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COLD ATOM NANOPHOTONICS GROUP

Cavity Ring-Down Spectroscopy

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June 17, 2015

Abstract

In the Cold Atom Nanophotonics group a Bose-Einstein condensate of photons is being realized using a dye-filled microcavity. One key element of this experiment is the high reflectivity of the mirrors forming the microcavity.

In this thesis we report on cavity ring-down spectroscopy. This technique was used to measure the reflectivity for a set of identical high reflectivity mirrors from CRD Optics. For wavelengths ranging from 560 nm to 580 nm, we measured reflectivities of $R \approx 99.8\%$. This is a factor 1000 less reflective than the expected reflectivity of R = 99.9998%. We review factors which could contribute to this difference, most noteworthy being the cleanliness of the mirrors.

Contents

1	Intr	roduction	1				
2	The	eory	2				
	2.1	Optical Cavity	2				
	2.2	Resonant Modes	4				
	2.3	Cavity Ring-Down	5				
3	Me	thods	7				
	3.1	Cavity Ring-Down Setup	7				
	3.2	Beam Focusing	9				
	3.3	Cavity Length Calibration	10				
4	\mathbf{Res}	ults	11				
	4.1	Ring-Down	11				
	4.2	Calibration Measurement	11				
	4.3	Reflectivity	12				
5	Dis	cussion	14				
6	Conclusion						
7	Ack	nowledgements	20				
\mathbf{A}_{j}	ppen	dix A Transmission Spectrum	21				
$\mathbf{A}_{]}$	Appendix B Python Code						

1 Introduction

In the field of nanophotonics the interaction of light and matter is studied on a fundamental level. One of the studied phenomena is the Bose-Einstein condensate (BEC). A BEC can be described as an overlap of bosonic matter waves in their quantum mechanical ground state. For massive bosons this is achieved by cooling them towards temperatures very close to absolute zero [1, 2]. Because photons are also bosons, it was theorized these could also form a BEC [3]. The first experimental observation of a photon BEC was made in 2010 [4–6]. Here, a laser was focused inside a dye-filled optical microcavity, which enabled number-conserving thermalization of photons. The cavity mirrors are used to provide a confining potential for the photonic BEC [4]. For these mirrors both the shape and their reflectivity determine the magnitude of the trapping potential. It is of importance to use high-reflectivity mirrors to reach the critical photon density where condensate formation occurs. Currently two photon BECs exist in the world [7,8]. The first uses a cavity formed by two identical spherical mirrors with a reflectivity of $R \approx 99.998\%$ for each individual mirror [9]. The second uses one planar and one spherical mirror with reflectivities of $R \approx 99.95\%$, as can be seen in Appendix A.1.

The cold atom nanophotonics group at Utrecht University is also working to realize a BEC of photons. The wavelength dependent reflectivities of the to-be-used mirrors determine which wavelength will be used for the photon BEC. It is therefore important to measure the reflectivity of these cavity mirrors for a certain spectrum. In this thesis a technique called Cavity Ring-Down Spectroscopy (CRDS) [10] is used to measure the reflectivity for a set of identical spherical mirrors from CRD Optics. This is performed for a wavelength spectrum ranging from 560 nm to 580 nm.

In ring-down measurements, first, a build up of monochromatic light is formed inside the cavity. After a certain power is reached, the laser focused inside the cavity is blocked. This way the exponential decay, or "ring-down", of the light inside the cavity can be measured. The ring-down time determines the light leaving the cavity per round trip, which is the exact quantity needed to determine the reflectivity of the cavity mirrors [11, 12].

We will first explain the basis of the theory needed to understand the behaviour of light within an optical cavity in Section 2, as well as the relation between ring-down time and reflectivity. We will then incorporate this theory with the description of the setup in Section 3. Finally, the results of our experiment are shown in Section 4, and thoroughly examined by discussing the possible elements which could interfere with a ring-down measurement in Section 5.

2 Theory

In this section we will first describe the necessary optics to understand how a cavity is formed and what its behaviour is. Then we will give a short insight into higher order modes, after which we will finish with the theory of cavity ring-down.

2.1 Optical Cavity

An optical cavity is created by placing two mirrors some distance from each other on the optical axis of a laser beam. These mirrors can be flat or curved, which leads to an arrangement of different light patterns inside the cavity.

The propagation of optical rays through several optical elements can be described by the ABCD matrix [13]

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}.$$
 (2.1.1)

In the case of light propagation inside a cavity, we are concerned with the following two ABCD matrices

• Propagation over a length L

$$\begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}. \tag{2.1.2}$$

• Mirror with curvature R

$$\begin{pmatrix} 1 & 0 \\ -2/R & 1 \end{pmatrix}. \tag{2.1.3}$$

The actual propagation of light in the cavity is now described by the product of the ABCD mactrices in Equations 2.1.2 and 2.1.3. Hence, for one round trip inside a cavity,

$$M = \begin{pmatrix} 1 & 0 \\ -2/R_1 & 1 \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -2/R_2 & 1 \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix},$$

$$= \begin{pmatrix} 1 - \frac{2L}{R_1} & 2L - \frac{2L^2}{R_2} \\ -\frac{2}{R_1} + \frac{4L}{R_1R_2} - \frac{2}{R_2} & -\frac{4L}{R_1} + \frac{4L^2}{R_1R_2} - \frac{2L}{R_2} + 1 \end{pmatrix}.$$
 (2.1.4)

To apply this formalism to gaussian beams, we introduce the q-parameter. This parameter defines the gaussian beam at a certain point z, i.e. the path along the optical axis, according to

$$\frac{1}{q(z)} = \frac{1}{R(z)} + \frac{i\lambda}{\pi w^2(z)},$$
(2.1.5)

where R(z) denotes the curvature of the beam, λ the wavelength, and w(z) the beam waist at a certain path length z. We will use the subscript 0 to denote z = 0, i.e. the center of the cavity.

If we choose $R_0 = \infty$ for q_0 , then

$$R(z) = z + \frac{z_R^2}{z},$$
(2.1.6)

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_R}\right)^2},$$
 (2.1.7)

and
$$z_R = \frac{\pi w_0^2}{\lambda}$$
. (2.1.8)

Where Equation 2.1.8 denotes the Rayleigh range of the beam. This range is the distance from z_0 for which the beam waist w_0 increases by a factor $\sqrt{2}$.

The ABCD parameters then determine the q-parameter after propagation via

$$q_{\rm f} = \frac{Aq_{\rm i} + B}{Cq_{\rm i} + D},\tag{2.1.9}$$

where the subscripts i and f denote initial and final, respectively, and the ABCD parameters are the same as used in the ABCD matrix formalism.

For a stable cavity it has to hold that $q_f = q_i$. In other words, the beam must remain identical after a round trip. This condition allows Equation 2.1.9 to be written as

$$Aq_{\rm f} + B - Cq_{\rm f}^2 - Dq_{\rm f} = 0. (2.1.10)$$

Another boundary condition is given by the curvature of the mirrors. For a stable cavity the curvature of the gaussian beam has to match the curvature of the mirrors. Thus, $R(z_1) = -R_1$ and $R(z_2) = R_2$, where z_1 and z_2 denote the positions of the respective mirrors along the optical axis as can be seen in Figure 2.1.

Solving this system of equations while defining the cavity length $z_2 - z_1 = L$, yields

$$z_1 = \frac{-L(R_2 - L)}{R_1 + R_2 - 2L}, \ z_2 = \frac{L(R_1 - L)}{R_1 + R_2 - 2L},$$
(2.1.11)

$$z_R^2 = \frac{L(R_1 - L)(R_2 - L)(R_1 + R_2 - L)}{(R_1 + R_2 - 2L)^2}.$$
(2.1.12)

Using Equation 2.1.8 we find

$$w_0^4 = \frac{\lambda^2 L(R_1 - L)(R_2 - L)(R_1 + R_2 - L)}{\pi^2 (R_1 + R_2 - 2L)^2},$$
(2.1.13)



Figure 2.1: A visual guide to the relation between beam waist, cavity length and position. The red lines denote the width of the laser beam, the curved black lines denote the cavity mirrors, and the curved dashed black line denotes the curvature of the beam.

$$w_1 = w_0 \sqrt{1 + \left(\frac{z_1}{z_R}\right)^2}$$
, and $w_2 = w_0 \sqrt{1 + \left(\frac{z_2}{z_R}\right)^2}$. (2.1.14)

Where w_1 and w_2 denote the beam waist on the first and second cavity mirror, respectively. For clarity, the implication of these equations is shown in Figure 2.1.

2.2 Resonant Modes

The electric field inside a cavity for higher order modes is given by [14]

$$E_{mn}(x,y,z) = \frac{E_0 w_0}{w(z)} H_m\left(\frac{\sqrt{2}x}{w(z)}\right) H_n\left(\frac{\sqrt{2}y}{w(z)}\right) e^{ikz - i(m+n+1)\phi(z)} e^{\frac{ikr^2}{2R(z)}} e^{-\frac{r^2}{w^2(z)}}.$$
(2.2.1)

Here, m and n denote the order of the mode, $r = \sqrt{x^2 + y^2}$, k the wavenumber of the transversal wave, and ϕ the phase. H_m and H_n denote the Hermite polynomials, and E_0 a normalization constant. The corresponding pattern for intensity is

$$I_{mn}(x,y,z) = I_0 \left[H_m\left(\frac{\sqrt{2}x}{w(z)}\right) e^{-\frac{x^2}{w^2}} \right]^2 \left[H_n\left(\frac{\sqrt{2}y}{w(z)}\right) e^{-\frac{y^2}{w^2}} \right]^2,$$
(2.2.2)

where I_0 denotes a normalization constant.

These different intensity patterns are called the Transverse Electromagnetic Modes (TEM_{mn}). In Figure 2.2 several TEM_{mn} intensity patterns are displayed. When



Figure 2.2: Different TEM_{mn} intensity patterns calculated with Equation 2.2.2, where m and n indicate the order of the mode.

both mirrors are aligned, and the laser is focused in between the mirrors, the only possible mode is TEM_{00} , the ground state. Due to inherent vibrations of the system, slight misalignment of the cavity occurs, which can result in higher order modes.

2.3 Cavity Ring-Down

The decay of light within a cavity is due to the non-perfect reflectivity of the mirrors when absorption is negligible ($\alpha \approx 0$). The beam power measured behind the cavity is given by [9]

$$I(t) = I_0 R_1^{\kappa} R_2^{\kappa}, \qquad (2.3.1)$$

where I_0 denotes the power of the laser, R_1 and R_2 the reflectivity of the respective mirror and κ the number of round trips in the cavity. Because κ is the number of round trips in the cavity, it can also be written as

$$\kappa = \frac{t}{t_r} = \frac{ct}{2L},\tag{2.3.2}$$

where t_r denotes the time needed for one round trip, c the speed of light and L the length of the cavity.

The power can then be written as

$$I(t) = I_0 e^{-\frac{t}{\tau}},$$
(2.3.3)

where

$$\tau = \frac{-2L}{c\ln R_1 R_2},\tag{2.3.4}$$

denotes the ring-down time.

There are two possible cases to determine the reflectivity. If both mirrors have the same specifications, their reflectivities should be the same. Thus, in our case, we have

• $R_1 = R_2 = R$ Rewriting Equation 2.3.4 gives

$$R = e^{-\frac{L}{c\tau}}.$$
(2.3.5)

If the mirrors are different, their reflectivities are presumably also difference. Then,

• $R_1 \neq R_2$ Again using Equation 2.3.4 gives

$$R_1 = \frac{1}{R_2} e^{-\frac{2L}{c\tau}}.$$
 (2.3.6)

3 Methods

In this chapter we will first provide an overview of the setup. We will start with discussing the laser and the electrical appliances. Next we will discuss the cavity mirrors and the lenses. We will finish by discussing the determination of the cavity length.

3.1 Cavity Ring-Down Setup



Figure 3.1: Setup schematic. The numbers right of the AOM denote the diffraction orders, the red lines denote the laser beam, the thinner black lines denote signal cables, and the L# the used lenses. Note the orientation of curvature of the lenses.

The setup used in this experiment is shown in Figure 3.1. For clarity, each individual aspect is described below in the order the laser passes through them.

First, a dye laser is used to couple light into the cavity. The laser used in this experiment is a coherent model 899-01 ring laser [15]. It is common for a dye laser to have a broad bandwidth. For CRDS it is important to reduce this to a minimum, to counteract the formation of multiple modes inside the cavity. For this purpose an intracavity element is used to narrow the bandwidth and thus make the laser more stable. When using a pump laser at a power of 5.5 W, the dye laser has a power output of ~800 mW. The laser is stable between 560 nm to 580 nm.

Next, the laser is sent through an acousto-optic modulator (AOM), this is a device which diffracts light into different orders using the acousto-optics effect [16]. In short, this effect changes the permittivity of the crystal inside due to a mechanical strain



Figure 3.2: Camera images of different TEM_{mn} modes on the back cavity mirror, imaged by the CCD shown in Figure 3.1.

caused by a Piezo-electric inducer. Here we use the first diffraction order of the laser at its highest power, which is 89.5% of the incoming power, while using a beam stop to block the other orders. The first order is then focused into the cavity using two lenses, L1 and L2 as shown in Figure 3.1.

The cavity itself is composed of the first mirror on a regular mirror mount, and the second mirror on a xyz-translation stage. A Piezo-element is embedded in the z-axis of the translation stage. By applying a voltage to the Piezo-element, using a tone-generator, nanoscale adjustments to the cavity length can be made. This allows us to achieve the perfect distance for the TEM₀₀ resonance [17].

The cavity mirrors are a set of curved 1 inch mirrors from CRD Optics, with a curvature of R = 1 m. Because the mirrors have the same curvature and should have the same reflectivity, Equation 2.3.5 can be used as the relation between reflectivity, ring-down time, and cavity length. The fundamental mode of the cavity can then be determined using Section 2.1. Using Equations 2.1.12 and 2.1.13, the Rayleigh range and beam waist in the middle of the cavity are found to be $z_R = 92$ mm and $w_0 = 130 \,\mu\text{m}$ respectively, for a cavity length of $L = 17 \,\text{mm}$ and a wavelength of $\lambda = 580 \,\text{nm}$. The beam waist on the mirrors is then $w_1 = w_2 = 131 \,\mu\text{m}$.

After the cavity, the beam is sent through a 45:55 (R:T) beam splitter, where one part of the beam is focused on a photodetector (using lenses L3 and L4), and the other part is enlarged and imaged on a CCD camera (using lenses L3 and L5). The CCD images the light on the second cavity mirror, thus telling us if the cavity is aligned properly when only the TEM₀₀ mode is observed. An example of different modes obtained with the CCD can be seen in Figure 3.2. From the comparison with Figure 2.2, the TEM_{$mn \in \{00,10,20\}$} can be identified. The used photodetector is an avalanche type¹, sensitive in the regime of 500 nm to 700 nm. An avalanche type was needed because its sensitivity enables us to measure the signal.

The signal from the photodetector is then sent to the comparator. When the signal

¹Thorlabs Temperature-Compensated Si Avalanche Photodetector, model APD130A2/M.

of the photodetector is above a tunable threshold, the comparator sends a signal to the AOM. This eliminates any diffraction order of the AOM. This stops light from entering the cavity, enabling us to measure the ring-down of the light inside the cavity. The resulting ring-down signal is then displayed on an oscilloscope, which we use to store the ring-down measurements.

Our measurements are performed for a wavelength range of $564 \,\mathrm{nm}$ to $579 \,\mathrm{nm}$, in intervals of $3 \,\mathrm{nm}$ for a fixed cavity length. For each wavelength a set of 500 ring-down measurements is performed.

3.2 Beam Focusing

To determine which lenses to use and which distances to put them at, the Python script in Appendix B was used in combination with a set of measurements of the beam waist on the optical axis. The beam waist along the optical axis is measured by fitting a gaussian over the CCD image of the beam, whose full width at half maximum is 2w(z). This is done for multiple positions on the optical axis to determine the curvature of the beam. This curvature describes the course of the beam. The script then determines the position and focal lengths for L1 and L2, using the measured beam waists, so the incoming beam is focused on the cavity. For a lens L1 with a focal length of $f_1 = 200 \,\mathrm{mm}$ and a lens L2 with a focal length of $f_2 = 50 \,\mathrm{mm}$, a beam waist of $w_0 \approx 160 \,\mu\text{m}$ was determined inside the cavity when the distance of L1 and L2 to the first cavity mirror is 325 mm and 50 mm respectively. The Rayleigh range of the laser inside the cavity, using Equation 2.1.8, is $z_R = 139 \,\mathrm{mm}$. Using Equation 2.1.6, the corresponding position of the cavity mirrors are calculated, this gives the cavity needed for the laser a length of $L \approx 40 \text{ mm}$. Because the difference between the fundamental position and the calculated position is a factor 5 smaller than the Rayleigh range, this difference shouldn't effect the mode formation inside the cavity. The beam waist is also a approximately a factor 160 smaller than the size of the mirrors, so no scattering due to boundary effects should therefore occur.

The lenses after the cavity were chosen such that light leaving the cavity is enlarged on the CCD and shrunk on the photodetector. Note that the magnification for two lenses is given by $\frac{f_x}{f_y}$ when the distances involved are a distance of f_x from the second lens to the image, a distance of $f_x + f_y$ between the two lenses, and a distance of f_y from the source to the first lens.

In this setup we used the lenses L3 with $f_3 = 75 \text{ mm}$, L4 with $f_4 = 15 \text{ mm}$, and L5 with $f_5 = 100 \text{ mm}$ as can be seen in Figure 3.1. This results in magnifications of 0.2 on the photodetector, and 1.33 on the CCD.

3.3 Cavity Length Calibration

A high accuracy determination of the cavity length is needed to most precisely determine the reflectivity of the mirrors. Because our cavity is mounted using two separate bases, we cannot use only the indicator on the z-axis of the translation stage. An adjustment is needed to provide the distance difference between the indicated and the actual length. After this difference is determined, the indicator can be used as it is then calibrated.

To determine this adjustment we use the relationship given by Equation 2.3.4, where we note that there is a linear dependence between cavity length L and ring-down time τ . When measuring ring-down times as a function of different cavity lengths, a linear fit on a scatter plot can be made to determine for which distance the ring-down time goes to zero, this point is also where the cavity has length L = 0 mm. The horizontal translation of the intersection of the fitted line with the horizontal axis to the origin is exactly the length difference with the indicator.

4 Results

We will first discuss our analysis of the acquired data, after which we will show the results of the cavity length calibration. Finally the resulting reflectivities for each wavelength measurement are shown.

4.1 Ring-Down

For each measured ring-down time, a fit was made using the exponential decay model from Equation 2.3.3. Both I_0 and τ are used as free parameters. The initial time from where the fit is started, corresponds to 70% of the maximum signal. One example of such a fit is shown in Figure 4.1.



Figure 4.1: The photodetector signal as imaged on the oscilloscope. The signal as function of time is visible as the blue line. An exponential fit according to Equation 2.3.3 is visible as the red dashed line, the τ in this equation denotes the ring-down time.

4.2 Calibration Measurement

Our calibration measurement was performed for lengths of 7, 14, and 20 mm of the second mirror, as indicated by the translation stage. The result and fit are shown in Figure 4.2. The fit shows a displacement of (18.07 ± 4.83) mm, this taken together with the used indicated translation length of 17 mm, forms a cavity with a total length of $L = (35.07 \pm 4.84)$ mm.



Figure 4.2: Calibration measurement of cavity length. Green dots denote the ring-down times measured by the photodetector. The blue line is a linear fit to these points. The intersection point at (-18.07 ± 4.83) mm, which determines the adjustment of the cavity length, is indicated by the red triangle.

4.3 Reflectivity

For each measured wavelength, the 500 resulting ring-down times are taken together in a histogram. An example of such a histogram is visible in Figure 4.3. The mean and standard deviations of the ring-down times are extracted from these histograms. Together with the determined cavity length from the calibration measurement, these are used to calculate the reflectivities of the mirrors using Equation 2.3.5.

Wavelength (nm)	Ring-down time (ns)	Reflectivity (%)
564.0 ± 0.5	58.9 ± 243678.53	99.8016 ± 820.0443
567.0 ± 0.5	57.94 ± 62306.29	99.7983 ± 216.7168
570.0 ± 0.5	59.58 ± 6.41	99.8038 ± 0.0342
573.0 ± 0.5	57.3 ± 5.8	99.796 ± 0.0348
576.0 ± 0.5	65.32 ± 4.93	99.821 ± 0.0281
579.0 ± 0.5	59.63 ± 3.69	99.804 ± 0.0295

The resulting reflectivities are shown in Table 4.1.

Table 4.1: Measured reflectivities for the used wavelengths.



Figure 4.3: An example of a histogram of determined ring-down times for 500 single measurement shots, as shown in Figure 4.1, for a single wavelength. Shown here is the histogram for a wavelength of $\lambda = (576.0 \pm 0.5) \text{ nm}.$

5 Discussion

The reflectivities shown in Table 4.1 are smaller by a factor 1000 than we would expect from the manufacturers data sheet shown in Appendix A.2. Furthermore, the measurements for a wavelength of $\lambda = 564$ nm and $\lambda = 567$ nm have deviations bigger than their mean value. This indicates something unexpected is happening in the fitting process. The cavity length is about twice as big according to the calibration measurement in respect to when guessed by eye, this will also be discussed.

An initial test to determine whether the determined reflectivities are of the right order is to measure the transmitted power for the individual mirrors. This provides a coarse indication for the reflectivity of the cavity because scattering effects are ignored. The transmitted power is first measured for a setup where no cavity is present and only a neutral density filter is placed before the photodetector, this filter entails a transmission of 0.001% of the incoming beam. The height of this signal is then compared to when a single cavity mirror is placed in the setup without the use of a filter. The measured signal when only a filter is placed is 340 mV, the signal is 275 mV and 380 mV for the first and second cavity mirror respectively. A lower signal means a higher reflectivity, while a higher signal means a lower reflectivity. An average for the factor between our mirrors and a neutral density filter is

$$\frac{\frac{275+380}{2}}{340} \approx 0.96$$

so the transmission is of the same order for both. Therefore the reflectivity should be of the order of 100 - 0.001% = 99.999%. This gives a hint that the measured reflectivities shown in Table 4.1 are lower than we expect them to be.

One occurrence we encountered was for the individual ring-down measurements, these would sometimes show a rise immediately after a ring-down as shown in Figure 5.1. Because these measurements don't follow the exponential decay specified in Equation 2.3.3, our fitting procedure wouldn't fit these correctly. We manually looked at all the measurements where such a rise occurred and concluded the only problem which occurred for our fitting procedure was this rise. We then determined the reflectivity again when discounting these measurements. The newly determined reflectivities are shown in Figure 5.2. For comparison, in Figure 5.3 histograms for before and after discounting are shown side by side for a single wavelength. Because the AOM is turned off for 100 µs by the comparator, no rise should be able to occur after this ring-down until the AOM is turned back on.

When comparing the measured reflectivities before and after the cleaning of outliers, a decrease in the uncertainty of the ring-down time is shown, while the reflectivity is of the same order for both. This is therefore no explanation for the magnitude of difference in reflectivity.

We also measured the time in which the first order of the AOM swtiches off by



Figure 5.1: An example of a quick rise of the power, immediately after a ring-down. The blue line is the photodetector signal, the exponential fit is visible as the red dashed line.



Figure 5.2: Reflectivity spectrum of Table 4.1 after discounting the measurements where no flat ending slope exists.



Figure 5.3: A visual comparison of the determined ring-down times before and after the discounting of outliers.

removing the cavity mirrors, and triggering the comparator manually. The measured decay is $\tau = (74.32 \pm 1.31)$ ns. This is of the same order as the measured ring-down times for different wavelengths. Because these times are so comparable, it is possible that the measured ring-down times are mostly due to the speed of the AOM. This doesn't explain the linear dependence between decay time and cavity length seen in Figure 4.2.

Next we look at the calibration measurement. It was discovered that the focus of beam was not directly in the center of the cavity, but misplaced towards the first mirror over the optical axis. The distance from the focus is $z_1 \approx -2.7 \,\mathrm{mm}$ and $z_2 \approx 17 \,\mathrm{mm}$, to the first and second cavity mirror respectively. To understand the scope this effect has on the cavity we looked at the Rayleigh range for the cavity, according to Equation 2.1.12. The Rayleigh range for these parameters is $z_R \approx 0.1 \,\mathrm{m}$, five times greater than the cavity length. Using Equation 2.1.7, $z_1 = -2.7 \,\mathrm{mm}$, and $z_2 = 17 \,\mathrm{mm}$, the beam waist on the mirrors is calculated to be

$$w(z_1) = \sqrt{1 + \left(\frac{z_1}{z_R}\right)^2} w_0,$$

$$\approx 1w_0,$$

$$w(z_2) = \sqrt{1 + \left(\frac{z_2}{z_R}\right)^2} w_0,$$

$$\approx 1.015w_0.$$

The curvature of the beam is then, using Equation 2.1.6 and $z_R = 139 \text{ mm}$ as calcu-

lated in Section 3.2,

$$R(z_1) = z_1 + \frac{z_R^2}{z_1},$$

 $\approx -7.12 \text{ m},$
 $R(z_2) = z_2 + \frac{z_R^2}{z_2},$
 $\approx 1.15 \text{ m}.$

While the curvature of the beam at z_1 is not close to unity, the beam waist is very small in comparison to the surface area of the mirrors as shown in Section 3.2. Therefore the effect of the misplaced focus should not interfere with the mode formation inside the cavity. The misplaced focus could explain the length from the calibration measurement fit. The cavity length should be twice the distance from the middle of the cavity to furthest cavity mirror. This would then be a cavity length of $L = 2 \times 17$ mm, which is in the same order of the relative cavity length offset. For completeness, we made a prediction for the actual length of the cavity, to show the effect of this distance modification. For a predicted physical cavity length of (19.7 ± 2.5) mm the reflectivities are shown in Table 5.1.

Wavelength (nm)	Ring-down time (ns)	Reflectivity (%)
564.0 ± 0.5	76.07 ± 18.98	99.9137 ± 0.0242
567.0 ± 0.5	72.45 ± 17.98	99.9093 ± 0.0253
570.0 ± 0.5	59.11 ± 5.04	99.8889 ± 0.017
573.0 ± 0.5	56.61 ± 1.34	99.884 ± 0.015
576.0 ± 0.5	65.35 ± 4.64	99.8995 ± 0.0146
579.0 ± 0.5	59.28 ± 2.64	99.8892 ± 0.0149

Table 5.1: Determined reflectivity for a predicted cavity length of $(19.7 \pm 2.5) \,\mathrm{mm}$ and after
discounting the measurements where no flat ending slope exists.

During the writing of this thesis, work was still being done on the cleaning procedure of the cavity mirrors. A method was developed where the cavity mirrors are cleaned by using First ContactTM Cleaning Solutions, these are quick drying polymer solutions. The solution is applied to one side of the mirror and removed after completely drying. For our used 1 inch optics, this time is approximately 15 minutes. The peeling of the polymerized plastic removes contaminations on the mirror and three times for the back, where the front is defined as the high-reflective surface. A new ring-down measurement was then performed immediately after cleaning the mirrors. This measurement was performed at a wavelength of $\lambda = (580.0 \pm 0.5)$ nm following the described method in this thesis. The only differences being a cavity length of $L = (100 \pm 20) \text{ mm}$ and an adjusted focus with a beam width of $w_0 = 200 \text{ µm}$ in the middle of the cavity.

An example of a single measurement for these mirrors is visible in Figure 5.4. In Table 5.2 the complete result of these measurements is shown. Differences between this measurement and the ones with the uncleaned mirrors are striking. We observe a higher signal, a longer slope, and a lower noise to signal ratio. A rise after a ring-down, as seen in Figure 5.1, is noticeably absent in these measurements. We therefore suggest the comparator triggers unreliably on low signals, this explains the absence of a rise for the higher power measurements.

Wavelength (nm)	Ring-down time (ns)	Reflectivity (%)
580.0 ± 0.5	2573.79 ± 58.92	99.987 ± 0.0026

Table 5.2: Ring-down time and reflectivity for a single wavelength for the CRD mirrors, immediately after the cleaning procedure. This measurement was done with a cavity length of $L = (100 \pm 20) \text{ mm}$.

This reflectivity is still a factor 100 smaller than the manufacturers data. It does however give an indication that the cleanliness of the mirrors is of the highest important in high-reflectivity mirrors. The assumption is that reflectivity can be increased further when the cleaning process is perfected and both the process and the ringdown measurements are performed in a lab that is significantly more dust free than the temporary lab that this work was carried out in.



Figure 5.4: Decay measurement for the cleaned mirrors with the revised cleaning procedure. The photodetector signal shown by the oscilloscope is visible as the blue line, the red dashed line is the exponential fit. The fit visible here has a τ of $(2.586 \pm 0.013) \,\mu s$.

6 Conclusion

We have been successful in performing cavity ring-down spectroscopy to determine the reflectivity of our mirrors. Both the modes, and shape of the decay coincide with what we expect from Section 2. However, when the reflectivity is compared to the manufacturers data shown in Appendix A.2, a discrepancy is visible. The expected reflectivity for these mirrors should lie in the order of R = 99.9998%, while we measure approximately R = 99.8%. This is an order difference of a factor 1000.

We investigated this reflectivity difference by discussing the speed of the first order elimination of the Acousto-Optics Modulator (AOM), the calibration measurement, and the cleaning process of the mirrors. Both the speed of the AOM and the calibration measurement can account for minor differences in reflectivities. These are not substantial enough to offer an explanation for the order of magnitude difference. The cleaning procedure is shown to have a greater impact on the reflectivity.

The reflectivity of the cavity mirrors after cleaning them according to our cleaning procedure, is $R = (99.987 \pm 0.026) \%$ for a wavelength of $\lambda = 580$ nm. This of the same order as the R = 99.95 % cavity mirrors used in the second photon Bose-Einstein Condensate (BEC). Our mirrors can therefore be used for the formation of a condensate in the measured regime of 560 nm to 580 nm.

Future research can be done on the cleaning process to provide insight in fluctuations of the reflectivity after cleaning. Using these high-reflectivity mirrors, more research can be done on the formation and interactions of a photon BEC.

7 Acknowledgements

My time spent working on my bachelors research is the highlight of my bachelors at Utrecht. I would like to thank Dries van Oosten for giving me this wonderful place to do my research, and for supervising my work. Your insights and knowledge never fail to inspire me. The time in the lab would have been a lot more boring if I was there by myself. I would like to thank Karindra Perrier for our time spent working together in the lab. The greatest help in finishing this thesis was Sebastiaan Greveling, thank you for putting up with my barrage of questions and the supervision of my work.

With failing equipment non of this would have been possible. I would therefore like to thank Dante Killian and Frits Ditewig for maintaining our electronics, Cees de Kok for keeping our laser running, and Paul Jurrius for helping with our hardware.

I would also like to thank Gordian, Robbert, James and Marcel for helping me find my way in the lab, distracting me with ping-pong breaks, going on lunches, but most of all for being such wonderful people to work besides. Furthermore I would also like to thank Ole Mußmann for providing the beam propagation Python code and Sandy Pratama for giving me tips on python and the writing process during our short breaks.

Getting to this point would not have been possible without the support of my parents, Joost van der Flier and Herma ter Hogt. Thank you for everything you have done for me.

Last but certainly not least, I would like to thank Fiona van den Heiligenberg for always supporting me and being there for me. You inspire me to keep improving in everything I do.



Appendix A Transmission Spectrum

Figure A.1: The measured transmission data sheet sent to us by Dr Nyman of Imperial College London. An estimate on the reflectivity was made on this basis. For a wavelength of $\lambda \approx 550 \,\mathrm{nm}$, an estimate reflectivity of $R \approx 99.95\%$ is determined.



Figure A.2: The reflectivity data sheet as sent to us by CRD Optics. The red line denotes the Transmission as a function of wavelength. The reflection is determined by neglecting absorption effects, $R = (1 - T) \times 100\%$.

Appendix B Python Code

This code was developed by Ole Mußmann. It is routinely used in the Nanophotonics group.

```
#!/usr/bin/env python
# -*- coding: utf-8 -*-
import numpy as np
import matplotlib.pyplot as plt
import pylab as pl
import itertools
from scipy.optimize import curve_fit
nanometer, micrometer, millimeter, centimeter, meter = \setminus
    1.E-9, 1.E-6, 1.E-3, 1.E-2, 1.
\# Optical_Elements can apply to the 's' or 'p' direction (i.e
   . cylindrical),
\# or 'all' (normal).
class Optical_Element(object):
    \# By inheritance from 'object', this is a new-style class
    \_registry = [] # iteration registry, make the class
       objects iterable
    def __init__(self, d, matrix, applies_to):
        self._registry.append(self) # add self to registry
        self.d = d # distance from reference point
        self.matrix = matrix \# abcd matrix
        self.applies_to = applies_to \# 's' or 'p' direction
           or 'all'
    def plotting (self, geometry, previous, q, d, n=1.):
        color = colors.next()
        axis = np.arange(previous, d, .1 * millimeter)
        beam, R_c = waist(q + axis - previous, n) \# radius
         print beam.shape
#
#
         print R_{-}c.shape
        pl.subplot(211)
        #pl.plot([0.0375, 0.05, 0.1, 0.15, 0.1875, 0.25,
            0.275, 0.325, 0.4 ] \setminus
        \#, [0.357, 0.347, 0.3375, 0.3465, 0.342, 0.348, \setminus
```

#

#

#0.355, 0.384, 0.39, 'ro ') # pl. plot([0.0375, 0.05, 0.1, 0.15, 0.1875, 0.25,0.275, 0.325, 0.4]#, [-0.69, -0.716, -0.72, -0.77, -0.755, -0.994,]#-0.815, -0.812, -0.84], 'ro')pl.plot([0.0375, 0.05, 0.1, 0.15, 0.1875, 0.25, 0.275, 0.325, 0.4][0.4935, 0.5015, 0.49875, 0.52825, 0.5185, 0.641,0.555, 0.568, 0.585, 'ro') pl.title("beam_waist") pl.ylabel("beam_waist_[m]") pl.xlabel("distance [m]")pl.plot(axis, beam / millimeter, color) pl.plot(axis, -beam / millimeter, color) if $(\min(\text{beam}) \text{ not in } [\text{beam}[0], \text{ beam}[-1]]): \# if$ there's a minimum ypos = max(beam) if (geometry == "p") else -max(beam) pl.text(axis[beam.argmin()], ypos / millimeter, $u^{\%}s \ = .\%.5 f micrometer \ d = .\%.5 f micrometer$ $nz_R = \%.5 fm$ " % (geometry, **min**(beam) / micrometer, axis $[\text{beam.argmin}()], \text{np.pi} * \min(\text{beam}) ** 2$ / lambda_laser)) global w_L $w_L = \min(beam)$ global z_wL $z_wL = axis [beam.argmin()]$ pl.subplot(212) pl.title("radius_of_curvature") pl.ylabel("radius_of_curvature") pl. xlabel ("distance_[m]") pl.plot(axis, 1 / R_c, color="black") pl.axhline(0) print $R_{-}c[0]$, $R_{-}c[-1]$, 1 / $R_{-}c[0]$, 1 / $R_{-}c[-1]$ #print "q :", q ###### THIS IS WHERE ALL Q FACTORS ARE PRINTED !!! **def** apply_element (self, geometry, previous, q): self.plotting(geometry, previous, q, self.d) # plot previous q q

```
q = apply_matrix(q, self.matrix)
        global q_end
        q_-end = q
        return self.d, q
class Lens(Optical_Element):
    def __init__(self, d=0., applies_to="all", f=float("inf")
       ):
        super(Lens, self).__init__(d, [[1., 0.], [-1. / f,
           1.]], applies_to)
class Slab(Optical_Element):
    def __init__ (self, d=0., applies_to="all", n1=1., n2=1.,
       t = 0.):
        super(Slab, self).__init__(d, [[[1., 0.], [0., n1 /
           n2]],
                                        [[1., 0.], [0., n2]
                                           n1]]],
                                    applies_to)
        self.n2 = n2
        self.t = t
    def apply_element(self, geometry, previous, q):
        self.plotting(geometry, previous, q, self.d) # plot
           previous q
        q += self.d - previous # apply distance to slab
        q = apply_matrix(q, self.matrix[0]) \# apply first
           interface
        global q_slab
        q_slab = q
        self.plotting(geometry, self.d, q, self.d + self.t, n
           = self.n2)
        q += self.t # apply slab thickness
        q = apply_matrix(q, self.matrix[1]) \# apply second
           interface
        global q_lens
        q_{lens} = q
        return self.d + self.t, q
```

```
def waist (q, n):
    z, z_r = q.real, q.imag
    w_0 = np.sqrt(z_r * lambda_laser / (np.pi * n))
    R_c = z * (1 + (z_r / z) * 2)
    return w_0 * np. sqrt (1 + (z / z_r) * 2), R_c
def apply_matrix (q, matrix):
    return (matrix [0] [0] * q + matrix [0] [1]) / \setminus
           ( matrix [1] [0] * q + matrix [1] [1] )
lambda_laser = 565 * nanometer
Lcav = 1.7 * centimeter
w_{laser} = 0.5 * millimeter
f_weak = 200 * millimeter
f_strong = 50 * millimeter
d_{slab} = 9 * millimeter
start = 0. * centimeter \# distance from reference point
end = 929*millimeter+ Lcav +0.02 # plot at least until end,
   distance from reference point
### or BEGIN set q parameters
w_0_cav = ((lambda_laser**2 / np.pi**2 * Lcav * (1-Lcav)))
   **(0.25) ) * meter
w_0_{initial_s} = 390 * micrometer
w_0_{initial_p} = 590 * micrometer
w_0_{initial} = 490 * micrometer
\#z_r_initial_s = np.pi * w_0_initial_s ** 2 / lambda_laser
\#z_r_initial_p = np.pi * w_0_initial_p ** 2 / lambda_laser
z_r_initial = np.pi * w_0_initial ** 2 / lambda_laser
z_{initial} = 1E-40
q_{initial_s} = z_{initial} + 1j * z_{r_{initial}}
q_{\text{initial}} = z_{\text{initial}} + 1j * z_{\text{r}_{\text{initial}}}
\#q_{-initial_{-}o = z_{-initial + 1j} * z_{-}r_{-initial}
```

```
### END set q parameters
```

Optical Elements # d = absolute distance to reference point # applies_to: 's' or 'p' direction (cylindrical) or 'all' (normal) slab1 = Slab(d=end-0.02-Lcav-d_slab, applies_to="all",n1=1., n2=1.458, t=9. * millimeter) len1 = Lens(f=f_weak, d=(-0.02+end-Lcav)-2*f_strong-f_weak -0.025, applies_to="all") len2 = Lens(f=f_strong, d=(-0.02+end-Lcav)-f_strong+0.0, applies_to="all") len3 = Lens(f=-1.5*meter, d =1E-24, applies_to="all")

dummy = Lens(d=end) # set endpoint

```
# sort elements by distance (d):
Optical_Element._registry.sort(key=lambda x: x.d)
colors = itertools.cycle(['b', 'g', 'r', 'c', 'm', 'y', 'k'])
```

```
### PLOT ALL Q FACTORS: ###
```

```
for geometry in ["p", "s"]: # calculating q, plotting w for
  "p" and "s"
  #print
  #print geometry
  q = q_initial_p if (geometry == "p") else q_initial_s
  previous = start # start drawing from "start" or
    previous lens position
  for element in Optical_Element._registry:
    if (element.applies_to == geometry) or (element.
        applies_to == "all"):
        #pl.vlines(lens.d, -25., 25.) # indicate lens
        positions
    previous, q = element.apply_element(geometry,
        previous, q)
```

```
#pl.plot(xdata, ydata / millimeter, 'o') # beam size
    measurements
```

```
#pl. plot (xdata, waist (xdata + popt[0] + 1j * popt[1], 1) /
millimeter) # fit
#pl. ylim (.0, 10.)
#pl. xlim (1.256, 1.26)
pl. show()
print "______initial_values_____"
print "lambda_laser_=_", lambda_laser*1E9, "nm"
print "Lcav_=_", Lcav*1E2, "cm"
print "f_weak_=_", f_weak*1E3, 'nm'
print "f_strong_=_", f_strong*1E3, 'mm'
print "w_0_cav_=_%.5f_micrometer" % (w_0_cav*1E6)
print "z_R_=_", z_wL
print "q_f, slab_=_", q_slab
```

References

- E. A. Cornell and C. E. Wieman, Nobel lecture: Bose-einstein condensation in a dilute gas, the first 70 years and some recent experiments, Rev. Mod. Phys vol. 74, no. 3, p. 875 (2002)
- [2] K. M. R. van der Stam, E. D. van Ooijen, R. Meppelink, J. M. Vogels, and P. van der Straten, *Large atom number bose-einstein condensate of sodium*, Rev. Sci. Instrum. vol. 78, no. 1, 013102 (2007)
- [3] Y. B. ZelDovich and E. Levich, Bose condensation and shock waves in photon spectra, Sov. Phys. JETP vol. 28, no. 1287-1290, p. 11 (1969)
- [4] J. Klaers, J. Schmitt, F. Vewinger, and M. Weitz, Bose-einstein condensation of photons in an optical microcavity, Nature vol. 468, no. 7323, pp. 545–548 (2010)
- [5] J. Schmitt, T. Damm, F. Vewinger, M. Weitz, and J. Klaers, *Thermalization of a two-dimensional photon gas in a polymeric host matrix*, New J. Phys. vol. 14, no. 7, p. 075019 (2012)
- [6] J. Klaers, J. Schmitt, T. Damm, F. Vewinger, and M. Weitz, Bose-einstein condensation of paraxial light, Appl. Phys. B vol. 105, no. 1, pp. 17–33 (2011)
- [7] R. Nyman and M. Szymańska, Interactions in dye-microcavity photon condensates and the prospects for their observation, Phys. Rev. A vol. 89, no. 3, p. 033844 (2014)
- [8] J. Marelic and R. Nyman, Experimental evidence for inhomogeneous pumping and energy-dependent effects in photon bose-einstein condensation, Phys. Rev. A vol. 91, no. 3, p. 033813 (2015)
- [9] C. Wahl, *Phase dynamics of a statistically flickering photon bose-einstien condensate*, Master thesis, Universität Bonn (2014)
- [10] M. Mazurenka, A. J. Orr-Ewing, R. Peverall, and G. A. Ritchie, *Cavity ring-down and cavity enhanced spectroscopy using diode lasers*, Ann. Rep. Progr. Chem. C vol. 101, pp. 100–142 (2005)
- [11] G. Sridhar, S. Agarwalla, S. Singh, and L. Gantayet, Cavity ring-down technique for measurement of reflectivity of high reflectivity mirrors with high accuracy, Pramana vol. 75, no. 6, pp. 1233–1239 (2010)
- [12] Y. Gong, B.-c. Li, and Y.-l. Han, Optical feedback cavity ring-down technique for accurate measurement of ultra-high reflectivity, Appl. Phys. B vol. 93, no. 2-3, pp. 355–360 (2008)

- [13] J. Dijkuis, E. van Faassen, and P. van der Straten, Lasers en coherente verschijnselen, Utrecht University (1996)
- [14] S. Hooker and C. Webb, Laser Pysics, Oxford University Press (2010)
- [15] T. Johnston, R. Brady, and W. Proffitt, Powerful single-frequency ring dye laser spanning the visible spectrum, Appl. Opt. vol. 21, no. 13, pp. 2307–2316 (1982)
- [16] B. Paldus, J. Harris Jr, J. Martin, J. Xie, and R. Zare, Laser diode cavity ringdown spectroscopy using acousto-optic modulator stabilization, J. Appl. Phys. vol. 82, no. 7, pp. 3199–3204 (1997)
- [17] G. Rempe, R. Lalezari, R. Thompson, and H. Kimble, Measurement of ultralow losses in an optical interferometer, Opt. Lett. vol. 17, no. 5, pp. 363–365 (1992)