# Influences on performance in higher order thinking skills 

Comparison of schools participating in the mathematics A-lympiad

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#### Abstract

Dutch mathematics education has made a shift towards higher order thinking skills. As a way of assessing these skills, the mathematics A-lympiad was created. In the course of the years, some schools would consistently perform better on this contest than others. I interviewed teachers from both high performing and low performing schools in an attempt to determine the factors that influence the performance of the students in the mathematics A-lympiad. The main differences were found in the amount of instruction about the ideas and goals of the mathematics A-lympiad and the stimulation of creative ideas. In the process, some difficulties that teachers experienced with the mathematics A-lympiad came to light. This included the role aesthetics plays in grading the essays of the mathematics A-lympiad, and how to grade the essays in general. Most of these difficulties are explainable and known, though difficult to resolve.


## Prologue

This research was performed as a final conclusion of my master Science Education and communication. The main question was posed by the Freudenthal Institute of Utrecht University, which is also responsible for the organization of the mathematics A-lympiad. The question itself is simple 'Why do some schools consistently score better on the mathematics A-lympiad?'. However, researching this is not trivial. There are many factors influencing the performance of schools on the mathematics A-lympiad, and I would not dare to claim I compared them all. I compared many however, and came across multiple relevant results. This thesis is intended for the parties involved in the mathematics A-lympiad. It may be of interest for any teacher partaking or planning to partake in the mathematics A-lympiad that is interested in which factors influence the performance of a school. Furthermore it is an external view of the process of the mathematics A-lympiad. I therefore end with some recommendations for both teachers and the organization of the mathematics A-lympiad.

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## 1 Introduction

### 1.1 Mathematics A

The Dutch schooling system went through several reformations in the last 30 years. In the 1980s, following up on the HEWET project, mathematics education on the preuniversity level (VWO) was split in two different courses: mathematics A and mathematics B. While mathematics B aimed at science students, mathematics A was designed for those students who will 'have little further education and mathematics in their academic studies, but who must be able to use mathematics as an instrument to a certain extent (de Haan \& Weijers, 2000). The main subjects of mathematics A were the basic principles of discrete mathematics, statistics and probability and some calculus. The emphasis lay in the application of mathematics, with a strong focus on the process of coming to an answer. However, mathematics A quickly reverted to something less idyllic, preparing for problems in the nationwide central examination. These problems generally required students to perform a calculation or drawing/interpreting a graph. Often, if not always, tasks that they practiced extensively in class. Though the students might be required to use their skills in a new context, the higherorder thinking skills as described by Bloom, Anderson and Krathwohl, (Krathwohl, 2002) which are related to being able to use and apply mathematics in a new context were not really tested in the central examination. This is mainly because nationwide central examination made it difficult to assess higher-order problem-solving skills, since a predetermined, reliable grading system is required for consistent grading throughout the nation.

Dutch mathematics education has seen some reformations in the years that follow. In 1998, Dutch mathematics education saw its first new form since the 1980s. Mathematics A1 (aimed at students preparing for cultural studies) and mathematics A2 (aimed at students preparing for economic studies) were derived from the original mathematics A, only to change it back to one altered form of mathematics A (which contained parts of A1 and A2) and mathematics C (which was derived from mathematics A1) in 2008. Despite the fact that mathematics A has seen three different forms since the 1980s, higher-order thinking and problem solving skills continued to be part of the written Dutch mathematics A examination requirements. The target group of mathematics A always stayed those students that were not going to need a deep understanding of mathematics for their following studies, but were required to have a set of mathematical tools at their disposal to solve problems in various applications.

The skills required to be able to solve problems, are difficult to define. Schoenfeld (1992) describes a variety of interpretations of problem solving. He categorized different goals of courses that train students in problem solving. These are train 'creative thinking', to prepare students for problem competitions, to learn standard techniques in mathematical modeling and to induce critical or analytical thinking. Such different goals lead to different ideas of what problem solving is, and on which skills lessons in problem solving should focus. The definition of problem solving on which the Dutch mathematics A educational system (or, if you wish, examination system) focusses is closely connected to the views that Hans Freudenthal had on real mathematics education (RME).

Realistic mathematics education does not necessarily imply that all the mathematics, or all the examples are from the real world. Rather, RME follows the view that mathematics education should be training the skills that are needed to solve problems one may stumble upon in daily life or work. Therefore, a focus on (mathematical) problem-solving is great part of that view. The skills needed to be able to handle realistic (mathematical) problems are skills to analyze a problem, come up with several strategies to solve it, and to be able to assess the value of each strategy (Doorman 2007). Testing these skills requires open ended problems that have no well-known procedure to solve them, since this forces students to come up with different strategies and evaluate each of them. These problems often have many possible solutions, and preferably no known best solution. Developing a decent test for this is no easy task, as developers have to find a balance: The problems should have no clear solution and require mathematical analysis, but should also be understandable and manageable for high school students. Furthermore, the criteria for assessing such exercises are diverse. Of course, the best solution is great, but in mathematics A, and especially in tests for these problem-solving skills the strategies used should be evaluated as well.

In this light, the mathematics A-lympiad was developed.

### 1.2 The mathematics A-lympiad

The mathematics A-lympiad is a contest for high school students, in which they face an open ended problem in groups of three or four. The assignments consist of two parts. The first part consists of some introductory exercises, explaining the context and some of the mathematics involved. The second (more important) part consists of an open ended assignment where students are stimulated to try and come up with original solutions to a given problem. During
a single day, students write an essay on their findings, on both the introductory exercises and the open ended final assignment. Students are allowed to use whatever means they have at their disposal to come up with solutions.

For example, the exercise in appendix A (A-lympiad preliminairy 2007-2008) is about breakschedules for workers. It starts with three exercises in which the concept of productivity of workers (and the decline of it if the workers work without breaks) is introduced. Students are challenged to find a way to determine the productivity of a worker, given a certain schedule. and students perform calculations on different schedules to see differences between them. The final exercise is more open ended: given certain limitations (rules set by the government, minimal number of productivity hours in total, etc.) determine two different working schedules, and describe any pros and cons as well as your motivation for why the company should use these schedules.

An exercise like this requires more from the students than an exercise that is a repetition of earlier work, as requiring two solutions makes clear that there is not a single, best solution. The essays are graded based on the creativity of the solutions and, more importantly, clearness and validity of the analysis of their solutions. Because of the strict time limit, students are forced to cooperate. This extensiveness of assessment, and with such a focus on process, is never reached with a three hour exam consisting of closed exercises.

The essays are often graded by the teacher of the students, who selects up to three to be sent in for the national contest. All the essays which are sent in are randomly sent back to four participating teachers, so each teacher will receive 8 to 10 essays, which they have to arrange from best to worst. This order is sent back to the committee, after which a ranking is made of all the essays that were sent in. The best teams are invited to a final, which exists of a new exercise. They have two days at a closed location to write their essay about this new exercise. These essays will then be graded by the committee.

In the last 25 years in which the mathematics A-lympiad was held, the number of schools participating steadily grew. There are some good reasons for this. Until 2007 schools were obliged to do practical assignments in their school exam to make sure there was some kind of testing of problem-solving skills of their students. Designing such practical assignments is difficult and time-consuming, and the mathematics A-lympiad provides a ready to go assignment, while there is no obligation to actually send in one or more essays for the contest.

If you pay the entrance fee, you are allowed to use the assignment, and if you don't send in an essay, you are not required to evaluate the assignments of other schools. However, the competing elements of the contest still stimulates a lot of schools to send in their essays.

Second, the one-day nature of the assignment makes it easy to fit in the already tightly packed curriculum. Finally, since a new assignment is designed every year, and all the schools participate on the same day, there is no risk of fraud, something that is a problem when students get a lot of time to do their assignment, or when assignments are reused over the years.

Is the mathematics A-lympiad a good test for mathematical problem solving skills? This question is quite difficult to answer. However, it offers truly open-ended problems that have no clear solution, making it a better test than the national exam questions that require the student to reproduce specific mathematical skills. The open ended nature of the assignments and the form in which they are answered (essays) makes objective grading difficult. However, if multiple 'experts' give their opinion, the cumulative results can be quite consistent, albeit not truly objective. Since the mathematics A-lympiad offers the required problems, and the grading system is consistent, the ranking can be considered as a valid test. Although it may be difficult to determinate whether in the overall ranking the number three is better or the number six, it is clear that a top 10 essay is better than an essay at the bottom of the rating.

Over the years, it turned out that some schools were often appearing in the finals. Therefore, if the A-lympiad is a decent test of the problem solving skills of students, the schools that are consistently reaching the finals might be delivering students that have developed these problem-solving skills better than schools that consistently fail to reach the final round. That is how my research-question came to be:

What do schools that consistently perform well at the mathematics A-lympiad do different from schools that consistently perform badly?

Differences between the schools, the teachers, and their students are expected to provide reasons for students to perform better or worse at the A-lympiad. Ideally, if we consider the mathematics A-lympiad to be a good test for problem-solving skills, this may imply that things that a high performing school does different from a low performing school, could have an influence on the problem-solving skills of their students. But this is something that is of
course very difficult to prove, since there are many different factors that are difficult to isolate. However, I intend to find differences between schools and teachers that may have some influence on the performance of the students on the mathematics A-lympiad, and my findings might help improve the mathematics A-lympiad itself or help schools to increase their performance on it.

## 2 Method

### 2.1 Selecting the schools

I started with a list of schools that participated in the mathematics A-lympiad since 2003. I reduced this to a list of the 91 schools that participated in the last 6 years (2007-2013). To be able to compare the schools properly, I had to create a profile for some of the best and some of the worst schools. To be able to do this, a ranking had to be made. Although the schools were ranked every year, there was no ranking containing the information of schools over several years.

To decide the difference between high-scoring and low-scoring schools, there were three aspects I considered. First, the school had to perform well over several years, because a school that only performed well during a single year is not a consistently high performing school. Second, the results had to be recent, because schools tend to change quite quickly, for example by changing their teachers or the courses they offer. Finally, I only wanted to investigate schools that had sent in plenty of essays, since a high average is easier to obtain if a school had only one or two participating teams.

To make the selection, I created a ranking based on the average score of each school (the sum of the scores divided by the number of essays a school sent in) of the last six years. The score of each essay is based on the positions they obtained while being ranked by the teachers. If a team was ranked first once, ranked third twice and ranked fourth once, the total score for this team would be $1+3+3+4=11$. Since teachers ranked eight to ten teams, the best possible score for a team was 4 , and the worst possible score was between 32 and 40 . Accumulating the results of the last six years, this led to a ranking of 91 schools that participated in the contest. However, in the resulting list, schools with very few participating teams were a little bit outstanding, because it was easy for them to get a high average score (so a low number of points). For example, the highest-scoring school only had one participating team which had done very well and reached the final, but this made the school unusable for my research since they didn't have consistently high scores.

Therefore, I made a second ranking, in which I divided the average score of the school squared by the number of teams with which they participated. $\left(\right.$ score $\left.=\frac{\text { average }^{2}}{\text { numberofteams }}\right)$. Basically, this adds a weight of the number of teams with which a school participated to the
score of the school. I also tried dividing the average score by the number of team per schools, but in that case the weight of the number of participating teams was too large, making schools with many teams rank at the top, just because they had many participating teams. In the new ranking, any teams that were ranked worse than 39th place had less than five teams participating. Some of the better ranked schools still had three or four teams, but in that case they performed particularly well.

I took the relative positions of the schools in each of these two rankings, and this became my final ranking. Note that only schools that were in the second ranking were selected for the final rank, since schools that scored worse than the 39th position in the second ranking had too little teams to be relevant for my research.

Of the schools, I selected the top four, the bottom four and three schools from the middle of the list. The schools were then approached to partake in my research. Six of these schools (three of the high-ranking, one of the middle-ranking and two of the low-ranking schools) were willing to participate.

### 2.2 Developing hypotheses and interview-instrument

Developing hypotheses directly lead to developing the interview instrument. Basically, every subject for which I hypothesized a possible influence on the results of a school lead to one or several questions for the interview-instrument. This makes it difficult to separate the the hypotheses from the development of the interview-instrument. Therefore, although perhaps somewhat unusual, I have combined these two subjects in this paragraph.

To get a clear image of the participating schools, I developed a semi-structured interviewinstrument which was divided into four different categories, one for each of the factors that I hypothesized as factors that may influence student results. Part of the interview was also dedicated to finding out the teachers opinion on the influential factors, since they might have some insight on factors of influence that I hadn't considered before.

I interviewed two teachers of every school, and if possible, also one or two students. The goal of the interviews was to get a clear picture of the different factors that could be of influence on the way students performed. Since the teachers were also the ones to determine the quality of an essay, there were three parties involved in the process: the students, the teachers, and the
school that brings them together. In this section, I will discuss some of the reasoning and the theoretical background that were decisive for the structure of the interview instrument.

### 2.2.1 The school

The first and most obvious difference between the schools is of course that they are in fact different schools. No matter how desirable it may be to use identical schools with only a couple of differences, this is clearly something that is impossible to achieve. Therefore, it is important to get a clear view of the properties of each the schools involved.

This leads to a lot of properties that could possibly differ. Some of these possible differences are very clearly defined, such as the level of students that is being taught at the school, which courses are offered (for example some schools offer Nature Life and Tech (NLT), a course which is based on multidisciplinary group projects), or in which grade the students participate in the mathematics A-lympiad. Other properties are less accurately measurable, such as school mentality and goals. Although the goals may be clear, it is unclear to which extent it is brought into practice.

Therefore, part of the interview was committed to establishing the properties of the school itself, and also the way it was brought into practice by the teachers. The main reason I also wanted to interview students was to check whether the teachers weren't just holding up appearances.

The way the school handles excellence of students should also be noted. Offering extra challenges and making a selection of excellent students should increase their performance (Rogers 2007, Lens \& Rand 2002).

Hence, the part of the interview that focused on the school profile, consisted of some questions about the school level and offered courses, but also focused on projects offered to their students that are similar to the A-lympiad, such as multidisciplinary, group or excellence projects and projects in which the students would do some kind of research.

### 2.2.2 Student preparation

If students of a school spent significant amounts of time with research exercises or group projects they might have more experience in collaboration, time management and the structure of the essays. But there are more ways the students could be prepared. The direct preparation of students shortly before they start the A-lympiad can differ. All previous exercises are viewable online, so students that have worked through these would clearly have a better idea of what to expect. Also, since schools participate for many years in a row, they could even study previous winning essays, if their school had these available.

Since the exercises are quite different every year, one might argue that viewing/working through old exercises might not be that beneficial for the student results. However, the main points on which the essays are graded remain practically the same over the different years. An instructional page is added to the exercises of the mathematics A-lympiad in which some notions are made about the points on which the essays will be graded. Since this doesn't change much over the years, one could analyze old essays on these points. Also, if they are better informed by their teacher, they could give more attention to these points. Tarmizi (1988) claims that being taught a structure of how to handle an exercise might even be a better way of preparing than looking at previous essays, since worked examples might produce a cognitive overload for the student. Note that previous essays aren't exactly worked examples, however, giving explicit points to pay attention to, which some teachers may do if they discussed the old exercises explicitly, may still have a bigger influence on the student results than just showing them the old essays.

So in short, students that were able to view old essays or that were able to spent some time with old exercises could have an advantage over students that didn't. Also, students with teachers that explicitly discussed the old exercises may also have an advantage.

Thus, the part of the interview that focused on student preparation explicitly focused on the way teachers prepared their students directly for the A-lympiad, while the student experience from previous essays or group projects was already discussed in the questions about the school program.

Finally, there was the question of student motivation. If students were not motivated to participate, they will probably put less effort into the essay and therefore make a worse essay.

However, students knew beforehand that if their essay was contesting, they could win a trip to the finals. Teachers had to select which essay they send in. Therefore, if the lack of motivation of a single group had a bad influence on their performance, they would probably not be selected. For this and the manageability of the research, only the overall motivation of students was questioned.

### 2.2.3 The teachers

For each teacher I started with some clear facts, such as level of experience and age. The teachers influence is split in two important roles in the process, one might even say three.

First of all they have to prepare students for the mathematics A-lympiad, and their teaching and teaching methods are something that is different for all the schools and teachers, so it's hard to pin down how far this influence goes. However, there is correlation between teacher motivation and student motivation, as well as teacher beliefs and practices (Stipek et al, 2001). So it is important to be informed of the teacher beliefs about mathematics and teaching, especially because the mathematics A-lympiad has a focus on doing proper research with mathematics instead of just reproducing mathematical skills.

Thus, part of the interview addressed to the teacher beliefs, their motivations and the way they bring this into practice. Also, some questions specifically focused on their motivations around the mathematics A-lympiad, since a teacher very motivated about the underlying motives of the A-lympiad, or very motivated about this specific contest for teams could bring extra motivation to his or her students and therefore increase their performance.

The second role teachers have in the participation of the mathematics A-lympiad, is that they have to grade the essays. This consists of two different parts: first of all, the teachers have to grade their own students and select which essays they want to send in to the (national) contest. The elements on which they grade the essays, or on which they select the essays for the contest determine what kind of essays they send in. This means that if a teacher thinks very differently than other teachers about what is important for the contest, their school will probably score bad.

Furthermore, the teachers have to grade essays from other schools, which determines the overall ranking. Since multiple teachers rank each essay there is some mediation in the
subjects on which the essays are rated. Although there is some instruction on the elements on the which the essays should be graded, it is unclear to what extent teachers bring this into practice and on which elements their focus lies while rating the essays. Hol (2000) describes the difficulties teachers experience while trying to rate the essays. The different ways of grading vary from very rough (no concrete criteria) to very detailed (all criteria are clear). This implies a very varied way of grading, with emphasis on many different aspects of the essays.

Fortunately, there are some instuctions for grading the essays. There is a yearly voluntary meeting for participating teachers in which they can come into contact and discuss their various methods, and the instructions for both students and teachers contains the following list of criteria for grading:

While grading the essay, attention may be payed to:

- Readability and clarity of the final exercise
- Completeness of the essay
- Use of mathematics
- Used argumentation and justification of made choices
- The depth of things that are done
- The way of presenting: including form, readability, structure, use and function of appendices.
- The (mathematical) creativity in the solutions of the exercises

Thus, the points of influence of the teacher are: The educating of the students, the influence of their beliefs about mathematics education and the A-lympiad, the selection of the essays for the national contest, and the grading of the essays of other schools.

So, a complete summary of the subjects on which the interviews were focused:

- School profile:
- Student level
- Student grade
- School mentality focus
- Multidisciplinary projects
- Research projects
- Group projects
- Student preparation
- Teacher instruction beforehand
- Practice with old exercises
- Viewing of old exercises and essays
- Student motivation
- Teacher motivation
- Age, teaching experience
- Research experience
- Motivation for being a teacher
- Motivation for the mathematics A-lympiad
- Visions on mathematics education
- Grading the essays
- On which elements lies their focus
- How do they do it
- How is their grading compared to other teachers

The first draft of the interview instrument was revised by an expert in the field of problem solving in mathematics education, after which some additions were made. These additions were mostly extending some questions with more specific information. For example the question about 'how does the school handle group projects?' was extended with questions about different parts to school (mathematics section, beta-cluster) and how they handled group projects. As another example, a question about whether or not students had been educated by the interviewed teacher was added.

After the revision of the questions of the teacher interview instrument, I developed the interview instrument for the students. The main purpose of the student interviews was to enrich and investigate whether the enacted preparation by the teacher is also experienced in that way by the students. Therefore, the student interview instrument consisted of questions addressing school environments and student preparation. I made a selection of relevant questions from the teacher interview instrument and edited them to better suit students. Some exemplary control questions are: 'How did you prepare for the mathematics A-lympiad' and 'have you had lessons on how to handle exercises?'

A pilot interview was performed. During this pilot it turned out I missed some questions about the environment in which the students took the mathematics A-lympiad. The questions about school environment before the pilot were mostly about the type of education the school provided, not so much about the direct environment in which the students made the assignment. It has to be part of the questions, for it is imaginable that the noise level of the environments can be of influence on the student results. A question regarding the envoironment was added to this interview, as well as add to the interview instrument. The pilot interview also lasted about 25 min . while I expected it to be about 45 min . This was not a problem, and the further interviews averaged at about 30 minutes.

### 2.3 Taking the interviews

As noted in the introduction, I visited six schools, three of which were high-scoring, one of which was average scoring and two of which were low scoring schools. At every school, I interviewed two teachers, and in two schools I was able to also interview students. These student interviews did not turn up any irregularities in the teacher interviews. The teachers did not seem to be very wary of what they said, they were open and honest. They regularly pointed out that things like the view of education the school has, is often 'big talk', and they were open about the fact that it has little influence on the way they teach. For teachers there was nothing to be gained by pretending to be better in the interviews, and if there were points where they were not satisfied with, they said so openly. The interviews were far from overly positive or negative but did addres both, which is an indication that the interviewed teachers were sincere. Since I was unable to interview students at all the schools, I will not go deeper into the meaning any of these interviews might have, except that they did not seem to imply irregularities within the teacher interviews.

### 2.4 Developing the profiles

After taking all the interviews, each interview was transcribed and coded according to the subjects that were discussed in section 3.2. Any mention of a subject that was part of the properties on which the interview was based were marked. If there was an explicit mention about anything that the teacher felt was relevant for the way students performed, it was also noted, though these instances were rare.

All mentioned subjects were summarized in a profile of each interview, separated in school, student preparation and teacher properties. Note that I attempted to use as many absolute facts
from the interview as possible, but some interpretation was inevitable. For example, on the question ' do you have any experience doing research?' One teacher would say: 'yes, I wrote a master thesis once' while another would say: 'no, I only wrote a master thesis once'. Of course, this is actually the same answer. As another example, the question 'why did you become a teacher?' Would often have an extensive answer, which sometimes could and had to be summarized to 'it was convenient' or 'I wanted to do something different, and this seemed fun'. Also, there might be a mention of the subject, while the mention is only 'no I didn't do anything with that', which is very valuable information, but makes it impossible to just take the number of mentions of a subject as a measure for the importance of a subject for the teacher. Thus, to some extent interpretation was required to create complete profiles.

Because of the subjective nature of this process, a second corrector was presented with six different profiles and two interviews and asked to match them together. The profiles only contained interpretive information, because things like the age of the teacher and the level of school would make the matching trivial. The matchups were successful, with the second corrector explicitly stating that it was doable because the focus points of the profile and the interview were so similar. Matching was noted to be difficult, because many profiles contained similar properties. The second corrector was also asked to look for shortcomings in the profiles, but the only shortcomings that were noted were missing facts, such as whether or not the mathematics A-lympiad essay was part of the school exam. All these were already in the profiles, but invisible for the second corrector.

The profiles were then compared to each other by looking at each of the different properties, comparing all statements and checking if there were similarities between different schools and teachers. If so, I would check if these different schools were all part of the high or low scoring schools.

## 3 Results

For the entire tables in which I compared the different profiles, see appendix B. In this section, I will discuss all properties I encountered shortly. The most noticeable are 3.2.1 teacher instruction beforehand and 3.4 grading the essay. All results were based on 6 different schools, with two teachers interviewed from each school.

### 3.1 School

### 3.1.1 Student level and grade

The level of students on each school varied. With the levels that were taught within each school varying from vmbo/havo/vwo, up to exclusive gymnasium, the students that are participating with the mathematics A-lympiad were either from 5 havo (h5), 5 vwo/gymnasium (v5) or 6 vwo/gymnasium (v6). There doesn't really seems to be a notable structure in scoring rates, with both high-performing and low performing schools that had students participate in both v5 and v6, and various teachers stating that they almost never send in an essay from h5. All studied schools had (during the studied years) all students of one or more yeargroups participate, with the teacher selecting which essays to send in.

### 3.1.2 School mentality focus

Whether the school focused on student performance or social development, the schools were evenly divided among high-scoring and low-scoring. All schools stated in their official vision that they strived to help to develop the students in many different areas, though some teachers stated that their school was more focusing on academic performance while others focus more on social development.

### 3.1.3 Multidisciplinary projects

All schools had some multidisciplinary projects over the years, mostly in the lower classes. Some schools have some projects for the upper classes, but those schools were evenly divided among high-scoring and low-scoring schools, and multiple teachers stated that it was difficult to find the time and to create good projects.

### 3.1.4 Research projects and excellence projects

Only two of the schools claimed to have a special program for excelling students, but only in the lower classes. Both these schools were from the high-scoring group. Only one of them claimed that the students were actually having an open research project, and this was also the only school that claimed they had students doing research projects. Although most teachers said there was attention for research in other beta courses, I would like to quote one of the teachers on this:
'Other courses do handle research, but that's not that what we would call research at the technasium. Often the research question is: 'What happens when you add stuff A to stuff B, heat stuff A, etc.' but that's using prescribed materials, the result is known and the students are just reproducing it.'

The point being made was that what most courses described as research, is often recollection of known facts. This has its uses, but is fundamentally different from the type of research that would be done in the technasium that the school is building, which is open-ended research, very similar to the mathematics A-lympiad. It should be noted that this particular school had better results on the mathematics A-lympiad with the students that followed the technasium curriculum then with the students that didn't. However, since there was only one group of students, and their essay wasn't from the years I considered, this group can be ignored for the sake of my conclusions.

### 3.2 Student preparation

### 3.2.1 Teacher instruction beforehand and viewing of old exercises and essays

The first real notable difference between high-scoring schools and low-scoring schools is in the amount of instruction that students receive beforehand. All students had some instruction on how the day that the mathematics A-lympiad was taken would go, when they had to be there, what they should bring, etc. However, since schools participates at the mathematics Alympiad for multiple years, and all the previous exercises are available on the website, some teachers decided to give a little bit more attention to the previous exercises. While some teachers would keep the instruction to some practical information for the day itself, other teachers would take previous exercises and point out what the important elements were and what to pay attention to in both the exercises and the group process.

Some examples of practical tips some teachers told students:
'Make sure you keep track of the time, and don't spend too much time on the starting exercises. The main focus of the assignment is the final open exercise.'
'When forming a group, make sure you have all the kinds of people you need: creative people, intelligent people, structured people and somebody that keeps a good atmosphere'
'Make sure that the essay is an ongoing story, don't just give answers to the exercises'

All of the high-performing schools had one or both of the teachers stating that they spent some time during their lesson before the mathematics A-lympiad on these kind of issues, while showing and discussing previous assignments. Two out of three high-scoring schools also stated they showed student essays from previous years.

None of the low-scoring schools would structurally do this. One teacher stated they sometimes showed old exercises when there is time, and the average score school stated that they didn't do this and just pointed out to the students that the old exercises were available on the website.

### 3.2.2 Practice with old exercises

None of the schools had students practice with old exercises. There were two schools that had students participate in v5 and v6, so the students of those schools did have a year to practice. One of the schools was a high-scoring school and one of the schools was a low-scoring school so this doesn't seem to signify any difference in performance.

### 3.2.3 Student motivation

For all the students of the schools, the mathematics A-lympiad was used as a practical assignment that accounted for $10 \%$ of the school exam. At some schools, this was once $20 \%$. But in the last couple of years, for all of the schools it was reduced to $10 \%$. This should make students motivated to perform well, which several of the interviewed teachers stated. Some of the high-scoring teachers made a clear point of motivating students by making them long for the finals weekend, or showing that it's possible to reach the finals by making a Hall of Fame in the school. Also sometimes brothers and sisters of students would have reached the finals,
which was according to a teacher also a motivating way of showing that the finals are reachable and that you should give it your best to win a nice weekend.

### 3.3 Teacher motivation

### 3.3.1 Age and experience

There was great variety in the age and teaching experience of the teachers I interviewed, with both young and older teachers in both high-scoring and low-scoring schools. The same could be said for teacher experience, there were quite some teachers that started teaching at a higher age, some of them having had quite some research experience in their previous jobs. But there seems to be no correlation between the performance of the students and the experience (in both teaching and research) of the teachers.

### 3.3.2 Motivation for becoming a teacher

Throughout the interviews, an often heard response on the question 'why did you become teacher?' was laughter followed by some form of 'coincidence' or 'it just happened'. Most of the teachers were unable to give a concrete reason for their career choice. Almost none of the interviewed teachers became teachers immediately after their studies, most of them seemed to have made a career change at some point in their life. This gave several teachers experience in doing research. However, there did not seem to be any difference between high-scoring and low-scoring schools and the amount of research experience they had.

### 3.3.3 Priorities in mathematics education

When asked about their priorities in mathematics education, there were many different reactions. However, while most teachers mentioned multiple elements that were important to mathematics education, there was a consensus, with all teachers stating in one way or another that teaching students some form of thinking, mathematical thinking, analytical thinking, abstract thinking or just thinking in general is their priority. About half of the teachers also mentioned that mathematics is necessary for the following life of the students, be it in their everyday business, or in their upcoming studies. Other mentions were teaching the students mathematics (as in the contents of the curriculum), teaching students a love for the beauty of mathematics, and focusing on a positive atmosphere in the mathematics classroom. There did not seem to be any correlation between the answers and the way the schools performed on the Mathematics A-lympiad.

### 3.4 Grading the essays

### 3.4.1 Elements of focus

There were five different factors mentioned by the teachers on which they grade the essays. Far from all teachers used all of these factors, and there were some remarkability's. The factors mentioned were:

- (mathematical) content
- aesthetics (layout, the overall look)
- the (clarity of) the structure of the contents
- creativity
- realism

The fifth factor, realism, was only mentioned by one teacher. Most notable were aesthetics and creativity. (Mathematical) content and structure were mentioned by most teachers, but there was no remarkable difference between high-scoring schools and low-scoring schools. Aesthetics was mentioned by almost every teacher, with many teachers noting that in practice it was a more important factor that it should be. Multiple teachers mentioned aesthetics as 'difficult to ignore' or 'what it often comes down to'. There was only one school where the teachers mentioned they 'hardly paid any attention to [aesthetics]' which was one of the lowscoring schools.

The most notable result on grading factors was on creativity. Five out of six teachers from high-scoring schools mentioned creativity as a factor that they took into account while grading, while one of the two average scoring teachers and none of the low-scoring teachers mentioned creativity at an element of their grading.

### 3.4.2 Way of grading

The way teachers grade the essays seem to differ a lot, especially for the national contest. Most of the teachers would use one of two methods for the local grading: they would either use a rubric to give a grading to the different factors, weighing each of them, or they would arrange them in order from best to worst after reading them (once or multiple times) and give them grades relative to each other. For the national grading, multiple teachers stated the way they arranged the essays, was similar to the way they graded their local essays. If teachers had
used a rubric, they could use it again for the national essays. Several teacher stated that they had little time to do this, because of their other work.

### 3.4.3 Grading compared to other teachers

Most teachers did not have an extensive view of the way other teachers would grade the essays, there seems to be limited communication about this. The voluntary teacher meeting is attended by teachers, but far from all participating schools attend. Also, most teachers seemed more interested in the quality of the assignment than communication about the way of grading. If they had a view of the methods of other teachers, it would usually be of other teachers at the same school. Several teachers stated that they would compare the eventual ranking of the national essays that they graded to their own ranking, and usually stated that if they did that, it would be similar. However, since their rating was one fourth of the influence for the national ranking, this is not very surprising.

## 4 Conclusions

The main reason for this research was to find out why some schools consistently performed better than others on the mathematics A-lympiad. There was a wide range of properties in which these schools, teachers and students could differ. This caused a wide variety of researched properties. Many of the researched properties did not lead to any notable differences between the high and low-scoring schools. It was often the case that there was little to no difference between the schools, or that any differences were equally divided between high-scoring schools and low-scoring schools. Most properties, such as participating classes, school mentality, teacher experience and teacher priorities in mathematics education did not lead to notable differences in the competition results. The lack of difference between schools was somewhat unexpected. I expected that schools from which students from 6 vwo participate would score higher than the onces with students from 5 vwo, but most schools would participate with 6 vwo students, and even having them participate in two years in a row happened in both high-scoring and low-scoring schools.

That school mentality did not bring up a notable difference in results is explainable, since for both social schools as well as schools focused on performance, arguments can be made for why they might score better. Since the mathematics A-lympiad is a group contest, it is social as well as result oriented.

Most of the teachers expressed the amount of project based education students had as their main expected result producing factor. This can not be concluded from the interviews, since the number of research (group) projects the different schools offered was often quite low. This made it difficult to distinguish differences between the schools. Though it should be noted that the only schools that offered excellence projects, were from high-scoring schools. Making it plausible that schools offering students more projects, could score better at a project-based contest.

Student motivation and the amount of practice students had with old exercises, was similar for all schools. All schools had the contest account for $10 \%$ of the school exam, with some of the higher scoring schools also stimulating and highlighting a social motivation among students as a result of winning in previous years. None of the students really practiced with old exercises, except for two schools where students will participate for two years in a row, but this is one high-scoring one low-scoring school, so this doesn't signify difference.

Conclusion: One previous experience with the mathematics A-lympiad does not increase performance.

The amount of instruction the teachers gave to their students differed very much between the high-scoring and low-scoring schools. Teachers at the high-scoring schools, or at least one of the teachers at every high-scoring school would take time to discuss the goals, the group process and the important grading factors of the mathematics A-lympiad with their students, often using old exercises as examples. The lower scoring schools did not structurally do this. This seems to imply that discussing the underlying ideas of the mathematics A-lympiad and instructing students on what's expected of them for the essays, and not only giving the practical information about the day, has a significant influence on the results of the students.

Conclusion: Discussing the underlying ideas of the mathematics A-lympiad with the students and instructing students on what's expected of them for the essays, improves the results of the students.

It should be noted that most of the teachers that did inform their students of the ideas and expectations of the mathematics A-lympiad, did not seem to spend very much time on this, usually about 30 minutes.

An aspect that was much mentioned in the interviews was the way that the essays looked, and how it influenced grading. Though most teachers claimed to not find this a high priority for grading, almost all of them stated that it was an aspect of the essays that was nigh impossible to ignore. I did not research whether good looking essays tend to score higher, so it is impossible to conclude anything about the influence aesthetics has on the scoring of an essay, it should be remarked that most teachers found it a troublesome aspect.

The final notability I found was among the different properties of the teachers. It was remarkable that almost all teachers from high scoring schools, and none of the teachers from the low scoring schools mentioned creativity as something they would take into regard while grading the essays. I do not say that teachers from the lower scoring schools did not take creativity into account while grading, but it was not something that they mentioned in the interviews, while teachers from high scoring schools did. This may indicate the creativity has a lower priority for the teachers for low-scoring schools. Therefore:

Conclusion: schools with teachers that think creativity is an important factor in the mathematics A-lympiad, tend to score better in the national contest.

## 5 Discussion

For this research it should be noted that because of the multiple conclusions, it is difficult to determine causality. Each of relevant differences between the schools that turned up, is unlikely to be the factor to cause the rest of the differences, but crossed influence is likely. For example, students that perform well on aesthetics may as well be creative in their solution. And if the teacher gives a more extensive instruction about the dos and don'ts of the mathematics A-lympiad, this may also stimulate its students more to come up with creative solutions, making them perform better. It is impossible to determine which factor is the main influence from the collected data. Therefore, I will discuss some of the implications the different results may have, and what teachers could do to increase this factor, perhaps increasing the results of their students on the mathematics A-lympiad.

### 5.1 Teacher instruction

First of all, quite likely to be of influence on the performance of the different students, is the amount of instruction the teachers gave to their students beforehand. It is plausible that even a short instruction on the goals and criteria of the mathematics A-lympiad to students beforehand could lead to a significant increase in their results. Students aware of the transcending goals of their work should be tempted to make use of higher order thinking skills, which the $\mathrm{A} / l \mathrm{lympiad}$ is trying to assess.

That better instruction leads to better results is hardly surprising, but it signifies that the difference between high-scoring schools and low-scoring schools may not lay on a skill level of the students, but more on the amount of knowledge a student has as to what is expected of them. Teachers participating in the mathematics A-lympiad may take this as an advice to take some time to inform their students, as it may increase their results on the mathematics Alympiad.

This result may lead to some undesirabilities in the mathematics A-lympiad. If the amount of preparatory instructions is the main influence on the way students perform on the mathematics A-lympiad, this leads to a bias in what the A-lympiad was meant to test: student problem solving skills. Therefore, the developers of the mathematics A-lympiad may want to stimulate a more equal amount of instructions on the different schools to level the playing field.

Otherwise, it could be the case that skilled students do not reach their proper results, because other students had a better view of what to do during the mathematics A-lympiad.

### 5.2 Aesthetics

A somewhat smaller issue is the way aesthetics seems to be an unavoidable part of the way teachers rate the essays. Although the instruction of the mathematics A-lympiad clearly states on what grounds the essay should be rated, and the only mention of something similar to aesthetics is 'The way of presenting: including form, readability, structure, use and function of appendices', multiple teachers did state that it was an important factor for the grading. Teachers try to avoid paying attention to it, but find this difficult. This problem is somewhat inherent to grading essays. This leads to a contradiction. On the one hand, teachers try not to pay too much attention to the way an essay looks, so it is not a major factor in assessing the essays. On the other hand, students should be aware of the criteria on which their essay is graded, and if aesthetics is part of this, it would be fair to include it in the criteria. However, this may lead to students paying overly much attention to it, which makes the essays stray from the desired criteria of the mathematics A-lympiad. Perhaps the organization could define the way students should present their essay a little more clear, for example by stating that their essay should be professional enough to be presented to an involved party, such as a local government or the board of an involved company.

### 5.3 Creativity

It seems to be that teachers who think creativity is an important factor for the mathematics Alympiad tend to produce higher scoring students. Creativity seems to be an important factor for producing a well performing essay (though of course, far from the only factor). This could be connected to the fact that teachers mentioned they sometimes find the mathematics Alympiad difficult to grade, as essays tend to be similar to each other. A creative idea can make an essay stand out, perhaps leading to a better rating. Also, there might be a connection between students that are creative in their solutions and students that deliver aesthetically pleasing essays.

### 5.4 Grading

There seems to be little communication between teachers about the way they grade the essays. This leads to a variety of grading systems, which is not necessarily a bad thing for the local
assessment, since they are not dependent on the assessment and other schools. However, in the national contest, this leads to different interpretations of what the essays should look like. Although the ranking in the national contest is based on the idea that multiple professional interpretations lead to a form of consensus about the quality of an essay, there seems to be some confusion about the criteria on which the ranking should be based. Because of the ideas behind the development of the mathematics A-lympiad, the organization might want to think about making the criteria for a good essay (i.e. an essay that shows the tested problem solving and communication skills) more concrete for the grading teachers. This might be done with the purpose of getting the teachers more in line with each other. However, this is somewhat in contrast with the idea that there is not one proper way of grading, and that multiple approaches lead to an acceptable assessment. However, multiple teachers claimed that they found grading difficult, and a bit more foothold handed to them by the organization might be appreciated.

### 5.5 Limitations of the study

There were some severe limitations to this study, which should be taken into account while considering the results and conclusions. First, the number of schools and teachers was quite limited. While for some of the smaller schools the two teachers I interviewed were all the teachers that were involved with mathematics A students, for the larger schools there were more teachers involved. One could even argue that every teacher that taught the students during their previous years was of influence on the results a school achieves on the mathematics A-lympiad. The number of schools (six) was also very limited. While more schools were selected, it was not that easy to find schools willing to cooperate. This was easier for the high-scoring schools, since they are involved and proud of their results on the mathematics A-lympiad. However, if this research could be done with double the amount of schools, results might be more reliable. Note that with 91 schools, and less than 39 schools that had decent results for more than 4 teams, the population to take the sample from wasn't all that large in the first place.

Second, the stability of which schools are high-scoring and which are low scoring is limited. Teachers, teachers that are responsible for the mathematics A-lympiad, school mentalities and participating schools change throughout the years, making it difficult to determine 'consistently high-scoring schools'. However, by limiting to the results of the last six years, a
significant difference between the schools became apparent. However, this is no guarantee that these schools will continue to perform like this in the upcoming years.

Furthermore, the amount of possible factors of influence was the main problem of this study. It is very desirable to isolate the different factors, and compare them one by one. Of course, this is impossible to achieve in this case. However, with this research, several possibilities of important factors were found, and others were debunked. This leaves room for further research that could be an in-depth investigation of different factors, for example comparing the way the instruction before the A-lympiad is offered. An independent observer would be able to make objective conclusions, while my conclusions are based on the subjective interpretations of the teachers that did the instruction.

## 6 Further recommendations

Finally, I will summarize my recommendations for both teachers and the organization of the mathematics A-lympiad. For teachers, when participating in the mathematics A-lympiad, be sure to prepare your students. It does not have to take enormous amounts of time, but spending 30 minutes on previous exercises and assignments, discussing the goals of the mathematics A-lympiad with them and helping them have a perspective in which they have an overview on what is important. This might increase the quality of the essays, and with that the quality of the essays of your participating teams. This will also help determining criteria for grading, as well as make them more clear to students.

Stimulating creativity might help as well, since one of the problems of the A-lympiad is students getting stuck. If you can teach your students that any idea might be a good idea, and writing your thought processes down helps to create an inspiring essay, whether it contains great results or only few, they might get over this bump a little bit easier. This can be done while discussing the goals, so this does not require a separate approach.

For the organization, giving the students a little more instruction on the way the essay should look, or who the target group should be, might give students a bit more clarity about the expected professionalism of their essay. This may make students aware of the role the presentation (and indirectly the aesthetics) plays in the contest. In many of the older exercises, this kind of instruction was already given, but in others, it was not. The organization should be aware of the relevance of this kind of instruction and try to include it.

Also, the organization could think about giving the teachers a little more instruction on the way the essay could (or should) be graded. Many teachers find grading the essays difficult and would probably appreciate some help. However, the organization may want to avoid having too much influence on the process, unless they have it very clear for themselves what they want the criteria for the essays to be. This is no easy task, and making use of the masses may be the only way to determine a proper grading. However, reconsidering or rethinking the criteria is never a bad idea.

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## 8 Appendices

### 8.1 Appendix A: Mathematics A-lympiad assignment 2007-2008

This is the entire exercise of the 2007-2008 Alympiad preliminary, except the appendices. This, and all other previous assignments can be found at http://www.fi.uu.nl/alympiade/en/welcome.html

## WORKING WITH BREAKS



Preliminary round for the $19^{\text {th }}$ Mathematics A-lympiad
23 November 2007


Texas Instruments

## Procedure for the preliminary round assignment for the 2007/2008 Mathematics A-lympiad

This Mathematics A-lympiad assignment contains one introductory assignment, two follow-up assignments and a final one.

## General advice for working on this assignment

- First read the complete text of the assignment, so that you know everything you have to do.
- Keep an eye on the time you spend on the introductory and follow-up assignments, be sure to have enough time left for the final assignment. Divide tasks where possible and discuss when needed.
- If you have divided tasks within your group, discuss the results of the previous assignments with each other, before you start working on the final assignment.
- In the final assignment it is important that you explain the two alternatives as clearly as possible and give an analysis of the advantages and disadvantages.
- The answers of the three introductory assignments do not belong in the elaboration of the final assignment. Add the results of the introductory and follow-up assignments in an appendix. Remember the worksheets containing the graphs!


## To be handed in:

- A complete description of two possible daily schedules, including an analysis of the advantages and disadvantages.
- As an appendix: the results of the introductory and follow-up assignments.

The jury will receive copies of your work; these copies must of course be legible. Therefore, use a black pen to write, only print on A4 paper, do not make drawings with a pencil. If in any doubt, make a test copy!

## Judging

Among other things, the jury will pay attention to:

- legibility and clarity of the final assignment,
- whether the work is complete,
- the use of mathematics,
- the arguments used and justification of choices (realism may play a part here),
- the level to which the assignment has been done,
- presentation: form, legibility, structure, use and function of appendices, etc.


## good luck and have fun

We dedicate the 19 edition of the Alympiad to Berend Wielens.
Berend has set the tone for this assignment, but he never heard the music.

## Introduction

Everybody has experienced that it's not possible to just keep on going at work. Leaving aside that you will, need to eat for example, most work will physically tire you and after some time your concentration will become less. A truck driver who goes on without a break for ten hours, a teacher who does eight hours of correction without stopping, or a journalist who sits at his computer continuously typing for nine hours ... fatigue and loss of concentration can cause big or small disasters in all these cases.
Besides, with most work there will be a loss of productivity. This can be measured most easily with production work in a factory.
So, taking a break is necessary, but... what is the best way to divide those breaks? A few breaks, but longer ones, or a lot of short breaks, or a mix of the two: how to make the optimal schedule.

## Introductory assignment

From experience and a study in a large German production firm the following global picture has emerged on the connection between productivity and the number of hours worked. (A more detailed graph can be found in Appendix 1)


The more hours you work, the lower your productivity will get. In the graph you can see, among other things, that after eight hours of working non-stop, so without any breaks, your productivity will have fallen to $50 \%$.

From other studies we know that taking a break raises productivity. Just after a break, a worker's productivity is higher than just before one. Or to put it another way: productivity is back at an earlier, higher level. The study has resulted in the following rules of thumb:

- After a break within the first five hours of working (that is pure working hours) productivity will be back at the level of the time that is 3.5 times the length of the break before the start of that break. An example helps to clarify the rule of thumb: if a worker starts at 08:00 and works until 11:25, by 11:25 his productivity has dropped to $90 \%$ of his maximum.

If the worker then takes a 40 minute break, by the end of that break he'll resume his work at a productivity level of $3,5 \times 40=140$ minutes before 11:25. Check, using the graph, what that new productivity level is.

- After a break that is taken after more than five working hours the effect is a bit smaller: in that case productivity after a break will be back at the level of three times the length of the break before the break.


## Assignment 1

In the company that was mentioned before, the working day starts at 8 in the morning and ends at 5 in the afternoon. At 12:00 there is a lunch break, which lasts an hour. So the working day lasts nine hours, eight of which are actual working hours.

A worker produces a maximum of 600 wpu (work production-units) per hour. This 600 wpu is the maximum productivity.

The company board of directors is mainly interested in workers' total productivity over the whole working day.
a Make an estimate, based on the graph of the total productivity for eight hours of continuous work. Use the graph in Appendix 1 for this.
b Also estimate the production for the given daily schedule, with a one hour break between 12:00 and 13:00, and work between 08:00 and 17:00. Clearly indicate how you have used the graph.

## Follow-up assignments

To make it easier to do the productivity calculations, a decision is made to simplify the model to a fitting linear model. You can find the graph for the adjusted model below, together with the graph for the original model:


As you can see, the new model assumes that, based on 8 hours of continuous work without a break, productivity will fall in a straight line from $100 \%$ to $60 \%$.

## Assignment 2

a Calculate total production per worker for eight hours of continuous work. Use the worksheets in Appendix 2
b Also calculate a worker's production for the original daily schedule (the one with working hours between 08:00 and 12:00 and between 13:00 and 17:00). Do not just give your calculations, but also explain using the graph on the worksheet in Appendix 2.
c Find out if productivity rises if you split the 1 hour break in a number of shorter breaks (which must all be the same length) with a total length of an hour. Where would you plan those breaks and what will be the maximum obtainable production in that case?

## Assignment 3

Most workers prefer to have as much (continuous) free time as possible. So for instance an extra (half) day off, or all working days shorter. If that isn't possible, most workers would prefer long breaks.
The company's board of directors agrees with all possible work- and presence scenarios, provided that a worker can organise his schedule so that he (or she) can bring in a productivity of at least 19.200 wpu per week. The factory is open every day between 07:30 and 18:30.

- Find out if this is possible for someone who wants to work four days.
- Find out attractive options for workers who stay on a five day week.

Give the accompanying daily schedules for all options and represent them graphically in one of the worksheets in Appendix 3.

## Final assignment

Of course it's not only the employer who determines rules for working hours. There are also health and safety (ARBO) rules that imposes all kinds of limitations on the daily schedule. These rules of course also help to protect the workers! On the next page you can find some of the health and safety rules that apply to the company.

The board of directors wants the highest possible production; the worker wants as much free time as possible.

Give at least two well-founded proposals for a daily schedule for the workers, that the works council and the board of directors can together make a choice from. Take into account:

- the interest of both employer and employee (worker)
- health and safety rules and
- the minimum of 19.200 wpu per week.

List the working hours and the daily schedule and determine the level of production that can be achieved with them. In any case, use graphics to support your proposal.

Also mention all considerations that have been taken into account, the advantages and disadvantages, and take care to give a clear motivation for the criteria used!

## Health \& Safety rules

Breaks: If you have a working day of:

- more than five and a half hours, you have at least 30 minutes of continuous rest break;
- more than eight hours, the break time will be at least 45 minutes, 30 of which will be continuous;
- more than ten hours, the break time will be at least 60 minutes, 30 of which will be continuous.


### 8.2 Appendix B: Tables of comparison

In this Appendix, you will an overview of the comparison between the different interviews. Note that these are not complete or absolute, since the interviews always contain more information than one is able to put in such short answers. For the readability: numbers refer to a position in importance, with 1 being the most important, 2 the next most important and so on. If a teacher did not order their answers, yes is filled in for all mentioned points.

| Teachers | Leeftijd | werkervaring | onderzoeks ervaring |
| :---: | :---: | :---: | :---: |
| High-scoring 1.1 | 44 | 8 | ja, veel |
| High-scoring 1.2 | 55 | 11 | nee |
| High-scoring 2.1 | 51 | 20 | ja |
| High-scoring 2.2 | 46 | 16 | nee |
| High-scoring 3.1 | 56 | 28 | nee |
| High-scoring 3.2 | 30 | 6 | ja |
| Average-scoring 1.1 | 27 | 4 | nee |
| Average-scoring 1.2 | 35 | 8 | nee |
| Low-scoring 1.1 | 48 | 16 | ja, adviseur |
| Low-scoring 1.2 | 64 | 23 | ja, laboratorium |
| Low-scoring 2.1 | 53 | 11 | beetje |
| Low-scoring 2.2 | 63 | 38 | nee |

Teachers

| Grading criterium | creativiteit | inhoud | uiterlijk | verslagsstructuur | realisme |
| :---: | :---: | :---: | :---: | :---: | :---: |
| High-scoring 1.1 | ja | als ze het maar goed uitleggen | ja | 1 |  |
| High-scoring 1.2 | ja | niet echt | ja, telt snel mee | ja | ja |
| High-scoring 2.1 | veelvuldig genoemd | 3 | 3 | 2 |  |
| High-scoring 2.2 | ja |  | belangrijke factor, landelijk |  |  |
| High-scoring 3.1 | ja | ja | komt vaak op neer | belangrijkst |  |
| High-scoring 3.2 | nvt | nvt | nvt | nvt | nvt |
| Average-scoring <br> 1.1 | ja |  | moeilijk door te kijken, vooral bij landelijk | 1 |  |
| Average-scoring $1.2$ |  | 1 |  | 2 |  |
| Low-scoring 1.1 |  | 1 | 2, moeilijk doorheen te kijken | 2 |  |
| Low-scoring 1.2 |  | ja | ja |  |  |
| Low-scoring 2.1 |  |  |  | ja |  |
| Low-scoring 2.2 |  |  |  |  |  |

Teachers

| Teaching priority | stof | sfeer | leren denken | wisk is overal/praktijk | passie voor vak |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { High-scoring } \\ & 1.1 \end{aligned}$ |  |  | ja, kaart denkstructuur aan |  |  |
| High-scoring <br> 1.2 |  |  | ja, kaart denkstructuur aan |  |  |
| High-scoring <br> 2.1 |  |  | ja |  |  |
| High-scoring <br> 2.2 |  |  | leerlingen moeten 'wiskunde taal' leren spreken | ja |  |
| High-scoring <br> 3.1 |  | ja | ja | ja |  |
| High-scoring <br> 3.2 |  |  | ja | ja |  |
| Average- <br> scoring 1.1 |  |  | ja |  |  |
| Averagescoring 1.2 | in de praktijk |  | ja | soms |  |
| Low-scoring 1.1 |  |  | ja | ja | ja |
| Low-scoring 1.2 | prioriteit <br> nummer 1 |  | achterliggende gedachte |  |  |
| Low-scoring 2.1 |  |  | ja |  |  |
| Low-scoring 2.2 |  |  | ja | ja |  |

Schools

|  | schooltype | pta? | $\underline{\text { niveaus }}$ | $\underline{\underline{\text { Deel- }}}$ | $\underline{\text { Excellentie }}$ | vakoverstijgende <br> projecten | Onderzoeks <br> projecten |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| High- <br> scoring 1 | sociale school | ja | hvg | v6,h5 | ja, <br> onderbouw | ja |  |

Student preparation by school

|  | Excelentieprojecten | vakoverstiigen de projecten | onderzoekstraining | van tevoren opdrachten laten zien | van te voren uitwerkingen laten zien |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Highscoring 1 |  | ja | ja | ja | ja |
| Highscoring 2 |  | af en toe, komen vaak niet van de grond. |  | ja, en de belangrijke punten worden toegelicht/aleen even ingelicht |  |
| Highscoring 3 | ja, onderbouw | ja, onderbouw |  | ja | ja, en leerlingen doen 2 jaar mee. |
| Averagescoring 1 |  | ja, onderbouw, <br> vooral op <br> samenwerking <br> gericht |  | verwezen naar de site | nee |
| Lowscoring 1 |  | ja, schoolbrede projectweken |  | soms |  |
| Lowscoring 2 |  | ja, brugklas, vooral sociaal |  | van vorig jaar | leerlingen doen 2 jaar mee |

