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Scriptie

Titel Scriptie: *	Vocational students' visual search patterns while solving a digital proportion task of the Mathematical Imagery Trainer For Proportions application.
Taal Scriptie: *	Engels
Samenvatting:	The aim of this study was to describe cognitive visual search patterns within the MIT-P app for embodied learning; an app based on a proportions task constructed by the Embodied Design Research Laboratory (EDRL), a research group of the university of Berkeley. The app stimulates the concept of proportion by means of visual and sensory motor interactions with four tasks, two parallel pluses or bars and two orthogonal pluses or bars with a 1:2 ratio. To describe visual search patterns, areas of interest and attentional anchors were used. The main research question was: *What visual search patterns and hand movements can be found from children in Dutch vocational education during a completed problem solving task in the MIT-P app for learning proportions?* The study used eye-tracking videos and log file data of Dutch vocational education students (n=21). The data was gathered in two Dutch vocational schools using a Tobii-x260 eye-tracker. The study focused on eye- and hand movements. Data was analysed by observing videos (n=4) made by the eye-tracker and analysis on areas of interest with Tobii studio (n=21). Results indicate that students have a linear and triangular attentional anchor for the parallel as well as the orthogonal task. Both tasks seem to have ten AOIs. The most important areas are the top of the bars, pluses, the area halfway the right bar, numbers and space in between pluses.
Trefwoorden: (gescheiden door ;)	Proportion; mathematics; vocational education; visual search; eye-tracking; attentional anchors; areas of interest
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Ingevuld op: * 19-06-2015
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MSc Thesis A.F.D (Anne-Ciska) Cuiper (3512339)
TITLE: VOCATIONAL STUDENTS' VISUAL SEARCH PATTERNS WHILE SOLVING A DIGITAL PROPORTION TASK OF THE MATHEMATICAL IMAGERY TRAINER FOR PROPORTIONS APPLICATION.
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Supervisors:
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Vocational students' visual search patterns while solving a digital proportion task of the Mathematical Imagery Trainer For Proportions application.

Abstract

The aim of this study was to describe visual search patterns within the MIT-P app for embodied learning, which is an app based on a proportions task constructed by the Embodied Design Research Laboratory; a research group of the university of Berkeley. The app stimulates the concept of proportion by means of visual and sensory motor interactions with four tasks, two parallel pluses or bars and two orthogonal pluses or bars with a 1:2 ratio. Areas of interest and attentional anchors were used to describe visual search patterns. The study used eye-tracking videos and log file data of Dutch vocational education students (n=21). The data was gathered in two Dutch vocational schools using a Tobii x2-60 eye-tracker. The study focused on eye- and hand movements. Data was analysed by observing videos (n=4) made by the eye-tracker and analysis on areas of interest with Tobii studio (n=21). Results indicate that students have a linear and triangular attentional anchor for the parallel as well as for the orthogonal task. Both tasks seem to have ten AOIs. The most important areas are the top of the bars, pluses, the area halfway the right bar, numbers and space in between pluses.

Keywords: Proportion, mathematics, vocational education, visual search, eye-tracking, attentional anchors, areas of interest.

1. Introduction

Many Dutch children in vocational education generally have a low motivation for mathematics and grades have declined in the last years (Inspectorate of Education of the Netherlands, 2014). The national Dutch curriculum emphasizes the understanding of mathematical concepts, for example through class discussion. However, since assignments in Dutch mathematical textbooks are abstract and focus less on understanding (Van Galen et al., 2005), many children in Dutch vocational education do not have a conceptual understanding of basic mathematic abstractions, such as proportionality and ratio (CITO, 2013; Lamon, 2012). According to Freudenthal (1973), proportional numerals and numerals for measurement are the numerals we use most in our daily live. Consequently, it is important that vocational students develop a conceptual understanding of these notions. To this end, the Mathematical Imagery Trainer For Proportions (MIT-P) has been constructed by the Embodied Design Research Laboratory research group at the university of Berkeley (Abrahamson & Lindgren, 2014; Abrahamson, 2014; Figure 1) and has been revised into an application for iPads (MIT-P) by the research group at the university of Utrecht. The apps' task is based on the ideas of embodied cognition (Barsalou, 2008; Abrahamson, 2014). According to the theory of embodied cognition, bodily states can be used as a ground for cognition and, therefore, conceptual understanding (Barsalou, 2008; Lakoff & Johnson, 1980). The MIT-P app contains an embodied-interaction activity that stimulates children to obtain a conceptual and embodied understanding of proportions and ratio (Abrahamson & Sánchez-García, 2014). Insight of the game is needed to improve training in students' proportion learning.

The aim of this study is to identify visual search patterns and bodily activities while solving proportion tasks with the app. The visual search patterns are seen as attentional anchors (AA; Abrahamson & Sánchez-Garcia, 2014) that are derived from areas of interest (AOI). The bodily activities consists of moving fingers on the iPad screen. The study employs an eye-tracking video system and a matching log file computer program to collect gaze and hand-movement data.

1.1. Theoretical Background: Embodied cognition

Embodied cognition is a form of grounded cognition (Barsalou, 2008), using bodily states, such as perceptual- or motor processes as a ground for cognition (Lakoff & Johnson 1980). According to the theory of grounded cognition, cognition can also be grounded in simulations and situated action. Barsalou (2008) defines a simulation as the re-enactment of perceptual-, motor- and introspective states acquired during experience with the world, body and mind. Mental imagery constitutes the best known case of these simulation mechanisms (Kosslyn, 2005). Wilson (2002) states that there are two processes of embodied cognition: online and offline. With online cognitive activities the student is physically involved in an actual bodily activity in the environment (Niedenthal, Barsalou, Winkielman, Krauth-Gruber & Ric, 2005). During an offline activity, the environment is conceived by the mind of the student, either from memory or even completely imagined. Wilson's (2002) offline activities are in accordance with the findings of several researchers who affirm that the student can simulate the environment and objects in the minds-eye (Gallese & Lakoff, 2005; Goldstone, Landy & Son, 2009).According to Abrahamson and Lindgren (2014) and Wilson (2002), within the theory of embodied cognition, conceptual reasoning originates in physical interaction and becomes internalised as simulated actions. This means students use their bodily movements to adapt to their environment and thereby gain tacit knowledge and understanding (Núñez, Edwards & Matos, 1999). Wilson (2002) and Barsalou (2008) argue that higher order cognitive processes, such as mathematics, are also rooted in the interaction between the student's body, and therefore perceptual and motor processes, and the environment. For embodied cognition, this means that students need conceptual knowledge of their surroundings to store in their memory. This makes it possible for them to re-enact the problem-solving as done in the environment without the environment present.

1.2 Embodiment and Mathematics

Research has shown that mathematics is made of mental abstractions such as abstract sign systems and inscriptional forms where there is no room for bodily movements (Núñez, 2004). However, Abrahamson and Lindgren (2014), Wilson (2002) and Núñez (2004) argue that concepts of mathematics are grounded in the situated, spatial-dynamical and somatic phenomenology of the person who is engaging in the activity. Also, it was found that children rely on visual-spatial structures to successfully solve mathematical problems (Presmeg, 1986). Therefore, mathematical themes, such as proportions and ratio, can be taught through activities wherein students use their body to acquire an understanding of mathematical concepts. In the MIT-P app, proportions can be taught through perceptual learning, in the form of visual perception of objects on the screen and bodily movements to change the objects. It enables the student to develop the visual-spatial structures needed for understanding proportionality. The app consists of four tasks; parallel with pluses, parallel with bars, orthogonal with pluses and orthogonal with bars (Figure 1). For every task, a grid or vertical number lines at the edges may also appear. Students need to move either bars or pluses up and down with their fingers on the screen to get the right proportion and a green screen or green bar (Figure 1). When the screen or bars are red, the right proportion is not yet achieved.

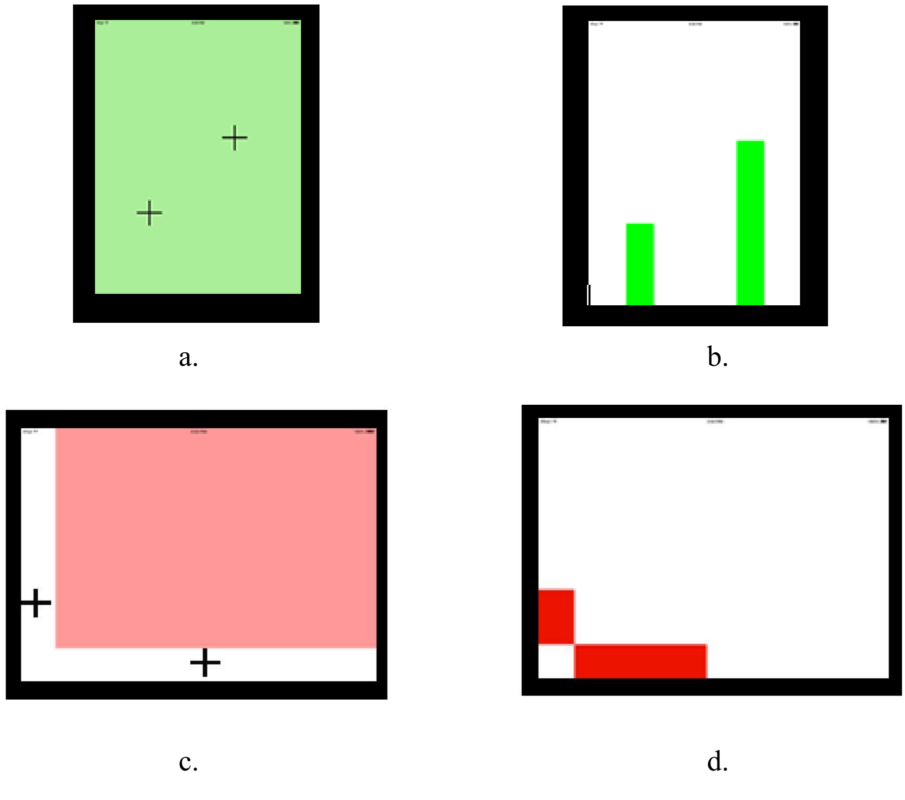


Figure 1. Sample screens from four modules of the Utrecht MIT-P touch-screen tablet. The four tasks were: parallel screen with pluses at finger point (a) or bars (b) and orthogonal full screen with pluses at finger point (c) or bars (d). Adopted from Shayan, Abrahamson, Bakker, Duijzer and Van der Schaaf (2014).

1.3. Research Question

According to Gibson (2000), visual search is a process of specifying significant information through selection from an array of information about objects from the environment (e.g. iPad screen), in relation to the perceiver's bodily activities. To identify visual search, search patterns can describe what objects contain significant information to the learner and are for that reason selected. The main research question is:

What visual search patterns and hand movements can be found from children in Dutch vocational education during a completed problem solving task in the MIT-P app for learning proportions?

To answer the main question, three sub questions were formulated.

1) *What gaze patterns can be found in tasks from the MITP-app and how do they relate to hand-movements?*
2) *What areas of interest are most important for the tasks in the MITP-app?*
3) *What attentional anchors can be derived from the search patterns?*

To answer these questions, this study uses gaze data and hand-movement data from vocational left and right handed students to map patterns of eye and finger movements.

1.4. Eye movements.

While playing with the MIT-P app, students are engaged in an online activity, being visually perceptive, which enables the creation of eye movement log file data. According to Lai et al. (2013), eye movements are in many research studies described as fixations and saccades (see also Drew et al., 2013). A fixation is a stable state of eye movement and a saccade is a rapid movement between fixations (Lai et al., 2013). In this study, eye movements will be referred to a set of parameters, in the form of fixation duration and fixation count on areas of interest. The fixation count can be used to show whether a target is important (Jacob & Karn, 2003), which makes it an area of interest. The fixation duration can show the relation between time and cognitive processes (Jacob & Karn, 2003). The provisional areas of interest will be based on former research on eye-movements and hand-eye coordination. Also the scan path, a spatial pattern of fixation sequences (Lai et al., 2013), is taken into account and described as an attentional anchor (AA).

1.4.1. Areas of interest.

Areas of interest are the visual objects, points or targets in an environment that are relevant to the subject in order to perform a task. Hayhoe and Ballard (2005) and Johansson, Westling, Bäckström and Flanagan (2001) found that the eye does not fixate on a visually salient target, but almost exclusively fixates on a target that is relevant for performing the task. Irrelevant targets, even when salient, are rarely fixated on (Land & Hayhoe, 2001; Hayhoe & Ballard, 2005). The AOIs and irrelevant targets can be identified by fixation count and fixation duration on visual objects, points or targets in the subjects' environment. In several studies (e.g. Johansson et al., 2001), eye movements in a number of diverse tasks on possible relevant areas have been monitored. While some researchers studied daily activities (Land & Hayhoe, 2001), others studied abstract hand-eye coordination (Johansson et al., 2001; Sailer, Flanagan & Johansson, 2005). In the current study, an abstract activity (i.e. without daily objects) will be monitored where the subjects have to manipulate a visual object on a screen.

Previous research on hand-eye coordination has shown that eyes fixate on a target before the hand moves (e.g. Carlton as cited in Binsted, Chua, Helsen & Elliott, 2001). Before the hand rests on the target, however, a second corrective saccade from the eye and a hand movement can take place (Helsen, Starkes, Elliott & Buekers, 1998). In that case, the eye leads the hand by providing the hand visual information about the target, so the hand will find the target at once (Binsted et al., 2001; Land & Hayhoe, 2001; Hayhoe & Ballard, 2005). Therefore, the manipulated object is only a relevant object before the hand moves, because the eye presents information about that object to the motor system to direct the hand. Because of this, the eyes will fixate on landmarks at which contact took place (Johansson et al., 2001). The manipulating movement is not fixated upon. During the manipulation, the eyes fixate on the area where the object will land, to inform the motor system.

Since the eye needs to provide specific information about a task to the motor system, the target that the eye fixates on depends on the task. Consequently, the eye is pro-active, with relevant fixations and saccades that are often made to a location in a scene in advance of an expected event (Land & Hayhoe, 2001; Johansson et al., 2001; Mcleod as cited in Hayhoe & Ballard, 2005). To know which targets are relevant for a task, or which events to expect, the student needs to have a learnt internal model of the event or task; something that can only be achieved by learning (Hayhoe & Ballard, 2005). An internal model is a mental image or simulation of a rule of a solved task. To identify task-related targets in the screen, and therefore AOIs, it is necessary to describe which targets in the screen are task related and could already be part of a learnt internal model by the student. In the Netherlands, children are taught quantitative proportions, though not yet as '1 to 2', but, for example, as '5 times as big' or '3 times as small'. Next, they are taught fractions like ½ and ¼, and ratio in the form of ratio tables (Van Galen et al., 2005).

When an activity involves two objects, where both objects are manipulated so that they make contact in an appropriate way, the eye movements differ from the eye movements when manipulating one object. With two objects, a number of fixations are made, often switching between the two objects, and the action is usually completed under visual control. (Land, 2001). When a preferred condition is met, the saccades stop. These checking operations require the eye to fixate on a relevant target position of an object, either in one long fixation or in a series of repeated fixations (Land, 2001). Because of the learnt internal model, Land and Hayhoe (2001) found little variation between subjects eye movements, only the order in which actions took place while making a sandwich or a cup of tea differed.

Former research shows that AOIs will be targets relevant to perform the task of the app. Due to the amount of numerals used in Dutch mathematical education, it is assumed that numerals in the app will be a relevant AOI. Other task related targets are assumed to be the interactive bars or pluses in the app, since these are 'objects' that need to be manipulated. With the bars, it is assumed that especially the top of the bar will be a target, since this part can be manipulated by hand movements. The empty space between the bars or pluses can be a relevant target for checking operations, since both bars/pluses (i.e. two objects) need to be manipulated. Irrelevant targets are expected to be the hands (Johansson et al., 2001; Land & Hayhoe, 2001) and the empty space that is not between the objects. Other salient targets, such as the grid lines, are uncategorised; neither as relevant or irrelevant.

Furthermore, due to the possible learnt internal model, little variation is expected to be found between subjects AOIs.

1.4.2. Attentional Anchors.

The concept of attentional anchors is relatively new. AAs show a sensorimotor coordination pattern coupled with distance (Abrahamson & Sánchez-Garcia, 2014). This means that the AA is a tool to connect latent phenomenal correlations in the perceptual field (Abrahamson & Sánchez-Garcia, 2015) Therefore, an AA makes it possible for the learner to coordinate sensorimotor actions. An example of an AA was given by Abrahamson & Sánchez-Garcia (2015) for the plus task (Figure 1a). They found that a subject made a vertical interval between the pluses to facilitate her performance of the task. In this article, AAs are search patterns in the form of scan paths that can take form as geometrical structures between AOIs, depending on the task.

1.5. Hand movements.

In addition to eye movements, hand movements are also present during the online activity with the MIT-P app. The student interacts with the app by moving fingers on the iPad screen. Finger movements will therefore also be a parameter. Even though limited research is done on the interaction between fingers and computers, research by Venjakob, Marnitz, Mahler, Sechelmann and Rötting (2012) explains how participants had to scroll through layers of radiology scans with their finger on a computer mouse to interact with a computer. This interaction between computer and finger is similar to the interaction of the MIT-P app, where two fingers interact with the app. The research of Venjakob et al. (2012) showed that runs and oscillations reflect similar patterns as eye tracking data. Runs are explained as large movements in one direction through a certain, large, amount of layers of scans. For the app it means that a run is defined as one large non-stop movement in one direction, from one finger for at least 2 centimetres. An oscillation is a smaller movement that goes back and forward within a modest range. For the app these are the movements that occur within 2 centimetres, in multiple directions. Runs and oscillations can go several directions in the app. However, solving the apps' game demands interactions like moving up and down the screen with the left and right finger.

Fingers can also move sideways, but this movement has no influence on the game in the app. Fingers can also stand still on the screen. This non-motion is also taken into account in this study.

2. Method

2.1. Participants

The participant group included 49 students in the ages 12-15 from all four levels of vocational education from two vocational schools in the Netherlands. Both schools had the same general educational approach. Two mathematic vocational teachers were approached by email. The teachers selected 49 students at random to participate in the research. None of the participants wore glasses or heavy make-up and motor control of their hands was normal. Participant data with an eye-tracking accuracy below 70% were excluded (*n*=25). Three participants did not state the answer to the task and were therefore excluded. The remaining 21 students were divided into left- (*n*=2) or right (*n*=9) handedness within the parallel task and left (*n*=3) or right (*n*=7) handedness within the orthogonal task.

2.1.1. Ethical approval.

Informed consent was obtained from all 49 students with a consent form (Appendix A) and both mathematical teachers. All students volunteered to participate and no compensation was offered.

2.2. Instrumentation

2.2.1. Eye tracker.

Equipment to be used in the study is the Tobii X2-60 eye-tracker (Figure 2), with a frequency of 60 Hz. Data from the eye-tracker was collected using Tobii studio 3.22. The eye-tracker made it also possible for an external camera to film the scene on the iPad. The eye tracker and external camera were connected to a laptop with the Tobii studio program, which enabled the eye-tracking data, hand movement data and student statements to be saved.

Figure 2. The eye-tracker is placed on the stand base, and the iPad is attached in the centre. Adopted from Shayan, Abrahamson, Bakker, Duijzer and Van der Schaaf (2014).

2.2.2. Tasks.

The Mathematical Imagery Trainer for Proportion app was designed for embodied cognition for mathematics and salient irrelevant information within the tasks was kept to a minimum. The tasks of the app existed of four conditions: Two tasks containing parallel pluses and parallel bars with a 1:2 ratio and two tasks containing orthogonal pluses and bars with a 1:2 ratio (Figure 1). With the right proportion between the pluses or bars, the bars turn green or the entire screen with the pluses turns green. The tasks consisted of three task steps: an empty screen with pluses/bars, a grid with pluses/bars and a screen with a grid and number lines along the edges, with pluses/bars. Also, phases of learning occurred that could be measured through talk-aloud data; the phase in which the student was looking for the answer (search phase), the phase in which the student stated the right proportion (answering phase) and the phase in which the student verified the answer (verifying phase).

2.2.3. Parameters.

Parameters for this study, as described from literature, are categorised in Table 1 and 2.

Table 1

Parameters for Eye Movements

Spatial eye fixation		
	Bars	
Areas of Interest (AOI)	Irrelevant points	Uncategorised
Left bar top	Fingertips	Left bar ground
Right bar top	White space	Right bar ground
Numbers left		Left bar middle
Numbers right		Right bar middle
White space		Grid lines bold
between objects		Grid lines small
	Pluses	
AOI	Irrelevant points	Uncategorised
Left +	Fingertips	Space under left +
Right +	White space	Space under right +
Numbers left		Grid lines bold
Numbers right		Grid lines small
Fixation duration		Time a fixation lasts on one point
Fixation count		Amount of fixation on one point

Note: The areas of interest and irrelevant regions were based on former research results from Hayhoe and Ballard (2005), Johansson et al. (2001), Land and Hayhoe (2001) and Mcleod as cited in Hayhoe and Ballard, (2005). The specific regions of interest for the task were based on known didactics in Dutch education (Van Galen et al., 2005).

Table 2

Parameters for Hand Movements

Hand movements
Hand moves before/during/after eye fixation
Left or right hand runs up
Left or right hand runs down
Hands run simultaneously
Left or right hand stands still
Both hands stand still
Left or right hand oscillates

Note: Hand movement parameters were based on former research by Venjakob et al. (2012).

2.3. Procedure

One student at a time sat in a room with two or three researchers, the iPad and the eye-tracker. Before starting the task, the student was asked for consent. The room was lid by artificial light. The child was to solve four problems in the app: 1:2 with pluses, 3:4 with pluses, 1:2 with bars, 3:4 with bars. With each level, the student had to move up and down the screen with both fingers, keeping the screen or bars green. The entire process took approximately 45 minutes, of which five or six times of approximately 20-70 seconds of moving fingers upwards on the screen. During the entire process, one researcher sat behind a laptop to monitor the eye tracker and video program. Another researcher sat next to the child and gave instructions. The second researcher was not to give an answer to the problem in the app, only instructions how to use the iPad or to repeat previous statements from the child to help them with the problem solving (Appendix B). The video recordings of the eye tracker were then analysed by the researcher.

2.4. Data analysis

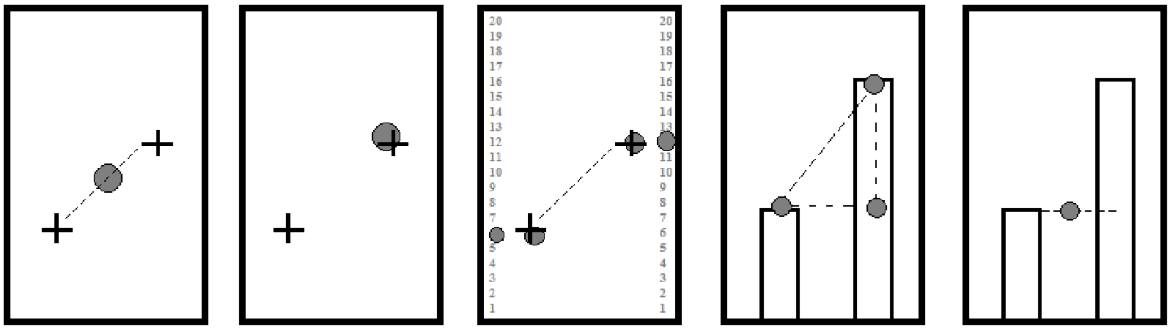
For the data analysis of the eye tracking data and hand movement data, the computer program Tobii Studio 3.22 was used. To start, videos of the 1:2 ratio task were selected with at least 70% of eye tracking data. For all 21 videos, the specific moments where the students move their hands simultaneously were noted as well as the moment the child stated the rule of the right proportion. To allocate the areas of interest, two videos of students with the parallel task and two videos of students with the orthogonal task were randomly selected per group and were observed during the simultaneous moments and coded using the AOIs based on former research. The moments were divided into phases by listening to the think-aloud data of the students: the search phase, the answering phase and the verifying phase; and task steps: empty screen, grid and numbers. Eye tracking movements, hand movements and thinking-aloud data were used. The sections of the task were then divided into relevant and irrelevant AOIs to compute the eye-tracking data log files in Tobii studio with the Tobii Fixation Filter. This was used for 11 students for the parallel task. For the other ten students, the data of the orthogonal task was not included due to measurement difficulties, as will be explained in the discussion. Mean fixation duration, fixation count and total fixation duration (fixation count times mean fixation duration per AOI) were computed.

3. Results

First, hand-movement data and eye tracking data of four students were observed to identify the areas of interest and to make a first draft of possible scan paths. Two students performed the parallel task, two students performed the orthogonal task. Second, the eye tracking data of al 21 students were computed using Tobii Studio. Relative fixation duration was calculated and analysed.

3.1. Preparatory exploratory observations parallel.

For the plus task, student 18, right handed, made saccadic movements between pluses and fixated in between pluses on the imaginary line or on a plus (Figure 3a).



3a. 3b. 3c. 3d. 3e.

Figure 3. Different fixation areas, saccades and scan paths as observed. Fixations in between pluses (a), fixations on the right plus (b), saccades to numbers (c), fixations and saccades in a triangular shape (d) and fixations in the bars' middle (e).

In the numbers task step, the numbers on both sides were included in the saccades as well (Figure 3b). The hand movements were simultaneous in the search phase, with both hand running up. At the beginning of the second task step, the hand-movements differed. The right hand ran up slowly while the left hand oscillated in order to make the screen green. When the screen turned green, both hands ran up again.

In the answering phase, the eyes fixated on the numbers eight and four at the edges of the screen (Figure 3c) before the hands moved. After running the hands to the numbers four and eight, the hands both did not move. At that moment, eye-tracking data disappeared because the student looked at the researcher to state the rule. During the verifying phase, the student fixated on the numbers and then made saccadic movements to the plus and the line corresponding with the correct number before running the hand to that line. This happened for both hands, until the screen turned green.

For the bars task, the student stated the answer after 16 seconds. Before that, during the search phase in the empty screen step, the student made saccadic movements between the middle of the right bar, and the tops of both bars in a triangular shape (Figure 3d).

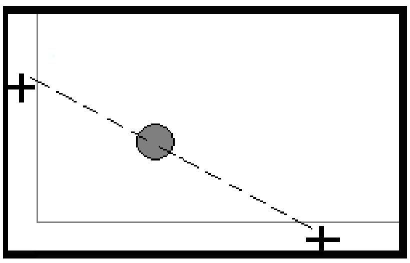
In the verifying phase, the student fixated on an imaginary line between the top of the left bar and the middle of the right bar (Figure 3e) while running with both hands. Only when the screen went red, both hands oscillated. In the numbers step, the eyes made saccadic movements between the top of the bars and both hands ran upwards.

Student 3, left-handed, also made saccadic movements between pluses in the plus task, search phase. This student, however, fixated on the right plus (Figure 3b), only to saccade to the left plus after the left hand started oscillating. In the number task, the student first fixates on the right plus while the hands stood still, then saccades from the pluses to the numbers (Figure 3c) while oscillating both hands to get a green screen. Then the student stated the rule. During the verifying phase, the eyes made saccadic movements from the right plus to the left and right numbers, to the left plus, while running with the right hand an oscillating the left hand.

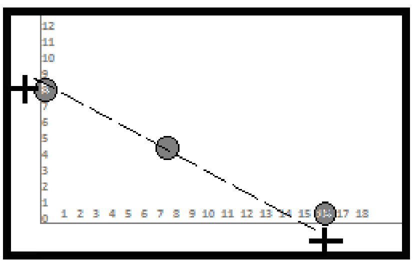
In the bars task, during the search phase, the eyes saccade between the top of the bars and the middle of the right bar (Figure 3d). The student oscillated both hands, first the right then the left. During the answering phase, the student did not state the rule literally. However, the student did state that there was a half and a whole. At this point, the eyes made saccadic movements between the top of the bars, and both hands were still. During the verifying phase, both hands ran, the eyes made saccadic movements between the top of the bars. The hands stood still when the students stated the complete rule, and oscillated after that, while the eyes made saccadic movements in the triangle shape again. To the end, the right hand ran and the left hand oscillated while the eyes fixated on the top of the right bar.

3.2. Preparatory exploratory observations orthogonal

In the first step of the orthogonal task, student 31 made saccadic movements between both pluses and fixated on a point in the middle of an imaginary line between the pluses (Figure 4a).



4a.



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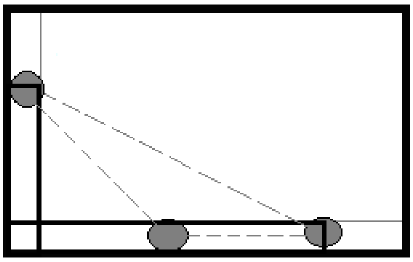

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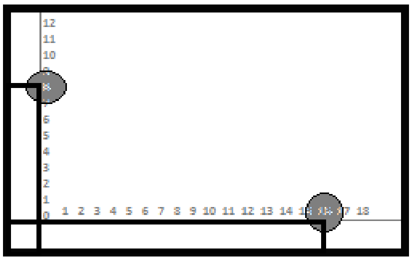
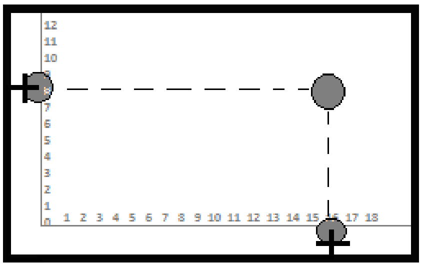


4d.



4e.

Figure 4. Different fixation areas, saccades and scanpaths as observed. Fixations and saccades were made in between pluses (a), in between pluses and numbers (b), in a triangle shape (c), at numbers or top of the bars (d) and in a rectangle shape (e).

The right hand ran while the left hand oscillated. During the grid step, the eyes made saccadic movements between pluses again and had a fixation in between the pluses, while both hands ran. The student stated the answer during the grid step, though not during the observed part of the video. Therefore, the student went from the search phase in the grid step to the verifying step in the numbers step. During the numbers step, the student made saccadic movements between the numbers left and right and the point in between the pluses (Figure 4b). During this time, the right hand ran and the left oscillated. Notably, the eyes fixated on the left numbers before moving the left hand, but the right hand moved before the eyes fixated on the right numbers.

During the bars task, in the search phase, the eyes started making saccadic movements from the top of the left bar, to the middle of the right bar (Figure 4c). Meanwhile, the right hand ran and the left hand oscillated. With the grid step, the hands both oscillated and moved before the eyes fixated on the top of the bar that was moving. Five seconds after this part of the task, the student stated the rule. The number step was therefore the verifying phase, in which the eyes made saccadic movements between the numbers left and right and fixated on the numbers left (Figure 4d). The eye movements occurred before the hands made running movements toward right number.

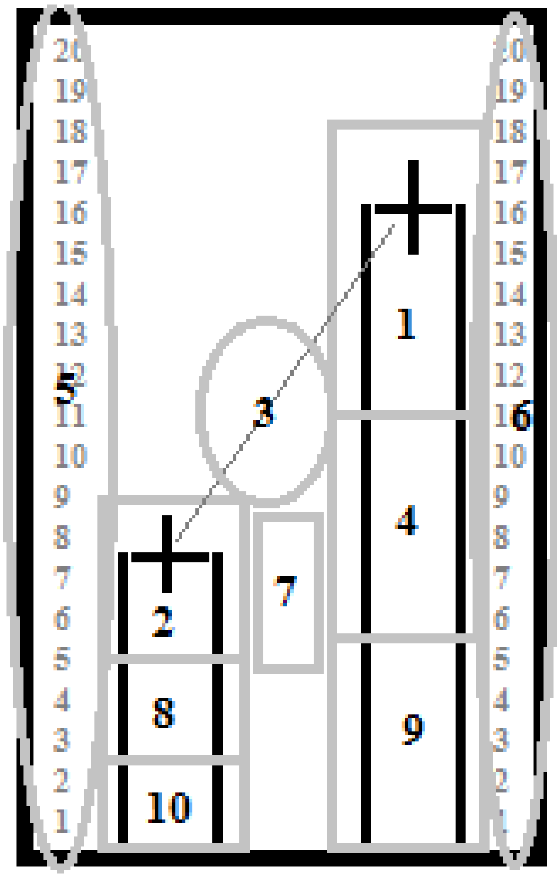
In the first task step of the plus task, student 45 fixated on the left plus and in between pluses and made saccadic movements between the left and right plus (Figure 4a). Both hands oscillated, but not at the same time. During the grid step, the eyes fixated in between pluses and made saccadic movements from the left plus to the right plus while both hands oscillated. During the first part of the numbers step, in the search phase, the eyes made saccadic movements between pluses, between numbers left and right and from the left plus, to an imaginary point at the height of both pluses (Figure 4e), to the right plus. Both hand kept oscillating. The student stated the answer during the number phase, thoguh not in the parts of the video that were observed. Therefore, there was a second time the numbers step was observed, but as the verifying phase. During the second numbers, the eyes fixated on the numbers left and right, before the hands oscillated to the numbers.

During the bar task, first step, the eyes made saccadic movements from the middle of the right bar, to the top of the left bar, to the top of the right bar and in between the top of both bars (Figure 4c). The eyes also fixated on the bottom of the right bar, while the student stated that he could not focus. The right hand made running movements during this step, the left hand oscillated. At the end of the empty screen step, the students stated the answer. The number step was therefore the verifying phase. During this last step, the eyes fixated in between bars and made saccadic movements from the top of the left bar, to the middle of the right bar, to the top of the right bar (Figure 4c). Both hands made running movements.

3.3. Areas of Interest

It was found that students looked at the pluses or top of the bars, the middle of the right bar and the numbers. Also, students tended to draw an imaginary line between pluses or top of the bars and made saccadic movements along that line or fixated in the middle of that line. The grid lines were looked at when they were accompanied by the numbers or once the student memorized the numbers and counted the lines. With the orthogonal task, it was found that both students looked at the top of the bars/pluses, the middle of the right bar and the numbers. Also, students made a imaginary line between the pluses/top of the bars and fixated in the middle of that line. With the bar task, both students fixated or made saccadic movements towards the middle of the right bar. One student made an imagined rectangular shape and made saccadic movements towards a point in the screen at the height of the top of both bars. Both students also made saccadic movements toward the bottom of the bars.

In Figure 5, the AOIs are presented as they appeared in the observations from the parallel task and in Figure 6, the AOIs are presented as they appeared in the observations from the orthogonal task. The right plus and left plus, also top of the right bar and top of the left bar are the areas that both students looked at most. Therefore, these are AOI 1 and AOI 2 in both Figures. The space between the pluses and the middle of the right bar were also fixated on, so they are defined as AOI 3 an AOI 4. Numbers on the left and right side were either fixated on or the eyes made saccadic movements to them. The left and right number lines are defined as AOI 5 and 6, respectively. With the parallel task, the space in between the top of the left bar and the middle of the right bar is AOI 7 (Figure 5). This AOI did not occur in the plus task, only in the bar task as with the triangle scan path (Figure 3d). In both tasks (Figure 5 and 6), AOI's 8 to 10 are uncategorised (Table 1) or not fixated on in the observations. These, however, are part of the salient objects with the bars and therefore taken into account. In the orthogonal task (Figure 4) the point in the screen at the height of both bar tops is AOI 11. AOI's number 1, 2, 3, 5, 6, 8 and 10 were used for the computing of eye tracking data for the parallel plus task, AOIs 1-10 were used for computing the eye tracking data of the parallel bars task (Figure 5). To compute eye tracking data for the orthogonal plus task, AOI 1, 2, 3, 5, 6, 9, 10 and 11 could be used and for the orthogonal bar task, the relevant AOIs are 1-11 (Figure 6).



Legend

1= RBT (right bar top or right plus)

2= LBT (Left bar top or left plus)

3= IBB (in between bars or pluses)

4= RBM (right bar middle)

5= NL (numbers left)

6= NR (numbers right)

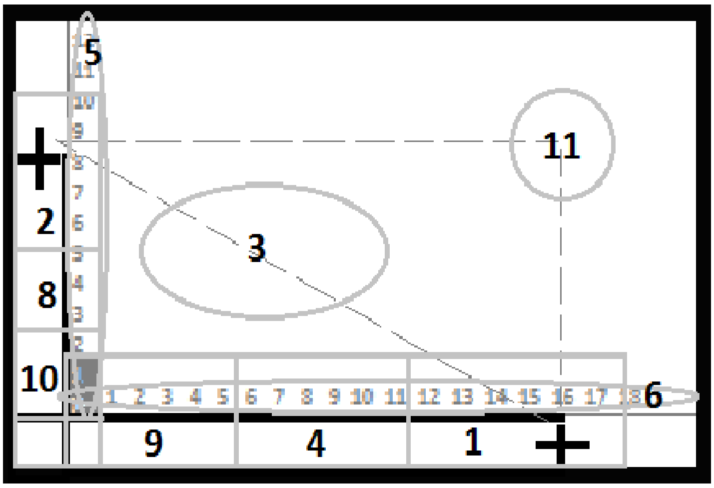
7= BB (between bars)

8= LBM (left bar middle)

9= RBB (right bar bottom)

10= LBB (left bar bottom)

Figure 5. AOIs and uncategorised areas according to observations.



Legend

1= RBT (right bar top or right plus)

2= LBT (Left bar top or left plus)

3= IBB (in between bars or pluses)

4= RBM (right bar middle)

5= NL (numbers left)

6= NR (numbers right)

8= LBM (left bar middle)

9= RBB (right bar bottom)

10= LBB (left bar bottom)

11= R (rectangle)

Figure 6. AOI's and uncategorised area's according to observations.

3.4. Total Fixation duration on AOIs Parallel.

To check the AOIs' importance in the parallel task, and to identify attentional anchors, data of eleven students was analysed for the parallel task using Tobii studios' metrics analysis. The AOIs were drawn into each video, for each task step. Per task step, there were 5 to 6 segments were students moved their fingers simultaneously while keeping the screen or bars green. The following metrics were calculated from the eye tracking data:

- Mean Fixation Duration (FD); average duration of a fixation per AOI.
- Fixation Count (FC); number of fixations per AOI
- Total Fixation Duration (TFD); total duration of all fixations per AOI
- Relative Total Fixation Duration (RTFD); TFD divided by the total time of the video segment. Early observations on the data showed that an RTFD of 15% was a suitable threshold to identify the most relevant AOIs, whereas an AOI with RTFD of <1% were considered irrelevant and are, therefore, excluded.

Table 3

Relative Total Fixation Duration per AOI per Student per Video Segment

Task step	AOI	3	27	7	10	12	14	17	18	20	24	25
Empty screen pluses	1	66.45%	59.02%	16.07%	48.91%	2.12%	5.40%	62.04%	36.05%	29.46%	25.28%	8.85%
	2	5.34%	4.44%	7.57%	3.82%	1.00%	8.28%	4.76%	0.78%	1.10%	2.57%	69.27%
	3	8.59%	3.13%	20.27%	6.64%	4.26%	0.56%	-	21.07%	1.90%	30.39%	0.51%
	9	-	-	-	-	-	-	-	-	-	-	-
	10	-	-	-	-	-	-	-	-	-	-	-
Grid pluses	1	25.88%	40.58%	43.89%	11.73%	23.00%	14.93%	57.37%	11.06%	23.27%	2.67%	11.65%
	2	5.42%	6.87%	-	9.67%	20.31%	2.77%	8.08%	21.28%	3.86%	7.42%	34.83%
	3	6.47%	4.44%	6.04%	-	24.74%	8.83%	19.79%	10.25%	12.61%	70.46%	3.30%
	9	-	-	-	-	-	-	-	-	-	-	-
	10	-	-	-	-	-	-	-	-	-	0.54%	-
Numbers pluses	1	47.19%	88.46%	3.08%	28.35%	30.61%	2.45%	20.60%	12.73%	20.56%	28.53%	26.52%
	2	9.64%	5.24%	3.12%	38.00%	3.93%	69.80%	30.08%	26.50%	17.06%	37.25%	20.93%
	3	1.94%	1.41%	0.52%	-	-	0.61%	0.25%	1.57%	1.83%	30.75%	0.19%
	5	34.17%	-	8.28%	15.29%	12.98%	1.98%	11.58%	8.27%	17.63%	6.05%	-
	6	2.97%	-	23.40%	0.12%	-	-	11.63%	1.27%	11.06%	7.73%	-
	9	-	-	-	-	-	-	-	-	-	-	-
	10	-	-	-	-	-	-	-	-	-	-	0.44%
Empty screen bars	1	66.67%	18.40%	29.84%	60.82%	16.77%	5.50%	49.50%	6.82%	8.89%	32.56%	40.38%
	2	26.76%	4.28%	16.24%	15.43%	26.97%	68.60%	0.10%	8.91%	0.54%	40.36%	23.10%
	3	-	9.70%	-	1.19%	1.55%	7.05%	1.40%	2.64%	-	2.28%	9.48%
	4	3.55%	-	9.12%	4.76%	36.81%	-	18.80%	22.64%	57.32%	5.75%	0.17%
	7	-	-	-	0.68%	0.29%	-	-	25.64%	1.57%	-	0.48%
	8	-	-	-	-	-	-	-	-	-	-	-
	9	-	-	-	-	-	-	-	-	-	-	-
	10	-	-	-	-	-	-	-	-	-	-	-
Grid bars	1	14.87%				62.10%	14.26%			6.14%	42.45%	
	2	18.84%				38.74%	68.39%			5.88%	51.44%	
	3	-				3.84%	1.09%			6.19%	0.95%	
	4	57.71%				11.03%	-			7.31%	1.14%	
	7	-				2.29%	-			9.45%	0.33%	
	8	-				0.32%	-			-	-	
	9	-				-	-			-	-	
	10	-				-	-			-	-	
Numbers bars	1	65.60%	23.18%	57.00%	16.69%	28.68%	24.43%	47.73%	26.12%	9.55%	38.77%	4.25%
	2	8.88%	32.75%	44.04%	12.6%	28.32%	18.70%	17.12%	25.28%	20.45%	45.83%	14.58%
	3	0.04%	2.05%	1.25%	-	0.54%	6.35%	0.67%	5.36%	0.45%	1.09%	0.42%
	4	-	-	-	-	-	-	0.52%	-	0.76%	-	-
	5	10.12%	1.13%	-	0.23%	15.64%	17.26%	12.63%	-	39.03%	1.60%	37.25%
	6	0.60%	2.55%	6.96%	0.22%	2.26%	-	19.04%	-	-	4.75%	-
	7	-	1.93%	1.00%	-	0.18%	-	-	-	-	1.42%	-
	8	-	-	-	-	-	-	-	-	-	-	-
	9	-	-	-	-	-	-	-	-	-	-	-
	10	-	-	-	-	-	-	-	-	-	-	-

Note. A dash indicates no fixations were made on that AOI. 10% of measurements were repeated by a second assessor and were found to be similar.

3.4.1. Plus task.

In the first task step, pluses with empty screen, AOIs 1, 2, 3, 9 and 10 were active. Within this step, only AOI 1, 2 and 3 were fixated on.

Table 4

Pluses with Empty Screen Longest Fixation on AOI

Student	Fixates >15% on AOI:	Fixates second >15% on AOI:	Fixates under 15% on AOI:
3	1		2, 3
27	1		2, 3
7	3	1	2
10	1		2, 3
12			3, 1, 2
14			2, 1, 3
17	1		2
18	1	3	2
20	1		3, 2
24	3	1	2
25	2		1, 3

Within this task step, AOI 1 was being fixated on by eight students for more than 15%. Three students fixated more than 15% on AOI 3. AOI 2 was only fixated upon by one student for more than 15%.

In the second task step, pluses with grid, AOI's 1, 2, 3, 9 and 10 were drawn. Only AOIs 1, 2 and 3 were fixated on.

Table 5

Pluses with Grid Longest Fixation on AOI

Student	Fixates >15% on AOI:	Fixates second >15% on AOI:	Fixates under 15% on AOI[a]:
3	1		3, 2
27	1		2, 3
7	3	1	2
10	1		3, 2
12			3, 1, 2
14			1, 2, 3
17	1	2	3
18	2		1, 3
20	1		3, 1
24	3		2, 1
25	2		1, 3

Note [a]: AOIs are in order of longest duration.

Within the second task step, AOI 1 was fixated on by six students above 15%. AOI 2 and 3 were equal, both with three students fixating for more than 15% of the total task time.

In the third task step, pluses with numbers, AOIs 1, 2, 3, 5, 6, 9 and 10 were drawn. AOIs 1, 2, 3, 5 and 6 were fixated on.

Table 6

Pluses with Numbers Longest Fixation on AOI

Student	Fixates >15% on AOI:	Fixates second >15% on AOI:	Fixates under 15% on AOI*:
3	1	5	2, 6, 3
27	1		2, 3
7	6		5, 2, 1
10	2	1, 5	
12	1		5, 2
14	2		1, 5
17	2	1	6, 5
18	2		1, 5, 3, 6
20	1	5, 2	3, 6
24	2	3, 1	6, 5
25	1	2	

Within the third task step, AOI 1 was fixated on by nine students for the duration of more than 15% of the total task step. Eight students fixated on AOI 2 for more than 15%. Four students fixated on

AOI 5 for longer than 15%. AOI 3 and 6 were only fixated by both one student for longer than 15% of the total time.

3.4.2. Bar task.

With the bar task, all three task steps were included. However, the step with the grid was only done by five students. The empty screen step and numbers step were done by all students.

In the first step, AOIs 1, 2, 3, 4, 7, 8, 9 and 10 were drawn. AOIs 8, 9 and 10 were not fixated on.

Table 7

Bars with Empty Screen Longest Fixation on AOI

Student	Fixates >15% on AOI:	Fixates second >15% on AOI:	Fixates under 15% on AOI:
3	1	2	4, 3
27	1		3, 2
7	1	2	4, 3
10	1	2	4, 3
12	4	2, 1	3
14	2		1, 3
17	1	4	3
18	7	4	2, 1, 3
20	4		1, 7
24	1	2	4, 3
25	1	2	3

Within this task step, AOI 1 was fixated on by eight students above 15% of the total task time. AOI 2 was fixated on by seven students for more than 15%. AOI 4 was fixated on by 4 students for over 15% and AOI 7 was fixated by one students for more than 15% of the total time. Five students fixated on AOI 7, but only one for a significant amount. Three students only fixated on the first three AOIs.

In the second step, AOIs 1, 2, 3, 4, 7, 8, 9 and 10 were drawn. AOIs 8, 9 and 10 were not fixated on. Not all eleven students participated in this step.

Table 8

Bars with Grid Longest Fixation on AOI

Student	Fixates >15% on AOI:	Fixates second >15% on AOI:	Fixates under 15% on AOI:
3	4	2	1
12	1	2	3, 4, 7
14	2		1, 3
18			1, 2, 3, 4, 5, 6, 7
24	2	1	4

Within this task step, AOI 2 was fixated on by four students for over 15% of the total time of the task. AOI 1 was fixated on by two students, and AOI 4 by one students for more than 15%.

Table 9

Bars with numbers longest fixation on AOI

Student	Fixates >15% on AOI:	Fixates second >15% on AOI:	Fixates under 15% on AOI:
3	1		2, 5
27	2	1	3, 5, 6, 7
7	1	2	3, 6, 7
10	1		2
12	1, 2	5	
14	1		
17	1	2, 6	5
18	1	2	3
20	5	2	1
24	2	1	3, 5, 6, 7
25	5	2	1

Within this task step, AOI 1 was fixated on by nine students for over 15% of the total time of the task. AOI 2 was fixated on by 8 students for more than 15%, AOI 5 was fixated on by four students and AOI 6 was fixated on by 1 student for 15 % of the total time of the task.

3.5. Summary of findings

Results from the video observations of the parallel plus task show that both students fixated on either the right or left plus or in between pluses. Eyes made saccadic movements from one plus to another, following an imaginary line. During the search phase, this was similar for the empty screen task step and grid task step. The left plus was fixated on when oscillating with the left hand. The right hand ran upwards. With the number step, both students made saccadic movements to the numbers as well.

Results of the video observations of the orthogonal plus task show that both students fixated in between pluses. Eyes made saccadic movements between pluses following an imaginary line. During the number phase, fixations were made on the numbers and in between pluses. Hands oscillated after eyes fixated on numbers. Only with one of the students, the right hand ran before the eyes fixated on the numbers right.

Video observations of the parallel bar task show that, during the search phase, one student fixated on top of both bars and the middle of the right bar. During the verifying phase, the fixation points remained the same. Hands ran upwards, only to oscillate when the task was incomplete. Eyes fixated on top of the bars while the hand ran.

Video observations of the orthogonal bar task show that both students fixated on the top of both bars and the middle of the right bar. For both students, the right hand ran, and the left hand oscillated. During the numbers phase, fixations were made by one students on the numbers left and in between bars. The other student fixated on the top of both bars and the middle of the right bar. Eyes fixated before the hands moved.

Subsequently, fixation points discussed in the method section were altered, having six fixation points (areas of interest) and five possible areas of interest in the form of other uncategorized points.

Relative total fixation duration of eleven videos for the parallel plus task showed that the right plus is the most important area of interest. The second most important area was the space in between the pluses. When adding numerals, especially numbers on the left side of the screen were an important area of interest. The bottom of the screen, AOI 9 and 10, is not fixated on.

For the parallel bar task, relative total fixation duration showed that the top of the right bar (AOI1) was the most important area of interest. Followed by the top of the left bar and the middle of the right bar and the left numbers. Results also show eye-movements trough the line between bar tops and the space between the bars at height of the left bar. This indicates that students use an attentional anchor in the shape of a triangle, or a line between bars, either between the top of both bars or a horizontal line at the height of the top of the left bar.

4. Discussion

The aim of this study was to identify cognitive visual search patterns linked to bodily activities for vocational students while solving the proportion tasks of the MIT-P app. It was proposed that these visual search processes could be divided into three concepts:

1. What gaze patterns can be found in tasks from the MITP-app and how do they relate to hand-movements?

2. What areas of interest are most important for the tasks in the MITP-app?

3. What attentional anchors can be derived from the search patterns?

4.1. Eye movements and their relation to hand-movements

Earlier research showed that eyes fixate on a target before the hand moves, to provide specific information to the motor system (Binsted et al., 2001; Land & Hayhoe, 2001; Hayhoe & Ballard, 2005). Results from the video observations of the parallel plus task show that students in this research fixated on the left plus while oscillating their left hand. Thus providing the motor system about the plus's movements. Also, during the verifying phase of both parallel and orthogonal tasks, eyes fixated on numbers and lines and made saccadic movements towards pluses before the hands moved the plus toward the line. This indicates that the eyes provided information about the known answer before the hand moved. These results indicate that students had an internal model during the verifying phase; they knew the right proportion (i.e. 1:2), and knew which numerals matched that proportion (i.e. 4 and 8). That means that they knew which numbers to hit to get a green screen. This was expected, since children in the Netherlands learn the 1:2 ratio as ½, and are thought the numerals that match this ratio (Van Galen et al., 2005).

Results of the video observations for the parallel bar task show that students' hands made running movements, while eyes where fixated on the top of both bars where the fingers were placed. According to Johansson et al. (2001) and Land and Hayhoe (2001), hands are irrelevant targets and eyes do not fixate on them, the manipulated object is only relevant to fixation before the hand moves.

Therefore, it is likely that students did not necessarily fixate on their hands. They did focus on the top of both bars. With the bar task, two objects needed to be manipulated. When two objects are to be manipulated, Land (2001) found that fixations are made between both objects and the action is completed under visual control, hence the fixations on the top of the bars.

Results of all tasks also showed no difference in used AOIs during the empty screen and grid task steps. Land and Hayhoe (2001) found little variation between subjects eye movements, because of the learnt internal models. However, during the empty screen and grid phase, the students had not stated the answer, and therefore an internal model was not likely to be present. A different explanation could be that students needed to perform the same task and knew which areas were to be manipulated. The grid lines could not be manipulated, and with a lack of numerals, students' did not see the relevance of the grid lines.

4.2. Relevant areas of interest

Areas of interest were important parts to measure the eye-tracking data and to see if attentional anchors occurred. For both parallel tasks the most important AOI that was the longest and most fixated on, was number 1, which was either the right plus or the top of the right bar. With the observation of the videos, it already seemed as if students fixated most on the right bar/plus, probably because this needed to be manipulated most, and had to be moved the quickest. Hayhoe and Ballard (2005) and Johansson et al. (2001) found that eyes only fixate on relevant targets for the task. The right plus/ top of bar therefore seems to be the most important target to play the tasks' game.

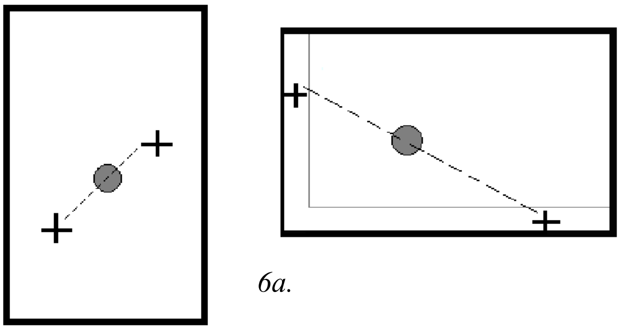
The second most important AOI was number 3, the space in between pluses for the plus task and number 2, the top of the left bar, for the bar task. Both are also relevant targets for the completion of the task. AOI number 2 was the third most important AOI for the plus task as well.

The numbers were also important AOIs. As already mentioned above, the numbers relate to internal models of students. However, students fixated more on the left number set than on the right. It is assumed that a practical reason for this is applicable. The right hand and arm seemed to block the right numbers, since the right plus/bar had to go highest. The left numbers were not blocked, and therefore, students had clear sight of the left numbers.

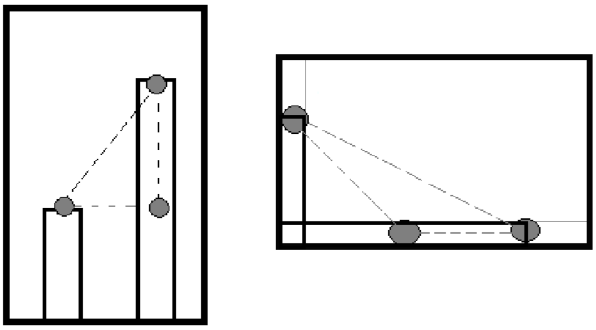
4.3. Attentional Anchors

During the plus task observations, an imaginary line seemed to appear between both pluses where the students fixated on for both the parallel and orthogonal plus task (Figure 6a). The results of the parallel relative total fixation duration showed that students only fixated on the pluses or the space in between, which confirms this finding for the parallel task. Both pluses were objects to be manipulated, which meant that, according to Land (2001), a number of fixations were made switching between the objects. This is also in accordance with the findings of Abrahamson & Sánchez-Garcia (2015), who found a similar vertical interval that facilitated subjects' performance of a task, making it less difficult to keep the screen green.

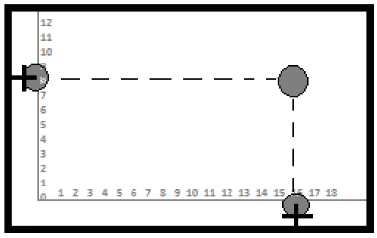
During the parallel and orthogonal bar task observations, all 13 students seemed to look along a triangular shape, fixating on the top of both bars and the middle of the right bar (Figure 6b). Also, the relative total fixation duration outcomes for the parallel task suggested fixations on top of both bars, in between the tops, on the middle of the right bar, and between bars at the height of the left bar. These findings correspond with findings of Shayan et al. (2015). Not only did they fixate on the two manipulated objects, they seemed to compare the height of the left bar to the middle of the right bar. This could indicate a simulated mental representation of the proportion. In the Dutch mathematical elementary school curriculum, students are taught 'half' (i.e. ½) , often with bar-like objects (Van Galen et al., 2005). This could mean that students have a mental representation of 'half', which could indicate a learnt internal model. Also, little variation was found, since all students seemed to use the triangle AA. This as well could indicate a learnt internal model (Land, Hayhoe, 2001). However, it remains unclear in this study whether students had an actual learnt internal model, since this study did not focus on the relation between think-aloud statements of the students and AAs.



6a.



6b.



6c.

Figure 6. Attentional anchors for parallel and orthogonal tasks in a linear shape (a) , a triangle shape (b) and a rectangle shape (c).

4.4. Limitations

The current study adds perspectives to research of visual search processes and embodied mathematical activities. However, a few remarks must be made toward the outcomes of this study. Firstly, due to issues with overlapping AOIs within the orthogonal tasks, the computed relative fixation durations of the ten videos of the orthogonal task were not reliable and, therefore, not taken into account in this study.

Secondly, the current study was an exploratory qualitative descriptive study and, as such, had a small participant group. The amount of participants was even smaller due to measurement difficulties for the orthogonal RTFD. Therefore, it must be taken into account that results of the orthogonal task in this study cannot be generalised. Results of the parallel task are supported by findings in other research, and can therefore be assumed to be right.

Furthermore, this study focused on the relevance of AOIs and the existence of AAs. The difference between students was only seen in terms of relevant AOIs and in sensorimotor activities during the small observation study. The difference in students learning outcomes or performance was not measured. Therefore, a fixed statement on the existence of learnt internal models cannot be made.

Finally, the participants in this study were divided into left- and right handedness. With the observations, no variation was found between participants. Due to the small amount of participants in the observations and the nature of the metrics of this study, the focus on left- and right handedness was discarded.

4.5. Future research

Following the limitations of this study, a follow-up quantitative research is suggested with a larger participant group. Within such a research, the focus can be laid on the relevance of the AOIs for the orthogonal task. For this focus, a solution needs to be provided for the measurement difficulties (i.e. the AOI overlap). The focus can also be on learning performance and learnt internal models, and a difference between left- and right handedness. To this goal, different relative metrics need to be used in order to be able to compare between subjects.

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Appendix A

Consent form for students (in Dutch)

Rechts/linkshandig Nummer:

Mijn naam is …………………………………………………..

Ik ben ……………………… jaar oud.

Ik zit in klas …………….. van VMBO basis/kader/gemengd/theoretisch.

Ik doe mee aan dit onderzoek en vind het goed als mijn gegevens anoniem worden gebruikt door de onderzoekers.

Handtekening Datum

…………………………………………………………………………………………………….

Appendix B

Interaction protocol students-researcher (in Dutch)

<u>Voor het begin</u>
Participant komt binnen en gaat zitten achter de iPad bij de eyetracker. De onderzoekers vertellen dat:

- De persoon een wiskunde app gaat uitproberen.
- Bij deze app moeten zij hun beiden vingers gebruiken om balken/plussen op het scherm te bewegen.
- Het doel is om erachter te komen wanneer de balken/het scherm groen wordt.
- Ze beginnen onderaan.
- Verder is het belangrijk dat zij rechtop zitten om de ogen te zien en de handen niet voor de eyetracker zitten.
- Ook moeten ze alles hardop vertellen.

Dan worden de ogen gekalibreerd van de participant en het programma op de iPad klaar gezet.

<u>Vanaf begin</u>

1. Vertellen waar ze moeten beginnen
   Je mag beginnen. Leg je vingers onderin het scherm.
2. Herhalen dat de opdracht is dat de balken groen worden en dat er een regel is waarom ze groen worden.
   Je mag de balken groen maken, en zorgen dat ze groen blijven als je beweegt.
3. Herhalen dat ze hardop moeten praten.
   Probeer alles hardop te zeggen wat je denkt.

4. Helpen met vinden van de interactie objecten (balk, plus). Aangeven waar hun vingers het scherm moeten raken om het interactie object te vinden.
5. Aangeven dat het de bedoeling is dat participanten in interactie blijven met de app. Door vingers op het scherm te houden.
6. Ervoor zorgen dat de eyetracker de participant kan blijven zien.
Je mag wat meer rechtop zitten. Kijk nog eens naar de eyetracker. Kan je je armen anders houden?

Niveau 2/3 (met lijnen en cijfers)

7. Aangeven dat de lijnen en cijfers er zijn voor gebruik.
8. Punt 2 en 3 blijven herhalen waar nodig.
9. Ter verduidelijking de participant vragen wat hij/zij aan het doen is en waarom. Allen open vragen stellen. (Zie hieronder voor vragen.)
10. Participant op weg brengen naar het juiste antwoord.

11. Bij alle niveaus wordt na een korte periode van ontdekken gevraagd of de participant over het gehele scherm kan bewegen met de vingers terwijl het scherm of de balken groen blijven.
Kun je nu helemaal omhoog gaan met de balken en ze groen houden? Kun je helemaal naar de zijkanten met de plussen en het scherm groen houden?

Einde
Wanneer de participant alle drie de niveaus heeft gedaan en kan verwoorden wat de regel achter de taak is (in eigen woorden), heeft de participant de taak volbracht. Degene vult dan nog een vragenformulier (waar de onderzoekers op noteren of de persoon links/rechtshandig is, een man of vrouw en het nummer van de participant) in en mag dan terug naar de groep.

Vragen
(Bij onderstaande vragen kan balken worden vervangen door plussen/kruisen)

Vragen die de onderzoekers kunnen stellen aan de participant om hen te begeleiden, zonder wat voor te zeggen zijn:

- **Voor het geven van aanwijzingen in het begin:**
  Kan je de balken ook stil houden en groen maken? (als ze teveel bewegen) Kan je nog meer groene balken vinden? (als ze niet bewegen) Kun je bovenin/onderin ook groene balken vinden?

- **Voor het verwoorden van de taak en hun acties:**
  Hoe doe je dat? Hoe heb je dat gedaan?

- **Algemene vragen om hen op een spoor te brengen:**
  Waarom is dat zo? Hoe komt dat? Kan je iets zeggen over…?

- **Om verder te laten ontdekken, als het niet vanzelf gaat:**
  Kan je je handen bewegen terwijl je de balken groen houdt? Is dit overal op het scherm hetzelfde? Kan je van beneden naar boven de balken groen houden? Probeer op het laagste punt groene balken te vinden en beweeg ze naar boven terwijl je ze groen houdt.

- **Om hen te laten verwoorden wat zij doen om het doel te ontdekken:**

Wat moet je doen om de balken groen te houden? Kan je laten zien wanneer de balken groen worden? Wat valt je op aan de balken? En als je dit onthoudt wanneer je de balken omhoog brengt. Klopt dit dan nog steeds?

- **Om hen het doel van de taak te laten ontdekken:**
  Wat moet je doen om de balken groen te maken? Wanneer worden de balken groen?

- **Om hen het doel van de taak te laten verwoorden:**
  Kan je uitleggen wat je nu hebt gedaan? Zou je aan iemand anders kunnen uitleggen wat de bedoeling is van deze app? Ben je erachter wat de regel achter de balken is om groen te maken?