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Inquisitive Semantics and Implicature

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Chapter 1

Introduction

Language has always been a source of interest for scientists. The human ability to produce and understand speech and writing is part of what differentiates us from other animal species, and therefore helps define what it is to be human. Linguistics and artificial intelligence researchers try to model language to better understand its mechanics, and to be able to reproduce it in a computer. Their goal is to have a computer understanding and producing linguistic expressions that are indistinguishable from those produced by humans.

Research on semantics has provided some insight into the way language could be modelled. Many ways exist to model all kinds of sentences in a way that is analyzable using logic, but many of those models are based on only the informative aspect of language. In these models, sentences are informative statements, ideal to model factual information like that given in a textbook. An example of such a sentence would be “Mr. Smith lives in London”. This sentence could then be modeled as something like $livesInLondon(Mr.Smith)$. We could then check whether Mr. Smith was in the set of entities living in London, and thereby would know whether the sentence is true or false.

One issue that is rarely addressed is one of questions: how to properly model a question? Some linguists suggest modeling a simple question “Where does Mr. Smith live?” like the following three statements: “I do not know where Mr. Smith lives, I want to know where Mr. Smith lives, and I believe you know where Mr. Smith lives.” (see Hamblin (1958)). While this method might provide a solution at least to some extent, this seems like a very unnatural way of thinking about questions. Ciardelli and Roelofsen (2011) and Groenendijk and Roelofsen (2009) describe another possible solution: a semantics more suited to natural conversation, and the logic accompanying it.

Inquisitive semantics was developed as an alternative to classical semantics, in which the interactive exchanges of information and questions that are real-world conversations could be modelled. Conversations play a big role in human society, and here we will only be able to consider conversations with the purpose of sharing information. Goals like convincing someone of something untrue or meaningless chit-chat will have to wait

This thesis will be an introduction to inquisitive semantics. In addition, we will look at how pragmatics can be handled in inquisitive semantics. We will consider this by looking at the work of Grice (1975) and an adaption of his maxims for this semantics by Groenendijk and Roelofsen (2009).

To start we will give a introduction to the definition of meaning and truth in inquisitive semantics. We will show how some important theorems from classical logic hold up in inquisitive logic. In the following chapter we will consider the way conversation is modelled using inquisitive semantics. In chapter four we will then look at Grice's conversational maxims and their translation to inquisitive semantics. This will then be followed by more analysis on inquisitive semantics' possibilities and application, both in science in general and in the field of artificial intelligence.

Chapter 2

Inquisitive Semantics

2.1 Sentence representation

In this chapter we will look at the logic behind inquisitive semantics. After some examples we will then continue with an explanation of how knowledge is represented in the semantics. Most of this chapter is based on the work of Ciardelli and Roelofsen (2011) and Groenendijk and Roelofsen (2009).

2.1.1 Support and meaning

Inquisitive semantics has a basis of indices and states. An index is a subset of the proposition letters $\mathcal{P}$. Sometimes indices are called "worlds", however, we will use the term indices. A state is a set of indices.
In this semantics, the language, and the sentences in it, are built up from $\bot$ by proposition letters and the connectives $\wedge$, $\vee$, $\rightarrow$, !, ? and $\neg$, where:

$$\neg p = p \rightarrow \bot$$
$$!p = \neg\neg p$$
$$?p = p \vee \neg p$$

Because we want the semantics to be able to deal with both statements and questions, the concept of meaning is slightly different from that in classical semantics. We say something is true in a state if the state supports it. Support by a state s is defined as follows:

Definition 1. *Support (for state s and index w)*
$s \vDash \perp \iff s = \emptyset$
$s \vDash p \iff \forall w \in s : p \in w$
$s \vDash p \wedge q \iff s \vDash p$ *and* $s \vDash q$
$s \vDash p \vee q \iff s \vDash p$ *or* $s \vDash q$
$s \vDash p \to q \iff \forall t \subseteq s : t \vDash p$ *implies* $t \vDash q$

Indices, or single-index states, behave classically. That means that for any index w and any formula ϕ:

$$\{w\} \vDash \phi \iff w \vDash \phi \tag{2.1}$$

Because of support being defined this way, meaning will also be different compared to classical semantics. A proposition ϕ will be considered true in a state s if s supports ϕ. With this definition, we can define multiple other terms that will come in useful later on.

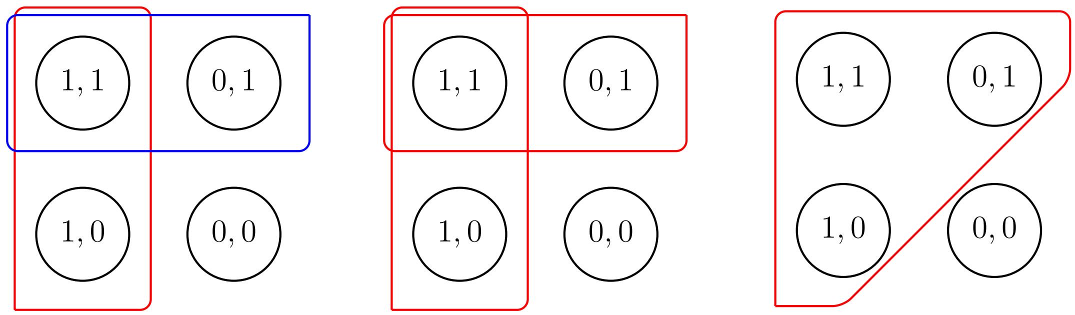
Definition 2. *For a formula ϕ:*

1. *A possibility for ϕ is a maximal state supporting ϕ. A state is a possibility for ϕ if it supports ϕ and it is not a subset of another state supporting ϕ.*

2. *A proposition of ϕ is the set of possibilities for ϕ.*

3. *The truth-set of ϕ is the set of indices where ϕ is true, or the maximal state where every index separately supports ϕ.*

The proposition for ϕ can be written as $[\phi]$, while the corresponding truth-set is written as $|\phi|$. Examples are given in figure 2.1 to clarify the differences between possibilities, propositions, and truth-sets. In this figure (and many of the ones following) the small circles containing numbers or symbols are depicting indices. The colored lines surrounding them show the states. Sometimes the line may surround all indices, in which case it shows the maximal state of all indices. We will use the terms illustrated in the figure to further define the different kinds of sentences in inquisitive semantics.



(a) The possibilities for $p \vee q$ (b) The proposition $p \vee q$ (c) Truth-set of $p \vee q$

Figure 2.1: Differences between possibility, proposition and truth-set of a formula

2.1.2 Classifying sentences

To help us talk about the different kinds of sentences in this semantics, we will introduce some new definitions. First we will define what a question is, and what an assertion is. To this end, we will also define inquisitiveness and informativity.

Definition 3. *Inquisitive, informative*

- *A sentence ϕ is inquisitive if the corresponding proposition $[\phi]$ contains more than one possibility.*
- *A sentence ϕ is informative if it proposes to remove at least one index from its state.*

Definition 4. *Questions & assertions*

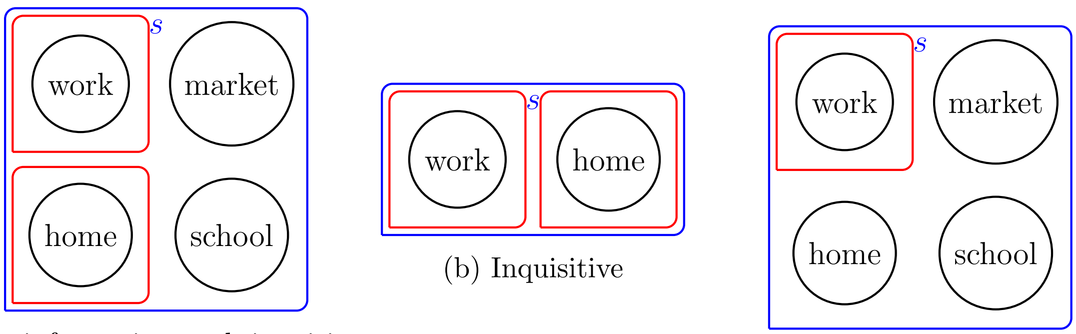
- *A sentence is a question if it is not informative.*
- *A sentence is an assertion if it is not inquisitive.*

Of course, a proposition can be both informative and inquisitive, for example if it proposes to eliminate one index, and to divide the remaining indices into groups.
Because of the definition of questions, it is possible for a question to not propose more than one possibility, for example if the question is meant to make sure everyone agrees with the common ground at a certain time. Similarly, it is possible for an assertion to not actually give new information. We will look at what this means for conversational implicature later. We will first show some examples of inquisitive and informative sentences.

Example 1.

*a) Is Ada at home or at work? (***Inquisitive (+ informative)***)*

*b) Ada is at work. (***Informative***)*



(a) informative and inquisitive

(b) Inquisitive

(c) Informative

Figure 2.2: Examples of inquisitive and informative sentences

In figure 2.2a we see a knowledge state containing three three possibilities, two of which are marked in red. When using this knowledge state, the sentence in (a) is both inquisitive (since there are still two possibilities that are possible answers to the question) and informative (since two indices are eliminated, because they are not contained in any possibility for (a)).
Figure 2.2b shows the purely inquisitive sentence. In this case, no indices are eliminated, but it is clear that the sentence outlines multiple possibilities. Figure 2.2c shows the proposal of sentence (b): only one possibility is left, and all indices not contained in that possibility will be discarded.

In most cases it will suffice to say that if a proposition contains a disjunction operator, it will be a question. If it does not contain a disjunction, it is an assertion. It is possible, however, to find examples in which this isn't the case. For those cases more specific definition will be sufficient.

To give the reader a better idea of what parts of inquisitive logic and classical logic are similar and different, we will look at some theorems and formulae and the way they are interpreted using inquisitive semantics.

2.2 Formulae in inquisitive semantics

As we have seen earlier, there is a difference in how truth is defined between inquisitive and classical semantics. To show what exactly the possibilities of inquisitive semantics are, I will here give some examples of possible statements in inquisitive semantics. These examples are of interest because they are true in classical logic, while this is not as trivial in inquisitive logic.
First we will show how the classical disjunction can be modelled in inquisitive semantics. We here define classical disjunction as $\forall w \in s : (p \in w \vee q \in w)$. So this is the possibility in s for which every index contains either p or q. For the following proof, recall that $!\phi$ is shorthand for $\neg\neg\phi$

Lemma 1. *Classical disjunction $(p \vee q)$ can be modelled in inquisitive semantics as $s \vDash !(p \vee q)$*

Proof.

$$
\begin{aligned}
s \vDash !(p \vee q) &\iff s \vDash \neg\neg(p \vee q) && (2.2)\\
&\iff s \vDash \neg(p \vee q) \rightarrow \bot && (2.3)\\
&\iff \forall t \subseteq s : t \vDash \neg(p \vee q) \text{ implies } t \vDash \bot && (2.4)\\
&\iff \forall t \subseteq s : t \nvDash \neg(p \vee q) && (2.5)\\
&\iff \forall t \subseteq s : t \nvDash (p \vee q) \rightarrow \bot && (2.6)\\
&\iff \forall t \subseteq s : \neg(\forall r \subseteq t : r \vDash (p \vee q) \text{ implies } r \vDash \bot) && (2.7)\\
&\iff \forall t \subseteq s : \neg(\forall r \subseteq t : r \nvDash (p \vee q)) && (2.8)\\
&\iff \forall t \subseteq s : \exists r \subseteq t : r \vDash (p \vee q) && (2.9)\\
&\iff \forall t \subseteq s : \exists r \subseteq t : r \vDash p \text{ or } r \vDash q) && (2.10)\\
& && (2.11)
\end{aligned}
$$

The final line of the proof says that in every subset t of s, there must be at least one subset r(this can be a single index) for which it is true that r contains either p or q or both. Because this is true for every subset, it must also be true for every index in s, if every index contains either p or q. So, the classical disjunction in inquisitive semantics can be modelled as the assertion $s \vDash !(p \vee q)$. □

Above proof is only shown using atoms. This is because inquisitive semantics is not closed under substitution. Therefore, it is hard to prove something using ϕ, ψ instead of p, q.

We will now show that inquisitive semantics is not closed under uniform substitution. We can't just replace a subformula with another formula and expect it to give comparable results. This is shown by proving that $p \equiv \neg\neg p$, while $p \vee q \not\equiv \neg\neg(p \vee q)$. Here $\phi \equiv \psi$ is defined as $\forall s : s \vDash \phi \Leftrightarrow s \vDash \psi$. Because the specific formulae used in this proof, we immediately prove that in inquisitive semantics double negation elimination does not apply.

Lemma 2. *Uniform substitution does not apply in inquisitive logic.*

Proof.

$$s \vDash p \iff \forall w \in s : p \in w \tag{2.12}$$

$$\begin{aligned}
s \vDash \neg\neg p &\iff s \vDash \neg p \to \perp && (2.13)\\
&\iff \forall t \subseteq s : t \vDash \neg p \to t \vDash \perp && (2.14)\\
&\iff \forall t \subseteq s : t \nvDash \neg p && (2.15)\\
&\iff \forall t \subseteq s : t \nvDash p \to \perp && (2.16)\\
&\iff \forall t \subseteq s : \neg(\forall r \subseteq t : r \vDash p \to r \vDash \perp) && (2.17)\\
&\iff \forall t \subseteq s : \neg(\forall r \subseteq t : r \nvDash p) && (2.18)\\
&\iff \forall t \subseteq s : \exists r \subseteq t : r \vDash p && (2.19)\\
&\iff \forall t \subseteq s : \exists r \subseteq t : \forall w \in r : p \in w && (2.20)\\
&\iff \forall w \in s : p \in w && (2.21)
\end{aligned}$$

Step 2.20 says that there in every subset t of s, there exists a subset r so that all indices in r contain p. If we take t to be the smallest possible subset, an arbitrary index, then r has to be that same index. So then we know that that index contains p. Because we chose this index arbitrarily the same is true for all indices in s. Thus we know that every index in s contains p, which is what is said in 2.21. and therefore that we can make the step from 2.20 to 2.21. This is also the definition of $s \vDash p$ in 2.12, so therefore in inquisitive semantics $p \equiv \neg\neg p$.

$$\begin{aligned} s \vDash p \vee q &\iff s \vDash p \text{ or } s \vDash q && (2.22)\\ &\iff \forall w \in s : p \in w \text{ or } \forall w \in s : q \in w && (2.23)\end{aligned}$$

$$\begin{aligned} s \vDash \neg\neg(p \vee q) &\iff s \vDash \neg(p \vee q) \rightarrow \bot && (2.24)\\ &\iff \forall t \subseteq s : t \vDash \neg(p \vee q) \rightarrow t \vDash \bot && (2.25)\\ &\iff \forall t \subseteq s : t \nvDash \neg(p \vee q) && (2.26)\\ &\iff \forall t \subseteq s : t \nvDash (p \vee q) \rightarrow \bot && (2.27)\\ &\iff \forall t \subseteq s : \neg(\forall r \subseteq t : r \vDash p \vee q \rightarrow r \vDash \bot) && (2.28)\\ &\iff \forall t \subseteq s : \neg(\forall r \subseteq t : r \nvDash p \vee q) && (2.29)\\ &\iff \forall t \subseteq s : \exists r \subseteq t : r \vDash p \vee q && (2.30)\\ &\iff \forall t \subseteq s : \exists r \subseteq t : r \vDash p \text{ or } r \vDash q && (2.31)\\ &\iff \forall t \subseteq s : \exists r \subseteq t : \forall w \in r : p \in w \text{ or } \forall w \in r : q \in w && (2.32)\end{aligned}$$

While at first glance this looks identical to the proof of $p = \neg\neg p$, there's an important difference. According to 2.24-32, $s \vDash \neg\neg(p \vee q)$ if every single index in s contains either p or q. Therefore every index can have a different valuation for p and q, as long as at least one of them is classically true.
According to 2.22-23, $s \vDash p \vee q$ if every index in s contains p, or every index contains q. This means that for $s \vDash p \vee q$ to be true, every index must have the same positive valuation concerning p or q. So, $p \vee q \not\equiv \neg\neg(p \vee q)$.
$s \vDash p \vee q$ is true when all indices contain p or all indices contain q. $s \vDash \neg\neg(p \vee q)$ is true when all indices contain either p or q. It is clear that while $s \vDash p \vee q \Rightarrow s \vDash \neg\neg(p \vee q)$, the opposite is not necessarily the case. Combined with the above proof of $p = \neg\neg p$, it is now clear that this system is not closed under uniform substitution.

□

From the above proof it is also clear that double negation elimination does not apply in inquisitive semantics, for the same reasons why $p \vee q \not\equiv \neg\neg(p \vee q)$.

Lemma 3. *Double negation elimination does not apply in inquisitive logic.*

Proof. If $s \vDash \phi$, then in every single index in s, ϕ is true. The same seems to be the case for $s \vDash \neg\neg\phi$: this can be written as in 2.21 ($\forall t \subseteq s : \exists r \subseteq t : r \vDash \phi$). This means that for every subset of s, there exists a subset in which every world contains ϕ. However alike the two seem, there is a crucial difference that makes double negation elimination a problem. An example situation is given in figure 2.3, for $\phi = p \vee q$. Here, in every subset there is an index in which $p \vee q$ is true. However, it is not the case that either all indices in s contain p or all indices contain q. Therefore, $s \nvDash p \vee q$ in the example. By this counterexample, we have shown that double negation elimination does not necessarily apply. □

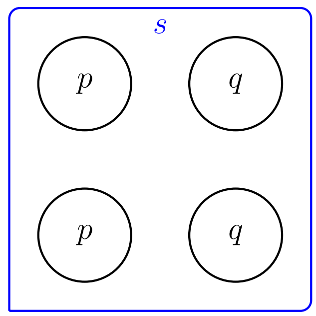


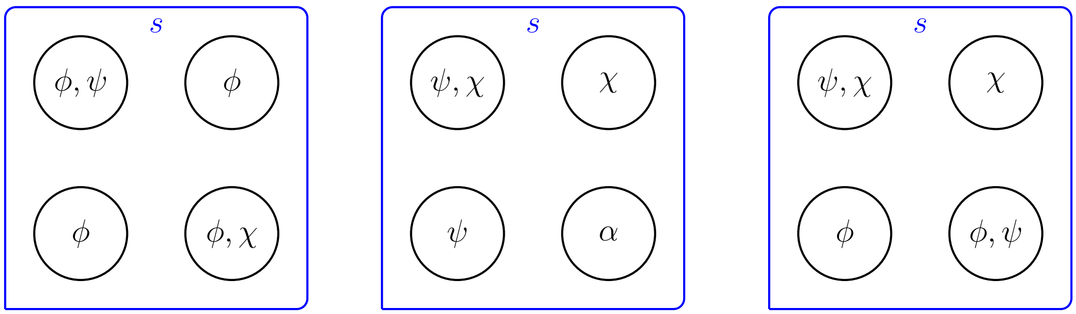
Figure 2.3: $s \vDash \neg\neg(p \vee q)$ but $s \nvDash p \vee q$

Note that double negation elimination does apply for atoms, since those cannot be interpreted differently in different indices to be supported by s. So $p = \neg\neg p$, but $\phi \neq \neg\neg\phi$. Another very important difference between classical and inquisitive logic is the latter's rejection of the law of the excluded middle. We will now look at the proof for this, based on Mascarenhas (2009). We show that $\phi \vee \neg\phi$ does not necessarily hold, thereby proving that the same is true for the law of excluded middle.

Lemma 4. *The law of no excluded middle does not apply in inquisitive logic.*

Proof. For $s \vDash \phi$ to be true if ϕ is an atom, every single world w in s must contain ϕ, or $\forall w \in s : \phi \in w$. Figure 2.4a shows this situation. If ϕ is not an atom, we know that we can reduce the formula using definition 1 defining support until only formulae of the form *state* $\vDash$ *atom* remain. For $s \vDash \neg\phi$ to be true, no world in s can contain ϕ (or all worlds contain $\neg\phi$, which would have the exact same result. Because worlds behave classically, for them not containing ϕ is equal to containing $\neg\phi$). This is formalized as $\forall w \in s : \phi \notin w$, shown in figure 2.4b.

Because of the way support is defined, it is possible that $s \not\models \phi \wedge s \not\models \neg\phi$. An example is given in figure 2.4c. When some world in s contain ϕ, while other worlds in s do not contain ϕ, it is not possible to either say $s \models \phi$ or $s \models \neg\phi$. This lead to $s \not\models \phi \vee \neg\phi$. It should be clear that because of this, $\phi \vee \neg\phi$ is not a tautology, and therefore the law of excluded middle does not apply in inquisitive semantics. □



(a) $s \models \phi$ (b) $s \models \neg\phi$ (c) $s \not\models \phi$ and $s \not\models \neg\phi$

Figure 2.4: Different situations of support in s

We have shown that some of the theorems used in classical logic cannot be applied when using inquisitive logic. It should be clear that these dissimilarities can create major differences in what is considered true in a system. This makes it possible for inquisitive logic to deal with questions in its related semantics. We will now proceed to give a little more information on how conversation can be modeled in inquisitive semantics.

2.3 Knowledge representation

Since we want to be able to use inquisitive semantics to model real conversations, we have to consider how a conversation is represented in the system that is used. This should keep track of what is said, what each participant of the conversation knows, and what every participant knows.

Groenendijk and Roelofsen (2009) and Ciardelli and Roelofsen (2011) describe a system that has its basis in a 'common ground': a state representing the common knowledge of the participants in a conversation. In other words, the common ground represents the knowledge that all participants have and of which they all know they all know it. At the beginning of a conversation, the common ground will generally be the maximal state, containing every single possible index. During the conversation, indices will be removed from the common ground, representing knowledge since fewer possibilities are left.

Each participant's personal knowledge is represented in the same way: a state representing what the participant knows and knows to know. The difference with the common ground is that this state generally already contains knowledge at the beginning of a conversation: indices have already been removed from the state. Therefore, a individual knowledge state is a maximal state that generally contains fewer indices than the state representing the common ground.

It is important to note that both the common ground and the individual knowledge states are maximal states: they contain all indices that are deemed possible by either the individual or the group. It is possible for these knowledge states to contain one or more smaller states. This could happen, for example, after an inquisitive sentence is uttered. This causes the knowledge state to 'divide' into several smaller states, the possibilities that are possible answers to the question. To avoid possible confusion, we will use 'knowledge state' to indicate the maximal state of one or more participants' indices. 'State' will be used as before, to indicate a set of indices.

Each sentence from a participant is viewed as a proposal to change the common ground. If accepted, it might also make a change to individual knowledge states of the participants. If any of the participants reject the proposal it is ignored, and no changes are made to any knowledge states.

During a conversation, states are removed from the common ground in accordance with the assertions and questions uttered. Assertions usually remove indices, while questions create a division between the indices in a knowledge state, inviting other participants to make an assertion confirming one of the groups. If the proposal is accepted, both the common ground and the personal knowledge states of the participants of the conversation are updated. The latter is done to ensure that the common ground really represents the knowledge of all participants.

Chronologically, the following is what happens when an proposal is voiced. First, every participant checks the proposal with their own knowledge state. If any participant finds the information given is inconsistent with their own knowledge, he isn't able to update his own knowledge state. Doing this would remove all indices from the knowledge state, since in the semantics something is only true if it is true in every index in a state: accepting something causing a contradiction as true would empty the knowledge state. Because of this, the participant has to announce his incapability to update his state. This cancels the updates of all participants' knowledge

states and the common ground. Of course, if the utterance is purely inquisitive, no participant should be able to reject the proposal. An inquisitive sentence does not yet propose to remove any indices (only its answer would do that), and so should always be consistent. The only situation in which an inquisitive proposal could be rejected is one concerning such a topic that no index in the participant's knowledge state contains this. This is hard to imagine and even harder to find an example of, therefore we will not consider it here.
However, if every participant finds the information in the utterance possible within their own knowledge, and no protest is made, all individual knowledge states and the common ground are adapted to the new information. If the utterance is informative, any indices that do not comply with the possibilities in the proposal are discarded. If the utterance is inquisitive, every knowledge state is divided into several states - the possibilities. A possible next utterance may be informative and, after acceptation, remove one or more of these possibilities.
After this step, no indices should remain in the individual knowledge states that have been removed from the common ground, since that would create inconsistencies.

Example 2. *To clarify this abstract idea, we will look at an example of two people, lets call them Andy and Bea, having a drink. In this example we will take an incredibly simple world, in which the only thing that matters is whether these people will drink coffee or tea. We also assume it is not possible to drink more than one beverage, so both participants will have to choose what drink they want.* C_n *will mean participant* n *is having coffee, while* T_n *will mean participant* n *is having tea.*

Our example conversation consist of the following utterances:

a) **A**: *"Would you like coffee or tea?"* $(C_2 \vee T_2\ )$

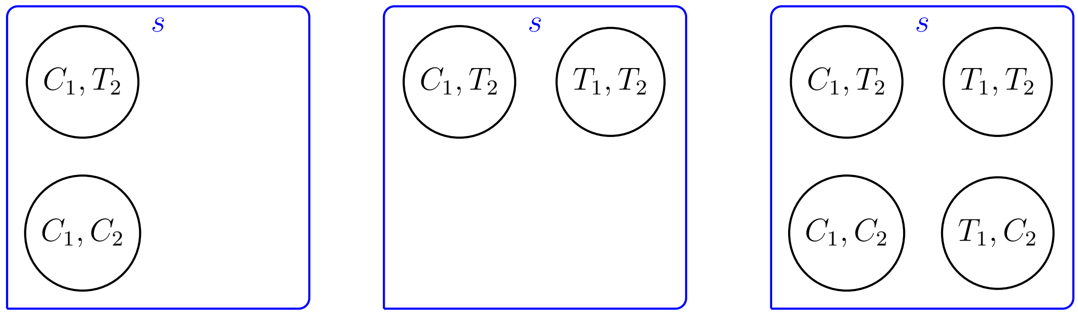
b) **B**: *"Tea, please."* (T_2)

c) **B**: *"How about you?"* $(C_1 \vee T_1)$

d) **A**: *"I'd like a coffee."* (C_1)

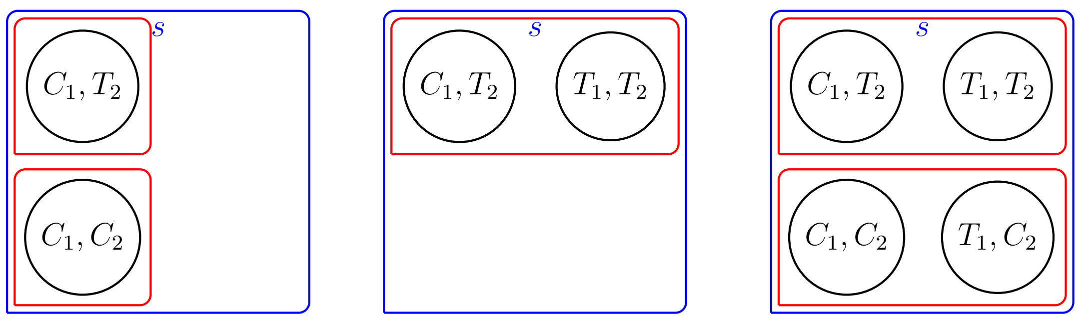
In figure 2.5 we see the knowledge states and common ground. Note that neither participant has every possible index represented in their knowledge states, because they already know what they themselves want to drink.

After the first sentence, Andy's question, the knowledge states will change. Because the question is inquisitive and not informative, no indices will be removed, but a



(a) Knowledge state of A (b) Knowledge state of B (c) Common ground

Figure 2.5: Situation at beginning of conversation

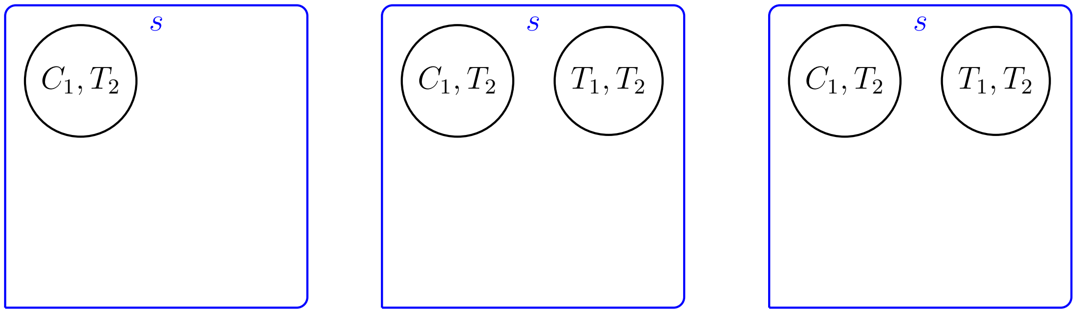


(a) Knowledge state of A (b) Knowledge state of B (c) Common ground

Figure 2.6: Situation after (a)

division is added as a sort of preparation for the answer. The result is shown in figure 2.6. After Bea's assertion that she will have tea (which is according to her knowledge state in which all indices contain her drinking tea), the indices in which this is not the case are removed from all three knowledge states. This means one of the previously selected groups is chosen, and every index outside of this group is removed from the knowledge states. This results in the knowledge states depicted in figure 2.7.

After the removal of indices from the states, it is obvious that Andy now knows the exact situation. After all, there is only one index left in his knowledge state. But, since Bea still does not know what Andy wants to drink, her knowledge state still contains multiple indices. The common ground, depicting the common knowledge, therefore also still contains multiple indices. For completeness, we will show what happens in step (c) and (d) of the conversation in figure 2.8. These steps follow

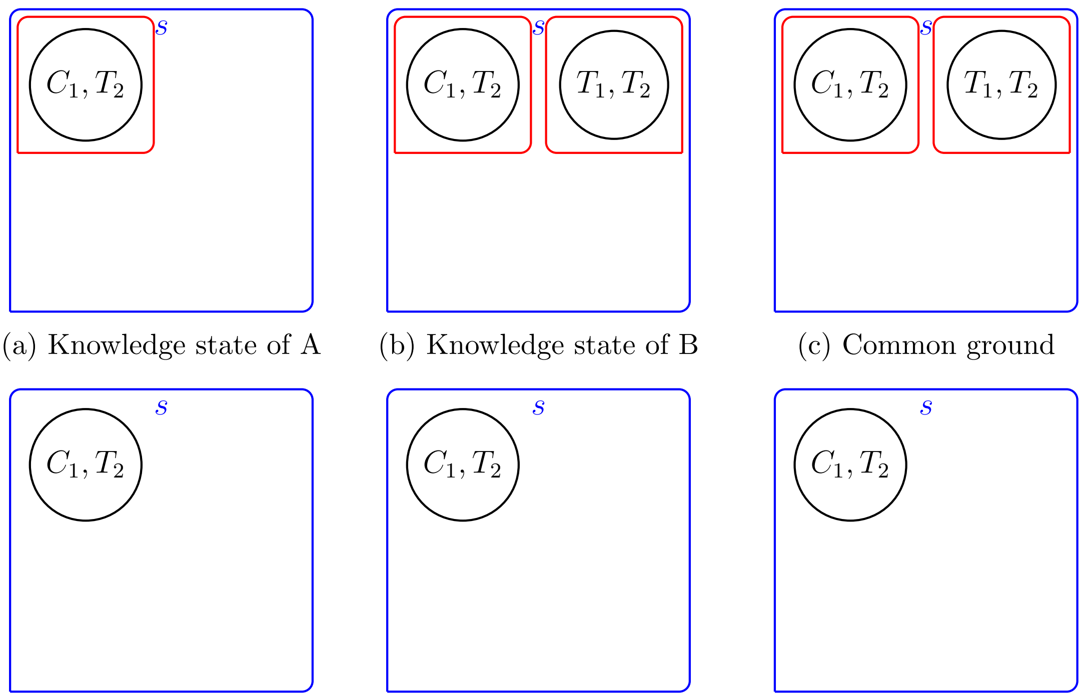


(a) Knowledge state of A (b) Knowledge state of B (c) Common ground

Figure 2.7: Situation after (b)

along the same lines as (a) and (b). After these steps, the common ground (and both individual knowledge states) only have one index left. This is, for this conversation, the end, since every participant knows everything there is to know. It is also possible that the conversation ends before there is only a single index remaining, depending on the goal of the participants. If Andy and Bea's only goal was to have it be known what Bea wanted to drink, the conversation could have ended after (b).

In this chapter, we have looked at the definition of support in inquisitive semantics. We defined possibilities, propositions, and truth-sets, along with other useful definitions, and we have seen how they can be used. Using our definition of support, we have also looked at several theorems from classical logic, and proved they do not all apply in inquisitive logic. Finally, we have seen how inquisitive semantics uses all of this to model conversations.
This will give us a basis for the following chapters, in which we will take a closer look at inquisitive logic and at Grice's maxims in inquisitive semantics.



(a) Knowledge state of A (b) Knowledge state of B (c) Common ground

(d) Knowledge state of A (e) Knowledge state of B (f) Common ground

Figure 2.8: Situation after (c) and (d)

Chapter 3

Inquisitive logic

In this chapter, we'll look at inquisitive logic, defined as the set of formulas valid in inquisitive semantics by Ciardelli and Roelofsen (2011). We will look at its disjunctive negative variant, and at the classification of the logic as an intermediate logic.

3.1 Entailment and the disjunction property

Inquisitive logic, InqL, is the set of formulas that are valid in inquisitive semantics. To properly define this logic, we'll first define entailment.
We define entailment in terms of support.

Definition 5. *Entailment*
A set of formula Θ entails a formula ϕ if and only if any state supporting all formulas in Θ also supports ϕ. ($\Theta \vDash_{InqL} \phi$)
A formula ϕ is valid iff it is supported by every state. ($\vDash_{InqL} \phi$)

We can say that two formulas are equivalent if they entail each other, or equivalently, if their sets of possibilities, their propositions, are equal.

Because a formula is valid iff it is supported by every state, any formula that is true in every state is in InqL. With this knowledge, we can see that InqL has the disjunction property. If $\phi \vee \psi$ is in InqL, then $\phi \vee \psi$ is supported by all possible states, and so by the maximal state I, containing every possible index ($(I) \vDash \phi \vee \psi$). But then, using our previous definition of support, we know that $(I) \vDash \phi$ or $(I) \vDash \psi$. We can then conclude that ϕ or ψ must also be in InqL, and therefore, that InqL has the disjunction property.

3.2 Disjunctive negative variant

Now we will look at the disjunctive negative variant of the logic. We will begin by giving the translation $dnt(\phi)$ of a formula ϕ. Then we will look at some examples to show that the disjunction of negations really is equivalent to the original formula.

Definition 6. *Disjunctive negative translation*

1. $dnt(p) = \neg\neg p$

2. $dnt(\bot) = \neg\neg \bot$

3. $dnt(\phi \vee \psi) = dnt(\phi) \vee dnt(\psi)$

4. $dnt(\phi \wedge \psi) = \bigvee\{\neg(\phi_i \vee \psi_j) | 1 \leq i \leq n, 1 \leq j \leq m\}$

   where:

   - $dnt(\phi) = \neg\phi_1 \vee \cdots \vee \neg\phi_n$
   - $dnt(\psi) = \neg\psi_1 \vee \cdots \vee \neg\psi_m$

5. $dnt(\phi \rightarrow \psi) = \bigvee_{k_1,\cdots,k_n}\{\neg\neg \bigwedge_{1\leq i\leq n}(\psi_{k_i} \rightarrow \phi_i) | 1 \leq k_j \leq m\}$

   where:

   - $dnt(\phi) = \neg\phi_1 \vee \cdots \vee \neg\phi_n$
   - $dnt(\psi) = \neg\psi_1 \vee \cdots \vee \neg\psi_m$

Now that we have these definitions, we will look at their working. One by one, we will give an example of how the disjunctive negative translation compares to the original formula.

Example 3. *dnt*(p) $s \vDash p \iff s \vDash dnt(p)$

$$s \vDash p \iff \forall w \in s : p \in w$$

$$\begin{aligned}
s \vDash dnt(p) &\iff s \vDash \neg\neg p \\
&\iff s \vDash \neg p \rightarrow \bot \\
&\iff \forall t \subseteq s : t \vDash \neg p \rightarrow t \vDash \bot \\
&\iff \forall t \subseteq s : t \nvDash \neg p \\
&\iff \forall t \subseteq s : t \nvDash p \rightarrow \bot \\
&\iff \forall t \subseteq s : \neg(\forall r \subseteq t : r \vDash p \rightarrow r \vDash \bot) \\
&\iff \forall t \subseteq s : \neg(\forall r \subseteq t : r \nvDash p) \\
&\iff \forall t \subseteq s : \exists r \subseteq t : r \vDash p \\
&\iff \forall t \subseteq s : \exists r \subseteq t : \forall w \in r : p \in w
\end{aligned}$$

It should be clear that both p *and* $dnt(p)$ *are only true if and only if every single index in* s *contains* p*. Therefore, they are equivalent.*

Example 4. $dnt(\phi \vee \psi)$ $s \vDash p \vee q \iff s \vDash dnt(p \vee q)$

$$\begin{aligned}
s \vDash p \vee q &\iff s \vDash p \text{ or } s \vDash q \\
&\iff \forall w \in s : p \in w \text{ or } \forall w \in s : q \in w
\end{aligned}$$

$$\begin{aligned}
s \vDash dnt(p \vee q) &\iff s \vDash dnt(p) \vee dnt(q) \\
&\iff s \vDash dnt(p) \text{ or } s \vDash dnt(q) \\
&\iff \forall t \subseteq s : \exists r \subseteq t : \forall w \in r : p \in w \text{ or } \forall t \subseteq s : \exists r \subseteq t : \forall w \in r : p \in w
\end{aligned}$$

Using example 3 for the last step of the proof we can see that $s \vDash p \vee q$ *is equivalent to* $s \vDash dnt(p \vee q)$*. Here, too, it is clear that both* $p \vee q$ *and* $dnt(p \vee q)$ *are true if every single index in* S *contains* p *or if every single index contains* q*.Do note that this* *example is relatively simple because p and q are atoms. However, for this equivalence it should not make a difference if they were formulas: it would merely make for a more complicated and confusing example.*

Example 5. *$dnt(\phi \wedge \psi)$ $s \vDash (p \vee q) \wedge a \iff s \vDash dnt((p \vee q) \wedge a)$*

$$
\begin{aligned}
s \vDash (p \vee q) \wedge a &\iff s \vDash (p \vee q) \textit{ and } s \vDash a \\
&\iff (s \vDash p \textit{ or } s \vDash q) \textit{ and } s \vDash a \\
&\iff (\forall w \in s : p \in w \textit{ or } \forall w \in s : q \in w) \textit{ and } \forall w \in s : a \in w
\end{aligned}
$$

$$
\begin{aligned}
s \vDash dnt((p \vee q) \wedge a) &\iff s \vDash \neg(\neg p \vee \neg a) \vee \neg(\neg q \vee \neg a) \\
&\iff s \vDash \neg(\neg p \vee \neg a) \textit{ or } s \vDash \neg(\neg q \vee \neg a) \\
&\iff s \vDash (\neg p \vee \neg a) \rightarrow\perp \textit{ or } s \vDash (\neg q \vee \neg a) \rightarrow\perp \\
&\iff \forall t \subseteq s : t \vDash (\neg p \vee \neg a) \rightarrow t \vDash\perp \textit{ or } \forall t \subseteq s : t \vDash (\neg q \vee \neg a) \\
&\qquad \rightarrow t \vDash\perp \\
&\iff \forall t \subseteq s : (t \vDash \neg p \textit{ or } t \vDash \neg a) \rightarrow t \vDash\perp \textit{ or } \forall t \subseteq s : \\
&\qquad (t \vDash \neg q \textit{ or } t \vDash \neg a) \rightarrow t \vDash\perp \\
&\iff \forall t \subseteq s : (t \nvDash \neg p \textit{ and } t \nvDash \neg a) \textit{ or } \forall t \subseteq s : (t \nvDash \neg q \textit{ and } t \nvDash \neg a) \\
&\iff \forall t \subseteq s : (t \nvDash p \rightarrow\perp \textit{ and } t \nvDash a \rightarrow\perp) \textit{ or } \forall t \subseteq s : \\
&\qquad (t \nvDash q \rightarrow\perp \textit{ and } t \nvDash a \rightarrow\perp) \\
&\iff \forall t \subseteq s : (\exists r \subseteq t : r \vDash p \textit{ and } \exists r \subseteq t : r \vDash a) \textit{ or } \forall t \subseteq s : \\
&\qquad (\exists r \subseteq t : r \vDash q \textit{ and } \exists r \subseteq t : r \vDash a)
\end{aligned}
$$

Here, we used $(p \vee q)$ for ϕ to show how the definition of ϕ is used in the disjunctive negative translation.
The transition from the second last to the last line is simplified. Used here in all places where it could be applied is:
$t \nvDash \phi \rightarrow\perp$
$\iff \neg(\forall r \subseteq t : r \vDash \phi \rightarrow r \vDash\perp)$
$\iff \neg(\forall r \subseteq t : r \nvDash \phi)$
$\iff \exists r \subseteq t : r \vDash \phi$
The last line, as interpreted using natural language, says that for every subset of s (with the smallest subset being an index), at least one index contains both p and a, or for every subset at least one index contains both q and a. This is the same as

every index in s containing both p and a or both q and a. Seeing how this is also the truth condition for $(p \vee q) \wedge a$, the two are equivalent.

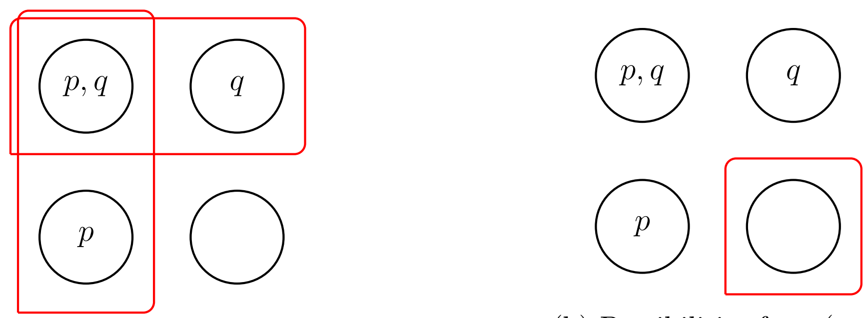
Example 6. $dnt(\phi \to \psi)$ $s \vDash (p \vee q) \to a \iff s \vDash dnt((p \vee q) \to a)$

$$
\begin{aligned}
s \vDash (p \vee q) \to a &\iff \forall t \subseteq s : t \vDash (p \vee q) \to t \vDash a \\
&\iff \forall t \subseteq s : (t \vDash p \text{ } or \text{ } t \vDash q) \to t \vDash a \\
&\iff \forall t \subseteq s : t \vDash p \to t \vDash a \text{ } and \text{ } t \vDash q \to t \vDash a \\
&\iff \forall t \subseteq s : \forall w \in t : p \in w \to \forall w \in t : a \in w \\
&\qquad\quad and \ \forall w \in t : p \in w \to \forall w \in t : a \in w
\end{aligned}
$$

$$
\begin{aligned}
s \vDash dnt((p \vee q) \to a) &\iff s \vDash \neg\neg((\neg a \to \neg p) \wedge (\neg a \to \neg q)) \\
&\iff s \vDash \neg((\neg a \to \neg p) \wedge (\neg a \to \neg q)) \to \perp \\
&\iff \forall t \subseteq s : t \vDash \neg((\neg a \to \neg p) \wedge (\neg a \to \neg q)) \to t \vDash \perp \\
&\iff \forall t \subseteq s : t \nvDash \neg((\neg a \to \neg p) \wedge (\neg a \to \neg q)) \\
&\iff \forall t \subseteq s : t \nvDash ((\neg a \to \neg p) \wedge (\neg a \to \neg q)) \to \perp \\
&\iff \forall t \subseteq s : \neg(\forall r \subseteq t : r \vDash ((\neg a \to \neg p) \wedge (\neg a \to \neg q)) \to r \vDash \perp) \\
&\iff \forall t \subseteq s : \neg(\forall r \subseteq t : r \nvDash ((\neg a \to \neg p) \wedge (\neg a \to \neg q))) \\
&\iff \forall t \subseteq s : \exists r \subseteq t : r \vDash ((\neg a \to \neg p) \wedge (\neg a \to \neg q)) \\
&\iff \forall t \subseteq s : \exists r \subseteq t : r \vDash (\neg a \to \neg p) \text{ } and \text{ } r \vDash (\neg a \to \neg q) \\
&\iff \forall t \subseteq s : \exists r \subseteq t : \forall c \subseteq r : c \vDash \neg a \to c \vDash \neg p \\
&\qquad\quad and \ \forall c \subseteq r : c \vDash \neg a \to c \vDash \neg q \\
&\iff \forall t \subseteq s : \exists r \subseteq t : \forall c \subseteq r : c \vDash (a \to \perp) \to c \vDash (p \to \perp) \\
&\qquad\quad and \ \forall c \subseteq r : c \vDash (a \to \perp) \to c \vDash (q \to \perp) \\
&\iff \forall t \subseteq s : \exists r \subseteq t : \forall c \subseteq r : \forall d \subseteq c : d \nvDash a \to \forall d \subseteq c : d \nvDash p \\
&\qquad\quad and \ \forall c \subseteq r : \forall d \subseteq c : d \nvDash a \to \forall d \subseteq c : d \nvDash q
\end{aligned}
$$

This example is long and somewhat hard to read. The last line says that for every subset (again, the minimal subset is an index) it has to be so that if the index does not contain a, then it also cannot contain p, and that if an index does not contain a, it also cannot contain q. This applies to every index in s. As we know, that is also the inverted definition of implication. So as we can see, $(p \vee q) \to a$ is equivalent to $dnt((p] \vee q) \to a)$.

The dnt is practical in inquisitive semantics because it matches the idea that a proposition is a set of possibilities. When translated to the disjunctive negative form, these separate possibilities are very easy to spot as the alternatives in the disjunction. In the same way, we know that all negative forms must be assertions, since for any formula ϕ, $\neg\phi$ cannot consist of more than one possibility. For an example of this, see figure 3.1. So the dnt can make it easier to see what possibilities



(a) Possibilities for $p \vee q$ (b) Possibilities for $\neg(p \vee q)$

Figure 3.1: Differences between positive and negative formulas

a formula embodies, which can be useful for more complex formulas.
We've looked at the disjunctive negative translation of the language, and seen some examples of how it is used. We will now go on to look at how inquisitive logic is related to other logics.

3.3 InqL and other logics

We will now look at InqL's position relative to other logics, starting with intuitionistic logic.

As Ciardelli and Roelofsen (2011) describe, there is a similarity between inquisitive and intuitionistic logic. Inquisitive logic evaluates the meaning of a formula through states. Whether a state supports a formula depends not only on what information is already available, but also on information that may become available later in time. Take, for example, a question, where the possibilities describe possible future knowledge.
In the same way, intuitionistic logic evaluates the meaning through a point in a Kripke model. Whether the formula is true depends then not only on that point, but also on the future accessible points, resembling future knowledge.

Ciardelli and Roelofsen (2011) prove that inquisitive support and satisfaction in a intuitionistic Kripke model M_I coincide, so that for every formula ϕ and for every non-empty state s:
$s \vDash \phi \iff M_I, s \Vdash \phi$
Because of this, we know that InqL contains the intuitionistic propositional logic, IPL. We also know that InqL is itself contained in classical propositional logic, CPL. So we know that $IPL \subseteq InqL \subseteq CPL$. We will now look at where exactly InqL falls between those two logics.

To do so, we introduce the notion 'intermediate logic'. An intermediate logic is defined as a set of formulas that contains IPL, does not contain $\bot$, and is closed under both modus ponens and uniform substitution. As we've seen earlier, InqL is not closed under uniform substitution. Therefore, it belongs to the class of weak intermediate logics, intermediate logics that aren't closed under uniform substitution. So, if we have a weak intermediate logic L, we can only say that ϕ and ψ are equivalent in L if $\phi \leftrightarrow \psi \in L$

Characterization of InqL

We now have enough notions to provide a characteristic description of inquisitive logic.

Lemma 5. *Weak intermediate logic L is InqL if:*

1. *for every formula ϕ, ϕ is in L equivalent to $dnt(\phi)$*

   and

2. *L has the disjunction property*

Proof. Let's first look at the first of those two conditions. Let ϕ be equivalent to $dnt(\phi)$ for all formulas in weak intermediate logic L. Then if $\phi \in L$, also $dnt(\phi) \in L$. A dnt-formula looks like $\neg x_1 \vee \cdots \vee \neg x_k$, and because InqL has the disjunction property we know InqL must contain at least one of $\neg x_1, \cdots, \neg x_k$. IPL and CPL agree on negations, and since weak intermediate logics are between IPL and CPL, we know that two weak intermediate logics will handle negations in the same manner. Because of this, we know that because $\neg x_i \in InqL$, $\neg x_i \in L$. So we know that $dnt(\phi) \in L$, and therefore $InqL \subseteq L$.

For the second part, let's assume that L also has the disjunction property. We can then use then same reasoning as above to show that $L \subseteq InqL$. If $\phi \in L$, then $dnt(\phi) \in L$, which mean $\neg x_1 \vee \cdots \vee \neg x_k \in L$. If L has the disjunction property, $\neg x_i \in L$ for some i. Since all weak intermediate logics treat negations the same, $\neg x_i \in InqL$, as well as $dnt(\phi)$ and ϕ. So, $L \subseteq InqL$, which means if the conditions are satisfied, L and InqL are equivalent. $\square$

We have looked at InqL's position relative to other logics, and singled out its characteristic features. We will now go on to look at conversational implication using the conversational maxims of Grice (1975) and adapted maxims of Groenendijk and Roelofsen (2009)

Chapter 4

Conversational Implicature

In this chapter we will look at Grice's maxims for conversational implicature, and how they translate to inquisitive semantics. Since inquisitive semantics has a very specific way of modelling conversation, we will see that some adaptations to the maxims have to be made for them to be fully translated.

First we will give a basic overview of Grice's maxims of conversation. Then we continue with a discussion of their function in inquisitive semantics.

4.1 Grice's maxims

In 1975, Grice published his article 'Logic and Conversation' (Grice, 1975). In this work he defined a set of implicated guidelines, the maxims, that people follow when participating in conversation. To be able to compose these maxims, Grice assumed that the purpose of conversation is an effective exchange of information. Though this is a very narrow specification, it will serve for our purposes, since inquisitive semantics is also used to model information exchange.

In his article Grice talks about what he calls conversational implicature. In human conversation, much can be said without actually telling someone. To take as an example the text from our introduction, if I were to ask "Where does Mr. Smith live?" in normal conversation, one would generally take it not just as a prompt for the listener to give the answer, but also as a declaration that I do not know where Mr. Smith lives. Grice therefore said that in conversation there always is some form of implicature. His maxims are guidelines that, when followed, would remove these implications, and in normal speech, we therefore do not always follow all maxims.

However, the maxims are useful when trying to be as clear and unambiguous as possible.

Grice's main principle of conversational implicature is called the Cooperative Principle. This principle, as defined by Grice (1975), is "Make your conversational contribution such as is required, at the stage at which it occurs, by the accepted purpose or direction of the talk exchange in which you are engaged." This principle can be divided in several categories: Quantity, Quality, Relation and Manner. Those categories are the basis for the actual guidelines or maxims. All maxims are formulated assuming that every participants goal is an effective exchange of information.

The Maxims

Definition 7. *Maxim of Quantity*

1. *Make your contribution to the exchange as informative as is required for its purpose.*

2. *Make your contribution to the exchange no more informative than is required.*

The maxim of Quantity makes sure that every participant in a conversation gives as much information as he can, keeping in mind the goal of the conversation. In addition, it prevents participants from giving too much information, which may be confusing or irrelevant for the current conversational goal. The second maxim is sometimes left out, because it is possible to get the same results using the maxim of Relation.

Definition 8. *Maxim of Quality*

1. *Do not say what you believe to be false.*

2. *Do not say what for which you don't have adequate evidence.*

The maxim of Quality's main purpose is to prevent people from giving information that may not be true. It is obvious how this would disrupt the conversation's goal, as untrue information can eventually lead to a contradiction in the knowledge of one or more of the participants. Here especially the assumption that every participant is after effective information exchange is important. While lying is relatively common in conversation, this is only the case when different conversational goals are to be reached than assumed here.

Definition 9. *Maxim of Relation*

1. *Be relevant.*

The maxim of Relation is the least well-defined maxim, as the definition of relevance can provide multiple problems and questions. For our purpose it is enough to define relevance as the effect it may have on the conversation: can it bring the conversation closer to its purpose, then a contribution is relevant. This maxim and the second maxim of quantity provide more or less the same guideline: don't say any more than you know is important, for it may only confuse matters.

Definition 10. *Maxim of Manner*

1. *Be perspicuous.*
   - (a) *Avoid obscurity of expression.*
   - (b) *Avoid ambiguity.*
   - (c) *Be brief.*
   - (d) *Be orderly.*
   - (e) *...*

The maxim of Manner has most to do with the manner in which participants give their information, making it as clear and precise as possible. This maxim as given by Grice is incomplete, as other maxims might be needed to completely cover every way in which a statement may be unclear, but the submaxims given are adequate for most purposes.

All maxims are needed for maximally effective exchange of information, but not all are always equally observed, and not all are equally important. For example, it is generally less important to be as brief as possible than it is to be truthful. Grice (1975) himself has suggested that the maxim of Quality might be the most important maxim for effective conversation. Other maxims could be added, especially considering the fact that not every human conversation has the purpose of information exchange, but some have the purpose of being funny or influencing people to act in a specific way. However, like stated above, the original Gricean maxims will do for our purposes.

4.2 Inquisitive Implicature

Now that we have given an overview of Grice's maxims, we will continue with their application to inquisitive semantics. In this analysis we will use the work of Groenendijk and Roelofsen (2009), who composed conversational rules for inquisitive semantics, based on Grice's maxims.

Maxim of Quantity

The maxim of Quantity says every contribution should be as informative as required for its purpose. In conversation, this means that the contribution should add something toward the conversational goal. To achieve this, a sentence should be either informative or inquisitive, both helping to (potentially) remove indices from the common ground and so increase knowledge.

The other submaxim of Quantity says a contribution should not be more informative than needed. This is already covered by the maxim of relation, and therefore, we will not use it here. Groenendijk and Roelofsen (2009) call this the Significance maxim.

Maxim of Quality

The maxim of quality says not to contribute anything that you believe may be false, or that you know is false. This, in inquisitive semantics, can be seen as never contributing a sentence that is not informative for the contributor. As something is only considered true by a participant if it is true for every index in their own knowledge state, he only knows something to not be false if it is contained in all indices. So, informative sentences should only be uttered when every index in one's knowledge state contains that information.

Since inquisitive sentences never eliminate indices from the common ground, the above doesn't apply for those sentences. Instead, a participant should not use an inquisitive sentence if the sentence is not inquisitive in their own knowledge state. This would mean that the participant could be more helpful by just providing the informative statement that is an answer to their own question. Since that would be inefficient, questions should only be uttered if it also adds a division in the speaker's own knowledge state. These rules together form the Sincerity maxim of Groenendijk and Roelofsen (2009).

Maxim of Relation

The maxim of relation only says "be relevant". This relevance can only be defined in relation to a former proposal or proposition. Following Groenendijk and Roelofsen (2009), we will look at its translation to inquisitive semantics using the notion of compliance. In conversation each participant should aim to be as compliant in their answers as possible.

Definition 11. *Compliance A proposal ϕ is in compliance with ψ if:*

1. *Every possibility in $[\phi]$ is the union of a set of possibilities in $[\psi]$.*
2. *Every possibility in $[\psi]$ restricted to $|\phi|$ is contained in a possibility in $[\phi]$.*

We will look at these one by one. We will only consider the situation where ψ, the first proposal, is a question. This isn't necessarily the case, but, as in natural conversation, a relevant answer to an assertion will usually be either agreement or rejection. After this, there is no special need for compliance. Therefore, we will mostly consider inquisitive proposals.

The first rule says that every possibility in a proposal should consist of one or more possibilities in the proposal it is a reaction to. Because of this rule, a proposal cannot add possibilities. Because it may contain multiple of the possibilities of its predecessor, it can be both an informative answer, eliminating indices, or another question proposing an easier to solve problem than the original question. This is an intuitive way to define compliance, since in natural human conversation these rules are usually followed. To clarify, here is an example.

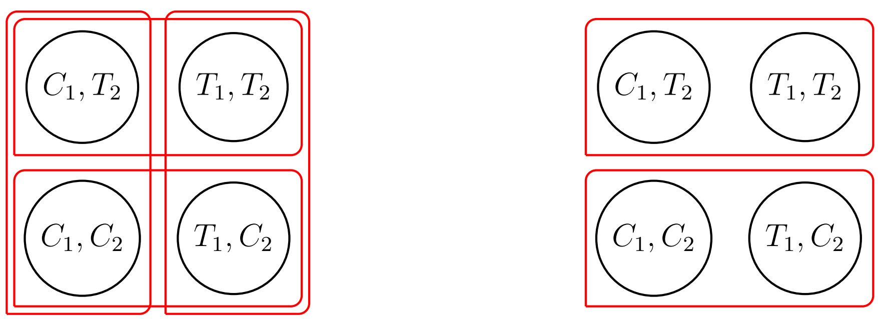
Example 7. *Let's look at the earlier example of Andy and Bea having a drink. Imagine there being a third person, Cate, who knows absolutely nothing yet about Andy and Bea's beverage preferences. Cate starts the conversation asking "Do both of you want coffee?". This is a relatively complex question to answer, seeing as the responder has to know both whether Andy wants coffee and whether Bea does.*

So now Andy has multiple options. He can answer that he would like a coffee, thereby eliminating all indices in which he'd want tea from the knowledge states. Or he can make the question less precise, to get an answer from Bea, which could help them answer the full question. In that case, he could say something like "Well, I don't know. Bea, do you want a coffee?" This would merge the four possibilities into just two: those in which Bea drinks coffee and those in which she drinks tea. Answering *in either of these ways would be considered compliant with Cate's question, based on the first rule.*

In the second rule, restriction of χ to $|\phi|$ is defined as the intersection of χ and $|\phi|$. If both proposals are questions, this can be simplified as "every possibility in $[\psi]$ is contained in a possibility in $[\phi]$". This rule exists to make sure no possibilities can be removed without reason. We can look at our example to see how this works.

Example 8. *This time, Cate asks "Does any of you want coffee?". This is a question of the form* $?C_1 \vee ?C_2$*. This means the proposal contains four distinct possibilities, like shown in figure 4.1a. Like in the previous example, we could have Andy answer "I don't know. Bea, do you want coffee?"* ($?C_2$)*. This would lead to the two possibilities shown in figure 4.1b. However, according to the second rule of compliance, this is not possible.*

As is clear in the pictures, there are two possibilities in the proposal of figure 4.1a that are not contained within any possibility of the second proposal. Because these possibilities are ignored, the second proposal is not considered in compliance with the first. Making the question more difficult to answer is not helpful or relevant in a conversation.



(a) $?C_1 \vee ?C_2$ (b) $?C_2$

Figure 4.1: Examples for the definition of compliance

As we can see the second rule prevents the first rule from being used too restrictively. In our first example, the first rule would make it a possibility for Andy to create less possibilities, thereby making it more difficult to answer (maybe not in this example, but in more difficult questions this could be a problem). The second rule prevents this by making it mandatory that every possibility in the first question should be in a possibility in the second. A last example (9) will show us a conversation in which both rules are applied to ask a second question.

Example 9. *Consider the following questions.*

a) What do you want to drink?

b) Do you want coffee, tea, or something else?

Sentence (a) divides several possibilities: tea, coffee, water, lemonade, orange juice. Then the following question (b) merges some of these possibilities, so that the only ones remaining are coffee, tea, and 'other'. This 'other' possibility contains all that fall neither within coffee nor within tea. No possibilities are ignored.

The two rules of compliance also help prevent over-informative answers, because an answer can never add indices to the knowledge states. Over-informative answers needs to be avoided, since the 'extra' information increases the chance of the proposal conflicting with the individual knowledge state of one of the participants, thereby being rejected.

Maxim of Manner

Grice's maxim of Manner says to be brief and orderly, and to avoid obscurity and ambiguity, and that more might be needed. Since not all of these submaxims can be translated to inquisitive semantics (ambiguity, for example, is difficult to convey when modelling), we will look at some of the rules introduced by Groenendijk and Roelofsen (2009) that have to do with the manner of speech.

The first is the rule that participants should "say more, and ask less", which could be classified as a rule about obscurity. The purpose of this rule is to have the participants reach their common goals faster, since in general more indices will be eliminated by assertions than by questions. Those eliminations enable more specific questions to be asked, and so for the conversation to reach its goal faster. So, we want participants to be as informative as possible. Participants should try and maximise elimination of indices.

Definition 12. *Degree of informativeness*
ϕ is at least as informative as ψ iff in every state where ψ is eliminative, ϕ is eliminative as well.

Another rule should be to properly update and maintain the knowledge states. Every individual should make sure to voice any contradictions of their own individual knowledge state with a proposal, to make sure none can be used to update the common ground. They should also update the common ground every time a proposal is accepted, as well as their individual knowledge states. If any of the states become inconsistent, it may well be impossible to reach the conversation goal.

The third rule is that every participant should strive to enhance the common ground to reach the conversational goal. This has much to do with the earlier discussed truthfulness and compliance, but it should be noted that if participants do not try to enhance the common ground, the conversation will likely end before the goal is reached.

We have seen both Grice's maxims and the adaptation for inquisitive semantics. Most maxims have a clear application in this semantics, although some, especially those of manner, remain a bit vague.

Chapter 5

Discussion

In the last chapters we looked at inquisitive semantics. More specifically, we looked how it can be used to model conversation, and how Grice's ideas on conversational implicature can be applied to it. We also looked at the accompanying logic and its classification as a weak intermediate logic. In this last chapter we will discuss how this system can contribute to further scientific research, both in general and in the field of artificial intelligence.

5.1 Inquisitive semantics' application in research

Since semantics can be seen as a branch of linguistics, most uses for inquistive semantics can be found in that field. Like mentioned in the introduction, some linguists say that questions can not be modeled in a intuitive way using classical semantics. Most existing techniques for modeling questions, and by extension, for modeling human conversation, can feel somewhat artificial. Inquisitive semantics can provide a new way to model language. This could help provide new insights on the way human language works. One could say that the classical way of modeling conversation is already sufficient. However, the intuitiveness of inquisitive semantics would make it more accessible for both laypeople and researchers from other fields, and easier to use in other applications.

Because inquisitive semantics partly depends on the notion of compliance, it would be interesting to look at the complexity of determining the compliance of a response ψ to a question ϕ. An algorithm to compute a set of compliant responses to a question has been described by Ciardelli et al. (2009). The algorithm given by them consists mainly of steps to syntactically change the formulas, with some steps for checking

classical entailment or satisfiability. This could mean that determining compliability is more complex than determining classical satifiability. A consequence could be that the way of modeling questions as a conjunction of three statements, for the time being, is more useful for practical applications.
Another issue can be seen when considering open-ended questions, which would be difficult to formulate in a open world. Only if the set of information is limited open-ended questions could be formulated, and a set of possible responses completed. A solution would have to be found for situations in which the information is not limited.

Further research could also focus on extending exisiting semantics to include things like storytelling, intentional lies, and convincing a listener to do something. So far semantics has mostly been about truthfulness of a statement. Inquisitive semantics changes this by placing the focus mostly on compliance, and in the same way, new or extended systems could be designed to deal with other uses of language.

5.2 Inquisitive semantics in AI

Inquisitive semantics could also play a role in artificial intelligence research and applications. One of the main issues in AI is one of language: the issue of how to get a computer to use language like a human would. Research in semantics in general has already made great progress in terms of language recognition and generation. A famous example would be Siri, Apple's virtual assistant application, that is able to recognize the user's questions and answer using proper English. Semantic models are used in many applications to help the computer analyze natural language input for meaning, and so make human-computer interaction more intuitive.

Since most human-computer interaction consists of questions and answers, inquisitive semantics would be a logical choice to help expand the current possibilities in natural language recognition. The way it can be used to model questions and statements could provide the basis for more natural use of language by computers. In such a program, syntax analysis could first recognize input as being a question. Then a model based on inquisitive semantics could use information from, for example, the internet, and combined with the right algorithm, pick the most compliant response to give to the user. However, like we explained in the section above, for now it is hard to imagine how any algorithm would work with an almost limitless source of information like the internet; the algorithm would have to be limited to a closed-information world, like a helpdesk for a commpany or a library database search.

Chapter 6

Conclusion

The purpose of this thesis was to give an introduction to inquisitive semantics. We first gave an overview at how a sentence is represented in the semantics using logical formulae, and we defined when to call a sentence inquisitive or informative.

This was followed by a comparison of inquisitive logic to other logics. We looked at some important proofs and examples to show how inquisitive logic differs from its classical counterpart. We showed that uniform substitution, doube negation elimination and the law of no excluded middle all do not apply in inquisitive semantics, even though they play a very important part in classical logic. We also saw that InqL can be classified as a weak intermediate logic. Specifically, as the weak intermediate logic that has the disjunction property and for which every formula is equivalent to its own disjunctive negative translation.

We then looked at how we can model knowledge and conversation using inquisitive semantics. This is done using individual knowledge states and an extra knowledge state called the common ground. We gave an overview of Grice's maxims for conversational implicature, and we saw how Groenendijk and Roelofsen (2009) proposed to adapt these for inquisitive semantics.

Finally we discussed the possibilities of inquisitive semantics in research and applications. It could help make semantic models more intuitive, but it may be a while before the difference in complexity between inquisitive and classical semantics will be irrelevant.

In this thesis we have only discussed the basics of inquisitive semantics. Of course, there is much more to be said about the semantics and its corresponding logic, especially about its relation to other logics. In the chapter before this conclusion we discussed some of the variations and extensions that would be interesting to look into. Other examples include unrestricted inquisitive semantics and extensions to a first-order logic.

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