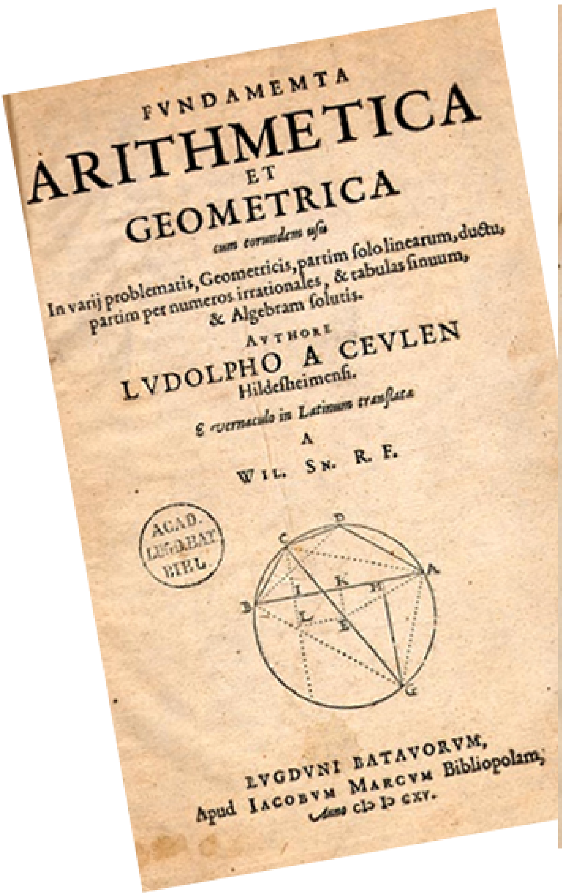
# Arithmetische en Geometrische Fondamenten

**by Ludolph van Ceulen**

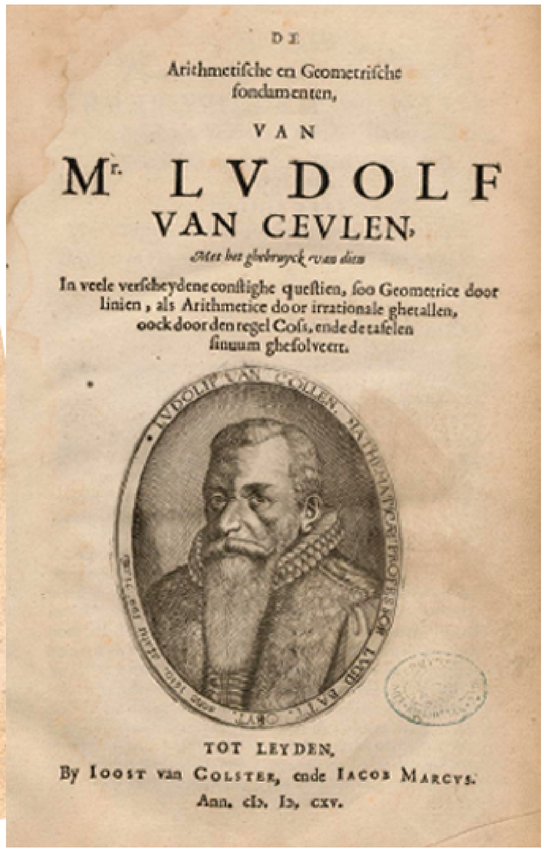
and its Latin translation

# Fundamenta Arithmetica et Geometrica

**by Willebrord Snellius**



FVNDAMEMTA ARITHMETICA ET GEOMETRICA
cum eorundem usu
In varij problematis, Geometricis, partim solo linearum ductu, partim per numeros irrationales, & tabulas sinuum, & Algebram solutis.
AVTHORE LVDOLPHO A CEVLEN Hildesheimensi.
E vernaculo in Latinum translata A WIL. SN. R. F.
LVGDVNI BATAVORVM, Apud IACOBVM MARCVM Bibliopolam, Anno cIɔ Iɔ cXV.



DE Arithmetische en Geometrische fondamenten,
VAN Mr. LVDOLF VAN CEVLEN,
Met het ghebruyck van dien
In veele verscheydene constighe questien, soo Geometrice door linien, als Arithmetice door irrationale ghetallen, oock door den regel Cos, ende de tafelen sinuum ghesolveert.
TOT LEYDEN, By IOOST van COLSTER, ende IACOB MARCVS. Ann. cIɔ. Iɔ. cxv.


**Universiteit Utrecht**

**Maartje van der Veen (3352064)**


Master thesis in Science Education and Communication (Mathematics)

Under the supervision of Steven Wepster

April 3, 2014

# Preface

This is my master thesis for the Master program Science Education and Communication at the University of Utrecht, Netherlands. This master thesis is a continuation of my research for my bachelor thesis and now, after another year of research, I am able to complete it.

I would like to thank my supervisor, Steven Wepster. I went to him since I wanted to write my thesis on the history of mathematics, a topic that really had my interest. He showed me an old mathematics textbook, the *Fundamenta Arithmetica et Geometrica*, which he was studying and suggested that I could make it my research for my bachelor thesis. When the research that was needed to fully grasp all the interesting content became too expansive, we decided to let my bachelor thesis be the start and continue on it for my master thesis. I gratefully made use of his outset which already contained some notes. These became the starting point for my research. Thanks for handing me this work, I really enjoyed studying it!

I also would like to thank my friends Kirsten de Hoog and Rianne Tel for correcting my English. Thanks for taking your time to help me!

Finally, I would like to thank my friend Chris Elsinga, who encouraged me to keep going and motivated me in times of setbacks. Thanks to you this thesis is finally finished!

I hope you will enjoy reading this thesis. If you discover any new information, don't hesitate to let me know!

Maartje van der Veen BSc

# Abstract

Here is a short summary of this master thesis. This text is used in the announcement for my graduation talk on October 9th, 2013. Since most of the audience were Dutch speaking, a Duth translation is added.

The *Arithmetische en Geometrische Fondamenten* (Arithmetic and Geometric Foundations) is a work on mathematics that was published in 1615, five years after the author, Ludolph van Ceulen, had passed away. In the same year, a Latin translation was published, translated by Van Ceulen's pupil Willebord Snellius. However, this Latin edition was not an exact translation. Snellius made the necessary improvements, changed structures and formulations, contributed his own mathematical discoveries and wrote comments on the problems presented in the work. In the research for my master thesis I have examined these changes and comments by Snellius. The goal was to determine the purpose of the work (both the original and the translation) and compare the mathematical styles of Van Ceulen and Snellius.

De *Arithmetische en Geometrische Fondamenten* is een wiskundig werk gepubliceerd in 1615, vijf jaar nadat de auteur, Ludolph van Ceulen, was overleden. In hetzelfde jaar werd een Latijn-se vertaling gepubliceerd, vertaald door Van Ceulen's leerling Willebord Snellius. Echter, deze Latijnse editie was geen exacte vertaling. Snellius maakte de nodige verbeteringen, veranderde structuren en formuleringen, voegde zijn eigen wiskundige vindingen toe en gaf veelvuldig commentaar. In het onderzoek voor mijn master scriptie heb ik deze aanpassingen en commentaren van Snellius onderzocht. Het doel was om het doel van het werk (zowel het origineel als de vertaling) te bepalen en de wiskundige aanpak van Van Ceulen en Snellius te vergelijken.

# Contents

# Introduction

For my bachelor thesis I began to study the *Arithmetische en Geometrische Fondamenten* by Ludolph van Ceulen and its Latin translation *Fundamenta Arithmetica et Geometrica* by Willebrord Snellius. In this thesis I will refer to these works as the *Fondamenten* and the *Fundamenta*. The *Fundamenta* is not only a translation, but contains multiple commentaries written by Snellius, a student of Van Ceulen. This makes the work a very valuable source for comparing the mathematical styles of these two mathematicians. But, what (kind of) commentaries did Snellius add to the *Fundamenta*? And how does Snellius approach the problems differently when he presents an alternative solution method? These questions lead to the main focus of this master thesis: the differences between the mathematical approaches of Van Ceulen and Snellius.

Translating the Latin commentaries that Snellius contributed to the *Fundamenta* proved more difficult than expected. I could not study all the commentaries in the time period I had for my bachelor thesis, so I restricted myself to the fifth book of the *Fundamenta*. In this master thesis I finally have had the time to study all commentaries and draw conclusions that are more substantiated.

In contrast to my bachelor thesis, I will not only focus on Willebrord Snellius and his translation of the *Fondamenten*, but also on the original work itself and its author Ludolph van Ceulen. By doing so, I will be able to give a more thorough comparison of the works and the differences in approaching mathematical problems between Van Ceulen and Snellius.

While I was studying different sources I stumbled upon many other questions that were not answered in any of the sources that I found. For example, Vlek and De Wreede both made suggestions to what the purpose of both works could be. Was the *Fondamenten* intended to be one book? And what were the intended audiences of Van Ceulen and Snellius. However, neither came to solid conclusions. How and where I found these questions can be found in chapter 2, where I discuss some of the main sources consulted for this thesis.

Furthermore, when investigating the original works, I stumbled upon a large variety of different editions of both the *Fondamenten* and the *Fundamenta*. This rose the question how many editions might exist. Is the *Fondamenten* only published in the year 1615 or are there other editions to be found with different publication years? In several old catalogues, there were different years of publication given, but were they correct? Also, some sources suggested that the *Fondamenten* could be the work on algebra promised by Van Ceulen, known as the 'Coss-book'. Finally, when studying other works by Van Ceulen, I noticed the similarities between the Latin translation of *Vanden Circkel*, titled *De Circulo*, and the *Fundamenta*. Could the *De Circulo* be regarded as a reprint of the *Fundamenta*? I have included all these questions in this thesis.

In the next chapter, biographies of Van Ceulen and Snellius are presented, together with a short introduction to the content of the works which is needed to understand the rest of this thesis. After discussing the earlier research that has been done on the works, the purpose of both works will be discussed. Thereafter a short detour reveals in detail how many publications were printed of the works and how they differed from each other. Subsequently, the main focus of this thesis is adressed. The commentaries which Snellius added to the *Fundamenta* will be discussed in general, after which some specific examples are given. Finally, the different mathematical approaches of Van Ceulen and Snellius are compared and conclusions are drawn.

# Chapter 1

# Background information

This chapter gives the background information for this master thesis and contains a summary of my bachelor thesis. The first two sections give short biographies of Ludolph van Ceulen and Willebrord Snellius. Section 1.3 describes briefly the content of the *Fondamenten* and the *Fundamenta*.

## 1.1 Ludolph van Ceulen

Ludolph van Ceulen was born January 28th 1540 in Hildesheim, Germany. He was a student of Jan Pouwelsz (Rijnierse and Wepster, 2010). Around 1578 he lived in Delft, where his main occupations considered arithmetic and mathematics. Soon he was known as reckoning and fencing master (Wepster, 2009, p. 98).


Figure 1.1: Ludolph van Ceulen.

From 1594 he was teaching the art of fencing in his own fencing school in Leiden (Hogendijk, 2006, p. 15). Besides these fencing lessons, he also taught his pupils mathematics. The parents of these pupils were mostly rich traders for whom he also did interest calculations. Here he became also known as someone who could solve mathematical problems and as reckoning master (Katscher, 1979, p. 103).

Van Ceulen could read German and Dutch, but not Latin or Greek. Therefore he needed a translator to consult old documents. Van Ceulens friend Jan de Groot (1554-1640) translated works by Archimedes which Van Ceulen needed for his *Vanden Circkel*. Van Ceulen owned a German edition of the *Elements* of Euclid, translated by Wilhelm Holtzman (Xylander), which he accurately studied and used for his own work, the *Fondamenten* (Katscher, 1979, p. 105) (see also section 3.3).

In 1600, at the age of 60, Van Ceulen became one of the first professors at the engineers school of Leiden, the *Duytsche Mathematique*, founded by Maurits van Nassau, where exclusively was being taught in Dutch. This school was connected to the University of Leiden (Katscher, 1979, p. 103). Van Ceulen taught arithmetic, fortifications and surveying. Several of these subjects can also be found in the *Fondamenten* (Wreede, 2007, p. 29). Van Ceulen taught until he passed away.

January 10th in the year 1600 Van Ceulen was titled by the curators of the university and the mayor of Leiden as professor, praised for his “high efficiency, experience and skills in these arts” (Katscher, 1979, p. 103).

One of the most important contributions from Ludolph van Ceulen to mathematics, were his calculations of the ratio between the length of de circumference of a circle and its diameter, which was for long called the “Ludolphian number” and is now known as $\pi$. His epitaph contains the first 35 decimal numbers of this number. These calculations were done by determining the lengths of the sides of in- and circumscribed equilateral polygons. He was also able to determine the sine of one degree at a precision of 36 decimals. For these calculations he must have had strong algorithms, which he, unfortunately, never published and got lost (Hogendijk, 2006, p. 17). With these achievements, Van Ceulen proved himself as reckoning master. This is also shown in the *Fondamenten* in which he also made a lot of complicated calculations.

Van Ceulen's main works were his *Vanden Cirkel* (1596) and *Arithmetische en Geometrische Fondamenten* (1615, posthumously). In *Vanden Cirkel* he explains his calculation on the ratio between the circumference of a circle and its diameter and his calculations on the length of the sides of equilateral polygons (3 to 80 angles). For this he cooperated with Adriaan van Roomen (Adrianus Romanus in Latin). They both understood the connection between these calculations and ‘cossic' equations (algebraic equations with a variable ‘coss'). His numeric approaches were exceptionally accurate, in which he proves again his skills as reckoning master. Next to these two major works he also wrote some smaller works which were reactions and corrections to his fellow mathematicians work:

- *Solutie ende werckinge op twee geometrische vraghen by Willem Goudaen inde jaeren 1580 ende 83 binnen Haerlem aenden kerckdeure ghestelt: mitsdadigers propositie van twee andere geometrische vraghen.* (1584)
  (A reaction to answers on two geometric questions by Willem Goudaen.)

- *Kort claer bewijs dat die nieuwe ghevonden proportie eens circkels iegens zyn diameter te groot is ende ouerzulcx de quadratura circuli des zeluen vinders onrecht zy...* (1585)
  (Short evidence that the found proportion of a circle to his diameter is too large and that the equation for calculating the quadrature of circles is incorrect.)

- *Proefsteen ende claerder wederleggingh dat het claarder bewijs (so dat ghenaempt is) op de gheroemde ervindingh vande quadrature de circkels een onrecht te kennen gheven, ende gheen waerachtich bewijs is:...* (1586)
  (Touchstone and reproof that *Clearer evidence* (so it is called) on the famous invention of the quadrature of circles is incorrect and is no real proof:...)

Next to *Vanden Circkel* is the *Fondamenten* the only other major work that Van Ceulen published (Bierens de Haan, 1878, p. 130-139). Van Ceulen died after enduring a long period of sickness on 31 December 1610 in Leiden (Katscher, 1979, p. 99) and was buried in the Sint Pieterskerk.

## 1.2 Willebrord Snellius

Snellius was probably[1] born on the 13th of June 1580 in Leiden. His original name is Willebrord Snel van Royen, but like his father Rudolph, professor mathematics on the university of Leiden, he used the Latin version of his name. He usually referred to himself as R.F. or Rudolphi Filius (son of Rudolph). His father taught him Latin and Greek and let him read philosophic authors. On his father's encouragement Snellius started a study law at the university of Leiden. However, Snellius chose mathematics, even though it was very difficult to get any recognition as specialized mathematician and a permanent position with a decent salary. Snellius was a pupil of Van Ceulen. At the age of 19 Snellius worked with Van Ceulen on the problem of triangle division. Some of Snellius' results were published in Van Ceulen's *Fondamenten*.

Figure 1.2: Willebrord Snellius.

In 1600 Snellius was allowed to teach on special days (Wednesday and Saturday when other teachers did not teach) at the university of Leiden. Later on, Snellius travelled to Germany where he met Adrianus Romanus (1561-1615) (see section 3.5). It is possible that Adrianus proposed to translate the Dutch works of Van Ceulen to Latin, to make them more accessible (Wreede, 2007, p. 47). Snellius also had contact with Tycho Brahe (1546-1601), Johannes Kepler (1571-1630) en Joannes Preatorius (1537-1616).

In the spring of 1602 Snellius came back to Leiden. He taught mathematics, arithmetic and astronomy at the university of Leiden and also gave private lessons. In the next year he prepared his first publication: a summary of the *Geometria* by Petrus Ramus (1515-1572), whose ideas had a lot of influence on the work of Rudolph and Willebrord Snellius. Willebrord dedicated his work, called *Petri Rami Geometriae Libri XXVII*, to his mentor Adrianus Romanus.

Next to the teaching at the university, Snellius also worked on several translations, publications and his own works. His first mathematical publications were reconstructions of three lost works of Apollonius of Perga (ca. 200 B.C.). In the preface of one of these publications he wrote that the content was complicated and without any structure, which he solved by re-ordering and shortening the content. He did the same later with the *Fondamenten*. Snellius was convinced that a mathematical publication should be concise:

> ..., the capability of teaching clear and perspicacious is the mark of an academic and intelligent man. And for this reason I am glad with short and exact answers when they are needed, which do not need to be long to be clear.[2]

Snellius developed with Apollonius' works his rhetoric skills and ideas about correct mathematics and at the same time he tried to win Maurits of Nassau as patron (Wreede, 2007, p. 63). He advised Maurits and corresponded with a lot of scientists. Meanwhile his status as

[1] For an explaination on the unsurtainty of the exact birth date, see (Wreede, 2007, p. 17).

[2] ...; plane autem, et perspicue expedire posse, docti et intelligentis viri. atque ideo, cum opus erit, consectaria me brevia et acuta delectant; quae, ut perspicua sint, ita longa esse non debent. (Snellius, 1608, p. 5-6), taken from (Wreede, 2007, p. 62).

an academic grew.

He desired an official recognition of his capabilities. In 1609 he was given an more official position by the senate. In this period his father was headmaster of the university. He worked a lot for little money, however he did not get his professor title because there was already a mathematics professor at the university. He did get the promise that he would be nominated to succeed his father after his retirement.

On February 8th, 1613, he finally got is desired title: associate professor as replacement for his aged and sick father. Two years later he became a regular professor. He taught different subjects of mathematics, like astronomy and optics. In 1617, Snellius published one of his most important works *Eratosthenes Batavus*, in which he presented his calculations of the circumference of the earth. He dedicated this work to the States General, probably after the example of Adriana Simons (widow of Van Ceulen), and received 200 guilders. Two years later he repeated this with the translation of Van Ceulen's *Vanden Circkel* and received 100 guilders. Snellius translated only a part of this work and published it as *De Circulo et Adscriptis Liber* along with a reprint of a major part of the *Fundamenta* (see section 4.2.1). He developed his own more direct approach for calculating $\pi$, however he did not take the effort to match Van Ceulen with the number of decimals (Snellius gave 'only' 34 decimals). Later Snellius wrote his own work about the quadrature of the circle, the *Cyclometricus*.

Snellius name was especially known for his discovery in the subject of refraction, which is now known as the "law of Snellius", although he did little research on this subject. This interest probably came from the relevance for astronomy and navigation, two subjects that received a lot of attention from Snellius. After a short time of sickness Snellius died at the age of 46 on October 30th 1626. He is praised for his contribution to arithmetic, astronomy, surveying and navigation.

## 1.3 The works

The publication *Fundamenta Arithmetica et Geometrica* is a translation by Willebrord Snellius of Ludolph van Ceulen's work *Aritmetische en Geometrische Fondamenten*. Both works were published in 1615, five years after Van Ceulen passed away. Commissioned by Adriana Simons, the widow of Van Ceulen, Snellius translated the work from Low German to Latin. However there are many differences between these publications. Snellius added several improvements, changed structures and formulations, contributed his own mathematical discoveries and many commentaries. In this section the content of the *Fondamenten* and the *Fundamenta* will be described and compared.

### 1.3.1 The *Fondamenten*

The complete title of the *Fondamenten* is:

> The Arithmetic and geometric basics, by Mr. Ludolph van Ceulen, with use of many different examples solved: geometrical with lines, arithmetical with irrational numbers, with cossic equations and sinus tables.[3]

[3] De Arithmetische en Geometrische fondamenten, van Mr. Ludolf van Ceulen, Met het ghebruyck van dien In veele verscheydene constighe questien, soo Geometrice door linien, als Arithmetice door irrationale ghetallen, oock door den regel Coss, ende de tafelen sinuum ghesolveert. (Ceulen, 1615a)

The *Fondamenten* contains six books, build up in chapters or just a list of various problems. Table 3.1 contains in the left column an overview of the layout of the work. The first book is build up in eight different chapters and explains basic arithmetic like addition, subtraction, multiplying and dividing. Van Ceulen starts with explaining (how to pronouns) whole numbers, followed by irrationals and proportions (ratios). After explaining the basics, two chapters with practical examples with irrational numbers follow. Van Ceulen uses a complete chapter for the explanation of the square root (chapter four). The first book ends with a chapter on binomial and residue numbers and a chapter about universal numbers. A binomial number is the addition of a rational and the square root of a rational number, a residue number is the difference between the two. A universal number is the square root of a binomial or residue number. Several examples are:

$$9+\sqrt{7}\ \textit{Binomial number}$$
$$\sqrt{19}-3\frac{1}{4}\ \textit{Residue number}$$
$$\sqrt{20+\sqrt{396}}\ \textit{Universal number}$$

Book two of the *Fondamenten* has many similarities with the *Elements* of Euclid. In the title, Van Ceulen indicates that in this book the basics of geometry is explained in its most simple way and 'uyt Euclides getrocken' (extracted from Euclid). Like the *Elements*, book two starts with definitions and axioms for geometry. The rest of this book contains 84 different propositions, all with a figure and construction. The only 'proofs' given by van Ceulen for his constructions are examples with numbers, and not generally geometric proofs. Overall, this book shows basic constructions and theories used in later books. Van Ceulen seems to have picked exactly those propositions from the *Elements* necessary for his work.

Book three contains many problems ordered by subject. First, Van Ceulen explains the theory of transforming figures to others (e.g. triangle to rectangle) with the same area and dividing figures by a certain ratio. After 34 problems Van Ceulen continues with theory regarding line segments. He explains addition, subtraction, multiplication and division of line segments with 15 problems. After that, general geometrical problems follow involving all the previously explained theory. Book three ends with a 'byvouch des derden Deels' (appendix to part three) which Van Ceulen begins as follows:

> Now follows the proof of previous propositions by numbers, to give the enthusiast the chance to practice, so he can gladly en freely work with numbers, and will not hesitate to make the examples of the following book.[4]

This appendix is a preparation for the next book of the *Fondamenten* in which many calculations are required to solve the geometric problems.

The last three books of the *Fondamenten* contain only geometric problems. Book four explains 57 problems in which Van Ceulen provides the solution (and solution method). He mainly explains problems about triangles. These problems were possibly meant for practice, like Van Ceulen wrote at the beginning of the appendix. Book five contains 47 problems on different subjects like triangles, circles and cyclic quadrilaterals, proven by numbers and geometry. The

[4] Volgt nu de bewijsinge etlicker voorgaender propositien door ghetallen, om des liefhebbers wille gestelt om hun daer na te oeffenen, op dat hij lustich ende seker met de ghetallen leert wercken, ende de exempels des volghenden deels niet schroomt te maecken. (Ceulen, 1615a, p. 156)

last problems are solved by van Ceulen using cossic equations. Book six ends the work with problems about in- and circumscribed (equilateral) polygons in and around circles. The 17th and last problem ends suddenly after the question without any answer.

### 1.3.2 The *Fundamenta*

The first obvious thing about the two works when comparing is that some chapters are missing in Snellius' translation. Table 3.1 shows which chapters are and are not used and where they are (re)located. Snellius leaves out a great part of chapter one and uses 31 pages for which Van Ceulen uses 68. In this book Van Ceulen explains basic arithmetic operations and theory about square roots and fractions. Snellius translation skips the first five chapters and starts with square roots. It is likely that Snellius rated these chapters low level. The chapter about square roots is in the translation divided in different chapters: addition, subtraction, multiplication and division with square root, which in the end makes chapter VI correspond with chapter VII of the original book. A note of Van Ceulen on the square root of a binomial or residue number is added as a separate chapter by Snellius. After this, the chapter numbering is equal again. It is clear that Snellius focuses mainly on the theory of square roots. He probably regarded this as hard to learn. In the final chapter Snellius is more extensive in his explainations compared to Van Ceulen (11 pages instead of 8).

The second book of the *Fondamenten* is copied one on one. However, Snellius does add the heading 'problema' to some of the propositions, hereby making a distinction between propositions and geometrical problems. For example, proposition 22, about the construction of a square, does get an extra heading 'problema', however, proposition 9, saying that opposite angles are equal, does not. Snellius missed proposition 82, which should also be included in the category problema's. Of the in total 87 propositions, 37 are given the extra heading 'problema'.

The dedicatory letter that Snellius placed before book three is a letter to Rosendalius (see section 3.1.2). Van Ceulen's widow, Adriana Simons, wanted to place her own dedicatory letter, which is why Snellius' letter is not placed at the beginning. The choice of this place is probably because as of book three the pre-work with propositions and constructions end and the problems begin. The appendix is interpreted as preparation for the problems of book four. Snellius moves this part to the beginning of book four. This is why chapter four is the first chapter where calculation examples are mixed with geometry problems. This makes a clear separation between basic mathematical operations, definitions and propositions, and geometric problems. The biggest change that Snellius made was the contribution of his own comments. After many problems Snellius writes his opinion about the solution of Van Ceulen and adds his own. In book five of the *Fundamenta*, Snellius comments a lot and presents regularly his own solution. It is remarkable that after the many commentaries in the books four and five, book six contains no commentaries at all. Except for the last problem since it was incomplete. He also moves the last three problems in a separate 'Appendicula' (appendix) as addition to book six. These commentaries are discussed in chapters 5 and 6.

# Chapter 2

# Earlier research

In this section three main sources that were used for this thesis are discussed and the conclusion of my bachelor research is summarised.

## 2.1 A Master thesis on Van Ceulen's *Fondamenten*

In 2008, Charlotte Vlek wrote her master thesis on the original work *Arithmetische en Geometrische Fondamenten* by Ludolph van Ceulen (Vlek, 2008). In her research she dedicated one section to the Latin translation of this work by the hand of Willebrord Snellius. Some conclusions made by Vlek will now be discussed.

The first remark she made is that Snellius left out a great part of the basic theory given in the first book of the work (see also chapters 1.3 and 3.4). According to Vlek, Snellius apparently saw this subject as hard to grasp. Vlek could not answer the question why Van Ceulen and Snellius disagreed on the importance of arithmetical subjects (Vlek, 2008, p. 52). In this thesis I will try to formulate an answer (see 7.1).

Another claim Vlek makes is that Snellius completely translated part two, in which Van Ceulen gives a selection of theorems from the *Elements* of Euclid. Indeed, Snellius translated every theorem in his *Fundamenta*, but he rephrased many theories to make the work more Euclid-like. Furthermore, she remarks that Snellius made a division between the first three books and the last three, by placing the appendix which Van Ceulen added to book three at the beginning of book four. This way, Vlek concludes, the work is divided in a part on theory and a part which mostly contains exercises (Vlek, 2008, p. 55). This division is, according to Vlek, reinforced by the fact that Snellius placed a preface before the start of book four. In fact, this preface (or better: introduction) is merely a translation of Van Ceulen's preface to the appendix of book three, to which Snellius added a quote from Eutocius:

> For these mathematical disciplines [arithmetic and geometry] seem to be sisters.[1] (Ceulen, 1615b, p. 137)

Hereby he points out his view on the relation of arithmetic and geometry. This quote links the part involving the theory of arithmetic to the part with geometric exercises. I assume Vlek meant the dedicatory letter written by Snellius, but this was placed before book III. This letter

[1] "*ταυτα γαρ τα μαθηματα δοκουντι ειμην αδελφα*." Snellius may have known this quote from his reconstructions of treatises by Appolonius of Perga, which was published as *Appolonius Batavus, seu, Exsuscitata Apolonii Pergaei* Περι *διωρισμενης τομης Geometria* in 1608.

intensified the separation between where the theory stops and the geometrical examples begin (see 3.1.2).

Furthermore, Vlek remarks that book five and six may not have been intended to be part of the original work. Her first argument for this is that the problems are of a too high level for Van Ceulen's target audience. Also, Van Ceulen repeated the theory about dividing figures from book three in book five, instead of combining all the question on this subject, which affirms the idea that book five was not originally part of the *Fondamenten*. Nevertheless, Vlek had found a reference in book five to the theorems from book two, hence concluding that it could not be completely independent from the earlier parts (Vlek, 2008, p. 55). I will discuss my position that the *Fondamenten* was indeed intended to be one combined work in section 3.2.

In Vlek's last chapter she tries to retrieve the purpose of the original work. The *Fondamenten* seems to be a textbook which was used in the engineering school where Van Ceulen was a teacher. But the large quantity of geometrical problems suggest that it is not intended as a textbook. This, she claims, also seems to be Snellius interpretation, since he omitted the parts where the basic theories were explained. This conclusion seems slightly premature since Vlek does not consider all the comments Van Ceulen and Snellius made throughout the books, which I intend to find. I will attempt to retrieve the purposes of both works and compare them in chapter 3.

## 2.2 De Wreede

Liesbeth de Wreede wrote her dissertation *Willebrord Snellius (1580-1626): a Humanist Reshaping the Mathematical Sciences* in 2007 at the University of Utrecht. She performed a very thorough research on the life and works of Snellius. Her work has been a great help for me in writing this thesis. I will discuss some of her remarks which I would like to work out in more depth.

In a long biographical chapter on the life of Snellius, De Wreede adds a section on the publication of the *Fundamenta*. She investigated a letter from Snellius to Rosendalius in which he tells his friend that he is asked by the widow of his deceased teacher Ludolph van Ceulen to translate his work into Latin. Later on in this letter, he writes that he demanded the right 'to enclose his own dedicatory letter in the middle of the book' (Wreede, 2007, p. 87). According to De Wreede, Snellius wanted to use this letter to rise up in his career and he was not hiding this motive from Rosendalius. De Wreede gave a very detailed discussion on the content of this dedicatory letter (see Wreede (2007, p. 188-201)), which I used to focus on the question what Snellius' purpose with the translation could have been (see section 3.1).

De Wreede wrote one subsection on the audiences of Snellius' works. Since almost all his work was written in Latin, Snellius' audience was limited to 'men of learning' (Wreede, 2007, p. 308). 'The core of his work was mainly meant for specialised mathematicians'. De Wreede suggests that Snellius' audience was probably very small since he did not have a large group of contacts among specialized mathematicians. Furthermore, Snellius had to take into account the wishes of his contacts as well as those of potential readers from the elite. De Wreede concludes by writing that 'Snellius' success was considerable, but not overwhelming' (Wreede, 2007, p. 309). This gave me a strong base while I was working on the question what target audience Snellius might have had in mind for the *Fundamenta* (see section 3.1.2).

De Wreede also writes about Snellius' irritation toward the publishing company for pressing him to finish his translation quickly. Snellius often complained about the lack of time to add contributions to problems or to have new figures cut (Wreede, 2007, p. 88-89). Nevertheless, Snellius did find the time to add some elaborations on Van Ceulen's work. De Wreede concludes in her book that Snellius did not have the opportunity to have new figures cut. Some figures still have Dutch words in them and Snellius complains many times that he was unable to provide a new figure. However, I did find some new figures in the Latin translation of the *Fondamenten*. I will present my findings in section 5.2.

De Wreede states that after the publication of the *Fundamenta*, there were two reprints (Wreede, 2007, p. 90). But she eventually only points out one, as being part of the Latin translation of *Vanden Circkel*, another work by Van Ceulen. I will discuss this *De Circulo* in section 4.2.1 and try to find out whether or not this can be presumed to be a reprint of the *Fundamenta*. In a footnote, De Wreede points to two other editions which were published in 1617 and 1618 (Wreede, 2007, p. 188). While working on this thesis I have seen the one from the Tresoar in Leeuwarden which was indeed published in 1618[2], but I was not able to consult the other since it can only be found abroad, in the Bibliothèque Nationale de France[3]. I did find a lot of different 1615 publications of the *Fondamenten* and the *Fundamenta*, of which some have a different dedicatory letter written by Adriana Simons. These editions will be presented in chapter 4.

At the end of her book, De Wreede compares the translation of the *Fondamenten* to its original. She points out the more general formulation which Snellius used to translate Van Ceulen's work, by using classical concepts. The example she gives is the following. Van Ceulen wrote:

> One wants to divide this triangle that is put here, drawn by $ABC$, into two equal parts, with a straight line drawn from vertex $C$. (Ceulen, 1615a, p. 119)

Whereas Snellius translated:

> To cut a given triangle in a given ratio with a straight line drawn from a vertex. (Ceulen, 1615b, p. 92)

Nevertheless, Snellius remained true to the original in his translation of the treatment following the problem. Furthermore, in several cases he made remarks at the end of a problem that the same treatment could be used for other situations as well. The fact that Snellius combined several problems in one also shows his effort to make the *Fundamenta* more generally applicable. De Wreede concludes:

> It seems that Snellius wanted to 'purify' Van Ceulen's work, which means to make it more Euclidean by imposing a clearer structure on it, by diminishing the role of numerical exemplary values in the geometrical problems and by having the correctness of all constructions proved. This purification would make the work fit better into the classical mathematical tradition. (Wreede, 2007, p. 240)

[2]This edition is published *Lugduni Batavorum, excudebat Georgius Abrahami A Marsse.*

[3]This edition is published *Amstelodami, apud H. Laurentium.*

In many commentaries, Snellius adds his own alternative solution method to the problem. According to De Wreede, Snellius did not intend to disregard the method which Van Ceulen had presented by replacing it with his own. Snellius could not prove whether his method was faster or more convenient (Wreede, 2007, p. 209). In a discussion on a commentary of Snellius to a problem that required to find a line segment with unit length, De Wreede concludes:

> There is a telling difference in method: the reader must be a good calculator with squares, preferably a virtuoso like Van Ceulen himself, to be able to use Van Ceulen's method efficiently, whereas Snellius used an Euclidean construction. His method could be called more geometrical. It was also more general, and therefore indeed easier, because his algorithm did not depend on the actual numbers. (Wreede, 2007, p. 209)

In chapter 6 I will discuss the differences in how Van Ceulen and Snellius presented different solution methods to the same problem.

## 2.3 Bierens de Haan

Bierens de Haan treats the life of Van Ceulen and the works written by Van Ceulen comprehensively in his *Bouwstoffen voor de geschiedenis der wis- en natuurkundige wetenschappen in de Nederlanden* (Building materials for the history of mathematics and physical sciences in the Netherlands) (Bierens de Haan, 1878, pp. 123–170). He also discusses *De Circulo*, the Latin translation of *Vanden Circkel* (more on this work see section 4.2.1). He is very detailed on what Snellius did and did not translate exactly. Bierens de Haan is very focussed on presenting the information he is giving correctly, especially when it involves the years of publication of works that have no date printed on them.

He also writes that he has seen three different publications of the *Fondamenten*, each with its own dedicatee and dedicatory letter (Bierens de Haan, 1878, p. 148). I also found that he had another edition of the Latin *Fundamenta* in his possession, printed by another publisher than the one mentioned on the titlepage of the edition I used for this thesis (Bierens de Haan, 1878, Note 19). I began to wonder if there could be more editions and began to search for them. Nevertheless, to answer this question was only an extra addition to this thesis and not the main focus, so I was unable to investigate this in great depth. My findings are detailed in chapter 4. Here you can also find his conclusion on whether or not the *Fondamenten* could be the promised *Coss-book* to which Van Ceulen refers in his preface of *Vanden Circkel* (see section 4.1.3).

Furthermore, Bierens de Haan also gives a lot of information about Snellius' own works. On the *Fundamenta* he gives a detailed description of the content and remarks the following:

> However, it were not pure translations he [Snellius] delivered, but rather remakes, or at least very free translations.[4]

In chapter 5 I will discuss the additions that Snellius added to the *Fundamenta* in more detail. The only thing Bierens de Haan further says about the original and its translation is this:

> Both editions of the works of Ludolph van Ceulen by Snellius (...), have had as a result, that the works of Van Ceulen abroad have become more known: but that the judgement of the foreigners on both work (...), was not fair. They wrote to Snellius

[4]Het waren echter geene zuivere vertalingen die hij leverde, maar eerder omwerkingen, althans zeer vrije vertalingen (Bierens de Haan, 1878, p. 149).

actually the method, which nevertheless, beyond all doubt Van Ceulen deserved. (...).[5]

## 2.4 Bachelor thesis

In my bachelor thesis I already discussed some commentaries added by Snellius, but I restricted myself to book V of the *Fondamenten* (Veen, 2011). In this master thesis, I will treat examples from the whole work. I will now give a short summary of the conclusion that I made at the end of my bachelor research.

In my bachelor thesis I concluded that Snellius had a broader mathematical knowledge than Van Ceulen, since he could read Latin and Greek and thus study the classical works. However, the exceptional computational skills of Van Ceulen are absent in Snellius, who simply uses a faster method. Snellius had a preference for the classical works and their traditional formulations of propositions, for short and concise mathematics, unlike Van Ceulen who seemed to have no trouble with cumbersome formulations.

The purpose that Snellius had with the *Fundamenta* remains unclear. He mentions the purpose of increasing Van Ceulen's fame abroad, but his commentaries are not all flattering about Van Ceulen's work (Veen, 2011, pp. 15–17). Snellius did have the goal to let the *Fondamenten* be more in line with the classical works. He also used the book to rise up in his career (Veen, 2011, pp. 7–9).

Much of the content of the *Fondamenten* appears to be meant as practise material for Van Ceulen's students. In many commentaries, Snellius complains about a lack of time. However, sometimes he presents an alternative method that is of a too high level to be used as lesson material. He seems more focused on finding a solution with a short and precise demonstration, than on giving a longer demonstration which serves as an example. In this, he differs from Van Ceulen who just stresses on the method and presents this thus very elaborate (Veen, 2011, p. 24). Nevertheless, this conclusion is only based on the fifth book of the *Fondamenten*. In chapter 5, I will discuss the content of commentaries throughout the work and in chapter 6 I will revisit the differences between the mathematical approaches of Van Ceulen and Snellius.

[5] Deze beide uitgaven van de werken van LUDOLPH VAN CEULEN door SNELLIUS, in verband met den eigen arbeid van SNELLIUS, dien wij straks zullen aanhalen, hebben tengevolge gehad, dat de werken van VAN CEULEN buitenslands meer bekend zijn geworden: maar ook, dat het oordeel dier buitenlanders over beider arbeid, hetgeen misschien uit den vorm der behandeling werd opgemaakt, niet rechtvaardig was. Men schreef aan SNELLIUS eigenlijk de methode toe, die toch buiten eenigen twijfel aan VAN CEULEN toekwam: en bij het naschrijven van deze meening kwam men er toe, om, zonder opzettelijke bestudeering der werken, SNELLIUS voor den fijneren analyticus, VAN CEULEN slechts voor een bloot onvermoeiden rekenaar te houden. (Bierens de Haan, 1878, pp. 150–151)

# Chapter 3

# The purpose of the *Fondamenten* and the *Fundamenta*

In the previous chapter, I have discussed Vlek's remarks on the purpose of the *Fondamenten*. Her conclusion was that it was not intended to be a textbook, in any case that was not Snellius' opinion (Vlek, 2008, p. 58). It might have been used as teaching material at the engineers school at which Van Ceulen was a teacher. Her final remark on the *Fondamenten* was that it is a treatise on the principles of arithmetic and geometry using problems and examples. In this chapter I will investigate the purpose of the original and the translated work more in depth. First I will discuss the intended target audience that Van Ceulen and Snellius might have had (see section 3.1). Secondly, I will discuss the remark made by Vlek whether Van Ceulen had intended his work to be one combined work (see section 3.2). Finally, I will try to answer the question what the purpose of both works might have been (see section 3.4 and 3.5).

## 3.1 The readers of the works

To answer the question as to what the purpose of the *Fondamenten* and the *Fundamenta* might have been, it is helpful to first discover the intended target audience the authors had in mind. This section discusses the target audience of both works by quoting remarks made throughout the works, but also by using the content of the dedicatory letters written by Adriana Simons and Willebrord Snellius.

### 3.1.1 Target audience of the *Fondamenten*

Since the *Fondamenten* was published *post mortem*, it is not easy to retrieve the motivation that Van Ceulen might have had with his work. For example, the preface of his work was not written by himself but by his widow. Therefore it only sheds some light on the purpose that Adriana Simons had with the publication of the *Fondamenten*. Nevertheless, a lot can be said by studying the content of the work, especially the introductions to new chapters and closing remarks to the books. The target audience that Van Ceulen had intended for his work can be retrieved by quoting some of his introductory remarks at the beginning of chapters. For example, in the third chapter of the first book of the *Fondamenten*, in which the theory of proportions is given, Van Ceulen remarks that before he will give the first example, he will first explain what a proportion is:

> Before I will describe the alleged foundations (...) it seems necessary to me first to

> write what proportion is, for the beginner of this art.[1] (Ceulen, 1615a, p.14)

From this quotation it follows that Van Ceulen had in mind a target audience of novices who were possibly not yet familiar with the concept of proportion. Another passage where Van Ceulen explicitly names his audience can be found in the introduction of the fourth chapter of book one:

> The operations of quotients by practice will come easily, as far as you have learned the previous with understanding. I will give you my method as elementary as I am accustomed to do when teaching my disciples. By practise comes the best understanding; not only in this, but in all arts [it] is found that the most diligent exceeds the slow significantly.[2] (Ceulen, 1615a, p.18)

From this passage it follows that Van Ceulen aims his message at an audience that is not (yet) familiar with the concepts of the most basic of operations, here applied to quotients. It is his goal to be as elementary as possible so that everyone can understand. That may be why he begins his *Fondamenten* with the very basics of theories that are needed to understand the last parts of the work.

The target audience that Van Ceulen had in mind for the second book of the *Fondamenten* is very clear in his statement at the end of the book:

> (...) this I wanted to teach for the beginner. I would herewith then close the second part of this book, in which I have laid a solid foundation for the pupils, and not for the scholars[3].

Here Van Ceulen mentions twice that he wrote this part of the book for the beginner. He is very clear that it is not meant for the more higher educated scholars.

In book three Van Ceulen does not make any of such remarks which could shed some light on what audience he intended for this part. It can only be uncovered by looking at the content of the book. This third book of the *Fondamenten* consists merely of geometrical examples on the transformation and division of figures, on line segments and on several geometrical problems such as the quadrature of the circle. Hence, the readers of this book had to be able to understand this theory. This book seems no longer to be addressed to beginners, but rather to average mathematics students.

The level of difficulty of the *Fondamenten* seems to climb when we go from book IV to VI. Vlek remarks that the examples and problems in book V seem to be of too high a level to be intended as exercises (Vlek, 2008, p. 55). Yet, I would argue that Van Ceulen *did* intend these problems to let his readers practise. This follows from a remark which he placed at the end of problem 32 of the fifth book[4]:

[1]Voor ende al eer ick de ghestelde fondamenten (...) dunckt my noodich na mijn simpel verstant, eerst te schrijven wat proportie is, voor den beginner deser const.

[2]De specien int gebroocken door practijck sullen licht vallen, so verre ghy met verstant de voorgaende geleert hebt, ick sal u mijn manier stellen opt eenvoudichste als ik gewoon ben, mijn Discipels te onderwijsen door de oeffeninghe comt het rechte verstant, niet allen hier in, maer in alle consten wert bevonden dat den vlijtighen, den traghen verre te boven gaet.

[3](...) dit heb ick willen bybrengen voor de beginner, etc. Wil hier mede dan het tweede deel deses boecks sluyten, daer inne ick opt eenvoudichste, voor de Leerlinghen, ende niet voor de Gheleerden, een vast fondament gheleyt hebbe. (Ceulen, 1615a, p. 114)

[4]This question was proposed and co-resolved by Snellius (Ceulen, 1615a, p. 232)

> This I wanted to prove extensively, such that the beginners can make and prove some [of the] following examples thenselves[5].

Here, Van Ceulen does not refer to beginners of the art of mathematics, but rather to beginners at this particular level of mathematics. In book six I could find only one remark in which Van Ceulen designates his audience:

> By these the experienced can find many cords,...[6]

This may show that Van Ceulen had a more experienced audience in mind for the last book of the *Fondamenten*. Overall, it can be said that Van Ceulen probably intended his work for an audience from beginners to academics.

### 3.1.2 Target audience of the *Fundamenta*

From the fact that the translation of the *Fondamenten* was in Latin, it can be retrieved that the target audience of the *Fundamenta* was a more learned audience, who were able to read the ancient language. Snellius adds no extra remarks which could indicate whether or not he had in mind the same audience as Van Ceulen. This can only be retrieved from the content of the translation, specifically in his choices to leave out a large part of the first book and later the first two books in his *De Circulo* (see also chapter 4.2.1). On the next page you can find a comparison of the structures of the *Fondamenten* and the *Fundamenta*. What immediately stands out is the fact that Snellius left out the first five chapters of the first book and starts his translation with taking square roots. The left out parts involve the basic theories on arithmetic like adding and subtracting (see also section 1.3.2). Perhaps Snellius deemed this theory of a too low level for his audience that he decided to leave it out. This is in line with his audience having a more learned background than Van Ceulen's audience (see section 3.1).

This becomes more apparent when we look at the letter Snellius wrote to Rosendalius and the added dedicatory letter. I will discuss the content of these letters to try to formulate an answer whether Snellius wrote his translation for another target audience.

#### A letter to a friend

In a letter to Rosendalius, possibly dated in 1615, Snellius tells his friend that he was asked by the widow of van Ceulen and some heirs to translate the *Fondamenten* into Latin "to make it accessible for an international learned audience" (Wreede, 2007, pp. 86–87). Snellius thus had a different target audience in mind then Van Ceulen.

Snellius replies to the widow that, although he feels himself burdened, he accepts the task in order to show that he held his friendship with Van Ceulen in high esteem and to extend Van Ceulen's fame abroad:

> Although I considered myself more burdened than honoured, I have nevertheless accepted to do it, in order to show that the memory of my deceased friend is very dear to me, and in order to extend Van Ceulen's fame abroad, which we have already acknowledged in the Netherlands in these sciences.[7] (Snellius, 1615, fol. $224^r$)

[5] Dit hebbe ick int langhe willen proeven, daer uyt de Beginners connen sommige volgende exempelen selver maecken ende proeven. (Ceulen, 1615a, p. 234)

[6] Door desen can den hervaren veel corden vinden,... (Ceulen, 1615a, p. 263)

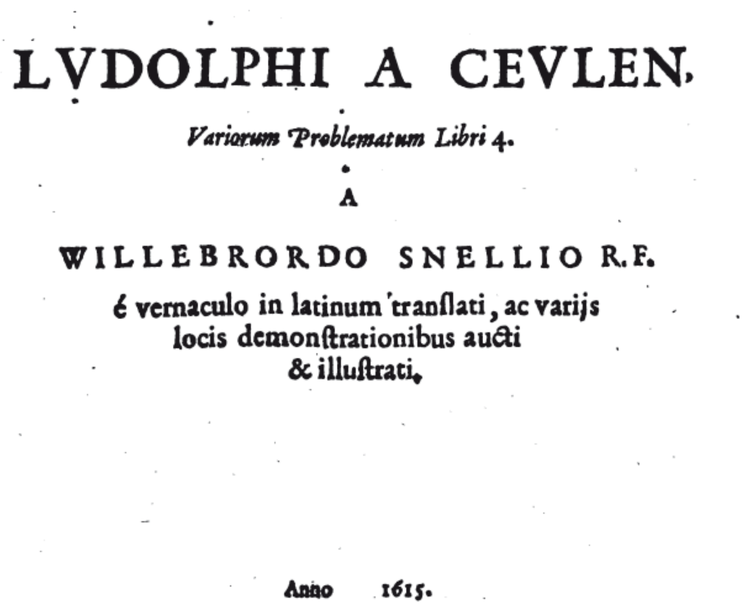
[7] Hic quamvis plus oneris quam honoris mihi imponi cernerem: tamen ut defuncti quondam amici recordationem mihi non ingratam ostenderem, et nomen atque faman, quam in his artibus in Belgio iam assecuti sumus irem amplificatum, facturum recepi. (Translation taken from (Wreede, 2007, p. 87))

| **Comparing the *Fondamenten* and the *Fundamenta*.** | | | | |
|---|---|---|---|---|
| Book | Title in *Fondamenten* | p. | Title in *Fundamenta* | p. |
| Preface | Preface by Adriana Simons | | Preface by Adriana Simons | |
| Book 1 | I Getallen algemeen | 1 | Left out | - |
| | II Van ghebroocken ghetallen | 7 | Left out | - |
| | III Van Proportie | 14 | Left out | - |
| | IV Al waer de voorgaende specien int ghebroocken, corter door practijck gheleerdt werden. | 18 | Left out | - |
| | V Daer in den reghel van drien in gheheel, en ghebroocken ghetallen geleert wert. | 37 | Left out | - |
| | VI Daer in de Extractie der quadraet wortel, ende de specien der Irrationale ghetallen geleert wert. | 45 | Cap. I De analysi lateris quadrati. | 1 |
| | - VIa Additie in ongheschicte getallen. | 47 | Cap. II De Additione irrationalium simplicium. | 4 |
| | - VIb Substractie van Irrationale getallen. | 51 | Cap. III De irrationalium simplicium subductione. | 8 |
| | - VIc Multiplicatie van Irrationale ghetallen. | 52 | Cap. IIII De irrationalium simplicium multiplicatione. | 11 |
| | - VId Divisie van Irrationale getallen. | 54 | Cap. V De irrationlium simplicium divisone. | 12 |
| | VII Al waer geleert werden, de specien van Binomische en Risidusche getallen. | 54 | Cap. VI De binominorum et residuorum, hoc est irrationalum compositorum notatione ac numeratione. | 13 |
| | - VIIa Additie van Binomische en Risidusche getallen. | 54 | Cap. VIa Irrationalium compositorum Additio. | 13 |
| | - VIIb Substractij van Binomische en Risidusche getallen. | 55 | Cap. VIb Irrationalium subductio. | 14 |
| | - VIIc Multiplicatie. | 56 | Cap. VIc Irrationalium compositorum multiplicatio. | 15 |
| | - VIId Divisie in Binomische ghetallen. | 58 | Cap. VId Irrationalium compositorum divisio. | 18 |
| | - VIIe vande Extractij der quadraet wortel uyt Binomische getallen. | 59 | Cap. VII De analysu lateris quadrati in irrationalibus compositis. | 19 |
| | VIII Al waer geleert werden, de specien van universale getallen. | 60 | Cap. VIII De numeratione conpositorum irrationalium universlium. | 21 |
| Book 2 | Diffinitien | 69 | Definitiones | 33 |
| | Volghen eenighe ghemeene bekentenissen, die wel te mercken zijn. | 72 | Axiomata, sive communes Notiones. | 36 |
| | Volghen eenige propositien ofte voorstellen Euclides tot dit werk dienende. | 72 | Propositiones quaedam ex euclide | 37 |
| | | | Preface by Willebrord Snellius. | 83 |
| Book 3 | Daer inne wt voorgaende gronde geleert wert de Figuere op menigerhande manieren te veranderen. | 115 | Figurarum transmutatione et sectione | 85 |
| | Van deelinge der Figuren | 119 | De Figurarum Sectione | 92 |
| | De Spetien in linien,(...) | 132 | [No subheadings, but comments in advance.] | 106 |
| | Volghen eenige Geometrische vragen,(...) | 139 | [No subheadings, but comments in advance.] | 115 |
| | Byvouch des derden Deels | 156 | [Added to problem in Book 4 as *Zetema* 1 t/m 4] | 137 |
| Book 4 | Daer in veel Constighe *Geometrische* exempelen ghestelt ende ghedolveert zijn. | 168 | $\Delta\varepsilon\delta\mu\varepsilon\nu\omega\nu$ Geometricorum per numeros solutione. [*Zetema* 5 t/m 57] | 137 |
| Book 5 | Van constige trecken, bewesen eensdeeld *Geometrici*, ende door getallen,(...) | 203 | Problematum miscellanearum liber quartus, (...) | 185 |
| Book 6 | Daer in eerst ghehandelt werdt, van de ghelijcksydighe figueren, in ende om de Circkels beschreven,(...) | 247 | de Figuris ordinatis circulo adscriptis,(...) | 241 |
| | [The last three problems of book VI] | 269 | Appendicula de circulo data ratione seconda. | 264 |

Table 3.1: Comparison of the structure of the *Fondamenten* and the *Fundamenta*.

### His own dedicatory letter

Although Snellius put a lot of effort in the translation of the *Fondamenten*, Van Ceulen's widow did not grant him the privilege to place a dedicatory letter at the beginning of the work. Instead, he was only given permission to place one in the middle of the work, which he did at the beginning of book three, right before the 'best and richest part of the whole work' destined for Rosendalius (Wreede, 2007, p. 87). Snellius added a title page before his dedicatory letter which reads 'Variorum Problematum Libri 4' (Four books of various problems) (see figure 3.1). Hereby, Snellius made a strong division between the first two books with theory and the last four books containing pratical problems[8]


LVDOLPHI A CEVLEN.

*Variorum Problematum Libri 4.*

A

WILLEBRORDO SNELLIO R.F.

é vernaculo in latinum translati, ac varijs
locis demonstrationibus aucti
& illustrati.

Anno 1615.

Figure 3.1: The titlepage in the *Fundamenta* placed before the dedicatory letter of Snellius to Rosendalius.

De Wreede summarises the content of this letter as follows:

> To summarize, the dedicatory letter to the *Fundamenta* shows that Snellius was a competent humanist, who mastered rhetoric well enough to be able to write a showpiece, starting with some common places, then addressing more controversial issues and showing the sharpness of his wit, and finally mitigating his tone again to show his reasonableness.[9]

It discusses the usefulness of mathematics in many different fields, including law since the dedicatee was a lawyer. Snellius writes a lot on the use of numbers in geometry and in the end discusses the content of Book X of Euclid's Elements (more on the connection between these two subjects can be found in Wreede (2007, pp. 192–201)). Since De Wreede already wrote an extensive exposition on Snellius' dedicatory letter, I will not repeat her work in this thesis. For more details on the content of the letter I would refer you to paragraph 5.4 of De Wreede's

[8]Another remarkable thing is that the translation of the title of the fifth book reads 'Problematum miscellaneaorum liber quartus,...' (Book four of mixed problems,...). But this seems to be an error. Maybe this print was meant to go in *De Circulo* where the second book is left out.

[9](Wreede, 2007, p. 201).

work[10].

The language used in the letter is that of a humanist scholar who 'elevates Van Ceulen's work'[11]. De Wreede suggests that Snellius 'may have used his own Latin translation of Van Ceulen's Fondamenten, a textbook for future engineers, for his university students' (Wreede, 2007, p. 113). The dedicatory letter is mainly used to indicate the value of the *Fundamenta* for an 'international learned audience'[12].

## 3.2 Intended to be one combined work?

A few years after Van Ceulen's death, Adriana Simons and some other heirs had decided to publish some of Van Ceulen's manuscripts. De Wreede suggests that some of these manuscripts were bundled together and published as the *Arithmetische en Geometrische Fondamenten* (Wreede, 2007, p. 86). Vlek questions whether the published *Fondamenten* were intended to be one combined work (see section 2.1) and Katscher seems to be convinced that it was not intended as a coherent textbook, but a composition of arithmetical, geometrical, trigonometrical and algebraic problems (Katscher, 1979, p. 119). He concludes this mainly by looking at the problems in the third book of the *Fondamenten*.

Since the target audiences of the different books of the *Fondamenten* vary, the question arises whether the work was intended to be one combined work. Since Van Ceulen was not alive when the work was being assembled, he could not clarify how the final work should be assembled.

Vlek concludes in the end of her master thesis that the first four parts of the *Fondamenten* seem to form a coherent set, but that books five and six deviate in subjects and level of difficulty (Vlek, 2008, p. 57). She suggests that these last two books might have been an amalgamation of manuscripts that were left behind. I found that the books in themselves do show proof of an intended order and coherency that contradicts this statement. I will subsequently show this in this section. Vlek also remarks that there are many references between books three to six of the *Fondamenten* that indicate that there is indeed some coherency between the books. After studying the work intensively, I have come to the conclusion that the *Fondamenten* was indeed intended to be one work. I base this conclusion mainly on the many references made throughout the work. Of these I will give some more details to uphold my statement.

To begin with, there are many references in book three to propositions in the second book or directly to the corresponding proposition in Euclid's *Elements*. This proves that book three was written with book two still in mind and the *Elements* near at hand.

Besides, the fourth book contains some evidence that it was not simply a sequence of separate examples of, among others, construction problems. There are many references made throughout the book to other propositions of the same book. For example, in problem 53 Van Ceulen refers to the thirteenth example[13]. The same can be said of book five, which contains numerous references to previous examples, such as the reference to problem 29 in problem 36[14]. I also

[10] More details on the content of the letter can be found in (Wreede, 2007, pp. 188-205).

[11] (Wreede, 2007, p. 191)

[12] (Wreede, 2007, p. 189)

[13] ...souckt de linien van noode zijnde, op de maniere ghedaen int 13ste exempel des vierden deels van desen,... (Ceulen, 1615a, p. 198).

[14] ..., door de maniere hier voor gheleert by den 29ste exempel,... (Ceulen, 1615a, p. 238).

found a reference in this fourth book to the third[15]. Furthermore, Van Ceulen writes after he answered only the first half of a question proposed in a problem in the fourth book:

> Hereafter I will answer the second question, which is on the size of triangle $BFP$[16].

This promised answer does not occur in the fourth, but in the fifth book of the *Fondamenten.* This might imply that Van Ceulen still remembered his promise while working on the fifth book and that he thus intended the fifth book to follow the fourth. Also in book five Van Ceulen makes a reference to the sixteenth example of the third book (Ceulen, 1615a, p. 209), to the 36th proposition of the second book (Ceulen, 1615a, p. 222) and to the eighth chapter of the first book (Ceulen, 1615a, p. 223). This refutes the hypothesis that the books were written separately and would be bundled together later.

In her master thesis, Charlotte Vlek states that book six of the *Fondamenten* stands out from the others, since it almost entirely contains calculations on the circumscribed figures in and around circles, which Van Ceulen treated comprehensively in *Vanden Circkel*[17]. Subsequently, she writes that this last book is the only place where calculations with an unknown ('Reghel Coss') occur. This, however, is not true, since the first calculation with coss is made in book four, proposition 26 (Ceulen, 1615a, p. 182), and the first use of the cossic symbols can be found in the first book (Ceulen, 1615a, p. 16). Furthermore, also in the fifth book Van Ceulen makes calculations using coss, for example in problems 39, 40, 42, 46 and 47 (Ceulen, 1615a, pp. 240–246). Five times, compared to the three times Van Ceulen uses coss in the sixth book (for problems 1, 10 and 13). Vlek's argument that book six would stand alone from the other books, based on the occurrence of calculations with coss, is therefore invalidated.

Finally, as early as the first problem of the sixth book Van Ceulen refers all the way back to the first book, chapter eight to be precise (Ceulen, 1615a, p. 249). Also, the first sentence in this last book reads as follows:

> In the second part it was learned how one constructs figures in and around circles. Here I will teach you how one shall find the sides and area of the in- and circumscribed figures[18].

Here he specifically refers back to the theory given in the second book. In the third problem he also refers to the second book, namely in proposition 58 (Ceulen, 1615a, p. 253). Hence Van Ceulen intended to place book two before this last, sixth book, pointing out that the books were not to be read apart from one another.

## 3.3 A selection from Euclid's *Elements*

The second book is a selection of propositions taken from the *Elements* of Euclid. The question remains why Van Ceulen added book two to his work, since most of his references made

[15]Dit condt ghy licht prouven: dese ende voorgaende mogen ghesolveertt werden, als int derde deel gheleert wert. (Ceulen, 1615a, p. 182).

[16]Hier naer sal ick de tweede vraghe beantwoorden, welck is naer de groote des Tryangles BFP. (Ceulen, 1615a, p. 192)

[17]Deel 6 springt eruit, doordat die bijna alleen maar over berekeningen aan in- en omgeschreven figuren in cirkels gaat, een onderwerp dan (sic) Van Ceulen ook in *Vanden Circkel* uitvoerig behandelt. (Vlek, 2008, pp. 9,10)

[18]Int tweede deel is gheleert, hoemen eenghe figuren in ende om de circkels beschrijven sal. Hier sal ick u leeren hoe men der in ofte omgeschreven gelijcksydige figuren, zijden, ende grootte vinden sal, ... (Ceulen, 1615a, p. 247).

throughout the work are directly to the corresponding propositions from the *Elements* of Euclid. But since we know that Van Ceulen started working on his *Fondamenten* around 1599, the year he mentioned that he was working on the seventh problem of book V[19], it becomes more clear. Namely, before 1606 there was no Dutch translation of the *Elements*[20], hence Van Ceulen made his own copy based on the German translation by Wilhelm Holtzman (also known as Xylander) (Vlek, 2008, p. 21). The second book of the *Fondamenten* might have been one of the first Dutch translations, though it was published after 1606.

Looking at the references made to the *Elements* throughout the work, there is reason for one last remark. Van Ceulen refers twice to the second proposition of the twelfth book of the Elements. The German translation of the *Elements* only consists of the first six books. Also, in the *Fondamenten*, Van Ceulen refers to a comment from the *Elements* made by Christoph Clavius (1538-1612) (see also section 6.2.3). Clavius wrote a Latin edition of the *Elements* containing all fifteen books[21] published in 1574. It contained a compilation of several commentaries on the *Elements* by different authors, including by Commandinus[22] and himself. Van Ceulen must have seen this translation where he found the propositions he needed from the twelfth book of Euclid. Van Ceulen thus possessed a German translation of Euclid's *Elements* by Xylander (Katscher, 1979, p.105) and a Latin translation by Clavius.

The last four propositions of the second book are not in a logical order (see also appendix E in which I have added an overview of the proposition from Euclid's *Elements* corresponding to book two). Almost all propositions are in the same order as Euclid's propositions. But, after Van Ceulen gives some from the sixth book of Euclid, he adds some from the first and second. The first[23] location where these propositions are used is in problems seven and eight of the addendum of book three, which Snellius placed at the beginning of the fourth book (Ceulen, 1615a, p. 160, 162). Later, in the end of the fourth and throughout the fifth book, he repeatedly uses the last two propositions from the second book[24]. Hence, book two may have been extended by adding four more propositions, numbered 80 till 84, which Van Ceulen needed to prove geometrical problems given in book three, four and five.

Another thing that stands out when looking at the order in which Van Ceulen poses the propositions taken from the *Elements*, is the fact that propositions 26 and 27 are taken from the sixth book of Euclid and propositions 28 till 31 are on the quadrature of the circle (not from the *Elements*), while all the previous ones were from the first and the following from the second book of the *Elements*. It is not true that these first two propositions from the *Elements* are used in the first problems of book three of the *Fondamenten*. I cannot find any explanation as to why Van Ceulen places these propositions in this order and adds some extras which are not to be found in the *Elements*.[25]

[19]...tot in dit tegenwoordighe jaer 1599... (Ceulen, 1615a, p. 212).

[20]The first Dutch translation of the *Elements* came out in 1606 by Jan Pieterszoon Dou (Heath, 1956a, p. 108).

[21]The last two books are not originally written by Euclid, but added as apocryphal books. These books were possibly written by Hypsicles and Isidore of Miletus (fl. ca. A.D. 532) respectively. (Boyer, 1991, pp. 118-119)

[22]More about Commandinus' comment in Clavius' translation of the *Elements* can be found in section 6.2.3.

[23]Van Ceulen uses proposition 83 already in book three (Ceulen, 1615a, p. 145). However, this is the only reference to one of these propositions throughout the whole third book. In the addendum to book three he uses propositions 80 and 82, which is simultaneously the only place where he ever uses these.

[24]Proposition 83 is used in problems 30, 48, 50 and 51 of book IV, and once in book five, proposition 20. Proposition 84, the last of book two, is used in problems 17, 43, 49 and 57, which is the last of book four. Proposition 84 is also used once in book three and three times in book five. (See also appendix E.)

[25]I also found a reference to the sixteenth proposition of the sixth book of Euclid in the fifth chapter of the first book of the *Fondamenten* (Ceulen, 1615a, p. 37). This proposition is not included in the second book.

## 3.4 The purpose of the *Fondamenten*

In order to retrieve the purpose of the *Fondamenten* I will first discuss the purpose of the original work in more detail (section 3.4). After that, I will give a view on Snellius' personal purpose with its translation (section 3.5).

Some of the subjects found in the *Fondamenten* were also being taught at the engineering school of Leiden (Wreede, 2007, p. 29), the *Duytsche Mathematique*, which was founded by Maurits van Nassau and where Van Ceulen was one of the first lecturers (Katscher, 1979, p.103) (see chapter 1). At this school students were exclusively taught in Dutch. This served Van Ceulen well, because he was not able to read Latin or Greek, he probably would not have received this position at another school. At the *Duytsche Mathematique* Van Ceulen taught arithmetic, field measurements and fortification. The first two subjects can also be found in the *Fondamenten*[26]. Whether the *Fondamenten* was meant to be used as a textbook at the engineering school is unclear, but it seems likely.

One of the purposes of the *Fondamenten* seems to be to teach different methods of calculations. All the given theory is immediately followed by numerous worked out examples where the theory is put into practise. For example, after a short instruction on how to work with the 'reghel van drien' (rule of three), Van Ceulen soon gives examples to practise (Ceulen, 1615a, pp. 37–44). He explicitly says that he gives the examples for the purpose of letting the 'lover' practice and learn by that exercise[27]. Further in the book he says again that he gives examples to 'let you practice'[28]. At the beginning of the appendix to book three (Snellius moved it to the beginning of book four) Van Ceulen writes the following:

> Now follow the proofs of several previous propositions by numbers, stated for the lover's wishes to let them practice, so that he lustily and certainly learns to work with numbers, and does not hesitate to make the examples of the next section.[29]

De Wreede also states that 'some of his (Van Ceulen) teaching material found a place in the *Fondamenten*' (Wreede, 2007, p. 29). Van Ceulen prepares his readers for the problems that follow and encourages them to work out the examples. It may be concluded that Van Ceulen was focused on training his target audience, not only giving theory. I will further discuss this statement in section 5.4.1.

## 3.5 Snellius' purpose with the *Fundamenta*

De Wreede suggests that perhaps Adrianus Romanus, who was a correspondent with Van Ceulen, had earlier proposed Snellius to translate Van Ceulen's works into Latin to make them more accessible for a learned reader (Wreede, 2007, p. 47) (see chapter 1). Snellius' translation

[26] Book one deals with the basics of arithmetic and book four starts with several problems on field measurements. Fields are mentioned for example in problems 1 and 14 (Ceulen, 1615a, p. 168 and p. 175).

[27] (...) ick hebbe dese ghedichtet, tot dien eynde day hem den Liefhebber oeffene, ende mede de ghebroockens leere in haer cleenste proportie brengen. (Ceulen, 1615a, p. 43)

[28] (...) om u te oeffenen, hebbe ick de volghende exempels van communicanten ghestelt. (Ceulen, 1615a, p .51)

[29] Volgt nu de bewijsinge etlicker voorgaender propositien door ghetallen, om des liefhebbers wille gestelt om hun daer na te oeffenen, op dat hij lustich ende seker met de ghetallen leert wercken, ende de exempels des volghenden deels niet schroomt te maecken. (Ceulen, 1615a, p. 156)

of the *Fondamenten* may be an answer to this request. In the letter to Rosendalius (see section 3.1.2), Snellius reveals his motives for helping with the translation. He wanted to rise up in his career, to become a regular professor and receive the same amount of salary his father had had before him. Snellius specifically asked Rosendalius to talk to the curators and plead for his case (Wreede, 2007, p. 87).

The purpose which Snellius had with the translation of the *Fondamenten* can also partly be derived from the dedicatory letter which he placed in the middle of the work. Snellius mainly used the letter as an advertisement for himself and as an instrument to receive promotion in the academic hierarchy[30], which could well have been Snellius' purpose for translating the whole work of the *Fondamenten*. Snellius was a junior academic at the time. If Van Ceulen had still been alive, Van Ceulen would have been a senior without any academic background, but nevertheless promoted to 'professor' at the *Duytsche Mathematique*. A title that Snellius also wanted to obtain.

Snellius also used the translation of the *Fondamenten* to publish his own findings. For example, his discovery of a theorem expressing the area of a cyclic quadrilateral in terms of its sides can be found in a long commentary on Van Ceulen's construction of quadrilaterals in the fourth book of the *Fundamenta* (Wreede, 2007, p. 279; Ceulen, 1615b, p. 188-190). He promises to give a better construction with figures in a next edition. One of his commentaries did indeed receive an elaboration in *De Circulo*, about which De Wreede wrote an elaborate discussion (Wreede, 2007, pp. 241–246). However, many promises made by Snellius to give better constructions have never been fulfilled. It can thus be concluded that Snellius used the *Fundamenta* for his own purpose of increasing his salary, besides the more noble case of spreading the work of his dear friend Van Ceulen.

[30](Wreede, 2007, pp. 200–201)

# Chapter 4

# Many different publications of the same work

As discussed in section 2.3, there are many different editions of both the *Fondamenten* and the *Fundamenta*. In this chapter, I will first discuss the editions of the *Fondamenten* (section 4.1). In the last section I will discuss the different editions of the *Fundamenta* and also give a description of the Latin translation of *De Circulo*, which seems to be a reprint of the *Fundamenta* (section 4.2).

## 4.1 Different editions of the *Fondamenten*

In this first section I will try to uncover the truth about the true publication year of the *Fondamenten* (see section 4.1.1). Then I will discuss the differences between the editions that I found of the *Fondamenten* (section 4.1.2). In this section I also discuss whether or not the *Fondamenten* could be the promised *Coss-book* to which Van Ceulen refers in his preface in *Vanden Circkel*.

### 4.1.1 Publication year

There is a great deal of uncertainty about the exact publication year of the *Fondamenten* and the *Fundamenta*. In my research for this thesis, I found many sources which presented different publication years. In this section I will describe the confusion on the exact date and try to determine, once and for all, the correct year.

According to Vorsterman van Ooijen (1868, p.9) the *Fondamenten* was published in 1595. Likewise, two sources that give 1595 as publication date are Fernand (1800, p. 105) and Wissenschaften (1871, p. 9), of which the latter also gives 1597 as the publication date for *Vanden Circkel* (this should be 1596). Bierens de Haan corrects Vorsterman van Ooijen by pointing out that in the preface of the *Fondamenten*, written by Van Ceulen's widow, the *Fondamenten* was referred to as the one promised in *Vanden Circkel*. However, *Vanden Circkel* was published in 1596, a year later than the presumed publication date for the *Fondamenten*[1]. This can also be

[1]Hebbe derhalven oock dese Aritmetische ende Geometrische Fondamenten van Mr. Ludoff (sic) van Colen mijn man sal: ged: de welcke al over lange jaren van den Autheur selve (in sijn boeck gheschreven vanden Circkel) zijn belooft geweest, doch van wegen zijn veelvoudige, soo publijcke als particuliere occupatien, tot noch toe inghehouden, int licht laten comen, ten dienste der nakomelinghen. Hieruit volgt, mijns inziens, dat de heer Vorsterman van Oyen zich moet vergist hebben, toen hij den datum der eerste uitgave van dit aangehaalde werk, de Arithmetische en Geometrische Fondamenten op 1595 vaststelt; zie diens Notice sur Ludolphe van Colen. (Bierens de Haan, 1878, pp. 144-145)

concluded by remarking that in the *Fondamenten* Van Ceulen gives several dates after 1595 for when he was working on a specific problem[2].

According to the Navorscher[3], the Dutch edition of the *Fondamenten* was published in 1616, a year after the Latin edition by Willebrord Snellius.[4] This seems to be incorrect since the titlepage states 1615. Another, more trustworthy source that agrees with the publication date of 1616 is the *Biographisch woordenboek der Nederlanden* (Biografic dictionary of the Netherlands)[5]. This source specifically says that the translation was published a year before the original Dutch work came out. With this finding I began to question whether there were more publications of the *Fondamenten* and if so, whether they might be different from the original. Since there are different editions of the *Fondamenten*, as will be discussed in section 4.1, it could be that a later published edition was meant, although, all editions I found were printed in 1615. I have not been able to discover which source quotes which and thus decide whether someone made a mistake which was then simply copied. The earliest source I found that gives 1616 as publication date for the *Fondamenten* is from 1728.[6]

It may be concluded that there are no editions from 1595, but there might be an edition from 1616. However, I could not find an edition with a publication date other than 1615.

### 4.1.2 Different dedications, different prefaces

The edition of the *Fondamenten* that I used for this thesis is dedicated to Count Maurits van Nassau. Bierens de Haan remarks in footnotes that he has two different editions of the *Fondamenten*, which only differ in the dedicatory letter. He described that his edition is dedicated to Count Ernest van Nassau, but that he also possesses an edition that is dedicated to Count Maurits van Nassau[7]. Furthermore, he writes to have seen a third edition in a library in Amsterdam.

Bierens de Haan continues that the prefaces following the dedications, all written by Van Ceulens's widow Adriana Simons, are completely different. The preface following the dedication

[2]See for example problem seven of the fifth book where he tells us that he was working on finding the solution in 1599. (Ceulen, 1615a, p. 212)

[3]The *Navorscher* was a Dutch magazine that helped readers to find answers to questions especially about history, genealogy and linguistics. One of the readers placed a question whether there was a list of Dutch works that where translated into another language and if such a list did not exist, he asked if others could help him form one (Loman, 1856, p. 360). In later editions of the *Navorscher* readers responded with giving titles of translated Dutch works, including the works *Fondamenten* and *Vanden Circkel* by Van Ceulen (see next footnote).

[4]Ludolf van Keulens *Arithmetische en Geometrische fondamenten* (Leiden, 1616), zijn door W. Snellius in't Latijn overgezet, o.d.t. *Fundamenta arithmetica et geometrica cum eorum usu, autore* Lud. A Ceulen*, ab Hildersheim, e vernaculo in latinum translata a* Willebrordo Snellio, R. F., Ludg. Bat., 1615. (Muller (ed.), 1860, p.72)

[5]*De Arithmetische en Geometrische Fondamenten* van Mr Ludolf van Keulen Leiden 1616 van hetwelk reeds een jaar vroeger eene Latijnsche vertaling van Snellius het licht zag onder den titel Fundamenta arithmetica et geometrica Leyd 1615 (Aa and Harderwijk, 1858, p. 93).

[6]I found four different sources giving 1616 as the publication date of the *Fondamenten*: (Visscher, 1852, p. 263), (Nienhuis, 1833, p. 12), (Lucius, 1728, p. 424), (Poggendorff, 1863, p. 213) and (Leibniz et al., 1916, p. 734). The first of these sources writes the following: "*Arithmetische en Geometrische Fondamenten van Mr. Ludolf can Keulen. Leiden, 1616.* De latijnsche vertaling door Snellius verscheen een jaar vroeger.", thus it specifically mentions that Snellius' translation appeared a year earlier.

[7]Wat het tweede werk betreft, dat van Noot (17): hierbij is eene andere, merkwaardige bijzonderheid te vermelden. Ik bezit toch daarvan een exemplaar, waarin de opdracht aan GRAEF ERNEST VAN NASSAV en de Edele Moghende, Hoochwijze, ghebiedende Heeren de Staten der Provintie van Gelderlandt, is weggelaten. Maar daarin wordt zij vervangen, door een opdracht (in verso van den titel) AEN DEN Hooch-gebooren Vorst ende Heere MAVRITS, mitsgaders de Edele, Hoochmogende, Wijze, Voorzienighe Heeren de Staten van Hollandt ende West-Vrieslandt. (Bierens de Haan, 1878, p. 148).

to Maurits contains many references to great achievements such as the building of the pyramids and the Tower of Babel. There are a lot of examples taken from the Bible, explicitly mentioning the chapter and book as if Simons wanted to prove her knowledge of the Bible. She states that mathematics is the most principal under the sciences[8]. After a lot of flatteries aimed at count Maurits, Simons writes that she could not bear to not bring out her husbands work for the ones who love the art of mathematics[9]. She ends her letter of dedication with the plea for money which she intends to use to publish more of Van Ceulen's work[10].

One edition dedicated to count Maurits is now in the possession of the *Scheepsvaartmuseum* (maritime museum) of Amsterdam. There are some remarkable facts about this particular edition from the museum. The work of Van Ceulen is bundled together with a work on navigation, titled *Stuermans Schoole* (Steersman's school) by Simon Pietersz., teacher of navigation at Medenblick (Pietersz., 1659). There are no dates to be found as to when this combined work was published. The publication date that is mentioned, namely 1658, is probably from the original first print, since the portrait drawing on the first page is dated 1659, one year later. It can thus be said that this collection was published after 1659. Since the *Fondamenten* was combined with another work it must have been requested specifically, which tells us that the *Fondamenten* was still known forty years after its publication.

In the library of the University of Amsterdam, I eventually found the third edition of the *Fondamenten* which was dedicated to the 'De Heeren Superintendenten' (The Admiralties; see figure 4.1). This copy contains a cachet which ascribes the work as being the property of the Mathematical Society of "Een onvermoeide arbeid komt alles te boven" (an untiring labour will overcome all). Bierens de Haan referred to this edition as being in the possession of this particular society, which proves that this is the very same edition he had found[11].


Aende Hoochweerdige Voorſienige, wijſe Heeren,
DE HEEREN SVPERINTEN-
denten ende Raden der Admiraliteyten van Hollandt
ende Weſt-Vrieſlandt.

Figure 4.1: The dedication in a third edition of the *Fondamenten*.

The preface following this dedication praises geometry by pointing out its use in building fortifications in times of war[12]. It also refers to a promise made by Van Ceulen in the preface

[8] *Waer onder datmen bekennen moet dat de wetenschappen der Mathematike onder andere mede de principalste sijn, want sy de andere niet alleen in subtiliteyt te boven gaen, maer oock nootwendich en profitabel sijn.* (Ceulen, 1615a)

[9] Soo en hebbe ick niet kunnen nalaten om de overgroote vlyt en arbeyt de welcke mijn Man saliger Meester Ludolh van Ceulen in deze heerlijcke wetenschap sijn leefdaghen aengewent en overgebracht heeft de selve nae sijn overlyden aen dach brengende den kunstlievenden te communiceren. (Ceulen, 1615a, Preface)

[10] Verhopende soo dit van Sijne Princelijcke Excellientie met goed oogen aengesien, ende van hare Mogentheden als aengenaem ontfangen wert, naer desen noch meer van sijnen arbeyt tot het gemeene nutt, en der subtijle geesten lust aen den dach te brengen. (Ceulen, 1615a, Preface)

[11] This edition can be found by the reference number OM 63-1164.

[12] want is het niet Geometria gheweest, door de welcke veele Steden ende Fortressen byna onoverwinnelijck sijn gemaeckt? (This quote can only be found in the preface of the edition of the *Fondamenten* dedicated to the

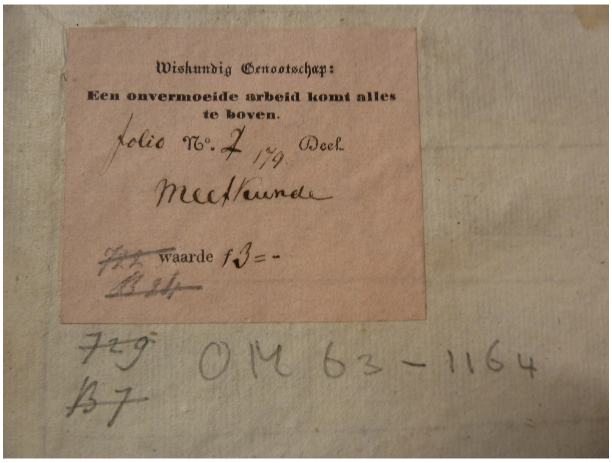



Figure 4.2: The cachet.

of his *Vanden Circkel*[13], that he would publish a work in which he would present problems involving the rule of Coss and many ingenious examples (see section 4.1.3).

I have searched for the edition dedicated to Count Ernest of Nassau in many libraries in the Netherlands, but could not find it. However, I did find two altogether different editions online. The fourth edition of the *Fondamenten* is dedicated to the States of Zeeland and is mainly focussed on philosophy. The fifth edition is dedicated to Count Willem of Nassau. The preface in this edition also contains the reference to the promise Van Ceulen made[14]. It seems that Adriana Simons has dedicated the *Fondamenten* to three men of Nassau. Willem was married to Maurits' sister and Ernest served under Maurits during the Eighty Years War in Holland.

Bierens de Haan wrongly remarks that the preface in the addition dedicated to Count Maurits was the most beneficial for Adriana Simons, since she received 72 guilders for it[15]. This amount of money was donated for the Latin edition, as is notated in the *Resoltutien der Algemeene Staten* of June 29th, 1615:

> On the request of Adriana Simons daughter, widow of the deceased Mr. Ludolff van Colen, during his life resident in Leiden, and has been there professor of mathematical sciences, is the suppliant dedicated of the sum of 72 guilders, because she dedicated and presented to the States General a certain book on geometry, titled: fundamenta arithmetica et geometrica cum eorundem usu etc.to the States General.[16]

Admiralties.)

[13]Only in the 1596 edition.

[14](het welcke hy al by zijn leven inde praefatie van zijn boeck geschreven vanden Cirkel heeft belooft, doch van weghen zijne groote ende meenighe occupatien, soo publijcke als particuliere tot noch toe ingehouden) (Ceulen, 1615a, Preface to Willem of Nassau).

[15]deze tweede opdracht is voor ADRIANA SYMONS van het meeste nut geweest: want het was zeker dientengevolge, dat zij haar wensch naar een subsidie bevredigd zag: daarop volgde toch denkelijk de resolutie der Algemeene staten van 29 Juni 1615, waarbij haar, op haar verzoek, eene som van 72 gulden werd toegekend. (Bierens de Haan, 1878, footnote no. 18)

[16]Op te requeste van Adriana Simons dochter, weduwe van wylen Mr. Ludolff van Colen, in sijn leven woonachtig tot Leyden, ende aldaer geweest zynde professor van de Mathematique scientie, is der suppliante toegeleyt de somma van 72 gulden, voordat sy haere Ho. Mo. heeft gedediceert ende gepresenteert seecker bouck in de geometrie, geintituleert: fundamenta arithmetica et geometrica cum eorundem usu etc. (Bierens de Haan, 1878, footnote no. 18)

This mistake of Bierens de Haan was also noted by De Wreede. She also finds it slightly unfair that Simons received the money instead of Snellius who put the most effort in the publication of the work.

In the appendix I have added the exact texts of all dedications and three of the four prefaces (see appendix A). All the editions I found were printed *By Ioost van Colster, ende Iacob Marcus, Anno 1615*. On the other hand, the editions of the *Fundamenta* seem to have been printed by at least four different publishers (see section 4.2).

### 4.1.3 The promised Coss-book

In the preface to his *Vanden Circkel*, published in 1596, van Ceulen writes a brief to the 'art-loving readers' in which he says:

> Insofar I will perceive gratitude, there will after this follow a greater work wherein among others the most-ingenious Rule Coss will be treated with many artful examples, which were send to me by many Masters of this art, with their solutions (...).[17]

Bierens de Haan at first thought that Van Ceulen meant by this other work his *Fondamenten*[18]. Indeed, in this work many examples of problems sent by different mathematicians are written which Van Ceulen solves by using algebra (coss). This assumption may be even more true when we look at the continuation of the quote presented above:

> (...) and that what is made and found thereon. With still the most necessary of the previous mentioned Rule Coss, which I found in Arnhem at the court of Gelderland anno 1589, by the help of God, by origin an artistic question, send to me by the highly educated doctor Johannes Wilhelmus Velsius, mathematician and physician in Leeuwarden. [19]

The question to which Van Ceulen refers here can indeed be found in the *Fondamenten*, namely in the 26th problem in the fifth book. Could it be that the *Fondamenten* was indeed the *Coss-book* that Van Ceulen had promised to write? Bierens de Haan suggests that Van Ceulen was unable to implement his intentions, since his widow, Adriana Simons, writes the following in the preface of the edition of the *Fondamenten* dedicated to Count Ernest:

> Therefore I shall publish to serve the descendants, this *Arithmetische and Geometrische Fondamenten* by Ludolph van Ceulen my husband may he rest in peace, which has been promised for many years by the author himself (in his work *Vanden*

[17] So verre ick danckbaerheyt mercke sal haest naer desen volghen een grooter werck daer inne onder andere ghehandelt sal werden van den alder-constighsten Regel Cos met veel konstighe Exempels my van veel Meesters deser konst te maken ghesonden met de beantwoordingh ende het gene daer op ghemaeckt ende ghevonden is Met noch het noodtwendighste der voornoemden Regel Cos welck ick tot Aernhem op 't Hoff van Gelder-landt Anno 1589 gevonden hebbe door de hulpe van Godt (Bierens de Haan, 1878, p. 323). Original quote can be found in the preface of (Ceulen, 1596).

[18] Tot nog toe meende ik, dat dit werk van VAN CEULEN was zijne Arithmetische en Geometrische Fondamenten, die het eerst [zie Aanteekening 17 bij No. VIII der Bouwstoffen] door zijne weduwe ADRIANA SYMONS in het licht is gegeven. Immers in dat werk ook verschillende personen genoemd, die hem vraagstukken ter oplossing hadden toegezonden, (...). (Bierens de Haan, 1878, p. 323)

[19] (...) ende het gene daer op ghemaeckt ende gevonden is. Met noch het noodt-wendighste der voornoemden Regel Cos, welck ick tot Aernhem op't Hoff van Gelder-landt Anno 1589 gevonden hebben, door de helpe van Godt, ter oorsake eener konstigher Vraghe, aen mijn gesonden door den hoogh geleerden D. Iohannes Wilhelmus Velsius, Mathematicus ende Medicus tot Leeuwaerden. (Ceulen, 1596, Preface).

*Circkel*), yet due to his many occupations, both publicly and privately, has been retained up to now.[20]

And in the edition dedicated to Count Willem of Nassau, Simons adds a similar remark:

> Therefore I have revealed this *Geometrische en Arithmetische fondamenten*, which already for a long time has been promised by the author Mr. Ludolf van Collen, my husband may he rest in peace, like he wrote in the preface of his work *Vanden Circkel*, but which was withheld up to now because of his large and many activities, because of his profession as well as other private foreclosures, which prevented him in his life[21].

However, Bierens de Haan mentioned a problem sent by Adrianus Romanus to Van Ceulen for which the latter promised to publish a solution in his 'great work'[22]. This 'great work' had to be the same as the one mentioned in the preface of *Vanden Circkel*. However, the problem involved can not be found in the *Fondamenten*.[23] After the last problem in book VI of the *Fondamenten*, Van Ceulen refers to this problem from Romanus.

> I could place here still different chords with other pieces, but it will come in more handy in my *Coss-book*, where I will place the findings of the highly learned Adrianus Romanus, whereby one can come to the equations of the sides of a variety of equilateral figures inscribed in the circle, and also the way through which means I have come to the value of 1ꝶ[24]. (Ceulen, 1615a, p. 269)

Remarkably, Snellius' translation of this passage leaves out the name of Adrianus Romanus[25]. Snellius translates the word *Cos-bouck* (sic) as 'Algebra'. From this passage, Bierens de Haan is able to conclude justly that the *Fondamenta* was not the promised *Coss-book*.[26]

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[20] Hebbe derhalven oock dese Aritmetische ende Geometrische Fondamenten van Mr. Ludoff (sic) van Colen mijn man sal: ged: de welcke al over lange jaren van den Autheur selve (in sijn boeck gheschreven vanden Circkel) zijn belooft geweest, doch van wegen zijn veelvoudige, soo publijcke als particuliere occupatien, tot noch toe inghehouden, int licht laten comen, ten dienste der nakomelinghen. (Ceulen, 1615a, preface to Count Ernest of Nassau.)

[21] Hebbe derhalven dese Geometrische en Arithmetische fondamenten, welcke al ouer lange Iaren vanden Autheur Mr. Ludolf van Collen mijnen Man saliger ghedachtenisse inde praefatie van sijn boeck gheschreven van den Circkel is belooft geweest, doch van weghen sijne groote ende veelvoudighe occupatien, waer mede hy in sijn leven, soo van weghen sijne Professie, als oock andere particuliere verhindernissen is belet tot noch toe achter ghehouden, in het licht laten komen (Ceulen, 1615a, preface to Admiralties).

[22] Van meer gewicht is de vermelding van zijn 'groote werk', waarin hij 'de voornoemde Quaestie gesolveert voor-dragen soude'. Dit moest wel dezelfde arbeid zijn, waarvan van CEULEN gewaagt in zijn voorbericht van zijn boek 'Van den Circkel', [zie Bouwstoffen No. VIII, §8.]. (Bierens de Haan, 1878, pp. 336–340)

[23] Maar in deze Fondamenten vond ik ons vraagstuk niet, en konde het ook daarin niet vinden, omdat het in geen der zes Deelen paste. Nu weet men, dat deze Fondamenten afbreken bij een voorstel 17 (blz. 271), waarbij wel eene figuur, maar geen antwoord of oplossing te vinden is. Het konde dus zijn, dat die Fondamenten slechts een brokstuk was, niet verder door VAN CEULEN bewerkt, en dientengevolge ook niet verder door zijne weduwe in het licht gegeven. (Bierens de Haan, 1878, p. 324)

[24] Ick wiste hier noch wel veelderhande corden te setten met ander stucken, maer tsal beter te passe comen in mijn Cos-bouc, daer ick de vindinge des Hooch-geleerden Adrianus Romanus sal stellen, daer door men can comen tot de verghelijckinge van alderhande zijden van figueren ghelijcksijdick inden circkel beschreven, ende mede de maniere door wat middel ick ghecomen ben tot de waerde van 1ꝶ. (Ceulen, 1615a, p. 269)

[25] Longe plures subtensas, aliaque huius generis hic in medium proferre possem: Verum omnia haec peculiarem sibi locum deposcunt in mea Algebra, quo in loco subtilem aequationum tabulam, in qua omnium polygonotum latera circulo in scripta Algebrica aequatione disposita et dictincta sint describam, et modum nostrum quo 1ꝶ valor investigetur explicabo. (Ceulen, 1615b, p. 263)

[26] Daaruit mag men dus besluiten, - afgescheiden van de vraag of de Arithmetische en Geometrische Fondamenten een brokstuk zijn of niet, - dat er nog een groot Werck over Cos door LUDOLF VAN CEULEN geschreven

The promise of the publication of a *Coss-book* found in *Vanden Circkel* is again present in the *Fondamenten*. Van Ceulen must have written the last unfinished problem in book VI shortly before he died, which is also suggested by Snellius as a reason for why the solution to the problem is missing. It is certain that the promised *Coss-book* was not published while Van Ceulen was still alive and might well have never been written.

## 4.2 Different editions of the *Fundamenta*

Just like there are several sources that give different publication dates for the *Fondamenten*, this also holds for its translation. According to a catalogue of a library in Russia and an article from the *Messenger of Mathematica*, a fourth edition of the Latin version was published in 1617 [27]. This last source also mentions that there must have been five editions. The assumption that there were several editions of the *Fundamenta* suggests that the work was well known and widely used.

According to De Wreede, there are two editions of the *Fundamenta* that were not published in 1615, but in 1617 and 1618 (see chapter 2.2). The last edition can be found in the Tresoar Library in Leeuwarden, the Netherlands. It was published by Georgius Abrahami A Marsse (the Latin name for Joris Abrahamsz. van der Marsce). The original 1615-edition can be found in the Bayerische Staatsbibliothek. I have not seen the edition from 1617, since it can only be found abroad, in the Bibliotèque Nationale de France.

All the other editions that I have consulted are from 1615, but some are from another publisher. These are the different publishers I found:

1. Apud Iacobum Marcum Bibliopolam, Anno 1615
2. Apud Iustum a Colster Bibliopolam, Anno 1615 (Apud J. a Colster et J. Marci, 1615)
3. Excudebat Georgius Abrahami A Marsse, Anno 1615

In the appendix B I have recorded some details about these editions. The first edition does not contain a dedicatory letter. The second is the one I used in this thesis. It has two title pages: on the first only the name of Joost van Colster is mentioned, on the second the name of Jacob Marcus is added[28]. The third appears to be a genuine reprint since the titlepage is corrected and a new figure is placed.

When I kept searching, I found an edition that contained some interesting notes[29]. One edition of the *Fundamenta* dedicated to Count Maurits contains many notes that may have been written for the purpose of publishing a new edition; many mistakes (both typographical

is; dat dit boek in Mei 1610 niet gedrukt was; dat zijne weduwe het niet heeft uit-gegeven; en dat het dus meer dan waarschijnlijk bij het over-lijden van VAN CEULEN niet genoegzaam voor de pers gereed was gemaakt. (Bierens de Haan, 1878, pp. 340–341)

[27](Struve and Lindemann, 1860, p. 33); *(...) the Cataloga of the Pulkowa (observatory) library gives the title "Ceulen L. a. Fundamenta Arithmetica et Geometrica. Amstel. 1617, no.4." That there should have been five editions of the Fundamenta or De Circulo (much alike in their contents) in the four years 1615-1619, shows the estimation in which Van Ceulen's works were held at the time.* (Glaisher, 1874, p. 28)

[28]There is an error on the first titlepage: it reads *Fundamemta*. On the second, this is corrected and a missing 's' is added. A figure is missing on the second titlepage.

[29]I found this edition on-line: `http://books.google.nl/books?id=1S8VAAAAQAAJ`. The original can be found in the library of the Université de Lausanne, France.

and mathematical), the page numbering and some headings (e.g. p. 142) are corrected. The person who probably wrote these notes has written his name on the title page, but it is not entirely readable; I deciphered it to be Gamaliel Curchod 1690 (who could be a french pastor). Curchod made comments on reordering the problems in book four[30] and added mathematical calculations[31]. He also made references to former problems more specific[32]. To make Snellius' sentences more clear, he added elaborations[33]. He adds a long commentary on page 79 at the end of the second book and in the sideline on page 152. On many occasions he corrects the text grammatically[34]. He changed the heading of book five by correcting 'quartus' to 'quintus' and added the number of the books in the headings of every page which strongly suggests that he intended a reprint of the work. He was not very careful with adding notation since he left a great number of ink stains. Since his notes are spread throughout the work, it could be argued that Curchod worked through the whole work. I concluded that Curchod must have had a new edition of the *Fundamenta* in his mind, but it is unclear whether this edition has ever been printed. What it does prove is that the *Fundamenta* was considered a work worthy of a reprint.

I found a total of seventeen digitalised versions of the *Fundamenta*; some can now be found in libraries abroad (for an overview of the locations, see appendix C). This proves that the work really did reach an international audience. Also, many editions contain notes and scribblings in the margins, which shows that the works were not only distributed, but also intensively used. By translating the work by Van Ceulen, Snellius did indeed make the content of the *Fondamenten* known throughout Europe.

### 4.2.1 De Circulo: a reprint of the *Fundamenta*

*Vanden Circkel* is another great work of Van Ceulen. It contains many calculations on in- and circumscribed regular polygons. The calculations involve the proportion between the circumference and the diameter of a circle, now known as $\pi$. In addition it also contains sine tables to be used by surveyors and a large part on interest.

In 1619, the Latin edition of *Vanden Circkel*, translated by Snellius, was published as *De Circulo et Adscripti Liber* (I will use the short reference *De Circulo*). This translation is a combination of the *Fundamenta* and *Vanden Circkel*. Furthermore, Snellius used this translation to publish elaborations on the problems of dividing triangles in an appendix. He complained many times in the *Fundamenta* that he was unable to add elaborations due to a lack of time and new figures. Now in this translation of *Vanden Circkel* he could finally add his contributions. The content of this added appendix is thoroughly discussed in Wreede (2007, pp. 241–246), so I will not repeat it here.

The *Fondamenten* and *Vanden Circkel* both deal with similar subjects. Especially the last chapter of the *Fondamenten*, which is mainly on calculations with circles, shows many similarities with its predecessor. I will point out the links that Van Ceulen himself made between these two works by referencing back and (possibly) forth. Finally, I will describe the translation of *Vanden Circkel* and explain why it can be seen as a reprint of the *Fundamenta*.

[30] Pages 153 and 159
[31] Pages 6, 19, 58, 145, 148, 149, 151, 152, 185, 186 and 268
[32] Pages 99, 118, 119 and 137
[33] Pages 4, 8, 14, 56 61, 68, 78, 123, 188, 216 and 258
[34] Pages 1, 72, 175, 189, 223 and 259

**References made back and forth**

According to Katscher, Van Ceulen must have started his *Fondamenten* as early as in 1596, since in his *Vanden Circkel* he writes:

> ...and in my *Fundamenten* [with u] (sic) in the second chapter proven. [35]

However, I think he interpreted this comment of Van Ceulen incorrectly. This quote can be found on Folio 18 of the 1596 edition of *Vanden Circkel.* In the second edition of 1615 I found this remark twice, namely on folio 33 and 36. Van Ceulen writes there in parenthesis *'(door mijn Fundament / in't tweede Capittel)'* (by my fundament in the second chapter) (Ceulen and Eycke, 1615). He uses an 'u' instead of an 'o' and he refers to his 'fundament' in singular. Thereby, this reference seems odd since the second chapter of the *Fondamenten* is a selection from the *Elements* of Euclid. Van Ceulen would have referred directly to the *Elements* if he needed to, but in this example he did not need any proposition to explain his calculations. This comment is simply a reference back to a proposition (or fundament) given in *Vanden Circkel* itself, namely the one mentioned in its second chapter.

In the *Fondamenten*, Van Ceulen does make several references back to *Vanden Circkel.* The first reference to *Vanden Circkel* in the *Fondamenten* is already in the first book. Van Ceulen remarks there that in his other work (i.e. *Vanden Circkel*) and in the next books of the *Fondamenten* that follow, the taking of square roots of irrational numbers is explained.

> (...), as in my work Vanden Circkel was being taught, and hereafter will follow.[36].

The second reference can be found at the beginning of chapter eight of the first book. Here Van Ceulen introduces the concept of universal numbers[37], which appear in calculations for finding the length of the sides of equilateral figures that are inscribed or circumscribed in a circle.

> (...) as is shown by many examples in my work Vanden Circkel.[38].

In the tenth proposition of the appendix to book III, Van Ceulen remarks that in *Vanden Circkel* he has calculated the ratio between the diameter and the periphery of a circle up to 22 decimals (in a quotient). Now in the *Fondamenten* he has, with the help of one of his pupils, continued his calculation up to the 32th decimal (Ceulen, 1615a, p. 163).

The most references to *Vanden Circkel* are found in the last book of the *Fondamenten*[39] which is mainly about circles, for example:

> ..., as is being taught in my book *Vanden Circkel*,... [40].

Hence he must have finished his *Vanden Circkel* before he laid his last hand on the last chapter of the *Fondamenten.* In section 4.1.3 it has already been shown that the reference in *Vanden Circkel* to a *Coss-book* was not a reference to the *Fondamenten.* It can be concluded that Van Ceulen did make many references in the *Fondamenten* back to his *Vanden Circkel*, but not the other way around.

[35] *...und aus meinen Fundamenten [mit u] im zweiten Kapitel bewiesen.* (Katscher, 1979, p. 119)
[36] (...), als in mijn boeck vanden circkel geleert wert, ende hier na volghen sal. (Ceulen, 1615a, p. 46)
[37] Numbers of the form $\sqrt{a+\sqrt{b}}$.
[38] (...) als door veel exempels in mijn boeck vanden circkel te sien is. (Ceulen, 1615a, p. 60)
[39] Pages 248, 255, 259, 261, 265, 269 and 270.
[40] ..., als in mijnen bouck van den circkel geleert wert,... (Ceulen, 1615a, p. 255)

### A reprint of the *Fundamenta*

In this section the differences between the 1619 edition of the *De Circulo* and the *Fundamenta* will be explained.

After a preface written by Snellius, *De Circulo* starts with the first chapter from the *Fundamenta*, thus beginning with the sixth chapter from the first book of the original *Fondamenten*. The translation then skips the second book, containing propositions taken from the *Elements*, and continues on the transformation and cutting of figures, i.e. book three from the *Fundamenta*. Some changes are made in the placing of figures[41] and numbering of lemmas[42]. Problema 44 is left out[43], probably because the question proposed in this problem did not fit in the category of problems surrounding it. Following comes book four, corresponding to book five of the *Fundamenta*, which was wrongly entitled as the fourth book which now does not need to be corrected. At the end of book five - the sixth from the *Fundamenta* - Snellius adds an appendix which contains an elaboration on his commentary to problems seven and eight of the fifth book. In his translation of the *Fondamenten*, Snellius was not able to present this elaboration due to the lack of new figures. Now in *De Circulo* he can place two new figures and add his promised demonstration[44].

After this almost exact copy of the *Fundamenta*, Snellius adds the translation of *Vanden Circkel*. From the 35 axioms given in the first chapter of *Vanden Circkel*, Snellius only kept four[45]. The fifth axiom in *De Circulo* is actually the proposition from the second chapter of *Vanden Circkel*, the 'fundament' to which Katscher referred. To three of the five axioms that Snellius gives in his first chapter, he added a long commentary with newly added figures to accompany his demonstrations. Furthermore, Snellius translated only the first fifteen chapters of *Vanden Circkel*; this is about a quarter of the original (Ceulen, 1596).

In conclusion, *De Circulo* is for the most part a reprint of the *Fundamenta*, because only the second chapter was left out and at the end only 54 pages of *Vanden Circkel* were added[46].

[41] See for example pages 48 and 101 of (Ceulen and Snellius, 1619).

[42] See for example page 71 of (Ceulen and Snellius, 1619).

[43] Compare (Ceulen, 1615b, pp. 133–134) with (Ceulen and Snellius, 1619, pp. 81–82) (note the sloppy numbering of the pages in *De Circulo*).

[44] See (Wreede, 2007, pp. 241–246) for a detailed discussion on these added appendix.

[45] The axioms left are 11, 14, 15 and 21 from *Vanden Circkel*.

[46] *De Circulo* has 280 pages, the 1596 edition of *Vanden Circkel* has 236 pages and the 1615 edition of *Vanden Circkel* has 346. The pages are numbered with Folio.

# Chapter 5

# The commentaries of Snellius

There are many differences between Van Ceulen's and Snellius' editions of the same work. The most significant alteration that Snellius made was adding his own contributions to the work in the form of commentaries on Van Ceulen's solutions to various problems. The commentaries are all printed in italics and are placed after or even interrupt the problems. These personal contributions of Snellius give us a unique opportunity to uncover his view on the work of his teacher. In my bachelor thesis, I had restricted myself to the commentaries of Snellius which he added to the fifth book of the *Fondamenten*. Now, I will treat examples from the whole work. In this chapter I will first address the contributions in general to give an overview of the different kind of commentaries which Snellius added to the *Fondamenten* (see section 5.1). On some of these contributions, I will discuss the commentary in more detail (see sections sections 5.2, 5.3, 5.4.1 and 5.4.2). Snellius did also find the time to add a lot of alternative solution methods to the *Fundamenta* (see problems 19, 21, 34, 36, 42 and 45 from book III; problems 31, 32, 36, 37, 42 and 57 from book IV; problems 2, 3, 4, 5, 8, 9, 16 and 47 from book V). I will address some of these later in this chapter and also in the next chapter (see section 5.5 and chapter 6).

## 5.1 The general idea behind the comments

Snellius attached many commentaries to several problems in his translation of the *Fondamenten*. In the appendix I have added a complete overview of the *Fundamenta* in which I included a detailed description on the contributions by Snellius (see appendix E). Here I will treat these commentaries more generally.

In the first book, Snellius mainly simplified the texts, corrected mathematical mistakes and added explanations to the theory about the basic operations with numbers[1]. For example, where Van Ceulen makes extensive use of the root sign to present a single number, Snellius explains the use of the root sign in more detail[2]. Snellius did not add any commentaries to the second book, which contains a Dutch translation of a selection of propositions taken from the *Elements* of Euclid[3]. Sometimes he added the subtitle 'problema' when a given proposition involved a construction. Likewise, the last book of the *Fundamenta* does not contain any contributions,

[1]See e.g. the very first example in the first chapter of the *Fundamenta* where Snellius clarifies the calculations; in chapter three on the notation of the root sign (Ceulen, 1615b, p. 10); in chapter eight explaining the reason behind the theory (Ceulen, 1615b, p. 22), the use of the root sign (Ceulen, 1615b, p. 23, 25 and 27) and simplifying the calculations (Ceulen, 1615b, p. 26).

[2]Another place where Snellius adds a comment on the use of the root sign can be found in the fifth book (Ceulen, 1615b, p. 222). See also (Wreede, 2007, p. 301) and (Veen, 2011, pp. 15-16)

[3]An overview on which propositions Van Ceulen selected can be found in the appendix of Vlek's master thesis (see (Vlek, 2008, p. 63)).

except for the elaboration of the last problem. This problem was lacking an answer due to Van Ceulen's passing away (Ceulen, 1615b, pp. 267–269).

The most and more interesting commentaries by Snellius can be found in books three to five, which contain a large variation of geometrical problems. He made many changes to the formulations of the problems, making it more general and thereby more Euclidean. In problem 15 of book three of the *Fondamenten*, presented as problema 36 in the *Fundamenta*, the formulations are as follows:

> **Problem 15 (Van Ceulen)**:
> One desires to draw a triangle, similar to this one drawn here, marked with $ABC$, and as large as the square $CDFE$ next to it.[4]
>
> **Problema 36 (Snellius)**:
> To find a triangle equal to a given square and similar to another given triangle.[5]

Where Van Ceulen referred specifically to presented figures, Snellius translation is more general and uses the classical concept of 'given' twice. Snellius' formulation of mathematics in the *Fundamenta* is often short and concise.

On many occasions Snellius complains about the lack of time he had for the translation of the work. Furthermore he claims that, due to this short time frame (and the limited amount of money), he was unable to present elaborations nor to have new figures cut to go with them[6].

> We were so much limited, by the pressing time and we also had to use deformed figures belonging to another statement; it is however not possible to alleviate this problem for the sake of the reader.[7]

Nevertheless, I will show that Snellius did find the time to add new figures to the *Fondamenten* (see section 5.2).

Besides complaining about a lack of time, Snellius also added extra explanations when he considered it necessary. For example, in proposition 20 of book V where he extended the explanation on the use of the notation of the root sign (see Veen (2011, pp. 15–17)). This contribution was necessary since the nested root that Van Ceulen presented was so large that it became almost impossible to conceive it correctly. Other examples are problem seven of book III, remarking that the problem can also be solved by steps following the previous problem (Ceulen, 1615a, p. 91); problem 28 of book IV, explaining a proportion which Van Ceulen neglected to mention (Ceulen, 1615a, p. 162); and problem two of book V, giving an analysis on Van Ceulen's construction which lacked a demonstration (Ceulen, 1615a, p. 186).

Snellius voices his opinion about several problems, especially when the problems involve multiplications of and division by line segments. For the multiplication of two line segments Van Ceulen sometimes considered the result to be another line segment, hereby disregarding the

[4] Men begheert te trecken eenen Tryangel, ghelijckformich desen hier ghetrocken, gheteeckent met ABC, ende soo groot als het bystaende quadraet CDFE. (Ceulen, 1615a, p. 155)

[5] Triangulum dato quadrato quidem aequale et alteri triangulo dato simile construere. (Ceulen, 1615b, p. 133)

[6] See e.g. (Ceulen, 1615b, p. 98, 135, 184, 205, 230, 233). For more information see (Veen, 2011, p. 15)

[7] Quamvis et temporis angustia circumscribamur, et figuris ad alienum arbitrium deformatis utamur, non possum tamen quin huic problemati in gratiam lectoris facem alliceam. 'Facem allicere' literally means 'allure light', which I translated to 'alleviate'. (Ceulen, 1615b, p. 135)

traditional geometric perspective which stated this product to be a two-dimensional rectangle (see (Ceulen, 1615a, p. 137)). At one place Van Ceulen explicitly describes the division of a rectangle by a line, hereby making a transition from a two-dimensional figure to a one-dimensional line:

> It may happen that some questions occur, which require for their artful answering that rectangular quadrilaterals be devided by lines, (...).[8]

Snellius was slightly agitated by these kind of statements, which becomes apparent when, before adding his own alternative method to find the solution evading this problem, he added his opinion in a commentary as follows:

> What this author claims, i.e. that the result of the geometrical multiplication of two lines is a line, is not supported by any authority, just as that which follows, i.e. that a line would result from the mutual division of two lines.[9].

More about this conflict between Van Ceulen and Snellius can be found in Wreede (2007, pp. 205–213).

Snellius also gives his opinion on the use of numbers in geometry[10]. Van Ceulen uses numerical examples to prove the geometrical problems. However, Snellius remarks that, although he does not consider this as a real proof, he follows the habit of Van Ceulen. He wants to offer the 'lovers of learning' the opportunity to practise with numbers given to line segments[11]. Snellius also gives a warning to Van Ceulen to fully simplify his notation of numbers[12]. This last example will be discussed in section 5.4.2.

There are several places where Snellius adds a theorem of his own. For example, he added a lemma which he needed in his alternative solution method[13]. Another theorem of Snellius involved Heron's theorem[14]. De Wreede already discussed these two in her work[15]. One contributed theorem that De Wreede did not write about can be found in problem 26 of book IV. This theorem can be used to facilitate the calculation needed to find the sides of a square inscribed in a triangle. I will elaborate on this addition by Snellius in section 6.2.3.

For some problems Snellius is able to shed some light on the origin of the question. Sometimes he can tell us that the presented problem was in fact proposed to Van Ceulen by himself[16] or he explains why one would like to solve the given problem[17]. I will elaborate on this explanation of the utility of two problems in section 5.3. Snellius also tells us about a problem

[8]Het ghebeurt wel dat eenighe vraghen voor-vallen, dat om de selve constich te beantwoorden, men rechthouckige viercanten moet divideren door linien, (...). (Ceulen, 1615a, p. 138)

[9]Namque quod hic autor postulat duarum linearum multiplicatione Geometrica lineam fieri, tam $\alpha\kappa\upsilon\rho o\nu$ est, quam id quod sequitur mutua duarum linearum divisione lineam existere. (Ceulen, 1615b, p. 113) Translation taken from (Wreede, 2007, p. 212)

[10](Ceulen, 1615b, pp. 105, 137 and 234)

[11]Ideoque haec quae sequuntur zetemata dedomenon formula concepi, ut philomatis occasionem subministrarem figuratorum numerorum affectionem et symptomata cognoscendi, inque istis sese exercendi, ut numeri isti quorum tractatio obscurior et intricatior hactenus habita suit ita minus aspera, au tab usu remota esse re ipsa comprobetur. (Ceulen, 1615b, p. 137)

[12](Ceulen, 1615b, p. 215)

[13](Ceulen, 1615b, p 123)

[14](Ceulen, 1615b, pp. 186–191)

[15]See Wreede (2007, pp. 271–278)

[16](Ceulen, 1615b, pp. 215–216)

[17](Ceulen, 1615b, pp. 126–127)

which Van Ceulen originally sent to Goudaen. I will reveal more interesting aspects about this particular problem in section 5.5. Finally, Snellius completes problems when Van Ceulen forgot to give the solution to the whole question[18], gives a generalisation of a problem[19] and adds a proof[20]. In addition, he quotes Eutocius[21] when he expands the preface to book IV and refers to the works of Commandinus[22] and Ptolemaeus[23].

In conclusion, Snellius corrected, explained, generalised and completed the *Fondamenten*, complained a lot, gave his opinion and added alternative solution methods and theorems of his own.

## 5.2 Adding new figures

Snellius complains many times about a lack of time because of which he is not able to add new figures ((e.g. Ceulen, 1615b, pp. 98, 230, 232)). De Wreede states that Snellius was not granted the time to have new figures cut, for both the *Fundamenta* and the *De Circulo et Adscripti Liber* (Wreede, 2007, p. 188). This, however, is not true. There are in fact some new figures in Snellius' commentaries, mostly when he needed them to accompany another solution method. For example, in a long commentary on the second problem in book V (see figure 5.1).

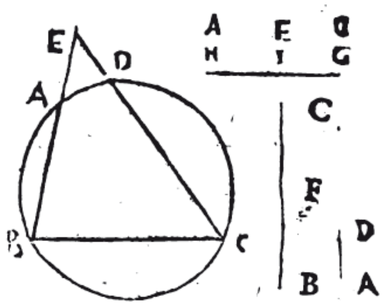
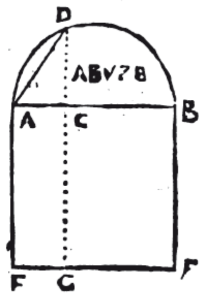



Figure 5.1: A figure that does not appear in the *Fondamenten* and must thus have been added by Snellius. (Ceulen, 1615b, p. 190)

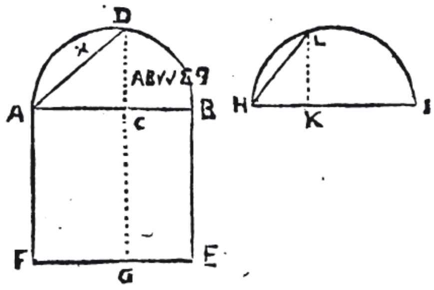
The problem prior to this commentary is about how to construct a quadrilateral inside a circle with four given line segments ($AB$, $BC$, $CD$ and $AD$). According to De Wreede, 'Snellius supplemented Van Ceulen's construction with an analysis, by which means he wanted to elucidate Van Ceulen's un-demonstrated construction' (Wreede, 2007, p. 282). The key to find the solution to the problem lies in determining the diagonal of the quadrilateral. After his analysis of the problem, Snellius gives an alternative way to determine the diagonal of the quadrilateral[24]. At the end of this three pages long commentary, Snellius attaches the demonstration on how to construct this cyclic quadrilateral. To explain his demonstration he placed a newly cut figure in the margin.

[18](Ceulen, 1615b, pp. 102, 156 and 209)
[19](Ceulen, 1615b, p. 163)
[20](Ceulen, 1615b, p. 205)
[21]See section 2.1 and (Ceulen, 1615b, p. 137).
[22](Ceulen, 1615b, p. 163)
[23](Ceulen, 1615b, p. 170)
[24]This problem and it's solutions by Van Ceulen and Snellius are thoroughly discussed in Wreede (2007, pp. 279–285).

There is even a place where Snellius adds two more figures. In a problem on adding line segments, problem 22 of book III of the *Fundamenta*, he proposes a different method to solve these kind of problems. To explain his method he made use of two new figures. This particular problem is treated in much detail by De Wreede (Wreede, 2007, pp. 205-213).




(a) The first newly added figure. (Ceulen, 1615b, p. 107)

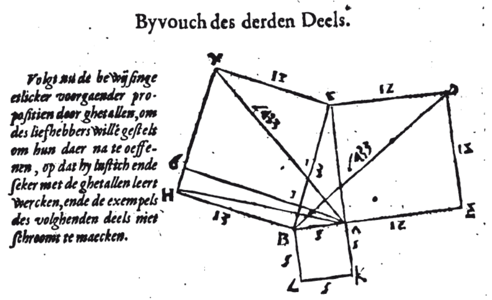



(b) A slightly different figure. (Ceulen, 1615b, p. 109)

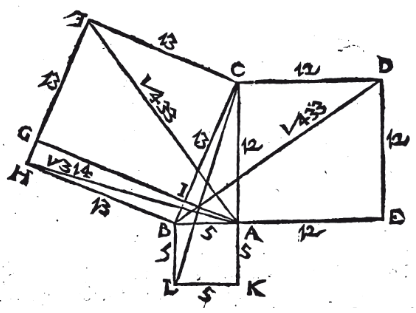
Figure 5.2: Two new figures Snellius added in a commentary on problem 21 of the third book of the *Fundamenta*.

The problem belonging to figure 5.2a asks to construct a line segment of length $\sqrt{28} + 3$ from a given line segment $AB = \sqrt{28}$. Snellius does this by constructing the mean proportional $AD = 3$ between $AB = \sqrt{28}$ and $AC = \frac{9}{28}\sqrt{28}$, where $AC$ is found by dividing $AB$ in 28 equal parts (using *Elements* VI,9) and taking 9 of them. In figure 5.2b the problem is to construct a line segment of length $\sqrt{\sqrt{192}} + 3$ for which Snellius applies his method twice (Ceulen, 1615b, pp. 107–109). To find the mean proportional between two lines, Snellius uses the construction of a half circle on $AB$ (and $HI$).

Finally, Snellius seems to add a new figure used to prove the theorem of Pythagoras (compare (Ceulen, 1615a, p. 156) with (Ceulen, 1615b, p. 137)). However, the figure Snellius uses is the same one that Van Ceulen used in his first treatment of this theorem as a proposition in the second book. Why Van Ceulen made this figure twice seems peculiar. Snellius chose to use the figure from book two a second time, probably because it is more accurate.




(a) The figure in the *Fondamenten.*




(b) The new figure in the *Fundamenta*, similar to the one used in book II (Ceulen, 1615a, p. 80).

Figure 5.3: A comparison of two figures both belonging to the same problem.

These examples prove that there was indeed some time to make new figures. The figures look slightly different from the original ones, maybe because they were cut by another artist. However, these are the only three new figures in the *Fundamenta*, showing that Snellius did have some, although not much, time to have new figures cut.

## 5.3 Explaining the utility of two problems

In this section I will discuss two commentaries which Snellius added in the third book of the *Fundamenta*. In these commentaries Snellius gives a reason as to why these particular problems were presented.

In problem 37 of the third book[25], Van Ceulen poses the question how to cut a given line segment $a$ in two parts such that the sum of the squares of the parts equals another given line segment $b$[26]. In this case the lengths are given to be $a = 20$ and $b = 16$. The subsequent problem is almost similar to this one, requiring the sum of the square of the smallest part with the square of line $b$ to equal the square of the largest part. However, the constructions to find this cut are very different in both cases.

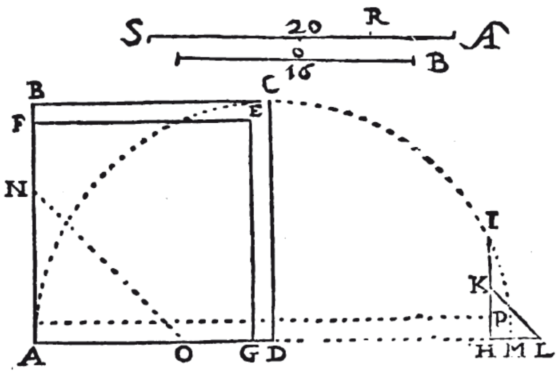



Figure 5.4: Figure belonging to problem 37 of book III.

The given line segment $a$ is presented in the figure as line segment $AS$ and the other given line segment $b$ as line segment $B$ (see figure 5.4). It is required to find the point $R$ on $a$ such that $AR^2 + RS^2 = b^2$. The construction accompanying problem 37 is as follows.

**Construction 5.3.1.**

1. Draw a square $ABCD$ with sides of length $b$.
2. Find $O$ on $AD$ and $N$ on $AB$ such that $AO = AN = \frac{1}{2}a$.
3. Draw a square $AFEG$ with $F$ on $AB$ and $G$ on $AD$ with sides of length $NO$.
4. Extend $AD$ in $M$ with $H$ on $AM$ such that $DH = AF$ and $HM = GD$.
5. Find $Q$ on $AB$ such that $AQ = GD$ and construct the rectangle $AQPH$ (its area equals the difference of the squares).

[25]This is proposition 16 on page 148 of the *Fondamenten.* Note that not all the proposition in this book are numbered.

[26]Van Ceulen uses capital letters for lines and vertices. I will use small letters for lines to differentiate from vertices. Note that this problem is also dimensionally inconsistent.

6. Construct the mean proportional $HI$ between $AH$ and $HM$.

7. Divide $HI$ equal in $K$.

8. Extend $DH$ in $L$ such that $HK = HL$.

9. Draw $LK$. Then $RS = \frac{1}{2}a + LK$ and $AR = \frac{1}{2}a - LK$ gives the desired cut in $R$.

I will give an algebraic formulation to this solution method.

**Solution method 5.3.2.**
To find $R$, one needs to calculate the length of $AR$ which is the mean proportional between $HM$ and $AH$. First calculate $HM = GD$, which is the difference between the length of the squares $ABCD$ and $AFEG$: $GD = AD - AG = b - NO = b - \frac{1}{\sqrt{2}}a$.
Next, calculate $AH$ which consists of three line segments $AH = AG + GD + DH$. By the construction (step 2) we know $AG = DH = NO = \frac{1}{\sqrt{2}}$, hence $AH = 2NO + GD = \sqrt{2}a + b - \frac{1}{\sqrt{2}}a = b + \frac{1}{\sqrt{2}}a$.
Now, to calculate the mean proportional, use $AH : HI = HI : GD$. This gives $HI^2 = AH \cdot GD = (b + \frac{1}{\sqrt{2}}a)(b - \frac{1}{\sqrt{2}}a) = b^2 - \frac{1}{2}a^2$, hence $HI = \sqrt{b^2 - \frac{1}{2}a^2}$.
Now $KL$ is the hypotenuse of the isosceles triangle with sides of $\frac{1}{2}HI$. Hence $KL^2 = \frac{1}{2}HI^2$, thus $KL = \sqrt{\frac{1}{2}b^2 - \frac{1}{4}a^2}$.
Add $KL$ to half of $a$, this gives the length of the largest part of $a$: $SR = \frac{1}{2}a + \sqrt{\frac{1}{2}b^2 - \frac{1}{4}a^2}$.

In this case $a = 20$ and $b = 16$, giving for the largest part $10 + \sqrt{28}$ and the smallest part $10 - \sqrt{28}$ which when squared and added gives 256, the square of $b$.

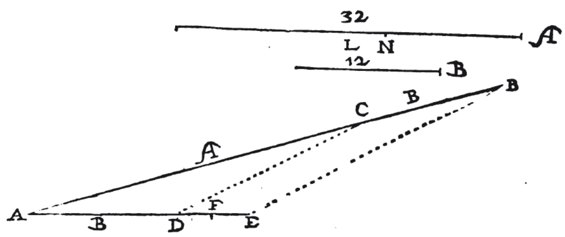



Figure 5.5: Figure belonging to problem 38 of book 3.

The construction belonging to problem 38 uses a different method to find the requested length. It is first required to find the line segment $DE$ such that $a : b = b : DE$. Then adding and subtracting $\frac{1}{2}DE$ to $\frac{1}{2}a$ gives the desired cut $N$. The construction is as follows (see also figure 5.5).

**Construction 5.3.3.**

1. Draw line $a = AC$ and $b = AD$ with common endpoint $A$ and an acute angle between them.

2. Extend $AC$ to $B$ with $CB = b$.

3. Draw $DC$ and a line through $B$ parallel to it.

4. Extend $AD$ until it cuts this last parallel line in $E$.

5. Then half of $DE$ gives the desired length.

Snellius wrote a commentary on each of these problems. First, he reveals that they are identical to the problem stated previously which asked to make a triangle from two given line segments with an inscribed circle with diameter equal to another line. He adds the calculations to find the radius of the inscribed circle, which is according to him the goal of these problems[27]. Indeed, looking at problem 37, when one wants to find the cut in line segment $a$ such that the squares of the parts add up to line segment $b$, this asks for a right-angled triangle with hypotenuse of length $b$ and other sides adding up to the length of line $a$. Likewise, problem 38 asks for a right-angled triangle with hypotenuse the largest part of line $a$ and others sides $b$ and the shortest part of line $a$. Why Snellius points to this 'practical' use of problems 37 and 38 remains unclear. Snellius does not explain the purpose of finding the inscribed circle to the right-angled triangles.

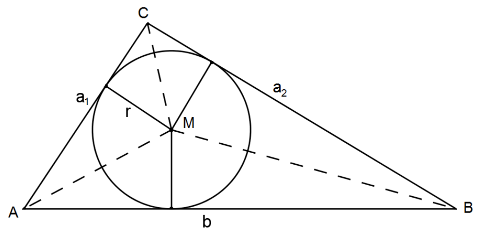



Figure 5.6: The area of a triangle is equal to $\frac{1}{2}(a_1 + a_2 + b)r$.

Snellius attaches some extra calculations to the problems. To find the radius of the inscribed circle, Snellius uses a consequence of Heron's theorem to first find the area of the triangle. Namely, let $a_1$, $a_2$ and $b$ be the sides of a right-angled triangle $ABC$ with an inscribed circle with radius $r$ and center $M$. Then, the area of triangle $ABM$ is equal to $\frac{1}{2}b \cdot r$. The areas of $AMC$ and $BMC$ follow similarly. Hence, when all three are added this gives $\frac{1}{2}(a_1 + a_2 + b)r$ for the area of the whole triangle (see figure 5.6). Now the area of the triangle $ABC$ can also be calculated by $\frac{1}{2}a_1 \cdot a_2$. This results in the following equality:

$$Area(\triangle ABC) = \frac{1}{2}(a_1 + a_2 + b)r \quad = \quad \frac{1}{2}a_1 \cdot a_2$$

Hence the radius of the inscribed circle belonging to the triangle of proposition 37 is equal to

$$\begin{aligned} r &= \frac{\frac{1}{2}a_1 \cdot a_2}{\frac{1}{2}(a_1 + a_2 + b)} \\ &= \frac{\frac{1}{2}SR \cdot RA}{\frac{1}{2}(SR + RA + b)} \ (see\ figure\ 5.4) \\ &= \frac{(10 + \sqrt{28})(10 - \sqrt{28})}{(10 + \sqrt{28}) + (10 - \sqrt{28}) + 16} \\ &= \frac{72}{36} = 2. \end{aligned}$$

[27] Ut lectorem benevolum hoc quoque ωροσθη κιδιω demerear, non quidem illam autoris in hac fabrica vestigia legendo, sed ad fontem ipsum digitum intendendo. Est inquam hoc problema re ipsa idem cum antecedente, sed alia formula propositum. saltem ad illud commodissime revocari potest. (Ceulen, 1615b, p.127)

In conclusion, Snellius explains in this commentary why one would like to find the desired cut, namely that the remaining parts form the sides of a right-angled triangle. Thereby, he explains partially the practical use of these two problems. However, Snellius does not explain why one would like to find the radius of the inscribed circle.

## 5.4 Discussion on explanation and notation

In this section I will give an example of a contribution by Snellius in which he provides an explanation on a method (section 5.4.1) and on the notation of numbers (section 5.4.2).

### 5.4.1 Extending Van Ceulen's explanation

Van Ceulen explains in book I, chapter eight, how to add $\sqrt{2+\sqrt{2+\sqrt{2}}}$ and $\sqrt{2-\sqrt{2+\sqrt{2}}}$. He uses this example to prove his method on how to add two nested roots, i.e. numbers of the form $\sqrt{a+\sqrt{b}}$ and $\sqrt{c+\sqrt{d}}$ with $a$, $b$, $c$ and $d$ rationals. Van Ceulen describes his method as follows:

> Add the squares of the numbers, then multiply the product of the squares with 4 and, finally, add the root of the result to the first result. The root from this sum is the sum of the roots. (Ceulen, 1615a, p. 61)

In modern notation, the result becomes:

$$\sqrt{a+\sqrt{b}}+\sqrt{c+\sqrt{d}}=\sqrt{(a+\sqrt{b})+(c+\sqrt{d})+\sqrt{4(a+\sqrt{b})(c+\sqrt{d})}}$$

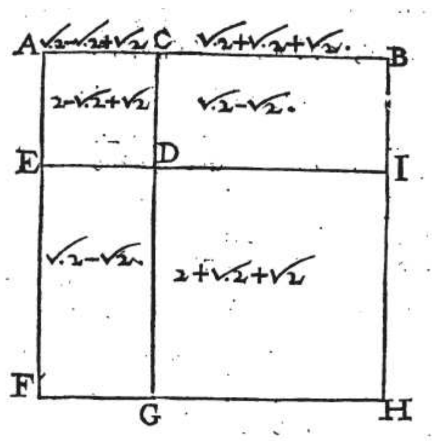



Figure 5.7: Proof corresponding to the addition of two nested roots (Ceulen, 1615a, p. 61).

Van Ceulen then gives an explanation for his method (see figure 5.7). Here, one wants to know the length of the side of the whole square, which will give the sum of the two nested roots ($AB = AC + CB = \sqrt{2-\sqrt{2+\sqrt{2}}}+\sqrt{2+\sqrt{2+\sqrt{2}}}$). Therefore, first calculate the area of the whole square and then take the root. The area of the square is the sum of the squares of the roots and *twice* their product.

Snellius, however, remarks justly that Van Ceulen's figure does not explain why in Van Ceulen's description the multiplication with 4 is needed. After giving another example, he continues with the remark that he wishes to further explain Van Ceulen's method, which he does with several calculations and the following comment:

> Caused by the duplication of the parts, then in this way under the influence of the root sign the first needs to be multiplied by 4 and the following by 16.[28]

He explains here that since the root needs to be multiplied by 2, the numbers under the root-sign need to be multiplied by 4 and the following with 16. This also follows easily when we notate the previous given root slightly different:

$$\sqrt{a+\sqrt{b}}+\sqrt{c+\sqrt{d}}=\sqrt{(a+\sqrt{b})+(c+\sqrt{d})+2\sqrt{a+\sqrt{b}}\sqrt{c+\sqrt{d}}}$$

Multiplication by 16 was needed in the double nested roots which Van Ceulen used in his 'proof', since they were of the form $\sqrt{a+\sqrt{b+\sqrt{c}}}$. Snellius thus extends Van Ceulen's explanation to make it more comprehensible and complete.

### 5.4.2 Notation of numbers

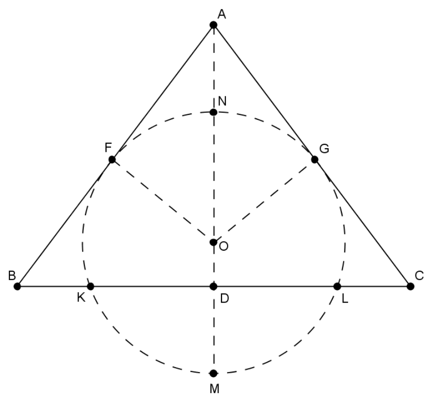



Figure 5.8: Problem fifteen of book V of the *Fundamenta*.

The next example comes from the fifth book and deals with a circle partly inscribed in a triangle[29] (see figure 5.8). Given is a circle $KNLM$ with given diameter $MN$ and chord $KL$. Furthermore, triangle $ABC$ is given with $B$ and $C$ on the line through $K$ and $L$. Line segments $AB$ and $AC$ are tangent to the circle in points $F$ and $G$ respectively. The lengths of line segments $BF$ and $CG$ are also given. The question is to find the lengths of the sides of the triangle $ABC$.
Van Ceulen immediately gives the answer without any calculations. Snellius comments that he proposed this question to Van Ceulen long ago. He also mentions that he remembers the numbers for the solution which he had found, but that these seem different from those Van Ceulen presented. After investigating the numbers he concludes that they are the same, yet that Van Ceulen's numbers are not expressed as simple as possible[30]. This shows that Snellius was more focused on simplicity than Van Ceulen. Snellius adds a warning to Van Ceulen that he had not expressed the numbers in the most simple way. The reason for the absence of a calculation may be, according to Snellius, that Van Ceulen found it too boring or he simply forgot[31]. Alternatively, Van Ceulen may have been satisfied by giving the numbers in the expression he wrote down, and thus not striving for the simplest expression. More on this commentary can be found in my bachelor thesis (Veen, 2011, p. 17).

[28]Factus a segmentis duplicandus, ergo hoc modo pro signorum affectione prior per 4 prostterior per 16 erit multiplicandus. (Ceulen, 1615b, p. 22)

[29]This is problem 23 in the *Fondamenten* (Ceulen, 1615a, p. 227) and problem 15 in the *Fundamenta* (Ceulen, 1615b, pp. 215–216). This last problem is wrongly numbered 14 for the second time.

[30]Cum olim hoc zetemation autori nostro proposuissem, memini hos numeros mihi ad quasiti solutionem ab ipso exhibitos, cumque istos a meis quos ex abaco meo de prompseram diversos viderem, e vestigio quidem ab ipso commissum putabam, quia bis, idque diversa via, eosdem numeros inveneram: tandem sensi novissimos eius numeros symmetros esse, quos ubi addidissem plane cum meis consentire deprehendi, sunt autem isti. (Ceulen, 1615b, p. 216)

[31]Quamobrem numeri illi non sex, sed quatuor tantum nominibus sunt compositi: atqui solaecismus est logisticus symmetros numeros divisos proponere, aut non protinus reducere: monui itaque statim autorem de numerorum suorum symmetria; verum taedia antesedetis abaci, aut memoriae aliquo lupsu id omissum existimo. (Ceulen, 1615b, p.216)

## 5.5 One problem, four mathematicians: Van Ceulen, Goudaen, Galilei and Snellius

In my bachelor thesis, I studied problem 9 from the fifth book of the *Fundamenta*[32] (see Veen (2011, pp. 22–23)). Snellius' commentary explains the origin of the problem and contains a new theory that leads to the solution of the missing part. The problem itself originated from an earlier work by Van Ceulen, his *Solutie ende Werckinghe* (Solutions and Demonstrations) (Ceulen, 1584). First I will disclose the situation of the problem as presented by Van Ceulen in his *Fondamenten.* This situation is sketched in figure 5.9. The construction of this figure is as follows.

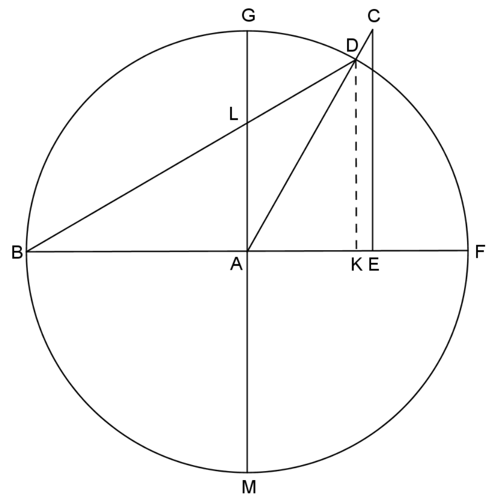



Figure 5.9: The figure belonging to Problema 9 of the *Fundamenta*

**Construction 5.5.1.**

1. Draw a circle with middlepoint $A$ and perpendicular diameters $BF$ and $GM$.
2. Let $E$ be a point on the diameter $BF$ such that $BE^2 = BF \cdot EF$.
3. In $E$ a perpendicular $EC$ is drawn with length equal to the radius of the circle.
4. Line $AC$ cuts the circumference in point $D$.
5. Finally line $BD$ is drawn cutting the vertical diameter $GM$ in point $L$.

The problem as formulated in the *Fondamenten* asks for the length of $BD$ and its parts $BL$ and $LD$, when the diameter $BF = 8$.

Snellius commentary to this problem is short, but remarkable, since he also gives the numbers for the lengths of $GL$ and $LM$. This seems a bit odd since this was not asked in the problem. Van Ceulen seems to have found a solution to find the lengths of $BL$ and $LD$, but does not give a solution in the *Fondamenten* to find $GL$ and $LM$. He does give the length for $LM$, but merely in a calculation to find $BL$. Now, Van Ceulen earlier published this problem,

[32] This is problem 16 in the *Fondamenten* (Ceulen, 1615a, pp. 222–223). The commentary can be found here (Ceulen, 1615b, pp. 209–210).

without the work or answers, in his *Solutie ende Werckinghe* (Ceulen, 1584) as one of two problems addressed to Willem Goudaen. Since the publication date of *Solutie ende Werckinghe* is 1584, it follows that Van Ceulen must have been working on this problem in or even before 1584.

The same problem occurs also in *Le Opere di Galileo Galilei* (Galilei and Saragat, 1968, pp. 32–33). It can be found in a letter from Michael Coignet (1549-1623) to Galileo Galilei (1564-1642). Coignet writes that due to the civil wars, it was hard to find a mathematician who still promotes the 'fine arts and studies'[33]. Nevertheless, he writes to have found one in Colonia, hereby misinterpreting the name of Ludolph van Ceulen (sometimes *von Collen*)[34]. The letter is dated 1588, hence Coignet may have encountered the problem four years after the first publication[35]. Coignet proposes the problem to Galileo to work on it[36]. Whether Goudaen or Galileo did indeed work on this problem is not known.

The problem presented in Coignet's letter is identical to the one found in the *Solutie ende Werckinghe*. However, the problem found in these two works contains slight differences with the one presented in the *Fondamenten*. The figure printed beside the problem in the letter and the *Solutie ende Werckinghe* contains different letters. Furthermore, here Van Ceulen does not specify the place of point $E$ on the diameter $BF$ as having proportions such that $BE^2 = BF \cdot EF$, but simply gives the numbers for these lines as being $BE = \sqrt{80} - 4$ and $EF = 12 - \sqrt{80}$. However, in this earlier presentation of the problem the required line segments are not only $BL$ and $LD$, but also $GL$ and $LM$, which are the line segments for which Snellius added the solution (but not the work) in the *Fundamenta*.

Now, the commentary that Snellius gives after this problem becomes more clear. After remarking that Van Ceulen's method is very elaborate and could be stated much more concise, Snellius points out that the calculations for the lengths of $GL$ and $LM$ are left out[37]. He probably knew it from the *Solutie ende Werckinghe* and remembered that the original question was longer and that there were more findings. Perhaps he worked on it in collaboration with Van Ceulen. Either way, Snellius must have worked on the question, for he gave us all the answers. Since Snellius says he has to postpone the work to another edition[38], he does not provide a demonstration or proof. Nevertheless, he does give some information as to how to find the answers. He says that $GL = BE - AC$ and $LM = EF + AC$. Snellius gives this statement without any proof. Still, Snellius statement resulted in the finding of a beautiful theorem:

**Theorem 5.5.2.**
*Given a circle with midpoint $A$ and horizontal diameter $BF$ and vertical diameter $GM$. Point $E$ on the diameter such that $BE^2 = BF \cdot EF$. A perpendicular $EC$, with length equal to the*

[33] Bella intestina miserabilis nostrae inferioris Germaniae adeo bonarum artium studia extinguerunt, ita quod vix apud nos aliquem invenies, qui his artibus et studiis favere videatur. (Galilei and Saragat, 1968, p. 32)

[34] Quidam Coloniensis tamen, nomine Ludolpho(1), nuper nobis proposuit aliqua problemata geometrica. The footnote (1) says: *Ludolpho van Ceulen* (germanicamente *von Collen*), che, equivocando sul cognomen, il *Coignet* tenne per nativo di Colonia, mentre era di Hildesheim. (Galilei and Saragat, 1968, p. 32)

[35] The date mentioned is the May 31th 1588: pridie calend. Aprilis, anno a Christo nato 1588. (Galilei and Saragat, 1968, p. 33)

[36] Hoc problema vero absolvimus adminiculo praeceptorum et regularum artis magnae, sive algebrae: quare si huius artis speculationes tibi cordi sint, poteris, si lubet, hoc praedictum problema tuo modo investigare. (Galilei and Saragat, 1968, p. 33)

[37] Operosissimam solvendi huius zetematis viam autor est secutus: potuit enim alia construxionis formula magna huius laboris pars declinari Sed illud silentio hic transmittere neque GL et LM segmenta diametri facillime absque ulla proportione dari, (...). (Ceulen, 1615b, p. 210)

[38] Demonstrationem in proxime editionem differre cogor. (Ceulen, 1615b, p. 210)

*radius, is drawn in E. Line AC cuts the circumference of the circle in point D. Line BD cuts GM in L. Then* $GL = BE - AC$ *and* $LM = EF + AC$.

We can prove this theorem using algebra.

*Proof.* Let $x := AE$ and $r$ be the radius of the circle.
From the construction we know that $GL = r - AL$ and $LM = r + AL$, hence we need to calculate $AL$. Since $\triangle ACE \sim \triangle ADK$ it follows that $\frac{DK}{CE} = \frac{AD}{AC}$. We know $CE = AD = r$ and using the theorem of Pythagoras we find $AC = \sqrt{AE^2 + EC^2} = \sqrt{x^2 + r^2}$. This gives $\frac{DK}{r} = \frac{r}{\sqrt{x^2+r^2}}$, hence $DK = \frac{r^2}{\sqrt{x^2+r^2}}$.
Since $\triangle BDK \sim \triangle BLA$ it follows that $\frac{AL}{BA} = \frac{DK}{BK}$. Again using the theorem of Pythagoras gives

$$AK = \sqrt{AD^2 - DK^2} = \sqrt{r^2 - (\frac{r^2}{\sqrt{x^2+r^2}})^2} = \sqrt{r^2 - \frac{r^4}{x^2+r^2}} = \sqrt{\frac{r^2x^2}{x^2+r^2}} = \frac{rx}{\sqrt{x^2+r^2}}.$$

We know $BA = r$ and $BK = BA + AK$, hence $BK = r + \frac{rx}{\sqrt{x^2+r^2}}$. This gives

$$\frac{AL}{r} = \frac{\frac{r^2}{\sqrt{x^2+r^2}}}{\frac{rx}{\sqrt{x^2+r^2}}} = \frac{\frac{r^2}{\sqrt{x^2+r^2}}}{\frac{r\sqrt{x^2+r^2}+rx}{\sqrt{x^2+r^2}}} = \frac{r^2}{r(\sqrt{x^2+r^2+x})} = \frac{r}{\sqrt{x^2+r^2}+x}.$$

Thus $AL = \frac{r^2}{\sqrt{x^2+r^2}+x}$.
Now multiplying both numerator and denominator with $\sqrt{x^2+r^2} - x$ gives

$$AL = \frac{r^2(\sqrt{x^2+r^2} - x)}{r^2} = \sqrt{x^2+r^2} - x = AC - AE.$$

Then $GL = r - AL = r - AC + AE = (r + AE) - AC = BE - AC$ and $LM = r + AL = r + AC - AE = (r - AE) + AC = EF + AC$. □

Since the construction for point $E$ such that $BE^2 = BF \cdot EF$ is not used in the proof, it is no requirement for this theorem. Thus, the actual theorem is more general then suggested in Snellius' formulation. The question that remains is whether Snellius indeed had a proof for his statement. Also, it remains unclear whether Van Ceulen had the answers for the missing parts himself and whether he was aware of the truth of the theorem given by Snellius. There is no work of Goudaen or Galileo to be found that refers to this problem. Whether this theorem and proof have been published before is unknown. However, the addition of Snellius shows his involvement with the problems that Van Ceulen placed in his *Fondamenten*. Snellius completed the problem and even took the trouble to find the solution to the missing parts.

# Chapter 6

# Concise vs. comprehensive

When I was studying all the commentaries that Snellius added to the *Fundamenta*, I eventually found a regularity in his additions. When Snellius gives an alternative solution method, he almost always first complains about the elaborate method that Van Ceulen used and then presents us with his own method which is often more intricate but above all more concise. In this chapter, I will show some examples to invigorate this observation. I will begin with describing Van Ceulen's general approach to his own problems (section 6.1). Next I will illustrate Snellius' concise approach by describing four different places where Snellius added his own solution methods (section 6.2).

## 6.1 Van Ceulen's approach: a large variation of solution methods

In section 3.4, I have already stated that Van Ceulen seems to be focused on training his target audience. This conclusion is also reinforced by his preference for giving different solution methods to similar problems. In the beginning of the *Fondamenten*, Van Ceulen was very extensive in his mathematical demonstrations and calculations. He seems to be focused on giving a large variation of solution methods, rather than one quick way to come to a solution. For example, in a chapter on the rule of three, Van Ceulen gives an extensive method to find the solutions to the question. Later on, he points out that it can also be found by using a more concise calculation[1]. Why would he first give a longer and more tedious method if he knows a shorter way to find the solution? This might suggest that Van Ceulen stresses on understanding a variety of different methods, rather than being able to find the solution in a concise manner.

In book four of the *Fondamenten* Van Ceulen repeatedly gives an alternative method to find a solution, for example:

> One could find the diameter BD by the previous method of the $43^{th}$ example. I will demonstrate another way that is very similar, by which the previous can also be found.[2]

From the fifth problem of book six:

[1]Sulcke Exempels, ende alle derghelijcke connen veel corter ontbonden werden, (...). (Ceulen, 1615a, p.38)
[2]Men soude de middellinie BD connen vinden door voorgaende maniere des $43^{sten}$ exempels. Ick sal hier een andere toonen dier seer naer ghelijck, daer door dat voorgaende mede te vinden is. (Ceulen, 1615a, p. 193)

> This can be answered in many ways, like this: (...). Another way, good for those who do not like to work with irrational numbers, (...). Again a different way using the sine tables. (...).[3]

It follows that Van Ceulen desired to give many different methods to solve the same problem. This might be because he wants the readers to build a broad repertoire of solution methods, or simply because he wanted to show off his abilities.

At some times, Van Ceulen seems to desire to give a shorter demonstration.

> This can be done in the same way as the previous examples. I will show you a more concise way like this:[4]

This shows that Van Ceulen was also interested in short construction methods, but he presented this concise method only after giving a more elaborate one. Furthermore, during the middle of the book Van Ceulen starts to use fewer words because he is convinced that the readers who have studied the previous content with understanding, will be able to understand the calculation without further explanation. He explains this choice twice:

> I have used few words in the work, the lover will understand everything, and find that these matters for themselves are not difficult, as long as that one knows how to properly work with numbers.[5]

> I will henceforth use as few words as possible (to be concise) and only show the contructions, the expert will understand the cause through the work and the preparation of figures, however where necessary words will not be lacking.[6]

He lives up to his promise for a few following problems, but soon starts giving very elaborately described solution methods again. It seems that, because he wants his demonstration to be understandable and clear, he needs a lot of words.

> ..., because this affair seems hard I will show my work below[7].

In conclusion it may be said that Van Ceulen was mainly focused on being as clear as possible, for which he needed a lot of words. With this, his demonstrations become long and tedious. Besides that, Van Ceulen often gives a large variation of solution methods to solve one problem. Snellius, on the other hand, stressed being short and concise and consequently has a preference for only one method.

---

[3]Dese can op veelderhande manieren beantwoort werden, als volcht: (...). Ander maniere, goet voor de ghene die geen lust hebben in de irrationale ghetallen te wercken, (...). Noch anders door de tafelen synuum. (Ceulen, 1615a, pp. 257–258)

[4]Dese is te maecken op de voorgaenden exempels. Ick sal u een naerder wech wijsen also: (Ceulen, 1615a, p. 199)

[5]Ick hebbe hiet int wercken weynich woorden ghebruyckt, den Liefhebber sal alles verstaen, ende mercken dat dese saecke in haer selven niet swaer (ghelijck schijnt) is, soo verde men gheschickt met de ghetallen weet te handelen. (Ceulen, 1615a, p. 221)

[6]Ick sal nu voortaen soo weynich woorden (om de corheyt) gebruycken als moghelick is, ende alleen het werck stellen, den hervaren in desen sal doort werck ende bereyden der figueren, de reden verstaen, doch daer de noot hier vordert, salt aen de woorden niet ghebreken. (Ceulen, 1615a, p. 195)

[7]..., om dat desen handel swaer schijnt sal ick u mijn werck onder tonen. (Ceulen, 1615a, p. 253)

## 6.2 Snellius' concise methods

Bierens de Haan writes about Snellius' contribution to the approximations of $\pi$:

> (...) his name and his fine analytic ingenuity is to be commemorated with high credit: not because he gave us the method of Van Ceulen, but because he delivered us another that led to a more rapid approach.[8]

Snellius was convinced that a mathematical work needed to be short and concise in order to be clear (Wreede, 2007, p. 62):

> (...), the ability to give clear and perspicuous explanations is the mark of a learned and intelligent man. And for this reason, I am delighted by short and sharp corollaries when they are needed, which must not be so long in order to be clear.[9]

In this section, I will give four examples of places where it becomes evident that Snellius was focused on giving short demonstrations which led to a concise solution method.

### 6.2.1 The first commentary: Ramus' method

The very first time Snellius adds a comment in the *Fundamenta* is after Van Ceulen explains his method for taking the square root of 'binomial' numbers[10]. A binomial number is the sum of a rational number and the square root of a rational, non-square number, for example $18\frac{1}{4}+\sqrt{308}$. Van Ceulen gives a rule for taking the square root of such a number:

**Theorem 6.2.1** (Van Ceulen's rule)**.**
*Subtract the squares of the parts from each other, from the remainder take the root, add this to the largest part of the binomial, the root from half of the sum is the first part, subtract the former half from the largest part, then the root from the remainder is the second part of the binomial.*[11]

In modern mathematical notation Van Ceulen takes the following steps:

**Solution method 6.2.2** (By Van Ceulen)**.**
Suppose we need to take the root of the binomial $a+\sqrt{b}$.

1. Subtract the squares: $a^2-b$
2. Take the root: $\sqrt{a^2-b}$
3. Add to largest part[12]: $a+\sqrt{a^2-b}$
4. Take the root of the half: $\sqrt{\frac{a+\sqrt{a^2-b}}{2}} =: \sqrt{c}$. This is the first part of the binomial.

[8](...) is zijn naam en zijn fijn analytisch vernuft met grooten lof te herdenken: niet omdat hij ons de methode van van ceulen gaf, maar omdat hij daarvoor eene andere leverde, die tot spoediger benadering voerde. (Bierens de Haan, 1878, p.172)

[9](...); plane autem, et perspicue expedire posse, docti et intelligentis viri. arque ideo, cum opus erit, consectaria me brevia et acuta delectant; quae, ut perspicua sint, ita longa esse non debent. (Snellius, 1608, p.5-6)

[10](Ceulen, 1615b, p. 20)

[11]**Regel**, Substraheert de quadraten der deelen van malcander, uyt Rest treckt den wortel, dese addeert tot het grootste deel des Binomiums, den wortel uyt de helft der somme is het eerste deel, de vorige helft substraheert wijders van het grootste deel, dan is den wortel, uyt de rest, het tweede deel des Binomium. (Ceulen, 1615a, p. 59)

[12]Van Ceulen seems to always consider $a$ as the largest part.

5. Subtract $c$ from $a$: $a - \frac{a+\sqrt{a^2-b}}{2}$

6. Take the root: $\sqrt{a - \frac{a+\sqrt{a^2-b}}{2}} =: \sqrt{d}$. This is the second part of the binomial.

7. Now $\sqrt{a+\sqrt{b}} = \sqrt{c} + \sqrt{d}$

Notice that $d$ can be rewritten as $\frac{a-\sqrt{a^2-b}}{2}$; the formula then becomes:

$$\sqrt{a+\sqrt{b}} = \sqrt{\frac{a+\sqrt{a^2-b}}{2}} + \sqrt{\frac{a-\sqrt{a^2-b}}{2}}.$$

Van Ceulen then shows how this method works by giving an example using numbers and several problems with their answers, and 'proves' his method by calculating the square of one solution and showing that this is indeed the binomial from which the root needed to be taken. Snellius, however, immediately gives Van Ceulens method, not by first stating it in words but with an example using numbers. He corrects one mistake Van Ceulen made in his calculations and then remarks that Petrus Ramus[13] (1515-1572) has a shorter method and describes this method:

**Solution method 6.2.3** (By Ramus)**.**

1. Subtract the squares of the halves: $\left(\frac{a}{2}\right)^2 - \left(\frac{\sqrt{b}}{2}\right)^2 = \left(\frac{a}{2}\right)^2 - \frac{b}{4}$

2. Take the root: $\sqrt{\left(\frac{a}{2}\right)^2 - \frac{b}{4}} =: c$

3. Now $\sqrt{a \pm \sqrt{b}} = \sqrt{\frac{a}{2} + c} \pm \sqrt{\frac{a}{2} - c}$

The complete formula becomes

$$\sqrt{a \pm \sqrt{b}} = \sqrt{\frac{a}{2} + \sqrt{\left(\frac{a}{2}\right)^2 - \frac{b}{4}}} \pm \sqrt{\frac{a}{2} - \sqrt{\left(\frac{a}{2}\right)^2 - \frac{b}{4}}}.$$

As can be seen, Ramus' method is much faster then Van Ceulen's; however, it requires more intricate steps. With this method, Snellius gives not only the solution for taking the root of a binomial number, but also for a residuum, which is the difference between a rational and the square root of a rational number. This method of Ramus can be found in his *Arithmeticae Libri Duo: Geometriae septem et viginti*[14]. Snellius copies the theorem of Ramus almost literally in the *Fundamenta* and gives the first of the five examples given by Ramus.

From this example of the commentaries of Snellius, we can conclude that Snellius preferred the concise and more intricate method of Ramus over the longer and simpler method of Van Ceulen.

[13]Ramus had a considerable influence on Snellius. More on his life and his connection to Snellius can be found in (Wreede, 2007, pp. 30–35).

[14]It is the second theorem in the fifth chapter of the first book on the analysis of square roots (see also (Ramus, 1627, p. 199)). This work by Ramus has many editions including an adaptation by Lazerus Schonerus (Ramus, 1627). This edition was used by Rudolph and Willebrord Snellius to write a commentary on the work of Ramus.

### 6.2.2 A triangle in a circle: two quicker methods

In problem 36 of the fourth book Snellius gives not one, but two alternative methods to find the solution[15]. The problem is the following: given an equilateral triangle ($\triangle FED$) with its base ($DE$) on the diameter ($CB$) of a circle and the top ($F$) on the circumference. If the area of the triangle is given, what is the length of the diameter? In this problem, the area of the triangle is 100.

Van Ceulen's method uses the rule of false position twice. First, he uses a smaller equilateral triangle with sides of $\sqrt{3}$ and calculates the area of this triangle to be $\sqrt{1\frac{11}{16}}$. Then he uses the rule of false position between the area of the triangle against the square of the length of its sides to find $\sqrt{53333\frac{1}{3}}$ for the length of the sides of a triangle with area of 100. The length of the sides of triangle $FED$ are thus $\sqrt{\sqrt{53333\frac{1}{3}}}$. Secondly, to find the diameter of the circle, Van Ceulen again uses the rule of false position by calculating the sides of a triangle when the diameter would be 2. He then finds $\sqrt{1\frac{1}{3}}$ for the sides of the triangle. Then using proportions between the length of the sides and the diameter ($\sqrt{1\frac{1}{3}}$:2=$\sqrt{\sqrt{53333\frac{1}{3}}}$:*diameter*) he finds $\sqrt{\sqrt{480000}}$.

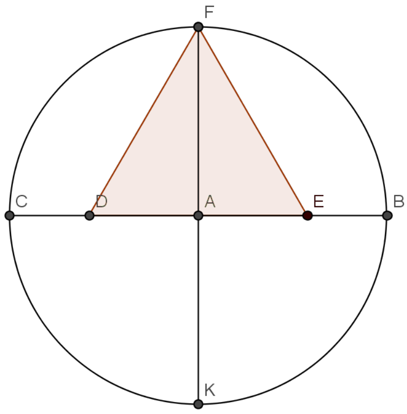



Figure 6.1: Figure from book IV, proposition 36.

Snellius uses the first part to find the sides of the triangle $FED$, but then offers two alternative methods in a comment on this problem. He first calculates the perpendicular $FA$ to be $\sqrt{\sqrt{30000}}$ by expressing the area as half the product of the base line and the perpendicular. Thus the diameter, being twice as much, is $\sqrt{\sqrt{480000}}$.

The second method uses the same principle as Van Ceulen, but with a triangle using simpler numbers, namely 2 for the sides and thus $\sqrt{3}$ for the area of the triangle. In this second method Snellius avoids needing to calculate the sides of the triangle first. He directly uses the proportions between the area of the triangle and the diameter of the circle: $\sqrt{3}:2=100:\sqrt{\sqrt{480000}}$.

Snellius' first method only requires the use of proportions once, as opposed to twice in Van Ceulen's solution. The second method requires fewer steps. Considering this, it can be said that Snellius' methods are quicker and easier to follow.

### 6.2.3 Constructing a square in a triangle

Problem 24 from the fourth book of the *Fondamenten* deals with the problem of a square inscribed in a triangle[16]. In the commentary to this problem one of the added theorems by Snellius can be found, of which De Wreede already discussed two (see section 5.1). I will discuss two more in the next section. The question here is: What is the length of the sides of a square, circumscribed by a triangle, when the lengths of the sides of the triangle are given?

[15] (Ceulen, 1615a, p. 188) and (Ceulen, 1615b, pp. 170–171).

[16] The corresponding problem in the translation *Fundamenta* is numbered 26 (Ceulen, 1615b, p. 161).

In his demonstration, Van Ceulen takes $AC$ as the base for the square and begins his answer with the construction needed to draw this square (see figure 6.2). Next, he calculates the length of the sides of the square (i.e. $HI$) using similarity of triangles. In the end, he also calculates $HI$ when $BC$ or $AB$ are the base for the square. He concludes that the largest inscribed square lies on the shortest base.

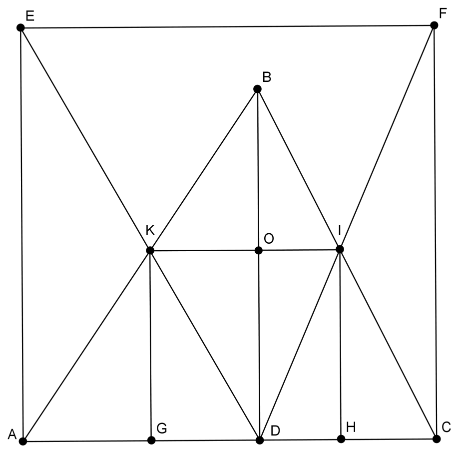



Figure 6.2: The construction belonging to problem 26 of book IV of the *Fondamenten.*

Snellius remarks that Van Ceulen's construction did not take into account the situation where the triangle has a right or obtuse angle[17]. Furthermore, he writes in his commentary on this problem that there is a shorter way to find the length of the sides of the square, namely by using a theorem derived from the subsequent problem 25[18]:

**Theorem 6.2.4.**
*Given a triangle with base b and perpendicular [height] p and an inscribed square with side s, we have* $(b+p) : b = p : s$.

In this particular problem, this results in the proportion $(BD + AC) : AC = BD : KI$. Since $BD = BO + OD$ and $OD = KI$ this theorem simply says that $BD : AC = BO : KI$. This is the proportion between the perpendicular and the base of the similar triangles $ABC$ and $KBI$ or $p_1 : b_1 = p_2 : b_2$ where $b_2 = s$ with $p_1$ and $b_1$ of $\triangle ABC$ and $p_2$ and $b_2$ of $\triangle KBI$. This can also be proven using the similarities of the triangles:

*Proof.* Since $\triangle BDA \sim \triangle BOK$, then $KO : BO = AD : BD$.
Since $\triangle BDA \sim \triangle BOI$, then $IO : BO = DC : BD$.
Hence, $(KO + IO) : BO = (AD + DC) : BD$ or $KI : BO = AC : BD$.
This gives also $BD : AC = BO : KI$ and adding 1 to both sides gives
$(BD + AC) : AC = (BO + KI) : KI$.
Since $KI = OD$ this results to $(BD + AC) : AC = (BO + OD) : KI = BD : KI$. □

[17] Cum in rectangulis et obtusangulis triangulis unicum duntaxat tale quadratum super recti vel obtuse base describe possit, consectariolum istud in illis locum non habet. (Ceulen, 1615b, p. 161)

[18] This is problem 27 in the *Fundamenta*. Snellius refers to problem 24 of the fifth book, but means to refer to problem 42 of the third book of the *Fundamenta* (which is problem 21 in the *Fondamenten*). Here he gives the proof to a similar problem, when a rectangle with sides with a given proportion needs to be inscribed in a triangle. This problem is also similar to problem 40 of book IV of the *Fondamenten*.

This theorem of Snellius indeed gives a much quicker way to find the length of the sides of the inscribed square. Snellius already used a derivative of this theorem in problem 42 of the third book of the *Fundamenta*[19]. Van Ceulen uses a variation of this theorem in problem 27, where he writes that Clavius (1537-1612) gave a general method from Commandinus' (1509-1575) translation of the *Elements*[20]. This commentary placed at the end of the sixth book of the *Elements* might be the original place where this method of constructing a square inside a triangle is given[21]. This also suggests that Van Ceulen may have known this Latin edition by Clavius, although he could not read it. (Perhaps Snellius had translated it for him.) Snellius modified the proportion given by Commandinus, that $BO : KI = BD : AC$ or $p_2 : s = p_1 : b_1$, such that he need not calculate the parts of the perpendicular. With his theorem he could immediately calculate the length of the sides of the inscribed square; all he needed was to find the perpendicular. In this commentary we, again, see that Snellius was driven to find the most direct approach possible.

### 6.2.4 Equiangular triangles: two new theorems

In the 30th problem of book IV of the *Fondamenten*[22], Van Ceulen describes a problem of finding the lengths of the sides of a small triangle ($\triangle KGL$) constructed inside another triangle ($\triangle ABC$) (see figure 6.3). Triangle $ABC$ is circumcribed by a circle with middlepoint $N$ and diameter $AH$. One of the sides of $\triangle KGL$ is part of the diameter of the circumscribed circle ($KL$), cut off in $K$ and $L$ by the intersections of the diameter with the altitudes of the large triangle from $B$ and $C$ ($BD$ and $CE$). The other two sides are parts of the altitudes of the large triangle ($LG$ and $KG$), with $G$ the intersection of all three altitudes.

Van Ceulen's demonstration is long and uses an abundance of similarities between triangles. With his method, which only leads to the length of one of the sides, it is needed to calculate almost every line segment of the figure.

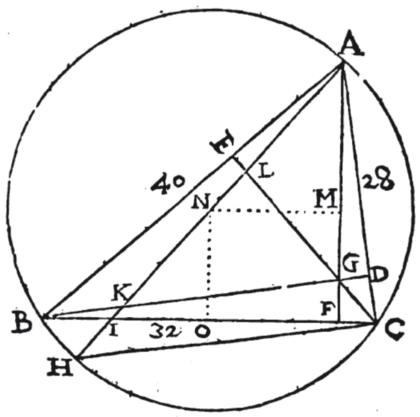



Figure 6.3: The construction belonging to problem 30 of book IV of the *Fondamenten*.

Snellius complains about the elaborate solution method and adds his own method in a long commentary (1,5 pages long). This one is distinct from his usual methods, since Snellius adds

[19]This is problem 21 in the *Fondamenten* (Ceulen, 1615a, p. 151).

[20]Clavius and Commandinus both published a Latin translation of Euclid's *Elements*. Commandinus in 1572 and Clavius in 1574. Clavius' edition is not only a translation, but contains a vast amount of notes collected from previous commentators and editors including some commentaries of his own. (Heath, 1956b, pp. 104–105)

[21]This commentary can be found on page 76 of (Commandino, 1572).

[22]This is problem 32-1 in the *Fundamenta* (Ceulen, 1615b, pp. 165–167). (Ceulen, 1615a, p. 185).

two new theorems which enable him to give a concise demonstration. The first of these theorems relates the sides of the large triangle and the altitudes to the diameter of the circle:

**Theorem 6.2.5.**
*The side of the given triangle added to the part of altitude from the opposite angle to the common intersection of the altitudes, possess as much as the diameter of the circumscribed circle.*[23]

At first, this theorem is somewhat vaguely described and seems to lack the remark that this theorem involves the squares of the lengths. Snellius' demonstration following this statement clarifies the theorem. Take, for example, side $AC$ and the altitude ($BD$) out of the opposite angle ($\angle ABC$) to the common intersection point ($G$), thus $BG$. Then Snellius claims that $AC^2 + BG^2 = AH^2$. He proves this by showing that $BG = HC$ and then using propositions 23 and 31 of the third book of the *Elements*, which are the theorems of Pythagoras and Thales[24]. The proof that $BG = HC$ of Snellius goes as follows:

*Proof.* Since $BD$ and $HC$ are both perpendicular to $AC$, $BG \parallel HC$.
Then $\angle GBC = \angle HCB$.
Also, $\angle HBC = \angle HAC$ (Eucl. III,21).
Since $\angle BFA = \angle HCA$ (right angle) and $\angle AHC = \angle ABC = \angle ABF$ (Eucl. III,21), also $\angle HAC = \angle BAF$.
Now, since $\angle ABC = \angle EBC = \angle ABF$ and $\angle BEC = \angle BFA$, also $\angle BAF = \angle BCE$.
It follows that $\angle HBC = \angle BCE = \angle BCG$.
Then $\triangle CGB$ and $\triangle CHB$ are equiangular and have side $BC$ in common. Therefore $\triangle HBC$ and $\triangle GCB$ are equilateral.
Hence $BG = HC$. □

This can also be proven in a shorter way:

*Proof.* Since $BD$ and $HC$ are both perpendicular to $AC$, $BG \parallel HC$.
By the theorem of Thales $\triangle ABH$ is right-angled, hence $BH \perp BA$.
Now with $EC \perp BA$ it follows that $EC \parallel BH$.
Thus $CGBH$ is a parallelogram, hence $BG = HC$. □

After Snellius' demonstration of this theorem, Snellius comments on Van Ceulen's 'rough and obliging' method which needs many calculations, and again poses a theorem which avoids this and shortens the solution method[25].

**Theorem 6.2.6** (Equiangular triangles)**.**
*Given $\triangle ABC$ with a diameter of its circumscribed circle drawn from $A$, and from $B$ and $C$ two altitudes intersecting the diameter in $K$ and $L$ and each other in $G$. Then $\triangle KGL$ is equiangular to $\triangle ABC$.*

*Proof.* Since $\angle ACB = \angle AHB$ (Eucl. III,21) and $BH \parallel EC$, also $\angle AHB = \angle ALE = \angle KLG$, thus $\angle KLG = \angle ACB$. Then $\angle LGK = \angle BAC$ follows from $LG \perp BA$ and $KG \perp AC$. Finally, $\angle ABC = \angle AHC$ (inscribed angle) $= \angle AKD = \angle LKG$ (this last angle can also be calculated by $180° - \angle KLG - \angle LGK$). □

[23]Latus dati trianguli cum segmento perpendicularis ab angulo opposito ad communem perpendicularium sectionem aeque possunt diametro circumscripti circuli. (Ceulen, 1615b, p. 166)

[24]Notice that sides $AH$, $HC$ and $AC$ form the right-angled triangle $AHC$.

[25]Caeterum scrupulosa et prolixa est autoris haec via, et incurrit in numerous nimium vastos, quibus hoc zetema interpolare minime opus fuit: quamobrem studioso lectori breviorem aliam subministrabo. (Ceulen, 1615b, p. 167)

With this theorem, the solution to the original problem can be found rather easily. Using the theory given in previous problems of the *Fondamenten*, one can find the lengths of the diameter, the altitudes and their parts. Using similarities, it follows that $\triangle AEL \sim \triangle AHB \sim \triangle AFC$, hence $AF : FC = AE : EL$ hereby finding the length of $EL$. Now $CG = BH = \sqrt{AH^2 - AB^2}$ which gives the length of one of the sides of the triangle: $LG = EC - EL - GC$. Now using the second theorem one finds that $AC : CB = LG : LK$ and $AC : AB = LG : KG$, which gives the lengths of the other sides $LK$ and $KG$.

This method evidently needs far fewer calculations and is much more concise than Van Ceulen's method. This commentary of Snellius shows us that he favoured concise solution methods. Adding two theorems of his own shows his knowledge of geometry.

# Chapter 7

# Conclusion

In this last chapter I will attempt to draw conclusions on the questions presented in the introduction. First, I will draw some conclusion from the first chapter by comparing the lives of Van Ceulen and Snellius and the choices they made for the publication of the *Fondamenten* and *Fundamenta*. Secondly, I will start by answering the second question on what the purpose of the works might by. Enclosed in finding the answer to this question are the subquestions of what the intended audience of the works could be and whether or not the *Fondamenten* was intended to be one combined work. Thirdly, I will present my findings on the many different editions of both works, their publications year and the peculiarities of the *Coss-book* and *De Circulo*. Finally, I will answer the main question on the different commentaries added by Snellius and how these shed some light on the meta mathematical differences between Van Ceulen and Snellius.

## 7.1 Difference in education

In the first chapter the lives of Van Ceulen and Snellius are disclosed. When comparing the two mathematicians, the difference in education is significant. Where Snellius was able to learn from the original Latin and Greek works, Van Ceulen only had knowledge on what was translated for him. Thereby, Snellius had enjoyed a sound education in mathematics, where Van Ceulen did not. However, the intensive training that Van Ceulen gave himself in working with numbers and developing intrinsic algorithm made him rise above the numerical skills of Snellius and many others.

In the *Fondamenten* Van Ceulen included the most basic of mathematics by beginning with explaining how to pronouns numbers. Snellius, however, skipped the first five chapters, hereby leaving out the basics and starting with what he probably regarded as more substantial mathematics. Snellius' choice is hereby consistent with his level of education in comparison with that of Van Ceulen.

## 7.2 Different audiences

From section 3.1 it may be concluded that Snellius and Van Ceulen had a different audience in mind for their works. From all the remarks Van Ceulen made to his readers, it follows that his audience probably consisted of beginners in mathematics. They required the basic theories given in the first two chapters, before they were able to work on the problems of book three. Nevertheless, the level of difficulty in chapters four to six may give rise to the question whether those chapters were intended for a similar audience. Evidently, Van Ceulen had a more learned audience in mind for the sixth book, which follows from the last quote given in section 3.1.1. Thus, Van Ceulen's audience consisted of readers from a broad range of mathematical levels, from pupils to scholars. The work is structured such that the beginner can eventually understand the final book when he has trained himself with the problems from all the previous books.

Snellius aimed his translation of the *Fondamenten* to a more educated audience. In the first place, they needed to be able to read Latin (and sometimes Greek). Second, since Snellius left out a great part of the basic theory from the first chapter (see chapter 3.4), he might have assumed that his readers already possessed the more basic knowledge. This also follows from his remark made in the letter to Rosendalius, where he specifically names his audience as being 'international learned' (see 3.1.2. From the dedicatory letter placed at the beginning of book three, De Wreede concluded that Snellius might have used the work for his students at the university. All this indicates that the audience Snellius had in mind for the *Fundamenta* was different from Van Ceulen's originally intended audience.

## 7.3 Different books

The *Fondamenten* was in fact intended to be one combined work. This follows from all the references made back and forth throughout the work from one book to the other. Van Ceulen's references are all made to a specifically numbered book, which reveal the order in which Van Ceulen had intended his work. Also the ordering of the content, from easy to difficult, suggests that it was intended to be bundled in that specific order. The fact that book two, containing propositions from Euclid's *Elements*, may have been extended to included propositions needed in later books suggests that Van Ceulen worked on the work as a whole (see section 3.3). Finally, with a problem left unfinished in book four and then finished in book five, it must be concluded that the *Fondamenten* was intended to be one combined work. With this conclusion, I stand against the conclusion made by Vlek that books five and six may not have been intended to be part of the work (see 2.1) and the similar suggestions made by De Wreede and Katscher (see section 3.2).

## 7.4 Different purposes

Since the intended audiences differ, it follows that the purposes which Van Ceulen and Snellius might have had must also be different. The original *Fondamenten* seems intended to be used as a text- and workbook at the engineering school where Van Ceulen taught mathematics. In many places Van Ceulen stresses his readers on the importance of practise. This also follows from the abundance of worked out examples and the ascending level in the content of the books.

While Van Ceulen intended his work to let his readers study the theory and work on the given problems, Snellius was more focused on presenting the most concise method possible. Therefore, Snellius was more adapted to the classic works of great mathematicians before him than Van Ceulen was. His purpose with the translation can be linked to various intentions. In the first place it could simply be regarded as an answer to the request of Romanus to translate the works of Van Ceulen to Latin. From the letter to Rosendalius and the extra dedicatory letter, we know that Snellius used the *Fundamenta* to 'spread the work of Van Ceulen' and also to rise up in his career. He might have used it for his own students at the university. The commentaries show that Snellius also used the *Fundamenta* to publish his own findings. With all these different intentions it is difficult to conclude what the main purpose of Snellius might have been with the *Fundamenta*.

## 7.5 Different editions

While working on this thesis, I became curious as to how many editions of the *Fondamenten* and the *Fundamenta* were published. From the research presented in chapter 4, we can now draw a conclusion on this subject.

As to the publication year of the *Fondamenten*, we can conclude that there is no edition printed in 1595. However, there might be some other than from 1615. The year 1616 is the most likely, since there are sources that give this year as the publication date, stating specifically that the *Fondamenten* was published a year after its Latin translation. The *Fundamenta* was published in 1615, 1618 and maybe in 1617.

There are at least five different editions of the *Fondamenten*, all printed in 1615, with the only difference being the dedicatory page and letter. Three of these were already mentioned in other sources; two I found myself. One of these two editions is dedicated to Willem of Nassau, the other to the States of Zeeland (section 4.1.2).

The *Fondamenten* is not the promised *Coss-book* to which Van Ceulen refers in his *Vanden Circkel*, since at the end of the *Fondamenten* the promise for this work is made again.

Of the *Fundamenta* I found four editions, of which one is printed in 1618, which is actually an exact copy of one of the three 1615 editions. The three editions from 1615 are identical except for the titlepage, since they have different publishers. There are two different dedicatory letters, for it is missing in one of the editions. Some sources suggest that there must be a fifth edition published in 1617. The Latin translation of *Vanden Circkel*, titled *De Circulo*, was mostly a reprint of the *Fundamenta*, hence it may be regarded as a sixth edition.

The large amount of different editions and the distribution of the works throughout all of Europe (see Appendix C) suggest that the *Fondamenten* and its translation were in high demand, and the notes found inside the works show that most of them were intensively used.

## 7.6 Different opinions

In chapter 5, the question of what (kind of) commentaries Snellius added to the *Fundamenta* is answered. Snellius corrected errors he found in the *Fondamenten* and simplified the theory given in the first book. He reformulated the propositions from the second book, so that they became more Euclidean (section 5.1). He complained about a lack of time to add elaboration and figures, but was able to have three new figures cut corresponding to his added alternative solution method (section 5.2). Sometimes he let the reader know that it had been in fact himself who proposed a question to Van Ceulen. After some problems, Snellius gave his own opinion on the approach of Van Ceulen, especially when it involved multiplications of and division by line segments. He expanded explanations to make them more comprehensible and complete, pointed out the utility of presented problems and completed work when parts of it were missing (sections 5.3 and 5.4.2). Furthermore, he added alternative solution methods and theorems of his own (section 5.5).

## 7.7 Different approaches

Van Ceulen preferred to present a large variation of solution methods to one or similar problems. He mainly uses many words to be as understandable as he wanted to be (section 6.1). Where Van Ceulen's method often are very long and tedious, Snellius sometimes adds a commentary in which he presents his own alternative solution method. From four different examples we have seen that Snellius preferred more intricate but specifically more concise methods. He favoured methods that need fewer steps or avoid weary calculations. His methods are often faster, more direct and easier to follow, although he often uses more complex steps (section 6.2). Finally, the addition of new theorems also shows Snellius' mathematical abilities (section 6.2.3 and 6.2.4).

## 7.8 Discussion and further research

In this master thesis I have examined all the contributions made by Snellius to the *Fundamenta*. Hereby, I have chosen not to skip those contribution on which other researchers already had given an extensive analysis. Also, to illustrate my observations I have selected contributions that best show the characteristics for a specific type. There are still several commentaries of Snellius that have not been thoroughly analysed. For a future study on this subject, it is recommended to collect all the research done by De Wreede, myself and others. This could present a complete overview of all the different contribution that Snellius added to the *Fundamenta*.

As for the algebraic proof given in section 5.5, it would be more suitable if there was a geometrical proof for the theorem of Snellius. After my presentation of this master thesis, one of the mathematicians in the audience came to me with his own 'proof without words' to the theorem. I have added his contribution in appendix D.

I really enjoyed working on this thesis. With all the sources handed to me I could continue my research for another two years. But for now, I am very proud of this final result. I hope you enjoyed reading this thesis and do not hesitate to contact me when you discover new information on this subject.

Maartje van der Veen BSc
mrtjvanderveen@gmail.com

# Appendix A

# Dedicatory pages of the *Fondamenten*

## A.1 Dedication to Count Mauritz of Nassau

AEN DEN
Hooch-gebooren Vorst ende Heere
MAURITZ
ghebooren Prince van Orangien, Grave
van Nassau, Catsenelleboge, Vyanden,
Dietz, Meurs, etc.
Marquis vander Veere ende Vlissingen, etc:
Heere van Polanen, Leck, Grave, Cuyck,
S. Vijt ende Doesburch, etc.
Ridder vande ordre des Causebants:
Gouverneur ende Capiteyn Generael over Gelderlant,
Hollant, Zeelant, West-Vrieslant, Zutphen, Utrecht,
ende Over-yssel; Admirael Generael vande
Nederlantsche Zee.
Mitsgaders, de Edele Hoochmogende, Wijze, Voor-
zienighe Heeren, mijn Heeren de Staten van
Hollandt ende West-Vrieslandt.

Mijn Genadige, Gunstige, ende Gebiedende HEEREN,

Daer sijn verscheyde oorsaken waer door de lofwaerdighe wetenschappen niet alleen by de gemene-man, maer oock insonderheyt by de groote in estijm en waerde ghehouden sijn. Want ettelijcke onder haer merckende de cortheyt deses levens hebben alleenlijck ghearbeyt om haer naem byhaere nasaten, en in toekomende tijden ruchtbaer te maken, en daerom groote wercken aengericht, de welcke niet lichtelijck vergaen, of oock naegedaen souden worden, als daer is geweest dat grouwelijck werck een Moeder van confusie, den Toren van Babel, de costelijcke en onnutte Pyramides by eenige Coningen van AEgypten opgebout, waer aen so meenich duysent menschen twintich Iaren lang gestadelijcken gearbeyt hebben, datse alleen in rhadijs, ajuyn en loock, verre over de dertichmael hondert duysent guldens verteert hebben: Item de onnutte doorgravinge van den berg Athos by Xerxes geattenteert, om dat men daer juyst met schepen soude, daer te vooren niet alleen lant maer oock hooge bergen lagen: S'glijks de Colossus ofte beelt staende schrijelinx over de haven van Rhodus, vijf en t'seventich cubiten hoog, alsoo dat het grootste Schip met staende masten daer onder deur passeren mocht: Ende noch meenich ander groot werck by de Romeynen aengeleyt: welcke alle tesamen anders geen wit en hadden, dan allen haer eygen naem groot en vermaert te maken, het welck een grouwel voor den Alderhoogsten is, gelijck wy expresselijck aen den Tooren van Babel, ende den hoogmoedighen roem van den Conink Nebucadneser sien moghen: want Godt uyt sijnen throon gint werck verstoort, en desen Conink uyt sijn heerlijcheyt verstoten ende het onvernuftige vee gelijck gemact heeft, het welcke ettelijcke onder de Heydenen selfs wel bemerkende, alsucke persoonen vergleken hebben by groote Reusen die de natuer bevechten en den Hemel bestormen wilden. Daerom andere dewelcke de sake een weynich nader hebben ingesien en nagedacht, hebben de wetenschappen bemint, ende de selve onderscheyden, en geestimeert nader selver subtiliteyt en geestichheyt, gelijck als daer sijn die verscheyden kunsten ende stucken by subtyle geesten geinventeert tot vermaeck des menschelijcken geslachts en ciraet van gansche Republijcken: de welcke men warlijc voor een singuliere gave van Godt den Heyligen-Geest aennemen en achten moet. Also sien wy by Mose int 36 cap. Exodi dat den Heere de subtile kunstenaers Betsaleel ende Aholiab verwect heeft om het werck van sijn heylige Tabelnakel van tapitserie, gout, en silver seer kostelijc te maken. Welcke liberaliteyt hy selfs den Heydenen niet onthouden en heeft, op dat sy uyt hare diepe afgodische slaep ontwakende den eenigen Godt soeden leeren erkennen, alsmen sien mach in de overtreffelijcke meesters Phidias, Apelles, Lysippus ende andere ontallickeCunstenaers meer, welker welken met anders als om haer grote kunst en subtiliteyt geacht sijn geweest, al hoe wel de selve dichmael geen sonderlinge gebruyck ofte nut en hadden. Daerom hebben sy ten laetsten als verstandige waerdeerders de sake selfs met de oogen des verstants afgemeten, wel wetende dat de grootheyt des wercks alleen een ijdele verwonderinghe; de subtiliteyt, een frayheyt en aerdichheyt mede brengt; maer beyde dickmaels sonder groot vordeel ofte proffijt als de noot het selve vereyst: hebben daerom de subtiliteyt tot nut en proffijt des menschelijcke gheslachts geimploijeert, ende haer recht ghebruyck aengewesen. Waer onder dat men bekennen moet dat de wetenschappen der Mathematike onder andere mede de principalste sijn, want sy de andere niet alleen in subtiliteyt te boven gaed, maer oock nootwendich en profitabel sijn: En glijck hare werckinge veelder handen en verscheyden is, so brengt sy mede verscheyde nutticheyt en vruchten voort, soo wel in tijden van oorloge, als oock in tyden van vrede: daerom sy mede tot allen tijden by alle treffelijcke Coningen en Princen een sonderlinge faveur hebben gemeriteert. Maer op dat ick nu verswyghe alle exemplen diemen tot desen eynde dienende soude mogen allegeren, wat soude wy doch voor een krachtiger ghetuygenisse voort mogen brengen ofte allegeren, anders als u Levendich voorbeelt o Ghy Fleur der Princen, de welcke niet alleen ale Coningen en Princen in't gebruyck van desen, maer oock inde subtielste speculatien alle te boven gaet die oyt in dese wetenschappen vermaert geweest sijn. Daerom bidde ick uwer E. Princelijcke Excellentie sijne ogen op hen selve als een volmaeckt exempel te willen staen: Ende dat hare Mogentheden gelieve hare ogen van mijne doode redenen tot dien actueusen en levendigen Prince te wenden, ende sijne Ridderlijke daden tot satisfactie van de defecten mijner woorden te ontfangen. Daerom dan nademael dese wetenschap (daer toe dit boeck mede is dienen) niet alleen vermakelijck is om de subtiliteyt der saken die daer in verhandelt werden, maer ooc mede dienstbaer om de nootwendige wetenschap der selven; soo en hebbe ick niet kunnen nalaten om de overgroote vlyt en arbeyt de welcke mijn Man saliger Meester Ludolf van Ceulen in dese heerlijcke wetenschap sijn leefdaghen aengewent en overgebracht heeft de selve nae sijn overlyden aen dach brengende den kunstlievenden te communiceren, En uwer E. Princelijcke Excellentie als aen het roer van dese Lnaden sittende, Midtsgaders uwe Hoogmogentheden als Vaderen des Vaderlantds onder wiens vleughelen en bescherminghe hy dit gheschreven heeft, te dediceren: om daer mede die onderlinge affectie de welke mijn Man salger uwe EE. altijt toe gedragen heeft, demoedelijcken te kennen te geven. Verhopende soo dit van Sijne Princelijcke Excellentie met goede oogen aengesien, ende van hare Mogentheden als aengenaem ontfangen wert, naer desen noch meer van sijnen arbeyt tot het gemeene nutt, en der subtijle geesten lust aen den dach te brengen.

uwe Princelijcke Excellentie,
en Uwer E.E. Mogentheden

Ootmoedighe
ADRIANA SIMONS
*Weeduwe van Ludolf van Ceulen.*

## A.2 Dedication to Ernest

AENDEN Edelen,
Doorluchtigen ende Hoochgebooren Grave,
GRAEF ERNEST van NASSAV,
Catsenelleboge, Vyanden, Diest, etc:
Heere tot Bilstein, Maerschalck du Camp, ende Gouverneur van Gelderlandt.
MITSGADERS De Edele, Moghende, Hoochvvijse, ghebiedende Heeren,
MIJN HEEREN DE STATEN DER PROVINTIE VAN GELDERLANT

I have not seen the original edition dedicated to Count Ernest. This is the only part I have seen from the dedicatory letter, taken from (Bierens de Haan, 1878, p. 144):

> Hebbe derhalven oock —— dese Aritmetische ende Geometrische Fondamenten van Mr. Ludoff (sic) —— van Colen mijn man sal: ged: de welcke al over lange jaren van den —— Autheur selve (in sijn boeck gheschreven vanden Circkel) zijn be- —— looft gevveest, doch van wegen zijn veelvoudige, soo publijcke als —— particuliere occupatien, tot noch toe inghehouden, int licht laten —— comen, ten dienste der nakomelinghen.
> ADRIANA SIMONS Weeduwe van Mr. Ludolf van Ceulen[1].

## A.3 Dedication to Willem of Nassau

AENDEN
Edelen, Hoochgebooren Vorst ende Grave
GRAEF WILLEM van NASSAU
Catsenelleboge, Vyanden, Diest, etx: Heere tot Bilstein,
Gouverneur van Vrieslant, Groeningen, ende de
Ommelanden.
Mitsgaders
De Edele, Hoochwijse, vermogende, gebiedende Heeren,
Mijn Heeren de Staten der Provintie van
Vrieslandt.

Mijn Genadige, gunstige, ende gebiedende Heeren,

[1](Bierens de Haan, 1878, footnote no. 17)

Naerdien Alexander de Groote, Edele ende Hoochmogende Heeren, verstaen hadde dat sijne Meester Aristoteles, zijn ganstsche Philosophie, de welcke hy uyt zijnen mont ghehoort hadde, int lichtende aende dach hadde ghebracht, so is het, dat hy hem hier over door brieven grootselijckx aen hem heeft beclaecht, om dat hy soodanighen wetenschap ende kennisse, die daer behoorde selfs boven de Konincklijcke kroone ende den Scepter geacht te worden, den kleynste ende gheringsten persoonen hadde gheopenbaert. Welcke daedt Alexandri, alhoewel dat zy in allen deelen niet en is te prijsen, om datse vande menschen schijnt te willen wech nemen, het geene hem aldermeest verciert. Soo ist nochtans dat wy hier connen sien ende bemercken, hoehooch hy de Philosophie heeft gheacht, te weten, dat oock selfs de Coninghen ende Princen, veel min andere liberalia ingania, hem niet en behoorden te schamen haer leven inde oeffeninge der selver te besteden. Doch naerdien het leven aller menschen hier beneden op der aerden cort is, en de dese wetenschap seer lang, alsoo datse in alle deelen naulickx van eenich mensche soude connen perfectelijck gheabsolveert werden: Soo en schijnen die geensins den minsten lof ende prijs waerdich te wesen, de welcke haer leven ghedurichlijck besteden inde oeffeninge van die deelen der Philosophie, waer door de Republijcken soo ten tijde van oorloch, als van vrede grootelijcks worden gheemendeert ende verbetert. Onder de welke geensins de laetste is Geometria, waer van (om niet verre te gaen) uwe Ed: Mog: als die gheene de welcke het selvighe, gheduerende den bloedigen ende swaeren oorloch in dese onse Vaderlanden, hebben bevonden, goede getuychenisse soude connen geven: ende daerom oock dese Scientie op alle manieren hebben ghepatrocineert, ende ghesocht te vorderen ende voort te planten. Hebben derhalven billick ende behoorijck gheacht te wesen dese Geometrische ende Arithmetische fondamenten, van Mr Ludof van Colen mijn man sal: ged: (het welcke hy al by zijn leven inde praefatie van zijn boeck geschreven vanden Cirkel heeft belooft, doch van weghen zijne groote ende meenighe occupatien, soo publijcke als particuliere tot noch toe ingehouden) in het licht te laten gaen, ten dienste der nacomelinghen, ende uwe Ed: Mog: te didiceren ende toe te schrijven. Voor eerst, om dat ick wiste ende seker was dat uwe Ed: Mog: Patronen ende voorstanders zijt liberalium artium ende Philosophiae, het welcke seer wel blijckt uyt de groote sorge ende acht die uwe Ed: Mog: over uwe Universiteyt ende Hooge Schole tot Franeker (die daer is een seminarium ende say-plaetse van soodanighe scientien) draecht. Ten tweeden, om dat het Princelijck huys van Nassau dese wetenschap altoos in grooter estime ende waerde heeft ghehouden, jae sich niet en heeft gheschaemt selfs verstant ende sinnen inde oeffeninghe der selver te besteden. Ten laetsten oock om datse een groot gebruyck heeft in uwe Ed: Mog. administratie ende bedieninge, soo gheduerende desen vrede, als insonderheyt ten tijden van oorloghe, de welcke ons (alsoo het blijckt) gestadich over het hooft hangt. Versoeckende ootmoedichlijck dat het uwe Ed: Mog: ghelieve desen arbeyt van Mr Ludolf van Colen sal: ged: in danck te willen aennemen, ende ghelijckerwijs een faetum posthumum te patrocineeren ende te beschermen.
Edele, Entseste, Hoochwijse, Vermogende, gebiedende Heeren, de Godt des Vredes beware uwe Ed: Mog: in lanckdurighen voorspoet, heyl, ende ghelucksalichheydt, tot bescherminghe van dese onse Vaderlanden, opbouweinghe ende voortplantighe van zijn Kercke ende Gemeynte, ende grootmakinge van zijnen heyligen name.

uwe Vorstelijcke ghenaden,
en Uwer E.E. Mogentheden

Ootmoedighe
ADRIANA SIMONS
*Weeduwe van Ludolf van Ceulen.*

## A.4 Dedication to the Admiralties

Aende Hoochweerdige Voorsienige, wijse Heeren,
DE HEEREN SVPEPINTEN-
denten ende Raden der Admiraliteyten van Hollandt ende West-Vrieslandt[2]

Aende Hoochweerdige Voorsienige, wijse Heeren,
DE HEEREN SUPERINTEN-
denten ende Raden der Admiraliteyten van Hollandt
ende West-Vrieslandt,
Mijn genadige, gunstige, ende gebiedende Heeren,

Naerdien de Philosophie, Hoochweerdige ende wijse Heeren, die daer is een kennisse ende wetenschap van Goddelijcke ende menschelijcke dinghen, niet en is ghevonden door het verstant ende de subtijlheydt der sterffelijcker menschen, maer een louter geschenk der onsterfelijcken Gods is: Het welcke oock der Philosophen voorloopers, de Poeten hebben willen te kennen gheven, wanneer sy hebben gedichtet dat Minerva de Goddinne ende Moeder der wijsheydt uyt de hersenen Iovis des alderoppersten Gods soude voort gekomen wesen. Soo en isser niet keerlijcker ofte waerdiger voor een mensche, die een redelijcke Creatuere van Godt geschapen is, dan dat hy sijn ghemoet ende sinnen gheduerichlijcken inde oefeninghe der Philosophie bestede, ende insonderheyt in die deelen der sever, de welcke hoewel sy de waerdichste ende alderuutnemenste sijn, nochtand door het verkeert oordeel veeler menschen, die welcke alle studien voor slecht ende ghering achten, die de menschen niet en verheffen tot groote digniteyten ende waerdicheden, ende de deure openen tot overvloeyende rijckdommen, worden veracht ende als met voeten getreden. Onder welcke wel de bysonderste is Mathematica, een wetenschap van kleynder waerden by den onverstandighen ende ongheleerden, maer van onwaerdeelijcken prijse by den verstandighen, ende onuutspreckelijcke costelicheyt in sich selven, alsoo dat die dese wetenschap wil wechnemen, de Sonne uut de werelt schijnt te willen wechnemen: want het is een wetenschap niet min ten tijde van oorloch, als van vrede nut ende hoochnodich, ende principalijck Geometria wesende een vande bysonderste deelen deser scientie. Het welcke, op dat ick niet te lang en sy in het verhalen van andere Coninckrijcken ende Landen, seer wel heeft ghebleken in deese onse Provincien gheduerende den lanckduerigen ende bloedighen oorloch: want is het niet Geometria gheweest, door de welkcke veele Steden ende Fortressen byna onverwinnelijck sijn gemaeckt? door de welcke nieuwe munimenta ende Schansen, dienende tot bescherminght van onse Vaderlanden ende Vryheden sijn begrepen? Door de welcke onser vyanden sterckten sijn beklommen, begraven ende beschanst? Ia is het niet dese wetenschap, door de welcke noch daghelijcks, staende desen Vrede, onse Steden bequamelijck worden vergrootet? onse Huysen ghebouwet? ende een yghelijck het sijne, als op het nauste wort toeghemeten? Daerom het goede opset ende voornemen der gheener seer hoochlijck is te prijsen de welcke dit studium vlytelijck oeffenen, ende nae lanckduerighen arbeydt ende oeffeninghe, eenighe monumenta haere nakomelinghen ten dienste nalaten. Onder de welcke hoewel wel de eerste ende voornaemste sijn Archimedes ende Prolomaeus, soo en achte ick nochtans niet dat daerom den arbeydt ende moeyte van anderen, die in desen oock haere nakomelinghen eenighen dienst hebben willen bewijsen, behoort veracht ofte verwerpen te worden. Hebbe derhalven dese Geomestriche en Arithmetische fondamenten, welcke al ouer lange Iaren vanden Autheur Mr. Ludolf van Collen mijnen Man saliger ghedachtenisse inde praefatie van sijn boeck gheschreven van den Circkel is belooft geweest, doch van weghen sijne groote ende veelvoudighe occupatien, waer mede hy in sijn leven, soo van weghen sijne Professie, als oock andere particuliere verhinderdissen is belet tot noch

[2] This dedication is also quoted in (Bierens de Haan, 1878, footnote no. 17 and p. 148)

toe achter ghehouden, in het licht laten komen: Ende u E. Hoochweerdighe ende Wijse Heeren willen dediceren ende opdraghen: Vooreerst om dat ick wel weiste ende versekert was dat ghy soodanige liberalia studia ende oeffeninghen van herten beminnet ende toeghedaen sijt. Ten tweeden om dat oock dese wetenschap seer noodich is, ende een groot ghebruyck heeft in u E. administratie ende bedieninge. Versoeckende ootmoedichlijck dat ghy tselfde in danck wilt aennemen, ende onder u patrocinie ende bescherminghe beschutten ende bewaren.

Uwer E.E.
Ootmoedighe
ADRIANA SIMONS
Weeduwe van Ludolf van Ceulen.

## A.5 Dedication to the States of Zeeland

Aende
Edele, Mogende ende Hooghwijse Heeren,
DE HEEREN STATEN
der Provincie van
ZEELANT.
De Hoochweerdige, Voorsienige, wijse Heeren,
DE HEEREN SUPERINTENDENTEN
ende Rade vande Admiraliteyt selver
Provincie.

Mijn gunstige, ende gebiedende Heeren,

Seer wel seyt den wijsen ende wijtberoemden Plato, Edele ende Hoochmoghende Heeren, datter geen heerlijcker ende uitnemender gave den sterfelijcken menschen vanden onsterfelijcken Godt en is gegeven, ofe oyt sal ghegeven werden, dan de Philosophie. Waer op oock diende den welsprekende Orateur Marcus Cicero, heeft met grooter verwondeinge uitgeroepen: O Philosophia, die daer zijt een leytsman van ons leven, een ondersoeckster der deuchden, ende verwerpster der ghebreken, wat souden niet alleen wy, maer het leven alles menschen hebben gheweest: Ghy zijt een laer-moeder der Steden, ghy hebt de woeste ende ongetemde menschen tot eene societeyt geroepen, ghy hebt se eerst door tsamenwonnighe, daer nae door het houwelijck ende de ghemeenschap van sprake tsamen verknocht, ghy zijt een vinster der wetten, een meestresse der seden ende discipline, tot u nemen wy ons toevlucht, van u vereyschen wy bystant. Seer heerlijcke ende overtreffelijcke woorden, de gantsche Philosophie op het hoochste verheffende, ende de oeffeninge der selve allen menschen aenprijsende. Doch nadien het leven der menschen op der aerden seer cort is, ende de Philosophie seer lang, alsoo datse naulicks van eenich mensche in alle haere deelen soude connen perfectelijck geadsolveert werden. Soo en schijnen die geensins den minsten loffende prijs te meriteren, die haer ghemoet ende sinnen geduerichlijck besteden inde oeffeninghe van die deelen der Philosphie, waer door de Republijcken ende Landen, soo ten tijde van vrede, als insonderheyt van oorloge, grotelijcks worden geemendeert ende verbetert. Onder de welcke gheensins de geringste is Geometria, waer van (om niet verre te gaen) uwe Ed: Mog: als de geene die welcke het selvighe, beneffens de andere geunieerde Provincien, geduerende den swaren ende bloedigen oorloch in dese onse Vaderlanden hebben bevonden, goede ghetuychnisse soudet connen gheven. Hebbe derhalve oock billick gheacht te wesen, dese Arithmetische ende Geometrische Fondamenten van Mr. Ludolff van Colen mijn man sal: gel: in het licht te laten comen, den nakomelinghen tot dienste ende uwe Ed: Mog:

als Maecenates wesende litterarum ende humanarum disciplinarum, de selfe te dediceren ende op te dragen: Ootmoedichlijck versoeckende dat ghy desen arbeyt van Mr Ludolph van Colen sal: ged: in danck wilt ontfanghen, ende meer in toecomende tijden van zijn werck verwachten.

Edele, Hoochmoghende ende Hoochwijse Heeren, de Godt des wijsheyts ende cloeck moedicheyts sy met u allen, ende segene uwe regeringhe, tot welstant van dijne Onderdanen, ende grootmakinge van zijnen grooten ende heylighen naeme.

uwe E.E. Mogent.

Ootmoedighe
ADRIANA SIMONS
*Weeduwe van Ludolf van Ceulen.*

# Appendix B

# Dedicatory pages of the *Fundamenta*

## B.1 Lacking a dedication

This edition was printed *Apud Iacobum Marcum Bibliopolam, Anno 1615* and reads on the last page *Ghedrucky to Leyden, By Ulderick Cornelis. ende Ioris Abramsz. Anno 1615.*
This edition can inter alia be found in the library of the University of Leiden, the Netherlands.

This edition of the *Fundamenta* published by Jacob Marcus lacks a dedicatory page. Two versions, which are digitalised for Google Books[1], have some interesting notes written on the titlepage (see section 4.2 and figure B.1).

## B.2 Dedication to Alberto, Ordinibus and Ambrosio Spinolae

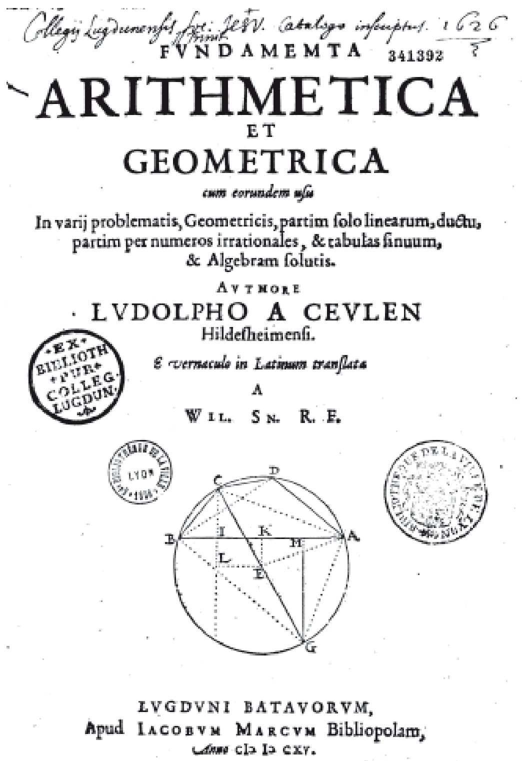
This edition is printed *Excudebat Georgius Abrahami A Marsse, Anno 1615.*
It is exactly the same as the edition printed in 1618, by the same publishers. This one can be found in the Tresoar library in Leeuwarden, the Netherlands.

The dedicatory page reads:

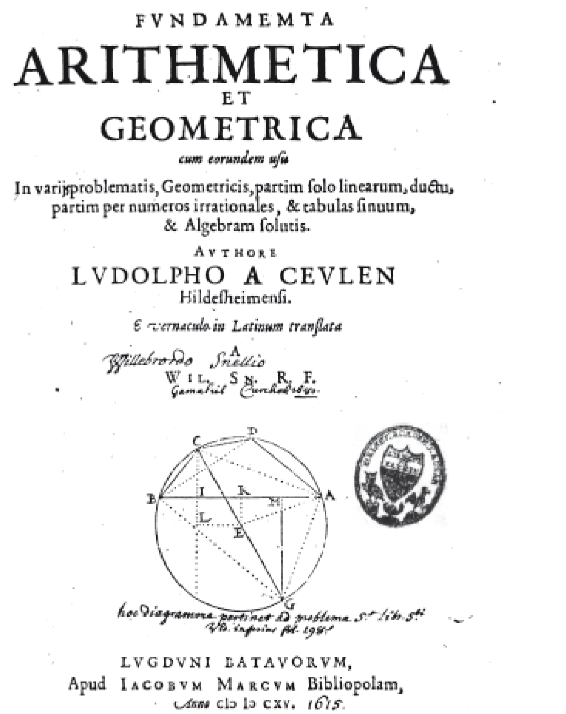
Illistrissimo, generosissimo, potentissimoque principi,
D. ALBERTO,
Dei tratia, Archiduci Austriae, Duci Burgundiae, Brabantiae, Lymburgiae, Lutzenburgiae, etc.
Comiti Flandriae, Artesiae, etc.
nec non
Nobilissimis, amplissimis, prudentissimisque
D.D. ORDINIBUS
Brabantiae
ut et
Splendidissimo, nobilissimo, fortissimoque heroi
D. AMBROSIO
SPINOLAE
Duci S. Severin, Principi Saravalae, Marchioni Benafrae: etc.

The dedicatory letter reads:

[1] See `http://books.google.nl/books?id=3kJ3YSKSOwgC` and `http://books.google.nl/books?id=1S8VAAAAQAAJ`.


FVNDAMEMTA
ARITHMETICA
ET
GEOMETRICA
cum eorundem usu
In varij problematis, Geometricis, partim solo linearum, ductu, partim per numeros irrationales, & tabulas sinuum, & Algebram solutis.
AVTHORE
LVDOLPHO A CEVLEN
Hildesheimensi.
E vernaculo in Latinum translata
A
WIL. SN. R. F.
LVGDVNI BATAVORVM,
Apud IACOBVM MARCVM Bibliopolam,
Anno CIƆ IƆ CXV.

(a) The edition can be find in the Public Library of Lyon, France.


FVNDAMEMTA
ARITHMETICA
ET
GEOMETRICA
cum eorundem usu
In varij problematis, Geometricis, partim solo linearum, ductu, partim per numeros irrationales, & tabulas sinuum, & Algebram solutis.
AVTHORE
LVDOLPHO A CEVLEN
Hildesheimensi.
E vernaculo in Latinum translata
WIL. SN. R. F.
LVGDVNI BATAVORVM,
Apud IACOBVM MARCVM Bibliopolam,
Anno CIƆ IƆ CXV.

(b) The edition can be find in the Bayerische Staatsbibliothek of München, Germany and in the National Library of Rome, Italy. With notes made by Curchod (see section 4.2).

Figure B.1: The titlepage of the *Fundamenta* published by Jacob Marcus.

Illustrissimi, Potensissimi, Amplissimique D.D.

Nullum à Deo munus Philosophia praestabilius mortalibus collatum, vel deinceps conferendum esse Divinus ille testatur Plato: quo aureo dicto innuere voluit nihil aequius nihilque dignius esse, quam ut animus hominis ad ardentem et sedulam Philosophiae meditationem sese omni studio as diligentia componat: nihil vero iniquius, nihilque indignius vel imprudentius quam mentem humanam (quae à natura novitatis cognitionisque studio trahitur, quaeque organa amplissima et aptissima in eum sinem adepta est) tam alto densoque constrictam et demersam esse veterno, ut morbum, quo periculosissimè affligitur, non agnoscat, vel apta saltem huic aguito et salutaria pharmaca omnino negligat comparare. Verumenimvero cum vita humana brevis admodum sit, Philosophia autem scientia longissima, adeo ut vix à quoquam quoad omnes partes perfectè absolvi possit, nequamquam inter postremos censendi mihi videntur ii, qui illam Philosophiae partem, quae circa corporum dimensiones occupata est, id est Mathematicam sedulo tractant, ut pote quae dignitatem summam summa cum utilitate coniunctam habet. Hinc enim domicilia exstruuntur, hinc urbes aedificantur, hinc omnia tam pacis quam belli tempore instrumenta machinaeque praeparantur, hinc non modo caelestis Civitatis viae exactissime, sed certis etiam domiciliis sidera describuntur, terra climatis, aliisque ad caeli formam partibus distinguitur. Quamobrem conatus eorum maximè laudandus est, qui hand Philosophiae partem ad Reip. usum diligenter excolunt, excultam posteritati consecratam volunt atque conservatam. Quorum in numero licet primas Archimedes atque Ptolomaeus merito atqne optimo iure sibi vendicent: non idcirco tamen, qui hac in re utilem quoque unquam collocarunt operam digno

laudis encomio defraudandi videntur. Ea propter cum M. LUDOLPHUS A COLLEN maritus meus (piae memoriae) in hoc studio totam vitam desudasset, ac nonnulla eius monumenta belgico idiomate conscripta ad Posterorum manus pervenissent, eqaue magno studio a pluribus in latinam linguam translata desiderarentur, facere non potui quin haec fundamenta Arithmetica ac Geometrica praelo subiicerem et pro virili Remp. litteratiam eius etiam opella iuvarem. Cur autem Illist. D.D.V. hosce M. LUDOLPHI labores offerre non dubitaverim, duae posissimum causae mihi occurrebant. Prima, quia vos benignissimos Patronos ac Mecaenates huiusmodi liberalium artium ac disciplinarum noveram. Altera, quia nemo erat cui in cognoscendis artibus tanta perspicacitas, in diiudicandis sinceritas, in defendendis potentia cum voluntate coniuncta esset. Itaque Illust. D.D.V. omni cum animi subiectione etiam atque etiam rogo atq; obtestor, ut hos M. LUDOLPHI A COLLEN mariti mei (piae memoriae) labores, clementi fronte excipiant, eorumque patrocinium ac tutelam suscipiant.
Deus Op. Max. Illust. D.D.V. omni rerum fortuna florentissimas quam diutissimè conservet.
Lugd. Bat. 1615.
Illist. D.D.V.
Subiectissima
ADRIANA SIMONIS.

## B.3 Dedication to Ordinibus

This edition is printed *Apud Iustum a Colster Bibliopolam, Anno 1615.* This edition can be found in the University libraries of Utrecht, Wageningen and Leiden. The first four pages are rather curious. On the first page one can find the titlepage with still some errors in it, which says it has been printed by Joost van Colster, as mentoined before. The second page contains the dedication to D.D. Ordinibus. The third is again a titlepage, but this time all the errors are corrected, the figure is missing and it says to have been printed *Apud J. a Colster et J. Marci, 1615.* Then follows again the page with the dedication, exactly the same as page two. Finally the dedicatory letter begins on page four.
This is the edition that I used to write this thesis. You can also find it online[2].
The dedicatory page, which is printed twice, reads:

Illustrissimis, magnificentissimis, potentissimis,
ac Amplissimis Dominis,
D.D. ORDINIBUS
Generalibus Faederatarum Belgij Provinciarum.

The dedicatory letter reads:

Illustrissimi Domini,

Cum Philosophia, Illustres et Magnifici Domini, verum divinarum atque humanaram scientia, non inventum sit hominum, sed splendissimum Dei donum: quod vetustissimi Philosophorum prodromi Poëte innuere voluerunt, dum sapientiae Praesidem Minervam ex cerebro Iovis diuumque hominumque Parentis prognatam, Musasque eiusdem et Mnemosynes esse filias commenti fuere: Nihil profecto aequius nihilque dignius esse videtur,quam ut animus hominis in pulcherrimum amplissimumq; huius mundi theatrum, tanquam in emporium demissus, ad sedulam et ardentem Philosophiae meditationem sese omni studio ac diligentia componat. Verumenim vero cum vita humana brevis admodum sit, scientia autem haec longissima, adeo ut vix

[2]Google Books says that this edition is published in 1617. The url is: `http://books.google.nl/books?id=l19wQwAACAAJ`

à quoquam quo ad omnes partes perfectè adsolvi possit, non postremam mihi laudem mereri identur ii, qui illam Philosophiae partem, quae circa corporum dimensiones occupata est, id est Mathematicam sedulo tractant: Que tametsi parum digitatis habere videatur, sorde atque iis ac fastidiatur, puibus in amore et delitiis ea potissimum habentur studiorum genera, quae ad quaestum ac magnificentiam comparata, quae in vulgas probantur, et quae amplissima proposita habere praemia putantur: Attamen si rerum aestimatores esse voluerimus Paulo aequiores, et dignitatem habere Mathematicas disciplinas, et quidem summam summa cum utilitate coniunctam re ipsa persentiscemus, adeo ut eas è societate humana qui tollat, solem ipsum de mundo tollere videatur. Quantum enim adiumenti cùm in agendo sive domi, sive foris, sive publicè sive privatim, tum in cognoscendo, vel sola numerandi scientia adferre potest? Quantus Geometriae cum usus, tum necessitas? Hinc domicilia ex struuntur, hinc urbes aedificantur, hinc Omnia tam belli quam pacis tempore instrumenta machinaeque praeparantur: Hinc hostilis illa in acies et oppida irruption, hinc globorum ignisque tremenda proiectio, hinc scalarum ad muros admotio et application. Pacis quoque tempore non modo cae lestis civitatis vias exactissimè, sed certis etiam domiciliis sidera describit, terram climatis aliisque ad caeli formam partibus distinguit. Quamobrem conatus corum maximè llaudandus videtur, qui hanc Philosophiae partem ad Reip: usum diligenter excolunt, excultam posteritati consecratam volunt atque conservatam. Quorum in numero licet primas Archimedes atque Ptolomaeus merito ac optimo iure sibi vendicent: non idcirco tamen, qui hac in re utilem quoque unquam collocarunt operam, digno laudis encomio defraudandi videntur. prropter ea cum Mr Ludolphus à Collen maritus meur (piae memoriae) in hoc studio totam vitam desudasset, ac nonnulla eius monumenta belgico idiiomate conscripta ad posteriorum manus pervenissent, eaque magno studio à pluribus in Latinam linguam translate desiderarentur, facere non potui quin fundamenta haec Arithmetica atque Geometrica praelo subiicerem, et pro virile Remp: litterariam eius opella iuvarem. Cur autem illustres ac Magnifici Domini hos Mr Ludolphi labores vobis offerre non dubita verim, duae potissimum causae mihi occurrebant. Quantam enim Illust,. D.D.V. hactenus expert fuerim clementiam, quam propense et liberale iuvandi stadium, res ipsa loquitur. Itaque ne ingratitudinis aliqua mihi macula inureretur, opusculum hoc levidense fostassis, nec tam Magnificis Dominis dignum, perpetuum tamen gratissimi subiectissimique animi symbolum, vobis omni cum animi submission inscribere atque dedicare visum est aequissimum. Altera causa est quod vos benignissimos Patronos ac Mecaenates huiusmodi liberalium atrium ac dissiplinarum agnoscerem, quorum in cognoscendis artibus perspicacitas, in diiudicandis sinceritas, in defendendis potential cum voluntate coniuncta perpetuò constaret. Itaque Illust: ac Magnif: Domini, omni cum animi subiectione etiam atque rogo et obtestor, ut hosce Mr Ludolphi à Collen mariti mei (piae mamoriae) labores clementi fronte excipiatis, et cum illos, tum memetipsam vobis clementissimè commendatan esse patiamini.

Deus opt. Max: Illust: D.D.V. omnium benedictionum genere cumulet, et quam diutissimè Reip: et Ecclesiis incolumes et florentes conservet. Ludg: Bat: Anno 1615.

Illust: D.D.V. Subiectissima
Adriana Simonis.

# Appendix C

# Locations of the works

In this appendix I have collected the details of several publication of the different editions of the *Fondamenten* and the *Fundamenta*. The edition that I used of the *Fondamenten* was dedicated to Count Maurits. The edition that I used of the *Fundamenta* was published by Colster (and Marcus).

| **Editions of the *Fondamenten*** | | | |
|---|---|---|---|
| **Library** | **Code** | **Dedication** | **Additional information** |
| University of Amsterdam (UvA) | OM 63-1787 | Maurits | Signed by *J. van Woestenberg 1803 VI*. Calculations in sideline. |
| University of Amsterdam (UvA) | OM 63-1164 | Admiralties | Signed by *Moses Lemant 1805*. Cachet reads *Wiskundig Genootschap: Een onvermoeide arbeid komt alles te boven.* |
| University of Amsterdam (UvA) | OM 63-1268 | Maurits | |
| University of Amsterdam (UvA) | OM 63-1688 | Maurits | Signed by *W$^s$. Holl.* and *M. Feller*. Imprint reads *NEDERL. Onderwijzers Genootschap.* On last page it reads *van Brandt, hoofdonderwijzer te Bellingwolde aan het N.O. Genootschap, overly April 1884*. Many corrections and calculations in the margins. |
| University Library (GM), Leiden | STA5 (671 A 14) | Unknown | |
| Special Collections (SZ) Library, Leiden | FILM 2000:12 | Same as above | Negative microfilm of STA (671 A 14). |
| Boerhaave Museam Library, Leiden | BOERH g 10000 | Unknown | |
| Bibliotheca Thysiana, Leiden | THYSIA 1584 | Unknown | |
| Het Scheepsvaartmuseum, Amsterdam | S.4793(140) | Maurits | |
| The Swiss Federal Institue of Technology, Zurich (ETH) | Rar 9077 | Willem | Digital edition: `http://www.e-rara.ch/doi/10.3931/e-rara-9135` |
| University Library, Cambridge | CCB.13.13:2 | Unknown | |
| Technological University, Delft | TR 506418 | Unknown | |
| Royal Library, Den Haag | 199 D 19 | States van Zeeland | `http://gateway.proquest.com/openurl?url_ver=Z39.88-2004&res_dat=xri:eurobo:&rft_dat=xri:eurobo:rec:ned-kbn-all-00002651-001` |

| **Editions of the *Fundamenta*** | | | |
|---|---|---|---|
| **Library** | **Code** | **Publisher** | **Additional information** |
| University of Utrecht (UB Uithof) | MAG : P qu 1032 | Colster | Prelims incomplete; lacks half-title and dedication L1,2 |
| University Library (GM), Leiden | STA3A (2360 C 18) | Marcus | |
| University of Groningen | OF- 1 | Unknown | |
| Tresoar, Leeuwarden | 69 Wk BB | Marsse | 1618 edition |
| Russian State Library | IV-lat. 4° | Marcus | Possible 1695 edition. Lacking pages 80-88. |
| University Library, Heidelberg | 83 H 503; MG/58013060,2 | Laurentium; Unknown | 1617 edition; |
| British Library, London | 530.k.7 | Colster | Lacks title-page and prelims. Also on microfilm: PB.Mic.20575. |
| Royal Library, Stockholm, Sweden | 143 A | Unknown | |
| National Library of France | FRBNF31528756 | Laurentium | 1617 edition. The first 79 pages (first two chapters) are wrongly connected at the end of the work. |
| National Library of France | FRBNF31528755 | Colster | |
| National Library, Firenze, Italy | MAGL.5.3.242 | Colster | |
| University Library, Cambridge | M.4.29 | Georgius | Lacks prelims |
| National Library of Rome, Italy | B 8.B.62 | Marcus | SBN: BVEE069955. With notes made by Curchod. |
| Ateneo Veneto, Venezia | 14. 3.P.20 | Marcus | Same as BVEE069955 |
| Biblioteca nazionale di torino | Q.VI.270 | Marcus | Same as BVEE069955 |
| Biblioteca Statale e Libreria Civica di Cremona | FA.Ingr.3.9.19 | Marcus | Same as BVEE069955 |
| University of Edinburgh Libraries | JA446 | Marsse | |
| University of Glascow Libraries | Sp Coll Ea5-b.3 | Marsse | |
| Bodleian Library, Oxford | Rigaud d.32 | Unknown | |
| Oxford University Libraries | OR.3.08; OR.3.8 (2) | Colster | |
| National Library of Scotland | Cn.2.8 | Colster | |
| ETH-Bibliothek Zrich | Rar 5306 | Marcus | Digital: `http://dx.doi.org/10.3931/e-rara-4244` |
| Gottfried Wilhelm Leibniz Bibliothek, Hannover | CD 1523:6 | Colster | Included in a collected work: Lesctiones opticae et geometricae, 1674 by Isaac Barrow. The work contains 33 books including by Willebrord Snellius, Adriaan van Roomen and Frans van Schooten. |
| Bayerische Staatsbibliothek, Mnchen | 4 Math.u. 14 | Marcus | Digital edition: `http://www.mdz-nbn-resolving.de/urn/resolver.pl?urn=urn:nbn:de:bvb:12-bsb10525442-0` |
| University Library, Erfurt | FBG MAG | Colster | |
| University Library, Tübinger | Bb 74.4 | Unknown | |

# Appendix D

# Proof without words

This ‘proof without words’ was thought out and send to me by Eisso Atzema after my presentation of this master thesis. It involves a geometrical proof for Snellius’ theorem corresponding to problem nine of book V of the *Fundamenta*. See for a discussion on this problem section 5.5.

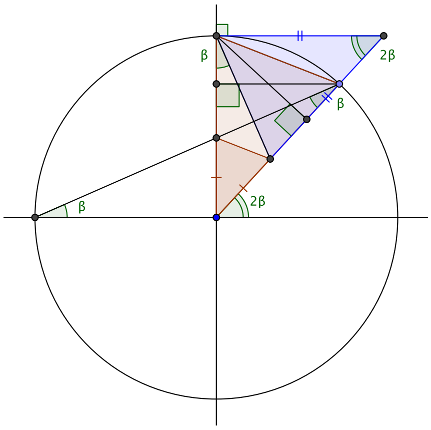



Figure D.1: Proof without words to problem nine of book V of the *Fundamenta*
.

# Appendix E

# Detailed comparison of the works

These tables contain the exact page numbers of each chapter in the *Fundamenta* and shows where it can be found in the original *Fondamenten.* Furthermore, all the commentaries which Snellius contributed are pointed out and briefly discussed. When a passage contained other interesting information, then this is mentioned in the last column. In these tables the names of Van Ceulen and Snellius are abbreviated to *vC* and *Sn*.

## E.1 Book I

p.: Pagenumber in the *Fundamenta*
Chap. Fu: Chapter in *Fundamenta*
Chap. Fo: Chapter in *Fondamenten*
Content: Description of the content found in the *Fondamenten*
Comments: Added commentaries by Snellius
Info: Additional information

| Fundamenta book I | | | | | |
|---|---|---|---|---|---|
| **p.** | **Chap. Fu** | **Chap. Fo** | **Content** | **Comments** | **Info** |
| 1 | I Surdorum Arithmetica. | VI | Introduction to the chapter on extracting square roots. | Snellius clarifies the first explanation. He places the calculations by the accompanying texts. His algorithm for extracting the root is slightly different from that of vC. His translation lacks some examples (from p. 50 of the *Fondamenten*). | |
| 4 | II De Additione irrationalium simplicium. | VIa | Adding simple irrationals. | | |
| 8 | III De irrationalium simplicium subductione. | VIb | Subtracting simple irrationals. | Sn adds an extra sentence in which he simplifies vC's description of the method to be used. On p. 10, Sn briefly explains the notation of the root sign with punctuation. | Some typing errors. |
| 11 | IIII De irrationalium simplicium multiplicatione. | VIc | Multiplication of simple irrationals. | | Sn simplifies one example, corrects one mistake and adds three more. |

| Fundamenta book I | | | | | |
|---|---|---|---|---|---|
| **p.** | **Chap. Fu** | **Chap. Fo** | **Content** | **Comments** | **Info** |
| 12 | V De Irrationalium simplicium divisone. | VId | Division of simple irrationals. | | Sn simplifies one example and corrects three mistakes. |
| 13 | VI De binominorum et residuorum, hoc est irrationalum compositorum notaione ac numeratione. | VIIa-d | On binomic and residual numbers. | Sn adds an extra sentence in which he explains the changing of signs after multiplication. | Sn again corrects and adds a lot of mistakes. |
| 19 | VII De analysi lateris quadrati in irrationalibus conpositis. | VIIe | Extracting the square root of a binomial number. | At the end of the chapter Sn gives his first comment. He says that Ramus' method is more elegant and faster and gives his method. | The translation lacks the general introduction to extracting square roots. One step is missing in the calculation. |
| 21 | VIII De numeratione conpositorum irrationalium universalium. | VIII | On universal numbers. | In the first problem, the figure is lacking and Sn adds a whole page with comments and calculations. He wanted to explain the reason behind the theory in more depth and why multiplication by four is needed (instead of the expected 2). | |
| 23 | " | " | " | Sn gives an explanation on the use of the root sign | |
| 25 | " | " | " | Sn gives an explanation on the use of the root sign | |
| 26 | " | " | " | Sn explains that the multiplication can be done easier by first making both numbers universal. He creates order in vC's chaos. | |
| 27 | " | " | " | Sn remarks that the long number can be seen as a binomial. He also gives another commentary on the notation of the root sign. | |

## E.2 Book II

p1.: Pagenumber in the *Fundamenta*
p2.: Pagenumber in the *Fondamenten*
No.: Number of the proposition, these are identical in the *Fondamenten* and the *Fundamenta*
Content: Description of the content found in the *Fondamenten*
Comments: Added commentaries by Snellius
Elements: The equivalent proposition from the *Elements* of Euclid. (III-4 means the fourth proposition of the third book. II-15* means that the proposition is not exactly the same, but a specific case or generalisation.)
Info: Additional information

| Fundamenta book II | | | | | | |
|---|---|---|---|---|---|---|
| **p1.** | **p2.** | **No.** | **Content** | **Comments** | **Elements** | **Info** |
| | | | **Definitions** | | | |
| 33 | 69 | 1 | Point | | | |

| Fundamenta book II | | | | | | |
|---|---|---|---|---|---|---|
| **p1.** | **p2.** | **No.** | **Content** | **Comments** | **Elements** | **Info** |
| | | 2 | Line | | | |
| | | 3 | Straight line | | | |
| | | 4 | Plane figure | | | |
| | | 5 | Curved surfaces | | | |
| | | 6 | Angle | | | |
| | | 7 | Rectilinear angle | | | |
| | 70 | 8 | Right angle | | | |
| 34 | | 9 | Perpendicular | | | |
| | | 10 | Obtuse angle | | | |
| | | 11 | Acute angle | | | |
| | | 12 | End | | | |
| | | 13 | Figure | | | |
| | | 14 | Circle | | | |
| | | 15 | Center | | | |
| | | 16 | Diameter | | | |
| | | 17 | Half circle | | | |
| | | 18 | Part of a circle | | | |
| | | 19 | Rectilinear figures | | | |
| | 71 | 20 | Triangle | | | |
| | | 21 | Quadrangles | | | |
| | | 22 | Polygons | | | |
| 35 | | 23 | Equilateral triangles | | | |
| | | 24 | Isosceles triangles | | | |
| | | 25 | Scalene triangles | | | |
| | | 26 | Right-angled triangles | | | |
| | | 27 | Obtuse-angled triangles | | | |
| | | 28 | Acute-angled triangles | | | |
| | | 29 | Square | | | |
| | | 30 | Rectangle | | | |
| 36 | | 31 | Rhombus | | | |
| | | 32 | Rhomboids | | | |
| | | 33 | Parallel lines | | | |
| | | | | | | |
| | | | **Axioms** | | | |
| | 72 | 1 | Similarity | | | |
| | | 2 | Adding equal parts to equals | | | |
| | | 3 | Subtracting equal parts from equals | | | |
| | | 4 | Subtracting equal parts from unequals | | | |
| | | 5 | Adding equal parts to unequals | | | |
| | | 6 | Equal multiplications | | | |
| | | 7 | Equal parts | | | |
| | | 8 | Whole larger than part | | | |
| | | 9 | Straight angles are equal | | | |
| | | 10 | Intersecting lines | | | |
| | | 11 | Two straight lines do not enclose | | | |
| | | 12 | Curved lines do enclose | | | |
| | | | | | | |
| | | | **Propositions and constructions** | | | |
| 37 | 73 | 1 | To make an equilateral triangle on a given line. | Problema 1 | I-1 | Sn has added the heading 'problema' to several propositions when it involves a construction. |
| | | 2 | Two triangles with two equal sides and one equal angle are equal. | | I-4 | |

| Fundamenta book II | | | | | | |
|---|---|---|---|---|---|---|
| **p1.** | **p2.** | **No.** | **Content** | **Comments** | **Elements** | **Info** |
| 38 | 74 | 3 | Isosceles triangles have equal angles. | | I-5 | |
| | | 4 | Triangle with equal angles has two equal sides. | | I-6 | |
| | | 5 | Bisector | Problema 2 | I-9 | |
| | | 6 | Division of a line | Problema 3 | I-10 | |
| 39 | 75 | 7 | Perpendicular | Problema 4 | I-11 | |
| | | 8 | To draw a perpendicular on a line trough a given point. | Problema 5 | I-12 | |
| | | 9 | Opposite angles | | I-15 | |
| 40 | | 10 | Opposite angles for more than two lines. | | follows from 9 | |
| | 76 | 11 | Three lines such that two added are larger than the third can form a triangle. | Problema 6 | I-22 | |
| | | 12 | To draw an angle equal to another angle in a given point on a line. | Problema 7-1 | I-23 | |
| | | 13 | Two lines intersected by a third with equal angles are parallel. | | I-27 | |
| 41 | 77 | 14 | Two lines intersected by a third with straight angles are parallel. | | I-28 | |
| | | 15 | When a line intersects two parallel lines, then the angles are equal. | | I-29 | |
| | | 16 | When two lines are parallel to another, they are also parallel to each other. | | I-30 | |
| | | 17 | To draw a parallel line through a given point above a given line. | Problema 7-2 | I-31 | |
| 42 | 78 | 18 | When the side of a triangle is extended, then the angle outside is larger than the two opposite angles inside the triangle. | | I-32* | |
| 43 | 79 | 19 | The longest side of a triangle is always opposite the largest angle. | | I-18 | |
| | | 20 | When the side of a triangle is extended, then the angle outside is equal to the two opposite angles inside the triangle. Moreover, the angles inside a triangle add up to 180 degrees. | | I-32* | |
| | | 21 | In a figure with two opposite sides which are parallel, the other two sides must also be parallel. | | I-33 | |
| | 80 | 22 | To construct a square on a given line. | Problema 8 | I-46 | |
| 44 | | 23 | Theorem of Pythagoras | | I-47 | |
| | 81 | 24 | Triangles between two parallel lines, with equal base on one of these lines, have an area equal to half a quadrilateral on that base between the parallel lines. | | I-37, 38. 41[1] | |
| 45 | 82 | 25 | The diagonal cuts a parallelogram in two equal parts and the opposite angles are equal. | | I-35 | |

[1] vC refers himself to 41.

| Fundamenta book II | | | | | | |
|---|---|---|---|---|---|---|
| **p1.** | **p2.** | **No.** | **Content** | **Comments** | **Elements** | **Info** |
| 46 | 83 | 26-1 | Triangles between parallel lines with equal base have equal area. With different base but between parallel lines, then they are proportional. | | I-37, 38 and VI,1 | In the *Fundamenta* this proposition is numbered as 25-2. |
| 47 | | 26-2 | A line drawn in a triangle parallel to a side cuts the other sides proportionally. And vice versa. | | VI-2 | In the *Fundamenta* this proposition is numbered as 27. |
| | 84 | 28 | To construct a rectangle with area equal to that of a given rectangle. | Problema 9 | | |
| | | 29 | To construct a rectangle with area equal to that of a given unfit quadrilateral (no right angles). | Problema 10 | | |
| 48 | 85 | 30 | To construct a rectangle with area equal to that of a given unfit pentagon. | Problema 11 | | |
| | | 31 | To construct a rectangle with area equal to that of a given unfit polygon. | Problema 12 | | |
| | | 32 | Line divided in parts: the rectangles of these parts and the line itself are equal to the square of the line itself. | | II-2 | |
| 49 | 86 | 33 | Line divided in two parts: the squares of the parts and two times the rectangles of the parts is equal to the square of the whole. | | II-4 | |
| 50 | | 34 | Line divided in two equal parts and two unequal parts: the rectangle of the unequal parts and the square of the difference between half the line and the largest part is equal to the square of half the line. | | II-5 | |
| | 87 | 35 | Line divided in two equal parts and with an extension: the rectangle of the line+extension with the extension and the square of half the line is equal to the square of half the line+extension. | | II-6 | |
| | | 36 | to cut a line such that the rectangle of the whole line with the smallest part is equal to the square of the largest part. | Problema 13 | II-11 | |
| 51 | 88 | 37 | To construct a square with area equal to a given figure. | Problema 14 | II-14 | |
| | | 38 | To construct a square with area equal to the areas of several given squares together. | Problema 15 | II-15 | |
| 52 | 89 | 39 | To construct a tangent to a circle in a given point. | Problema 16 | III-17 | |
| 53 | | 40 | When on a cord in a circle is drawn a triangle with opposite angle on the centre of the circle, then this angle is twice as when the angle is on the circumference of the circle. | | III-20 | |

| Fundamenta book II | | | | | | |
|---|---|---|---|---|---|---|
| **p1.** | **p2.** | **No.** | **Content** | **Comments** | **Elements** | **Info** |
| | 90 | 41 | All angles on the same cord and on the circumference of the circle are equal. | | III-21 | |
| 54 | | 42 | The opposite angles of a circumscribed quadrilateral are equal to two right angles. | | III-22 | |
| | 91 | 43 | In equal circles, equal angles are always on equal cords (in centre or on circumference). | | III-26 | |
| | | 44 | A perpendicular on a cord always goes through the centre. | | III-1, III-3* | |
| | | 45 | To divide an arc in two equal parts. | Problema 17 | III-30 | |
| 55 | | 46 | A triangle in a circle with middle line as base and top angle on the circumference has a right top angle. When the base is smaller, then the angle is larger and vice versa. | | III-31 | |
| 56 | 92 | 47 | Given a tangent to a circle. When from the tangent point is drawn a line that cuts the circle, then the angle at the tangent point is equal to the angle at the cutting point. | | III-32 | |
| 57 | 93 | 48 | To construct part of a circle on a line with an angle equal to a given angle. | Problema 18 | III-33 | |
| | | 49 | To cut a circle such that the angle is equal to a given angle. | Problema 19 | III-34 | |
| 58 | 94 | 50 | When two lines cut inside a circle, then the rectangles of the parts are equal. | | III-35 | |
| | | 51 | Given a circle and a point outside the circle. When a line is drawn from the point cutting the circle and one tangent to the circle, then the quadrilateral from the cutting line and the part from the point to the first intersection with the circle is equal to the square of the tangent line. | | III-36 | |

| Fundamenta book II | | | | | | |
|---|---|---|---|---|---|---|
| **p1.** | **p2.** | **No.** | **Content** | **Comments** | **Elements** | **Info** |
| 59 | 95 | 52 | All cutting lines from the same point outside the circle have equal rectangles with the line in the circle and the part up to the circle. **Definitions on in- and circumscribed figures:** A rectilinear figure is inscribed in a rectilinear figure when all angles of the inner figure hit all the sides of the outer figure. A rectilinear figure circumscribes another when all sides hit the angles of the inner figure (when they have equal amount of sides). A rectilinear figure circumscribes a circle when all sides hit the circumference. A rectilinear figure is inscribed in a circle when all angles hit the circumference. A right line is inscribed in a circle when the endpoints hit the circumference. | | Follows from 51 | |
| 60 | 96 | 53 | To construct a triangle inscribed in a circle with angles equal to a given triangle. | Problema 20 | IV-2 | |
| | | 54 | To construct a triangle circumscribing a circle which is similar to a given triangle. | Problema 21 | IV-3 | |
| 61 | 97 | 55 | To construct a circle in a triangle. | Problema 22 | IV-4 | |
| 62 | | 56 | To construct a circle circumscribing a triangle. | Problema 23 | IV-5 | |
| | 98 | 57 | To construct a square in and around a triangle. | Problema 24 | IV-6, VI-7 | |
| 63 | | 58 | To construct an inscribed equilateral pentagon. | Problema 25 | IV-11 | |
| 64 | 99 | 59 | To construct a circumscribed equilateral pentagon. | Problema 26 | IV-12 | |
| | | 60 | To construct an inscribed equilateral hexagon. | Problema 27 | IV-15 | |
| 65 | 100 | 61 | A bisector cuts the opposite side of a triangle in the proportion that the other two sides have to each other. | | VI-3 | |
| | | 62 | Triangles with equal angles have equal proportions. | | VI-4, VI-5 | |
| 66 | 101 | 63 | The perpendicular through the right angle cuts the triangle into two triangle both similar to the original and which are middle proportional. | | VI-8 | |
| | | 64 | To construct a middle proportional line to two given lines ($A : B = B : C$). | | VI-13 | |
| 67 | 102 | 65 | To cut a line such that another line (shorter than half the original line) stands in the middle of the proportions of the parts (golden ratio). | Problema 28 | inverse of 64 | |
| | | 66 | To cut a line in a given amount of parts. | Problema 29 | VI-9 | |
| 68 | | 67 | To cut a line in the same proportion as another cut line. | Problema 30 | VI-10 | |

| Fundamenta book II | | | | | | |
|---|---|---|---|---|---|---|
| **p1.** | **p2.** | **No.** | **Content** | **Comments** | **Elements** | **Info** |
| | 103 | 68 | To find a third line such that the proportion of the first to the second is equal to the proportion of the second to the third. | Problema 31 | VI-11 | |
| 69 | | 69 | Given three lines, to find a fourth line with the same proportion. | Problema 32 | VI-12 | |
| | 104 | 70 | Given three line, to find a fourth line such that the proportion of the first to the second is equal to the proportion of the third to the fourth. | Problema 33 | VI-12 | |
| 70 | | 71 | Given two rectangles with equal areas, then the sides are wrongly proportioned ($a : c = b : d$) and vice versa. Equal quadrilaterals with acute or obtuse angles have at equal angles wrongly proportioned sides. | | VI-14 | |
| 71 | 105 | 72 | Given equal triangles with equal angles, then the opposite sides are wrongly proportioned and vice versa. | | VI-15 | |
| | 106 | 73 | To construct a figure on a given line which is similar to another figure. | Problema 34 | VI-18 | |
| 72 | | 74 | All similar figures have a proportion equal to the squares of the sides. | | VI-19* | In the *Fundamenta* this proposition is wrongfully numbered as 47. |
| 73 | 107 | 75 | Given three proportional lines and two similar figures on the first and second line. Then the proportion of the first line to the third line is equal to the proportion of the first figure to the second figure. | | follows from 74 | |
| | 108 | 76 | Parallelograms with equal angles have the same proportion to each other as their sides. | | VI-23 | |
| 74 | | 77 | All quadrilaterals enclosed by parallel sides, the quadrilaterals where the diagonal goes through are similar to each other and the whole quadrilateral. | | VI-24 | |
| | 109 | 78 | To construct a rectilinear figure equal to a given figure and similar to another. | | VI-25 | |
| 75 | | 79 | Given three similar figures (including curvilinear) with bases the sides of a triangle. Then the area of the figure on the side opposite the right angle is equal to the areas of the figures on the other two sides. | Problema 35 | VI-31 | |
| 76 | 110 | 80 | To construct a parallelogram on a given angle with area equal too a given triangle. | Problema 36 | I-42 | |
| | 111 | 81 | In all parallelograms, the quadrilaterals through which the diagonal goes are similar. | | follows from 77 | |

| Fundamenta book II | | | | | | |
|---|---|---|---|---|---|---|
| **p1.** | **p2.** | **No.** | **Content** | **Comments** | **Elements** | **Info** |
| 77 | | 82 | To construct a parallelogram on a line with a given angle with area equal to a rectilinear figure. | | I-44 | |
| | 112 | 83 | In an obtuse triangle the square of the side opposite to the obtuse angle is larger than the sum of the squares of the other two sides. | | II-12 | |
| 78 | 113 | 84 | In an acute triangle the square of the side opposite the acute angle is smaller than the squares of the sum of the other two sides. | | II-13 | |

## E.3 Book III

p.: Pagenumber in the *Fundamenta*
No. Fu: Number of the proposition in the *Fundamenta*
No. Fo: Number of the proposition in the *Fondamenten*
Content: Description of the content found in the *Fondamenten*
Comments: Added commentaries by Snellius
Info: Additional information
Refs: References made to the equivalent proposition from the *Elements* of Euclid (III-4 means the fourth proposition of the third book), book 2 of the *Fondamenten* (II:13 means the thirteenth proposition) and sometimes other books.

| Fundamenta book III | | | | | | |
|---|---|---|---|---|---|---|
| **p.** | **No. Fu** | **No. Fo** | **Content** | **Comments** | **Info** | **Refs** |
| 85 | - | | Preface | Sn explains why he added a different subdivision of the propositions. He says to add the subtitle 'problema' to propositions that involve constructions. | | |
| | | | | | | |
| | | | **Transformations** | | | |
| 85 | 1 | | Transformation of quadrilateral to triangle. | | | II:27 |
| 86 | 2 | | Transformation of a pentagon to a triangle. Transformation of a hexagon to a triangle. | | | |
| 87 | 3 | | Transformation of a hexagon to a triangle with a specified base. Transformation of a hendecagon to a triangle. | | | 2:27, I-37 |
| 88 | 4 | | Transformation of a decagon to a equilateral triangle. | | | |
| 89 | 5 | | Transformation of a triangle to triangle with a given height (smaller). Transformation of a triangle to triangle with a given height (larger). | | | |
| 90 | 6 | | Adding triangles to one rectangular with e gives height. | | | |

| Fundamenta book III | | | | | | |
|---|---|---|---|---|---|---|
| **p.** | **No. Fu** | **No. Fo** | **Content** | **Comments** | **Info** | **Refs** |
| 91 | 7 | | Transformation of a parallelogram with a given height and angle. | Sn remarks that if this problem is too hard to understand, it can also be constructed in steps by using the same constructions as the previous problems. | | |
| | | | | | | |
| | | | **Division** | | | |
| 92 | 8 | | Division of a triangle in two equal parts. Division of a triangle in three equal parts. | | | I-36, VI-1 |
| | 9 | | Cutting 1/3 of a triangle from a given point on an edge. Cutting 2/4 of a triangle from a given point on an edge. Cutting 4/7 of a triangle from a given point on an edge. | | | I-23 |
| 94 | 10 | | Cutting 2/3 of a triangle with a line parallel to a given edge. | | | |
| | 11 | | Cutting a part from a triangle that equals a quadrilateral. | | | I-43 |
| 95 | 12 | | Cutting a part from a triangle that equals another triangle with a line parallel to a given edge (all three edges demonstrated). | | | VI-15 |
| 96 | 13 | | Cutting a part from a triangle that equals another triangle from a given point on an edge. Division of traingle in two parts with the same proportions as two other triangles. Division of triangle in two parts with the same proportions as two quadrilaterals. | | | I-37, VI-1, VI-10 |
| 98 | 14 | | Cutting 1/4 of a quadrilateral from a given vertex. | Sn remarks that there are situations where vC's construction does not work, but that he can not give a figure to show the problem. | | |
| | 15 | | Division of a quadrilateral in two equal parts from a given point on an edge. | | | |
| 99 | 16 | | Cutting 1/3 of a quadrilateral with a line parallel to a given edge. | | | II:72, II:75, VI-1, VI-15 |

| Fundamenta book III | | | | | | |
|---|---|---|---|---|---|---|
| **p.** | **No. Fu** | **No. Fo** | **Content** | **Comments** | **Info** | **Refs** |
| 100 | 17 | | Cutting a part from a quadrilateral equal to another quadrilateral through a given point on an edge. Cutting a part from a quadrilateral equal to another quadrilateral nd 1/4 of the original quadrilateral through a given point on an edge. Cutting a part from a pentagon equal to a given triangle. Cutting a part from a hexagon equal to another hexagon nd 1/5 of the original hexagon. | | | |
| 102 | 18 | | Cutting 4/9 of a hexagon through a given point on an edge. Division of a hexagon in three equal parts from a given point on an edge. | | vC's construction is not finished. Sn corrects and completes it. | |
| 103 | 19 | | Cutting a part from a pentagon equal to 3/2 of another pentagon with a line parallel to an edge. Division of a quadrilateral in two equal parts with a line parallel to an edge. Cutting a part from an hexagon equal to a given quadrilateral with a line parallel to an edge. Cutting a part from an hexagon equal to a given quadrilateral nd 1/6 of the hexagon with a line parallel to an edge. | Sn gives an alternative solution and gives his opinion about giving numbers to line segments. | | I-28 |
| | | | | | | |
| | | | **Line segments** | | | |
| 106 | 20 | 1 | Theory on adding line segments. | | | |
| | 21 | 2, 3, 4, 5 | $24+\sqrt{13}$. $\sqrt{28}+4$. $\sqrt{19}+\sqrt{14}$. $\sqrt{\sqrt{15}+3}$. (Requires the construction of a line segment with unit length.) | Sn adds a long commentary (including two new figures). He given an alternative solution method which he found 'both very elegant and extremely easy to perfrom' (Wreede, 2007, p. 209) and presents the method which Euclid used. He claims that his 'little theorem' could help to avoid vC's troublesome method (Wreede, 2007, pp. 205-213). | The first figure is wrongfully placed. It belongs to problem 23. | Book 2, prop 22; I-47 |
| 109 | 22 | 6 | Theory on subtracting line segments. | | Figure is lacking. | I-3 |

| Fundamenta book III | | | | | | |
|---|---|---|---|---|---|---|
| **p.** | **No. Fu** | **No. Fo** | **Content** | **Comments** | **Info** | **Refs** |
| | 23 | 7, 8, 9, 10 | $10-\sqrt{7}$. $(\sqrt{13}+\sqrt{17})-\sqrt{13}$. $8-\sqrt{7+\sqrt{18}}$. $\sqrt{7}+\sqrt{3}-\sqrt{\sqrt{7}-\sqrt{3}}$. | Sn changes the first problem to $10-\sqrt{6}$ to simplify the problem, but now the figure does not represent the construction (see p.111). Sn re-interpreted vC's problem in exact geometrical terms (Wreede, 2007, p. 210). | Question is proposed by Simon Stevin in 1583. | |
| 112 | 24 | 11, 12 | $3\cdot\sqrt{19}$. $\sqrt{3}\cdot\sqrt{5}$. | Sn adds a long commentary on the fact that vC implies that the product of two line segments is itself a line segment (instead of a parallelogram). He rephrased the problem such that it became geometrically valid. A detailed discussion on this problem can be found in (Wreede, 2007, pp. 210-213). | | II:51, III-35 |
| 114 | 25 | 13, 14, 15 | $24\div 3$. $\sqrt{19}\div\sqrt{2}$. Dividing a rectangular by a line. | | | II:55 |
| | | | | | | |
| | | | **Geometrical problems** | | | |
| 115 | 26 | - | To find the square that is $n\in\mathbb{N}$ times as large as a given polygon. | | Sn clarifies the construction. | II:38, II-15 |
| 116 | 27 | 2 | To find the square that is $\frac{1}{n}$ with $n\in\mathbb{N}$ as large as a given square. | | Sn explains the construction and skips a trivial part. | |
| | 28 | 3 | To find a circle that is $n\in\mathbb{N}$ times as large as a given circle. | | | XII-2 |
| 117 | 29 | 4 | To find a circle that is $\frac{1}{n}$ with $n\in\mathbb{N}$ as large as a given circle. | | | |
| | 30 | 5 | Quadrature of the circle. (Archimedes) | | Continued in zetema 2 in the fourth book of the *Fundamenta*. (p. 144) | |
| 118 | 31 | 6 | To construct a circle equal to a given square. (Archimedes) | | | |
| 119 | 32 | 7 | To construct a circle with a circumference equal to a given line segment. (Cardinales Cusani) | | Continued in zetema 3 in the fourth book of the *Fundamenta*. (p. 145) | |
| | 33 | 8, 9 | Quadrature of the circle. To construct a line segment as long as the circumference of a given circle. (Viète) | | Continued in zetema 4 in the fourth book of the *Fundamenta*. (p. 146) | II-11 |
| 120 | 34 | 10 | To find the diameter of a circle inscribed in a given triangle. | Sn gives an alternative solution that is faster. He promises to give an elaboration on his own invention in a later publication (Wreede, 2007, p. 89, 187). | | II:55, II:61, II:84 |

| Fundamenta book III | | | | | | |
|---|---|---|---|---|---|---|
| **p.** | **No. Fu** | **No. Fo** | **Content** | **Comments** | **Info** | **Refs** |
| 121 | Lemma 1 | 1 | When four lines (or numbers) are proportional, then the rectangle of the first and last is equal to the rectangle of the second and third. | | | |
| | Lemma 2 | 2 | When four numbers are proportional, then the product of the square of the first with the fourth is equal to the product of the two middle numbers with the first. | | Sn uses different numbers to proof this lemma. He focusses on when $a : b = c : d$ then $ad = bc$ instead of $a^2d = abc$. | |
| | Lemma 3 | 3 | The root of the product of two squares is the product of the numbers. | | | |
| 122 | 35 | 14 | To find the area of a triangle with given edges. (Heron's theorem) | Sn gives his own opinion about multiplying up till the fourth dimension and gives his own method with an added lemma. For a detailed discussion on this two pages long commentary see (Wreede, 2007, p. 271–278). | | II:12, II:13 |
| 125 | 36 | 15 | To construct a triangle with a perimeter equal to a given line and the inscribed circle with a diameter equal to another given line. | Sn gives an alternative solution. | | II:55, I-43, II-5 |
| 126 | 37 | 16 | To cut a given line such that the squares of the parts added equals the square of another given line. | Sn gives the reason why one would like to find this, namely to find a rectangular triangle with base $B$ in which the inscribed circle has a diameter of $A - B$. | | |
| 127 | 38 | 17 | To cut a given line such that the square of the smallest part added to another given square is equal to the square of the largest part. | Sn explains the purpose of this problem as being the same as the one before. | | VI-2 |
| 128 | 39 | 18 | To cut a given line such that the square of the largest added to line $C$ equals the square of the smallest added to line $B$. | | | I-15 |
| 129 | 40 | 19 | To construct a triangle similar to a given triangle with a proportion equal to the proportion of two given lines. | | | VI-12, VI-19 |
| | 41 | 20 | To cut two lines such that their rectangles are equal. | | | |
| 130 | 42 | 21 | To construct a rectangle inscribed in a given triangle such that the sides have a given proportion. | Sn gives a more faster solution and corrects some mistakes. | | |

| Fundamenta book III | | | | | | |
|---|---|---|---|---|---|---|
| **p.** | **No. Fu** | **No. Fo** | **Content** | **Comments** | **Info** | **Refs** |
| 131 | 43 | 22 | To construct a line $C$ such that the rectangle of $B + C$ and $C$ equals the square of a given line $A$. | | | III-36 |
| | 44 | 23, 25 | Two problems on finding two lines such that they are perpendicular and when extended form a triangle with one side equal in length to a given line. | Sn combines problem 23 and 25 . | Sn corrects a lot of mistakes. | I-47 |
| 133 | 43-2 | 24 | To construct a line from the end of the diameter of a circle such that $CD$ (with $D$ the intersection with the circle) has a given proportion to the perpendicular on the diameter. | | | |
| | 44-2 | 26 | To construct a triangle similar to a given triangle and equal to a given square. | | | VI-25 |
| | 45 | 27 | To construct a rectangle in a given rectangle such that the differences of the sides are equal. | Sn adds a final remark to this chapter. He complains about the lack of time and figures, nevertheless, he gives a description of his own method. | | |

## E.4 Book IV

p.: Pagenumber in the *Fundamenta*
No. Fu: Number of the proposition in the *Fundamenta*
No. Fo: Number of the proposition in the *Fondamenten*
Content: Description of the content found in the *Fondamenten*
Comments: Added commentaries by Snellius
Info: Additional information
Refs: References made to the equivalent proposition from the *Elements* of Euclid (III-4 means the fourth proposition of the third book), book 2 of the *Fondamenten* (II:15 means the 15th proposition) and sometimes other books.

| Fundamenta Book IV | | | | | | |
|---|---|---|---|---|---|---|
| **p.** | **No. Fu** | **No. Fo** | **Content** | **Comments** | **Info** | **Refs** |
| 137 | **Preface** | **Byvouch** | | Sn expands the preface. He quotes Eutocius and gives his view on giving numbers to figures. | | |
| | 1 | - | Proof with numbers for the Pythagorean theorem and several theorems on the area of triangles and rectangles from the second book. | | Sn combines the first nine examples. | II:23, II:24, II:26, II:71, II:80, II:82 |

| Fundamenta Book IV | | | | | | |
|---|---|---|---|---|---|---|
| **p.** | **No. Fu** | **No. Fo** | **Content** | **Comments** | **Info** | **Refs** |
| 144 | 2 | - | The quadrature of the circle by Archimedes. | | The figure belonging to the proposition to which is referred is added again. Reference to *Vanden Circkel* and Pieter Cornelsz. | |
| 145 | 3 | - | The quadrature of the circle by Cusanus. | | Again the figure is added. | |
| 146 | 4 | - | The quadrature of the circle by Viète. | | | |
| 147 | 5-1 | - | An instrument for the quadrature of the circle. | | | |
| 149 | 5-2 | 1 | To calculate the area of a land with known sides. | | First proposition of the fourth book of the *Fondamenten.* The numbering uses 5 three times. | |
| 151 | 5-3 | 2,3 | To calculate the quadrature of a triangle-shaped land. | | Corrections of vC on wrong examples he found in a book printed in Antwerpen in 1547. | I-38, I-40 |
| 152 | 6 | 4 | To calculate the sides of a triangle when the area is given. | | This is again a correction on a wrongful example from the same book. The figure placed beside the work is wrong. | |
| 153 | 7 | 5 | To calculate the area of a triangle when the sides are given. | | Again a correction and a warning. | |
| | 8 | 6 | To calculate the height and parts of the base of a triangle when the sides are given. | | | II:63, VI-8 |
| | 9 | 7 | To calculate the parts of the base of a triangle cut by the bisector. | | | II:61 |
| 154 | 10 | 8 | To calculate the parts of the lines cut by the bisector in a triangle. | | | |
| | 11 | 9 | To calculate a perpendicular in a triangle. | | | |
| 155 | 12 | 10 | To calculate lines in a triangle. | | | VI-3 |
| | 13 | 11 | To calculate lines cut by the medians of a triangle. | vC did not calculate the asked parts of the medians, which Sn adds in a short commentary. | | I-38, VI-1 |
| 156 | 14 | 12 | To calculate lines in a triangle. | | This is the same example as in proposition 51 of book two of the *Fondamenten.* | III-3, III-36 |
| | 15 | 13 | To calculate lines in a triangle. | | | I-13, I-32, VI-8 |

| Fundamenta Book IV | | | | | | |
|---|---|---|---|---|---|---|
| **p.** | **No. Fu** | **No. Fo** | **Content** | **Comments** | **Info** | **Refs** |
| 157 | 16 | 14 | To calculate distances from the corners of a triangle-shaped field to the top of a scarecrow placed in the middle of the field. | | | I-39, III-21, III-31 |
| 158 | 17 | 15(16) | To calculate the diameter of a circle with a given inscribed triangle. | | | II-13, III-21 |
| | 18 | 16 | To calculate the sides of a triangle when the base and height are given. | | | II-5, VI-8 |
| 159 | 19 | 17(27) | To calculate the sides of a equilateral triangle when the area is given. | | | VI-19 |
| | 20 | 18 | To calculate the sides of a triangle with a given proportion to each other when the are is given. | | | |
| 160 | 21 | 19 | To calculate lines in a triangle. | | | |
| | 22 | 20 | To calculate lines in a triangle. | | | |
| 159-1 | 23 | 21 | To cut a part of a triangle with given area. | | | |
| | 24 | 22 | To calculate a side of a triangle with given area and other sides. | | | I-38, VI-1 |
| 160-1 | 25 | 23 | To calculate the sides of a triangle with given largest inscribed square. | | | |
| | 26 | 24 | To calculate the sides of an square inscribed in a triangle. | Sn refers to problem 24 of book 3 of the *Fondamenten.* He says that like the base added to the perpendicular is against the base, so is the perpendicular against the side of the inscribed square. | | |
| 161 | 27 | 25 | To calculate the sides of a rectangle inscribed in a triangle with given proportions. | | | Ref to third book of *Fondamenten.* |
| 162 | 28 | 26 | To calculate the sides of an square inscribed in a triangle and a triangle | Sn explains a proportion that vC neglected to mention. | | |
| 163 | 29 | 27 | To calculate the largest square inscribed in a triangle. | Sn gives a generalisation of this problem. He also refers to Comandinus and problem 42 of book three. | Reference to commentary by Frederico Comandum at the end of the sixth book of the *Elements*, edition by Clavius. | |
| 164 | 30 | 28 | To calculate lines in a triangle. | | | II-12 |

| Fundamenta Book IV | | | | | | |
|---|---|---|---|---|---|---|
| **p.** | **No. Fu** | **No. Fo** | **Content** | **Comments** | **Info** | **Refs** |
| | 31 | 29 | To calculate the sides of an inscribed triangle. | Sn gives a faster method to find the solution. | | |
| 165 | 32-1 | 30 | To calculate the sides of an inscribed triangle. | Sn remarks that he is not sure of the solutions given by vC. He gives two theorems (with proof) which lead to a faster method to find the solutions. This commentary is 1,5 pages long. | | III-21, III-31 |
| 168 | 32-2 | 31 | To calculate the area of an extended triangle. | Sn gives an 'easier and quicker' way. | | |
| | 33 | 32,33 | To calculate the side of an extended triangle. (two examples) | | Sn combines two examples to one. | I-47 |
| 169 | 34 | 34 | To calculate the side of a triangle. | | | |
| | 35 | 35 | To calculate sides of a triangle. | Sn remarks that in book 1 of Ptolemaeus (with commentary of Theone) another calculation can be found. | This problem was proposed by Simon Stevin in 1582 and vC first solved it with algebra. | |
| 170 | 36 | 36 | To calculate the diameter of a circle when the area of an inscribed equilateral triangle is known. | Sn gives two alternative methods to find the solution (a lot faster). | | |
| 171 | 37 | 37 | To calculate the sides of an circumscribed triangle. | Sn gives a quicker method. | | I-47, III-21, III-31 |
| 172 | 38 | 38 | To calculate the sides of a triangle with known area and proportions. | | | VI-19 |
| | 39 | 39 | To calculate the sides of a circumscribed triangle. | | | III-1, III-35 |
| 173 | 40 | 40 | To calculate the sides of a rectangle inscribed in a triangle with given proportions. | | This problem is the same as problem 25 (zetema 27 in *Fundamenta*). | |
| | 41 | 41 | To calculate sides of a quadrilateral. | | | VI-13 |
| 174 | 42 | 42 | To calculate the sides of a circumscribed triangle. | Sn remarks that this problem is similar to zetema 37 and solves this problem with the same method. | This problem was handed to vC by an experienced master. | I-47 |
| 175 | 43 | 43 | To calculate the lengths of the diagonals of a quadrilateral. | | | II-13, VI-8 |
| 176 | 44 | 44 | To construct and calculate the sides and areas in a triangle. | | | I-37, II-13 |
| | 45 | 45 | To calculate the sides of a quadrilateral. | | | VI-2 |
| 177 | 46 | 46 | To calculate the sides of a triangulated quadrilateral. | | | |

| Fundamenta Book IV | | | | | | |
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| **p.** | **No. Fu** | **No. Fo** | **Content** | **Comments** | **Info** | **Refs** |
| 178 | 47 | 47 | To calculate sides in a quadrilateral. | | vC remarks that he will use lesser words in the following examples. | |
| | 48 | 48 | To calculate the side of a triangulated quadrilateral. | | | II-12 |
| 179 | 49 | 49 | To calculate sides of two intertwined triangles. | | This problem was proposed to vC by Adriaen Ockers. | II-13 |
| 180 | 50 | 50 | To calculate the diagonal of a quadrilateral. | | vC says to calculate the area later, which he does in proposition 32 of book five. | II-12 |
| | 51 | 51 | To calculate the side of a triangle. | | | II-12, VI-2 |
| 181 | 52 | 52 | To calculate sides of a triangle. | | | |
| | 53 | 53 | To calculate the sides of a triangle with an inscribed circle. | | vC remarks that this problem is the 57th question of the *Geometria* by Symon Iacobi, but the method given here is different. | IV:13 |
| 182 | 54 | 54 | To calculate the area of a triangle. | | | |
| | 55 | 55 | To calculate the sides of a triangle when parts of the sides are given. | | vC gives a general rule to proof the calculation. | |
| 183 | 56 | 56 | To calculate the sides of a triangle when the base and the sum of two sides is given. | | | |
| | 57 | 57 | To calculate sides in a triangle. | Sn remarks that since there is no more money, he is not able to place more lines. Nevertheless, he gives his own method to solve the last problem. | | I-47, II-13 |

## E.5 Book V

p.: Pagenumber in the *Fundamenta*
No. Fu: Number of the proposition in the *Fundamenta*
No. Fo: Number of the proposition in the *Fondamenten*
Content: Description of the content found in the *Fondamenten*
Comments: Added commentaries by Snellius
Info: Additional information
Refs: References made to the equivalent proposition from the *Elements* of Euclid (III-4 means the fourth proposition of the third book), book 2 of the *Fondamenten* (II:15 means the 15th proposition) and sometimes other books.

| Fundamenta book V | | | | | | |
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| **p.** | **No. Fu** | **No. Fo** | **Content** | **Comments** | **Info** | **Refs** |
| 185 | 1 | 1 | To construct a cyclic quadrilateral with four given lines. (by arithmetic) | | Given by Iohan Pouwelsz. twenty years ago. | I-15, VI-21 |
| 186 | 2 | 2 | To construct a cyclic quadrilateral with four given lines. (by geometry) | Sn gives an analysis of vC's not demonstrated construction. He concludes that it is easy to follow the synthesis now. He remarks that 'if you consider line, the construction of this problem is rather laborious, but it is very easy when dealt with by means of numbers.' (Ceulen, 1615b, p. 189), translation taken from (Wreede, 2007, p. 284). Sn uses four-dimensional magnitudes, but states that this can be avoided which he promises to show in a second edition with suitable figures. Sn gives another way to determine the diagonal $AC$ in geometrical terms.(Wreede, 2007, p. 279–285) Sn writes down a 'little theorem' analogous to Heron's theorem (see (Wreede, 2007, pp. 285-287)). At the end, Sn gives one of his own solution with an added figure. | Given by Cornelis Pietersz. four years ago. | |
| 191 | 3 | 3,4 | To construct a cyclic quadrilateral with four given lines. (by geometry and arithmetic) | Sn rewrote the solution to be more similar to Viète's work. He adds a short commentary in which he explains his changes and adds more calculations. (I have discussed this problem in detail in my bachelor thesis, see (Veen, 2011).) | Taken from Viète (1595). | I-33, I-37, I-47, II-13, III-22 |
| 194 | 4 | 5 | To calculate the line that divides the circumscribed quadrilateral in two equal parts. | Sn gives a more easier and shorter way to test by numbers whether the part that was cut off is indeed half of the quadrilateral. | | III:16 |
| 196 | 5 | 6 | To calculate parts of a line of a circumscribed quadrilateral cut by the diameter. | Sn gives an alternative and easier way to find the solution. | Reference to the 16th example of book three. | |
| 197 | 6 (5) | 7a | To calculate the areas of parts of a circumscribed quadrilateral cut by the diameter. | | | III-36, VI-28 |

| **Fundamenta book V** | | | | | | |
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| **p.** | **No. Fu** | **No. Fo** | **Content** | **Comments** | **Info** | **Refs** |
| 198 | 7 | 7b,8,9,10,11,12 | To cut $\frac{1}{4}$, $\frac{1}{3}$ (twice), a given proportion and $\frac{1}{2}$ of a triangle by a line through a point outside the triangle. | Sn writes that he combined problems which involved cutting triangle with a line through a point outside the triangle. He refers to his translation of a work by Appolonius where he presented a more elegant method and promises to provide an elaboration in a second edition. For the exact translation of this commentary see (Wreede, 2007, pp. 240–241). | Sn bundled problems with the point $O$ outside the triangle together. | I-47, II-14, II-15b, III-36, VI-14, VI-16 |
| 205 | 8 | 13,14,15 | To divide a triangle in two equal parts by a line through a point inside the triangle. To calculate parts of a side cut by a line dividing a triangle. To divide a triangle by a given proportion | Sn provided vC's construction with a proof (without numbers), which he 'could have provided much easier and more elegant if a figure to his own liking had been available' (Wreede, 2007, p. 240) . At the end of the problem Sn remarks that $BF$ does not have to be regarded in two parts and gives an alternative calculation. | Sn bundled problems with the point $O$ inside the triangle together. | |
| 209 | 9 | 16 | To calculate the length of a line in a circle. | Sn remarks that the given construction is complicated and gives an easier way. He also remarks that $GL$ and $LM$ are parts of the diameter which vC neglected to mention. He promises to give a proof in a next edition. (I have discussed this problem in detail in my bachelor thesis, see (Veen, 2011).) | | I:8, II:36 |
| 210 | 10 | 17 | To calculate lines of a triangle inscribed in a square in a circle. | | | I-47 |
| 211 | 11 | 18 | To calculate the diameters of circles and the sides of a triangle inscribed in a parallelogram. | | | VI-8 |
| 213 | 12 | 19 | To calculate the sides of a quadrilateral with one side of a hendecagon and another the diameter of a circle. | | Publicly accosted on June 8th, 1598 in Leiden. | II-13 |
| | 13 | 20 | To calculate lines of a triangle partly inscribed in a circle. | | | II-12, VI-3 |

| Fundamenta book V | | | | | | |
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| **p.** | **No. Fu** | **No. Fo** | **Content** | **Comments** | **Info** | **Refs** |
| 214 | 14 | 21,22 | To calculate lines of a triangle partly inscribed in a circle with numerical example. | | | |
| 215 | 15 (14) | 23 | Another example of the previous with different numbers. | vC only gives answers without a calculation. Sn remarks that at first his answer seemed different, but that in fact it was equal to vC's. Sn uses a sine table to show this. His calculations are two decimal places more accurate. Sn warns vC that he should wrote down the numbers as simple as possible (Wreede, 2007, p. 301). | | |
| 216 | 16 | 24 | To calculate a line inside two circles partly inscribed in a triangle. | Sn remarks that vC did not show how he found that $DI = 6$. Sn shows how to find this in four different ways (with a nice little theorem). However, Sn uses that $BQ = 7$ instead of $BQ = 6\frac{1}{2}$. | | III-36 |
| 217 | 17 | 25 | To calculate a diameter inside two circles partly inscribed in a triangle. | Sn adds the calculation to $BZ$. | | |
| 218 | 18-I | 26,27,28 | To calculate two sides of a triangle when the basis, the perpendicular and the sum of the two sides are known without algebra. (with two solutions from Sn) | | Send by Iohannes Wilhelmi Velsius, given to Sn. The figure of the third problem (on p. 220) has corrected letters. | II-5, II-13, VI-8 |
| 220 | 18-II | 29,30 | To calculate two sides of a triangle when the basis, the area (or perpendicular) and the proportion between the two sides are known. | Sn remarks that there is a situation in which vC's method does not work (when the angle of $A$ is obtuse). He says it is easy to solve, but does not give the calculations. In one sentence (p. 222) Sn remarks there is a risk to find other solutions and gives those. | | I-4, I-47, VI-1 |
| 222 | 19 | 31 | To calculate the area of a triangle when the basis and the sum of the two other sides are known. | | | |
| | 20 | 32 | To calculate a line inside a circle. | Sn gives comments on the use of the root sign twice. | This question was proposed and co-resolved by Sn. In the proof it uses the table of Valentinus Otto. | I-47, II-5 |
| 224 | 21 | 33 | To calculate lines and areas of a circle inscribed in a triangle. | | | II-12, III-35, VI-19 |

| Fundamenta book V | | | | | | |
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| **p.** | **No. Fu** | **No. Fo** | **Content** | **Comments** | **Info** | **Refs** |
| 226 | 22 | 34 | To calculate the two sides of a triangle with given basis, area and sum o the two sides by algebra and the sine table. | | Same as Problema 18-I. First problem in book V solved with algebra. | |
| 227 | 23 | 35 | To calculate the two sides of a triangle with given basis, area and sum o the two sides by the sine table. | | Same as Problema 18-I. | I-47 |
| 228 | 24 | 36 (37) | To calculate two sides of a triangle with given angle and area. | | | I-47 |
| | 25 | 37 (36) | To calculate two sides of a triangle with given angle and area. | | Reference back to 29th example. | I-32, III-36 |
| 229 | 26 | 38 | To calculate the angles in a triangle with given sides. | | | I-32 |
| | 27 | 39 | To calculate angles and sides of a triangle. | | | I-47 |
| 230 | 28 | 40 | To calculate two sides and angles of a triangle with given basis, square of one of the sides and proportion. | Sn remarks that this problem (solved with coss) can easier be solved with geometry, but he does not have a figure and is retained to give this solution. | | |
| 231 | 29 (26) | 41 | To calculate the diameters of three circles with center points of a triangle tangent to the inscribed circle. | | | |
| | 30 | 42 | To calculate parts of a line in a circle which intersects with a given proportion with the diameter. | | | III-35 |
| 232 | 31 (33) | 43 | To calculates lines in a quadrilateral. | Sn remarks that vC invites as it were to find the other methods, but that he does not have the time or figure. vC wrote that the answers were too complicated to notate, but Sn gives them in his comment. | Solved by Sn and Nathaniel Claeszoon. | |

| Fundamenta book V | | | | | | |
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| **p.** | **No. Fu** | **No. Fo** | **Content** | **Comments** | **Info** | **Refs** |
| 234 | 32 | 44 | To calculate the are of a triangle when sides of adjacent triangles are known. | Again vC states that the numbers that were to solve the problem were too complicated. Sn criticized him on not being exact and denied his statement asserting that either vC made a calculation mistake or had chosen a less suitable construction (Wreede, 2007, p. 301). He gives the numbers and his opinion on using numbers in geometry. | Send by Adrianus Ockersz.. This problem is the continuation of problem 50 of book four. Pieter Cornelisz. helped vC to find the solution. | I-32, IV:50 |
| 235 | 33 | 45 | To calculate the perpendicular in a quadrilateral. | This problem is again solved using a sine table. Sn gives the exact numbers that solve the problem. | Send by Nicolaes Pietersz. of Deventer, printed in Amsterdam in 1584 and recommended to Willem Goudaen. Solved by Samuel Krop van Doevers in 1599 (the preceding year). vC refers to his work published against Willem Goudaen. | |
| 237 | 34 (30) | 46 | To calculate lines in two intertwined triangles. | | Send by a good friend. | |
| 238 | 35 | 47 | To calculate lines in two intertwined triangles (with different numbers). | First, Sn criticizes vC on being too long and complicated. Then he gives an alternative method to find the solution to problem 34 (previous), using arithmetical progression. Finally, he gives two alternative methods to problem 35. | Answer to previous sender. | |

## E.6 Book VI

p.: Pagenumber in the *Fundamenta*
No. Fu: Number of the proposition in the *Fundamenta*
No. Fo: Number of the proposition in the *Fondamenten*
Content: Description of the content found in the *Fondamenten*
Comments: Added commentaries by Snellius
Info: Additional information
Refs: References made to the equivalent proposition from the *Elements* of Euclid (III-4 means the fourth proposition of the third book), book 2 of the *Fondamenten* (II:15 means proposition 15) and *Vanden Circkel*.

| Fundamenta book VI | | | | | | |
|---|---|---|---|---|---|---|
| **p.** | **No. Fu** | **No. Fo** | **Content** | **Comments** | **Info** | **Refs** |
| 241 | Introduction | | | | | Book 2 |
| | 1 | 1 | Calculations on the 4, 8, 16 and 32 inscribed and circumscribed equilateral polygons. | | | *Vanden Circkel*; I:8 |

| Fundamenta book VI | | | | | | |
|---|---|---|---|---|---|---|
| **p.** | **No. Fu** | **No. Fo** | **Content** | **Comments** | **Info** | **Refs** |
| 245 | 2 | 2 | Calculations on an inscribed and circumscribed triangle. | | | |
| 247 | 3 | 3 | Calculations on an inscribed and circumscribed pentagon. | | | II:58 |
| 249 | 4 | 2,4 | Calculations on an inscribed and circumscribed quindecagon. | | Sn reverses the order of parts of problem 2 and 4. Also the missing pages in the *Fondamenten* (p. 253-254) are present in the *Fundamenta*. | *Vanden Circkel* |
| 251 | 5 | 5,6 | Calculations on the area and diameter of a circumscribed pentagon with given sides. | | Sn adds 'Reghel 6' to problem 5. | I-15, I-32, VI-19, *Vanden Circkel* chapter 8 |
| 253 | 6 | 7 | Calculations on the length of the sides of a circumscribed pentagon with given diameter. | | | |
| | 7 | 8 (not numbered) | Calculations on the length of the sides of a circumscribed pentagon with given area of the circle. | | | |
| 254 | 8 | 9 | Calculations on a circumscribed hexagon. | | | |
| | 9 | 10 | Calculations on a circumscribed heptagon. | | | I-47, *Vanden Circkel* chapter 1 |
| 255 | 10 | 11 | Calculations on a circumscribed quatuordecagon. | | | |
| 256 | 11 | 12 | Calculations on a circumscribed octagon. | | | |
| | 12 | 13, 14 (not numbered) | Calculations on a circumscribed nonagon. | | | VI-4, *Vanden Circkel* |
| 264 | A1 | 15 | To find the length of a line which cuts a circle in three equal parts. | | | *Vanden Circkel* |
| 265 | A2 | 16 | To find the length of a line which cuts a circle in four or five equal parts. | | | |
| 267 | A3 | 17 | To find the sine and the area of part of a circle which is cut with a given proportion. | Sn adds the missing answer, remarking that it possibly lacks due to vC's passing. | | |

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