Skyrmion Dynamics in Thin Films with Perpendicular Magnetic Anisotropy

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Abstract

In this thesis we consider dynamics of skyrmions driven by the Spin Hall effect, in particular in finite systems. We look at the interaction of a skyrmion with the boundary of the system. We find that the magnetization at the edge is tilted, which is expected to lead to a repulsive potential that tends to push the skyrmions away from the edge.

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1 Introduction

Do you also dislike it when your smartphone runs low on battery that quick? Topologically stable skyrmions may possibly be the answer for us! Skyrmions can be used for a more efficient way of data storage and data processing. As they have many advantages; like their size, it is only a few nanometres in diameter, so the information density in a square area can be very high. Another very important feature is the low power consumption that is needed for spin currents to drive the skyrmions. Lower power consumption means longer battery life! We first give an introduction to the topics involved, i.e. magnetism in materials in general and an introduction to skyrmions. An excellent textbook on magnetic materials is 14.

1.1 Magnetic materials

Let us start with the general subject of magnetism in materials. We can distinguish three different kinds of magnetism:

1) Diamagnetism; all materials have this magnetic property, it counters the externally applied magnetic field (to some extent) and usually it is very weak, however there are some materials with strong diamagnetism, for example superconductors. Their ability to levitate above a permanent magnet comes from this diamagnetism.

2) Paramagnetism; in materials with this type of magnetism, magnetization is created directional and proportional to the externally applied field. After switching the field off, the material will not retain the magnetic properties it had with the field still on. This is because of thermal fluctuations which randomize the spin orientation, so the magnetic properties of material will be similar to the state prior to turning the field on.

3) Ferromagnetism; applying a magnetic field on materials with this kind of magnetism will align all magnetic domains in the same direction, and by doing so creating a very strong (ferro)magnet, sometimes the magnetization of this material will be far greater than the externally applied magnetic field. Even after removing the external field the material will be magnetized. You have to apply a magnetic field in the opposite direction if you want to demagnetize it, as can be seen in the hysteresis loop figure 1.

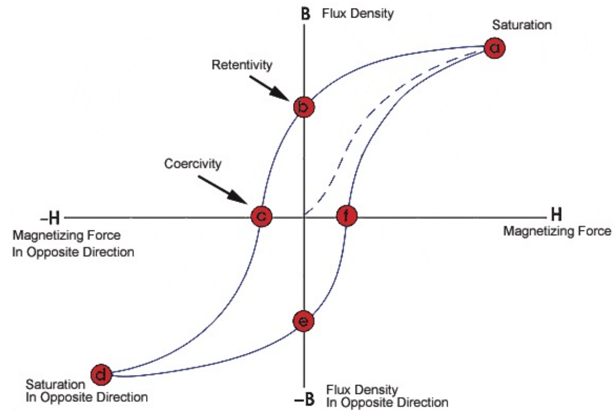

Figure 1: *Hysteresis loop; starting in point a) we see that the magnetization B can only reach zero if the applied field is taken in the -H direction. Image taken from 13.*

The amount of magnetic force necessary to demagnetize the material is called the coercivity. In general the coercivity is directly correlated with magnetic anisotropy. So the higher the magnetic anisotropy of the material, the more effort it takes to demagnetize it. In ferromagnets we can have all kinds of spin excitations, i.e the spins[1] can be oriented in a certain way. An interesting type of such an excitation is a domain wall. The domain wall corresponds to a gradual transition between two magnetic domains, see figure 2. The size of the domain wall depends on the ratio between the exchange energy and anisotropy of the material and usually the size is in the order of 10^2 atoms.

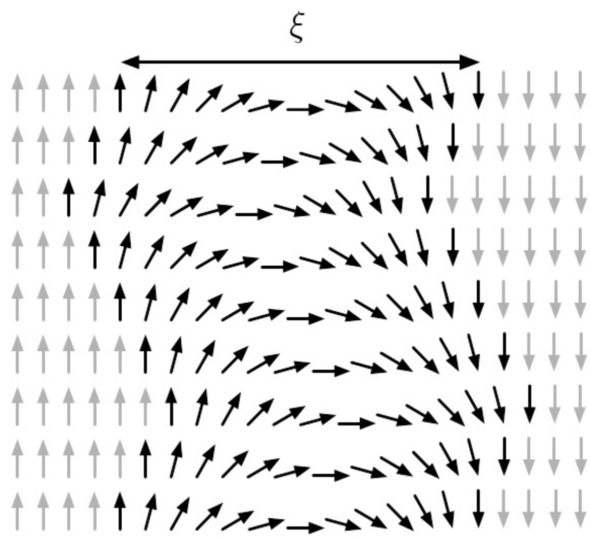

Figure 2: *Domain wall; a transition between a magnetic domain with spin-up and a magnetic domain with spin-down.*

Another type of excitations are spin-waves. To illus-

[1]The magnetization and spin are directly related through some proportionality factors e.g. the Bohr-magneton

trate this consider a system where all the spins point in the same direction, now we flip the spin of one atom, but then it tends to go back, because all of his neighbours are pointing in the other direction, so it gradually flips at the expense of his neighbours. On their turn they act on their neighbours, making them flip etc. These spin-flips propagate like a wave through the material, and hence the name, spin-waves. These propagating deviations of the spin orientations are also known as magnons.

Another form of excitations in magnetic materials are skyrmions. And that is exactly what we will investigate in the following sections.

1.2 Skyrmions

The subject of this Thesis is a skyrmion in a ultra thin magnetic film. Other good articles on this subject are.[4, 15, 18] To begin with we must answer a couple of the questions that might have emerged at this point: e.g. "What are skyrmions exactly?" and "Why are we interested in them?". In this section we will try to answer these questions.
*A **skyrmion** is a topological structure of magnetization vectors.* These vectors are standing in a spiral-like configuration (see figure 3). The magnetization vector field $\boldsymbol{\Omega(x)}$ describes this profile of the skyrmion. The magnetization orientation of an atom in a point $\boldsymbol{x}$=(x,y,z), is mathematically given by:

$$\begin{aligned}\boldsymbol{\Omega(x)} &= sin\big(\theta(\rho,\varphi,z)\big)cos\big(\phi(\rho,\varphi,z)\big)\hat{\rho}\\ &\quad + sin\big(\theta(\rho,\varphi,z)\big)sin\big(\phi(\rho,\varphi,z)\big)\hat{\varphi}\\ &\quad + cos\big(\theta(\rho,\varphi,z)\big)\hat{z},\end{aligned}$$

where θ is the polar angle and ϕ the azimuthal angle.

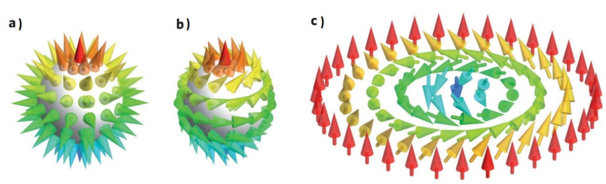

Figure 3: *a) and b) illustrate 3D skyrmions that can be found in some bulk materials, c) illustrates a 2D skyrmion with the magnetization in the center pointing down. Image taken from 16.*

We will not look at 3D skyrmions, but only at ultra thin films - which are comparable with 2D skyrmion structures. To be more specific, we look at layered magnetic systems with perpendicular magnetic anisotropy, e.g. Pt/CoFe/MgO multilayer as in 18.

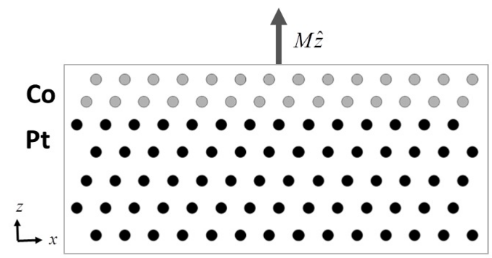

Figure 4: *Platinum-cobalt interface, with a thickness of only a couple of atoms, the gray and black dots represent the cobalt and platinum atoms.*

We define θ to be zero when the magnetization is in the +z-direction, aligned with the external field (clearly most of the magnetization will be in this direction), and in the center of the skyrmion, θ will be π. The skyrmion is rotationally symmetric. The azimuthal angle ϕ of the magnetization vectors has a constant value ϕ_0, depending on your system's materials. In ferromagnets the magnetization rotation has no preferred handedness, but inversion symmetry breaking can lift the chiral degeneracy.[2] The Dzyaloshinskii-Moriya interaction favours magnetization rotation with a fixed chirality, hence ϕ_0 will be fixed. Meaning that the direction of the magnetization vectors will only depend on the polar angle θ. For the system we consider ϕ_0 will be zero.

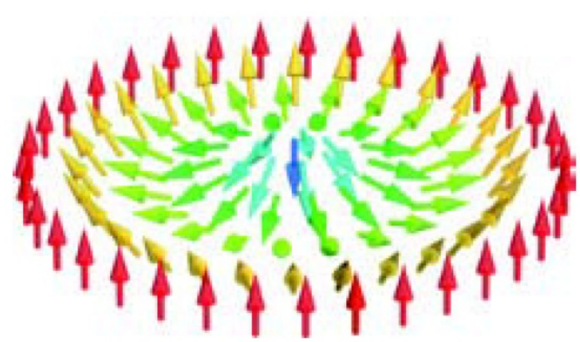

Figure 5: *Skyrmion with ϕ_0= 0, also called the "Hedgehog"-formation. Image taken from 9.*

Since the discovery that skyrmions can be manipulated with very low currents in 2009,[17] active research has been undertaken in this field. Skyrmions can be driven under smaller (about an order 10^5) currents than ferromagnetic domain walls. This can be explained by observing that domain walls have intrinsic pinning sites whereas skyrmions do not. Furthermore skyrmions can avoid impurities in a material, contributing a lot to their movement freedom;[5] this can lead to faster execution time in devices. This raises potential to a new generation of devices for information storage. Therefore it is very important to understand as much as possible about skyrmions, and thus lots of research is and must be conducted. Currently individual skyrmions can be created and destroyed, this has been done by a number of people in a number of different ways: it has experimentally been found that a single skyrmion can be created via local spin-polarized currents from a scanning tunnelling microscope,[3] and that skyrmions can be created dynamically by destabilizing the ferromagnetic background state through a spin-polarized current.[10] Some work has already been done on the skyrmion dynamics in an infinite system.[6,12] We will look at the static skyrmion profile in Chapter 2. In Chapter 3 we consider the effect of an electric current in the platinum layer, this in turn will affect the skyrmion via the Spin Hall effect. Then we continue to find the boundary conditions for a finite system, and finally attempt to find out more about the associated skyrmion dynamics.

2 Static skyrmion profiles

In this chapter we find the magnetization profile of a skyrmion, and its dependence on different parameters. We follow the discussion in 12. Our starting point is the energy of the system as a function of the magnetic orientation vector $\boldsymbol{\Omega}(\boldsymbol{x})$,

$$\begin{aligned}E(\boldsymbol{\Omega}) = t_{FM}\int dx\Bigg\{ &-\frac{J_s}{2}\boldsymbol{\Omega}\cdot\nabla^2\boldsymbol{\Omega} + K(1-\Omega_z^2)\\ &+\frac{C}{2}\left(\hat{y}\cdot\left(\boldsymbol{\Omega}\times\frac{\partial\boldsymbol{\Omega}}{\partial x}\right) - \hat{x}\cdot\left(\boldsymbol{\Omega}\times\frac{\partial\boldsymbol{\Omega}}{\partial y}\right)\right)\\ &+\mu_0 HM(1-\Omega_z) - \mu_0 M\boldsymbol{\Omega}\cdot\boldsymbol{H}_d\Bigg\},\end{aligned}$$

with t_{FM} the thickness of the ferromagnet, which in our case, as said, is very thin. Furthermore J_s the spin-stiffness due to exchange-interaction, K the anisotropy constant and C the strength of the Dzyaloshinskii-Moriya interactions (DMI) that arise due to the spin-orbit coupling in combination with lack of inversion symmetry.

The exchange-interaction is a short range effect and tends to align all neighbouring spins in the same direction. The DMI is also a short range effect, but whereas the exchange interaction wants to align the spins, the DMI contributes to making the neighbouring spins stand orthogonal to each other. This competition between these two terms results in the chirality of the skyrmion. Furthermore we have the uniaxial anisotropy in the z-direction, applicable to a (e.g. Pt/Co) layered system. The term $\mu_0 HM$ comes from the Zeeman energy density, which gives the coupling of the system to the external magnetic field H, and μ_0 is the magnetic permeability in free space.

In order to calculate the integral in the above equation, we transform $E[\Omega]$ to polar coordinates $(\theta(\rho,\varphi,z),\phi_0,z)$ and obtain:

$$\begin{aligned}\frac{E(\theta)}{2\pi t_{FM}} = \frac{J_s}{2}\int d\rho\rho\Bigg\{ &\left(\frac{d\theta}{d\rho}\right)^2 + \frac{sin^2(\theta)}{\rho^2} + 2C_2(1-cos(\theta))\\ &+cos(\phi_0)\left(\frac{d\theta}{d\rho}+\frac{sin(\theta)cos(\theta)}{\rho}\right)\\ &+(C_1+C_3cos^2(\phi_0))sin^2(\theta)\Bigg\},\end{aligned}$$

where we rescaled the constants by bringing some factors like J_s outside of the integral, leaving us with the three dimensionless parameters; $C_1 = \frac{2J_sK}{C^2}$ $C_2 = \frac{\mu_0 HMJ_s}{C^2}$ $C_3 = \frac{2\mu_0 M^2 J_s}{C^2}$. Different values of these parameters will be compared, but first we need to vary the energy to get a differential equation for θ, i.e.

$$E(\theta) \to E(\theta+\delta\theta).$$

Varying the energy results in:

$$E(\theta+\delta\theta) = E(\theta) + \int d\rho(**)\delta\theta,$$

where (**) is:

$$\begin{aligned}&\frac{d^2\theta}{d\rho^2} + \frac{1}{\rho}\frac{d\theta}{d\rho} - \frac{sin(\theta)cos(\theta)}{\rho^2} + cos(\phi_0)\frac{sin^2(\theta)}{\rho}\\ &-\Big(C_1+C_3cos^2(\phi_0)\Big)sin(\theta)cos(\theta) - C_2 sin(\theta) = 0.\end{aligned}$$

As we have seen, the skyrmion profile only depends on the polar angle θ. So what we want to do next is to solve this differential equation for θ. Note however, that this cannot be done analytically. Therefore we have to solve it numerically. Numerically we find the skyrmion solution θ_{Sk} for different values of C_1, C_2 and C_3. We have plotted these in figure 6.

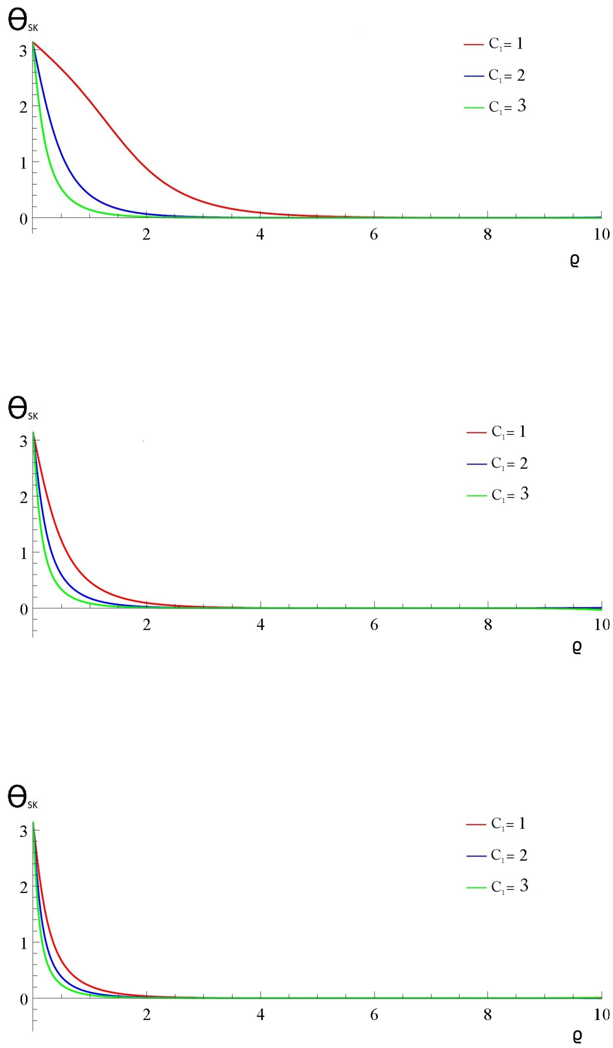

Figure 6: *Three plots with different values of C_2, respectively C_2= 0, 1 and 2. C_3=0.*

3 Skyrmion dynamics in an infinite system

In this chapter we will take a look at the dynamic behaviour of a skyrmion in an infinite system, that is, a system without boundaries. In the first paragraph we will find that a skyrmion will not move if there is not an electric field to drive it. Thereafter we will apply an in-plane electric field and observe the skyrmion's movement. We follow the discussion in 12.

3.1 Dynamics

We start with the Landau-Lifshitz-Gilbert equation (LLG), which gives a good description of the magnetization dynamics of the skyrmion.

$$\frac{\partial \mathbf{\Omega}}{\partial t} = -\frac{\gamma}{M}\mathbf{\Omega} \times \frac{\delta E[\mathbf{\Omega}]}{\delta \mathbf{\Omega}} - \alpha_G \mathbf{\Omega} \times \frac{\partial \mathbf{\Omega}}{\partial t}, \tag{3.1}$$

where γ is the gyromagnetic ratio, M the magnetization and α_G the Gilbert damping constant. Furthermore $\frac{\delta E[\mathbf{\Omega}]}{\delta \mathbf{\Omega}} = 0$ for the skyrmion solution, shown in Appendix A, and we'll be using this in Chapter 4 to find the boundary conditions. So the LLG equation reduces to:

$$\frac{\partial \mathbf{\Omega}}{\partial t} = -\alpha_G \mathbf{\Omega} \times \frac{\partial \mathbf{\Omega}}{\partial t}. \tag{3.2}$$

To represent the skyrmion's dynamic motion, it is necessary to add a time dependence, such that the magnetization has a time dependence term, i.e.,

$$\Omega(x) \to \Omega(x - X_{sk}(t)).$$

This transforms the LLG equation into:

$$\frac{\partial \mathbf{\Omega}(x - X_{sk}(t))}{\partial t} = -\alpha_G \mathbf{\Omega}(x - X_{sk}(t)) \times \frac{\partial \mathbf{\Omega}(x - X_{sk}(t))}{\partial t},$$

Now we take the crossproduct between $\Omega(x - X_{sk}(t))$ and the LLG equation. This leads to:

$$\mathbf{\Omega}(x - X_{sk}(t)) \times -\nabla\mathbf{\Omega} \cdot \frac{\partial \boldsymbol{X}_{sk}(t))}{\partial t} + \mathbf{\Omega}(x - X_{sk}(t)) \times \left(\alpha_G \mathbf{\Omega}(x - X_{sk}(t)) \times -\nabla\mathbf{\Omega} \cdot \frac{\partial \boldsymbol{X}_{sk}(t))}{\partial t}\right) = 0.$$

Next we take the dot product with the partial derivatives and integrate over all space. In order to calculate this integral we change our coordinate system from Cartesian to polar coordinates.
As we took $\phi_0 = 0$, we consequently have

$$\Omega(\theta) = sin\theta\hat{\rho} + cos\theta\hat{z}, \tag{3.3}$$

resulting in:

$$\epsilon_{ij} 4\dot{x}_j - \alpha_G A \dot{x}_j = 0, \tag{3.4}$$

with ϵ_{ij} the Levi-Civita symbol. This tells us that $\dot{x} = \dot{y} = 0$ which means that the velocity of the skyrmion is zero in the absence of driving forces.

3.2 Moving skyrmion

To get the skyrmion moving, we add an in-plane current J_ϵ in the x-direction. To take this into account we need to modify the LLG equation and we add a term which describes this current. It is given by $\beta\Omega \times (I_s \times \Omega)$, with I_s the spin current and

$$\beta = \frac{\gamma\hbar\theta_{SH} J_\epsilon}{2e\mu_0 M t_p}.$$

Here θ_{SH} the effective spin-Hall angle of the normal-metal layer. Going through the same procedure as above, this new term becomes:

$$-\int d\boldsymbol{x} \frac{\partial \boldsymbol{\Omega}}{\partial x_i} \cdot (\boldsymbol{\Omega} \times (\beta\boldsymbol{\Omega} \times (I_s \times \boldsymbol{\Omega})))$$
$$= -\epsilon_{ij}\pi\beta I_{x_i} \int d\rho \bigg(\rho\frac{d\theta}{d\rho} + sin\theta cos\theta\bigg).$$

The integral on the right hand side can be evaluated numerically, we just call it a constant B. Putting it all together we get:

$$\epsilon_{ij} 4\dot{x}_i - \alpha_G A\dot{x}_j = -\epsilon_{ij}\beta B I_{x_i}, \tag{3.5}$$

From this we can find the skyrmion's velocity. It is remarkable that the $\dot{y}$ component of the velocity has a non-zero value, so the movement will not be only in the direction of the applied electric field, (see 12).

4 Skyrmion dynamics in a finite system

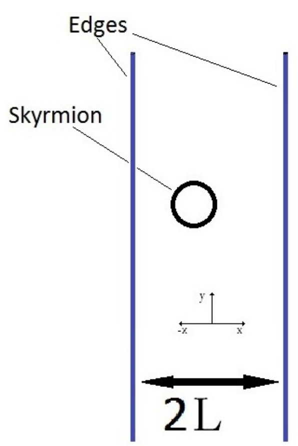

Figure 7: *Skyrmion in a finite system with length $2L$.*

4.1 Boundary Conditions

Our purpose in this section is to find how the magnetization vector is oriented near the edge of a finite system, and how this will influence the motion of the skyrmion. We will find that the magnetization at the edge is tilted, presumably resulting in a repulsive force on the skyrmion. To find the boundary condition for a finite system we begin with demanding that the Landau-Lifshitz-Gilbert equation on the boundary should be zero.

$$\frac{\partial \boldsymbol{\Omega}}{\partial t} = -\frac{\gamma}{M}\boldsymbol{\Omega} \times \frac{\delta E[\boldsymbol{\Omega}]}{\delta \boldsymbol{\Omega}} = 0. \tag{4.1}$$

Now, we first plug in $\frac{\delta E[\boldsymbol{\Omega}]}{\delta \boldsymbol{\Omega}}$

$$-\frac{\gamma}{M}\boldsymbol{\Omega} \times \Big(\nabla^2\boldsymbol{\Omega} + \hat{y}\frac{C}{2}\frac{\partial \boldsymbol{\Omega}}{\partial x} - \hat{x}\frac{C}{2}\frac{\partial \boldsymbol{\Omega}}{\partial y}\Big) = 0.$$

Next we perform some calculations and (we use the third term in appendix A to) rewrite the equation to:

$$-\frac{\gamma}{M}\int \frac{J}{2}\boldsymbol{\Omega} \times \nabla^2\boldsymbol{\Omega} + \boldsymbol{\Omega} \times C\delta\boldsymbol{\Omega} \cdot \bigg(\frac{\partial \boldsymbol{\Omega}}{\partial x} \times \hat{y}\bigg) = 0,$$

and obtain with some more mathematical steps (via appendix B) the boundary condition for Ω:

$$\frac{\partial \boldsymbol{\Omega}}{\partial x} = \frac{1}{\xi}\bigg(\hat{y} \times \boldsymbol{\Omega}\bigg)\bigg|_{Edge}. \tag{4.2}$$

This is in agreement with the findings of previous studies.[11] The above result gives us the opportunity to find the boundary conditions in terms of θ. To do this we simply fill in Ω in the above equation, from which, we distill:

$$\frac{\partial \theta}{\partial x} = \frac{1}{\xi}\big(cos(\phi)\big). \tag{4.3}$$

For $\phi_0 = 0$ this means

$$\frac{\partial \theta}{\partial x} = \frac{1}{\xi}. \tag{4.4}$$

4.2 Varying the polar angle theta

In this section we vary the energy with respect to θ. (Varying with respect to ϕ gives no contribution.) Varying θ as said leads to:

$$E[\theta + \delta\theta, \phi] \simeq E[\theta] + \int \frac{\delta E}{\delta \theta}\delta\theta,$$

With

$$\frac{\delta E}{\delta \theta} = -J_s\frac{\partial^2\theta}{\partial x^2} + 2K\sin(\theta)\cos(\theta)$$
$$+ C\sin^2(\theta)\sin(\phi)\frac{\partial \phi}{\partial x} - \mu_0 HM\sin(\theta) = 0.$$

We now consider θ to be small, so that: $\cos(\theta) \approx 1$, $\sin(\theta) \approx \theta$, $\mathcal{O}(\theta^2) = 0$ and $\phi = \phi_0 = 0$. After using these approximations, a lot of terms drop out and we remain with a second order differential equation:

$$\frac{\partial^2\theta}{\partial x^2} = \alpha\theta, \tag{4.5}$$

where we defined $\alpha := \frac{1}{J_s}(\mu_0 HM - 2K)$.

4.3 Solving the polar angle theta on the edge

The general solution to this second order differential equation is of the form:

$$\theta(x) = Ae^{i\sqrt{\alpha}x} + Be^{-i\sqrt{\alpha}x}. \tag{4.6}$$

Using $\theta(0) = 0$ we have A = - B and this in combination with the boundary condition from section 4.1, evaluated at the edge, gives us:

$$\left.\frac{\partial\theta}{\partial x}\right|_L = i\sqrt{\alpha}Ae^{i\sqrt{\alpha}L} + i\sqrt{\alpha}Ae^{-i\sqrt{\alpha}L} = \frac{1}{\xi}. \tag{4.7}$$

Now we can solve for A and insert it in equation (4.6), which results in:

$$\theta_{edge}(x) = \frac{1}{\sqrt{\alpha}\xi cos(\sqrt{\alpha}L)} sin(\sqrt{\alpha}x), \tag{4.8}$$

with $\alpha = \frac{1}{J_s}(\mu_0 HM - 2K)$ and $\xi = \frac{J_s}{2C}$. Just to get a feeling on how the magnetization in the edge depends on the length of the system we have made a few plots, see figure 8.

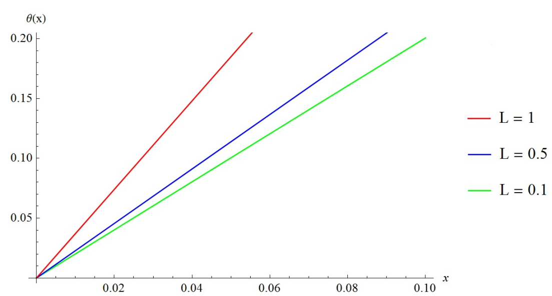

Figure 8: *A plot of θ_{edge} for different sizes of the system, we see that the curve gets steeper for bigger length scales.*

So we have found that the magnetization at the edge is described a sine, but for small x it behaves linearly, as can be seen clearly in figure 8. Therefore we will introduce a linear function $f(x)$ which represents the magnetization at the edge as follows:

$$f(x) = \begin{cases} \frac{c(x-(L-\xi))}{\xi} & \text{if } x > L - \xi \\ 0 & \text{if } x \leq L - \xi \end{cases}$$

for some arbitrary constant c. We just want to understand the system qualitatively, so for simplicity we take c=1. Now the total magnetization of our finite system is given by adding the skyrmion profile to the magnetization profile at the edge, shown by figure 9.

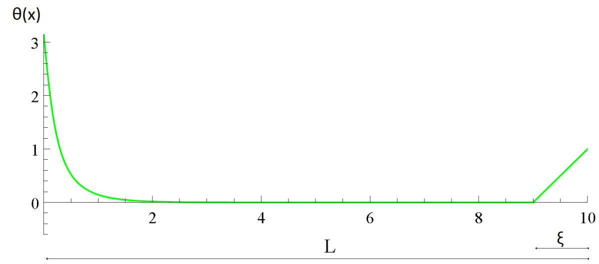

Figure 9: *A plot of $\theta_{total}(x) = \theta_{Sk}(x) + f(x)$. In this figure we see the total magnetization profile of our system, i.e., the sum of the skyrmion profile θ_{Sk} plus the magnetization profile of the edge f(x).*

4.4 Equations of Motion

In a similar way as in section 3.2, we will apply an electric field, which will induce a motion of the skyrmion as we saw earlier. But now our system has boundaries, which means that there will be an interaction between the skyrmion and the edge when the skyrmion approaches this boundary. We expect that the edge will act as a bumper, so when the skyrmion comes in, it collides with the edge and bounces back. To make it more visual take a look at figure 10b,c. The closer the skyrmion gets to the edge the stronger the repulsive force. And for sufficient high velocities the skyrmion will probably break through and escape. To check these statements we want to calculate the system's potential energy profile. Doing this is beyond the scope of this thesis, but nonetheless we will provide a starting point to do this.

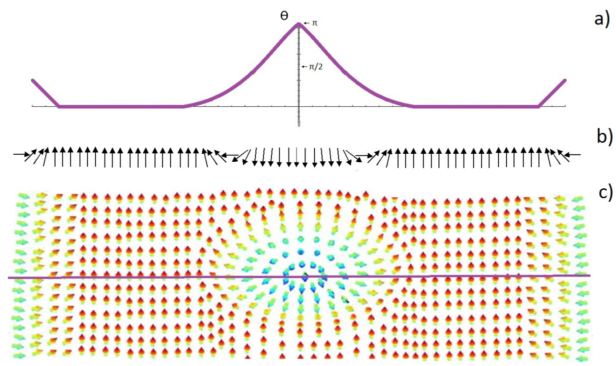

Figure 10: *In these figures we see the total profile of our system, a) Total profile cross-section in the x-direction. b) Orientation of the magnetization vectors schematically. c) (semi-)topdown-view on a hedgehog-skyrmion and edge profile.*

So first we need to have θ_{total}(x) = θ_{Sk}(x) + f(x) explicitly, where f(x) as before and we will use an approximation for the numerical skyrmion solution θ_{Sk} namely θ_{Sk}(x)= πe^{-x}. The skyrmion profile in Cartesian coordinates, which we need to evaluate the energy in a finite system, is given by:

$$\Omega_{sk}(x,y,z) = \begin{pmatrix} sin(\theta(\sqrt{x^2+y^2}))\frac{x}{\sqrt{x^2+y^2}} \\ sin(\theta(\sqrt{x^2+y^2}))\frac{y}{\sqrt{x^2+y^2}} \\ cos(\theta(\sqrt{x^2+y^2})) \end{pmatrix}$$

Now to find the potential energy we define a potential V(x) by means of

$$V[\mathbf{\Omega}(\theta_{total})]. \tag{4.9}$$

Numerical evaluation of this potential is beyond the scope of this thesis.

5 Conclusion

In conclusion, after introducing the topic of magnetism in materials, we studied the profile of a static skyrmion, as well as the dynamics of a skyrmion in an infinite system. Also we have found the magnetization at the edge of a finite system. We found that the Dzyaloshinskii–Moriya interaction is responsible for the magnetic tilt at the edge. Furthermore we made a start to use this to describe the dynamics of a skyrmion in such a finite system. Hopefully we are now one step closer to skyrmion-based magnetic information storage. The boundary repulsion can possibly be used to control skyrmion dynamics in nanowires, from which we may build efficient magnetic memory with very low energy consumption. This results in a longer discharge cycle for your smartphone's battery, which can contribute to your daily happiness. Above all it will reduce the amount of energy required in data storage worldwide! But we are not there yet, research must be continued to realize skyrmion based data carriers.

Future research on this topic can be performed by calculating how the interaction between the skyrmion and edge magnetization goes. We have investigated a system with easy-axis anisotropy, it should be noted that some recent study,[1] finds enhanced stability for the skyrmion with an easy-plane anisotropy instead of easy-axis, so this may be a field to endeavour. There is also research being conducted on creating ("writing") skyrmions, but not so much research on efficiently and quickly detecting ("reading") skyrmions. Research in this field is encouraged.

6 Appendices

6.1 Appendix A

The $\frac{\delta E[\mathbf{\Omega}]}{\delta \mathbf{\Omega}}$ term.
First we use some variational calculus

$$E[\mathbf{\Omega}] \to E[\mathbf{\Omega} + \delta\mathbf{\Omega}]. \tag{6.1}$$

After doing this we will get something in the form

$$E[\mathbf{\Omega} + \delta\mathbf{\Omega}] = E[\mathbf{\Omega}] + \int dx \frac{\delta E[\Omega]}{\delta \mathbf{\Omega}} \delta\Omega. \tag{6.2}$$

The calculation is done below.
Our expression for the energy is

$$\begin{aligned} E(\mathbf{\Omega}) =& t_{FM} \int dx \Bigg\{ -\frac{J_s}{2} \mathbf{\Omega} \cdot \nabla^2 \mathbf{\Omega} + K(1 - \Omega_z^2) \\ &+ \frac{C}{2}\left(\hat{y} \cdot \left(\mathbf{\Omega} \times \frac{\partial \mathbf{\Omega}}{\partial x} \right) - \hat{x} \cdot \left(\mathbf{\Omega} \times \frac{\partial \mathbf{\Omega}}{\partial y} \right) \right) \\ &+ \mu_0 H M (1 - \Omega_z) - \mu_0 M \mathbf{\Omega} \cdot \vec{H}_d \Bigg\} \end{aligned}$$

We vary the energy, to keep things clear and transparent we'll do this term by term.

1) First term $-\frac{J_s}{2}\mathbf{\Omega} \cdot \nabla^2 \mathbf{\Omega}$;

$$\mathbf{\Omega} \cdot \nabla^2 \mathbf{\Omega} \to (\mathbf{\Omega} + \delta\mathbf{\Omega}) \cdot \nabla^2 (\mathbf{\Omega} + \delta\mathbf{\Omega}) \tag{6.3}$$

$$= \mathbf{\Omega} \cdot \nabla^2 \mathbf{\Omega} + \delta\mathbf{\Omega} \cdot \nabla^2 (\mathbf{\Omega}) + \mathbf{\Omega} \cdot \nabla^2 \delta\mathbf{\Omega} + \delta\mathbf{\Omega} \cdot \nabla^2 \delta\mathbf{\Omega}. \tag{6.4}$$

2) Second term $K(1 - \Omega_z^2)$;

$$K(1 - \Omega_z^2) \to K(1 - (\Omega_z + \delta\Omega_z)^2) \tag{6.5}$$

$$= K(1 - (\Omega_z^2 + \delta\Omega_z^2 + 2\Omega_z \delta\Omega_z)) \tag{6.6}$$

$$\approx K(1 - (\Omega_z^2 + 2\Omega_z \delta\Omega_z)) \tag{6.7}$$

$$= K(1 - \Omega_z^2) + -K2\Omega_z \delta\Omega_z). \tag{6.8}$$

3) Third term

$$\begin{aligned} &\frac{C}{2}\left(\hat{y} \cdot \left(\mathbf{\Omega} \times \frac{\partial \mathbf{\Omega}}{\partial x} \right) - \hat{x} \cdot \left(\mathbf{\Omega} \times \frac{\partial \mathbf{\Omega}}{\partial y} \right) \right) \\ &\to \frac{C}{2}\left(\hat{y} \cdot \left((\mathbf{\Omega} + \delta\mathbf{\Omega}) \times \frac{\partial (\mathbf{\Omega} + \delta\mathbf{\Omega})}{\partial x} \right) - \hat{x} \cdot \left((\mathbf{\Omega} + \delta\mathbf{\Omega}) \times \frac{\partial (\mathbf{\Omega} + \delta\mathbf{\Omega})}{\partial y} \right) \right) \\ &= \frac{C}{2}\hat{y} \cdot \left(\left(\mathbf{\Omega} \times \frac{\partial \mathbf{\Omega}}{\partial x} \right) + \left(\mathbf{\Omega} \times \frac{\partial \delta\mathbf{\Omega}}{\partial x} \right) + \left(\delta\mathbf{\Omega} \times \frac{\partial \mathbf{\Omega}}{\partial x} \right) + \left(\delta\mathbf{\Omega} \times \frac{\partial \delta\mathbf{\Omega}}{\partial x} \right) \right) \\ &- \frac{C}{2}\hat{x} \cdot \left(\left(\mathbf{\Omega} \times \frac{\partial \mathbf{\Omega}}{\partial y} \right) + \left(\mathbf{\Omega} \times \frac{\partial \delta\mathbf{\Omega}}{\partial y} \right) + \left(\delta\mathbf{\Omega} \times \frac{\partial \mathbf{\Omega}}{\partial y} \right) + \left(\delta\mathbf{\Omega} \times \frac{\partial \delta\mathbf{\Omega}}{\partial y} \right) \right) \\ \\ &= \frac{C}{2}\hat{y} \cdot \left(\left(\mathbf{\Omega} \times \frac{\partial \mathbf{\Omega}}{\partial x} \right) + \left(\delta\mathbf{\Omega} \times \frac{\partial \mathbf{\Omega}}{\partial x} \right) + \left(\mathbf{\Omega} \times \frac{\partial \delta\mathbf{\Omega}}{\partial x} \right) \right) \\ &- \frac{C}{2}\hat{x} \cdot \left(\left(\mathbf{\Omega} \times \frac{\partial \mathbf{\Omega}}{\partial y} \right) + \left(\delta\mathbf{\Omega} \times \frac{\partial \mathbf{\Omega}}{\partial y} \right) + \left(\mathbf{\Omega} \times \frac{\partial \delta\mathbf{\Omega}}{\partial y} \right) \right), \end{aligned}$$

we do the part with the $\hat{y}$ term first. The $\hat{x}$ is done in a similar fashion.
Partial integration gives us:

$$\int \frac{C}{2}\hat{y}\cdot\left(\mathbf{\Omega}\times\frac{\partial\delta\mathbf{\Omega}}{\partial x}\right). = \frac{C}{2}\hat{y}\cdot\left(\mathbf{\Omega}\times\delta\mathbf{\Omega}\right)\Bigg|_S - \int\frac{C}{2}\hat{y}\cdot\left(\frac{\partial\mathbf{\Omega}}{\partial x}\times\delta\mathbf{\Omega}\right) \tag{6.9}$$

Cyclic permuting, i.e. $(a\cdot(b\times c) = b\cdot(c\times a))$:

$$\hat{y}\cdot\left(\frac{\partial\mathbf{\Omega}}{\partial x}\times\delta\mathbf{\Omega}\right) = \delta\mathbf{\Omega}\cdot\left(\hat{y}\times\frac{\partial\mathbf{\Omega}}{\partial x}\right). \tag{6.10}$$

Using the above gives:

$$= \frac{C}{2}\hat{y}\cdot\left(\delta\mathbf{\Omega}\times\frac{\partial\mathbf{\Omega}}{\partial x}\right) - \frac{C}{2}\delta\mathbf{\Omega}\cdot\left(\hat{y}\times\frac{\partial\mathbf{\Omega}}{\partial x}\right) + \frac{C}{2}\hat{y}\cdot\left(\mathbf{\Omega}\times\delta\mathbf{\Omega}\right)\Bigg|_S \tag{6.11}$$

$$= \frac{C}{2}\delta\mathbf{\Omega}\cdot\left(\frac{\partial\mathbf{\Omega}}{\partial x}\times\hat{y}\right) - \frac{C}{2}\delta\mathbf{\Omega}\cdot\left(\hat{y}\times\frac{\partial\mathbf{\Omega}}{\partial x}\right). \tag{6.12}$$

$$= C\delta\mathbf{\Omega}\cdot\left(\frac{\partial\mathbf{\Omega}}{\partial x}\times\hat{y}\right). \tag{6.13}$$

Doing the same for $\hat{x}$ and obtaining:

$$= -C\delta\mathbf{\Omega}\cdot\left(\frac{\partial\mathbf{\Omega}}{\partial y}\times\hat{x}\right). \tag{6.14}$$

4) The fourth term is simply:

$$\mu_0 HM(1-\Omega_z)\rightarrow\mu_0 HM(1-\Omega_z) - \mu_0 HM\delta\Omega_z. \tag{6.15}$$

5) And the fifth:

$$\mu_0 M\mathbf{\Omega}\cdot\vec{H}_d\rightarrow\mu_0 M\mathbf{\Omega}\cdot\vec{H}_d + \mu_0 M\delta\mathbf{\Omega}\cdot\vec{H}_d. \tag{6.16}$$

Now we collect all terms with $\delta\theta$ and equal it to zero, we neglect higher orders of $\delta\theta$.

$$\delta\mathbf{\Omega}\cdot\nabla^2(\mathbf{\Omega}) - K2\Omega_z\delta\Omega_z + C\delta\mathbf{\Omega}\cdot\left(\frac{\partial\mathbf{\Omega}}{\partial x}\times\hat{y}\right) - C\delta\mathbf{\Omega}\cdot\left(\frac{\partial\mathbf{\Omega}}{\partial y}\times\hat{x}\right) - \mu_0 HM\delta\Omega_z + \mu_0 M\delta\mathbf{\Omega}\cdot\vec{H}_d = 0.$$

6.2 Appendix B

We start with

$$-\frac{\gamma}{M}\int \Omega \times \frac{\delta E}{\delta \Omega} = -\frac{\gamma}{M}\int \frac{J}{2}\mathbf{\Omega} \times \nabla^2 \mathbf{\Omega} + \mathbf{\Omega} \times C\delta\mathbf{\Omega} \cdot \left(\frac{\partial \mathbf{\Omega}}{\partial x} \times \hat{y}\right), \tag{6.17}$$

Integrating over a box with length d on the edge:

$$-\frac{\gamma}{M}\int_{-d/2}^{d/2} dx \Big\{ \frac{J}{2}\mathbf{\Omega} \times \nabla^2 \mathbf{\Omega} + C\mathbf{\Omega} \times \left(\frac{\partial \mathbf{\Omega}}{\partial x} \times \hat{y}\right)\Big|_a \Big\} = 0. \tag{6.18}$$

Rewriting by taking a partial derivative out in front of the first term we want to do this for the second term as well:

$$-\frac{\gamma}{M}\int_{-d/2}^{d/2} dx \Big\{ \frac{J}{2}\frac{\partial}{\partial x}\left(\mathbf{\Omega} \times \frac{\partial \mathbf{\Omega}}{\partial x}\right) + C\mathbf{\Omega} \times \left(\frac{\partial \mathbf{\Omega}}{\partial x} \times \hat{y}\right)\Big|_a \Big\} = 0, \tag{6.19}$$

This can be done because:

$$\frac{\partial}{\partial x}\left(\mathbf{\Omega} \times \left(\mathbf{\Omega} \times \hat{y}\right)\Big|_a\right) = \frac{\partial \mathbf{\Omega}}{\partial x} \times \left(\mathbf{\Omega} \times \hat{y}\right)\Big|_a + \mathbf{\Omega} \times \left(\frac{\partial \mathbf{\Omega}}{\partial x} \times \hat{y}\right)\Big|_a \tag{6.20}$$

and

$$\frac{\partial \mathbf{\Omega}}{\partial x} \times \left(\mathbf{\Omega} \times \hat{y}\right)\Big|_a = \mathbf{\Omega}\Big(\frac{\partial \mathbf{\Omega}}{\partial x} \cdot \hat{y}\Big) - \hat{y}\Big(\frac{\partial \mathbf{\Omega}}{\partial x} \cdot \mathbf{\Omega}\Big) = 0. \tag{6.21}$$

So using the previous we have:

$$-\frac{\gamma}{M}\int_{-d/2}^{d/2} dx \Big\{ \frac{J}{2}\frac{\partial}{\partial x}\left(\mathbf{\Omega} \times \frac{\partial \mathbf{\Omega}}{\partial x}\right) + C\frac{\partial}{\partial x}\left(\mathbf{\Omega} \times \left(\mathbf{\Omega} \times \hat{y}\right)\Big|_a\right). \tag{6.22}$$

And now we can take out the partial derivative:

$$-\frac{\gamma}{M}\int_{-d/2}^{d/2} dx \frac{\partial}{\partial x}\Big\{ \frac{J}{2}\mathbf{\Omega} \times \frac{\partial \mathbf{\Omega}}{\partial x} + C\mathbf{\Omega} \times \left(\mathbf{\Omega} \times \hat{y}\right)\Big|_a \Big\} = 0, \tag{6.23}$$

$$-\frac{\gamma}{M}\int_{-d/2}^{d/2} dx \frac{\partial}{\partial x}\Big\{ \frac{J}{2}\mathbf{\Omega} \times \left(\mathbf{\Omega} \times \frac{\partial \mathbf{\Omega}}{\partial x}\right) + C\mathbf{\Omega} \times \left(\mathbf{\Omega} \times \left(\mathbf{\Omega} \times \hat{y}\right)\Big|_a\right)\Big\} = 0. \tag{6.24}$$

The integral is zero only if:

$$\frac{J}{2}\left(\frac{\partial \mathbf{\Omega}}{\partial x}\right) + C\left(\mathbf{\Omega} \times \hat{y}\right)\Big|_a = 0, \tag{6.25}$$

$$\frac{J}{2}\left(\frac{\partial \mathbf{\Omega}}{\partial x}\right) = -C\left(\mathbf{\Omega} \times \hat{y}\right)\Big|_a, \tag{6.26}$$

$$\frac{\partial \mathbf{\Omega}}{\partial x} = -C\frac{2}{J}\left(\mathbf{\Omega} \times \hat{y}\right)\Big|_a. \tag{6.27}$$

Renaming the prefactor,

$$\frac{\partial \mathbf{\Omega}}{\partial x} = -\frac{1}{\xi}\left(\mathbf{\Omega} \times \hat{y}\right)\Big|_a \tag{6.28}$$

$$= \frac{1}{\xi}\left(\hat{y} \times \mathbf{\Omega}\right)\Big|_a. \tag{6.29}$$

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