# Evaporative Cooling of a Bose Gas and Interacting Spinor Condensates

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#### Abstract

This thesis reports on the two projects I have completed during my Bachelor's research at the Bose Einstein Condensate (BEC) research group of Peter van der Straten: optimizing evaporative cooling and measuring interactions between spinor condensates.

To create a BEC, the process of evaporative cooling is used. This process is modeled and optimized using a Monte-Carlo optimization, which shows that the lab procedure can be improved to gain up to 13% particles at  $T_c$  in a specific situation. The optimization also shows that expanding the trap volume during the cooling process is less efficient than keeping the volume at the smallest achievable value in our setup, since the densities are not high enough for the loss processes to dominate.

To measure spin drag in BEC's, a measurement is prepared in the far off resonance trap in which two spinor condensates are accelerated towards a third condensate, where all three condensates are in different spin states. The measurement shows that under the specific conditions of the experiment, the condensates collide inelastically. It was hypothesized that this could be explained by the immiscibility of different spin condensates. This is refuted, since the force due to immiscibility is one order of magnitude smaller than necessary to explain the motion of the condensates.

**Keywords:** Bose Einstein Condensation, evaporative cooling, monte-carlo optimization, spinor condensates, spin drag, domain walls, immiscibility

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# Chapter 1

# Introduction

A Bose Einstein Condensate (BEC) is a state of matter with very unusual properties; it is for example superfluid and coherent matter. This makes it a very interesting research subject and also a perfect setting for research into phenomena such as atomic (spin) interactions, spin waves and density waves. It is, however, very complicated to create a BEC: it can exist only under very specific conditions, which are impossible to create without specialized equipment and detailed knowledge of physical processes on the atomic level. In a way, this adds to the appeal of BEC research: one has to do a lot of work to discover the properties of BEC's. The difficulty of creating a BEC is also reflected in the fact that it took 70 years from its first theoretical prediction to the actual creation of a BEC in the laboratory in 1995. Since then the amount of research into BEC's has increased rapidly.

In this bachelor's thesis, I will report on two projects I have completed while working for the BEC research group at the Debye Institute at Utrecht University. In the following two sections, I will briefly explain the nature of a BEC and the experimental apparatus in which the BEC's are created and studied. Chapters 2 and 3 discuss a simulation and optimization program I have created to improve one of the procedures that is used to create a BEC, namely evaporative cooling of a sodium gas. Chapter 4 discusses a measurement of interactions between condensates with different spins, so called spinor condensates.

## 1.1 Bose Einstein Condensate

All particles can be classified into two classes: fermions and bosons. Systems of fermions are described by a wave function, which is antisymmetric with respect to the exchange of two particles, while the wave function for a boson system is always symmetric with respect to two particle exchange([7], p.4). A consequence of the antisymmetry of the fermion wave function is that fermions cannot occupy the same state. Bosons, on the other hand, can occupy the same state. This explains the difference in behaviour between bosons and fermions at low temperature: while bosons can accumulate in the lowest energy state, fermions can not and are still distributed among many energy levels. When the occupation of the ground state by bosons becomes macroscopic, it is called a Bose Einstein Condensate and can be described by a single wave function. Additionally, the formation of the BEC is a phase transition.

This phenomenon can also be approached from the wave description of atoms: when a gas is cooled, the de Broglie wavelength  $\Lambda = \sqrt{\frac{h^2}{2\pi m k_b T}}$  of the atoms increases. If the average spacing between the atoms becomes smaller than the de Broglie wavelength, the atoms start to overlap. Ultimately, the overlapping waves combine into a giant matter wave or macroscopic wavefunction, the BEC.

In experiments, BEC's are created in dilute gases of for example sodium or rubidium atoms, by cooling the gas to extremely low temperatures. This was first accomplished in 1995. In the experimental setup in the BEC-lab in Utrecht, the temperature where the condensation begins lies usually around 1  $\mu$ K, while the typical density of a condensate is 10<sup>20</sup> m<sup>-3</sup>.

### **1.2** Experimental setup

The experimental setup consists of many components, such as lasers, coils with a cooling system, an ultra high vacuum chamber, wiring and various computer-controlled switches<sup>1</sup>. All these components together form a machine that can cool a gas to very low temperatures (~ 1  $\mu$ K), in this case a gas of sodium atoms. To create a BEC, solid sodium is brought into the gas state by heating it in an oven to approximately 570 K. The atoms then escape the oven through a nozzle to create an atomic beam.

<sup>&</sup>lt;sup>1</sup>This section is based on discussions between Pieter, Alexander and myself, but the information can also be found in for example [6] or [7]

These atoms are cooled in a so-called Zeeman slower: in a tube in which a magnetic field is present, a laser beam hits the atoms in the direction opposite to the atom beam's motion. The laser is near-resonant to a transition of the sodium atoms and due to the Doppler shift, is only resonant for atoms with a specific velocity. These atoms interact with photons from the laser, which has two consequences: firstly, they pick up momentum from the photon, which slows them down. Secondly, the photon excites the atom. After a very short time (several nanoseconds), the atom will emit the photon, in a random direction. This means that the atom gains some velocity, but in a random direction. Therefore, when averaged over many atom-photon interactions, the change in the atom's velocity due to the emission of the photon averages to zero. The initial interaction that excites the atoms does slow the atoms down and therefore the beam is effectively cooled by this process. After an atom has been slowed down, the laser is no longer resonant with the transition. To be able to cool atoms with many different velocities, the inhomogeneous magnetic field is present: the magnitude of the magnetic field varies along the axis of the tube, in such a way that at the beginning of the tube, atoms with a large velocity are resonant with the laser and at the end of the tube, only atoms with a very low velocity are resonant. This is a consequence of the Zeeman effect, which shifts the resonance required to excite the atoms. At the end of the Zeeman slower, the gas has a temperature in the order of a few milliKelvin.

The atoms are then loaded into a magneto-optical trap (MOT), where the atoms are trapped as well as cooled. Six near resonant and circularly polarized laser beams shine on the gas in the origin of a coordinate system, with the lasers along the axes. A spatially varying magnetic field is also present, which Zeeman shifts the resonance of the atoms. If an atom moves away from the trap center, the absorption rates for photons from all the laser beams change, inducing a restoring force towards the origin. This traps the atoms at the center of the six laser beams. In addition, the atoms are cooled by the same mechanism that cools the atoms in the Zeeman slower, down to approximately 500  $\mu$ K.

The gas is loaded into a magnetic trap (MT), which is a spatially varying magnetic field that creates a three dimensional harmonic potential. In this trap, the atoms are cooled using evaporative cooling until they condense into a BEC at around 1  $\mu$ K. This process will be discussed in detail in the next chapter. The MT only traps atoms in the  $|F = 1, m_f = -1\rangle$  state. For experiments where different spin species are studied, the gas can also be loaded into a far off resonance trap (FORT). This trap is induced by a sharply focussed laser beam, which creates a potential using the AC Stark effect, which can be well approximated by a harmonic potential. In the FORT, atoms in the  $|F = 1, m_f = -1\rangle$ ,  $|F = 1, m_f = 0\rangle$  and the  $|F = 1, m_f = 1\rangle$  state are all trapped.

As a final step, the gas or condensate can be viewed using absorption imaging. A laser beam shines on the atoms, which absorb photons from the laser beam. A camera is placed in the laser beam after the atom cloud, such that on the camera the shadow of the atom cloud is cast. Usually, the atom cloud is imaged in time of flight (tof), which means that the trap is switched off, typically 10 to 30 milliseconds before the images are made. This allows the gas to expand and the shadow of the expanding gas gives information about the velocity distribution of the gas. From the total absorption one can derive the number of particles in the atom cloud and from the velocity distribution its temperature. Absorption imaging has two important limitations: firstly, due to the photon absorption, the BEC is destroyed when a picture is taken, making it impossible to take several pictures of the same condensate. Secondly, when the density of the atom cloud is too high, it is no longer possible to extract the exact velocity distribution from the image, since the uncertainty in the measured density increases exponentially and can for large densities even become larger than the density itself.

# Chapter 2

# **Evaporative Cooling**

In order to create a Bose Einstein Condensate in a gas, one has to increase the phase space density of the gas, given by ([7], p.22):

$$N(\frac{\hbar\bar{\omega}}{k_bT})^3,\tag{2.1}$$

where  $\hbar$  is the Planck constant divided by  $2\pi$ ,  $\bar{\omega} = (\omega_x \omega_y \omega_z)^{1/3}$  is the geometric mean of the trap frequencies, T is the temperature and  $k_b$  is the Boltzmann constant. This means in practice that the particle density of the gas has to be increased and the temperature has to be lowered. There are not many processes that increase phase space density, but laser cooling, as explained in the previous chapter and evaporative cooling are two of the few processes that do. In this section, the evaporative cooling process is discussed.

One can cool a classical gas by making use of the fact that in such a gas, the energy distribution of the atoms satisfies a Maxwell-Boltzmann distribution with a finite width ([3], p.182). By removing atoms with an energy above the average energy, the temperature of the remaining atoms is lowered. Although removing atoms will initially change the energy distribution, after a certain period of time (the thermalization time  $\tau_{th}$ ), due to collisions in the gas, the gas will again be in equilibrium and the distribution will regain its Gaussian shape.

In our setup the trap is a harmonic potential, induced by a magnetic field. The magnitude of the magnetic field increases with the distance from the center of the trap and shifts the transition energy between the spin states of the atom according to the Zeeman effect. This creates a potential minimum for weak field seekers, atoms in the  $|F = 1, m_f = -1\rangle$  state. Atoms in the  $|F = 1, m_f =$  1) state are high field seekers and are therefore accelerated away from the trap. Atoms in the  $|F = 1, m_f = 0\rangle$  state are not magnetically trapped and fall due to gravity. One can remove the  $|F = 1, m_f = -1\rangle$  atoms in the trap using a radio-frequency field, which flips the spins of these atoms to the  $|F = 1, m_f = 0\rangle$  state and the  $|F = 1, m_f = 1\rangle$  state.

The energy available to an atom determines the amplitude of the oscillatory motion of the atom in the trap. This means that atoms with little energy stay closer to the trap center, while 'hot' atoms are often found further away from this potential minimum. Atoms whose temperature is above average can thus feel a higher magnetic field and the energy gap between their spin states can be larger. By choosing the radio-frequency field such that it is resonant with this transition at a certain magnetic field strength, only the 'hot' atoms get excited to another spin state and are removed from the trap, decreasing the energy per particle of the trapped gas. By applying a radio-frequency field with a decreasing frequency in time, the temperature of the atoms which spins are flipped by the field, is also decreasing in time. If this process takes place on a timescale which is long enough to allow the gas to thermalize, only atoms with an above average energy are expelled, effectively cooling the trapped gas.

### 2.1 Theoretical modeling

In the limit that the gas is in equilibrium, the following two differential equations hold for N(t)and T(t), the number of particles and the temperature as a function of time during the evaporative cooling process ([2], p.19):

$$\frac{dN}{dt} = -N\langle n \rangle v_r \sigma e^{-\eta} \tag{2.2}$$

$$\frac{dT}{dt} = -\frac{T}{3}(\eta - 2)\langle n \rangle v_r \sigma e^{-\eta}$$
(2.3)

where  $v_r = 4\sqrt{\frac{k_b T(t)}{\pi m}}$  is the mean speed of the atoms in the gas, m is the mass of a sodium atom,  $k_b$  is the Boltzmann constant,  $\sigma = 8\pi a^2$  is the cross section with  $a = 2.8 \cdot 10^{-9}$  being the scattering length of sodium and  $\eta = \frac{\epsilon_t}{k_b T}$ , with  $\epsilon_t$  the trap depth, making  $e^{-\eta}$  a Boltzmann factor.  $\langle n \rangle$  is the average density of the gas, which will be given more attention in the next section.

However, these equations do not take into account all relevant processes in the evaporative cooling procedure. I will now discuss which processes also contribute significantly in our experiment and how they can be incorporated in the model. Firstly, it is impossible to create a perfect vacuum, allowing for interactions between particles from the background at room temperature and particles in the gas at a temperature below 500  $\mu$ K. When a background particle collides with a cooled atom, this atom will be knocked out of the trap, due to the enormous velocity difference between the background and cooled particles ([9], p.1). This process is always present in the experiments and puts an upper boundary on the duration of the experiment and can be expressed as the lifetime  $\tau$  of the trapped gas, which is typically around 250 s. The contribution of this effect is estimated by observing the number of particles in the trap as a function of time: the number of particles in the trap decays exponentially, thus the total contribution of this process can be written as  $\frac{dN}{dt} = -\frac{1}{\tau} \cdot N$ .

At high densities (above  $10^{20} \text{ m}^{-3}$ ), the contribution of another process, three-body decay (or three-body recombination) becomes the dominant one ([8], p.2029). In this process two atoms bind together to form a molecule, while a third atom carries the released energy, which is orders of magnitude larger than the temperature, away and escapes from the trap ([1], p.2). The loss rate of this process is described by  $\frac{dN}{dt} = -c_2 \cdot \langle n^2 \rangle$ , where  $c_2 = 1.1 \cdot 10^{-42} \text{ m}^6 \text{s}^{-1}$  ([8], p.2029). The highly energetic atom can, on its way out of the cold gas, collide with many cold atoms, transferring a lot of momentum to other atoms, which in turn also collide with many other atoms. This can lead to an avalanche of collisions, which can destroy the cold gas ([9], p.1). In our experimental setup, the effect of avalanches is limited by the elongated shape of the trap: the typical length of the trajectory of an atom on its way out of the gas is very short, since in two dimensions the gas is very small, and can collide with only a few atoms on its way out. I have not been able to include the effects of avalanches in the model, since it is very difficult to model and my time on this project was limited. Also, the above argument shows that the effect of avalanches is not very large in our setup.

During cooling, it can be beneficial to expand the volume in which the gas is trapped adiabatically. Indeed, expanding a gas will lower its density, causing the loss rate for three-body decay to drop, but also lower its temperature. In our setup, adiabatic expansion of the trap means expanding at a rate which is slower than the trap frequencies. If the expansion is faster than the trap frequencies, the phase space density decreases due to this expansion. Since a BEC requires an increase of the phase space density, it is clear that the best way to expand is adiabatically, since this will keep the phase space density constant. It is also evident that cooling using expansion cannot create a BEC. I can, however, use the fact that adiabatic processes do not change the phase space density to calculate the effect of an adiabatic expansion on the temperature of a gas: the phase space density of the trapped gas is given by formula 2.1. Additionally, in an ideal adiabatic expansion, N is constant and in this model, only the axial trap frequency  $\omega_z$  will change in time, thus I can postulate that  $\frac{T}{\omega_z^{1/3}}$  is constant. In the simulation, time steps are discrete, thus it is easy to find the change in T,  $\Delta T = T(t) - T(t - \Delta t)$ , from the following statement:

$$\frac{T(t)}{\omega_z(t)^{\frac{1}{3}}} = \frac{T(t - \Delta t)}{\omega_z(t - \Delta t)^{\frac{1}{3}}},$$
(2.4)

Thus we have

$$T(t) = T(t - \Delta t) \left(\frac{\omega_z(t)}{\omega_z(t - \Delta t)}\right)^{\frac{1}{3}}.$$
(2.5)

### 2.2 Density calculations

The differential equations for N and T contain the average density  $\langle n \rangle$  of the gas and the factor for three-body recombination contains the average squared density  $\langle n^2 \rangle$ . To completely determine the time dependence of N and T, it is therefore important to have expressions for these quantities. In the next subsection, I will calculate both these quantities using Boltzmann statistics.

#### 2.2.1 Boltzmann statistics

For a classical gas in a harmonic trap induced by the potential  $V(\vec{x}) = \frac{m}{2}((\omega_x x)^2 + (\omega_y y)^2 + (\omega_z z)^2)$ , the number density n(x, y, z) is given by

$$n(\vec{x}) = N \frac{e^{-\beta V(\vec{x})}}{\int d\vec{x} \ e^{-\beta V(\vec{x})}} = N \frac{e^{-\beta V(\vec{x})}}{V_e}.$$
(2.6)

To clean up notation, I have introduced the effective volume  $V_e$ , as

$$V_e = \int d\vec{x} \ e^{-\beta V(\vec{x})} = \left(\frac{2\pi k_b T}{m\bar{\omega}^2}\right)^{3/2},$$
(2.7)

which follows from Gaussian integrals<sup>1</sup>. Now I have the following integral for the ensemble average of the density:

$$\langle n \rangle = \frac{\int d\vec{x} \ n(\vec{x})e^{-\beta V(\vec{x})}}{V_e} = N \frac{\int d\vec{x} \ e^{-2\beta V(\vec{x})}}{V_e^2}$$
(2.8)

<sup>&</sup>lt;sup>1</sup>See the appendix for a more detailed calculation

which is a Gaussian integral in three dimensions and gives for the average density:

$$\langle n \rangle = N \frac{\left(\frac{2\pi k_b T}{2m\bar{\omega}^2}\right)^{3/2}}{V_e^2} = N \frac{V_e}{2^{3/2} V_e^2} = \frac{1}{2\sqrt{2}} \frac{N}{V_e}.$$
(2.9)

In the same way, I can calculate  $\langle n^2 \rangle$ :

$$\langle n^2 \rangle = \frac{\int d\vec{x} \ n(\vec{x})^2 e^{-\beta V(x)}}{V_e} = N^2 \frac{\int d\vec{x} \ e^{-3\beta V(\vec{x})}}{V_e^3}$$
(2.10)

gives

$$\langle n^2 \rangle = N^2 \frac{\left(\frac{2\pi k_b T}{3m\bar{\omega}^2}\right)^{3/2}}{V_e^3} = N \frac{V_e}{3^{3/2}V_e^3} = \frac{1}{3\sqrt{3}} \frac{N^2}{V_e^2}.$$
 (2.11)

From these expressions it follows that there is in fact a great difference in the magnitude of the naive average density  $\frac{N}{V_e}$  and the exact expression, namely a factor of almost 3.

### 2.2.2 Bose Einstein statistics

The results from the previous section are applicable for a non-interacting gas. Bose Einstein condensation, however, exists only in bosonic gases, due to the nature of bosons. It is well-known that bosons are described by a different distribution function than the usual Boltzmann-distribution, namely the Bose Einstein function  $f_{BE} = \frac{1}{e^{\beta(H-\mu)}-1}$  ([2], p.144), where *H* is the total energy of the system and  $\mu$  is the chemical potential of the gas. For high temperatures, this function reduces to the Boltzmann function, since then  $e^{\beta(H-\mu)} \gg 1$  and  $f_{BE} = \frac{1}{e^{\beta(H-\mu)}} = e^{-\beta(H-\mu)}$ . Since the evaporative cooling process cools a Bose gas to the regime where its bosonic nature becomes apparent, it makes sense to investigate how the expressions for the average density change when calculated with Bose-Einstein statistics compared to the original Boltzmann statistics.

To start, I use that the number of particles N is given by ([7], p.27)

$$N = \int \frac{d\vec{p} \, d\vec{x}}{(2\pi\hbar)^3} \frac{1}{e^{\beta(\frac{p^2}{2m} + V(\vec{x}) - \mu)} - 1},\tag{2.12}$$

where p is momentum. This means that the density  $n(\vec{x})$  is given by

$$n(\vec{x}) = \int \frac{d\vec{p}}{(2\pi\hbar)^3} \frac{1}{e^{\beta(\frac{p^2}{2m} + V(\vec{x}) - \mu)} - 1},$$
(2.13)

which can be made dimensionless by introducing  $z(\vec{x}) = e^{\beta(\mu - V(\vec{x}))}$  and  $y = \frac{p^2}{2mk_b T}$ :

$$n(\vec{x}) = \frac{2}{\sqrt{\pi}} \Lambda^3 \int_0^\infty dy \frac{\sqrt{y}}{z^{-1} e^y - 1},$$
(2.14)

where  $\Lambda = \sqrt{\frac{\hbar^2}{2\pi m k_b T}}$  is the de Broglie wavelength ([2], p.144). This integral can be expressed in terms of a polylogarithmic series

$$g_{3/2}(z(\vec{x})) = \sum_{n=1}^{\infty} \frac{e^{n\beta(\mu - V(\vec{x}))}}{n^{3/2}},$$
(2.15)

and is actually equal to  $\Gamma(\frac{3}{2})g_{3/2}(z(\vec{x}))$ , where  $\Gamma$  is the Euler-Gamma function. By combining this integral with equation 2.14, the number density follows as

$$n(\vec{x}) = \frac{1}{\Lambda^3} g_{3/2}(z(\vec{x})).$$
(2.16)

The ensemble average density and the ensemble average of the squared density then follow  $as^2$ 

$$\langle n \rangle = \frac{1}{\Lambda^3} \frac{\sum_{n,m=1}^{\infty} \frac{e^{(n+m)\beta\mu}}{(nm(n+m))^{3/2}}}{\sum_{n=1}^{\infty} \frac{e^{n\beta\mu}}{n^3}}$$
(2.17)

$$\langle n^2 \rangle = \frac{1}{\Lambda^6} \frac{\sum_{n,m,l=1}^{\infty} \frac{e^{(n+m+l)\beta\mu}}{(nm(n+m+l))^{3/2}}}{\sum_{n=1}^{\infty} \frac{e^{n\beta\mu}}{n^3}}$$
(2.18)

Now there are two sets of expressions for the density terms in the differential equations. At high temperatures, these two sets agree, as could be expected, because the distribution functions also coincide at high temperatures. There is a difference at low temperatures, which is illustrated in figure 2.1. The figure shows the density during a simulation of the cooling process, according to both Boltzmann and Bose Einstein statistics. The difference between the two quantities becomes as large as 20%. In the next chapter, I will discuss the aforementioned simulation.

<sup>&</sup>lt;sup>2</sup>See the appendix for a more detailed calculation



Figure 2.1: The density in a cooling simulation, calculated using the two different sets of expressions. The red line belongs to the Bose-Einstein density, the blue one to the Boltzmann density. The temperature at 60 seconds, where the two lines start to differ, is approximately 20  $\mu$ K.

# Chapter 3

# Modeling

In this chapter I will explore how one can use the equations that were derived in the last chapter to simulate the evaporative cooling process. In particular, this simulation can be used to find the conditions in which the critical temperature for condensation is reached with as many particles as possible.

As was shown in the last chapter, the process can be modeled with two equations for the time derivatives of N and T:

$$\frac{dN}{dt} = -N\langle n \rangle v_r \sigma e^{-\eta} - \frac{1}{\tau} \cdot N - c_2 \cdot \langle n^2 \rangle$$
(3.1)

$$\frac{dT}{dt} = -\frac{T}{3}(\eta - 2)\langle n \rangle v_r \sigma e^{-\eta} + g(T, \omega_z), \qquad (3.2)$$

where  $\tau$  is the typical 1/e lifetime of the thermal cloud,  $c_2 = 1.1 \cdot 10^{-42}$  and  $g(T, \omega_z)$  is a function that accounts for the effects of adiabatic cooling. Notice that  $v_r$  is also temperature dependent, as well as  $\eta = \frac{\Delta RFh}{k_b T}$ , where  $\Delta RF$  is the difference between the radio frequency field RF(t) that radiates through the thermal cloud and the trap bottom in MHz. The trap floor is the minimum amount of energy that a particle in the trap has, expressed in Hz. The fact that T(t) is present in equation 3.1 through  $v_r$  and N(t) in equation 3.2 through  $\langle n \rangle$  shows that these two equations are in fact coupled differential equations.

In the experiment, one can change the time dependence of the RF-field and the trap frequencies, which can vary as a function of time. Lowering the frequency of the RF-field lowers the energy of the atoms that get spin flipped by this RF-field and therefore has to decrease as a function of time during the cooling process. By lowering the trap frequencies, the volume, in which the particles are confined, is enlarged and thus the density is lowered. As can be seen in the density equations 2.9, the three trap frequencies are always present as the geometric mean of the frequencies,  $\bar{\omega}$  and therefore all the volume changes can be made by changing only one of the frequencies during the cooling process. I will from now on set  $\omega_x = \omega_y = \omega_r = 143$  Hz and change  $\bar{\omega}$  (and thus the trap volume) solely by changing  $\omega_z$ . Also, lowering  $\omega_z$  while keeping  $\omega_x$  and  $\omega_y$  constant will ultimately give the trap a cigar shape, which is suitable for our experiments.

### **3.1** Solving first order coupled differential equations

I have written C-scripts<sup>1</sup> that solve equations 3.1 and 3.2 numerically given a set of initial conditions, such as  $N_0$  and  $T_0$ . Also the time dependence of RF(t) and  $\omega_z(t)$  has to be specified. In this section I will explain how one can solve the aforementioned equations 3.1 and 3.2 numerically.

The most important idea for solving first order differential equations numerically lies in transforming the problem from a continuous problem to a discrete problem. This can be done by noticing that

$$\frac{dN(t)}{dt} = \lim_{\Delta t \to 0} \frac{N(t + \Delta t) - N(t)}{\Delta t}.$$
(3.3)

By approximating the limit by a small but nonzero  $\Delta t$ , we can express  $N(t + \Delta t)$  in terms of N(t)and T(t). Indeed, by equating the righthand side of 3.1 to the righthand side of 3.3, we find

$$N(t + \Delta t) = N(t) - \left( N(t) \langle n(t) \rangle v_r(t) \sigma e^{-\eta(t)} - \frac{1}{\tau} \cdot N(t) - c_2 \cdot \langle n(t)^2 \rangle \right) \Delta t, \qquad (3.4)$$

where all time dependencies are made explicitly. In the same fashion, we can express  $T(t + \Delta t)$  in terms of T(t) and N(t):

$$T(t+\Delta t) = T(t) - \left(-\frac{T(t)}{3}(\eta(t)-2)\langle n(t)\rangle v_r(t)\sigma e^{-\eta(t)}\right)\Delta t + T(t)\left(\left(\frac{\omega_z(t+\Delta t)}{\omega_z(t)}\right)^{\frac{1}{3}} - 1\right).$$
(3.5)

The last term in the previous equation follows from equation 2.5 and accounts for the effects of adiabatic cooling. These equations now determine uniquely the time evolution of N(t) and T(t), given  $N(0) = N_0$  and  $T(0) = T_0$ . Notice that it is not problematic that  $\omega_z$  at time  $t + \Delta t$  has to be known to calculate  $T(t + \Delta t)$ , since this time dependence is known in advance. Another problem does exist, however: if one wants to use the expressions for average density which are

<sup>&</sup>lt;sup>1</sup>The name of these scripts are coolingsimple.c and cooling.c. Coolingsimple.c uses Boltzmann statistics to calculate the densities, while cooling.c uses Bose Einstein statistics.

derived from the Bose-Einstein distribution, the time evolution of  $\langle n(t) \rangle$  is not known before hand, since it contains the chemical potential, which cannot be expressed in terms of known quantities when T is close to  $T_c$ . In the next section I will explain a method of finding the chemical potential from known quantities.

Of course,  $\Delta t$  has to be very small to minimize the difference between the approximation and the exact limit, without lengthening the calculation time of the script to unacceptable values. Since in an optimization it is necessary to simulate the cooling cycle for many different - in the order of thousands of - sets of input parameters, it seems reasonable to require that calculating the time dependence for one cooling cycle cannot exceed 0.1 second. For this requirement, the total number of time steps that can be taken is around 100000, as tests have shown. This also defines the value of  $\Delta t$  as  $\frac{t_n}{100000}$ , where  $t_n$  is the total duration of the cooling cycle, which is typically between 40 and 120 seconds.

The aforementioned time constraint for the optimization has another consequence: the most accurate expressions for the density are the ones that were calculated using Bose Einstein statistics, but calculating the density using these expressions takes a lot of time and causes the simulation to take approximately 40 times as long. This would mean that the total run time of all the relevant optimizations would take over 500 hours, which is even longer than the total time usually spent on a Bachelor's thesis! So, although the expressions derived from Boltzmann statistics may differ from the actual values in the order of 20%, as was shown in the last chapter, they are the only feasible option to use in an optimization and therefore I will use them from now on.

## 3.2 Chemical potential of a Bose gas

To find the chemical potential of a Bose gas, one can use an expression for the phase space density ([7], p.22):

$$\left(\frac{\hbar\bar{\omega}}{k_bT}\right)^3 N = \sum_{n=1}^{\infty} \frac{e^{\beta\mu n}}{n^3}.$$
(3.6)

At every stage of the simulation, N and all the quantities in the bracket are known, such that only the chemical potential  $\mu$  is unknown. It is unfortunately not possible to invert this equation into an equation for  $\mu$ , but this does not mean that  $\mu$  cannot be extracted from this equation. At a given moment in time, the left hand side of equation 3.6 is known, thus all that needs to be done is finding a value of  $\mu$  that matches both sides. It is known that  $\mu$  is negative for a Bose gas and goes to zero as it approaches  $T_c$  and does so in a monotonous way. This means that the right hand side of equation 3.6 is a strictly monotonously increasing function of  $\mu$ , with its maximum at  $\mu = 0$ . By choosing a sensible lower bound for  $\mu$  (in the order of  $10^{-26}$ ), the search domain is a bounded interval, which can be scanned for the correct value. In my script, I have used a binary search to determine the value of  $\mu$ , which can be described by the following algorithm:

- 1. Define the search interval by setting its begin point  $x_b$  and end point  $x_e$
- 2. Define  $\mu = \frac{x_b + x_e}{2}$  and calculate  $f(\mu) = \sum_{n=1}^{\infty} \frac{e^{\beta \mu n}}{n^3}$

3. If 
$$f(\mu) > \left(\frac{\hbar\bar{\omega}}{k_bT}\right)^3 N$$
, set  $x_e = \mu$ , otherwise, set  $x_b = \mu$ .  
4. Return to step 2.

By repeating this algorithm, the left hand side of equation 3.6 can be approximated to arbitrary

By repeating this algorithm, the left hand side of equation 3.6 can be approximated to arbitrary precision. In the script, the precision is set such that the error is smaller than  $10^{-8}$ . This requires approximately 75 iterations of the algorithm.

## 3.3 Optimization

As said before, a simulation of the cooling procedure can be used to optimize this process for given starting conditions. For our purposes, this means that the total number of particles at the condensation temperature  $T_c$  has to be as large as possible.

Meeting this condition can be done for various parameters: for different  $N_0$ ,  $T_0$ , vacuum lifetime  $\tau$  and for different geometries in the trap at  $t = t_n$ . It is important to be able to optimize for different environments, since in practice, the properties of all the aforementioned quantities can change from day to day. If at a given day, conditions are far from ideal, it is useful to know fast how the parameters can be changed to get the best results possible. Also, in different experiments, different trap geometries are used.

#### **3.3.1** Monte Carlo optimization

To find the optimal values for all parameters, I have written a optimization using the Monte Carlo method. In this section, I will explain how this optimization is constructed, which problems may arise and the limitations of this method.

The problem can be formulated as follows: I want to find the global maximum of a function F, which

- 1. takes certain functions RF(t) and  $\omega_z(t)$  from the domain, which I will call A
- 2. simulates the cooling process using the simulation
- 3. and returns the total number of particles if  $T_c$  is reached and zero if  $T_c$  is not reached.

The way that the functions for the RF-field and the axial trap frequency are inserted in the function F is done in a way that mimics the actual input in the experiment: the function values at a very limited number of points in time are given and the program calculates all the other function values, using a linear interpolation method that is embedded in the program. In the program, the standard number of input variables is 20, 10 to specify each function, although the script allows any number of defining values. The values for the RF-field lie between 0 and 60 MHz and the values for  $\frac{\omega_z(t)}{2\pi}$  lie between 0.01 and 16 Hz. The domain A can therefore be described as a box in multidimensional Euclidean space.

It is not easy to further specify the domain of F, but it is clear that A, in the way as just described, contains points that should not be considered: there are for example points in A which specify the RF-sweep to be constant at 60 Mhz. It was already argued in the last chapter that such a sweep does not cool the gas effectively and in particular will not help it reach  $T_c$ . It is therefore necessary to specify as a constraint that only points in A should be considered which parameterize a cooling cycle that cools to  $T_c$ . In the next section the precise implications of this constraint are researched.

To find the global maximum of F, I programmed the following procedure: the algorithm starts at a point  $x_0 \in A$ , which is chosen by the user. It then calculates  $F(x_0)$  and (pseudorandomly) chooses a new point  $x_1 \in A$ . It calculates  $F(x_1)$  and compares it to  $F(x_0)$ . If  $F(x_1) < F(x_0)$ , the procedure starts again. If, however,  $F(x_1) > F(x_0)$ , the algorithm redefines  $x_0 = x_1$  and starts the procedure again. In this way, as the number of iterations increases, also the value of  $F(x_0)$  increases, provided that  $F(x_0)$  is not already the maximum value of F. Much freedom lies in the precise rules that determine the new point  $x_1$ : one can allow the algorithm to pick any point in the entire set A, but usually, some constraints are imposed. In the next section, I will discuss the rules I have developed for choosing new points and the constraints these rules imply.

#### 3.3.2 Taking domain steps

The most important constraint on the search algorithm is that it should only consider points for which the cooling simulation reaches  $T_c$ . Therefore, the algorithm always starts from a reference point which does lead to  $T_c$  and takes steps from this point. Additionally, it is important that the algorithm can find the global maximum of F, which can be difficult: there can exist local maxima, which can act as a trapping point for the stepping algorithm. Also, if the allowed steps are not chosen carefully, other objects, such as ridges and saddle points may be problematic. As an example, I will explain why a saddle point may be problematic. Consider the following case, illustrated by figure 3.1: from the center, the only allowed steps are along the grid lines, indicated by the red lines. However, movement along these lines is always downward, such that, due to the stepping limitations, the center has become a local maximum. Notice that the center is most definitely not the global maximum, since at the left and right edge, the function has a higher value.

The stepping algorithm I developed consists of two types of steps, which I will call type-I and type-II: a type-I step is a step in a random direction along the grid lines and a type-II step is a step in a completely arbitrary direction. Type-I steps are generated by randomly selecting a direction from the approximately 20 possible ones and randomly selecting a magnitude for the step, depending on the chosen direction. One can see that the magnitude of the step should depend on the chosen direction by comparing some points on the RF-sweep: the value of the RF-sweep varies between 60 MHz at the beginning of the sweep to approximately 1.80 MHz at the end. To reach  $T_c$ , the final value of the sweep cannot be too large, so values close to the end of the sweep are constrained to the range between approximately 1.7 and 2.5 MHz, a total range of less than 1 MHz. The begin points of the sweep can vary on a much larger scale, making differentiated step sizes a necessity for convergence. Type-II steps are generated by selecting a step size for each of the possible directions, resulting in a step in a random direction. Since the stepping direction is random, type-II steps make sure the algorithm cannot get stuck in a saddle point.



Figure 3.1: If the algorithm can only generate steps along the red lines, the center of this figure becomes a local maximum.

In the previous paragraph, the necessity for the usage of the type-II step in the algorithm is already explained, but it is yet to be discussed why it is not a sufficient. This can be explained using the following case: suppose the algorithm has already found values for all points on the RF-sweep which are close to the optimal values, except for one or two points, which are still pretty far from their optimal value. To improve on the existing sweep, which is already fairly accurate, the algorithm has to generate a step which leaves almost all points unchanged and takes a big step in the direction of the two points which are far away from their optimal value. The chance for this to happen is very small and lies between  $10^{-3}$  and  $10^{-4}$ , depending on the step sizes in the various directions. This would therefore require a lot of iterations of the algorithm. Using the type-I step, the chance for a step in a good direction is  $\frac{1}{10}$ , a much larger chance. Thus type-I steps are a necessary part of the algorithm, if one wants to shorten the run time.

#### 3.3.3 Summary of the algorithm

Now that it is clear what the components of the optimization algorithm are, the complete program can be summarized: at the beginning of an optimization run, the user can input the conditions under which the RF-sweep and trap volume should be optimized, by setting  $N_0$ ,  $T_0$ , the lifetime  $\tau$  and possibly the trap shape after cooling. This last condition in practice means presetting the variables for the trap volume and not allowing them to vary. Additionally, the user should check that the initial reference point, which is also specified, models an RF-sweep and trap volume function that cool the gas towards  $T_c$ . To optimize, the algorithm starts by setting the total cooling time at n = 40 s. It then optimizes using the algorithm that was discussed in the preceding sections and after 4000 iterations writes the best result for N at  $T_c$  to a file, along with the optimal values for all the variables and the used cooling time. It then increments the cooling time with 10 seconds and repeats the same procedure. The program is written in such a way that it stops 2 steps after the decline in the maximum number of particles has set in or after a maximum cooling time of 150 seconds.

The created file can be read and contains the optimal cooling time and values for RF(t) and  $\omega_z(t)$ .

### 3.4 Results

This section will now present the results of the optimization and is split in two parts. In the first part, evidence is presented that shows that the optimization works and converges to a maximum within 4000 iterations<sup>2</sup>. The second part will present the the optimization results for a specific case, as an example.

<sup>&</sup>lt;sup>2</sup>This takes approximately 4 minutes on the university server

#### 3.4.1 Convergence

To show that the optimization can effectively find the maximum of the function F and both step types work as expected, I will show the convergence curves of the algorithm for several different cases. A convergence curve shows the best result found for F as a function of the number of iterations. I will first show that the algorithm converges to a global maximum and then analyze the effects of the two different step types.

It is in general impossible to prove that the algorithm finds the global maximum, since to do this one needs to check the convergence for all the points in the domain, which are infinitely many points. The strongest corroboration that the found maximum by the optimization is the global maximum, however, is by showing that it finds the same maximum from many starting points in the domain. In figure 3.2, the convergence curves for 10 randomly selected starting points are plotted. The right



Figure 3.2: The convergence curves for the algorithm for 10 different starting points. The left figure shows only the result of the first 400 steps, the right figure shows the entire curve (4000 iterations). For this case,  $N_0 = 2 \cdot 10^{10}$  and  $T_0 = 450 \ \mu \text{K}$ .

figure shows that the curves find approximately the same maximum. The standard deviation of the found maxima is  $2.2 \cdot 10^6$ , which is approximately 0.1% of the mean value. This corroborates the hypothesis that the optimization finds the global maximum.

In the previous section, it was argued that the algorithm required two types of steps, namely type-I and type-II steps. I will now show that this is indeed the case, by looking at two examples. Firstly, I want to show that the algorithm needs the type-II steps. As discussed before, the most important property of type-II steps is that the algorithm cannot get stuck in a saddle point or on a ridge. It is, however, extremely difficult to find a saddle point or a ridge in the domain of F, because it is

20 dimensional and its shape is unknown. This makes it impossible to explicitly demonstrate this property of the type-II step. I can show, however, that the algorithm converges better when the



Figure 3.3: The convergence curves for the algorithm when it uses only type-II steps (blue) and when it uses only type-I steps (red). For this case,  $N_0 = 2 \cdot 10^{10}$  and  $T_0 = 450 \ \mu\text{K}$ .

steps are of type-II as compared to type-I and can also get closer to the global maximum. This is shown in figure 3.3, from which it is also clear that the type-II step makes the algorithm much faster: the type-II step reaches the best result of the type-I step after only 500 steps.

To show that the type-I step is necessary, I have prepared the algorithm in such a way that it starts from a point in which all parameters are really close to their optimal value, except for one parameter. As argued before, the algorithm should converge much faster using the type-I step than using the type-II step. The results (see figure 3.4) show that the type-I step converges faster and closer to the maximum of F than the type-II step. However, the convergence of the algorithm using only type-II steps is stronger than expected. The figure shows approximately 20 successful steps using the type-II step, which is a success rate of  $10^{-2}$ , much higher than the expected  $10^{-3}$  to  $10^{-4}$ . A possible explanation is that small changes in the parameters that model the start of the RF-sweep have very little influence on the total number of particles at  $T_c$ : for example, changing the value of the RF(10) by 2 MHz (or 5%) changes the total number of particles at  $T_c$  by less than 0.1%. Changing a parameter at the end of the RF-curve by the same relative amount has a larger



Figure 3.4: The convergence curves for the type-I step (red) and the type-II step (blue), starting at a point in which only 1 parameter needs to change to reach the global maximum. Here,  $N_0 = 2 \cdot 10^{10}$  and  $T_0 = 450 \ \mu \text{K}$ .

effect, in the order of 1%. This means that the algorithm can take a type-II step in more directions than I assumed in the section on domain steps: steps, which hardly change the end of the RF-curve and make a reasonable change to the parameter which is still far away from its ideal value, will all improve the result at  $T_c$ . This enhances the chance of a step towards the maximum and could explain high success rate of the type-II step.

#### 3.4.2 Example

Now that is is clear that the optimization is working properly, I will present the results of the optimization of a specific case. Also, in this section I will try to answer another interesting question: in the optimization, there is a competition between two processes: to cool as efficiently as possible, the collision rate should be as high as possible and therefore, also the density should be at its maximum value. At very high densities, however, the loss processes, especially three-body recombination, contribute significantly more than they do at low densities. From this it is obvious that the density cannot increase indefinitely, since at some point, the loss processes will dominate, which gives suboptimal results. The question is what the best strategy is to follow in the experiment: expanding the trap volume to minimize the losses, making the trap volume as small as possible in our setup to maximize the collision rate, or a strategy somewhere in between.

In a paper [10] published in 2003, Yamashita et al. conclude that to create the largest possible condensate using evaporative cooling, one should not expand the trap volume to avoid heavy losses, but instead cool as fast as possible through the high density regime . In practice, this means cooling normally until the density is of the order  $10^{20}$  m<sup>-3</sup> and then quickly lowering the RF-sweep to the value of the trap bottom. However, in the experimental setup in Utrecht, a different strategy is applied: at the final stages of cooling, the trap volume is expanded, to avoid densities above  $10^{20}$  m<sup>-3</sup> and therefore the loss processes cannot play a significant role. The optimization program should be able to answer which of the two strategies works the best to create the largest condensate possible.

The initial conditions I chose are close to the values used in the experiment at the time of writing (June 2012):  $N_0 = 1.5 \cdot 10^{10}$ ,  $T_0 = 450 \ \mu\text{K}$ ,  $\tau = 270 \ \text{s}$  and  $\frac{\omega_r}{2\pi} = \frac{900}{2\pi} \approx 143 \ \text{Hz}$ . I have not given a constraint on the trap shape, such that the program could decide if increasing the trap volume would yield better results than a closed trap.

The optimal cooling time according to the program is approximately 80 s. This is in good agreement with the cooling time that was used in the lab until recently, which was 85 s. The result for the optimized RF-sweep is shown in figure 3.5. As can be seen, the two curves are pretty similar, which means that the used RF-curve in the experiment is already really close to the optimal one. According to the optimization, the trap volume should stay as minimal as possible, which is in sharp contrast with the original function  $\omega_z(t)$ . Both curves are shown in figure 3.6. The maximum number of particles at  $T_c$  is  $1.29 \cdot 10^9$ , while the original parameters yield  $1.14 \cdot 10^9$  particles at  $T_c$ . This means that the optimization has found parameter values that increase the maximum number of particles with 13%. If the original RF-sweep is used with a closed trap, the maximum is  $1.20 \cdot 10^9$ . From this I can estimate that the improvement stems both from changes to the RF-curve as well as the trap volume.

In particular, from this data, I can draw the conclusion that the trap volume should remain as small as possible in our setup throughout the entire cooling cycle. An analysis of the density throughout the cooling cycle (see figure 3.7) shows that the density in fact never reaches the regime where the loss processes become dominant. Therefore, from this data no conclusion can be drawn concerning



Figure 3.5: The original RF-curve (red) and the optimized RF-curve (blue).



Figure 3.6: The original  $\omega_z(t)$  (red) and the optimized  $\omega_z(t)$  (blue).

the question if opening the trap at high densities is more efficient than cooling fast through the high density regime. In addition, for all achievable values of  $N_0$  and  $T_0$ , which means  $N_0 \leq 5 \cdot 10^{10}$  and  $T_0 \geq 350 \ \mu\text{K}$ , the density during cooling never exceeds  $10^{20} \text{ m}^{-3}$ , while the trap volume in



Figure 3.7: The density throughout the cooling cycle.

the optimal case is still at its smallest achievable value. This suggests that the high density regime lies for most cases in the stage that the condensate grows, when  $T < T_c$ . In this regime the model that is used for this simulation is no longer valid and I can therefore not conclusively state which strategy is better in that regime. It could also be the case that the effect of avalanches due to threebody recombination is already significant at lower densities. This could mean that the regime in which three-body recombination has a significant influence lies lower than estimated before, thus at densities lower than  $10^{20}$  m<sup>-3</sup>.

## 3.5 Discussion

As I have shown in the previous section, the program succeeds in optimizing the parameters according to the criteria. Particularly, in the example an increase of 13% in the maximum number of particles has been realized, compared with the results of the original parameters. The most interesting conclusion is that expanding the trap volume during the cooling cycle is less effective than keeping the trap volume at its minimal value, because the density never grows to values that lie in the regime where the loss processes dominate. Unfortunately, this conclusion is valid only in the regime where the model is valid, which is when  $T > T_c$ . To find out which strategy is better in the  $T < T_c$  regime, a more complete model should be used, which should for example contain Bose-statistics and avalanches.

Indeed, as shown in chapter 2, the density calculated using classical Boltzmann statistics is up to 20% lower than the density calculated using Bose-Einstein statistics when T is close to  $T_c$ . This could mean that the experimental density does lie in the regime where three-body recombination is a dominant process. Also, avalanches have not been included, which could augment the effect of three-body recombination and thus lower the lower bound of the regime where the loss processes dominate.

Another limitation of this optimization program lies in the fact that the optimized parameters depend heavily on the initial conditions with which they were found. In principle this is not a problem, but in practice it can be, since some of the initial conditions ( $N_0$  and  $T_0$ ) can change significantly between measurements and are also hard to determine with accuracy. This limits the usability of the results in the experiment, although conceptual questions, such as which strategy is more efficient, can still be answered using the program.

# Chapter 4

# **Interacting Spinor Condensates**

### 4.1 Motivation and preparation of the experiment

In the fall of 2011, a measurement was done on spin drag in a Bose gas [5]. This measurement effectively gave some insight in the interactions between cold Bose atoms in different spin states; it measured the drag rate that exists due to the interactions between atoms in different spin states. To further investigate this phenomenon, we wanted to continue to measure the effect of spin drag for interacting BEC's with different spins, or spinor condensates. One could expect a different behaviour for spinor condensates, since BEC's are superfluids. In May and June 2012, I have worked together with Pieter and Alexander on the preparations for this measurement. In the following few paragraphs, I will give an overview of the steps we took to prepare the setup. After that, I will discuss the first measurement.

Firstly, because only atoms in the  $|F = 1, m_f = -1\rangle$  state are trapped in the magnetic trap (MT), we have prepared the far off resonance trap (FORT), by making sure it was properly aligned with the MT. Bad alignment leads to oscillations in the FORT, which is undesirable. In a spin drag measurement it is important to know the forces that act on the particles, which is much more difficult if the interacting particles are also oscillating.

Secondly, to measure interactions of different spin species, it is necessary to create different spin states in the trap. After loading the atoms from the MT to the FORT, there are still only atoms in the  $|F = 1, m_f = -1\rangle$  state, since that is the only state trapped in the MT. To create other spin species, we used the same method that was used to expel atoms from the trap in the evaporative

cooling process (see chapter 2): a radio frequency field flips the spins of the atoms. To make sure that all the spin species have approximately the same number of particles and temperature, it is important to apply an RF-field with varying frequency, since atoms with different energies have a different transition energy to other spin states.

In trying to make sure that the different atom clouds contained approximately the same number of atoms, we encountered a problem. We estimated the total number of atoms from our pictures, taken in absorption imaging, by fitting a Gaussian distribution. In principle this should give a reliable value for the number of particles. Using this estimates, we calculated a flip ratio as the total number of particles in the  $|F = 1, m_f = -1\rangle$  state divided by the total number of particles in the  $|F = 1, m_f = 0\rangle$  state<sup>1</sup>. The clouds were spatially separated by the magnetic field of the MT, that was switched on during time of flight. Initially, with any configuration for the RF-field we could not find values below 1.7 for the flip ratio, while an equal distribution of the particles over all the available states should yield a flip ratio of 1. Additionally, we found that giving the RF-field more time to flip atoms did not change this ratio, which is really unexpected: the energy that is supplied by the RF-field allows atoms in the  $|F = 1, m_f = -1\rangle$  state to go to a  $|F = 1, m_f = 0\rangle$ state, but also vice versa, with equal probability. This suggests that if enough time is given, an equilibrium will arise with both states equally populated.

We have found that this problem really was an optical illusion; the different atom clouds actually contained approximately the same number of atoms. By doing a measurement of the flip ratio for different polarizations of the probe beam that shines on the atoms to take the pictures, we found that the polarization has a very large influence on the absorption of each spin species. Luckily, there was another way of determining the amount of atoms in the spin clouds. Instead of taking pictures of the different thermal clouds, we cooled the spin flipped clouds further in the FORT, until BEC's had formed in the clouds. The idea was that, using the size of the BECs and the known trap frequencies we could determine the flip ratio, using the formula for the Thomas-Fermi radii,  $R = \sqrt{\frac{2\mu}{m\omega^2}}$ .

The first pictures of the BEC's in the FORT showed two condensates, one in the  $|F = 1, m_f = -1\rangle$ state and one in the  $|F = 1, m_f = 0\rangle$  state, of approximately the same size and thus the flip ratio

<sup>&</sup>lt;sup>1</sup>The  $|F = 1, m_f = 1\rangle$  atoms were not visible in the images, most likely because they were pulled away from view by the magnetic field that was present before imaging.

equalled the desired 1. An interesting feature of these condensates was that they did not mix, as can also be seen in figure 4.1: the pictures show clear domain walls between the condensates. We did not understand why this was the case, but a possible explanation will be given in the analysis section of this chapter. We expected that the strong magnetic gradient from the MT that separated the two BEC's for imaging might be an important factor, since it was also present in the stage that the BEC's were formed, so we tried to lower the strength of this magnetic field. This caused the different condensates to be closer together in the images, but we were pleasantly surprised to also find a condensate of atoms in the  $|F = 1, m_f = +1\rangle$  state. Of course, in principle it makes sense that all three spin species were present in the trap, but due to the initial problems with the flipping process, we had forgotten this spin species.

In the pictures with the three condensates (see figure 4.1 for an example), we could still clearly see



Figure 4.1: Three condensates with different spins in the FORT. The time of flight is 26 ms domain walls. Additionally, we did not manage to influence the relative position of the different

condensates: the  $|F = 1, m_f = -1\rangle$  condensate was always the left one, the  $|F = 1, m_f = 0\rangle$  the middle one and the  $|F = 1, m_f = 1\rangle$  BEC the right one. This provided a new problem: to measure spin drag, it is necessary to be able to move the different condensates through each other, but the mechanism that create the magnetic fields in the setup only allowed us to pull the condensates further away from each other. The easiest way to move the condensates through each other is then to make use of a property the harmonic trap: its restoring force towards the center: one pulls the  $|F = 1, m_f = -1\rangle$  and  $|F = 1, m_f = 1\rangle$  condensates from the center using the magnetic gradient of the MOT and then simply switches off the magnetic field, so the condensates try to return to the trap center and have to interact, by moving through each other or colliding. On the basis of the fact that the spinor condensates behave as if they are immiscible, it can be expected that their interaction is not be a simple damped oscillation due to spin drag.

In the next section, I will discuss the measurement of the time evolution of the different condensates, when prepared in the configuration described in the previous paragraph.

## 4.2 Measurement on interacting spinor condensates

We measured the interactions of the three BEC's by following their position in the trap through time. As a quick reminder, I will first restate which steps were taken to create the three condensates, starting from the MOT, where the atoms are caught after being slowed down in the Zeeman slower. The atoms were loaded to the MT, where they were cooled to a temperature just above  $T_c$  using evaporative cooling. Then the atoms were loaded into the FORT, where they were spin flipped, by applying an RF-field for 50 ms with a frequency that was linearly ramped from 1.6 to 1.8 MHz. In the following step, the atoms were further cooled to BEC by opening and closing the trap in a nonadiabatic way. This procedure gives the configuration depicted in figure 4.1. The  $|F = 1, m_f = -1\rangle$ and  $|F = 1, m_f = 1\rangle$  condensates were then pulled away from the trap center using the magnetic field of the MOT coils at only 10% of its normal magnitude. After this procedure, the waiting period was scheduled, which was incremented from 0 to 600 ms in steps of 50 ms. To take a picture of the condensates, the MT gradient at 5% of its usual magnitude was applied to spatially separate the condensates in the vertical direction. The time of flight used was 26 ms.

The results of this measurement can best be summarized in a figure (see figure 4.2). It is clear



Figure 4.2: The first six images in the series. The time runs in the direction of reading and the time elapsed between each picture is 50 ms. It is clear that the condensates collide.

from the selection of images shown in figure 4.2 that the condensates collide inelastically, instead of moving through each other to perform a damped oscillation. The  $|F = 1, m_f = 0\rangle$  condensate gets compressed by the collision with the two other condensates, which seem to contain shock waves. This is one of the possible loss channels for the energy that dissipates from this inelastic collision.

The question now is what causes the repulsion of the two condensates. One possible answer is that it is energetically unfavourable for the condensates to mix. More precise, the interaction energy of a system in which  $|F = 1, m_f = -1\rangle$  and  $|F = 1, m_f = 0\rangle$  condensate are mixed is higher than the interaction energy of a system in which the two sets of condensates are separated ([4], p.9). This is caused by the fact that the scattering lengths for collisions between the three different spin species are different and the interaction energy goes as a power law of the scattering length, as can be seen in equation 4.3. I have investigated the influence of this effect by estimating the magnitude of the repulsive force of the immiscibility and comparing this to the force that is necessary to cause the motion of the condensates. In the next sections, I will discuss how I have determined the motion of the condensates from the pictures and how I have determined the magnitude of the influence of the immiscibility.

#### 4.2.1 Analysis of the motion

To extract the force that acts on the  $|F = 1, m_f = -1\rangle$  and  $|F = 1, m_f = 1\rangle$  condensates from the image series, the easiest method is to use Newton's second law. All one needs is the motion of the condensates, which gives the acceleration, which in turns gives the force.

To find the motion of the condensates, I have written a IDL-script (threeBECs.pro) to find the center of mass motion for the three condensates. The images are loaded into this script as a matrix, where each entry corresponds to a pixel on the camera. The value of each entry represents how much light shines on the corresponding pixel, as compared to the case where there is no atom cloud at all. The matrix is cut into three strips, one for each of the condensates. Next, to avoid the influence of background particles and more importantly the influence of the thermal cloud, the image strips are filtered. This filter changes the entries in the matrices by resetting every pixel with a value higher than 0.7 to 0 and resetting every other pixel by the total number of atoms in that pixel. To do this, I used the fact that the value of the pixel p relates to the optical density as  $p = e^{-A}$ , where A is the optical density and the total number of particles can be found using

$$N = \frac{2\pi}{3\lambda^2} \frac{18}{5} (M \cdot d)^2 A, \tag{4.1}$$

where  $\lambda$  is the wavelength for this imaging transition, M = 2.9 is the magnification factor,  $d = 8 \cdot 10^{-6}$  is the size of a pixel in m<sup>2</sup>. I determined the threshold value 0.7 by manually checking if this value gave a good representation of the condensates.

By assuming all the motion was in the horizontal direction, which seems sensible, I could further simplify the data. By summing all the entries in the columns of the matrix, the resulting system is one dimensional, which has a simple center of mass formula. By setting the position of the  $|F = 1, m_f = 0\rangle$  condensate as the origin of the coordinate system, the relative position of the other two condensates can easily be extracted(see figure ??). The motion seems to be a reasonably well behaved periodic function, as could be expected.

This method of finding the center of mass could contain uncertainties. When the condensates are compressed, such as during the collision, the density might locally exceed the maximum density that can properly be measured using absorption imaging.

If the local density exceeds this maximum, the uncertainty in the optical density has become very large, which significantly increases the uncertainty in the center of mass. However, this effect was



Figure 4.3: The relative position of the  $|F = 1, m_f = 1\rangle$  (upper) and  $|F = 1, m_f = -1\rangle$  (lower) condensates.

strongest for the  $|F = 1, m_f = 0\rangle$  condensate, which was compressed approximately symmetrically by the two other condensates, and therefore the center of mass of this condensate will not have changed substantially.

The acceleration of the condensates can be found, using that  $a = \frac{d^2s}{dt^2}$ . To find the force, all

one needs is the mass. The mass of a sodium atom is well known, namely  $m_s = 3.82 \cdot 10^{-26}$  kg. Finding out how many atoms are in the condensates is, however, not a trivial task. Due to the effect explained in the previous paragraph, the number of particles cannot be extracted from every image of the series. This illustrated in figure 4.4, from which one can clearly see that the aforementioned effect has a strong effect



Figure 4.4: The total number of particles for the three condensates as a function of time.

on the number of particle: the total number of particles even increases at two stages (between 60 ms and 160 ms and between 440 ms and 560 ms), which is impossible. The simplest way to

continue is to assume that the number of particles in the BEC's is constant in time and extract the number of particles from the separated BEC's in the first image (the first image of figure 4.2), which is precisely what I have done. Different measurements of the number of particles from this image give consistent results and correspond to the values in figure 4.4. Assuming constant particles numbers is probably a simplification, but as can be seen in figure 4.4, the total loss in particles is certainly not larger than 25%. This is small enough for the analysis I will do in the next section. The force can be calculated using  $F = Ma = Nm_s a$  and is shown in figure 4.5. This force has



Figure 4.5: The total force that acts on the  $|F = 1, m_f = -1\rangle$  condensate.

several components, including the force arising from the harmonic potential and the force due to the interactions. In the trap center, the trapping force is absent, which is exactly where the two condensates interact. These interaction points correspond to the peaks below the x-axis in figure 4.5. Therefore, the force due to the interaction has a magnitude in the order of  $10^{-20}$  N.

### 4.2.2 Magnitude of the immiscibility effect

I wish to compare the force that was found in the previous section with the force that arises from the immiscibility of the condensates. To do this, I will analyze the situation that is illustrated in figure 4.6: I will restrict this analysis to the interaction between the  $|F = 1, m_f = -1\rangle$  and the  $|F = 1, m_f = 0\rangle$  condensates. Using a computer simulation, I have determined what the extra amount of energy is that is needed to overlap the two condensates as a function of the total overlap,



Figure 4.6: Two overlapping condensates. The red condensate is in the  $|F = 1, m_f = -1\rangle$  state, the blue one is a  $|F = 1, m_f = 0\rangle$  condensate. The overlap is parameterized using dx.

parameterized by dx. Note that in principle it is also possible to find the force on the condensates by looking at the thickness of the domain walls. Unfortunately, I could not do this in this specific case, because the imaging did not allow me to resolve the domain wall, such that I only have an upper bound for the thickness of the domain wall.

The interaction energies of the condensates satisfy the following formula ([7], p.156):

$$E = \int_V n(\vec{x})(\mu - V(\vec{x}))d\vec{x}, \qquad (4.2)$$

where  $V(\vec{x})$  is the harmonic potential of the trap,  $n(\vec{x})$  is the density and  $\mu$  is given by

$$\mu = \frac{h}{4\pi} \bar{\omega} \left(\frac{15Na}{\bar{a}}\right)^{2/5},\tag{4.3}$$

where  $\bar{\omega}$  is the geometric mean of the trap frequencies,  $\bar{a} = \sqrt{\frac{\hbar}{m\bar{\omega}}}$  is the harmonic oscillator strength and a is the scattering length of the atoms in the condensate.

In particular, the scattering lengths of the condensates depend on the local densities of the different spin species, because they are related to the scattering channels of the different spin species. When a particle in the  $|F = 1, m_f = 0\rangle$  is surrounded primarily by  $|F = 1, m_f = -1\rangle$  particles, the dominant scattering process is the collision of the atom in the  $|F = 1, m_f = 0\rangle$  state with an atom in the  $|F = 1, m_f = -1\rangle$  state and the scattering length is equal to  $a_{-1,0} = 54.54a_0$ , which is the scattering length of the  $|F = 1, m_f = \pm 1\rangle + |F = 1, m_f = 0\rangle$  and of the

 $|F = 1, m_f = -1\rangle + |F = 1, m_f = -1\rangle$  channel ([4], p.9). Here  $a_0$  is the Bohr radius. If on the other hand, a  $|F = 1, m_f = 0\rangle$  particle is surrounded primarily by other atoms in the  $|F = 1, m_f = 0\rangle$ 

state, its scattering length should equal the scattering length of the  $|F = 1, m_f = 0\rangle + |F = 1, m_f = 0\rangle$  channel, which is  $a_{0,0} = 52.66a_0$ .

From this the following can be concluded: the energy of the  $|F = 1, m_f = -1\rangle$  condensate does not change when it overlaps with the  $|F = 1, m_f = 0\rangle$  condensate, since the scattering lengths for both possible collisions are the same. The entire energy difference thus comes from the increase of the energy of the  $|F = 1, m_f = 0\rangle$  condensate. Also, the scattering length depends on position, since the densities of the two condensates are not homogeneous and the collision rates are proportional to the densities. For the scattering length  $a_0$  of the atoms in the  $|F = 1, m_f = 0\rangle$  condensate, I can write:

$$a_0(\vec{x}) = \frac{n_{-1}(\vec{x})a_{-1,0} + n_0(\vec{x})a_{0,0}}{n_{-1}(\vec{x}) + n_0(\vec{x})},\tag{4.4}$$

where  $n_{-1}(\vec{x})$  is the density of the  $|F = 1, m_f = -1\rangle$  BEC and  $n_0(\vec{x})$  is the density of the  $|F = 1, m_f = 0\rangle$  BEC.

To be able to perform the integral in formula 4.2, I have to make an additional assumption: I assume that the density profiles of the condensates are Thomas-Fermi distributions. This is a commonly used distribution that follows from the same approximation from which formula 4.2 is derived, the Thomas-Fermi approximation ([7], p.154). The non-normalized density profiles can be expressed in terms of the Thomas-Fermi radii  $R_i = \sqrt{\frac{2\mu}{m\omega_i^2}}$ :

$$n_1(x, y, z) = 1 - \frac{x^2}{R_{1,x}^2} - \frac{y^2}{R_{1,y}^2} - \frac{z^2}{R_{1,z}^2}$$
(4.5)

$$n_2(x, y, z) = 1 - \frac{x^2}{R_{2,x}^2} - \frac{(y + R_{1,y} + R_{2,y} - dx)^2}{R_{2,y}^2} - \frac{z^2}{R_{2,z}^2}.$$
(4.6)

The index 1 belongs to the  $|F = 1, m_f = 0\rangle$  condensate, which is located in the trap center. The index 2 belongs to the  $|F = 1, m_f = -1\rangle$  condensate, which is located at a different value on the axial axis y. Its center is located at  $y = -R_{1,y} - R_{2,y} + dx$ , such that for dx = 0, the two condensates just touch. By increasing the parameter dx, the two condensates overlap more and more, to complete overlap when  $dx = R_{1,y} + R_{2,y}$ . The Thomas-Fermi radii can be calculated using known trap frequencies and the number of particles of the condensates, which can be determined from figure 4.4.

Assuming that the condensates satisfy the Thomas-Fermi distribution has its limitations: in the experiment the condensates are clearly deformed due to compression, such that they not satisfy the

Thomas-Fermi distribution. This will not be a problem in this analysis, because it gives only an order of magnitude of the forces. Also, when the condensates start to interact, their distribution really is a Thomas-Fermi distribution.

Using all the information above, it is possible to calculate the interaction energy of the  $|F = 1, m_f = 0\rangle$  condensate using equation 4.2. I have performed the integral numerically using



Figure 4.7: Left: The dimensionless interaction energy as a function of dx. Right: The force that can be derived from the energy function.

a Mathematica-script for 50 different values of the overlap parameter dx, ranging from no overlap to full overlap. The resulting data is presented in the left image of figure 4.7, which gives the interaction energy as a function of dx. By using  $F = -\frac{dU}{dx}$  I have calculated the force that acts on the  $|F = 1, m_f = -1\rangle$  condensate, which is depicted in the right image of figure 4.7. It is clear that the force never exceeds  $10^{-21}$  N. The irregularities in the data is probably caused by rounding errors in the numerical integration, but this will play no role in the analysis, since I am only interested in an order of magnitude of the force.

### 4.3 Discussion

The two interaction forces can now be compared and it is immediately clear that the force that is calculated using the model in the previous section is too small to account for the motion of the condensates; it is one order of magnitude smaller than the actual force on the condensates  $(10^{-21} \text{ versus } 10^{-20})$ . As said before, the uncertainty in the magnitude of the actual force due to the decreasing number of particles is much smaller than the difference between the two forces. Although no stringent statistical analysis has been done, I think that the made assumptions are solid enough to conclude that the mechanism is more complicated than supposed by the hypothesis that the forces due to the immiscibility of the condensates account for the motion.

To understand what the mechanism is that is responsible for the fact that the condensates collide in the way they do, more research should be done on this phenomenon. It could for example be interesting to see if the interactions are different when the velocities of the condensates are different; at low velocities, there will probably be less disturbances due to shock waves. Also, repeating the same measurement with condensates at lower densities could also be interesting; perhaps in that case, the domain walls can be resolved, which would allow for comparison between the two methods.

Additionally, the compression of the condensates suggests that there might be a lot of interesting hydrodynamics involved in this interaction. The compression can also store energy, which will probably partially dissipate due to shock waves, but can also be transformed in kinetic energy of the  $|F = 1, m_f = -1\rangle$  condensate. Possibly, this could explain the motion of the condensates.

# Reflection

At the moment of writing, I am at the end of a four month period working for the BEC research group. In this period, I have done a lot of different things, such as studying relevant theory, learning how to work with the experimental setup and helping Pieter and Alexander with daily activities, which ranged from filling the nitrogen tank to cleaning mirrors for the laser beams. I think it has given me a good insight in the way experimental research is done. One thing that I will remember in particular is the wide variety of things you need to know if you want to do research on BEC's: not only is it important to have a good grasp on the physics of BEC's, but also on other aspects of physics such as laser optics, quantum mechanics and atomic physics. In addition, there are also various skills you need to have: for example, you have to be able to use lasers and how to program scripts to help you analyze experimental data.

Although I have not learned everything there is to know about BEC research - which is of course simply impossible -, I think that I have still learned a lot in the past four months. For example, I have learned how to program in the programming language C and also a bit in IDL, which was completely new to me. I really enjoyed programming the evaporative cooling simulation and it has given me a basis on which I can build for future programming projects. Also, the weekly meetings with the entire research group were really stimulating and helped me understand the type of problems that occur in the lab. Although during the first few meetings I sometimes had trouble understanding what the discussion was about, I was pleased to notice that as timed went by, I could understand more and more of the detailed discussions and actually try to contribute.

In my time in the lab, I have also done various measurements, such as trap frequency measurements, measurements to optimize the MT to FORT overload and of course the measurement of interacting spinor condensates as discusses in chapter 4. It was really interesting to get to know the various aspects of the setup better and I have learnt a lot from the discussions that arose during the measurements.

In the last month, I have also learned a skill that is important for every scientist, namely communicating about my research in various ways: by making a poster, writing a report and talking for an audience of other physicists. As a byproduct, I also experienced what it is like to work under pressure, for example in my preparation for the talk for the BONZ seminar: the analysis I talked about had been done only two days prior to the talk!

In summary, I can safely say that I have seen all the aspects that are important in experimental research. My enthusiasm for the field of BEC research has only increased, due to the fascinating things I have seen in the lab and I hope that I might have a chance to contribute to research into BEC's in the future.

# Appendix

As a reference, I will in this appendix explicitly calculate the integral expressions from chapter 2. First the effective volume:

$$\begin{split} V_e &= \int d\vec{x} \ e^{-\beta V(\vec{x})} \\ &= \int d\vec{x} \ e^{-\frac{m}{2k_b T}((\omega_x x)^2 + (\omega_y y)^2 + (\omega_z z)^2)} \\ &= (\sqrt{\pi})^3 \left(\frac{2k_b T}{m\omega_x^2}\right)^{1/2} \left(\frac{2k_b T}{m\omega_y^2}\right)^{1/2} \left(\frac{2k_b T}{m\omega_z^2}\right)^{1/2} \\ &= \left(\frac{2\pi k_b T}{m(\omega_x \omega_y \omega_z)^{2/3}}\right)^{3/2}, \end{split}$$

using Gaussian integrals. The ensemble average density can be calculated as follows:

$$\begin{split} \langle n \rangle &= \frac{1}{\Lambda^3} \frac{\int d\vec{p} \, d\vec{x} \, g_{3/2}(z(\vec{x})) \frac{1}{e^{\beta(\frac{p^2}{2m} + V(\vec{x}) - \mu)} - 1}}{\int d\vec{p} \, d\vec{x} \frac{1}{e^{\beta(\frac{p^2}{2m} + V(\vec{x}) - \mu)} - 1}} = \frac{\Gamma(\frac{3}{2})}{\Gamma(\frac{3}{2})\Lambda^3} \frac{\int d\vec{x} \, g_{3/2}(z(\vec{x})) g_{3/2}(z(\vec{x}))}{\int d\vec{x} \, g_{3/2}(z(\vec{x}))} \\ &= \frac{1}{\Lambda^3} \frac{\int d\vec{x} \sum_{n,m=1}^{\infty} \frac{e^{(n+m)\beta(\mu - V(\vec{x}))}}{(nm)^{3/2}}}{\int d\vec{x} \, g_{3/2}(z(\vec{x}))} = \frac{1}{\Lambda^3} \frac{\sum_{n,m=1}^{\infty} \frac{e^{(n+m)\beta\mu}}{(nm)^{3/2}} \int d\vec{x} \, e^{-(n+m)\beta(V(\vec{x}))}}{\sum_{n=1}^{\infty} \frac{e^{n\beta\mu}}{n^{3/2}} \int d\vec{x} \, e^{-n\beta(V(\vec{x}))}} \\ &= \frac{1}{\Lambda^3} \frac{\sum_{n,m=1}^{\infty} \frac{e^{(n+m)\beta\mu}}{(nm)^{3/2}} \frac{V_e}{(n+m)^{3/2}}}{\sum_{n=1}^{\infty} \frac{e^{n\beta\mu}}{n^{3/2}} \frac{V_e}{n^{3/2}}}{\sum_{n=1}^{\infty} \frac{e^{(n+m)\beta\mu}}{n^{3/2}}} \\ &= \frac{1}{\Lambda^3} \frac{\sum_{n,m=1}^{\infty} \frac{e^{(n+m)\beta\mu}}{(nm(n+m))^{3/2}}}{\sum_{n=1}^{\infty} \frac{e^{n\beta\mu}}{n^{3}}} \end{split}$$

The ensemble average of the squared density follows in almost the same way:

$$\begin{split} \langle n^2 \rangle &= \frac{1}{\Lambda^6} \frac{\int d\vec{x} \; g_{3/2}(z(\vec{x}))^2 g_{3/2}(z(\vec{x}))}{\int d\vec{x} \; g_{3/2}(z(\vec{x}))} = \frac{1}{\Lambda^6} \frac{\int d\vec{x} \sum_{n,m,l=1}^{\infty} \frac{e^{(n+m+l)\beta(\mu-V(\vec{x}))}}{(nml)^{3/2}}}{\int d\vec{x} \; g_{3/2}(z(\vec{x}))} \\ &= \frac{1}{\Lambda^6} \frac{\sum_{n,m,l=1}^{\infty} \frac{e^{(n+m+l)\beta\mu}}{(nml)^{3/2}} \int d\vec{x} \; e^{-(n+m+l)\beta(V(\vec{x}))}}{\sum_{n=1}^{\infty} \frac{e^{(n+m+l)\beta\mu}}{n^{3/2}} \int d\vec{x} \; e^{-n\beta(V(\vec{x}))}} = \frac{1}{\Lambda^6} \frac{\sum_{n=1}^{\infty} \frac{e^{(n+m+l)\beta\mu}}{(nml)^{3/2}} \frac{V_e}{(n+m+l)^{3/2}}}{\sum_{n=1}^{\infty} \frac{e^{(n+m+l)\beta\mu}}{(nm(n+m+l))^{3/2}}}{\sum_{n=1}^{\infty} \frac{e^{n\beta\mu}}{n^3}} \end{split}$$

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