The feasibility of many-worlds as the leading interpretation of Quantum Mechanics

Bachelor Thesis

Martijn Hidding,
Utrecht University

Supervisor: prof. dr. D. Dieks

January 14, 2014

Abstract

This thesis takes a look at the many-worlds interpretation (MWI) and discusses if it can become the leading interpretation of quantum mechanics. Firstly the Copenhagen interpretation is outlined and its issues relating to the measurement problem are discussed while taking recent insights in decoherence theory into account. Establishing that wave-function collapse still yields interpretational issues, I discuss the ideas of Hugh Everett, who proposed the many-worlds interpretation, and David Wallace who is a prominent advocate of the MWI. In particular I discuss and review Wallace's arguments in favor of viewing the wave-function and the many worlds realistically. The next part shortly focuses on the testability of the many-worlds interpretation and the philosophical and theoretical issues relating to probability in the MWI. I end by concluding that the no-collapse interpretations are preferable to the Copenhagen interpretation, and I argue in favor of a realistic view towards all of the worlds. Furthermore, I conclude that standard textbooks on quantum mechanics should be updated to introduce students to the insight that wave-function collapse isn't needed in forming a complete theory of quantum mechanics.

Contents

1 Introduction

In this thesis we will take a look at the interpretation of quantum mechanics. In particular we'll examine the many-worlds interpretation (MWI) as a possible solution for the philosophical problems that the Copenhagen interpretation raises. I will discuss the MWI for the most part as it is explained by David Wallace in his book "The Emergent Multiverse". It is not necessary to have read his book in order to understand this thesis as we briefly explain most concepts in the corresponding chapters of this paper.

I will assume the reader has at least an undergraduate knowledge of quantum mechanics and its mathematical formalism. When introducing new concepts they will be illustrated with additional examples. We'll discuss the many-worlds debate from a wide range of standpoints, also including philosophical arguments. However care is taken to separate philosophical arguments from the more rigorous mathematical ones whenever possible so that the mathematical treatment can be considered independently.

At the end of the paper I will summarize the current situation in theoretical physics that is relevant to the MWI-debate and propose a few directions of possible future research. But firstly I want to devote a small section to explaining why I am interested in the many-worlds interpretation and why I believe the Copenhagen interpretation has some fundamental problems.

1.1 The motivation behind this thesis

As a third-year undergraduate student of physics and mathematics my first course on quantum mechanics was just a year ago. It used the book "An introduction to quantum mechanics", by David J. Griffiths and gave a first look into the world at the microscopic scales. The book adopted the general terminology of the Copenhagen interpretation and wave-function collapse was interpreted as a statistical tool for giving the outcome of measurements.

During this course I did not learn about other interpretations of quantum mechanics, and besides that, I wasn't fully aware there were others. The good things about the course and book were the explanation of the most important concepts in quantum mechanics, but not much time was spent on the paradoxes like Schrödingers cat and the EPR-paradox. Also the Schrödinger equation was just stated without a justification and at the time it felt like it just fell out of the air.[1]

So, like many students, I couldn't help but wonder why exactly the world works the way it does. There was a certain vagueness to the whole thing that I couldn't

---

[1] I don't want to imply that Griffiths or the course was to blame here, in fact I believe both were quite good. But when teaching quantum mechanics there is always the dilemma about which one to explain first: the mathematics of quantum mechanics or its physical implications for the world? Griffiths explains in his book that he first wants the reader to understand what quantum mechanics says and then consider the issue of what it means. I feel positive that in the future we can do it the other way around and I believe the MWI or at least the modal interpretation of quantum mechanics, which we'll also briefly discuss, points us in the right direction for doing this.

blame on the course, but only on quantum mechanics itself. Thus when I started researching my subject for my thesis I quickly knew that I wanted to do research in the interpretation of quantum mechanics, since I don't think there has been another subject in physics yet that has interested me as much as.

Eventually I ended up with the many-worlds interpretation as it seemed the most promising one to solve the measurement problem. I was skeptical at first, wondering if the many-worlds interpretation was merely the result of some physicists' love of science-fiction, but my attitude towards it has become pretty favorable. For this David Wallace of Oxford is mostly to blame, as I believe he makes good points on why the Everett interpretation is (currently) the best to consider.

At first the idea of many-worlds seemed far-fetched to me. I got the idea that quantum theory simply gives probabilistic statements for the outcomes of certain measurements, and that the many-worlds interpretation was simply saying without justification that all the other outcomes exist too, but in other universes.

Of course if this was the sole element of the many-worlds interpretation there could be no reason to take it seriously scientifically. We could just as well say there is another universe out there with all the magical elements we dreamed of as kids. It's not that I don't want to believe in those things, and who knows, maybe they do exist in another universe! But such a thing isn't provable and unjustified speculation is not why I started studying physics.

But what I instinctively missed was that quantum mechanics is not only about giving probabilistic statements for measurements, but that quantum interference/entanglement play a big role in quantum physics. Claiming that quantum interference is merely a mathematical tool which aids in interpreting calculations for giving probabilistic outcomes of measurements is in my view not justifiable. I would like to give a proper explanation for these phenomena and describe them realistically as parts of the world, just as we consider particles to be.

So when working on this thesis I found out that many-worlds gives exactly this explanation. It gives us a theory consisting of a deterministic universe with local interactions. Furthermore it resolves all interpretational issues relating to non-local wave-function collapse, indeterminacy of a particles properties or quantum entanglement. Now then this is something worth considering!

But now that I am more favorable towards the idea, I still see the doubts other students or physicists have with accepting or at least considering many-worlds, since I had them myself too. Some students already shiver on the idea of many-worlds and feel that it is mostly a pop-science name to attract attention from journalists. Others feel instinctively that because the other worlds are immeasurable, there is thus no reason to believe in them.

Lastly what I noticed the most, to my personal surprise, is that many students adopt a pragmatic view on quantum mechanics. I believe too much emphasis is placed on teaching students that quantum mechanics has properties that completely defy classical logics. Widely known is Feynman's quote "If you think you understand quantum mechanics, you don't understand quantum mechanics."

But I am afraid this will cause people to think that they are somehow being good physicists by acknowledging that quantum mechanics isn't understandable. That a good physicist doesn't go into a proper discussion about why the world works the way it does and that so-called "philosophical-alternatives" to orthodox quantum theory are merely speculation and not proper physics. The mentality seems common that one simply shouldn't care which way it turns out to be.[2]

But I do care about this and I believe that people like Deutsch and Wallace who are researching this area of physics are doing an important job for the physics community. Even if they turn out to be wrong, I still believe they are justified in seeking a better alternative for the Copenhagen interpretation.[3] Though I don't agree with everything that Wallace writes, I believe some of his arguments are put aside too quickly and for no good reason but bias against many-worlds. Thus I personally hope that more emphasis would be put on the discussion in universities, and that the initial skepticism of many students could be lessened.

I firstly want to take a look at 'orthodox' quantum mechanics as it is still taught to students nowadays and address the problems it brings up. I believe that the Copenhagen interpretation has fundamental flaws that cannot be resolved. Furthermore I will compare Copenhagen with the many-worlds interpretation and eventually conclude that many-worlds is currently the better alternative. Also I will focus critically on Wallace's views as he is the latest and most prominent advocate of the many-worlds interpretation and therefore his statements, especially the ones that could be wrong, are important to the many-worlds debate.

I believe that any student of quantum mechanics should ask himself the question why the world works the way it does. And if every student does this, I certainly believe that many-worlds would become an accepted and possibly the preferred interpretation of quantum mechanics. But there is still a lot to justify for the many-worlds interpretation. I hope at least that my previous discussion could provide a motivation to look into it. So let's do that.

---

[2] I don't want to go over my head here, and I don't want to imply that I am an authority on the subject. But I have noticed how easily good students seem to ignore this area of physics and I believe I am justified, even with my lack of knowledge, in saying that they have no good reason for doing this.

[3] And I do believe many physicists will agree with me on this point. The problem of wave-function collapse needs to be resolved one way or another.

2 'Orthodox' Quantum Mechanics

2.1 Introduction

Let's start with a quick review of what we could call, 'orthodox' quantum mechanics, given by the Copenhagen interpretation. Firstly it is not my intention to give a complete overview of the Copenhagen interpretation, as that is beyond this thesis. In this section we'll give a simple and quick outline of the key points that are relevant to our discussion. Furthermore we'll address some parts of the famous discussion between Einstein and Bohr, because it is still relevant to our current views on quantum mechanics and gives a context for addressing a phenomenon called decoherence.[4]

Because I will not outline the Copenhagen interpretation fully, this adds the small danger of giving a biased view by highlighting its failures and ignoring its successes. I want to make the reader aware that this section should mostly be viewed as a discussion of the problems and difficulties with the Copenhagen interpretation relevant to our debate, than as an attempt to give a proper description of it.

Having said that we'll start with the basics by considering the double-split experiment, which according to Feynman has the heart of quantum mechanics in it.[5]

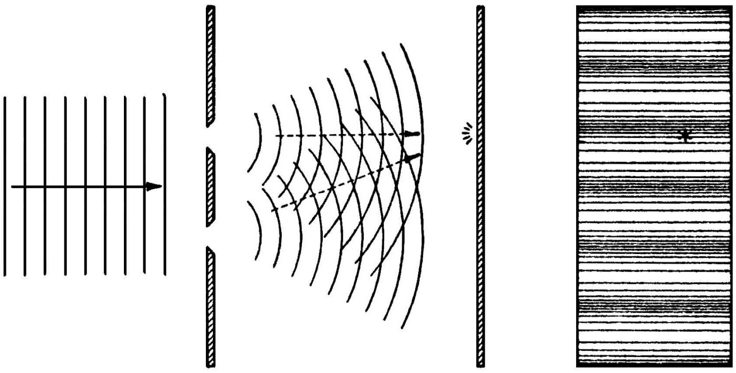

Figure 1: A version of the double split experiment using electrons.

From the double-split to the Copenhagen interpretation The double split experiment is famous for showing the wave-particle duality. Before the twentieth century the experiment was thought to show the wave-nature of light. Light that was send into a double split was shown to exhibit an intensity pattern on a screen that was specific to wave-interference. This result caused the Newtonian theory that light consisted of particles to become abandoned and the theory that light was composed of waves to be favored.

---

[4] I'll assume the reader is not yet familiar with decoherence.

[5] It seems that in some physics texts quoting Feynman has become a popular way of grabbing the readers attention, so let's follow the trend.

However Einstein showed in 1905 that light could transfer energy in discrete chunks as if it did consist of particles, by his photoelectric effect. The situation turned out to be problematic when particles like electrons seemed to exhibit wavelike-behavior in a double split experiment. Individual electrons would end up at a definitive place, but the distribution of many electrons would form an interference pattern just like that of interfering waves. It turned out that this seemed to hold for all elementary particles. (Currently the biggest object shown to exhibit the wave-particle duality is buckminsterfullerene.)

Eventually it was postulated that the position where individual particles are detected cannot be predicted, but that the distribution of many particles follows a well-defined statistical distribution. This insight gave rise to quantum mechanics as we know it. In stark contrast to the processes described in classical mechanics, probability became an intrinsic part of nature and this was the start of a long chain of philosophical problems posed by quantum mechanics.

Uncertainty only plays a role in classical mechanics if a complete description of all degrees of freedom of a system is not given. However it is assumed that a complete description can in principle be given to the system, even if it requires a computational capacity that is not currently available to us. For example we cannot calculate the trajectories of all the atoms of a gas in a box, but the macroscopic behaviour of the gas can be given by statistical approximations.

In quantum mechanics the situation was different and uncertainty seemed inherit in the underlying nature of the processes. For this the wave-function was introduced as the central concept to describe the quantum state of a particle. Niels Bohr was the first to express his believes that no completer description could be given to quantum systems than the wave-function and its probabilistic predictions for the outcomes of measurements. His ideas, together with Heisenberg's ideas eventually evolved into what is now known as the Copenhagen interpretation of quantum mechanics.

Von Neumann's Postulates The principles of quantum mechanics as developed at the time and still taught today can be stated in its postulates, formulated by Dirac and Von Neumann. These postulates are specific to the Copenhagen interpretation, but other interpretations of quantum mechanics do largely the same predictions[6] and therefore follow very closely related mathematical rules.

As an example of the similarities in the mathematics between interpretations we can consider the MWI. Although the Born postulate and projection postulate are not needed in the MWI, the interpretation predicts that quantum measurements give the impression to be in perfect agreement with these postulates.

The wording in the postulates is characteristic of the Copenhagen interpretation. It is common in the Copenhagen interpretation to say that a system is in a state $|\psi\rangle$. Other interpretations might say that $|\psi\rangle$ is the wave-vector that describes

---

[6] Any predicted deviation from orthodox quantum mechanics is still too small to be measurable.

the system, but remain agnostic about the system actually being in the state of $|\psi\rangle$.[7]

For simplification we give the postulates in their discrete form. Things become a bit more subtle mathematically when working with an infinite basis like the position basis, but the essentials of the postulates remain.

1. **State postulate** Every physical system has a corresponding Hilbert space $\mathcal{H}$. The states of the system are completely described by unit vectors in $\mathcal{H}$. A composite physical system corresponds to the direct product of the Hilbert spaces of the subsystems.

2. **Observables postulate** Every physical quantity $\mathcal{A}$ of the system corresponds to a self-adjoint operator $\hat{A}$ in $\mathcal{H}$, the so-called 'observables'.

3. **Spectrum postulate** The only possible outcomes which can be found upon measurement of a physical quantity $\mathcal{A}$, corresponding to an operator $\hat{A}$, are values from the spectrum of $\hat{A}$.

4. **Born postulate** If the system is in a state $|\psi\rangle \in \mathcal{H}$, and a measurement is made of a physical quantity $\mathcal{A}$, corresponding to an operator $\hat{A}$ with a discrete spectrum $\mathrm{Spec}(\hat{A})$, the probability to find the outcome $a_i \in \mathrm{Spec}(\hat{A})$, is equal to

   $$P^{|\psi\rangle}(a_i) = \langle\psi|P_{a_i}|\psi\rangle\,,$$

   where P_{a_i} is the projector from the spectral decomposition of $\hat{A}$.

5. **Schrödinger postulate** As long as no measurements are made on the system, the time evolution of the system is described by a unitary transformation

   $$|\psi(t)\rangle = U(t,t_0)\,|\psi(t_0)\rangle \tag{2.1.1}$$

6. **Projection postulate** If the system is in a state $|\psi\rangle \in \mathcal{H}$ and a measurement is made on a physical quantity $\mathcal{A}$ corresponding to an operator $\hat{A}$ with discrete spectrum, and the outcome of the measurement is the eigenvalue $a_i \in \mathrm{Spec}(\hat{A})$, the system is, immediately after the measurement, in the eigenstate

   $$|\psi\rangle \approx \frac{P_{a_i}\,|\psi\rangle}{|P_{a_i}\,|\psi\rangle\,|} \tag{2.1.2}$$

These postulates describe a wealth of experience, but imply a radical new viewpoint of nature. Bohr introduced the concept of complementarity to clarify a wide range of interpretational issues relating to the statistics. Although the exact explanation of Bohr's complementarity principle is sometimes a matter of debate, a reasonably good one about the role of the complementarity principle in quantum mechanics stated by philosopher Phillipp Frank is:

---

[7] Within the Copenhagen interpretation this choice of words is sometimes used too. Heisenberg and Bohr differed with each other on this point. Heisenberg saw the wave-function more as representing knowledge about a system, though Bohr felt more easy to state a system was actually in the state $|\psi\rangle$ and that this was all there was to describe about the system.

> Quantum mechanics speaks neither of particles the positions and velocities of which exist but cannot be accurately observed, nor of particles with indefinite positions and velocities. Rather, it speaks of experimental arrangements in the description of which the expressions "position of a particle" and "velocity of a particle" can never be employed simultaneously.

According to Bohr complementarity was a widespread phenomenon. To see an example of this let's focus on the decay time of atomic nuclei.

Even though the same kind of wave-function can be attributed to the same kind of atomic nuclei, their measured decay times can be different and their decay products can be send in different directions. An intuïtive explanation is that the wave-function gives only a statistical description of the properties of a large group of similar particles, but that individual particles have extra degrees of freedom not described in the wave-function. This would then explain why it is observed that individual particles deviate from average behaviour.

However Bohr resisted such views and claimed that the idea itself, that an individual nucleus had a definite time of decay, was incorrect. He explained that decay time was a classical concept that only acquired meaning in a measurement setup, because a measurement would invoke a disturbance to the nucleus. For this reason Bohr explained there could not be spoken about the concept of decay time outside the context of a measurement, as he believed that classical properties acquired meaning exactly because of the specific interaction between the system and the measurement device.

Let's consider a few other questions that quantum mechanics brought up:

- Should we consider the electron as a wave or a particle?
- Is the uncertainty in measurements a result of the limits of our measurement apparata, or of the world itself?
- How does the electron's wavefunction collapse when a measurement is performed?

One can tackle the first question by noting that the wave and particle description of an electron are complementary. In a certain set-up it might seem that an electron is a wave, and in another set-up it might seem to be a particle. It is not necessary to describe the electron as either a particle or as a wave, since these are classical concepts that only acquire context in the process of a measurement.

Furthermore complementarity implies that one can not speak of a well-defined position and momentum at the same time. If you would arrange the measurement setup to yield what we would classically describe as position, one is not able to properly describe momentum as it acquires no meaning in this measurement context. So you can not really speak of the concept of momentum outside a measurement context, and since measuring position excludes measuring momentum, position and momentum exclude each other.[8] Thus the uncertainty

---

[8] There seems to be a kind of instrumentalist viewpoint underlying this. Some followers of the Copenhagen interpretation explain complementarity in a different way and Bohr's exact view about this is sometimes a matter of debate. However it seems generally accepted that

principle in the Copenhagen interpretation is seen as a result of complementarity which results in an essential limit to the precision of measuring apparata.

Lastly wave-function collapse was postulated as an axiom and a fundamental property of the world. Thus a measurement of a an observable $\hat{A}$ of a particle that happens to yield its eigenvalue α_i causes the wave-function to collapse into an eigenstate $|a_i\rangle$. However the physical nature of this process is not explained by the Copenhagen interpretation.

Wave-function collapse is currently the main interpretational problem of quantum mechanics. The most notable opponent to wave-function collapse was of course Einstein who tried to disproof Bohr's reasoning by introducing the EPR-correlation paradoxes. Besides that Einstein did not believe that the world was indeterministic, and he famously remarked that 'God does not play dice'.

2.2 Einstein's objections

Most of Einstein's criticism focused on the peculiar nature of wave-function collapse. For example at the 5th Solvay conference in Brussels, Einstein talked about two different ways of viewing the wave-function. The first one, supported by Bohr and Heisenberg, saw the wave-function as a description of a system which is as complete as possible. The other way of viewing it was that the wave-function gave a statistical average of the properties of identically prepared systems.

Einstein argued that the first view couldn't be complete. After encountering the double-split the wave-function would spread out but the particle would only be measured at one particular place after it hit the screen. Thus he argued the wave-function must suddenly disappear everywhere else in space, leading to a non-local 'action at a distance', something he was against. Einstein argued however that this did not apply to the second view of the wave-function as a statistical function.

Later on Einstein together with Boris Podolsy and Nathan Rosen introduced the EPR-paradox to show more problems with the non-locality of quantum mechanics. We'll give a stylized version of their thought experiment, using spins instead of position and momentum.[9]

One can bring two particles into a combined state in which the sum of both spins is equal to zero. After this one can bring them apart for an arbitrary distance. Now if let's say observer A measures the spin of particle A to be +1/2, he immediately gains knowledge that the spin of particle B is -1/2. So if quantum mechanics is consistent it must be that observer B (and A) will always measure the spin of particle B to be -1/2.

Now this brings up problems with Bohr's reasoning. If particle B has an indeterminate spin before measurement, it shouldn't be indeterminate anymore after measurement of particle A, since one knows what the outcome will be.

Bohr implied that it is not possible to regard objects governed by quantum mechanics as having intrinsic properties independent of their determination with a measuring device.

[9] This version of the EPR-paradox is probably the most well-known and was introduced by David Bohm.

Thus EPR reasoned that either some non-local interaction must have occurred such that measurement of particle A would collapse the wave-function of particle B, or the particle already had definite properties from the beginning on. They argued therefore that the second interpretation was to be favored as the first one seemed to conflict with special relativity since nothing can travel faster than the speed of light. They concluded by saying quantum mechanics was still an incomplete theory.

However it turned out that this phenomenon which is now widely now as quantum entanglement did not do damage to Bohr's interpretation. It was shown that relativity was not violated by the apparent non-local interaction. Firstly even if observer A measures the spin of his particle, he cannot communicate this result to observer B faster than the speed of light. This immediately resolves any concerns relating to violation of causality. Furthermore the interaction that occurs between the particles because of their entanglement does not rely on the transmission of particles and thus there is no violation of the speed of light limit anyhow.

The situation turned even more into Bohr's favor when John Bell showed with his inequalities that no physical theory of local hidden variables can ever reproduce all of the predictions of quantum mechanics. Because of Bell's argument the situation turned around and quantum entanglement actually became a prime support for Bohr's theorem. Thus Einstein's hope that there was a completer description of reality hidden underneath quantum mechanics was largely dismissed.

Many scientists believed that Bohr had won the debate and that classical intuïtions did not apply to the quantum world. Wave-function collapse did look peculiar, but it was in no conflict with the existing theories and could explain the observed results of quantum experiments. Therefore most scientists settled with the Copenhagen interpretation, or at least used the Copenhagen interpretation in a pragmatic way to explain the outcomes of measurements.

Current situation Although most scientists settled with the Copenhagen interpretation, more attempts were made at finding alternative interpretations of quantum mechanics. Notable is the de Broglie-Bohm theory which succeeds in building a hidden variable theory underneath quantum mechanics. The theory is not in conflict with Bell's inequalities because it relies on non-local interactions. However it has turned out that this is also the downfall of the theory as it has not succesfully been adopted into the domain of special relativity, something that other interpretations of quantum mechanics have been.

Another few interpretations coined are the transactional interpretation, information-theory based interpretations and the many-minds interpretations.[10] These interpretations are currently seen as less likely contenders for the Copenhagen interpretation and we will not discuss them in this thesis. The interpretation we discuss in this thesis is of course the many-worlds interpretation in which the projection postulate corresponding to wave-function collapse is thrown away and all measurement outcomes are considered real but in equidistinct world.

---

[10] A variant of the many-worlds interpretation.

The Copenhagen interpretation itself has also changed a bit in recent times because physicists have acquired a better understanding of a quantum phenomenon called decoherence, which arguably helps explain how wave-function collapse comes forth.[11] We'll start discussing what decoherence is and the role it plays in the Copenhagen interpretation. As we will later see decoherence plays an even more important role in the many-worlds interpretation.

2.3 Decoherence

The important insight of decoherence theory is that quantum systems are never isolated but continuously interact with their environment. Because of this both system and environment should be viewed as a single quantum system. It can then be shown that states emerge from the Schrödinger postulate alone where observers and measurement devices do measurements as if the system has suddenly collapsed into a single definite state.

Thus by decoherence it can be clarified why observers see something that resembles an instantaneous wave-function collapse and decoherence lessens criticism on behalf of quantum mechanics relying on 'spooky' action a distance. However this does not mean that decoherence solves the measurement problem completely, and there is still a debate on how decoherence should be interpreted in the Copenhagen interpretation.[12] To illustrate this we consider an example.

Let's say we do an experiment where we measure the z-spin of a particle. We expect that we get an outcome spin-up/spin-down with 50% probability. The role decoherence plays in our experiment is that it describes how two states emerge from the dynamics: one in which the observer measures spin up and another in which the observer measures spin down. But the fate of the resulting superposition remains an interpretational issue and thus the measurement problem is not solved. The reader might correctly predict that the many-worlds interpretation resolves the issue by explaining every quantum outcome is equally real in a distinct world.

I will loosely base the following section on the paper "Decoherence and the transition from quantum to classical" from Wojciech H. Zurek, who originally proposed that environmental induced decoherence provides insight into the limit from quantum to classical. On the one hand I have added additional information on the use of density matrices to make the discussion more accessible for undergraduate students of quantum mechanics. On the other hand I approach the issue less rigorously and from a more qualitative point of view, trading in some of the mathematics for a qualitative explanation of the ideas at stake.

The subject of enviromental induced decoherence can be given a much more complete description than described below and I would advice the reader to

---

[11] Some physicists believe that decoherence solves the measurement problem, but leading adherents of decoherence theory generally disagree. A good and thorough discussion about the role of decoherence in the measurement problem can be found in the article "Decoherence, the measurement problem, and interpretations of quantum mechanics" by Maximilian Schlosshauer. [5]

[12] The measurement problem is the problem of how and if wave-function collapse occurs.

read Zurek's paper on his/her own later on. However I have tried to make the following description of decoherence good enough to serve as an introduction in describing the role of decoherence in the measurement problem.

The mathematical formulation of decoherence relies on the use of density operators to describe so-called mixed states. One can use a mixed state to describe a situation wherein an experimenter has insufficient knowledge about the experiment and only knows the probabilities p_i that a quantum system will be found in one of the pure states $|\psi_i\rangle$.[13]

We consider a Hilbert space $\mathcal{H}$ with a set of arbitrary pure states $\{\psi_i\} \subset \mathcal{H}$. The density operator of a mixed state is given by

$$\hat{\rho} = \sum_i p_i \left|\psi_i\right\rangle \left\langle\psi_i\right|, \tag{2.3.1}$$

where p_i denotes the probability to find the system in the pure quantum state $|\psi_i\rangle$. Choosing an arbitrary basis of $\mathcal{H}$, which we denote by $\{f_1, f_2, \ldots\}$ and which is normalized but not necessarily orthogonal, one can write the density operator as a matrix using the expression

$$\hat{\rho}_{mn} = \sum_i p_i \left\langle f_n|\psi_i\right\rangle \left\langle\psi_i|f_m\right\rangle \tag{2.3.2}$$

There is also an analogue to Schrödinger's equation for mixed states, which is called the Von Neumann equation:[14]

$$i\hbar\frac{d}{dt}\hat{\rho} = [\hat{H}, \hat{\rho}], \tag{2.3.3}$$

where $\hat{H}$ is the Hamiltonian of the system.

Furthermore one can generalize the the expectation value of an operator $\hat{A}$ in a pure quantum state to mixed quantum states. For a pure state $|\psi\rangle$ we have that $\langle\hat{A}\rangle = \langle\psi|\,\hat{A}\,|\psi\rangle$. Thus for mixed states one can use that $\langle\hat{A}\rangle = \sum_i p_i \langle\psi_i|\,\hat{A}\,|\psi_i\rangle$, which can be simplified to $\langle\hat{A}\rangle = \mathrm{Tr}(\hat{\rho}\hat{A})$.

As a concrete example of a density operator, let's consider two orthogonal states $|\uparrow\rangle$ and $|\downarrow\rangle$ which correspond to the z-spin of a particle.[15] We span our basis straightforwardly by

$$\left\{ |\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}.$$

Now a particle that is either in the pure state $|\uparrow\rangle$ or the pure state $|\downarrow\rangle$ with (classical) probabilities 1/2 and 1/2 can be described by a density matrix[16]

$$\hat{\rho} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}. \tag{2.3.4}$$

---

[13] Thus a mixed state is a classical probability function over pure quantum states.

[14] This form of Von Neumann's equation is only valid in the Schrödinger picture.

[15] Or in general to any two-state quantum system.

[16] So this is different from the particle being in a pure state $|\psi\rangle = \alpha\,|\uparrow\rangle + \beta\,|\downarrow\rangle$.

We see that the diagonal elements of the matrix represent the classical probability of finding the particle in state $|\uparrow\rangle$ or $|\downarrow\rangle$, and indeed $\langle \hat{A}\rangle = \mathrm{Tr}(\hat{\rho}\hat{A})$ as can be seen by filling in $\hat{A} = |\uparrow\rangle, |\downarrow\rangle$.

For another example we consider a particle in the pure state $|\psi\rangle = \alpha |\phi_1\rangle + \beta |\phi_2\rangle$, where the $|\phi_i\rangle$ are normalized but not necessarilly orthogonal. Its density operator is given by

$$\rho = |\psi\rangle \langle\psi| = |\alpha|^2 |\phi_1\rangle \langle\phi_1| + |\beta|^2 |\phi_2\rangle \langle\phi_2| + \alpha\beta^* |\phi_1\rangle \langle\phi_2| + \beta^*\alpha |\phi_2\rangle \langle\phi_1| \,, \tag{2.3.5}$$

and its density matrix in the $\{|\phi_1\rangle, |\phi_2\rangle\}$ basis is

$$\hat{\rho} = \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{pmatrix}. \tag{2.3.6}$$

We see again that the expectation values are $\mathrm{Tr}(\rho\,|\phi_1\rangle) = |\alpha|^2$ and $\mathrm{Tr}(\rho\,|\phi_2\rangle) = |\beta|^2$ as expected. Because this density matrix describes a quantum mechanical superposition between the states in the basis it has its off-diagonal elements nonzero and equal to $\alpha\beta^*$ and $\alpha^*\beta$.

Generally the off-diagonal elements of a density matrix provide a measure of the amount of quantum interference between the states in the basis (i.e. quantum entanglement).[17] A completely mixed state will always have its off-diagonal elements to zero.

We can use a so-called reduced density operator to describe a subsystem of a bigger quantum system. To illustrate this we take a look at an EPR-spin pair of two particles, with a wave-function described by:[18]

$$|\psi\rangle = \frac{1}{2}\sqrt{2}\,(|\uparrow\rangle_1 |\downarrow\rangle_2 + |\downarrow\rangle_1 |\uparrow\rangle_2)\,. \tag{2.3.7}$$

Straightforwardly the two subsystems of the EPR-pair correspond to particle 1 and particle 2. The density operator of the whole system is given by

$$\begin{aligned}\hat{\rho} = |\psi\rangle \langle\psi| = \frac{1}{2}\big(& |\uparrow\rangle_1 \langle\uparrow|_1 |\downarrow\rangle_1 \langle\downarrow|_2 + |\uparrow\rangle_1 \langle\downarrow|_1 |\downarrow\rangle_2 \langle\uparrow|_2 + \\ & |\downarrow\rangle_1 \langle\downarrow|_1 |\uparrow\rangle_2 \langle\uparrow|_2 + |\downarrow\rangle_1 \langle\uparrow|_1 |\uparrow\rangle_2 \langle\downarrow|_2\big),\end{aligned} \tag{2.3.8}$$

which written down in the basis of particle 1 becomes the density matrix

$$\hat{\rho} = \begin{pmatrix} \frac{1}{2}|\downarrow\rangle_2 \langle\downarrow|_2 & \frac{1}{2}|\downarrow\rangle_2 \langle\uparrow|_2 \\ \frac{1}{2}|\uparrow\rangle_2 \langle\downarrow|_2 & \frac{1}{2}|\uparrow\rangle_2 \langle\uparrow|_2 \end{pmatrix}. \tag{2.3.9}$$

From the ket $|\psi\rangle$ we see that the spin of particle 1 is completely determined by the spin of particle 2 and vice versa. Because of entanglement between the particles we generally cannot attribute a pure state to each one of the particles individually. We can only attribute a pure state to the composite system of the two particles which resides in the tensor product of their Hilbert spaces.

---

[17] To get an understanding of why the off-diagonal elements provide a measure of the interference I want to advice the reader to work out a few concrete examples for himself. Consider for example the pure state $\frac{1}{2}\sqrt{2}(|\uparrow\rangle + |\downarrow\rangle)$ and write the density matrix down in the $\{|\uparrow\rangle, |\downarrow\rangle\}$ basis and in the $\{\frac{1}{2}\sqrt{2}(|\uparrow\rangle + |\downarrow\rangle), \frac{1}{2}\sqrt{2}(|\uparrow\rangle - |\downarrow\rangle)\}$ basis. Also consider a partially mixed state with probability $\frac{1}{2}$ of being in state $|\uparrow\rangle$ and probability $\frac{1}{2}$ of being in a state $\frac{1}{2}\sqrt{2}(|\uparrow\rangle + |\downarrow\rangle)$.

[18] For simplicity we write $|\uparrow\rangle_1 \otimes |\downarrow\rangle_2$ as $|\uparrow\rangle_1 |\downarrow\rangle_2$ and so on.

However what we can do is attribute a so-called (reduced) density operator to particle 1. We know that particle 1 will be found in either the state $|\uparrow\rangle_1$ or $|\downarrow\rangle_1$ and thus by ignoring our knowledge about the entanglement of the two particles, we can use a classical uncertainty to describe the expected state of particle 1.

We define the reduced density operator $\hat{\rho}_1$ of particle 1 straighforwardly by taking the partial trace of $\hat{\rho}$ over the basis of particle 2, which corresponds to summing over the possible states of particle 2:

$$\begin{aligned} \hat{\rho}_1 = \mathrm{Tr}_{P_2}(\hat{\rho}) = \sum_{|i\rangle \in \{|\uparrow\rangle_2, |\downarrow\rangle_2\}} \langle i| \left(|\psi\rangle \langle\psi| \right) |i\rangle &= \frac{1}{2}\left(|\downarrow\rangle_1 \langle\downarrow|_1 + |\uparrow\rangle_1 \langle\uparrow|_1 \right) \\ &= \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}. \end{aligned} \tag{2.3.10}$$

As expected the reduced density matrix $\hat{\rho}_1$ describes a mixed state where the probability of particle 1 being in either the spin up or down state is 50%.

We are going to use reduced density operators to understand how the combined quantum state of a quantum system, its measurement device[19] and the environment can reduce to a mixed state over pure quantum states which describe measurement devices/observers doing measurements as if sudden wave-function collapse has occured.

To illustrate this we consider the quantum state of a spin-1/2 particle and its measurement device. We denote the particle as the system S, and we denote the detector of the particle's spin as system D. We let the Hilbert space H_s of the particle be spanned by the kets $|\uparrow\rangle$ and $|\downarrow\rangle$, which correspond to the spin up and down state of the particle. Furthermore we let the Hilbert space of the detector H_d be spanned by $|\uparrow_d\rangle$ and $|\downarrow_d\rangle$. Generally we assume our particle is in a pure state $|\psi\rangle = \alpha |\uparrow\rangle + \beta |\downarrow\rangle$.

Denoting the unitary time operator during measurement as U, we can describe the dynamics of the system as:

$$\begin{aligned} &U(\alpha |\uparrow\rangle |\downarrow_d\rangle) = \alpha |\uparrow\rangle |\uparrow_d\rangle \\ &U(\beta |\downarrow\rangle |\downarrow_d\rangle) = \beta |\downarrow\rangle |\downarrow_d\rangle \end{aligned}$$

and in general:

$$U(|\psi\rangle |\downarrow_d\rangle) = \alpha |\uparrow\rangle |\uparrow_d\rangle + \beta |\downarrow\rangle |\downarrow_d\rangle = |\phi\rangle \tag{2.3.11}$$

We see that a spin-up state always corresponds to the detector pointing up and similarly for the spin down state. However, we never see a measurement device yield two values at the same time, and we will find the system in either $|\uparrow\rangle |\uparrow_d\rangle$ or $|\downarrow\rangle |\downarrow_d\rangle$. Thus we need some kind of process that explains how the particle and measurement apparatus randomly end up in one of the two states, with probabilities $|\alpha|^2$ and $|\beta|^2$ respectively. Normally we would invoke wave-function for this[20], but we are not going to do this now.

---

[19] The measurement device can also be interpreted as an observer. It just depends on how many elements one includes in the chain.

[20] We might say wave-function collapse occurs when an observes sees the system, but the Copenhagen interpretation remains vague on when wave-function collapse occurs. Does it occur in the measurement device, or does only a conscious observer cause the collapse? The question remains unanswered.

As an alternative we look for a process that yields a mixed state where the chance of getting $|\uparrow\rangle|\uparrow_d\rangle$ is given by $|\alpha|^2$, and the chance of getting $|\downarrow\rangle|\downarrow_d\rangle$ is $|\beta|^2$. Such a mixed state does not resolve the measurement problem completely, because we will only see the system in one state, but it will explain that the possibilities we have for the future both concern states where it looks as if the particle has suddenly collapsed into a spin-up or down state.

Thus we hope to find a classical probability mixture, represented by a density matrix

$$\begin{pmatrix} |\alpha|^2 |\uparrow\rangle\langle\uparrow| & 0 \\ 0 & |\beta|^2 |\downarrow\rangle\langle\downarrow| \end{pmatrix}, \tag{2.3.12}$$

in the $\{|\uparrow_d\rangle, |\downarrow_d\rangle\}$ basis, which has its off-diagonal elements equal to zero. But currently the density matrix of the measurement device is given by:

$$\hat{\rho} = |\phi\rangle\langle\phi| = \begin{pmatrix} |\alpha|^2 |\uparrow\rangle\langle\uparrow| & \alpha\beta^* |\uparrow\rangle\langle\downarrow| \\ \alpha^*\beta |\downarrow\rangle\langle\uparrow| & |\beta|^2 |\downarrow\rangle\langle\downarrow| \end{pmatrix}, \tag{2.3.13}$$

where there are off-diagonal elements corresponding to quantum interference.

It seems we're looking for a process that sets the off-diagonal elements of our density matrix to zero. However this approach runs into problems since one can always choose an arbitrary basis for the density operator. Suppose we choose a basis $\{\frac{1}{2}\sqrt{2}(|\uparrow\rangle|\uparrow_d\rangle - |\downarrow\rangle|\downarrow_d\rangle), \frac{1}{2}\sqrt{2}(|\uparrow\rangle|\uparrow_d\rangle + |\downarrow\rangle|\downarrow_d\rangle)\}$. Simply letting the off-diagonal elements of $\hat{\rho}$ go to zero in this basis gives us a probabilistic mixture of two states that have no immediate physical meaning.

Because of such basis ambiguity we want to find a process for which the off-diagonal elements of our density matrix only go to zero for a preferred-basis corresponding to what we classically expect. We will find this process by the insight that in physically realistic scenarios a system always gets entangled with its environment. As it turns out the environment picks out a preferred basis of classically well-defined states, which do not interfere with each other, resulting in a classical probability mixture.

So let's include the environment in the discussion, which we denote by $\mathcal{E}$ and for which we choose the basis $\{|\epsilon_0\rangle, |\epsilon_\uparrow\rangle, |\epsilon_\downarrow\rangle\}$. The dynamics becomes:

$$U(\alpha|\uparrow\rangle|\downarrow_d\rangle|\epsilon_0\rangle) = \alpha|\uparrow\rangle|\uparrow_d\rangle|\epsilon_\uparrow\rangle$$
$$U(\beta|\downarrow\rangle|\downarrow_d\rangle|\epsilon_0\rangle) = \beta|\downarrow\rangle|\downarrow_d\rangle|\epsilon_\downarrow\rangle$$

and in general:

$$U(|\psi\rangle|\downarrow_d\rangle|\epsilon_0\rangle) = \alpha|\uparrow\rangle|\uparrow_d\rangle|\epsilon_\uparrow\rangle + \beta|\downarrow\rangle|\downarrow_d\rangle|\epsilon_\downarrow\rangle = |\phi\rangle \tag{2.3.14}$$

Using this our density matrix becomes:

$$\hat{\rho} = |\phi\rangle\langle\phi| = \begin{pmatrix} |\alpha|^2 |\uparrow\rangle\langle\uparrow| |\epsilon_\uparrow\rangle\langle\epsilon_\uparrow| & \alpha\beta^* |\uparrow\rangle\langle\downarrow| |\epsilon_\uparrow\rangle\langle\epsilon_\downarrow| \\ \alpha^*\beta |\downarrow\rangle\langle\uparrow| |\epsilon_\downarrow\rangle\langle\epsilon_\uparrow| & |\beta|^2 |\downarrow\rangle\langle\downarrow| |\epsilon_\downarrow\rangle\langle\epsilon_\downarrow| \end{pmatrix}. \tag{2.3.15}$$

At first it might seem that adding the environment to the experiment does not do us any good, as the procedure is similar to adding the measurement apparatus to our system. However we have made an important step, because it turns out we can reasonably assume our environmental states $|\epsilon_\uparrow\rangle$, $|\epsilon_\downarrow\rangle$ become orthogonal after a small amount of time.

I will not quantify this (though one can) but give a few qualitative arguments to make this intuïtive.[21] We can include all the interactions that take place between the measurement apparatus and an observer reading the state of the apparatus in the environment. This information gets transferred through billions of atoms and photons jiggling and flying in all kinds of directions. This means that a change in the pointer of a measurement apparatus quickly grows out to a completely different macroscopic quantum wave for the environment.

It can be shown that the interaction between states of the environment fades exponentially and extremely fast for macroscopic objects. Thus it is not reasonable to assume that the macroscopic states $|\epsilon_\uparrow\rangle$, $|\epsilon_\downarrow\rangle$ interfere with each other noticeably and we can let $\langle\epsilon_\uparrow|\epsilon_\downarrow\rangle \approx 0$.

Let's calculate the reduced density matrix for the system, where we trace out the environment and use the orthogonality of the environmental states. The result is a density operator with its off-diagonal elements equal to zero, just as we wanted:

$$\hat{\rho}_r = \mathrm{Tr}_{\mathcal{E}}(\hat{\rho}) = \begin{pmatrix} |\alpha|^2 \left|\uparrow\right\rangle \left\langle\uparrow\right| & 0 \\ 0 & |\beta|^2 \left|\downarrow\right\rangle \left\langle\downarrow\right| \end{pmatrix}. \tag{2.3.16}$$

We see that the quantum system reduces to statistical behaviour that predicts we will find ourself in one of the states $\left|\uparrow\right\rangle \left|\uparrow_d\right\rangle$ and $\left|\downarrow\right\rangle \left|\downarrow_d\right\rangle$ where it looks as if our particle has suddenly collapsed to it's spin-up or down state. More importantly, for this we don't have to resolve to using wave-function collapse. A massively entangled quantum state remains, but because it is entangled with the environment, measurements on a subsystem without taking the environment into account quickly reduce to classical statistical behaviour.

We might still fear the issue of basis ambiguity is at stake. What if we had picked a different basis, would we then have ended up with different states at the diagonal? The answer is negative, because the environmental states only become orthogonal for different pointer states of the measurement apparatus. This should be intuïtive considering our previous discussion, but it can be rigorously proven in a model that considers the interaction Hamiltonian between the measurement apparatus and the environment. However I won't show how to do this as it is not directly relevant for our discussion.[22]

Concluding our discussion we have now seen the key element of decoherence theory, namely that environmental interactions cause a classical probability distribution to emerge over an environmentally induced basis that corresponds to our classical intuïtion. However the question of how to interpret this probability distribution remains, and because of this the measurement problem is not completely solved.[23]

---

[21] An example of a mathematical model for the environment is a system comprised of a very big number of quantum harmonic oscillators. Zurek uses this example in his paper.

[22] The reader can read this for himself in Zurek's papers.

[23] In the Copenhagen interpretation we can view a mixed state as a distribution over pos-

2.4 Paradoxes and problems with Copenhagen

We have now summarized the Copenhagen interpretation and the role decoherence plays in explaining wave-function collapse. As the section about Einstein's criticism should have shown, the Copenhagen interpretation has not been without its opponents. However we concluded the discussion with Bohr as the winner.

Though Einstein did not win the debate, the Copenhagen interpretation has endured more criticism over the years. Looking back at the Einstein-Bohr debates I believe Einstein was right for pointing out conceptual difficulties with the Copenhagen interpretation, but it turned out he defended his views in the wrong way. Entanglement like in the EPR-paradox turned out to be a key element of quantum mechanics and the EPR-'paradox' actually backfired on those who held similar views to Einstein, because Bell's inequalities showed that no hidden-variable theory of quantum mechanics could produce such results. (Einstein had already passed away before Bell's inequalities were derived.)

In the following section we are going to discuss which problems are still at hand with the Copenhagen interpretation. There is no general consensus among all physicists whether the problems we'll discuss are actual problems or a misunderstanding of quantum theory, resulting from applying classical logics to the non-classical quantum mechanics. However during my research for this thesis I have noticed that most leading physicists would agree with me that the old and naive notion of the Copenhagen interpretation that uses Bohr's complementarity and which postulates wave-function collapse as an axiom has become dated.

Even though, the Copenhagen interpretation is probably still the most popular among physicists. The naive notion of wave-function collapse has partly been replaced with the insights of decoherence theory and it seems less emphasis is put on Bohr's complementarity. Furthermore the Copenhagen interpretation is popular because of it's pragmatic nature. You can remain agnostic about whether wave-function collapse actually occurs but at least accept it as a very useful tool in understanding quantum phenomena. But even if that seems fine for doing calculations, to get farther ahead in science we must get a proper understanding of what quantum mechanics means for the world.

The following points are usually brought up as problems with the Copenhagen interpretation:

1. The measurement problem
2. The special place of measurement apparata and of the observer
3. Indeterminism as a fundamental part of nature

We will firstly discuss these points without taking decoherence into account. Not many physicists completely stick to this view anymore but wave-function collapse is still used as a tool in teaching students quantum mechanics. Therefore by considering the problems without taking decoherence into account, I want

---

sible worlds. The many-worlds interpretation claims that the probability distribution is a distribution over worlds that are equally real.

to make clear that naive wave-function has no other place than to serve as a simplification of quantum theory.

After that we will briefly discuss how decoherence has changed our picture of the Copenhagen interpretation, but also argue that it is not enough to solve the measurement problem. Since we will only consider a small part of the debate, I will again recommend the reader to look into the discussions a bit more by his/herself.

The measurement problem The problem with wave-function collapse as an axiom of quantum theory is that it seems to contradict the Schrödinger postulate. On the one hand we have a perfectly deterministic time-evolution of the wave-function, but on the other hand 'measurement' collapses the wave-function indeterministically. This causes a number of conceptual problems. Firstly measurement devices and observers are given a special place in the theory. Secondly we know nothing about the physical nature of wave-function collapse, and we have no good answer to the question of how and why wave-function collapse occurs. Until now we have always been able to find deeper and deeper physical laws and there should not be a reason to assume we can't do that for wave-function collapse.

Putting measurement devices on a special place in the theory also causes an interpretational challenge. Measurement devices are made up of the same atoms and elementary particles as the rest of the universe and should therefore be governed by the same quantum mechanical rules. It seems difficult to accept that the laws of quantum mechanics, which work at a fundamental level of the world, do describe the laws of elementary particles but not the laws of objects made out of those particles!

To resolve this there have been a number of attempts to describe consciousness as the cause for wave-function collapse. Von Neumann was the first to come up with the idea and a modern proponent of the theory is sir Roger Penrose. Though the theory forms an excellent basis for quantum mysticism, the mainstream scientific community has been very critical of such attempts. Through decoherence theory it has been shown that the neural processes in the brain can only remain in a superposition for a time too small to actually effect consciousness or the environment. [7]

There are also attempts to postulate hard-criteria for when wave-function collapse happens, which are the so-called objective collapse interpretations. As an example one could postulate a theory wherein wave-function collapse repeatedly occurs throughout the universe after the quantum wave reaches a certain size. However, objective-collapse theories have brought up problems on their own and always involve a slight, but theoretically measurable, modification of quantum theory. Currently there is no good reason to assume the mathematics behind quantum theory needs to be modified as it has been tested to an amazing precision.[24]

---

[24] But maybe in the future we will measure deviations from linear quantum theory. There is the problem that general relativity has not been unified with quantum theory, but if a modification to quantum theory will resolve this problem, it must be a very small one that is not currently measurable. If that were true quantum mechanics could still be linear to an

If we don't give measurement devices or observers a special place in the theory one runs into other interpretational problems. Well known is the example of Schrödingers cat that evolves into the states dead and alive and "collapses" to one of these after the observer opens the box. How should such a superposition and its collapse be interpreted? Is the cat really in a superposition and does that mean the act of observing the cat's superposition could kill it?[25] One can also extend Schrödingers cat to an experiment called Wigner's friend.

In that experiment a friend of us performs the Schrödingers cat experiment in a room and we walk into the room after a while. Before we have walked into the room, we can describe it with a wave-function. According to the unitary dynamics of quantum mechanics, the wave-function evolves into a superposition of a sad friend/dead cat pair and a happy friend/alive cat pair. So again we run into trouble trying to interpret this superposition. Do we actually collapse our friend to a certain state after entering the room?

Stated like this it looks as if 'orthodox' quantum theory runs into big interpretational issues when the wave-function grows out into the macroscopic domain. Luckily for the Copenhagen interpretation the situation is not as bad as I portray it here. By decoherence it can be shown that macroscopic superpositions evolve into a classical state in an extremely small time-frame. Thus Schrödingers cat will never be found in a superposition and neither will we interfere with ourselves when we walk into a two-door bedroom.

Decoherence also partially solves the issue of when wave-function collapse occurs. We don't have to put measurement apparata or observers in a special place in the theory. Thus decoherence can partially resolve a lot of issues with the naive notion of wave-function collapse as it was first introduced in the Copenhagen interpretation. We also have less need for Bohr's complementarity principle, because we don't have to state that measurement 'defines' classical meaning. However we already made clear, decoherence does not resolve the measurement problem completely.

Even though a macroscopic object quickly decoheres into a statistical ensemble over classical states, we still need to interpret this ensemble in some way. There is a simple way to do this: we don't necessarily have to consider the wave-function or wave-function collapse as a 'real'[26] part of nature. Heisenberg for example considered the wave-function as representing our knowledge of a quantum system and he explained collapse as a change in knowledge of the experimenter about a quantum system.[27]

In this sense one can say that quantum mechanics predicts either a dead cat or an alive cat state, but that connecting the mathematics directly to the world is a misinterpretation of quantum theory. An instrumentalist view of quantum

---

extremely good approximation that could still allow macroscopic superpositions to emerge (though they would of course quickly decohere as usual).

[25] This is not an explanation that many physicist would want to take seriously, but it does show that one can easily draw strange conclusions from the Copenhagen interpretation.

[26] We will expand more on the discussion about realism when we start debating the many-world interpretation.

[27] Thus Heisenberg deviated in his views from Bohr. In literature there is sometimes confusion in describing the Copenhagen interpretation, partly because of this. One text might say measurement defines classical meaning, and another text might be agnostic about this.

mechanics also saves one from the measurement problem. As long as the wave-function is seen as a tool to give probabilities for measurements, there are no questions to ask.

However later on, in accordance with the views of David Wallace, we will argue that an instrumentalist view does not help in getting a realistic and proper understanding of the world. Furthermore we will try to show that when only considering the Schrödinger postulate, there is almost no room left to interpret the wave-function as only a mathematical tool. We will postpone this discussion for later, after we have discussed Wallace's arguments in favor of the many-worlds interpretation.

Therefore we conclude a bit prematurely by saying that when the wave-function is treated as a 'real' phenomena, in the sense that the wave-function exists in this world in some form and is directly (we could say one-to-one) related to the mathematics, the Copenhagen interpretation brings up difficult interpretational problems.[28]

---

[28] Of course the discussion is bigger than presented here, but personally I have come to the conclusion, influenced by Wallace's views, that arguments in favor of the Copenhagen interpretation always discard some sense of scientific realism. This will be clarified when we discuss the many-worlds interpretation later on. I don't expect the reader to take my word for it, but nonetheless I feel positive that you will not resolve for yourself all the conceptual issues with the Copenhagen interpretation. It is not without reason that so many alternatives have been coined over the years.

3 The Many-Worlds Interpretation

Hugh Everett III proposed in his 1957 Phd. thesis an alternative for the Copenhagen interpretation which he called "The Theory of The Universal Wave Function", and also the "Relative-State Formulation" [4] of quantum mechanics. Everett showed that, in theory, quantum mechanics could do without wave-function collapse by taking the whole universe as a universal quantum wave. Unfortunately Everett's early work was largely ignored, although his Phd. supervisor John Wheeler, a prominent physicist, arranged a few opportunities for Everett to explain his work to a bigger audience.

Everett's ideas came more into the mainstream after Bryce DeWitt popularized the term many-worlds in the early seventies, by publishing a few articles largely based on Everett's work. DeWitt's popularization and later talks by Everett finally brought way to new proponents of many-worlds, of whom David Deutsch from Oxford University stands out, who among other things advocates the many-worlds interpretation in his popular book 'The Fabric of Reality'. Deutsch' work has been expanded on in the last decade by David Wallace, also from Oxford, and Wallace's work is by some believed to contain the most detailed account of the Everett interpretation to date.

In the following section we'll focus largely on the book "The Emergent Multiverse" by David Wallace, that advocates the Many-Worlds interpretation of Quantum Mechanics. [8] In Wallace's words "The basic thesis of this book is that there is no quantum measurement problem." Like the many slightly different versions of the MWI coined over the years, Wallace follows the key idea that unitary quantum mechanics is sufficient to provide an interpretation of quantum mechanics on its own and there is no need to postulate wave-function collapse.

But firstly we'll take a look into Everett's relative-state formulation to lay down the basic insights of what would become the many-worlds interpretation. Although Everett did not talk about other worlds explicitly in his earlier articles, it is well believed that Everett did believe in the existence of other worlds where different quantum outcomes have occurred.

3.1 Everett's insight

In the following section we are going to discuss Everett's 1957 short-paper titled "Relative State Formulation of Quantum Mechanics". We are not going to discuss the whole paper and cut in some of its mathematics. Nonetheless we'll treat it thoroughly enough to get an introductory grasp of the many-worlds interpretation. Later we'll learn even more when we start taking a look at Wallace's book.

Those who are interested can read the original paper in a book by Bryce DeWitt, that contains a handy compilation of articles about many-worlds. The book is straightforwardly called "The Many-Worlds Interpretation of Quantum Mechanics". I have updated the notation Everett used for consistency with the rest of this thesis, and picked out the parts I believe are most important.

Relative-state formulation Firstly Everett outlines that the conventional or "external observation"[29] formulation of quantum mechanics is essentially the following: a physical system is completely described by a state function $|\psi\rangle$, which is an element of a Hilbert space, and which only gives information to the extent of specifying the probabilities of the results of various measurements which can be made on the system by external observers. Everett outlines two fundamentally different ways in which the state function can change:

Wave-function collapse The discontinuous change brought about by the observation of a quantity with eigenstates $|\phi_1\rangle, |\phi_2\rangle, \ldots$, in which the state $|\psi\rangle$ will be changed to the state $|\phi_j\rangle$ with probability $|\langle\psi|\phi_j\rangle|^2$.

Unitary dynamics The continuous, deterministic change of state of an isolated system with time according to a wave equation $\frac{\partial|\psi\rangle}{\partial t} = \hat{A}|\psi\rangle$, where $\hat{A}$ is a linear operator.

Everett claims that when there is only an approximate measurement the theory does not clearly specify what happens and what kind of a mixture between wave-function collapse and unitary dynamics should be used. Everett wants to work out the dilemma and states the goal of his paper: to regard pure wave mechanics, without wave-function collapse, as a complete theory of quantum mechanics.

In order to deduce how this is possible he first introduces the concept of relative states. Everett considers a composite system S, composed of two subsystems S_1 and S_2, with associated Hilbert spaces $\mathcal{H}_1$ and $\mathcal{H}_2$. If $\{|\xi_i^{S_1}\rangle\}$ and $\{|\eta_j^{S_2}\rangle\}$ are complete orthonormal sets of states for S_1 and S_2, respectively, then the general state of S can be written as a superposition:

$$|\psi^S\rangle = \sum_{i,j} a_{ij} |\xi_i^{S_1}\rangle |\eta_j^{S_2}\rangle \tag{3.1.1}$$

Although by their entanglement the subsystems S_1 and S_2 do not possess definite states independently of one another (except when all but one of the a_{ij} are zero), we can uniquely assign for any choise of state in one subsystem a corresponding relative state in the other subsystem. Working out the relative state of system S_2 for a state $|\xi_k^{S_1}\rangle$ becomes

$$|\psi^{S_2}(\text{rel.}\,|\xi_k^{S_1}\rangle)\rangle = \langle\xi_k^{S_1}|\psi^S\rangle = N_k \sum_j a_{kj} |\eta_j^{S_2}\rangle\,, \tag{3.1.2}$$

where N_k is a normalization constant. In the Copenhagen interpretation the relative state in S_2 for a state $|\xi_k^{S_1}\rangle$, is interpreted as the conditional probability distribution for the results of all measurements in S_2, given that S_1 has been measured and found to be in state $|\xi_k^{S_1}\rangle$.

We can write down the original wave-function using relative states:

$$|\psi^S\rangle = \sum_i \frac{1}{N_i} |\xi_i^{S_1}\rangle |\psi^{S_2}(\text{rel.}\,|\xi_k^{S_1}\rangle)\rangle \tag{3.1.3}$$

Subsequently Everett makes a proposal to how this relates to observation. He denotes an observer whose memory contains representations of the events

---

[29] The Copenhagen interpretation.

$A, B, \ldots, C$ by $|\psi^0_{[A,B,\ldots,C]}\rangle$. The symbols $A, B, \ldots, C$ are assumed to be ordered time-wise and can be considered as punches in a paper tape, impressions on a magnetic reel, or configurations of brain cells.

Everett considers a "good" observation of a quantity $\hat{A}$ of a system S in an eigenstate $|\phi_i\rangle$ by an observer with initial state $|\psi^0\rangle$, as an interaction that transforms each

$$|\psi^{S+0}\rangle = |\phi_i\rangle\, |\psi^0_{[\ldots]}\rangle \tag{3.1.4}$$

into a new state

$$|\psi^{S+0'}\rangle = |\phi_i\rangle\, |\psi^0_{[\ldots\alpha_i]}\rangle \tag{3.1.5}$$

where α_i is the eigenvalue corresponding to $|\phi_i\rangle$. The state $|\psi^{S+0'}\rangle$ is characterized by α_i, such that $|\psi^0_{[\ldots\alpha_i]}\rangle$ is a *different* state for each i. If the system S is not in an eigenstate, but a general state $\sum_i a_i\, |\phi_i\rangle$, the final total state will have the form

$$|\psi^{S+0'}\rangle = \sum_i a_i\, |\phi_i\rangle\, |\psi^0_{[\ldots\alpha_i]}\rangle \tag{3.1.6}$$

Using the previous results Everett derives two straightforward rules.

Rule 1 The observation of a quantity $\hat{A}$, with eigenfunctions $|\phi_i^{S_1}\rangle$, in a system S_1 by the observer, transforms the total state according to:

$$|\psi^{S_1}\rangle\, |\psi^{S_2}\rangle \ldots |\psi^{S_n}\rangle\, |\psi^0_{[\ldots]}\rangle \to \sum_i a_i\, |\phi_i^{S_1}\rangle\, |\psi^{S_2}\rangle \ldots |\psi^{S_n}\rangle\, |\psi^0_{[\ldots,a_i]}\rangle\,, \tag{3.1.7}$$

where $a_i = \langle\phi_i^{S_1}|\psi^S\rangle$.

Rule 2 Rule 1 may be applied separately to each element of a superposition of total system states. Thus a determination of $\hat{B}$ with eigenfunctions $|\eta_j^{S_2}\rangle$, on S_2 by the observer transforms

$$\sum_i a_i\, |\phi_i^{S_1}\rangle\, |\psi^{S_2}\rangle \ldots |\psi^{S_n}\rangle\, |\psi^0_{[\ldots]}\rangle \tag{3.1.8}$$

into the state

$$\sum_{i,j} a_i b_j\, |\phi_i^{S_1}\rangle\, |\eta_j^{S_2}\rangle\, |\psi^{S_3}\rangle \ldots |\psi^{S_n}\rangle\, |\psi^0_{[\ldots\alpha_i,\beta_j]}\rangle\,, \tag{3.1.9}$$

where $b_j = \langle\eta_j^{S_1}|\psi^S\rangle$, which follows straightforwardly from the application of Rule 1 to each element $|\phi_i^{S_1}\rangle\, |\psi^{S_2}\rangle \ldots |\psi^{S_n}\rangle\, |\psi^{S_0}_{[\ldots\alpha_i]}\rangle$ and then superposing the results with the coefficients a_i.

Now that the formal notation is laid down Everett proposes how it can be used to analyze general measurement scenario's. For this we again consider an observer system S_0 in the initial state $|\psi^{S_0}_{[\ldots]}\rangle$. Furthermore we arrange our experiment such that we measure the same quantity $\hat{A}$ in a number of separate but identical systems, which are initially in the same state given by

$$|\psi^{S_1}\rangle = |\psi^{S_2}\rangle = \ldots = |\psi^{S_n}\rangle = \sum_i a_i\, |\phi_i\rangle \tag{3.1.10}$$

The initial state function of the measurement setup becomes

$$|\psi_0^{S_1+S_2+\ldots+S_n+0}\rangle = |\psi^{S_1}\rangle \ldots |\psi^{S_n}\rangle\, |\psi^0_{[\ldots]}\rangle \tag{3.1.11}$$

We assume that the measurements are performed in the order $S_1, \ldots, S_n$. The state after the first measurement is then given by Rule 1

$$|\psi_1^{S_1+S_2+\ldots+S_n+0}\rangle = \sum_i a_i\, |\phi_i^{S_1}\rangle\, |\psi^{S_2}\rangle \ldots |\psi^{S_n}\rangle\, |\psi^0_{[\ldots,\alpha_i^1]}\rangle \tag{3.1.12}$$

In general we can apply Rule 2 to get the final state after r measurements, given by

$$|\psi_r\rangle = \sum_{i,j,\ldots,k} a_i a_j \ldots a_k\, |\phi_i^{S_1}\rangle \ldots |\phi_k^{S_r}\rangle\, |\psi^{S_{r+1}}\rangle \ldots |\psi^{S_n}\rangle\, |\psi^0_{[\ldots,\alpha_i^1,\alpha_j^2,\ldots,\alpha_k^r]}\rangle \tag{3.1.13}$$

This result gives way to Everett's final conclusion. Firstly we note that $|\psi_r\rangle$ consists of a superposition of states

$$|\psi'_{ij\ldots k}\rangle\, |\psi^0_{[\alpha_i^1,\alpha_j^2,\ldots,\alpha_k^r]}\rangle = |\phi_i^{S_1}\rangle\, |\phi_j^{S_2}\rangle \ldots |\phi_k^{S_r}\rangle \otimes |\psi^{S_{r+1}}\rangle \ldots |\psi^{S_n}\rangle\, |\psi^0_{[\alpha_i^1,\alpha_j^2,\ldots,\alpha_k^r]}\rangle\,, \tag{3.1.14}$$

where each of the states $|\psi^0_{[\alpha_i^1,\alpha_j^2,\ldots,\alpha_k^r]}\rangle$ describes an observer state with a definite memory sequence $[\alpha_i^1, \alpha_j^2, \ldots, \alpha_k^r]$. Furthermore, if we take into account that these observer states are orthogonal because of their macroscopic size[30] the relative state of any observer state $|\psi^0_{[\alpha_i^1,\alpha_j^2,\ldots,\alpha_k^r]}\rangle$ in the final superposition is given by

$$|\psi'_{ij\ldots k}\rangle = |\phi_i^{S_1}\rangle\, |\phi_j^{S_2}\rangle \ldots |\phi_k^{S_r}\rangle \otimes |\psi^{S_{r+1}}\rangle \ldots |\psi^{S_n}\rangle \tag{3.1.15}$$

Thus relative to an observer with a memory sequence $[\alpha_i^1, \alpha_j^2, \ldots, \alpha_k^r]$, the (observed) system states are the corresponding eigenfunctions $|\phi_i^{S_1}\rangle$, $|\phi_j^{S_2}\rangle$, …, $|\phi_k^{S_r}\rangle$.

We see that any substate of the total superposition $|\psi_r\rangle$ describes a perfectly realistic measurement scenario, where the observer has a memory sequence that is in agreement with the state of the measured system relative to him/her. To get to this result we didn't need to invoke wave-function collapse and everything followed from the unitary dynamics.

Everett argues that if we would take the place of the observer in any of these substates, we would be in a state of affairs wherein we have perceived an apparently random sequence of definite results for the measurements we performed. Of course every outcome has a place in the final superposition, but to any of the observer states it seems as if some kind of wave-function collapse has occured which has led to their particular measured sequence of results.

Now with our previous discussion in mind the road to regarding pure wave-mechanics as a complete theory has almost been fully traveled. Lastly we have to show why we measure some results with higher probability than others. Everett

---

[30] Which we can rigorously prove using decoherence

closes his paper by seeking a general scheme to assign a measure $m(a_i)$ to the elements in a superposition of orthogonal eigenstates $\sum_i a_i |\phi_i\rangle$.

Everett argues that we cannot measure the phase factors of the complex amplitudes a_i, and that we must have $m(a_i) = m(|a_i|)$. Furthermore Everett assigns an additivity requirement such that

$$m(\alpha) = \sum_{i \in \alpha} m(a_i) \tag{3.1.16}$$

He then shows that the only measure satisfying this criteria has the property $m(a_i) = c a_i^* a_i$. Thus the only measure possible is a square amplitude measure with an arbitrary constant that may be normalized. Having deduced this we see that this measure assigns to the $i, j, \ldots k$th element of $|\psi_r\rangle$, the measure weight

$$M_{ij\ldots k} = (a_i a_j \ldots a_k)^* (a_i a_j \ldots a_k) \tag{3.1.17}$$

and this gives the same prediction as the Copenhagen interpretation: the probability of measuring an eigenvalue α_i of an observable $\hat{A}$ is weighted by the square amplitude $|a_i|^2$ of the eigenstate $|\phi_i\rangle$ in the superposition.

It is important to note that Everett does not prove that such a probability measure should actually be introduced. However he does show that if a probability measure is to be introduced, it must be a square amplitude measure. Because of this Everett has not shown that pure wave mechanics is indeed a complete theory, though he does make it very plausible. In this thesis I will not spend much time discussing the problem of probability in the no-collapse interpretations, though I will briefly discuss how David Wallace attempts to solve the problem later on.

Everett's paper is the first of the so called no-collapse interpretations of quantum mechanics, which view every physical system, including the whole universe, as a single massively entangled quantum system. Everett's paper is generally interpreted in two different ways. Firstly there is the branch of the so called 'modal' interpretations which claim that every term in the superposition represents a possible world, of which only one will truly come to existence. Secondly there is a branch of many-worlds interpretations which assign realism to every term in the superposition and claim they are all equally real.[31]

Everett himself doesn't speak about the ontology of the states in $|\psi_r\rangle$. This might be explained because it was safer for Everett to remain agnostic about the actual existence of the states in a quantum superposition. In that way his paper couldn't easily be discredited for making speculative claims. However, it is generally believed from discussions later in his life that Everett favored the idea that other worlds, where different quantum outcomes have occured, do exist.

In the next section we'll take a look at Wallace's attempt to build on the ideas introduced by Everett. Wallace believes that realism is a decisive element in

---

[31] There is really only one many-worlds interpretation, but there are also the closely related consistent-histories interpretation and the many-minds interpretation.

choosing between the modal interpretations and the many worlds interpretation.[32]

3.2 Wallace's views in his 'Emergent Multiverse'

We are going to take a look at Wallace's book 'The Emergent Multiverse', which can be currently be considered as the biggest attempt in advocating the many-worlds interpretation to the general physics community. It is important to note that this section is not meant as a discussion about Wallace's argument, but is meant to lay down what Wallace's views are. In that way I hope you can get a reasonable idea of Wallace's arguments so that we can critically discuss them later on in the section 'The Many-Worlds Debate and Wallace'.

Wallace's book is structured in three major parts. Part 1 starts by laying out the general framework and giving a philosophical account of why quantum mechanics leads to 'many worlds'. Part 2 is exclusively concerned with issue of probability in the MWI, which is the issue of why the squared amplitudes of the terms in the wave function give probabilities for measurements/worlds and how such probabilities should be interpreted. After these foundational works have been laid down, Part 3 lastly focuses on some of the consequences of the MWI for our scientific world view, such as our views on uncertainty, identity and the quantum state in space time.

When we take a critical look at the MWI and Wallace's arguments in particular we will mostly focus on Part 1 of Wallace's book, and only briefly on Part 2. The reason for this is that Part 2, where Wallace derives the Born probability rule from unitary dynamics, is currently considered the most controversial in the physics community. However it is generally agreed upon that a complete notion of the MWI must somehow be able to derive the Born-rule from unitary dynamics, which is why Wallace's spends so much of the book in trying to prove it. Though part 3 contains a number of interesting philosophical outlooks on reality, we are also not going to discuss it any further in this thesis.

Nonetheless, if we can justify at least Part 1 of Wallace's book we already provide a good motivation for pursuing further research into the MWI, which I believe is currently the most important. Let's therefore start by taking a look at Part 1 of Wallace's book, which composes the first three chapters of Wallace's book.

The Paradox of Measurement In the first chapter, "The Paradox of Measurement", Wallace sets out the general framework for his book. He begins by laying down his views of science, which make him believe that the many-worlds interpretation is the most straightforward interpretation of quantum theory.

Wallace's view of science is a species of realism. He argues that the point of molecular biology, of high-energy astrophysics, of economics, or of any other

---

[32] Here realism is understood as a concept in the philosophy of science, where it is usually described as the view that the world is objective and independent of how we think of it, or how we describe it.

sort of science is to understand the way the world is and to give us information about the structure of the world. A bit tongue-in-cheek, he makes a comparison between quantum physics and palaeontology.

He explains that very little has ever been written, and very little needs to be written, on the interpretation of palaeontology. There is simply no serious scientific debate on the ontology of dinosaurs. Wallace argues that this is because nobody distinguishes the theory that dinosaurs existed from the 'theory' that dinosaurs didn't exist but fossils are exactly the way they would have been if dinosaurs had existed. (pg. 11)

Wallace wants the situation to be the same for quantum mechanics, and he believes the MWI can take a similar place in quantum theory as dinosaurs can in palaeontology. However he argues the problem with quantum mechanics in its current form is that it fails in telling us objective facts about the world. At best, it seems to be telling us about the results of experiments we can perform, and this in an ad hoc way. (pg. 13)

Wallace argues that 'measurement' cannot be represented physically. To illustrate this he considers the system

$$U\;|+\rangle\otimes|\text{'ready'}\rangle=|\text{'up'}\rangle\qquad U\;|-\rangle\otimes|\text{'ready'}\rangle=|\text{'down'}\rangle\,,\tag{3.2.1}$$

where U denotes the unitary dynamics that work on the system. By the linearity of unitary quantum mechanics we must have that

$$U\left(\alpha\,|+\rangle+\beta\,|-\rangle\right)\otimes|\text{'ready'}\rangle=\alpha\,|\text{'up'}\rangle+\beta\,|\text{'down'}\rangle\tag{3.2.2}$$

We know that an actual measurement of the system can yield only one result, so we need an explanation for the fact that only one result is observed. Furthermore we need to explain why the observed results are probabilistic and depend on the mod-square amplitude of α and β. (pg. 23)

This is of course the measurement problem for which numerous solutions have been proposed. Wallace gives a few short arguments for why the other interpretations of quantum mechanics[33] should be put aside, but he doesn't spend much time on it as it would distract from the crux of the matter: unmodified unitary quantum mechanics (without wave function collapse) can yield an interpretation on its own. According to Wallace the only catch is that the universe turns out to consist of many parallel worlds.

Wallace then begins exploring the 'Everett interpretation' of quantum mechanics. Using Wallace's exact words, he summarizes Everettian quantum mechanics by saying: it consists of two very different parts: a contingent physical postulate, that the state of the Universe is faithfully represented by a unitary evolving quantum state; and an a priori claim about that quantum state, that if it is interpreted realistically it must be understood as describing a multiplicity of approximately classical, approximately non-interacting regions which look very much like the classical world'. (pg. 38)

Wallace goes on to outline the main issues with such an interpretation. He makes a distinction between two problems, which he calls the *ontological* and

---

[33] e.a. objective-collapse, information theoretic interpretations, de Broglie-Bohm theory.

the *propability* problem. (pg. 39) The first one addresses the issue of regarding quantum superpositions as describing multiplicity. The second one is concerned with the concept of probabilities in the Everretian interpretation.

To consider the issue of ontology Wallace makes a quick comparison to classical mechanics. He imagines a situation where we have two electromagnetic fields $\boldsymbol{F_1}(x,t)$ and $\boldsymbol{F_2}(x,t)$, that represent a pulse of ultraviolet light zipping between Earth and Moon, and a pulse of ultraviolet light between Venus and Mars respectively. He goes on to consider the superposition

$$\boldsymbol{F}(x,t) = 0.5\boldsymbol{F_1}(x,t) + 0.5\boldsymbol{F_2}(x,t) \tag{3.2.3}$$

and argues in a way similar to the Copenhagen interpretation: What weird sort of thing is this? Must it not represent a pulse of ultraviolet light that is in a superposition of traveling between Earth and Moon, and of traveling between Mars and Venus? How can a single pulse of ultraviolet light be in two places at once? (pg. 36)

Wallace explains that such reasoning is of course nonsense. There is a perfectly good description of $\boldsymbol{F}(x,t)$: it does not describe a single ultraviolet pulse in a weird superposition, it just describes two pulses. Wallace wants to argue that this is what the Everett interpretation claims about macroscopic quantum superpositions: they are just states of the world in which more than one macroscopically definite thing is happening at once.

Lastly Wallace briefly touches on some other issues with the MWI. He responds to the question of the preferred basis: if superpositions are supposed to represent distinct worlds, with respect to which basis are theses superpositions defined? (pg. 39) But he argues that asking 'which basis' distracts from the real question: what justifies regarding a quantum state as a collection of quasi-classical worlds in the first place? Wallace explains the justification must come from within quantum theory itself and in the next chapters he will show how to do this.

The Emergence Of Multiplicity In the next chapter 'The Emergence Of Multiplicity', Wallace addresses the ontology problem from a few different angles. Firstly he discusses whether the other worlds should explicitly be postulated from the theory. One might argue that either the axioms of quantum theory must be modified to include explicit mention of 'multiple physical worlds', or the existence of these worlds must be some kind of illusion. (pg. 47)

But Wallace argues there is no need for this. He believes it is untrue that any entity not directly represented in the axioms of a theory is an illusion. Rather, science is full of entities which are nowhere to be found in the underlying microphysics. Wallace explains that the generic philosophy-of-science term for such entities is that they are emergent. They are not directly definable in the language of microphysics, but do emerge from those underlying laws.

As an example Wallace considers the Bengal tiger, which consists of many quarks, electrons and the like in the Standard Model, even though the Standard Model doesn't contain direct references to tigers. (pg. 48) According to Wallace tigers can be understood as patterns, or structures, that emerge from states of that microphysical theory.

Furthermore we could wonder about the 'reality' of such higer-level descriptions of the world, but Wallace answers that this depends on the usefulness of the pattern identified. He supports his view by stating a position of philosopher Daniel Dennett that he can relate to:

Dennett's criterion A macro-object is a pattern, and the existence of a pattern as a real thing depends on the usefulness–in particular, the explanatory power and predictive reliability–of theories which admit that pattern in their ontology. (pg. 50)

Wallace explains we cannot really understand a tiger by looking at it from the viewpoint of molecular physics, or by viewing it in isolation without its environment and the other animals. Therefore we generally seek to describe the tiger at a more functional level, for example by describing the behaviour of the tiger using zoology. In Wallace's terminology this means that molecular physics *instantiates* zoology in the case of the Bengal tiger.[34] (pg. 53)

Now similar to what we're used to in classical physics we can use our understanding of instantiation and higher-level descriptions of physical systems to make claims about Schrödingers cat in the box. Wallace explains that thermodynamics predicts the entropy of the box will go up as long as it's isolated, from cat psychology we know the cat will likely fall asleep[35], and from cat physiology we know that the cat won't grow another tale. (pg. 59)

Wallace argues that the explanatory power of these higher-level descriptions give way to regarding the cat as an objectively real structure instantiated by the box's constituents. He concludes by saying that if we apply the same principles to quantum mechanics as we apply in general through science to identify higher-level ontology, we find that when both the histories $|\mathrm{cat}_l(t)\rangle$ and $|\mathrm{cat}_d(t)\rangle$ represent a state of affairs where the system in question is structured like a cat, they represent a state of affairs where the system in question is a cat. (pg. 60)

This also makes us understand Wallace's claim that there is no preferred basis problem. Following his reasoning it doesn't matter if we can write the quantum state using any other decomposition, because in the end that decomposition will just be another form of writing the same structure that instantiates the classical description of a cat.

At the end of the chapter Wallace does point out a crucial detail: neither in the case of electromagnetism or quantum theory does the presence of multiplicity follow merely from linearity. He explains that way lies contradiction if

$$|{\uparrow}\rangle \otimes |\mathrm{cat}_d(t)\rangle + |{\downarrow}\rangle \otimes |\mathrm{cat}_l(t)\rangle \tag{3.2.4}$$

---

[34] Wallace gives a big and delicate exploration of what instantiation is, and the explanation in this paragraph is simplified. Nonetheless I feel personally that the whole debate about emergence and instantiation distracts from the real discussion about the ontology of the quantum state itself. However we will postpone critically assessing Wallace's arguments on that subject for the next chapter.

[35] Actually experiences with my own cats make me believe the cat will try to get out of the box, but maybe Wallace's cats are more lazy than mine.

instantiates both a live cat and a dead cat merely because it is a superposition of a live-cat state and a dead-cat state. Then so presumably does

$$(|\uparrow\rangle \otimes |\text{cat}_d(t)\rangle + |\downarrow\rangle \otimes |\text{cat}_l(t)\rangle) + (|\uparrow\rangle \otimes |\text{cat}_d(t)\rangle - |\downarrow\rangle \otimes |\text{cat}_l(t)\rangle) \quad (3.2.5)$$

But this (up to normalization) is just another way of writing the dead-cat state. (pg. 62)

Wallace resolves the issue by saying that there is in general no interference between macroscopic states. For this he introduces the phenomenon of decoherence which in general prevents macroscopic degrees of freedom of quantum systems from interfering. If he can motivate decoherence, it is guaranteed that structures instantiated by the macroscopic degrees of freedom of quantum systems are not erased when those systems are in superpositions of macroscopically definite states. (pg. 62) He devotes the next chapter to that task.

Chaos, Decoherence and Branching In the third chapter 'Chaos, Decoherence and Branching' Wallace makes his final steps in his solution to the ontological problem. This chapter of Wallace's book is the last chapter we will explicitely treat for our later discussions. Firstly Wallace summarizes the findings of Chapter 2 that

- Certain quantum-mechanical histories of certain systems instantiate– simulate, if you like– a quasi-classical history.
- Superpositions of those histories then instantiate multiple quasi-classical histories– always assuming that interference between histories can be neglected.

He then explains that the purpose of this chapter is to go from the rather hand-waving notion of the emergence of worlds to something much more quantitative and precise.

First he shows how quasi-classicality appears in simple isolated systems. Wallace shows that the quantum phase-space has the property of basis-preservation for reasonably localized quantum states. Because of this, various classical histories instantiated by different wave-packet states can coexist. (pg. 65-68)

However not all macroscopic states remain reasonably localized, with the exception being chaotic systems. We can show that the time t at which a chaotic system starts behaving non-classically is given by:

$$t > \tau_L \ln\left(L\sqrt{\frac{m}{\hbar\tau_L}}\right), \quad (3.2.6)$$

where τ_L is the Lyapunov exponent, depending on the dynamics of the chaotic system. A quantitative example yields that a dust mote of mass 10^{-12} kg which experiences chaotic dynamics, has a Lyapunov timescale of approximately 10 seconds and will cease to behave classically after 250 seconds. (pg. 73)

Thus Wallace argues there remain problems with the naïve recovery of quasi-classicality. He summarizes the three biggest problems as: (pg. 76)

1. It is inaccurate to treat the macroscopic degrees of freedom of a system as dynamically isolated from its residual degrees of freedom.[36]
2. Chaotic quantum systems do not behave quasi-classically over sufficiently long timescales.
3. In situations where the dynamics are not even approximately classical, we have no reason to assume a macroscopic quantum system can still be viewed as a collection of quasi-classical systems.

After pointing out these problems Wallace argues that the theory of environmental induced decoherence will solve all these problems. Though we will not discuss Wallace's theoretical treatment of decoherence, as we already discussed decoherence in the chapter about the Copenhagen interpretation, we will focus on a number of important conclusions Wallace draws from decoherence theory.

Firstly Wallace shows by a quantitative example that Schrödingers cat would endure in a macroscopic superposition for only approximately 10^{-35} seconds. Because of this we are justified in considering different states of macroscopic objects as orthogonal and interference terms disappear. (pg. 80) Furthermore Wallace goes on by showing rigorously how a branching structure emerges from quantum mechanics by environmentally induced decoherence. (pg 87-99)[37]

After having developed this branching structure, he can focus on some important questions relating to the number of other worlds. Wallace makes a distinction between three kind of processes that cause branching: deliberate human experiments, natural quantum measurements such as when radiation causes cell mutation, and classically chaotic processes. (pg. 99) Wallace explains there is no sense in which these phenomena lead to a *discrete* branching process.

This is an important insight for physicists just introduced to the MWI, who are tempted to ask how many branches there are. Wallace explains that while a branching structure can be discerned, it has no natural 'grain'. We can just continue in fine-graining our decoherent history space[38] until a point comes where interference between branches ceases to be negligible. Thus we cannot ask how much branches there are where it is sunny, but we could say the combined weight of the sunny branches is for example 0.7. (pg. 100)

The Born Rule and Beyond We have now summarized Wallace's arguments for resolving the ontological problem. The biggest part of Wallace's book is subsequently focused on resolving the probability problem. The probability problem is the last significant issue that remains in the many-worlds interpretation, and also partly in the modal interpretations.[39]

---

[36] As an example Wallace considers it question-begging to consider a 'rigid body' as 'rigid', because the very claim that the body is 'rigid' should be derivable from the underlying physics of its constituents.

[37] I will leave it too you to read this by yourself if you are interested.

[38] Wallace introduces this terminology to describe the branching structure as part of a so called decoherent history space, which roughly said contains all the possible histories of quantum events.

[39] In the modal interpretations we don't have the problem that very low-probability outcomes are considered true, but the problem of justifying the use of the Born postulate does remain.

Response to Wallace's derivation of the Born-rule[40] in the scientific community has been mixed, as we'll see later on. To go into a proper discussion of Wallace's arguments would require a much bigger thesis, and I will not give a summary of Wallace's arguments on the probability problem in this section. However, at the end of the next chapter I will give a quick outline of how Wallace tries to resolve the probability problem and I will briefly discuss some of the criticism that has been brought up against his derivation over the years. In the end I do not believe the probability problem is big enough to discard the many-worlds interpretation.

---

[40] By which we mean the Born postulate from the first chapter.

4 The Many-Worlds Debate and Wallace

Now that we have given a brief account of the views expressed in Wallace's book, it is time to take a critical look at his arguments and reasoning. We'll firstly discuss the ontological problem and focus on his realist views and the role they play in the many-worlds interpretation. Furthermore we will discuss the role of indeterminism in quantum mechanics and clarify the link between realism and (in)determinism.

After that we will attribute a short section to describing the probability problem and some criticism that has come up against Wallace's derivation of the Born rule. We'll conclude that the probability problem is the only major obstacle left for the many-worlds interpretation.

Sometimes we will accompany the discussion with examples and thought experiments. Though there is not always a single best way to debate a theory I have tried choosing the examples discussed and the counterarguments against Wallace in such a way to highlight most of the issues that are at stake with Wallace's interpretation.

4.1 The ontological problem

To discuss the ontological problem we'll focus on Wallace's realist stance in physics, which can be seen as the most philosophical part of justifying the many-worlds interpretation. Even though, it forms an important part in clarifying why one would opt for a many-worlds interpretation. Especially considering that an instrumentalist view of quantum mechanics is complete enough to give precise predictions of microscopic experiments.

According to Wallace, the position of realism is strong enough to justify the many-world interpretation. I will argue that this isn't the case, but that realism combined with the a priori requirement of having a step-by-step reductive and deterministic description of the multiverse does provide a strong motivation for the MWI. For this I will argue that Wallace's arguments for emergence also rely implicitely on this assumption. Wallace does claim in his book that many-worlds is a natural consequence of a realist stance in physics, but I will try to put some doubts on this claim by arguing how ones views of realism could differ from those of his.

I suspect that Wallace does not bring up a deterministic multiverse as an a priori argument in his book because he wants to find a single simple and clear motivation for the MWI. To dare and try coming up with such a view is respectable in itself, and furthermore I believe he does a very good job at making a case for many-worlds. However, since his book has raised plenty of doubts in the scientific community too, going deeper into the ontological problem and questioning some of his realist ideas can make the discussion clearer for physicists outside of the MWI debate. Therefore approaching the problem from a few different angles actually adds to the theory instead of making it less elegant.

4.1.1 Realism of the wave-function

In 'orthodox' quantum theory different stances are taken towards the reality of the wave-function. Firstly there is a group of physicists that consider the wave-function as a real element of reality and its collapse as a physical process. Off course there is still an ambiguity as to what is considered 'real', but usually it is meant that the mathematics that describes the wave-function can be one-to-one connected to our universe. Then there is another group of physicsts which see the wave-function more as a statistical tool or as a way of representing our knowledge of a quantum system. This has the advantage of evading interpretational problems with for example macroscopic superpositions.

I will argue in favor of Wallace and the first group that the wave-function should be considered as a real part of nature and not as a mathematical tool. However, I am a bit afraid that speaking about what is 'real' and what is not could scare off physicists who have no interest in philosophy. A physicist that purely focuses on mathematics can already use quantum mechanics to explain what he sees in the world, without relying on Copenhagen or many-worlds. Therefore he might wonder why this is not enough to satisfy all physicists.

It might seem that these unsatisfied physicists merely take the role of a classical philosopher by claiming we need a 'more complete' picture of the world[41], but in fact the decision between many-worlds and Copenhagen or other interpretations is not much more philosophical than other areas of science. Nothing stops us from asking questions in classical physics like 'is a force real?', or 'what makes the properties of an elementary particle real?'

However, usually we take ontology of classical objects for granted. This might be related to the fact that the epistemological problem, of how much knowledge we can acquire of the world, is less of a threat in classical mechanics than in quantum mechanics. The position and momentum of classical objects are definitely measurable and if it wasn't for quantum mechanics, we wouldn't have a strong reason to question the existence of these properties even when they aren't observed.

Now in quantum mechanics the ontological and epistemological problem become a bit entangled[42]. Because of this the position has arisen that nature doesn't lend itself to a complete description and that indeterminism is a fundamental part of the way the universe is. However, the many-worlds interpretation has the power to put a different perspective on this debate. Though it doesn't let us predict the exact outcomes of quantum phenomena, just like 'orthodox' quantum mechanics can't do that, it does provide an answer to ontogical questions about the universe.

---

[41] This could explain why many physicists and physics students seem reluctant to accept the many-worlds interpretation. To them it might look as if a preference for an interpretation of quantum mechanics is only part of a philosophical discussion which is not directly related to the physics. It doesn't seem directly obvious that changing wave-function collapse to a scenario consisting of infinitely many other worlds that are non-interacting with ours makes our theory any simpler. Besides, one might get the idea that the discussion is about whether this is a 'good' or 'bad' thing, which makes things even more vague. However, as we will see luckily, the discussion is much richer than this.

[42] Pun intended.

A difference between quantum physics and classical physics is of course that the wave-function can't be measured directly like the objects in classical physics. I consider that an epistemological difference, and except for that the discussion about realism in quantum physics is very much alike the discussion in classical physics.[43] Therefore when we argue if the wave-function is 'real', we can use the word 'real' in the most intuïtive way. We don't ask ourselfs, is it real in our mind, is it real as a concept, we ask ourselves: is there really, let's call it physically, a wave in the world that follows the Schrödinger equation?

Wallace proposes that the position of realism can justify the many-worlds interpretation, and he gives arguments for this from a few different angles. Firstly he talks about why one wants a realist interpretation of the world. Secondly he talks about why a realistic interpretation of the world must imply interpreting a quantum mechanical superposition as yielding multiplicity instead of indeterminicity. We are going to put some doubts on these claims, but also expand on them to fill possible gaps in Wallace's arguments. Let's discuss first why we want a realistic interpretation of the world.

Realism and the purpose of science In the section 'Problems with the Copenhagen interpretation' we tried to argue that quantum theory as it is most commonly taught nowaday does not provide a sufficient picture of reality. Despite this, the formalism of quantum mechanics is currently complete enough for a pragmatic use of the theory regardless of our interpretation. Therefore we will discuss why a physicist should still consider other interpretations like many-worlds.

Furthermore we will motivate Wallace's view that science is there to give a description of the world and not of the outcomes of measurements. Thus we'll motivate asking questions about the mathematics of quantum mechanics, even if it might not give us new answers to practical problems.[44] Later on we will discuss whether this position will make us draw the same conclusions as Wallace.

Of course opinions about why or why not to pursue (certain areas of) science cannot be arguments for physical theories, but they can serve as motivations for pursuing them. Having said that, I personally just like doing physics exactly because I want to get an understanding of the world.[45] That's why I seek out and search for the answers, and whether that is a result of evolution or a higher sense of curiosity that exists in the universe does not concern me.

---

[43] In this stance I am influenced by Wallace and loosely follow his views in the first chapters of his book.

[44] And that what might not seem of practical use at first might turn out to be of use later, since we are unable to predict the future. It could very well be that the many-worlds interpretation yields insights that help in resolving other problems with quantum mechanics, like the unification of QM and GR.

[45] I cannot motivate why I like this other than that I 'like' this. In the same way I cannot describe to you my quale of the color red. But what I can say to you is that when I want to discover something, I want to try and form a coherent theory of it that does not rely on my brain splitting in two parts: one of which believes A and one of which believes B without me trying to make A and B consistent with each other. I fear the Copenhagen interpretation does rely on such inconsistencies, for example in the case of wave-function collapse that completely contradicts the unitary postulate.

If you only need quantum mechanics in a pragmatic way, then use it that way. Many people don't know about the theory of relativity and they don't need to, because it is not important for their daily life. But I believe a (theoretical) physicist should not only be driven by what is practical for him/her, but also by a curiosity about the world. This doesn't mean the physicist has to resort to unjustified speculation, but we'll find out that for the MWI no such thing is needed.

There is however a complication with Wallace's stance that the purpose of science is to give an understanding of the world. We can ask, what kind of understanding is meant? Our world-view doesn't have to be much different from instrumentalism if it's meant that we can describe all the properties of the world. In fact, we might wonder what there could possibly be more to the world than the properties itself? If we could somehow get a hold of all the properties of the world in any exact situation there would be nothing more to add. So if the world has indeterministic properties, could we then conclude a probabilistic theory gives a realistic view of the world?[46]

The position of scientific realism encompasses a large range of philosophical issues on which different stances can be taken. Therefore, I do not believe realism can serve as a direct justification for the many-worlds interpretation. The fact that we want to interpret quantum mechanics realistically does not imply we have to interpret the wave-function realistically. We will expand on this issue in the section 'Realism and the MWI'.

To justify seeking another leading interpretation for quantum mechanics we must also ask ourselves: where do the differences lie in the variants of quantum mechanics? Certainly it is not in the mathematics, although the answer to the question 'why does the world work like this?' can be answered in completely different ways. This might make us conclude that bowing towards any particular interpretation can only be decided upon by 'subjective' speculation, unless maybe a theory pops up that provides us with a distinct description of a particular experiment so that the difference can be finally be decided upon by measurement.

Furthermore it seems that in the many-worlds interpretation such a thing is futile at the present. So what then? Are we just left with an interpretation that predicts the same as the Copenhagen interpretation but has a different internal structure?[47] We could argue that Wallace's realist world-view is right or wrong, or (in)comparable with the role of realism in classical mechanics, but in the end any such analysis might fall completely to the ground if many-worlds would make a testable and unique prediction in the future that distinguishes it from the other interpretations. Therefore we could wonder if giving arguments for the MWI has any purpose at all!

However we will try to show that the MWI has a number of important theoretical advantages over the Copenhagen interpretation. Furthermore we will

---

[46] Personally I don't believe we could state it this bluntly, but to say that there is only one way to realistically view the world wouldn't be right. This is something I fear Wallace does a bit in his book, by using realism selectively in such a way to support his own views.

[47] Not exactly, there are physicists who argue that the MWI makes distinct predictions from the collapse interpretations, but such arguments are still speculative. As an example, a proposal from David Deutsch relies on a self-aware computer with artificial intelligence.

dedicate a later section on outlining some possibilities for doing measurements that distinguish the many-worlds interpretation from others. In doing that we will also discuss how certain experiments have an arguably better explanation in the many-worlds interpretation (e.g. quantum computing is sometimes cited by MWI-proponents).

Let's accept Wallace's realist viewpoint insofar that the scientific enterprise is to give a realist understanding of the world. By accepting realism we devote ourselves to a mentality that will make us ask questions about the world itself and not only about the measurements we do.[48] Whether that will make us draw the same conclusions about the world as Wallace is the subject of the next section.

Realism and the MWI By acknowledging that the purpose of science is to give a realistic description of the world, we don't have to follow all the conclusions that Wallace draws. Wallace not only claims that the goal of science is to give a realistic description of the world, he also seems to believe that any proponent of scientific realism is obliged to interpret the wave-function itself realistically.

But this is a point where a sceptic's views of realism could significantly differ from those of Wallace as there seems no a priori reason to interpret the wave-function this way. If we don't interpret the wave-function realistically, we can say the elements in a quantum state merely represent possibilities, which is what the modal interpretations do.

Wallace opposes views where the wave-function is not being interpreted realistically in a short way by drawing a parallel between paleontology and quantum theory. As he points out, dinosaurs are not a calculational tool for explaining fossils, but it is assumed they were actually there one day and are real parts of the world.

But there is an obvious counterargument to this. Indeed fossils are interpreted as a confirmation of dinosaurs even if they are not a direct observation of dinosaurs, just like we want the measurements of quantum mechanical experiments to show the reality of the wave-function. But a big difference between a theory of dinosaurs and quantum mechanics is that dinosaurs are 'physical' in our world(s).

We could still imagine a scenario wherein we extract some DNA and give a rebirth to the dinosaurs in present time and view them with our own eyes. Because we are ourselves part of the quantum wave in the many-worlds interpretation, there is no way for us to take a direct look at this wave, as we can do with dinosaurs. Thus we might still question the ontology of quantum states.

---

[48] When a century ago we knew only some macroscopic but definitive properties of a gas, we did not resign but we asked the question 'why is it this way?'. And then when a better understanding of the underlying atomic nature of the gas came up we asked ourselves again: why is it this way? In this way every question lead to another question. This chain still continues and where it will stop is of importance for quantum mechanics. We might wonder whether the Copenhagen interpretation has assumed that the chain has stopped. If we believe the Copenhagen interpretation then there is no more to ask. The world is indeterministic and this is a fact. However we cannot be sure yet if this is the case, which is why a realist mentality points in favor of researching other interpretations for QM such as the MWI.

Let's however put our discussion a bit into historical perspective. The whole reason quantum mechanics gave problems in the first place is because we could not explain how a single particle could interfere with itself after the double-split and still end up as a single particle on the screen. This is what made us agnostic about the nature of the wave-function.

But now we know that decoherence theory can exactly explain why we only see a single particle. The only catch is that it predicts other structures in the universe, corresponding to worlds where different outcomes occur. Is this so much different from classical physics where we gained an understanding of how big the universe actually was by assuming the dots in the sky were actually stars made up of similar stuff as our planets? The other worlds are not 'postulated' but merely a result of the unitary dynamics of quantum physics. Considering this we might have a bit more reason to believe the wave-function is a real part of nature.

However, though this makes it plausible, it does not completely justify that the wave-function should be considered a real part of nature. Because of this it can again be said that Wallace's claim that the position of realism is enough to justify interpreting a quantum state as yielding multiplicity is not satisfying enough, and it looks like he is sneaking something in here. In the next section we will discuss that making determinism an a priori requirement for the multiverse/universe provides a way to fill the gap in Wallace's argument.

Determinism of the multiverse In this section we'll discuss (causal) determinism. By this I mean that everything that happens in the world can be related to a cause, and that processes can be described in a reductive step-by-step manner.[49]

Indeterminism in quantum mechanics was one of the issues for Einstein who believed that God does not play dice. Though Einstein's hopes for the existence of hidden variables were definitely proved wrong by Bell's inequalities, Everett has at least shown that a form of determinism is still within reach in quantum mechanics, even if it doesn't help us predict the exact outcomes of measurements.

I want to argue that determinism is a requirement for a complete and consistent theory of the world. In any theory in classical physics we can describe processes causally and reductively in a step-by-step manner. In logics and mathematics it's the same. Conclusions follow from a well defined procedures and equations that will lead to a definite outcome.

Because of this we can answer question like 'what is a tree?'. We can describe the structure of the tree and its leaves by biology and ask more questions, like what is the wood made up off? By continually asking such question we end up with

---

[49] So I deviate a bit from the usual definition of determinism as a view that given a certain set of conditions for a system its past and future are exactly predictable. Another point worth noting is that even if everything has a cause, we might still not be able to measure this cause and describe processes step-by-step. As an example we would have more difficulty deducing the big bang if the sky turned out to be dark. But I'll adopt a positive view and assume we are still long from the point where we don't have enough information to construct a proper model of reality, or of quantum physics in particular.

our usual worldview consisting of atoms and of the interactions between those atoms, which we can currently describe with a fair degree of approximation. Furthermore we used to assume that those interactions can be approximated better by finding even more elementary structures of nature. But as it has turned out the chain stops and small scale interactions don't leave themselves to be described deterministically when quantum physics comes into play.

However I want to argue that assuming that this probability is a true element of reality runs into problems, because this also implies that reality at this level can not be described by (step-by-step) logics. What would that mean? We can understand how our tree emerges from atomic physics, in a step-by-step way, but where does atomic physics emerge from?

The thing it emerges from is certainly not completely random, as some outcomes of measurements are more likely than others. Thus is it then half-random? Can we describe the emergence of quantum phenomena out of something in a 'somewhat step-by-step' way? That doesn't make sense either, and this breaks our whole pattern of thought. It almost seems if the question itself 'why do atoms behave the way they do' has become nonsensical. And it's not only atoms, as it is assumed that quantum physics applies to systems of any size!

Applying logics is in many cases just putting an isomorphism between the world and something which we describe in words or on paper[50]. What does saying that this is not possible mean? Honestly I don't have any idea how to address that question, and I wonder if Bohr and Heisenberg where able to understand the issue without evading it somehow. They just managed to ease it a bit by putting any part of randomness (and weirdness) in that one process of wave-function collapse. Logics and determinism would apply just fine all the time - except for those cases where the Schrödinger wave would instantly collapse.

Summarizing this we can say that by assuming indeterminism we claim the world does in essense not lend itself to a reductive step-by-step description, or what is closely related: computability. Let's focus on this computability and remark that the indeterminism we find in quantum physics can always be modelled by deterministic (computable) models.

Surely Bell's theorem sets some hard bounds on what these deterministic descriptions can be, but the quest is not over. We can for example imagine an extremely capable computer running a simulation of a quantum world.[51] Let's put ourselves in the mind of a programmer with a deterministic machine that has a computational capacity to simulate a whole world. We wonder, how can we

---

[50] I write 'many cases' because I am afraid infinities and things like Gödels theorem might obscure the picture.

[51] I am afraid that physicists/philosophers who have philosophized about simulating reality in a computer feel that discussing computability makes our problem more difficult. As an example we use real numbers which have an infinite decimal expansion in mathematical models for the world. One might rightfully wonder if such numbers are computable, since they would require and infinite number of steps of computation. I don't want to get into a discussion on this topic, but my personal opinion is that this is not a problem in the same way that determinism implies that the past and present are already laid down. One might claim our world still needs to 'calculate' into the future, but this seems like an unnecessary addition to the theory only because we want to see ourselfs at the center of time (and it does not have to be like this.) Thus I believe the concept of 'computability' can be applied to the world, even if it requires an infinite number of steps of computation.

simulate quantum mechanical phenomena on our deterministic machine?

At first we might think that we can just simulate the Copenhagen model of quantum mechanics and invoke wave-function collapse now and then when needed.[52] For this we could use a deterministic random number generator relying on degrees of freedom in our world that are out of reach for the simulated world.

But this approach will quickly run into problems: We cannot find any criteria for when to invoke wave-function collapse. All objective-collapse interpretations invoke some kind of non-linearity into the wave-function. Thus, assuming the linearity of quantum mechanics holds for the foreseeable future, the simulation would not be a perfect simulation of quantum physics and have noticeable and testable differences from quantum theory.

So if Copenhagen doesn't work we need some other way of simulating the quantum physics. We could try to do some kind of simulation of many-worlds wherein we simulate only the most probable world and let out the others, but this again isn't possible. For this we would have to simulate the wave-function and eventually cut off some parts corresponding to less-probable worlds. But these parts still interfere with the other worlds, even if the interference is really small. Thus again, in fact in the same way as when we tried it with the Copenhagen interpretation, our simulation would not follow the unitary dynamics of quantum physics.

Thus and here comes the crux: the only way to perfectly simulate a quantum mechanical world on a deterministic computer would be to just use the unitary dynamics and let every structure flow from it. If we do that the wave-function is definitely real in this simulated univere, in the sense that there is a one-to-one connection between the mathematics and the simulated wave on the computer. Thus we need to simulate every world, and we also see all the worlds exist and are just as real.

Now with our discussion at the beginning I tried to make a connection between logics, determinism and computability. I believe a computable world is the only logical world and that our world is computable. I don't see how our universe can follow the laws of logics in describing how classical structures emerge from a decohering wave-vector, but can be completely non-logical about the nature of the wave-vector itself. A deterministic universe is completely describable, and a deterministic universe can always mimic an indeterministic universe. So I believe there is no good reason to assume our universe is indeterministic. Determinism gives us a lot more insight into the world and the nature of reality itself.

Wrapping up our discussion on computationability I want argue that Wallace's arguments on how classical worlds emerge from quantum mechanics fall flat if my previous discussion is not taken into consideration. As an example Wallace describes about Schrödingers cat: *if we apply the same principles to quantum mechanics as we apply in general through science to identify higher-level ontology, we find that, since both the histories $|cat_l(t)\rangle$ and $|cat_d(t)\rangle$ represent a state*

---

[52] Indeed as explained newer versions of the Copenhagen interpretation rely on decoherence which arguably helps in understanding wave-function collapse, however the measurement problem remains such that we will for simplicity talk about wave-function collapse in the same way as the old Copenhagen theory would.

of affairs where the system is structured like a cat, the system in question is a cat.

Indeed if we assume our wave-function is real there is no reason to prefer any world over another. We can show in a step-by-step way (thus by doing the calculations) how different classical worlds emerge from the quantum mechanical wave-function. But if we don't assume the wave-function is real this argument just falls flat. Even when adopting a realist mentality, the fact that we want to give a complete description of the world does not justify assuming the wave-function is real.

But I believe when we take our previous discussion into consideration, and note that there is no universe imaginable where quantum mechanics exists in another way then the MWI (and I wonder if any universe that is not computable is really imaginable to us) that the wave-function must inevitably be interpreted realistically.

Though the above dialog does not serve as a solid proof that determinism should apply to the world I hope it does make it more intuïtive. If the reader still has doubts I want to remind him/her that our universe before quantum mechanics has always proven out to be deterministic. Only when we didn't find any alternative for explaining our measurements in quantum mechanics did we start considering an indeterministic universe. But the alternative is there now and it's a deterministic multiverse given by the MWI.

Even though many-worlds still does not predict definite outcomes for microscopic experiments from our perspective as observers, at least it succeeds in putting a deterministic and explanatory theory underneath the apparant uncertainties in quantum experiments, and that should definitely be a plus.

4.1.2 Filling in some gaps

In the previous section we have given a complete motivation for the many-worlds interpretation. In this section I will discuss a few issues that I believe a sceptic might point out to discredit Wallace's claims. I believe Wallace is right in his conclusions but his arguments are sometimes too short to be completely satisfying. Therefore I will try to complement Wallace's arguments with the ideas we build in the previous section.

A problem is that Wallace sometimes writes with a bit too much confidence. I'm not sure whether this is because he wants to write a consistent book or because he has become too comfortable with his own arguments. In the end I suspect it is a combination of both of these things. To tone this down a bit I want to point out some subtle issues in his arguments. While doing this I will try my best to stick closely to Wallace's words and describe his views neutrally.

I'll base the following section on the discussions I had with people at the institute for the history and foundations of science[53] and I will try to answer most of the questions I have heared other people ask, or have asked myself, about Wallace's book.

---

[53] Though I have discussed these matters with other people, the 'interpretation' of Wallace's book and arguments I describe here is of course my interpretation and my responsibility.

Firstly we'll focus on the preferred basis problem. The corresponding question is: even if there is no interference between the terms in the superposition and we write it in a particular independent basis, why are we justified in viewing the individual components of the wave-vector as describing real worlds? Could we try writing our system as a superposition of different basis elements? Would then these basis elements be real?

Wallace briefly mentions the preferred basis problem in his book and writes: *[...] a realist interpretation of the quantum state compels us to understand it as being (or better, as instantiating) multiple classical worlds. So I resist the 'preferred basis' terminology in this book.*

Furthermore at the end of chapter 2 he writes about Schrödingers cat: *if we apply the same principles to quantum mechanics as we apply in general through science to identify higher-level ontology, we find that, since both the histories* $|cat_l(t)\rangle$ *and* $|cat_d(t)\rangle$ *represent a state of affairs where the system is structured like a cat, the system in question is a cat.*

Arguably 'resisting' the preferred basis problem is not the most subtle move in a book that tries to convince sceptics/physicists that the MWI is the most straightforward interpretation of quantum mechanics. Even if there turns out to be no problem relating to basis ambiguity, the question does pop up in literature and it deserves a proper answer. Therefore we are going to describe what Wallace means by the above, although I hope that you can already make this up for yourself after having read our previous discussion about realism and computability.

By decoherence we can show that a quantum state that evolves according to the unitary dynamics of quantum mechanics instantiates a structure representing multiple classical worlds. If we interpret the quantum state realistically so that the emergent branching structure is not seen as a mathematical set of possible worlds, we are done. Of course for this we have to be sure that the quantum state should actually be interpreted realistically, which I believe Wallace does not fully motivate, but in the previous section we showed that we consider our universe is computable we have no other room than to interpret the quantum state realistically.

Surely we could write the quantum state down in a different basis, but that does not change its physical meaning. By decoherence we can show that properties which resemble a classical world emerge from the quantum state, and whether or not we write the quantum state down in a different basis, these properties are still the same. Since these properties make up what we call a classical world we could say there is no preferred basis problem (like Wallace does): the physical significance of the quantum state is independent of its basis.

Furthermore something that Wallace could have mentioned too is that this situation is not different from 'orthodox' quantum mechanics. Suppose Schrödingers cat evolves to a state

$$|\uparrow\rangle \otimes |\mathrm{cat}_d(t)\rangle + |\downarrow\rangle \otimes |\mathrm{cat}_l(t)\rangle \,. \tag{4.1.1}$$

If the cat system would collapse to one of these states, they better both represent a classical cat (so the quantum states instantiate the cat) as else we might look at the cat and it would collapse into a dog.

Something Wallace often does in his book is to draw parallels between general science and quantum physics. For example he claims that a realistic interpretation of quantum mechanics is no different from a realistic interpretation of fossils as a proof for dinosaurs. But such arguments are only partially satisfying. In general through science we are dealing with structures that are directly measurable. The wave-function is never measured, only the probabilities it predicts. So in that sense it would be intuïtive to see the other worlds in the many-worlds interpretation as probable worlds and not as real worlds.

Furthermore I believe Wallace puts too much value on emergence. As an example he writes about how the principles of thermodynamics, solid-state physics, animal physiology and cat psychology show us that the quantum state that represents Schrödingers cat is actually structured like a cat. It seems that what he is trying to say is: if we apply this reasoning to understand how the cat we see in our world emerges from a quantum state, we are equally justified in applying this reasoning to all components of that quantum state. Thus all predicted quantum states are equally real.

I have noted the problem with this already a few times: there is no reason to interpret a mathematical structure that represents the properties of a cat as a real cat. It is not obvious that we should link mathematics to the real world like this, even if we want to give a realist view of the world. Because of this I have heared people stating the concern that Wallace sneaks something in to motivate the many-worlds interpretation, especially when comparing it to other non-collapse interpretations like the modal interpretations.

I want to conclude this by saying again that the position of scientific realism cannot serve as a complete justification for the many-worlds interpretation. Wallace's arguments on emergence are only valid if we can justify that the wave-function is not only an element of mathematics but something that truly exists in the world in the way the mathematics predicts. However, when we accept that the only way we can truly understand quantum mechanics, in the sense that we have the knowledge to program a perfect simulation of it on a computer, we see that there is no room left for any other interpretation than the many-worlds interpretation.

4.2 Testing the Many-Worlds theory

Wallace argues in his book that any test of unitary quantum mechanics is in fact a test of the Everett interpretation. (pg. 104) We will shortly discuss this idea and how literally it can be taken. For this we'll find that some of the topics we previously discussed will come up again. However this time we will go through the arguments rather quickly, just to get an outline of the debate, and we will not go into a deep discussion about every step we take.

Let's first try to argue in favor of Wallace and compare the situation to classical mechanics. In classical mechanics we intuïtively adhere to the position of realism, such that we believe unobservables have the same ontological status as observables. We don't expect the moon to disappear when we stop looking at it. Furthermore it is hard to think of any scenario in which it does, because any other observer that is looking at the moon will see it disappear and can tell us this result.[54] Even if all people in the world don't look at the moon, there is still large theoretical evidence of its existence simply by its gravitational force on the earth.[55]

The reason is that the moon is never completely isolated from its surroundings, so it's sudden absence will be noticed one way or another. Let's therefore adopt the position that we only question the ontology of things that are completely isolated from their surroundings theoretically. That is different from them being isolated pragmatically, where they do interact with the environment but we don't have the necessary tools to measure this interaction.[56] Since in classical physics everything continually interacts with everything else, by gravity, the emission of photons etc, we don't usually doubt the very existence of object even when they remain unobserved. In order for our theories to be consistent there is always some way that the influence of an object through environmental interaction reaches us, and therefore we believe things are really out there, regardless of whether they are being directly observed or not.

Now let's compare this to the realm of quantum mechanics and the MWI. A quantum system[57] is never isolated from its surroundings, just like the situation in classical mechanics. We might therefore be able to theoretically measure this interaction with the environment, and the interaction itself predicts a branching structure that resembles multiple classical worlds. It all falls into place, and measurements of quantum theory must be measurements direct measurements of the MWI.

But hold on! There are also many differences to spot! We never 'directly' observe the wave-function, but only the probabilities it predicts. I would agree, but expanding on those differences will boil down to a discussion of realism again. So then what should we make of Wallace's claim that any test of unitary quantum mechanics is in fact a test of the Everett interpretation? Well it is unjustified in the sense that we can surely put another interpretation behind our measurement results and it will be equally effective. However if we take into account that the most explanatory explanation is given by the MWI, we might agree with Wallace's claim.

To illustrate this we will discuss an experiment proposed by Avshalom Elitzur and Lev Vaidman that shows what is known as counterfactual measurement. We will compare the theoretical explanation of the experiment in the Copenhagen interpretation to the many-worlds interpretation, and see the benefits the many-worlds interpretation gives us in the interpretation of the experiment.

The Elitzur-Vaidman bomb tester The Elitzur-Vaidman bomb tester is the solution to a problem proposed by Avshalom Elitzur and Lev Vaidman. [3]

---

[54] Off course we can question if those other observers are 'reliable' but let's keep it simple.

[55] If we want to make things more difficult we can now state that we only know there is a gravitational force on the earth, but we don't know for example if the moon still emits light. However let's not go into an endless discussion to make the argument 'perfect'.

[56] We draw our conclusions quickly here, for simplicity. Bohr might not have agreed because he turned the pragmatic problem of measuring a particle's properties into a discussion about the ontology of these properties themselves, by introducing his complementarity principle.

[57] Actually a quantum system can be any kind of system according to the MWI, but we add the word 'quantum' because we can

The problem is stated as follows: *Consider a stock of bombs with a sensor of a new type: if a single photon hits the sensor, the bomb explodes. Suppose further that some of the bombs in the stock are out of order: a small part of their sensor is missing so that photons pass through the sensors hole without being affected in any way, and the bomb does not explode. Is it possible to find out bombs which are still in order?*

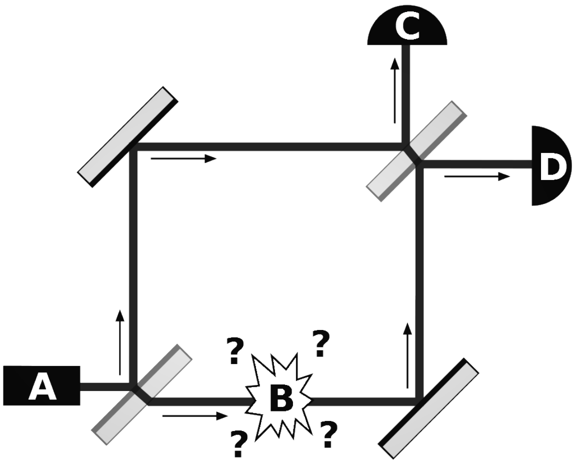

Figure 2: The setup for the Elitzur-Vaidman bomb-tester.

[58]

As it turns out the answer is affirmative, by essentially considering a setup like the Mach-Zender interferometer with a bomb on one of the paths.[59] We align the apparatus in such a way that there is complete destructive interference at detector D and constructive interference at detector C. Thus when no bomb is put at the lower path the photons will be consistently measured at detector C. Furthermore the half-silvered plane mirrors are constructed in such a way that the photon has a 50% chance of being reflected/transmitted. (Fig. 2).

We will first analyze the apparatus step-by-step in the Copenhagen interpretation and see how well that fares. After this we will analyze the experiment in the Many-Worlds interpretation and see that it provides us with some important benefits compared to Copenhagen.

Emission

- A photon is emitted from the photon source at A and passes the half-silvered plane mirror. From there the wavefunction of the photon will progress as an equal superposition between the lower and the upper path.

---

[58] Figure edited from Wikimedia, under the Creative Commons License.

[59] Our apparatus is a bit simplified compared to the one described in their paper, but the main outline is the same.

If the bomb has a sensor

- Because the bomb has a sensor and thereby serves as a measuring device, the wave-function of the photon collapses.
- Thus the photon must go through either the lower path and explode the bomb, or through the upper path.
- If the photon goes through the upper path it will be equally likely to end up in C or D as we can be sure there is no quantum interference from the lower path.

If the bomb doesn't have a sensor

- Now our apparatus has the same form as a Mach-Zender interferometer. Because of the way our sensors are aligned the photon will be measured at C.

Thus we will only measure a photon at detector D if the bomb contains a sensor. Using our scheme repeatedly, we can identify 25% of the working bombs. Another 25% of the workings bombs will remain unidentified and the other 50% will explode. By repeating the procedure using the 25% of unidentified bombs, we can eventually identify

$$\sum_{n=1}^{N} \frac{1}{4^n} = \frac{1}{3} \tag{4.2.1}$$

of the working bombs, without interacting with them! Thus, in terminology of the Copenhagen interpretation, this experiment supposedly shows the non-local character of quantum mechanics which is caused by the wave-function collapse. It seems that we are able to measure a quality of the bomb, without interacting with it (at least not in a sense that involves direct interaction with the photon.) These kinds of measurements are sometimes called interaction-free measurements.

We will now analyze our experiment using the many-worlds interpretation and we see our analysis becomes quite different. After the photon is emitted and encounters the first half-silvered mirror, its wave-vector will again be in a quantum superposition of a photon that goes through the upper path and a photon that goes through the lower path.[60] Now let's distinguish between cases again.

If the bomb has a sensor

- The part of the wave-function of the photon that goes through the lower path will be absorbed by the sensor of the bomb and detonate it. The explosion will become entangled with the environment through the means of decoherence, which leads to a classical world in which it is observed that the bomb has exploded.
- The part of the photon's wave-function that goes through the upper path will simply arrive at the last half-silvered plane mirror and because there is no more interference from the wave-function of the

---

[60] We don't have to speak of the world splitting already because there is no decoherence. Note that a split of the world into other worlds is merely a quantum superposition where the correlations between the individual elements in the basis has been lost.

photon in the lower path, there will be no destructive interference. Thus the photon will be in an equal superposition between detector C and D and because of decoherence the worlds splits into a world in which an observer measures the photon at detector C and a world where he measures it at detector D.

If the bomb doesn't have a sensor

- Our photon will interfere with itself always causing us ending up in a world where detector C measures the photon.

Our analysis of the experiment in the MWI context seems a lot more satisfying than in the Copenhagen interpretation. Firstly we don't have to invoke wave-function collapse when the photon reaches a bomb with a sensor. Thus we don't have a non-local theory, that suddenly makes the photon decide to be in the upper or lower path, after it found out that the bomb had a sensor without interacting with it in any intuïtive way.[61]

We see that according to the MWI we always explode the bomb in some world, however we sometimes find ourselves not to be in the world in which this happened. When the bomb has not exploded in the other universe we measure interference of the photon in that universe, but when we don't measure interference we know the bomb has exploded in the other universe and our photon has been absorbed.

However we must note that the experiment could have just as well been explained by the Copenhagen interpretation, and in fact Elitzur and Vaidman use Copenhagen terminology in their article, to show the supposedly non-local characteristics of quantum mechanics.[62] But this experiment is in fact a good hint towards the many-worlds interpretation. We don't have to invoke wave-function collapse and there is no 'spooky' action at a distance. We could however still try to find a number of arguments to justify other interpretations.

1 Firstly in favor of the Copenhagen interpretation, we could adopt a sort of instrumentalist view that the wavefunction only reflects our knowledge of the system, not its workings per se. So the wave-function predicts that 25% of the working bombs will explode, another 25% will be identified and so on, but we remain somewhat agnostic (if we could say so) about how this information is received. By this we try to follow Bohr's reasoning that: *"There is no quantum world. There is only an abstract quantum physical description. It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature…"*

2 Secondly to get around the non-locality completely we could also invoke some kind of super-determinism to explain the apparently non-local effects. For this we have to argue that it was somewhere in time and space predetermined that eventually in the future the photon would go the upper path and end up at detector D exactly in such a way that we were also

---

[61] Maybe proponents of the Copenhagen interpretation would say that it doesn't have to be intuïtive. However we can be certain that no physical interaction occurs that would be describable with special relativity as then the non-locality would violate the speed of light limit. Thus if we would say that interaction between the photon and the bomb does occur it must be in quite a peculiar way.

[62] Nonetheless Vaidman himself is a proponent of the many-worlds interpretation.

> predetermined by this to have a working bomb. Thus the photon already 'knew', somewhere hidden in the dynamics of the world, that we would end up with a working bomb. And then lastly we have to explain why this happens for only 25% of the working bombs, even though our setup looks almost the same in each case that we test a different bomb. (Though our setup is arguably not the same in super-determinism as we have a different bomb every time and different interactions with the environment will have occured while switching the bomb that will eventually make us end up with a possibly different outcome for the path the photon takes.)

The first strategy bottles down to our debate about realism and we already concluded that we seek to find a realist description of the world so that we try giving the world a definitive state even when it is not directly observable or interacting with us. Therefore considering this the MWI seems the preferable interpretation. The second kind of reasoning seems implausible as it is not clear at all how for example tiny fluctuations in the big bang would eventually predetermine the dynamics to make our photon end up at detector D if and only if our bomb is real. [63]

Thus the explanation of the Elitzur-Vaidman bomb-tester seems to be the most satisfactory in the many-worlds interpretation and the experiment could come very close to what Wallace described: that any test of unitary quantum mechanics is in fact a test of the Everett interpretation.

Lastly we discuss a (mostly hypothetical) way of distinguishing between the many-worlds interpretation and the other interpretations of quantum mechanics by considering the quantum suicide experiment.

Quantum suicide A version of the quantum suicide experiment is stated by Tegmark [6] which he describes as a repeated Schrödingers cat experiment, with the experimenter as the cat. The experimenter builds an apparatus consisting of a machinegun which is connected to a device that every second measures the spin of a photon in a state $\frac{1}{\sqrt{2}}\left|\uparrow\right\rangle + \frac{1}{\sqrt{2}}\left|\downarrow\right\rangle$. If the device measures spin-up the machine gun fires and if it measures spin-down an audible click is heared but no bullet is fired.

The experimenter first asks his assistent to aim the device at a plain wall and to turn the device on. Every second a click or shot will be heared, with an approximately equal distribution of both of them. Now that the experimenter knows the device works he asks his assistent to aim the device at him and turn it on. Every time the spin of the photon is measured the world will branch into a superposition

$$\left|\overset{\cdot\cdot}{\_}\right\rangle \otimes \left|\text{apparatus}\right\rangle \longrightarrow \left|\overset{\cdot\cdot}{\smile}\right\rangle \otimes \left|\text{click}\right\rangle + \left|\overset{\times\,\times}{\_}\right\rangle \otimes \left|\text{shot}\right\rangle \tag{4.2.2}$$

If the bullet is fired fast enough such that the experimenter is unaware of being hit and killed by the bullet, the experimenter can make no other observation

---

[63] However we should be carefull to dismiss the idea completely. Some research has been done, in particular by Gerard 't Hooft, who has worked on simplified superdeterministic classical models that show some of the qualities of quantum mechanics. However the arguments are still controversial and these models have not yet been succesfully extended towards describing concrete experiments like the double-split experiment or the EPR-paradox.

than to hear the click and not be hit by a bullet. Thus when the experimenter is in front of the presumably deadly apparatus all he will hear is a click every second. Tegmark notes the experimenter can even test if the apparatus is doing fine by stepping away from it and hearing a few bullets go, and by stepping in front of it again and hearing only clicks.[64]

Thus the quantum suicide experiment seems to provide a definite way for the experimenter to test the many-worlds hypothesis. The downside is that in almost any branch the assistent will experience having killed the experimenter, and therefore the experimenter will have a hard time to prove the validity of the MWI to others than himself. There is however the possibility of changing the experiment to include a second gun which fires at the same time as the first gun. The experimenter can then ask a friend to do the experiment with him, and they can both agree with each other that the MWI is true.[65]

The quantum suicide experiment is closely related to the concept of quantum immortality. The idea is that since we can never experience ourselves being dead, we must always find ourselves in a world where we have survived despite considerable odds against us. Quantum immortality is generally not taken literally as it is well understood that dying is a gradual process as a result of a huge number of quantum events. Therefore we normally lose consciousness gradually and the universe will branch out in other worlds where our consciousness becomes less and less.

However, the issue is not completely resolved by this, since the MWI predicts that even extremely unlikely worlds are real. There are always worlds imaginable with an immensely small but non-zero probability wherein the process of aging reverses or halts in a thermodynamically very unrealistic, but possible way. We'll discuss this problem among other things in the next section.

4.3 Derivation of the Born rule

When I started working on this thesis I hoped to provide a full review of the ideas in Wallace's book, but I found out that discussing the probability problem requires at least as much work as discussing the ontological problem. Resolving the ontological problem is the biggest step towards getting physicists interested in the many-worlds interpretation, but resolving the probability problem is the most important for the completion of the interpretation.

Because of this I have decided to focus for the most part on clarifying the ontological problem and motivating the realism of the wave-function. In this section we will shortly focus on the probability problem and I will point out some criticism that has been brought up against Wallace. We wil not go into a thorough discussion of how this criticism might be resolved, because it would require a much bigger thesis.

For a proper explanation of quantum mechanics one must obviously explain why and how probabilities arise over different measurement outcomes. Everett

---

[64] Of course he must step away fast enough to secure that his ear is not blown off.

[65] Though the tragedy in most of the universes will be bigger, where there are now two mourning families.

showed that if we impose a reasonable measure over orthogonal eigenstates in a quantum superposition, it must be a square amplitude measure. However, this doesn't resolve the question of why we should introduce such a probability measure in the first place.

We could resolve the probability problem simply by adding an extra postulate to the MWI that states a probability measure should be introduced. But this would lessen the simplicity of many-worlds, which resulted from removing wave-function collapse from quantum theory. Because of this there are many attempts to derive the Born rule directly from the unitary postulate. Wallace has worked on his derivation over the course of a decade, inspired by Deutsch, and Wallace's derivation is currently the most promising to solve the probability problem.

First we will illustrate the probability problem with a few simple examples. Suppose we flip a 'quantum'-coin that branches the world into the worlds H (for heads) and T (tails). Since we can expect to find ourselves in either world H or T, we'll have the subjective experience of getting either heads or tails with 50% probability after flipping the quantum-coin. Thus so far probability can be explained as a consequence of the subjective experience of the observer, who in every branch has the experience of ending up in that branch by mere chance.

However, problems start arising if we consider a scenario in which the quantum-coin is loaded:

$$|\text{Initial}\rangle \Longrightarrow 0.8\,|\text{Heads}\rangle + 0.6\,|\text{Tails}\rangle \tag{4.3.1}$$

By simple branch-counting we'd still expect to have a 50% chance of ending in either of these worlds, although the Born-rule tells us we will find ourselves in the H world approximately $0.8^2 = 64\%$ of the times we flip the quantum-coin.

Furthermore branch-counting also runs into problems when considering repeated splits into worlds with equal branch-weights. Let's consider an experiment in which we flip a coin repeatedly, but we stop when we get heads. Furthermore we also stop if we get tails a thousand times (which should be very unlikely). The wave-function will now evolve into a state:

$$\begin{aligned}
|\text{Initial}\rangle &\Longrightarrow \frac{1}{2}\sqrt{2}\,|\text{H}\rangle + \frac{1}{2}\sqrt{2}\,|\text{T}\rangle \\
&\Longrightarrow \frac{1}{2}\sqrt{2}\,|\text{H}\rangle + \frac{1}{2}\sqrt{2}\left(\frac{1}{2}\sqrt{2}\,|\text{TH}\rangle + \frac{1}{2}\sqrt{2}\,|\text{TT}\rangle\right) \\
&\Longrightarrow \frac{1}{2}\sqrt{2}\,|\text{H}\rangle + \frac{1}{2}\sqrt{2}\left(\frac{1}{2}\sqrt{2}\,|\text{TH}\rangle + \frac{1}{2}\sqrt{2}\left(\frac{1}{2}\sqrt{2}\,|\text{TTH}\rangle + \frac{1}{2}\sqrt{2}\,|\text{TTT}\rangle\right)\right) \\
&\Longrightarrow \ldots
\end{aligned}$$

Naive branch-counting yields a lot more worlds in which the initial flip gave tails, and by that we might conclude that it's more probable to find ourselves in a world in which the initial flip gives tails. Off course such a thing isn't true because the Born rule tells us we have a 50% probability of ending in the world $|\text{H}\rangle$, and a 50% probability of ending in all of the other worlds combined.

Thus we need some way of getting to the Born-rule in general measurement scenario's. You might have some ideas of how to do this, especially in the example above[66], but we will not try coming up with a solution on our own. Wallace notes similar problems with branch-counting in his book and a large amount of Wallace's work is on circumventing these problems and still letting the Born-rule arise from quantum theory without wave-function collapse.

Wallace uses a game theoretic strategy to explain why a rational observer should bet in accordance with the Born rule on the probable outcomes of quantum-experiments. Explaining the Born rule as a consequence of rational betting behavior allows Wallace to sidestep the issue of introducing probability as a seperate element into the theory. Furthermore, this betting behaviour allows for more complexity than naive branch-counting, so that the branch-weights can serve the same purpose as they do in orthodox quantum mechanics.

However Wallace's attempt to derive the Born Rule has also not been without criticism. Most of the criticism focuses on the argument being circular with respect to decoherence. An example of criticism regarding possible circularity is given by Richard Dawid [2]. Dawid writes that for Wallace's game theoretic derivation to work, he first works out that quantum mechanics develops a natural branching structure. This branching structure emerges because of decoherence and the fast disappearance of the off-diagonal elements in a density matrix.

However Dawid notes that we can only give an understanding of the elements in the density matrix by considering them as probabilities. Thus the emergent branching structure of the MWI can only be established by assuming that branch weights play the role of probabilities. Wallace's eventual derivation of the Born-rule is again based upon this branching structure, and this is where the circularity comes in.

This same problem has also been noted by David Baker [1]. He notes that Everettians like Wallace and Deutsch talk about probability by considering subjective experiences of people. However to consider people in different branching contexts one first has to constitute what a measurement is and how the preferred basis can be established. The proponent of Everettian QM will invoke decoherence to establish the branching structure, but this can only be done by acknowledging that the off diagonal elements become of low amplitude and are therefore negligible. However, to say that these low amplitude elements can indeed be neglected, one has to invoke the Born rule. Baker concludes that deriving the Born rule from decoherence is thus circular.

If it's not directly obvious that claiming the low branch-weights are negligible is equal to invoking the Born-rule, we can consider another thought-experiment. Suppose we introduce some kind of inverse probability measure over the branches such that low-probability outcomes are actually highly probable. Then the branches with high amplitude can be ignored and we will not end up with a nice branching structure. Thus there must be some kind of justification in the theory of QM itself that justifies that low branch-weights are negligible and this is given by the Born-rule.

---

[66] e.g. one could propose to take time into consideration while branch-counting.

Luckily the situation is partly in favor of the MWI, because Gleason's theorem shows that the only possible probability measure over quantum states is the Born-rule.[67] However, this does not resolve the problem of why one should introduce a probability measure in the first place. Wallace's gives a partial response on this at page 254 of his book, and he argues that probabilities themselves are also emergent:

Sceptic [...] What makes perturbations that are small in Hilbert-space norm 'slight', if it's not the probability interpretation of them?

Wallace Lots of dynamical features of the theory. Small changes in the energy eigenvalues of the Hamiltonian, in particular, lead to small changes in quantum state after some period of evolution. Sufficiently small displacements of a wavepacket lead to small changes in quantum state too. [...]

Dawid explicitely mentions Wallace's response to the circularity problem in his paper, but he remains unconvinced. Since we will not go into a further discussion on this topic, we can conclude for now that Wallace's derivation of the Born-rule still raises some doubts at best and is plainly circular in the worst case.

Lastly we want to point out a problem that we already touched upon while discussing the quantum suicide experiment. The MWI presumes that every world, regardless of its branch-weight is actually real. This implies that there are also branches, with a very low branch weight, in which an observer spontaneously disintegrates as his atoms move apart. Wallace counters in his book that the existence of unrealistic worlds with low branch weights should not be a bigger problem in the Everett interpretation than in classical physics, since we also dismiss scenario's with very low probabilities in statistical mechanics.

Whether or not Wallace's argument is completely satisfying, we can unfortunately not just resolve the issue by dismissing branches with very low branch-weight. Suppose we throw our quantum-dice a million times. According to the MWI the very improbable branches wherein we only throw heads or tails a million times exist like any other world. Whether we like this or not, it will be a very tricky business to find criteria of when the branches become too improbable. Everytime we flip the quantum-coin the result is independent of the previous throws. Thus having thrown tails a hundred times doesn't make throwing heads the next time any less likely, assuming the quantum-coin is fair. Therefore we can't justify adding a cutoff point for the improbable branches somewhere in the chain of throwing coins.

We'll conclude our discussion for now by acknowledging that there are still issues that need to be resolved in the probability problem of many-worlds. Personally I believe the issues with probability in the no-collapse interpretations are less severe than the problems with wave-function collapse in orthodox quantum mechanics. Because of this I have the hope that the probability problem will be completely resolved in the future. The probability problem can be considered the last obstacle for the MWI in becoming the leading interpretation of quantum mechanics.

---

[67] Gleason's theorem only applies to Hilbert-spaces of dimensionality higher than three, but this includes any realistic measurement scenario in quantum mechanics where lots of particles get entangled.

5 Conclusion: MWI vs Copenhagen

In the beginning of this thesis I wrote down my motivation for looking into the many-worlds interpretation. I claimed that if every student asks himself the question of why the universe works the way it does, the many-worlds interpretation could very well become the preferred interpretation of quantum mechanics. In this section I am going to do a short recap of the ideas discussed in this thesis and we'll discuss how justified I was in claiming that many-worlds has the potential of replacing the Copenhagen interpretation.

We started by laying down the Copenhagen interpretation as it was introduced by Bohr and Heisenberg around the late 1920's. We then took a look at the issues with the Copenhagen interpretation. I hope that I have given the reader sufficient doubts on this, especially considering the measurement problem, which I argued is unresolved to this date even when taking decoherence into consideration.

So if the Copenhagen interpretation is actually insufficient, we need to look into an alternative. We started to consider the relative-state formulation, which was proposed by Hugh Everett. Everett's claim was that quantum theory, without wave-function collapse, is complete enough to yield an interpretation on its own. This idea was pretty radical at the time, and it took a while until Bryce DeWitt popularised Everett's ideas and renamed them to the many-worlds interpretation.

Having discussed Everett's ideas there was certainly still some work to do. We started focussing on Wallace's book 'The Emergent Multiverse', as Wallace is currently the most prominent advocate of the many-worlds interpretation. I decided against diving too deep into the probability problem, but instead tried to focus on the ontological problem. The reason for this is that a proper discussion of the ontological problem motivates the decision between many-worlds and the modal interpretations, which are also no-collapse interpretations, but which question the ontology of the other worlds.

I started by arguing that Wallace's arguments, relying on realism and emergence are not sufficient in motivating the idea that the wave-function yields multiplicity instead of indeterminacy. However, I tried to make plausible that a step-by-step description of the world, where the universe/multiverse is considered deterministic leaves little room for other interpretations than the MWI. This does not mean the worlds we find ourselves in are actually deterministic, in the sense that we can exactly predict the outcomes of measurements, but it does mean we have sufficient knowledge about the mechanics of the universe so that we can describe the universe deterministically and simulate it on a computer.

After having this discussion we took a look into the Elitzur-Vaidman bomb-tester and concluded that the explanation of counterfactual measurements is a lot more satisfying in the many-worlds interpretation. Though the idea of many-worlds is radical in itself, I hoped that discussing the experiment could put some of the intuïtive classical logics back into our description of the way the universe works.

We then took a look at the quantum suïcide experiment, which led way to discussing problems with probability in the many-worlds interpretation. We showed that there is still criticism on Wallace's arguments and that the probability problem has not been completely resolved. In fact the probability problem can be considered the biggest obstacle left in the theoretical formulation of the many-worlds interpretation.

The leading interpretation of QM Having summarized our discussion I want to draw a final conclusion on the possibility of having the many-worlds interpretation as the leading interpretation of QM.

Let's ask ourselves: why has the Copenhagen interpretation not already been replaced by other interpretations? There is certainly more and more consensus among scientists that the notion of wave-function collapse needs to be revised. Furthermore we have seen that simply taking wave-function collapse away from quantum theory makes room for a different, but equally predictive interpretation of quantum mechanics.

There are a few points that can explain why we haven't yet replaced the Copenhagen interpretation with something else:

1. **Pragmatics of the CI** Wave-function collapse is an easy tool in understanding quantum experiments.
2. **Historical perspective** The Copenhagen interpretation was conceived before the many-worlds interpretation. At the time it was conceived, it was the only interpretation that could explain all the observed phenomena in quantum mechanics. Therefore we might argue that physicists have become more accustomed to the problems with the CI than to those with the MWI, simply because of the historical sequence of events.
3. **The ontological problem** The proponents of many-worlds certainly have some explaining to do to the layman in physics. Even though the theoretical simplicity of the MWI is something beautiful to look at, the idea of parallel universes might still be considered too radical to believe in.
4. **The probability problem** If every outcome is equally real, we must acknowledge there are some really 'weird' worlds out there even if they are very improbable. Furthermore we have to justify why certain quantum worlds are actually more likely than others, which conflicts with the intuïtive practice of branch-counting.

I personally believe the first two points are the ones keeping us from accepting many-worlds the most. It's the pragmatics that make the Copenhagen interpretation so powerful. But even though that's the case, I don't think that explaining a branching universe to fresh students of quantum mechanics will be more difficult than explaining wave-function collapse. In fact, I've seen a lot of students take pride in understanding the 'weirdness' of quantum mechanics. Maybe in the future we can let them take pride in understanding the world is actually branching all around us.

Secondly the historical sequence of events poses a problem. All the best textbooks on quantum mechanics currently rely on the Copenhagen interpretation.

I very much hope that the smart physicists who are capable of writing such books will spend a section on laying down the no-collapse interpretations in the future. I believe it will be a good practice to introduce new students to the idea that wave-function collapse isn't needed in forming a complete theory of quantum mechanics.

Once this idea settles into the scientific community the debate on many-worlds will become a lot more fruitful, and maybe some other good ideas will even pop up because of it. I don't expect the authors of textbooks to claim the other worlds are really out there, but even if they describe the modal interpretation of quantum mechanics and remain agnostic about the existence of other worlds, they have at least made a first step in slowly replacing the Copenhagen interpretation.

Furthermore, I hope I have provided the reader with enough material to form an opinion on the ontological problem. I believe that we should view the other worlds as being just as real as ours, even if we cannot interact with them. This is because viewing the quantum wave-function as a real element of reality, and not only as a part of the mathematics, gives us the power of providing a complete step-by-step reductive description of the universe. However, even if the ontological problem of the MWI remains under discussion, the theoretical simplicity of the no-collapse interpretations is already a big advantage compared to the Copenhagen interpretation.

Because of this it's already a major victory to make the a form of the modal interpretation of quantum mechanics the standard interpretation. A problem that might still threaten many-worlds is the possibility that quantum mechanics actually turns out to be a linear approximation of a non-linear theory. If that were true it might be that macroscopic superpositions do not exist which would discredit the many-worlds interpretation.[68] However, the modal interpretations would in that case still be a very workable model for quantum mechanics, since macroscopic quantum states would be seen as mere possibilities.

Finally, the last major obstacle for the MWI is then the probability problem. Since we have not fully discussed the problem, we cannot form a definite prediction of its severity in the future. However, we did see that explaining probability in a simple scenario wherein the world splits into two equal parts is very much possible. Therefore I want to conclude on this optimistically by saying that the probability problem has the potential of being fully solved in the future.

Taking all of this into account I feel safe to say that quantum mechanics has become a lot less 'weird' over the last decades, even it involves the universe branching into parallel worlds. There is certainly lots of research left to be done, but we should currently be wise enough to acknowledge that the no-collapse interpretations are a more explanatory and simpler way of viewing quantum mechanics. I want to conclude by saying this is certainly an insight that deserves more recognition in the mainstream scientific community.

---

[68] I am not claiming that this will actually be the case if quantum theory turns out to be non-linear, since we cannot know what the consequences will be for our interpretations until we actually measure such a non-linearity. Maybe the linear approximation is good enough to still yield parallel universes.

References

[1] David Baker. Measurement Outcomes and Probability in Everettian Quantum Mechanics. *Studies in History and Philosophy of Modern Physics*, 2006.

[2] Richard Dawid. Many Worlds: Decoherent or Incoherent? (Unpublished). *Studies in History and Philosophy of Modern Physics.*

[3] Avshalom C. Elitzur and Lev Vaidman. Quantum Mechanical Interaction-Free Measurements. *Foundations of Physics*, 1993.

[4] Hugh Everett III. "Relative State" Formulation of Quantum Mechanics. *Reviews of Modern Physics*, 1957.

[5] Maximilian Schlosshauer. Decoherence, the measurement problem, and interpretations of quantum mechanics. 2005.

[6] Max Tegmark. The Interpretation of Quantum Mechanics: Many Worlds or Many Words? *Institute for advanced study, Princeton, NJ 08540*, 1997.

[7] Max Tegmark. The importance of quantum decoherence in brain processes. *Phys. Rev. E 61*, 2000.

[8] David Wallace. The Emergent Multiverse. *Oxford University Press*, 2012.