The Measurement Problem, Decoherence and Education

Considering the debate around the collapse of the wave function as a result of environment-induced decoherence, and teaching the theory to the early quantum mechanic.

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Abstract

A study is made of the status of measurement, in particular the projection postulate, in quantum mechanics. Part of this will be a historical outlook: how was the concept of measurement viewed by the quantum architects? Then a more systematic investigation is made on what has been called the measurement problem which poses the question why superpositions of quantum states are never observed. Reasons are given why the projection postulate does not from a satisfactory solution, namely the non-unitary evolution of time and the lack of a precise definition of the word 'measurement'. Since the 1980's active research has been undertaken in the field of decoherence phenonema. Environment-induced decoherence has been proposed to solve the measurement problem by looking at open quantum systems instead of closed ones. It is inquired upon to what extent decoherence is indeed a solution. It has been shown that the fundamental problem survives, but a justification is found for the appearance of classical properties in a universal quantum world. Although the discussion on the decoherence solution has arrived at an approximate equilibrium since the first few years of this century, the terms 'decoherence' and 'collapse' are still used interchangeably in a vast amount of literature. This is pointed out to be an uncareful use of words. However, the decoherence program has inspired a new way of thinking about interpretations, giving preference to some modern interpretations - the non-collapse theories - over the early Copenhagen or subjective interpretations. It seems that the ideas behind decoherence theories have seeped through to the most modern textbooks. However, the textbooks currently in use mostly date back to orthodox times of the late 1980's. According to the recent textbooks, it seems that a shift in teaching interpretational quantum mechanics is to be expected, similar to the impact Bell's inequalities made on education in the early 1980's.

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“From what source shall I, as a partial layman in the realm of pedagogy, derive courage to expound opinions with no foundation except personal experience and personal conviction? If it were really a scientific matter, one would probably be tempted to silence by such considerations. However, with the affairs of active human beings it is different. Here knowledge of truth alone does not suffice; on the contrary this knowledge must continually be renewed by ceaseless effort, if it is not to be lost. It resembles a statue of marble which stands in the desert and is continuously threatened with burial by the shifting sand. The hands of service must ever be at work, in order that marble continue lastingly to shine in the sun.”

—Albert Einstein, 1954 [i]

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[i] Albert Einstein, “On Education” in *Ideas and Opinions* (Avenel: Wings Books, 1954), p. 59.[18]

Contents

Introduction

"It would seem that the theory is exclusively concerned about results of measurements, and has nothing to say about anything else. [...] In the beginning natural philosophers tried to understand the world around them. Trying to do that they hit upon the great idea of contriving artificially simple situations in which the number of factors involved is reduced to an minimum. Divide and conquer. Experimental Science was born. But experiment is a tool. The aim remains: to understand the world. To restrict quantum mechanics to be exclusively about piddling laboratory operations is to betray the great enterprise."

—John Stewart Bell, 1990 [ii]

1 Prologue

1.1 Points of view and the importance of the measurement problem

Making measurements is one of the most important things in the progress of empirical science. We are interested in the properties of Nature. We find these by observation through the senses by singling out properties that belong to objects, such as weight or length. But most importantly, we use the scientific method to build physical theories. This is done by deliberate experiment. The interaction between measuring properties in experiments, creating theories, predicting new values for these properties under different circumstances and performing this new measurement is what has lead science to its succes today. Theories rest on empirical evidence, wherein measurement has the central role. In classical physics the theory does not differentiate between physical processes and typical 'acts of measurement'. This is important, because any interaction between two physical systems thus can act as a measurement. But what happens when the theory starts attributing properties to the measurement process? What if measuring a quantity of a system is not an external intervention, but part of the system and deforms the property of interest? This is what happens in quantum physics.

In the foundations of the theory of quantum physics, there linger two main philosophical problems that give rise to many conceptual misinterpretations and confusions. The first one concerns the abandonment of the causal laws of classical physics and the introduction of probability laws. The laws of quantum mechanics predict different outcomes with corresponding probabilities for repetitions of the same event. This is due to the so-called superposition of states, that is: a state that in a sense represents a multitude of values for one physical property at the same time. Second, there is the problem of how we should talk about the object of observation before and after this measurement. One difficulty here is to find a criterion to define what 'a measurement' is. That criterion is not incorporated in the theory. It is this second problem, that will be the subject of this thesis and it is called the 'measurement

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[ii] John Stewart Bell, "Against Measurement," Physics World, 1990. [10]

problem of quantum mechanics.'

On the one hand, what makes these concepts in quantum mechanics problematic is that they are counter-intuitive, in the sense that they do not resemble our experiences of everyday life. These are the above mentioned superposition and measurement, in combination with wave-particle duality and Heisenberg's uncertainty principle. I do not see any object at more than one position at the time, along with it's position I can certainly also measure it's velocity without essentially disturbing it. Waves are very different from balls. At first sight there seem to be contradictions between what we see in the quantum world compared to our world. But I feel that the apparent contradictions vanish when we begin to understand that there is no logic that compels us to impose principles from macroscopic physics on the microscopic world: they do not *have* to resemble each other. Although we might argue about which world is more fundamental and must desire to find their relations or pinpoint a transition, we cannot deny that microscopic entities may behave differently.

On the other hand, there is a much more fundamental issue at stake: we would like to know what it *is* that our theory is describing. We want to interpret it and understand what is really going on. If there are signals that would lead us to reject or embrace the notion of causality; or if it tells us that Nature has reserved a special role for the measuring process, we want to investigate and understand these signals; why they are there and what they mean. We would want quantum mechanics to be about things that really exist in the world, instead of what we find in the laboratory. We want it to be independent of results of measurement.[iii]

In the narrow sense, these are not really physical problems, since quantum mechanics has proved itself to work flawlessly in predicting the outcome of measurements. For the pure instrumentalist then, there is no *real* problem. Still, I feel that these question really are problems and that they are significant not only to the philosopher of science, but also to the physicist. The instrumentalist may not require his theory to resemble what the world is really like, but he should at least feel uncomfortable with the fact that in practicing quantum mechanics he has retreated from the possibility of making certain predictions, whereas predictions are the only goal of his undertaking. If the *pure* instrumentalist (if he exists) does not feel this discomfort in any way, he too should want to understand why this is so, for maybe the predictions could be better, by which I mean deterministic rather that indeterministic, with a different theory at his disposal. More importantly the instrumentalist too wants to include all processes into his theory, including the measurement process. And it is precisely this point, that the measurement problem brings up. On he other hand the *realist* practitioner[iv] of quantum physics surely wants to truly understand the theory he uses every day. He would like to know if the abstractions he makes in his head while solving a problem correspond to something in reality. In the words of David J. Griffiths: “Èvery competent physicist can ‘do’ quantum mechanics, but the stories we tell ourselves about what we are doing are

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[iii] John Stewart Bell, “The Theory of Local Beables,” in *Foundations of Quantum Mechanics* p. 50-61, (Singapore: World Scientific Publishing, 2001), p. 54.[9]

[iv] In general the community of physicists consists mainly of realists, at least as far as the well considered descision has been made to take a standpoint between realism and instrumentalism.

as various as the tales of Scheherazade, and almost as implausible."[v].

In 2013, Maximilian Schlosshauer, Johannes Kofler and Anton Zeilinger published the article *"A Snapshot of Foundational Attitudes Toward Quantum Mechanics."* In the article they put forward the results of a poll taken out among 33 participants of a conference on the foundations of quantum mechanics. In his blog Sean Carroll dubbed the results as "The Most Embarrassing Graph in Modern Physics," see Figure 1.[vi]

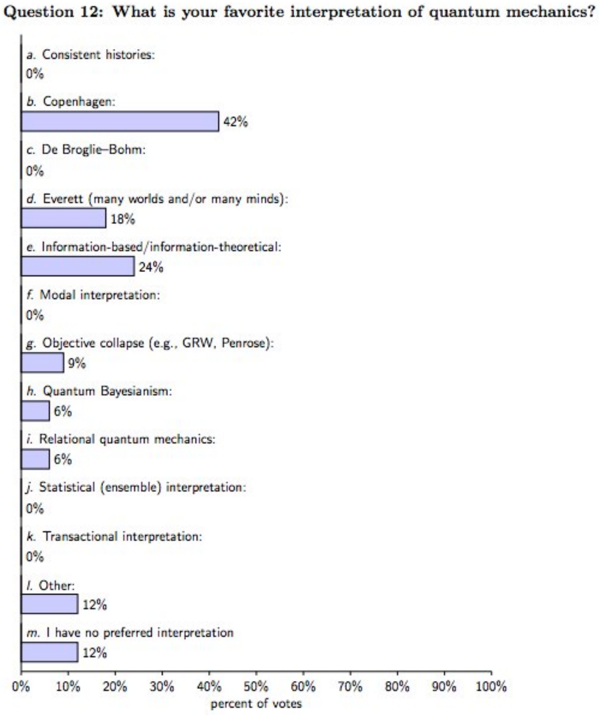

Figure 1: "A Snapshot of Foundational Attitudes Toward Quantum Mechanics," by Maximilian Schlosshauer, Kofler and Zeilinger.

The graph is embarrassing because even though we have a firm sense of what quantum mechanics does, we don't really know how to interpret it: "Every student who gets a degree in physics is supposed to learn QM above all else. There are a variety of experimental probes, all of which confirm the theory to spectacular precision. And yet — we don't understand it. Embarrassing."[vii] The trouble all revolves around the measurement problem and how to interpret it.

So here I want to stress that in a broader sense, the problems that surround the projection postulate are as real for the physicist as they are to the philosopher. The numerous proposed solutions to the measurement problem indicate it to be a large gap in our understanding of the theory. The measurement problem is not just a philosophical nuance that can be waved away by the physicist. Even the very word

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[v] David Griffiths, *An Introduction to Quantum Mechanics.* (New Jersey: Pearson Education, 1995), p. vii. [23]

[vi] Sean Carroll, "The Most Embarrassing Graph in Modern Physics," `http://www.preposterousuniverse.com/blog/2013/01/17/the-most-embarrassing-graph-in-modern-physics/`(consulted on April 4th 2014). [14]

[vii] Carroll, "Embarrassing."

measurement itself is ambiguous and its meaning is not so self-evident as the physicist makes it sound. Clarification and understanding of what makes these problems to be problems, how we can interpret them and what consequences arise for different solutions would bring us many steps closer to understanding Nature.

1.2 Motives for the Research

When I was just about to start with my second year in physics as an undergraduate student I was very excited. After a year of classical physics I was going to my first lecture on quantum mechanics. We learned about wave functions, Hilbert space and operators, but we were never given a complete motivation of what these entities meant. My teacher told me: "This is the theory that gives the correct prediction of measurements. We will forgo the fundamental questions about interpretations to metaphysics, which is not part of this course. Now here is the Schrödinger equation..." At the time I believed this to be very disappointing. I was ready to tackle these metaphysical problems as well. But there might be a good point in his words. There is a massive amount of literature surrounding philosophical interpretations, for which much knowledge is needed about the theory itself, setting up experiments and interpreting information philosophically. So it seems it is not advisable to go through all this in a introductory physics course, where the goal is to teach the way to *do* quantum mechanics in only one semester. Still, there could be dangerous consequences of completely neglecting questions of interpretations. And as David Griffiths warns in his imagery above: a physicist might teach him or her self an inconsistent physical interpretation. This, I think, must be prevented at all cost. Besides I cannot but emphasize that a better understanding of the foundations of a theory helps the physicist and the student in his or her work. I think working on the interpretation and contemplation of the problems should start at the very beginning, in the undergraduate years.

This reveals the underlying motive for this research: to *encourage* the investigation in order to understand quantum physics and what the foundations of the theory imply philosophically. We will do this by focusing on the measurement problem introduced above.

In a later course on Advanced Quantum Mechanics there was another interesting remark from another teacher, which had to do with the projection postulate. My teacher had to go into the subject briefly, but he was also eager to tell us that this postulate wasn't really needed in practice. It was not more than a remark, though he told us it was decoherence that solved the problem. This struck me as profoundly interesting. Is the problem solved? and how is it done? and mostly, why didn't I hear about it sooner? Students can become very confused by not being able to survey where different interpretations have their roots, which is something that might trouble them for the rest of their careers and lives.

Decoherence is the loss of coherence of phase angles of components in a superposition. From this dephasing, properties of classical probabilities arise, i.e. interference terms die out very quickly. This process has been known since the beginning of the theory, for it is already there in Von Neumann's *Mathematical foundations of Quantum Mechanics*, where he sets forth the theory in an axiomatic manner. Active research on the idea that this mechanism can be used in finding an answer to the measurement problem though, has only been going on since the 1980's.[viii] Solving the measurement problem through decoherence has been the program of many, including Dieter Zeh, Erich Joos, Wojciech Zurek, John Wheeler nad Max Tegmark. Decoherence is a prominent feature in an interpretation that Jeffrey Bub called the 'New Orthodoxy.'[ix] Whereas the old orthodoxy surely is a version of the Copenhagen interpretations, the New Orthodoxy is a colourful mix of (i) environment-induced decoherence introduced by Zurek, Joos and Zeh in the eighties, (ii) some notions of Everett's 'many-worlds'-interpretation (1957) and (iii) 'consistent histories' developed by (Robert)[x] Griffiths.

1.3 Aims and Outlines of the Research

From the motives listed above I want now to discuss the goals I want to reach here. My first goal will be to explain the measurement problem to a greater extent and discuss what it is that makes it to be a problem. This will be done by means of contrasting early solutions with modern ideas, as well as putting forth the difference between collapse and non-collapse theories. Furthermore, I want to expound on environment-induced decoherence in relation to the measurement problem and to what extent it is a solution. Next an indication will be given of the 'new orthodoxy' that is formed significantly by the ideas behind decoherence. One of final tasks I set myself is to investigate to what extend this new orthodoxy -that strengthens the non-collapse theories- have seeped through to textbooks on quantum mechanics.

To do this effectively, I start with an outline of what measurement means in classical and in quantum theory. The double-slit experiment will be an example throughout; this will function to reflect the ideas to. Further on I present the Von Neumann postulates. Here we attend to the projection postulate and why it entered the theory. This will be done by considering the Compton-Simon experiment of X-ray scattering. A discussion will be devoted to the status of the projection postulate and its interpretations, especially the 'old orthodoxy': subjective and Copenhagen interpretations. Problems with the unclear distinction between measurements and physical processes will be discussed, as well as the notion of time-assymetry. In the third part mixed states will be introduced as a part of the quantum theory of ensembles, leading to the explanation of the effect of decoherence. Ideas will be put forth why environment-induced decoherence seems to solve -but does not solve- the measurement problem. In the last chapters a selection of modern textbooks -which are used in universities now- will be studied with an eye to how they deal with the measurement problem. Is there indeed a new orthodoxy, besides orthodox interpretations of the 1930's and 1940's? Is this dealt with in modern textbooks

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[viii] Maximilian Schlosshauer, "Decoherence, the measurement problem, and interpretations of quantum mechanics," Reviews of Modern Physics 2004: **76**, p. 1267-305. [47]

[ix] Jeffrey Bub, *Interpreting the Quantum World* (Cambridge: Cambridge University Press, 1997), Chapter 8: The new orthodoxy, p. 212.[13]

[x] The consistent histories approach is developed largely due to the work of Robert Griffiths. His namesake: David Griffiths is also mentioned frequently here, he is the author of the textbook *Introduction to Quantum Mechanics* (1995).

on quantum mechanics? Are there are textbooks that consider decoherence as an interpretational arguement, maybe raising the hope that the projection postulate is not fundamental? Most textbooks that are in use seem to date back to the 1980's and 1990's and since the conceptual decoherence debate settled in the past decade no hints about decoherence are expected there. But maybe there textbooks that adher to the new orthodoxy, indicating a significant shift in our view towards interpreting quantum mechanics. This well then also be available to students in a similar way as Bell's inequalities appeared throughout text books in the 1970's.

2 Touching the Measurement Problem

2.1 Classical Measurements

In classical physics, we are interested in the properties that adhere to a physical system. We perform measurements on the system in order to find out these properties. If not directly using our senses to measure colours or sounds, we use instruments, e.g. we use a ruler to measure lengths and a balance to measure weights. In classical mechanics, essentially all physical influences between systems can be considered as measurements. The theory itself tells us which interactions are possible and what effects they have, i.e. the theory provides us with an understanding of the notion of measurement. Moreover, when we use a ruler to measure lengths and a balance to measure weights, we do not believe that our measurements have effected these properties. Three grams of sugar remain to weigh three grams whether they are on my balance or on the kitchen sink. Often it is stated that it is typical of a classical measurement that it has a neglectible effect on the system, and that this is what distinguishes classical from quantum measurements. But this cannot be the case, since there are many examples from classical physics where the influence of the measuring instrument plays an important disturbing role. Imagine measuring the temperature of a thimble of nearly freezing water with a large mercury thermometer at room temperature.[xi], or finding the position of a vase in a dark room with a stick. A measurement finds a specific value that the system had in the past. If we can make inferences to use this past value -in combination with the effect of measurement- to predict the future, we are doing proper classical physics. So a measurement can influence the system of interest, and in general it does. The important thing is that there is nothing that makes a distinction of principle between the act of measurement and other interactions in the theory, i.e. measurements are ordinary physical processes. That is what a classical measurement essentially is: *a physical process that gives us information about one or more of the properties of a system at interest, through interaction also described by the theory.* That a measurement might disturb the system of interest is not a problem *per se*, because we can include the instrument into the theory. We can use inferences to establish a state in which the classical system was before measurement. Also, the disturbance of the system can be made as small as the experimentator wishes. A measurement that does not disturb the system is called an *ideal measurement* or a measurement of the *first kind*. In reality, such a measurement cannot strictly exist, since there is always some recoil from the interaction between system of interest and instrument.

[xi] Hans Reichenbach, *Philosophic Foundations of Quantum Mechanics* (Mineola: Dover Publications, 1998 (1^{st} ed. 1944)), 16: modified version of the thermometer example.[38]

2.2 The Double Slit Experiment and Interference

Two roads diverged in a yellow wood
And sorry I could not travel both
And be one traveler, long I stood
And looked down one as far as I could
To where it bent in the undergrowth..

—Robert Frost, *The Road not Taken*, 1916

The well known double-slit (or two-slit) experiment is what Richard Feynman called the "one experiment, which has been designed to contain all of the mystery of quantum mechanics."[xii] And he is quite right. Whenever you get stuck in interpreting the predictions that quantum physics makes, you can think back to this experiment and find the same problems there. From quantum mechanics we know that sometimes systems behave like waves and sometimes like particles, but generally as both. The Schrödinger equation describes a particle that is also smeared out over space. Important for this research is to understand what happens when 'measuring' a property that determines the particleness or waviness of the system; in this case the presence or absence of interference.

The double-slit experiment can be done with different constituents.[xiii] It can be done with light for example, but also with electrons, or neutrons. Light was first discovered as a wave, later as a particle and finally as something of both. With the electron it is the other way around, i.e. it was first discovered as a particle, later as a wave and shortly after as both. Photons and electrons both behave in this weird way, but luckily they behave the same regarding their weirdness. As electrons have electric charge which makes them easier to detect, we will stick with them in this experiment.[xiv] It consists of a mono-energetic beam of electrons shining on a wall with two openings in it a little distance apart (the openings are small compared to the wavelength of the electrons). The electrons go through the slits and can be caught by a detector, say an electron multiplier, which measures the number of electrons. The detector can move in the direction parallel to the wall (say the x-direction), so that a distribution of electrons arriving at a (imagined) second wall can be constructed. See Figure **2**.

When the detector is put at a certain position x we can speak of the probability amplitude that an electron will arrive at that position. This probablity amplitude is what we want to construct. We can close one of both slits to obtain information about the probability distribution of the separate slits. Then we can open both slits, find the distribution that the two slits create together; and compare.

[xii] Richard Feynman, *The Character of Physical Law* (London: Penguin, 1965), p. 130. [20]

[xiii] The experimental set-up, or scale, has to be suitable, of course, to detect single particles. This set-up is different for different constituents. For photons and electrons though, it is very much the same.

[xiv] Richard Feynman, Robert Leighton and Matthew Sands, *The Feynman Lectures on Physics* (New York: Basic Books, 2010), p. 1-4.[19]

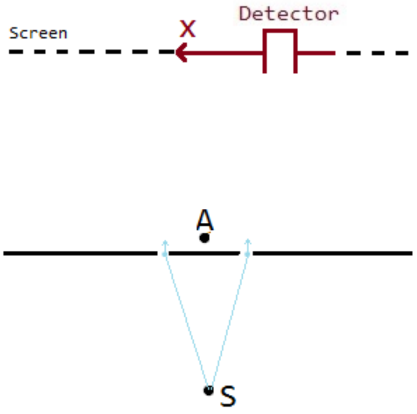

Figure 2: The double-slit experiment that Feynman dubbed as "the experiment that contains all quantum mysteries". Electrons are fired (one by one) from source S, through the two slits. A detector can be put at variable positions x and function to construct a 'screen'. The screen displays an intensity ($\propto$ probability) distribution showing additive behaviour or interference.

If this experiment is performed with either particles or with waves is -of course- important for the result. The distribution for one slit is the same in both cases. It is an intensity peak P_1 (or P_2) just behind slit 1 (or 2) and falls off smoothly, see Figure **3a**. When this experiment is performed with particles the distribution P_{12} created with both slits open would have one maximum just behind the middle of the two slits, which is just $P_1 + P_2$. This is since particles should really behave like (indestructible) bullets, i.e. in discrete lumps and without interference, see Figure **3b**.

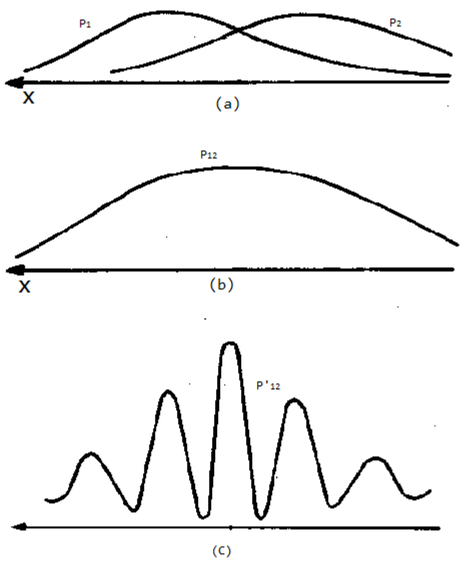

Figure 3: The probability distributions for 3a): slits opened separately, 3b): both slits open and additive -that is particle-like- behaviour, 3c): both slits open and interference patterns.

When the experiment is performed with waves (say water waves), we find an interference pattern P'_{12}, as in Figure **3c**). This interference pattern is due to the 'cross terms' when calculating the intensity (height squared) of the (water) waves. The height H_1 of wave 1 is the real part of $H_1 e^{i\phi t}$, with $H_1 \in \mathbb{C}$, with ϕ the phase of the wave; similar for wave 2. The total height arriving at the detector is just the sum $H = H_1 + H_2$. The intensity gives the probability distribution (since it is proportional), but the intensity doesn't simply add:

$$P'_{12} = |H|^2 = |H_1 + H_2|^2 = P_1 + P_2 + 2\sqrt{P_1 P_2} \cos \delta, \tag{1}$$

with δ the phase differences. The last term is due to interference.

Now when we analyse the results of the experiment performed with electrons instead of bullets or water, we find the following. (i) The electrons always arrive in a discrete way at the detector. The electron multiplier either 'clicks' or not. There is no such thing as half a click. Also when lowering the intensity of the incoming electrons, these clicks do not get dimmer. We are assured now that the electrons are particles or discrete packages. Our expectation of the probability distribution would be P_{12}. But what we find is (ii) the interference pattern P'_{12}. This means that there are particular positions x, where the number of arriving electrons is not the same as the number of the arriving electrons from the slits separately. We conclude that the electrons arrive in a discrete way, but also behave wavy, due to the presence of the interference term. Thus the electrons display wave-particle duality.[xv]

Now, since the number of electrons doesn't just add, it might help to see through which hole the electrons really go. If a single electron either goes through slit 1 *or* slit 2, we would have found the distribution P_{12}. But we do not find P_{12}, so the electrons do maybe not go through only one slit. This is weird because electrons come in lumps and do not split in half, so we should investigate its path. Electrons carry charges and these scatter light, so we place a light source in between of the two slits (at point A a in Figure 2). Now a flash[xvi] of light should appear along the path that the electron follows.[xvii] The results of this experiment state that *every* time there is a click from the electron multiplier, we also see a flash of light near slit 1 *or* slit 2, never at both at once. This is remarkable, for that would mean the number of electrons do actually go through only one slit, which means we should get distribution P_{12}. And in fact, when we do this experiment watching the flashes ánd counting the arriving electrons at the detector, we really do find P_{12} instead of P'_{12}. This is the essential quantum paradox. Somehow the interference is lost when we get the information about the path of the electrons, i.e. when we performed a measurement. Feynman concludes: "[...] when we look at the electrons the distribution of them on the screen is different than when we do not look."[xviii]

[xv] The particles could also be assumed to move in oscillating line, but then we encounter greater anomalies. For a discussion about these anomalies, Reichenbach (*Philosopical Foundations, 32*) refers to chapter 9 of Louis de Broglie's *Introduction à l'Etude de la Mécanique ondulatoire* (Paris: [Hermann], 1930).

[xvi] The flash of light is the amplification of the photon.

[xvii] Feynman, *Lectures*, 1-7.

[xviii] Ibid., 1-8.

This loss of interference is essential. It gives rise to the projection postulate which deems this feature fundamental. Quantum systems seem to be able to be in more states at once, like an electron going through both holes. But when performing an experiment to find out which state it is really in (which path the electron really takes), this characteristic is lost. Somehow we change the system in an essential way, merely by observing it. As Brukner and Zeilinger put it: “The observer can decide whether or not to put detectors into the interfering path. That way, by deciding whether or not to determine the path through the double-slit experiment, he can decide which property can become. If he chooses not to put the detectors there, then the interference pattern will become reality; if he does put the detectors there, then the beam path will become reality.”[xix]

2.3 Quantum Mechanical Measurements and Entanglement

The double-slit experiment above is an historic example that showes peculiarities of the quantum world: wave-particle duality and the loss of interference through the act of measurement. When unobserved, a quantum state is free to be smeared out over multiple classical properties at the same time: superposition. These superposed states are not to be interpreted as if the particle in question is in just one term of the superposition and not in the others. It is most definitely nót to be interpreted as an *ensemble* of the various terms in the superposition, as we would do in classical statistical physics. Instead, superposed states form entirely different new states that can be experimentally distinguished from the eigenstates, as we have seen in the double-slit experiment. When measured however, the system reveals indeterministically only one classical property; as if it is forced to choose one. A superposition is never presented to us, thus the interference is lost through the act of measurement. We have seen that in classical mechanics there are many measurement processes imaginable that do change the properties of the system, but these disturbances may be made smaller and smaller by choosing a smarter experimental set-up. In quantum mechanics it appears there is in general an intrinsic mechanism that guarantees us to change the system as a result of the measurement, no matter how smart the experimentator. Also, we cannot make the change in the system arbitrarily small as we can do in classical mechanics. In quantum measurements -most of the time- it is like we are always wielding sticks, shattering vases.

A measurement is always performed by some sort of apparatus. And the result is always represented as a number corresponding to some variable. This number is realized through physical interaction, indeed it is precisely this sensitivity to a variable of interest that serves as the criterion for choosing an appropriate measuring device. So a physical coupling between the system of interest and the apparatus is required. In classical physics this coupling is easily understood as the measurement and can be explained in terms of the theory. In quantum mechanics this is not so easily understood. In general, it is hard to define what a measurement is. We can choose to decribe our apparatus A (which is generally macroscopic) in terms of a set

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[xix] Caslav Brukner and Anton Zeilinger, “Young’s experiment and the finiteness of information,” Philosophical Transactions of the Royal Society **360** (2002), p. 1061–9. [8]
Note that the use of the word ‘reality’ has Copenhagen-like connotations.

of eigenstates $\{|a_i\rangle\}$ in the corresponding Hilbert space $\mathcal{H}_A$, where i is an index referring to the possible 'pointer states' of the apparatus.[xx] This pointer state is sensitive to the physical quantity that we want to measure, like mercury in a thermometer is sensitive to a change in temperature. The physical interaction is described when this state of the apparatus is coupled to the state of the system of interest $\mathcal{S}$ described by the wave function $|\Psi_\mathcal{S}\rangle$ in the corresponding Hilbert space $\mathcal{H}_\mathcal{S}$. The two physical systems $\mathcal{S}$ and A interact with each other through an interaction Hamiltonian.[xxi] The Hilbert spaces of the two systems then form a direct product space $\mathcal{H}_S \otimes \mathcal{H}_A$. This is what Schrödinger called 'entanglement' (Verschränkung[xxii]). Now, when the system of interest is in a superposition of eigenstates, which it generally is, we have: $|\Psi_\mathcal{S}\rangle = \sum_i c_i |\psi_i\rangle$. Then the combined system is represented by a direct product state:

$$\sum_i c_i |\psi_i\rangle \otimes |a_0\rangle \rightsquigarrow \sum_i c_i |\psi_i\rangle \otimes |a_i\rangle \,, \tag{2}$$

where $|a_0\rangle$ is the state of the apparatus before measurement, i.e. the 'neutral' pointer position when there is no interaction. The evolution of the combined state as a result of the entanglement governed by the interaction Hamiltonain is denoted by '$\rightsquigarrow$', which is short for the physical interaction between apparatus and system of interest. Once the interaction has taken place, for the apparatus is pointing at a value corresponding to the i^{th} eigenstate. By Eq (2) we have a dynamical theory of measurement: the *physical* interaction with the measurement apparatus defines an *observable*. The question is how to interpret Eq (2).[xxiii] Here lie all interpretational difficulties in quantum mechanics. Since they form a total state, we know that all terms in this superposition are all actually present, but when making a measurement -if measurements are assumed to give definite results, which they should- we obtain one particular result $|\psi_p\rangle \otimes |a_p\rangle$. How this transition from superposition to unique outcome exactly happens is the dilemma that has been called the *quantum measurement problem.* Is the measuring device for us (observers outside of the system) now in a superposition with a distribution of possible pointer values? Or does a 'classical world' emerge from the macroscopics of the measuring device, destroying interference? When discussing the postulates we can put this into further context.

What happens when we make a second measurement? Measuring through which slit the photon actually goes is a first measurement. Finding out that the interference pattern is lost happens at the screen and is a second measurement. It seems we

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[xx] The set of pointer states form a basis of kets $\{|a_i\rangle\}$ that are all mutually orthogonal since pointing at one number is clearly distinquishable from pointing at another. Note furthermore that we are assuming a universal validity of quantum theory, since we subscribe a quantum mechanical state to the macroscopic measuring apparatus. This is common fashion and found with Von Neumann (1932), but would not be accepted by supporters of the Copenhagen interpretation.

[xxi] Schmidt's theorem ensures us that there is a very general Hamiltonian that describes this process and that it is possible to decompose the space into two subspaces spanned by two uncorrelated sets of orthogonal eigenstates $\{|\psi_i\rangle\}$ and $\{|a_i\rangle\}$. As we will see when discussing environment-induced decoherence, for three or more decompositions this Hamiltonian gets more and more restricted. Also, see Appendix B for Schmidt's theorem.

[xxii] Erwin Schrödinger, "Die gegenwärtige Situation in der Quantenmechanik," *Naturwissenschaften* **23**, p. 807-12 (1935), translation in Wheeler and Zurek, *Quantum Theory and Measurement*, "The present situation in Quantum Mechanics" 152-67 (Princeton: Princeton University Press, 1983), p. 162.[48]

[xxiii] Dennis Dieks, "Quantum Mechanics without the Projection Projection Postulate," in *Foundations of Physics*, **19**, No. 11, p. 1397-1423 (Plenum Publishing, 1989), p. 1404.[16]

have to conclude that the wave function of the photon has 'collapsed' into one of the eigenstates (one slit) instead of maintaining its superposition. This is what is called "collapse of the wave function." It describes the loss of interference as a result of observing the system. This becomes apparent when we observe via the second measurement. This principle was introduced for the first time by Werner Heisenberg in his 1927 paper "*Über den anschaulichen Inhalt der quantentheoretischen Kinematic und Mechanik*," although Heisenberg did not use the term "collapse", but spoke of a "reduction of the wave-packet to a new state" whereas the information we have about the system fundamentally changes through observation.[xxiv] It was first incorporated into the mathematical framework of the theory by John Von Neumann in 1932.[xxv] He calls it the "reduction of the state vector". The term 'projection postulate' is due to the great physicist Henry Margenau in 1936.[xxvi]

The next part of the research will be devoted to some of the arguments that lead to projection and putting it in the mathematical perspective of the Von Neumann postulates.

[xxiv] Werner Heisenberg "Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik," *Zeitschrift für Physik* **43**, 172-98 (1927), translation in Wheeler and Zurek, *Quantum Theory and Measurement*, "The Physical Content of Quantum Kinematics and Dynamics" p. 152-67 (Princeton: Princeton University Press, 1983), p. 74. [24]

"[The] determination of the position selects a definite "q" from the totality of possibilities and limits the options for all subsequent measurements. [...] [T]he results of later measurements can only be calculated when one again ascribes to the electron a "smaller" wavepacket of extension λ (wavelength of the light used in the observation). Thus, every position determination reduces the wavepacket back to its original extension λ."

[xxv] John Von Neumann, *Mathematische Grundlagen der Quantenmechanik.* (Berlin: Springer, 1932),
Page numbers refer to the english translation:
English translation: *The Mathematical Foundations of Quantum Mechanics*, by Robert T. Beyer (Princeton: Princeton University Press, 1955).[36]

[xxvi] Henry Margenau, "Quantum-Mechanical Description." *Physical Review* **49**: p. 240-2, 1936. [32]

Part I

The Postulates of Quantum Mechanics

The aim of the research was to discuss the status of projection among the postulates. To keep the aims in sight we need a large amount of the formalism of quantum mechanics. The postulates are mathematical, so we were inclined to enter the formalism at some point. This does avoid some problems though. In a sense the mathematics is clearer than words, because it does not have explicit philosophical connotations. The philosophical problems arise when interpreting the formalism. But in another sense -and for the same reason- it is less clear, because it is more abstract (since as a good physicist you always still have some interpretation left to do when you finish your calculations and stare at the result). Speaking of quantum physics is very sensitive to the interpretation you are using. Next to speaking of the theory it will also not be helpful to paste all mathematical cracks when they do not add in an important way to the purpose but do claim a lengthy explanation. Mathematical postulates bypass the first complication and can serve to ignore the second if they are stated somewhat less general, i.e. we will only use the discrete and non-degenerate spectra of operators. This avoids a long discussion on the mathematics of functional analysis instead of just linear algebra and the technical complication that an eigenvalue can belong to more than one eigenstate.[xxvii] Without these philosophical and mathematical constrictions a set of the axioms based on empirical evidence will help to expose problems concerning foundations. The interpretational issue and the status of the postulates however, need to be supported by argument. These arguments will be given afterwards; discussing their status -especially the projection postulate- is the main subject of this research. The probability distribution can be discrete or continuous depending on the operator at hand and the system in which it is yielded.

3 The Von Neumann Postulates

The formulations of quantum mechanics consists of abstract mathematical structures, mostly from linear algebra and functional analysis. These structures are constructive and complete. It was John Von Neumann who was the first to axiomatize quantum mechanics.[xxviii] In this way he established a connection between the physical and the mathematical concepts that had arisen in de late 1920's. First of all we find ourselves outside the familiar three-dimensional real Cartesian vectorspace and into a complex Hilbert space. The classical states {x,p} of a particle are represented by two generalized coordinates: its position x and its momentum p. In quantum mechanics the states $|\Psi\rangle$ span the Hilbert space, i.e. states are abstract vectors or rays called states.[xxix]

First of all it is important to mention that quantum mechanics is not always presented as if it is based on a set of unanimously accepted postulates. Most intro-

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[xxvii] These simplification do not effect the arguments that are considered, but rather narrow the scope of physical systems we are looking at, while also keeping the physics in sight.

[xxviii] Von Neumann, *Grundlagen*.

[xxix] For a relatively quick, but good overview I refer to the online Stanford Encyclopedia(online): `http://plato.stanford.edu/entries/qm/`.

ductionary textbooks do not approach the theory in an axiomatic manner.[xxx] Even the books that do, which are most often members of the great variety of 'Quantum Mechanics for Mathematicians'-textbooks, still do not completely agree on the starting points. However, the Von Neumann postulates are the first complete description of the theory in a rigorous mathematical structure, fully consistent with the results of experiment. Also, to see where the problems lie in interpretational issues, a set of well-defined axioms will be helpful to spot the difficulties. Before discussing them in some detail, I will first give them here:

1. The State postulate

   Every physical system is described in terms of unit elements of a Hilbert space $\mathcal{H}$. The description of the system in a linear combination of these unit vectors spanning $\mathcal{H}$ is called the state $|\Psi\rangle$.

2. The Observables postulate:

   Every physical quantity $\mathcal{A}$ of the system is represented by a corresponding Hermitian operator A operating in $\mathcal{H}$.

3. The Measurement postulate(s)[xxxi]

   If a physical quantity $\mathcal{A}$, with corresponding operator A, is being measured of a system in a state $|\Psi\rangle \in \mathcal{H}$, the outcome of the measurement will yield an eigenvalue from the spectrum of A, i.e. from the eigenvalue equation: $A\,|a_i\rangle = \alpha_i\,|a_i\rangle$ (where α_i and $|a_i\rangle$ are the eigenvalues and eigenstates of operator A). Which value from the spectrum is obtained is given by a probability:

   $$P(a_i) = |\,\langle a_i \,|\, \Psi\rangle\,|^2. \tag{3}$$

4. Schrödinger postulate:

   The time-evolution of the state is unitary:

   $$|\Psi(t)\rangle = U(t,t_0)\,|\Psi(t_0)\rangle\,, \tag{4}$$

   where $U(t,t_0)$ is a unitary transformation of the state.

[xxx] Mukul Agrawal, "Axiomatic / Postulatory Quantum Mechanics," CA 94305 (Stanford: Stanford University, 2002).[2]

[xxxi] To cover the confusion of words: The 'measurement postulate' is an axiom describing which values will be obtained upon measurement, i.e. those which belong to the spectrum of the operator of the physical quantity, with a certain probability. It is not to be confused with the 'measurement problem', which is about the disturbing character of a measurement on the system and thus only important when doing repeated measurements.

5. Projection postulate:

   If a measurement on a physical quantity $\mathcal{A}$ is performed on a physical system in a state $|\Psi\rangle \in \mathcal{H}$, and the result is α_k, then immediately after the measurement the system will be in an eigenstate which has α_k as its eigenvalue. Thus:

   $$|\Psi\rangle = \sum_i c_i |\alpha_i\rangle \xrightarrow{a_k} |\alpha_k\rangle\,, \tag{5}$$

   where $\xrightarrow{a_k}$ denotes a measurement with result a_k.

3.1 The State and Observable Postulates

That a physical system is described by states in an Hilbert space that are complex-valued is not evident from first glances. There are many ways to find these kind of structures arising from the experiments. This is what has been done in the late twenties and was united by Von Neumann in 1932 as the underlying structure of the results. As a side-remark, there is a very clear explanation of why complex numbers appear in the first chapter of *Modern Quantum Mechanics*[xxxii] by Sakurai and Napolitano, who use an analogy with the polarization of light.

A Hilbert space seems natural to account for superposition of the states as quantum waves. This is because the abstract vectors naturally obey the addition principle inherent in vector spaces, but -when added- also do not change the size of the system as would happen in classical physics, since the elements of the space allow for contructive and destructive interference. Superposition means there are (equally real) superposed states as linear combinations of the eigenstates. If $|1\rangle$ and $|2\rangle$ are two states the theory tells us $a|1\rangle + b|2\rangle\,, \{a, b\} \in \mathbb{C}$ is also an allowed physical state. In other words: *the quantum mechanical whole is more than the sum of its parts.*

The observables postulate ensures physical quantities to be represented by *Hermitian* operators. These operators working on the state vectors produce the eigenvalues from the spectrum of A that then serve as the physical values of the quantity $\mathcal{A}$. Results of measurement obviously can not be complex, but in quantum mechanics, it is also not possible to measure the real and imaginary parts of a complex dynamical variable separately. In classical mechanics this can easily be done when a variable is represented by real and imaginary parts, but in quantum mechanics two measurements in general interfere with each other. So the first measurement usually will disturb the state and therefore affect the second.[xxxiii] So the operator is demanded to be Hermitian so that the eigenvalues are automatically real. This postulate thus defines how the value for a physical measuring process comes from the mathematical theory.

Also, two states can be considered the same when they differ only by a 'phase factor' $e^{i\phi}$ with $\phi \in \mathbb{R}$. Consider system 1 described by the state $|\Psi\rangle$ and system 2

[xxxii] J.J. Sakurai, J. Napolitano, *Modern Quantum Mechanics* (San Francisco: Pearson Education, 1994), p. 6. [46]

[xxxiii] Paul Dirac, *The Principles of Quantum Mechanics* 4^{th} ed. (London: Oxford University Press, 1958), p. 34, 1^{st} ed. 1930. [17]

described by a state that differs from this by a phase factor: $|\Psi'\rangle = e^{i\phi}|\Psi\rangle$. Then the probability of measuring eigenvalue a_i of the physical quantity $\mathcal{A}$ for system 1 is:

$$P_1(a_i) = \langle\Psi\,|\,a_i\rangle\,\langle a_i\,|\,\Psi\rangle = |\,\langle\Psi\,|\,a_i\rangle\,|^2.$$

For system 2 this is:

$$\begin{aligned} P_2(a_i) &= \langle\Psi'\,|\,a_i\rangle\,\langle a_i\,|\,\Psi'\rangle \\ &= \left\langle e^{i\phi}\Psi\,\middle|\,a_i\right\rangle\left\langle a_i\,\middle|\,e^{i\phi}\Psi\right\rangle \\ &= \langle\Psi\,|\,a_i\rangle\,(e^{i\phi})^\dagger e^{i\phi}\,\langle a_i\,|\,\Psi\rangle \\ &= |\,\langle\Psi\,|\,a_i\rangle\,|^2 \\ &= P_1(a_i), \end{aligned}$$

using the measurement postulate.

In a physical theory it is the outcome of measurements that determines the physical status of a system. So both $|\Psi\rangle$ and $|\Psi'\rangle$ describe the same physical state. Indeed, the results of measurements are ensured to be the same through the Born postulate. The phase factor will play an important role when discussing decoherence.

3.2 The Measurement Postulate

What is expressed through the measurement postulate (or Born postulate[xxxiv]) is a quantum mechanical property that was exposed when discussing the double-slit problem: superposition, or interference. This is probably the most important feature that makes quantum mechanics counter-intuitive, i.e. the possibility that a physical quantity can have more values at the same time. This postulate relates our abstract mathematical entities to a concrete translation into terms we are used to: unique properties of a system at one time.

The measurement postulate ensures that we will not measure a superposition of eigenvalues, even though it is there. This is reasonable from our point of view. It would be hard to imagine to find the pointer of a Volt meter for 40% on 4 Volts and for 60% on 10 Volts, as in a macroscopic superposition. The actual outcome of the experiments is 'simply' given by just one of eigenvalues, as is familiar to us. This does not mean though, that when there are two systems described by the same state $|\Psi\rangle$ (and are thus two copies of exactly the same physical system) it will produce the same value for the same physical quantity. It is here that we find the characteristic indeterminism of quantum mechanics. It tells us that only the probability of finding

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[xxxiv] The measurement postulate is actually a combination of the original formulation of two separate postulates: the Spectrum postulate and the Born postulate, where the former limits the outcomes of measurement to eigenvalues from the spectrum of the physical operator, and the latter adhers to this a probability distribution. However, it seems clearer to me to state these two notions that link the mathematical entities to observable quantities as if they are one: 'the outcome of a measurement is one of the eigenvalues obtained from a probability distribution.' Even more important, to explain the reason that physical observables are represented by operators, one first needs the Born postulate anyway, since it follows from there that the probability of finding an eigenstate outside the spectrum of the operator is zero.

a specific outcome of a measurement can be predicted. Only when we are sure that the system is in an eigenstate of the to-be-measured quantity can this probability be equal to 1. Note however, that it is generally not clear what is meant by the word 'measurement'.

3.3 The Schrödinger Postulate: Time Evolution

Just as Newton's 2^{nd} law governs the time evolution of the vectors that describe the position of a particle in a Cartesian vector space, there is a quantummechanical law that governs the time evolution of a state-vector in Hilbert space. The quantum[xxxv] version of Newton's 2^{nd} law is Schrödinger's equation. It describes a linear time-evolution, i.e. the evolution of the state vector is smooth through time.

If the system exhibits time invariance, the time transformation will only depend on the time difference, thus $U(t, t_0) = U(t - t_0)$. Since the symmetry of the single time parameter has a corresponding symmetry group and does not change the states to a vector space outside of $\mathcal{H}$, there is a group generator H defined as: $U(t - t_0) = e^{\frac{i}{\hbar}H\cdot(t-t_0)}$, with H an Hermitian operator so that U is unitary. We can set $t_0 = 0$ as well, for it is an arbitrary starting point in time: so $U(t - t_0) = U(t) = e^{\frac{i}{\hbar}Ht}$.

When we plug this into Schrödinger's equation and take the derivative to time:

$$i\hbar\frac{d}{dt}\left|\Psi(t)\right\rangle = i\hbar\frac{d}{dt}U(t)\left|\Psi_0\right\rangle = HU(t)\left|\Psi_0\right\rangle = H\left|\Psi(t)\right\rangle. \tag{6}$$

This is more common way to write Schrödinger's equation. H is the Hamiltonian of the system. It determines how the system will evolve over time.

3.4 The Projection Postulate: Collapse of the Wave Function

Consider the double-slit experiment again. The fact that we observed an interference pattern P'_{12} means that one electron should be described as going through both slits at the same time. Interference is described as several orthogonal component states comprising one final state, namely the one in superposition. In this case there are two eigenstates: $|\psi_1\rangle$ is the electron going through hole 1 and $|\psi_2\rangle$ is the electron going through hole 2 . Together they form the superposed electron state: $|\Psi_e\rangle = \frac{1}{\sqrt{2}}|\psi_1\rangle + \frac{1}{\sqrt{2}}|\psi_2\rangle$. Denote the operator to measure the 'which path'-information by $\hat{X}$ and we find for the expectation value:

$$\langle\hat{X}\rangle = \langle\Psi_e|\hat{X}|\Psi_e\rangle = \frac{1}{2}\{\langle\psi_1|\hat{X}|\psi_1\rangle + \langle\psi_2|\hat{X}|\psi_2\rangle + \langle\psi_1|\hat{X}|\psi_2\rangle + \langle\psi_2|\hat{X}|\psi_1\rangle\}. \tag{7}$$

But when introducing the light in the experimental setup that enables us to find information about the path the electron takes, this state changes. We lose the interference pattern and instead get normal additive behaviour P_{12}. This distribution

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[xxxv] It is rather the Hamilton equations that formulate Newton's laws in a more generalized (generalized coordinates) fashion that corresponds better to the quantum mechanical language, but the physics is still Newtonian.

is described by 50% of the electrons in $|\psi_1\rangle$, and the other 50% in $|\psi_2\rangle$. So by measuring the path of a single electron -say it went through slit 1- the superposed state got projected onto its eigenvector corresponding to slit 1:

$$|\Psi_e\rangle = \frac{1}{2}\,|\psi_1\rangle + \frac{1}{2}\,|\psi_2\rangle \xrightarrow{Projection} |\psi_1\rangle\,. \tag{8}$$

Invoking this projection is the standard way to cope with the experimental results. Now the electron really goes through one slit. This is equivalent to the disappearance of the interference terms, of course:

$$\langle\hat{X}\rangle = \langle\Psi_e|\hat{X}|\Psi_e\rangle = \frac{1}{2}\{\langle\psi_1|\hat{X}|\psi_1\rangle + \langle\psi_2|\hat{X}|\psi_2\rangle\}. \tag{9}$$

So next to Schrödinger evolution, quantum mechanics seems to exhibit another kind of evolution through time: a projection as a result of measurement. This is expressed through Von Neumann's projection postulate, it was first mentioned by Heisenberg in 1927 parallel to his discovery of the uncertainty principle.[xxxvi] It tells us what happens with a system after measuring it. Dirac phrased it: "A measurement always causes the system to jump into an eigenstate of the dynamical variable that is being measured."[xxxvii] See Figure **4**.

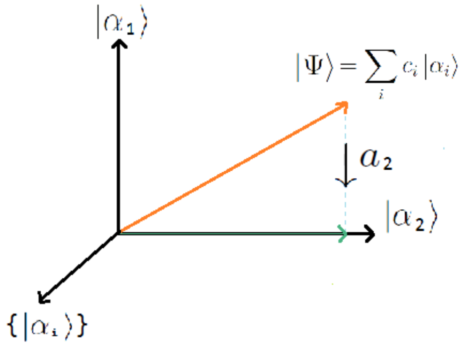

Figure 4: The state that is in a superposition of the eigenstates of some observable A with eigenstates $\{|\alpha_i\rangle\}$ is projected onto one of the eigenstates as a result of measurement. An immediately repeated measurement will thus be predictable with certainty. The set of orthogonal states is represented as $\{|\alpha_i\rangle\}$.

Note that projection is of importance only when we make a second measurement. In the double-slit case: measurement 1 is on the path and measurement 2 is on the screen. Suppose you look at an object, thus measuring physical quantities, and you look away for the briefest moment. When you look back at the system you'd expect that (as long as there are no huge external forces) the system is still in the same as it was before. This expectation follows from everyday experience. This is also what we find in quantum mechanics. After a measurement the system does not jump back into the superposition, it stays in the measured eigenstate. The projection postulate ensures this mathematically. A stronger version of the postulate was

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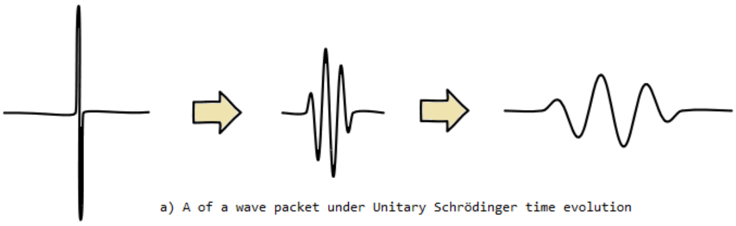
[xxxvi] Heisenberg, "quantentheoretischen Kinematik," 74.

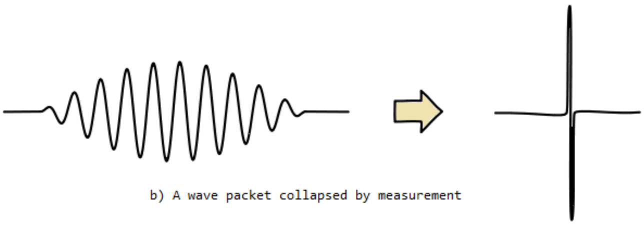
[xxxvii] Dirac, *Principles*, 36.

given by Lüders[xxxviii] and it ensures that the state is also normalized after projection: $|\Psi\rangle = \sum\limits_i c_i \, |\alpha_i\rangle \xrightarrow{a_p} \frac{|\alpha_p\rangle}{|\langle a_p|a_p\rangle|}$.

It is clear that projection exhibits non-unitary behaviour. This means information is lost. By measuring value a_k there is no way to construct the wave function as it was before measurement. That information is lost forever by the fundamental measurement disturbance. Thus we really have two processes of evolution through time: (i) Schrödinger evolution and (ii) Projection. As depicted for wave packets in Figure **5**.



a) A of a wave packet under Unitary Schrödinger time evolution



b) A wave packet collapsed by measurement

Figure 5: Two types of evolutions through time depicted by wave-packets. Upon measurement a wave-packet is localized. When a state (in this case an eigenstate) is left alone it spreads out under Schrödinger evolution.

3.5 Von Neumanns Empirical Argument for Collapse: X-ray-electron collisions

Von Neumann proposed the projection postulate in his axiomatic approach to quantum mechanics. Since it is this postulate that reveals the source of the measurement problem, we must look into the arguments why it was proposed. In his *Mathematical Foundations of Quantum Mechanics* Von Neumann offered an empirical argument for his projection postulate based on the experiment performed by Compton and Simon in 1925.[xxxix] This experiment gives empirical justification for applying the classical laws[xl] of elasticity and momentum conservation by scattering X-rays of electrons. Von Neumann uses this result and adds an interpretation of the concept of measurement during this experiment, from which he motivates the concept of projection by measurement. I will first discuss the original experiment before going into Von Neumanns argument.

[xxxviii] Gerhart Lüders, "Über die Zustandsänderung durch den Meßprozeß," *Annalen der Physik* 1951: **443**, p. 322-8.[31]

[xxxix] A.H. Compton and A.W. Simon, "Physical Review," 1925: **25**, p. 289.[15]

[xl] When speaking of classical laws, I include relativistic corrections. In this case, since it involves a scattering of X-rays, we are dealing with relativistic effects.

We will consider the Compton-Simon experiment in its simplest form. It consists of a (monochromatic) X-ray beam passing through a cloud chamber. There is a gas in this chamber from which the X-rays scatter through interaction with the orbital electrons of the atoms of the gas. The tracks of electrons that get knocked out of their orbits are observed by making cloud chamber photographs. The observed patterns are sketched in Figure **6**.

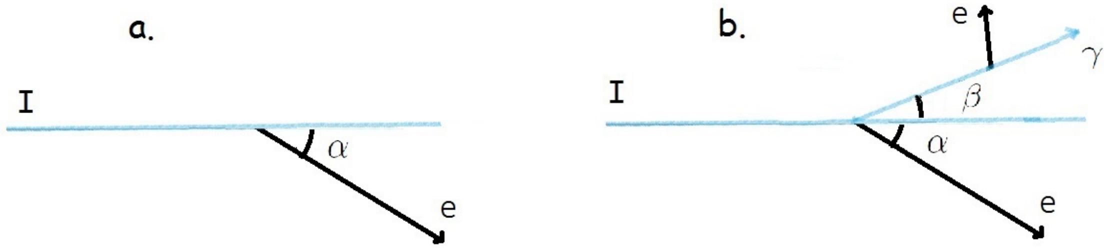


Figure 6: Possible results from the Compton-Simon experiment, published in 1925. I indicates the direction of the incident beam of photons, e's are recoiling electrons and γ is a scattered photon. α and β are the angles of the scattered particle with the incident beam I. Keep in mind that in a cloud chamber only the paths of charged particles are observed. Since photons are neutral, their paths are not observed in the experiment; indicated with blue. 6a: An electron is knocked out of its orbit by an incoming photon. 6b: The photon scattered by an event as in 6a knocks another electron from its orbit.

In Figure 6a) we see an electron being knocked out of his orbit -under an angle α with the incident beam- from the collision with a photon from the incident beam. In Figure 6b) the photon from this previous event (which changed direction due to the interaction with the electron) goes on to scatter of yet another electron.

Compton and Simon took out only the photographs which displayed the events of Fig. 1-b. Their objective was to see how the directions of the scattered electrons and scattered photons correlated to the initial collision. They did so by measuring the angles α and separating them. Then they examined those photographs where β was equal to the angle α. They found that there is a unique direction of the recoiling photon corresponding to each direction of recoil for the electron.[xli] Now when we regard the X-ray photons as particles with a momentum $p = \frac{hf}{c}$, use the relativistic (classical) equations of momentum conservation and assume the electrons initially at rest (since $v_{electron} \ll v_{x-ray}$), then the paths of the scattered photon and the scattered electron are uniquely related by:

$$\cot\frac{\beta}{2} = -(1 + \frac{hf}{mc^2})\tan\frac{\alpha}{2}. \tag{10}$$

The angles as measured from the photographs obey this relation. In that way Compton and Simon showed that the classical laws of (elastic) collisions hold and that X-rays can be regarded as particles.

This is where Von Neumann takes up the thread and considers an experiment[xlii] similar to that of Compton and Simon. In this experiment the paths of the electron

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[xli] Joseph Sneed, "Von Neumann's Argument for the Projection Postulate," Philosophy of Science 1966: **33**, p. 25.[50]

[xlii] Von Neumann, *Grundlagen*, 211.

and the photon before the collision are known *a priori*.[xliii] The observations of the particles are carried out by Geiger counters placed along the angular degree of freedom, as in Figure 7.

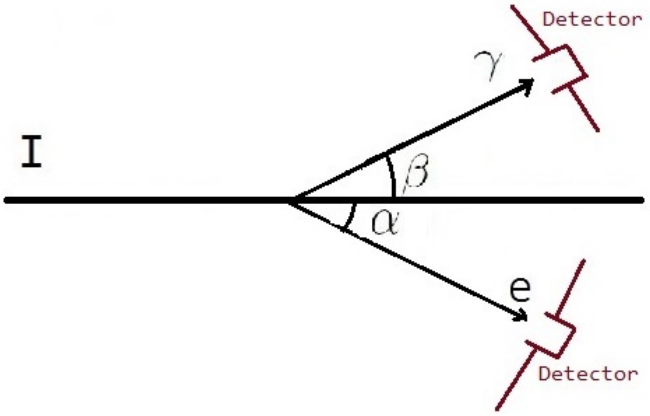

Figure 7: A *gedanken* experiment similar to the experiment of Compton and Simon as envisioned by Von Neumann in 1932.

The conclusions of the Compton and Simon make it possible to use the classical machinery of momentum conservation. That includes the designation of the central line of the collision. This is the same line before and after the collision.[xliv] So apart from the magnitude of the momenta (which is not relevant for the direction of the paths), we are dealing with 5 degrees of freedom, i.e. the two directions of the paths taken by the particles before the collision, the two paths after collision, and the central line. The last three of these are initially unknown, but it is enough to measure one of them in order to calculate the last two. Von Neumann considers the possibility that we are interested in the central line, which he calls R. He states that a measurement of the path of either recoiling particle is enough to calculate R, and can therefore also be regarded as a measurement of R. He continues by emphasizing that even though we do not know R before measurement, we are not completely unable to say something about its properties, because we might be able to calculate a probability distribution for the position of the central line from the initial *magnitudes* of the momenta of the particles. Also, because the detectors can be moved closer to or further away from the collisions, we can control if we want to detect the photon first and the electron later, or *vice versa*. Then, he starts his argument:

(*i*) Before we make any measurement of R we can only make statistical statements about what R will be when we measure it.

(*ii*) After a measurement M_1 (say, of the photon) yielding a particular value r_p for R (the central line), but before M_2 (measurement of the electron) occurs, we can say that r_p will be measured at the moment when M_2 occurs.

[xliii] Von Neumann remains unclear on how exactly the incident paths can be known before a measurement is carried out. This, however, does not change the validity of his argument.

[xliv] The central line is the line of the sum of the particle's momentum vectors. By invoking conservation of momentum it follows that this line must be the same line before and after collision.

(*iii*) Therefore, the state of the system changed by the measurement of the photon M_1 from a state about which we can only speak of as a probability distribution for values of R to a state where we can assign one particular value r_p for the outcome of a measurement.

Von Neumann doesn't explicitely put this in quantum mechanical terms, but his point is clear: (*iii*) gives rise to a projection from a statistical set of statements to a statement about a specific value. It is important, however, that the statistical interpretation is being used here, as was usual at the time. This is interesting because it shows which premises are implied here through the interpretation of the probability distribution of a system.[xlv] These implied premisses are to finish the deductive validity of conclusion (*iii*). Outside of the statistical interpretation we will need additional premisses. But as it stands, in agreement with other experiments conducted at time, Von Neumann's argument gives a strong justification for introducing the projection postulate in quantum mechanics. In quantum mechanical terms, the wave function $|\Psi\rangle = \sum_i \langle r_i \,|\, \Psi\rangle \,|r_i\rangle = \sum_i c_i \,|r_i\rangle$[xlvi] allows us to make statements -although statistical- about the value of R after measurement M_1, whereas the measurement M_1 of a specific value r_p gives rise to a collapse of the wave function, as depicted in Figure 8.

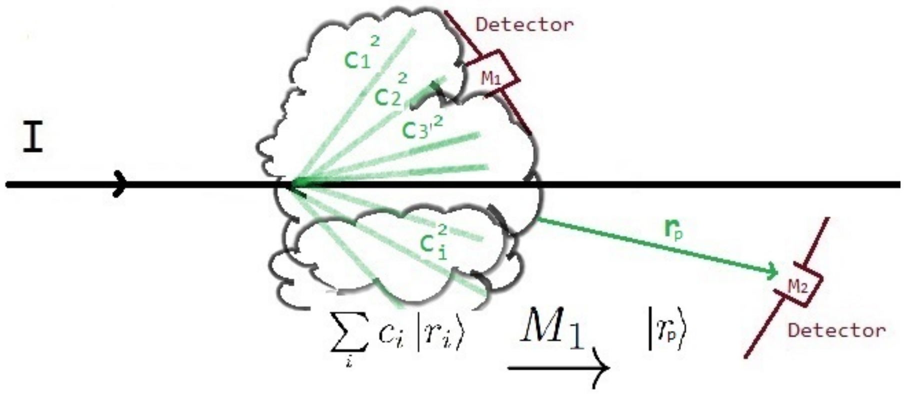

Figure 8: Collapse/Projection of the wave function as a result of measurement of R. The measurement M_1 reduces the wave function from a superposition of states, with probability c_i^2, to a determined state $|r\rangle$, justifying the projection postulate.

The wave function undergoes the following projection:

$$|\Psi\rangle = \sum_i c_i \,|r_i\rangle \xrightarrow{\textit{Projection}\, M_1} |r_p\rangle \,. \tag{11}$$

So the wave function has changed. Before measurement it was in a superposition of possible values, and predicted to find a state $|r_p\rangle$ with a probability $c_{r_p} = |\,\langle r_p \,|\, \Psi\rangle\,|^2$. After measurement it is projected to one of the eigenstates. Now the wave function is no longer in superposition and it predicts a certain value r_p with probability $|c_{r_p}|^2 = |\,\langle \Psi \,|\, r_p\rangle\,|^2 = |\,\langle r_p \,|\, r_p\rangle\,|^2 = 1$, i.e. the value r_p is predicted

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[xlv] The interpretations of probabilities *an sich* will not further be discussed, although it is wildly fascinating. See for example J. Bricmont, *Chance in Physics*, eds.: J. Bricmont, G. Ghirardi et al.. [12]

[xlvi] where $|r_i\rangle$ denotes an orthonormal basis of possible values r_i of R. This variable is continuous, but for simplicity we imagine that we take only a few possible paths, consistent with the simplifications before.

with certainty upon another measurement. So now we predict M_2 with certainty, since $|r_p\rangle \xrightarrow{M_2} |r_p\rangle$.

This is how Von Neumann sketched the motivation for invoking collapse. Already here we might have the impression that we are dealing with what we know of the system, the way we can describe it. The question is if there is a fundamental change through the process of measurement. Although Von Neumann did not prove the necessity of the projection postulate, he did give a motivation in this way, which is consistent with all the experiments at the time.

Part II

The Measurement Problem

“I would by all means have men beware, lest Æsop’s pretty fable of the fly that sate [sic] on the pole of a chariot at the Olympic races and said, ‘What a dust do I raise,’ be verified in them. For so it is that some small observation, and that disturbed sometimes by the instrument, sometimes by the eye, sometimes by the calculation, and which may be owing to some real change in the heaven, raises new heavens and new spheres and circles.”

—Francis Bacon, 1625 [xlvii]

[xlvii] Francis Bacon, ‘Of Vain Glory,’ *The Works of Francis Bacon (1887-1901)* **6**, James Spedding, Robert Ellis and Douglas Heath (eds.), (Bristol: Longmans and Co., 1858), p. 503. [7]

4 Uneasiness about Projection

In his *Scientific Autobiography* Max Planck wrote:

> "An experiment is a question which science poses to Nature, and a measurement is the recording of Nature's answer." [xlviii]

In classical physics, it seems that this poetic phrasing by Planck suffices to explain why experiments are conducted. We want our questions to be answered and to do that, we construct artificially simple situations from which we try to extract answers. In quantum mechanics we do nothing different of course, but it is Nature's answer that changes every time we ask the same question. Quantum mechanics is governed by indeterministic laws. But after we do have a definite recording of Nature's answer, a repeated measurement on the same system of the same variable deterministicly yields the same answer. The measurement problem is about explaining how this works. Why is it that the superposition is invisible? What is the mechanism behind this? It seems reasonable from empiric results to invoke the projection postulate, and that is exactly what happened. In a sense, the projection postulate is a solution to the measurement problem. Indeed, it is the most obvious solution since it accords for all empirical data. All experiments are consistent with the projection postulate. It has long since been the most common solution.

But physicists seem always to have expressed some unease about this solution, for it has consequences that might be hard to accept. Von Neumann himself already expresses this uneasiness and devotes the last chapter of his book to the measuring process. He says this discussion is necessary because we "found a peculiar dual nature of the quantum mechanical procedure which could not satisfactorily be explained. Namely, we found that a state ϕ is transformed into the state ϕ' under the action of the energy operator H [...] which is purely causal." On the other hand, "ϕ undergoes in a measurement a non-causal change." So even the great quantum architect has expressed his uneasiness about having a Schrödinger equation on the one hand, and a projection postulate on the other, that governs the same physical system.

Probably the most problematic is the notion of measurement. What distinguishes a measurement from an ordinary physical process? This concept has been the source of the discussions between Einstein and Bohr in the 1930's out of which the Einstein-Podolsky-Rosen paradox and the Schrödinger's cat paradox arose. These were answered by Bohr in his version of the Copenhagen interpretation, but in a manner that has increasingly been unsatisfactory. This will be discussed later on.

The most important problems with the projection postulate I would judge the non-unitary time evolution on the one hand, which results in the loss of information, and the ill-defined concept of measurement on the other. We will treat these problems in the following secitions.

[xlviii] Max Planck, *The Meaning and Limits of Exact Science*, Science 30 Sep 1949: No. 2857, p. 325. Advance reprinting of chapter from Planck's *Scientific Autobiography* (1949), p. 110. [43]

4.1 The Asymmetry of Time

The Moving Finger writes; and, having writ,
Moves on: nor all thy Piety nor Wit
Shall lure it back to cancel half a Line,
Nor all thy Tears wash out a Word of it.

—Umar Khayyám, 11th century,
translation by Edward Fitzgerald.

As we have seen a second kind of time evolution -besides evolution through the time-symmetrical Schrödinger wave equation- is introduced to the theory via the projection postulate. This has been done to cope with the discrepancy between superpositions and unique results of measurement, i.e. the measurement problem. This is a break with classical physics, which only has one sense of evolving physical systems, i.e. Newtons 2nd law, which is unitary. So with two evolution postulates the theory seems less elegant than it was with one. But the problem is not so much the amount of postulates. It is the unitary *contra* the non-unitarity, say the question of symmetry or assymmetry in time. An important feature of classical (and relativistic) mechanics is that the laws of Nature are time-reversible. So regardless of the sign of t, the behaviour is the same. This is found in the even powers[xlix] of the time variable in the classical differential equations, like Newtons law $F = m\frac{d^2x}{dt^2}$ or the classical wave equation $\frac{d^2u}{dx^2} = \frac{1}{c^2}\frac{d^2u}{dt^2}$ with dispersion corrections only in powers of 4, 6, ..., i.e. never odd, so effectively the sign of t is always positive.[1]

Where the Schrödinger equation is time-symmetrical, the projection postulate -if it is to be deemed fundamental- says that events played back in time do not follow the same rules as those played out forward, making the theory time-irreversible due to the non-linear behaviour of the projection operator. The reverse of the projection postulate does not happen. A state in a superposition of eigenstates collapses to a single eigenstate upon measurement, but a single eigenstate only evolves to a superposition through linear Schrödinger behaviour where no collapse (or 'recollapse') occurs. Thus there is an asymmetry. Through the collapse process we lose information of the system which can never be reconstructed. This assymmetry is a problem, because we do not understand how the mechanism works. This inevitably leads to a fundamental irreversibility that is different from the thermodynamic irreversibility that we know from classical physics, which results from our ignorance of single particles in ensembles. If a fundamental non-unitarity is not bad enough, we also have no explanation of how this process works. David Albert put the problem quite accurately:

The dynamics and the postulate of collapse are flatly in contradiction with

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[xlix] There is of course this other concept that gives rise to a notion of a prefered direction of time: Entropy. This is the tending of statistical systems to decrease order, minimizing it's energy and maximizing the entropy. This is what is called the thermodynamic arrow of time by Arthur Eddington in 1927.[21] For further reading I would recommend Sean Carroll's *From Eternity to Here: The Quest for the Ultimate Theory of Time*, Dutton, 2010.

[1] Max Planck, *A Survey of Physical Theory* (New York: Dover publications, 1993), p. 8, 1st ed. 1925.[44]

> one another [...] the postulate of collapse seems to be right about what happens when we make measurements, and the dynamics seems to be bizarrely *wrong* about what happens when we make measurements, and yet dynamics seems to be *right* about what happens whenever we *aren't* making measurements. [li]

Anyone would agree there lies the problem of what the concept of measurement really is. It is the act of measurement itself that is causing trouble here. Is there a way to catch the measurement process in physical terms? or is it something that is intrinsic to our consciousness? Maybe it is as Pauli said that the act of measurement is "outside the laws of nature." The problem is how to reconcile the Schrödinger evolution and the apparent notion of collapse. The answer must lie in the answers surrounding the question 'what is a measurement?'

4.2 What is a 'Measurement'?

Obviously, human senses are inadequate to perceive microscopic phenomena directly. In order to investigate the behaviour of such a system, we need a measuring device, or apparatus, which is macroscopic. We have seen that somewhere during the process of the coupling of the macroscopic device to the (microscopic) system of interest and our realization that the device displays some particular pointer value, the state of the system changes. This was a conclusion justified by making a second measurement, which showed always to be in accordance with a state projected to the eigenstate corresponding to the result of the first measurement.

Think of the double-slit experiment. We have seen that the probability distribution P'_{12} collapses into P_{12} by simply looking at which path the electrons take. But what is it really that triggers collapse here? Can we not just say that the recoil of the scattering photons causes the electrons to change their behaviour. Well yes, this is true: by trying to watch which path the electrons take we have bombarded them with photons. So we want this effect to be smaller. The momentum recoil the electron experiences scales with the wavelength of the light via De Broglie's formula $p = \frac{h}{\lambda}$. Thus light of a larger wavelength will effect the electron less. By increasing the wavelength of the light the interference does get less and P'_{12} indeed starts moving towards P_{12}. The problem is that when we use light of a wavelength that is too large, the flashes due to two passing electrons will overlap, and we will not be able to differentiate between them anymore. It is just at these wavelengths of light that the recoil is small enough to see interference and obtain the 'which path'-information. This is exactly what Heisenberg proposed as a general principle that would hold the laws of quantum mechanics together. *There is no experiment that can determine the path the electron takes, without destroying interference effects.*[lii] In our case, for Δx the uncertainty in the position, λ the wavelength and p the momentum:

$$\Delta x \Delta \lambda \geq 1 \Rightarrow \Delta x \Delta p \geq h. \tag{12}$$

This is the Heisenberg uncertainty principle.[liii] We cannot make the wavelength arbitrarily small, without making it harder and harder to see which path has been taken

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[li] David Albert, *Quantum Mechanics and Experience* (Cambridge, MA: Harvard University Press, 1992), p. 79. [3]

[lii] Feynman, *Lectures*, 1-9.

[liii] Heisenberg, "quantentheoretischen Kinematik," 76.

by the electron. It seems that we can find no way to measure which path is taken by the electrons while also preserving the interference pattern. This seems to hold in all experiments, and many different experiments have been contrived to circumvent this problem. It is a consequence of the projection postulate in that it is impossible to make two simultaneous measurements of two observables if the operators do not commute. Thus it seems that projection is part of the deal. When we measure a quantum system, we destroy interference. But this is not entirely so. There is also the possibility of describing the measurement scheme without the projection postulate: the so-called non-collapse theories will be treated in chapter 6. First we will discuss some early interpretations that dealt with the measurement problem: subjective interpretations and Copenhagen interpretations. For these the projection postulate is fundamental. This fundamental status is then justified through the interpretation.

5 Early Interpretations and Criticism

5.1 Consciousness contra Physical Processes

“If we imagine that there is a machine whose structure makes it think, sense, and have perceptions, we could conceive it enlarged, keeping the same proportions, so that we could enter into it, as one enters into a mill.”

—Gottfried Leibniz, 1714[liv]

Even Von Neumann himself did not manage to find a way to catch the measuring process in physical terms. He proposed that the solution lies in a psycho-physical parallelism, which he says is a fundamental requirement of the scientific viewpoint, since “it must be possible to describe the extra-physical process of the subjective perception as if it were in reality in the physical world.”[lv] The point is that projection ought to happen somewhere along the chain of events that lead to the registration of the measurement in our minds. This projection is defined to happen at the moment of measurement, but this moment can be taken at every intermediate physical process such as the interaction of the measuring device with the system of interest; or the needle pointing at some particular value; or the photon scattering of this needle falling on the eye of the observer; or the signal that our eye sends to the brain in the form of chemical reactions. This is called the Von-Neumann chain. Since at some moment we do register a unique value, this process has to stop somewhere. There is nothing that automatically saves this from infinite regress. Somewhere this chain of events has to terminate by using the projection postulate. The problem is at which stage this collapse occurs. Von Neumann shows that a Hamiltonian governing the process from the entangled state of neutral pointer position $|a_0\rangle$ to a particular pointer value $|a_p\rangle$ corresponding to the measured system always exists, such that Eq. (2) : $\sum_i c_i \left|\psi_i\right\rangle \otimes |a_0\rangle \rightsquigarrow \sum_i c_i \left|\psi_i\right\rangle \otimes |a_i\rangle$ is quite general.[lvi] Through this he shows that applying the collapse postulate to any of the mentioned intermediary processes, leads to the same statistics via the Born postulate. To solve this Von Neumann proposes the extra-physical idea, i.e. the consciousness of the observer. For when we are considering a measurement we never make a statement about a certain value of a physical quantity, but we say “[...] an observer has made a certain (subjective) observation.”[lvii] Quantum mechanics describes the physical world, but when this is coupled to the psychological world, i.e. when a measurement is made, we experience an indeterministic jump: collapse. This is a very personal affair, since “subjective perception leads us into the intellectual inner life of the individual, which is extra-observational by its very nature (since it must be taken for granted by any conceivable observation or experiment).”[lviii] This is the explanation Von Neumann gives to link the world of superpositions and the world of measurement; projection

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[liv] Gottfried Wilhelm von Leibniz, *The Principles of Philosophy, or, the Monadology*, paragraph 17.

[lv] Von Neumann, *Foundations*, 419.

[lvi] Von Neumann proves this via the bi-orthonormal decomposition (or Schmidt’s theorem), which allows one to write the entangled system in two orthonormal basis sets corresponding to the subsystems while keeping these stes independent of each other.

[lvii] Ibid., 420.

[lviii] Ibid., 418.

happens and when we are personally aware of the observation.

In 1939 London and Bauer give a similar explanation. They consider the right hand side of Eq (2) and stress that so far there is only a coupling between two systems, which is not yet a measurement. "A measurement is achieved only when the position of the pointer has been observed. It is precisely this increase in knowledge, acquired by observation, that gives the observer the right to choose among the different components [...] and to attribute thenceforth to the object a new wave function. [...] We note the essential role played by the consciousness of the observer [...]. Without this effective intervention, one would never obtain a new ψ function." It is argued that the observer has a special relation towards the observed, which is the "faculty of introspection" allowing him to say 'I am in a certain state'. In this way the observer creates his own objective reality and by introspection breaks the Von Neumann chain.[lix]

These subjective views of the measurement problem has been popular in the 1930's and 40's, but later became a minority view. Although occasionally defended by important physicists, e.g. Geitler or Wigner, holding the human consciousness responsible for collapse has proven to be very hard to except for the physical community.[lx] A measurement is generally regarded as a recording of some sort, rather than the observation of a conscious mind. About Von Neumann's explanation that it is the conscious mind that collapses the wave function Nico van Kampen says: "I find it hard to understand that someone who arrives at such a conclusion does not seek the error in his argument. Quantum mechanics is not a theory of the mind of an observer, but of physical, objectively recorded phenomena [...]. The question is how ψ relates to atomic spectra or specific heats; the mind of the observer is irrelevant."[lxi] Nevertheless, the subjectivists arguments given -e.g. by London, or Wigner- seem plausible, and there is no logic that compels us to believe the physical world is not coupled to the non-physical in this way. At least it is very interesting to see that physicists take refuge to this psychological world. But as far as it is plausible, it is also arbitrary. Where Von Neumann ends the chain of processes at the point where it enters our consciousness, the state could be projected anywhere along the chain, not just at the end. This would result in the same statistics, via the Born postulate. So the existence of the Hamiltonian satisfying the process of Eq (2) does not prove projection has to happen at consciousness. As before, the debate revolves around the interpretation of the right hand side of (2). But if we would find a physical way, i.e. a way to explain collapse in a quantative way, that would settle the debate.

That measurements should be explainable in physical terms appeals to our intuition. It is hard to imagine a measurement being something 'outside the laws of nature.' Of course this is hard, because we are used to describe everything we use as

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[lix] Fritz London and Edmond Bauer, "La théorie de l'observation en mécanique quantique," *Actualités scientifiques et industrielles: Exposés de physique générale* **775**, translation in Wheeler and Zurek, *Quantum theory and Measurement*, "The theory of observation in quantum mechanics," p. 217-58 (Princeton: Princeton University Press, 1982). [30]

[lx] Osvaldo Pessoa, "Can decoherence help to solve the measurement problem?" *Synthese* **113**, p. 323-46 (Dordrecht: Kluwer Academic Publishers, 1988), p. 324.[41]

[lxi] Nico van Kampen, "Ten theorems about quantum mechanical measurements," *Physica* **A153**, p. 97-113 (Haarlem: Elsevier Science Publishers, 2002), p. 101.[29]

a measuring device in terms of physical processes. Classically a thermometer points at a value because the mercury interacts with the molecules of the system that is being measured. Intuitively, quantum mechanics shouldn't make a difference. Take for example the double-slit experiment with the light source that enables us to tell whether an electron has gone through slit 1 or slit 2. Imagine the travelling electron going through a slit, scattering a photon towards our eye. If we take Dirac and Von Neumann seriously -which one should always do- on the topic of projection through measurement, we have to ask a very important question. When do we speak of a measurement and when do we speak of a physical interaction? It seems very difficult to assign a line of demarcation between a physical process and a process of measurement. Even more strongly, it seems desirable to describe the whole measurement process in terms of Schrödinger's unitary evolution. Hans Reichenbach puts this opinion more clearly:

> Some philosophers, and some physicists as well, have interpreted Heisenberg's statement as the confirmation, in terms of physics, of traditional philosophical ideas concerning the influence of the perceiving subject [...] that the subject cannot be strictly separated from the external world [...]; or that the subject creates the object in the act of perception; or that the object seen is only a thing of appearance, whereas the thing in itself forever escapes human knowledge; or that the things in nature must be transformed according to certain conditions before they can enter into human consciousness, etc.[sic] *We cannot admit that any version of such a philosophical mysticism has a basis in quantum mechanics.* Like all other parts of physics, quantum mechanics deals with nothing but relations between physical things; all its statements can be made without reference to an observer." [lxii]

Reichenbach touches a few important points of criticism to some of the ideas proposed in his time. He attacks the idea that those things from outside physics have any place in the physical theory. Physics is about physical entities. Consciousness can play no role in quantum mechanics, since it is not a physical entity. Most of his critisism though, seems to be directed at the Copenhagen interpretation. The subject creating the object by measuring it is a feature first introduced by Heisenberg in 1927.[lxiii] The 'transforming of things according to certain conditions' hints at Bohr insisting on the idea that we can only understand a measurement in classical terms, by which he means that our language and minds cannot grasp the quantum world. The problem with Reichenbach's statement is that as long as we still don't know how to describe the mechanism of collapse in physical terms, we have no explanation how it happens that a superposition transforms to a single realized state, whatsoever. Still, from a physical point of view, the objective approach should be desired.

[lxii] Reichenbach, *Foundations*, 15. [my italics].

[lxiii] Heisenberg, "quantentheoretischen Kinematik," 73.

5.2 The Copenhagen Interpretation

The Copenhagen interpretation is most commonly attributed to Bohr. In 2004 though, Howard raised the issue that it is hard to really trace some of the arguments back to the person who first contrived it or even to trace it back to one period in time and that the 'Copenhagen interpretation' is a vague construct of mostly Bohr's and Heisenberg's arguments and later philosophical developments.[lxiv] I encourage all to read Howards article, but to not go into all this here, we will confine the Copenhagen interpretation to Bohr's views.

These views are more or less the following. The concepts of the classical world determine how we think (about the physical world). Our minds are 'wrapped up' in such a way that we can only think in the concepts that classical physics hands us, including out language. The quantum state $|\Psi\rangle$ is not visualisable (*anschaulich*) and therefore it is only a mathematical tool to represent overlapping classical concepts. This overlapping is what Bohr calles 'complementarity', i.e. multiple classical concepts complement each other. Important is that for Bohr it was the classical world that was prior to the quantum world.[lxv]

Bohr emphasizes that we need a measurement device (an 'amplifier') to make a translation from quantum physics to classical concepts. Heisenberg takes this further by stating that we shouldn't try to talk about the quantum phenomena. We should just describe things with the mathematical theory we have, but not try to interpret these purely quantum mechanical entities. Prior to measurement, we have no words to describe what the state of a system looks like and we will fail in trying to do so. When measuring the orbit of an electron Heisenberg describes the measurement process as if this orbit is created or produced by the act of observation: "The 'orbit' comes into being only when we observe it." [lxvi] Before observation we can simply not attribute any physical values to the system. The quantum world is described by the mathematics of the wave function and this world is sharply divided from our unique classical world. This is what Primas called the 'Heisenberg Cut'.[lxvii] See Figure **9**. [lxviii]

For Bohr and Heisenberg the split between the quantum world and the classical world is purely conceptual. For it is not so much a cut between microscopic and macroscopic, it is a cut between object and instrument. The Heisenberg cut is not a physical cut, it is more a conceptual split between object and instrument. It is an epistemological cut on how to talk about quantum mechanics. When we consider a measuring device or instrument getting entanglement with the to-be-measured object it forms a new object and we lose the ability to make an observation. In order to treat an instrument *as* an instrument you have to think of it as something not

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[lxiv] Don Howard, "Who Invented the "Copenhagen Interpretation"? A Study in Mythology," *Philosophy of Science* **71/5**, p. 669-82 (The University of Chicago Press, 2002). [25]

[lxv] Guido Bacciagaluppi, "The Role of Decoherence in Quantum Mechanics," *The Stanford Encyclopedia of Philosophy*, ed.: Edward Zalta, `http://plato.stanford.edu/archives/win2012/entries/qm-decoherence/`. [6]

[lxvi] Heisenberg, "quantentheoretischen Kinematik," 73.

[lxvii] Hans Primas "The Cartesian cut, the Heisenberg cut, and disentangled observers," in *Symposia on the Foundations of Modern Physics* 1992, K. Laurikainen, C. Montonen (eds.), *World Scientific* (Singapore: Editions Frontières, 1993).[45]

[lxviii] Figure 'The Borderland' from Wojciech Zurek, "Decoherence and the Transition from Quantum to Classical-*Revisited*," *Los Alamos Science* 2002: **27**, p. 2. [56]

entangled. The apparatus doesn't have to be 'special' in a sense that it is outside of the laws of Nature. It is fine to treat a measuring device quantum mechanically, at least in principle. But to preserve the ability of observation, we have to *think of* the apparatus as if it is not entangled. [lxix]

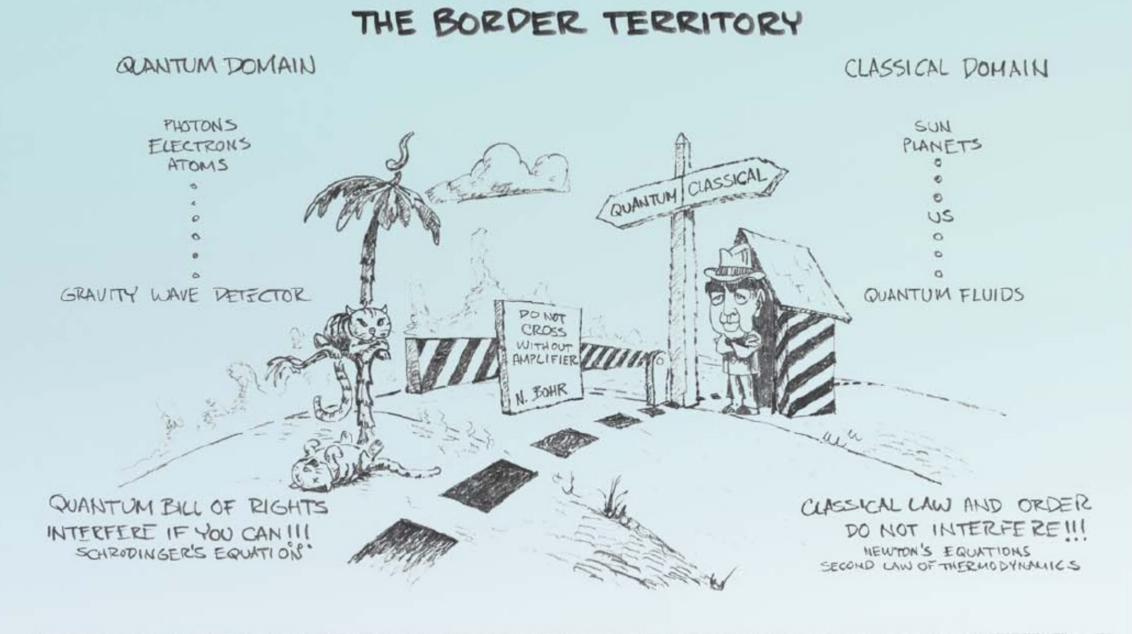

Figure 9: The Heisenberg cut. A sharp border between the quantum world and the world of familiar classical observations. The border is not a physical cut, but rather a conceptual split between the quantum system and the classical properties after amplification, that is measurement.

In any case, the Copenhagen approach remains somewhat fuzzy, or at least involved through the notions of complementarity – and correspondence, which originate from the early days of the theory and whose definitions seem not to paste the cracks of the modern theory anymore. I would say that instead of solving the measurement problem, the Copenhagen interpretation really tries to dissolve it. It leaves us with an indeterministic non-local theory in which it is not clear how a measurement changes the non-locality. The transition from quantum systems to a local classical measurement outcome describes no more than the projection postulate does. Zeh expresses this idea clearly:

> The Copenhagen Interpretation is often hailed as the greatest revolution in physics, since it rules out the general applicability of the concept of objective physical reality. I am instead inclined to regard it as a kind of "quantum voodoo:" irrationalism of dynamics.[lxx]

And once more by Tegmark and Wheeler: "Rampant linquistic confusion may contribute to [this undecidedness of interpretation]. It is not uncommon for two physicists who say that they subscribe to the Copenhagen interpretation [...] to find

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[lxix] Based on the Portland lecture "The Decoherence Prism," by Maximillian Schlossauer at the QFQI Conference, Decoherence & Friends, May 2013.

[lxx] H.D. Zeh, *Decoherence and the Appearance of a Classical World in Quantum Theory*, eds E. Joos et al., (Heidelberg: Springer, 2003), p. 27. [28]

themselves disagreeing about what they mean."[lxxi]

Nevertheless, the Copenhagen interpretation has inspired the community of physicists. It was regarded as the 'official' interpretation since the Solvay Congres in 1927, when the consensus was that Einstein lost the debate with Bohr.[lxxii] It is the interpretation that is generally discussed in every introductionary quantum mechanics textbook. And for good reason, since it supplies the early quantum mechanic with the possibility to bypass difficult questions interpreting phenomena at the quantum level and 'just do the math', because -according to the Copenhagen monarchy- it makes no sense to speak of these phenomena. More discussion on textbooks will be made in the last part.

6 Modern Interpretations

6.1 Non-Collapse theories

Since the projection postulate seems to express uneasiness among physicists it is worthwhile to find out if we can do without it. Consider again the measuring process described by Eq (2):

$$\sum_i c_i \, |\psi_i\rangle \otimes |a_0^{(0)}\rangle \rightsquigarrow \sum_i c_i \, |\psi_i\rangle \otimes |a_i^{(0)}\rangle .$$

We take into account another measurement -say $A^{(1)}$ with neutral pointer state $A_0^{(1)}$ and corresponding Hilbert space $\mathcal{H}_{A^{(1)}}$ - that measures system $\mathcal{S}$ a second time. Including this secondary measuring device into the entanglement leads to an extension of Eq (2), namely:

$$\begin{aligned} &\sum_i c_i \, |\psi_i\rangle \otimes |a_0^{(0)}\rangle \otimes |a_0^{(1)}\rangle \\ \rightsquigarrow &\sum_i c_i \, |\psi_i\rangle \otimes |a_i^{(0)}\rangle \otimes |a_0^{(1)}\rangle \\ \rightsquigarrow &\sum_i c_i \, |\psi_i\rangle \otimes |a_i^{(0)}\rangle \otimes |a_i^{(1)}\rangle , \end{aligned} \tag{13}$$

where each $\rightsquigarrow$ represents going to a later time interval, i.e. the measurements are consecutive and are governed by some (in general different) interaction Hamiltonian between the individual systems. By the first measurement, $A^{(0)}$ became entangled with the superposed state of the system. We see that by the second measurement $A^{(1)}$ is measuring the entangled system $\mathcal{S} + A^{(0)}$ and gets entangled to this. Now we have found that the superposition of system $\mathcal{S}$ simply carries over upon the measuring devices. It shows to be contagious. Continuing along this path, a third measuring device $A^{(2)}$ would couple in the same way, and so on for n measuring devices (surpressing the direct products):

[lxxi] Max Tegmark and John Wheeler, "100 Years of Quantum Mysteries," *Scientific American* 2001: **284**, p. 68-75.[51]

[lxxii] Bub, *Interpreting*, 190.

$$\sum_i c_i \left|\psi_i\right\rangle \left|a_0^{(0)}\right\rangle \left|a_0^{(1)}\right\rangle \left|a_0^{(2)}\right\rangle ... \left|a^{(n)}\right\rangle \rightsquigarrow ... \rightsquigarrow \sum_i c_i \left|\psi_i\right\rangle \left|a_i^{(0)}\right\rangle \left|a_i^{(1)}\right\rangle \left|a_i^{(2)}\right\rangle ... \left|a^{(n)}\right\rangle . \tag{14}$$

This represents an enormous entangled superposition of macroscopic measuring devices in a very large Hilbert space $\mathcal{H}_S \otimes \mathcal{H}_{A^{(0)}} \otimes \mathcal{H}_{A^{(1)}} \otimes ... \otimes \mathcal{H}_{A^{(n)}}$.

The difference is now that we have not invoked the projection postulate after $A^{(0)}$ measured the system. The measurement has left the superposition intact. And the statistics remains the same, since according to the Born postulate, the probability of finding the particular value $a_p^{(1)}$ when making a measurement with $A^{(1)}$ is $|c_p|^2$. The key advantage of this is that the concept of measurement does not have a special status in this picture. Measurements in this way are on the same foot with ordinary physical processes and only the Schrödinger evolution has been used.

Still a problem remains: the pointer values of the emasurement devices are now also in superposition. From experience we know that only a single term from this superposition is presented to us. So this leaves the measurement problem mostly intact. The general conclusion from this is that although we have tried to describe the measurement process as purely quantum mechanical interactions ruled by unitary evolution, it still seems there is a special role for the act of measurement: namely that it causes the termination of the chain. So it seems an additional concept is needed to interpret this non-collapse idea. Appropriately, interpretations that approach the problem via this way are called *non-collapse theories*. For example, Everett's 'relative state'- or 'many-world'-interpretation proposes that every term in the superposition of the total state of the universe lives on -relative to the other terms- shut off from each other. Every term represents an equally 'real' physical state.[lxxiii]

6.2 Paths Towards a Solution

Besides the solutions proposed by Von Neumann or Bohr, more modern interpretations exist that also rely on the fundamental status of collapse. Such a collapse theory seeks to modify the Schödinger equation with non-unitary terms as a fundamental master equation of Nature. The most famous is GRW-theory, developted in 1986 by the Italian physicists Ghirardi, Rimini and Weber. They proposed to adjust the Schrödinger equation with a term that produced "spontaneous collapse". They say every quantum system undergoes collapse spontaneously every once in a while, i.e. something like $\frac{10^{15}}{N}$ seconds for a N-particle system.[lxxiv] In any entangled system the collapse of one subsystem will drag the whole system with it. Since everyday objects are entangled systems of a very large number of subsystems. Therefore the system as a whole is practically continuously collapsing and thus never observed in

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[lxxiii] N. David Mermin, "The Ithaca Interpretation of Quantum Mechanics," Pramana **55**: p. 549-65, at Golden Jubilee Workshop on Foundations of Quantum Theory (Bombay: 1998). [33]

[lxxiv] David Albert, "On the Collapse of the Wave-Function" in *Sixty-Two Years of Uncertainty*, ed. Arthur Miller, p.153-165 (New York: Plenum Press, 1990), p. 154.[4]

a superposition.[lxxv]

The measurement problem is now divided in two optional paths to a solution:

1. Collapse Theories: If projection is fundamental, at what stage of the measurement does the projection actually occur and why? Can this be experimentally verified?
2. Non-collapse Theories: If projection is not fundamental, how can the transition from the superposition to the unique state be explained by unitary evolution only?

Thus the divide is between those who deem collapse fundamental and believe the world is non-unitary (1) and those who do not and try to explain 'apparent' collapse in physical terms (2). Solving the former problem has the constraint of experimental verification for it introduces a change in the fundamental laws. Solving the latter problem is to seek a way to explain the collapse of the wave function by some -supposedly complex- physical process and back this up by quantative research. Attemps and proposals to solve the measurement problem have been attempted via both paths.[lxxvi]

If a solution is to be found via way (2) this must come from the theory itself without the projection postulate. In the case of measurements therefore, we are left only with the measurement postulate. A modern theory which seems to go a long way in providing a solution via path (2), that is in the form of a complicated process– is decoherence.

[lxxv] Giancarlo Ghirardi, Alberto Rimini and Tullio Weber, "Unified dynamics for microscopic and macroscopic systems," *Physical Review* 1986: **34**, p. 470-91. [22]

[lxxvi] This is as far as we will go into modern collapse-theories in this research.

Part III

Environment-Induced Decoherence

“A ‘collapse’ in the traditional sense is no longer necessary — in effect, it has already happened. [...] [I]mplications of decoherence for the origin of quantum probabilities and to the role of information processing in the emergence of ‘objective existence’ [as a result of decoherence] significantly reduces and perhaps even eliminates the role of the ‘collapse’ of the state vector.”

—Wojciech Zurek, 1998 [lxxvii]

[lxxvii] Wojciech H. Zurek, “Decoherence, Einselection and the Existential Interpretation (The Rough Guide),” *Philosophical Transactions of the Royal Society* **A356**, p. 1793-821, (London: n.p. 1998), p. 1793.[57]

7 The Hope to Resolve the Measurement Problem

7.1 An Historical Accident

In 1970 Heinz-Dieter Zeh published the paper *On the Interpretation of Measurement in Quantum Theory* in which he expressed critisism of the statistical interpretation of wave functions and -more importanty- gave a hint towards a thermodynamical solution of wave function collapse considering the handedness of a sugar molecule. The interference of left-handedness and right-handedness was argued to decrease fast with time as a result of interactions with the 'environment' in a thermodynamical irreversible way.[lxxviii] The environment was considered as some large amount of degrees of freedom that where not considered in the original model of the system (they can be internal or external). The idea therefore was that we should look at open systems instead of closed ones if we want to understand why objects become classical at larger than nanoscopic scales. This is important in everyday observation, because there is no practical system that can really be stripped of its environment. In classical physics the environment is normally taken as a disturbance or as something we have to leave out of consideration if we want to understand the nature of the system. The idea is that in quantum physics this environment is not just noisy as it is in classical physics, but brings about a significant 'tweaking' (called localization) of the wave function. This idea was fully recognized at the end of the 1970's and has been a field of active research since the 1980's. It turns out that these formerly ignored degrees of freedom have a significant effect on the ordering of phase angles (coherence) in a superposition, hence the term dephasing (or decoherence). This dephasing of interference terms seems to lead to the same consequences as wave function collapse. Collapse 'instantaneously' puts the interference terms to zero as a result of measurement, while decoherence describes the damping of these terms on very tiny time scales. Thus decoherence drew on the idea that a physical process was found to describe collapse, hence resolving the measurement problem via path (2) and finding an explanation why a universal world appears classical to us.

In a vast amount of physics literature -especially in solid state physics- the terms collapse and decoherence are being used interchangeably. For example, Nobel Price laurate Philip Anderson speaks of this explicitly as he states that decoherence is "the process that used to be called 'collapse of the wave function'. The concept is now experimentally verified by beautiful atomic beam techniques quantifying the whole process."[lxxix] Erich Joos calls it an "historical accident" that it wasn't recognised for so long that the effect of environmental interactions leading to decoherence could have important consequences for the measurement problem.[lxxx]

Resolution of the measurement problem would seem to be preferable via a physical route, as opposed to the route to consciousness or introspection taken by Von Neumann, London and Bauer (and Wigner, at first), but also to the route taken by Bohr and Heisenberg who deem the classical world more fundamental. In short, a

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[lxxviii] Heinz-Dieter Zeh, "On the Interpretation of Measurement in Quantum Theory," in *Foundations of Physics* **1**, p. 69–76 (n.p. Springer, 1970), p.72. [53]

[lxxix] Philip Anderson, "Reply to Cartwright", in reply of "Science: 'A Dappled World' or a 'Seamless Web'?" by Nancy Cartwright in *Studies in History and Philosophy of Modern Physics,* **32**, p. 499-500.[5]

[lxxx] Erich Joos, *Decoherence: Theoretical, Experimental and Conceptual Problems*, Lecture Notes in Physics **538**, P. Blanchard, D. Giulini, J. Kupsch, I.O. Stamatescu (eds.), (New York: Springer, 2000), p. 13.[27]

route less orthodox. Decoherence seems to offer hope of such routes. It seems to indicate that classical concepts arise from the formalism through interaction with the environment. Here we will use the term ‘environment’ for the degrees of freedom outside of the system and the apparatus, i.e. *external* as opposed to internal. As an opening remark: decoherence comes from quantum mechanics itself, so without philosophical (or ‘interpretational’) input it is not an interpretation of the theory. Thus decoherence alone can’t contradict Von Neumann or Bohr. What it can do though, is strengthen several interpretations over others and shed light on the path to the solution.

7.2 Ensembles and the Density Operator

Decoherence is a phenomenon arising from statistical physics applied on quantum mechanical wave functions. As thermodynamical properties such as temperature have not been defined in the fundamental quantum theory we need to consider *ensembles* of quantum systems. To handle them effectively we will also introduce the density matrix.

An ensemble is a collection of physical systems. For example, one electron in an electron/Fermi gas -a model for a metal- is such a system. If all electrons are characterized by the same spin state, as a result of a strong external magnetic field[lxxxi] in the ‘up’-direction, say spin up $|\uparrow\rangle$, we call the ensemble *pure*. In the absence of such a field we expect the spin states to be randomly distributed over the states $|s_i\rangle$ with i indicating the possible spin directions, i.e. there is no preferred direction of spin. In such a system we have a statistical distribution of spin states where each particular state $|s_p\rangle$ has equal weights, thus we have a *random ensemble*. Of course, by varying the strength of the external field one can control the population of these states as to put 40% of the spins in the state $|\uparrow\rangle$ and 60% in $|\downarrow\rangle$. Such an ensemble is referred to as *mixed*,[lxxxii] with weights w_i and $\sum_i w_i = 1$. Note that with the concept of mixed ensembles we have introduced a classical sense of probability through the coefficients w_i, which are not to be confused with the Born coefficients $|c_i|^2$. The difference lies in the fact that quantum superpositions are new physical states through interference. All these quantum states actually exist, whereas in classical distributions only one of the terms is real. Classical distributions are a pragmatic consequence of our ignorance. So a mixed ensemble is a mixture of pure ensembles. Recall that the quantum mechanical whole is more than the sum of its parts due to superposed states, whereas we see here that the classical whole is exactly the sum of its parts.

Now we consider the expectation value of a physical operator A with eigenstates $|a_i\rangle$ corresponding to the physical quantity $\mathcal{A}$ when we make a measurement on a mixed ensemble. Now the spin states $|s_i\rangle$ are replaced by general states specifying a single quantum entity: $|\Psi_i\rangle$. This is the classical average of finding the quantum mechanical expectation:

[lxxxi] Consider the electons non-interacting and very tightly bound to their corresponding ions in the metal.

[lxxxii] In many papers a mixed ensemble is often referred to as a mixed state. This is already confusing in keeping apart the quantum and the classical. Here we will use ensemble for a statistical system of a collection of particles and state for the state vector that describes an individual of the collection.

$$
\begin{aligned}
\langle A\rangle &= \sum_i w_i \langle\psi^{(i)}|A|\psi^{(i)}\rangle \\
&= \sum_i w_i \sum_{j,k} \langle\psi^{(i)}|a_j\rangle \langle a_j|A|a_k\rangle \langle a_k|\psi^{(i)}\rangle \\
&= \sum_i w_i \sum_{j,k} a_k \langle\psi^{(i)}|a_j\rangle \,\delta_{jk}\, \langle\psi^{(i)}|a_k\rangle^\dagger \\
&= \sum_i w_i \sum_j a_j |\langle\psi^{(i)}|a_j\rangle|^2,
\end{aligned} \tag{15}
$$

projecting the spin states at the eigenstates $|a_i\rangle$ of A by inserting the identity operator twice and using the orthogonality of the eigenstates. This result expresses the probability of finding the system in an eigenstate of A and *also* the probability of finding this particular eigenstate in the population fraction w_i. Now we write the same result in general (complete orthonormal) bases $|\mu_i\rangle$ and $|\nu_i\rangle$:

$$
\begin{aligned}
\langle A\rangle =&\sum_i w_i \sum_{j,k} \langle\psi^{(i)}|\nu_j\rangle \langle\nu_j|A|\mu_k\rangle \langle\mu_k|\psi^{(i)}\rangle \\
=&\sum_{j,k} \langle\mu_k| \sum_i w_i \,|\psi^{(i)}\rangle \langle\psi^{(i)}|\, |\nu_j\rangle \langle\nu_j|A|\mu_k\rangle \\
=&\sum_{j,k} \langle\mu_k|\, \hat\rho \,|\nu_j\rangle \langle\nu_j|A|\mu_k\rangle \\
=&Tr[\hat\rho A],
\end{aligned} \tag{16}
$$

for $\hat\rho = \sum_i w_i |\psi_i\rangle\langle\psi_i|$, the density operator. It is the weighted sum over the projection operators $P_i[\psi_i] = |\psi_i\rangle\langle p|$, which are just dyadic products of the eigenstates, thus $\hat\rho = \sum_i w_i P_i[\psi_i]$. The density operator gives us all the physical information about the system. We can contruct the matrix representation, i.e. the density matrix, for a basis that diagonalizes it. For a pure ensemble the density matrix is clearly idempotent: $\hat\rho^2 = \hat\rho \to \hat\rho(\hat\rho - 1) = 0$. Thus for a pure ensemble it will be zero everywhere except for one eigenvalue being 1. For a completely random ensemble it will be the identity matrix normalized by the number of populations. For a mixed ensemble the diagonal will be mixed. The density matrix is of course not diagonal when we change bases. *The off-diagonal terms in that case denote fractions of the wave function being in superpositions of the eigenstates of that basis.*

These off-diagonal terms are claimed to vanish due to decoherence. This happens over time. How does the density matrix behave in time? As an answer, we know that the weights w_i do not change over time when we leave them alone. The only time evolution is due to the quantum states themselves, which obey Schrödinger's Eq (5):

$$
\begin{aligned}
i\hbar\frac{d}{dt}\hat{\rho} &= \sum_i w_i\{\hat{H}\left|\psi_i(t)\right\rangle\left\langle\psi_i(t)\right| - \left|\psi_i(t)\right\rangle\left\langle\psi_i(t)\right|\hat{H}\} \\
&= -[\hat{\rho},\hat{H}], \qquad (17)
\end{aligned}
$$

which is the quantum Liouville equation. This is the Schrödinger's equation for mixtures, where the system is characterized by $\hat{\rho}$ instead of Ψ.

7.3 Environment-Induced Decoherence as a Solution to the Measurement Problem

Generally, all our systems are interacting with the surrounding environment. It is only in very special cases, that we can look at individual quantum systems. To be free of the environment, these are typically systems on the scale of atoms. Making a measurement always involves a measuring device generally in contact with the environment $\mathcal{E}$ (eigenstates $\{|\epsilon_i\rangle\}$). We devide the world into three Hilbert spaces, that of the system S of a pure state (generally in superposition), of the apparatus A with pointer states a_i, and of the environment: $\mathcal{H}_\mathcal{S}\otimes\mathcal{H}_\mathcal{A}\otimes\mathcal{H}_\mathcal{E}$. The Von Neumann measurement scheme (dropping direct products) for an environmentally perturbed system becomes:[lxxxiii]

$$
\sum_i c_i\left|\psi_i\right\rangle\left|a_0\right\rangle\left|\epsilon_0\right\rangle \xrightarrow{H_1} \sum_i c_i\left|\psi_i\right\rangle\left|a_i\right\rangle\left|\epsilon_0\right\rangle \xrightarrow{H_2} \sum_i c_i\left|\psi_i\right\rangle\left|a_i\right\rangle\left|\epsilon_i\right\rangle, \qquad (18)
$$

where H_1 and H_2 indicate (this time explicitely) different interaction Hamiltonians that govern the entanglement. The density matrix of the total system is

$$
\hat{\rho}_{\mathcal{SAE}} = \sum_{jk} c_j c_k^\dagger\left|\psi_j\right\rangle\left|a_j\right\rangle\left|\epsilon_j\right\rangle\left\langle\psi_k\right|\left\langle a_k\right|\left\langle\epsilon_k\right|. \qquad (19)
$$

Now we introduce the concept of the reduced density matrix:[lxxxiv]

$$
\hat{\rho}_{\mathcal{SA}} = Tr_\mathcal{E}[\hat{\rho}_{\mathcal{SAE}}] = \sum_{jk} c_j c_k^\dagger\left|\psi_j\right\rangle\left|a_j\right\rangle\left\langle\psi_k\right|\left\langle a_k\right|\left\langle\epsilon_j|\epsilon_k\right\rangle. \qquad (20)
$$

The reduced density matrix -as can be seen from the formula- is 'tracing away' (equivalently: 'integrating out') degrees of freedom of the environment $\mathcal{E}$. This is because we are entirely uninterested in $\mathcal{E}$ and we are in almost every real situation completely ignorant of it. What we are interested in is the pointer status of the apparatus. We assume this pointer state is only sensitive to the system, thus $A = A_{\mathcal{SA}}\otimes\mathbb{1}_\mathcal{E}$. When we accept the measurement postulate and use the reduced density matrix instead of the full density matrix in finding the expectation value of such a quantity through (14), we obtain the same statistics as we would have done using the full $\hat{\rho}_{\mathcal{SAE}}$:

[lxxxiii] We are still regarding ideal measurements, i.e. the system is not essentially disturbed or destroyed by interaction with the environment.

[lxxxiv] Schlosshauer, "Decoherence and interpretations," 1275.

$$\langle A \rangle = Tr[\hat{\rho}_{\mathcal{SAE}} A] = Tr_{\mathcal{E}}[\hat{\rho}_{\mathcal{SA}} A_{\mathcal{SA}}]. \tag{21}$$

This is the justification of using $\hat{\rho}_{\mathcal{SA}}$ instead of $\hat{\rho}_{\mathcal{SAE}}$, as a calculational tool. For calculating probabilities this is as Bell said a good solution "For All Practical Purposes" (FAPP).[lxxxv] In interpreting the reduced density matrix -as we will see- we have to be very careful, since the density matrix of the entire system still displays coherence.

To this point the interference that is present in the system is also still present in the entanglement: $|\psi_j\rangle |a_j\rangle \langle\psi_k| \langle a_k|$, for $j \neq k$. The crucial claim that proponents of the decoherence solution make is that the environmental basis vectors $|\epsilon_i\rangle$ are approximately orthogonal or very rapidly become so.[lxxxvi] Thus,

$$\langle\epsilon_i|\epsilon_j\rangle \approx \delta_{ij}, \tag{22}$$

leading us to a completely 'decohered' system:

$$\hat{\rho}_{\mathcal{SA}} = Tr_{\mathcal{E}}[\hat{\rho_{\mathcal{SAE}}}] = \sum_j |c_j|^2 \Big(|\psi_j\rangle \langle\psi_j| \otimes |a_j\rangle \langle a_j| \Big). \tag{23}$$

The claim of Eq (22) has to be shown by the interaction Hamiltonian of course, but it can be

Eq (23) shows us that interaction with the environment has lead the initial pure state exhibiting interference (coherent state) to a reduced mixture (decoherent). The final state does not display interference and can therefore be interpreted as a *classical mixture*. Hence the title of the book by Joos & Zeh: *Decoherence and the Appearance of a Classical World in Quantum Theory.*[lxxxvii] The idea is that the physics behind the projection postulate has been explained. On the level of the observer the environment has already bombarded the system and the apparatus with random phase correlations and has long since carried away the coherence and distributed it over the large amount of degrees of freedom. Thus the observer only experiences a classical world. This explains why we do not see objects at more than one place at the same time, that means there is 'apparent collapse'.

Also it seems that we have not used any notion of *fundamentally* non-unitary behaviour and therefore reinstalled time-symmetry into the theory as discussed in section 4.1. This will be discussed further in chapter 7. Global phase coherence however, is not lost. Only locally we find decoherence, but globally the interference is carried of through the environment. In practice on the other hand it is usually very hard to disentangle a system from its environment after interaction has taken place. It is here we find the source for the assymmetry of time experienced when making

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[lxxxv] Bell, "Against Measurement," 34.

[lxxxvi] Schlosshauer, "Decoherence and interpretations," 1277.
How rapid is rapidly? "Many orders of magnitude larger than the thermal relaxation time", therefore not resolvable by far in any experimental set-up.

[lxxxvii] Joos et al., *Appearance.*

measurements, but now we have found it in a thermodynamic irreversibility, instead of an obscure quantum irreversibility. The difference is that the thermodynamic irreversibility is interpreted as a consequence of our ignorance of the many degrees of freedom, while with projection we are dealing with a law-like irreversibility that is fundamental to Nature.

7.4 Decoherence and the Double-Slit experiment

Consider again the double-slit experiment. This is an experiment that lead us to the idea collapse occurs during a measurement. The photon is described by $|\Psi_e\rangle = \frac{1}{\sqrt{2}}\big(\,|\psi_1\rangle + |\psi_2\rangle\,\big)$. Originally one would want to know through which slit the electron goes. For this we use a light source at point A. The photon field (uncoupled to the electrons) will be described by $|\xi_0\rangle$ and can be seen as the environment $\mathcal{E}$ of the system. See Figure **10**.

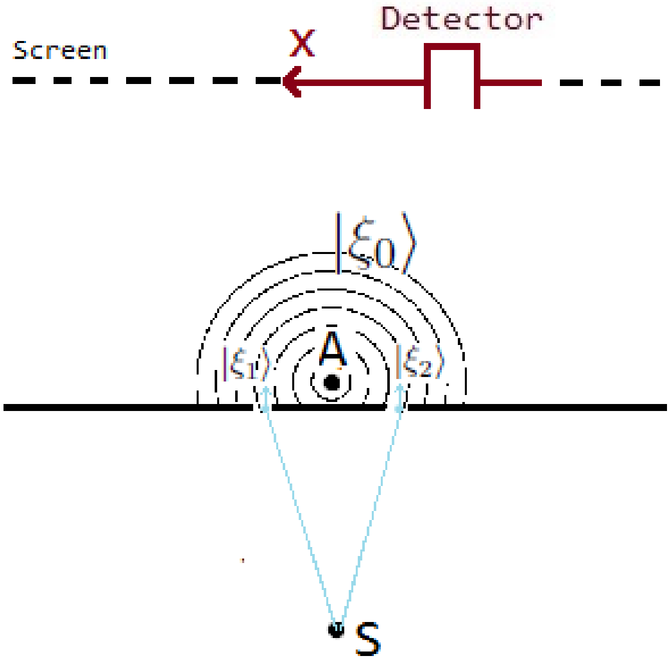

Figure 10: The double-slit experiment with a light source at point A. This light source creates a photon field $|\xi_0\rangle$ that couples to the electrons going through slit $|\psi_1\rangle$ and slit two $|\psi_2\rangle$ as $|\xi_1\rangle$ and $|\xi_2\rangle$, respectively. According to Eq. (25) it is now the term $\langle\xi_2|\xi_1\rangle$ that determines if an interference pattern is observed or not.

We know from the laws of optics that the photon field is sensitive to electrons. Through entanglement the photon field now couples to the electronsystem:

$$|\Psi_e\rangle = \frac{1}{\sqrt{2}}\big(\,|\psi_1\rangle + |\psi_2\rangle\,\big) \otimes |\xi_0\rangle \rightsquigarrow \frac{1}{\sqrt{2}}\big(\,|\psi_1\rangle \otimes |\xi_1\rangle + |\psi_2\rangle \otimes |\xi_2\rangle\,\big). \tag{24}$$

For the expectation value of the 'which path'-information

$$\langle\hat{X}\rangle = \frac{1}{2}\{\langle\psi_1|\hat{X}|\psi_1\rangle + \langle\psi_2|\hat{X}|\psi_2\rangle + \langle\psi_1|\hat{X}|\psi_2\rangle\,\langle\xi_1|\xi_2\rangle + \langle\psi_2|\hat{X}|\psi_1\rangle\,\langle\xi_2|\xi_1\rangle\}. \tag{25}$$

Now the term $\langle\xi_1|\xi_2\rangle$ determines the interference between $|\psi_1\rangle$ and $|\psi_2\rangle$. By Eq (22) we already expect the environmental states to be orthogonal. In principal, this has to be determined by calculating the how $|\{Psi_e\rangle \otimes |\xi_0\rangle$ evolves according to the quantum Liouville equation. For that we need the interaction Hamiltonian between photon and electron and this can of course be done. But we already know the behaviour of $\langle\xi_1|\xi_2\rangle$, because they are clearly distinguishable from each other. We know this from the laws of optics. And since they are distinguishable they are automatically orthogonal. Therefore the last two terms in Eq (25) vanish, the coherence is lost. In a certain sense *the light is continuously 'measuring' the electrons.* When the interference terms vanish we obtain the same result as we did by invoking the projection postulate. Only this time we have only used Schrödinger's equation.

The interaction between electron and photon certainly depends on the wavelength of the light used. When increasing the wavelength of the light as in section 4.2 the distinquishability of $|\xi_1\rangle$ and $|\xi_2\rangle$ decreases as the signals tend to overlap. This effects their orthogonality. A continuous increase in the wavelength continuously inreases the inner product $\langle\xi_1|\xi_2\rangle$ from 0 to 1, i.e. from the situation that the flash clearly belongs to one path to the situation that the flash is too large to assign one of the two paths to.

8 Why Decoherence does not solve the Measurement Problem

8.1 Non-unitary elements in Partial Tracing

We have seen what decoherence is: a dynamical process resulting from the entanglement of the system $\mathcal{S}$ with the environment $\mathcal{E}$. Practically irreversibly the "environmentally traced over" density matrix transforms a pure state to a reduced mixed state, giving at least the appearance of wave function collapse.

In interpreting this however, there is a catch when considering the ensemble of states and the single pure state. Where collapse of the single pure state was denoted by Eq (7) as $|\Psi\rangle = \sum_i c_i |\psi_i\rangle \rightsquigarrow |\psi_k\rangle$ this is not exactly what is described by decoherence. What decoherence describes is rather a statistical version of projection:[lxxxviii]. For a pure state:

$$\begin{aligned}\hat{\rho}[\Psi] &= P[\sum_k c_k\psi_k] \\ &= \sum_{jk} c_k c_j^\dagger |\psi_k\rangle\langle\psi_j| \\ &= \sum_j |c_j|^2 |\psi_j\rangle\langle\psi_j| + \sum_{j\neq k} c_k c_j^\dagger |\psi_k\rangle\langle\psi_j| \\ &\xrightarrow{Projection} \sum_j |c_j|^2 P[\psi_j], \end{aligned} \tag{26}$$

[lxxxviii] Pessoa, "Decoherence," 326.

where in the last step the interference terms are surpressed by invoking a *statistical* projection postulate. This is the process that decoherence describes, since the right hand side is the same as in Eq (23). Thus the same result is obtained by either using this statistical version of projection or by taking the partial trace over the environment. That these two mathematical operations lead to the same outcome implies that the physical process behind this might just be exactly the same thing.

We see that even though the state Ψ of the total system $\mathcal{S} + A + \mathcal{E}$ was pure this does not guarantee that the state of the subsystem $\mathcal{S} + A$ is pure. Indeed in general it isn't pure, but an *improper* mixture. This transition can't be explained by mere unitary evolution. Information is lost by tracing out the environmental degrees of freedom. The time evolution of the subsystem is not unitary. therefore, the measurement problem for individual systems is still real.

The point is that if Eq (8) is really fundamental and it happens to each single system in the ensemble it does indeed explain the result found by decoherence given by Eq (23) or (26) as a statistical generalization, but the converse is not true. If Eq (26) is fundamental is has no explanatory power over the collapse of individual quantum systems. In the words of Osvaldo Pessoa: "*an explanation for collapse implies an explanation for decoherence, but an explanation for decoherence doesn't imply an explanation for collapse.*"[lxxxix] Decoherence does not solve the measurement problem for it does not provide a mechanism for individual collapse.

8.2 New Orthodoxy

The discussion if decoherence solves the measurement problem is somewhat overdue. In 2003 Omnes remarks: "Many of its consequences have been obtained theoretically, but its foundation, the range of its validity, and its full meaning are still rather obscure. This is due most probably to the fact that it deals with deep aspects of physics, not yet fully investigated."[xc] But at the same time Zeh writes "Environment-induced decoherence by itself does not yet solve the measurement problem, since the pointer states $[|a_i\rangle]$ may be assumed to *include* the total environment (the 'rest of the world'). Identifying the thus arising global superposition with an *emsemble* of states [...] would beg the question. This argument is nonetheless found wide-spread in the literature.[xci] Adler adds: "these striking comments to the contrary, I do not believe that either detailed theoretical calculations or recent experimental results show that decoherence has resolved the difficulties associated with quantum measurement theory."[xcii] These remarks show that the hope to resolve the measurement problem through the decoherence program has been on the wane since the beginning of the past decade. The knowledge of decoherence however has made tremendous progress and it is a very important feature when looking at interpretational problems. On the other hand it seems to need input from somewhere else if it is really

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[lxxxix] Ibid., 325.

[xc] Roland Omnès, "Decoherence, irreversibility, and selection by decoherence of exclusive quantum states with definite probabilities," *Physical Review* A, 2003: **65**, p. 1. [40]

[xci] Zeh in Joos et al., *Appearance of a Classical World*, 21.

[xcii] Stephen Adler, Why decoherence has not solved the measurement problem: a response to P.W. Anderson, *Studies in History and Philosophy of Modern Physics* **34**, p. 135-142, (Elsevier Science, 2002), p. 136.[1]

to represent an answer.

We have seen that decoherence is a physical phenomenon from quantum physics itself. It can therefore not serve as an interpretation of the theory. Although it gives us a justification in using the projection postulate after a 'measurement', it does not solve the measurement problem, since it does not explain how individual collapse really happens. As discussed, this is because the environmental tracing is just the statistical version of the projection operator. What decoherence can do though, is justify why the world appears classical to us, since it tells us why macroscopic interference dies out extremely fast. Furthermore, although it does not explain the fundamental problem it can highlight certain paths towards the solution, it strengthens some interpretations and weakens others. Examples of interpretations that are weakened would be the Copenhagen interpretation, since the decoherence program presupposes that quantum mechanical instead of classical properties are fundamental, and the consciousness interpretation, because decoherence implies collapse has occured long before the measurement interaction reaches our eyes, brains and awareness. Interpretations that are strengthened by the decoherence program contain the many-worlds interpretation, the modal interpretation and the consistent histories interpretation. In other words the non-collapse theories are favoured by decoherence or at least they are made more plausible. Also, as experimental techniques extent their reach some fundamental features of quantum measurements get more testable. Examples are the Aharonov-Bohm effect, the Quantum Zeno effect and Quantum Non-Demolition Measurements. As these techniques improve theoretical speculations get more limited, thus also bringing interpretations closer together. The measurement problem is still alive, thus the question about the fundamental status of the projection postulate is not adressed and remains open even today.

Jeffrey Bub points out that nevertheless the majority of physicists in practice adher to a kind of *new orthodoxy.*[xciii] This would consist of some blend of the 'consistent histories' approach and the many-worlds interpretation,[xciv] but relies heavily on environmental induced decoherence. Omnès calles it '*the* interpretation of quantum mechanics'.[xcv] It is a combination of elements from interpretations of which most of them are just those elements that decoherence strengthens believe in.

Wheeler and Tegmark also express similar observations: "An information poll taken in July 1999 at a conference on quantum computation at the Isaac Newton institute in Cambridge, England, suggests that the prevailing viewpoint is shifting. Out of 90 physicists polled, only eight declared that their view involved explicit wave function collapse.' " [xcvi]

It shows that although decoherence has not solved the measurement problem, it has brought a significant change in the thinking of physicists in the field. The decoherence debate has settled only in the previous decade. It is clearly an important step forward and should be passed on to the new generations. An important question

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[xciii] Bub, *Interpreting*, 212.

[xciv] An explanation of these interpretations falls outside the scope of this thesis.

[xcv] Roland Omn'es, *The Interpretation of Quantum Mechanics* (Princeton: Princeton University Press, 1994).[my italics][39]

[xcvi] Tegmark and Wheeler, "Quantum Mysteries," 78.

to be asked when dealing with such a shifting view of the interpretation of our most succesful theory is: ‘has this new orthodoxy seeped through to the institutions and universities where quantum mechanics is taught to fresh minds?’

Part IV

The Collapse Debate and Education

"[I]t is time to update the quantum textbooks: although these books, in an early chapter, infallibly list explicit nonunitary collapse as a fundamental postulate, [...] today many physicists —at least in the burgeoning field of quantum computation— do not take this seriously. The notion of collapse will undoubtedly retain great utility as a calculational recipe, but an added caveat clarifying that it is probably not a fundamental process violating the Schrödinger equation could save astute students many hours of confusion."

—Tegmark and Wheeler, 2001 [xcvii]

[xcvii] Tegmark and Wheeler, "Quantum Mysteries," 79.

9 Why I think Education on the Measurement Problem is Important

In the introduction I put a remark about my teacher dismissing questions on the foundations of quantum mechanics to metaphysics. At that point, I felt disappointed, so I took some metaphysics courses and -among many other things- I did learn about the many different interpretations of quantum mechanics. Now maybe my teacher had a point, the fundamentals of quantum mechanics (and of physics generally) tend to fall outside of the scope of physics. This is because there is a great number of different philosophical viewpoints that would indeed deserve a course of their own. In my first textbook David Griffiths wrote: "*I do not believe one can intelligently discuss what quantum mechanics means, until one has a firm sense of what quantum mechanics does*"[xcviii]

This put me at ease for some time. Now an important question arises: To what extent should a textbook on quantum mechanics treat the discussions on what quantum mechanics *means*? I want to investigate this question by looking at differences in how the measurement problem is dealt with in textbooks today. Of particular importance in this is the concept of decoherence and the shift in view claimed to exist. Zeh expressed that quantum textbooks are even irrational in their treatment of the collapse process, as "'the pragmatic irrationalism' that is common is quantum textbooks (complementarity, dualism, uncertainty etc.)"[xcix] When the physicists in the field adher to a new orthodoxy and the students learn quantum mechanics from dated textbooks (and at the same time from these physicists), an *update* is indispensible. The new paradigm Zurek spoke of can not be inquired upon if students do not have a firm knowledge of what the problem is and what the alternatives are. Interpreting the quantum world proves to be a field of widespread controversy. At the foundations of this controversy really lies the measurement problem. It is important to not just teach the (orthodox) theory -including the projection postulate- as if it is as uncontroversial as Maxwell's laws. It is not. And students ought not to be confused by this if this confusion can be prevented.

We can't push this idea to far though. The first and foremost goal of a textbook is to teach the student the necessary skills to apply in later research. In addition, there isn't really time in the curriculum to give much attention to interpretational problems. That is why it might be advised to postpone most questions to later on in the career. What should not happen though, is that the student has no idea about what these conceptual problems are and how they arise. The measurement problem is not a 'special' topic on the same foot with say Berry phases or the Born approximation. It is rather a fundamental topic needed for clarity, not profundity. Besides being interested in calculational challenges the student also wants to know about the causes of interpretational challenges. Since interpretation challenges arise from the measurement problem, it should be discussed in every quantum textbook. I don't wish to imply this is something that absolutely does not happen in textbooks. The textbooks that are being used today do discuss the status of measurement -at

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[xcviii] Griffiths, *Introduction to QM*, p. vii.

[xcix] Heinz-Dieter Zeh, "Roots and Fruits of Decoherence," Poincaré seminar 2005: Quantum Decoherence p. 151-75 (Basel: Birkhäuser Verlag, 2006), p. 174.[54]

least briefly- as will be shown in the next section. The problem might lie in the update of quantum textbooks to describe the new orthodoxy, thus also introducing this update to students, as will be discussed in the section after that.

There is another development that we can compare the rise of decoherence to: *the Bell inequalities.* It was typical for early textbooks to adhere to the mentality of ignoring interpretational questions, because it was dubbed metaphysical, therefore unscientific. Thus is was deemed useless to ask questions towards or example hidden variable theories. The Bell-inequalities changed this by making these questions experimentally verifiable. And it has been observed that textbooks have taken in sections about topics such as the EPR-paradox, the no-clone theorem and Schrödingers cat. Thus interpretational problems were re-introduced to students. Something similar has seems to have happened with the decoherence program: interpretational problems have been brought closer to us via physical terms, i.e. decoherence can be calculated, as do problems using the Bell inequalities. This has had the effect we have discussed. A similar change in textbooks is indeed to be expected.

10 Textbooks

The search is now for textbooks that satisfy this demand of discussing the measurement problem in a modern light. How has the decoherence debate influenced the books? It is often implied in the literature that 'decoherence' and 'collapse' are synonymous and -although this is not true- this could have as an effect that a textbook might not mention explicit wave function collapse. Postulating collapse might be on the same foot with making an argued case of loss of coherence through the macroscopic sizes of the system.

In investigating this an immediate problem pops up. There is a difference between the most modern textbooks and the textbooks that are being used in the universities today. Most of the books that are in use in quantum mechanics courses date back to the late eighties and early ninetees. Judging by the years of release these textbooks will not contain anything about the projection/decoherence debate, since this debate only cemented around the years 2003-2005. To get an idea from a small (and slightly arbitrary) sample, textbooks that are being used in courses right now include: at Cambridge University: *Introduction to Quantum Mechanics*, by B. H. Bransden and C.J. Joachain, 1989. At the Massachusetts Institute of Technology (MIT) and Utrecht University it is *Introduction to Quantum Mechanics*, by D. J. Griffiths, 1995. At Yale University it is *Principles of Quantum Mechanics*, by R. Shankar, 1980. Also the book *Modern Quantum Mechanics* by J.J. Sakurai and J. Napolitano is often recommended as a supplementary textbook. These books are widely used in the world over many other universities as well. We will discuss their approach to teaching the measurement problem.

10.1 Popular Today

The mentioned books are very similar in their approach to the education of the measurement problem. They either evade it altogether or postpone it to a brief discussion at the end of the book. If the theory is discussed conceptually most attention is given to the indeterministic character of the theory, i.e. the Born postulate and not to how the act of measurement affects the system. The projection postulate/collapse of the wave function is mentioned quickly afterwards as a well-known fact.

As in Bransden the only direct argument given is: "In such a case [the ideal measurement case], it is reasonable to expect that if a particular result a_n is obtained in the first measurement, the same result will be obtained if the measurement is repeated immediately. Since the result of the second measurement can be predicted with certainty, we deduce that after the first measurement the state of the system is described by the eigenfunction [...]. Hence, in the case the act of measurement has a 'filtering' effect so that whatever the state of the system before the measurement, it is certainly in an eigenstate of the measured quantity immediately afterwards."[c] Thus projection is given as a fundamental face of the theory. Harmfully, this is mentioned in the chapter where the postulates are discussed and this projection is not mentioned as a separate postulate. Griffiths devotes more pages to the interpretational challenges of the theory. Already at the second page he starts with a section on the statistical interpretation the hidden variable hypothesis, the Copenhagen interpretation and an agnostic approach. He remarks about the Copenhagen interpretation that "[a]mong physicists it has always been the most widely accepted position. Note however, that if it is correct there is something very peculiar about the act of measurement – something that over half a century of debate has done precious little to illuminate."[ci] But this is when discussing the indeterminacy of the theory; the measurement postulate. According to Griffiths, these positions somehow have nothing to do with the collapse of the wave function. But without (apparent) collapse, how does any interpretation explain a 'classical appearance' of the world? Griffiths devotes 15 lines in total to collapse, among which: "the first measurement radically alters the wave function, so that it is now sharply peaked around C."[cii] He does give arguments to explain this: 'On this question everyone is in agreement" and "it would be tough to prove that the particle was really found at C in the first instance, if this could not be confirmed by immediate repetition of the measurement."[ciii] Both Bransden and Griffiths devote the last pages of the books to interpretational related issues such as Bell's inqualities, the no-clone theorem, and Bransden even mentions the measurement problem. This is a perfect solution in my eyes. Waiting for the student to be in a position to appreciate the argument. What is bypassed by this trick however, is that it anchors on the discussions in the early chapters, i.e. it assumes collapse. Thereby the arguments for collapse are not given when discussing the real problems, or only partially (even vaguely) given in the early chapters.

Sakurai solves the intuitive problem by example. The Stern-Gerlach experiment (of ideal measurements) is used throughout the book as the analogy for all interpretational problems. When discussing measurements in general there is an honest remark: "This is not a particularly easy subject for beginners, so we first turn to

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[c] B. H. Branden and C. J. Joachain, *Introduction to Quantum Mechanics* (Harlow: Longman, 1989), p. 196.

[ci] Griffiths, *Introduction to QM*, 4.

[cii] Ibid., 5.

[ciii] Ibid., 4.

the words of the great master, P. A. M. Dirac, for guidance: ‘A measurement always causes the system to jump into an eigenstate of the dynamical variable that is being measured.’”[civ] This is just arguing by intimidation. Of course every student (or every person) should be inclined to take Dirac extremely seriously, but he or she does need a justification for this statement. The implicit promise made by the word ‘first’ is never fully realized.

Shankar mentions the projection postulate for the first time when discussing the postulates. Interestingly enough, he combines the projection postulate and the Born postulate as if it is one: “If the particle is in a state $|\Psi\rangle$, [ideal] measurement of the variable (corresponding to) Ω will yield one of the eigenvalues ω with probability $|\langle\omega\,|\,\Psi\rangle|^2$. The state if the system will change from $|\Psi\rangle$ to $|\omega\rangle$ as a result of measurement.”[cv] Apart from that peculiarity the motivation for projection after that is extensively discussed and exemplified with discussions around Compton scattering not unsimilar to Von Neumann’s argument or Feynman’s discussion of the double-slit problem. These really aid the student’s understanding. In any case projection is fundamental.

As a reminder, the textbooks just discussed are some of the most widely used in the world at the present moment. All these text books at some point do mention the EPR paradox and the Bell-inequalities, especially Bransden and Griffiths. There are no words on non-collapse theories or decoherence. What the books have in common is that they belong to old orthodoxy. They take the projection postulate as an obscure, but fundamental fact about Nature. This obscurity means that it cannot be adequately explained what a measurement is and why it leads to wave function collapse. The measurement problem remains unanswered in the new orthodoxy as well, but now we are armed with the process of decoherence. As discussed, decoherence provides a justification for apparent collapse. This justification leads us to more modern interpretations. We should teach accordingly.

10.2 More Modern Textbooks

New textbooks on quantum mechanics appear every year. The expectation is that conceptual decoherence, -just as Bell’s inequalities- could have found its way into modern textbooks. The debate has only settled down in the beginning of the past decade, so we should look at books from this period to now. In these books we might expect ‘new orthodoxy’ ideas, i.e. strengthened believe towards non-collapse theories and elements of decoherence.

At Oxford University quantum physics is taught by James Binney from the book *The Physics of Quantum Mechanics* written by him and David Skinner. It was first published in 2008 and is being revised every year the following three years. Here we find many modern elements. In the first chapters of the book collapse of the wave function is introduced as something that seems to happen, but that we have to interpret it: “What happens when the “wavefunction collapses”? It is tempt-

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[civ] Sakurai, *Modern QM*, 21.

[cv] Ramamurti Shankar, *Principles of Quantum Mechanics* 2^{nd} ed. (New York: Plenum Press, 1980), p. 116.[49]

ing to suppose that this event is not a physical one but merely an updating of our knowledge of the system: that the system was already in the state before the measurement, but we only became aware of this fact when the measurement was made. It turns out that this interpretation is untenable, and that wavefunction collapse is associated with a real physical disturbance of the system."[cvi] But it comes with an disclaimer: "It's time to examine this collapse hypothesis critically."[cvii] It is followed by an attack on orthodox ideas "So this Copenhagen interpretation of quantum mechanics implies that every measurement leads to a momentary suspension of the equations of motion, so the system can be steered, by forces unspecified, into a randomly chosen state! This is not serious physics." [cviii] There is a chapter on the density matrix, the reduced density matrix and environmental decoherence. They combine the conceptual ideas with practical ones like quantum computing, but most importantly it is argued that closed quantum systems are limited: "In practice a system under study will sooner or later become entangled with its environment, and once it has, we will be obliged to treat the system as one for which we lack complete information. That is, we will have to predict the results of measurements with a non-trivial density operator. The transition of systems in this way from pure states to impure [mixed] ones is called quantum decoherence."[cix] And to crown it all, there is a hint to future research and a final attack on orthodox collapse: "Unfortunately, we probably need an extension to quantum mechanics to take this step, because a conventional quantummechanical theory of the measuring instrument will require us at some point to 'observe' the instrument using the collapse hypothesis, from which we are trying to escape: *quantum mechanics is a theoretical arena from which the only exit to the real world is through the turnstile of the collapse hypothesis.*"[cx] Obviously, Binney and Skinnes's book is one belonging to the new orthodoxy. This might indicate something is indeed changing in quantum education.

For this we will need more examples. And there are many. For example there is the book by Jochen Pade: *Quantum Mechanics for Pedestrians* (two volumes) used at Oldenburg University in Pade's own class, published this very year. It contains three whole chapters that deal with interpretational question -albeit in the second volume- namely one about the EPR-paradox and Bell-inequalities and one about decoherence. This coincides with the hypothesis that decoherence will bring about a similar change in view regarding these issues. Pade is very similar in his remarks as Binney is. There is an attack at the old orthodoxy: "For decades, the attitude of the 'old' Copenhagen school was authoritative; it claimed that the physical analysis of the measurement process in quantum-mechanical terms would be a pointless undertaking [...]. However, many people found it quite unsatisfactory to 'split' the world into a quantum realm dominated by the SEq [Schrödinger Equation] and a separate realm of classical instruments. Where and by which criteria should one draw the line?"[cxi] Zeh's 1970 article is mentioned as a first step towards modern ideas. It is followed by the characteristic new orthodox phrasing: "the theory of decoherence is considered an important element that can contribute to the explanation of the

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[cvi] James Binney and David Skinner,*The Physics of Quantum Mechanics* (Malvern: Cappella Archive 2008), p. 15.[11]

[cvii] Binney, *Physics of QM*, 132.

[cviii] Ibid., 133.

[cix] Ibid., 126.

[cx] Ibid., 135. [my italics]

[cxi] Jochen Pade, *Quantum Mechanics for Pedestrians* (Heidelberg: Springer, 2014), p. 161. [42]

measurement problem."[cxii]

Similarly, in a (more graduate) textbook *Spectral Theory and Quantum Mechanics* by Valter Moretti we find a chapter "The phenomenon of decoherence as a manifestation of the macroscopic world."[cxiii] In which we find the familiar approach of environmental tracing and the remarks such as: "it could elucidate why certain macroscopic objects behave classically; perhaps it could explain, alternatively, what in the common interpretation of the formalism goes under the name of collapse of the state (which in reality would never occur) even though it is not clear how to justify the apparent violation of locality."[cxiv]

There are also books that are very similar to those discussed in the previous section. These books I reckon among the 'old orthodoxy'-textbooks. They maintain the attitude of cleverly avoiding interpretational issues and focus on the calculational problems. Examples are *Quantum Mechanics, Concepts and Applications* by Nouredine Zettili in 2009,[cxv] *A Modern Approach to Quantum Mechanics* by John Townsend in 2012,[cxvi], *Quantum Mechanics, a Textbook for Undergraduates* by Makesh Jain in 2007,[cxvii] and *Quantum Mechanics for Scientists and Engineers* by David Miller in 2008.[cxviii]

10.3 Conclusions on Education

Three important observations need to be made explicit. By looking at textbooks the indication is that (i) there are indeed textbooks written in the years 2007-2014 that exhibit the ideas of the new orthodoxy, i.e. these devote a large part of the book to interpretational questions through non-collapse considerations and decoherence. Furthermore (ii) either the books are very similar to the textbooks I dubbed 'old orthodox' (for simplicity), or the books contain discussions about both the measurement problem ánd decoherence. Many books incorporate the *combination*, and mostly the combination only. However, (iii) there are also books that do not adher to this at all. And finally, (iv) the books that do exhibit these ideas are not widely used. They are mostly used at the universities where the writer(s) teaches the course himself.

Mostly observation (ii) may be leading us to the hypothesis that a shifting view is indeed taking place. The idea that the attention spend on decoherence goes hand-in-hand with the attention spend on the measurement problem is a good indication that there is happening more than just teaching more topics.

[cxii] Pade, *Pedestrians*, 162.

[cxiii] Valter Moretti, *Spectral Theory and Quantum Mechanics* (Milan: Springer, 2013), p. 659.[34]

[cxiv] Moretti, *Spectral Theory*, 660.

[cxv] Nouredine Zettili, *Quantum Mechanics, Concepts and Applications* (Chichester: Wiley and Sons, 2009).[55]

[cxvi] John Townsend, *A Modern Approach to Quantum Mechanics* (N.p. [United States], University Science Books, 2012).[52]

[cxvii] Makesh C. Jain, *Quantum Mechanics, a Textbook for Undergraduates* (New Delhi: Learning private limited, 2007). [26]

[cxviii] David A.B. Miller, *Quantum Mechanics for Scientists and Engineers* (New York: Cambrdige University Press, 2008).[35]

It needs to be said that the selection made can be seen as arbitrary and indeed it would need more research to take much more universities and textbooks into account. This might give a statistical idea of the balance between (i) and (iii). A problem with this might be that these modern textbooks are not available at university libraries and therefore hard to come by.

Furthermore, via (iv) the hypothesis might be put forward that -even though modern books are available through (i)- there exists *niches* of teachers within the repecting univeristies that do not see the reason to update the quantum text books. This would involve to recoordinate the entire course and enter unfamiliar terrain, which demands a great deal of the time of the researcher.

In the quote at the opening of this chapter Tegmark and Wheeler are right that collapse is explicitely mentioned in the early chapters of all quantum introductionary textbook and that this is a source of confusion. They are wrong in the statement that these textbooks state explicit collapse 'infallibly'. But after all, this was in 2001 so the collapse/decoherence-debate hadn't settled yet. We see by observation (i) that the update they are asking for is indeed taking place. Signs of change regarding pre-21^{st} textbooks are taking place. At least the ships carrying this update a clearly visible at the horizon.

Part V

Conclusion

We have discussed the measurement problem on the leading example has been the double-slit experiment. The question is why superpositions of a quantum nature are never observed. The projection postulate has been described as the most evident solution to it. When a measurement is made a non-unitary ‘jump’ is made to an eigenstate corresponding to the measured eigenvalue. Accordingly, uneasiness about this postulate has been discussed. First there is the problem of having two processes that describe the evolution of a sytem through time: Schrödinger’s unitary evolution and the non-unitary collapse of the wave function. Collapse brings the problem of fundamental irreversible processes, which are different from thermodynamical irriversibility that is only a consequence of our ignorance of the system. This leads to time asymmetry and the loss of information through the act of measurement. The second problem has been the vain definition of the concept of measurement: it is unclear what the line of demarcation should be between ordinary physical processes and ‘measurements’.

After that the problem was discussed that it was also unclear when projection really happens. This was depicted by the conundrum of where to cut the Von Neumann chain. Von Neumann, London, Bauer, Weitner and Wigner proposed that the solution lies outside the physical world, that it is our consciousness that collapses the wave function. This solution was unsatisfactory for most physicists and the need began to rise for a solution that only needs Schrödinger evolution. Thus two paths to a solution were tread upon: that of the supporters of collapse theories and that of the supporters of non-collapse theories.

Environmentally induced decoherence initially seemed to shed light on a solution that only considers unitary evolution. This becomes through looking at open systems instead of closed ones. In an ensemble of quantum states the environment -which are the internal degrees of freedom in a system, the air between apparatus and the system or even the cosmic microwave background- carries away the coherence of the system and distributes it over its many components. By ‘tracing out’ the unobserved degrees of freedom of the environment it is clear that the off-diagonal terms in the density matrix of the system go to zero rapidly. Thus the system experiences decoherence. This is then identified with an ‘apparent’ collapse, since it explains why the world ‘appears’ classical to us. However, we have seen that the fundamental problem is not adressed, since the collapse of the single quantum system could not be explained. This is because taking the partial trace seems to correspond to introducing a statistical version of the projection postulate. Also, by introducing a third set of eigenstates to the entangled system (i.e. that of the environment) the Schmidt decomposition no longer garantees that the eigenstates are uncorrelated. Thus the Hamiltonian that governs this entanglement must be of a special from. It seems that individual collapse is able to explain decoherence, but decoherence is not enough to explain individual collapse. therefore, interpreting decoherence by itself is unsuccesful in resolving the measurement problem. On the other hand decoherence does have the ability to highlight certain paths towards a solution. It justifies why the world appears classical to us, i.e. why we do not observe superpositions. It strengthens certain interpretations and weakens others. This effect has been noticed by many in observing the popularity of the orthodox interpretations. In any case, a cornerstone achievement of decoherence is the recognition that considering open systems instead of closed systems plays an important in foundational issues.

According to Bub's new orthodoxy, which is supposed to represent the majority of physicists, it is generally believed that among physicists and philosophers of science collapse is either a physical phenonemon, or an epiphenonemon that is only apparent but not fundamental. It is apart from our consciousness and explainable on physical terms. Also there is no clear border between our world and the world of the microscopic; the transition is smooth, not sharp. Decoherence is a discovery that supports this view in every way, since it gives us a justification to postulate the collapse of the wave function. Interpretations of quantum mechanics grow closer together due to the need to take decoherence into account. This strengthened the non-collapse theories in particular.

In undergraduate education, there are on the one hand lectures in which the fundamental status of collapse of the wave function is denied and there is a literature of publications that interchangeably (uncarefully) use the terms 'decoherence' and 'collapse'. On the other hand there are the textbooks -used by the students to get familiar with the theory- that explicitely denote collapse fundamental. This is because the textbooks that are being used were written before the debate settled as described in part IV, i.e. the process of decoherence is a justification, but only partial explanation of the use the projection postulate. The answer is only partial, but the term 'new orthodoxy' implies that decoherence has outfashioned the orthodox approach. The orthodox Copenhagen interpretation seems to be out of fashion. On the one hand the student learns quantum mechanics in a way the orthodox ideas are presented as fundamental -particularly the projection postulate- where the opinion of the working class physicists have more modern ideas (and hopes). On the other hand there are already more modern textbooks that are also sporadically in use. But these textbooks are mostly used in the universities where the writer teaches. This might give rise to the hypothesis that quantum courses in universities are taught by the *niche* of teachers corresponding to the university, where no need to update the course with a new textbook. Maybe it is time to advertise this need.

Projection is in any case a calculational tool that we must use whatever interpretation we choose. It might even be fundamental. But although there are modern text books, most of the fresh minds learn quantum mechanics from textbooks dating back to the eighties and early ninetees. The early quantum mechanic is confronted with the orthodox ideas, while we are living in a non-orthodox world. Einstein's "On Education" tells us we must guard over the marble that is our knowledge, our hands must be at work if we want it to shine forever.

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