Current-induced torques and transport in Rashba ferromagnets

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Abstract

Motivated by recent experiments, in this thesis we study the effect of spin orbit coupling on current induced torques. To do so, we first introduce a simple model for ferromagnets in which spin-orbit coupling is not taken into account and we motivate the need to include it in the theory. After that we introduce the Rashba Hamiltonian including spin-orbit coupling and we use the Boltzmann equation to obtain the electron distribution function in the presence of external fields, from which the spin density, spin torques, and transport properties are determined. We find expressions for the current and heat current induced torques, to first order in the strength of the spin-orbit coupling and the external fields.

Contents

1 Introduction and Motivation

In this section I give some theoretical background to fully understand the rest of this thesis and I motivate the interest of it. I first explain what ferromagnetic materials are and the role that domain walls play in them. I also discuss some of the applications that can be given to these ferromagnets (many of them are already been used in the industry). In the end, I describe an experiment which motivates the need to use spin-orbit coupling in the theory to fully understand the physical properties and phenomena of these materials. More information can be found in [7].

1.1 Ferromagnetic materials

In a few words, ferromagnetic metals are those metals that can be magnetized very easily (iron, nickel, cobalt...). That is, they can behave as elementary (temporal and permanent) magnets under the influence of a magnetic field. Ferromagnetism is then the phenomenon by which a metal (or some electron system) becomes spontaneously spin polarized.

The main source of this phenomenon can be found in the electrons in atoms of these materials. The spin and the charge of an electron combined create a (small) magnetic dipole moment which, in principle, creates a small magnetic field. When these tiny magnetic dipoles are aligned in the same direction, they create a measurable macroscopic field by adding their individual magnetic fields. Those materials that have a filled electron shell will have a total dipole moment zero because the spin of the electrons are in up and down pairs and they cancel each other, so that there is no contribution at all to the total magnetic field. This is why ferromagnetism only occurs in materials with partially filled shells. Their atoms can have a net magnetic moment because there are unpaired electrons whose contribution is not canceled out by other electrons.

One can think that, according to classical electromagnetism, two nearby magnetic dipoles will tend to align in opposite directions, so their magnetic fields will oppose one another and cancel out. However, this effect is very weak, because the magnetic fields generated by individual spins are small and the resulting alignment is easily destroyed by thermal fluctuations. In ferromagnetic materials, a much stronger interaction between spins arises because the change in the direction of the spin leads to a change in electrostatic repulsion between neighboring electrons. The exchange interaction is much stronger than the dipole-dipole magnetic interaction. This is why nearby spins tend to align in the same direction in ferromagnetic materials. Therefore, when the orbitals of the unpaired outer valence electrons from adjacent atoms overlap, the distributions of their electric charge in space are further apart when the electrons have parallel spins than when they have opposite spins. The parallel-spin state is more stable because the electrostatic energy of the electrons is reduced when their spins are parallel compared to their energy when the spins are anti-parallel. In other words, the electrons can move 'further apart' by aligning their spins, so the spins of these electrons tend to line up. All of the dipoles in a ferromagnetic material will be aligned below the Curie temperature. As the temperature increases, thermal motion competes with the ferromagnetic tendency for dipoles to align. The Curie temperature is a critical point. When the temperature rises beyond it there is a second-order phase transition and the system can no longer maintain a spontaneous magnetization, although it still responds paramagnetically to an external field. Below that temperature, there is a spontaneous symmetry breaking and magnetic domains form. The Curie temperature itself is a critical point, where the magnetic susceptibility is theoretically infinite and, although there is no net magnetization, domain-like spin correlations fluctuate at all length scales.

A bulk piece of ferromagnetic material is divided into tiny magnetic domains. Within each domain, the spins are aligned, but the spins of separate domains point in different directions and their magnetic fields cancel out, so the object has no net large scale magnetic field (see Fig. 1). This is why a piece of ferromagnetic material does not have a strong magnetic field, and these pieces are usually found in an "unmagnetized" state. The exchange interaction of ferromagnetic materials

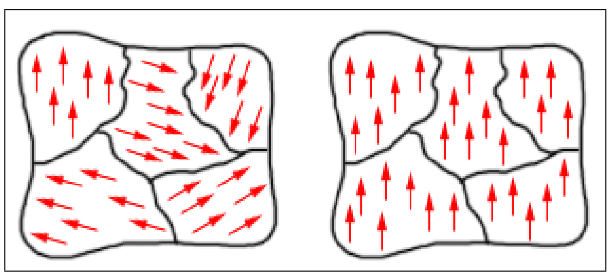


Figure 1: Domain walls (black lines) separate tiny magnetic domains within the ferromagnet. The image shows a magnetized (unmagnetized) ferromagnet in the right (left).

is a short-range force, and this is why these materials spontaneously divide into magnetic domains. The magnetic dipoles reduce their energy by orienting in opposite directions over large distances. If all the dipoles in a piece of ferromagnetic material are aligned parallel, it creates a large magnetic field extending into the space around it. This contains a lot of magnetostatic energy. The material can reduce this energy by splitting into many domains pointing in different directions, so the magnetic field is confined to small local fields in the material, reducing the volume of the field. The domains are separated by thin domain walls a number of molecules thick, in which the direction of magnetization of the dipoles rotates smoothly from one domain's direction to the other. Thus, a piece of iron in its lowest energy state ("unmagnetized") generally has little or no net magnetic field.

However, if it is placed in a strong enough external magnetic field, the domain walls will move, reorienting the domains so more of the dipoles are aligned with the external field. The domains will remain aligned when the external field is removed, creating a magnetic field of their own extending into the space around the material, thus creating a "permanent" magnet. The domains do not go back to their original minimum energy configuration when the field is removed because the domain walls tend to become 'pinned' on defects in the crystal lattice, preserving their parallel orientation (Barkhausen effect). Although this state of aligned domains found in a piece of magnetized ferromagnetic material is not a minimal-energy configuration, it is metastable, and can persist for long periods.

1.2 Domain-wall motion

A domain wall (DW) is basically an interface separating magnetic domains. A magnetic domain is a part of the ferromagnet where the majority of the spins are pointing in a particular direction (see Fig. 1). As a basic example (see Fig. 2), one can consider that at some point in the ferromagnet there are many spins pointing up, and at some other point spins pointing down, then somewhere in between there must be a domain wall; a gradual reorientation of the spins across the space. If a current is applied to this domain wall, it will suffer torques which will make it move. These are called the spin-transfer torques. See [8] for further information.

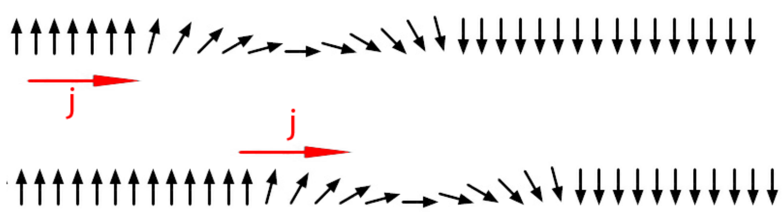
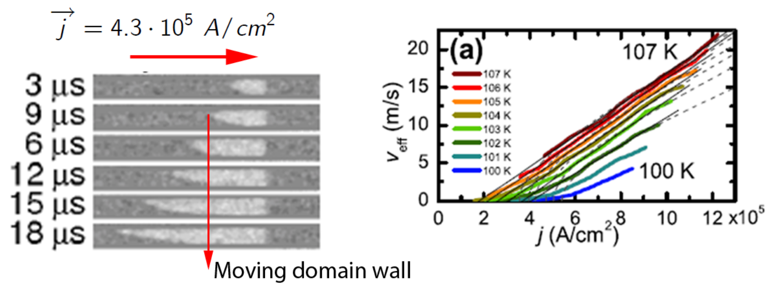


Figure 2: The domain wall moves due to the torques applied by the current.

In [2] they measure the velocity and the position of a DW when they apply a flow of electric current to the system. In the Figure 3 one can see the position of the DW at different times, and also a plot where it is shown that the speed of the DW increases with the temperature.

Domain walls are then small magnetic objects (their width is usually below 10 nm), that can propagate with large speeds ($\sim$ 100 m/s^{-1}), and can therefore be used to transmit or store information.



M. Yamanouchi, et al. (2006), expts. on magnetic semiconductor GaMnAs

Figure 3: Left: DW moves in the opposite direction to current independent of the initial magnetization orientation, and that DW displacement is proportional to pulse duration. Right: DW velocity as a function of current at various device temperatures. Thin line and broken thin line show fitted linear and square root dependencies of velocity on current, respectively.

1.3 Applications

One of the reasons why we want to study the properties of ferromagnetic materials is because many electrical and electromechanical devices and magnetic storage components are based on ferromagnetism. As an example, below I describe the functioning of racetrack memories [4]:

Up to date we can find two main means of storing digital information, the solid-state random access memories (RAMs) and magnetic hard disk drives (HDDs). Storing data in HDDs is much cheaper than storing data in a RAM memory, but the problem with HDDs is that they are very slow compared to RAMs because the consume a lot of energy and time rotating the massive disk. This is why nowadays computers use both types of memories, even though the architecture of computing systems would be greatly simplified if there were a single memory storage device with the low cost of the HDD but the high performance and reliability of solid-state memory. Normally the way to design cheaper and faster devices is to reduce the size of individual memory elements or data storage bits (HDDs and RAMs are basically two-dimensional arrays of magnetic bits and transistors, respectively). An alternative approach is to consider constructing truly three-dimensional devices.

One such approach is "racetrack" memory (see Figure 4), in which magnetic domains are used to store information in tall columns of magnetic material arranged perpendicularly on the surface of a silicon wafer.

The idea is that there are two types of magnetic domains that are magnetized in opposite directions (either up or down) along the racetrack memory, and magnetic domain walls (DWs) (see section 1.2) are formed at the boundaries of such magnetic domains. Each domain has then a head (positive pole) and a tail (negative pole). The DWs along the racetrack alternate between head-to-head and tail-to-tail configurations, and by pinning sites along the racetrack one can control the spacing that separates two consecutive domain walls. That is, one can control the length of a bit creating such pinning sites. These pinning sites also give the DWs enough stability to resist external perturbations (stray magnetic fields from nearvy racetracks or thermal fluctuations, for instance).

The data bits (that is, the DWs) can be moved along the racetrack and they intersect writing and reading elements integrated in the racetrack. These data bits can be read using magnetic tunnel junction magnetoresistive sensing devices, if they are close to or in contact with the racetrack. Writing bits can be done by using the self-field of currents passed along neighboring metallic nanowires or by using the spin-momentum transfer torque effect derived from current injected into the racetrack from magnetic nanoelements, for instance.

One important thing that must be taken into account is that one cannot use uniform magnetic fields, because neighboring DWs would move in opposite directions and so eventually annihilate each other. One has to use non-uniform local magnetic fields to manipulate DWs, and the process to store memory becomes much more complex and expensive.

1.4 Experimental motivation

In [1] they study Pt/Permalloy bilayer films with a microwave-frequency (RF) charge current applied in the film plane. They generate an oscillating transverse spin current in the Pt layer and inject it in the Py layer, and this spin current generates an oscillating spin torque. This torque induces a precession in the magnetization of the Py layer. By adjusting the appropiate frequency and field bias they can obtain a strong resonant precession of the magnetization. Due to the anisotropic magnetoresistance of the Py film, the precession of the magnetization results in an oscillation of the bilayer resistance. This effect generates a DC voltage signal across the sample. They can measure directly the absorbed spin current by the Py layer once they know these resonance properties. Their measurement setup is shown in Fig. 5:

We have shown this experiment as a motivation to introduce spin-orbit coupling. In the next section we introduce the standard spin torques theory which cannot completely explain the physics of this experiment in the absence of SO coupling.

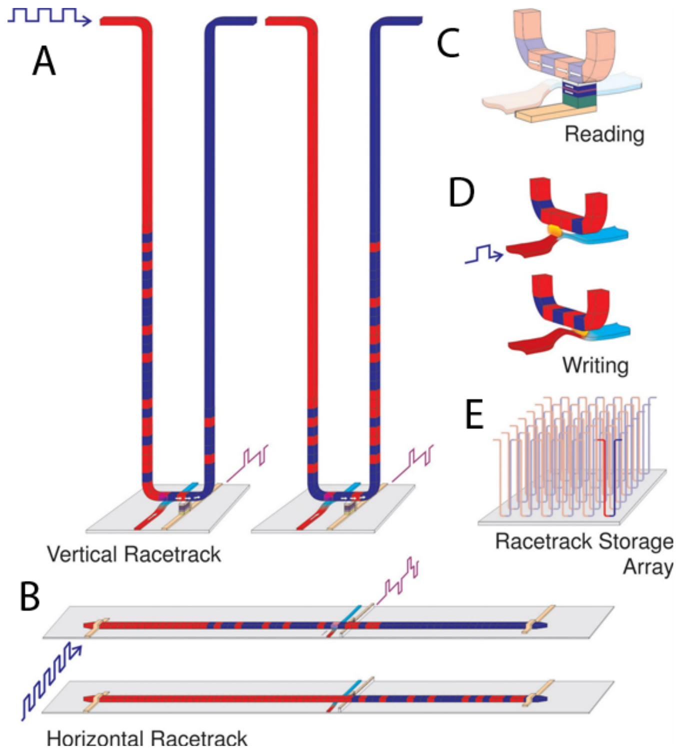


Figure 4: Racetrack memory. (A) The highest storage density is achieved by storing the pattern in a U-shaped nanowire normal to the plane of the substrate. (B) A horizontal configuration uses a nanowire parallel to the plane of the substrate. (C) Reading data is done by measuring the tunnel magnetoresistance of a magnetic tunnel junction element connected to the racetrack. (D) Writing data can be accomplished by the fringing fields of a DW moved in a second ferromagnetic nanowire oriented at right angles to the storage nanowire. (E) Arrays of racetracks are built on a chip to enable high-density storage.

2 Conventional Spin Torques

In this chapter I give a brief explanation of spin torques and I explain the (toy) model that one can use to study ferromagnetic materials and spin-torques. Also, in the end of this section I motivate the need to include spin-orbit coupling in the theory.

2.1 Spin Torques

A ferromagnet has a specific (local) magnetization, and if an electric current $\overrightarrow{j}$ is driven through the ferromagnet, the expectation value of the spins of the conduc-

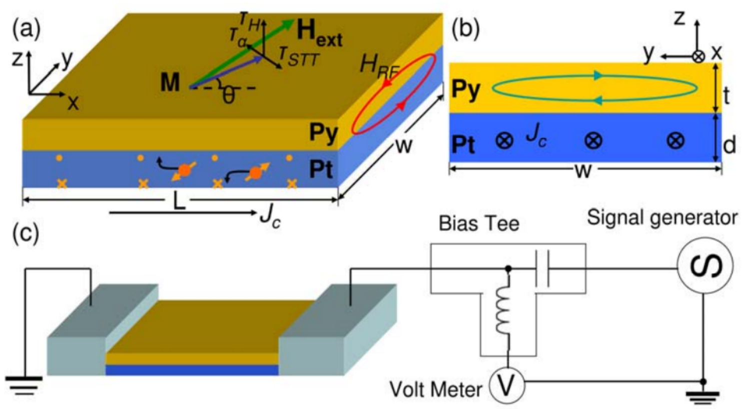


Figure 5: (a) Pt/Py bilayer thin film with the spin transfer torque τ_{STT}, the torque τ_H induced by the Oersted field H_{RF} , and the direction of the damping torque τ_α. θ is the angle between the magnetization $\mathbf{M}$ and the microstrip. $\mathbf{H}_{ext}$ is the applied external field. (b) Left side view of the Pt/Py system, with the solid line showing the Oersted field generated by the current flowing just in the Py layer. (c) Schematic circuit for the measurement.

tion electrons will point in the direction of such magnetization. If the sample of the ferromagnet contains two magnetic domains with different magnetization, the current will first polarize in the direction of the first domain it encounters[1]. After that the current will flow into the second magnetic domain and will adjust itself to the local magnetization of the second domain. The magnetization of the conduction electrons will change, and this change is transferred to the local magnetization of the ferromagnet. This means that the polarized current can make a torque on the local magnetization of the ferromagnet. In the simple case of two singular domains separated by a domain wall, the first domain will grow and the second domain will shrink. This means that the domain wall has moved due to the torques provoqued by the current. This can be used to excite oscillations or even flip the orientation of the magnet. These effects are usually only seen in nanometer scale devices.

The principle of spin transfer is illustrated in the previous figure, where a spin polarized current, with spins pointing up in the z direction is propagating in the $\overrightarrow{x}$ direction from the left to the right. This current enters a ferromagnetic zone with a local magnetization $\overrightarrow{M}$ (θ is the angle between $\overrightarrow{M}$ and $\overrightarrow{z}$). The spin current will have a transverse and a perpendicular component to the local magnetization. The spins of the conduction electrons start to precess around $\overrightarrow{M}$, and, on average,

[1]An electrical current is generally unpolarized (consisting of 50% spin-up and 50% spin-down electrons); a spin polarized current is one with more electrons of either spin, or with different spins flowing in different directions.

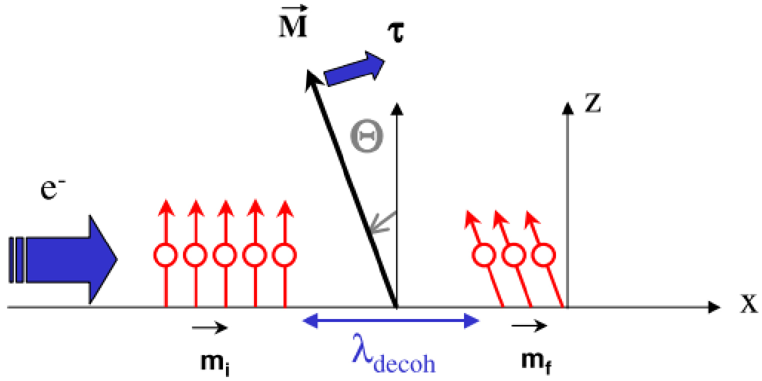


Figure 6: Image taken from [3]

the spins are aligned with $\overrightarrow{M}$ after a few rotations, when the coherence is lost due to the diffuse transport in ferromagnetic metals, meaning that the transverse spin component has been lost. In this process a torque has been transferred to the local magnetization due to the conservation of angular momentum. This torque tends to rotate the magnetization vector towards the $\overrightarrow{z}$ axis. The direction and amplitude of this torque depends on the current density, the degree of polarization of the spin current and on the sign of the injected current. The decoherence process occurs in a very small lengthscale λ_{decoh}.The first experiments clearly proving the existence of the spin transfer effect date from the year 2000 [5-6]. One possible application of this effect is, for instance, to use spin torques to manipulate the local magnetization of a ferromagnet, so that it is possible to control the position of a magnetic domain wall.

2.2 Toy model for ferromagnets

To study theoretically ferromagnetic materials we initially consider a simplified model where the localized electrons and atomic nuclei are described by a continuous magnetization density $\overrightarrow{M}(\overrightarrow{x},t)$, which in principle can depend on position and time. We also focus in the regime far below the Curie temperature, in such a way that we can describe this magnetization by the saturation magnetization density M_s and a unitary vector pointing in the direction of the magnetization $\overrightarrow{\Omega}(\overrightarrow{x},t)$.

$$\overrightarrow{M}(\mathbf{x},t) = M_s \overrightarrow{\Omega}(\overrightarrow{x},t)\,. \tag{1}$$

Within this model, the dynamics of the magnetization (that is, the equation of motion for the magnetization direction $\overrightarrow{\Omega}$ in a ferromagnet) can be described using the Landau-Lifshitz-Gilbert equation

$$\frac{\partial \overrightarrow{\Omega}}{\partial t} = \gamma \overrightarrow{\Omega} \times \overrightarrow{H}_{eff} - \alpha \overrightarrow{\Omega} \times \frac{\partial \overrightarrow{\Omega}}{\partial t} + \frac{\Delta}{2} \overrightarrow{\Omega} \times \overrightarrow{S}. \tag{2}$$

This is a differential equation that describes the precessional motion of the magnetization $\overrightarrow{M}$ of the ferromagnet. Here, $\overrightarrow{H}_{eff}$ is the effective field, containing energy contributions to the magnet, such applied fields, impurities, etc. α is the Gilbert damping parameter, which we neglected in our study to keep things simple. And $\overrightarrow{S}$ is the spin density of the conduction electrons of the ferromagnet.

In order to describe the physical properties of this system, we can consider the following simple Hamiltonian for the electrons in a ferromagnet:

$$H = \underbrace{\frac{\hbar^2 \overrightarrow{k}^2}{2m}}_{\text{Free part}} \underbrace{- \frac{\Delta}{2} \overrightarrow{\Omega} (\overrightarrow{x}, t) \cdot \overrightarrow{\tau}}_{\text{Exchange coupling}}. \tag{3}$$

We are interested in the dynamics of the magnetization, that is, in the behaviour of $\overrightarrow{\Omega} (\overrightarrow{x}, t)$ under certain circumstances. To do that, we can first consider the easiest special case in which we consider homogeneous magnetization ($\overrightarrow{\nabla} \overrightarrow{\Omega} = 0$) and we do not apply any kind of external field ($\overrightarrow{E} = 0$, $\overrightarrow{\nabla} T = 0$). We also assume that, when evaluating $\overrightarrow{S}$, the electrons are much faster than $\frac{\partial \overrightarrow{\Omega}}{\partial t}$. We can then find that $\overrightarrow{S} = \frac{m\Delta}{2\pi\hbar^2} \overrightarrow{\Omega}$ [2]. If we plug this solution in the Landau-Lifshitz-Gilbert equation 2, we find that the contribution of the electrons to the motion of the magnetization is absent:

$$\left. \frac{\partial \overrightarrow{\Omega}}{\partial t} \right|_{electrons} = \frac{\Delta}{2} \overrightarrow{\Omega} \times \overrightarrow{S} \sim \overrightarrow{\Omega} \times \overrightarrow{\Omega} = 0. \tag{4}$$

We can now switch on the electric field, $\overrightarrow{E} \neq 0$, and consider non-homogeneous magnetization, $\overrightarrow{\nabla} \overrightarrow{\Omega} \neq 0$. One can solve the equation of motion (see [9]) and see that the spin density behaves as $\overrightarrow{S} \sim \overrightarrow{\Omega} \times \left(\left(\overrightarrow{E} \cdot \overrightarrow{\nabla} \right) \overrightarrow{\Omega} \right)$. This implies that

$$\begin{aligned} \left. \frac{\partial \overrightarrow{\Omega}}{\partial t} \right|_{electrons} &\sim \overrightarrow{\Omega} \times \overrightarrow{S} \sim \left(\overrightarrow{E} \cdot \overrightarrow{\nabla} \right) \overrightarrow{\Omega} \equiv - \left(\overrightarrow{V_S} \cdot \overrightarrow{\nabla} \right) \overrightarrow{\Omega} \\ \left. \left(\frac{\partial \overrightarrow{\Omega}}{\partial t} + \overrightarrow{V_S} \cdot \overrightarrow{\nabla} \right) \overrightarrow{\Omega} \right|_{electrons} &= 0. \end{aligned} \tag{5}$$

This equation of motion for the magnetization implies that, if we know that the magnetization $\overrightarrow{\Omega} (\overrightarrow{x}, t) = \overrightarrow{\Omega}_o (\overrightarrow{x})$ when $\overrightarrow{E} = 0$, then $\overrightarrow{\Omega} (\overrightarrow{x}, t) = \overrightarrow{\Omega}_o \left(\overrightarrow{x} - t\overrightarrow{V_S} \right)$

[2] For more details, see section 3.2.

when we switch on the electric field $\overrightarrow{E}$ at $t = 0$. ($\overrightarrow{V_S}$ is a vector proportional to the electric field which describes the speed at which this magnetization moves in space). This means that domain walls move with a certain velocity, just like it was shown in the experiment in section 1.2.

2.3 Outline

The previous model is not a complete model. First of all, in some experiments they have see the influence of currents even when the magnetization is homogeneous, and it is not possible to predict this phenomenon within this model. Also, back in section 1.4 we can see the inversion assymetry in Fig. 5. There is a need to include this effect in the Hamiltonian of the system in order to derive all the physical properties of the system.

A possible solution is to include spin-orbit coupling in the Hamiltonian. In section 3 we introduce the Rashba Hamiltonian taking into account such interaction and in sections 5 and 6 we investigate the effect of spin-orbit coupling on current-induced magnetization dynamics.

3 Spin-orbit coupling and equilibrium properties

In quantum physics, the spin–orbit interaction is basically any interaction of a particle's spin with its motion. Spin-orbit coupling makes the spin degree of freedom respond to its orbital environment. In solids this yields such intriguing phenomena as a spin splitting of electron states in inversion-asymmetric systems even at zero magnetic field and a Zeeman splitting that is significantly enhanced in magnitude over that for free electrons.

3.1 Rashba Hamiltonian

Spin-orbit coupling for electrons arises as a relativistic correction to the Schrödinger equation that can be derived from the Dirac equation. Consider an electron moving in an electric field $\overrightarrow{E}(\overrightarrow{x})$. This electric field can be derived from an electrostatic potential $V(\overrightarrow{x})$ via $\overrightarrow{E}(\overrightarrow{x}) = -\overrightarrow{\nabla}V(\overrightarrow{x})$. Given that the electron is moving with a velocity $\overrightarrow{v}$, this electric field can be Lorentztransformed ito a magnetic field in the rest frame of the electron, such that $\overrightarrow{B} = \overrightarrow{E} \times \overrightarrow{v}/c$, with c the velocity of light. The spin of the electron is given by $\overrightarrow{S} = \hbar\overrightarrow{\tau}/2$, where $\hbar$ is Planck's constant and $\overrightarrow{\tau}$ the vector of Pauli matrices. This spin couples to the magnetic field via the Zeeman interaction in such a way that we have to consider the Hamiltonian $H_Z = \frac{g_e\mu_B}{2}\overrightarrow{B}\cdot\overrightarrow{\tau}$. μ_B is the Bohr magneton and g_e the gyromagnetic ratio of the electron. We can express this Hamiltonian in terms of the electrostatic potential and the momentum of the electron given by $\overrightarrow{v} = \overrightarrow{p}/m_e$, where m_e is the mass of the electron, so that the hamiltonian that couples the electron's momentum to its spin is given by $H_{SO} = \frac{g_e\mu_B}{2cm_e}\left(\overrightarrow{\nabla}V(\overrightarrow{x}) \times \overrightarrow{k}\right)\cdot\overrightarrow{\tau}$.

In solid-state materials the potential $V(\overrightarrow{x})$ arises from an ionic lattice. Typically in semiconductor heterostructures we can consider that the electron's motion is confined to a two-dimensional $x-y$ plane, so that the wave function of the electron can be written as $\Psi(x) = \varphi(x,y)\chi_o(z)$. $\varphi(x,y)$ is the effective wave function of the electron that moves in two dimensions, and $\chi_o(z)$ is the ground-state wave function of the potential $V_c(z)$ that confines the electrons to the two-dimensional plane at $z = 0$. If there is inversion assymetry in the system (that is, if $V_c(z) = V_c(-z)$), the wave function obeys $\chi_o(z) = \chi_o(-z)$. In case where the electric field is perpenticular to the $x-y$ plane (the system is homogeneous in this plane), the effective Hamiltonian becomes $H_{SO} = \lambda\left(\overrightarrow{\tau}\times\overrightarrow{k}\right)\cdot\overrightarrow{z}$.

We can therefore include this spin-orbit interaction in the Hamiltonian introduced in section 2.2 obtaining then the Rashba Hamiltonian, given by

$$H_R = \underbrace{\frac{\hbar^2\overrightarrow{k}^2}{2m}}_{\text{Free part}} + \underbrace{\lambda\left(\overrightarrow{\tau}\times\overrightarrow{k}\right)\cdot\overrightarrow{z}}_{\text{Spin-orbit coupling}} - \underbrace{\frac{\Delta}{2}\overrightarrow{\Omega}\left(\overrightarrow{x},t\right)\cdot\overrightarrow{\tau}}_{\text{Exchange coupling}}. \tag{6}$$

One can now see that there is a preferred direction ($\overrightarrow{z}-$direction), such that we take into account the inversion assymetry in the experiment explained in section 1.4 (see Fig. 5). Within this extended model with spin-orbit coupling one can now explain the results of that experiment.

3.2 The Boltzmann equation

The Boltzmann equation 7 describes the statistical behavior of a system. It is an equation for f_k, which is the matrix (in spin space) distribution function, that encodes up and down spin-density contributions in its diagonal elements and transverse spin-density contributions (coherence between up and down spins) in its off-diagonal elements. $\overrightarrow{F}$ represents all external force fields applied to the system. The right hand side of the equation corresponds to collision term resulting from collisions between particles that are assumed to be uncorrelated prior to the collision.

$$\frac{\partial f_k}{\partial t} + \frac{\partial f_k}{\partial \overrightarrow{x}} \frac{\overrightarrow{k}}{m} + \frac{\partial f_k}{\partial \overrightarrow{k}} \overrightarrow{F} = \left(\frac{\partial f_k}{\partial t} \right)_{\text{coll}} . \tag{7}$$

One can solve this equation for the special case where $\overrightarrow{F} = 0$, that is, no external forces are applied to the system. Considering steady-state solutions, and neglecting collision terms, one can find the following result:

$$f_{eq} = \underbrace{\frac{1}{1 + e^{\beta\left(\varepsilon_{\mathbf{k},1} - \mu\right)}}}_{\text{Fermi-Dirac distribution}} \left|\chi\right\rangle_1 \left\langle\chi\right|_1 + \frac{1}{1 + e^{\beta\left(\varepsilon_{\mathbf{k},2} - \mu\right)}} \left|\chi\right\rangle_2 \left\langle\chi\right|_2 . \tag{8}$$

where $\left|\chi\right\rangle_1$ and $\left|\chi\right\rangle_2$ are the eigenvectors of the Hamiltonian and $\varepsilon_{\mathbf{k},1}$ and $\varepsilon_{\mathbf{k},2}$ the corresponding eigenvalues:

$$\begin{array}{ll} \varepsilon_{\mathbf{k},1} = \frac{\hbar^2 \overrightarrow{k}^2}{2m} - \frac{\eta}{2} & \left|\chi\right\rangle_1 = \frac{1}{\sqrt{2\eta(\eta + \Omega_z \Delta)}} \begin{pmatrix} \Omega_z \Delta + \eta \\ \Delta(\Omega_x + i\Omega_y) + 2\lambda(ik_x - k_y) \end{pmatrix} \\ \varepsilon_{\mathbf{k},2} = \frac{\hbar^2 \overrightarrow{k}^2}{2m} + \frac{\eta}{2} & \left|\chi\right\rangle_2 = \frac{1}{\sqrt{2\eta(\eta - \Omega_z \Delta)}} \begin{pmatrix} \Omega_z \Delta - \eta \\ \Delta(\Omega_x + i\Omega_y) + 2\lambda(ik_x - k_y) \end{pmatrix} \end{array} , \tag{9}$$

where $\eta = \sqrt{\Delta^2 \overrightarrow{\Omega}^2 + 4\Delta\lambda \left(k_x \Omega_y - k_y \Omega_x\right) + 4\lambda^2 \overrightarrow{k}^2}$.

With this matrix distribution function one can calculate some physical quantities. As an example, below I show the results for the spin density of the conduction electrons, $\overrightarrow{S}$, defined as $\overrightarrow{S} = \int \frac{d^2\mathbf{k}}{(2\pi)^2} \cdot Tr\left[f_{eq} \overrightarrow{\tau}\right]$:

$$\overrightarrow{S} = \frac{m\Delta}{2\pi\hbar^2} \overrightarrow{\Omega} . \tag{10}$$

One can see that it is the same result obtained in section 2.1, where SO interaction was not taken into account. Now the question that arises is: What happens when we apply an electric field (or any other kind of external field) taking into account spin-orbit coupling?

We will address this question in the following chapters.

4 Overview of magneto-thermo-electric transport phenomena

In this chapter we discuss some of the transport phenomena that occur in the Rashba ferromagnet, and will be encountered in the microscopic calculations discussed later on.

4.1 Hall effect

In 1879 Edwin Hall made discovered that, when a current-carrying conductor is placed in a magnetic field, the Lorentz force presses its electrons against one side of the conductor. This leads to a voltage difference across the electrical conductor perpendicular to the current. This means that if the direction of motion of moving charges is not parallel to the magnetic field, these charges experience the so called Lorentz force. When such a magnetic field is absent, the charges follow approximately straight, 'line of sight' paths between collisions with impurities, phonons, etc. However, when a magnetic field with a perpendicular component is applied, their paths between collisions are curved so that moving charges accumulate on one face of the material. On the other face there is the sanme number of opposite charges, resulting in an asymmetric distribution of charge density across the Hall element that is perpendicular to both the 'line of sight' path and the applied magnetic field.This separation of charges creates an electric field that opposes the migration of further charge in such a way that, as long as the charge is flowing, a steady electrical potential is established accross the conductor. This effect is the basis of semiconductor physics and solid-state electronics.

4.2 Anomalous Hall effect

One year later, Hall found that this 'pressing electricity' effect was a few times larger in ferromagnetic iron than in non-magnetic conductors. This stronger effect in ferromagnetic conductors is known as the anomalous Hall effect (AHE). It takes place in solids with broken time-reversal symmetry, typically in a ferromagnetic phase, as a consequence of spin-orbit coupling. This effect does not arise from the contribution of the ferromagnet magnetization to the total magnetic field.

4.3 Electric conductivity

The conductivity matrix σ_e determines how the charge current behaves depending on the electric field $\mathbf{E}$ applied to the ferromagnet. It is a property of the material and it measures its ability to conduct the flow of electric current.

4.4 Seebeck effect

This effect can be seen when temperature differences create electricity. The voltage created by this effect is of the order of several microvolts per kelvin difference. When heat is applied to a semiconductor, heated electrons flow toward the cooler part, generating then an electric current in the semiconductor. That is, a temperature gradient can create an electric current.

4.5 Peltier effect

When a flow of electric current passes through a semiconductor, there will be a difference of temperature in different parts of the semiconductor, depending upon the direction in which an electrical current passed through them. This effect is analogous to the Seebeck effect. This means that and electric field can create a heat current.

4.6 Thermal conductivity

Analogous to the electric conductivity, the thermal conductivity is the property of a material's ability to conduct heat.

5 Currents and spin density

In this chapter I explain briefly how to obtain a solution for the Boltzmann equation under the linear response approach, and I use it to calculate the dependence of the spin density on the external electric field. I also introduce the charge current and the heat current. We can study the transport properties of the ferromagnet by studying the dependence of these two currents on the external fields.

5.1 Boltzmann equation and spin density

When some external fields (ie., $\overrightarrow{E}$ and $\frac{\overrightarrow{\nabla} T}{T}$) are applied to the system and spin-orbit interactions are taken into account, we need to solve again the Boltzmann equation for the matrix distribution function $f_{\mathbf{k}}\left(\overrightarrow{E}, \frac{\overrightarrow{\nabla} T}{T}\right)$. This matrix distribution function will of course depend now on the external fields. However, solving the Boltzmann equation in this case is non-trivial, so we consider the linear response approach. This means that we separate the matrix distribution function in two parts, $f_{\mathbf{k}}\left(\overrightarrow{E}, \frac{\overrightarrow{\nabla} T}{T}\right) = f_{\mathbf{k}}^{(0)} + f_{\mathbf{k}}^{(1)}$, where $f_{\mathbf{k}}^{(0)}$ is the equilibrium distribution function f_{eq} that we found in section 3.2 and $f_{\mathbf{k}}^{(1)}$ is supposed to be linear in $\overrightarrow{E}$ and $\frac{\overrightarrow{\nabla} T}{T}$.

Under this apporach, one can derive the following equation for $f_{\mathbf{k}}^{(1)} = f_{\mathbf{k}}$ (see appendix A):

$$e\overrightarrow{\mathbf{E}}\frac{\partial f_{eq}}{\partial \overrightarrow{\mathbf{k}}} + \frac{\overrightarrow{\boldsymbol{\nabla}} T}{T}\frac{\partial f_{eq}}{\partial \overrightarrow{\mathbf{k}}}\left(\varepsilon_{\mathbf{k}} - \mu\right) + \frac{i}{\hbar}\left[H_R, f_{\mathbf{k}}\right] = -\frac{1}{\tau} f_{\mathbf{k}}. \tag{11}$$

We included the collision term $\left(\frac{\partial f_k}{\partial t}\right)_{\text{coll}} = -\frac{1}{\tau} f_{\mathbf{k}}$ where τ is the relaxation time (we consider that the electrons are much faster than the change in the magnetization). One can solve this equation for $f_{\mathbf{k}}$ and calculate the spin density. Below I show the result (when $\overrightarrow{\boldsymbol{\nabla}} T = 0$ for simplicity):

$$\begin{aligned} \overrightarrow{S} \;=\; & \frac{m\Delta}{2\pi\hbar^2}\overrightarrow{\Omega} + \frac{em\lambda}{\pi\hbar\Delta}\left(\overrightarrow{E} \times \overrightarrow{z}\right) \times \overrightarrow{\Omega} + \\ & + \frac{em\lambda\tau}{\pi\hbar^2}\left(\left(\overrightarrow{E} \times \overrightarrow{z}\right) \times \overrightarrow{\Omega}\right) \times \overrightarrow{\Omega}. \end{aligned} \tag{12}$$

When spin orbit coupling is not taken into account (that is, when $\lambda = 0$), one recovers the result obtained in section 2.2. One can think of this result as follows: If we take the Fermi sphere in the steady state, when no external fields are applied, we could see that there number of electrons with an arbitrary momentum k is the same as number of electrons with momentum $-k$, so that they cancel out and there is no contribution to the spin-density. However, when one applies an external field to the system, the Fermi sphere is shifted in such a way that there is a non-zero contribution to the spin density.

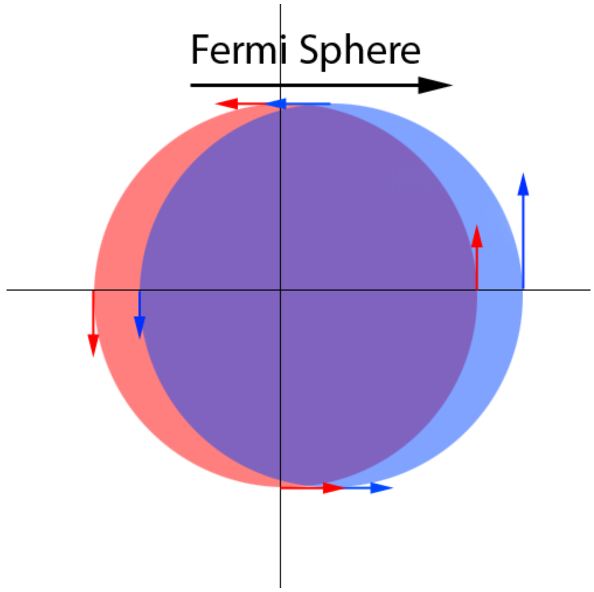


Figure 7: A particle with momentum $\overrightarrow{k}$ has a spin pointing perpendicular to it.

5.2 Charge current

In this thesis we focus on the calculation of two currents, the first one of them being the charge current, given by

$$\mathbf{j}_c = \frac{e}{\hbar}\int \frac{d^2\mathbf{k}}{(2\pi)^2} Tr\left[f_{\mathbf{k}}\frac{\partial H_R}{\partial \mathbf{k}}\right]. \tag{13}$$

This current basically describes the flow of electric charge carried by the conduction electrons through the ferromagnet.

5.3 Heat current

The second current that we calculate is the heat current, given by[3]

$$\mathbf{j}_q = \frac{1}{\hbar}\int \frac{d^2\mathbf{k}}{(2\pi)^2} Tr\left[f_{\mathbf{k}}\frac{1}{2}\left\{\frac{\partial H_R}{\partial \mathbf{k}}, H_R - \mu\right\}_+\right]. \tag{14}$$

Analogous to the charge current, the heat current describes how heat flows through the ferrommagnet.

[3]For any two matrices A and B, by definition $\{A, B\}_+ = AB + BA$.

6 Current-induced torques with SO coupling

Below I show the results obtained for the charge current, the heat current, the spin density and the spin torques.

6.1 Currents

The charge current and the heat current can be expressed as follows:

$$\mathbf{j}_c = \sigma_e \overrightarrow{\mathbf{E}} + \sigma_e S T \left(\frac{-\overrightarrow{\boldsymbol{\nabla}} T}{T} \right) \quad (15)$$

$$\mathbf{j}_q = e\sigma_e P \overrightarrow{\mathbf{E}} + K' T \left(\frac{-\overrightarrow{\boldsymbol{\nabla}} T}{T} \right), \quad (16)$$

where σ_e is the electric conductivity matrix, S represents the Seebeck effect, P the Peltier effect and K' contains the information of the thermal conductivity. These four matrices depend of course on the magnetization $\overrightarrow{\Omega}$, the spin-orbit coupling λ and the exchange coupling Δ, and they contain the information of the transport phenomena in Rashba ferromagnets.

The form of these matrices is rather complicated and the results are still not clear (see section 7). We know that the off-diagonal elements of the conductivity matrix behave as $\sim \lambda^2 \Delta \tau$. One remarkable consequence is that when $\lambda = 0$, that is, when there is no spin-orbit coupling, the off-diagonal elements of the conductivity matrix σ_e are zero, meaning that there is no anomalous Hall effect. Also when $\Delta = 0$ (no exchange coupling) the conductivity matrix becomes diagonal and the AHE is not present. Similar results can be derived for the other matrices.

6.2 Spin density

The spin density depending on the electric field and the temperature gradient behaves as

$$\begin{aligned} \overrightarrow{S} = & \frac{m\Delta}{2\pi\hbar^2} \overrightarrow{\Omega} + \frac{em\lambda}{\pi\hbar\Delta} \left(\overrightarrow{E} \times \overrightarrow{z} \right) \times \overrightarrow{\Omega} + \frac{em\lambda\tau}{\pi\hbar^2} \left(\left(\overrightarrow{E} \times \overrightarrow{z} \right) \times \overrightarrow{\Omega} \right) \times \overrightarrow{\Omega} + \\ & + \frac{m\lambda\tau\,(8\mu_F + 5\Delta)}{4\pi\hbar^2} \left(\frac{\overrightarrow{\nabla} T}{T} \times \overrightarrow{z} \right) + \frac{m\lambda\mu_F}{\Delta\pi\hbar} \left(\frac{\overrightarrow{\nabla} T}{T} \times \overrightarrow{z} \right) \times \overrightarrow{\Omega} + \\ & + \frac{m\lambda\tau\,(8\mu_F + 5\Delta)}{4\pi\hbar^2} \left(\left(\frac{\overrightarrow{\nabla} T}{T} \times \overrightarrow{z} \right) \times \overrightarrow{\Omega} \right) \times \overrightarrow{\Omega}. \end{aligned} \quad (17)$$

One can see that when there are no external forces (that is, $\overrightarrow{E} = \frac{\overrightarrow{\nabla} T}{T} = 0$), then one recovers the solution $\overrightarrow{S} = \frac{m\Delta}{2\pi\hbar^2}\overrightarrow{\Omega}$, found in section 2.2. It is the same just because the spin density does not depend on the spin orbit coupling parameter λ when no external forces are applied, so that one can use the toy model to find that the spin density is proportional to the magnetization of the system. We can also find that we have some similar terms for both external fields; $\left(\left(\overrightarrow{F} \times \overrightarrow{z}\right) \times \overrightarrow{\Omega}\right) \times \overrightarrow{\Omega}$ and $\left(\left(\overrightarrow{F} \times \overrightarrow{z}\right) \times \overrightarrow{\Omega}\right) \times \overrightarrow{\Omega}$ (where $\overrightarrow{F}$ is either $\overrightarrow{E}$ or $\frac{\overrightarrow{\nabla} T}{T}$). We also find the extra term $\sim \frac{\overrightarrow{\nabla} T}{T} \times \overrightarrow{z}$, that does not appear for the electric field.

6.3 Spin torques

These external forces will also produce some torques in the magnetization. These torques are given by $\overrightarrow{\tau}_T = \frac{\Delta}{\hbar}\overrightarrow{S} \times \overrightarrow{\Omega}$. We found the following result:

$$\begin{aligned} \overrightarrow{\tau}_T &= -\frac{em\lambda\Delta\tau}{\pi\hbar^3}\left(\overrightarrow{E} \times \overrightarrow{z}\right) \times \overrightarrow{\Omega} + \frac{em\lambda}{\pi\hbar^2}\left(\left(\overrightarrow{E} \times \overrightarrow{z}\right) \times \overrightarrow{\Omega}\right) \times \overrightarrow{\Omega} + \\ &\quad + \frac{m\lambda\mu_F}{\Delta\pi\hbar\tau}\left(\frac{\overrightarrow{\nabla} T}{T} \times \overrightarrow{z}\right) \times \overrightarrow{\Omega} + \frac{m\lambda\mu_F}{\pi\hbar^2}\left(\left(\frac{\overrightarrow{\nabla} T}{T} \times \overrightarrow{z}\right) \times \overrightarrow{\Omega}\right) \times \overrightarrow{\Omega}. \end{aligned} \tag{18}$$

The first thing to notice is that when there is no spin orbit coupling ($\lambda = 0$), there are no spin torques in the system. We also find that there are two types of torques, $\left(\overrightarrow{F} \times \overrightarrow{z}\right) \times \overrightarrow{\Omega}$ and $\left(\left(\overrightarrow{F} \times \overrightarrow{z}\right) \times \overrightarrow{\Omega}\right) \times \overrightarrow{\Omega}$ (where $\overrightarrow{F}$ is an external field), so that we would get the same kind of torques if we intechange $\overrightarrow{E} \longleftrightarrow \frac{\overrightarrow{\nabla} T}{T}$. There is still some work to be done to fully understand the physical meaning of these results.

7 Conclusion/Discussion/Outlook

In summary, we started introducing a 'toy' model for ferromagnets in which in principle we only took into account the exchange interaction. It was found that this model could not completely explain all the phenomena that can be measured through experiments in ferromagnets. In order to solve this problem, we introduced the Rashba Hamiltonian, in which spin-orbit coupling was also taken into account, and we derived a theory from scratch that allowed us to calculate some physical quantities such as the charge and the heat currents, the spin density and the spin-torques. We found these quantities depending on the external field forces (we considered an external electric field and a gradient of temperature) and also on the (homogeneous, time independent) magnetization of the system.

There is more work to be done. First of all, there is a need to understand every term appearing in the spin-torques and the spin density and their physical meaning, and see the differences between the torques produced by the electric field and the ones produced by the temperature gradient. The results for the matrices that describe the transport phenomena in section 6.1 still need to be simplified and evaluated. Also, new results could be derived for a (specific) non-homogeneous magnetization. The idea is to solve the equation of motion for the magnetization under certain conditions and study its behaviour.

Appendix A

The Boltzmann equation is given by (see section3.2)

$$\frac{\partial f_k}{\partial t}+\frac{\partial f_k}{\partial \overrightarrow{\mathbf{x}}}\frac{\overrightarrow{\mathbf{k}}}{m}+\frac{\partial f_k}{\partial \overrightarrow{\mathbf{k}}}\overrightarrow{\mathbf{F}}=\left(\frac{\partial f_k}{\partial t}\right)_{\text{coll}}. \tag{19}$$

The matrix distribution function can be written as

$$f_k\left(t\right)=\left\langle c_k^{\dagger}\left(t\right)c_k\left(t\right)\right\rangle, \tag{20}$$

where $c_k\left(t\right)$ is the Fourier component of the spinor $\psi\left(x,t\right)$:

$$c_k\left(t\right)=\int d^3\overrightarrow{\mathbf{x}}\,\psi\left(x,t\right)e^{-i\overrightarrow{\mathbf{k}}\overrightarrow{\mathbf{x}}}.$$

The spinor satisfies $\widehat{H}\psi\left(x,t\right)=E\psi\left(x,t\right)$ where $\widehat{H}$ is the Hamiltonian of the system. We also know from quantum mechanics that

$$-\frac{\partial c_k\left(t\right)}{\partial t}=\frac{1}{i\hbar}\left[c_k\left(t\right),\widehat{H}\right]. \tag{21}$$

To derive the "linearized quantum kinetic equation" for the matrix distribution function we first derive 20 with respect to time:

$$\frac{\partial f_k}{\partial t}=\left\langle\frac{\partial c_k^{\dagger}\left(t\right)}{\partial t}c_k\left(t\right)\right\rangle+\left\langle c_k^{\dagger}\left(t\right)\frac{\partial c_k\left(t\right)}{\partial t}\right\rangle. \tag{22}$$

Using 21 and 22 and setting $\hbar=1$, we can rewrite the previous equation as follows:

$$\frac{\partial f_k}{\partial t}=-i\left\langle\left[c_k^{\dagger}\left(t\right),\widehat{H}\right]c_k\left(t\right)\right\rangle-i\left\langle c_k^{\dagger}\left(t\right)\left[c_k\left(t\right),\widehat{H}\right]\right\rangle.$$

Using the commutation rule $A[B,C]+[A,C]B=[AB,C]$ the previous equation can be simplified to

$$\frac{\partial f_k}{\partial t}=-i\left\langle\left[c_k^{\dagger}\left(t\right)c_k\left(t\right),\widehat{H}\right]\right\rangle=-i\left[\left\langle c_k^{\dagger}\left(t\right)c_k\left(t\right)\right\rangle,\widehat{H}\right]=-i\left[f_k\left(t\right),\widehat{H}\right]. \tag{23}$$

Finally, using 19 and 23, taking into account that we can rewrite $f_k\left(t\right)$ in an external field expansion, $f_k\left(t\right)=f_k^{(0)}\left(t\right)+f_k^{(1)}\left(t\right)$, that $f_k\left(t\right)$ does not depend on $\overrightarrow{\mathbf{x}}$ (homogeneous case), and that the collision term is given by $\left(\frac{\partial f_k}{\partial t}\right)_{\text{coll}}=-\frac{1}{\tau}f_k^{(1)}$, we obtain:

$$\frac{\partial f_k^{(1)}\left(t\right)}{\partial t}+\overrightarrow{\mathbf{F}}\frac{\partial f_k^{(0)}\left(t\right)}{\partial \overrightarrow{\mathbf{k}}}+i\left[\widehat{H},f_k^{(1)}\left(t\right)\right]=-\frac{1}{\tau}f_k^{(1)}. \tag{24}$$

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