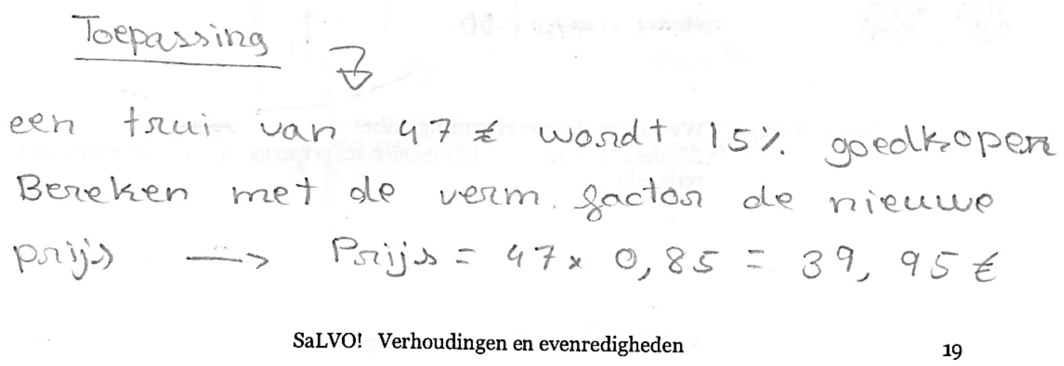
4. **Wednesday September 23th 21 minutes**
   Discussing the “§3 Entry level” and the concept of using the multiplication factor for calculating percentages. Then how to calculate a percentage change. The students make this note in their booklets.

Procentuele toenamen

oud	erbij	nieuw	k = verm. factor
100%	34%	134%	1,34
100%	7,5%	107,5%	1,075
100%	0,3%	100,3%	1,003

Procentuele afname

oud	eraf	nieuw	k= verm. factor
100%	32%	68%	0,68
100%	7%	93%	0,93

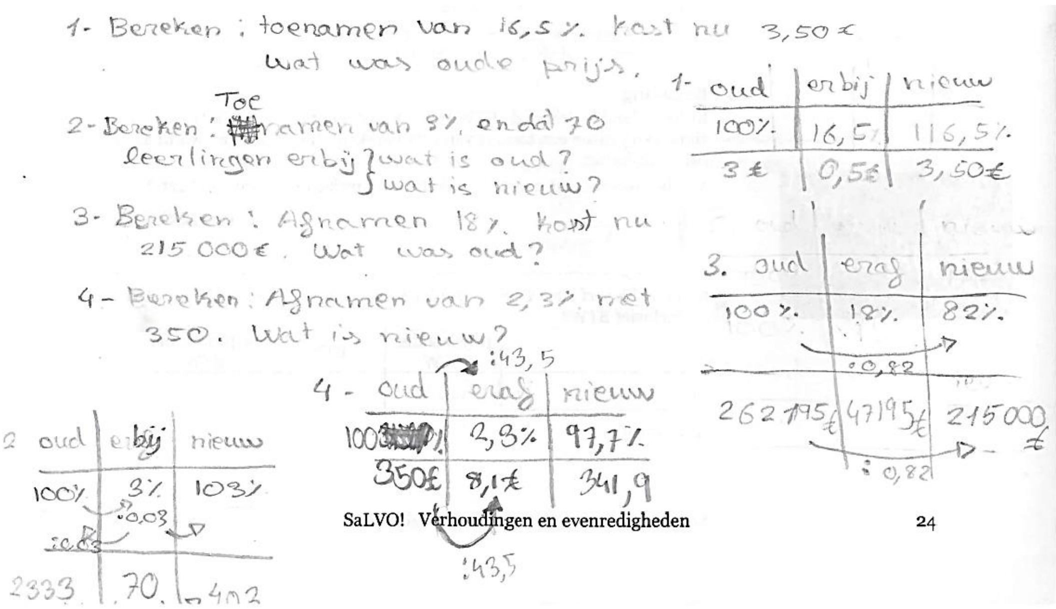


Students' homework: exercise 18, 19, 20 again but with multiplication factor, "instap" blz. 20 and exercise 21, 22.

** Thursday 24th the students had a test on the previous chapter.*

5. **Friday September 25th 12 minutes**
Discussing the "Entry level" on page 20, and the concept of backwards calculations. The students get a lot of time to do their homework.
Students' homework: 23 – 26.

6. **Wednesday September 30th 16 minutes**
The teacher starts with four example test exercises. The students get 9 minutes to do these exercises and afterwards the teacher explains them. The students all have to do the exercise with multiplication tables.



After this the teacher introduces the subject of enlarging and decreasing surfaces, by asking what will happen if you set the copy machine on 50%. The students do the "Instap" on page 15 and afterwards discuss their methods with the teacher.
Students' homework: 11 – 17 from booklet 3.

7. **Thursday October 1st** **12½ minutes**

Explaining the concept of multiple reductions. How many percent is left when you decrease with 30% for two times? This means two times calculating with 70% thus with 0.7. After this explaining the concept of backwards calculation again. If you know that a decrease of 2.3% represents 350, how many is left? The students have to draw multiplication tables to find out. Then the teacher and students work on exercise 18. If you know k^2 how do you calculate k? And what happens if k^2 is smaller than 1?
Students' homework: 19 – 28 (excl. 23).

8. **Friday October 2nd** **12 minutes**

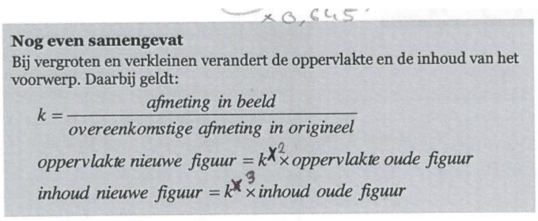
Discussing the homework problems. Introducing the concept of the multiplying factor with volumes. The teacher brought Duplo building blocks to explain.
Students' homework: "Entry level" page 36/37, exercise 36 and 38b

9. **Wednesday October 7th** **4 minutes**

The teacher explains the experiment that the students will perform this lesson shortly. She discusses the general rule that the students had to fill in in exercise 36c: If all dimensions become k times as big, then the volume will become k^3 times as big. The rest of the time the students work on the experiment. They work in groups of 4 or 5 students. They have to weigh how much rice fits in the cones they brought to class. They also have to measure and weigh clay balls to find the relationship they all filled in in exercise 36c.
Students' homework: finishing the calculations on the experiments (exercise 37 and 38).

10. **Thursday October 8th** **7 minutes**

Discussing the results of the experiment done yesterday. The teacher remarks that there is a severe mistake in the booklet on page 43 and she lets the students find it.
Students' homework: 39 – 43.

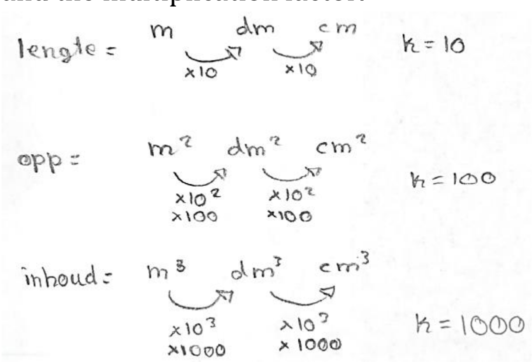

Nog even samengevat
Bij vergroten en verkleinen verandert de oppervlakte en de inhoud van het voorwerp. Daarbij geldt:

$$k = \frac{\textit{afmeting in beeld}}{\textit{overeenkomstige afmeting in origineel}}$$

oppervlakte nieuwe figuur = k^2 × *oppervlakte oude figuur*
inhoud nieuwe figuur = k^3 × *inhoud oude figuur*

11. **Friday October 9th** **7 minutes**

Discussing the homework problems. The teacher lets the students make a note about length/surface/volume: the relation between "adding 2 zeroes" when going from m^2 to cm^2 and the multiplication factor.



The students do exercise 44 in class.

** Wednesday October 14th the students made the survey test.*

Numbers & Space group

In total: 127.5 minutes of explaining.

1. **Wednesday September 16th** **15 minutes**

   The teacher explained that the class is part of a study and that they will work with their normal textbooks. The students start with percentages. The teacher explained the link between percentages and decimal numbers (pre-knowledge of students). How to calculate 20% of 400 with using a decimal number? The students come up with other ways, the teacher emphasizes that a decimal number needs to be used. The students work on exercises 2, 3, 4 and 5. After 10 minutes the teacher discusses exercises 2 and 3.

   Students' homework: 4, 5, 8.

2. **Monday September 21st** **5 minutes**

   The teacher starts with discussing the answers on exercises 4, 5, and 8 and links this to the new theory: calculating the new value with a given percentage change. For example, an amount of 85 increases with 21%. The rest of the lesson the teacher talks about the previous chapter.

   Students' homework: 10, 11, 12

** Tuesday 22nd the students had a test on the previous chapter.*

3. **Wednesday September 23rd** **10 minutes**

   The teacher starts with the answers for the questions 10, 11, 12. Afterwards they talk on percentage change when the amount is decreasing. The teacher also explains how the students should write their answers. They should use decimal numbers for percentages. The students work on their homework the rest of the lesson.

   Students' homework: 15, 16, 17, read theory on p. 127, 20ab, 21, 22

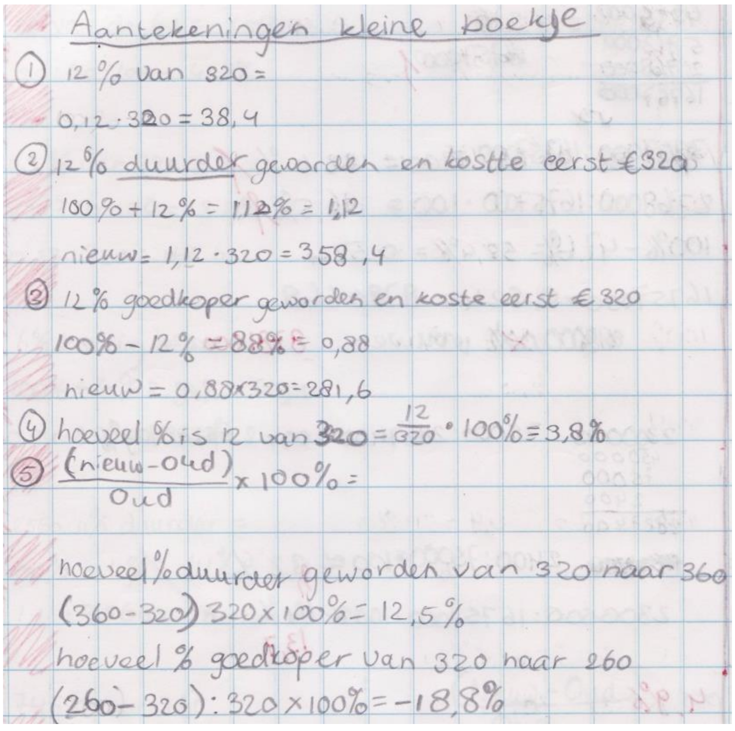
4. **Monday September 28th** **6 minutes**

   The teacher checks the exercises with the students by talking about the answers. The teacher gives the student an overview of the things they have learned so far, supplemented with the new concept of percentage change. The students learn *(new – old) / old x 100%*.

   The students write the note below in their notebooks.

   Afterwards, students work on their homework.

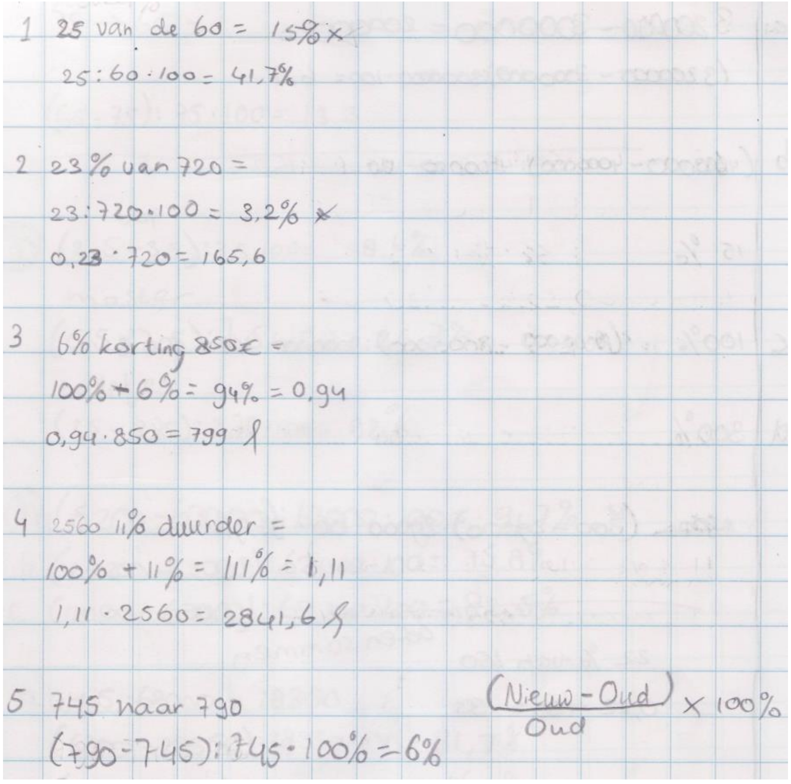
   Students' homework: 23, 24, theory blz. 129, 26



Aantekeningen kleine boekje
① 12 % van 320 =
0,12 · 320 = 38,4
② 12 % duurder geworden en kostte eerst €320
100 % + 12 % = 112% = 1,12
nieuw = 1,12 · 320 = 358,4
③ 12 % goedkoper geworden en koste eerst € 320
100% − 12 % = 88% = 0,88
nieuw = 0,88×320 = 281,6
④ hoeveel % is 12 van 320 = 12/320 · 100% = 3,8%
⑤ (nieuw − oud) / oud × 100% =
hoeveel % duurder geworden van 320 naar 360
(360−320) 320 × 100% = 12,5 %
hoeveel % goedkoper van 320 naar 260
(260−320) : 320 × 100% = −18,8%

5. **Tuesday September 29th** **9 minutes**

The teacher checks the exercises with the students. After this she gives the students five exercises to practice. The students make the exercises themselves. Afterwards the teachers asks a student for each question to give the answers. She writes this, with the required notation, on the whiteboard. After this note the students work on exercises 27, 28 and 29.



You can see in this student note that this student made mistakes in the first two questions and wrote the right way underneath her own calculations.

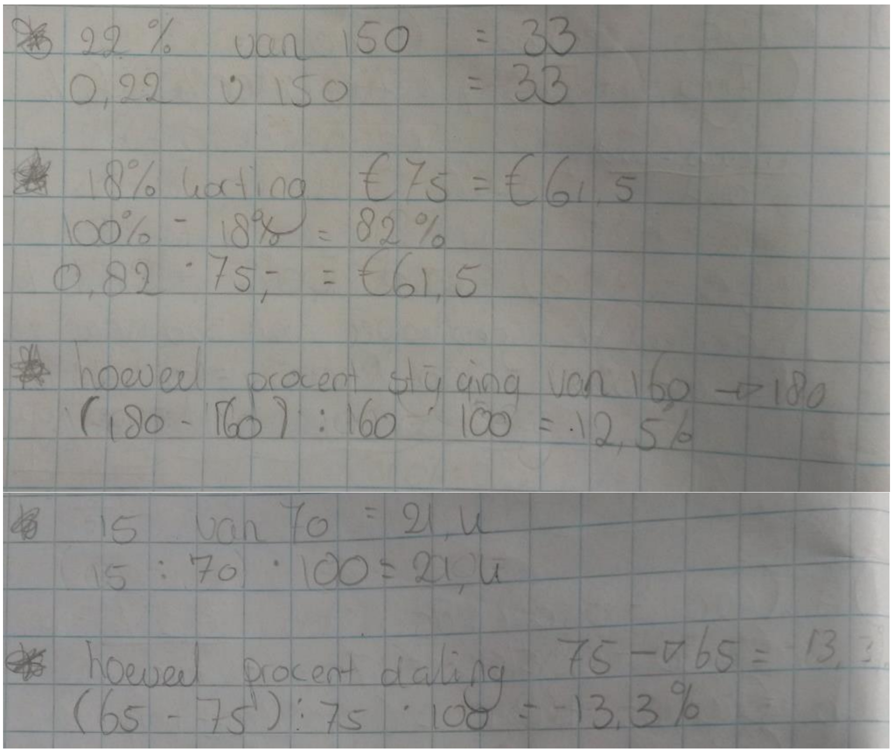
Students' homework: 27, 28, 29, read theory between on p. 130

6. **Wednesday September 30th** **15 minutes**

The teacher starts with discussing the answers on questions 27, 28 and 29. The rule *(new – old) / old x 100%* is discussed; and the teacher makes a remark about rounding percentages on 1 decimal. The teacher gives again five exercises to practice, but now demands that the students calculate it by typing in it in their calculators in once. For example, when calculating how much you have to pay when you get 18% discount on €75, she wants the students to type in 0.82×75 and not first $75 : 100 \times 18$ and then $75 - Ans$. Regarding the last exercise the teacher makes the remark that the calculator says -13.3 so that this means a discount of 13.3%. See the note on the next page.

The rest of the lesson students' work on exercises 30 – 33.

Students' homework: 30, 31, 32, 33



7. Monday October 5th **20 minutes**

Today the teacher starts with the second part of the booklet, the part about enlarging figures and other objects. The teacher explains the concept of enlargement factor and that they have to measure the original and the image. In class the students work on exercise 29, 30, 31, 32 (of chapter 8).
Students' homework: finish exercise 32, do 33, 34, 35

8. Tuesday October 6th **15 minutes**

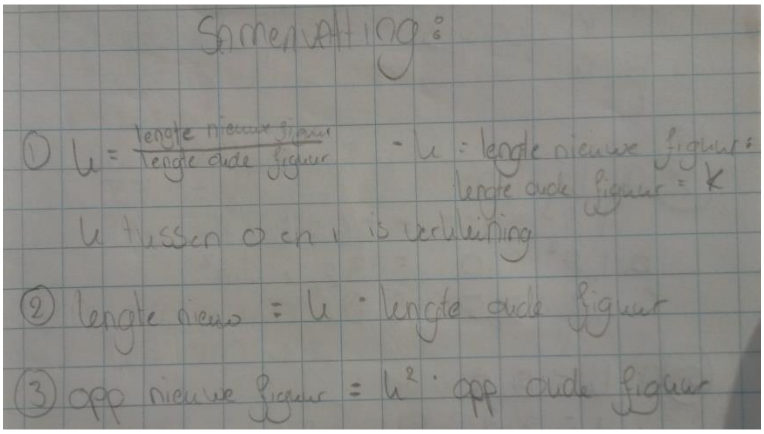
The teacher starts with repeating yesterdays information about the enlargement factor. Which letter do we use? What do we know when $0 < k < 1$? What do we call the 'original'? After this the teacher gives the answers on questions 32, 33 and 34 so the students can check these. Then the students have to draw a rectangle of 1×4 cm and draw the rectangle that has been enlarged with factor 3. Then they have to calculate the surface and see if they see a relation between those surfaces. They learn that $surface\ old \times k^2 = surface\ new$. They check if this also holds when $k < 1$. It does. The students have to start working on exercise 38 but they do not know how to proceed. The teacher explains that they need to measure the images for calculating k. There still is some time left to work on exercise 39 and 40.
Students' homework: 39 – 43

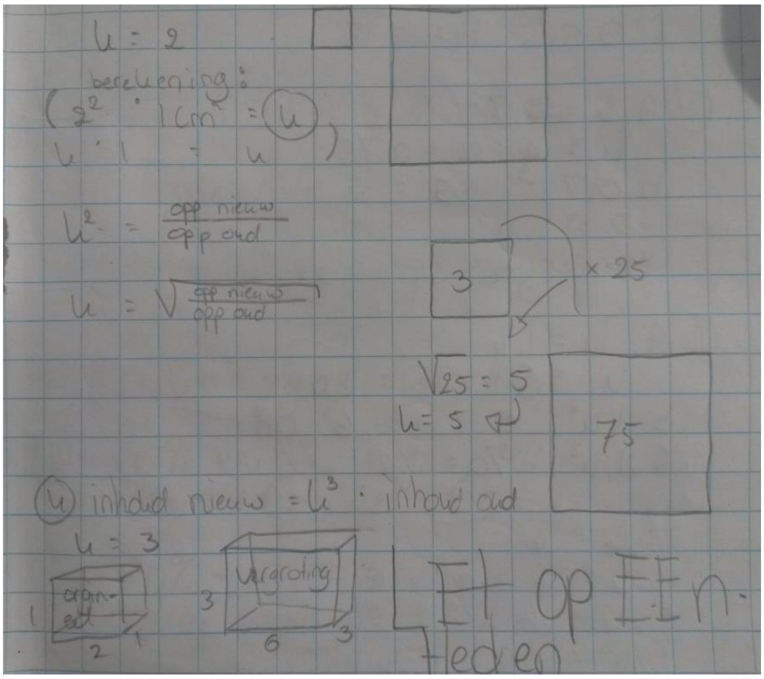
9. Wednesday October 7th **7½ minutes**

The teacher starts with discussing the answers of 38 and 43. She makes some notes about notation; write down your measurements and the dimension (cm, cm^2). She explains that in exercise 43 you know that scale $1 : 250$ means that $k = 250$ and that you have to convert cm^2 to m^2 by removing four zeroes.
After this the teacher writes two rectangles on the whiteboard, one with surface 3 and one with surface 75. She tells the students they have to be able to calculate the enlargement

factor. One student immediately says $k = 5$. Together with the class they see that $k^2 = \frac{75}{3}$ so this means that $k^2 = 25$ and that $k = \sqrt{25} = 5$. Afterwards, the students read the theory on page 107 on their own and start working on exercises 47 and further.
Students' homework: 47 – 51

10. Monday October 12th **20 minutes**
The teacher starts with discussing the answers of the homework. After this she gives a summary that the students make a note of. In class students work on exercises 50 and 51.





Students' homework: 52, 53, 57ab

11. Tuesday October 13th **5 minutes**
In the last lessons the students can ask their questions and work on more exercises to practice with chapter 8. The teacher gives five questions on the topic percentages to practice this again.

** Wednesday October 14th the students made the survey test.*

H. Comparability the SaLVO group with the N&S Group

Heterogeneity

The graph in Figure 8 shows the amount of students in respectively the SaLVO and Numbers & Space group that was last year in a specific 7th grade class. There are six different 7th grade classes: 1A, 1B, 1C, 1D, 1E and 1F. Some students enrolled in the 8th grade coming from another school, these are labeled 'extern'.

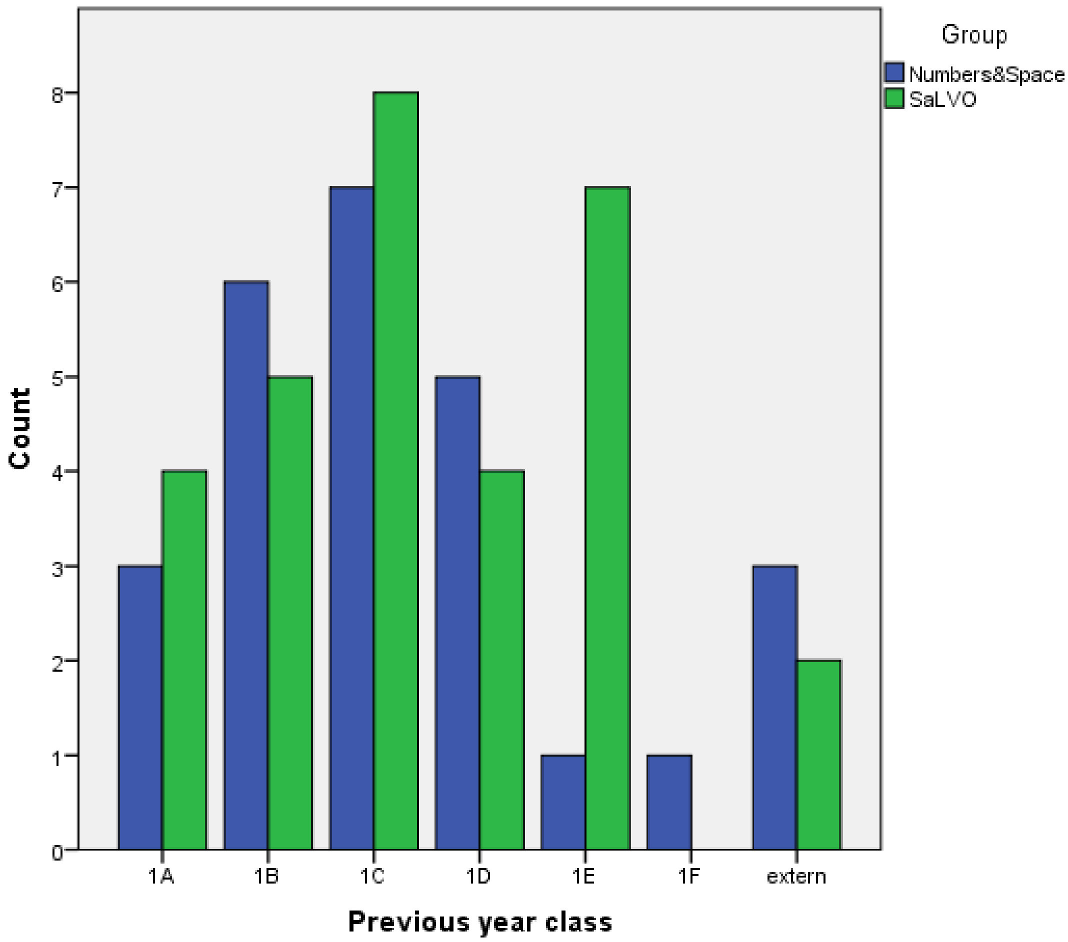


Figure 8 – In which class were the students the previous year?

I. APPROACHES USED PER STUDENT PER QUESTION

The tables below show how many students used which approach for every question in the SaLVO group (Table 11) and the Numbers & Space group (Table 12). The letters below the question numbers refer to the question topic: P for percentages and E for enlarging.

Table 11 – Approaches used by the SaLVO group

	Question						
Approach used	**1 E**	**2 E**	**3 E**	**4 E**	**5 E**	**6 P**	**7 P**
SaLVO				4 [13.8%]	2 [6.9%]	25 [86.2%]	26 [89.7%]
Non-discriminating	29 [100%]	29 [100%]	29 [100%]		1 [3.4%]	4 [13.8%]	2 [6.9%]
Numbers & Space				25 [86.2%]	26 [89.7%]		1 [3.4%]

	Question					
Approach used	**8 E**	**9 P**	**10 P**	**11 P**	**12 P**	**13 P**
SaLVO		13 [44.8%]	13 [44.8%]	17 [58.6%]	2 [6.9%]	13 [44.8%]
Non-discriminating	15 [51.7%]	9 [31.0%]	8 [27.6%]	5 [17.2%]	27 [93.1%]	11 [37.9%]
Numbers & Space	14 [48.3%]	7 [24.1%]	8 [27.6%]	7 [24.1%]		5 [17.2%]

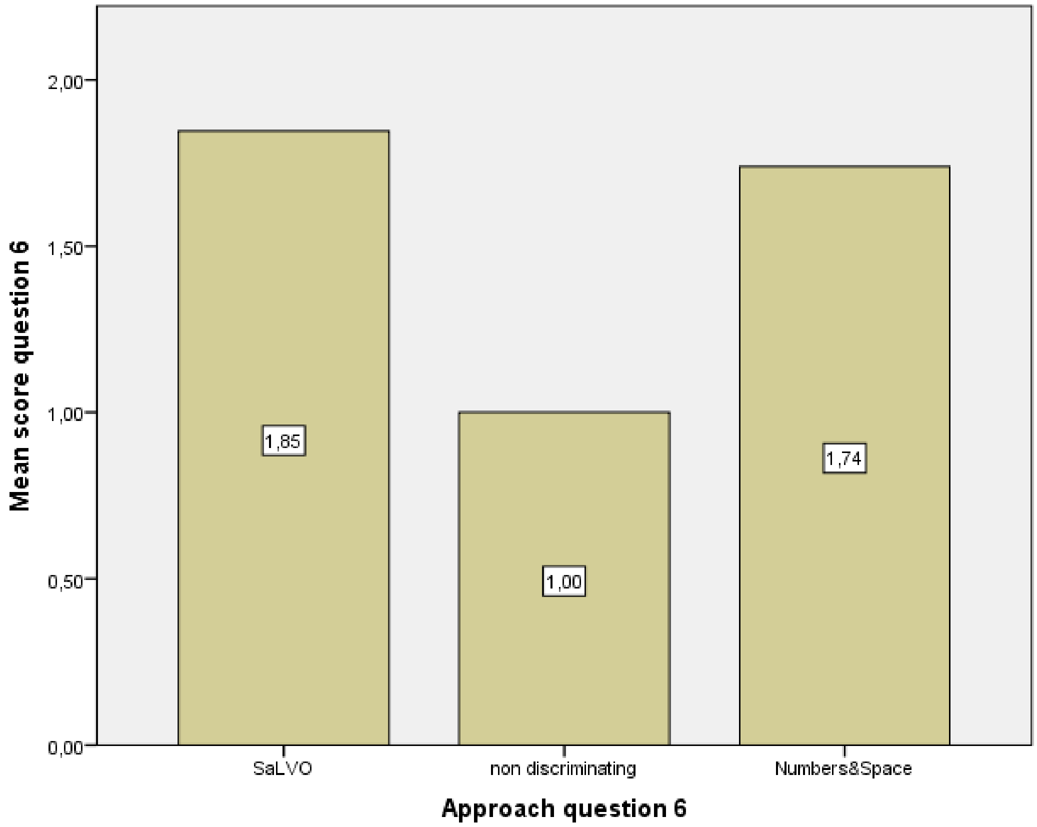
Table 12 – Approaches used in the Numbers & Space group

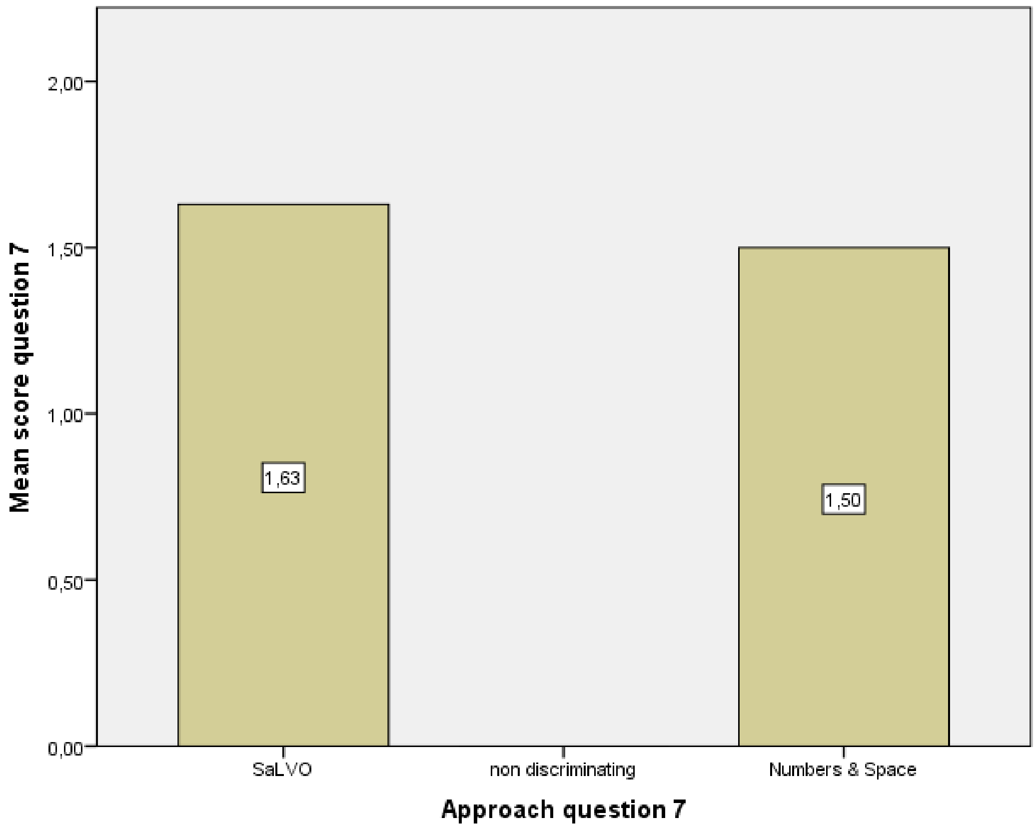
	Question						
Approach used	**1 E**	**2 E**	**3 E**	**4 E**	**5 E**	**6 P**	**7 P**
SaLVO						1 [4%]	1 [4%]
Non-discriminating	25 [100%]	25 [100%]	25 [100%]			1 [4%]	1 [4%]
Numbers & Space				25 [100%]	25 [100%]	23 [92%]	23 [92%]

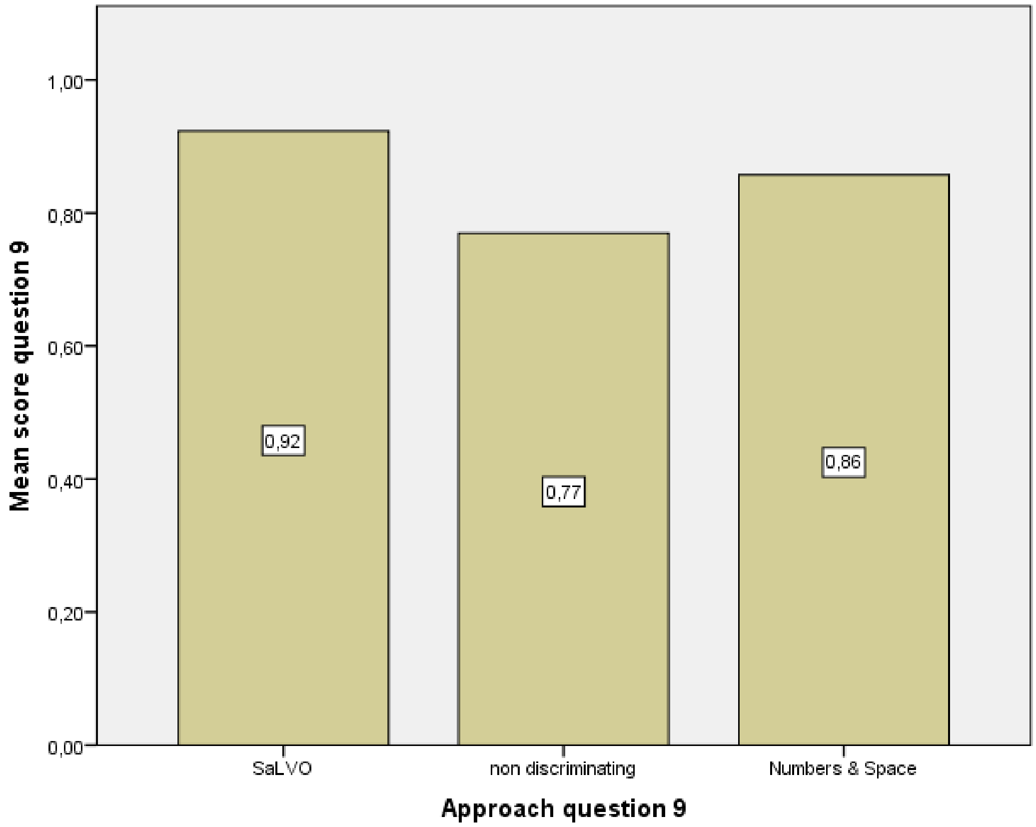
	Question					
Approach used	**8 E**	**9 P**	**10 P**	**11 P**	**12 P**	**13 P**
SaLVO						1 [4%]
Non-discriminating	2 [8%]	4 [16%]	3 [12%]	4 [16%]	20 [80%]	14 [56%]
Numbers & Space	23 [92%]	21 [84%]	22 [88%]	21 [84%]	5 [20%]	10 [40%]

J. POINTS GAINED PER APPROACH PER QUESTION

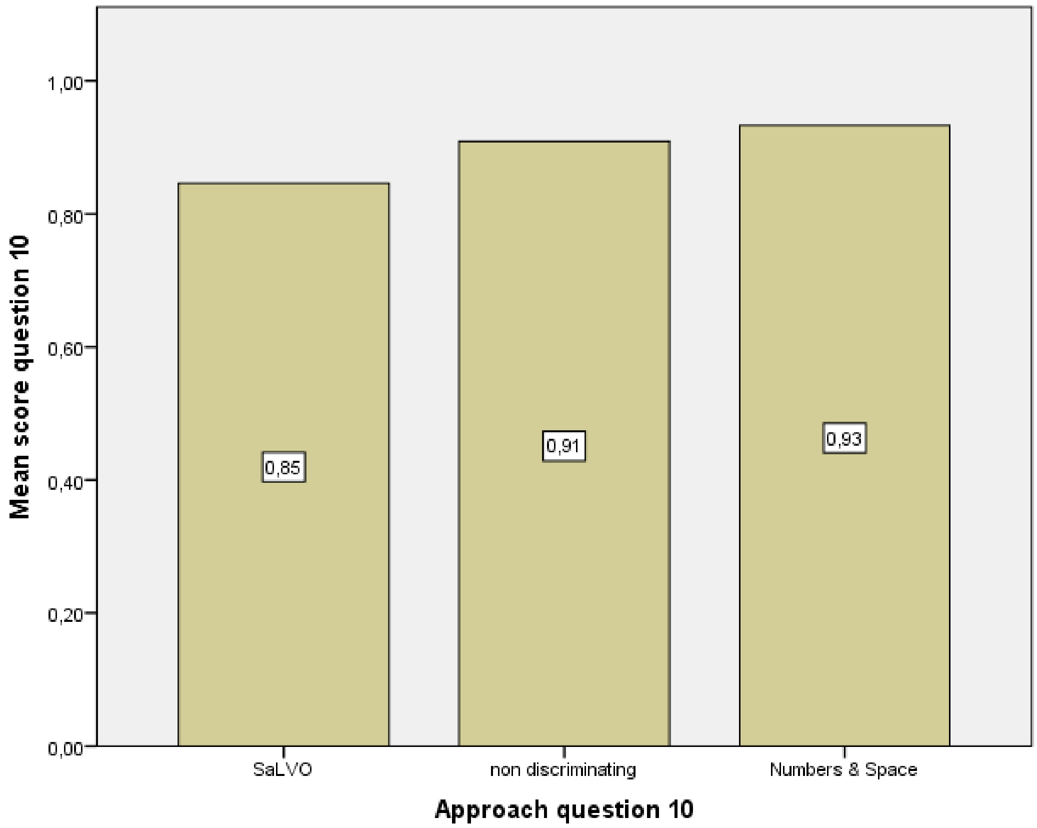
Below are six histograms that show how many points were gained on average on the questions 6, 7, 9, 10, 11, and 13; by using the SaLVO, non-discriminating or Numbers & Space approach.



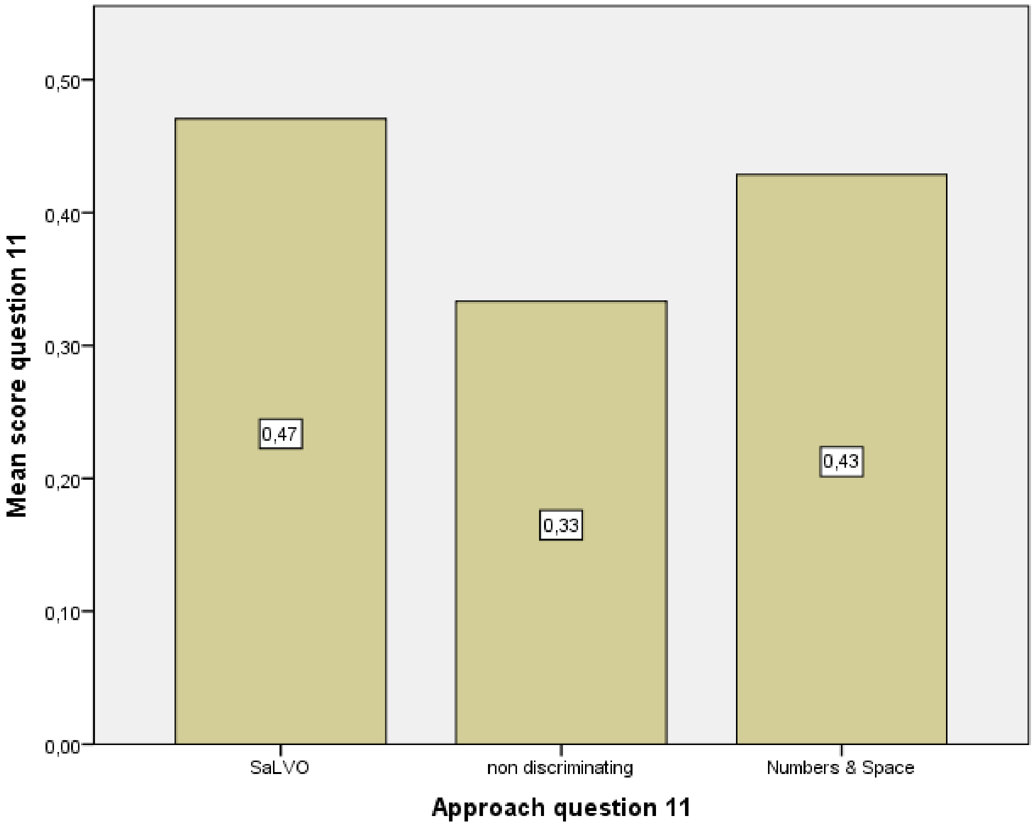


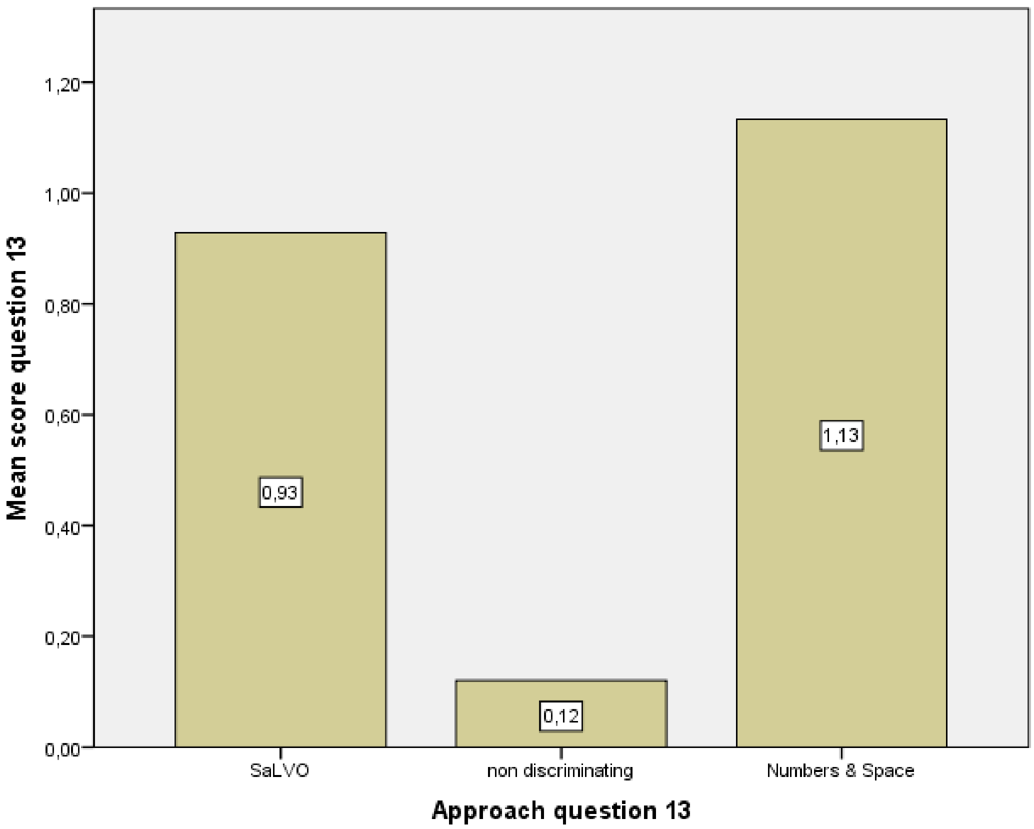


1,00
0,80
0,60
0,40
0,20
0,00
Mean score question 9
0,92
0,77
0,86
SaLVO
non discriminating
Numbers & Space
Approach question 9



1,00
0,80
0,60
0,40
0,20
0,00
Mean score question 10
0,85
0,91
0,93
SaLVO
non discriminating
Numbers & Space
Approach question 10


Mean score question 11
0,50
0,40
0,30
0,20
0,10
0,00
0,47
0,33
0,43
SaLVO
non discriminating
Numbers & Space
Approach question 11


Mean score question 13
1,20
1,00
0,80
0,60
0,40
0,20
0,00
0,93
0,12
1,13
SaLVO
non discriminating
Numbers & Space
Approach question 13